Uncertainty and the Term Structure of Interest Rates

Jamie Cross, Aubrey Poon and Dan Zhu
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Jamie L. Cross† ‡ Aubrey Poon§ Dan Zhu¶

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Abstract

We present a new stylized fact about the link between uncertainty and the term structure of interest rates: Unexpectedly heightened uncertainty elicits a lower, steeper, and flatter yield curve. This result is established through a Yields-Macro model that includes dynamic Nelson-Siegel factors of U.S. Treasury yields, and accounts for endogenous feedback with observable measures of uncertainty, monetary policy, and macroeconomic aggregates. It is also robust to three distinct measures of uncertainty pertaining to the financial sector, the macroeconomy and economic policy. An efficient Bayesian algorithm for estimating the class of Yields-Macro models is also developed.

JEL classification: E43, E52, G01, G12.

Keywords: Yield curve, Uncertainty, Monetary Policy, Dynamic Nelson-Siegel model, Bayesian estimation.

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†Melbourne Business School, University of Melbourne.
‡Centre for Applied Macroeconomics and Commodity Prices, BI Norwegian Business School.
§University of Kent.
¶Monash University.
1 Introduction

The term structure of interest rates (also known as the yield curve) refers to the range of bond yields at different terms to maturity. Financial market participants and central bankers pay close attention to the term structure for several reasons. First, the Federal Funds Rate represents the instantaneous rate of return on overnight lending, making short-term yields closely connected to monetary policy. Second, long-term interest rates reflect information about market expectations of future monetary actions (Hansen et al., 2019). Thirdly, the slope of the yield curve, i.e., the difference between long- and short-term rates, provides a robust leading indicator of future macroeconomic activity (Estrella and Hardouvelis, 1991).

In this paper we investigate how the term structure reacts to unexpectedly heightened uncertainty of various forms. Our investigation is motivated by the desire of market participants and policy makers to understand the drivers of yield curve dynamics, together with a vast literature that finds various types of uncertainty have notoriously contractive macroeconomic effects (e.g., Bloom, 2009; Mumtaz and Zanetti, 2013; Jurado et al., 2015), and a burgeoning literature exploring how uncertainty impacts the term structure (Castelnuovo, 2019; Tillmann, 2020; Hansen et al., 2019; Shang, 2022). Using local projections regressions, Castelnuovo (2019) finds that heightened financial uncertainty reduces both short- and long-term interest rates. Tillmann (2020) and Shang (2022) use distinct term-structure models with uncertainty interactions to independently show that high uncertainty about monetary policy decreases the central banks ability to impact the term structure.

We differ from these recent studies in two ways. First, we propose the use of a conceptually simple Yields-Macro (YM) model to examine the effects of uncertainty shocks on the yield curve while controlling for broader macroeconomic dynamics. Originally proposed by Diebold et al. (2006), the YM model decomposes the associated yield curve into level, slope, 1Castelnuovo (2022) provides a recent review of the literature on the economic effects of uncertainty shocks.
and curvature factors using a dynamic Nelson-Seigel representation (Diebold et al., 2006). Dynamic interactions between these factors, uncertainty, monetary policy, and the broader macroeconomy are jointly modeled with a structural vector autoregression (SVAR) model. Controlling for simultaneity in this nexus is potentially important given the aforementioned literature on uncertainty shocks. Second, we study the effects of multiple three distinct types of uncertainty: the financial uncertainty measure of Ludvigson et al. (2021), the macroeconomic uncertainty measure of Jurado et al. (2015), and, the U.S. economic policy uncertainty (EPU) measure of Baker et al. (2016). Each of these uncertainties have been shown to have significant macroeconomic effects in a variety of empirical studies. Exploring whether these uncertainties elicit similar or different yield curve dynamics therefore has important theoretical consequences. In this sense, the paper is also related to very recent studies on the use of structural macroeconomic models to examine the effects of uncertainty on the yield curve (Amisano and Tristani, 2023; Bianchi et al., 2023; Leippold and Matthys, 2022). We view these study as complementary in that we use a relatively agnostic and simple econometric model to examine these uncertainty effects. At the very least our results provide a set of stylized facts that can be used to evaluate the fit of structural models with both uncertainty and the term structure of interest rates.

Our main general insight is that unexpectedly heightened uncertainty elicits a temporary reduction in all three yield curve factors, resulting in a lower, steeper and flatter yield curve. The reduced yield curve level and slope factors are found to be consistent with theories of expectations about the future path of interest rates and output (Estrella, 2005), and are also in line with what financial practitioners refer to as a ‘Bull Steepening’ of the yield curve, i.e., a situation in which short-term interest rates fall faster than long-term rates. The finding that higher uncertainty elicits a credible decrease in the curvature factor reflects a relative decline in medium-term yields. This is particularly notable since this factor has been shown to be

\[2\text{A related approach by Shang (2022) proposes a Yields-Only model of Diebold and Li (2006) in which exogenously identified regime-dependent monetary policy uncertainty shocks impact the yield curve factors.}

\[3\text{This is contrast to a ‘Bear Steepening’ in which long-term rates rise faster than short-term rates.}\]
relatively unresponsive to other types of shocks within the broader macro-finance literature. Uncertainty shocks are also found to simultaneously deteriorate macroeconomic conditions. This evokes a monetary expansion and subsequent overshooting in capacity utilization that is in line with research on uncertainty driven ‘wait-and-see’ business cycles (Bachmann and Bayer, 2013). Over time, however, the economy recovers, expectations about compensation for longer-term securities improve, and the yield curve reverts to its normal state. The results are found to be robust to all three distinct uncertainty measures. This suggests the existence of a general mechanism in which uncertainty is being transmitted to the yield curve through expectations of future monetary policy actions.

