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Multi-item dynamic lot sizing with multiple transportation modes and item fragmentation

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ABSTRACT

This paper addresses a tactical joint inventory and transportation planning problem for multiple items with deterministic and time-varying demand, considering different transportation modes and item fragmentation. The latter corresponds to the splitting of the same item ordered quantity between several trucks or containers. On the one hand, fragmenting the items potentially reduces the number of containers used. On the other hand, loading the item lot fragments on several containers may negatively impact the handling and shipping operations. This new problem is proposed as a way to tackle such conflict. Several Mixed Integer Linear Programming models are proposed for the problem, which rely on two multi-item lot-sizing models with mode selection and two bin-packing models with item fragmentation. A relax-and-fix heuristic is also proposed. Using realistic instances, computational experiments are first conducted to identify the most efficient model in terms of computational time, to study the impact of key parameters on the computational complexity and to analyze the efficiency of the heuristic. Then, managerial insights are derived through additional computational experiments, in particular, to identify contexts requiring joint optimization of lot-sizing and bin-packing decisions, as well as the impact of item fragmentation constraints. Directions for future research are finally proposed.

1. Introduction

Inventory management models support decisions on how to procure or produce goods and in what quantity, in anticipation of future demand, by balancing the inventory holding and fixed setup costs on the planning horizon. According to Mosca et al. (2019), the decision-making practices in industry shift from isolated, unilateral department decisions to multiparty supply chain planning, and integrated modeling techniques have risen in popularity. Recent research stresses the economic benefits and needs to integrate inventory management decisions with other supply chain decisions, such as transportation planning, although the organizational and computational complexity of those compared to sequential solutions of simpler hierarchical problems might increase (Hrabec et al., 2022). For example, increased logistics costs can arise when simplifying transportation costs and ignoring the availability of multiple transportation modes (Engebrethsen and Dauzère-Pérès, 2022). In practice, long-term and short-term supply chain decisions may be interrelated and impact the overall performance of the supply chains (Jalal et al., 2022). In this paper, we integrate inventory and transportation mode selection decisions together with item fragmentation, i.e. the splitting of the same item between several trucks or containers. Improving the loading configuration may help to better satisfy practical constraints and policies for loading and transportation operations of multiple products and to reduce logistics costs.

As in Grunewald et al. (2018), the focus of our paper is on direct links connecting a single source, which could be a supplier or a central warehouse, with a single sink (a production site, distribution center, or sales outlet). Such links represent, for example, vendor-managed inventory systems, store deliveries, or buyer-driven inbound logistics. The downstream supply chain member makes replenishment decisions for multiple items with dynamic, i.e. deterministic and time-varying, demand over a finite planning horizon. The logistics costs include the inventory holding and ordering costs for each item, along with the transportation cost for shipping the items from the vendor using various transportation modes. The objective of the problem is to minimize the total cost while fully satisfying the customer demand. When explicitly considering transportation capacities for various modes, one of the possible extensions of the inventory models highlighted in Engebrethsen and Dauzère-Pérès (2019) is the integration of more operational decisions related to the container loading configurations. Such extension is common for other supply chain related decisions, such as

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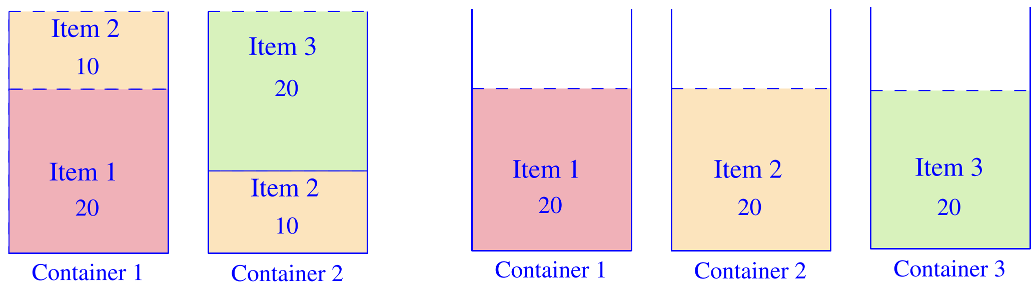


(a) Fragmentation-allowed policy (b) Fragmentation-forbidden policy

Fig. 1. Best truck loading given the fragmentation-acceptance policy.

vehicle routing problems or when dealing with several products having different configurations and container utilization rates (Mosca et al., 2019). In this work, we assume that the items are palletized in standard euro-pallets without any geometrical or weight differences. The only attributes of the items determining the loading configuration are the lot sizes expressed as the number of items/pallets.

