WISEflow
Europe/Oslo(CEST)
13 Jun 2023

Handelshøyskolen BI

GRA 19703 Master Thesis

Thesis Master of Science 100% - W

Predefinert informasjon

Startdato:	09-01-2023 09:00 CET	**Termin:**	202310
Sluttdato:	03-07-2023 12:00 CEST	**Vurderingsform:**	Norsk 6-trinns skala (A-F)
Eksamensform:	T		
Flowkode:	202310||11184||IN00||W||T		
Intern sensor:	(Anonymisert)		

Deltaker

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Informasjon fra deltaker

Tittel *:	Inflation and Its Distributional Effects on Wealth
Navn på veileder *:	Federico Gavazzoni

Inneholder besvarelsen konfidensielt materiale?:	Nei	**Kan besvarelsen offentliggjøres?:**	Ja

Gruppe

Gruppenavn:	(Anonymisert)
Gruppenummer:	136
Andre medlemmer i gruppen:	

BI Norwegian Business School
Masters of Science in Finance

Masters Thesis

Inflation and Its Distributional Effects on Wealth

Supervisor:
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Oslo, June 13, 2023

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1 Abstract

This paper examines the impact of inflation on wealth distribution in the United States. Through a synthesis of empirical findings and economic theory, we undertake a comprehensive analysis of how inflation influences the balance sheets across the wealth distribution in the US. Our study entails the determination of inflation sensitivity across various wealth percentiles, followed by a comparative assessment. The relative magnitude of this measure serves as a determinant of the resulting shape assumed by the wealth distribution in the face of inflationary pressures. Our investigation reveals a U-shaped pattern in the aggregate duration across the wealth distribution. Specifically, it indicates that individuals at both ends of the wealth spectrum, namely the wealthiest and the poorest, exhibit the highest degree of sensitivity to changes in inflation. In contrast, the middle percentiles demonstrates comparatively lower sensitivity to inflationary fluctuations.

2 Introduction

How does the distribution of wealth evolve in the presence of inflation? The dynamics of wealth distributions encompass intricacies that are both multifaceted and occasionally contentious. Numerous perspectives have emerged regarding the drivers and ramifications of wealth inequality, as well as the potential measures to constrain or regulate shifts in wealth distribution. However, we contend that the fundamental prerequisite for effectively managing wealth distributions lies in attaining a comprehensive understanding of the underlying mechanisms and rationales behind its fluctuations. Our research aims to contribute (works towards contributing) to this endeavor by exploring the distributional ramifications of inflation on wealth.

Our inquiry revolves around the notion that various components of household balance sheets respond differently to inflationary pressures, thereby exerting distinct influences on a household's overall wealth. We assert that the composition of a household's balance sheet should determine the net impact of inflation on their wealth. Given that the aggregate balance sheet composition varies across different wealth percentiles, the repercussions on individual balance sheet items should manifest in varying degrees of inflation sensitivity across different wealth groups. Consequently, our study seeks to observe the changes in the distributional shape induced by inflation while holding all other factors constant. The resulting findings will be compared to existing literature on the intersection of wealth distribution and inflation.

3 A Review of The Literature

3.1 Summary of Related Work

Since the 1980s, wealth inequality has exhibited an upward trend in the United States, United Kingdom, and France, particularly among the top 1 percentile, coinciding with a decline in interest rates (Greenwald et al., 2021). This phenomenon can be attributed to bond pricing theory, wherein lower interest rates necessitate higher levels of wealth to maintain future consumption at a constant level. As interest rates decrease, individuals must increase their invested wealth to sustain their desired level of consumption. Furthermore, bonds with fixed yields experience appreciation as interest rates decline, resulting in increased wealth for bondholders. The Gordon Growth Model also highlights the influence of interest rate changes on stock prices (Asness, 2003). However, market pricing mechanisms often seem to overlook the impact of inflation and interest rate changes on expected future cash flows, leading to somewhat irrational pricing.

Real estate has also witnessed substantial appreciation since the 1980s. As described by (Adkins et al., 2021), the suppression of wage inflation has led to a reduction in the required rate of return on financing, making higher mortgage burdens more manageable for consumers. Consequently, housing prices have surged, contributing to a widening wealth gap between homeowners and renters. These dynamics are evident in the Survey of Consumer Prices (Bricker et al., 2020), which provides insights into the wealth distribution across different percentiles. The wealth of top percentiles is predominantly concentrated in private businesses and financial assets, explaining the disproportionate wealth accumulation among asset owners compared to non-asset owners. Despite some inconsistencies in the literature regarding the impact of monetary policies on wealth distribution and concentration, ownership of assets remains a significant explanatory factor (Colciago et al., 2019).

To gain a comprehensive understanding of the effects of inflation on different financial instruments, (Fooladi et al., 2021) propose a method for extracting bond sensitivities to inflation beyond solely interest rates. This approach enables a more nuanced exploration of the inflationary implications for bond investments. Additionally, (Leibowitz et al., 1989) introduce the concept of equity duration, which disentangles the duration arising from inflation and real interest rates, providing valuable insights into the sensitivity of equity investments to these factors.

3.2 Empirical Evidence of Wealth Inequality

It is widely acknowledged in the literature that there is substantial empirical evidence pointing to an increase in wealth inequality. Piketty's work, along with the World Inequality Database, are frequently cited as primary sources for examining wealth inequality trends.

(Greenwald et al., 2021) demonstrate that wealth inequality among the top 10% of wealth holders in the US has increased in tandem with the required investments for sustaining a $1 level of consumption over the same period. Similar findings are observed for the top 1% percentile in the US, as well as in the UK and France, among both the top 10% and top 1% wealth holders in each respective country. The study further suggests a negative correlation between financial wealth and real market rates, supported by empirical evidence.

In contrast, (Adkins et al., 2021) argue that the primary driver of wealth inequality, particularly in Australia, is the significant growth in asset prices, with real estate prices experiencing an average annual growth rate twice that of wages. This has created a significant wealth divide between those who possess assets and those who do not. The study highlights the challenge faced by ordinary families in acquiring a share of these assets, given the rapid increase in real estate prices. Consequently, those who already own assets benefit disproportionately from the wealth accumulation. This finding aligns with the wealth growth observed in Piketty's research, where housing price appreciation plays a crucial role. It suggests that factors beyond asset class characteristics, such as monetary policies, contribute to the widening wealth gap.

However, (Colciago et al., 2019) present a contrasting perspective, citing studies by (Piketty & Goldhammer, 2014) and (Atkinson, 2015) that highlight an increase in wealth inequality. Nevertheless, based on the evidence examined in their study, they find mixed results regarding the role of monetary policies as a significant driver of wealth inequality.