In addition to establishing our main empirical result, we also make two improvements to the state-of-the-art algorithm for estimating YM models (Diebold et al., 2008). First, we make the algorithm fully Bayesian. Since Nelson and Siegel (1987) it has been common to fix the exponential decay rate parameter of the yields before estimating the remaining model parameters (e.g., Diebold and Li, 2006; Diebold et al., 2006; Afonso and Martins, 2012; Byrne et al., 2019, among others). For instance, Diebold and Li (2006) use the value that maximizes the curvature loading at 30 months to maturity. In contrast, we propose a fully Bayesian approach that estimates this parameter with a Griddy-Gibbs algorithm (Ritter and Tanner, 1992). Second, we show how the computationally intensive Kalman filtering and smoothing recursions for sampling the factors can be replaced by a faster, more efficient, and conceptually simpler, precision sampling algorithm (Chan and Jeliazkov, 2009), which has been shown to speed up computations in a variety of state space models (Chan and Strachan, 2023).

The rest of the paper is organized as follows. Section 2 introduces the empirical methodology and algorithm. Section 3 presents the empirical results, and Section 4 concludes.

\[\text{Shang (2022) recently proposed a precision-based algorithm to sample the factors in a non-linear dynamic Nelson-Siegel model with a Metropolis-Hastings step based on the integrated likelihood. In contrast, our method directly samples the factors from their conditional posterior density. The two methods are related in that they are precision-based, but are numerically distinct in that our sampler is direct while theirs is indirect.}\]
2 Empirical Methodology

In this section, we briefly introduce the YM model and discuss how we use this framework to address our research question. We then present an efficient Gibbs-Sampling algorithm for estimating the yield curve factors and exponential decay rate parameter. Finally, we present the data that we use to estimate the model.

2.1 The Yields Macro (YM) Model

Following Nelson and Siegel (1987) and Diebold and Li (2006), we specify a three-component dynamic exponential approximation to the cross-section of treasury yields over time as

\[
y_t(\tau) = L_t + S_t \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) + C_t \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - \exp^{-\lambda \tau} \right),
\]

where \(L_t, S_t,\) and \(C_t,\) are respectively interpreted as time-varying level, slope, and curvature yield curve factors, and \(\lambda\) is an exponential decay rate parameter that also governs where the loading on \(C_t\) obtains its maximum. A theoretical foundation for this class of Nelson-Siegel yield curve models is provided by Krippner (2015). Econometrically, the factor dynamics are jointly modeled as a first-order vector autoregressive (VAR) process of the form

\[
\begin{bmatrix}
L_t \\
S_t \\
C_t
\end{bmatrix} =
\begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix} +
\begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix}
\begin{bmatrix}
L_{t-1} \\
S_{t-1} \\
C_{t-1}
\end{bmatrix} +
\begin{bmatrix}
\omega_{tL} \\
\omega_{tS} \\
\omega_{tC}
\end{bmatrix}.
\]

(2)

This ‘Yields-only’ (YO) model can thus be viewed as a state space model with (1) and (2) representing the measurement and state equations, respectively. Letting \(Y_t = \left[y_t(\tau_1), y_t(\tau_2), \ldots, y_t(\tau_N)\right]'\) where \(y_t(\tau_i)\) is a bond yield with \(\tau_i\) months to maturity, and \(f_t = (L_t, S_t, C_t)'\), allows the mea-
surement equation to be expressed more compactly as

\[ Y_t = \Lambda(\lambda) f_t + \varepsilon_t, \varepsilon_t \sim N(0, \Sigma), \quad (3) \]

where \( \Sigma = \text{diag}(\sigma_1^2, \sigma_2^2, ..., \sigma_N^2) \), and the factor loading is a matrix variate function such that

\[
\Lambda(\lambda) = \begin{bmatrix}
1 & \frac{1-e^{-\tau_1 \lambda}}{\tau_1 \lambda} & \frac{1-e^{-\tau_1 \lambda}}{\tau_1 \lambda} & e^{-\tau_1 \lambda} \\
1 & \frac{1-e^{-\tau_2 \lambda}}{\tau_2 \lambda} & \frac{1-e^{-\tau_2 \lambda}}{\tau_2 \lambda} & e^{-\tau_2 \lambda} \\
... & ... & ... & ... \\
1 & \frac{1-e^{-\tau_N \lambda}}{\tau_N \lambda} & \frac{1-e^{-\tau_N \lambda}}{\tau_N \lambda} & e^{-\tau_N \lambda}
\end{bmatrix}.
\]

Similarly, the state equation in (2) can be written as

\[ f_t = b + B f_{t-1} + \omega_t, \quad \omega_t \sim N(0, \Omega), \quad (4) \]

where \( \Omega = \text{diag}(\omega_1^2, \omega_2^2, ..., \omega_N^2) \). The model is completed by noting that

\[
\begin{bmatrix}
\omega_t \\
\varepsilon_t
\end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega & 0 \\ 0 & \Sigma \end{bmatrix} \right). \quad (5)
\]

Following Diebold et al. (2006) the above YO model can be extended to a ‘Yields-Macro’ (YM) model that incorporates information from macroeconomic indicators. This is done by letting

\[ f_t = (f_{1t}', f_{2t}')' \text{ in which } f_{1t} = (L_t, S_t, C_t)' \text{ is a vector of yield curve factors, } f_{2t} \text{ is an } m \times 1 \text{ vector of macroeconomic indicators, and the definitions of the model parameters } \theta = \{ \lambda, \Sigma, \mu, B, \Omega \} \text{ in (3) and (4) are modified appropriately.} \]

The macroeconomic indicators used in this study are: uncertainty, capacity utilization, unemployment, the annual inflation rate, and the Federal Funds Rate. Specified in this manner the YM model facilitates a joint examination of how the yield curve and macroeconomic variables respectively respond to both yield curve shocks and macroeconomic shocks. Since our focus is
on examining the effects of an uncertainty shock, we follow the common practice of identifying this shock via recursive identification in which the uncertainty measure is placed first in the vector of macroeconomic indicators (see, e.g., Caggiano et al., 2014; Leduc and Liu, 2016, among others).

