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Navigating the Path to Achieving the Paris Agreement: Insights from a Python-based Climate Change Model

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Authors: Emilie Befring Mørk & Jone Omdal Bø
Supervisor: Alfonso Irarrazabal

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Contents

1 Introduction

In recent years, empirical research on the economic impacts of climate change has witnessed a significant surge. The pressing issues of global warming and climate change have garnered substantial attention due to their profound implications for the planet, ecosystems, and the global economy. Human-induced carbon emissions resulting from economic activities have been identified as the primary drivers of these phenomena, with varying regional and local economic impacts. (NASA, 2023)

Our study draws inspiration from the work of Cruz and Rossi-Hansberg (2022), titled "The Economic Geography of Global Warming," published as a working paper by the University of Chicago's Becker Friedman Institute for Economics. Building upon their research, we aim to replicate and further investigate their model by translating their codes from Matlab into Python. This endeavor enables us to delve deeper into their analysis, explore alternative scenarios, and contribute to the understanding of climate change dynamics and policy implications.

Global warming refers to the sustained increase in Earth's surface temperature observed since the pre-industrial era. The combustion of fossil fuels is primarily responsible for this phenomenon, resulting in an estimated average global temperature rise from pre-industrial times of approximately 1.01 degrees Celsius. Alarmingly, the pace of temperature increase is currently accelerating at a rate exceeding 0.2 degrees Celsius per decade. (NASA, 2023)

Climate change includes long-term alterations in weather patterns, influencing local, regional, and global climates. The significant increase in greenhouse gas emissions, particularly from extensive fossil fuel consumption since the mid-20th century, has elevated heat-trapping gas levels and subsequently raised the Earth's average surface temperature. The consequences of climate change pose substantial risks to the global economy. (NASA, 2023)

Elevated temperatures and its consequences have the potential to cause extensive damage to infrastructure and property, negatively affect human health and productivity, and disrupt crucial sectors such as agriculture. Developing countries, often characterized by weaker infrastructure, limited technological advancements, and fewer resources for climate adaptation and mitigation, face heightened vulnerability to these impacts. (WHO, 2021) Furthermore, climate change exacerbates migration patterns, particularly in regions that are more vulnerable and less developed, such as Asia and Africa.

Our research focuses on exploring scenarios that prioritize avoiding climate overshoot, wherein global temperatures surpass the 2-degree Celsius threshold before moving back and stabilizing below the 2-degree treshold. Climate overshoot poses severe risks, including irreversible environmental damage, intensified extreme weather events, and disruptions to ecosystems and biodiversity. By identifying pathways that remain below this critical threshold, our aim is to contribute to the formulation of sustainable and resilient strategies that mitigate the most severe consequences of climate change. (Climate Overshoot Comission, 2022)

Through comprehensive analysis and modeling, we investigate alternative pathways and policies that offer potential solutions to the challenges posed by climate change. By understanding the complexities associated with climate change risks, including the implications of climate overshoot, we aspire to guide policymakers and stakeholders in making informed decisions that promote climate stability and sustainability.

By unraveling the details of climate change dynamics and exploring strategies to mitigate associated risks, we believe our research can play a role in shaping a more sustainable and resilient future for our planet and its inhabitants.

2 Objective

The objective of this study is to investigate and analyze strategies aimed at preventing the global average temperature from surpassing a 2-degree Celsius increase above pre-industrial levels, in line with the goals set forth by the Paris Agreement. By examining the existing literature and utilizing a Python-based model based on the framework proposed by Cruz & Rossi-Hansberg (2022), we will explore various parameters and their effects on key variables such as temperature, CO2 emissions, clean energy utilization, GDP, and population density.

Through our research, we aim to contribute to the understanding of the actions and policies necessary to fulfill the commitments of the Paris Agreement. The findings of this study will provide valuable insights for policymakers, researchers, and stakeholders, aiding them in the development of effective strategies for climate change mitigation and adaptation. By integrating climate considerations into decision-making processes, businesses and organizations can play a crucial role in working towards a sustainable and resilient future. Ultimately, this research supports global efforts to combat climate change and foster a better world for current and future generations.

3 Literature review

By using Cruz and Rossi-Hansberg (2022) as a starting point and replicating their model, which was implemented using MATLAB, we can dive deeper into the topic of climate change and its effects on the economy at both local and global levels. The model served as a tool to study various aspects, including abatement technologies, the impact of carbon taxes, and clean energy subsidies. Replicating their model allows us to build upon their research and conduct further analysis, exploring numerous issues related to climate change in greater depth.

By quantifying the model, Cruz and Rossi-Hansberg (2022) were able to simulate the economy forward over several centuries in order to evaluate the consequences that global warming has on the economy. The phenomenon is expected to have heterogeneous effects over space according to their study, where colder regions such as Alaska, Northern Canada and Siberia can expect welfare gains up to 11%, while the hotter regions such as South America, India and Australia are expected to experience welfare losses up to 20%. Their findings show that the world on average will face a welfare loss of 6%, with implications of poorer and less developed regions in the world facing the highest warming losses.

Deschênes and Greenstone (2007) are some of the pioneers of the empirical papers on the topic of climate change and its consequences on economic and social outcomes. Their paper investigated the economic impact of climate change on agricultural land in the U.S. by estimating the impact of presumably random year-to-year variations in weather and temperature on agricultural profits. The study showed quite different effects depending on the states being studied. While California was estimated to have an impact of approximately -50% of state agricultural profits, other states were estimated to have an increase in their agricultural profits. This methodology has been used to study various weather effects, including the effects on morality, crime and conflict, migration, and GDP and GDP growth.

In recent years, estimates have been incorporated in economic models of global warming, known as Integrated Assessment Models (IAM). Several papers have been using these core models to study topics such as the role of clean technology investments and innovations in mitigating climate damages, migration, and capacity to meet food demand for different changes in climate conditions. Cruz and Rossi-Hansberg (2022) tries to contribute to the development of IAMs by incorporating recent development in spatial quantitative models.

In Desmet et al. (2018), they develop a spatial growth theory at a fine level of geographical resolution which is used to analyse the evolution of the economy over several centuries. Some of their findings includes that relaxing migration restrictions can lead to large welfare gains but also that the world economy will concentrate in very different sets of regions and nations depending on migratory frictions. Cruz and Rossi-Hansberg (2022) builds on this framework, but they also incorporate local fertility and population dynamics, energy use, fossil fuels extraction costs, and effects of temperature on productivity and amenities.

Environmental questions have been addressed through the lens of spatial dynamic models in the incipient literature of spatial IAMs. One paper by Balboni (2019) looks at the costs of road investment at the coast of Vietnam in the event of a rise in the sea level. This is also the subject matter in Desmet et al. (2021), where they measure the spatial shifts in population and economic activity due to sea level rise and coastal flooding. They used a spatially disaggregated model of the world economy, estimating that by the year 2200, a projected 1.46 percent of the global population will be displaced. The model also estimates a loss in real GDP by up to as much as 4.5 percent in 2200.

A variety of different papers have evaluated the impact that global warming has across different economic sectors, something that Cruz and Rossi-Hansberg (2022) does not incorporate in their model or paper. The author uses a dynamic economic model to study global warming and labour market reallocation. Some of the findings concludes that agricultural workers face welfare losses three times as large as the average worker. They do this by quantifying the model they use for 6 sectors and 287 countries and subnational units.

The Intergovernmental Panel on Climate Change (IPCC) plays a crucial role in assessing scientific information related to climate change. Its comprehensive reports provide valuable insights into the current state of the climate system, the impacts of human activities, and projected future scenarios. The IPCC report highlights the undeniable influence of human activities on the climate system, leading to widespread and rapid consequences for the atmosphere, oceans, cryosphere, and biosphere. The last four decades have successively been the warmest since 1850, with human-induced CO2 emissions identified as the primary cause. Concentrations of CO2, CH4, and N2O have reached unprecedented levels, affecting the open ocean's acidification and causing global retreat of glaciers and Arctic sea ice. Moreover, climate change is already impacting various weather and climate extremes globally, including heatwaves, heavy precipitation, droughts, and tropical cyclones. Under all emission scenarios considered, global surface temperature is projected to continue rising until at least the mid-21st century. Without substantial reductions in CO2 and greenhouse gas emissions, global warming is expected to surpass the 1.5°C and 2°C thresholds. The projected temperature increase by the end of the century ranges from 1.0°C to 1.8°C with very low emissions, 2.1°C to 3.5°C with medium emissions, and around 3.3°C to 5.7°C with high emissions. Limiting global warming to a specific level requires achieving net zero CO2 emissions and significant reductions in other greenhouse gases. (Schulz, 2022) The IPCC report relies on the the representative concentration pathways (RCP) scenarios, which depict a range of greenhouse gas emissions pathways, as inputs for assessing future climate change. These scenarios, including RCP2.6 (low emissions), RCP4.5 and RCP6 (medium emissions), and RCP8.5 (high emissions), explore different trajectories of greenhouse gas concentrations. (Van Vuuren, D.P, 2011) By utilizing these scenarios, the IPCC assesses and projects the potential impacts of climate change under various emissions pathways, enabling policymakers to understand the consequences of different policy choices and the urgency of climate action.

The Paris Agreement, adopted in 2015 under the United Nations Framework Convention on Climate Change (UNFCCC), represents a landmark international effort to combat climate change and mitigate its impacts. At its core, the agreement aims to limit global warming well below 2 degrees Celsius above pre-industrial levels and pursue efforts to limit the temperature increase to 1.5 degrees Celsius. This temperature threshold has been identified as critical for avoiding catastrophic consequences and safeguarding the planet's ecological systems, vulnerable communities, and future generations. The commitment to preventing global average temperature from surpassing a 2-degree Celsius increase is rooted in extensive scientific research and assessment reports, such as those by the Intergovernmental Panel on Climate Change (IPCC). These reports have unequivocally demonstrated the severe risks associated with higher temperature increases, including more frequent and intense extreme weather events, rising sea levels, biodiversity loss, and disruptions to ecosystems and human livelihoods. (United Nations Climate Change, 2023).

4 Methodology: The Model

The model we are going to replicate by Cruz & Rossi-Hansberg (2022) is an extended version of what Desmet et al. (2018) did by incorporating several dimensions in the economic component. Firstly, an endogenous law of motion for global population have been included. Secondly, the model accounts for the use of labour, land, and energy as necessary inputs for production, with energy being derived from either fossil fuels or clean sources. The use of fossil fuels results in CO2 emissions, while clean sources do not. Thirdly, we have accounted for the spatial heterogeneity of fundamental amenities, productivities, and natality rates due to local climate conditions. To achieve this, we have utilized reduced-form models from IPCC (2013) for the carbon cycle and global temperature models and have projected from global to local temperature using the statistical down-scaling approach by Mitchell (2003).

Subsections 4.1 to 4.5 of this study draw inspiration from Cruz and Rossi-Hansberg (2022), specifically pages 7 to 14, where they provide insightful analysis on various aspects of the model. These subsections delve into the intricate details of the model's components, examining topics such as the dynamics of population, the role of labor, land, and energy in production, and the spatial variations driven by local climate conditions. The findings and insights presented in these subsections contribute to our understanding of the economic implications of climate change mitigation strategies.

Additionally, subsection 4.6 is based on Cruz and Rossi-Hansberg (2022) from the Supplementary Materials section, specifically pages 1 to 8. This subsection explores specific details and results related to our analysis, building upon the supplementary materials provided by Cruz and Rossi-Hansberg (2022).

Moreover, it is worth noting that the translation of this model from MATLAB to Python required a significant amount of time and effort. We invested considerable resources in adapting and implementing the model in Python to ensure its compatibility and accessibility. This translation process involved precisely converting the mathematical equations and computational algorithms from the original MATLAB codebase into Python syntax. The effort invested in this translation was necessary to harness the capabilities of the Python-based model and derive meaningful insights based on our research objectives.

In the appendix section of this study, we provide the Python codes that correspond to the model implementation. These codes serve as a valuable resource for readers interested in replicating or further exploring our analysis. The availability of the Python codes enhances transparency, reproducibility, and the opportunity for researchers to build upon our work.

4.1 Endowment and Preferences

The global economy can be mapped onto a two-dimensional surface S, where each location is defined as a point $r \in S$ with corresponding land density$H(r)$. In each period t, the world economy comprises L_t agents, and global population changes over time due to endogenous natality rates. Agents derive utility every period from consuming a set of differentiated varieties $c_t^{\omega}(r)$, which are aggregated according to a CES utility function, as well as from local amenities $b_t(r)$, and their idiosyncratic preference for their location of residence, $\epsilon_t^i(r)$. Should agents choose to move from r to s at time t, they incur mobility costs $m(r, s)$, which act as a permanent flow cost from that time onwards. The period utility of agent i residing in location r at time t and with a location history $r_- = (r_0, ..., r_{t-1})$ is determined by:

$$u_t^i(r_-, r) = [\int_0^1 c_t^\omega(r)^\rho d\omega]^{\frac{\sigma}{\rho}} b_t(r) \epsilon_t^i(r) \prod_{s=1}^{t} m(r_{s-1}, r_s)^{-1} \tag{1}$$

Labor gives the agents income, which they supply inelastically at a rate of one unit, and for which they receive a wage $w_t(r)$. Additionally, they receive a share of land rents, $H(r)R_t(r)$, which is uniformly distributed among all residents of the location. Hence, the per capita real income $y_t(r)$ can be expressed as $(w_t(r) + R_t(r)/L_t(r)/P_t(r)$, where $L_t(r)$ represents the local population density (population per unit of land) and $P_t(r)$ is the local ideal CES price index. The parameter σ governs the curvature in the utility function, thereby determining the elasticity of utility to real income. Local amenities, $b_t(r)$, are impacted by congestion in a way that can be characterized as $b_t(r) = \bar{b_t}(r)L_t(r)^{-\lambda}$, where $b_t\bar{(r)}$ denotes the fundamental amenities of a location and λ is the congestion elasticity of amenities to population density. Local climate conditions can affect fundamental amenities through the use of the *damage* function $\Lambda^b(.)$, which indicates the percentage change in fundamental amenities when local temperatures rises from $T_{t-1}(r)$ in period $t-1$ to $T_t(r) = \Delta T_t\bar{(r)} + T_{t-1}(r)$ in period t. This is given by

$$\bar{b}_t(r) = (1 + \Lambda^b(\Delta T_t(r), T_{t-1}(r)))\bar{b}_{t-1}(r). \tag{2}$$

From this equation, we get that if $\Lambda^b(\Delta T_t(r), T_{t-1}(r))$ is negative, the amenities in location r will experience damage as a result of local temperature increases. On the contrary, if it is positive, the amenities in the location will experience improvement. The damage function's sensitivity to temperature levels, rather than just temperature changes, accounts for the diverse spatial impacts that are anticipated to result from global warming. For example, amenities in hot regions such as African countries are expected to decline as temperature rise further, while those in cold areas such as Siberia are expected to benefit from a warmer climate.

Additionally, households are subject to idiosyncratic taste shocks, $\epsilon_r^i(r)$, which are assumed to be independent and identically distributed across households, locations, and time, following a Fréchet distribution with a scale parameter of 1 and a shape parameter of $1/\Omega$. The degree of dispersion in agent preferences across location is determined by the value of Ω, which acts as a second congestion force.

The cost of moving from location r to s is modeled as the product of an origin-specific cost, $m_1(r)$, and a destination-specific cost, $m_2(s)$, such that

$m(r,s) = m_1(r)m_2(s)$. Since remaining in the same location has no cost, $m(r,r) = 1$, and the origin costs are simply the inverse of the destination costs, i.e., $m_1(r) = 1/m_2(r)$. Thus, the permanent utility cost of entering a location is offset by a permanent utility benefit when leaving, and agents only incur the flow cost while residing there. This method of modeling migration costs implies that migration decisions are reversible, and as a result, agents' location choices are solely determined by current variables rather than past or future ones. As is typical in discrete choice models with idiosyncratic preferences, the proportion of households residing in location r at period t is given by:

$$\frac{L_t(r)H(r)}{L_t} = \frac{u_t(r)^{1/\Omega} m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega} m_2(v)^{-1/\Omega} dv} \tag{3}$$

The term $u_t(r)$ refers to the portion of local utility that is not specific to an individual's preferences, i.e., it is independent of idiosyncratic taste shocks. This is given by:

$$u_t(r) = b_t(r)y_t(r)^{\sigma} = b_t(r)\left[\int_0^1 c_t^{\omega}(r)^{\rho} dw\right]^{\sigma/\rho} \tag{4}$$

At the end of each period t, households have a net offspring count of $n_t(r)$, which influences local natality rates. These rates, denoted by $n_t(r) = \eta(y_t(r), T_t(r))$, are exogeneous to the individual, but endogenous to the location's real income and temperature. Therefore, before migration decisions are made in the next period $t+1$, local population density $L'_{t+1}(r)$ is determined by the equation $L'_{t+1}(r)H(r) = (1 + n_t(r))L_t(r)H(r)$. It is important to note that global population not only depends on the distribution of natality rates across space and time, but also on the spatial distribution of population in the previous period.

4.2 Technology

In this model, each cell contains a continuous spectrum of firms that produce distinct product varieties represented by $\omega \in [0,1]$. The firms utilize a technology with constant returns to scale that involves labor, land, and energy. The output produced per unit of land for a particular for a particular product variety, ω, is a function of the production workers, $L_t^w(r)$, and the energy used, $e_t^{\omega}(r)$, both per unit of land. It is important to note that since land is fixed factor with a share of $1-\mu$, agglomerating labor and energy in a given location will lead to decreasing returns, which acts as a third congestion force. The output per unit of land of variety ω is then given by

$$q_t^{\omega}(r) = \phi_t^{\omega}(r)^{\gamma_1} z_t^{\omega}(r)(L_t^{\omega}(r)^{\chi} e_t^{\omega}(r)^{1-\chi})^{\mu}, \tag{5}$$

The productivity of a firm is determined by two factors: its innovation decision, $\phi_t^\omega(r) \geq 1$, and an idiosyncratic location-variety productivity shifter, $z_t^\omega(r)$. Investing in innovation incurs a cost of $v\phi_t^\omega(r)^\xi$ per unit of land, expressed in units of labor. The exogeneous productivity shifter is a random variable that follows a Fréchet distribution, with a cumulative distribution function $F(z, a) = e^{-a_t(r)z^{-\theta}}$, and is independently and identically distributed across varieties and time. The scale parameter at(r) determines the level of productivity in a location and is influenced by agglomeration externalities arising from high population density and past innovations. Specifically, we set $a_t(r) = \bar{a}_t(r)L_t(r)^\alpha$, where α determines the strength of the first agglomeration force. In turn, the endogenous dynamic process that determines the fundamental productivity, $\bar{a}_t(r)$, is given by the following equation:

$$\bar{a}_t(r) = (1+\Lambda^\alpha(\Delta T_t(r), T_{t-1}(r))) \left(\phi_{t-1}(r)^{\theta\gamma_1} [\int_S D(v, r)\bar{a}_{t-1}(v)dv]^{1-\gamma_2} \bar{a}_{t-1}(r)^{\gamma_2} \right), \tag{6}$$

Equation (6) comprises four distinct elements. Firstly, the term $\phi_{t-1}(r)^{\theta\gamma_1}$ stands for the shift in the local distribution of shocks due to previous innovation decisions of firms, which are now assumed to be integrated into the local technology. The second element, $\left[\int_S D(v, r)\bar{a}_{t-1}(v)dv\right]^{1-\gamma_2} \bar{a}_{t-1}(r)^{\gamma_2}$, captures the impact of past technology on the current production function, including both the location's own technology level $\bar{a}_{t-1}(r)$ and the diffusion of technology from other locations. This component is based on Desmet et al. (2018) and generates a spatial endogenous growth model. The third component, $\Lambda^a(\cdot)$, reflects the effect of temperature on local productivity in cell r at time t. Finally, since $\Lambda^a(\cdot)$ depends on temperature levels, it can account for the diverse spatial impacts of global warming on productivity.

In contrast to Desmet et al. (2018), this model incorporates energy as a factor of production, in addition to land and labor. Golosov et al. (2014), Hassler et al. (2019), and Popp (2006), among others, propose that energy and other factors should be combined using a Cobb-Douglas production function, where $(1-\chi)\mu$ represents the share of energy in the production process. Furthermore, energy is a CES composite of clean soruces, $e_t^{c,\omega}(r)$, and fossil fuels, $e_t^{f,\omega}(r)$, with the elasticity of substitution being ϵ. The use of fossil fuels leads to CO_2 emissions, which contribute to the greenhouse effect by accumulating in the atmosphere. However, the use of clean energy does not lead to these negative externalities. Specifically, the model assumes the following:

$$e_t^\omega(r) = \left(\kappa e_t^{f,\omega}(r)^{\frac{\epsilon-1}{\epsilon}} + (1-\kappa)e_t^{c,\omega}(r)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{7}$$

The relative productivity of both technologies in producing energy is determined by κ. It is necessary to make the assumption of competitive local energy markets where the price of each energy type is equivalent to its marginal production cost. To produce one unit of energy of type j, where j is either f for fossil fuels or c for clean sources, $Q_t^j(r)$ units of labor is required. The cost of energy differs depending on the source, location, and time, a following the next equation:

$$Q_t^f(r)O = \frac{f(CumCO2_{t-1})}{\zeta_t^f(r)} and Q_t^c(r) = \frac{1}{\zeta_t^c(r)}. \tag{8}$$

The cost of fossil fuel extraction, $Q_t^f(r)$, is determined by two factors. The first factor, in the numerator, is the cost of extracting fossil fuels from the ground, which is assumed to convex and increasing in the total cumulative CO_2 emissions in the world, as suggested by Nordhaus and Boyer (2002). This is denoted by the term $CumCO2_{t-1}$. As the world's cumulative emissions increase, carbon reserves shrink, which in turn increases the cost of fossil fuel extraction. Cumulative emissions are calculated as the sum of cumulative emissions in the previous period and the global CO_2 emissions released in the current period, denoted by E_t^f. Namely, we have the equation:

$$CumCO2_t = CumCO2_{t-1} + E_t^f = CumCO2_{t-1} + \int_S \int_0^1 e_t^{f,\omega}(v)H(v)d\omega dv. \tag{9}$$

It is assumed that the pace at which technology advances over time in the fossil fuel and clean energy sectors are linked to global real GDP, y_t^w, which is an endogeneous variable in this model and depends on firms' investment decisions. Specifically, it is assumed that a one percent increase in global real GDP leads to a log-productivity increase in energy generation of type j, denoted by $\zeta_t^j(r)$, by v^j. This elasticity is allowed to vary across different types of energy. Therefore, the denominator of the energy price is related to the energy generation productivity, $\zeta_t^j(r)$. Therefore, the magnitude of the externality on energy productivity investments generated by firms' innovations depends on the evolution of real GDP. This is shown by the following equation:

$$\zeta_t^j(r) = \left(\frac{y_t^w}{y_{t-1}^w}\right)^{v^j} \zeta_{t-1}^j(r), where y_t^w = \int_S \left(\frac{L_t(v)H(v)}{L_t}\right) y_t(v)dv. \tag{10}$$

In this model it is assumed that land markets are competitive, where firms compete to secure the right to produce in a parcel of land through a bidding process. This is important because past innovations, which are embedded in the local idiosyncratic distributions of productivities, benefit all potential entrants. This implies that the optimal solution for firms' dynamic innovation problem is to choose the level of innovation that maximizes their current profits (or their bid for land) as all future gains from current innovation will accrue to the fixed factor, which is land. Since future firms' profits are zero, they do not affect a firm's decisions. As there is a continuum of potential entrants, firms bid all of their profits after covering innovation costs, resulting in zero profits for firms. Therefore, in this economy, the maximum bid for land is the local land price, $R_t(r)$, every period. This is proven in Desmet and Rossi-Hansberg (2014). In summary, firms in r maximize according to:

$$\max_{q,L,\phi,e^f,e^c} p_t^\omega(r,r)q_t^\omega(r) - w_t(r)L_t^\omega(r) - w_t(r)v\phi_t^\omega(r)^\xi - w_t(r)Q_t^f(r)e_t^{f,\omega}(r)$$

$$-w_t(r)Q_t^c(r)e_t^{c,\omega}(r) - R_t(r) \tag{11}$$

We can obtain the total energy cost in labor units by using the first-order conditions with respect to fossil fuel and clean energy. This expression can be rewritten as the energy composite, $e_t^\omega(r)$, times its ideal price index, $Q_t^f(r)$. Specifically, $Q_t(r)e_t^\omega(r) = Q_t^f(r)e_t^{f,\omega}(r) + Q_t^c(r)e_t^{c,\omega}(r)$, where $Q_t(r)$ is defined as $Q_t(r) = \left(\kappa^\epsilon Q_t^f(r)^{1-\epsilon} + (1-\kappa)^\epsilon Q_c^f(r)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$. Because the technology is Cobb-Douglas, the firm's energy costs are proportional to labor costs. Thus, we can express $Q_t(r)e_t^\omega(r) = \frac{1-\chi}{\chi}L_t^\omega(r)$. This simplifies the firm's problem to a form like the one presented in Desmet et al. (2018), and thus all their finding are applicable.