(Bricker et al., 2020) provide additional evidence of the increasing share of total wealth held by the top 1% and top 10% wealth holders in the US. The share of total wealth held by the top 1% has risen steadily from 25% in 1989 to approximately 34% in 2019. Similarly, the next top 9% of wealth holders have experienced a slight increase from 37% to 39%, while families outside the top 10% have seen a decline in their share of total wealth from 35% to 27%. These findings highlight the persistent growth in wealth concentration among the top percentiles, indicating a widening wealth gap.

3.3 Asset Portfolios in Different levels of Wealth

(Bricker et al., 2020) provide detailed insights into wealth concentration and its distribution across different wealth levels, namely the top 1 percentile, the next 9 percent, the next 40 percent, and the bottom 50 percentile. The authors examine wealth allocation across six categories: business (privately held businesses), houses (home equity in primary residences), Other, Financial (non-retirement financial assets), DC (retirement assets held in accounts), and DB (pension assets). The data highlights not only the increase in wealth allocation but also the substantial variations in asset class weights. Notably, business and financial assets exhibit higher weights as the wealth percentiles increase, while other asset categories show the opposite effect.

In their study, (Bricker et al., 2020) define wealth as "marketable" wealth, which represents the difference between a household's assets and debts. To enhance the information provided by the Federal Reserve Bulletin (Bhutta et al., 2020), the authors include defined benefit pension assets and the wealth of Forbes 400 families in their wealth distribution measure. Defined benefit pension assets constitute a significant portion of household wealth in the United States. By incorporating this information, the median household wealth increases from approximately \$121,000 to \$172,000. Although the original Survey of Consumer Finances (SCF) measure does not include the wealth of Forbes 400 families due to disclosure concerns, Bricker et al. employ a method described by Bricker, Hansen, and Volz (2019) to incorporate their wealth, resulting in an additional \$3 trillion added to the total wealth in 2019.

Furthermore, (Bricker et al., 2020) compare their SCF data with the World Inequality Database (WID) and a methodology employed by Smith, Zidar, and Zwick (2020). While there are slight differences in the estimation methods used across the three measures, they all generally agree on the wealth accumulation at the top end of the distribution until the 2007 financial crisis. However, the measures diverge in their findings in the post-crisis period.

(Greenwald et al., 2021) utilize data from the World Inequality Database (WID) to analyze wealth inequality. They define wealth as the sum of financial and non-tradeable wealth, with non-tradeable wealth representing the discounted value of future labor income. Financial wealth approximates the aforementioned "marketable" wealth mentioned earlier.

3.4 Valuation of Assets

(Greenwald et al., 2021) employ a reduced-form asset pricing model to achieve several objectives, including the computation of long-term real bond yields and the costs of a 30-year real annuity. The model is capable of calculating the McCauley duration of assets such as stocks and real estate, which are fitted to historical prices. It takes into account both overall consumption and total labor income when determining the total cost and length of a claim.

To price bonds, (Greenwald et al., 2021) present the following formula for nominal bond yields with a maturity of τ:

$$y^{\$}_{t,\tau} = \frac{-1}{\tau} A^{\$}_{\tau} - \frac{1}{\tau} (B)^{\$\prime}_{\tau} z_{\tau}$$

In this equation, the scalar $A^{\$}_{\tau}$ and vector $B^{\$}_{\tau}$ are determined by ordinary difference equations (ODEs), which are influenced by the characteristics of the state vector and market prices of risk. Real bond yields exhibit similar characteristics and are also exponentially affine, with coefficients determined by their own set of ODEs.

For pricing equity and real estate, (Greenwald et al., 2021) utilize a VAR (vector autoregressive) model that incorporates the log price-dividend ratio and log-dividend growth of various market segments, including the aggregate stock market, small stocks, growth stocks, value stocks, infrastructure stocks, and real estate. The value of an asset is determined by discounting its future cash flows. In the case of unpredictable dividends carrying zero-risk price risk, the dividend strips are evaluated as zero-coupon bonds. The dividend-price ratios act as yields on real bonds, with the coupon adjustments accounting for deterministic coupon bonds. The data analysis conducted by the authors reveals substantial price movements associated with shocks to dividend growth that are orthogonal to shocks to bond yields.

(Asness, 2003) acknowledges the negative correlation between financial assets and inflation rates, but challenges the prevailing reasoning. He argues that while investors focus on the impact of inflation on discount rates, they often overlook the earnings growth component. Asness illustrates a strong correlation between the earnings yield (inverted P/E ratio) and the ten-year Treasury yield during the period from 1965 to 2003, indicating that earnings tend to increase with inflation.

4 Balance-Sheets Across the Wealth Distribution

In this section of the study, we examine the distribution of balance sheet items across different segments of the wealth distribution, including the bottom 50%, next 40%, next 9%, next 0.9%, and the very top 0.1% (The Federal Reserve, 2022). To simplify our analysis, we categorize the balance sheet items into three groups: equity-like items, bond-like items, real estate, and others.

Regarding real estate, we specifically consider residential home equity as defined in the balance sheet. Other types of real estate assets are included within the equity-like items category. Home mortgages and consumer credit are identified as clear candidates for being classified as bond-like items due to their similarities with traditional bonds. Additionally, we classify pension entitlements as bond-like items due to their predetermined cash flows, which are typically received in the slightly distant future.

Within the equity-like items category, we include private businesses, actual equities (which are broadly defined as businesses), and other miscellaneous assets. Furthermore, we neglect consumer durable goods, such as cars, refrigerators, and washing machines, to the "other assets" category, as they do not fit within the previously defined traditional asset classes, and we do not consider them a considerable part of most households net worth. It is important to note that consumer durable goods are not the primary focus of this thesis.

Table 1 presents the grouping of balance sheet items into real estate, bond-like items, and equity-like items. The specific items listed within each category are provided to offer clarity and categorization consistency.

The purpose of this analysis is to investigate the distribution of balance sheet items among different wealth groups. By exploring the composition of these assets across various segments of the wealth distribution, we aim to shed light on the variations and disparities in wealth accumulation and allocation.

Utilizing data from the Survey of Consumer Finances and Financial Accounts, we can conduct an empirical analysis of the allocations of equity, bond, and real estate holdings across different wealth groups. This examination allows us to get a sense of the varying composition of balance sheet items within each group. The tables presented below provide the percentages of these balance sheet items after they have been categorized into their respective groups.