2.2 Bayesian Estimation

The YM model in (3) and (4) falls into the class of linear Gaussian state space models. Following Diebold and Li (2006) it has become common to estimate YM models using a two-step procedure. In step 1, the exponential decay rate parameter, $\lambda$, is first estimated by finding the value that maximizes the medium-term factor loading at some date between two- or three-year maturities. In step 2, $\lambda$ is fixed at this value, and the Kalman filter is used to compute the optimal yield predictions and associated prediction errors, and remaining parameters are estimated by maximizing the Gaussian likelihood function. Diebold et al. (2008) propose to replace the classical estimation in step 2 with a 2-Block Gibbs sampling algorithm is used to sample the model parameters $\theta$ from their respective (conditional) posterior distributions, and the latent factors are sampled via the multi-move Kalman filter based algorithm of Carter and Kohn (1994). However, they still fix the value of $\lambda$ as in step 1. This approach has been widely employed in the subsequent studies that estimate YM models (see, e.g., Mumtaz and Surico, 2009; Bianchi et al., 2009; Zantedeschi et al., 2011; Byrne et al., 2019, among others).

We propose two refinements to this algorithm. First, we integrate the two steps into a fully Bayesian algorithm thereby allowing estimation uncertainty in step 1 to impact the estimates in step 2. This is done by introducing a new block that samples the exponential decay rate parameter $\lambda$ as part of the Gibbs sampling algorithm via a Griddy-Gibbs step (Ritter and Tanner, 1992). Second, we show that the computationally intensive Kalman filter recursions can be replaced with an efficient precision sampling algorithm (Chan and Jeliazkov, 2009) that has been shown to substantially speed up computations in a variety of (conditionally)
Gaussian state space models (see Chan and Strachan (2023), and references therein).

To that end, we specify standard independent priors for elements of $\theta$. Specifically

$$\beta = \text{vec}(B) \sim N(0, V_\beta),$$
$$\Omega \sim IW(\nu_0, S_0),$$
$$\sigma_i^2 \sim IG(\nu_i, S_i) \text{ for } i = 1, \ldots, N,$$

and set the prior for the initial condition of the factors to be $f_0 \sim N(0, V_f)$. The hyperparameters for these distributions are chosen so that the resulting prior distributions are noninformative, i.e., $V_\beta = 10I_k$, $\nu_0 = G$, $S_0 = 10I_G$, $\nu_i = 5$, $S_i = .04$, $i = 1, \ldots, N$, $a = 0$, $b = 0.1$, and $V_f = 10I_n$.

Since we are the first to estimate the rate parameter using Bayesian methods, we specify an uninformative uniform prior over a bounded support

$$\lambda \sim U(a, b),$$

where $0 < a < b$.

**2.2.1 Posterior Distributions**

In this section, we present details of an efficient precision-based algorithm to draw $f_{t2}$ and a Griddy-Gibbs step for drawing $\lambda$. The conditional posteriors for the remaining parameters in the model are easily derived and we omit them for brevity.

**Sampling $f_{t1}$** First, note that the log-likelihood implied by (3) is given by

$$\log p(y|\theta, f_{t1}) = -\frac{T}{2} \log(2\pi|\Sigma|) - \frac{1}{2} \left\{ (y - I_T \otimes \Lambda(\lambda)f_{t1})'(I_T \otimes \Sigma)^{-1} (y - I_T \otimes \Lambda(\lambda)f_{t1}) \right\},$$

where $0 < a < b$. 

where \( y = [Y'_1, Y'_2, ..., Y'_T]' \) and \( f_1 = [f'_{11}, f'_{21}, ..., f'_{T1}]' \). Similarly, we can rewrite (4) as

\[
Hf = \alpha + \omega, \omega \sim N(0, S),
\]

(9)

where \( f = [f'_{11}, f'_{12}, f'_{21}, f'_{22}, ..., f'_{T1}, f'_{T2}]', \alpha = [b' + f'_0B', b', ..., b]'', \omega = (\omega_1, ..., \omega_T)', S = I_T \otimes \Omega \) and

\[
H = \begin{bmatrix}
I_n & 0 & 0 & \cdots & 0 \\
-B & I_n & 0 & 0 \\
0 & -B & I_n & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & -B & I_n
\end{bmatrix}
\]

where \( b = [b'_1, b'_2]' \), \( B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \). Note here \( H \) is a band matrix and it is therefore simple to show that \(|H| = 1\). Thus, \( H \) is invertible, and by a change of variable, (9) can be interpreted as

\[
f \sim N(\bar{\alpha}, (H'S^{-1}H)^{-1}),
\]

(10)

in which \( \bar{\alpha} = H^{-1}\alpha \).

Using (8) and (10), along with the fact that

\[
f = \mathbb{I}_T \otimes [\mathbb{I}_3, O_{3 \times d}]'f_1 + \mathbb{I}_T \otimes [O_{d \times 3}, \mathbb{I}_d]f_2,
\]

(11)

it is straightforwardly to show that the conditional posterior of \( f_1 \) is given by

\[
(f_1|\theta, f_2, y) \sim N(\mu_f, K_f^{-1})
\]

(12)
in which
\[ K_f = I_T \otimes \Lambda(\lambda) \Sigma^{-1} \Lambda(\lambda) + I_T \otimes [I_3, O_{3 \times d}] H' S^{-1} H I_T \otimes [I_3, O_{3 \times d}]' \]
\[ \mu_f = K_f^{-1} \left( (I_T \otimes \Lambda(\lambda) \Sigma^{-1}) y + I_T \otimes [I_3, O_{3 \times d}] H' S^{-1} H \tilde{\alpha} \right). \]

Since the precision matrix $K_f$ is a band matrix, we sample from the distribution efficiently using the precision-sampling algorithm of Chan and Jeliazkov (2009).