4.3 Prices, Export Shares, and Trade Balance

The market for goods is characterized by competition, and therefore firms sell their products at the marginal cost, which includes transportation costs. The trade cost of shipping a good from location r to s is represented by the iceberg cost function, denoted by $\varsigma(s,r) \geq 1$. Thus, the price of the good at location s and time t, $p_t^\omega(s,r)$, is equal to the product of the iceberg cost, marginal input cost at location r, and the inverse of the productivity level of energy source at location r, i.e., $p_t^\omega(s,r) = \varsigma(s,r)mc_t(r)/z_t^\omega(r)$. Since all firms face the same input prices, the marginal input cost is identical across firms, and is given by $mc_t(r) = MQ_t(r)^{1-\chi)\mu w_t(r)^{1-\mu+\gamma_1/\xi}R_t(r)^{1-\mu-\gamma_1/\xi}}$, where M is a constant of proportionality that is determined by the production parameters.

Following the standard trade structures based on Eaton and Kortum (2002), we can express the probability, denoted as $\pi_t(s,r)$, that a good produced in location r is consumed in location s as a gravity equation, which can be represented as:

$$\pi_t(s,r) = \frac{a_t(r)\left[mc_t(r)\varsigma(r,s)\right]^{-\theta}}{\int_S a_t(v)\left[mc_t(v)\varsigma(v,s)^{-\theta}dv\right]}. \tag{12}$$

The price index of a location, denoted by $P_t(r)$, is given by the Gamma function as shown in equation (13):

$$P_t(r) = \Gamma\left(\frac{-\rho}{(1-\rho)\theta+1}\right)^{-\frac{1-\rho}{\rho}}\left[\int_S a_t(v)\left[mc_t(v)\varsigma(r,v)\right]^{-\theta}dv\right]^{-1/\theta} \tag{13}$$

To ensure the trade balance in each cell over the long run, they impose that total income, which is the sum of labor income and land rents, at location r equals the total expenditure on goods from r, which is given by:

$$w_t(r)L_t(r)H(r) = \int_S \pi_t(v,r)w_t(v)L_t(v)H(v)dv. \tag{14}$$

4.4 Climate and the Carbon Cycle

The combustion of fossil fuels and other activities, such as deforestation, results in the emissions of carbon dioxide into the atmosphere. The carbon cycle outlines how carbon accumulates in the atmosphere. The IPCC (2013) proposes the dynamics that dictate the evolution of atmospheric CO_2, where the atmospheric carbon stock, St, changes over time according to the following equation:

$$S_{t+1} = S_{pre-ind} + \sum_{l=1}^{\infty}(1-\delta_\ell)\left(E^f_{t+1-\ell} + E^x_{t+1-\ell}\right). \tag{15}$$

As shown in equation (9), E^f_t represents the CO_2 emissions from fossil fuel combustion that arise endogenously. Additionally, E^x_t accounts for exogeneous CO_2 emissions from non-fuel combustion based on the IPCC scenario RCP 8.5 or 6.0. The parameter $S_{pre-ind}$ indicates the CO_2 stock in the pre-industrial era (year 1800), while $(1-\delta_\ell)$ represents the fraction of CO_2 emissions that remain the atmosphere after l period. Greater concentrations of carbon dioxide increase the global radiative forcing, F_{t+1}, which measures the net inflow of energy and is approximated using the method of Myhre et al. (1998):

$$F_{t+1} = \varphi log_2(S_{t+1}/S_{pre-ind}) + F^x_{t+1}, \tag{16}$$

The parameter φ in the equation above represents the forcing sensitivity, which is the increase in the radiative force when the carbon stock doubles compared to its pre-industrial level. The radiative forcing from non-CO2 greenhouse gases, such as methane and nitrous oxide, is denoted as F^x_t and is obtained from the RCP 8.5 or 6.0 scenarios. The global temperature rises when the inflow of energy from the sun exceeds the outflow of energy exiting the planet, according to a process defined by the following equation:

$$T_{t+1} = T_{pre-ind} + \sum_{\ell=0}^{\infty} \zeta_\ell F_{t+1-l}, \tag{17}$$

The worldwide temperature over land in the pre-industrial era is denoted by $T_{pre-ind}$ in the equation above, while ζ_ℓ represents the current temperature response to radiative force increase that occurred ℓ periods ago. Since carbon emissions affect global temperature and not local temperatures directly, they constitute a global externality. Nevertheless, to quantify the model at a fine geographical resolution, it is necessary to assume a relationship between global and local temperatures. Following Mitchell (2003), a linear down-scaling relationship is adopted, which provides accurate results. Specifically, we get the following relationship:

$$T_t(r) - T_{t-1}(r) = g(r) \cdot (T_t - T_{t-1}), \tag{18}$$

The temperature in cell r changes by $g(r)$ ℃ when global temperature changes by one ℃, where $g(r)$ is a coefficient that depends on the local physical characteristics of the location. To quantify the model at a fine geographical resolution, it is assumed that $g(r)$ remains fixed over time.

4.5 Competitive Equilibrium and Balanced Growth Path

A dynamic competitive equilibrium can be defined by the conditions presented in previous sections. It makes it possible to simplify the system of equations that defines a spatial equilibrium in a given period to a system of equations for population and wages in each location. Using these equations, it is possible to directly compute all other variables, including firm investments. A unique solution to the system of equations exists if two conditions are met. First, either the elasticity of substitution between fossil fuels and clean energy is one (Cobb-Douglas) or the innovation elasticity with respect to global real income growth is the same across energy types ($\epsilon = 1$or $v^f = v^c$). These assumptions maintain the log-linear structure of the model. Second, the static agglomeration economies associated with local production externalities (α/θ) and the degree of returns to innovation (γ_1/ξ) must not dominate the three congestion forces. These congestion forces are governed by the value of the negative elasticity of amenities to density adjusted by the elasticity of utility to real income (λ/σ), the share of land in production which determines the degree of local decreasing returns ($1-\mu$), and the variance of taste shocks adjusted by the elasticity of utility to real income (Ω/σ). The second condition generalizes the condition in Desmet et al. (2018).

In the model, a spatial equilibrium in a given period determines firm innovation, energy use, and carbon emissions. The resulting temperatures and next period's amenities and productivities are then computed using equations (2), (6), and the climate and carbon cycle model. This allows to compute the dynamic equilibrium forward, period by period, for as many years as needed. Eventually, the distribution of population across space and the world real output growth rate converge to a Balanced Growth Path (BGP) if certain conditions are met. These conditions include: (i) total natality rates $(1 + n_t(r))$ converge to one as income per capita grows; (ii) the stock of carbon is finite and certain variables, such as $(1 - \delta_\ell)$, ζ_ℓ, E^x_ℓ and F^x_ℓ converge to constant values, eventually stabilizing temperatures; (iii) ($\epsilon = 1$or $v^f = v^c$); and (iv) $\alpha/\theta + \gamma_1/\xi + \gamma_1/\xi(1 - \gamma_2)) \leq \lambda/\sigma + (1 - \mu) + \Omega/\sigma$. This states that the agglomeration forces, including dynamic agglomeration forces through innovation, $\gamma_1/\xi(1-\gamma_2)$, are weaker than the three congestion forces. Although the dynamics of the model are protracted, convergence to a Balanced Growth Path is not fully achieved for the two-century horizon considered in analysis.

4.6 Forward Solution

In this section we reverse-engineer the model solution by tracing back in time the system of equations that determine the levels of essential resources, productivity, and migration expenses. The subsection Forward Solution focuses on computing the forward solution of the model when ad-valorem carbon taxes, $\tau_t(r)$, and clean energy subsidies, $s_t(r)$, are present. These taxes are levied on the firm and are uniformly rebated to the households living in region r through a lump-sum transfer, $\Phi_t(r)$. The firm's optimization problem, which involves minimizing costs with respect to fossil fuels and clean energy, can be expressed as follows:

$$w_t(r)Q_t(r)e^w_t(r) = \underset{e^f,e^c}{min} w_t(r)(1+\tau_t(r))Q^f_t(r) + w_t(r)(1-s_t(r))Q^c_t(r)e^{c,w}_t(r) \tag{19}$$

$$st \left(\kappa e^{f,w}_t(r)^{\frac{\epsilon-1}{\epsilon}} + (1-\kappa)e^{c,w}_t(r)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} = e^w_t(r)$$

If we take the first order condition with respect to $e^{f,w}_t(r)$and $e^{c,w}_t(r)$ yield the following relationships:

$$\frac{e^{c,w}_t(r)}{e^{f,w}_t(r)} = \left(\frac{1-\kappa}{\kappa}\frac{1+\tau_t(r)}{1-s_t(r)}\frac{Q^f_t(r)}{Q^c_t(r)}\right)^{\epsilon} \tag{20}$$

$$Q_t(r) = \left(\kappa^{\epsilon}(1+\tau_t(r)^{1-\epsilon}Q^f_t(r)^{1-\epsilon} + (1-\kappa)^{\epsilon}(1-s_t(r))^{1-\epsilon}Q^c_t(r)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \quad (21)$$

The lump-sum transfer per unit of land can be defined following the derivation below:

$$\Phi^{\omega}_t(r) = \left(\tau_t(r)Q^f_t(r)e^{f,\omega}_t(r) - s_t(r)Q^c_t(r)e^{c,\omega}_t(r)\right) \quad (22)$$

$$= Q_t(r)e^{\omega}_t(r) - \left(Q^f_t(r)e^{f,\omega}_t(r) + Q^c_t(r)e^{f,\omega}_t(r)\right)$$

$$= Q_t(r)e^{\omega}_t(r) - \tilde{Q}_t(r)^{1-\epsilon}Q_t(r)^{\epsilon}e^{\omega}_t(r),$$

$$\tilde{Q}_t(r) = \left(\kappa^{\epsilon}(1+\tau_t(r))^{-\epsilon}Q^f_t(r)^{1-\epsilon} + (1-\kappa)^{\epsilon}(1-s_t(r))^{-\epsilon}Q^c_t(r)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \quad (23)$$

By using equation (21), it is possible to reduce the firm's problem to the following maximization problem

$$\max_{q,L,\phi,e} p^{\omega}_t(r,r)\phi^{\omega}_t(r)^{\gamma_1}z^{\omega}_t(r)\left(L^{\omega}_t(r)^{\chi}e^{\omega}_t(r)^{1-\chi}\right)^{\mu}$$

$$-w_t(r)\left[L^{\omega}_t(r) + v\phi^{\omega}_t(r)^{\xi} + Q_t(r)e^{\omega}_t(r)\right] - R_t(r), \quad (24)$$

By taking the first order conditions of this problem with respect to $e^{\omega}_t(r)$ and $L^{\omega}_t(r)$, it is possible to derive the following equation:

$$Q_t(r)e^{\omega}_t(r) = \left(\frac{1-\chi}{\chi}\right)L^{\omega}_t(r). \quad (25)$$

The previous equations makes it possible to collapse the maximization problem of the firm to the following problem:

$$\max_{L,\phi} p^{\omega}_t(r,r)\left(\frac{1-\chi}{\chi}\frac{1}{Q_t(r)}\right)^{(1-\chi)\mu}\phi^{\omega}_t(r)^{\gamma_1}z^{\omega}_t(r)L^{\omega}_t(r)^{\mu} - \frac{w_t(r)L^{\omega}_t(r)}{\chi}$$

$$-w_t(r)v\phi^{\omega}_t(r)^{\xi} - R_t(r), \quad (26)$$

By taking the first order conditions of the maximization problem above with respect to $\phi^{\omega}_t(r)$ and $L^{\omega}_t(r)$, we get:

$$w_t(r)L^{\omega}_t(r) = \mu\chi(p^{\omega}_t(r,r)q^{\omega}_t(r)), \quad (27)$$

$$\chi\mu v\phi^{\omega}_t(r)^{\xi} = (\gamma_1/\xi)L^{\omega}_t(r). \quad (28)$$

Fossil fuel use is calculated by using the following equation, where $e^{f,w}_t(r)$

denotes fossil fuel use:

$$e_t^{f,w}(r) = \left(\frac{(1-\chi)\mu}{\mu+\gamma_1/\xi}\right)\left(\frac{\bar{L}_t^w(r)}{\varphi_t(r)Q_t(r)}\right)\left(\frac{\kappa Q_t(r)}{(1+\tau_t(r))Q_t^f(r)}\right)^{\epsilon} \tag{29}$$

Clean energy use:

$$e_t^{c,w}(r) = \left(\frac{(1-\chi)\mu}{\mu+\gamma_1/\xi}\right)\left(\frac{\bar{L}_t^w(r)}{\varphi_t(r)Q_t(r)}\right)\left(\frac{(1-\kappa)Q_t(r)}{(1+s_t(r))Q_t^c(r)}\right)^{\epsilon} \tag{30}$$

The forward simulations uses equation (31) in order to retrieve $u_t(\cdot)$ from equation (32):

$$F_t^L(r)\hat{u}_t(r)^{\frac{1}{\sigma}\left(\frac{\flat_L}{\Omega/\sigma}+\frac{\theta(1+\theta)}{1+2\theta}\right)} = \kappa\int_S F_t^R(v)\hat{u}_t(v)^{\frac{1}{\sigma}\left(\frac{\flat_R}{\Omega/\sigma}-\frac{\theta^2}{1+2\theta}\right)}\varsigma(r,v)^{-\theta}dv, \tag{31}$$

$$U_t = \left(\frac{L_t}{\int_S[u_t(v)/m_2(v)]^{1/\Omega}dv}\right)^{\frac{1}{\frac{1}{\Omega/\sigma}-\frac{\theta}{\flat_R-\flat_L}}} = \left(\frac{L_t}{\int_S[\hat{u}_t(v)/m_2(v)]^{1/\Omega}dv}\right)^{-\frac{\flat_R-\flat_L}{\theta}}; \tag{32}$$

The law of motion is rewritten in order to simplify the evolution of carbon stock. This gives the following equations, that are being used to compute S_{t+1}:

$$S_{t+1} = S_{0,t+1} + \sum_{i=1}^{3} S_{i,t+1}, with S_{0,t+1} = S_{0,t} + a_0(E_t^f + E_t^x), \tag{33}$$

$$S_{i,t+1} = (e^{-1/b_i})S_{i,t} + a_i(E_t^f + E_t^x), i \in \{1,2,3\}. \tag{34}$$

The global temperature module can be rewritten analogously to the carbon circulation, given by the next equation. The values of T_{t+1} is calculated by:

$$T_{t+1} = T_{1,t+1} + T_{2,t+1}, with T_{j,t+1} = (e^{-1/d_j})T_{j,t} + \frac{c_j}{d_j}F_{t+1}, j \in \{1,2\}. \tag{35}$$

The forward_climate function continues to compute the damage functions, $\Lambda^a(\cdot)$ and $\Lambda^b(\cdot)$, by setting the damage function level by confidence interval and setting the sources of the damage functions. Next, the damages on amenities, $\bar{a}_{t+1}(\cdot)$,and on productivities, $\bar{b}_{t+1}$, are being updated. These are computed from equations (2) and (6) described ealier on. The function finishes off by computing and updating the global population, L_{t+1}.

5 Analysis

Our research is driven by the urgent need to address climate change and prevent global temperatures from surpassing a critical threshold. The Paris Agreement sets the objective of limiting global warming to within 2 degrees Celsius above pre-industrial levels. This temperature goal is crucial in mitigating the severe consequences of climate change, including irreversible environmental damage, extreme weather events, and disruptions to ecosystems. (United Nations Climate Change, 2023).

In this study, we aim to investigate the implications of the 2-degree Celsius temperature goal and analyze the strategies required to achieve it. By utilizing a Python-based model built upon the framework proposed by Cruz & Rossi-Hansberg (2022), we will examine various parameters and their impacts on key variables such as temperature, CO2 emissions, clean energy utilization, the evolution of GDP, and population density.

Through our analysis, we seek to deepen the understanding of the time required to achieve the 2-degree Celsius goal and the specific parameters that influence its attainment. By identifying the factors that contribute to climate overshoot and the pathways that allow us to remain below the critical threshold, we can contribute to the formulation of effective and sustainable strategies for climate change mitigation and adaptation.

The findings of this study will have significant implications for policymakers, researchers, and stakeholders involved in climate change mitigation efforts. By integrating these insights into decision-making processes, businesses and organizations can play a vital role in driving the transition to a sustainable and resilient future. Our research aligns with global efforts to combat climate change and fosters the hope of creating a better world for present and future generations.

5.1 The Baseline Scenario

In this section, our analysis begins by adopting the baseline scenario proposed by Cruz & Rossi-Hansberg (2022) as our starting point. They set the following parameters to specific values in their baseline scenario: $\varepsilon = 1.6$, maxCO2 = 19,500, and RCP = 8.5. The RCP 8.5 scenario represents a future trajectory characterized by high challenges for mitigation, with a reliance on fossil fuel-intensive practices, and low adaptation measures due to rapid development. (Van Vuuren, D.P, 2011) By utilizing this parametrization, we provide a comprehensive framework to examine the long-term effects on the global economy over a span of 200 years, from the year 2000 to 2200. Our primary focus is to investigate key factors such as temperature trends at both global and local scales, emissions patterns, the utilization of clean energy sources, and population density. Using the baseline scenario as a reference, we can compare and contrast the outcomes with alternative scenarios where we manipulate various parameters to ensure adherence to the temperature target established by the

United Nations. Through this analysis, we aim to gain valuable insights into the potential pathways and policies required to mitigate the risks associated with climate change while promoting sustainable development.

5.1.1 Global CO2 emissions

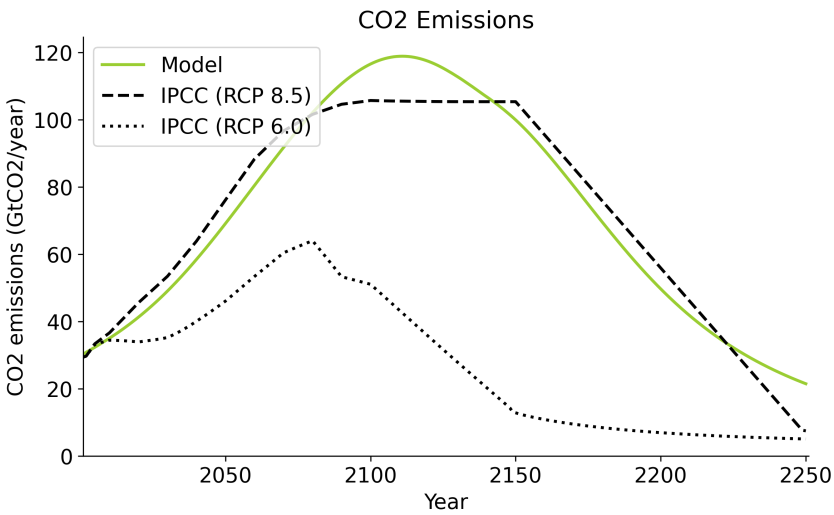

Figure 1: CO2 Emissions - Baseline scenario

Figure 1 illustrates CO2 emissions from year 2000 to 2250 and provides a visual representation of the projected CO2 emission path, comparing it with the pessimistic scenarios, RCP 8.5 and 6.0, outlined in the IPCC (2013) report. Our model indicates that carbon dioxide emissions from fossil fuel combustion are expected to grow during the current century, which can result from economic growth and advancements in fossil fuel technology. However, emissions are anticipated to decline towards zero after reaching a peak in 2110, as extraction costs increase sharply with the depletion of fossil fuel resources. The bell-shaped emission trajectory observed in the model aligns with the exogenous abatement process outlined by the IPCC (2013), which ultimately leads to declining emission projections.

5.1.2 Global Temperature

The rise in the concentration of greenhouse gases increases global temperatures, as depicted in Figure 2, which represents the global temperature relative to the pre-industrial level. To highlight the temperature goal set by the Paris agreement, a red line has been added to the figure at the 2-degree target. According to our model, the projected trajectory indicates that the global temperature is expected to reach the 2-degree line by the year 2039. This alignment with the central objective of the agreement underscores the significance of adhering to the temperature limit to mitigate the potentially severe consequences

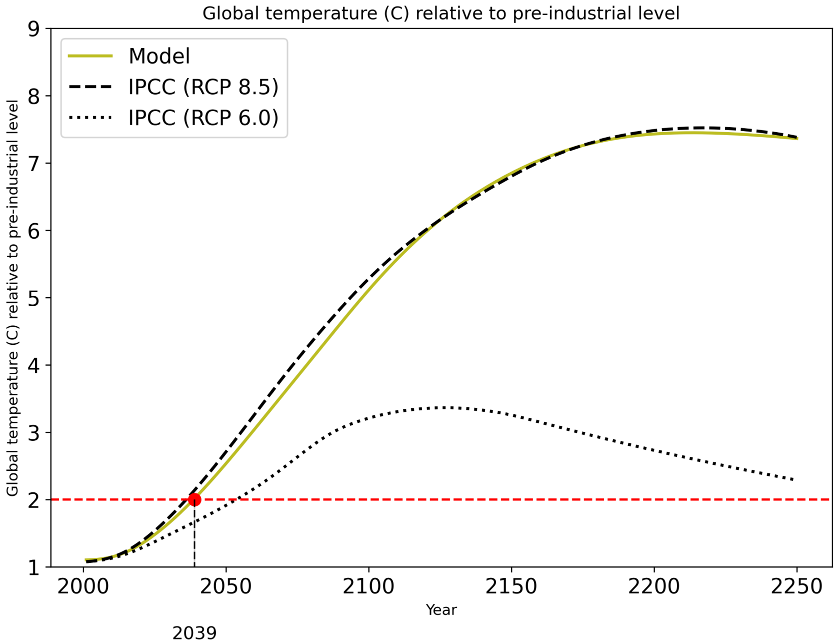

Figure 2: Global Temperature - Baseline scenario

associated with surpassing the 2-degree Celsius threshold.

Looking further into the future, by the end of this century, the global temperature is estimated to rise by approximately $5°C$ compared to the pre-industrial level, according to our model. By the year 2200, the projected increase in global temperatures reaches roughly $7°C$. As the consumption of carbon dioxide declines towards zero, the global temperature gradually approaches a long-run equilibrium between $6°C$ and $7°C$ above the pre-industrial level. It is worth noting that our model's parametrization of the carbon cycle and its close alignment with the emission trajectory depicted in the RCP 8.5 scenario results in an almost exact match between our temperature evolution and that of the RCP 8.5 scenario.

The projected rise in global temperatures, as depicted by our model, has significant consequences across various dimensions. This alarming trajectory has implications such as an increased risk of extreme weather events, disruption of ecosystems and biodiversity, sea-level rise and coastal flooding, impacts on human health, and substantial economic implications. These consequences highlight the urgency of taking effective measures to reduce greenhouse gas emissions and address climate change to avoid the potentially devastating effects on the environment, societies, and the global economy.

5.1.3 Log CO2 emissions

Two color-coded maps presented in Figure 3 provide insightful visual representations of the relative changes in CO2 emissions over time. The first map depicts the relative change between the years 2000 and 2110, as we were particularly interested in observing the changes in 2110 since the emissions are reaching a peak. Additionally, the second map represents the relative change between the years 2000 and 2200, providing insights into the extended time period and its implications for CO2 emissions and climate change. These maps offer valuable insights into the spatial distribution of CO2 emissions based on the simulated dataset generated by our model. The color scale utilized in the maps ranges from red, indicating regions with high emissions, to blue, representing areas with low emissions. Notably, the analysis takes into account emissions per land area, measured in GtCO2 per land, allowing for a comprehensive understanding of distribution patterns. Examining these maps provides a clearer understanding of the geographical disparities in CO2 emissions and their potential implications for climate change mitigation and adaptation strategies.