As previously stated, the objective of this analysis is to explore how the distribution of asset classes moves with inflationary shocks among different segments

Table 1: Balance sheet items grouped

Real Estate	Bond-like	Equity-like
Residential Real Estate	Home Mortgages	Private Businesses
	Consumer Credit	Corporate Equities
	Other Liabilities	Other assets
	DB Pension Entitlement	
	DC Pension Entitlement	

Notes: The balance-sheet has been simplified to consist of real estate, bond-like items and equity-like items. This table shows how the more detailed balance sheet composition has been divided into the aforementioned groups.

of the wealth distribution. By examining the percentage composition of equities, bonds, and real estate within each wealth group, we intent to observe how the aggregate duration for the balance sheet items add up within each wealth group.

The data employed in this thesis are derived from the Survey of Consumer Finances and Financial Accounts, which is a reputable governmental source. These data sources offer comprehensive and reliable information on the financial holdings of households across various income and wealth groups.

Table 2: Compositions of assets on balance sheet

Assets	Bottom 50%	Next 40%	Next 9%	Next 0.9%	Top 0.1%
Bond-like	(10.74%)	(29.52%)	(26.50%)	(4.86%)	(1.10%)
Equity-like	(19.00%)	(29.62%)	(48.67%)	(78.20%)	(90.45%)
Real Estate	(70.25%)	(40.86%)	(24.83%)	(16.94%)	(8.46%)
	(100%)	(100%)	(100%)	(100%)	(100%)
Total	8.28T	45.00T	54.02T	26.04T	17.31T

Notes: In this table you can find how large a percentage the bond-like, equity-like, and real estate makes up of the total asset side of the balance sheet, for each wealth group.

The findings extracted from the tables underscore the substantial variations in the composition of balance sheet items observed among the five distinct wealth groups. These disparities in balance sheet structures, coupled with the varying sensitivity to inflation exhibited by the different balance sheet items, form the fundamental basis upon which our analytical conclusions are constructed. The pronounced divergence in the asset allocation patterns across wealth strata

Table 3: Compositions of liabilities on balance sheet

Liabilities	Bottom 50%	Next 40%	Next 9%	Next 0.9%	Top 0.1%
Mortgages	(49.04%)	(73.40%)	(85.61%)	(77.48%)	(44.84%)
Credit	(50.96%)	(26.60%)	(14.39%)	(22.52%)	(55.16%)
	(100%)	(100%)	(100%)	(100%)	(100%)
Total	6.05T	7.92T	6.67T	0.69T	0.14T

Notes: In this table you can find how large a percentage the mortgages and consumer credit makes up of the total liability side of the balance sheet, for each wealth group.

serves as a crucial determinant of the outcomes and implications that arise from our research endeavor.

5 The Analysis

Given the complexity of the subject matter under analysis, it becomes necessary to introduce certain simplifying assumptions to enhance the comprehensibility of the process. These assumptions are designed to streamline the analysis without compromising its overall integrity. The key simplifying assumptions made in this study are as follows:

Only government issues non-mortgage debt: To facilitate a more focused analysis, we exclude the consideration of corporate bonds and other forms of non-mortgage and credit debt. As government debt constitutes a substantial portion of the total debt in the United States and exhibits a relatively standardized nature, we utilize the aggregate marketable government debt as a suitable proxy to measure the aggregate household sensitivity to inflation. This approach aligns with the primary objective of examining the differential impact of inflation across various wealth percentiles.

Government debt categorization: To simplify the representation of government debt, we classify it into three main categories: Treasury Bills, Treasury Bonds, and Treasury Notes. This categorization provides a reasonable approximation of the overall level of government debt. Given the focus on analyzing wealth percentiles' responses to inflation, a more detailed breakdown of the debt structure is deemed unnecessary.

Household balance sheet liabilities: For the purpose of this analysis, we consider the liabilities of the household balance sheet to primarily consist of mortgages and consumer credit debt. The category of ”other liabilities” is negligible in terms of its contribution to total liabilities and is thus included within the consumer credit component.

Neglecting convexity: To maintain simplicity in our analysis, we disregard the concept of convexity. As the primary objective is to examine the directional and shape changes in the distribution of wealth resulting from inflation, the precise magnitude of change for a specific security in relation to inflation becomes immaterial.

Equity-like items represented by the S&P 500 index: In line with the aforementioned rationale, equity-like items are simplified by considering the S&P 500 index as a representative sample. This approach adequately captures the general effects of inflation on equities, which aligns with the objective of analyzing the transformation of the wealth distribution caused by inflation. The assumption is strengthened by the fact that an increasing amount of investors are relying on passive funds which is widely proxied by the S&P 500 index.

Continuous-compounding Fisher Equation: To facilitate the deployment of expected inflation-duration on bond-like items, we assume the validity of a continuous-compounding version of the Fisher Equation.

$$(1+n) = (1+\pi)(1+r)$$

where,

n = nominal rate,

π = inflation rate,

r = real rate,

This assumption allows for the separation of the sensitivity of these items to changes in real interest rates from their sensitivity to inflation. While the literature on the Fisher effect displays mixed findings and some disagreement, we rely on the long-run validity of the Fisher equation to approximate the division between real and inflationary sensitivities.

Despite the controversy surrounding the Fisher Equation, we believe that this assumption is appropriate for the purposes of our analysis. The ultimate goal is to observe the reformation of the wealth distribution resulting from inflation. Even if the Fisher effect fails to precisely capture the relationship between inflation and interest rates, it still provides a valuable approximation of how various assets and liabilities would respond to changes in inflation.

Real Estate Assumptions: The framework around this thesis builds upon the use of financial models. Specifically, the Dividend Discount Model (DDM), to real estate investing. The thesis draws the parallel between real estate and other income producing assets such as equities, the intrinsic value of residential real estate can be obtained by discounting future rental income. This approach is grounded in the notion of market equilibrium between professional actors and resident focused actors converge. This allows a model similar to the DDM to explain the price investors and regular residents are willing to pay for a residential property. With discounting future values of rental income, non-professionals consider the opportunity cost of renting their residential estate and professionals the opportunity cost of other financial assets. To encapsulate the perceived market risk and returns, most of the publicly listed residential REITs will be used as a proxy.

6 Bond-like items

Given that home mortgages, consumer credit, pension fund entitlements, and other liabilities on the household balance sheet exhibit characteristics akin to bonds, we employ a consistent methodology to ascertain their respective sensitivities to fluctuations in inflation. However, we employ distinct datasets that capture the unique attributes of each item. For home mortgages, we utilize a US mortgage-backed security index sourced from Bloomberg, encompassing 936 mortgage-backed securities. To account for the convexity arising from the potential for early mortgage repayment, we calculate the weighted average option-adjusted duration (OAD). This approach, utilizing a mortgage-backed security index, allows for a closer approximation of the aggregate mortgage market in the US. A household with a mortgage is presumed to hold a short position in this index.