To draw the initial condition, $f_0$, we note that setting $t = 1$ in (4) implies that
\[ f_1 = b + B f_0 + \omega_1, \omega_1 \sim N(0, \Omega). \] (13)

Combining this with the prior for $f_0$ in (6), the conditional posterior for $f_0$ is
\[ p(f_0 | y, Z, \Sigma, \Omega, \beta) \propto -\frac{1}{2} \left\{ (f_1 - b - B f_0)' \Omega^{-1} (f_1 - b - B f_0) \right\} - \frac{1}{2} \left\{ f_0' V_f^{-1} f_0 \right\}. \] (14)

Thus, using standard linear regression results it follows that
\[ (f_0 | y, Z, \Sigma, \Omega, \beta, f) \sim N(\hat{f}_0, K_{f_0}), \] (15)
in which
\[ K_{f_0} = (B' \Omega^{-1} B + V_f^{-1})^{-1}, \quad \hat{f}_0 = K_{f_0} \{ B' \Omega^{-1} (f_1 - b) \}. \]

**Sampling $\lambda$** As mentioned above it has so far been common practice to fix $\lambda$ to be a particular value. We on the other hand are agnostic and estimate $\lambda$ given the data. Given a uniform prior for $\lambda$, $p(\lambda) \sim U(a, b)$, the conditional posterior of $(\lambda | y, f_1 f_0, \theta)$ is given by
\[ (\lambda | y, f_1, f_0, \theta) \propto p(y | \theta, f_1) p(\lambda), \]
where $a < \lambda < b$. Since the support of this conditional density is bounded and non-standard, we draw from this conditional density using a Griddy-Gibbs step (Ritter and Tanner, 1992). This is done using the following steps:

1. Construct a grid with grid points $\hat{\lambda}_1, \ldots, \hat{\lambda}_R$, where $\hat{\lambda}_1 = a$ and $\hat{\lambda}_R = b$.

2. Compute $F_i = \sum_{j=1}^{i} p(\hat{\lambda}_j | \bullet)$.

3. Generate $U$ from a standard uniform distribution.

4. Find the smallest positive integer $k$ such that $F_k \geq U$ and return $\lambda = \hat{\lambda}_k$.

### 2.3 Data

We use monthly data from 1971M8 to 2020M12. Following Diebold and Li (2006), the yield curve is modeled using consider zero-coupon U.S. Treasury maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months. However, the data that we use is from a recently constructed dataset by Liu and Wu (2021), who show that their yield curve dataset provides more of an accurate representation of the raw data compared to existing measures.

A three-dimensional plot of the yield curve data is shown in Figure 1, and Table 1 contains associated descriptive statistics. The data provide a few stylized facts about empirical yield curves that are consistent with alternate datasets (e.g., Diebold and Rudebusch, 2013; Diebold et al., 2006). First, the yield curve exhibits a large degree of temporal variation, and takes on a variety of shapes: upward sloping, downward sloping (inverted), concave and convex. Second, the yield curve is increasing and concave on average.$^5$ Third, short rates are typically more volatile, less persistent, and have a greater range, than long rates.$^5$

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$^5$We emphasize that the literature tends to define the empirical proxy of the yield curve slope as $S_t = y_t(3) - y_t(24)$. Thus, $S_t > 0$ implies a downward sloping yield curve and vice versa.
Figure 1: Yield Curves from August 1971 to December 2020 at maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months

We consider three extremely popular types of uncertainty: macroeconomic uncertainty as measured by Jurado et al. (2015), the financial uncertainty measure of Ludvigson et al. (2021) and the three-component U.S. economic policy uncertainty (EPU) measure of Baker et al. (2016). These metrics are shown in Figure 2. The economic policy uncertainty measure commences from January 1985, however the other two uncertainty measure date back to the start of our sample in August 1971. It is immediately evident that these measures are positively correlated, and tend to increase during periods of recession. They also have independent variation. For instance, financial uncertainty exhibits a distinct spike during the Black Monday event of October 1987, while the EPU index spikes in September 2001 and the 2011 debt ceiling crisis.

Finally, data on four macroeconomic variables: Capacity utilization (CU), the unemployment rate (UE), CPI inflation rate (CPI), and the Federal Funds rate (FFR), was sourced from the FRED-MD database.
### Table 1: Descriptive statistics of yield curves