In general, the ordering, transportation, and receiving costs grow with the variety of products (Chopra and Meindl, 2007). Reducing the number of products loaded in a container may have a positive impact on handling and shipping costs. In practice, companies may have different preferences related to fragmentation for an order with multiple items, i.e., if the product can be split among various containers or trucks or not. Heßler et al. (2021) describe some policies used in the retail industry. For some companies, avoiding item fragmentation may lead to less labor- or equipment-intensive handling operations. For example, if products are handled separately for every truck, then the processing effort may depend on the number of different items. This assumption may be related to additional quality control and inspections, load securing, labeling, space reservation and customs clearance activities (typically per truck or container) required at the origin or the destination. In such cases, it is cost-efficient to minimize the overall number of splits, i.e., it is preferable to pack the transported quantity of an item in two trucks than in three trucks, or even to forbid item fragmentation for some or all items. However, a loading policy that allows item fragmentation may better utilize the transportation capacity and decrease its costs.

It is therefore important to investigate how the tactical lot-sizing decisions can be optimized to lessen such conflict between the efficiency of transportation and handling operations. In this work, we investigate such a question by considering a policy where the buyer sets a limit on the allowed number of fragmentations per period. Note that neither pallets nor packages on the pallets are split. Only the order can be split so that the pallets of one product are transported using more than one truck. The problem studied in this paper extends the classical dynamic multi-item lot sizing problem, by considering multiple items, multiple transportation modes (as in Engebrethsen and Dauzère-Pérès (2022)), and operational considerations related to item fragmentation (as in the bin-packing problem of Heßler et al. (2021)). The main contributions of this paper include the following:

- We propose four Mixed Integer Linear Programming (MILP) formulations for the problem.
- Using realistic instances of different sizes, we compare the formulations in terms of computational efficiency and identify the problem parameters impacting the problem complexity. The performance of a relax-and-fix heuristic is also investigated when applied to this new problem.
- Using the results of several computational experiments, we identify the contexts in which it is cost beneficial to solve the proposed problem in an integrated manner, and we investigate the cost impact of fragmentation constraints.

The paper is organized as follows. Section 2 exemplifies and motivates the integration of fragmentation decisions in a multi-item dynamic lot sizing problem. We review the relevant literature in Section 3. The MILP model of the investigated problem is presented in Section 4. Due to the complexity of the problem, four formulations are proposed using reformulations of the lot sizing and bin packing problems in Section 5. The relax-and-fix heuristic is presented in 6. Section 7 describes the computational results of the different formulations and the heuristic, analyzing their performance and the impact of specific problem parameters on computational complexity. In Section 8, we investigate the cases where the proposed integrated model may lead to significant cost savings and analyze how imposing stricter item fragmentation policies may increase the total costs. Finally, we summarize the conclusions and present managerial implications and future research suggestions in Section 9.

2. Problem motivations

Let us consider one supplier and one customer that use a direct shipment transportation policy, and assume that a sequence of operations is triggered each time a shipment is dispatched and received. A realistic setting arising in the food and beverage industry that motivates this work is described in Heßler et al. (2021), where multiple products are shipped directly from a factory to individual warehouses using Full-truckload (FTL) trucks. While minimizing the number of trucks that can transport the ordered quantities, the proposed bin-packaging problem should decide on the assignment of the different products' lots to the truck by taking into account different policies of the warehouses motivated by ensuring the efficiency of their operations per truck. Under the *fragmentation-forbidden policy*, a warehouse forces the supplier to load each product lot on the same truck, and therefore forbids any fragmentation. Under the *fragmentation-allowed policy*, the warehouse does not forbid fragmentation, but the supplier has to minimize their number. The main motivation of our work is the argument that taking both transportation and handling operations into account in lot-sizing models can tackle the conflict highlighted above. Let us consider the following illustrative example.

Example. Let us consider a supplier shipping three products, each with an order of 20 pallets. The transportation is ensured by trucks with a maximum capacity of 30 pallets. Fig. 1 shows the best loading under the two policies. Two trucks are used when fragmentation is allowed, and there is necessarily one fragmented item. When fragmentation is forbidden, the supplier must load each item in a separate truck resulting in three trucks instead of two.

To illustrate the fact that lot sizes play a critical role in the use of transportation resources and the benefit of item fragmentation, let us consider a lot-sizing problem with three items and a planning horizon of two periods. Table 1 presents the demand for the three items and two feasible replenishment plans. Assuming the availability of one FTL mode with a capacity of 30 pallets, the quantities ordered in the second

Table 1
Illustrative example data.

