CENTER FOR RESEARCH IN ECONOMICS AND MANAGEMENT

CREAM Publication No. 10 - 2010

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March 29, 2010

Abstract

Local network externalities are present when the utility of buying from a firm not only depends on the number of other customers (global network externalities), but also on their identity and / or characteristics. We explore the consequences of local network externalities within a framework where two firms compete offering differentiated products. We first show that local network externalities, in contrast to global network externalities, don't necessarily sharpen competition. Then we show that the equilibrium allocation is inefficient, in the sense that the allocation of consumers on firms does not maximize social surplus. Finally we show that local network externalities create a difference between the marginal and the average consumer, which gives rise to inefficiently high usage prices and too high level of compatibility between the networks.

Key words: Local network externalities, differentiated products, competition, efficiency

JEL codes: D 43, D 62

1 Introduction

Network externalities are present when a user's utility of consumption of a good depends on the set of other users that are consuming the good. In the economics literature on network externalities, Rohlfs (1974), Katz and Shapiro (1985), Arthur (1989), Farrell and Saloner (1985, 1986), and Katz and Shapiro (1992), network externalities are primarily captured by the unidimensional variable size. In reality the composition of the network may also matter.

*We highly appreciate comments from Michael Katz, participants at Society of Economic Dynamics annual meeting, and participants on seminars at UC Berkeley, UC Irvine, University of Oslo, and University of Toronto. Financial support from Telenor is gratefully acknowledged.

Consumers may have preferences for the type (or identity) of the consumers in a network as well as their numbers. This is referred to as local network externalities. The examples of local network externalities are abound.

The identity of consumers is important in classical network industries, such as telecommunication, when service compatibility is imperfect. Some telecommunication firms (particularly mobile phone operators) set different on-and off net prices. As a result, consumers prefer subscribing to the same service as the people with whom they communicate. There are similar effects in choice between platform providers. It is convenient to use the same system as colleagues and business partners. In addition, increasing returns to scale in providing applications imply that the availability of applications for a platform will depend on the preferences of its adopters and hence customers will tend to choose a platform where the preferences of the other customers match their own.

Other examples can be found in the financial service industry, i.e. credit card and other bank services. In choosing a credit card, the trading habits of the other customers matter because they influence vendor acceptance of cards. In banking, direct and indirect transaction costs may be lower if trading partners use the same bank. In addition, a bank's customer base is a source of information that can benefit customers within the bank's area of specialization Fjeldstad and Sasson (2010).

The examples don't stop with the classical network industries. For consumption goods or services that involve social interaction, consumers generally have preferences for the identity of other customers. Obvious examples are clubs and social networking sites. For schools and universities, other customers (students) form a pool both for social interaction and a basis for a future professional network. There may be similar effects in employment decisions if the attractiveness of an employer is a function of the set of current employees.

In the present paper we analyze competition in the presence of local network externalities. Two firms supply horizontally differentiated products. As in the standard model, agents have preferences over product varieties, referred to as their technological preference. In addition they have preferences over the size and composition of the customer base of the firms. This is modeled by attributing to each consumer a "social location" on a circle, and letting consumers have a preference for using the same service as consumers to whom they are closely located on the circle. Finally, social location and technological preferences are assumed to be (imperfectly) correlated. In the case of services that facilitate customer exchange, correlated preferences may relate to mode of exchange. With respect to platforms, users that are socially close may have similar technological needs.

Our paper makes four contributions to the literature on network externalities. The first is methodological. We propose a model of competition with local network externalities, and show that if the social preferences are not too strong relative to the technological preferences, then the model has a unique equilibrium. We characterize this equilibrium and show how it depends on the fundamental parameters of the model, the nature of the network externalities, and the relative strength of the technological versus social preferences.

Our second contribution regards the effects of network externalities on competition intensity. It is a celebrated result that network externalities may stiffen competition between firms (Gilbert 1992, Farrell and Saloner 1992, Foros and Hansen 2001, Laffont et al. 1998, Shy 2001), as network externalities increase the elasticity of the demand function. Surprisingly, we find that with local network externalities this effect may be weakened or even eliminated, even if the marginal consumers highly value an increase in the network size. The reason is that after a price change, the previously marginal consumer is inframarginal and the new marginal consumer has different social preferences.

Third, we analyze the welfare effects of the model and show that the equilibrium is not socially optimal. Compared with the planner's solution, consumers put too much emphasis on their technological preferences and too little emphasis on their social preferences.

Finally, we show that local network externalities create systematic differences between the average and the marginal consumers. In expected terms, the inframarginal consumer has shorter social distance to the average consumer in the network than has the marginal consumer. We show that if firms offer two-part tariffs for connection and usage, then they will set usage price above marginal costs in order to extract rent from the inframarginal consumers. When investing in enhanced one-way compatibility, firms will overinvest, because the marginal agent will have stronger social ties to the customers in the other network than has the average customer.

There is empirical evidence that local network externalities are important. Birke and

Swann (2005) study individual consumers' choice of mobile operators in the U.K. They find that individual choices are heavily influenced by the choices of others in the same household. Tucker (2008) analyzes the introduction of a video-messaging technology in an investment bank. She finds that adoption by either managers or workers in boundary spanner positions has a large impact on the adoption decisions of employees who wish to communicate with them. Adoption by ordinary workers has a negligible impact. Corrocher and Zirulia (2009) survey Italian students' choice of mobile operator and find that local network effects (the choice made by friends and family members) play an important role, although the strength of the effects is heterogeneous.

Some of the seminal contributors on network externalities were aware that network externalities need not be spillovers. Rolphs (1974) pointed out that there may be "communities of interest groups" where the members care mostly about the behavior of the other members in the group. Farrel and Klemperer (2007) note that "A more general formulation (of network externalities) would allow each user i to gain more from the presence of one other user j than of another k", and refers to this as local network externalities without pursuing it further. Swann (2002) assumes that different groups differ in diffusion rates and communication patterns, and on this basis show that network effects hardly will be linear in the size of the network.

Banerji and Dutta (2009) analyze an adoption model where the agents form groups, and the members of each group communicate more with the other members of the group than with members of other groups. Firms compete in prices and offer identical products, and there is an equilibrium where the market is segmented. If one firm reduces prices marginally below the other, it may not attract a group as the members are not able to coordinate their decisions. Hence the market is segmented. Sundararajan (2007) analyzes consumers' decision to adopt a network when network externalities are local and the agents have incomplete information about the structure and strength of adoption complementarities. Galeotti and Goyal (2009) study optimal strategies for influencing the behavior of a group of people who are socially connected, and how this depends on the dispersion of social connections. Finally, our paper is tangent to a literature on coordination and formation of, as well as exchange in networks, of people, see Kranton and Minehart (2001) and Bala and Goyal (2000), and

Ballester et al (2006).

In contrast with the contributions cited above, we assume that although network externalities are local, the number of connections of each person is large (infinite), so that the law of large number applies. Our assumptions better reflect sociological accounts of networks showing that people and firms maintain a combination of a limited number of strong, often clustered, ties with closely associated others and a much larger number of weak ties (Granovetter 2004).

In addition, we introduce sufficiently strong regularity conditions on the model so that we obtain a unique equilibrium, with a structure that is similar to the structure in models with spillovers network externalities. Hence our model may bridge a gap between the literature on adoption in small networks and the literature on competition with spillovers network externalities.

The outline of the paper is as follows: In the next section we formalize local network externalities, and set up the competitive framework. We define equilibrium and show existence and uniqueness in section 3, and in section 4 we study how local and spillovers network externalities influence competition intensity. We then move on to analyzing the welfare properties of the model in section 5. In section 6 we study how consumer heterogeneities that endogenously arise with local network externalities may influence pricing decisions, while section 7 concludes. Proof are relegated to the appendix.

2 Modeling local network externalities

We analyze competition between two networks, supplied by firm A and firm B.

The innovation in this paper is our modeling of consumer preferences, which have two parts. First, the consumers' social preferences are represented by a Salop circle, with circumference equal to two.[1] Each consumer has a *social location* (or just location) on this circle. Denote by $z_i \in \Omega$ agent i's social location, where $\Omega = [-1, 1]$. We refer to the location $z = 0$ as the *north pole* and $|z| = 1$ as the *south pole*. Finally, let d denote a distance measure on

[1] The motivation behind letting agents be distributed on the circle is to avoid the asymmetry associated with consumers on the end of a line that only communicate in one direction.

Ω, defined as

$$d(z_i, z_j) = \min[|z_i - z_j|, 2 - |z_i - z_j|]$$

Thus $d(z_i, z_j)$ is the shortest distance between the two agents along the circle.

Let us give some examples. If the application at hand relates to membership in clubs, social location reflects status and foci. If it relates to the choice of platform, e.g. Apple or Windows based computers, the social location will be influenced by occupation and education. If the application at hand relates to banking, social location may reflect industry and business niche, while in mobile telephony it may be related to friends and family.

The second step regards the utility obtained by interaction with peers choosing the same supplier. The function $g : [0, 1] \to R^+$ shows agent i's preference for being in the same network as an agent at social distance d. We assume that g is strictly decreasing in d, reflecting that agents gain more from "being together" with people that are socially close than socially distant.

