# *Management compensation and market timing under portfolio constraints*

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# Management Compensation and Market Timing under Portfolio Constraints *

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**Abstract**

This paper shows that portfolio constraints have important implications for management compensation and performance evaluation. In particular, in the presence of portfolio constraints, allowing for benchmarking can be beneficial. Benchmark design arises as an alternative effort inducement mechanism vis-a-vis relaxing portfolio constraints. Numerically, we solve jointly for the manager's linear incentive fee and the optimal benchmark. The size of the incentive fee and the risk adjustment in the benchmark composition are increasing in the investor's risk tolerance and the manager's ability to acquire and process private information.

**Keywords:** Market Timing, Incentive Fee, Benchmarking, Portfolio Constraints

**JEL Classification Numbers:** D81, D82, J33.

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**Management Compensation and Market Timing under Portfolio Constraints**

# 1 Introduction

In this paper, we study the effect of relative (to a benchmark) performance evaluation on the provision of incentives for the search for private information under moral hazard when managers face exogenous portfolio constraints that limit their ability to sell short and purchase on margin.

The Investment Company Act of 1940 restricts the investment activities of mutual funds.[1] Beyond regulation, there exist statutory restrictions on the fund's leverage. The general consensus is that "the maximum leverage ratio allowed for mutual funds is 1.5 to 1 and most operate with less. Funds are strictly limited in the extent to which they can borrow, sell securities short, purchase securities on margin, or invest in certain derivatives."[2] In this paper we focus on the two most prevalent restrictions: short-sales and margin purchases. Almazan et al (2004) document that, according to the self-reported information that funds must submit to the SEC in Form N-SAR, approximately 70% of mutual funds explicitly state that short-selling is not permitted. This figure rises to above 90% when the restriction is on margin purchases.[3]

Constraints on short-selling and margin purchases are exogenous in our model.[4] We claim that portfolio constraints have important implications for management compensation and performance evaluation. Our main contribution is to show that, in the presence of portfolio constraints, allowing for benchmarking can be beneficial. Benchmark design arises as an alternative effort inducement mechanism vis-a-vis relaxing portfolio constraints. Numerically, we solve jointly for the manager's linear incentive fee and the optimal benchmark. As an additional result, the paper shows that when the benchmark composition is endogenously determined, the principal's optimal benchmark choice will not necessarily coincide with the benchmark that maximizes the fund's Information Ratio (excess return per unit of tracking error volatility).

We propose a two-period, two-asset (the market and a risk-less bond) model. The principal in our model represents the fund investors. The agent would be the fund management company. The management company is hired by investors to take portfolio decisions. In exchange, it receives a management fee. The management fee includes a basic fee and a performance based incentive fee, possibly benchmarked to a given portfolio return. The basic fee should be interpreted as a flat percentage fee which depends in a predictable way on the fund's size (assets under management) and past performance. It is, therefore, implicitly and explicitly unrelated

[1] Another historically relevant regulation, repealed in the Taxpayer Relief Act of 1997, was the "short-short" rule that indirectly limited funds' ability to use short sales and derivatives. Regulation T by the Fed limits the initial margin to 50% of the purchase price of securities that can be purchased on margin.

[2] "Why Mutual Funds Do Not Pose Systemic Risks," Paul Schott Stevens President and CEO Investment Company Institute (ICI), 16th Annual Investment Company Directors Conference, 2009.

[3] Of course, investors can effectively leverage their portfolios above those limits by investing in derivatives, hence limiting the impact of explicit constraints on short-selling and margin purchase. According to the evidence reported in Almazan et al (2004) for funds domiciled in the US, from 1994 through 2000, on average, less than 1% of funds who could invest in options and less than 15% of funds who could invest in futures did actually invest in the corresponding derivatives. Although these percentages vary across years and fund's age, the general tenor is that unconstrained funds made limited use of derivatives investment during this period.

[4] The literature review in section 2 describes briefly several papers that explain empirically and theoretically how portfolio constraints may arise optimally.

to future performance. In other words, a higher basic fee implies that managers with higher reservation utility (arguably, those with a better record and/or working for bigger funds) will receive a higher compensation.[5] According to the current US regulation (see Thomas and Jaye (2006)) the incentive fee adjustment is symmetric around the benchmark; it is also expressed as a percentage of the fund's assets under management. In our model, both the incentive fee and the benchmark composition will be determined endogenously.

A number of new insights arise after introducing portfolio constraints. First, looking at the manager's effort and portfolio choice problem, we show that the active portfolio and effort decisions (hence, performance) depend on both the incentive fee and the benchmark composition. The relationship between the manager's effort and the incentive fee has been studied by Gómez and Sharma (2006). The relationship between the effort decision and the benchmark composition, however, contrasts with the well-known "irrelevance result" in Stoughton (1993) and Admati and Pfleiderer (1997): the manager's effort is independent of the incentive fee and the benchmark composition; it only depends on the manager's effort disutility. Under portfolio constraints, we show that benchmarking turns out to be equivalent to making portfolio constraints effectively less binding, increasing the marginal utility of effort. We derive explicitly the effort maximizing benchmark's composition as a function of the market moments, the portfolio constraints, and the manager's risk-aversion coefficient. We show that the irrelevance result in Admati and Pfleiderer (1997) arises only in the limit, when there are no portfolio constraints.

To understand the model's intuition, consider a manager who is totally constrained in her ability to sell short and purchase at margin. Under moral hazard, the manager's optimal portfolio can be decomposed in two components: her unconditional risk-diversification portfolio plus her active or "timing" portfolio.[6] Benchmarking will be immediately reflected in the manager's unconditional portfolio: the manager replicates the benchmark, effectively, the "riskless" asset in relative terms. The timing portfolio depends on the manager's costly effort to improve her timing ability through superior information. For an uninformed manager, this portfolio would be zero. For a hypothetical perfectly informed manager, it would push the optimal total portfolio to either boundary: 100% in the risky asset if the market risk premium is forecasted to be positive; 100% in the bond otherwise. As effort increases, and depending on the composition of the benchmark, either the lower bound (limiting short selling) or the upper bound (limiting margin purchases) will be, marginally, more likely to be binding. Ex-ante, when the manager takes his optimal effort decision, he will take into account this likelihood. Why would he exert more (costly) effort when he cannot trade accordingly due to the existing portfolio constraints? The amount of effort, relative to the unconstrained case, will be lower. What is the role of the benchmark and how how can it help to alleviate the effort underinvestment? By choosing the appropriate benchmark composition, portfolio constraints are effectively relaxed. Intuitively, the benchmark

[5] The basic fee may have sometimes two components, depending on the average fund family size and the fund's specific size. Moreover, this fee may decrease in a concave way with the size of the fund. For an empirical investigation of the effect of this concavity on the fund's performance and volatility, see Deli (2002) and Massa and Patgiri (2009).

[6] Since the manager needs to choose between a well-diversified market portfolio and the risk free asset, active management in our model is analogous to timing ability. Hence, we use the two terms interchangeably. Appendix A extends the model by introducing a second risky asset and stock picking or selectivity ability on the side of the manager.

that leaves, in expected terms, both constraints equidistant would maximize the marginal utility of effort, hence increasing effort expenditure. This midpoint is shown to depend on the portfolio constraints and the usual components in the manager's portfolio decision: the stock return moments, the manager's risk aversion and the contract's incentive fee. When the portfolio space is unconstrained, so is the timing portfolio. The unconstrained manager's effort decision is, in fact, independent of the incentive fee since he controls the actual size of the portfolio investment; the manager's optimal effort depends only on his effort disutility. Benchmarking the manager's incentive fee fails to induce any additional effort on the unconstrained manager.

Turning to the investor's problem, she has to decide the benchmark composition and the fee structure. The investor is confronted with a trade-off: on the one side, by benchmarking the manager's compensation and increasing the explicit incentive fee, she may increase the manager's effort expenditure, as discussed before. On the other side, benchmarking the manager distorts the optimal risk-sharing properties of the original, unconstrained first-best contract where only risk sharing dictates the optimal compensation. The investor's optimal decision regarding the size of the incentive fee and the benchmark composition will depend crucially on her risk aversion relative to the manager's risk aversion. The more risk tolerant the investor, relative to the manager, the higher the incentives of the former to induce higher effort by the latter, even if it is at the expense of forcing the manager to take more risk than it would be, otherwise, optimal. This is consistent with the intuitive idea of avoiding "closeted" passive managers who "peg" their portfolios to the benchmark without exerting enough effort. Obviously, the more risk tolerant the investor relative to the manager, the more relevant this problem becomes.

If portfolio constraints are removed, we converge to the standard Admati and Pfleidederer (1997) irrelevance result: benchmarks are suboptimal. This is due to the inability of the investor to induce higher effort on the manager by changing either the incentive fee or the benchmark composition. In practical terms, this means that we should expect lower or no benchmarking at all when the manager's investment options are largely unconstrained. The numerical results for the optimal contract under moral hazard and portfolio constraints confirm this intuition. The optimal incentive fee and benchmarking, relative to the first-best, unconstrained case, increase as the investor becomes less risk averse than the manager. At the same time, the effort under-investment and the utility loss (in the form of variation in the Certainty Equivalent Wealth) if the investor keeps the first best, zero-benchmark contract under portfolio constraints increases when the manager becomes relatively more risk averse than the investor.

The empirical implications of our model are consistent with the evidence documented for mutual funds and pension funds. Elton, Gruber and Blake (2003), find evidence of superior performance among US mutual funds with explicit incentive fees, as compared with similar funds without explicit incentive fees. This is consistent with the incentives for active management (higher effort) provided by explicit incentive fees for constrained, risk-averse mutual fund managers. Deli (2002) studies the advisory contracts of a sample of over 5,000 funds in the US. Only 365 of those funds use contracts that contain some adjustment besides the percentage of assets (flat fee) provision. Among them, funds with explicit incentive fees linked to relative performance. He finds that funds that invest primarily in equity securities and that show higher turnover are more likely to have an incentive fee adjustment. This is consistent with our model.

Arguably, funds investing primarily in equity attract less risk averse investors. Our model predicts that as the investor (relative to the manager) becomes less risk averse, incentives fees and relative (benchmarked) performance adjustment should become more relevant. Deli (2002) argues that higher portfolio turnover may reflect more information-based trading. Our numerical results suggest that, other things equal, as the manager's effort disutility decreases (reflecting greater managerial ability in information acquisition), incentive fees and benchmarking should increase. Finally, Blake, Lehmann, and Timmermann (2002), show that UK pension funds are largely unconstrained in their portfolio choice. At least in the short term, their fees are directly related to the fund value they achieve in absolute terms, and not relative to any predetermined benchmark. This is consistent with the suboptimality of benchmarks in the absence of portfolio constraints. It is important to notice that, according to our model, the different typology of contracts (incentive fee and benchmark composition) in response to risk aversion and managerial ability would not arise without the concurrence of moral hazard and portfolio constraints.

The rest of the paper is organized as follows. Next we review the related literature. Section 3 introduces the model. The standard unconstrained results are reviewed in section 3.1 while the effect of portfolio constraints are analyzed in section 3.2. In section 4, we derive the composition of the effort-maximizing benchmark portfolio. Section 5 studies the principal's problem. A numerical solution to the investor's optimal contract (including the benchmark) is presented in section 6. Section 7 concludes the paper. Appendix A introduces a second asset and allows for stock-picking ability on the manager's side. Qualitatively, our results are shown to be robust to this extension. All proofs are presented in Appendix B.

## 2 Related Literature

The extant literature has tried to understand why portfolio constraints exist in the first place. In a model with asymmetric information and moral hazard, Dybvig, Farnsworth, and Carpenter (2010) show that trading constraints (albeit of a different form to those studied in this paper, as explained below) may be necessary to elicit truthful revelation of the manager's private information. Almazan et al (2004) present evidence consistent with portfolio constraints being used as an alternative to monitoring the manager's activities when other mechanisms (outside directors on the board, less experienced managers or when the fund is managed by an individual rather than a team) are absent. Interestingly, they find no significant portfolio performance across funds with different levels of portfolio restrictions. They claim that this is evidence in favor of portfolio constraints as part of the optimal compensation contract. Grinblatt and Titman (1989) and Brown, Harlow, and Starks (1996) argue that cross-sectional differences in constraint adoption might be related to characteristics that proxy for managerial risk aversion. Agarwal, Boyson, and Naik (2009) show that hedged mutual funds mimicking hedge funds' investment strategies perform better than traditional mutual funds, on account of having more flexibility due to lesser portfolio constraints. Finally, portfolio constraints have been shown to be important in explaining the cross-sectional stock return anomalies (see Nagel (2005)).

The design of fund management compensation schemes has elicited interest amongst both practitioners and researchers. The academic literature has focused on two broad areas: how

contract design affects the risk-taking behavior of managers and their incentives to gather private information. Roll (1992) was the first to illustrate the undesirable effect of relative (i.e., benchmarked) portfolio optimization in a partial equilibrium, single-period model. In particular, he shows that the manager's active portfolio is independent of the benchmark composition and that this leads the manager to take systematically more risk than the benchmark. Despite the sub-optimal risk allocation, the portfolio optimization literature takes as given that the manager minimizes tracking error volatility subject to an excess return and studies how different constraints on the portfolio's total risk (Roll (1992)), tracking error (Jorion (2003)), and Value-at-Risk (VaR) (Alexander and Baptista (2008)) may help to reduce excessive risk taking. Bajeux et al (2007) study the interaction between tracking error and portfolio weight constraints. Interestingly, Jorion (2003) writes (footnote 7, page 82): "in practice, the active positions will depend on the benchmark if the mandate has short-selling restrictions on total weights." Our model formalizes this intuition and shows that, in the presence of portfolio constraints, the manager's active portfolio depends on the benchmark composition. More importantly, for the constrained manager, the tracking-error minimization mandate arises endogenously through the manager's relative incentive fee.

Another strand of the literature has focused on the effect of a performance-related incentive fee on managers' incentive to search for private information. Examples include Bhattacharya and Pfleiderer (1985), Stoughton (1993), Heinkel and Stoughton (1994) and Gómez and Sharma (2006). In particular, our paper makes a contribution to the literature on optimal benchmarking. We do so by imposing certain limits to the scope of our contract. For instance, inspired by the current regulation for mutual funds incentive fees in the US, we take as given the nature of the fee structure, a linear fulcrum (symmetric) structure around the benchmark, and concentrate on the benchmark design under portfolio constraints. After the seminal work of Starks (1987) comparing symmetric and asymmetric incentive fees, several papers have dealt with the question of the optimal fee structure. Das and Sumdaram (2002), for instance, follow a different approach to our paper: the benchmark is exogenously given and the object of study is the design of the fee structure (fulcrum versus asymmetric). They show that, as expected, asymmetric incentive fees induce adverse incentives for extra risk taking, consistently with their convex design. On the other side, in a context with asymmetric information about the manager's quality (adverse selection problem), asymmetric fees may prove to be a less onerous mechanism to screen out more skilled managers. Similarly, Cuoco and Kaniel (2011) take the benchmark portfolio composition as given and study the impact of both fulcrum and asymmetric fees on the price and volatility of assets included in the benchmark. Fulcrum fees induce a positive price effect and a negative Sharpe ratio effect on the assets included in the benchmark. The effect of asymmetric contracts on both prices and Sharpe ratios is more ambiguous. Other papers studying the effect of portfolio delegation on equilibrium prices include Brennan (1993) and Gómez and Zapatero (2003).