Item	Demand		Plan I		Plan II	
	$t = 1$	$t = 2$	$t = 1$	$t = 2$	$t = 1$	$t = 2$
1	20	20	20	20	20	20
2	15	15	20	10	30	0
3	10	10	20	0	10	10

period fit in one container for both plans. The quantities ordered in the first period of Plan I correspond to the figures used in the bin packing problem illustrated in Fig. 1. As shown above, the ordered quantities induce either a single fragmentation or require one additional container depending on the fragmentation-acceptance policy. However, the quantities ordered in the first period in Plan II can fit in two containers without any fragmentation: the ordered quantity of the second item fills a container, and the quantities of the two remaining items can be loaded in another container.

As shown in the example of Fig. 1, there is a conflict between the efficiency of transportation operations and the efficiency of handling operations under the prism of item fragmentation. On the one hand, fragmenting the items potentially reduces the number of containers used, motivating, for example, the study of the vehicle routing problem where split delivery is allowed (Archetti et al., 2014a). On the other hand, loading the item lot fragments on several containers may negatively impact the handling and shipping operations.

The illustrative example shows that the right choice of the lot sizes can help to balance between the efficient usage of transportation resources and the handling operations. Therefore, optimization models supporting the supplier's packing decisions for a specific fragmentation policy can also benefit the buyer by integrating such decisions with transportation and inventory planning. Note that fragmentation is only relevant when several items are jointly replenished. As it is difficult to accurately estimate the actual cost related to additional activities imposed by fragmentation, we implement a hard constraint on the maximum allowed number of possible fragmentations for all items in each period.

All these reasons described in this section motivate our choice of studying a multi-item dynamic lot-sizing problem that explicitly models transportation capacities and indirectly takes into account the efficiency of handling operations through the notion of fragmentation. To the best of our knowledge, no integrated dynamic inventory lot-sizing model with transportation mode selection considers the fragmentation policies.

3. Literature review

This section briefly reviews the relevant literature. First, relevant lot-sizing problems focusing on integrating transportation decisions are reviewed. Next, the literature dealing with the bin backing problem and its extensions considering fragmentation is briefly summarized.

Lot sizing literature. Dynamic inventory lot-sizing models have been studied by researchers for many decades. For a literature review on dynamic lot-sizing models, we refer the reader to for example Brahimi et al. (2017) for single item lot-sizing problems, or Robinson et al. (2009) for multi-item lot-sizing problems. Inventory lot-sizing problems that are coordinated with the planning of transportation operations did not receive much attentions according to Büyükkaramikli et al. (2014). A review of integrated inventory-transportation models, that simultaneously address transportation operations (routing, mode and policy selection, etc.) and various inventory decisions, can be found in Mosca et al. (2019). The authors stress the need to consider multiple items and transportation modes to propose more realistic models that better reflect the industry needs. For example, Ertogral (2008) proposes a Lagrangian relaxation procedure to solve a multi-item uncapacitated dynamic lot-sizing problem with piecewise linear LTL transportation costs. Venkatachalam and Narayanan (2016) propose efficient formulations and heuristics for the multi-item lot-sizing problem with transportation costs.

Engebrethsen and Dauzère-Pérès (2019) provide a detailed review of transportation mode selection decisions in inventory lot-sizing research and the industrial practices. The authors pinpoint the main shortcomings of the existing models, such as the consideration of a single mode and the simplification of the transportation cost functions, not reflecting realistic piece-wise linear price schedules such as Full-Truckload (FTL) and Less-than-Truckload (LTL), or mode capacities. Only few papers consider dynamic deterministic demands for multiple items and mode selection decision assuming simplified transportation costs or restricted mode usage. Rizk et al. (2006a) and Rizk et al. (2006b) consider multiple items and transportation cost with discounts as a part of the purchasing cost, so-called unit replenishment cost, for a dynamic deterministic demand. These models apply a pre-processing approach for replenishment cost modeling, where general cost functions with the lowest unit cost among all modes for each quantity have been generated, and different modes could not be combined for the same shipment.