Suppose a fraction $H(z)$ of the agents of social location z belongs to network A (or, alternatively, the probability that a person located at z chooses the A-network).[2] We assume that the value of interaction is additive, in the following sense: Then the social utility of joining firm A and B for a person of location z_i, denoted by $\mathbf{g}_A(z_i)$ and $\mathbf{g}_B(z_i)$, respectively, can be written as

$$\begin{aligned} \mathbf{g}_A(z_i) &\equiv \int_\Omega g(d(z, z_i))H(z)dz \\ \mathbf{g}_B(z_i) &\equiv \int_\Omega g(d(z, z_i))\,[1 - H(z)]\,dz. \end{aligned}$$

We refer to this as the network utility of an individual associated with joining firm A and firm B, respectively. For notational simplicity, the subscript Ω is dropped in all integrals from now on. Finally, define $\mathbf{g}$ as

$$\mathbf{g} \equiv \int g(d(z, z_i))dz = \mathbf{g}_A(z_i) + \mathbf{g}_B(z_i) \tag{1}$$

Note that $\mathbf{g}$ denotes the maximum network utility a consumer can get, the same for all agents, obtained if all agents in the economy is with the same supplier.

[2] At this point our model allows for two different interpretations. Either there may be one person located at each z, in which case $H(z)$ is a probability. Or it may be a continuum of agents with measure 1 at each z, in which case $H(z)$ is a fraction. We will use the two interpretations interchangeably.

We do not allow g to be negative. Hence there is no crowding-out effects of membership. This seems to be a reasonable assumption for platforms, banks, and telephony, but maybe less so for social clubs, where the average member "type" may matter. Note also that this additivity property gives rise to increasing return to scale on the demand side, and thus brings in an element of spillovers network externalities.[3]

The two rivaling suppliers A and B offer horizontally differentiated products. We model technological preferences by the Hotelling line, where the suppliers are located at the end points of a line of unit length, while the consumers are located between them. Technological differences may reflect pure technological features, user-friendliness, and design. Apple and Microsoft have chosen different solutions, as have Playstation and X-box. Different mobile phone operators also offer services with different features that appeal to different segments of the market. Finally, schools may offer different curricula and students may differ in their preference for these.

A driving assumption in our analysis is that social and technical preferences may be related. We assume that people who are socially close are more likely to share the same technological preferences. For instance, when choosing between Apple and Windows-based computers, the technological solutions of the respective platforms may be better suited for some professional tasks than others, and thus be preferable by members of certain professions. People that one would prefer to co-affiliate with may have similar interests as oneself regarding curriculum (schools), activities (clubs), and calling plans (e.g. different relative pricing of messaging and voice in mobile phone services). More specifically, let y denote the location of a consumer on the technology line, with firm A located at $y = 0$ and firm B at $y = 1$. Consider a consumer who has social location at z_i. We assume that this consumer's location in technology space is stochastic and drawn from a distribution given by

$$y_i = a|z_i| + (1 - a)\varepsilon \tag{2}$$

Here ε is drawn from a uniform distribution on $[0, 1]$, i.i.d. for all agents, and the parameter a satisfies $0 \leq a \leq 1$. If $a = 0$ then y and z are independent. If $a = 1$, then the two variables

[3] We will sometimes refer to $\mathbf{g}$ as the total number (measure) of "friends" that an individual has, with the value of being in the same network as a friend normalized to 1. With this interpretation, $g(d)$ may be interpreted as the probability density that a person has a friend (or the number of friends) at distance d.

are perfectly correlated.

The expected technological preference (conditional on z) can be written as

$$Ey|z = a|z| + (1-a)/2$$

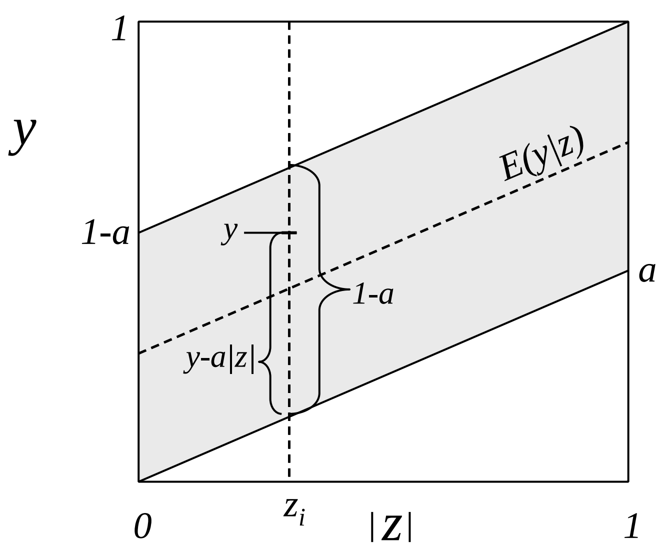
Thus $Ey|0 = (1-a)/2$ and $Ey|1 = (1+a)/2$, while $Ey|1/2 = 1/2$. Note the symmetry around 1/2. The cumulative distribution function of y conditional on z, $F(y|z)$ can be written as

$$\begin{aligned} F(y|z) &= 0 && \text{if } y < a|z| \\ &= \frac{y-a|z|}{1-a} && \text{if } a|z| \leq y \leq a|z| + 1 - a \\ &= 1 && \text{if } y > a|z| + 1 - a \end{aligned} \quad (3)$$

Or, more compactly,

$$F(y|z) \equiv \max\left[\min\left[\frac{y-a|z|}{1-a}, 1\right], 0\right]$$

The distribution is illustrated in the following figure;



The support of the conditional distribution $y|z$ is indicated by the shaded area.

Let $F(y) = \int_0^1 F(y|z)dz$ denote the unconditional distribution of y, and $f(y)$ the associated density. The conditional density $f(y|z)$ is $1/(1-a)$ if $a|z| \leq y \leq a|z| + 1 - a$ and zero otherwise, hence the unconditional density at $y = 1/2$ is[4]

$$f(1/2) = \frac{1}{1-a} \quad \text{if } a \leq 1/2 \tag{4}$$

$$f(1/2) = \frac{1}{a} \quad \text{if } a \geq 1/2 \tag{5}$$

The uncontingent distribution of y is thus only uniform in the special cases with $a = 0$ or $a = 1$. As will be clear below, this is not important for our analysis.

The utility of an agent with characteristics (y_i, z_i) by joining network A at price p_A, and network B at price p_B, is given by

$$u^A(y_i, z_i) = \alpha - ty_i + \mathbf{g}_A(z_i) - p_A \tag{6}$$

$$u^B(y_i, z_i) = \alpha - t(1 - y_i) + \mathbf{g}_B(z_i) - p_B \tag{7}$$

The parameter t reflects the intensity of technological preferences, below referred to as the "transportation cost" per unit of technological distance, while α denote the intrinsic value of being connected to a platform. In what follows we assume that α is sufficiently big so that the entire market is covered. We require that $t(1 - a) > \mathbf{g}$ (see below).

The timing of the model goes as follows:

1. The two firms A and B simultaneously and independently choose prices p_A and p_B, respectively. The firms are not able to price discriminate by setting different prices for agents with different locations at the circle.

2. The agents independently decide which firm to go to, given the prices and given their expectations about the choice of the other agents in the economy. In equilibrium, expectations are rational.

As a benchmark case, we derive the equilibrium of the model with pure global network externalities, i.e., where $g(d)$ is independent of d. More specifically, $g(\delta) = \mathbf{g}/2 \quad \forall \delta$, in

[4] To see this, note that

$$f(1/2) = f(1/2|z) \Pr[a|z| \leq \frac{1}{2} \leq a|z| + 1 - a]$$

which after some manipulation gives the equation.

which case $\int g(d(z, z_i))dz = \int \mathbf{g/2}dz = \mathbf{g}$. Let $\overline{y}^m$ denote the technological preference of a consumer that is indifferent between the two networks (independent of z). It follows that

$$\begin{aligned} u^A(y_i) &= \alpha - ty_i + \mathbf{g}F(\overline{y}^m) - p_A \\ u^B(y_i) &= \alpha - t(1 - y_i) + \mathbf{g}(\mathbf{1} - F(\overline{y}^m)) - p_B \end{aligned}$$

Hence

$$\overline{y}^m = \frac{1}{2} + \frac{p_B - p_A}{2t} + \frac{\mathbf{g}(\mathbf{2}F(\overline{y}^m) - 1)}{2t} \tag{8}$$

Taking the derivative with respect to p_A gives

$$\frac{d\overline{y}^m}{dp_A} = -\frac{1}{2(t - \mathbf{g}f(\overline{y}^m))}$$

Suppose the firms have equal costs c. Firm A maximizes $\pi_A = (p_A - c)F(\overline{y}^m)$, with first order condition $F(\overline{y}^m) - (p_A - c)f(\overline{y}^m)\frac{d\overline{y}^m}{dp_A} = 0$. For firm B, the first order condition reads $(1 - F(\overline{y}^m)) - (p_B - c)f(\overline{y}^m)\frac{d\overline{y}^m}{dp_A} = 0$. In the symmetric equilibrium with $\overline{y}^m = F(\overline{y}^m) = 1/2$ it follows that[5]

$$p_A = p_B = c + \frac{t}{f(1/2)} - \mathbf{g}$$

Thus, from (4) and (5),

$$p_A = c + ta - \mathbf{g} \quad \text{if} \quad a \geq 1/2 \tag{9}$$

$$p_A = c + t(1 - a) - \mathbf{g} \quad \text{if} \quad a \leq 1/2. \tag{10}$$

We have thus reiterated the well-known result that global network externalities reduce equilibrium prices in a symmetric equilibrium. The point is that global network externalities make demand more price sensitive: A reduction in price brings in new agents. This makes the network even more attractive, and even more agents are attracted to the network, and it is the existence of transportation costs that keep demand from exploding.