<table>
<thead>
<tr>
<th>Maturity (months)</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Minimum</th>
<th>Maximum</th>
<th>$\hat{\rho}(1)$</th>
<th>$\hat{\rho}(12)$</th>
<th>$\hat{\rho}(30)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.609</td>
<td>3.501</td>
<td>0.024</td>
<td>15.946</td>
<td>0.989</td>
<td>0.858</td>
<td>0.656</td>
</tr>
<tr>
<td>6</td>
<td>4.767</td>
<td>3.552</td>
<td>0.049</td>
<td>16.133</td>
<td>0.990</td>
<td>0.867</td>
<td>0.674</td>
</tr>
<tr>
<td>9</td>
<td>4.881</td>
<td>3.565</td>
<td>0.082</td>
<td>16.107</td>
<td>0.990</td>
<td>0.873</td>
<td>0.690</td>
</tr>
<tr>
<td>12</td>
<td>4.967</td>
<td>3.562</td>
<td>0.103</td>
<td>15.962</td>
<td>0.990</td>
<td>0.878</td>
<td>0.705</td>
</tr>
<tr>
<td>15</td>
<td>5.040</td>
<td>3.558</td>
<td>0.109</td>
<td>15.901</td>
<td>0.990</td>
<td>0.883</td>
<td>0.720</td>
</tr>
<tr>
<td>18</td>
<td>5.105</td>
<td>3.556</td>
<td>0.114</td>
<td>15.943</td>
<td>0.991</td>
<td>0.887</td>
<td>0.733</td>
</tr>
<tr>
<td>21</td>
<td>5.160</td>
<td>3.544</td>
<td>0.120</td>
<td>15.910</td>
<td>0.991</td>
<td>0.890</td>
<td>0.743</td>
</tr>
<tr>
<td>24</td>
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<td>3.520</td>
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<td>15.719</td>
<td>0.991</td>
<td>0.892</td>
<td>0.752</td>
</tr>
<tr>
<td>30</td>
<td>5.299</td>
<td>3.475</td>
<td>0.114</td>
<td>15.545</td>
<td>0.991</td>
<td>0.897</td>
<td>0.767</td>
</tr>
<tr>
<td>36</td>
<td>5.400</td>
<td>3.442</td>
<td>0.123</td>
<td>15.569</td>
<td>0.991</td>
<td>0.900</td>
<td>0.776</td>
</tr>
<tr>
<td>48</td>
<td>5.580</td>
<td>3.370</td>
<td>0.172</td>
<td>15.475</td>
<td>0.992</td>
<td>0.902</td>
<td>0.790</td>
</tr>
<tr>
<td>60</td>
<td>5.713</td>
<td>3.286</td>
<td>0.231</td>
<td>15.195</td>
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<td>0.903</td>
<td>0.799</td>
</tr>
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<td>72</td>
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<td>3.233</td>
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<td>14.989</td>
<td>0.992</td>
<td>0.905</td>
<td>0.804</td>
</tr>
<tr>
<td>84</td>
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<td>3.169</td>
<td>0.382</td>
<td>14.950</td>
<td>0.992</td>
<td>0.902</td>
<td>0.806</td>
</tr>
<tr>
<td>96</td>
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<td>3.117</td>
<td>0.445</td>
<td>14.941</td>
<td>0.992</td>
<td>0.905</td>
<td>0.808</td>
</tr>
<tr>
<td>108</td>
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<td>3.067</td>
<td>0.488</td>
<td>14.945</td>
<td>0.992</td>
<td>0.905</td>
<td>0.810</td>
</tr>
<tr>
<td>120</td>
<td>6.163</td>
<td>2.996</td>
<td>0.530</td>
<td>14.939</td>
<td>0.992</td>
<td>0.900</td>
<td>0.809</td>
</tr>
<tr>
<td>Level</td>
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<td>3.292</td>
<td>0.249</td>
<td>15.278</td>
<td>0.992</td>
<td>0.895</td>
<td>0.758</td>
</tr>
<tr>
<td>Slope</td>
<td>-1.554</td>
<td>1.385</td>
<td>-4.358</td>
<td>3.756</td>
<td>0.948</td>
<td>0.484</td>
<td>-0.098</td>
</tr>
<tr>
<td>Curvature</td>
<td>-0.360</td>
<td>0.970</td>
<td>-2.695</td>
<td>2.765</td>
<td>0.927</td>
<td>0.637</td>
<td>0.377</td>
</tr>
</tbody>
</table>

Note: Descriptive statistics are for monthly yields at different maturities. Following Diebold et al. (2006), the empirical yield curve level, slope and curvature are defined as (Level)

$$L_t = \frac{y_t(3) + y_t(24) + y_t(120)}{3},$$

(Slope) $S_t = y_t(3) - y_t(24)$, and (Curvature) $C_t = 2y_t(24) - y_t(3) - y_t(120)$.

The final three columns are sample autocorrelations at lag lengths of 1, 12 and 30 months.
3 Empirical Results

We present the results across four subsections. First, we discuss posterior estimates of the yield curve. Second, we investigate whether the lagged coefficient on the uncertainty term for each variable in $\mathbf{B}$ is significantly different from zero. Third, we discuss the impact of an uncertainty shock on the yield curve and macroeconomy. Finally, we discuss the impact of an uncertainty shock on the entire term structure. All results were obtained using 50,000 posterior simulations after discarding the first 10,000 draws as a burn-in period.

3.1 Posterior Estimates of the Yield Curve

The posterior median estimate for the exponential decay parameter in each model is provided in Table 2. Overall, we find the results to be extremely robust to the choice of uncertainty indicator used in the YM model. This is despite the fact that the model including the EPU
index is estimated on a shorter sample than the alternative uncertainty measures. The median estimate of $\hat{\lambda}$ is found to be within the range of 0.039-0.045. This means that the data suggests that the curvature factor is maximized at, on average, around 48 months to maturity. The small range of the 90 percent credible interval given by at most $[0.03, 0.05]$ suggests that the parameter is precisely estimated. In contrast, Diebold and Li (2006) and Diebold et al. (2006) respectively calibrate $\hat{\lambda}_{DRA} = 0.0609$ and $\hat{\lambda}_{DRA} = 0.077$, which are associated with maturities of 30 and 24 months. Both of these values are beyond the empirical range of our estimated posterior distribution, and highlight the importance of estimating the $\lambda$ in empirical work, as opposed to fixing it a priori.

The posterior estimates of the yield curve factors are also found to be extremely robust to the choice of uncertainty metric used in the model. We therefore present results obtained using the financial uncertainty indicator here and defer remaining results to the Online Appendix. The estimated posterior median of the factors (solid line) and associated 95 percent credible intervals (shaded region) are provided in Figure 3. The narrow credible intervals around these factors indicate that they are precisely estimated. We find that the factors reflect the various stylized facts about empirical yield curves discussed in Section 2.3. We also observe that the slope factor becomes positive before each recession, which is in line with the idea that yield curve inversions are a leading indicator of the business cycle (Estrella and Hardouvelis, 1991).