Jaruphongsa et al. (2007) and Palak et al. (2018) model two modes with an FTL-like multiple set-up structure, assuming that the capacities of the two modes are integers of each other and allowing the two modes to be combined for the same order. Ekşioğlu (2009) proposes an extended model with more than two FTL-like modes. Kopanos et al. (2012) model decisions on the procurement of additional FTLs from an external transportation company in addition to an internal fleet charging unit transportation costs every period. Hammami et al. (2012) and Mogale et al. (2017) assume constant unit costs for transportation when modeling multiple transportation modes. Absi et al. (2016) propose a dynamic inventory model with multiple replenishment modes, each having a fixed cost and a unit cost and carbon emission parameter for both transportation and production. In this model the modes can be combined, but transportation capacity limits are not considered. Cheng and Li (2021) extend the previous model by considering multiple products, heterogeneous truck fleet and proposing a simulated annealing algorithm, constraining the transportation mode usage to a single truck type per period, and analyzing mode usage for various carbon cap constraints. Hwang and Kang (2016) propose a two-phase algorithm for a lot-sizing problem with backlogging and stepwise transportation cost without speculative motives, considering a single FTL model and a single LTL mode with linear unit cost available, assuming that carriers could vary over periods. Akbalik and Rapine (2018) study a single-item uncapacitated inventory problem with FTL-like multiple replenishment modes and batch deliveries incurring fixed cost per batch. The authors show that this problem is NP-hard even for a single period, and propose dynamic programming algorithms and heuristics. Engebrethsen and Dauzère-Pérès (2022) propose a single-item lot-sizing problem with more than two FTLs, LTLs and realistic transportation costs and capacities, allowing various modes to be combined.

Bin Packing literature. The Bin Packing Problem is a well-studied combinatorial optimization problem in the literature. For a review of solution methods for this problem and its variants, we refer the reader to Coffman et al. (2013). In its basic variant, item fragmentation (splitting an item into two or more parts and packing each part into different bins) is not allowed. However, in some applications, this assumption is too restrictive, see for example Archetti et al. (2014b) for vehicle routing where split delivery is allowed. Fragmenting items has the potential to reduce the number of bins or containers used. This variant is called the Bin Packing Problem with Item Fragmentation, which is a trivial problem if there are no restrictions on the number of fragmentations (Ekici, 2021). Several variants of the bin packing problem with item fragmentation are discussed in the literature. Concerning the objective function, two variants have been proposed.

In the first variant called *bin-minimization*, an upper bound on the number of fragmentations is given, and the number of bins is minimized. For given purchased quantities, a subproblem of the problem explored in this work is the variable cost and size bin packing problem with item fragmentation corresponding to the bin-minimization variant. As demonstrated by the numerical experiments in Section 7, the adaptation of the chain formulation proposed in Casazza (2019) improves the efficiency of the overall optimization model. More details about the chain formulations are provided in Section 5.2. In the second variant of bin packing problems with fragmentation called *fragmentation-minimization*, a maximum number of bins is given, and the number of fragmentations is minimized. This variant is used in Section 8.2 when exploring the impact of the tightness of the fragmentation constraints on the total cost. Heßler et al. (2021) solve a practical bin-packing problem and optimize several objective functions in lexicographic order, including the minimization of fragmentations. In the bin-packing sub-problem of our model, it is possible to use and combine heterogeneous FTL modes or containers of different sizes and costs. Heßler et al. (2021) consider a homogeneous truck fleet in terms of size but different in terms of the presence or absence of a cooling system defining the ability to transport standard, cooled or frozen products.

Integrated lot sizing and bin-packing literature. Molina et al. (2016) propose to integrate production lot-sizing decisions with pallet loading decisions, under constraints of limited weight, volume or load value in pallets. The authors consider a capacity reservation type of transportation contract, proposed by van Norden and van de Velde (2005), and pre-process the so-called Manufacturer's Pallet Loading Problem for loading the items onto pallets without mixing them, providing the input into lot sizing problem. However, no item fragmentation policy or any information on which item is being loaded onto what truck is considered. Despite the increasing interest for integrated bin packing and lot-sizing problems among researchers (see Melega et al., 2018), transportation planning is never considered. To our knowledge, no authors have previously integrated fragmentation policies into a lot-sizing model.

4. Problem modeling

We assume that the buyer has to satisfy the demand of I items without shortage in a planning horizon of T periods, with the possibility of using up to M different FTL modes. There are no constraints on the quantity ordered in each period and backlogging is not allowed. Each item i is associated with a demand d_{it} in period t, an inventory holding cost h_i and a fixed ordering cost s_i. Each FTL mode m has a capacity c_m (e.g., number of pallets, tonnes) per container and a freight rate per container f_m. Each item has a weight w_i, that corresponds to the proportion of a container capacity unit occupied by one unit of item i. For example, if the container capacity is expressed in terms of number of pallets, w_i is the proportion of one pallet occupied by one unit of item i. In the computational experiments of Sections 7 and 8, we use $w_i = 1$ for all the items as the demand is large enough to be expressed in terms of number of pallets.