[5] From the first order condition for p_A and symmetry it follows that

$$p_A = c + \frac{1}{2}[f(\frac{1}{2})\frac{d\overline{y}^m}{dp_A}]^{-1} = c + \frac{t}{f(\frac{1}{2})} - \mathbf{g}$$

3 Equilibrium

In this section we derive the equilibrium of the model with local network externalities, that is, when $g(\delta)$ is a strictly decreasing function of δ at some intervals. We first solve the second stage of the game, which we refer to as the assignment game. Then we solve for the optimal prices given the equilibrium of the assignment game.

3.1 The assignment game

In this section we focus on the agents choice of network for given prices p_A and p_B. The cost to a firm of handling a customer is $c_j, j = A, B$.

The attractiveness of a given network depends both on how many other agents that chose the network, and on their social location. Let $H_0(z)$ denote the fraction of the agents located at $z \in [-1, 1]$ that are customers of firm A. For any z_i at which there is an indifferent agent, let $y^m(z_i)$ denote the technological preference of that agent. Note that y^m depends on z_i, since the social position of the agent influences the distribution of the agent's friends on the two networks. From (6) and (7) it follows that

$$u^A(y^m(z_i), z_i) = u^B(y^m(z_i), z_i)$$

$$\Longleftrightarrow$$

$$y^m(z_i) = \frac{p_B - p_A + \mathbf{g}_A(z_i) - \mathbf{g}_B(z_i) + t}{2t} \tag{11}$$

Let $H_1(z)$ denote the fraction of agents at social localization z that prefers the A-network given H_0, and write $H_1(z) = \Gamma H_0(z)$. In order to characterize Γ we use that $H_1(z) = F(y^m(z)|z)$. From (3) it thus follows that

$$\begin{aligned} \Gamma H(z_i) &= 0 && \text{if } \ y^m(z_i) < a|z_i| \\ &= \frac{y^m(z_i) - a|z_i|}{1-a} && \text{if } a|z_i| \leq y \leq a|z_i| + 1 - a \\ &= 1 && \text{if } \ y^m(z_i) > a|z_i| + 1 - a \end{aligned} \tag{12}$$

Or, more compactly,

$$H_1(z_i) = \Gamma H_0(z_i) = \max\left[\min\left[\frac{y^m(z_i) - a|z_i|}{1-a}, 1\right], 0\right] \tag{13}$$

Since (6) and (7) are continuous in y, it follows that $y^m(z_i)$ and thus $H_1(z_i)$ are continuous. From (11) and the definitions of $\mathbf{g}_A(z_i)$, $\mathbf{g}_B(z_i)$ and $\mathbf{g}$ it follows that

$$y^m(z_i) = \frac{\int g(d(z,z_i))H_0(z)dz + \frac{p_B - p_A - \mathbf{g} + t}{2}}{t}$$

Inserted into (13) this gives

$$\Gamma H_0(z_i) = \max\left[\min\left[\frac{\int g(d(z,z_i))H_0(z)dz + \frac{p_B - p_A - \mathbf{g} + t}{2} - ta|z_i|}{t(1-a)}, 1\right], 0\right] \quad (14)$$

For given prices p_A and p_B, an *equilibrium distribution function* $H^e(z)$ is a fixed-point satisfying

$$H^e(z) = \Gamma H^e(z)$$

Proposition 1 *Suppose* $\mathbf{g} < t(1-a)$*. Then* Γ *is a contraction mapping with modulus* $\frac{\mathbf{g}}{t(1-a)}$*. Hence, for any given prices* p_A *and* p_B*, the fixed point* $H(z) = \Gamma H(z)$ *exists and is unique.*

Thus, whenever $\mathbf{g} < t(1-a)$, the coordination game between the agents has a unique solution. In order to understand the result, note that the assumption on parameter values implies that the technology preferences are strong compared with the network effect. Assume for the moment that $H(z) < 1$ for all z and suppose as an example that all types increase their threshold value $y^m(z)$ with Δ units. This increases H with $\Delta/(1-a)$ units. The increased utility of joining network H due to network externalities is thus $\Delta\mathbf{g}/(1-a)$. The increase in transportation cost for the marginal agent however is Δt, which is greater than $\Delta\mathbf{g}/(1-a)$ by assumption.

As a result, self-fulfilling prophesies is not an issue in this model: an increase in the number of agents going to one network increases the attractiveness of the network, but not sufficiently much to compensate for the increased transportation costs for the new agents.

Given proposition 1, we can easily show that $H^e(z)$ has the following properties:

Lemma 1 *The equilibrium function* $H^e(z)$ *has the following properties*

i) $H^e(z)$ *is symmetric around* $z = 0$, $H^e(z) = H^e(-z)$*. If* $p_A = p_B$ *then* $H^e(z) = 1 - H^e(1-z)$, $0 \le z \le 1/2$ *(with the analogous property for* $z < 0$*).*

ii) For all values of z where $0 < H^e(z) < 1$, $H^e(z)$ is strictly decreasing in z for $z > 0$ and strictly increasing in z for $z < 0$ (except in the special case where $H^e(z) = 0.5$ everywhere, see below).

iii) H can be written as a function of $p_B - p_A$ and is increasing in $p_B - p_A$ for all z

iv) With $p_A = p_B$, the following holds:

a) An increase in $\mathbf{g}$ or a decrease in t increases $H^e(z)$ for $|z| < 1/2$, and the decrease is strict if $H^e(z) < 1$. The opposite holds for $|z| > 1/2$.

b) An increase in a (a reduction in $1 - a$) increases $H^e(z)$ for $|z| < 1/2$, and the increase is strict if $H^e(z) < 1$. The opposite holds for $|z| > 1/2$.

It is possible to show that for the case with $p_A = p_B$, $H(z)$ is concave on $z \in (-1/2, 1/2)$ and convex on the complementary interval (the proof is available upon request).

3.2 Equilibrium prices

In this section we derive the equilibrium prices p_A and p_B. Let N_A and N_B denote the total number of agents in network A and B, respectively. Then

$$\begin{aligned} N_A(p_B - p_A) &= \int H(z; p_B - p_A) dz \\ N_B(p_B - p_A) &= \int \left[1 - H(z; p_B - p_A)\right] dz = 2 - N_A(p_B - p_A) \end{aligned}$$

The profit of firm A and B can be written

$$\begin{aligned} \pi_A &= (p_A - c_A) N_A(p_B - p_A) \\ \pi_B &= (p_B - c_B)\left[2 - N_A(p_B - p_A)\right] \end{aligned}$$

with first order conditions

$$N_A(p_B - p_A) - (p_A - c_A) N'_A(p_B - p_A) = 0 \tag{15}$$

$$2 - N_A(p_B - p_A) - (p_B - c_B) N'_A(p_B - p_A) = 0 \tag{16}$$

With identical costs, the unique solution to the two equations is given by[6]

$$p_A = p_B = c + \frac{1}{N'_A(0)} \tag{17}$$

The second order condition for firm A reads

$$-2(p_B - p_A)N'_A(p_B - p_A) + (p_A - c_A)N''_A(p_B - p_A) < 0 \tag{18}$$

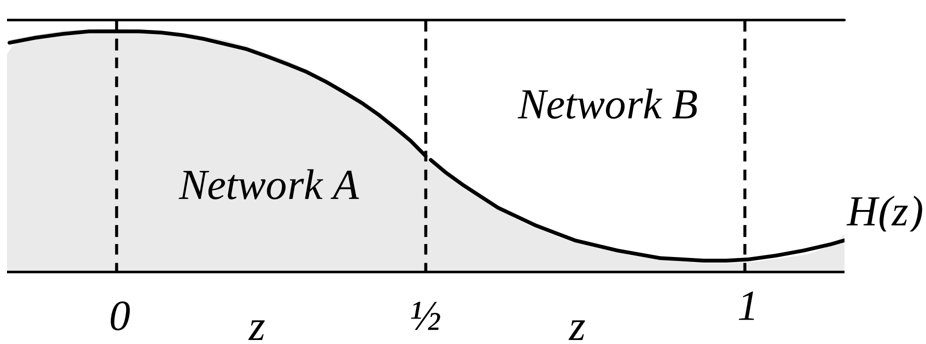
The second order condition for firm B is defined analogously. Due to symmetry, $N_A(\cdot)$ is odd, and thus has an inflection point at zero. Hence $N''_A(0) = 0$, and the second order conditions are satisfied locally.