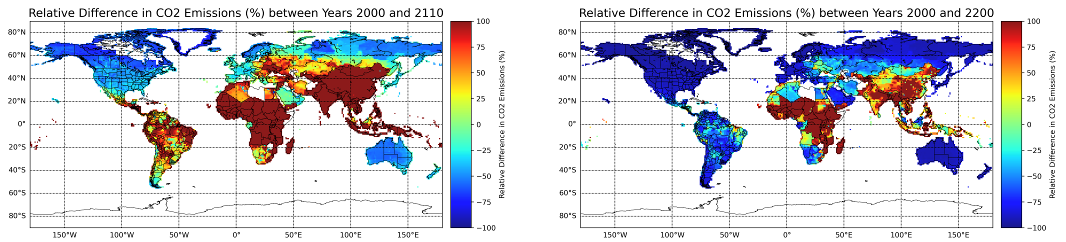

Figure 3: Relative change: CO2 emissions

Analyzing the map representing the relative change between 2000 and 2110, we observe that certain regions in South America, Africa, and Asia are depicted in red, indicating increased CO2 emissions compared to the year 2000. Conversely, North America, Europe, and Oceania are displayed in blue, signifying a reduction in emissions. Several factors may contribute to this pattern. Firstly, regions that have implemented and enforced strict environmental regulations and policies, along with investments in renewable energy sources and energy-efficient technologies, may have successfully curbed their CO2 emissions. Additionally, favorable geographical characteristics, such as the abundance of renewable energy resources, may have facilitated the transition to cleaner energy sources in these regions. However, a detailed analysis is required to validate these explanations and understand the specific dynamics influencing emission reductions in these areas.

Turning our attention to the map representing the change between 2000 and 2200, we observe an overall positive change characterized by a predominantly blue color scheme, indicating a decrease in CO2 emissions. However, certain regions, such as parts of Central Africa and East Asia, still exhibit higher emissions. Several factors could contribute to this discrepancy. In Central Africa, challenges related to economic development, limited access to clean energy technologies, and dependence on carbon-intensive industries may impede significant emission reductions. (Gujba, H., et al, 2012) In East Asia, rapid industrialization and a growing population might result in increased energy demand and associated CO2 emissions. Addressing these challenges would require a combination of factors, including the implementation of sustainable development policies, increased access to clean energy technologies, and supportive international collaborations. (Liu, X., & Bae, J. (2018)

It is important to note that these are potential explanations based on the observed patterns and require further research and analysis to establish causal relationships and fully understand the dynamics influencing CO2 emissions in these regions. Nonetheless, these visual representations highlight the spatial variations in emissions and emphasize the importance of targeted strategies to mitigate emissions and promote sustainable practices on a global scale.

5.1.4 Log clean energy

The two maps presented in Figure 4 provide a visual representation of global clean energy adoption in the years 2000 and 2200. The color scale ranges from red to blue, indicating high and low usage of clean energy, respectively. In the year 2000, regions such as Europe, parts of America, and East Asia demonstrate significant clean energy adoption, represented by red areas. However, Algeria shows lower utilization of clean energy, depicted by predominantly blue areas. This discrepancy may be attributed to factors such as limited access to renewable resources or a greater reliance on traditional energy sources.

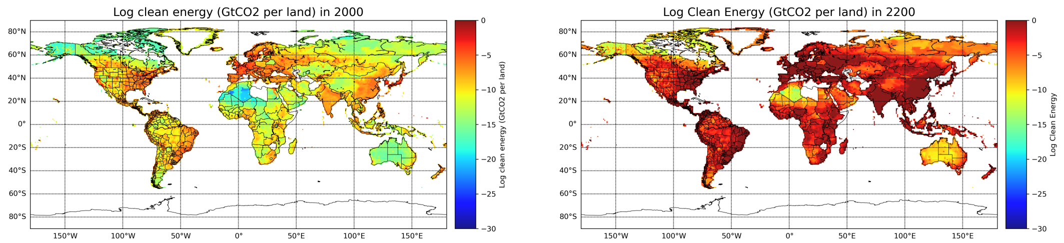

Figure 4: Comparison of Log clean energy

Comparing the map for the year 2200 to that of 2000, a notable global shift towards increased clean energy adoption is evident. The majority of the world is depicted in red, indicating higher utilization of clean energy sources. This shift can be attributed to advancements in renewable energy technologies, supportive policy frameworks, and a growing recognition of the importance of sustainable energy practices. These findings highlight the significance of ongoing efforts to promote and expedite the global transition to clean energy sources, ensuring a sustainable and low-carbon future.

5.1.5 January-July temperature

Understanding and monitoring temperature patterns is crucial for assessing the impact of climate change. Figure 5 aims to visually analyze and compare the relative change at local levels between Janyary and July around the world between the years 2000 and 2200 using a color-coded map. The map represents the relative change in temperatures. Dark red indicates a positive relative change, suggesting an increase in temperatures. Green indicates a near-zero relative change, while dark blue represents a negative relative change, indicating a decrease in temperatures.

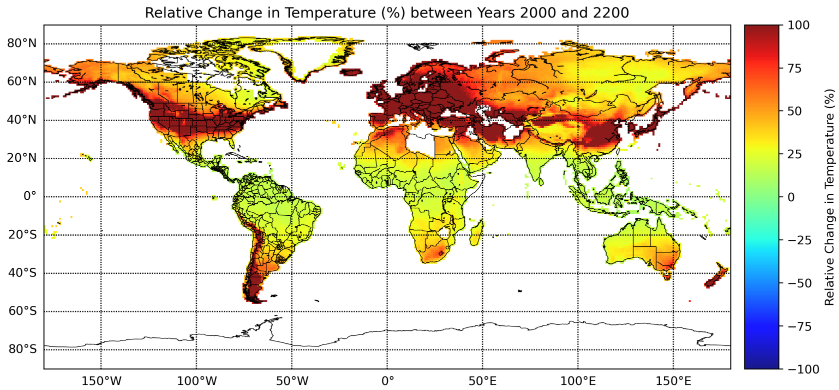

Figure 5: January-July Temperature - Relative change

Several notable observations can be made from the map. Firstly, regions such as Europe, the Middle East, the United States, parts of China, and the west coast of South America are prominently displayed in dark red hues. This indicates a substantial relative change in temperatures, nearing or exceeding 100%. These regions are expected to experience a significant increase in temperatures compared to the baseline year of 2000.

Conversely, a large portion of Africa and the rest of South America appears in shades of yellow, representing a positive relative change in temperatures of approximately 25%. While not as pronounced as the regions mentioned above, these areas are still expected to undergo a noticeable increase in temperatures during the January-July period.

In contrast, regions such as Russia, Australia, and Canada exhibit a combination of yellow and orange shades on the map. This suggests a positive relative change in temperatures ranging from 25% to 50%. These areas are projected to experience a moderate increase in temperatures compared to the year 2000.

The observed variations in temperature increases across different regions can be attributed to a multitude of factors. Geographical location, local climate dynamics, and regional differences in climate change impacts all contribute to the diverse distribution of temperature changes seen on the map.

The maps depicting the relative change of CO2 emissions (Figure 3) and the relative change in temperatures (Figure 5) reveal intriguing and seemingly contradictory results. Figure 3 illustrates that the regions with the highest relative change in CO2 emissions are primarily observed in Africa, Asia, and South America, suggesting an increase in greenhouse gas emissions in these areas over the simulated 200-year period. Conversely, Figure 5 displays the relative change in temperatures, indicating that the regions experiencing the most significant temperature variations are predominantly located in Europe, the Middle East, the United States, parts of China, and the west coast of South America.

This apparent contradiction can be attributed to the complex interplay of numerous factors influencing climate change. The relative change in CO2 emissions can reflect alterations in the release of greenhouse gases into the atmosphere, influenced by factors such as population growth, economic development, energy consumption patterns, and policy interventions. Therefore, while regions with the highest relative change in CO2 emissions may experience increased greenhouse gas emissions, the resulting temperature changes may be influenced by other factors, such as regional climatic characteristics and interactions between various components of the climate system. Thus, the observed disparity between the maps underscores the complexity of climate change and the need for a comprehensive understanding of the underlying mechanisms driving both CO2 emissions and temperature variations.

5.1.6 Population density

Understanding population density patterns is essential for studying human geography and exploring the interplay between population distribution, environmental factors, and socio-economic dynamics. This subsection aims to analyze the observed changes in population density between the year 2000 and year 2200 as visualized in Figure 6. The maps employed a color scale, where warmer colors represented higher population density and colder colors indicated lower density. The change in population density was derived by comparing the two datasets.

In the initial year of 2000, the global population density map depicted a predominantly green color, indicating no changes in population distribution. Figure 6 provides insights into the projected changes in population density over a 200-year period. Notably, as we move away from the equator, the map exhibits a progressively warmer color scheme, suggesting an increase in population density. Several factors could contribute to this observed pattern. Firstly, regions located farther from the equator often offer more favorable environmental conditions, such as milder climates and abundant natural resources,

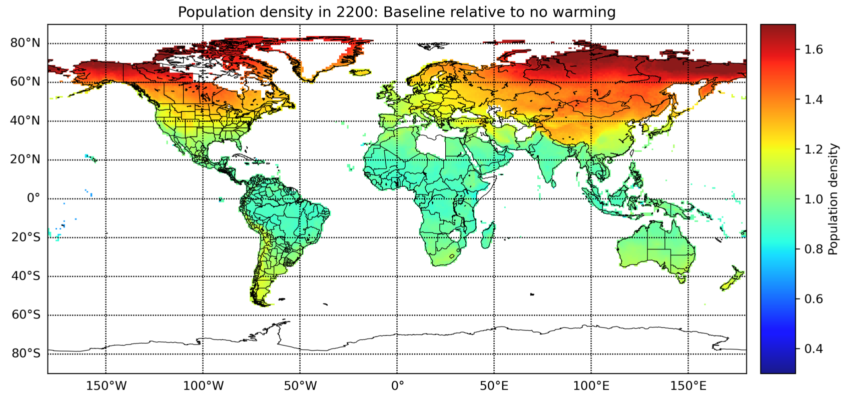

Figure 6: Population Density - Baseline Scenario

making them attractive for human settlement and economic activities, when the temperature increases. Additionally, the availability of land and the potential for economic development might encourage migration and population concentration in these areas. Moreover, factors such as access to amenities, and employment opportunities may also influence population distribution patterns. Nevertheless, further research and analysis are necessary to comprehensively understand the underlying dynamics driving the observed increase in population density away from the equator.

5.2 Adjusting the elasticity of substitution between fossil fuel and clean energy sources

In this subsection, we focus on examining the elasticity of substitution (ε) between fossil fuels and clean energy sources in the context of the baseline model. By adjusting the value of ε, specifically increasing it to 2.6 and decreasing it to 0.6, we aim to assess the potential changes and impacts on the baseline model's dynamics.

5.2.1 Global CO2 emissions

Figure 7 provides a visual representation of the relationship between CO2 emissions and different values of epsilon (ε). The green line corresponds to the baseline value of epsilon, set at 1.6, while the red line represents a decreased epsilon of 0.6, and the blue line represents an increased epsilon of 2.6. The plot clearly demonstrates that CO2 emissions are influenced by the value of epsilon. When epsilon is decreased to 0.6, as depicted by the red line, CO2 emissions increase. This can be attributed to the lower elasticity of substitution between fossil fuels and clean energy sources associated with a lower epsilon value. With a limited ability to substitute fossil fuels with cleaner alternatives, the baseline model exhibits higher CO2 emissions. Conversely, when epsilon is in-

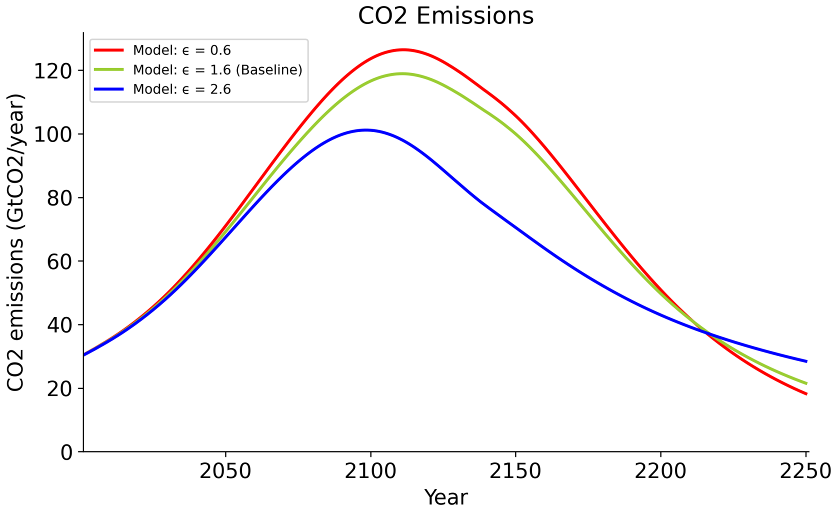

Figure 7: CO2 Emissions - Different values for epsilon

creased to 2.6, as shown by the blue line, CO2 emissions decrease. This is due to the higher elasticity of substitution, indicating a greater ease in transitioning to cleaner energy technologies. The increased substitution of fossil fuels with cleaner alternatives results in a reduction in CO2 emissions. Therefore, the observed relationship between epsilon and CO2 emissions underscores the importance of promoting a higher elasticity of substitution to facilitate the transition to cleaner energy sources and mitigate climate change effectively.

5.2.2 Global Temperature

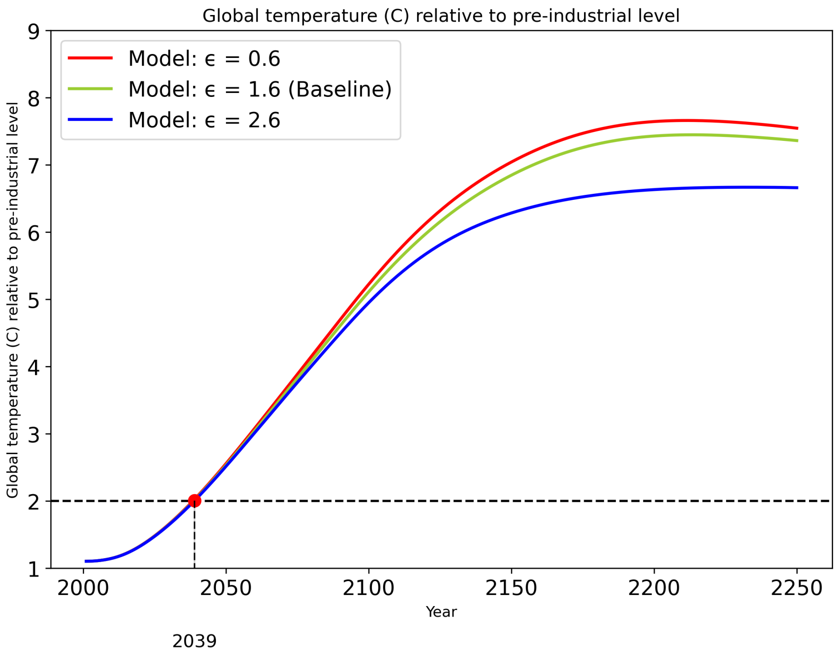

Figure 8: CO2 Emissions - Different values for epsilon

Moving on to Figure 8, which illustrates the global temperature relative to the pre-industrial level from 2000 to 2250, we can observe the impacts of varying ε on the temperature trajectory. Despite the different ε values, all three lines in the plot reach the 2-degree threshold around the same year as the baseline model (green line). This suggests that changes in the value of ε have a delayed effect on global temperature changes. The delayed effect can be attributed to several factors. Firstly, the inertia in the climate system, along with the cumulative nature of CO2 emissions, means that temperature responses to changes in ε take time to manifest. Secondly, other influential factors, such as CO2 sinks, feedback mechanisms, and external forcings, can contribute to the overall temperature trajectory, potentially mitigating or delaying the effects of changes in ε.

5.2.3 Fossil Fuel and Clean Energy Usage

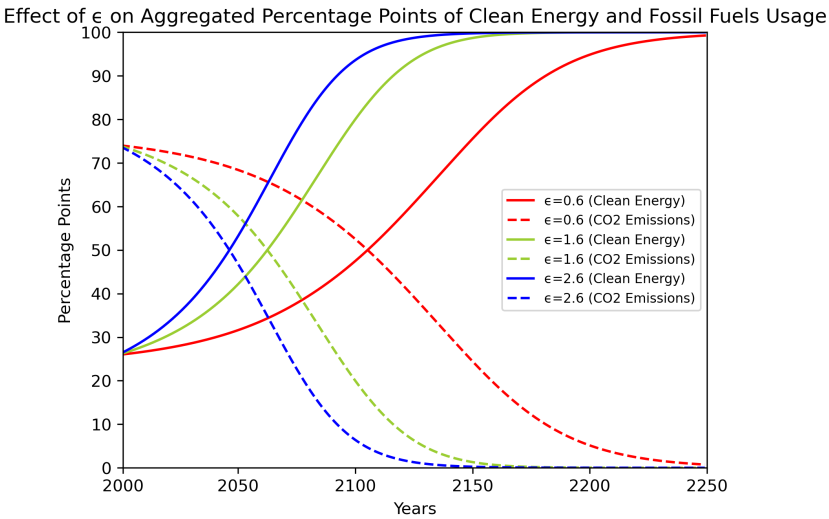

Figure 9: Fossil fuels and clean energy usage - Different values of Epsilon

Figure 9, which illustrates the effect of ε on the aggregate demand percentage points of clean energy and fossil fuel usage, provides valuable insights into the dynamics of energy consumption and the role of ε in driving the transition towards cleaner energy sources. The solid line represents clean energy usage, while the dashed line represents fossil fuel usage, with the curves depicting the movement in percentage points from the year 2000 to 2250. The observed trend of the clean energy curve increasing and the fossil fuel curve decreasing indicates a shift in aggregate demand towards cleaner energy sources over the specified time period. This trend aligns with global efforts to reduce greenhouse gas emissions, mitigate climate change, and foster sustainable energy practices. The contrasting movement of the two curves underscores the substitution effect between clean energy and fossil fuels, highlighting the changing preferences and choices of consumers and policymakers. A notable pattern emerging from Figure 9 is the relationship between the magnitude of ε and the steepness of the curves. It is evident that as the value of ε increases, the clean energy curve exhibits a steeper slope, indicating a more significant increase in aggregate demand for clean energy. Conversely, as ε decreases, the clean energy curve becomes less steep, indicating a slower rate of growth in clean energy usage. This relationship between ε and the steepness of the curves can be attributed to the concept of substitutability between clean energy and fossil fuels. When ε is larger, it implies a higher degree of substitutability, meaning that clean energy sources are considered more effective substitutes for fossil fuels. This perception may stem from advancements in clean energy technologies, improved cost competitiveness, supportive policies, and growing public awareness of environmental concerns. The greater substitutability encourages consumers to shift their demand towards clean energy sources, resulting in a steeper curve that reflects a more substantial increase in aggregate demand for clean energy. Conversely, when ε is smaller, it suggests a lower degree of substitutability between clean energy and fossil fuels. In such cases, clean energy sources may be perceived as less viable substitutes, possibly due to limitations in technology, higher costs, or inadequate infrastructure. The lower substitutability hinders the rate of transition towards clean energy, resulting in a less pronounced movement in aggregate demand for clean energy over the specified time period. Consequently, the curve representing clean energy usage exhibits a gentler slope.

In conclusion, the examination of ε in our model reveals its significant impact on various aspects of energy consumption and transition to cleaner energy sources. The findings demonstrate that the value of ε influences CO2 emissions, global temperature, and aggregate demand for clean energy and fossil fuels. A higher value of ε indicates a greater substitutability between clean energy and fossil fuels, leading to reduced CO2 emissions, a more pronounced shift towards clean energy, and a steeper increase in aggregate demand for clean energy. Conversely, a lower value of ε signifies lower substitutability and results in slower changes in CO2 emissions, less pronounced shifts towards clean energy, and gentler movements in aggregate demand. However, it is worth noting that while ε plays a significant role in energy dynamics, it did not have a substantial influence on the ability to stay below the two-degree threshold for global temperature increase. Regardless of the specific value of ε, the modeled scenarios still reached the two-degree threshold within a similar timeframe. This suggests that achieving climate targets relies on other factors such as renewable energy deployment, energy efficiency measures, and decarbonization policies. Thus, while ε impacts energy consumption and transition, its direct effect on temperature outcomes appears relatively limited compared to other critical determinants in the energy system.

However, it is important to acknowledge the challenge of determining the an optimum elasticity that would effectively facilitate reaching the temperature target. This observation suggests that solely increasing the elasticity of substitution between fossil fuels and clean energy, through means such as technological advancements, removing market barriers, policy support for clean energy, and addressing vested interests, may have limited direct impact. Therefore, it becomes imperative to explore and evaluate other mitigation measures, which is done in sections 5.3 and 5.4 of our study.

5.3 Adjusting the total stock of carbon dioxide available for energy production on the planet

In this subsection, we explore the parameter maxCO2, which represents the total stock of carbon dioxide available for energy production on the planet. In this section, the value of ϵ has been reset to its initial value of 1.6. The baseline model initially set maxCO2 at 19,500 GtCO2.2, based on the IPCC's most pessimistic scenario with RCP level of 8.5 (Cruz & Rossi-Hansberg, 2022). To investigate its implications, we examined the effects of significantly increasing it to 50,000 and decreasing it to 2,650, aligning with the target set by the Paris Agreement to limit global warming to below 2 degrees Celsius. It is important to note that reducing the value to 2,650 represents a substantial decline and may be viewed as an extreme scenario in practice. However, this analysis highlights the magnitude of the measures required to achieve the 2-degree target, emphasizing the need for ambitious and transformative actions to effectively address climate change.

While adjusting the actual total stock of carbon dioxide available for energy production may not be possible, there are indirect ways to influence this parameter through policy interventions and measures. One such approach is the implementation of stringent regulations and policies that limit the extraction and usage of fossil fuels, such as prohibiting the pumping of oil or placing strict restrictions on coal mining. By reducing the reliance on carbon-intensive energy sources and promoting the adoption of cleaner alternatives, such policies can effectively limit the total stock of carbon dioxide available for energy production over time.

Additionally, investments in research and development of carbon capture and storage (CCS) technologies can also indirectly impact the total stock of carbon dioxide. CCS technologies aim to capture and store carbon dioxide emissions from industrial processes, preventing them from being released into the atmosphere. By advancing these technologies and their widespread deployment, it becomes possible to remove carbon dioxide from the energy production process and reduce the overall stock of carbon dioxide available for emissions.

Furthermore, encouraging international cooperation and collaboration on climate change mitigation efforts can lead to collective actions that indirectly influence the total stock of carbon dioxide. Cooperation between countries in adopting renewable energy sources, sharing clean energy technologies, and implementing emission reduction targets can collectively contribute to limiting the overall carbon dioxide emissions and effectively adjust the available stock for energy production.

While these approaches may not directly manipulate the total stock of carbon dioxide available for energy production, they serve as viable strategies for reducing carbon dioxide emissions and mitigating the adverse effects of climate change. By implementing policies that discourage the use of fossil fuels, promoting the development and adoption of clean energy technologies, and fostering global cooperation, we can effectively work towards achieving the temperature targets set by the Paris Agreement and ensure a more sustainable and resilient future.

5.3.1 Global CO2 emissions

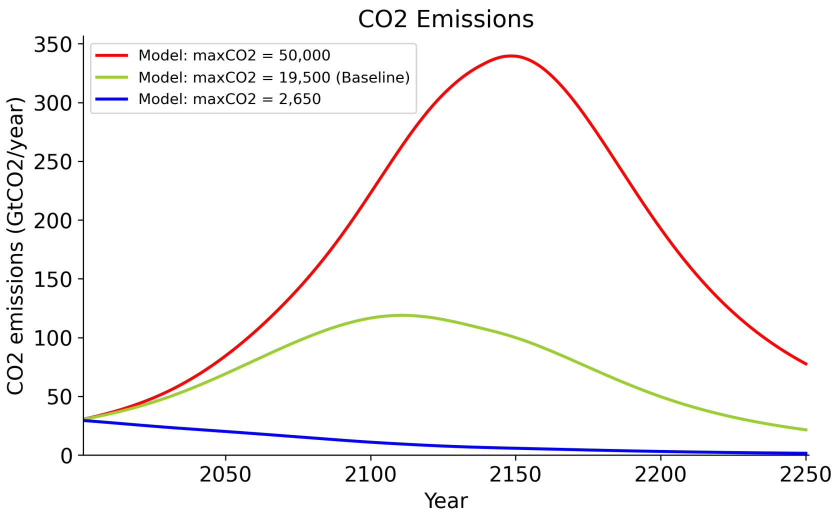

Figure 10: CO2 Emissions - Different values for MaxCO2

Figure 10, depicting CO2 emissions over time, showcases distinct patterns for the different maxCO2 values. The red line, representing the increased maxCO2 of 50,000, exhibits a concave shape with significantly higher CO2 emissions. This occurs because with a larger available carbon dioxide stock, there is less incentive to limit emissions, resulting in higher levels of CO2 released into the atmosphere. The concave shape suggests that the rate of CO2 emissions increases at an accelerating pace over time, potentially leading to a more severe climate impact. On the other hand, the blue line, reflecting the decreased maxCO2 of 2,650, demonstrates a declining trend in CO2 emissions. When the total stock of carbon dioxide available for energy production is limited, there is a heightened focus on reducing emissions and transitioning to cleaner energy sources.