In addition to the lot sizing problem with transportation mode selection, the problem modeling aims at proposing a container loading that can facilitate handling operations. As the lot sizing and container loading decisions are not at the same level, the model aims more specifically at ensuring that it is possible to find container loading patterns that ensure the efficiency of the handling operations given the produced optimal lot sizing decisions. To do so, we assume that loading the shipped quantity of one item in different containers negatively impacts the efficiency of handling operations. In this work, *fragmentation* accounts for the situation where a shipped quantity of an item is dispersed in different containers while it can be packed in one container. We consider a hard constraint on the number of possible fragmentations for all items, denoted F. This additional constraint requires the detailed modeling of the item assignment to containers. In the following, k is used as an index of the container of a given FTL mode m.

To explain the model in greater detail, we present below two distinct sub-problems: The uncapacitated multi-item lot-sizing problem in Section 4.1 and the variable cost and size bin packing problem with item fragmentation in Section 4.2. First, we present in (1) the objective function that minimizes the ordering costs (OC), the inventory holding costs (IC), and the transportation costs (TC) over the finite horizon. As different formulations of the subproblems are proposed, the terms of this generic objective function are calculated through equality constraints using the appropriate decision variables and parameters of the associated formulation. More precisely, OC and IC are detailed in Sections 4.1 and 5.1 respectively, and TC is detailed in Sections 4.2 and 5.2 respectively.

$$\min OC + IC + TC \tag{1}$$

4.1. Lot-sizing problem

This section presents the constraints of the lot sizing problem. The resulting formulation is quite intuitive and is often referred to in the literature as the aggregate formulation (AGG). In this formulation, for each period t, O_{it} is a binary variable that models whether item i is ordered in period t, Q_{it} is the quantity of item i ordered in period t, and I_{it} is the inventory level of item i at the end of period t. We assume, without loss of generality, that the inventory levels at the beginning ($I_0 = 0$) and at the end of the planning horizon are zero.

$$OC = \sum_{i=1}^{I} s_i \sum_{t=1}^{T} O_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{2}$$

$$IC = \sum_{i=1}^{I} h_i \sum_{t=1}^{T} I_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{3}$$

$$I_{it} = I_{i(t-1)} + Q_{it} - d_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{4}$$

$$Q_{it} \le \left(\sum_{t'=t}^{T} d_{it'}\right) O_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{5}$$

$$O_{it} \in \{0,1\} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{6}$$

$$Q_{it}, I_{it} \ge 0, \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{7}$$

Constraints (2) and (3) compute, respectively, the total ordering cost and the total inventory cost in the objective function (1). Constraints (4) are the classical inventory balance equations, expressing that the inventory $I_{i(t-1)}$ of item i at the end of period $t-1$ added to the shipment Q_{it} received in period t are used to satisfy demand d_{it} of item i at period t. What remains is kept in stock at the end of the period (I_{it}). Constraints (5) ensure that the fixed ordering cost of item i is incurred each time there is a positive quantity of i ordered in period t. $\sum_{t'=t}^{T} d_{it'}$ is the value of the big-M parameter and represents the maximum quantity that can be optimally ordered. Constraints (6) define O_{it} as binary and Constraints (7) define Q_{it} and I_{it} as non-negative decision variables.

4.2. Variable cost and size bin packing problem with fragmentation

Let us first introduce the decision variables to model the different constraints related to this problem. A_{mt} denotes the number of containers of FTL mode m used in period t. By introducing the constraints on the number of allowed fragmentation, one must explicitly model the assignment of the item shipped quantities to the containers of the different modes. Therefore, for each period t, the necessary variables for such explicit modeling of the assignment of item shipped quantities to FTL containers are the quantity of item i loaded on container k of mode m denoted by X_{itkm}, the binary variable modeling if item i is loaded in container k of FTL mode m denoted by L_{itkm} and the binary variable U_{tkm} modeling if container k of mode m is used in period t.

If the shipped quantity of an item cannot fit in the container with the largest capacity, forbidding fragmentations leads to the infeasibility of the model. For example, if 70 pallets of an item have to be shipped in one period while the largest container has a capacity of 30 pallets, at least 3 containers must be used to transport the ordered quantity. To ensure the model always has feasible solutions, the constraints on the number of fragmentations do not consider what can be called *mandatory fragmentations*. Therefore, we introduce the additional integer variable F_{it} modeling the number of mandatory fragmentations of item i in period t. Considering the previous example, there are two mandatory fragmentations. Using the defined notations, the constraints of the variable cost and size bin packing problem are defined below.