4 Characterizing equilibrium

In what follows we want to characterize the equilibrium in some detail. To simplify the exposition we assume that $c_A = c_B = c$ in which case the equilibrium is symmetric. In general, it is hard to characterize equilibrium. However, for some sets of parameters the equilibrium take particularly simple forms. We refer to these as open and closed equilibria.

4.1 Open equilibrium

We say that the equilibrium is open if $0 < H^e(z) < 1$ for all z, in which case there are marginal agents for all locations z.



[6] To show uniqueness, note that it follows from (15) and (16) that $\frac{p_A - c}{p_B - c} = \frac{N_A(p_B - p_A)}{2 - N_A(p_B - p_A)}$. If $p_A > p_B$, the left hand side exceeds one whereas the right hand side is strictly below one, a contradiction.

Consider an agent located at $z = 0$ with the largest technological preference for the B-network relative to the A-network, obtained for $\varepsilon = 1$ (see equation 2). This agent prefers the B-network if

$$\mathbf{g}_A(0) - t(1-a) < \mathbf{g}_B(0) - ta$$

or

$$\mathbf{g}_A(0) - \mathbf{g}_B(0) < t(1-2a)$$

As $\mathbf{g}_A(0) > \mathbf{g}_B(0)$, a necessary condition for open equilibrium is that this person has a technological preference for the B-network, i.e. that $1 - 2a > 0$ or

$$a < 1/2$$

A sufficient condition is that

$$\mathbf{g} < t(1-2a)$$

The left-hand side is an upper bound on the social gain of being in the A-network rather than the B network. The condition requires that the maximum technological preference for the B-network (the right-hand side) outweighs this upper bound on the social gain from being in the A-network. Clearly this ensures $H^e(0) < 1$, and hence that $H(z) < 0$ for all z.

An open equilibrium is more likely if g is close to the uniform distribution on $[0, 1]$, in the sense that a bigger set of other parameter values will lead to an open equilibrium (social location does not matter for interaction). It is trivial to show that if g uniform on $[0, 1]$ (global network externalities) the equilibrium is open whenever $a < 1/2$.

Lemma 2 *Suppose* $0 < H^e(z) < 1$ *for all* z. *Then*

$$N'_A(\cdot) = -\frac{1}{t(1-a) - \mathbf{g}} \tag{19}$$

Inserted into (17) this immediately gives us our next proposition:

Proposition 2 *In an open equilibrium, prices are given by (with topscript O indicating open equilibrium)*

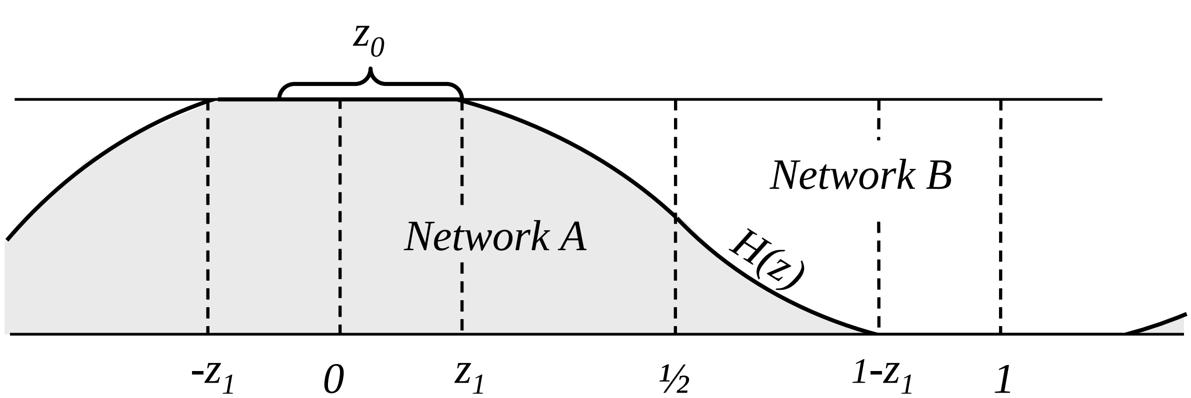
$$p_A^O = p_B^O = c + t(1-a) - \mathbf{g} \tag{20}$$

If we compare (20) and (10) (since $a < 1/2$) we see that they are identical. The existence of network externalities increases competition and decreases prices. Furthermore, the shape of g does not influence network pricing, only $\mathbf{g}$. *Thus, in the open equilibrium, only the global properties of the network externalities, measured by* $\mathbf{g}$, *influences prices. The network structure, defined by the shape of* g*,plays no role.*

4.2 Closed equilibrium

If the equilibrium is open, all the agents in the economy influence each other through friends of friends effects, in the following sense: Suppose $H(z)$ shifts up on an interval around an arbitrary z_i. This will make it more attractive to enter the $A-$network for all the agents who have friends on this interval. This again makes it more attractive to join the $A-$network for agents who have friends who have friends on the interval, and so on. In the end $H(z)$ increases for all z.

However, if $H = 1$ (0) on sufficiently large intervals around the north (south) pole, this chain may be broken. To be more specific, let z_1 denote the highest value of z such that $H(z_1) = 1$, hence $H(z) = 1$ on the interval $[-z_1, z_1]$. Define $z_0 > 0$ to be the smallest value such that $g(z_0) = 0$. It follows that if $z_0 < 2z_1$, an increase in $H(z_i)$ for $z_i > z_1$ will not increase H for negative values of z, i.e., changes in H in the western hemisphere do not influence the value of H in the eastern hemisphere. In this case we say that the equilibrium is closed.



Note that z_0 is exogenously determined by the shape of g. For the equilibrium to be closed, the agent located at $\frac{z_0}{2}$ with the largest technological preference for the B-network relative to the A-network, must strictly prefer the A-network.

$$\mathbf{g}_A(\frac{z_0}{2}) - t\left[1 - a + a\frac{z_0}{2})\right] > \mathbf{g}_B(\frac{z_0}{2}) - t\left[1 - (1-a) - a\frac{z_0}{2}\right]$$

or

$$\mathbf{g}_A(\frac{z_0}{2}) - \mathbf{g}_B(\frac{z_0}{2}) > t\left[1 - 2a(1 - \frac{z_0}{2})\right]$$

The left-hand side of the equation is positive, hence a sufficient the equilibrium to be closed is that $1 - 2a(1 - \frac{z_0}{2}) < 0$. The latter can be rewritten as $az_0/2 + (1-a) \leq 1/2$. Note that this condition can only be true if $a > 1/2$, and it is always satisfied if a is sufficiently large. Furthermore, as $\mathbf{g} \to 0$, a necessary condition for the equilibrium to be closed is that $a > 1/2$.

When the equilibrium is closed, the equilibrium distribution H has some remarkable properties. Define $\Delta p := p^B - p^A$, let $H^{\Delta p}(z)$ denote the equilibrium distribution of customers given Δp (hence $H^0 = H^e$ denotes the distribution when $\Delta p = 0$).

Lemma 3 *Suppose the equilibrium is closed.. Let $\delta = \frac{\Delta p}{2ta}$, and let Δp be sufficiently small so that $z_1 > \frac{z_0 + |\delta|}{2}$. Then the following holds*

a) For $z > 0$, then $H^{\Delta p}(z_i) = H^0(z_i - \delta)$.

b) For $z < 0$, then $H^{\Delta p}(z_i) = H^0(z_i + \delta)$

c) The derivative of $N_A(\Delta p)$ at $\Delta p = 0$ gives

$$N_A'(0) = -\frac{1}{ta} \tag{21}$$

This is possible also for $z_i < \delta$ since by assumption $H(z) = 1$ on $[-\delta, \delta]$. The analogous result holds for $\Delta p < 0$.

By inserting (21) into (17) it follows that (with topscript C indicating that the equilibrium is closed):

Proposition 3 *Suppose the equilibrium is closed. Then*

$$p_A^C = p_B^C = c + at \tag{22}$$

If we compare (22) and prices without network effects (9) with $a \geq 1/2$, we see that prices are identical to a situations without network externalities (with $\mathbf{g} = 0$). *When the equilibrium is closed, network externalities do not influence prices!* Hence in this regime, neither the size of the global network effect $\mathbf{g}$ nor the underlying structure matters for pricing decisions.

To gain intuition for the proposition, first note that global network externalities tend to increase price competition, because they increase the price elasticity of demand. Reducing the price then increases the size of the network, and this will make the network even more attractive. This mechanism does not hold in the closed equilibrium. A reduction in price will increase the network size, and this increases the social value of the network for the agents that previously were marginal. However, these agents are now inframarginal. The now marginal customers do not have more friends in the network than the previous marginal customers.

To be more precise, note that from lemma (3), a decrease in say p_A shifts the $H(z)$ function to the right with δ units. Hence the marginal customer at $z + \delta$ obtains exactly the same social utility as did the previously marginal consumer at z before the shift. Hence, the multiplier effect associated with global network externalities is defused. This is possible as long as none of the agents communicate with people on the "opposite" side of the polar points ($z = 0$ or $|z| = 1$).

4.3 Hybrid equilibrium

A hybrid equilibrium is an equilibrium that is neither open nor closed. i.e., when $H(z) = 1$ for $|z|$ close to the polar points while $H(z_0) < 1$. Hybrid equilibria may exist for a wide range of parameter values. A sufficient condition for the existence of hybrid competition is that $a > 1/2$ (which rules out an open equilibria) and $z_0 \geq 1$ (which rules out closed equilibria). The pricing formulas (20) and (22) give a lower and upper bound on prices in equilibria with hybrid competition.