5.3.2 Global Temperature

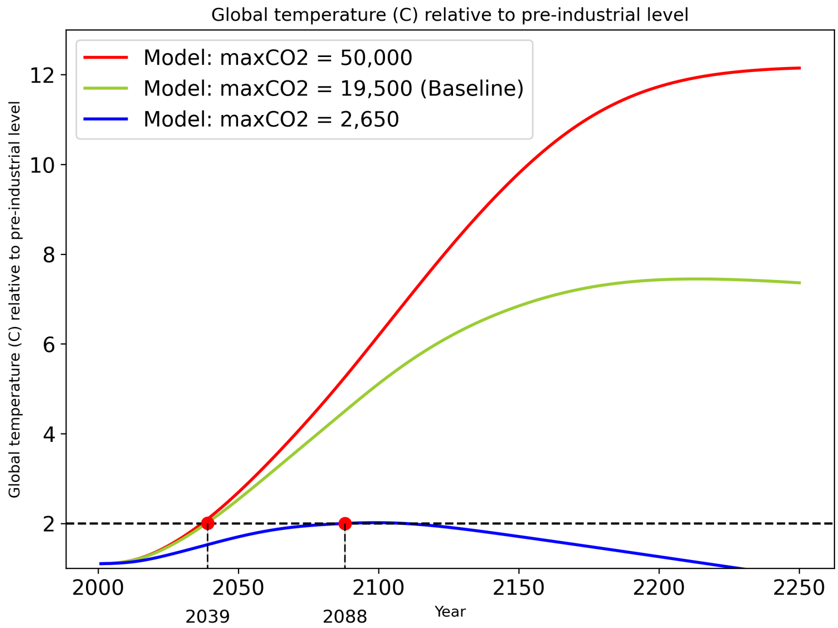

Figure 11: Global temperature - Different values for MaxCO2

Moving to Figure 11, which illustrates global temperature relative to pre-industrial levels, we can observe the consequences of different maxCO2 values on temperature outcomes. The high maxCO2 value of 50,000 results in a steep temperature increase of over 12 degrees. This is due to the significant amount of CO2 emissions associated with a larger available carbon dioxide stock, leading to a higher concentration of greenhouse gases in the atmosphere and consequently driving up global temperatures. Both the baseline model (green line) and the red line cross the critical 2-degree threshold by 2039, indicating that even with the baseline scenario or an increased maxCO2, the temperature target is surpassed relatively early. In contrast, the blue line exhibits a concave curve, peaking at 2 degrees in 2088 and then declining. This suggests that reducing the maxCO2 value to 2,650 allows for a slower temperature increase and potentially enables a better chance of achieving the temperature target.

Overall, the behavior of the figures can be attributed to the interplay between the available carbon dioxide stock, CO2 emissions, and their impact on global temperatures. Higher maxCO2 values lead to concave-shaped emissions curves with higher CO_2 levels and more pronounced temperature increases, while lower maxCO_2 values result in declining emissions curves and slower temperature trajectories. These insights emphasize the importance of managing and reducing carbon dioxide stocks to mitigate climate change and work towards a sustainable future.

5.4 Adjusting Carbon Taxes

In our quest to limit global temperature rise to below 2 degrees Celsius, as mandated by the Paris Agreement, it is imperative to examine the efficacy of different carbon tax values. This subsection delves into the implications of various carbon tax schemes on CO_2 emissions and global temperatures within the context of our overarching goal. In this section, the value of ϵ and maxCO_2 have been reset to its initial value. The baseline model, represented by the green line, does not incorporate any taxes. Additionally, we investigate the effects of ad-valorem carbon taxes set at a constant rate of 200% (red line) and a carbon tax scheme set at 50%, with an annual growth rate of 3.63% (blue line), specifically designed to align with the 2-degree Celsius temperature target. It is important to note that achieving the 2-degree threshold requires significant adjustments to carbon tax levels, which would likely entail a substantial shock to current economic systems and policy frameworks.

5.4.1 Global CO2 emissions

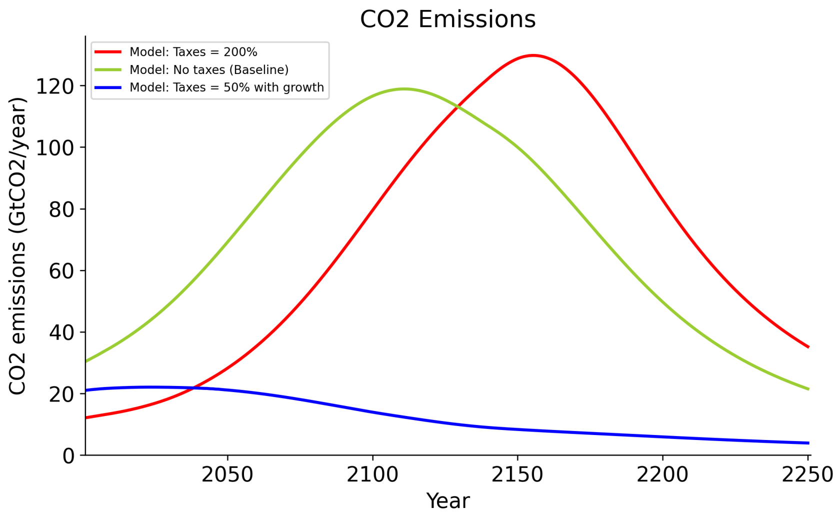

Figure 12: CO2 emissions - Different values for taxes

Figure 12 depicts the evolution of CO_2 emissions from 2000 to 2250. Increasing taxes by 200% (red line) demonstrates that carbon taxes effectively reduce the present consumption of fossil fuels, resulting in a significant initial decrease in carbon emissions. However, over time, the red line exhibits a counterintuitive pattern where it surpasses the emissions of the baseline model, utilizing more CO2 emissions than when no taxes were imposed. This suggests that carbon taxes primarily delay the use of carbon resources rather than diminishing their overall utilization. Several reasons may account for this rebound effect, including increased efficiency in carbon extraction or a delayed transition to cleaner energy sources. Contrasting the baseline and red lines, the blue line representing excise carbon taxes with a gradually increasing rate over time showcases a distinct trajectory in Figure 12. Commencing at a higher value than the red line due to a 50% tax rate increase (instead of 200% in the case of the red line), the blue line steadily declines over time. By employing a tax mechanism that grows consistently, the carbon tax policy depicted by the blue line achieves long-term emission reductions. The continuous escalation in the tax rate ensures that the cost of fossil fuels relative to clean energy alternatives rises progressively, incentivizing a transition to cleaner options. Consequently, CO2 emissions exhibit a persistent decline, offering a more favorable trajectory for mitigating climate change.

5.4.2 Global Temperature

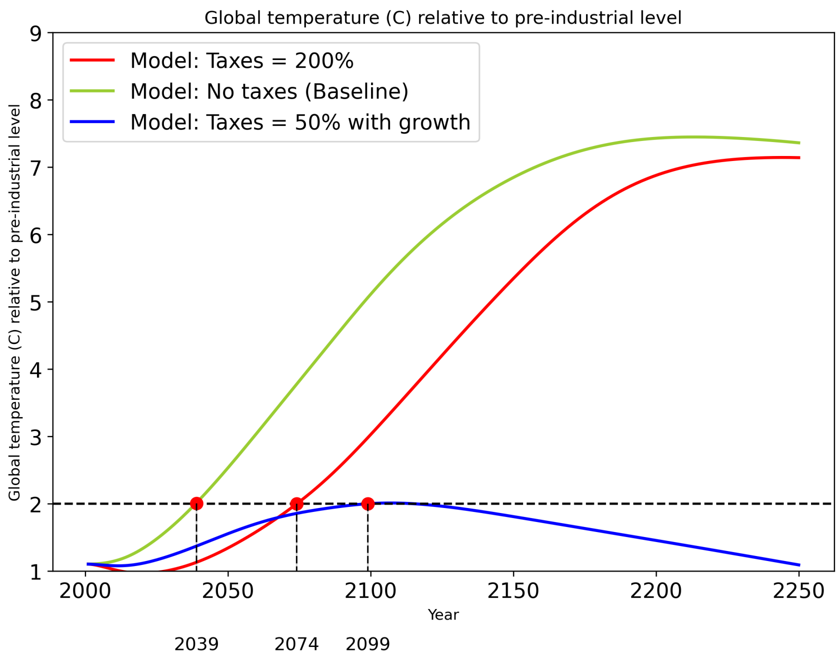

Figure 13: Global temperature - Different values for taxes

Figure 13 illustrates the global temperature relative to pre-industrial levels from 2000 to 2250. In the absence of taxes, the baseline model shows the highest temperature rise, reaching 2 degrees Celsius by 2039. Introducing a 200% carbon tax (red line) delays the temperature increase, but eventually aligns with the green line by 2250, yielding a comparable outcome. This delay in temperature rise implies that carbon taxes alone merely postpone the inevitable without providing a long-term solution. In contrast, the blue line presents a more promising outcome, with the highest temperature point remaining within the desired 2-degree Celsius target. This achievement is facilitated by implementing a gradually increasing carbon tax that incentivizes the adoption of clean energy alternatives, promotes long-term planning, and stimulates investment. This tax policy drives economic restructuring, compelling industries to embrace energy-efficient technologies and transition to low-carbon practices.

Carbon taxes play a vital role in our journey towards achieving the temperature target set by the Paris Agreement. While the 200% ad-valorem carbon tax delays emissions but eventually leads to a rebound effect, the gradually increasing excise tax (blue line) presents a more favorable trajectory with persistent emission reductions. However, it is crucial to recognize that carbon taxes alone are insufficient for comprehensively addressing climate change. They must be complemented with incentives to foster technological advancements and encourage sustainable practices.

5.5 Visualizing Climate Success

This subsection delves into the vision of a world that successfully avoids surpassing the critical 2-degree Celsius temperature threshold set by the Paris Agreement. By adjusting the total stock of carbon dioxide available for energy production on the planet to 2650 GtCO2 and implementing a carbon tax scheme set at 50% with an annual growth rate of 3.63%, two scenarios were created. Surprisingly, these scenarios yielded identical results 200 years later, prompting a merging of their analyses. This subsection examines the implications of this convergence and discusses the findings in detail.

Upon analyzing the results of the adjusted carbon stock and carbon tax scenarios, it became evident that both scenarios led to the same outcomes after a span of 200 years. This unexpected convergence raises intriguing questions regarding the underlying dynamics and assumptions of the models. The likelihood of two distinct scenarios yielding identical results over a long time period calls for further investigation and critical evaluation of the models employed.

5.5.1 Log CO2 emissions in year 2200

In Appendix 8.1.1, titled "Local CO2 Emissions," two maps are presented to examine the log CO2 emissions in the year 2200. Figure 15 represents the baseline model, while Figure 16 visualizes the scenario under climate success. Despite making adjustments to the total stock of carbon dioxide available for energy production (set at 2650 GtCO2) and implementing a carbon tax scheme (with a rate of 50% and an annual growth rate of 3.63%), the maps exhibit a surprising lack of noticeable differences.

The absence of discernible disparities between the two maps raises intriguing questions and prompts further analysis. It is possible that the adjustments made, although intended to address CO2 emissions and promote climate success, were not significant enough or comprehensive in nature to result in observable variations in the log CO2 emissions map. Climate change is a complex issue that requires the consideration of multiple interconnected factors, and achieving substantial changes may necessitate a more holistic approach.

Furthermore, the efficacy of interventions such as carbon taxation and adjustments to carbon dioxide stocks can be influenced by various contextual factors and their interactions within the model. Complex dynamics involving economic considerations, technological advancements, and behavioral patterns could have contributed to the lack of visible disparities in the log CO2 emissions map.

These findings underscore the intricate nature of addressing climate change and highlight the importance of adopting a comprehensive approach that encompasses a range of strategies and policies beyond singular interventions. While carbon taxation and carbon dioxide stock adjustments are vital elements in climate mitigation, their effectiveness may be contingent upon complementary actions that address various aspects of the climate challenge.

5.5.2 Log clean energy in year 2200

Figure 18 in appendix depicts the distribution of logarithmic clean energy adoption in the year 2200 in the case of climate success. Upon closer examination, the map reveals striking similarities to the corresponding baseline model at the same time period, figure 17 in appendix, indicating a lack of substantial progress in clean energy adoption despite successfully avoiding the 2-degree Celsius threshold. This observation raises important questions about the effectiveness of the adjustments made, specifically in relation to the parameter "maxCO2" and the implementation of spatially uniform excise carbon taxes.

The map shows that the adoption does not exhibit significant differences. This indicates that adjusting the "maxCO2" parameter to 2,650 might not have effectively encouraged the transition to clean energy sources. Although the intention was to restrict the total amount of carbon dioxide available for energy generation, it seems that this adjustment did not result in a noteworthy increase in the adoption of clean energy technologies. Therefore, it is necessary to carefully examine the assumptions and mechanisms used in the model and investigate potential obstacles that prevented the desired shift towards cleaner alternatives.

The absence of notable variations in the map of log clean energy adoption also raises questions about the impact of implementing spatially uniform excise carbon taxes. While these taxes were intended to incentivize regions globally to transition to cleaner energy sources, the lack of significant progress suggests that additional factors may be at play. It is possible that the uniform tax distribution failed to account for regional variations in resource availability, technological capabilities, or policy frameworks, which could have hindered the widespread adoption of clean energy technologies.

Despite successfully avoiding the 2-degree threshold, the analysis of Figure 18 reveals a concerning lack of substantial progress in clean energy adoption. Both the adjustment of the "maxCO2" parameter and the implementation of spatially uniform excise carbon taxes did not appear to have significantly influenced the transition to clean energy sources.

5.5.3 January-July Temperature in year 2200

Figure 20 in appendix portrays the distribution of local temperature anomalies during the months of January to July in the year 2200 in the case of climate success. Notably, the map exhibits a striking similarity to the corresponding baseline model at the year 2000, figure 19 in appendix, indicating that the implemented adjustments in the model have successfully mitigated significant temperature rise.

The similarity between the map of local temperatures in 2200 and the results from the baseline model suggests that the adjustment of the "maxCO2" parameter to 2,650 has effectively curtailed the temperature rise. By limiting the total stock of carbon dioxide available for energy production, the adjustment has prevented excessive emissions and minimized the associated warming effect. This highlights the importance of proactive measures in controlling carbon dioxide emissions and their subsequent impact on global temperatures.

The similarity in the distribution of local temperatures to the baseline model also reflects the role of implementing spatially uniform excise carbon taxes. The uniform tax distribution incentivizes regions worldwide to transition to cleaner energy sources, thereby reducing greenhouse gas emissions and mitigating temperature rise. The absence of significant temperature anomalies in the map suggests that the carbon tax scheme has been successful in promoting the adoption of low-carbon practices globally. This underscores the collective effort and shared responsibility required to address climate change effectively.

Figure 20 demonstrates that the adjustments made in the model, including the "maxCO2" parameter adjustment and the implementation of spatially uniform excise carbon taxes, have effectively prevented a significant rise in local temperatures by the year 2200. The similarity between the map and the baseline model at the year 2000 indicates that the implemented measures have successfully mitigated the warming effect. This underscores the importance of proactive policies and international cooperation in achieving climate goals.

5.5.4 Population density in year 2200

Figure 21 in appendix, depicting a constant population density in the year 2200 after implementing the adjustments to the model, provides an optimistic outlook in terms of climate success. The absence of significant changes in population density compared to the baseline model indicates a level of stability and equilibrium in human settlements, reflecting a scenario where the world successfully manages to limit global temperature rise to within the 2-degree threshold set by the Paris Agreement.

This outcome suggests that the measures taken, including adjusting the "maxCO2" parameter and implementing spatially uniform excise carbon taxes, have effectively stabilized the adverse impacts of climate change on population distribution, since it looks just like it did in year 2000. The stability in population density across different regions implies a balanced and sustainable development trajectory, where people are able to maintain their livelihoods and well-being without facing significant disruptions due to climate-related factors. In this scenario, the model showcases the successful coexistence of a habitable environment and a thriving human population. By effectively mitigating climate change and reducing carbon emissions, the world has managed to avoid the more severe consequences of global warming, such as displacement, resource scarcity, and environmental degradation. This achievement reflects a collective effort to preserve the Earth's ecosystems, protect vulnerable communities, and ensure a sustainable future for generations to come.

However, it is important to note that the model's representation of constant population density in Figure 21 should be interpreted within the context of the specific factors considered in the model. While it demonstrates the potential outcome if climate success is achieved, it might not capture all the complexities and intricacies of real-world population dynamics. Other socio-economic, political, and demographic factors that influence population distribution may not be fully captured in the model, resulting in a simplified representation of the relationship between climate success and population density.

Nevertheless, Figure 21 provides a visual representation of the desirable outcome when climate success is realized and the 2-degree threshold is not exceeded. It highlights the importance of proactive and effective climate mitigation strategies, emphasizing the need for global cooperation, policy interventions, and sustainable practices to ensure a world where human populations can thrive in harmony with a stable and resilient environment.

5.5.5 Real Gross Domestic Product

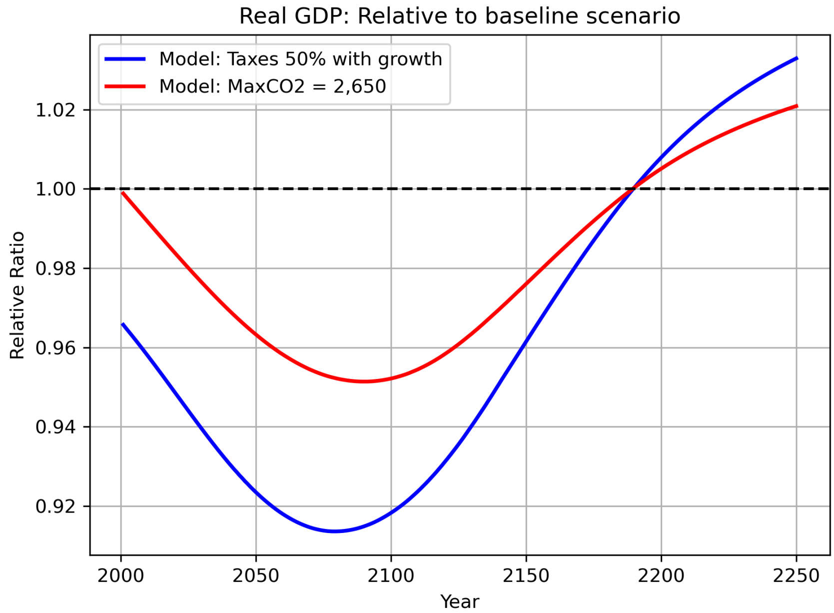

Figure 14: Real GDP: relative to baseline scenario

Figure 14 provides valuable insights into the economic repercussions of the adjustment made in the model to mitigate climate change. The aim was to understand the long-term implications of this adjustment on real GDP (Gross Domestic Product), which serves as a key indicator of economic performance.

The red line in the figure represents the scenario where the total stock of carbon dioxide available for energy production is adjusted to 2,650 GtCO2. Interestingly, the red line starts at the baseline level (1.00) in the year 2000, suggesting that the adjustment does not immediately impede economic growth. This initial observation indicates that the chosen adjustment in the model does not have an immediate adverse impact on the overall GDP trajectory. In contrast, the blue line represents the scenario where a carbon tax scheme is implemented, with a tax rate set at 50% and an annual growth rate of 3.63%. The blue line starts at a slightly lower value of 0.97 in the year 2000, indicating a minor deviation from the baseline GDP level due to the introduction of carbon taxes. This initial decrease in GDP can be seen as a reflection of the economic adjustments and transition costs associated with implementing the carbon tax scheme.

As we observe the movement of the lines over time, their convex shape captures the complex relationship between climate change mitigation measures and economic growth. This curvature suggests that, despite the challenges and adjustments faced in transitioning to a low-carbon economy, both the adjusted carbon stock and carbon tax scenarios ultimately lead to economic growth. This finding indicates that efforts to mitigate climate change do not necessarily impede long-term economic prosperity. The convergence of the red and blue lines, intersecting at a value of 1 between the years 2150 and 2200, highlights an important observation. It implies that, in the long run, the economic impact of the adjustment in the model and the implementation of the carbon tax scheme can be comparable, resulting in a similar level of real GDP. This suggests that the chosen adjustment and the carbon tax scheme can effectively balance climate change mitigation with economic growth objectives. However, the steeper curve of the blue line in the later years, particularly noticeable towards the year 2250, indicates that the carbon tax scheme leads to a higher value of real GDP compared to the adjusted carbon stock scenario. This finding suggests that the carbon tax scheme, with its increasing tax rate over time, not only supports climate change mitigation but also promotes greater economic growth. It underscores the potential economic benefits of aligning environmental objectives with market-based mechanisms.

In summary, Figure 14 provides valuable insights into the economic repercussions of the adjustment made in the model to mitigate climate change. It demonstrates that the chosen adjustment and the implementation of a carbon tax scheme can yield positive economic outcomes while addressing environmental concerns. These findings emphasize the importance of considering both economic and environmental factors when formulating climate change mitigation strategies.

6 Conclusion

In conclusion, this study aimed to investigate and analyze strategies to prevent the global average temperature from surpassing a 2-degree Celsius increase above pre-industrial levels, in alignment with the objectives of the Paris Agreement. By examining existing literature and utilizing a Python-based model based on the framework proposed by Cruz & Rossi-Hansberg (2022), we explored various parameters and their effects on key variables such as temperature, CO2 emissions, clean energy utilization, GDP, and population density.

Through our research, we aimed to contribute to the understanding of the actions and policies necessary to fulfill the commitments of the Paris Agreement. The findings of this study provide valuable insights for policymakers, researchers, and stakeholders, aiding them in the development of effective strategies for climate change mitigation and adaptation. Integrating climate considerations into decision-making processes is crucial for businesses and organizations to play a vital role in working towards a sustainable and resilient future.

Our baseline model provides a perspective on the potential trajectory of global climate change if no significant actions are taken to mitigate greenhouse gas emissions and limit temperature rise. Based on the model's findings, there is a notable risk of surpassing the critical 2-degree Celsius temperature threshold. The limited progress in emissions reduction observed in certain regions, along with the intensification of warmth at the equator, suggests that without effective intervention, the world may exceed the desired temperature targets outlined in the Paris Agreement. These results emphasize the urgent need for decisive and comprehensive strategies to address climate change and prevent the potential consequences associated with surpassing the 2-degree threshold. It is imperative for policymakers, researchers, and stakeholders to collaborate and take swift action to reduce emissions, transition to clean energy sources, and implement sustainable practices to ensure a sustainable and resilient future for generations to come.

Our analysis examined the impact of several adjusted parameters on the transition to cleaner energy sources and the mitigation of climate change. The elasticity of substitution (ε) between fossil fuels and clean energy, investigated through our model, revealed its significant influence on energy consumption patterns and the shift towards clean energy. Higher values of ε indicated a greater substitutability between the two types of energy, leading to reduced CO2 emissions, a more prominent adoption of clean energy, and increased demand for sustainable alternatives. However, our model indicated that the value of ε did not have a substantial impact on the ability to remain below the 2-degree threshold for global temperature increase.

Our investigation into the total stock of carbon dioxide available for energy production demonstrated its significant role in shaping emissions and temperature trajectories. Modifying this parameter in our model revealed distinct patterns. Higher maximum cumulative CO2 values resulted in emissions curves with a concave shape, characterized by higher CO2 levels and more pronounced temperature increases. Conversely, lower values yielded declining emissions curves and slower temperature trajectories. This highlights the critical importance of managing and reducing carbon dioxide stocks to effectively mitigate climate change. Moreover, it is worth noting that the substantial decline to a maximum cumulative CO2 value of 2,650, as observed in our model, represents a significant reduction that may be challenging to achieve in practice, considering its potential unrealistic nature.

The implementation of a carbon tax scheme emerged as a vital component in achieving the temperature target of the Paris Agreement. Our findings suggest that a carbon tax with a gradually increasing rate over time proved more effective in achieving long-term emission reductions, compared to a fixed 200% ad-valorem carbon tax that led to a rebound effect. However, it is important to note that carbon taxes alone are insufficient and should be complemented by incentives for technological advancements and sustainable practices. Furthermore, it is crucial to acknowledge that the significant difference between the proposed carbon tax schemes and the current policy landscape represents a considerable challenge in terms of transitioning to such approaches.

In our baseline model, we chose the RCP 8.5 scenario, considered the most challenging in terms of mitigation efforts and adaptation requirements. Despite the severity of this scenario, our analysis demonstrates the effectiveness of the selected mitigation policies, namely the adjustment of the total stock of carbon dioxide available for energy production and the implementation of a carbon tax scheme. While it is true that substantial adjustments were required to achieve the 2-degree target within the context of RCP 8.5, it is important to consider that a lower RCP scenario may have necessitated less drastic changes in these policies. Nevertheless, the results highlight the crucial role of proactive and ambitious actions in reducing emissions, managing carbon dioxide stocks, and implementing effective economic incentives to steer the trajectory towards climate stabilization, even in the face of the most challenging scenarios.