The question of benchmark optimality is present in Ou-Yang (2003). This paper studies a standard portfolio delegation problem in continuous time. The author derives the optimal benchmark endogenously. He shows that, contrary to the usual convention of a static, buy-and-hold benchmark, the optimal benchmark is dynamic and actively managed. Unlike in our model, there is no moral hazard problem and the manager's portfolio choice is unconstrained.

Li and Tiwari (2009) also deal with the question of optimal benchmarking. These authors study the optimal compensation of a manager in a portfolio delegation problem under moral hazard including, potentially, a non-linear option-type incentive fee. The fundamental difference between their model and ours is the existence of the option-like compensation component. This component may help to overcome the effort under-investment problem documented by Stoughton (1993) and Admati and Pfleiderer (1997). The benchmark design is shown to be crucial for this result. In particular, the benchmark has to be chosen to accurately reflect the investment style of the manager. Unlike in their paper, in our model there is no asymmetric component. The reason for the benchmark design in our model comes from the existence of portfolio constraints, which are absent in Li and Tiwari (2009).

There are two closely related papers in this literature that deserve special attention. The first paper, Basak, Pavlova, and Shapiro (2008) focuses on incentives arising from performance-flow relation. In their dynamic model, the manager's compensation is a function of the fund's flow. The adverse risk incentives arise due to the documented convex relationship between the fund's relative performance and net flows.[7] If the fund underperforms a reference benchmark by more than a given threshold (the benchmark restriction), the contract is terminated. This is akin to a portfolio insurance constraint against unbounded portfolio losses. Provided that the benchmark is not too risky, increasing the manager's maximum allowed loss is shown to be effective in curbing the manager's adverse incentives for excessive risk taking and aligning the interest of the manager with that of the investor. In our model, the manager is compensated with an explicit, linear incentive fee proportional to the portfolio's relative performance. Our focus is not on curbing adverse risk incentives but on providing the manager with incentives for collecting unobservable information. Thus, we do not model explicitly convex payoffs but rather impose exogenous portfolio constraints that may arguably arise endogenously in a model with limited liability like in Basak, Pavlova, and Shapiro (2008).

The second paper is Dybvig, Farnsworth, and Carpenter (2010). In their model, the manager is offered a finite menu of allowable portfolio strategies and sharing rules for each possible signal realization. Further, they assume a mixture model whereby the joint density function of the manager's signal and the market state are affine in the manager's effort choice. This setting allows the authors to solve the problem, in principle, for any general sharing rule, not just affine rules. We, instead, take the linear contract observed in practice as given. The fundamental difference between their paper and ours hinges on the nature of the constraints. In their model, trading constraints arise when the signal realization is not observable by the investor. Truthful revelation is at the core of their trading constraints. The optimal contract rewards the manager for reporting "extreme signals." In other words, it is necessary to induce the manager to act aggressively on extreme information. In our model, explicit portfolio constraints (in the form of observable short-selling and margin purchase limits) are exogenous. They are motivated by adverse risk-incentives and monitoring costs. More importantly, constraints have the opposite effect: they limit the manager's incentive to exert effort. The benchmark's role (ultimately,

[7] See, for instance, Gruber (1996), Chevalier and Ellison (1997), Sirri and Tufano (1998), Basak, Shapiro, and Tepla (2006) and Basak, Pavlova, and Shapiro (2007). Del Guercio and Tkac (2002) show that the sensitivity of flow to performance in the pension fund industry is less convex than in the mutual fund industry.

its composition) is to alleviate such a limitation by indirectly relaxing the impact of those constraints. The tradeoff between this effort enhancing mechanism and the distortion in risk sharing determines the optimal benchmark.

# 3 The model

The manager and the investor have preferences represented by exponential utility functions: $U_a(W) = -\exp(-aW)$ and $U_b(W) = -\exp(-bW)$, respectively. Throughout the paper, we will use $a > 0$ ($b > 0$) to denote the manager (investor) as well as his (her) *absolute* risk aversion coefficient. The investment opportunity set consists of two assets: a risk-free asset with gross return $R$ and a stock with stochastic excess return $x$ normally distributed with mean excess return $\mu > 0$ and volatility $\sigma$. These two assets can be interpreted as the usual "timing portfolios" for the active manager: the bond and the stock market portfolio (or any other stochastic timing portfolio). Appendix A extends the model by including a second risky asset.

The investment horizon is one period. Payoffs are expressed in units of the economy's only consumption good. All consumption takes place at the end of the period. The manager's compensation has two components: a basic fee, $F$, defined as a percentage of the fund's assets under management $W_0$, and typically known as "fraction of the fund;" and an explicit performance-based incentive fee, $A \in (0, 1]$, also defined as a percentage of funds under management and related to the fund's return relative to that of a predefined benchmark portfolio.

After learning the contract, the manager decides whether to accept it or not. If rejected, the manager gets his reservation value. If he accepts the contract, then he puts some (unobservable) effort $e > 0$ in acquiring private information (not observed by the fund's investor) that comes in the form of a signal

$$y = x + \frac{\sigma}{\sqrt{e}}\epsilon,$$

partially correlated with the stock's excess return. The noise term has a standard normal distribution $\epsilon \sim \mathcal{N}(0, 1)$. For simplicity, we assume $E(x\epsilon) = 0$.

The greater the effort the more precise the manager's timing information. Conditional on the manager's effort, the stock's excess return is normally distributed with conditional mean return $E(x|y) = \frac{\mu+ey}{1+e}$ and conditional precision $\text{Var}^{-1}(x|y) = \frac{1}{\sigma^2}(1 + e)$. Hence, $e$ can also be interpreted as the percentage (net) increase in precision induced by the manager's private information. Notice that, in case $e = 0$, the conditional and unconditional distributions coincide: there is no relevant private information.

Effort is costly. The monetary cost of effort disutility is a percentage $V(D, e) = \frac{D}{2}e^2$ of the fund's net asset value $W_0$. $D > 0$ represents a disutility parameter.[8]

## 3.1 Unconstrained Portfolio Choice

Based on the conditional moments, the manager makes his optimal portfolio decision: he will invest a percentage $\theta$ in the stock and the remaining $1 - \theta$ in the risk-free bond. Therefore, the

[8] Appendix B shows that the results in the paper generalize to a broader set of convex disutility functions.

portfolio's return will be $R_p = R + \theta x$. Define the benchmark's return as $R_H = R + Hx$ with $H$ as the *benchmark's policy weight*: the proportion in the benchmark portfolio invested in the risky stock. For analytical tractability, the parameter $H$ can take any value in the real line. It should be interpreted as the relative weight of the risk-adjustment in the benchmark used for measuring the fund's performance in the explicit incentive fee. $H = 0$ would be equivalent to no benchmarking. The higher $H$ in absolute value, the bigger the case for benchmarking the manager's performance.

Given a contract $(F, A, H)$, the conditional end-of-the-period wealth for the manager and the investor are given, respectively, by $W_a(\theta) = W_0(F + A(R_p - R_H))$ and $W_b(\theta) = W_0 R_p - W_a(\theta)$. The portfolio's *net return* is given by $R_p - R_H = \bar{\theta}x$ with $\bar{\theta} = \theta - H$, the net investment in the risky stock.[9]

Rearranging terms, we can write the manager and the investor final wealth as follows:

$$W_a(\bar{\theta}) = W_0(F + A\bar{\theta}x), \tag{1}$$
$$W_b(\bar{\theta}) = W_0(R_H + (1 - A)\bar{\theta}x - F). \tag{2}$$

After these definitions, the conditional utility function for the manager and the investor can be expressed, respectively, as

$$U_a\left(W_a(\bar{\theta})\right) = -\exp\left(-aW_a(\bar{\theta}) + W_0 V(D, e)\right),$$
$$U_b\left(W_b(\bar{\theta})\right)\big) = -\exp\left(-bW_b(\bar{\theta})\right).$$

In this setting, the Arrow-Pratt risk premium for the manager will be, $AW_0\frac{aAW_0}{2}\bar{\theta}^2\sigma^2$. Thus, $aAW_0$ represents the manager's *relative* risk aversion coefficient. For simplicity, and without loss of generality, we normalize $W_0 = 1$.

We shall proceed backwards. First, for every signal realization $y$, we will obtain the optimal portfolio choice $\theta(y)$. Then, after recovering the manager's unconditional indirect utility function, we will tackle the manager's effort decision. Given $y$, the unconstrained manager's optimal net portfolio solves

$$\bar{\theta}(y) = \text{argmax}_{\bar{\theta}}\left\{E(W_a(\bar{\theta})|y) - (a/2)\text{Var}(W_a(\bar{\theta}|y))\right\},$$

which yields the optimal portfolio

$$\theta(y) = H + \frac{\mu}{aA\sigma^2} + \frac{ey}{aA\sigma^2}. \tag{3}$$

The manager's optimal portfolio has three components: the total benchmark's investment in the risky stock, $H$; the unconditional optimal risk-return trade-off, $\frac{\mu}{aA\sigma^2}$ and, depending on

[9] Sometimes the benchmark may include a minimum excess return $\tau > 0$ such that $R_H = R + \tau + Hx$. Notice that this is equivalent to defining $F = F' - A\tau$ in equations (1) and (2). Solving for $F$ and $A$, $F'$ is obtained as a function of $\tau$.

the manager's signal $y$ and his effort expenditure, $e$, the *timing portfolio*, $\frac{ey}{aA\sigma^2}$.[10] Replacing $\theta(y)$ in the manager's expected utility function and integrating over the signal $y$ we obtain the manager's (unconditional) expected utility:

$$EU(W_a(e)) = -\exp\left(-(1/2)(\mu^2/\sigma^2) - aF + V(D,e)\right) g(e), \tag{4}$$

with $g(e) = \left(\frac{1}{1+e}\right)^{1/2}$. At the optimum, the marginal utility of effort must be equal (first-order condition) to its marginal disutility:[11]

$$V_e(D, e_{SB}) = \frac{1}{2(1+e_{SB})}. \tag{5}$$

We call this solution the *second best effort*.[12] The convexity of the manager's disutility of effort function guarantees that the necessary condition (5) is also sufficient for optimality. Clearly, the manager's second best effort choice (hence the quality of his private information) is independent of the benchmark composition, $H$, and percentage incentive fee $A$. This is the same result as in Proposition 3 in Admati and Pfleiderer (1997). Effort only depends on the manager's disutility coefficient, $D$.

### 3.2 Constrained Portfolio Choice

We depart now from the Admati and Pfleiderer (1997) setting by introducing portfolio constraints explicitly in the model. Assume that the manager cannot short-sell or purchase on margin. Let $m \geq 1$ denote the maximum trade on margin the manager is allowed: $m = 1$ means that the manager is not allowed to purchase the risky stock on margin; for any $m > 1$ the manager can borrow and invest in the risky stock up to $m - 1$ dollars per dollar of the fund's current net asset value. Let $s \geq 0$ denote the short-selling limit: $s = 0$ means that the manager cannot sell short the risky stock; for any $s > 0$ the manager can short up to $s$ dollars per dollar of the fund's current net asset value. In terms of the manager's portfolio choice problem, this implies $m \geq \theta \geq -s$ or, equivalently, $m - H \geq \bar{\theta} \geq -(H + s)$.

The manager then solves the following constrained problem

$$\bar{\theta}(y) = \text{argmax}_{m-H \geq \bar{\theta} \geq -(H+s)} \left\{ E(W_a(\bar{\theta})|y) - (a/2)\text{Var}(W_a(\bar{\theta})|y) \right\}.$$

Call $\lambda_m \leq 0$ and $\lambda_s \leq 0$ the corresponding Lagrange multipliers, such that $\lambda_m(m - H - \bar{\theta}) = \lambda_s(\bar{\theta} + H + s) = 0$. There are three solutions. If neither constraint is binding, $\lambda_m = \lambda_s = 0$, then the interior solution follows: $\bar{\theta}(y) = \frac{\mu + ey}{aA\sigma^2}$. Alternatively, there are two possible corner solutions: first, if the short-selling limit is binding, $\lambda_m = 0$ and $\lambda_s = E(x|y) + aA(H + s)\text{Var}(x|y) < 0$. In such a case, $\bar{\theta} = -(H + s)$. In the second corner solution, the margin purchase bound is hit: $\lambda_s = 0$ and $\lambda_m = -E(x|y) + aA(m - H)\text{Var}(x|y) < 0$. In such a case, $\bar{\theta} = m - H$.

[10] Notice that this corresponds exactly to the optimal portfolio choice (4) in Admati and Pfleiderer (1997) in the presence of a risk-free asset ($\theta_c = 0$).

[11] The subscripts $e$ and $ee$ denote, respectively, first and second derivative with respect to effort.

[12] The first best effort is the effort the unconstrained manager would exert under no asymmetric information, that is, in the absence of moral hazard.

Solving for the optimal portfolio $\theta(y)$ as a function of the signal realization we obtain that, in the case of no timing ability ($e = 0$), $\theta = H + \frac{\mu}{aA\sigma^2}$ provided $-\left(s + \frac{\mu}{aA\sigma^2}\right) \leq H \leq m - \frac{\mu}{aA\sigma^2}$. For the case when $e > 0$ we obtain:

$$\theta(y) = \begin{cases} -s & \text{if } y < -\frac{\mu}{e} L_s, \\ H + \frac{\mu}{aA\sigma^2} + \frac{ey}{aA\sigma^2} & \text{otherwise,} \\ m & \text{if } y > \frac{\mu}{e} L_m. \end{cases} \tag{6}$$

We call

$$\begin{aligned} L_s(H) &= 1 + (H + s)\left(\frac{\mu}{aA\sigma^2}\right)^{-1}, \\ L_m(H) &= (m - H)\left(\frac{\mu}{aA\sigma^2}\right)^{-1} - 1, \end{aligned}$$

the *leverage ratios*. These ratios represent the *net* (relative to the benchmark) maximum leverage from selling short $(H+s)$ or trading at margin $(m-H)$ as a proportion of the manager's optimal unconstrained portfolio when $e = 0$ and $H = 0$.

Looking at the way the leverage ratios change with benchmarking, we observe that $\frac{\partial}{\partial H} L_s = \left(\frac{\mu}{aA\sigma^2}\right)^{-1} > 0$ and $\frac{\partial}{\partial H} L_m = -\left(\frac{\mu}{aA\sigma^2}\right)^{-1} < 0$. That is, $L_s$ ($L_m$) increases (decreases) with $H$. Moreover, given the (risk-adjusted) market premium $\mu/\sigma^2$, the marginal change in $L_s$ ($L_m$) increases (decreases) with the manager's relative risk aversion $aA$.