5 Efficiency

In this section we analyze the efficiency properties of equilibrium, independently of which of the classes (open, closed or hybrid) it belongs to. First we derive the optimal distribution of agents over networks, and refer to this as composition efficiency. Recall that $g_A(z_i)$ denotes the social value of an agent at z_i of joining network A. At any given social location z_i, a fraction $H(z_i)$ of the agents join network A, hence the *total social value* created in network A, V_A, is

$$V_A = \int \mathbf{g}_A(z_i)H(z_i)dz_i$$

Analogously, denote the total social value created in network B by V_B . Then

$$V_B = \int \mathbf{g}_B(z_i)(1 - H(z_i))dz_i$$

In the appendix we characterize the allocations of agents on networks that give the highest and the lowest total social value, given that the two networks are equally large. The total social value is *minimized* if $H(z) = 0.5$ for all z, in which case each agent can communicate with exactly half of her friends. The social value is *maximized* if $H(z)$ equals 1 on an interval with measure 1, and is zero on the complementary interval. However, the allocation that maximizes total social value implies that some of the agents are allocated to a network with a technology they disfavor. Hence there is a trade-off between the social benefits of increasing the number of connections and costs associated with not allocating consumers according to technological preferences.

For a given distribution $H(z)$ let $T(z)$ denote aggregate transportation cost for agents located at z. Recall that the technological preference of the marginal consumer is given by (from 2)

$$y^m(z) = a|z| + (1 - a)H(z).$$

(By definition this is also the technology preference for the marginal customer in firm B). It follows that

$$T(z) = \int_0^{y^m(z)} tyf(y|z)dy + \int_{y^m(z)}^1 t(1 - y)f(y|z)dy$$

Taking derivatives with respect to $H(z_i) \in (0,1)$, and utilizing that $f(y|z) = 1/(1-a)$ (from 3), gives

$$\frac{dT(z_i)}{dH(z_i)} = 2t[y^m(z) - \frac{1}{2}]$$

Finally, aggregate transportation costs are given by $\mathbf{T} = \int T(z)dz$

A *composition efficient* distribution, denoted by $H^*(z)$ maximizes social welfare defined as

$$\begin{aligned} W &= V_A + V_B - \mathbf{T} \\ &= \int [\mathbf{g}_A(z_i)H(z_i) + \mathbf{g}_B(z_i)(1 - H(z_i)) - T(z)]dz \end{aligned} \tag{23}$$

We want to maximize W point-wise. In the appendix we show that with an interior solution, this first order condition can be written as

$$H^*(z_i) = \frac{\int g(d(z, z_i))H^*(z)dz + \frac{\frac{t}{2} - \mathbf{g}}{2} - \frac{t}{2}a|z_i|}{\frac{t}{2}(1-a)}$$

If the right-hand side exceeds 1, then $H^*(z_i) = 1$. If the right-hand side is below 0, then $H^*(z_i) = 0$. Thus $H^*(z)$ is a fixed-point to the mapping Γ^g given by

$$\Gamma^g H^*(z_i) = \max\left[\min\left[\frac{\int g(d(z, z_i))H^*(z)dz + \frac{\frac{t}{2} - \mathbf{g}}{2} - \frac{t}{2}a|z_i|}{\frac{t}{2}(1-a)}, 1\right], 0\right] \tag{24}$$

If we compare (14) and (24) for $p_A = p_B$ we see that the only difference between Γ and Γ^g is that t in Γ is replaced with $t/2$ in Γ^g. Hence the following proposition is immediate

Proposition 4 *The equilibrium distribution is not composition efficient. The social efficient composition profile $H^*(\cdot)$ is steeper than the equilibrium profile $H(\cdot)$. Thus, for $|z| < 1/2$ it follows that $H^*(z) \geq H(z)$ with strict inequality whenever $H(z) < 1$. The opposite is true for $|z| > 1/2$.*

The result follows from Lemma 1 iv b) and the fact that the planner's solution is equivalent with the market solution with t replaced by $t/2$.

The efficiency result is intuitive. The consumers, when choosing between suppliers, trade off transportation cost and social gains. However, the social gain is matched by an equally large externality on the other agents in the network. The transportation cost, by contrast, is carried by the agent in its entirety. As a result, the planner puts twice as much weight on social value relative to transportation cost as the market, or equivalently half as much weight on transportation costs.

For $|z_i| < 1/2$, $H^e(z_i) > 1/2$. Thus, the agent located at z_i obtains more social value by joining the A-network than the B-network. For the same reason, the positive externality of joining the A-network is larger than the positive externality associated with joining the $B-$network, and it follows that $H^*(z_i) > H^e(z_i)$ on the entire northern hemisphere. The opposite holds on the southern hemisphere

Put differently, the net externalities associated with increasing $H(z)$ at $z = z_i$ in the market solution $H^e(z)$ is $\mathbf{g}_A(z_i) - \mathbf{g}_B(z_i)$ where $\mathbf{g}_A(z_i)$ and $\mathbf{g}_B(z_i)$ are evaluated for the equilibrium distribution H^e. Again observe that the net externality is positive if the marginal agent at z_i has a majority of friends in the A-network. An agent at the northern hemisphere has more friends connected to the A-network than the B-network. Hence if she chooses firm A, the net externality is positive. Thus, compared to first best composition efficiency, too many agents at the northern hemisphere choose network B, and too many agents at the southern hemisphere choose network A. The welfare maximizing distribution $H^*(z)$ is thus steeper than the equilibrium distribution $H^e(z)$.

6 Endogenous agent heterogeneity

Differences in preferences between marginal and average agents may give rise to distortions. This was first explored in Spence's (1975) model of a monopolist's choice of quality. If marginal and average consumers value quality differently, the quality level chosen by the monopolist will not be socially optimal.

Local network externalities, in contrast with global externalities, give rise to a difference between the marginal and the average agents in a network, as the former in average obtains less utility from interacting than the latter. This is true both in the closed, open and hybrid equilibrium. In slightly extended versions of the model this may lead to new distortions, which come in addition to and may exacerbate the composition inefficiencies analyzed above.

6.1 Communication intensity

In this subsection we assume that consumers, when connected to a network, choose how much to use it. This is clearly an important aspect in communication platforms, which we use as our example. However, it is also relevant for clubs (where agents choose how much to use it) and platforms like game consoles (where the agents choose how many applications to buy).

We assume that the utility a consumer obtains from communication within a relationship is endogenous and given by $\omega(x)$, where x is usage. We let $\mathbf{g}_A(z_i)$ and $\mathbf{g}_A(z_i)$ denote the number of friends (or connections) in the A and the B network, respectively, for a person located at z_i. For simplicity, we assume that only communication paid by the agent gives rise to utility.[7] Finally, an agent can only communicate with the agents in the same network. Compatibility is discussed in the next section.

Firms compete by offering two-part tariffs (p_j, q_j), $j = A, B$, where q is the cost of using the network and p is a fixed fee. The net surplus $v(q_A)$ per friend for a consumer in network A is

$$v(q_A) = \underset{x}{Max}[\omega(x) - q_A x]$$

We write the optimal usage as a function of q_A, $x(q_A)$. Note that $x(q_A) \equiv -v'(q_A)$.

The timing of the game is exactly as before, the only difference is that firms now advertise a pair (p_j, q_j). The utility for a agent (z_j, y_j) of joining the A network is

$$u^A(y_i, z_i) = \alpha - ty_i + v(q_A)\mathbf{g}_A(z_i) - p_A \tag{25}$$

and similarly, the utility of joining the B network is

$$u^B(y_i, z_i) = \alpha - t(1 - y_i) + v(q_B)\mathbf{g}_B(z_i) - p_B \tag{26}$$

The expressions are identical with the corresponding expressions for u^A and u^B in (6) and (7) except for the multiplicative terms $v(q_A)$ and $v(q_B)$. By doing exactly the same exercise as above when deriving (14), it follows that for given prices, the equilibrium distribution $H^x(z)$ is the fixed point to the mapping Γ^x given by

[7] Note that the social externality identified in the previous section is still present: If a person joins a network, her "friends" in that network obtains utility from having one more person to communicate with.

$$\Gamma^x H(z_i) = \max\left[\min\left[\frac{\frac{v(q_A)+v(q_B)}{2}\int g(d(z,z_i))H(z)dz + \frac{p_B-p_A-v(q_B)\mathbf{g}+t}{2} - ta|z_i|}{t(1-a)}, 1\right], 0\right] \quad (27)$$

Note that for given q_A and q_B, $v(q_A)$ and $v(q_B)$ are constants, hence we can show existence and uniqueness of the fixed point in exactly the same way as above.

Define $\mathbf{G}_A \equiv \int \mathbf{g}_A(z_i)H(z_i)dz_i$ as the total number of connections or friends in the network.[8] The profit of firm A is given by

$$\pi_A = (p_A - c)N_A + (q_A - c_x)x(q_A)\mathbf{G}_A \quad (28)$$

It follows that $x(q_A)\mathbf{G}_A$ shows aggregate usage of the network, while $(q_A - c_x)$ is the mark-up per unit of usage. Note that the firm not only care about the size of its network, but also its composition (the social location of its customers), as this influences $\mathbf{G}_A$.