In conclusion, this study highlights the urgency and complexity of addressing climate change to prevent the adverse impacts of exceeding the 2-degree Celsius threshold. It emphasizes the need for immediate and collaborative actions by stakeholders to reduce emissions, transition to clean energy sources, and implement comprehensive policies that consider the interplay of various parameters. By pursuing these strategies and promoting sustainable practices, we can strive for a resilient and sustainable future for generations to come.

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8 Appendix

8.1 Maps

8.1.1 Local CO2 emissions

Baseline in year 2200

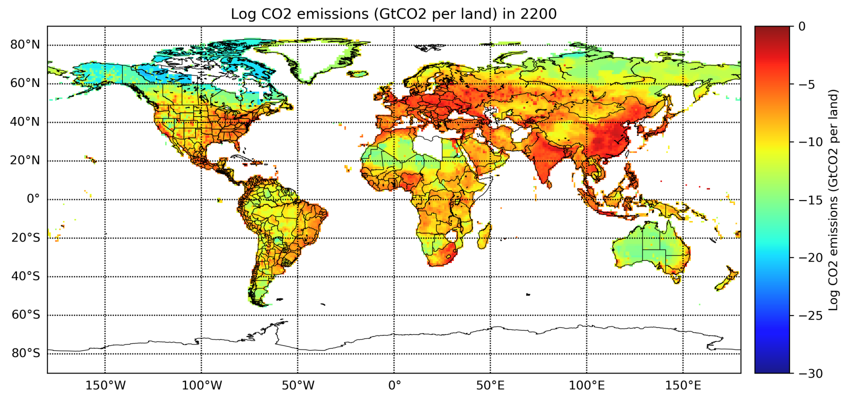

Figure 15: Local CO2 emissions: Baseline

Climate success in year 2200

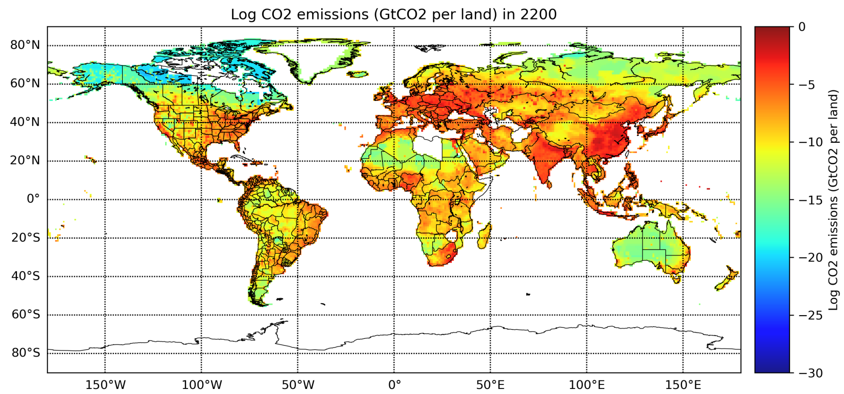

Figure 16: Local Log CO2 emissions: Climate Success

8.1.2 Local clean energy use

Baseline in year 2200

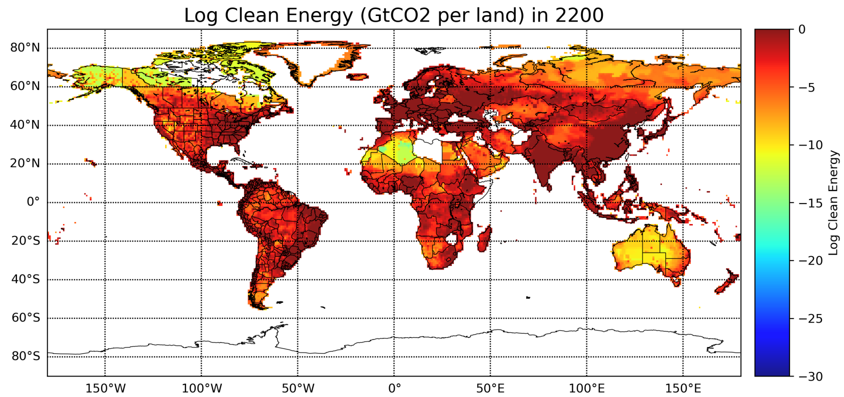

Figure 17: Local clean energy: Baseline

Climate success in year 2200

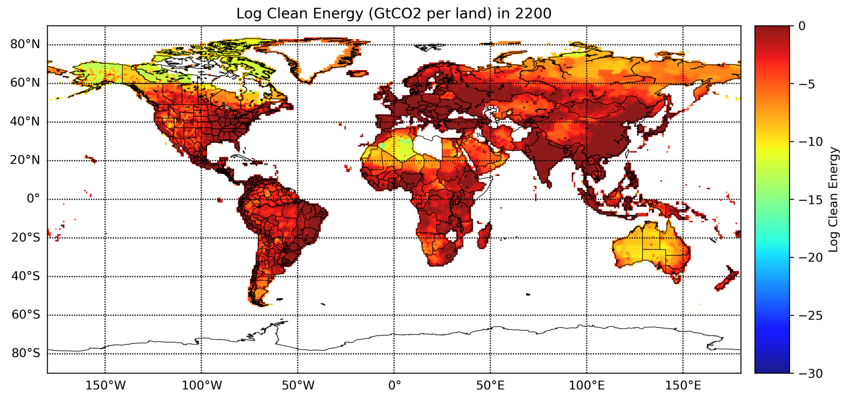

Figure 18: Local clean energy: Climate Success

8.1.3 January-July temperature

Baseline in year 2000

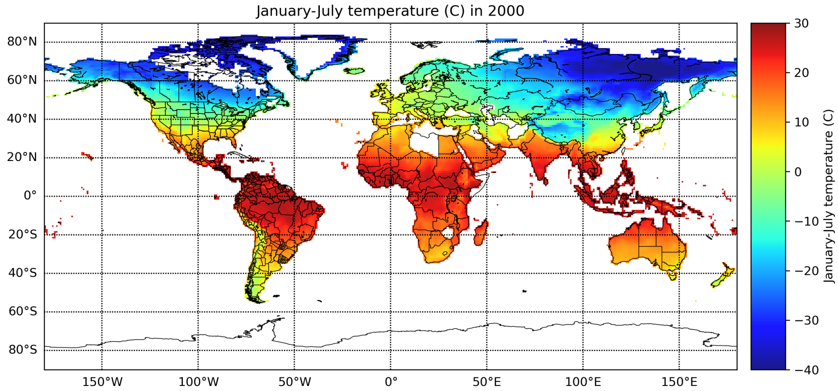

Figure 19: Local temperature January-July: Baseline

Climate success in year 2200

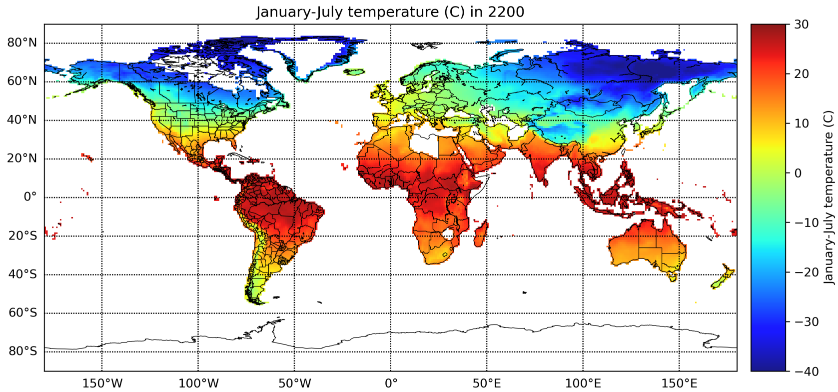

Figure 20: Local temperature January-July: Climate success

8.1.4 Population density

Clinate success in year 2200

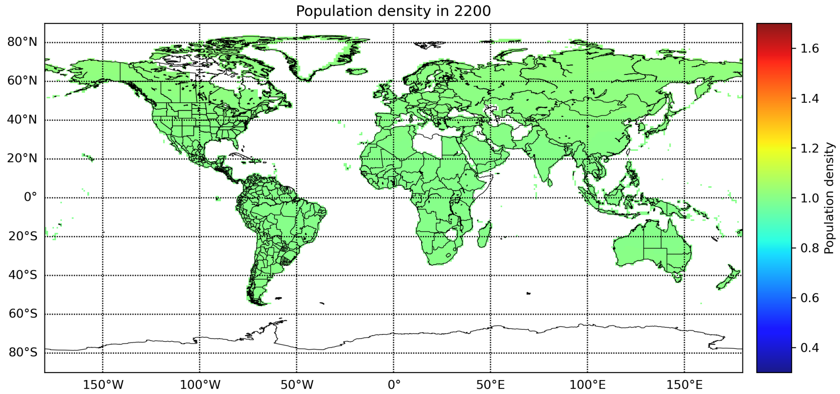

Figure 21: Population density: Climate Success

8.2 Python Code

```
# Initializing - Function that loads the data and
# generates the global variables of the model. import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import math
import h5py
import scipy.io
import csv
import scipy.optimize as optimize
from scipy.optimize import root
#datapath = "/Users/jonebo/Documents/BI/Masteroppgave/Code/Data"
def initialize(ind_RCP, maxCO2, eps,datapath =
"/Users/jonebo/Documents/BI/Masteroppgave/Code/Data"):

        max_cumCO2 = maxCO2 = 19500
        ind_RCP = 8.5
        eps = 0.6
        lbar = 5.9174e+09    # total population
        lambda1 = 0.32                      # congestion externalities
        gamma1 = 0.319      # elasticity of tomorrow's productivity relative to today's innovation
        gamma2 = 0.99246  # elasticity of tomorrow's productivity relative to today's productivity
        eta = 1        # parameter driving scale of technology diffusion
        mu = 0.8      # labor share in production
        nu = 0.15     # intercept parameter in innovation cost function
        ksi = 125  # elasticity of innovation costs relative to innovation
        psi_util = 0.045    # wellbeing parameter
        beta = 0.965     # discount factor for present discounted values
        alpha = 0.06        # agglomeration externalities
        theta = 6.5          # variance productivity shocks
        Omega = 0.5          # variance taste shocks
        tCO2_toe = 2.8466     # conversion factor: GtCO2 per Gtoe
        price_fossil0_world = 72.99*1000000000    # price of fossil fuels in usd/GtCO2
        price_clean0_world = 1.15*76.34*10**9/tCO2_toe   # price of clean energy in usd/GtCO2

        H0_d5py = h5py.File(datapath+'H0_areal.mat','r')
        data=H0_d5py.get('H0')
        H0=np.array(data).T
        H=H0.T[np.nonzero(H0.T)]
        n = len(H)   #Numbers of cells with positive land

    # The ratio of amenities to utility.
        amen_util_H0_d5py = h5py.File(datapath+'Derived/amen_util.mat','r')
        data = amen_util_H0_d5py.get('amen_util_H0')
        amen_util_H0=np.array(data).T
        amen_util0 = amen_util_H0[ :,2]  #Data from year 2000

    # Productivities in 2000
        prod_H0_d5py = h5py.File(datapath+'Derived/prod.mat','r')
        data = prod_H0_d5py.get('prod_H0')
        prod_H0 = np.array(data).T
        prod0 = prod_H0[ :,2]  #Data from year 2000

        #Population
        data_pop_d5py= h5py.File(datapath+'pop_Gecon.mat','r')
        data_pop=data_pop_d5py.get('pop')
        pop_aux=np.array(data_pop).T
        pop0=pop_aux[:,:,2]  #Population in 2000
        pop0_dens = pop0.T[H0.T>0]/H0.T[H0.T>0]
        pop5=pop_aux[:,:,3]   #Population in 2005
        pop5_dens = pop5.T[H0.T>0]/H0.T[H0.T>0]
        pop1_dens= (4/5) * pop0_dens + (4/5) * pop5_dens

         #Wages
        data_wage_d5py = h5py.File(datapath+'wage_Gecon.mat','r')
        data_wage=data_wage_d5py.get('wage')
        wage_aux=np.array(data_wage).T
        W0 = wage_aux[:,:,2] #Wage in 2000
        w0 = W0.T[H0.T>0]

        #Agriculture index
        share_agri_grid = pd.read_csv(datapath+'share_agri_grid.csv', header=None)
        s_array = np.array(share_agri_grid)
        share_agri= s_array.T[np.nonzero(H0.T)]
        agri_index = 10 * np.round(share_agri, 1) + 1

        #HDI in 2000
        data_HDI_GDPpc = pd.read_csv(datapath+'HDI_GDPpc.csv', sep=',')
        data_HDI0 = data_HDI_GDPpc.loc[:n-1,"HDI_17048"]

        HDI0 = np.empty((180, 360))   #Human Development Index in 2000
        HDI0.T[H0.T>0] = data_HDI0
```

```
        u0 = np.exp( HDI0.T[H0.T>0]**3 / psi_util ) #Utility in 2000
        a_norm = amen_util0*u0  #Amenities in 2000

        #Bilateral trade cost on Earth cells at t=0 only
        trmult_reduced = np.array(h5py.File(datapath+'trmult_reduced.mat')['trmult_reduced']).T

        # National demarcations
        C = pd.read_csv(datapath+'C.csv', sep=',',header=None)
        arr_c_vect = C.to_numpy()
        C_vect = arr_c_vect.T[H0.T>0]
        uni_c = np.unique(C_vect)
        length_C_vect = len (uni_c)

        # Developed and developing world
        D = pd.read_csv(datapath+'D.csv', sep=',',header=None)
        arr_d_vect = D.to_numpy()
        D_vect = arr_d_vect.T[H0.T>0]
        uni_d = np.unique(D_vect)
        length_D_vect = len (uni_d)

        #Africa and rest of the world
        Africa = pd.read_csv(datapath+'Africa_map.csv',header=None)
        arr_africa_vect = Africa.to_numpy()
        Africa_vect = arr_africa_vect.T[H0.T>0]
        uni_africa_vect = np.unique(Africa_vect)
        length_Africa_vect = len (uni_africa_vect)

        # Global CO2 emissions from fossil fuels from 2000 to 2600, by IPCC
        emi_ff_RCP = pd.read_csv(datapath+'CO2_ff.csv',header=None)
        emi_ff = emi_ff_RCP[5-int(np.floor(ind_RCP/2))-1]
        emi0_ff = emi_ff[0]

        # Global CO2 emissions from NON fossil fuels from 2000 to 2600, by IPCC
        emi_no_ff_RCP = pd.read_csv(datapath+'CO2_noff_smooth.csv',header=None)
        emi_no_ff = emi_no_ff_RCP[5-int(np.floor(ind_RCP/2))-1]
        emi0_no_ff = emi_no_ff[0]      emi_no_ff = emi_no_ff[1:]

        #Total CO2 emissions for 2000
        emi0 = emi0_no_ff + emi0_ff     #Total CO2 emissions for 2000

        #CO2 emissions at cell level for 2000
        data_emi_d5py = scipy.io.loadmat(datapath+'CO2_EDGAR.mat')
        data_emi_d5py.keys()
        emi_cell = data_emi_d5py["CO2_EDGAR"]
        emi_cell_two = emi_cell[:,:,2]
        emi0_cell = emi_cell_two.T[H0.T>0]

        # Match CO2 emissions and clean energy at the country level for 2000
        for i in range(1, length_C_vect+1):
                emi0_ff_cell[C_vect==i] = emi0_cell[C_vect==i]*emi0_ff_country[i-1]/   \
                np.sum(emi0_cell[C_vect==i])
                clean0_cell[C_vect==i]  = emi0_cell[C_vect==i]*clean0_country[i-1] /  \
                np.sum(emi0_cell[C_vect==i])

        emi0_ff_cell = emi0_ff_cell*emi0_ff/np.sum(emi0_ff_cell)
        emi0_ff_cell = emi0_ff_cell/H     clean0_cell = clean0_cell/H
        #Define global use of fossil fuels and clean energy
        clean0 = (clean0_cell * H).sum()
        fossil0 = emi0_ff.copy()   #global use of of fossil fuels

        #Construct share of fossil fuels in energy composite
        fossil_share = (1+(price_clean0_world/price_fossil0_world)*(clean0/fossil0)**(1/eps))**(-1)

        #Construct share of energy in production, equations (7) and (25)
        price_energy0_world = ((fossil_share**eps) * (price_fossil0_world**(1-eps)) + \
        (1-fossil_share)**eps*price_clean0_world**(1-eps))**(1/(1-eps))
        energy0 = (fossil_share*fossil0**((eps-1)/eps) + \
    (1-fossil_share)*clean0**((eps-1)/eps)**(eps/(eps-1))

        GDP0 = (H * pop0_dens* w0).sum()
        energy_share = price_energy0_world * energy0 * (mu + (gamma1/ksi))/GDP0
        chi = 1 - energy_share/mu
        #Read cost extraction data
        cost_CO2_data=pd.read_csv(datapath+'CO2_cost.csv', header=None)

        #Define cost extraction function
        def costCO2_fct_aux(cumCO2,*cost_emi_param_aux):
                f = (cost_emi_param_aux[0])/(cost_emi_param_aux[1] + \
        np.exp(-  cost_emi_param_aux[2]*(cumCO2-cost_emi_param_aux[3]))) - \
            (cost_emi_param_aux[4]*19500/(cumCO2-19500))**3
            return f

        #Find parameters that better fit the data
```

```
        cost_emi_param_i = np.array([2.0, 1, 1e-4,5000,1])       # intial guess
        xdata = cost_CO2_data[1]
        ydata = cost_CO2_data[0]
        lb    = [1, 0, 0, 0, 0]
        ub    = [np.inf, np.inf, 5*1e-4, np.inf, np.inf]
        tol   = 1e-12

        res = optimize.curve_fit(costCO2_fct_aux, xdata, ydata, p0=cost_emi_param_i, \
        bounds=(lb,ub),method='trf',xtol=tol,ftol = tol, gtol=tol,verbose=1)

        #Redefine extraction
        cost_emi_param_f = res[0]
        cost_emi_param_f[4] = cost_emi_param_f[4]**3
        cost_emi_param = np.concatenate((cost_emi_param_f, [maxCO2, 3]))

        #Define normalization relative to Princeton wage
        GDPpc0 = GDP0 / (w0[3198] * lbar)
        cost_emi_param[0] = cost_emi_param[0] / GDPpc0
        cost_emi_param[4] = cost_emi_param[4] / GDPpc0
        price_clean0_world = price_clean0_world / GDPpc0
        conv_usd = 1 / GDPpc0

        def costCO2_fct(cumCO2):
                f = cost_emi_param[0] / (cost_emi_param[1] + np.exp(-cost_emi_param[2] * \
                (cumCO2 - cost_emi_param[3]))) - cost_emi_param[4] * \
                (cost_emi_param[5] / (cumCO2 - cost_emi_param[5])) ** cost_emi_param[6]
                return f

        # Construct energy use at the cell level, equation (7)
        fossil0_cell = emi0_ff_cell.copy()
        energy0_cell = (fossil_share * (fossil0_cell ** ((eps-1) / eps)) +  (1-fossil_share) * \
        (clean0_cell ** ((eps-1) / eps))) ** (eps / (eps-1))

        # Construct energy price at the cell level by source, equations (29), (30) and (25)
        const_energy = (1-chi) * mu / (mu + gamma1/ksi)
        price_clean0 = const_energy * (1-fossil_share) * \
        (pop0_dens / energy0_cell) * ((energy0_cell / clean0_cell) ** (1/eps))
        price_fossil0 = const_energy * fossil_share * \
        (pop0_dens / energy0_cell) * (energy0_cell / fossil0_cell) ** (1/eps)
        price_energy0 = ((fossil_share ** eps * price_fossil0 ** (1-eps)) +  \
        ((1-fossil_share) ** eps * price_clean0 ** (1-eps))) ** (1/(1-eps))
        price_fossil0_avg = sum(price_fossil0 * fossil0_cell * H) / sum(fossil0_cell * H)
        price_fossil0_adj = price_fossil0_world / (1e9 * price_fossil0_avg)

        # Construct initial energy productivities
        zeta_clean0 = price_clean0_world / price_clean0
        zeta_fossil0 = costCO2_fct(0) / price_fossil0

        #Set parameters regarding depreciation of CO2 in the atmosphere
        a0 = 0.2173        #share of CO2 emissions remaining in the atmosphere forever
        a1 = 0.2240        #share of CO2 emissions associated with the timescale b1
        a2 = 0.2824
        a3 = 0.2763
        b1 = 394.4
        b2 = 36.54
        b3 = 4.304
        S_preind = 2200   #CO2 stock in pre-industrial era

        #Read total CO2 emissions from 1999 to 1945
        emiCO2_data_mat = pd.read_csv(datapath+'CO2_hist.csv',sep='\t',header=None).iloc[0:25]

        emiCO2_data =  np.flipud(emiCO2_data_mat.to_numpy().flatten())
        length_emi_data = len (emiCO2_data)
        sum_emiCO2_data = emiCO2_data.sum()
        vec=np.arange(start=0, stop=length_emi_data,step=1)
        S0= a0 * sum_emiCO2_data + S_preind
        S1= a1 * np.sum(np.exp(-(vec) / b1) * emiCO2_data)
        S2= a2 * np.sum(np.exp(-(vec) / b2) * emiCO2_data)
        S3= a3 * np.sum(np.exp(-(vec) / b3) * emiCO2_data)

        # Set forcing parameters
        forc_sens = 5.35

        # Read non CO2 radiative forcing
        forc_noCO2_RCP = np.genfromtxt(datapath+'Forcing_noCO2_smooth.csv', delimiter=",")
        if ind_RCP != 7.25:
                forc_noCO2 = forc_noCO2_RCP[:, 4 - int(np.floor(ind_RCP/2))]
        else:
                forc_noCO2 = 0.5*(forc_noCO2_RCP[:, 4 - int(np.floor(8.5/2))] + \
        forc_noCO2_RCP[:, 4 - int(6/2)])

        # Set global temperature parameters
        c1 = 0.631
```

```python
c2 = 0.429
d1 = 8.4
d2 = 409.5
temp_preind = 8.1  # temperature in pre-industrial era

# Read forcing from 2000 to 1825
forc_data = np.flipud(np.genfromtxt(datapath+'Forcing_hist.csv', delimiter=","))
length_forc_data = len(forc_data)

# Initialize temperature layers, equation (35)
temp1 = (c1/d1)*np.sum(np.exp(-(np.arange(0,length_forc_data,1))/d1)*forc_data)
temp2 = (c2/d2)*np.sum(np.exp(-(np.arange(0,length_forc_data,1))/d2)*forc_data) + \
temp_preind
temp0_global = temp1 + temp2

data_temp = scipy.io.loadmat(datapath+'temp.mat')
temp0_local_long = data_temp['temp0_local_10y']
temp0_local = data_temp['temp0_local_10y_mean']
Delta_temp1 = data_temp['Delta_temp1']
temp_past = data_temp['temp_10y_past']
Delta_temp = data_temp['Delta_temp']

# Read temperature scaler computed in temp_downscaling.do
scaler_table = pd.read_csv(datapath+'/Derived/scaler_temp.csv')
lat_scaler = scaler_table.iloc[:, 0].values
lon_scaler = scaler_table.iloc[:, 1].values
scaler_data = scaler_table.iloc[:, 4].values
length_scaler = len(lon_scaler)

# Arrange temperature scaler in a grid
scaler_grid = np.full((180, 360), np.nan)
for i in range(0, length_scaler):
    lat_index = int(90.5 - lat_scaler[i]-1)
    lon_index = int(lon_scaler[i] + 180.5-1)
    scaler_grid[lat_index, lon_index] = scaler_data[i]
scaler_grid[H0 == 0] = np.nan

scaler_grid_long = np.hstack((scaler_grid[:, 340:360], scaler_grid, scaler_grid[:, 0:20]))
H0_long = np.hstack((H0[:, 340:360], H0, H0[:, 0:20]))

#for i in range(1,12):
#    for k in range(5):
i=1
k=0
indi_scaler = (H0_long > 0) & np.isnan(scaler_grid_long)
lat_scaler, lon_scaler =np.where(indi_scaler == True)
cent_scaler = (lon_scaler > 20-1) & (lon_scaler < 381-1)
lat_scaler = lat_scaler[cent_scaler]
lon_scaler = lon_scaler[cent_scaler]
length_scaler = len(lat_scaler)

for j in range(length_scaler): #length_scaler
    slice_vec = scaler_grid_long[lat_scaler[j]-i:lat_scaler[j]+i+1,\
    lon_scaler[j]-i:lon_scaler[j]+i+1]
    if np.count_nonzero(~np.isnan(slice_vec)) > 0:
        scaler_grid_long[lat_scaler[j], lon_scaler[j]] = np.nanmean(slice_vec)
scaler_grid_long = np.hstack((scaler_grid[:, 340:360], scaler_grid, scaler_grid[:, 0:20]))
H0_long = np.hstack((H0[:, 340:360], H0, H0[:, 0:20]))

for i in range(1,12):
    for k in range(5):
        indi_scaler = (H0_long > 0) & np.isnan(scaler_grid_long)
        lat_scaler, lon_scaler = np.where(indi_scaler == True)
        cent_scaler = (lon_scaler > 20-1) & (lon_scaler < 381-1)
        lat_scaler = lat_scaler[cent_scaler]
        lon_scaler = lon_scaler[cent_scaler]
        length_scaler = len(lat_scaler)

        for j in range(length_scaler):

slice_vec = scaler_grid_long[lat_scaler[j]-i:lat_scaler[j]+i+1, lon_scaler[j]-i:lon_scaler[j]+i+1]

            if np.count_nonzero(~np.isnan(slice_vec)) > 0:
                scaler_grid_long[lat_scaler[j], lon_scaler[j]] = np.nanmean(slice_vec)
total_sum2 = np.nansum(scaler_grid_long)
scaler_temp2 = scaler_grid_long[:, 20:380]
scaler_temp = scaler_temp2.T[H0.T>0]

# Read coefficients from damage function estimated in damage_function.do
theta_amen_logi_vect = pd.read_csv(datapath+'Derived/amen_coeff_10y_1h_20b_550d.csv')
theta_amen_logi_vect = theta_amen_logi_vect.to_numpy()
theta_prod_logi_vect = pd.read_csv(datapath+'Derived/prod_coeff_10y_1h_20b_550d.csv')
theta_prod_logi_vect = theta_prod_logi_vect.to_numpy()
```

```
        theta_amen_scen = theta_amen_logi_vect[:, :9]
        theta_prod_scen = theta_prod_logi_vect[:, :9]
        theta_amen_min = theta_amen_scen[0, 8]
        theta_amen_max = theta_amen_scen[1, 8]
        theta_amen_center = theta_amen_scen[2, 8]
        theta_amen_steep = theta_amen_scen[3, 8]

        def theta_amen_temp(temp):
                return theta_amen_min + (theta_amen_max - theta_amen_min) /(1 + \
                np.exp(theta_amen_steep * (temp - theta_amen_center)))

        theta_prod_min = theta_prod_scen[0, 8]
        theta_prod_max = theta_prod_scen[1, 8]
        theta_prod_center = theta_prod_scen[2, 8]
        theta_prod_steep = theta_prod_scen[3, 8]

        def theta_prod_temp(temp):
                return theta_prod_min + (theta_prod_max - theta_prod_min) /(1 + \
                np.exp(theta_prod_steep * (temp - theta_prod_center)))

        # Display results for damage function estimates
        temp_cold = -37.79
        temp_hot = 25.77
        sol_amen = root(theta_amen_temp, 0)
        sol_prod = root(theta_prod_temp, 0)

        theta_amen_logi_vect_agri =
        pd.read_csv(datapath+'derived/amen_coeff_full_agri_10y_1h_20b_550d.csv')
        theta_amen_scen_agri = theta_amen_logi_vect_agri.values

        theta_prod_logi_vect_agri =
        pd.read_csv(datapath+'derived/prod_coeff_full_agri_10y_1h_20b_550d.csv')
        theta_prod_scen_agri = theta_prod_logi_vect_agri.values

        # Read net birth historical data
        birth_death = np.genfromtxt(datapath+'birth_death_pop.csv', delimiter=',')
        country_data = birth_death[:,0]
        year_data = birth_death[:,1]
        natal_data = birth_death[:,2]
        pop_data = birth_death[:,3]

        # Define natality rates of 2000 and 2020
        natal0 = sum(natal_data[year_data==2000] * pop_data[year_data==2000]) /sum(pop_data[year_data==2000])
        natal20 = sum(natal_data[year_data==2020] * pop_data[year_data==2020]) / sum(pop_data[year_data==2020])

        # Keep data for years < 2001
        natal_data = natal_data[year_data < 2001]
        country_data = country_data[year_data < 2001]
        pop_data = pop_data[year_data < 2001]
        year_data = year_data[year_data < 2001]

        # Read UN population projections
        pop_hist = np.genfromtxt(datapath+'pop_uncert.csv', delimiter=',')
        pop_low95 = pop_hist[0, 50:]
        pop_low80 = pop_hist[1, 50:]
        pop_med = pop_hist[2, 50:]
        pop_high80 = pop_hist[3, 50:]
        pop_high95 = pop_hist[4, 50:]
        pop_low95_hist = pop_hist[0, :]
        pop_low80_hist = pop_hist[1, :]
        pop_med_hist = pop_hist[2, :]
        pop_high80_hist = pop_hist[3, :]
        pop_high95_hist = pop_hist[4, :]
        emi_ff_RCP = np.genfromtxt(datapath+'CO2_ff.csv', delimiter=',')
        emi_no_ff_RCP = np.genfromtxt(datapath+'CO2_noff_smooth.csv', delimiter=',')
        emiCO2_RCP = emi_ff_RCP + emi_no_ff_RCP
        stockCO2_layers_RCP = np.zeros((4, 600, 4))
        forc_RCP = np.zeros((600, 4))
        temp_layers_RCP = np.zeros((2, 600, 4))

         # Construct CO2 stock, forcing and temperature, using data on CO2 emissions and non-CO2
# forcing, equations (16) and (33)-(35)
        for i in range(4):
        #i=0
                stockCO2_layers_RCP[0, 0, i] = S0 + a0 * emiCO2_RCP[0, i]
                stockCO2_layers_RCP[1, 0, i] = (np.exp(-1 / b1)) * S1 + a1 * emiCO2_RCP[0, i]
                stockCO2_layers_RCP[2, 0, i] = (np.exp(-1 / b2)) * S2 + a2 * emiCO2_RCP[0, i]
                stockCO2_layers_RCP[3, 0, i] = (np.exp(-1 / b3)) * S3 + a3 * emiCO2_RCP[0, i]

                forc_RCP[0, i] = forc_sens * np.log(np.sum(stockCO2_layers_RCP[:, 0, i]) / S_preind) \
                + forc_noCO2_RCP[0, i]

                temp_layers_RCP[0, 0, i] = (np.exp(-1 / d1)) * temp1 + (c1 / d1) * forc_RCP[0, i]
```

```
        temp_layers_RCP[1, 0, i] = (np.exp(-1 / d2)) * temp2 + (c2 / d2) * forc_RCP[0, i]


        for t in range(599):
            stockCO2_layers_RCP[0, t + 1, i] = stockCO2_layers_RCP[0, t, i] + \
            a0 * emiCO2_RCP[t +  1, i]
            stockCO2_layers_RCP[1, t + 1, i] = (np.exp(-1 / b1)) * \
                stockCO2_layers_RCP[1, t, i] + a1 * emiCO2_RCP[t + 1, i]
            stockCO2_layers_RCP[2, t + 1, i] = (np.exp(-1 / b2)) * \
                stockCO2_layers_RCP[2, t, i] + a2 * emiCO2_RCP[t + 1, i]
            stockCO2_layers_RCP[3, t + 1, i] = (np.exp(-1 / b3)) * \
                stockCO2_layers_RCP[3, t, i] + a3 * emiCO2_RCP[t + 1, i]
            forc_RCP[t+1, i] = forc_sens * np.log(np.sum(stockCO2_layers_RCP[:, t+1, i]) \
            / S_preind)/np.log(2) + forc_noCO2_RCP[t+1, i]


            temp_layers_RCP[0, t+1, i] = (np.exp(-1 / d1)) * temp_layers_RCP[0, t, i] + \
            (c1 / d1) * forc_RCP[t+1, i]
            temp_layers_RCP[1, t+1, i] = (np.exp(-1 / d2)) * temp_layers_RCP[1, t, i] + \
            (c2 / d2) * forc_RCP[t+1, i]

    stockCO2_RCP = np.sum(stockCO2_layers_RCP, axis=0)
    temp_global_RCP = np.sum(temp_layers_RCP, axis=0) # Historical clean energy use
    clean_energy_data = pd.read_csv(datapath+'clean_energy_hist.csv')
    clean_energy_data = clean_energy_data.iloc[:-1,:] # from 1965 to 1999

    # Historical fossil fuel CO2 emission and its trend
    emi_ff_all = np.genfromtxt(datapath+'CO2_hist_ff.csv', delimiter=',')
    emi_ff_data = emi_ff_all[:, 0]
    emi_ff_data_tend = emi_ff_all[:,1]
    map_data = scipy.io.loadmat(datapath+'map_grid.mat')
    map0 = map_data['map'] # map of size 2700x5400 for a better resolution of plots
    map_lat, map_lon = map0.shape
    aux_lon = np.linspace(-180,180,map_lon+1)
    aux_lon = (aux_lon[:-1]+aux_lon[1:])/2
    aux_lat = np.linspace(-90,90,map_lat+1)
    aux_lat = (aux_lat[:-1]+aux_lat[1:])/2
    aux_kron = np.ones((map_lat//180,map_lat//180))
    b0y_max = 0.045
    b1y_min = 0.01
    b1y_max = 0.03
    b2y_min = -0.00090
    b2y_max = -0.00050
    b2T_max = 0.015
    bsy_min = 0
    bsy_max = 8
    bsT_min = 14
    bsT_max = 22
    natal_param = [b0y_max, b1y_min, b1y_max, b2y_min, b2y_max, b2T_max, bsy_min, \
    bsy_max, bsT_min, bsT_max]

    color_olive = [128/255, 128/255, 0/255]
    color_darkgreen = [0/255, 100/255, 0/255]
    color_darkcyan = [0/255, 139/255, 139/255]
    color_yellowgreen = [154/255, 205/255, 50/255]
    color_greenyellow = [173/255, 255/255, 47/255]
    color_darkseagreen = [143/255, 188/255, 143/255]
    color_limegreen = [50/255, 205/255, 50/255]

    name_type_vect = ['Warm', 'WarmAm', 'WarmPr', 'WarmRCP6', 'WarmRCP7.25', 'WarmAg']
    name_long_type_vect = ['', 'onlyAm_', 'onlyPr_', 'RCP6', 'RCP7.25', 'agri']
    name_maps_type_vect = ['', 'effect␣only␣on␣amenities', 'effect␣only␣on␣productivity', \
    'RCP␣6.0', 'RCP␣7.25', 'effects␣by␣industry']


    name_dam_vect = ['low95', 'high95', 'low90', 'high90', 'low80', 'high80', 'low60', 'high60', 'med']

    name_dam_long_vect = ['Low␣95%', 'High␣95%', 'Low␣90%', 'High␣90%', 'Low␣80%', \
    'High80%', 'Low␣60%', 'High␣60%', 'Baseline']

    table_prop = ['␣', 'BGP␣Real␣GDP', 'PDV␣Real␣GDP', 'BGP␣Welfare', 'PDV␣Welfare']

    return {'H':H, 'amen_util0':amen_util0, 'a_norm':a_norm, 'prod0':prod0, 'n':n,
'lbar':lbar, 'lambda1':lambda1,'gamma2':gamma2, 'eta':eta, 'mu:mu,␣'ksi':ksi,␣'alpha':alpha,
'theta':theta,␣␣'omega':omega,␣'trmult_reduced':trmult_reduced,
'gamma1':gamma1,␣'eta':eta,␣'emi0_ff':emi0_ff,␣'emi_no_ff':emi_no_ff,␣'emi0':emi0,␣'chi':chi,
'cost_emi_param':cost_emi_param,␣'a0':a0,␣'a1':a1,␣'a2':a2,␣'a3':a3,␣'b1':b1,␣'b2':b2,␣'b3':b3,
'S0':S0,␣'S1':S1,␣'S2':S2,␣'S3':S3,␣␣'S_preind':S_preind,␣'forc_sens':forc_sens,
'forc_noCO2':forc_noCO2,␣'c1':c1,␣'c2':c2,␣'d1':d1,␣'d2':d2,␣'temp1':temp1,␣'temp2':temp2,
'temp0_global':temp0_global,␣'temp0_local':temp0_local,␣'scaler_temp':scaler_temp,
'theta_amen_scen':theta_amen_scen,␣'theta_prod_scen':theta_prod_scen,
'theta_amen_scen_agri':theta_amen_scen_agri,␣'theta_prod_scen_agri':theta_prod_scen_agri,
'name_dam_vect':name_dam_vect,␣'price_clean0_world':price_clean0_world,
```

```
'zeta_clean0':zeta_clean0,␣'zeta_fossil0':zeta_fossil0,␣'fossil_share':fossil_share,
'D_vect':D_vect,␣'Africa_vect':Africa_vect,␣'length_D_vect':length_D_vect,
'length_Africa_vect':length_Africa_vect,␣'agri_index':agri_index,'natal_param':natal_param,
'natal0':natal0,␣'natal20':natal20,␣'name_type_vect':name_type_vect,
'name_long_type_vect':name_long_type_vect,␣'name_maps_type_vect':name_maps_type_vect,
'H0':H0,␣'pop0_dens':pop0_dens,␣'C_vect':C_vect,␣'tCO2_toe':␣tCO2_toe,
'fossil0_cell':fossil0_cell,␣'clean0_cell':clean0_cell,␣'aux_lon':aux_lon,␣'aux_lat':aux_lat,
'aux_kron':aux_kron,␣'emiCO2_RCP':emiCO2_RCP,␣'stockCO2_RCP':stockCO2_RCP,
'temp_preind':temp_preind,␣'temp_global_RCP':temp_global_RCP,␣'pop_low95':pop_low95,
'pop_low80':pop_low80,␣'pop_med':pop_med,␣'pop_low95':pop_low95,␣'pop_high95':pop_high95,
'pop_high80':pop_high80,␣'map0':map0,␣'color_darkcyan':color_darkcyan,
'color_olive':color_olive,␣'color_darkgreen':color_darkgreen,
'color_yellowgreen':color_yellowgreen,␣'conv_usd':conv_usd,␣'price_fossil0':price_fossil0,
'price_clean0':price_clean0,␣'emiCO2_RCP':emiCO2_RCP,␣'eps':eps,␣'maxCO2':maxCO2,
'ind_RCP':ind_RCP}
```