Equation (6) shows how the constraints and benchmarking interact to provide incentives for effort expenditure. To see the intuition, let us focus first on the short-selling constraint. Let us assume for the moment that there exists no limit to margin purchases ($m \to \infty$) and that no short position can be taken ($s = 0$). Under these assumptions, and after exerting effort $e$, the manager receives a signal $y$ and makes his optimal portfolio choice:

$$\theta(y) = \begin{cases} 0 & \text{if } y < -\frac{\mu}{e} L_s \\ H + \frac{\mu + ey}{aA\sigma^2} & \text{otherwise,} \end{cases}$$

with $L_s = 1 + H\left(\frac{\mu}{aA\sigma^2}\right)^{-1}$. When $H = 0$, all signals $y < -\frac{\mu}{e}$ lead to short-selling. Imagine now that the manager is offered a benchmarked contract, with $H > 0$ the benchmark's proportion invested in the risky stock. In this case, the short-selling bound is only hit for *smaller* signals $y < -\frac{\mu}{e} L_s$. In general, increasing $H$ leads to a "wider range" of implementable signals relative to the case of no benchmarking ($H = 0$). Since the effort decision is taken prior to the signal realization, the fact that more signals are implementable under benchmarking ($H > 0$) increases the marginal *expected* utility of effort. The size of this *incremental area* grows with $HaA$. Hence, we expect *the impact of benchmarking to be relatively higher for more risk averse investors.*

Alternatively, assume there is no benchmarking ($H = 0$) but the short-selling limit is expanded from $s = 0$ to $s = H$. Figure 1 shows that, *ceteris paribus*, the effort choice of the manager will coincide with the effort put under benchmarking: given that $s = 0$, benchmarking

the manager's portfolio return ($H > 0$) is, in terms of effort inducement, *equivalent to relaxing the short-selling bound from* 0 *to* $H$. In other words, in the absence of margin purchase constraints, the manager's effort depends on $s + H$; benchmarking the manager's performance and relaxing his short-selling constraints are *perfect substitutes for effort inducement*. The higher $s$, the lower the marginal expected utility of effort induced by benchmarking. In the limit, when the short-selling bounds vanish ($s \to \infty$), we converge to the unconstrained scenario in Section 3.1 where benchmarking was shown to be irrelevant for the manager's effort decision.

These two new results show that when the manager is constrained, intuitively, effort will suffer. Leaving aside the risk-sharing argument and focusing on effort inducement, we show that the investor has two options: either to relax the constraint or to modify the benchmark composition. The former may not be an option, due to regulation or to other concerns that justify the existence of the constraint in the first place. Our model shows that the benchmark design offers the investor an additional "degree of freedom:" she may keep constraints in place while partially alleviating the effort-underinvestment problem. In particular, if the manager's short-selling ability is restricted, the investor may find it convenient to "neutralize" in part this constraint by increasing the benchmark's risk. How responsive the manager's effort decision is with respect to changes in the benchmark composition will depend on how close his unconditional portfolio is to the benchmark. This is why we expect that any change in the benchmark will have a greater effect the more risk averse the manager is.

Let us focus now on the margin purchase constraint. Assume $s \to \infty$ and $m = 1$. This implies that the manager can short any amount but cannot trade on margin: for "very good" signals the manager can only invest up to 100% of the fund's net asset value in the risky stock. His optimal portfolio (as a function of the signal) will be:

$$\theta(y) = \begin{cases} 1 & \text{if } y > \frac{\mu}{e} L_m, \\ \\ H + \frac{\mu + ey}{aA\sigma^2} & \text{otherwise,} \end{cases}$$

with $L_m = (1 - H)\left(\frac{\mu}{aA\sigma^2}\right)^{-1} - 1$. $L_m$ is decreasing in $H$. Decreasing $H$ in the manager's compensation just makes the portfolio constraint "less binding," i.e., binding only for bigger signals. For instance, moving from a benchmarked contract ($H > 0$) to a non-benchmarked contract ($H = 0$) would increase the manager's effort: signals that were not implementable under benchmarking become now feasible. Symmetric to the short-selling constraint, the expected impact on effort expenditure would be analogous if benchmarking were not removed ($H > 0$) and the constraint on margin purchases made looser: from $m = 1$ to $m = 1+H$. Therefore, in the absence of short selling constraints, the manager's effort depends on $m - H$: benchmarking the manager and tightening the margin purchase constraint are perfect substitutes for the manager's effort (dis)incentive. Again, the impact of benchmarking increases, in absolute terms, with the manager's relative risk aversion, $aA$. In the limit, when the manager faces no margin purchase constraint ($m \to \infty$), the benchmark composition is irrelevant for the manager's effort decision.

Notice that the intuition in the case of constraints to margin purchases works symmetrically to the short selling case: the investor may now give more incentives to the manager by either

relaxing the constraint or, alternatively, reducing the risk exposure of the benchmark. By tilting the benchmark towards the risk free bond, the manager gets extra incentives to play more aggressively on "positive" (excess return) signals that would, otherwise, hit the margin constraint. This increases the marginal utility of effort, hence, effort expenditure.

In summary, by modifying the benchmark portfolio composition we observe two opposing effects: for the short selling constrained manager, increasing the benchmark's percentage invested in the risky stock ($H$) induces the manager to put more effort. In contrast, for the manager constrained in his ability to purchases at margin, increasing that percentage lowers the effort incentives. Thus, when (as for most mutual fund managers) both short selling and margin purchase are constrained, the trade-off between these two effects yields the *effort-maximizing benchmark*. This is the question we investigate in the next section.

# 4 The effort-maximizing benchmark

To analyze the composition of the effort-maximizing benchmark, we proceed as follows. Proposition 1 introduces the manager's unconditional expected utility under short selling ($0 \leq s < \infty$) and margin purchase ($1 \leq m < \infty$) constraints for all possible values of $H$ in the real line. In Proposition 2, we show the existence of a continuous and differentiable effort function, $e(H)$, that yields a unique effort choice for each value of $H$. The function attains a global maximum at $H^* = \frac{m-s}{2} - \frac{\mu}{aA\sigma^2}$.

Before introducing the constrained manager's unconditional expected utility we need some notation. Let $\Phi(\cdot)$ denote the cumulative probability function of a Chi-square variable with one degree of freedom: $\Phi(x) = \int_0^x \phi(z)\, dz$, with

$$\phi(z) = \begin{cases} \frac{1}{\sqrt{2\pi}}\, z^{-1/2} \exp(-z/2) & \text{when } z > 0; \\ 0 & \text{otherwise.} \end{cases}$$

**Proposition 1** *Given the finite portfolio constraints $s \geq 0$ and $m \geq 1$, the risk-averse manager's expected utility is $EU_a\left(W_a(e)\right) = -exp(-(1/2)\mu^2/\sigma^2 - aF + V(D,e)) \times g(e, L_s, L_m)$ with the function $g(e, L_s, L_m)$ defined in Appendix B.*

The manager's expected utility function is weighted sum of his unconstrained expected utility (4), independent of $H$, and his expected utility function when the portfolio hits either the short-selling constraint bound, $\exp\left(\frac{\left(\frac{\mu}{\sigma} L_s\right)^2}{2}\right)$, or the margin purchase bound, $\exp\left(\frac{\left(\frac{\mu}{\sigma} L_m\right)^2}{2}\right)$. When the manager is constrained, the benchmark composition (i.e., the value of the parameter $H$) affects the quality of the timing signal through the effort choice.

**Corollary 1** *The function $g(e, L_s, L_m)$ is decreasing with respect to effort for every contract $(F, A, H)$.*

Notice that functions $g(e, L_s, L_m)$ and $g_e(e, L_s, L_m)$ are symmetric with respect to $H$ around $H^* = \frac{m-s}{2} - \frac{\mu}{aA\sigma^2}$, the center of the interval $[-(s + \frac{\mu}{aA\sigma^2}), m - \frac{\mu}{aA\sigma^2}]$. To see this, let $\delta$ represent the deviation in the benchmark portfolio's percentage invested in the risky asset above ($\delta > 0$)

or below ($\delta < 0$) the reference value $H^*$. It can be shown that $L_s(H^* + \delta) = L_m(H^* - \delta)$ for all $\delta \in \Re$. Replacing the latter equality in the functions $g$ and $g_e$, the symmetry is proved.

We call $e_{TB}$ the *third best effort* that maximizes the constrained manager's expected utility function in Proposition 1:

$$e_{TB} = \text{argmax}_e \, -(1/2)\exp(-(1/2)\mu^2/\sigma^2 - aF + V(D,e)) \; \times g(e, L_s, L_m). \tag{7}$$

From (7) it is obvious that, unlike in the unconstrained scenario, the manager's optimal effort depends on $H$ (through $L_s$ and $L_m$). We want to study how the third best effort changes with $H$. In particular, whether there exist an effort-maximizing compensation benchmark.

The following proposition presents general conditions on the effort disutility function and the range of the parameter $H$ for which there exists a well behaved effort function, that is, a function that yields, for each $H$, the utility maximizing third best effort (7). More importantly, the same conditions are shown to be sufficient for the existence of $H^*$ that elicits the highest effort from the manager. The value of $H^*$ is explicitly derived as a function of the manager's portfolio constraints on short selling, $s$, and margin purchase, $m$; his relative risk aversion, $aA$; and the market portfolio moments, $\mu$ and $\sigma^2$.

**Proposition 2** *For all $H \in [-(s + \frac{\mu}{aA\sigma^2}), m - \frac{\mu}{aA\sigma^2}]$ there exists a unique function $e(H)$, continuous and differentiable, such that $e(H) = e_{TB}$. Let $H^* = \frac{m-s}{2} - \frac{\mu}{aA\sigma^2}$. Then, $e(H^*) > e(H)$ for all $H \neq H^* \in [-(s + \frac{\mu}{aA\sigma^2}), m - \frac{\mu}{aA\sigma^2}]$.*

**Corollary 2** *Provided it exists, the effort function $e(H)$ is increasing in $H$ for all $H < -(s + \frac{\mu}{aA\sigma^2})$ and decreasing in $H$ for all $H > m - \frac{\mu}{aA\sigma^2}$. Moreover, the effort function is symmetric in $H$ around $H^*$, i.e., $e(H^* + \delta) = e(H^* - \delta)$ for all $\delta \in \Re$.*

Another way to interpret the effort maximizing benchmark composition $H^*$ is by looking at the fund's Information Ratio. The Information Ratio (relative performance per unit of tracking error volatility) increases with the manager's effort for every signal $y$.[13] Figure 2 shows the Information Ratio as a function of the signal $y$ and given effort $e$. Notice that, when $e = 0$, the Information Ratio coincides with the Sharpe Ratio for every signal $y$. When $e$ increases, the slope increases in absolute value, making the Information Ratio greater for every signal $y$. As $e \to \infty$, in the limit, the Information Ratio also tends to infinity. For $y = \mu$, the Information Ratio becomes $\frac{\mu}{\sigma}\sqrt{1+e}$. Averaging across $y$, the expected Information Ratio is greater than $\frac{\mu}{\sigma}\sqrt{1+e}$ since for all $y < -\frac{\mu}{e}$, the Information Ratio "bounces back": the manager would short the risky asset. Proposition 2 shows that given the contract $(F, A)$, the constrained manager's expected Information Ratio reaches a maximum at $H^*$. Whether this level of effort that maximizes the Information Ratio is optimal or not for the investor will be analyzed in section 6.

So far we have shown that under portfolio constraints, by choosing the appropriate benchmark, effort expenditure can be maximized. The following proposition shows that, for any given

[13] For a given signal $y$, the Information Ratio is defined as $IR(y) = \frac{E(\bar{\theta}(y)x|y)}{SD(\theta(y)x|y)}$. The unconditional Information Ratio will be $IR(e) = \int_{\Re} IR(y)dF(y) = \int_{\Re} \frac{|\mu+ey|}{\sigma\sqrt{1+e}}dF(y) > \frac{\mu}{\sigma}\sqrt{1+e}$, with $F(\cdot)$ the normal distribution function for the signal $y$.

contract and any portfolio composition, the effort choice for the constrained manager is smaller than for the unconstrained manager.

**Corollary 3** *For any given contract $(F, A, H)$ and finite manager's risk aversion, a, the constrained manager's third best effort $e_{TB} < e_{SB}$. In the limit, when the portfolio constraints vanish, the third best effort and the second best effort coincide.*

In other words, the model predicts that, other things equal, unconstrained managers will outperform constrained managers regardless of the composition of the benchmark used in the compensation of constrained managers. This prediction is consistent with Agarwal, Boyson, and Naik (2009) who find that relatively unconstrained hedged mutual funds and hedge funds outperform constrained traditional mutual funds. However, the prediction of our model is in contrast with the effect of the benchmarking restriction documented in Basak, Pavlova, and Shapiro (2008). In their model, constraining the manager may be beneficial for the investor in curbing adverse risk incentives. In our case, these constraints do not arise endogenously. A crucial difference between their model and ours lies in the convexity of the manager's compensation. We conjecture that this convexity may play a crucial role in deriving explicit portfolio constraints endogenously.

We conclude this section by studying two special cases of the more general constrained problem. As illustrated in the examples in Section 3.2, when the manager is only short selling constrained (i.e., unlimited margin purchases), increasing the benchmark investment in the risky asset, $H$, gives the manager more incentives to exert greater effort. In the case of unlimited short selling and constrained margin purchases, the result is symmetric: effort decreases with $H$. In either case, there is no effort maximizing benchmark composition. The following corollary summarizes these findings.

**Corollary 4** *When the manager can purchase at margin with no limit but faces a short selling bound, the effort function is monotonically increasing in $H$. Symmetrically, when the manager can sell short with no restriction but faces limited margin purchase, the effort function is monotonically decreasing with $H$.*

# 5 The principal's problem

The investor's optimal contract $(F, A, H)$ maximizes her expected utility subject to the manager's incentive compatibility and participation constraints. For simplicity, and without loss of generality, we normalize the manager's reservation value to $-\exp(-(1/2)\mu^2/\sigma^2)$. For a given contract $(F, A, H)$, the manager's (conditional) wealth is given as a percentage, equation (2), of the fund's net asset value.

The constrained manager, after accepting the contract, decides how much effort to exert. Subsequently, he receives the signal $y$ and invests a proportion $\theta(y)$ as in (6) in the risky asset. Let $t(A) = \frac{b(1-A)}{aA}$ and $T(A) = (2 - t(A))t(A)$. The investor's expected utility is introduced in the following proposition.

**Proposition 3** *Given the portfolio constraints $s \geq 0$ and $m \geq 1$, the expected utility of the risk-averse investor is $EU_b(W_b(e)) = -exp(b(F-R) - (1/2)\mu^2/\sigma^2) \times v(e, L_s, L_m)$ with the function $v(e, L_s, L_m)$ defined in Appendix B.*

The investor must choose the optimal linear contract, which includes the optimal flat fee and the incentive fee, $F$ and $A$, respectively, and the optimal benchmark, $H$, subject to the participation constraint $-\exp(-(1/2)\mu^2/\sigma^2 - aF + V(a,e)) \times g(e, L_s, L_m) \geq -\exp(-(1/2)\mu^2/\sigma^2)$. Clearly, neither effort nor $H$ or $A$ are a function of $F$. This, along with the fact that the left-hand side is increasing in $F$ and the investor's utility is decreasing in $F$, implies that under the optimal contract, the participation constraint is binding. In other words, managers with higher reservation utility (arguably, with a better record and/or working for bigger funds) will receive a higher flat fee. The investor's expected utility thus can be expressed as a function of the contract $(A, H)$, and the manager's level of effort, $e$:

$$EU_b(W_b(e)|A,H) = -exp(-bR - (1/2)\mu^2/\sigma^2 + (b/a)V(D,e)) \times g(e, L_s, L_m)^{b/a} v(e, L_s, L_m). \quad (8)$$

We want to study how the portfolio constraints *and* the presence of moral hazard affect the investor's optimal contract. We distinguish four cases depending on whether the manager's effort is publicly observable or not (moral hazard) and whether the manager is constrained or unconstrained in his portfolio choice.