For the remaining bond-like items, we employ US government debt as a proxy. Specifically, for consumer credit and other liabilities, we employ short-term government debt with maturities of less than one year. Conversely, for pension entitlements, we employ longer-term government debt with maturities exceeding five years. Hence, consumers are presumed to hold a short position in short-term government bonds, while pension holders are assumed to maintain a long position in longer-term bonds.

While the comprehensive composition of government debt encompasses various instruments such as bonds, notes, bills, federal financing, floating rate notes, and Treasury Inflation-Protected Securities (TIPS), we have opted to streamline the dataset by focusing solely on bonds, notes, and bills. Consequently, the ensuing table presents the adjusted marketable debt after implementing these simplifications. For a more comprehensive understanding of the methodology employed in this process, a detailed explanation can be found in the appendix.

Table 4: Adjusted marketable US. Government debt pr. 30/04/2023

Security Type	Total Amount (in millions)	Percent
Bonds	4,094,321	18.9%
Notes	13,627,978	62.9%
Bills	3,942,645	18.2%
Total Marketable Debt	21,664,944	100%

Notes: This is an overview of the US outstanding marketable debt, after we have simplified it. A more correct and detailed overview of the debt composition can be found in the appendix.

6.1 Deriving inflation duration

In order to investigate the sensitivity of bond-like items on households' balance sheets to inflation, we adopt the methodologies outlined by (Fooladi et al., 2021) In their work, they derive the expected inflation duration through the following process:

The value of an indexed instrument with π, i and r in order of occurrence as inflation rate, nominal rate and real rate paying C_t real dollars at time t:

$$C_t e^{-rt} = (C_t e^{\pi t})e^{-it}$$

The present value of the instrument with multiple cash flows:

$$V = \Sigma C_t e^{-rt}$$

In terms of discounted nominal expected cash flow:

$$V = \Sigma(C_t e^{\pi t})e^{-it} = \Sigma C_t e^{(\pi - i)t}$$

Real duration from (Macaulay, 1938) duration:

$$D_r = -\frac{1}{V}\frac{dV}{dr} = \Sigma\frac{C_t e^{-rt}}{V}t$$

Assuming Fisher equation holds, the real duration can be expressed as:

$$D_r = -\frac{1}{V}\frac{dV}{dr} = \Sigma\frac{C_t e^{(\pi-i)t}}{V}t$$

Nominal duration: Sensitivity with respect to changes in nominal rate.

$$D_i = -\frac{1}{V}\frac{dV}{di} = \frac{\delta r}{\delta i}\Sigma_{t=1}^{T}\frac{C_t e^{-rt}}{V}t = \frac{\delta r}{\delta i}D_r$$

Here, when the source of change in nominal rate comes from the real rate $\frac{\delta r}{\delta i} = 1$, and nominal duration is equal to the real duration. On the other hand, when $\frac{\delta r}{\delta i} = 0$, the nominal duration of the instrument is equal to expected inflation duration. (For a fully inflation adjusted asset, this should be 0)

$$D_\pi = -\frac{1}{V}\frac{dV}{di} = (1-\frac{\delta r}{\delta i})\Sigma_{t=1}^{T}\frac{C_t e^{\pi-i}t}{V}t = (1-\frac{\delta r}{\delta i})D_r$$

6.2 Obtaining Inflation Duration

Similar to the approach employed in (Fooladi et al., 2021), we estimate the sensitivity of real bond yields to changes in nominal yields by performing a regression analysis. Specifically, we regress the continuously compounded real yield ($r_{j,m}$) of inflation-linked government bonds with different maturities (m) on their corresponding continuously compounded nominal yields ($i_{j,m}$). The regression equation is represented as follows:

$$r_{j,m} = \alpha + \beta i_{j,m} + \epsilon_{j,m}$$

Here, α represents the intercept term, β denotes the slope coefficient, and $\epsilon_{j,m}$ represents the error term. The slope coefficient $\beta = \frac{\delta r}{\delta i}$ captures the relationship between nominal and real duration, providing insights into the nominal duration of a government-issued bond relative to its real duration, expressed as $D_i/D_r = \frac{\delta r}{\delta i}$.

To derive the inflation duration, we subtract β from 1 (1-β) and multiply the resulting difference by the duration of the respective bonds and notes. This calculation allows us to quantify the sensitivity of bond prices to changes in inflation expectations. The outcomes of the regression analysis can be observed in Table 5, presented below.

Table 5: Regression Results

Time	TTM	Intercept	β	Adj. R-squared	Weight
Bonds	30Y	-1.34 (0.017)	0.74 (0.006)	84.1%	81%
	20Y	-1.21 (0.022)	0.72 (0.006)	83.5%	19%
Notes & Bills	10Y	-1.65 (0.023)	0.85 (0.006)	84.5%	26%
	7Y	-1.44 (0.023)	0.81 (0.007)	80.5%	26%
	5Y	-1.33 (0.020)	0.77 (0.007)	79.3%	48%

Notes: This table contains the results of the regression analysis with equation $r_{j,m} = \alpha + \beta i_{j,m} + \epsilon_{j,m}$ Standard errors are in the parentheses. All results shows large statistical significance, which makes sense considering the only difference between the dependent and independent variable is the inflation protection. The important takeaway from these regressions are the β results. They will serve the purpose of helping us extract inflation duration from nominal duration.

Utilizing the bond weights for 30-year and 20-year maturities, we can compute the weighted beta (β) for bonds. Similarly, we can repeat this process for notes and bills. However, it is important to note that there are no inflation-protected securities available for maturities below 5 years. Consequently, we apply the same 5-year beta for all maturities below 5 years. Additionally, for Mortgage-Backed Securities (MBS), we employ the same beta as for bonds, given that mortgages often exhibit characteristics more akin to long-term debt.

By incorporating the weighted betas for bonds, notes, bills, and mortgage backed securities (MBS), we capture the inflation sensitivity of these respective fixed-income securities across various maturity profiles. These results allow us to acquire knowledge about the potential impact of inflation on government debt with various time to maturities.