To investigate the empirical reliability of these estimated factors, in Figure 4, we plot the posterior median estimates of the yield curve factors (blue line) against their empirical counterpart (black line). Consistent with Table 1, the empirical counterparts are respectively calculated as (Level) $L_t = \frac{y_t(3)+y_t(24)+y_t(120)}{3}$, (Slope) $S_t = y_t(3) - y_t(24)$, and (Curvature)

<table>
<thead>
<tr>
<th>Uncertainty Measure</th>
<th>Posterior median</th>
<th>95% credible interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial</td>
<td>0.043</td>
<td>(0.03,0.05)</td>
</tr>
<tr>
<td>Macroeconomic</td>
<td>0.045</td>
<td>(0.04,0.05)</td>
</tr>
<tr>
<td>Economic Policy</td>
<td>0.039</td>
<td>(0.03,0.05)</td>
</tr>
</tbody>
</table>
\( C_t = 2y_t(24) - y_t(3) - y_t(120) \). The results show that our estimated factors track their empirical counterpart reasonably well. The estimated correlation coefficients between each factor and its empirical counterpart are 0.91, 0.96 and 0.90, respectively. These high correlations are in line with estimates in both Diebold and Li (2006) and Diebold et al. (2006), and suggest a very high degree of co-movement between the factors and their empirical counterparts. The descriptive statistics in Table 3 are also close to those reported in Table 1, thus lending further support towards their credible interpretation as level, slope and curvature factors.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Minimum</th>
<th>Maximum</th>
<th>( \hat{\rho}(1) )</th>
<th>( \hat{\rho}(12) )</th>
<th>( \hat{\rho}(30) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>6.725</td>
<td>2.716</td>
<td>0.949</td>
<td>14.073</td>
<td>0.989</td>
<td>0.878</td>
<td>0.760</td>
</tr>
<tr>
<td>Slope</td>
<td>-2.010</td>
<td>2.070</td>
<td>-6.308</td>
<td>6.151</td>
<td>0.962</td>
<td>0.584</td>
<td>-0.041</td>
</tr>
<tr>
<td>Curvature</td>
<td>-1.017</td>
<td>2.570</td>
<td>-9.818</td>
<td>6.608</td>
<td>0.936</td>
<td>0.591</td>
<td>0.439</td>
</tr>
</tbody>
</table>

Note: Descriptive statistics are for the estimated factors (posterior median). The final three columns are sample autocorrelations at lag lengths of 1, 12 and 30 months.

As a final determination of the validity of the factors interpretation as level, slope and curvature, we also investigate whether important cross-variable correlations are in line with the-
Figure 4: Estimated yield curve factors (blue) and empirical proxies (black)

Economic theory suggests that the yield curve level is related to the level of expected inflation, and that the yield curve slope is linked to real activity (Estrella, 2005). We therefore calculated the correlations between $L_t$ and the University of Michigan’s 1-year ahead inflation expectations, and between $S_t$ and capacity utilization as an indicator of macroeconomic activity. The correlation between $L_t$ and inflation expectations is 0.49 suggesting a link between the movements in the yield curve level and inflation expectations that is consistent with the Fisher equation. The correlation between $S_t$ and capacity utilization is 0.59, which suggests that the slope yield curve closely follows the economy’s cyclical behavior. Similar values for these correlations are reported in Diebold et al. (2006).

3.2 Does uncertainty impact the Yield curve?

To formally assess whether uncertainty impacts the Yield curve we utilize the Savage-Dickey density ratio (SDR); a variant of the Bayes Factor (BF) that is useful when testing the credibility of equality constraints on a subset of one or more of the parameters in a model. Here
we test whether or not the lagged coefficients on the uncertainty term for all the variables (except for uncertainty) are jointly different from zero. Let the vector of lagged coefficient on the uncertainty term for all the variables be defined as \( b_{UNC} = (B_{21}, B_{31}, B_{41}, B_{51}, B_{61}, B_{71})' \). The SDR associated with this restriction is given by

\[
BF = \frac{p(b_{UNC} = 0)}{p(b_{UNC} = 0|y)},
\]

where \( BF \) denotes the Bayes factor. The numerator is the marginal prior density of \( b_{UNC} \) evaluated at zero, and the denominator is the marginal posterior density evaluated at zero. If \( b_{UNC} \neq 0 \), then the numerator will be larger than the denominator, implying that larger values of (16) provide substantial evidence that \( b_{UNC} \) is jointly different from zero, and vice-versa.

The resulting Bayes Factors are reported in Table 4. Since the SDR is a version of the Bayes Factor, a numerical interpretation of the results can be made via the popular rules of thumb proposed by Kass and Raftery (1995). According to their interpretation, the strength of evidence provided by the \( BF \) for values between 1–3.2 are ‘not worth more than a bare mention’, 3.2–10 are ‘substantial’, 10–100 are ‘strong’, and anything greater than 100 is ‘decisive’. Our results show that there is ‘strong’ evidence that macroeconomic uncertainty has a credible impact on the yield curve, ‘substantial’ evidence for financial uncertainty, and ‘decisive’ evidence for economic policy uncertainty. While the general conclusion that uncertainty has a credible impact on the yield curve is novel, our result that economic policy uncertainty has a relative large impact on the yield curve is in line with recent results that high policy uncertainty may mute the Federal Reserve’s ability to impact bond yields (Tillmann, 2020; Shang, 2022).

<table>
<thead>
<tr>
<th>Uncertainty Measure</th>
<th>BF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial</td>
<td>37.10</td>
</tr>
<tr>
<td>Macroeconomic</td>
<td>14.36</td>
</tr>
<tr>
<td>Economic Policy</td>
<td>129.57</td>
</tr>
</tbody>
</table>
3.3 How does uncertainty impact the yield curve?

To assess how an uncertainty shocks impact the yield curve we examine impulse response functions (IRFs) to a one standard deviation uncertainty shock from each of the uncertainty measures: financial, macroeconomic and economic policy. The resulting IRFs are presented in Figures 5-7. In each case the thick line represents the posterior median and the shaded region is the associated 68 percent credible interval.