Assume first that the manager's portfolio is unconstrained. If the manager's effort decision is observable, the investor maximizes her expected utility with respect to $A$, $H$, and effort. We call this the *first best scenario*. We show then that the optimal contract is given by the first best incentive fee, $A_{FB} = b/(a+b)$, and zero benchmarking, $H = 0$. Notice that this corresponds exactly to Proposition 1 in Admati and Pfleiderer (1997) in the presence of a risk-free asset. The function $v(e, L_s, L_m)$ becomes $g(e)$. The investor chooses the first best effort level, $e_{FB}$, that solves

$$\max_e EU_b(W_b(e)|A_{FB}, 0) = -\exp\big(-bR - (1/2)(\mu/\sigma)^2 + (b/a)V(D,e)\big) g(e)^{\frac{a+b}{a}}.$$

This results in the first order condition:

$$V_e(D, e_{FB}) = \frac{1 + a/b}{2(1 + e_{FB})} = \frac{1/A_{FB}}{2(1 + e_{FB})}.$$

Notice that the higher the manager's risk aversion (relative to the investor's risk aversion), $a/b$, the lower the optimal incentive fee, $A_{FB}$, and, consequently, the higher the investor's participation in the fund's return, $1 - A_{FB}$. Hence, the investor becomes more interested in the manager's signal precision: the marginal utility of effort increases and so does $e_{FB}$.[14]

In the case when the manager's effort decision is not observable, the investor's problem consists in finding the optimal split that maximizes (8) subject to the manager's optimal effort

[14] Here we are assuming that $D$ is independent of $a$. In case effort's marginal disutility were increasing in $a$, the net effect on the first best effort would be unclear.

condition. Assume first that there exist no portfolio constraints. We call this scenario the *second best*. As shown in Section 3.1, the manager's second best effort, $e_{SB}$, is independent of $A$ and $H$. This result is consistent with Stoughton (1993) and Admati and Pfleiderer (1997). The investor will choose the same contract as in the first best case: $(A_{FB}, 0)$. The second best effort satisfies the optimality condition (5):

$$V_e(D, e_{SB}) = \frac{1}{2(1 + e_{SB})}.$$

Comparing the latter two conditions, it is obvious that $e_{FB} > e_{SB}$ for all $a/b > 0$. That is, the second best effort coincides with the first best effort the investor would choose herself in the limit when $b \to \infty$ (or, $a \to 0$) and, consequently, $A_{FB} \to 1$. This would be equivalent to a swap contract between the manager (who takes all portfolio risk) and the investor (who gets, in exchange, a flat fee, $F < 0$, from the manager). Notice that the manager's marginal utility of effort is, in the second best case, independent of $a/b$, $A$ or $H$. Moreover, the cost (in terms of effort expenditure) of moral hazard increases with $a/b$: the investor would want to increase the manager's effort but the second best contract fails to induce it. This failure will be partially offset in the presence of portfolio constraints where both the incentive fee and the benchmark composition play a role in inducing greater effort by the manager.

Intuitively, looking at the first best effort choice (observable by the investor, hence enforceable) we see the investor's tradeoff: risk-sharing versus effort inducement. The more risk-averse the investor is relative to the manager (higher $A_{FB}$) the more concerned she (the investor) is about risk-sharing relative to effort inducement. The manager's marginal utility of effort (a function of $1/A_{FB}$) decreases, and so does effort expenditure. On the contrary, as the investor's risk aversion decreases relative to the manager's (lower $A_{FB}$), the more focused the investor becomes on the effort inducement problem relative to risk sharing. In this case, the manager's marginal utility of effort increases, and so does effort.

In the second best case, the investor cannot observe the effort choice anymore. The manager is unconstrained in his portfolio choice. This means that he (the manager) decides the scale of the final investment, regardless of how much he participates in the final output (he can always sell short or purchase at margin as much as needed to "accommodate" any sharing rule $A$). Hence, the investor has lost her ability to leverage the manager's effort up to the first best level. Notice that the marginal utility of effort in the second best case is a percentage $0 < A_{FB} < 1$ of the first best marginal utility. The more risk tolerant the investor with respect to the manager, the more severe the latter's effort underinvestment relative to the first best and, as shown next, the stronger the case for incentive fees and benchmarking in the contract design.

We turn now to the case in which the manager's portfolio is constrained. Assume first that the manager's effort is observable. We show that the contract $(A_{FB}, 0)$ is still optimal. The function $v(e, L_s, L_m)$ becomes $g(e, L_s(0), L_m(0))$. In this *constrained first best scenario*, the investor chooses the constrained first best effort level, $e^c_{FB}$, that maximizes $EU_b(W_b(e)|A_{FB}, 0) = -\exp\big(-bR - (1/2)(\mu/\sigma)^2 + (b/a)V(D, e)\big)g(e, L_s(0), L_m(0))^{\frac{a+b}{a}}$:

$$V_e(D, e^c_{FB}) = (1 + a/b)\,\frac{g_e}{g}(e^c_{FB}, L_s(0), L_m(0)).$$

Notice that, as expected, portfolio constraints decrease the optimal effort choice: $e_{FB} > e^c_{FB}$.

Assume now that the manager's effort is not observable. We call this scenario the *third best*. The manager's third best effort satisfies (7). Section 3.2 shows that effort is increasing in $A$ and reaches an absolute maximum at $H^*$. We show that the contract $(A_{FB}, 0)$ is no longer optimal. These results are presented in the following proposition.

**Proposition 4** *Absent any portfolio constraint, the contract* $(A_{FB}, 0)$ *is optimal, both for the public information case as well as under moral hazard.*

*Under portfolio constraints and no moral hazard, the contract* $(A_{FB}, 0)$ *is still optimal. When the effort decision is not observable by the investor and hence there exists moral hazard, the contract* $(A_{FB}, 0)$ *is suboptimal.*

The implication of this proposition is that, to justify a benchmark different from the risk-free asset (or, in its absence, the minimum variance portfolio), *both* moral hazard and portfolio constraints must coexist. The following table summarizes the four possible scenarios and the optimal contract $(A, H)$ in each of them:

| | Effort observable | Effort unobservable |
|---|---|---|
| Unconstrained portfolio | FIRST BEST (FB) $(A_{FB}, 0)$ | SECOND BEST (SB) $(A_{FB}, 0)$ |
| Constrained portfolio | CONSTRAINED FB (CFB) $(A_{FB}, 0)$ | THIRD BEST (TB) $(A_{TB}, H_{TB})$ |

We are interested in studying the optimal contract in the third best scenario, $(A_{TB}, H_{TB})$. In spite of the simplifications, we cannot solve analytically for the general optimal contract under moral hazard and portfolio constraints. In the next section, we present a numerical solution to the problem.

# 6 A numerical solution of the third best contract

Let us recall that the effort disutiliy function is given by $V(D, e) = \frac{D}{2}e^2$. Throughout the numerical analysis, we take the market excess return $\mu = 6\%$ and the market volatility $\sigma = 18\%$, both on an annual basis. We assume the manager is fully constrained, that is, $s = 0$ and $m = 1$. For this exercise, the manager's risk aversion coefficient $a$ takes values $a = \{2, 3, 4, 5, 6\}$; the investor's risk aversion coefficient $b = 4$. The effort disutility parameter $D = 1$. All the results, as percentage figures, are reported in Table 1.

[INSERT TABLE 1 HERE]

Panel A shows, for comparison purposes, the first best incentive fee, $A_{FB} = b/(a + b)$, and effort, $e_{FB}$, for each value of the manager's risk aversion coefficient. Recall that the first best contract is unbenchmarked ($H_{FB} = 0$). The first best scenario assumes no portfolio constraints

and no moral hazard. As expected, the incentive fee decreases as the manager's risk aversion increases. On the other side, the first best effort increases: as the investor retains a higher percentage of the portfolio's return, the marginal utility of effort increases. The second best effort is constant for all $a$ and equal to $e_{SB} = 36.60\%$. Notice that, as predicted in section 5, the effort underinvestment increases as the investor becomes more risk tolerant relative to the manager.

Panel B presents the optimal third-best contract that maximizes the investor's certainty equivalent wealth (CEW) under portfolio constraints. This contract is obtained as follows. For each value of the manager's risk aversion coefficient, $a$, we calculate the manager's third best effort effort (7) and the investor's CEW corresponding to the expected utility (8) for a grid of values for $A$ and $H$. $A$ changes from 0.01 through 1 at intervals of length 0.01. Likewise, $H$ changes from $-(\frac{0.06}{aA0.18^2})$ through $1 - \frac{0.06}{aA0.18^2}$ at intervals of length 0.01.

As expected, the incentive fee $A_{TB}$ decreases as the manager's risk aversion increases. More interestingly, for any value of the manager's risk aversion coefficient $a$, the constrained third best incentive fee is always higher than the corresponding unconstrained, first best incentive fee, $A_{FB}$. This confirms that under both moral hazard and portfolio constraints, the explicit incentive fee, $A$, plays an additional role beyond pure risk sharing: namely, inducing the manager to put more effort. The optimal benchmarking, $H_{TB}$, is higher than zero and increasing in $a$; the higher the manager's risk aversion relative to the investor's, the more relevant benchmarking becomes. The intuition for this result is as follows: when the investor's risk aversion is relatively low with respect to the manager's risk aversion ($b$ lower than $a$), the investor keeps a higher percentage of the portfolio's relative performance. In other words, the effort inducement argument becomes relatively more important for the investor. Under portfolio constraints, benchmarking may help to alleviate the manager's underinvestment in effort. At the same time, recall from section 3.2 that the effort-inducement impact of benchmarking as an alternative to relaxing portfolio constraints becomes stronger as the manager's risk aversion increases. Therefore, we expect the efficacy of benchmarking in effort inducement to increase as the investor becomes more risk-tolerant relative to the manager.

This is confirmed in Panel C where we report the third best effort and the percentage increase in the third-best effort relative to the effort under the suboptimal first best contract (the effort the manager would put if he is offered the first best contract). This percentage can be interpreted as the increase in the signal precision due to optimal benchmarking; it more than doubles from 6.61% for $a = 2$ to 15.03% for $a = 6$. Intuitively, constrained, risk-averse managers have incentives to passively (i.e., zero effort) track the benchmark portfolio at the expense of the investor's utility. Our model suggests that, under portfolio constraints, benchmarked incentive fees are a useful mechanism to turn the passive manager into an active (i.e., positive effort) portfolio manager. As the investor's risk aversion decreases relative to the manager's (that is, for higher $a$), the higher the impact of incentive fees and optimal benchmarking on effort expenditure.

Panel D studies the effect of changing the manager's risk aversion on the investor's CEW.[15] This percentage can be interpreted as the excess risk-free return that would yield the same expected utility to the investor as granting the manager the third best contract and letting him choose the random portfolio. We observe that the investor's expected utility increases with the manager's risk aversion (effectively as the investor becomes more risk tolerant relative to the manager). More interestingly, the increase in utility relative to the suboptimal, unbenchmarked first best contract more than triples from 0.5% for $a = 2$ to 1.74% for $a = 6$. This can be interpreted as a cost (forgone return) of the suboptimality of the first best contract under moral hazard and portfolio constraints.

In section 4, we defined $H^*$ as the effort maximizing benchmark composition under portfolio constraints. Panel E studies whether this benchmark is optimal for the investor. We report $H^*$ as in Proposition 2 evaluated at the third best incentive fee $A_{TB}$. Notice that the effort maximizing benchmark is different from the third best optimal benchmark $H_{TB}$. In our numerical example, $H^*$ implies shorting the risky asset. This is a direct consequence of the manager's preferences: the unconditional (zero information) portfolio $\mu/aA\sigma^2$ involves investing above 50% in the risky stock for all the risk aversion coefficients under consideration. The portfolio is bounded between zero and one. To induce the highest effort incentives on the manager, the benchmark has to "compensate" for the high risk exposure of the unconditional portfolio, hence resulting in an effort-maximizing benchmark that shorts the risky stock. As the investor becomes relatively less risk averse with respect to the manager, the investor's share in the portfolio payoff increases and the distance between the effort maximizing benchmark and the optimal third best benchmark shrinks. This is an important implication of our model: depending on the investor's risk aversion relative to the manager, the investor may optimally forgo higher effort inducement on the manager by moving away from the highest effort benchmark $H^*$ and therefore, the highest Information Ratio. The intuition behind this result lies in the balance between the incentives for effort expenditure (which increases the investor's expected utility) and the distortion that benchmarking introduces by leading to a suboptimal risk sharing (Roll's critique to benchmarking). When the investor is more risk averse than the manager (in our example, for $a = 2$ and $a = 3$), the investor's part in the portfolio's return $(1 - A)$ decreases and so does her marginal utility from the manager's effort. The investor's concern about risk sharing dominates the role of the benchmark in providing managerial effort incentives. As the manager becomes more risk averse than the investor ($a = 5$ and $a = 6$), the difference between the optimal third best benchmark and the effort maximizing benchmark decreases. As a consequence, given the third best incentive fee $A_{TB}$, the third best effort approaches the highest effort under portfolio constraints $e(A_{TB}, H^*)$, also reported in Panel E.

In unreported exercises, we repeat Table 1 for values of the effort disutility parameter $D = 2$ and $D = 3$. Qualitatively, the results and conclusions are the same. Holding the manager's risk

[15] Given the investor's utility function, $U_b(W) = -\exp(-bW)$, the certainty equivalent wealth of the expected utility $u$ is given by the inverse of this function, $C(u) = -\ln(-u)/b$. Clearly, for any two values of the investor's expected utility, $u_1$ and $u_2$, $u_1 > u_2$ if and only if $C(u_1) > C(u_2)$. In concrete, given equation (8), for a given expected utility value $u = -\exp(-bR - (1/2)\mu^2/\sigma^2 + (b/a)V(D,e)) \times g(e, L_s, L_m)^{b/a} v(e, L_s, L_m)$. Then, $C(u) - R = (1/2b)\mu^2/\sigma^2 - (1/a)(V(D,e) + \ln g(e, L_s, L_m)) - (1/b)\ln v(e, L_s, L_m)$. We call $CEW(u) = C(u) - R$, the *excess* risk-free return (above the bond's return, $R$) that leaves the investor indifferent.

aversion constant, effort and utility (CEW), as expected, decrease as $D$ increases. The increment of the incentive fee with respect to the unconstrained first best share is lower the higher $D$. At the same time, benchmarking (the size of $H_{TB}$) decreases with $D$ while the distance with respect to the effort maximizing benchmark $H^*$ widens up. All these results, intuitively suggest that as the manager's ability to obtain and process private information decreases (higher $D$), incentive fees and benchmarking become more costly and less relevant in effort inducement. The role of the incentive fee as a risk sharing mechanism grows relative to effort inducement.

# 7 Conclusion

This paper investigates the effort inducement incentives of (potentially benchmarked) linear incentive fee contracts. Incentives arise explicitly via the compensation of the manager. The investor has to decide simultaneously the incentive fee (the manager's participation in the delegated portfolio's return) and the benchmark composition.

The contribution of our paper to the literature on management compensation comes from the fact that we incorporate portfolio constraints explicitly in the model. These constraints are exogenous in our model and could be motivated by regulation or, as suggested by Almazan et al (2004), as alternative monitoring mechanism in a broader equilibrium model.