This result provides a factor that helps determine the extent of change in the value of bond-like balance sheet items in response to fluctuations in inflation. The equation can be expressed as:

Table 6: Results of duration analysis

Item type	Weighted β	Weighted Duration*	Infl. Duration*
Total Bonds	0.73	15.8	4.21
Total Notes	0.78	3.2	0.70
Total Bills	0.77	0.2	0.05
Total MBS	0.73	6.2	1.66

Notes: This table shows the weights of the β for bonds, notes and bills respectively. They have been calculated taking the value-weighted average of each tenor when the debt was first issued. The weighted duration of each debt type can be multiplied with (1-β) resulting in the inflation duration.

$$\Delta B = -D^* \times \Delta i \times B$$

Here, ΔB represents the change in the value of bond-like balance sheet items, D^* denotes the modified duration (which incorporates the inflation duration calculated using the weighted beta), Δi represents the change in inflation, and B represents the initial value of the bond-like balance sheet items. By multiplying the modified duration, the change in inflation, and the initial value of the bond-like items, this equation quantifies the potential impact of inflation on the value of these assets.

7 Equity-like items

Equity-like items encompass assets that generate cash flows through business operations. Within the balance sheet framework, these items primarily include Private Businesses and Corporate Equities. These assets involve the production of goods or provision of services in exchange for monetary returns and are typically valued using discount models.

To capture a comprehensive view of the business landscape and investor sentiment, the S&P 500 index is commonly employed. This index is widely recognized for its representation of various business sectors and its extensive historical performance data. Scholars and researchers consider the S&P 500 as a suitable proxy for the American business market and an indicator of how investors perceive the overall business environment.

Table 7: S&P 500 Sector Weights as of 28/04/2023

Sector	Market Cap. (in millions)	Percent
Information Technology	9,455,836.53	25.80%
Health Care	5,277,676.20	14.40%
Financials	4,801,219.32	13.10%
Consumer Discretionary	3,628,402.39	9.90%
Industrials	3,078,644.45	8.40%
Communication Services	3,041,993.92	8.30%
Consumer Staples	2,712,139.16	7.40%
Energy	1,722,574.87	4.70%
Utilities	1,062,865.35	2.90%
Materials	952,913.76	2.60%
Real Estate	916,263.23	2.50%
Sectors Totalled	36,650,529.19	100%

Notes: This is an overview of the respective sector weightings of the S&P 500

Under the premise that the selected index accurately reflects the overall condition of businesses in the United States, a simplifying assumption is made regarding the resemblance of private businesses to the index weightings. This assumption justifies the inclusion of both Private Businesses and Corporate Equities within the category of equity-like items.

7.1 Equity-data

In this section, we utilize the dataset sourced from Shiller's seminal work "Irrational Exuberance," (Shiller, 2016) which is publicly available at Yale University's website. The dataset comprises monthly observations of prices, dividends, and earnings from the S&P 500 index, as well as Consumer Price Index (CPI) data and 10-Year Treasury yields. The temporal coverage spans from 1871 to the present day, thereby providing a comprehensive historical record for conducting our computations and analyses.

7.2 Methodology

7.2.1 Inflation Duration

To estimate the equity sensitivity to inflation, we adopt the approach outlined in (Leibowitz et al., 1989), which decomposes the equity duration model into real rate sensitivity and inflation sensitivity. We begin with the equation:

$$\frac{\Delta P}{P} = -D_{DDM}\left(1 - \gamma + \frac{\delta h}{\delta r}\right)\Delta r - D_{DDM}\left(1 - \lambda + \frac{\delta h}{\delta i}\right)\Delta i$$

Here, the variables are defined as follows:

ΔP = Changes in equity prices,

D_{DDM} = Equity duration,

γ = Earnings growth sensitivity to changes in real rates,

δ = Earnings growth sensitivity to changes in inflation rates,

$\frac{\delta h}{\delta r}$ = The changes in equity market risk premium with respect to changes in real rates,

$\frac{\delta h}{\delta i}$ = The changes in equity market risk premium with respect to inflation,

Δr = Changes in real rates,

Δi = Changes in inflation

In this equation, the first part,

$$-D_{DDM}\left(1 - \gamma + \frac{\delta h}{\delta r}\right)\Delta r$$

represents changes in equity prices resulting from changes in real rates, while the second part,

$$-D_{DDM}\left(1 - \lambda + \frac{\delta h}{\delta i}\right)\Delta i$$

represents changes in equity prices resulting from changes in the inflation rate, which is the focus of this thesis. This equation endeavors to capture the price sensitivity of assets to fluctuations in inflation by examining how the market incorporates both the risk premium $\frac{\delta h}{\delta i}$ and the price sensitivity λ. A substantial flow-through rate indicates a diminished influence on equity risk premium, as anticipated earnings rise in tandem with higher premiums.

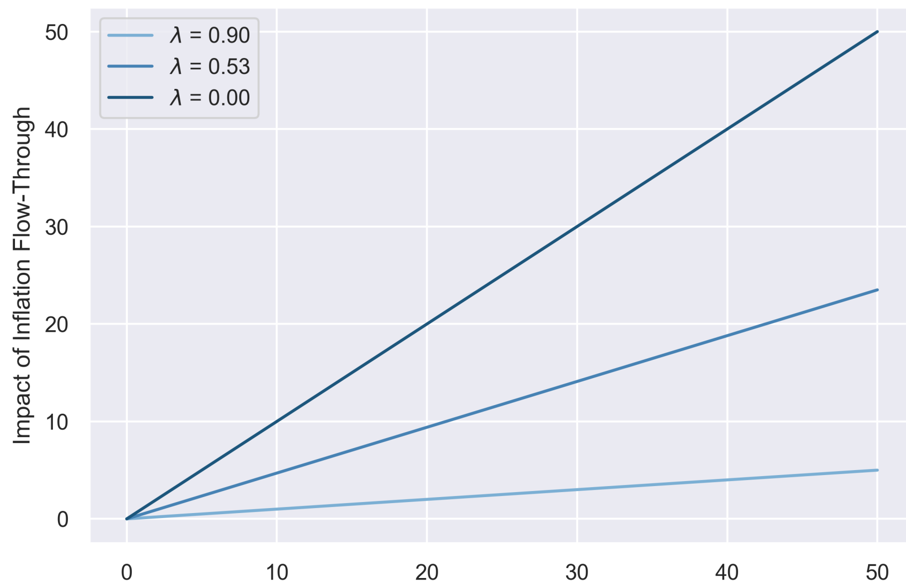

Figure 1: Impact of Flow-Through to Duration

Notes: This curve shows impact of inflation at different flow-throughs, where a higher flow-through indicate a lower impact in duration.

To obtain D_{DDM}, we employ the Dividend Discount Model (DDM):

$$P = \frac{D_0(1+g)}{k-g}$$

where D_{DDM} can be derived as:

$$D_{DDM} = -\frac{\delta lnP}{\delta k} = \frac{1}{k-g}$$

with k representing the discount rate and g representing the long-term growth rate.