Listing 1: Python code: Initialize Function

```
# Forward Climate Function: Simulation

# Importing needed libaries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import math
import h5py
import scipy.io import csv
import scipy.optimize as optimize from scipy.optimize
import root
import quantecon as qe from numba
import jit from numba.typed
import Dict
from initializer34 import initialize # Importing the initialize function - previous code
datapath = "/Users/jonebo/Documents/BI/Masteroppgave/Code_EGGW/Data/"

init_re=initialize(8.5,19500,0.6,datapath=datapath) #Callin the initialize function
init_re.keys()
# Loading values in init_re H, amen_util0, a_norm, prod0, n, lbar, lambda1, gamma2, eta, mu, \
ksi, alpha, theta, Omega, trmult_reduced, gamma1, nu, emi0_ff, emi_no_ff, emi0, chi, \
cost_emi_param, a0, a1, a2, a3, b1, b2, b3, S0, S1, S2, S3, S_preind, forc_sens,         \
forc_noCO2, c1, c2, d1, d2, temp1, temp2, temp0_global, temp0_local, scaler_temp, \
theta_amen_scen, theta_prod_scen, theta_amen_scen_agri, theta_prod_scen_agri, \
name_dam_vect, price_clean0_world, zeta_clean0, zeta_fossil0, fossil_share, D_vect, \
Africa_vect, length_D_vect, length_Africa_vect, agri_index, natal_param, natal0, natal20, \
name_type_vect, name_long_type_vect, name_maps_type_vect, H0, pop0_dens, C_vect, \
tCO2_toe, fossil0_cell, clean0_cell, aux_lon, aux_lat, aux_kron, emiCO2_RCP, \
stockCO2_RCP, temp_preind, temp_global_RCP, pop_low95, pop_low80, pop_med, \
pop_high95, pop_high80, map0, color_darkcyan, color_olive, color_darkgreen, \
color_yellowgreen, conv_usd, price_fossil0, price_clean0, eps, maxCO2, ind_RCP =
init_re.values()


# Loading results from the estimates for natality function, migration costs,
# and energy productivities
results20_d5py = h5py.File(datapath+'Derived/results20_med.mat','r')
l0_model = results20_d5py.get('l0_model')
arr_l0_model=np.array(l0_model).T
l20 = results20_d5py.get('l20') arr_l20=np.array(l0_model).T
realgdp0_model = results20_d5py.get('realgdp0_model') arr_realgdp0_model=np.array(realgdp0_model).T
realgdp20 = results20_d5py.get('realgdp20') arr_realgdp20=np.array(realgdp20).T
temp20 = results20_d5py.get('temp20') arr_temp20=np.array(temp20).T
natal_d5py = h5py.File(datapath+'Derived/natal_migr_Warm_med.mat','r')
coeff_pop_i = natal_d5py.get('coeff_pop_i')
arr_coeff_pop_i=np.array(coeff_pop_i).T
m2_i = natal_d5py.get('m2_i')
arr_m2_i=np.array(m2_i).T
upsilon_clean_i = natal_d5py.get('upsilon_clean_i') arr_upsilon_clean_i=np.array(upsilon_clean_i).T
upsilon_fossil_i = natal_d5py.get('upsilon_fossil_i') arr_upsilon_fossil_i=np.array(upsilon_fossil_i).T

# Loading results from the backward simulation
backward_d5py = h5py.File(datapath+'Derived/results_backward_Warm_med.mat','r')
amen_Warm_b = backward_d5py.get('amen_Warm_b') arr_amen_Warm_b=np.array(amen_Warm_b).T
clean_Warm_b = backward_d5py.get('clean_Warm_b')
arr_clean_Warm_b=np.array(clean_Warm_b).T
emiCO2_ff_Warm_b = backward_d5py.get('emiCO2_ff_Warm_b')
arr_emiCO2_ff_Warm_b=np.array(emiCO2_ff_Warm_b).T
l_Warm_b = backward_d5py.get('l_Warm_b')
arr_l_Warm_b=np.array(l_Warm_b).T
net_births_Warm_b = backward_d5py.get('net_births_Warm_b')
arr_net_births_Warm_b=np.array(net_births_Warm_b).T
price_clean_Warm_b = backward_d5py.get('price_clean_Warm_b')
arr_price_clean_Warm_b=np.array(price_clean_Warm_b).T
```

```
price_emi_Warm_b = backward_d5py.get('price_emi_Warm_b')
arr_price_emi_Warm_b=np.array(price_emi_Warm_b).T
prod_Warm_b = backward_d5py.get('prod_Warm_b')
arr_prod_Warm_b=np.array(prod_Warm_b).T
realgdp_Warm_b = backward_d5py.get('realgdp_Warm_b')
arr_realgdp_Warm_b=np.array(realgdp_Warm_b).T
temp_local_Warm_b = backward_d5py.get('temp_local_Warm_b')
arr_temp_local_Warm_b=np.array(temp_local_Warm_b).T
u_Warm_b = backward_d5py.get('u_Warm_b')
arr_u_Warm_b=np.array(u_Warm_b).T

# Runs model forward -- baseline estimation
# Set features of the simulation
T = 250   # number of periods - we chose 250
ind_dam = 8    # damage function level: baseline
name_dam = name_dam_vect[ind_dam] # name of damage function level
ind_exo = 0    # exogenous temperature and population path
taxCO2 = np.zeros((n,T)) # No taxes
#taxCO2 = 0.5*(np.ones((n,T)))     # Path for carbon taxes
#taxCO2_growth = 1.0363            # Growth rate of taxes for each year.
#taxCO2 = taxCO2* np.tile(taxCO2_growth**np.arange(T), (n, 1)) # Updating taxCO2.
subclean = np.zeros((n,T))        # path for clean energy subsidies
abat = np.zeros((n,T))            # path for abatement
val_adap = np.ones((4,1))         # degree of adaptation: baseline
migr_exp = np.ones((2,1))         # border frictions
ind_agri = 0                      # sectoral decomposition
ind_clim = 1 # source of damages, so that a value of 0 indicates no damages,  1 damages \
# on both amenities and productivities, 2 damages only on amenities,  and \
# 3 damages only on productivity (= 1 in baseline).

# Initialize Output Variables
# Creating matrixes for relevant output and variables.
l = np.zeros((n, T))
u = np.zeros((n, T))
prod = np.zeros((n, T))
phi = np.zeros((n, T))
realgdp = np.zeros((n, T))
realincome = np.zeros((n, T))
realgdp_w = np.zeros((T, 1))
price_fossil = np.zeros((n, T))
price_clean = np.zeros((n, T))
price_energy = np.zeros((n, T))
price_energy_tilde = np.zeros((n, T)) v
arphi = np.zeros((n, T))
zeta_fossil = np.zeros((n, T))
zeta_clean = np.zeros((n, T))
clean = np.zeros((n, T))
cumCO2_ff = np.zeros((T, 1))
emiCO2_ff_abat = np.zeros((n, T))
emiCO2_total = np.zeros((T, 1))
stockCO2_layers = np.zeros((4, T))
forc = np.zeros((T, 1))
temp_layers = np.zeros((2, T))
temp_local = np.zeros((n, T))
amen = np.zeros((n, T))
net_births = np.zeros((n, T))

if ind_exo == 0: # endogenous CO2 emissions, temperature and population
        emiCO2_ff = np.zeros((n, T))
        temp_global = np.zeros((T, 1))
        lbar_time = np.zeros((T+1, 1))
elif ind_exo == 1:   # exogenous CO2 emissions, temperature and population
        if ind_clim == 1:
                data_Warm = np.load(datapath+'derived/results_forward_Warm_' + \
                name_dam_vect[ind_dam] + '.npz')
                emiCO2_ff = data_Warm['emiCO2_ff_Warm']
                temp_global = data_Warm['temp_global_Warm']
                l_Warm = data_Warm['l_Warm']
                lbar_time = np.sum(l_Warm * np.tile(H, (T, 1)), axis=0)
        else:
                data_noWarm = np.load(datapath+'derived/results_forward_noWarm.npz')
                emiCO2_ff = data_noWarm['emiCO2_ff_noWarm']
                temp_global = data_noWarm['temp_global_noWarm']
                l_noWarm = data_noWarm['l_noWarm']
                lbar_time = np.sum(l_noWarm * np.tile(H, (T, 1)), axis=0)
elif ind_exo == 2:
        lbar_time = np.tile(lbar, (T+1, 1)).T
elif ind_exo == 3:
        lbar_time = np.tile(lbar, (T+1, 1)).T
        if ind_clim == 1:
                data_Warm = np.load(datapath+'derived/results_forward_Warm_' + \
                name_dam_vect[ind_dam] + '.npz')
                emiCO2_ff = data_Warm['emiCO2_ff_Warm']
```

```
                temp_global = data_Warm['temp_global_Warm']
        else:
                data_noWarm = np.load(datapath+'derived/results_forward_noWarm.npz')
                emiCO2_ff = data_noWarm['emiCO2_ff_noWarm']
                temp_global = data_noWarm['temp_global_noWarm']

# Initialize Parameters and variables of initial period
# Read parameters
data_natal = h5py.File(datapath+'derived/results20_med.mat','r')
m2 = np.array(data_natal.get('m2_i')).T
m2 = m2.reshape(-1)
coeff_pop = np.array(data_natal.get('coeff_pop_i')).T
coeff_pop = coeff_pop.reshape(-1)
upsilon_fossil = data_natal['upsilon_fossil_i'][0, 0]
upsilon_clean = data_natal['upsilon_clean_i'][0, 0]
data_20 = h5py.File(datapath+'derived/results20_med.mat','r')
l0_model = data_20['l0_model'][:].T
l0_model = l0_model.reshape(-1)
realgdp0 = data_20['realgdp0_model'][:].T
realgdp0 = realgdp0.reshape(-1)
realgdp0_w = np.sum(realgdp0 * l0_model * H) / lbar

# Adjust migration costs
if migr_exp[0] != 1:
        length_border_vect = length_D_vect
        border_vect = D_vect
        border_migr_exp = migr_exp[0]
elif migr_exp[1] != 1:
        length_border_vect = length_Africa_vect
        border_vect = Africa_vect
        border_migr_exp = migr_exp[1]
if np.sum(np.abs(migr_exp - 1)) != 0:
        n2 = np.zeros(n)
        for i in range(length_border_vect):
                n2_aux = np.sum(m2[border_vect==i] * l0_model[border_vect==i] * H[border_vect==i]) / \
                  np.sum(l0_model[border_vect==i] * H[border_vect==i])
                n2[border_vect==i] = n2_aux
        m2 = (m2 / n2)
        m2 = m2 / np.min(m2)
        n2 = n2 / np.min(n2)
        m2 = m2 * (n2**border_migr_exp)

# Update adaptation parameters
trmult_reduced_aux = trmult_reduced ** val_adap[0]
m2_aux = m2 ** val_adap[1]
gamma1_aux = gamma1 * val_adap[2]
Omega_aux = Omega * val_adap[3]

# Update productivity and amenities
avgprod = np.mean(prod0)
const_phi = ((gamma1_aux / ksi) / (nu * (mu + gamma1_aux / ksi)))**(1/ksi)
const_energy = (1 - chi) * mu / (mu + gamma1_aux / ksi)
phi0 = const_phi * l0_model**(1/ksi)
prod[:,0] = eta * prod0**gamma2 * avgprod**(1-gamma2) * phi0**(gamma1_aux*theta)
amen[:,0] = a_norm

# Set CO2 stock, forcing and temperature
if ind_exo == 0 or ind_exo == 2:
        stockCO2_layers[0,0] = S0 + a0*emi0
        stockCO2_layers[1,0] = (np.exp(-1/b1))*S1 + a1*emi0
        stockCO2_layers[2,0] = (np.exp(-1/b2))*S2 + a2*emi0
        stockCO2_layers[3,0] = (np.exp(-1/b3))*S3 + a3*emi0
        forc[0] = forc_sens*np.log(np.sum(stockCO2_layers[:,0])/S_preind)/np.log(2) + forc_noCO2[0]
        temp_layers[0,0] = (np.exp(-1/d1))*temp1 + (c1/d1)*forc[0]
        temp_layers[1,0] = (np.exp(-1/d2))*temp2 + (c2/d2)*forc[0]
        temp_global[0] = np.sum(temp_layers[:,0])

temp_local[:,0] = temp0_local.flatten() + scaler_temp.squeeze()*(temp_global[0]-temp0_global)
temp_local_aux = temp_local[:,0]
Delta_temp = temp_local[:,0] - temp0_local.flatten()

# Set damage function level by confidence interval
if ind_clim != 0:
        if ind_agri == 0:
                theta_amen_min = theta_amen_scen[0, ind_dam]
                theta_amen_max = theta_amen_scen[1, ind_dam]
                theta_amen_center = theta_amen_scen[2, ind_dam]
                theta_amen_steep = theta_amen_scen[3, ind_dam]
                theta_amen_temp = lambda temp: theta_amen_min + (theta_amen_max - \
                        theta_amen_min) / (1 + np.exp(theta_amen_steep * (temp - theta_amen_center)))

                theta_prod_min = theta_prod_scen[0, ind_dam]
                theta_prod_max = theta_prod_scen[1, ind_dam]
```

```
                theta_prod_center = theta_prod_scen[2, ind_dam]
                theta_prod_steep = theta_prod_scen[3, ind_dam]
                theta_prod_temp = lambda temp: theta_prod_min + (theta_prod_max - theta_prod_min) / \
                        (1 + np.exp(theta_prod_steep * (temp - theta_prod_center)))

        else:
                theta_amen_min = theta_amen_scen_agri[0, :]
                theta_amen_max = theta_amen_scen_agri[1, :]
                theta_amen_center = theta_amen_scen_agri[2, :]
                theta_amen_steep = theta_amen_scen_agri[3, :]
                theta_amen_temp = lambda temp, agri_index: theta_amen_min[agri_index] + \
                        (theta_amen_max[agri_index] - theta_amen_min[agri_index]) / (1 + \
                        np.exp(theta_amen_steep[agri_index] * (temp - theta_amen_center[agri_index])))

                theta_prod_min = theta_prod_scen_agri[0, :]
                theta_prod_max = theta_prod_scen_agri[1, :]
                theta_prod_center = theta_prod_scen_agri[2, :]
                theta_prod_steep = theta_prod_scen_agri[3, :]
                theta_prod_temp = lambda temp, agri_index: theta_prod_min[agri_index] + \
                        (theta_prod_max[agri_index] - theta_prod_min[agri_index]) / (1 + \
                        np.exp(theta_prod_steep[agri_index] * (temp - theta_prod_center[agri_index])))

# Set damage function sources: amenities or productivities
if ind_clim == 1:      # damages on both amenities and productivities
        if ind_agri == 0:
                amen[:,0] = (1 + theta_amen_temp(temp_local_aux) * Delta_temp) * amen[:,0]
                prod[:,0] = (1 + theta_prod_temp(temp_local_aux) * Delta_temp) * prod[:,0]
        else:
                amen[:,0] = (1 + theta_amen_temp(temp_local_aux,agri_index) * Delta_temp) * amen[:,0]
                prod[:,0] = (1 + theta_prod_temp(temp_local_aux,agri_index) * Delta_temp) * prod[:,0]
elif ind_clim == 2:   # damages only on amenities
        if ind_agri == 0:
                amen[:,0] = (1 + theta_amen_temp(temp_local_aux) * Delta_temp) * amen[:,0]
        else:
                amen[:,0] = (1 + theta_amen_temp(temp_local_aux,agri_index) * Delta_temp) * amen[:,0]
elif ind_clim == 3:   # damages only on productivities
        if ind_agri == 0:
                prod[:,0] = (1 + theta_prod_temp(temp_local_aux) * Delta_temp) * prod[:,0]
        else:
                prod[:,0] = (1 + theta_prod_temp(temp_local_aux,agri_index) * Delta_temp) * prod[:,0]

# Defining the natality function
def natal_fct(logrealgdp, temp, logrealgdp_w, coeff_pop_d):
        data_20 = h5py.File(datapath+'derived/results20_med.mat','r')
        l0_model = data_20['l0_model'][:].T
        l0_model = l0_model.reshape(-1)
        realgdp0 = data_20['realgdp0_model'][:].T
        realgdp0 = realgdp0.reshape(-1)
        l20 = data_20['l20'][:].T
        l20 = l20.reshape(-1)
        realgdp20 = data_20['realgdp20'][:].T
        realgdp20 = realgdp20.reshape(-1)
        temp20 = data_20['temp20'][:].T
        temp20 = temp20.reshape(-1)
        logrealgdp0 = np.log(realgdp0)
        realgdp0_w = np.sum(realgdp0 * l0_model * H) / lbar
        logrealgdp0_w = np.log(realgdp0_w)
        logrealgdp20 = np.log(realgdp20)
        logrealgdp20_w = np.log(np.sum(realgdp20 * l20 * H) / np.sum(l20 * H))
        pop0_sh = l0_model * H / lbar     pop20_sh = l20 * H / np.sum(l20 * H)
        # Read numerical restrictions on natality functions
        b0y_max, b1y_min, b1y_max, b2y_min, b2y_max, b2T_max, bsy_min, bsy_max, bsT_min, \
        bsT_max = natal_param
        logi_b2T_fct = lambda x: b2T_max / (1 + np.exp(-x))
        logi_b0y_fct = lambda x: b0y_max / (1 + np.exp(-x))
        logi_b1y_fct = lambda x: b1y_min + (b1y_max - b1y_min) / (1 + np.exp(-x))
        logi_b2y_fct = lambda x: b2y_min + (b2y_max - b2y_min) / (1 + np.exp(-x))
        logi_bsy_fct = lambda x: bsy_min + (bsy_max - bsy_min) / (1 + np.exp(-x))
        logi_bsT_fct = lambda x: bsT_min + (bsT_max - bsT_min) / (1 + np.exp(-x))
    natal_fct_logrealgdp = lambda logrealgdp, coeff_pop: (logi_b0y_fct(coeff_pop_d[0]) + \
            (logi_b2y_fct(coeff_pop_d[3])-logi_b0y_fct(coeff_pop_d[0])) *  np.exp(- \
                logi_b1y_fct(coeff_pop_d[1]) * np.square(logrealgdp - logi_bsy_fct(coeff_pop_d[4]))) * \
            (logrealgdp < logi_bsy_fct(coeff_pop_d[4]))) + (0 + (logi_b2y_fct(coeff_pop_d[3])-0) * \
            np.exp(-np.exp(coeff_pop_d[2]) * np.square(logrealgdp - logi_bsy_fct(coeff_pop_d[4]))) * \
            (logrealgdp >= logi_bsy_fct(coeff_pop_d[4])))



        # Call the function
        result = natal_fct_logrealgdp(logrealgdp, coeff_pop_d)

        # Define b0T
        b0T = 2 * natal0 - logi_b2T_fct(coeff_pop_d[6]) * np.sum(np.exp(-np.exp(coeff_pop_d[5]) \
        *(temp0_local - logi_bsT_fct(coeff_pop_d[7])) ** 2) * pop0_sh) - 2 \
```

```python
        * np.sum(natal_fct_logrealgdp(logrealgdp0, coeff_pop_d) * pop0_sh)

        # Construct bw
        term1 = b0T + np.sum(logi_b2T_fct(coeff_pop_d[6]) * np.exp(-np.exp(coeff_pop_d[5]) * \
        (temp20 - logi_bsT_fct(coeff_pop_d[7])) ** 2) * pop20_sh)
        term2 = natal20 - np.sum(natal_fct_logrealgdp(logrealgdp20, coeff_pop_d) * pop20_sh)
        term3 = logrealgdp20_w - logrealgdp0_w
        bw = np.log(-1 + (term1 * np.ones_like(term2)) / term2) / term3

        # Construct numerator of temperature component of natality rate function.
        natal_fct_temp_num = b0T + logi_b2T_fct(coeff_pop[6]) * np.exp(-np.exp(coeff_pop[5]) \
        * (temp_local_aux - logi_bsT_fct(coeff_pop[7])) ** 2)

        # Construct denominator of temperature component of natality rate function.
        natal_fct_temp_denom = (1 + np.exp(bw * (logrealgdp_w - logrealgdp0_w)))


        # Construct natality functions
        natal_fct_val = np.real(natal_fct_temp_num / natal_fct_temp_denom + \
        natal_fct_logrealgdp(logrealgdp, coeff_pop_d))

        return natal_fct_val


# Calling the natality function
logrealgdp0 = np.log(realgdp0)
logrealgdp0_w = np.log(realgdp0_w)
if ind_exo == 0:
        net_births0 = natal_fct(logrealgdp0, temp0_local, logrealgdp0_w, coeff_pop)
        pop_prev = H * l0_model * (1 + net_births0)
        lbar_time[0] = round(sum(pop_prev))

# Define function for extraction cost function
def costCO2_fct(cumCO2):
        return cost_emi_param[0]/(cost_emi_param[1] + np.exp(-cost_emi_param[2]*(cumCO2- \
                cost_emi_param[3]))) - cost_emi_param[4]*(cost_emi_param[5]/(cumCO2- \
                cost_emi_param[5]))**cost_emi_param[6]

# Precomputing auxiliary variables
denom = 1 + 2 * theta
squ = (alpha - 1 + (lambda1 + gamma1_aux / ksi - (1 - mu)) * theta)  # term in square brackets \
# of flat_R and flat_L

flatL = lambda1 - squ / denom
flatR = 1 - lambda1 * theta + (1 + theta) * squ / denom
flat = flatR - theta * flatL
exp_uhatL = flatL / Omega_aux + (1 + theta) / denom
exp_uhatR = flatR / Omega_aux - theta**2 / denom

FL_H_m2 = (np.power(H, (flatL-1/denom)/exp_uhatL) * np.power(m2_aux, \
flatL/(Omega_aux*exp_uhatL)))
FR_H_m2 = (np.power(H, -flatR+theta/denom) * np.power(m2_aux, -flatR/Omega_aux))

# Set guesses for the simulation.
uhat_i = np.ones(n) # guess for uhat
if ind_exo == 0 or ind_exo == 2:
        emi_ff_i = emi0_ff  # guess for global CO2 emissions
realgdp_growth_i = 1.017  # guess for global realgdp growth
uhat_i = uhat_i

# Set numerical parameters
updatee = 1    # speed of update when iterating over CO2 emissions
updater = 1
tol = 1e-2  # tolerance for error when iterating over uhat
tol_e = 1e-2   # tolerance for error when iterating over CO2 emissions
tol_realgdp = 1e-2

# Simulating the model
i_1st = 0
i_2nd = 0
i_3rd = 0

# Creating dictionary of input variables for the simulation
my_dict = {
        'zeta_fossil0': zeta_fossil0,
        'zeta_clean0': zeta_clean0,
        'realgdp0_w': realgdp0_w,
        'emi0_ff': emi0_ff,
        'zeta_fossil': zeta_fossil,
        'zeta_clean': zeta_clean,
        'realgdp_w_prev': realgdp_w,
        'cumCO2_ff': cumCO2_ff,
        'H': H,
```

```
        'cost_emi_param': cost_emi_param,
        'costCO2_fct': costCO2_fct,
        'amen': amen,
        'theta': theta,
        'denom': denom,
        'exp_uhatL': exp_uhatL,
        'prod': prod,
        'FL_H_m2': FL_H_m2,
        'FR_H_m2': FR_H_m2,
        'tol_realgdp': tol_realgdp,
        'i_1st': i_1st,
        'i_2nd': i_2nd,
        'i_3rd': i_3rd,
        'tol_e': tol_e,
        'price_clean0_world': price_clean0_world,
        'fossil_share': fossil_share,
        'taxCO2': taxCO2,
        'eps': eps,
        'subclean': subclean,
        'mu': mu,
        'chi': chi,
        'gamma1_aux': gamma1_aux,
        'ksi': ksi,
        'uhat_i': uhat_i,
        'exp_uhatR': exp_uhatR,
        'trmult_reduced_aux': trmult_reduced_aux,
        'Omega_aux': Omega_aux,
        'm2_aux': m2_aux,
        'lbar_time': lbar_time,
        'const_phi': const_phi,
        'flat': flat,
        'u': u,
        'lambda1': lambda1,
        'eta': eta,
        'gamma2': gamma2,
        'avgprod': avgprod,
        'const_energy': const_energy,
        'ind_exo': ind_exo,
        'T': T,
        'abat': abat,
        'emiCO2_ff': emiCO2_ff,
        'emi_no_ff': emi_no_ff,
        'updatee': updatee,
        'emi_ff_i': emi_ff_i,
        'realgdp_growth_i': realgdp_growth_i,
        'updater': updater,
        'stockCO2_layers': stockCO2_layers,
        'a0': a0,
        'b1': b1,
        'a1': a1,
        'a2': a2,
        'a3': a3,
        'b2': b2,
        'b3': b3,
        'forc_sens': forc_sens,
        'S_preind': S_preind,
        'forc_noCO2': forc_noCO2,
        'temp_layers': temp_layers,
        'c1': c1,
        'c2': c2,
        'd1': d1,
        'd2': d2,
        'temp0_global': temp0_global,
        'temp0_local': temp0_local,
        'scaler_temp': scaler_temp,
        'ind_clim': ind_clim,
        'ind_agri': ind_agri,
        'theta_amen_temp': theta_amen_temp,
        'temp_local_aux': temp_local_aux,
        'theta_prod_temp': theta_prod_temp,
        'agri_index': agri_index,
        'natal_fct': natal_fct,
        'coeff_pop': coeff_pop,
}

# Defining the function for the simulation
def my_function(my_dict):
        zeta_fossil0 = my_dict['zeta_fossil0']
        zeta_clean0 = my_dict['zeta_clean0']
        realgdp0_w = my_dict['realgdp0_w']
        emi0_ff = my_dict['emi0_ff']
        zeta_fossil = my_dict['zeta_fossil']
        zeta_clean = my_dict['zeta_clean']
```

```
        realgdp_w_prev = my_dict['realgdp_w_prev']
        cumCO2_ff = my_dict['cumCO2_ff']
        H = my_dict['H']
        cost_emi_param = my_dict['cost_emi_param']
        costCO2_fct = my_dict['costCO2_fct']
        amen = my_dict['amen']
        theta = my_dict['theta']
        denom = my_dict['denom']
        exp_uhatL = my_dict['exp_uhatL']
        prod = my_dict['prod']
        FL_H_m2 = my_dict['FL_H_m2']
        FR_H_m2 = my_dict['FR_H_m2']
        tol_realgdp = my_dict['tol_realgdp']
        i_1st = my_dict['i_1st']
        i_2nd = my_dict['i_2nd']
        i_3rd = my_dict['i_3rd']
        tol_e = my_dict['tol_e']
        price_clean0_world = my_dict['price_clean0_world']
        fossil_share = my_dict['fossil_share']
        taxCO2 = my_dict['taxCO2']
        eps = my_dict['eps']
        subclean = my_dict['subclean']
        mu = my_dict['my']
        chi = my_dict['chi']
        gamma1_aux = my_dict['gamma1_aux']
        ksi = my_dict['ksi']
        uhat_i = my_dict['uhat_i']
        exp_uhatR = my_dict['exp_uhatR']
        trmult_reduced_aux = my_dict['trmult_reduced_aux']
        Omega_aux = my_dict['Omega_aux']
        m2_aux = my_dict['m2_aux']
        lbar_time = my_dict['lbar_time']
        const_phi = my_dict['const_phi']
        flat = my_dict['flat']
        u = my_dict['u']
        lambda1 = my_dict['lambda1']
        eta = my_dict['eta']
        gamma2 = my_dict['gamma2']
        avgprod = my_dict['avgprod']
        const_energy = my_dict['const_energy']
        ind_exo = my_dict['ind_exo']
        T = my_dict['T']
        abat = my_dict['abat']
        emiCO2_ff = my_dict['emiCO2_ff']
        emi_no_ff = my_dict['emi_no_ff']
        updatee = my_dict['updatee']
        emi_ff_i = my_dict['emi_ff_i']
        realgdp_growth_i = my_dict['realgdp_growth_i']
        updater = my_dict['updater']
        stockCO2_layers = my_dict['stockCO2_layers']
        a0 = my_dict['a0']
        b1 = my_dict['b1']
        a1 = my_dict['a1']
        a2 = my_dict['a2']
        a3 = my_dict['a3']
        b2 = my_dict['b2']
        b3 = my_dict['b3']
        forc_sens = my_dict['forc_sens']
        S_preind = my_dict['S_preind']
        forc_noCO2 = my_dict['forc_noCO2']
        temp_layers = my_dict['temp_layers']
        c1 = my_dict['c1']
        c2 = my_dict['c2']
        d1 = my_dict['d1']
        d2 = my_dict['d2']
        temp0_local = my_dict['temp0_local']
        temp0_global = my_dict['temp0_global']
        scaler_temp = my_dict['scaler_temp']
        ind_clim = my_dict['ind_clim']
        ind_agri = my_dict['ind_agri']
        theta_amen_temp = my_dict['theta_amen_temp']
        temp_local_aux = my_dict['temp_local_aux']
        theta_prod_temp = my_dict['theta_prod_temp']
        agri_index = my_dict['agri_index']
        natal_fct = my_dict['natal_fct']
        coeff_pop = my_dict['coeff_pop']

        for t in range(T):
                if t == 0:
                        zeta_fossil_prev = zeta_fossil0
                        zeta_clean_prev = zeta_clean0
                        realgdp_w_prev = realgdp0_w
                        cumCO2_ff[t] = emi0_ff
```

```
            else:
                zeta_fossil_prev = zeta_fossil[:, t-1]
                zeta_clean_prev = zeta_clean[:, t-1]
                realgdp_w_prev = realgdp_w[t-1]
                cumCO2_ff_aux = cumCO2_ff[t-1] + sum(emiCO2_ff[:, t-1] * H)
                ind_cumCO2 = (cumCO2_ff_aux >= cost_emi_param[5] - 0.5)
                if ind_cumCO2:
                    cumCO2_ff_aux = cost_emi_param[5] - 0.5
                cumCO2_ff[t] = cumCO2_ff_aux

            # Compute extraction cost
            costCO2_avg_i = costCO2_fct(cumCO2_ff[t])


            # Pre-compute auxiliary variables to equation (31)
            FL_aux = amen[:,t]**((1+theta)/(denom*exp_uhatL)) * prod[:,t]**(1/(denom*exp_uhatL)) * \
            FL_H_m2  # outside the integral

            FR_aux = amen[:,t]**(theta**2/denom) * prod[:,t]**((1+theta)/denom) * FR_H_m2  \
            # inside the integral

            error_realgdp = 1 + tol_realgdp
            i_1st = 0
            while error_realgdp > tol_realgdp:
                i_1st += 1
                # Update energy productivity, equation (10)
                zeta_fossil[:,t] = zeta_fossil_prev*(realgdp_growth_i)**(upsilon_fossil)
                zeta_clean[:,t] = zeta_clean_prev*(realgdp_growth_i)**(upsilon_clean)

                error_e = 1 + tol_e
                i_2nd = 0
                while error_e > tol_e:
                    i_2nd += 1

                    # Construct energy price, equations (8), (21), (23)
                    price_fossil[:,t] = costCO2_avg_i/zeta_fossil[:,t]
                    price_clean[:,t] = price_clean0_world/zeta_clean[:,t]
                    price_energy[:,t] = (fossil_share**eps*(1+taxCO2[:,t])**(1-eps) \
                        *price_fossil[:,t]**(1-eps)+ (1-fossil_share)**eps*(1-subclean[:,t]) \
                        **(1-eps)*price_clean[:,t]**(1- eps))**(1/(1-eps))
                    price_energy_tilde[:,t] = (fossil_share**eps*(1+taxCO2[:,t])**(-eps) \
                        *price_fossil[:,t]**(1-eps) + (1-fossil_share)**eps*(1-subclean[:,t])** \
                        (-eps)*price_clean[:,t]**(1-eps))**(1/(1-eps))
                    varphi[:,t] = (mu*chi + gamma1_aux/ksi + mu*(1-chi)*(price_energy_tilde[:,t] \
                        /price_energy[:,t])**(1-eps))/\(mu+gamma1_aux/ksi)

                    # Precompute auxiliary variables to equation (31)
                    FL = FL_aux * price_energy[:,t]**(-theta*mu*(1-chi)/(denom*exp_uhatL)) * \
                        varphi[:,t]**(theta*(mu+gamma1_aux/ksi)/(denom*exp_uhatL))
                        # outside the integral

                    FR = FR_aux * price_energy[:,t]**(-theta*(1+theta)*mu*(1-chi)/denom) * \
                        varphi[:,t]**(theta*(1+theta)*(mu+gamma1_aux/ksi)/denom)
                        # inside the integral

                    # Iterate uhat, equation (31)
                    error = tol + 1
                    i_3rd = 0
                    while error >= tol:
                        i_3rd += 1
                        integral = FR * np.power(uhat_i, exp_uhatR)
                        rhs = np.matmul(trmult_reduced_aux, integral)
                        uhat_f = FL * rhs**(1/(theta*exp_uhatL))
                        error = np.sum((uhat_i - uhat_f)**2)
                        uhat_i = uhat_f.copy()

                    # Solve for population, equation (3), and scale it to add up to lbar_time(t)
                    l[:,t] = H**(-1) * uhat_i**(1/Omega_aux) * m2_aux**(-1/Omega_aux)
                    l[:,t] = l[:,t] / np.sum(H*l[:,t]) * lbar_time[t]

                    # Calculate innovation
                    phi[:,t] = const_phi * (l[:,t]/varphi[:,t])**(1/ksi)

                    # Retrieve utility, equation (71)
                    u[:,t] = uhat_i * (lbar_time[t]/np.sum(uhat_i**(1/Omega_aux)*m2_aux** \
                            (-1/Omega_aux)))**(flat/theta)

                    # Calculate real income and real GDP per capita, equation (4)
                    realincome[:,t] = u[:,t] / amen[:,t] * l[:,t]**lambda1
                    realgdp[:,t] = (1 + (mu+gamma1_aux/ksi)*(varphi[:,t]-1)) * realincome[:,t]

                    realgdp_w[t] = sum(realgdp[:,t] * H * l[:,t]) / sum(H * l[:,t])