![Figure 5: Impulse Response to a Financial Uncertainty Shock](image)

The main general insight from this analysis is that each of the uncertainty shocks elicits a similar transmission mechanism. In each case, unexpectedly heightened uncertainty has a credibly negative impact on the yield curve’s level, slope, and curvature factors.

The negative response of the yield curve level is in line with the notion that market participants expect the Fed to respond to the deterioration of macroeconomic conditions — as seen by the initial reduction in capacity utilization and associated heightened unemployment — via expansionary monetary policy by reducing the Federal Funds Rate. A notable differ-
Figure 6: Impulse Response to a Macroeconomic Uncertainty Shock

The difference between the level responses across uncertainty measures is that the level response to a macroeconomic uncertainty shock quickly reverts to its pre-shock value, while the decline in the level factor following the financial uncertainty and EPU shocks are extremely persistent. These differences can be explained by the Fisher equation. According to this equation, a prolonged decrease in the level factor is consistent the interpretation of market participants decreasing their expectations of future inflation, and vice-versa. Indirect evidence of such behavior is provided in the CPI inflation response. In that case, the financial uncertainty and EPU shocks elicit an initial temporary increase in inflation followed by persistent decrease, while the macroeconomic uncertainty shock has a more persistent increase in inflation. Both of these responses lend empirical support to the Fisher effect. This raises the question of why inflation responds differently to the various types of uncertainty shock. As discussed in the recent survey by Castelnuovo (2022), the inflationary effect of uncertainty shocks is uncertain, and relates to a debate surrounding the theoretical classification of uncertainty in terms of
aggregate demand and supply. If unexpectedly heightened uncertainty moves inflation and output (here proxied by capacity utilization) in the same direction then it is classified as aggregate demand shock. In contrast, if the uncertainty shock moves inflation and output in opposite directions then it is classified as an aggregate supply shock. Using this classification our results are broadly in line with the financial and EPU shocks falling into the category of aggregate demand shocks, however the classification of macroeconomic uncertainty is less clear. To our mind, this is not too surprising given that financial markets and economic policy are generally believed to be closely related to demand conditions in the economy, given that the primary origins of business cycle fluctuation have been debated for over a century. On the one hand, the neoclassical perspective of the business cycle is that macroeconomic fluctuations are primarily driven by supply-side shocks. On the other hand, the New Keynesian perspective is that demand side factors play a more pertinent role. Given the relatively modest response of inflation to the macroeconomic uncertainty shock, the results here are more in
line with results from New-Keynesian models which predict that macroeconomic uncertainty has a demand side effect on inflation due to its origins in countercyclical markups and sticky prices (e.g., Leduc and Liu, 2016; Basu and Bundick, 2017; Bianchi et al., 2023).

Since the slope factor is here defined as long-rate minus short-rate, this decrease in slope factor suggests the slope of the yield curve has increased, or steepled, in response to the uncertainty shock. Such a steepening can be caused by short-term interest rates falling more quickly than long-term rates. Alternatively, a steepening may also occur if long-term interest rates rise faster than short-term interest rates. Financial practitioners often refer to these two distinct scenarios as a ‘Bull Steepening’, and ‘Bear Steepening’, respectively, since their occurrence is indicative of overall market sentiment.\(^6\) Given that both short-term and long-term rates decline following an uncertainty shock, our results suggests that an uncertainty shock induces a ‘Bull Steepening’ of the yield curve, as opposed to a ‘Bear Steepening’. Over time, however, the decrease in the policy rate causes macroeconomic conditions to improve. As the economy recovers, expectations about compensation for longer-term securities improve, and the slope of the yield curve gradually reverts to its normal state. The resulting effect is a U-shaped response in the slope factor. This result is also in line with recent results in Amisano and Tristani (2023) and Bianchi et al. (2023) who respectively find that macroeconomic uncertainty shocks steeple the slope of the yield curve, albeit with different methodologies to the one used here. Similar results for financial uncertainty shocks are also reported in Castelnovo (2019).

Another novel result in this paper is that the uncertainty shock has a significant negative impact on the yield curve’s curvature factor. A decrease in the curvature factor implies a relative reduction in medium-maturity yields, and a flatter shape of the yield curve. Since medium-term interest rates are reflective of expected future short-term rates, a reduction in the curvature factor is indicative of an expectations channel. As discussed earlier, following

\(^6\)Practitioners also often discuss the existence of so called ‘Bear Flattening’: When short-term rates rise faster than long-term interest rates, and ‘Bull Flattening’: When long-term rates fall faster than short-term rates. However neither of these scenarios are consistent with a steepening of the yield curve. The term ‘flattening’ here should also be interpreted with respect to the relative slope of the yield curve as opposed to its curvature.
the uncertainty shock, rational investors will expect that the deterioration of macroeconomic conditions will be met by a monetary expansion as the Fed aims to stimulate the economy. Expectations of further reductions in the interest rate result in lower medium-term rates, and associated flattening of the yield curve. This result is especially notable since this factor has been shown to be relatively unresponsive to other types of macroeconomic shocks (Diebold et al., 2006). To the best of our knowledge, the subsequent macro-finance literature is also yet to establish any credible association between the curvature of the yield curve and shocks from any macroeconomic variable.

A possible explanation for our general result that uncertainty elicits a lower, steeper, and flatter yield curve is that uncertainty is being transmitted to the yield curve through market perceptions and expectations of future monetary policy actions. This idea is not only consistent with the idea that monetary policy is a primary driver of term structure dynamics, but also suggests that the relative strength of this mechanism may be influenced by the Fed’s policy communication. For instance, using UK data from the Bank of England (BoE) Inflation Report, Hansen et al. (2019) find that uncertainty in the BoE’s communication of economic conditions plays an important role in moving long-term interest rates. A similar result for the European Central Bank (ECB) has recently been established by Leombroni et al. (2021). It is therefore likely that such a result may also hold true for the Fed, however we leave this to future research.