Under portfolio constraints and moral hazard, our model derives a new set of predictions. Incentive fees should be higher in the presence of portfolio constraints. Moreover, the optimal composition of the benchmark is not exogenous to those constraints.[16] When the benchmark design is endogenous, maximizing the Information Ratio may turn suboptimal for the fund investor: depending on the investor's risk aversion (relative to the manager's) the increase in the manager's timing ability may not compensate for the excessive risk exposure. Only when the investor is sufficiently risk tolerant and the manager's ability high enough, maximizing the Information Ratio becomes optimal for the investor. These results are in contrast with the predictions from the unconstrained setting in Adamati and Pfleiderer (1997), where benchmarking is suboptimal. When portfolio constraints are removed, the model predicts that the manager's effort is unrelated to the incentive fee and the benchmark composition, a well-known result in the literature. These results are consistent with the prevalence of absolute return (non-benchmarked) compensation schemes among hedge fund managers, arguably much less constrained than mutual fund managers. They are also consistent with the prevalence of incentive fees among funds that invest predominantly in equity versus debt (catering to more risk tolerant investors) and funds that show higher turnover (managers trading more aggressively on their private information).

There are two possible, related extensions of our model worth mentioning. The first extension would include implicit incentive fees. To what extent implicit incentives (driven by flow-performance sensitivity) and explicit incentives act as complements or substitutes is still an open question. Del Guercio and Tkac (2002) compare the flow-performance sensitivity across mutual

[16] Almazan et al (2004) find that portfolio constraints are not relevant in explaining the cross section performance of mutual funds. There is, however, no distinction in their tests between funds with and without explicit incentive fees. Elton, Gruber, and Blake (2003) do find superior performance among incentive fee funds, however, they do not study how constrained the funds are.

and pension funds in the US. They find important differences between both industries: relative performance measures like tracking error and style-adjusted returns are more closely related to flow among pension funds; flow in the mutual fund industry is likely to be more closely related to absolute returns. Regarding the flow-performance sensitivity, they find a lower explanatory power of quantitative performance measures in explaining flow in the pension fund industry, relative to the mutual fund industry. Taking this evidence together, if implicit and explicit incentive fees act as substitutes, we would expect a higher weight of implicit, performance-flow driven incentives in the mutual fund industry than in the pension fund industry. In other words, explicit incentive fees might play a bigger role in pension fund industry relative to the mutual fund industry. Additionally, and by the same substitution argument, explicit benchmarking (as studied in our model) would be more prevalent in the mutual fund industry than in the pension fund industry where performance-flow incentives are more sensitive to relative performance rather than absolute performance. Consistently, the evidence reported in Deli (2002) and Elton, Gruber, and Blake (2003) confirms that only a small percentage of mutual funds use explicit incentive fees, all of which are benchmarked. Golec (1992) and Golec and Starks (2004) mention that explicit incentive fee contracts are more prevalent among pension funds than mutual funds, although there is no reference to the structure, benchmarked or not, of these incentive fees.[17]

The second extension is related to the assumed piece-wise linearity of the incentive fee. The interaction between convex implicit or explicit fees for the manager and portfolio constraints is left for future research.

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[17] As an alternative explanation to the limited use of incentive fees in the mutual fund industry, Golec and Starks (2004) document that in 1971, the US Congress banned asymmetric incentive fees for mutual funds. As a consequence, many mutual funds dropped incentive fees totally. On the other side, Coles et al (2000) claim that in practice there are "few, if any, regulatory limitations on the type of compensation contracts that are legally acceptable," implying that the role of regulation in explaining the limited use of incentive fees among mutual fund managers may be only partial.

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**Estimates from the Numerical Exercise**

| | Manager's risk aversion coefficient $a$ | | | | |
|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 |
| | Panel A: The first best contract | | | | |
| $A_{FB}$ | 66.67 | 57.14 | 50.00 | 44.44 | 40.00 |
| $e_{FB}$ | 50.00 | 56.06 | 61.80 | 67.26 | 72.47 |
| | Panel B: The third best contract | | | | |
| $A_{TB}$ | 73.66 | 65.14 | 58.00 | 54.44 | 51.00 |
| $H_{TB}$ | 06.31 | 07.24 | 10.17 | 10.97 | 11.48 |
| | Panel C: The third best effort | | | | |
| $e_{TB}$ | 10.96 | 13.77 | 15.53 | 17.36 | 18.75 |
| $\Delta e_{TB}$ | 06.61 | 08.42 | 08.52 | 12.36 | 15.03 |
| | Panel D: The third best CEW | | | | |
| $CEW(A_{TB}, H_{TB})$ | 01.65 | 02.35 | 02.67 | 02.85 | 02.97 |
| $\Delta CEW(A_{TB}, H_{TB})$ | 00.49 | 00.75 | 00.91 | 01.39 | 01.74 |
| | Panel E: Effort maximizing $H^*$ | | | | |
| $H^*$ | -75.70 | -44.76 | -29.82 | -18.03 | -10.52 |
| $e(A_{TB}, H^*)$ | 12.56 | 14.85 | 16.41 | 17.97 | 19.19 |

Table 1: Panel A reports the unconstrained, first best incentive fee $A_{FB}$ and effort $e_{FB}$ in the absence of moral hazard. $(A_{TB}, H_{TB})$ in Panel B represents the optimal third best contract. Panel C shows $e_{TB}$, the constrained, third best effort exerted by the manager; $\Delta e_{TB} = e_{TB}/e(A_{FB}, 0) - 1$ represents the percentage increase in effort under the optimal third best contract relative to the suboptimal first best contract $(A_{FB}, 0)$. $CEW(A_{TB}, H_{TB})$ in Panel D is the investor's Certainty Equivalent Wealth (CEW) under the third best contract; $\Delta CEW_{TB} = CEW(A_{TB}, H_{TB})/CEW(A_{FB}, 0) - 1$ is the corresponding increase in CEW under the optimal third best contract relative to the suboptimal first best contract $(A_{FB}, 0)$. The effort maximizing benchmark composition $H^*$ is presented in Panel E together with $e(A_{TB}, H^*)$, the highest effort under portfolio constraints when the manager receives an incentive fee $A_{TB}$. The unconstrained, second best effort under moral hazard is independent of $a$ and equal to $e_{SB} = 36.60$. The investor's risk aversion coefficient is $b = 4$. $\mu = 0.06$, $\sigma = 0.18$, $D = 1$, $m = 1$, $s = 0$. All figures as percentages.

## Appendix A: Introducing stock-picking ability

In this paper, for the sake of simplicity, we have assumed that there exists one single risky asset and that the manager can only show timing ability.

In this section we will explore how the introduction of a second risky asset and the existence of stock-picking ability may affect our main conclusions. Assume that there are two risky assets and a risk-less bond. Let the risky assets excess return be denoted by $(x, z)$. $x$ denotes the excess return on a well diversified, timing portfolio representing the fund's investment objective. $z$ is the excess return on another security or portfolio with unconditional risk premium $\mu_z$ and volatility $\sigma_z$. By investing in security $z$ the manager expects to obtain a risk-adjusted excess return $\mu_z$ at the expense of increasing the portfolio's diversifiable risk by $\sigma_z$. We follow the "portfolio model" of timing and selectivity in Admati et al (1986). Assets $x$ and $z$ are independent and normally distributed:

$$\begin{pmatrix} x \\ z \end{pmatrix} \sim \mathcal{N}\left( \begin{pmatrix} \mu_x \\ \mu_z \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_z^2 \end{pmatrix} \right).$$

The manager and the investor have the same preferences as in the simpler single risky asset model. Likewise, the contract offered to the manager has the same structure and parameters, namely, $(F, A, H)$. After learning the contract, the manager puts some non-observable effort. In this setting, we distinguish between two types of effort and, accordingly, two types of managerial skills. On the one side, the manager may invest effort to learn about the performance of the securities in the fund's objective investment opportunity set. This effort is represented by $e_x \geq 0$. If the manager puts some effort $e_x > 0$, according to the model in Section 3.1, the manager's investment in the timing portfolio in $t-1$ should increase with the portfolio's realized performance in $t$. Independently, the manager may invest to identify a mispriced security. The more he learns about this security, the higher the excess return per unit of idiosyncratic risk. Notice that this formulation is equivalent to the factor structure in Admati and Pfleiderer (1997) with the timing portfolio playing the role of the common factor and the selectivity portfolio representing the idiosyncratic term.[18]

If the manager accepts the contract, he decides both levels of effort in acquiring private (non observable) information that materializes in two independent signals:

$$\begin{aligned} y_x &= x + \tfrac{\sigma_x}{\sqrt{e_x}}\epsilon_x, \\ y_z &= z + \tfrac{\sigma_z}{\sqrt{e_z}}\epsilon_z. \end{aligned}$$

Noise terms follow a standard normal distribution. Following the traditional approach in the literature, we assume that selectivity information is independent of market timing information. This implies that both noise terms are orthogonal to $x$ and $z$ and uncorrelated. In other words, by observing the market timing private signal, the manager learns nothing about stock picking and vice-versa. Moreover, assume that $E(i\ \epsilon_i) = E(\epsilon_x\, \epsilon_z) = 0$ for $i = \{x, z\}$.

The greater the effort, the higher the corresponding's signal's precision. Conditional on the

[18] In their model, the manager only obtains information (puts effort) about the idiosyncratic term.

manager's effort and the signal realization, the timing portfolio's excess return is normally distributed with mean excess return $E(x|y_x) = \frac{\mu_x + e_x\, y_x}{1+e_x}$ and conditional precision $\text{Var}^{-1}(x|y_x) = \frac{1}{\sigma_x^2}(1+e_x)$. Analogously, the conditional excess return on the selectivity security will be normally distributed with mean excess return $E(z|y_z) = \frac{\mu_z + e_z\, y_z}{1+e_z}$ and conditional precision $\text{Var}^{-1}(z|y_z) = \frac{1}{\sigma_z^2}(1+e_z)$. Given our assumptions, the conditional returns are uncorrelated.

Effort is costly. We redefine the effort disutility function $V(D,e) = \frac{D_x}{2}e_x^2 + \frac{D_z}{2}e_z^2$ to accommodate the manager's timing and selectivity abilities. This implies that market timing and selectivity effort disutility are independent.[19]

We now revisit the unconstrained portfolio choice in Section 3.1. Based on the conditional moments, the manager decides what percentage of the fund's net asset value to invest in the timing portfolio, $\theta_x$ and what percentage to invest in the individual security $\theta_z$; the remaining, $1 - \theta_x - \theta_z$ is invested in the risk-free bond. Let $\theta = (\theta_x, \theta_z)'$. Therefore, the portfolio's return will be $R_p = R + (x,z)\theta$.

The benchmark is defined as a portfolio of the risk-free rate, the timing portfolio and the selectivity security with proportions $(1 - H_x - H_z, H_x, H_z)$ and return $R_H = R + H_x\, x + H_z z$.

The portfolio's net return over the benchmark is given by $R_p - R_H = (x,z)\bar{\theta}$ with $\bar{\theta} = (\theta_x - H_x, \theta_z - H_z)'$. Given the signal realization $(y_x, y_z)$ and following the same procedure as in Section 3.1, we obtain the unconstrained conditional portfolio $\bar{\theta}(y) = \frac{1}{aA}\left(\frac{\mu_x + e_x\, y_x}{\sigma_x^2}, \frac{\mu_z + e_z\, y_z}{\sigma_z^2}\right)'$ and the manager's unconditional expected utility:

$$EU(W_a(e)) = -\exp\left(-(1/2)(\mu_x^2/\sigma_x^2) - (1/2)(\mu_z^2/\sigma_z^2) - aF + V(D,e)\right) g(e_x) g(e_z),$$

with $g(e_i) = \left(\frac{1}{1+e_i}\right)^{1/2}$. Equation (5) shows the the optimal second best effort $e_i$ with $i = \{x, z\}$. Timing and selectivity effort choices are independent of the contract and the benchmark composition. They only depend on the effort disutility parameters in $D$. This is the well-known non-incentive results in Admati and Pfleiderer (1997).

We now tackle the constrained portfolio choice. Let $s_i$ and $m_i$, with $i = \{x, z\}$ denote the corresponding portfolio constraints for the timing and selectivity assets, respectively. The portfolio constraints need not coincide for both assets. Let us recall that portfolio constraints are exogenous in our model. They are motivated by regulatory or statutory constraints aiming at protecting investor's risk-exposure or, alternatively, to costly monitoring mechanisms over the manager's unobservable actions. The percentage invested in each asset, $\theta_i(y_i)$, will be a function of the timing signal $y_i$ for $i = \{x, z\}$. $\theta_i(y_i)$ coincides with portfolio (6) in Section 3.2 where, given our assumptions on the information structure, the portfolio constraints $(m_i, s_i)$, the asset moments $(\mu_i, \sigma_i)$, the signal $y_i$, the effort choice $e_i$ and the benchmark composition $H_i$ are now asset specific.

Averaging across $y_x$ and $y_z$, we obtain the manager's unconditional expected utility $EU(W_a(e)) = -\exp(-(1/2)(\mu_x^2/\sigma_x^2) - (1/2)(\mu_z^2/\sigma_z^2) - aF + V(D,e))\, g(e_x, L_{s_x}, L_{m_x})\, g(e_z, L_{s_z}, L_{m_z})$, with $g(e_i, L_{s_i}, L_{m_i})$ the asset specific equivalent to Proposition 1. Proposition 2 and corollaries 2 through 4 hold for

[19] See Van Nieuwerburgh and Veldkamp (2008) and Kacperczcyk, Van Nieuwerburgh, and Veldkamp (2009) for a model with strategic choice of learning and its implications for timing and selectivity skills.

each asset. That is, both for timing and selectivity skills, the unbounded second best effort is greater than the constrained third best effort and they coincide in the limit, as the corresponding portfolio limits vanish. Notice that the explicit effort maximizing benchmark composition $(H_x^*, H_z^*)$ will depend on the asset moments and the specific portfolio constraints on each asset. Moreover, the effort function for each signal will be a function of the proportion held in the same stock in the benchmark: $e_x(H_x)$ for the timing portfolio and $e_z(H_z)$ for the selectivity asset.

Let us turn now to the principal's problem. Analogously to the reformulation of the manager's expected utility function in the presence of selectivity ability, the investor's expected utility function in Proposition 3 becomes $EU_b(W_b(e)) = -exp(b(F-R) - (1/2)\mu_x^2/\sigma_x^2 - (1/2)\mu_z^2/\sigma_z^2) \times v(e_x, L_{s_x}, L_{m_x})\, v(e_z, L_{s_z}, L_{m_z})$ with the function $v$ as defined in the Appendix B.

Appendix B shows that the results in Proposition 4 hold in the presence of selectivity information. Concretely, the contract $(A_{FB}, 0)$ is shown to be suboptimal. Since the third best effort function for each signal is independent of the benchmark component for the other signal, the numerical results with respect to the optimal benchmark composition and its relation to the portfolio constraints in Section 6 will remain qualitatively unchanged.