To determine the parameter λ, we follow the methodology outlined in (Asness, 2003). We run the regression:

$$G_{EPS} = \alpha + \lambda i$$

where G_{EPS} represents nominal earnings growth and i represents inflation. We replicate Asness's method by calculating the smoothed monthly 10-year trailing growth in earnings estimates and CPI data. Extending the dataset until the present year (2022) allows for more recent results. Our regression results yield:

$$G_{EPS} = 0.021 + 0.9114_{CPI}$$

Additionally, we obtain $\frac{\delta h}{\delta i}$ by regressing both real rates and inflation rates on the implied equity risk premium of the S&P 500:

$$h = \alpha + \beta_1 r + \beta_2 i$$

Understanding how inflation affects the equity risk premium is crucial, as large increases in the equity risk premium can lead to price decreases and negative returns. This analyzis paints a clearer picture of how inflation impact risk.

7.2.2 Equity Risk Premium

Furthermore, we recognize the equity risk premium as the inverse of duration. This implied premium can be derived from the dividend discount model as:

$$P = \frac{D_0}{k - g}$$

Solving for the discount factor, we have:

$$k - g = \frac{D_0}{P}$$

According to (Damodaran, 2019), if the risk-free rate (rf) is considered as the constant growth rate, then the dividend yield represents the equity risk premium (ERP):

$$\frac{D}{P} = k - g$$

$$\frac{D}{P} = k - rf$$

$$\frac{D}{P} = ERP$$

This notion is supported by (Rozeff, 1984). Damodaran further highlights the importance of considering stock repurchases alongside dividends to obtain a more accurate metric in modern times, as there has been a shift towards stock buybacks in recent years. The findings of our analysis are summarized in the table presented below.

Table 8: Equity Results

Variable	Result
λ	0.91
$\frac{\delta h}{\delta i}$	0.4270
D_{DDM}	22.54
Inflation duration	-11.62

It should be noted that the interpretation of a 1% change in inflation leading to a 11.62% decrease in the S&P 500, ceteris paribus, is not intended to predict exact stock price changes resulting from an inflation shock. While this interpretation has been criticized by (Blitzer et al., 2010), it serves a meaningful purpose in our analysis, as our main objective is to examine the relative magnitudes of wealth changes in response to inflation.

There are certain caveats to consider. The flow-through data is based on the Ibbotson Period, while other measurements are derived from the past 10 years to capture recent market behavior. Given the increasing prevalence of stock buybacks as opposed to solely dividend payments, we have incorporated the (dividend + stock buyback)/price yield to obtain a more accurate payout ratio. For the duration calculation, we have utilized the geometric average equity premium over the past ten years.

8 Real Estate

Real estate stands out as the asset class that constitutes the most substantial component within individual portfolios. Consequently, fluctuations in real estate values exert the most substantial influence on wealth variations for the majority of individuals, particularly those with lower levels of wealth. This asset class is typically represented on balance sheets as equity in primary residences. To assess the inflationary impact on real estate equity, we adopt the techniques commonly employed in analyzing the equity section of financial statements. By employing these techniques, we seek to measure and evaluate the effects of inflation on real estate equity in a rigorous and comprehensive manner.

8.1 Data

The Residential Real Estate Investment Trusts (Residential REITs) sector and residential real estate generate income through rental properties. Investors acquire properties and rent them out for profit. Similarly, residential REITs acquire and manage portfolios of residential properties for profit. Through publicly traded REITs, investors are able to gain exposure indirectly to real estate. These publicly traded equities usually hold portfolios diversified across geographical locations as well, capturing most of the American market. Therefore, the thesis suggests a residential REIT index as a good proxy for the sector.

We propose creating a new market capitalization-weighted index with a specific focus on residential REITs listed on the New York Stock Exchange. The aim is to provide a more accurate representation of the residential real estate sector compared to existing indices, such as the FTSE Nareit All Equity REITs index, which includes real estate assets beyond the residential sector.

The index is constructed with inspiration from Robert Shiller's monthly data for the S&P 500, with prices and dividends. In addition, market capitalization is added to obtain market weights of the respective equities. A similar structure will ensure consistency and comparability with previous works in the thesis. The inclusion criteria require a minimum 50% of assets invested in residential real estate in the United States, excluding mortgages. This definition adheres to the balance sheet definition of residential real estate and ensures a sole focus on residential real estate sector.

8.2 Data Source

The selection of companies has been derived from Yahoo Finance's screener of residential REITs listed on the NYSE (Finance, 2023). The necessary data is collected from Financial Modeling Prep's API, which provides historical data for the companies extending back to the 1980s, while most had data covering their entire lifetime. This data enables the examination of long-term performance and behavior of the sector as a whole for our index. The data gathering process is in the appendix.

8.3 Methodology

In our real estate analysis, we employ a methodology that match the approach used for equities. However, we substitute the S&P 500 dataset with the Residential Index, and additionally incorporate the Consumer Price Index (CPI) for the rent equivalent of primary residence (U.S. Bureau of Labor Statistics, 1982) to estimate rental income growth.

Drawing upon the explanations provided in the equity section, we delve into the investigation of empirical evidence concerning the influence of inflation on real estate price movements. Given our premise that discounted future rental income determine the intrinsic value of residential properties, it becomes imperative to discern the flow-through effects of inflation on rental income. Notably, shelter expenses account for 34% of the CPI, while the rent equivalent of primary residence represents 24% of the CPI (U.S. Bureau of Labor Statistics, 2023). Consequently, we anticipate a close correlation between rent and the inflation rate. To mitigate multicollinearity between the dependent and independent variables, we employ the CPI less shelter (U.S. Bureau of Labor Statistics, 1947) as the explanatory variable. This approach parallels that used for equities; however, due to the comparatively smaller number of observations in real estate data, we opt for a five-year smoothed annualized growth rate rather than a ten-year smoothed rate. The flow-through effects of inflation on rental income can be expressed as follows:

$$G_s = \alpha + \lambda G_i$$

where G represents growth, s denotes rent, and i signifies inflation.

The results reveal the inflation flow-through effects as:

$$G_s = 0.018 + 0.6556 G_i$$

yielding an Adjusted R^2 value of 0.382.

To assess the impact of inflation on the implied risk premium, we study the relationship between inflation and the Residential REIT index' implied equity risk premium. Building upon the work of (Leibowitz et al., 1989), we adopt their approach of regressing the inflation and real rates against the implied equity risk premium. The regression model takes the form:

$$h = \alpha + \beta_1 r + \beta_2 i$$

This framework allows us to dissect the effects of interest rates on the risk premium, separating them into real rates and inflation rates. The regression analysis encompasses the entire lifespan of the index, spanning from January 1989 until the end of 2022, with monthly observations. The regression results yield the following equation:

$$h = \alpha + 0.5290r + 0.2842i$$

By employing this regression model, we gain insights into the relationship between inflation, real rates, and the implied equity risk premium of the Residential REIT index.