We only consider symmetric equilibria. Since optimization with respect to p_A corresponds to the simpler case above, we focus on the choice of usage price q_A. In the appendix we derive the optimal q_A, given the constraint that p_A is adjusted in such a way that the market share of firm 1 stays constant at 1/2. The first order condition for q_A can be written as

$$[1-\gamma]x(q_A) + (q_A - c_x)x'(q_A) + \frac{x(q_A)(q_A - c_x)el_q\mathbf{G}_A}{q_A} = 0 \quad (29)$$

where

$$\gamma := \frac{\mathbf{g/2}}{\mathbf{G}_A}$$

and $el_q\mathbf{G}$ is the elasticity operator. The variable γ shows number of friends that the marginal customers have in the network relative to the number of friends the average customer has in the network. To see this, first note that in the symmetric equilibrium, the agent located at $z = 1/2$ has half of its friends in both networks. Marginal customers north of equator have more, and south of equator less than half of their friends in the A network. Due to symmetry, it follows that in average the marginal consumers have exactly $\mathbf{g}/2$ friends in the A-network. The denominator shows the total number of "friends" in the network. Since each network in the symmetric equilibrium obtains a measure of 1 customers (the measure

[8] Each pair of friends counts as two friends, as person i is friend with person j and person j is friend with person i.

of consumers in the economy is 2) this is also the average number of friends per customer in the network. With pure global network externalities $\gamma = 1$, in all other cases $\gamma \in (1/2, 1)$.

The first term in (29) thus represents rent extraction from the inframarginal types. Since inframarginal customers on average have higher communication intensity than the marginal customers, increasing the usage price increases total payments from existing customers, even though the fixed price p_A is reduced so that the market share of the firm stays constant.

The second term in (29) is self-explanatory. The last term shows the change in incomes from usage fees caused by changes in the composition of the network. In the appendix we show that $el_q\mathbf{G}_A < 0$: A higher usage price hurts the marginal agents with many friends in the network (z low) more than those with a few friends in their network (z high). A higher q_A thus implies that H becomes flatter, and hence that total traffic falls (even though the market share stays constant).

However, with marginal cost pricing, $q_A = c_x$, the last term in (29) is zero. Hence with marginal cost pricing, the left-hand side of (29) is strictly positive as long as $\gamma < 1$. The next proposition is thus immediate

Proposition 5 *The firms set the communication price q_k, $k = A, B$ above marginal cost. Thus, the communication price exceeds the price level that induces a static first best level of traffic represented by marginal cost pricing (provided that $\gamma < 1$).*

The finding contrasts the standard result that two-part tariff induces marginal cost pricing on usage and therefore efficient usage in the standard model without local network externalities (Farrel and Saloner 1992). Local externalities create agent heterogeneity, and since marginal customers on average have lower usage than inframarginal customers traffic price can be used as a rent extraction device. The firm thus trades off efficiency and rent extraction for the inframarginal ("high-type") agents.

The network owner prices internal traffic as if he had some degree of market power, where the degree of market power is captured by the relative deviation between the marginal and the average intensity of exchange. With global network externalities, symmetry between agents prevails (hence $\gamma = 1$), which means that the network adopts marginal cost pricing.

Note that γ decreases as the spread of g decreases, and approach $1/2$ when the support of g converges to zero.

It can be shown that the result does not depend on our limitation of the contract space to two-part tariffs. With an optimal general contract, increasing the usage price for marginal agents relaxes the incentive compatibility constraint of the inframarginal consumers, and hence enables the firm to extract more rents from the latter. Finally, the effect is weakened by the negative effect that increased usage price has on the composition of the network. As long as the network has a positive margin on usage, the resulting reduction in traffic is costly for the network.

Define the *constrained efficient* usage price as the usage price that maximizes net welfare given that agents are distributed according to individual optimization (i.e., the price that emerges if a planner could set the usage price but make no other decisions). Then the following holds:

Lemma 4 *The constrained efficient usage price is below marginal cost*

The lemma follows directly from proposition 4. There are no externalities related to communication intensity (since only the payer gets utility from communication). It is trivial to show that $H^*(z)$, the socially optimal distribution function H (for given v) solves (27) with $t/2$ substituted in for t. Hence the socially optimal distribution H^* is steeper than the equilibrium distribution function H^e.

As we have seen, a higher usage price hurts the marginal agents with many friends in the network (z low) more than those with a few friends in their network (z high).The H function thus decreases for values of z above $1/2$ (with many friends) and decreases for $z > 1/2$ (with few friends in the network). It follows that by subsidizing usage, the planner can make the distribution function steeper and thus closer to the socially optimal distribution.

The market solution for usage pricing thus distorts the distribution of H^e by making it flatter, and this leads to a distribution of agents on the networks that are even further away from the optimal distribution.

6.2 Compatibility

We will now discuss the firms' incentives to undertake investments in order to make the networks compatible. We focus on the situation with one-way compatibility. Thus, network A may give its members (improved) access to network B by undertaking an investment. Let $\theta_A \leq 1$ denote the degree at which the agents in network A can utilize network B, and write the cost of compatibility as $C(\theta_A)$. We only include connection pricing (no two-part tariffs). The degree of compatibility is set independently and simultaneously by the two firms at stage 1, together with prices p_A and p_B. In other respects the timing is unchanged.

We assume that compatibility from the A network to the B network only benefits the consumers in the A network (consistent with the assumption above that only the caller receives utility). The utilities of an agent (y_i, z_i) in network A and and B, respectively, are given by

$$\begin{aligned} u^A(y_i, z_i) &= \alpha - ty_i + \mathbf{g}_A(z_i) + \theta_A \mathbf{g}_B(z_i) - p_A \\ u^B(y_i, z_i) &= \alpha - t(1-y_i) + \mathbf{g}_B(z_i) + \theta_B \mathbf{g}_A(z_i) - p_B \end{aligned}$$

By reasoning exactly as when deriving (14), it follows that for given prices, the distribution $H(z)$ is defined by the fixed point to the mapping Γ^C defined as

$$\Gamma^C H(z_i) = \max\left[\min\left[\left(1 - \frac{\theta_A + \theta_B}{2}\right)\int g(d(z, z_i))H(z)dz + \frac{p_B - p_A - (1-\theta_A)\mathbf{g} + t}{2}, 1\right], 0\right]$$

Network A's net profit equals

$$\pi_A = p_A \int H(z)dz - C(\theta_A)$$

In the appendix we show that the firms will choose a degree of compatibility such that the marginal customers' valuation of compatibility equals marginal costs. Recall from the last section that the marginal customers on average have half of their friends in the other network. First order conditions for θ_A is thus

$$C'(\theta_A) = \frac{\mathbf{g}}{2} \tag{30}$$

The socially efficient degree of compatibility (contingent on equal market shares), by contrast, maximizes welfare W defined by (23) less the costs $C_A(\theta_A) + C_B(\theta_B)$, and where V_A now reads

$$\begin{aligned} V_A &= \int [\mathbf{g}_A(z_i) + \theta_A \mathbf{g}_B(z_i)] H(z_i) dz dz_i \\ V_B &= \int [\mathbf{g}_B(z_i) + \theta_B \mathbf{g}_A(z_i)] H(z_i) dz dz_i \end{aligned}$$

Maximizing W w.r.t. θ_A at $H = H^*$ (the socially optimal distribution) gives the first order condition

$$C_A'(\theta_A) = \int \mathbf{g}_B(z_i) H^*(z_i) dz_i \tag{31}$$

The right-hand side of (31) is the total number of "friends" that the members of network A have in network B. Since the measure of agents in network B is 1 (due to symmetry) this is also the average number of friends members of network A has in network B. This is less than $\mathbf{g}/2$ - the density of customers in network A is larger on the northern than the southern hemisphere, while the opposite is true in the B network.

Proposition 6 *The firms have too strong incentives to make the networks (one-way) compatible.*

The result emerges despite the fact that there are no externalities associated with compatibility in itself, as compatibility is one-way. With local network externalities, the marginal agents value compatibility higher than the average agents, since the marginal agents communicate more with the agents in the other network than does the average agent. Since firms compete for the marginal agents, it is his/her preferences that governs the choice of compatibility. Hence too much resources are spent on making the systems compatible compared with the socially optimal level.[9]

[9] Farrel and Saloner (1992) find in a model with global network externalities that firms chose an optimal level of compatibility. Our result show that their result is not robust when allowing for local network externalities.

The comparison above is between the compatibility in the market solution and first best compatibility. If we instead use the constrained efficient compatibility level as the benchmark, this will actually strengthen our results. First, note that if the planner takes the equilibrium distribution H^e as given, equation (31) with H^* substituted out with H^e defines the contingent optimal compatibility level. Since H^e is flatter than H^*, the right-hand side of equation (31) then decreases, and the constrained efficient value of θ_A becomes even lower (and thus further away from the equilibrium level).