$$TC = \sum_{m=1}^{M}\sum_{t=1}^{T} f_m A_{mt} \qquad t = 1,\dots,T,\ m = 1,\dots,M \tag{8}$$

$$w_i Q_{it} = \sum_{m=1}^{M}\sum_{k=1}^{K} X_{itkm} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{9}$$

$$L_{itkm} \le O_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M,\ k = 1,\dots,K \tag{10}$$

$$X_{itkm} \le \min\{c_m, w_i \sum_{t'=t}^{T} d_{it'}\} L_{itkm} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M,\ k = 1,\dots,K \tag{11}$$

$$L_{itkm} \le U_{tkm} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M,\ k = 1,\dots,K \tag{12}$$

$$\sum_{i=1}^{I} X_{itkm} \le \min\{c_m, \sum_{i=1}^{I} w_i \sum_{t'=t}^{T} d_{it'}\} U_{tkm} \qquad t = 1,\dots,T,\ m = 1,\dots,M,\ k = 1,\dots,K \tag{13}$$

$$A_{mt} = \sum_{k=1}^{K} U_{tkm} \qquad t = 1,\dots,T,\ m = 1,\dots,M \tag{14}$$

$$F_{it} \le \frac{w_i Q_{it}}{\max_{m=1}^{M} c_m} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{15}$$

$$F_{it} \ge \frac{w_i Q_{it}}{\max_{m=1}^{M} c_m} - 1 + \epsilon \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{16}$$

$$\sum_{i=1}^{I}\left(\sum_{m=1}^{M}\sum_{k=1}^{K} L_{itkm} - O_{it}\right) - \sum_{i=1}^{I} F_{it} \le F \qquad t = 1,\dots,T \tag{17}$$

$$\epsilon O_{it} \le Q_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{18}$$

$$U_{t(k+1)m} \le U_{tkm} \qquad t = 1,\dots,T,\ m = 1,\dots,M,\ k = 1,\dots,K-1 \tag{19}$$

$$A_{mt}, F_{it} \in \mathbf{N} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M, \tag{20}$$

$$L_{itkm}, U_{tkm} \in \{0,1\} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M,\ k = 1,\dots,K \tag{21}$$

$$X_{itkm} \ge 0 \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M,\ k = 1,\dots,K \tag{22}$$

In addition to the ordering and inventory costs, the objective function (1) includes a term related to the transportation costs TC computed by Constraint (8). Constraints (9) convert the ordered quantity Q_{it} to the consumption of the transportation capacity and enforce the acquisition of such required capacity. Constraints (10) avoid the loading decisions of item i in period t to be considered for an item that is not ordered. Constraints (11) ensure that the binary variable modeling the use of an FTL mode container by an item is equal to 1 as soon as there is at least one item shipped quantity that is loaded in the container. Constraints (12) state that an FTL mode container is used as soon as an item requires it. Constraints (13) state that if an FTL mode container is used, the total shipped quantity of all items should not exceed its maximum capacity. Constraints (14) count the number of containers of an FTL mode m used in each period. Constraints (15) and (16) compute for each item i and each period t the number of mandatory fragmentations where ϵ is a very small number. The number of mandatory fragmentations is computed using the floor function on the ratio between the required capacity of the total quantity to be shipped in t and the capacity of the largest container. Constraints (17) impose a limit on the total number of fragmentations. For each ordered item (i.e., $O_{it} = 1$), the number of fragmentations is computed as the sum of the number of used FTL mode containers, from which the minimum number of required containers (number of mandatory fragmentations plus 1) is subtracted. To avoid incurring an ordering cost without ordering any quantity, Constraints (18) ensure that "$O_{it} = 1 \implies Q_{it} > 0$" (the converse of the implication modeled by Constraints (5)). Constraints (19) are used to eliminate symmetries and reduce the search space. Constraints (20), (21) and (22) define the integer, binary, and non-negative variables respectively. This formulation of the bin packing subproblem is called in the remainder of the paper *straightforward formulation* and denoted S.

5. Problem reformulation

The model studied in this work and defined in Section 4 comprises two NP-hard problems: The dynamic multi-item lot sizing problem and the variable cost and size bin packing problem with item fragmentation in each planning period. This section proposes reformulating the studied problem using known reformulations for the two subproblems. Therefore, instead of the aggregate formulation for the lot sizing problem of Section 4.1, Section 5.1 presents a known disaggregate formulation. Section 5.2 adapts the formulation proposed in Casazza (2019) to model the problem of transportation mode selection and container loading in each period. In total, given the formulation used for each subproblem, four formulations are compared in Section 7. As the different cost components are computed through constraints, the objective function in (1) is valid regardless of how the sub-problems are formulated.