```

```
                # Update productivity, equation (6)
                if t < T-1:
                avgprod = np.mean(prod[:,t])
                prod[:,t+1] = eta * prod[:,t]**gamma2 * avgprod**(1-gamma2) * \
                                phi[:,t]**(gamma1_aux*theta)

                # Calculate clean energy use, equation (30)
                clean[:,t] = const_energy * (1 / varphi[:,t]) * (l[:,t] / price_energy[:,t]) * \
                ((1-fossil_share) * price_energy[:,t] / ((1-subclean[:,t]) * price_clean[:,t]))**eps

                # Calculate CO2 emissions, equation (29)
                if ind_exo == 0 or ind_exo == 2:
                   emiCO2_ff[:,t] = const_energy * (1 / varphi[:,t]) * (l[:,t] / price_energy[:,t]) * \
                   fossil_share * price_energy[:,t] / ((1+taxCO2[:,t]) * price_fossil[:,t]))**eps

                # Update CO2 emissions by abatement
                emiCO2_ff_abat[:,t] = (1 - abat[:,t]) * emiCO2_ff[:,t]
                emi_ff_f = np.sum(emiCO2_ff[:,t] * H)
                emi_ff_f_abat = np.sum(emiCO2_ff_abat[:,t] * H)
                emiCO2_total[t] = emi_ff_f_abat + emi_no_ff[t]

                # Compare global CO2 emissions
                if ind_exo == 0 or ind_exo == 2:
                        error_e = np.abs(emi_ff_f - emi_ff_i)
                        emi_ff_i = updatee * emi_ff_f + (1 - updatee) * emi_ff_i
                        costCO2_avg_i = np.mean(costCO2_fct(cumCO2_ff[t] + \
                                np.linspace(0,emi_ff_i,100)))
                else:
                        error_e = 0

        # Compare growth rate realgdp
        realgdp_growth_f = realgdp_w[t] / realgdp_w_prev
        error_realgdp = abs(realgdp_growth_f - realgdp_growth_i)
        realgdp_growth_i = updater * realgdp_growth_f + (1 - updater) * realgdp_growth_i

# Set growth rate of previous period
realgdp_growth_i = realgdp_growth_f

if t<T-1:
# Update CO2 stock, forcing and temperature, equations (9), (18) and (35)
if ind_exo == 0 or ind_exo == 2:
        stockCO2_layers[0,t+1] = stockCO2_layers[0,t] + a0*emiCO2_total[t]
        stockCO2_layers[1,t+1] = np.exp(-1/b1)*stockCO2_layers[1,t] + a1*emiCO2_total[t]
        stockCO2_layers[2,t+1] = np.exp(-1/b2)*stockCO2_layers[2,t] + a2*emiCO2_total[t]
        stockCO2_layers[3,t+1] = np.exp(-1/b3)*stockCO2_layers[3,t] + a3*emiCO2_total[t]
        forc[t+1] = forc_sens*np.log(np.sum(stockCO2_layers[:,t+1])/S_preind)/np.log(2) + \
                forc_noCO2[t+1]

        temp_layers[0,t+1] = np.exp(-1/d1)*temp_layers[0,t] + (c1/d1)*forc[t+1]
        temp_layers[1,t+1] = np.exp(-1/d2)*temp_layers[1,t] + (c2/d2)*forc[t+1]

        temp_global[t+1] = np.sum(temp_layers[:,t+1])
        temp_local[:,t+1] = temp0_local + scaler_temp*(temp_global[t+1]-temp0_global)

        # Update local temperature
        if ind_clim != 0:
                temp_local_aux = temp_local[:,t+1]
                Delta_temp = temp_local[:,t+1] - temp_local[:,t]

        # Update damages on amenities and productivities, equations (2) and (6)
        if ind_clim == 1: # damages on both amenities and productivities
                if ind_agri == 0:
                        amen[:,t+1] = (1+theta_amen_temp(temp_local_aux)*Delta_temp)*amen[:,t]
                        prod[:,t+1] = (1+theta_prod_temp(temp_local_aux)*Delta_temp)*prod[:,t+1]
                else:
                        amen[:,t+1] = \
                                (1+theta_amen_temp(temp_local_aux,agri_index)*Delta_temp)*amen[:,t]
                prod[:,t+1] = \
                                (1+theta_prod_temp(temp_local_aux,agri_index)*Delta_temp)*prod[:,t+1]
        elif ind_clim == 2: # damages only on amenities
                if ind_agri == 0:
                        amen[:,t+1] = (1+theta_amen_temp(temp_local_aux)*Delta_temp)*amen[:,t]
                else:
                        amen[:,t+1] = \
                                (1+theta_amen_temp(temp_local_aux,agri_index)*Delta_temp)*amen[:,t]
        elif ind_clim == 3: # damages only on productivities
                if ind_agri == 0:
                        prod[:,t+1] = (1+theta_prod_temp(temp_local_aux)*Delta_temp)*prod[:,t+1]
                else:
                        prod[:,t+1] = \
                                (1+theta_prod_temp(temp_local_aux,agri_index)*Delta_temp)*prod[:,t+1]
                amen[:,t+1] = amen[:,t]
        elif ind_clim == 0:  # No damages
```

```
                                        amen[:,t+1] = amen[:,t]

                        # Update global population
                        log_realgdp = np.log(realgdp[:,t])
                        log_realgdp_w = np.log(realgdp_w[t])
                        if ind_exo == 0:
                                net_births[:,t] = natal_fct(log_realgdp, temp_local_aux, log_realgdp_w, coeff_pop)
                                pop_prev = (1+net_births[:,t]) * l[:,t] * H
                                lbar_time[t+1] = round(np.sum(pop_prev))

        # Sum CO2 stock layers, equation (34)
        stockCO2 = np.sum(stockCO2_layers, axis=0)

        # Assign values to the variables
        l_Warm = l
        u_Warm = u
        prod_Warm = prod
        realgdp_Warm = realgdp
        amen_Warm = amen
        emiCO2_ff_Warm = emiCO2_ff
        emiCO2_total_Warm = emiCO2_total
        stockCO2_Warm = stockCO2
        temp_global_Warm = temp_global
        temp_local_Warm = temp_local
        price_emi_Warm = price_fossil
        clean_Warm = clean
        price_clean_Warm = price_clean
        net_births_Warm = net_births

        return (l_Warm, u_Warm, prod_Warm, realgdp_Warm, amen_Warm, emiCO2_ff_Warm,
            emiCO2_total_Warm, stockCO2_Warm, temp_global_Warm, temp_local_Warm,
            price_emi_Warm, clean_Warm, price_clean_Warm, net_births_Warm)

# Call the function and run the simulation
l_Warm, u_Warm, prod_Warm, realgdp_Warm, amen_Warm, emiCO2_ff_Warm,
emiCO2_total_Warm, stockCO2_Warm, temp_global_Warm, temp_local_Warm,
price_emi_Warm, clean_Warm, price_clean_Warm, net_births_Warm = my_function(my_dict)

# Saving the output of the simulation
# Create a dictionary with desired variable names
data = {
        'l_Warm': l_Warm,
        'u_Warm': u_Warm,
        'prod_Warm': prod_Warm,
        'realgdp_Warm': realgdp_Warm,
        'amen_Warm': amen_Warm,
        'emiCO2_ff_Warm': emiCO2_ff_Warm,
        'emiCO2_total_Warm': emiCO2_total_Warm,
        'stockCO2_Warm': stockCO2_Warm,
        'temp_global_Warm': temp_global_Warm,
        'temp_local_Warm': temp_local_Warm,
        'price_emi_Warm': price_emi_Warm,
        'clean_Warm': clean_Warm,
        'price_clean_Warm': price_clean_Warm,
        'net_births_Warm': net_births_Warm,
}

# Save the variables
save_path = '/Users/jonebo/Documents/BI/Masteroppgave/Code_EGGW/Data/Derived/'
np.savez(save_path + 'results_forward_Warm_EPS06.npz', **data)
```