# Appendix B: Proofs

## Proof of Proposition 1

Replacing (6) in the manager's utility function:

$$
\begin{aligned}
EU\left(W_a(y)\right) = &-\exp(-aF + V(D,e)) \times \\
&\begin{cases}
\exp\left((H+s)aAE(x|y) + (1/2)((H+s)aA)^2 \mathrm{Var}(x|y)\right) & \text{if } y < -\frac{\mu}{e} L_s \\
\exp\left(-(1/2)E^2(x|y)/\mathrm{Var}(x|y)\right) & \text{otherwise} \\
\exp\left(-(m-H)aAE(x|y) + (1/2)((m-H)aA)^2 \mathrm{Var}(x|y)\right) & \text{if } y > \frac{\mu}{e} L_m.
\end{cases}
\end{aligned}
$$

Multiplying the previous expression by the density function of the signal variable, $y$, we obtain:

$$
\begin{aligned}
&-\exp(-(1/2)(\mu^2/\sigma^2) - aF + V(D,e)) \left(\frac{e}{1+e}\right)^{1/2} \frac{1}{\sqrt{2\pi}\sigma} \times \\
&\begin{cases}
\exp\left(\frac{\left(\frac{\mu}{\sigma} L_s\right)^2}{2}\right) \exp\left(-(1/2)\frac{e}{1+e}\left(\frac{y}{\sigma} - \frac{\mu}{\sigma} L_s\right)^2\right) & \text{if } y < -\frac{\mu}{e} L_s \\
\exp\left(-(1/2)e\left(\frac{y}{\sigma}\right)^2\right) & \text{otherwise} \\
\exp\left(\frac{\left(\frac{\mu}{\sigma} L_m\right)^2}{2}\right) \exp\left(-(1/2)\frac{e}{1+e}\left(\frac{y}{\sigma} + \frac{\mu}{\sigma} L_m\right)^2\right) & \text{if } y > \frac{\mu}{e} L_m.
\end{cases}
\end{aligned}
$$

Replace $k = \frac{e}{1+e}\left(\frac{y}{\sigma} - \frac{\mu}{\sigma}L_s\right)^2$ if $y < -\frac{\mu}{e}L_s$; $k = \frac{e}{1+e}\left(\frac{y}{\sigma} + \frac{\mu}{\sigma}L_m\right)^2$ if $y > \frac{\mu}{e}L_m$, and $k = e\left(\frac{y}{\sigma}\right)^2$ otherwise. Integrating over $k$ and given the definition of $\Phi(\cdot)$, we obtain the the manager's unconditional expected utility function $g(e, L_s, L_m) = (1/2)\times$

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{2}\right)\left[1 + \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s\right)^2\right)\right] +$$

$$\left(\frac{1}{1+e}\right)^{1/2}\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right) - \Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)\right] +$$

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{2}\right)\left[1 - \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m\right)^2\right)\right]$$

if $H < -\left(s + \frac{\mu}{aA\sigma^2}\right)$;

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{2}\right)\left[1 - \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s\right)^2\right)\right] +$$

$$\left(\frac{1}{1+e}\right)^{1/2}\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right) + \Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)\right] +$$

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{2}\right)\left[1 - \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m\right)^2\right)\right]$$

if $-\left(s + \frac{\mu}{aA\sigma^2}\right) \leq H \leq m - \frac{\mu}{aA\sigma^2}$;

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{2}\right)\left[1 - \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s\right)^2\right)\right] +$$

$$\left(\frac{1}{1+e}\right)^{1/2}\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right) - \Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)\right] +$$

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{2}\right)\left[1 + \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m\right)^2\right)\right]$$

if $H > m - \frac{\mu}{aA\sigma^2}$. *QED*

## Proof of Corollary 1

The first derivative of the function $g(e, L_s, L_m)$ with respect to effort is $g_e(e, L_s, L_m) = -\frac{1}{4}\left(\frac{1}{1+e}\right)^{3/2} \times$

$$\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)-\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)\right] \quad \text{if } H<-\left(s+\frac{\mu}{aA\sigma^2}\right)$$

$$\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)+\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)\right] \quad \text{if } -\left(s+\frac{\mu}{aA\sigma^2}\right)\leq H\leq m-\frac{\mu}{aA\sigma^2}$$

$$\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)-\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)\right] \quad \text{if } H>m-\frac{\mu}{aA\sigma^2},$$

By definition, $|L_m|>|L_s|$ for all $-\infty<H<-\left(s+\frac{\mu}{aA\sigma^2}\right)$ such that $\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)-\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)\right]>0$; likewise $|L_s|>|L_m|$ for all $\infty>H>m-\frac{\mu}{aA\sigma^2}$ such that $\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)-\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)\right]>0$. *QED*

## Proof of Proposition 2

Let us define $\mathcal{J}(e,L_s,L_m)=V_e(D,e)\times g(e,L_s,L_m)+g_e(e,L_s,L_m)$. The function $\mathcal{J}\in\mathcal{C}^1$ for all $(e,H)$. The third best effort in (7) satisfies:

$$\mathcal{J}(e_{TB},L_s,L_m)=0, \tag{B1}$$
$$\mathcal{J}_e(e_{TB},L_s,L_m)>0. \tag{B2}$$

The implicit function theorem allows us to solve "locally" the equation; that is, for all $(\hat{e},\hat{H})$ that satisfy (B1) and (B2), effort $e$ can be expressed as a function of $H$ in a neighborhood of $(\hat{e},\hat{H})$.

More formally: for all $(\hat{e},\hat{H})$ that satisfy (B1) and (B2) there exists a function $e(H)\in\mathcal{C}^1$ and an open ball $B(\hat{H})$, such that $e(\hat{H})=e_{TB}$ and $\mathcal{J}(e(H),L_s,L_m)=0$ for all $H\in B(\hat{H})$.

Taking the derivative of $\mathcal{J}(e_{TB},L_s,L_m)$ with respect to $H$:[20]

$$e_H(H)=-\mathcal{J}_H(e_{TB},L_s,L_m)\times\mathcal{J}_e^{-1}(e_{TB},L_s,L_m).$$

Taking the second derivative of (B1) with respect to $e$:

$$g_{ee}(e,L_s,L_m)=\frac{1}{2}\left(\frac{1}{1+e}\right)^{3/2}\left\{\frac{3}{2}\left(\frac{1}{1+e}\right)\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)+\Phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)\right]+\right.$$
$$\left.\frac{1}{e^2}\left[\phi\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{e}\right)\times\left(\frac{\mu}{\sigma}L_s\right)^2+\phi\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{e}\right)\times\left(\frac{\mu}{\sigma}L_m\right)^2\right]\right\}>0.$$

Condition (B2) can be written as $V_{ee}(D,e)>-\frac{g_e}{g}(e,L_s)\times V_e(D,e)-\frac{g_{ee}}{g}(e,L_s)$. $-\frac{g_e}{g}(e,L_s)<\frac{1}{2(1+e)}$ and $\frac{g_{ee}}{g}(e,L_s)\geq 0$. In our case, $V_{ee}(D,e)=D$, $V_e(D,e)=De$. Then, (B2) is satisfied for all $H\in\left[-\left(s+\frac{\mu}{aA\sigma^2}\right),m-\frac{\mu}{aA\sigma^2}\right]$.[21]

[20] The subscript $H$ denotes first derivative with respect to $H$. The subscript $eH$ denotes cross derivative with respect to $e$ and $H$.

[21] In general, this condition is satisfied for all disutility functions $V(D,e)$ convex enough.

The sign of $e_H(H)$, therefore, depends on the sign of $\mathcal{J}_H(e, L_s, L_m) = V_e(D, e) \times g_H(e, L_s, L_m) + g_{eH}(e, L_s, L_m)$.

From Corollary 1,

$$g_{eH}(e, L_s, L_m) = -\left(\frac{1}{1+e}\right)^{3/2} e^{-1/2} \frac{aA\sigma}{\sqrt{2\pi}} \left[\exp\left(\frac{-\left(\frac{\mu}{\sigma}L_s\right)^2}{2e}\right) - \exp\left(\frac{-\left(\frac{\mu}{\sigma}L_m\right)^2}{2e}\right)\right] \tag{B3}$$

for all $H \in \Re$.

Let us define the gamma function $\Gamma(u) = \int_0^\infty t^{u-1}\exp(-t)dt$ for $u > 0$. The *incomplete* gamma function is given by $\Gamma(u, v) = \int_v^\infty t^{u-1}\exp(-t)dt$ for $v > 0$. From (B1),

$$\begin{aligned} &g_H(e, L_s, L_m) = \\ &\quad \frac{aA\mu}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}, \frac{1+e}{e}\right)\left(L_s\exp\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{2}\right) - L_m\exp\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{2}\right)\right) - \\ &\quad \left(\frac{e}{1+e}\right)^{1/2} \frac{2aA\sigma}{\sqrt{2\pi}}\left[\exp\left(\frac{-\left(\frac{\mu}{\sigma}L_s\right)^2}{2e}\right) - \exp\left(\frac{-\left(\frac{\mu}{\sigma}L_m\right)^2}{2e}\right)\right]. \end{aligned} \tag{B4}$$

By definition, $L_s(H^* + \delta) = L_m(H^* - \delta)$, for all $\delta \in \Re$. For all $0 < \delta < \frac{m+s}{2}$, $L_s(H^* - \delta) < L_m(H^* - \delta)$ and $L_s(H^* + \delta) > L_m(H^* + \delta)$. Let $L_s^* = L_s(H^*)$ and $L_m^* = L_m(H^*)$. For $\delta = 0$, $L_s^* = L_m^*$. Therefore, $e_H(H) > 0$ for all $-\left(s + \frac{\mu}{aA\sigma^2}\right) \leq H < H^*$ and $e_H(H) < 0$ for all $H^* < H \leq m - \frac{\mu}{aA\sigma^2}$; $e_H(H^*) = 0$. Since the function $e(H)$ is continuous and differentiable, it follows that $H^*$ is a local maximum in the interval $\left[-\left(s + \frac{\mu}{aA\sigma^2}\right), m - \frac{\mu}{aA\sigma^2}\right]$. *Q.E.D.*

## Proof of Corollary 2

Let $H < -\left(s + \frac{\mu}{aA\sigma^2}\right)$. Then, $L_s < 0$ and $L_m > 0$ and $|L_s| < |L_m|$. From (B1),

$$\begin{aligned} &g_H(e, L_s, L_m) = \\ &\quad aA\mu L_s\exp\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{2}\right)\left[1 + \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s\right)^2\right)\right] - \\ &\quad aA\mu L_m\exp\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{2}\right)\left[1 - \Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m\right)^2\right)\right] - \\ &\quad \left(\frac{e}{1+e}\right)^{1/2} \frac{2aA\sigma}{\sqrt{2\pi}}\left[\exp\left(\frac{-\left(\frac{\mu}{\sigma}L_s\right)^2}{2e}\right) - \exp\left(\frac{-\left(\frac{\mu}{\sigma}L_m\right)^2}{2e}\right)\right] < 0 \end{aligned} \tag{B5}$$

From (B3), $g_{eh}(e, L_s, L_m) < 0$. It follows that $e_H(H) > 0$ for all $H < -\left(s + \frac{\mu}{aA\sigma^2}\right)$.

Let $H > m - \frac{\mu}{aA\sigma^2}$. Then, $L_s > 0$ and $L_m < 0$ and $|L_s| > |L_m|$. From (B1),

$$
\begin{aligned}
&g_H(e, L_s, L_m) = \\
&\quad aA\mu L_s \exp\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s\right)^2\right)\right] - \\
&\quad aA\mu L_m \exp\left(\frac{\left(\frac{\mu}{\sigma}L_m\right)^2}{2}\right)\left[1+\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m\right)^2\right)\right] - \\
&\quad \left(\frac{e}{1+e}\right)^{1/2}\frac{2aA\sigma}{\sqrt{2\pi}}\left[\exp\left(\frac{-\left(\frac{\mu}{\sigma}L_s\right)^2}{2e}\right)-\exp\left(\frac{-\left(\frac{\mu}{\sigma}L_m\right)^2}{2e}\right)\right] > 0.
\end{aligned}
$$

From (B3), $g_{eh}(e_{TB}, L_s, L_m) > 0$. It follows that $e_H(H) < 0$ for all $H > m - \frac{\mu}{aA\sigma^2}$. *Q.E.D.*

**Proof of Corollary 3**

Let $H \in \left[-\left(s+\frac{\mu}{aA\sigma^2}\right), m-\frac{\mu}{aA\sigma^2}\right]$. We re-write the function $\mathcal{J}(e, L_s, L_m)$ as:

$$
\begin{aligned}
\mathcal{J}(e, L_s, L_m) = &\left[V_e(D,e) - \frac{1}{2(1+e)}\right]\left(\frac{1}{1+e}\right)^{1/2}\left[\Phi\left(\frac{\left(\frac{\mu}{\sigma}(L_s)\right)^2}{e}\right)+\Phi\left(\frac{\left(\frac{\mu}{\sigma}(L_m)\right)^2}{e}\right)\right] \\
&+V_e(D,e)\left\{\exp\left(\frac{\left(\frac{\mu}{\sigma}(L_s)\right)^2}{2}\right)\times\left[1-\Phi\left(\frac{\left(\frac{\mu}{\sigma}(L_s)\right)^2}{e}(1+e)\right)\right]\right. \\
&\left.+\exp\left(\frac{\left(\frac{\mu}{\sigma}(L_m)\right)^2}{2}\right)\times\left[1-\Phi\left(\frac{\left(\frac{\mu}{\sigma}(L_m)\right)^2}{e}(1+e)\right)\right]\right\}.
\end{aligned}
$$

Evaluating this function at the second best effort and given (5) we obtain

$$
\begin{aligned}
&\mathcal{J}(e_{SB}, L_s, L_m) = \\
&\quad V_e(D, e_{SB})\left\{\exp\left(\frac{\left(\frac{\mu}{\sigma}(L_s)\right)^2}{2}\right)\times\left[1-\Phi\left(\frac{\left(\frac{\mu}{\sigma}(L_s)\right)^2}{e_{SB}}(1+e_{SB})\right)\right]\right. \\
&\quad \left.+\exp\left(\frac{\left(\frac{\mu}{\sigma}(L_m)\right)^2}{2}\right)\times\left[1-\Phi\left(\frac{\left(\frac{\mu}{\sigma}(L_m)\right)^2}{e_{SB}}(1+e_{SB})\right)\right]\right\} > 0.
\end{aligned} \tag{B6}
$$

This implies that $E_e U_a(W_a(e_{SB})) = -\exp(-(1/2)\mu^2/\sigma^2 - aF + V(D, e_{SB})) \times \mathcal{J}(e_{SB}, L_s, L_m) < 0$.

Therefore, for the constrained manager, the marginal utility of effort at $e_{SB}$ is negative. Since $e_{TB}$ is unique and the function is continuous in $e$, given conditions (B1) and (B2), it follows that $e_{SB} > e_{TB}$ for all $H \in \left[-\left(s+\frac{\mu}{aA\sigma^2}\right), m-\frac{\mu}{aA\sigma^2}\right]$. Given Corollary 2, this result holds for all $H \in \Re$. Next we show that

$$
\lim_{z\to\infty}\left[\exp(z/2)\times\left(1-\Phi\left(z\,\frac{1+e}{e}\right)\right)\right] = 0. \tag{B7}
$$

Re-writing (B7) and applying L'Hôspital's rule we get:

$$\lim_{z\to\infty}\frac{1-\Phi\left(z\,\frac{1+e}{e}\right)}{\exp\left(-z/2\right)}=\lim_{z\to\infty}\frac{\exp(-z/2e)}{z^{1/2}}=0.$$

Therefore, given (B6) and (B7), $\mathcal{J}(e_{SB},L_s,L_m)$ tends to zero when $m$ and $s$ tend to infinity. In the limit, the constrained manager's marginal expected utility of effort becomes zero at $e_{SB}$, $E_eU_a(W_a(e_{SB}))=0$. *Q.E.D.*

## Proof of Corollary 4

**Lemma 1** *For all* $0<x<\infty$, $\frac{1}{2}\left(1-\Phi(x)\right)-\phi(x)<0$.

**Proof:** See Lemma 1 in Gómez and Sharma (2006)

Let $m\to\infty$ and $0\leq s<\infty$. We call $g_H(e,L_s)=\lim_{m\to\infty}g_H(e,L_s,L_m)$ and $g_{eh}(e,L_s)=\lim_{m\to\infty}g_{eh}(e,L_s,L_m)$. From (B5), $g_H(e,L_s)<0$ for $H<-\left(s+\frac{\mu}{aA\sigma^2}\right)$. For $H>-\left(s+\frac{\mu}{aA\sigma^2}\right)$, $g_H(e,L_s)=2a\mu L_s\times\exp\left(\frac{\left(\frac{\mu}{\sigma}L_s\right)^2}{2}\right)\left\{\frac{1}{2}\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s\right)^2\right)\right]\right.$
$\left.-\phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s\right)^2\right)\right\}<0$, given Lemma 1.