The preceding findings provide valuable insights into the impact of inflation on movements in residential real estate prices. By considering both expected earnings growth and the effects on the discounting rate, we can comprehensively analyze these effects. The formula capturing these dynamics is represented as follows:

$$-D_{DDM}\left(1 - \lambda + \frac{\delta h}{\delta i}\right)\Delta i$$

This equation encapsulates the relationship between changes in inflation (Δi) and movements in residential real estate prices. It highlights the combined influence of factors such as expected income growth, and the implied risk premium. The resulting coefficient of -10.81 indicates the magnitude of the impact of changes in inflation on residential real estate price movements.

To provide a concise summary of these findings, the relevant data is presented below:

Table 9: Real Estate Results

Variable	Result
λ	0.66
$\frac{\delta h}{\delta i}$	0.28
D_{DDM}	23.03
Inflation duration	-14.47

Notes: This table serves as an overview for the results of the Real Estate section of the thesis.

9 What have we achieved?

In this analysis, we have explored the intricate relationship between inflation and the distribution of wealth. Our investigation utilizes both balance sheet analysis and duration analysis techniques, allowing us to gain insights into the shape and trajectory of wealth distribution under inflationary conditions. Specifically, we examine the inflation duration of various asset classes, including bond-like and equity-like instruments, as well as real estate. By reviewing the composition of these assets across different wealth percentiles, we are able to ascertain the extent to which inflation affects the net worth of each wealth group.

Our findings reveal that the wealthiest individuals exhibit a fairly high degree of sensitivity to inflation, corroborating the earlier research conducted by (Greenwald et al., 2021), which suggests that individuals with greater wealth tend to hold portfolios with longer duration. However, our analysis indicates that the bottom 50% of the wealth distribution display the highest susceptibility to inflation. This phenomenon can be attributed to their substantial reliance on housing assets, as our results demonstrate that housing exhibits a higher inflation duration when compared to bonds.

10 Conclusion

Through an analysis of the sensitivities of balance sheet components to inflation and discerning the aggregate sensitivity to inflation within each wealth group, as determined by the composition of these components in their balance sheets, we can acquire valuable insights into the impact of inflation on the configuration of wealth distribution in the United States. The ensuing table presents the net

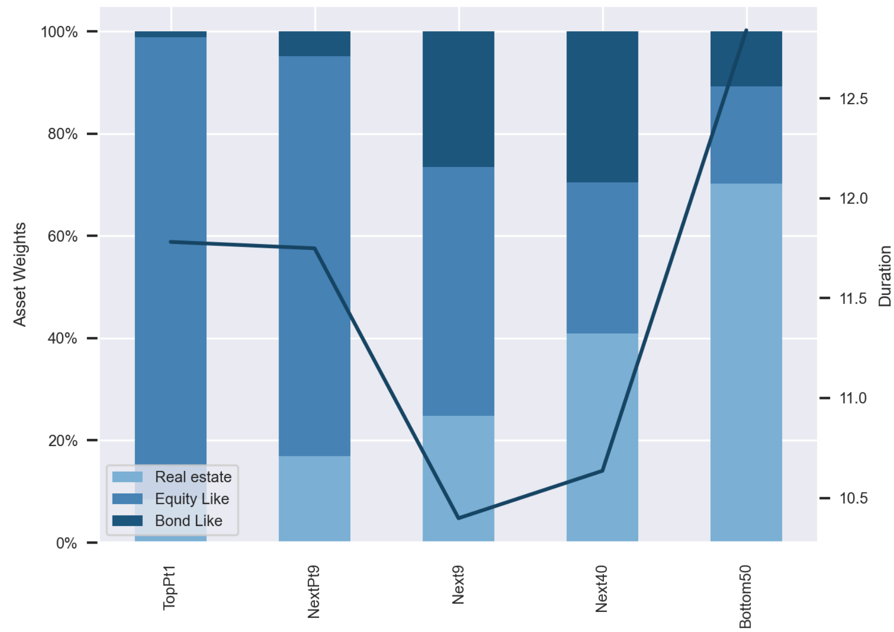

Figure 2: Impact of Flow-Through to Duration

Notes: This curve shows the net duration across different wealth groups in the United States. It displays a U-shape, meaning the bottom 50%, accompanied by the wealthiest has the highest sensitivity to inflation.

impact resulting from a one percentage point increase in inflation.

The most pronounced impact is observed within the bottom 50% of the wealth distribution, primarily stemming from their considerable relative investments in housing, an asset class also marked by a substantial inflation duration measure. The second most impacted is observed within the top 0.1% bracket, given their substantial equity holdings, which correspondingly entail a substantial inflation duration measure. Following suit, the subsequent 0.9% segment shoulders a significant burden of inflation duration, also primarily due to the elevated proportion of equity in their balance sheets. The "Next 40%" and "Next 9%" groups occupy the lowest ranks, as their balance sheets exhibit a more equitable distribution across various items, with higher concentrations of bond-like items, or more specifically pension plans.

In summary, the impact of inflation on wealth inequality does not yield an overtly discernible outcome. While inflation exerts negative effects on both the wealthiest and the poorest segments of the economy, there exists a marginal increase in wealth inequality attributable to inflation. This is owing to the slightly greater negative impact on the bottom 50% group in comparison to the

Table 10: Summarized Results

Percentile	Net Factor
Top 0.1%	-0.1101
Next 0.9%	-0.1045
Next 9%	-0.0897
Next 40%	-0.0940
Bottom 50%	-0.1201

richest. The middle class, however, seems to be the group least affected by the negative effects ultimately arising from inflation, emerging as the beneficiaries within this context.