Second, consider the constrained efficient compatibility level when we take into account that the compatibility level influences the equilibrium distribution $H^e(z)$, in an analogous way to usage prices. Increasing θ_A has a negative effect on composition efficiency, since it attracts agents that communicate intensively with the other network (that is types $z_i > 0.5$) and punish agents with most of their friends in the A-network (types $z_i < 0.5$). Hence, a high level of compatibility makes the equilibrium distribution $H^e(z)$ flatter. However, we have already seen that the efficient distribution $H^*(e)$ is steeper than the equilibrium distribution H^e. Hence, in the constrained efficient solution (where the planner could set the level of compatibility but nothing else), the planner would reduce compatibility further in order to obtain a more efficient composition of consumers on networks.

7 Concluding remarks

Network externalities are important in a several markets, particularly related to ICT. In the economics literature, the focus has been on global network externalities, where the network effects are related solely to size. In the present paper we argue that the network effects not only work through the size of the customer base, but also through its composition, i.e., the attributes of the customers in the customer base and in particular their exogenously given relationships to each other. We refer to this as local network externalities.

We propose a way of modeling local network externalities, which is sufficiently rich to capture the main attributes of network composition and still sufficiently simple to make the analysis tractable, and which embodies global externalities as a special case. We do this by using a two-dimensional spatial model. Consumers have a location in a social space, and interact mostly with people located closely to them in this space. In addition, consumers' technological preferences are represented by a location in technological space. Finally, the consumers' location in the two spaces may be correlated in the sense that if two agents are close in the social space they are also likely to be close in the technological space.

Two firms that are horizontally differentiated in technology compete for customers. We show that as long as social preferences are not too strong relative to technological preferences, the model has a unique equilibrium. The equilibrium has several interesting properties. First, the well known result that network externalities stiffen competition may not hold when network externalities are local. Second, the allocation of consumers on networks is not efficient, as there is a social externality associated with the choice of network that the customers do not take into account when choosing between networks. Third, local network externalities create a difference between average and marginal consumers, and this lead to inefficiently high usage prices and too high levels of (one-way) compatibility.

8 Appendix

Proof of proposition 1

We apply Blackwell's sufficient condition[10]. It follows from Blackwell's sufficient condition that Γ is a contraction and thus has a unique fixed-point if it satisfies i) a monotonicity condition, and ii) discounting. Denote by S the set of all bounded continuous functions on $[-1, 1]$. Then Γ is a mapping from S into S. It is bounded above by 1 and below by 0, and continuous as $H(z)$ is continuous. The monotonicity condition requires that if $H_i, H_j \epsilon S$ and $H_i(z) \leq H_j(z)$ all z, then $\Gamma H_i(z) \leq \Gamma H_j(z)$ for all z. Since the RHS of (14) is increasing in $H(z)$ for all z, the monotonicity condition is satisfied. Consider next the discounting condition. The discounting condition requires that there exists some α in $(0, 1)$ such that for all H_i in S, all $v \geq 0$, and all z_i we have $\Gamma(H_i + v)(z_i) \leq \Gamma(H_i)(z_i) + \alpha v$. It follows from

[10] See e.g. Sydsæter, Strøm and Berck (2005) or Stokey and Lucas (1989).

(14) that

$$\Gamma(H_i+v)(z_i) = \max\left[\min\left[\frac{\int g(d(z,z_i))(H_i(z)+v)dz + \frac{p_B-p_A-\mathbf{g}+t}{2} - a|z_i|}{t(1-a)}, 1\right], 0\right]$$
$$= \max\left[\min\left[\frac{\int g(d(z,z_i))H_i(z)dz + \frac{p_B-p_A-\mathbf{g}+t}{2} - a|z_i|}{t(1-a)} + v\frac{\mathbf{g}}{t(1-a)}, 1\right], 0\right]$$

Hence, if neither the requirement that $H \leq 1$ (the minimum operator) or the requirement that $H \geq 0$ (the max operator) binds, it follows that $\Gamma(H_i+v)(z_i) = \Gamma(H_i)(z_i) + v\frac{\mathbf{g}}{t(1-a)}$. If either the minimum operator or the maximum operator strictly binds, then $\Gamma(H_i+a)(z_i) < \Gamma(H_i)(z_i) + v\frac{\mathbf{g}}{t(1-a)}$. It follows that Γ is a contraction mapping with modulus $\frac{\mathbf{g}}{t(1-a)}$.

Proof of Lemma 1

i) Suppose the equilibrium is not symmetric around 0. Then there exists a strictly positive number z' such that $H(z') \neq H(-z')$. But since the model is symmetric, there must exists another equilibrium distribution H' defined as $H'(z') = H(-z')$ and $H'(-z') = H(z')$. Since the equilibrium is unique we have thus derived a contradiction. The claim that if $p_A = p_B$ then $H(z_i) = 1 - H(1-z_i)$ for all $z_i \epsilon [0,1]$ can be proved by exactly the same argument

ii) Suppose $H(z)$ is strictly increasing in z at an interval in $[0,1]$. It follows that $H(z)$ has a local maximum for some $z^* \neq 0$ on this interval. Define z' as the highest value of z less than z^* such that $H(z') = H(z^*)$. If z^* is also a global maximum, then $z' = -z^*$. Now define a new distribution function $\widetilde{H}(z)$ such that $\widetilde{H}(z) = H(z^*)$ on $[z', z^*]$ and $\widetilde{H}(z) = H(z)$ otherwise. Since, by construction, $\widetilde{H}(z) \geq H(z)$ for all z and strictly greater on the interval (z', z^*) it follows that $\Gamma\widetilde{H}(z) \geq \widetilde{H}(z)$, with strict inequality on $[z', z^*]$. Since $H(\cdot)$ is a contraction there exists a fix point $H_2(z) = \lim_{T\to\infty} \Gamma^T \widetilde{H}(z) \geq \widetilde{H}(z)$, which is a contradiction due to uniqueness.

Finally, suppose H is decreasing but not strictly, and constant at some interval $[z_1, z_2]$, and strictly decreasing otherwise. This cannot be an equilibrium either. The agent localized at z_1 obtains stronger network effects than one localized at z_2, hence $y^m(z_1) > y^m(z_2)$. From equation (12) it then follows that $\Gamma H(z_1) > \Gamma H(z_2)$, and H cannot be a fixed point

iii) From (14) it follows that Γ, and thus the fixed-point H, depends on the difference $p_B - p_A$. Let $H^*(z)$ denote the initial equilibrium, and consider an increase in $p_B - p_A$. It follows that $\Gamma H^* \geq H^*$, with strict inequality for all z where $1 > H^*(z) > 0$. The result thus follows from monotonicity, see proof Proposition 1.

iv) We first prove the following lemma

Lemma 5 *Assume that the distribution $H_0(z)$ satisfies $H(z) = 1 - H(1-z)$ for $0 \leq z \leq 1/2$. Suppose further that $H_0(z) \leq \Gamma H_0(z)$ for all $|z| < 1/2$ with strict inequality for some z. Then $H_0(z) \leq \Gamma H_0(z) < \Gamma^{T-1} H_0(z) < \Gamma^T H_0(z) < H^e(z)$ and $\Gamma^\infty H_0(z) = H^e(z)$. For $|z| > 1/2$the inequalities are reversed.*

First note that it follows from (14) that if $H(z) = 1 - H(1-z)$ then $\Gamma H(z) = 1 - \Gamma H(1-z)$. Hence this symmetry property is preserved. Furthermore, since by assumption $\Gamma H_0(z) \geq H_0(z)$ at the interval $z \epsilon (-\frac{1}{2}, \frac{1}{2})$, it follows from (14) that $\Gamma^2 H_0(z) > \Gamma H_0(z)$ for all z in the interval and vice versa on the complimentary interval. This holds for each step Γ^T. Since the mapping is bounded, it must converge, and since the equilibrium is unique it must converge to the equilibrium distribution. QED

Let $H_0^e(z)$ denote the initial equilibrium, and consider an increase in $\mathbf{g}$, a decrease in t or an increase in a. Then it follows from (14) that $\Gamma H_0(z) > H_0(z)$ for all $|z| < 1/2$ and vice versa for $|z| > 1/2$, hence lemma 5 applies. Property iv) thus follows.

Proof of lemma 2

Suppose $dH^e(z)/dp_A$ is independent of z. In the open equilibrium, the definition of Γ given by (14) reads

$$\Gamma H_0(z_i) = \frac{\int g(d(z, z_i)) H_0(z) dz + \frac{p_B - p_A - \mathbf{g} + t}{2} - ta|z_i|}{t(1-a)}$$

Differentiating both sides of the fixed-point equation $\Gamma H^e(z) = H^e(z)$ with respect to dp_A, assuming that $dH^e(z) = dH^e$ independent of z thus gives that for any z_i,

$$H^e(z_i) = \frac{\boldsymbol{g} dH^e(z_i) - dp_A/2}{t(1-a)d}$$

or

$$\frac{dH^e(z_i)}{dp_A} = -\frac{1}{2} * \frac{1}{t(1-a) - \boldsymbol{g}}$$

independent of z_i. By construction, $H^e + dH$ is thus an equilibrium distribution, and as the equilibrium distribution is unique it is also the only one. As the social circle has a circumference of 2, $dN_A = 2dH$, and this gives (19).