Listing 2: Python code: Forward Climate Function

8.3 Forward Iteration

8.3.1 Input to the code

The following variables are required as inputs for the forward climate fucntion, in order to set the parameters of the simulation and provide the necessary data for the calculations.

- T: is the number of time periods for which the economy will be simulated.

- ind_clim: the source of damages, which can take on values from 0 to 3. A value of 0 indicates no damages, while a value of 1 indicates damages to both amenities and productivities. A value of 2 indicates damages only to amenities, and a value of 3 indicates damages only to productivity.

- ind_dam: the level of the damage function, which can take on values from 1 to 9. Values of 1 and 2 correspond to the lower and upper curves of the 95% confidence interval, respectively. Values of 3 and 4 correspond to the lower and upper curves of the 90% confidence interval, respectively. Values of 5 and 6 correspond to the lower and upper curves of the 80% confidence interval, respectively. Values of 7 and 8 correspond to the lower and upper curves of the 60% confidence interval, respectively. A value of 9 corresponds to the baseline estimate.

- ind_exo: an indicator variable that can take on values of 0 or 1. A value of 0 denotes that CO2 emissions, temperature, and population are endogenously computed, while a value of 1 denotes that these variables are exogenously taken from the baseline scenario.

- taxCO2: the path of carbon taxes for each cell and time period.

- subclean: the path of clean energy subsidies for each cell and time period.

- abat: the share of CO2 emissions abated in each cell and time period.

- val adap: a 4x1 vector which determines the cost of trade, migration, innovation, and the inverse of migration elasticity.

- migr exp: a 2x1 vector which sets the border costs.

- ind agri: an indicator variable that takes on either 1 or 0. When ind agri equals 1, the damage function accounts for the share of value added in agriculture. When ind agri is 0, the damage function ignores agriculture.

In the next sections, a brief explanation of what is being done in the different parts of the code is being explained. The headlines corresponds to the headlines in the code marked with "#".

8.3.2 Initialize parameters and variables

Adjust migration costs

Function $m(r, r')$,

$$m_2(r) = \tilde{m}_2(r)/n_2(D(r))$$

Update adaptation parameters

Ω: A greater value of Ω implies more dispersion in agent's preferences across regions, so that mobility responses are mostly driven by idiosyncratic motives, rather than spatial differences in utility adjusted for the migration cost of entering the region.

Update productivity and amenities

The world's average relative expenditure in fossil fuels and clean energy, and the ratio of energy expenditures to the wage bill, are given by equation 19 at page 16. We believe the second one could correspond to the variable "conts_energy":

$$\left(\frac{Q_0^f}{Q_0^c}\right)\left(\frac{E_0^f}{E_0^c}\right)^{\frac{1}{\epsilon}} = \frac{\kappa}{1-\kappa},$$

and

$$\frac{w_0 Q_0 E_0}{w_0 L_0} = \frac{\mu(1-\chi)}{\mu+\gamma_1/\xi}$$

Set CO2 stock, forcing and temperature

Rewriting the law of motion:

$$S_{t+1} = S_{0,t+1} + \sum_{i=1}^{3} S_{i,t+1}, with S_{0,t+1} = S_{0,t} + a_0(E_t^f + E_t^x),$$

$$S_{i,t+1} = (e^{-1/b_i})S_{i,t} + a_i(E_t^f + E_t^x), i \in \{1,2,3\}.$$

Measures the net inflow of energy:

$$F_{t+1} = \varphi log_2(S_{t+1}/S_{pre-ind}) + F_{t+1}^x,$$

The global temperature module:

$$T_{t+1} = T_{1,t+1} + T_{2,t+1}, with T_{j,t+1} = (e^{-1/d_j})T_{j,t} + \frac{c_j}{d_j}F_{t+1}, j \in \{1,2\}.$$

Set damage function level by confidence interval

Building initial value for Λ^b and Λ^a use in amenities and productivities:

$$\Lambda_t^a(r) = \Lambda^a(\Delta T_t(r), T_t(r))$$

$$\Lambda_t^b(r) = \Lambda^b(\Delta T_t(r), T_t(r))$$

Set damage function sources: amenities or productivities

Start with $a_0 = a_{norm}$ and $b_0 = \eta prod_0^{\gamma_2} aprod^{(1-\gamma_2)} \phi_0^{\gamma_{aux}\theta}$?

$$\bar{b}_t(r) = (1 + \Lambda^b(\Delta T_t(r), T_{t-1}(r)))\bar{b}_0(r).$$

$$\bar{a}_t(r) = (1 + \Lambda^\alpha(\Delta T_t(r), T_{t-1}(r)))\bar{a}_0(r),$$

Update population size

Relation between natality rates, real GDP, and temperature:

$$n_t(r) = \eta(y_t(r), L_t(r)$$

From the the natality function, the following equations are being used:

$$\eta^y(log(y_t(r))) = B(log(y_t(r)); b^\ell)\cdot 1(log(y_t(r)) < b^*) + B(log(y_t(r)); b^h)\cdot 1(log(y_t(r)) \geq b^*),$$

$$b_0^T(\flat, x_0) = 2n_0^w - \frac{1}{L_0}\int_S \left(2\eta^y\left(log(y_0(v); b^\ell, b^h) + b_2^T e^{-b_1^T(T_0(v)-b^{*T}}\right)^2\right) L_0(v)H(v)dv.$$

$$b_w(\flat, x_0, x_{20}) = \frac{1}{log(y_{20}^w/y_0^w)} log\left(\frac{\int_S b_0^T(\flat, x_0) + b_2^T e^{-b_1^T(T_{20}(v)-b^{*T})^2)L_{20}(v)H(v)dv}}{n_{20}^w L_{20} - \int_S \eta^y(log(y_{20}(v)); b^\ell, b^h)L_{20}(v)H(v)dv}\right)$$

$$\eta^T(T_t(r), log(y_t^w)) = \frac{B(T_t(r); b^T)}{1 + e^{b_w}[log(y_t^w) - log(y_0^w)]}$$

Define function for extraction cost function

$$f(CumCO2_t) = \left(\frac{f_1}{f_2 + e^{-f_3(CumCO2_t - f_4)}}\right) + \left(\frac{f_5}{maxCumCO2 - CumCO2_t}\right)^3$$

Precompute auxiliary variables

Defining "denom" and "squ" which are just parts of the equations for $\flat_i$and F_t^i. Then they compute $\flat_R$ and $\flat_R$.

$$\flat_L = \frac{\lambda\theta}{\sigma} - \frac{\theta}{1+2\theta}\left[\alpha - 1 + \theta\left(\frac{\lambda}{\sigma} + \frac{\gamma_1}{\xi} - (1-\mu)\right)\right],$$

$$\flat_R = 1 - \frac{\lambda\theta}{\sigma} + \frac{1+\theta}{1+2\theta}\left[\alpha - 1 + \theta\left(\frac{\lambda}{\sigma} + \frac{\gamma_1}{\xi} - (1-\mu)\right)\right].$$

Precompute auxiliary variables to equation

Precomputing some of the parts of the following equations in this section.

$$F_t^L(r)\hat{u}_t(r)^{\frac{1}{\sigma}\left(\frac{\flat_L}{\Omega/\sigma} + \frac{\theta(1+\theta)}{1+2\theta}\right)} = \kappa\int_S F_t^R(v)\hat{u}_t(v)^{\frac{1}{\sigma}\left(\frac{\flat_R}{\Omega/\sigma} - \frac{\theta^2}{1+2\theta}\right)}\varsigma(r,v)^{-\theta}dv$$

$$F_t^L(r) = \bar{b}_t(r)^{-\frac{1}{\sigma}\frac{\theta(1+\theta)}{1+2\theta}}\bar{a}_t(r)^{-\frac{\theta}{1+2\theta}}\hat{Q}_t(r)^{\frac{(1-\chi)\mu\theta^2}{1+2\theta}}H(r)^{\frac{\theta}{1+2\theta}-\flat L}\varphi_t(r)^{-\frac{(\mu+\gamma_1/\xi)\theta^2}{1+2\theta}}m_2(r)^{-\frac{1}{\sigma}\frac{\flat L}{\Omega/\sigma}}$$

$$F_t^R(v) = \bar{b}_t(v)^{\frac{1}{\sigma}\frac{\theta^2}{1+2\theta}} \bar{a}_t(v)^{-\frac{1+\theta}{1+2\theta}} \hat{Q}_t(v)^{-\frac{(1-\chi)\mu\theta(1+\theta)}{1+2\theta}} H(v)^{\frac{\theta}{1+2\theta}-\flat R} \varphi_t(v)^{-\frac{(\mu+\gamma_1/\xi)\theta(1+\theta)}{1+2\theta}} m_2(v)^{-\frac{1}{\sigma}\frac{\flat R}{\Omega/\sigma}}$$

8.3.3 Simulating the Model

First while loop Then extractions costs and auxiliary variables are being computed. Then the initial energy productivities is given by equation the equation below. These are used to update energy productivity in the first while loop in the simulation.

$$\zeta_t^j(r) = \left(\frac{y_t^w}{y_{t-1}^w}\right)^{v^j} \zeta_{t-1}^j(r), where y_t^w = \int_S \left(\frac{L_t(v)H(v)}{L_t}\right) y_t(v)dv.$$

Second while loop Right after, in the second while loop in the simulation, the energy price is constructed. These equations corresponds to the variables "price_fossil" and "price_clean", "price_energy", "price_energy_tilde" and "varphi".

$$Q_t^f(r)O = \frac{f(CumCO2_{t-1})}{\zeta_t^f(r)} and Q_t^c(r) = \frac{1}{\zeta_t^c(r)}.$$

$$Q_t(r) = \left(\kappa^\epsilon(1+\tau_t(r))^{1-\epsilon} Q_t^f(r)^{1-\epsilon} + (1-\kappa)^\epsilon(1-s_t(r))^{1-\epsilon} Q_t^c(r)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

$$\widetilde{Q_t}(r) = \left(\kappa^\epsilon(1+\tau_t(r))^{-\epsilon} Q_t^f(r)^{1-\epsilon} + (1-\kappa)^\epsilon(1-s_t(r))^{-\epsilon} Q_t^c(r)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

$$\varphi_t(r) = \frac{\mu\chi + \gamma_1/\xi + \mu(1-\chi)(\widetilde{Q_t}(r)/Q_t(r))^{1-\epsilon}}{\mu + \gamma_1/\xi}$$

Some of the parameters used for these equation include $\chi = 0.96$, which is the share of labor in labor-energy composite; $\epsilon = 1.6$, which is the elasticity of substitution between energy sources; $\kappa = 0.89$, which is the share of fossil fuels in energy composite; $f(\cdot)$, which is extraction costs; $\zeta_0^c(\cdot)$ and $\zeta_0^f(\cdot)$, which is initial energy productivities based on current energy use; $v^f = 1.16$, which is the elasticity of fossil fuel productivity growth to global real GDP per capita growth; and $v^c = 1.16$, which is the elasticity of clean energy productivity growth to global real GDP per capita growth.

Third while loop Next in the second while loop, they precompute some of the auxiliary variables, before they iterate $\hat{u}_t(\cdot)$. This is done in the third while loop, where the following equation is being used.

$$F_t^L(r)\hat{u}_t(r)^{\frac{1}{\sigma}\left(\frac{\flat_L}{\Omega/\sigma}+\frac{\theta(1+\theta)}{1+2\theta}\right)} = \kappa \int_S F_t^R(v)\hat{u}_t(v)^{\frac{1}{\sigma}\left(\frac{\flat_R}{\Omega/\sigma}-\frac{\theta^2}{1+2\theta}\right)}\varsigma(r,v)^{-\theta}dv$$

$$F_t^L(r) = \bar{b_t}(r)^{-\frac{1}{\sigma}\frac{\theta(1+\theta)}{1+2\theta}}\bar{a}_t(r)^{-\frac{\theta}{1+2\theta}}\hat{Q}_t(r)^{\frac{(1-\chi)\mu\theta^2}{1+2\theta}}H(r)^{\frac{\theta}{1+2\theta}-\flat L}\varphi_t(r)^{-\frac{(\mu+\gamma_1/\xi)\theta^2}{1+2\theta}}m_2(r)^{-\frac{1}{\sigma}\frac{\flat L}{\Omega/\sigma}}$$

$$F_t^R(v) = \bar{b_t}(v)^{\frac{1}{\sigma}\frac{\theta^2}{1+2\theta}}\bar{a}_t(v)^{-\frac{1+\theta}{1+2\theta}}\hat{Q}_t(v)^{-\frac{(1-\chi)\mu\theta(1+\theta)}{1+2\theta}}H(v)^{\frac{\theta}{1+2\theta}-\flat R}\varphi_t(v)^{-\frac{(\mu+\gamma_1/\xi)\theta(1+\theta)}{1+2\theta}}m_2(v)^{-\frac{1}{\sigma}\frac{\flat R}{\Omega/\sigma}}$$

This concludes the third while loop. Continuing, still being inside the second while loop, they solve for population. This is then scaled up to lbar_time(t).

$$L_t(r) = \frac{1}{H(r)}\frac{u_t(r)^{1/\Omega}m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega}m_2(v)^{-1/\Omega}dv}L_t$$

The next step is calculating innovation. These equations are equalled to:

$$\chi\mu v\phi_t^\omega(r)^\xi = (\gamma_1/\xi)L_t^\omega(r)$$

$$\overline{L}_t^w(r) = (\frac{\mu+\gamma_1/\xi}{\mu\chi})\varphi_t(r)L_t^w(r)$$

The next step is to retrieve utility, which is given by:

$$U_t = \left(\frac{L_t}{\int_S[\hat{u}_t(v)/m_2(v)]^{1/\Omega}dv}\right)^{-\frac{\flat_R-\flat_L}{\theta}}$$

After this, real income and real GDP per capita is being calculated, which is done by using:

$$u_t(r) = b_t(r)y_t(r)^\sigma = b_t(r)\left[\int_0^1 c_t^\omega(r)^\rho dw\right]^{\sigma/\rho}$$

Next, productivities are being updated using:

$$\bar{a}_t(r) = (1+\Lambda^\alpha(\Delta T_t(r),T_{t-1}(r)))\left(\phi_{t-1}(r)^{\theta\gamma_1}[\int_S D(v,r)\bar{a}_{t-1}(v)dv]^{1-\gamma_2}\bar{a}_{t-1}(r)^{\gamma_2}\right)$$

After this, they calculate clean energy use and CO2 emissions, which are done by using:

$$e_t^{c,w}(r) = \left(\frac{(1-\chi)\mu}{\mu+\gamma_1/\xi}\right)\left(\frac{\bar{L}_t^w(r)}{\varphi_t(r)Q_t(r)}\right)\left(\frac{(1-\kappa)Q_t(r)}{(1+s_t(r))Q_t^c(r)}\right)^{\epsilon}$$

$$e_t^{f,w}(r) = \left(\frac{(1-\chi)\mu}{\mu+\gamma_1/\xi}\right)\left(\frac{\bar{L}_t^w(r)}{\varphi_t(r)Q_t(r)}\right)\left(\frac{\kappa Q_t(r)}{(1+\tau_t(r))Q_t^f(r)}\right)^{\epsilon}$$

Next, they update CO2 emissions by abatement. The variable $v_t(r)$ is the share of CO2 emissions abated in region r at period t, the evolution of atmospheric CO2:

$$S_{t+1} = S_{pre-ind} + \sum_{l=1}^{\infty}(1-\delta_\ell)\left(E'^{f}_{t+1-\ell} + E^x_{t+1-\ell}\right)$$

where

$$E'^{f}_{t} = \int_s\int 0(1-\nu_t(v))e^{f,\omega}(v)H(v)d\omega dv.$$

Lastly in the second while loop, they update CO2 emissions and compare global CO2 emissions. This is followed by the last part of the first while loop, which includes comparing the growth rate of real GDP.

After the loops, different variables are being updated. CO2 stock, forcing and temperature are being updated using:

$$CumCO2_t = CumCO2_{t-1} + E_t^f = CumCO2_{t-1} + \int_S\int_0^1 e_t^{f,\omega}(v)H(v)d\omega dv.$$

$$T_t(r) - T_{t-1}(r) = g(r)\cdot(T_t - T_{t-1}),$$

$$S_{t+1} = S_{0,t+1} + \sum_{i=1}^{3} S_{i,t+1}, with S_{0,t+1} = S_{0,t} + a_0(E_t^f + E_t^x),$$

$$S_{i,t+1} = (e^{-1/b_i})S_{i,t} + a_i(E_t^f + E_t^x), i \in \{1,2,3\}.$$

$$T_{t+1} = T_{1,t+1} + T_{2,t+1}, with T_{j,t+1} = (e^{-1/d_j})T_{j,t} + \frac{c_j}{d_j}F_{t+1}, j \in \{1,2\}.$$

$$S_{t+1} = S_{pre-ind} + \sum_{l=1}^{\infty}(1-\delta_\ell)\left(E^f_{t+1-\ell} + E^x_{t+1-\ell}\right).$$

$$F_{t+1} = \varphi log_2(S_{t+1}/S_{pre-ind}) + F^x_{t+1},$$

$$T_{t+1} = T_{pre-ind} + \sum_{\ell=0}^{\infty} \zeta_\ell F_{t+1-l},$$

Amenities and Productivity. The following equations allow to update ameneties and produtivity.

$$\bar{b}_t(r) = (1 + \Lambda^b(\Delta T_t(r), T_{t-1}(r)))\bar{b}_{t-1}(r). \tag{36}$$

$$\bar{a}_t(r) = (1+\Lambda^\alpha(\Delta T_t(r), T_{t-1}(r))) \left(\phi_{t-1}(r)^{\theta\gamma_1} [\int_S D(v,r)\bar{a}_{t-1}(v)dv]^{1-\gamma_2} \bar{a}_{t-1}(r)^{\gamma_2} \right), \tag{37}$$

Some of the parameters for productivity evolution is that $\gamma_2 = 0.993$, which is the relation between population and growth; $\xi = 125$, which is the elasticity of bid rents to investments in technology; and $v = 0.15$, which is the level of innovation costs that yields an initial growth rate of real GDP of 1.75%.