Therefore, $g_H(e,L_s)<0$ for all $H\in\Re$. From (B3), $g_{eh}(e,L_s)<0$ for all $H\in\Re$. Thus, $e_H(H)>0$ for all $H\in\Re$. Following the same procedure, it is trivial to show that $e_H(H)<0$ for all $H\in\Re$ when $s\to\infty$ and $1\leq m<\infty$. *Q.E.D.*

## Proof of Proposition 3

Replacing (6) in the investor's utility function:

$$EU\left(W_b(y)\right)=-\exp(b(F-R))\times$$
$$\begin{cases}\exp\left(-b(H-(1-A)(s+H))E(x|y)+(b^2/2)(H-(1-A)(s+H))^2\mathrm{Var}(x|y)\right) & \text{if } y<-\frac{\mu}{e}L_s\\ \exp\left(-b\left(H+(1-A)\frac{\mu+ey}{aA\sigma^2}\right)E(x|y)+(b^2/2)\left(H+(1-A)\frac{\mu+ey}{aA\sigma^2}\right)^2\mathrm{Var}(x|y)\right) & \text{otherwise}\\ \exp\left(-b(H+(1-A)(m-H))E(x|y)+(b^2/2)(H+(1-A)(m-H))^2\mathrm{Var}(x|y)\right) & \text{if } y>\frac{\mu}{e}L_m.\end{cases}$$

Multiplying the previous expression by the density function of the signal variable $y$, we obtain:

$$EU\left(W_b(y)\right) = -\exp(b(F-R) - (1/2)\mu^2/\sigma^2)\left(\frac{e}{1+e}\right)^{1/2}\frac{1}{\sqrt{2\pi}\sigma}\times$$

$$\begin{cases} \exp\left(\frac{\left(\frac{\mu}{\sigma}(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\times & \\ \exp\left(-\frac{e}{2(1+e)}\left(\frac{y}{\sigma}-\frac{\mu}{\sigma}\left(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu}\right)\right)^2\right) & \text{if } y<-\frac{\mu}{e}L_s \\ & \\ \exp\left(\frac{1}{2}\frac{1}{1+e}\left(\frac{\mu}{\sigma}\left(t(A)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right)\exp\left(\frac{\mu^2}{2\sigma^2}\frac{e}{1+e}\frac{\left(T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}\right)^2}{1+eT(A)}\right)\times & \\ \left(\frac{1}{1+T(A)e}\right)^{1/2}\exp\left(-\frac{e(1+eT(A))}{2(1+e)}\left(\frac{y}{\sigma}+\frac{\mu}{\sigma}\frac{T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}}{1+eT(A)}\right)^2\right) & \text{otherwise} \\ & \\ \exp\left(\frac{\left(\frac{\mu}{\sigma}(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\times & \\ \exp\left(-\frac{e}{2(1+e)}\left(\frac{y}{\sigma}+\frac{\mu}{\sigma}\left(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right) & \text{if } y>\frac{\mu}{e}L_m. \end{cases}$$

$$\text{Replace } k=\frac{e}{1+e}\begin{cases} \left(\frac{y}{\sigma}-\frac{\mu}{\sigma}\left(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu}\right)\right)^2 & \text{if } y<-\frac{\mu}{e}L_s \\ (1+eT(A))\left(\frac{y}{\sigma}+\frac{\mu}{\sigma}\frac{T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}}{1+eT(A)}\right)^2 & \text{otherwise} \\ \left(\frac{y}{\sigma}+\frac{\mu}{\sigma}\left(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2 & \text{if } y>\frac{\mu}{e}L_m. \end{cases}$$

Integrating over $k$ and given the definition of $\Phi(\cdot)$, the unconditional utility function becomes $EU_b(W_b(e)) = -exp(b(F-R)-(1/2)\mu^2/\sigma^2)\times v(e,L_s,L_m)$ with $v(e,L_s,L_m) = (1/2)\times$

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1+\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(1+\frac{1+t(A)e}{1+e}(L_s-1)-\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right]+$$

$$\exp\left(\frac{1}{2}\frac{1}{1+e}\left(\frac{\mu}{\sigma}\left(t(A)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right)\exp\left(\frac{\mu^2}{2\sigma^2}\frac{e}{1+e}\frac{\left(T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}\right)^2}{1+eT(A)}\right)\left(\frac{1}{1+T(A)e}\right)^{1/2}$$
$$\left[\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(\frac{1+T(A)e}{1+e}(1+L_m)-1-\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)-\right.$$
$$\left.\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(1+\frac{1+T(A)e}{1+e}(L_s-1)+\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)\right]+$$
$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(\frac{1+t(A)e}{1+e}(1+L_m)-1+\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right].$$

if $H<-\left(s\frac{aA+eb(1-A)}{A(a-eb)}+(1+e)\frac{\mu}{A(a-eb)\sigma^2)}\right)$;

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(1+\frac{1+t(A)e}{1+e}(L_s-1)-\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right]+$$

$$\exp\left(\frac{1}{2}\frac{1}{1+e}\left(\frac{\mu}{\sigma}\left(t(A)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right)\exp\left(\frac{\mu^2}{2\sigma^2}\frac{e}{1+e}\frac{\left(T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}\right)^2}{1+eT(A)}\right)\left(\frac{1}{1+T(A)e}\right)^{1/2}$$
$$\left[\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(\frac{1+T(A)e}{1+e}(1+L_m)-1-\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)-\right.$$
$$\left.\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(1+\frac{1+T(A)e}{1+e}(L_s-1)+\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)\right]+$$
$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(\frac{1+t(A)e}{1+e}(1+L_m)-1+\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right].$$

if $-\left(s\frac{aA+eb(1-A)}{A(a-eb)}+\frac{(1+e)\mu}{A(a-eb)\sigma^2)}\right)\leq H<-\left(s\frac{aA+eb(1-A)(2-t(A))}{aA+eb(1-A(2-t(A)))}+\frac{(1+e)\mu}{(aA+eb(1-A(2-t(A))))\sigma^2}\right)$;

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(1+\frac{1+t(A)e}{1+e}(L_s-1)-\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right]+$$

$$\exp\left(\frac{1}{2}\frac{1}{1+e}\left(\frac{\mu}{\sigma}\left(t(A)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right)\exp\left(\frac{\mu^2}{2\sigma^2}\frac{e}{1+e}\frac{\left(T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}\right)^2}{1+eT(A)}\right)\left(\frac{1}{1+T(A)e}\right)^{1/2}$$
$$\left[\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(1+\frac{1+T(A)e}{1+e}(L_s-1)+\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)+\right. \tag{B8}$$
$$\left.\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(\frac{1+T(A)e}{1+e}(1+L_m)-1-\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)\right]+$$
$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(\frac{1+t(A)e}{1+e}(1+L_m)-1+\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right].$$

if $-\left(s\frac{aA+eb(1-A)(2-t(A))}{aA+eb(1-A(2-t(A)))}+\frac{(1+e)\mu}{(aA+eb(1-A(2-t(A))))\sigma^2}\right)\leq H<m\frac{aA+eb(1-A)(2-t(A))}{aA+eb(1-A(2-t(A)))}-\frac{(1+e)\mu}{(aA+eb(1-A(2-t(A))))\sigma^2}$;

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(1+\frac{1+t(A)e}{1+e}(L_s-1)-\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right]+$$

$$\exp\left(\frac{1}{2}\frac{1}{1+e}\left(\frac{\mu}{\sigma}\left(t(A)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right)\exp\left(\frac{\mu^2}{2\sigma^2}\frac{e}{1+e}\frac{\left(T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}\right)^2}{1+eT(A)}\right)\left(\frac{1}{1+T(A)e}\right)^{1/2}$$
$$\left[\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(1+\frac{1+T(A)e}{1+e}(L_s-1)+\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)-\right.$$
$$\left.\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(\frac{1+T(A)e}{1+e}(1+L_m)-1-\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)\right]+$$
$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(\frac{1+t(A)e}{1+e}(1+L_m)-1+\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right].$$

if $m\frac{aA+eb(1-A)(2-t(A))}{aA+eb(1-A(2-t(A)))}-\frac{(1+e)\mu}{(aA+eb(1-A(2-t(A))))\sigma^2}\leq H<m\frac{aA+eb(1-A)}{A(a-eb)}-\frac{(1+e)\mu}{A(a-eb)\sigma^2}$;

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(1+t(A)(L_s-1)-bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(1+\frac{1+t(A)e}{1+e}(L_s-1)-\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right]+$$

$$\exp\left(\frac{1}{2}\frac{1}{1+e}\left(\frac{\mu}{\sigma}\left(t(A)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right)\exp\left(\frac{\mu^2}{2\sigma^2}\frac{e}{1+e}\frac{\left(T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}\right)^2}{1+eT(A)}\right)\left(\frac{1}{1+T(A)e}\right)^{1/2}$$
$$\left[\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(1+\frac{1+T(A)e}{1+e}(L_s-1)+\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)-\right.$$

$$\left.\Phi\left(\frac{1+e}{e(1+T(A)e)}\left(\frac{\mu}{\sigma}\left(\frac{1+T(A)e}{1+e}(1+L_m)-1-\frac{e}{1+e}bH(t(A)-1)\frac{\sigma^2}{\mu}\right)\right)^2\right)\right]+$$

$$\exp\left(\frac{\left(\frac{\mu}{\sigma}(t(A)(1+L_m)-1+bH\frac{\sigma^2}{\mu})\right)^2}{2}\right)\left[1+\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}(\frac{1+t(A)e}{1+e}(1+L_m)-1+\frac{e}{1+e}bH\frac{\sigma^2}{\mu})\right)^2\right)\right].$$

if $H > m\frac{aA+eb(1-A)}{A(a-eb)} - \frac{(1+e)\mu}{A(a-eb)\sigma^2}$. *Q.E.D.*

## Proof of Proposition 4

Assume first that the manager's effort choice is publicly observable. Given equation (8), the investor chooses the contract $(A, H)$ that satisfies the first order optimality condition:

$$\frac{\partial}{\partial i}EU_b(W_b(e)|A,H) = -\exp\big(-bR-(1/2)(\mu/\sigma)^2+(b/a)V(D,e)\big)\times \tag{B9}$$
$$\left(\frac{b}{a}g(e,L_s,L_m)^{b/a-1}g_i(e,L_s,L_m)v(e,L_s,L_m)+g(e,L_s,L_m)^{b/a}v_i(e,L_s,L_m)\right)=0,$$

for $i = \{A, H\}$. We distinguish two cases: with and without portfolio constraints.

Without portfolio constraints, $s \to \infty$ and $m \to \infty$. The manager's expected utility (4) is independent of $A$ and $H$. The investor's expected utility in (B8) becomes:

$$v(e) = \exp\left(\frac{1}{2}\frac{1}{1+e}\left(\frac{\mu}{\sigma}\left(t(A)-1+bH\frac{\sigma^2}{\mu}\right)\right)^2\right)$$
$$\exp\left(\frac{\mu^2}{2\sigma^2}\frac{e}{1+e}\frac{\left(T(A)-1-bH(t(A)-1)\frac{\sigma^2}{\mu}\right)^2}{1+eT(A)}\right)\left(\frac{1}{1+T(A)e}\right)^{1/2}.$$

By definition, $t(A_{FB}) = T(A_{FB}) = 1$; $t_A(A_{FB}) = \frac{a+b}{ab}$; $T_A(A_{FB}) = 0$. Then, it follows immediately that $v_i(e|A_{FB}, 0) = 0$, $i = \{A, H\}$, for any effort $e$. Hence, the contract $(A_{FB}, 0)$ is (first order condition) optimal.

With portfolio constraints, notice first that $g(e, L_s, L_m|A_{FB}, 0) = v(e, L_s, L_m|A_{FB}, 0)$. Let us analyze now the partial derivatives of function $v$ and $g$ with respect to $A$ and $H$. Taking the derivative of (B8) with respect to $H$ and evaluating it at the contract $(A_{FB}, 0)$ yields:

$$
\begin{aligned}
v_H(e, L_s, L_m|A_{FB}, 0) = & -2\frac{b^2}{a+b}\mu \\
& \left\{ \exp\left( \frac{\left(\frac{\mu}{\sigma}L_s(0)\right)^2}{2} \right) L_s(0) \times \right. \\
& \left( \frac{1}{2}\left[ 1 - \Phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2 \right) \right] - \phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2 \right) \right) + \\
& \exp\left( \frac{\left(\frac{\mu}{\sigma}L_m(0)\right)^2}{2} \right) L_m(0) \times \\
& \left. \left( \frac{1}{2}\left[ 1 - \Phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2 \right) \right] - \phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2 \right) \right) \right\}.
\end{aligned}
$$

Equation (B4) evaluated at $(A_{FB}, 0)$ becomes:

$$
\begin{aligned}
g_H(e, L_s, L_m|A_{FB}, 0) = & 2\frac{ab}{a+b}\mu \\
& \left\{ \exp\left( \frac{\left(\frac{\mu}{\sigma}L_s(0)\right)^2}{2} \right) L_s(0) \times \right. \\
& \left( \frac{1}{2}\left[ 1 - \Phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2 \right) \right] - \phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2 \right) \right) + \\
& \exp\left( \frac{\left(\frac{\mu}{\sigma}L_m(0)\right)^2}{2} \right) L_m(0) \times \\
& \left. \left( \frac{1}{2}\left[ 1 - \Phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2 \right) \right] - \phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2 \right) \right) \right\}.
\end{aligned}
$$

Taking the derivative of (B8) with respect to $A$ and evaluating it at the contract $(A_{FB}, 0)$, we obtain:

$$
\begin{aligned}
v_A(e, L_s, L_m|A_{FB}, 0) = & \\
& -2bs\mu \exp\left( \frac{\left(\frac{\mu}{\sigma}L_s(0)\right)^2}{2} \right) L_s(0) \times \\
& \left( \frac{1}{2}\left[ 1 - \Phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2 \right) \right] - \phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2 \right) \right) \\
& -2bm\mu \exp\left( \frac{\left(\frac{\mu}{\sigma}L_m(0)\right)^2}{2} \right) L_m(0) \times \\
& \left( \frac{1}{2}\left[ 1 - \Phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2 \right) \right] - \phi\left( \frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2 \right) \right).
\end{aligned}
$$

Equation (B4) at $(A_{FB}, 0)$ can be rewritten as follows:

$$
\begin{aligned}
&g_A(e, L_s, L_m|A_{FB}, 0) = \\
&\quad 2as\mu\exp\left(\frac{\left(\frac{\mu}{\sigma}L_s(0)\right)^2}{2}\right)L_s(0)\times \\
&\quad \left(\frac{1}{2}\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2\right)\right]-\phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_s(0)\right)^2\right)\right)+ \\
&\quad 2am\mu\exp\left(\frac{\left(\frac{\mu}{\sigma}L_m(0)\right)^2}{2}\right)L_m(0)\times \\
&\quad \left(\frac{1}{2}\left[1-\Phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2\right)\right]-\phi\left(\frac{1+e}{e}\left(\frac{\mu}{\sigma}L_m(0)\right)^2\right)\right).
\end{aligned}
\tag{B10}
$$

From the former equations, $g_i(e, L_s, L_m|A_{FB}, 0) = -\frac{a}{b}v_i(e, L_s, L_m|A_{FB}, 0)$ for $i = \{A, H\}$ Evaluating the optimality condition (B9) at $(A_{FB}, 0)$ and given the partial derivatives for $v$ and $g$, it follows that the contract $(A_{FB}, 0)$ satisfies the first order optimality condition in the absence of moral hazard.