Proof of lemma 3

a and b) Define the function $H^{\Delta p}(z_i) \equiv H^0(z_i - \delta)$ for $z_i \geq z_1 - \frac{z_0}{2}$, and $H^{\Delta p}(z_i) = H^0(z_i + \delta)$ for $z_i \leq -z_1 + \frac{z_0}{2}$ and analogously around the south pole. The two definitions are consistent even if $z_1 - z_0 - \delta < 0$, since $H^0(z) = 1$ on $[-z_1, z_1]$ and $z_1 \geq z_0/2 + \delta/2$ by assumption.

Denote by $\Gamma^{\Delta p}$ the mapping defined by (14) given the price difference $\Delta p = p_B - p_A$. We want to show that $H^{\Delta p} \equiv H^0(z - \delta)$ solves $H^{\Delta p} = \Gamma^{\Delta p} H^{\Delta p}(z)$, or equivalently that $\Gamma^{\Delta p} H^{\Delta p}(z) = \Gamma^0 H^0(z - \delta)$. From (14) (recall that $g(z_0) = 0$),

$$\Gamma^{\Delta p} H^{\Delta p}(z_i) = \max\left[\min\left[\frac{\int_{z_i - z_0}^{z_i + z_0} g(d(z, z_i)) H^{\Delta p}(z) dz + \frac{\Delta p - \mathbf{g} + t}{2} - taz_i}{t(1-a)}, 1\right], 0\right]$$

Suppose $z_i \geq z_1$. Inserting $H^{\Delta p}(z) \equiv H^0(z - \delta)$ and $\delta = \frac{\Delta p}{2ta}$gives

$$\Gamma^{\Delta p} H^{\Delta p}(z_i) = \max\left[\min\left[\frac{\int_{z_i - z_0}^{z_i + z_0} g(d(z, z_i)) H^0(z - \delta) dz + \frac{\mathbf{g} + t}{2} - ta(z_i - \delta)}{t(1-a)}, 1\right], 0\right]$$

or

$$\begin{aligned}\Gamma^{\Delta p} H^{\Delta p}(z_i) &= \max\left[\min\left[\frac{\int_{z_i - z_0 - \delta}^{z_i + z_0 - \delta} g(d(z, z_i - \delta)) H^0(z) dz + \frac{\mathbf{g} + t}{2} - ta(z_i - \delta)}{t(1-a)}, 1\right], 0\right] \\ &= H^0(z_i - \delta) \qquad (32)\end{aligned}$$

where the last equality follows by definition. Note that at z_1, $H(z_1 - \delta) = 1$. For $0 < z_i < z_1$, it follows that $H^{\Delta p}(z_i) \geq H^{\Delta p}(z_1) = 1$, and thus that $H^{\Delta p}(z_i) = 1 = H^0(z_i - \delta)$. The same argument holds for $z < 0$ and around the south pole. The result thus follows.

c) The number of customers for supplier A can be written as

$$\begin{aligned}N(\Delta p) &= 2\left[\int_0^{z_1 - \frac{\Delta p}{2ta}} 1dz + \int_{z_1 - \frac{\Delta p}{2ta}}^{1 - z_1 + \frac{\Delta p}{2ta}} H(z - \frac{\Delta p}{2ta}) dz + \int_{1 - z_1 + \frac{\Delta p}{2ta}}^{1} 0dz\right] \\ &= 2\left[\int_0^{z_1 - \frac{\Delta p}{2ta}} 1dz + \int_{z_1}^{1 - z_1} H(z) dz + \int_{1 - z_1 + \frac{\Delta p}{2ta}}^{1} 0dz\right]\end{aligned}$$

Differentiating with respect to Δp yields

$$N'(0) = -\frac{1}{ta}$$

as stated.

Proofs regarding efficiency

Maximizing and minimizing social value.

With two symmetric networks this is equivalent to maximizing V_A with respect to the distribution $H(z)$ subject to $\int H(z)dz = 1$, that is

$$\max_{H(z_i)} \iint g(z_i - z)H(z)H(z_i)dzdz_i \quad \text{s.t.} \quad \int H(z_i)dz_i = 1$$

with the associated Lagrangian

$$L = \int \left[\int g(z_i - z)H(z)dz - \lambda \right] H(z_i)dz_i$$

Point-wise maximization yields the first order condition

$$\begin{aligned}
\int g(z_i - z)H(z)dz - \lambda &> 0 \rightarrow H(z_i) = 1 \\
\int g(z_i - z)H(z)dz - \lambda &< 0 \rightarrow H(z_i) = 0 \\
\int g(z_i - z)H(z)dz - \lambda &= 0 \rightarrow H(z_i) \text{ undetermined}
\end{aligned}$$

Obviously there are two solutions satisfying the first order conditions, either $H(z) = 0.5$ all z, or $H(z) = 1$ for all $z\epsilon[z', -(1 - z')]$ where z' is arbitrary, and $H(z) = 0$ otherwise.[11] The two solutions are referred to as the maximum and minimum solutions respectively.

First order conditions with interior solution

We maximize (23) point-wise with respect to $H(z_i)$. When doing so, we have to take into account that an increase in $H(z_i)$ influences $\mathbf{g}_A(z_j)$ for all z_j, and likewise for $\mathbf{g}_B(z_j)$. More specifically, a one unit increase in H on an interval δz around z_i increases social value for an agent at z_j if joining the network by $g(d(z_i, z_j))\delta z$ units. The aggregate effect is thus

[11] Observe from the first order conditions that the number of friends in the A network, $\int g(z_i - z)H(z)dz$, must be equal for all z_i at which $H(z_i)$ is strictly between 0 and 1. Then it follows trivially that H can be interior on an interval only if $H = 0.5$ everywhere.

$\int g(d(z_i, z_j)dzdz_j = g_A(z_i)dz$. This comes in addition to the direct effect of increasing H in (23). The derivative of the integrand in (23) with respect to $H(z_i)$ is thus

$$\begin{aligned} & 2\mathbf{g}_A(z_i) - 2\mathbf{g}_B(z_i) - t(1 - 2y^m(z_i)) \\ = \ & 4\mathbf{g}_A(z_i) - 2\mathbf{g} + t - 2t[a|z_i| + (1-a)H^*(z_i)] \end{aligned} \tag{33}$$

where we have used that $\mathbf{g}_A(z_i) + \mathbf{g}_B(z_i) = \mathbf{g}$. With an interior solution for H^*, this derivative is zero, in which case (33) reads

$$H^*(z_i) = \frac{\int g(d(z, z_i))H^*(z)dz + \frac{\frac{t}{2} - \mathbf{g}}{2} - \frac{t}{2}a|z_i|}{\frac{t}{2}(1-a)}$$

as stated in the text.

Two part tariff - first order conditions

In any equilibrium, the combination of p_i and q_i maximizes the profit of firm i given its market share (Armstrong and Vickers 2001). In a symmetric equilibrium, consider an increase in q_A combined with a decrease in p_A such that $-p_A + v(q_A)\mathbf{g}/2$ is constant. It is trivial to show that the new distribution function $\widetilde{H}$ will satisfy $\widetilde{H}(1/2) = 1/2$ and $\widetilde{H}(|z|) = 1 - \widetilde{H}(1-z)$. Hence the market share of firm A stays constant. A scale neutral change in p_A, q_A thus requires

$$\frac{dp_A}{dq_A} = \frac{v'(q_A)\mathbf{g}}{2} = \frac{-x(q_A)\mathbf{g}}{2} \tag{34}$$

Maximizing (28) with respect to q_A subject to (34) yields the first order condition

$$\begin{aligned} -N_A\frac{x(q_A)\mathbf{g}}{2} + [x(q_A) + x'(q_A)(q_A - c)]\,\mathbf{G}_A \\ +x(q_A)(q_A - c)\frac{\partial \mathbf{G}_A}{dq_A} = 0 \end{aligned}$$

or (since $N_A = 1$ in the symmetric equilibrium)

$$[1-\gamma]x(q_A) + (q_A - c_x)x'(q_A) + \frac{x(q_A)(q_A - c_x)el_q\mathbf{G}_A}{q_A} = 0$$

where $\gamma := \frac{1}{2}\mathbf{g}/\mathbf{G}_A$ and el_q is the elasticity operator.

Consider a person located at z_i. The derivative of u_A and u_B given by (25) and (26) wrt q_A given (34) gives

$$\frac{du_A(z_i)}{dq_A} = -x(q_A)(\mathbf{g}_A - \mathbf{g}/2)$$

Since $\mathbf{g}_A$ is decreasing in z it follows that an increase in z makes H less steep, and as a result $\mathbf{G}_A$ decreases. Hence $el_q G_A < 0$.

Compatibility - first order conditions

We reason in exactly the same way as when deriving first order conditions with two-part tariffs. Firm A chooses a combination of compatibility and prices p_A that maximizes profit for a given market share, i.e., solves (assuming symmetry)

$$\max_{\theta_A, p_A} p_A - C(\theta_A) \qquad \text{s.t.} \qquad\qquad - p_A + \theta_A \mathbf{g}/2$$

with first order condition (assuming interior solution)

$$C'(\theta_A) = \mathbf{g}/2$$

as stated in the text.

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ISSN 1891-599X