11 APPENDIX

11.1 Figures

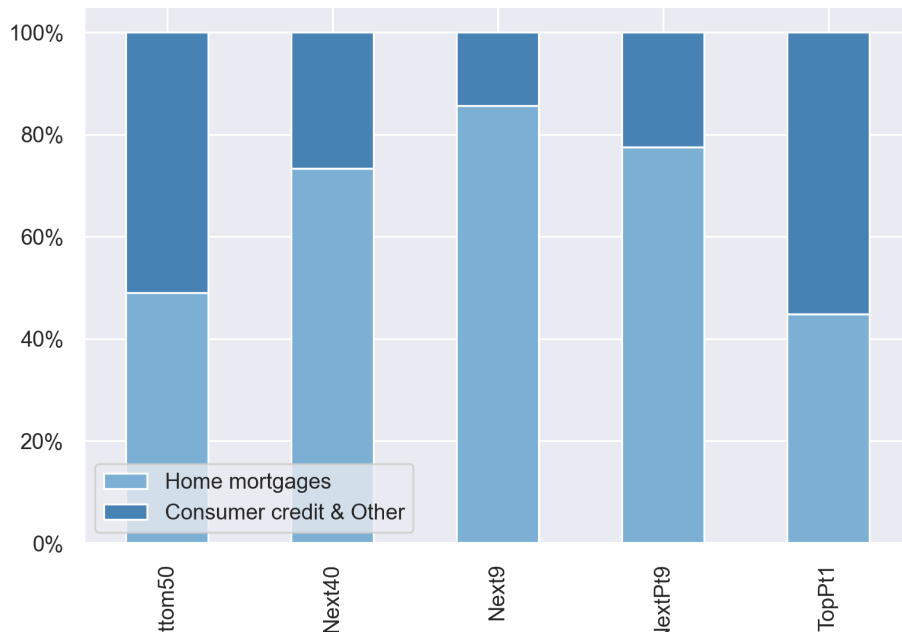

Figure 3: Liability Weightings

Notes: Liability Balance Sheet Items

11.2 Bonds Data

To estimate the inflation sensitivity of bond-like items, we collect data on government debt from (Treasury, 2023). The data consists of every current individual outstanding marketable Bill, Note, Bond, Federal Financing Bank, Floating Rate Note Inflation-Protected Security, as of the end of April 2023. We then sort the data based on security type and compute the weighted average duration of the total marketable US. government debt over time.

To simplify, we neglect Federal Financing Bank, Floating Rate Notes, and Inflation Protected Securities. Federal Financing Bank, can be neglected because it makes up such a small part of the debt, Floating Rate Notes, because their duration is effectively zero, and Inflation-Protected Securities makes sense to exclude because we want to measure the debts' inflation sensitivity in the end, and inflation protected securities are by nature not sensitive to inflation. In the table 4 below, you can find a full list of securities that makes up the government debt as of late April 2023, and the simplified version is found in table 11.

Table 11: Marketable US. Government debt pr. 30/04/2023

Security Type	Total Amount (in millions)	Percent
Bonds	4,094,321	17.26%
Notes	13,627,978	57.45%
Bills	3,942,645	16.62%
Federal Financing Bank	4,847	0.02%
Floating Rate Notes	516,059	2.18%
Inflation-Protected Securities	1,537,127	6.46%
Total Marketable Debt	23,722,978	100%

Notes: The United States marketable government debt consists of Bonds, Notes, Bills, Federal Financing Bank, Floating Rate Notes and Inflation-Protected Securities.

Table 12: Adjusted marketable US. Government debt pr. 30/04/2023

Security Type	Total Amount (in millions)	Percent
Bonds	4,094,321	18.9%
Notes	13,627,978	62.9%
Bills	3,942,645	18.2%
Total Marketable Debt	21,664,944	100%

Notes: The marketable government debt after is has been simplified.

The remaining composition of government debt can be further grouped based on the time to maturity of the securities when they were first issued. We can then calculated the value-weight of 20Y and 30Y securities in the bond category and similarly, compute the value weights of 2Y, 3Y, 5Y, 7Y and 10Y notes. Bills have a 100% weight of below 1Y securities. These weights, can be used later to compute the value-weighted average duration of the respective debt categories.

For mortgage backed securities, we use the (Bloomberg, 2023) Bloomberg US MBS Index as proxy for the mortgage market in the US. The data set consists of the 936 individual mortgage backed securities that makes up the index and their respective price, yield to worst (YTW), option adjusted duration (OAD), par value, market value and index weight. The option adjusted duration of the index as a whole is simply the weighted average of the OAD reported in the bloomberg dataset.

$$\Sigma_{i=1}^{n}(w_i \times OAD_i)$$

11.3 Real Estate Data

In accordance with the S&P data, the methodology for constructing the index involved collecting historical prices, market capitalization, and dividends.

Regarding dividends, it is common practice in the industry to consider quarterly payouts, although they may be irregular. To address this, the ex-dividend date data was utilized for achieving more accurate results. As this information was not directly available from the profit and loss statements, the dividends were indexed based on their ex-dividend date. The current dividend was then combined with the dividends paid within the previous year to determine the twelve-month trailing dividends of each company.

To consolidate the data, a date range was established from the start of 1980 to the present day, encompassing all relevant data points. This range was utilized as an index for each individual DataFrame, and the most recent available data was used to fill any missing values. This approach ensured a dataset where all data points were consistent, facilitating subsequent trimming of the dataframes to the last business day of each month and yielding a comprehensive dataset of monthly values.

To derive the appropriate dividend yield for the entire sector, consideration was given to the market capitalization weights of the index. This involved summing up the dividend yields of all companies weighted by their respective market capitalizations:

$$\frac{D}{P}_{Mkt,t} = \sum \frac{D_{i,t}}{P_{i,t}} W_{i,t}$$

This calculation was performed for each month, with W_i recalculated to account for the availability of companies at each respective time point, ensuring a comprehensive data. A comprehensive list of the REITs included in the index can be found in table 13.

Table 13: List of REITs used in the making of the index

Ticker	Name	Market Cap
AVB	AvalonBay Communities, Inc.	25.581B
EQR	Equity Residential	23.666B
UDR	UDR, Inc.	22.556B
INVH	Invitation Homes Inc.	21.23B
MAA	Mid-America Apartment Communities, Inc.	18.067B
SUI	Sun Communities, Inc.	17.586B
AMH	American Homes 4 Rent	14.626B
ESS	Essex Property Trust, Inc.	14.32B
ELS	Equity LifeStyle Properties, Inc.	13.515B
MAA-PI	Mid-America Apartment Communities, Inc.	12.058B
CPT	Camden Property Trust	11.826B
AMH-PH	American Homes 4 Rent	8.678B
AMH-PG	American Homes 4 Rent	8.477B
AIRC	Apartment Income REIT Corp.	5.346B
IRT	Independence Realty Trust, Inc.	3.956B
VRE	Veris Residential, Inc.	1.66B
ELME	Elme Communities	1.434B
AIV	Apartment Investment and Management Company	1.256B
NXRT	NexPoint Residential Trust, Inc.	1.069B
UMH	UMH Properties, Inc.	963.562M
UMH-PD	UMH Properties, Inc.	452.745M
BRT	BRT Apartments Corp.	327.784M
CLPR	Clipper Realty Inc.	220.212M

Notes: CSR was not included as it created unknown problems in the script.

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