We turn now to the case of moral hazard. Without portfolio constraints (second best scenario), the manager's effort (5) is independent of $A$ and $H$. Hence, as we just showed, the contract $(A_{FB}, 0)$ is optimal. Under portfolio constraints (third best scenario), the third best effort, $e_{TB}$, is a function of $A$ and $H$. The first order condition for optimality requires that

$$
\frac{\partial}{\partial i}EU_b(W_b(e_{TB})|A, H) = \frac{\partial}{\partial i}EU_b(W_b(e_{TB})|A, H) + \frac{\partial}{\partial e}EU_b(W_b(e)|A, H)\frac{\partial}{\partial i}e_{TB}(A,H) = 0,
\tag{B11}
$$

for $i = \{A, H\}$. We have just proved that $\frac{\partial}{\partial i}EU_b(W_b(e)|A_{FB}, 0) = 0$ for all effort. By definition, $\frac{\partial}{\partial e}\ EU_b(W_b(e)|A_{FB}, 0)|_{e=e_{TB}} = -\exp\big(-bR - (1/2)(\mu/\sigma)^2 + (b/a)V(D, e_{TB})\big)\frac{b}{a}g(e_{TB}, L_s(0), L_m(0))^{b/a}$ $\times\big[\mathcal{J}(e_{TB}, L_s(0), L_m(0)) + \frac{a}{b}g_e(e_{TB}, L_s(0), L_m(0))\big]$. Given (B1), the later equation can be rewritten as: $-\exp\big(-bR - (1/2)(\mu/\sigma)^2 + (b/a)V(D, e_{TB})\big)g(e_{TB}, L_s(0), L_m(0))^{b/a}g_e(e_{TB}, L_s(0), L_m(0)) > 0$ given Proposition 1 and Corollary 1.

From Proposition 2 and Corollary 2, for all $A \in (0, 1]$, $\frac{\partial}{\partial H}e_{TB}(A, H) > 0$ $(< 0)$ for $H < H^*$ $(H > H^*)$; $\frac{\partial}{\partial H}e_{TB}(A, H) = 0$ for $H = H^*$. Hence, $\frac{\partial}{\partial H}e_{TB}(A_{FB}, 0) = 0$ only for $H^* = 0$. We investigate now whether $\frac{\partial}{\partial A}e_{TB}(A_{FB}, H^* = 0) = 0$.

$\mathcal{J}_A(e_{TB}, L_s, L_m|A_{FB}, H^* = 0) = V_e(D, e_{TB})g_A(e_{TB}, L_s, L_m|A_{FB}, H^* = 0) + g_{eA}(e_{TB}, L_s, L_m|A_{FB}, H^* = 0)$. $V_e(D, e_{TB}) > 0$. From (B11) and given Lemma 1, $g_A(e_{TB}, L_s, L_m|A_{FB}, H^* = 0) < 0$. From Corollary 1, $g_{eA}(e_{TB}, L_s, L_m|A_{FB}, H^* = 0) < 0$. Given (B2), $\mathcal{J}_e^{-1}(e_{TB}, L_s, L_m|A_{FB}, H^* = 0) > 0$. Therefore, $\frac{\partial}{\partial A}e_{TB}(A_{FB}, H^* = 0) = -\mathcal{J}_A(e_{TB}, L_s, L_m|A_{FB}, H^* = 0)\times\mathcal{J}_e^{-1}(e_{TB}, L_s, L_m|A_{FB}, H^* = 0) > 0$. Thus, the contract $(A_{FB}, 0)$ is suboptimal.

When we introduce selectivity information in Appendix A, the first order condition (B9) under public information becomes:

$$\frac{\partial}{\partial i}EU_b(W_b(e)|A,H_x,H_z) = -\exp\big(-bR-(1/2)(\mu_x/\sigma_x)^2-(1/2)(\mu_z/\sigma_z)^2+(b/a)V(D,e)\big)\times$$
$$\big[\big(\tfrac{b}{a}g(e_x,L_{s_x},L_{m_x})^{b/a-1}g_i(e_x,L_{s_x},L_{m_x})v(e_x,L_{s_x},L_{m_x})+g(e_x,L_{s_x},L_{m_x})^{b/a}v_i(e_x,L_{s_x},L_{m_x})\big)$$
$$g(e_z,s_z,m_z)^{b/a}v(e_z,L_{s_z},L_{m_z})+\big(\tfrac{b}{a}g(e_z,L_{s_z},L_{m_z})^{b/a-1}g_i(e_z,L_{s_z},L_{m_z})v(e_z,L_{s_z},L_{m_z})+$$
$$g(e_z,L_{s_z},L_{m_z})^{b/a}v_i(e_z,L_{s_z},L_{m_z})\big)\,g(e_x,s_x,m_x)^{b/a}v(e_x,L_{s_x},L_{m_x})\big]=0,$$

for $i=\{A,H_x,H_z\}$. Notice that $g(e_j,L_{s_j},L_{m_j}|A_{FB})=v(e_j,L_{s_j},L_{m_j}|A_{FB})$ for $j=\{x,z\}$. Moreover, the cross derivatives of $g(e_x,\cdot)$ -alternatively $g(e_z,\cdot)$- and $v(e_x,\cdot)$ -alternatively, $v(e_z,\cdot)$- with respect to $H_z$ -alternatively, $H_x$- are zero. Hence, the same arguments used above to prove the (first order) optimality of the contract $(A_{FB},0)$ when effort is observable hold, both with and without portfolio constraints, in the presence of selectivity information.

In the case of moral hazard, when both effort choices are not observable, the first order condition (B11) becomes $\frac{\partial}{\partial i}EU_b(W_b(e_{TB})|A,H_x,H_z)=\frac{\partial}{\partial i}EU_b(W_b(e_{TB})|A,H_x,H_z)+$
$\frac{\partial}{\partial e}EU_b(W_b(e)|A,H_x,H_z)\left(\frac{\partial}{\partial i}e_{x_{TB}}(A,H_x),\frac{\partial}{\partial i}e_{z_{TB}}(A,H_z)\right)=0$, for $i=\{A,H_x,H_z\}$ and $e_{TB}=(e_{x_{TB}},e_{z_{TB}})$. Evaluated at the first best contract $(A_{FB},0)$, $\frac{\partial}{\partial i}EU_b(W_b(e_{TB})|A,H_x,H_z)=$

$$-\exp\big(-bR-(1/2)(\mu_x/\sigma_x)^2-(1/2)(\mu_z/\sigma_z)^2+(b/a)V(D,e_{TB})\big)g(e_{x_{TB}},L_{s_x},L_{m_x})^{b/a}g(e_{z_{TB}},L_{s_z},L_{m_z})^{a/b}\times$$
$$\big[g_{e_x}(e_{x_{TB}},L_{s_x},L_{m_x})g(e_{z_{TB}},L_{s_z},L_{m_z})\tfrac{\partial}{\partial i}e_{x_{TB}}(A,H_x)+g_{e_z}(e_{z_{TB}},L_{s_z},L_{m_z})g(e_{x_{TB}},L_{s_x},L_{m_x})\tfrac{\partial}{\partial i}e_{z_{TB}}(A,H_z)\big],$$

for $i=\{A,H_x,H_z\}$. Following the same arguments as in the case without selectivity we conclude that the first best contract is suboptimal. *Q.E.D.*

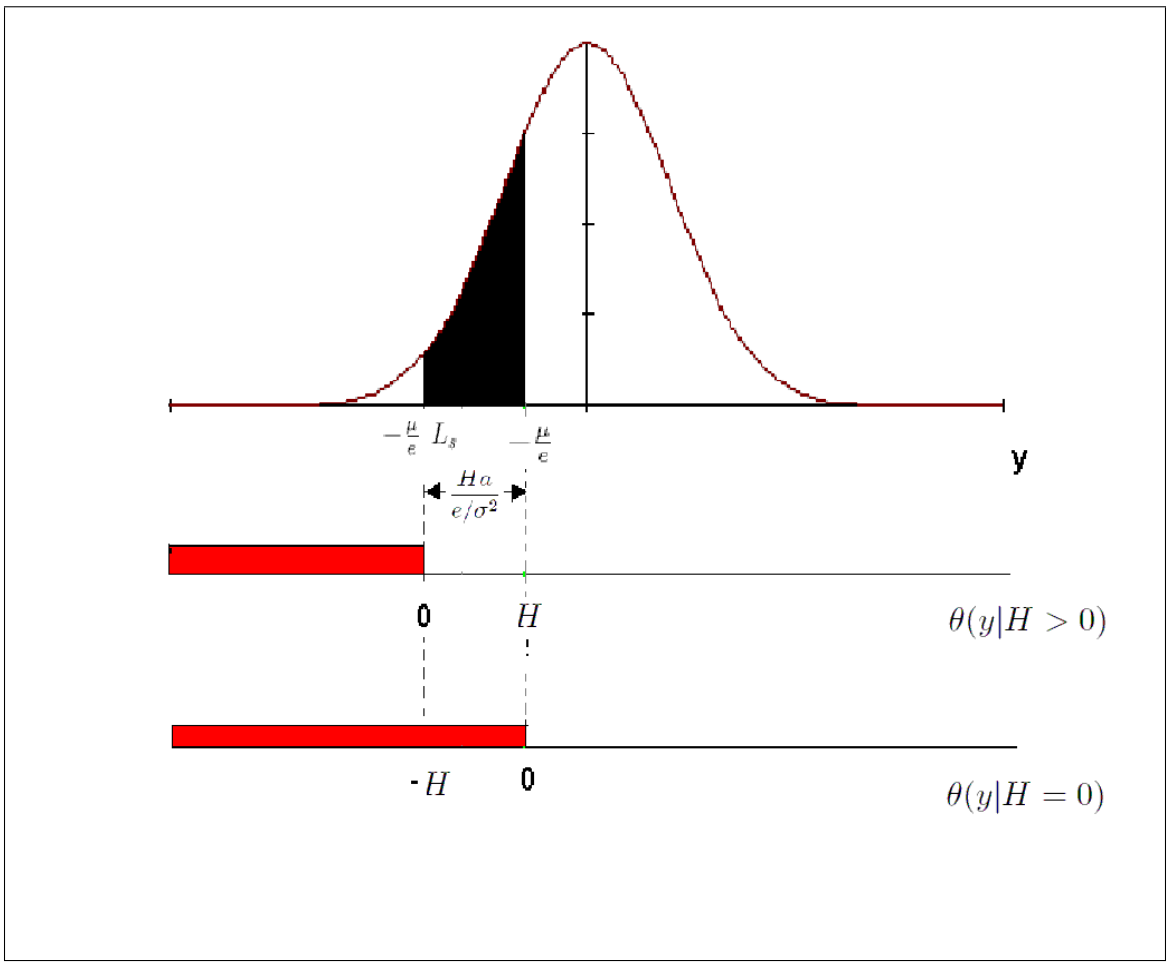


Figure 1: We assume that short-selling is totally forbidden ($s = 0$) and there is no limit to margin purchase ($m \to \infty$). For simplicity, let $A = 1$. After putting effort $e$ the manager receives a signal $y$ and makes her optimal portfolio $\theta$. When $H = 0$ (bottom portfolio line), all signals $y < -\frac{\mu}{e}$ lead to short-selling. When $H > 0$ (upper portfolio line), the short-selling bound is hit for signals $y < -\frac{\mu}{e}L_s$. In both cases, the region of these non-implementable portfolios is marked by the thick line. Under benchmarking ($H > 0$) there is an *incremental* area for implementable signals relative to the case of no benchmarking. The size of this area, $\frac{Ha}{e/\sigma^2}$, increases with benchmarking ($H$) and the manager's risk aversion ($a$); it has probability mass equal to the shaded area in the density function plot.

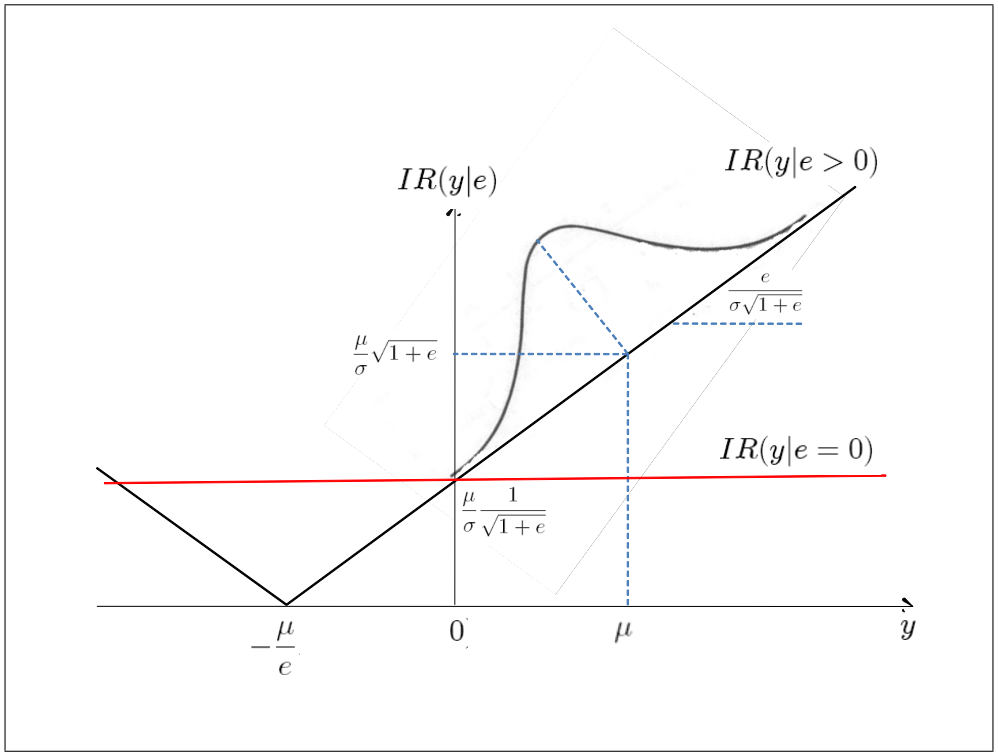


Figure 2: This figure represents the Information Ratio as a function of the signal $y$ and given effort $e$. Notice that, when $e = 0$, the Information Ratio coincides with the Sharpe Ratio for all signal $y$. When $e$ increases the slope increases in absolute value, making the Information Ratio higher for all signal $y$. As $e \to \infty$, in the limit, the Information Ratio also tends to infinity. For $y = \mu$, the Information Ratio becomes $\frac{\mu}{\sigma}\sqrt{1+e}$. Averaging across $y$, the expected Information Ratio is higher than $\frac{\mu}{\sigma}\sqrt{1+e}$ since for all $y < -\frac{\mu}{e}$, the Information Ratio "bounces back:" the manager would short the risky asset.