5.1. Lot-sizing problem reformulation

This section focuses on the multi-item lot-sizing sub-problem. There are very few reformulations for such a complex problem, so it is common to use reformulations for uncapacitated single-item lot sizing problems. The formulation presented in Section 4.1 is called aggregate in contrast with disaggregate formulations such as the facility location problem-based formulation (Krarup and Bilde, 1977) and the shortest path problem-based formulation (Evans, 1985). These two disaggregate formulations are considered as tight formulations because their LP relaxations have optimal solutions in which the binary ordering variables O_{it} are integer. We present below the facility location problem-based formulation. New variables are defined along additional parameters to provide the facility-location formulation, denoted *FAL*. Let $Q_{it't}$ denote the number of units shipped in period t' to satisfy the demand of period t. To facilitate the computation of the inventory costs in this new formulation, let us denote the holding cost for supplying one unit for an item i in period t from replenishment period t': $h_{it't} = h_i(t-t')$. For modularity, variables Q_{it} from the aggregate formulation, i.e., the quantity of item i ordered in period t, are used again. Indeed, variables Q_{it} are required by both formulations of the bin packing problem. Instead of considering the constraints in Section 4.1, the FAL formulation includes the constraints below.

$$OC = \sum_{i=1}^{I} s_i \sum_{t=1}^{T} O_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{23}$$

$$IC = \sum_{i=1}^{I} \sum_{t'=1}^{T} \sum_{t=t'}^{T} h_{it't} Q_{it't} \qquad t = 1, \dots, T, \; i = 1, \dots, I \tag{24}$$

$$\sum_{t'=1}^{t} Q_{it't} = d_{it} \qquad t = 1, \dots, T, \; i = 1, \dots, I \tag{25}$$

$$Q_{it't} \leq d_{it} O_{it'} \qquad i = 1, \dots, I, \; t' = 1, \dots, T, \; t = t', \dots, T \tag{26}$$

$$Q_{it'} = \sum_{t=t'}^{T} Q_{it't} \qquad i = 1, \dots, I, \; t' = 1, \dots, T \tag{27}$$

$$O_{it} \in \{0, 1\} \qquad i = 1, \dots, I, \; t = 1, \dots, T \tag{28}$$

$$Q_{it't}, Q_{it'} \geq 0 \qquad i = 1, \dots, I, \; t' = 1, \dots, T, \; t = t', \dots, T \tag{29}$$

Constraints (23) and (24) compute the total ordering cost and the total inventory cost, respectively, in objective function (1). Constraints (25) are the new inventory balance constraints, expressing that the demand at period t must be satisfied by the total quantities shipped in the previous or current period. Constraints (26) ensure that the fixed ordering cost of item i is incurred each time there is a positive quantity of item i ordered in period t'. These constraints also define an upper bound on the optimal total shipped quantity $Q_{it't}$. Constraints (28) and (29) define the binary and continuous non-negative variables, respectively.

5.2. Bin-packing problem reformulation

The transportation mode selection sub-problem described in Section 4.2 can be qualified as the variable cost and size (depending on the mode) bin (container) packing problem with item fragmentation (VCSBF). Contrary to the classical bin packing problem, items can be fragmented and fractionally assigned to different bins. The second difference lies in the fact that the bins might be heterogeneous (different cost and capacity). The straightforward formulation provided in Section 4.2, despite being easier to understand, is not the most efficient one, as shown in the numerical results, due to its large number of binary variables and constraints. For example, binary variables are introduced for each possible assignment of an item to the available number of containers of each mode. Instead of the straightforward formulation, this section adapts a recent reformulation of VCSBF to model the problem of transportation mode selection and container loading in each period. This formulation, called *chain* formulation and referred to by C, is proposed by Casazza (2019).

The chain formulation reduces the problem of packing items into a minimum-cost set of chains instead of a minimum-cost set of bins, where each chain corresponds to a subset of the bins. To illustrate this, Fig. 2 compares the straightforward and chain representations of a feasible packing solution. In Fig. 2(a), five items are packed in four bins, three with the largest capacity and a smaller one. Items 2 and 3 are fragmented as their quantities exceed the largest capacity. In the chain representation (Fig. 2(b)), the same solution is represented by two chains. The first chain is basically the first three bins that are stacked, while the second chain corresponds to the last bin. The three first bins are stacked, or chained, as the two first bins share item 2, while item 3 is fragmented between the second and third bins. So, instead of assigning items to bins, the chain formulation assigns items to chains, each with a capacity and a cost that are, respectively, equal to the sum of the capacities and costs of its bins. By doing so, the chain formulation involves significantly fewer variables than the straightforward formulation. For more details regarding the chain formulation, the reader is referred to Casazza and Ceselli (2014), Casazza (2019). Beyond its impact on the computational complexity of the studied model, the chain formulation perfectly fits the buyer's perspective considered in this work. By integrating the fragmentation constraints in the lot-sizing model, the buyer only ensures that the supplier can ship the ordered quantity with the lowest transportation costs and comply with the preferred fragmentation-acceptance policy. Knowing in which container each item is loaded is irrelevant to the buyer, at least in the planning stage.