Finally, we highlight that all of the uncertainty shocks elicit an increase in unemployment and an overshooting effect within capacity utilization. Both of these results are consistent with the findings of numerous studies in the broader literature on the macroeconomic effects of uncertainty shocks (e.g., Bloom, 2009; Bachmann and Bayer, 2013; Caggiano et al., 2014, 2022; Basu and Bundick, 2017). The general mechanism is that an increase in uncertainty impacts the real option value of waiting on investments. This results in firms temporarily pausing their hiring and investment decisions while they wait-and-see the Fed’s policy response, and the broader macroeconomic effects. The reduction in job creation relative to the steady state
results in increased unemployment, and an initial reduction in capacity utilization, followed by an overshooting as the economy recovers. Such precautionary behaviour by firms has been identified as a major cause of U.S. business cycles (Bachmann and Bayer, 2013).

3.4 How does uncertainty impact the term structure?

To further investigate how the yield curve responds to uncertainty shocks, we also examine how the entire term structure reacts to such shocks. Computationally, this is done by premultiplying the impulse response vector by the factor loading $\Lambda(\lambda)$ in (3). The IRFs associated with a one standard deviation uncertainty shock are shown in Figures 8-10. Since the impulse responses are of the entire term structure, they can be viewed as functional IRFs in the spirit of Inoue and Rossi (2021). The main difference is that Inoue and Rossi (2021) investigate the response of a functional variable (i.e., the term structure) to a functional monetary policy shock, while we consider the response of a functional variable to a scalar uncertainty shock.

In line with our results on the yield curve factors, we see that the level of the term structure is lowered after the shock. There is also a general monotonic impact in terms of the relative magnitude in the level effects at different dates-to-maturity, with short-term yields falling much more than long-term yields. This result provide further support for our aforementioned finding that uncertainty elicits a general steepening of the yield curve, and confirms our interpretation of the uncertainty shock generating a ‘bull steepening’ of the yield curve, as opposed to a ‘bear steepening’. The results also show that the short end follows exhibits a U-shaped pattern with strong convexity, while the long end is relatively flatter. Thus, while uncertainty shocks decrease the relative curvature across the entire yield curve, on average, there is important heterogeneity in the responses of yields at different dates-to-maturity.

Another area of heterogeneity in the responses of yields at different dates-to-maturity occurs across the various uncertainty measures. While we find that each of the uncertainty shocks has a similar impact on the yields at the short-end — a sharp drop, followed by a gradual
increase — we find a large degree of heterogeneity in the responses at the long-end of the curve. Financial uncertainty results in a gradual decline in uncertainty that is extremely persistent, macroeconomic uncertainty elicits more of a horizontally translated S-shaped response, and economic policy uncertainty elicits a U-shaped response; albeit with a long right-tail due to the high degree of persistence in the shocks effects. These results are again consistent with the idea that the relative effects of uncertainty at the long-end of the yield curve may be influenced by the Fed’s policy communication (Hansen et al., 2019). This is because the Fed is likely better able to communicate away uncertainty relating to future paths of policy, relative to the future paths of macroeconomic and financial conditions. Policy uncertainty may therefore have a relatively smaller impact on long-rates compared to other forms of uncertainty.

Figure 8: Functional Impulse Response Functions to a Financial Uncertainty Shock
Figure 9: Functional Impulse Response Functions to a Macroeconomic Uncertainty Shock

Figure 10: Functional Impulse Response Functions to an Economic Policy Uncertainty Shock
4 Conclusion

How does the the yield curve react to unexpectedly heightened uncertainty? To address this question we have proposed a simple Yields-Macro model that jointly estimates the dynamic effects of uncertainty, the yield curve, and the macroeconomy. Two simple extensions of the state-of-the-art Bayesian algorithm for estimating such models were also provided. First we showed how the exponential decay rate parameter of the yields can be sampled as part of the algorithm using a Griddy-Gibbs step. Second, we showed that the latent yield curve factors can be sampled simply and efficiently using a modern precision sampling algorithm in place of the conventional Kalman filter.

Our results provided new insights on the empirical impact of uncertainty on the yield curve. First, results from the Savage-Dickey density ratio provided strong evidence of a credible link between the yield curve and three distinct measures of uncertainty: financial, macroeconomic and policy related uncertainties. Next, when investigating the impact of unexpectedly heightened uncertainty on the yield curve, we found the general insight that each type of uncertainty shock results in a lower, steeper and flatter yield curve, along with a deterioration in macroeconomic conditions. This evokes a monetary expansion and subsequent overshoooting effect in capacity utilization that is in line with studies on uncertainty driven ‘wait-and-see’ business cycles. Our finding that higher uncertainty generally elicits a credible decrease in the curvature factor is especially notable since this factor has been shown to be relatively unresponsive to other macroeconomic shocks within the broader macro-finance literature. Given the fact that each uncertainty shock elicits a similar transmission mechanism, a possible explanation for this general result is that uncertainty is being transmitted to the yield curve through market perceptions and expectations of future monetary policy actions. This calls for the need to better understand the theoretical connections between uncertainty, the yield curve and the broader macroeconomy.
References


A Posterior estimates using different uncertainty measures

We here present the posterior estimates of the yield curve where we use the macroeconomic uncertainty measure and EPU in place of the financial uncertainty measure which was used in the main text. The estimated yield curve factors are respectfully presented in Figures 11-12. We find the results are extremely robust to the choice of uncertainty indicator used in the YM model, despite the fact that the EPU is estimated on a shorter sample than the other two uncertainty measures.

![Graphs of Level, Slope, and Curvature]

Figure 11: Posterior estimates of the Yield Curve factors: Macroeconomic uncertainty
Figure 12: Posterior estimates of the Yield Curve factors: Economic policy uncertainty
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