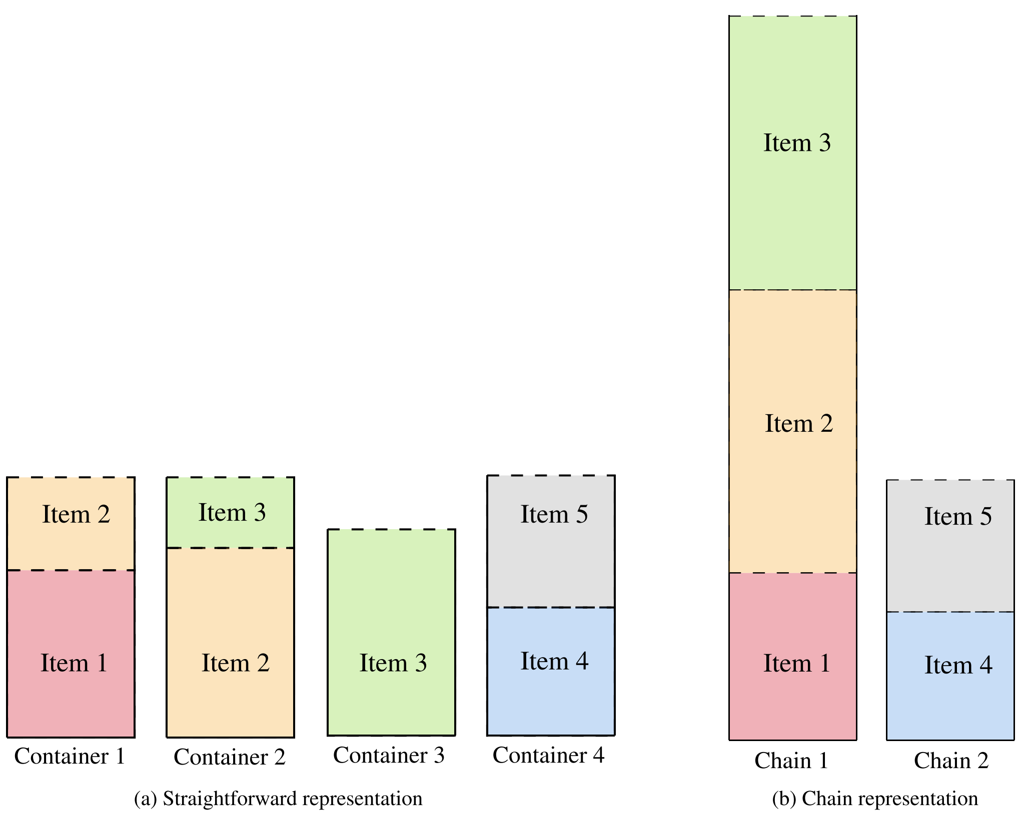


Fig. 2. Example of a feasible packing solution.

Table 2
List of the problem formulations.

Formulation	Lot sizing problem		Bin packing problem	
	Aggregate (AGG) Section 4.1	Facility location (FAL) Section 5.1	Straightforward (S) Section 4.2	Chain (C) Section 5.2
AGG-S	✓		✓	
AGG-C	✓			✓
FAL-S		✓	✓	
FAL-C		✓		✓

To use the chain formulation to model the problem of transportation mode selection and container loading in each period, let K be a sufficient number of containers. This number can also be considered as the maximum number of chains since each chain corresponds to a set of at least one container, and therefore no solution using more than K chains is feasible. However, using a number of chains as small as possible shows a significant improvement in the efficiency of the mathematical model. As the chain formulation constrains each item to be assigned to at most one chain, the number of chains is also bounded by the number of items. Let us consider $C = \min\{K, I\}$ as the maximum number of chains, and let c be the chain index. Besides C and c, several variables must be redefined or introduced. The number of containers of a FTL mode m in a chain c in period t is denoted by A_{mtc}. Other binary variables required by this new formulation are the variables that take value 1 when item i is assigned to chain c in period t denoted L_{itc} and the variable modeling the use of a chain c in a period t denoted by U_{tc}. Finally, X_{itc} is the quantity of item i ordered in period t and loaded in chain c. The formulation is presented below.

$$TC = \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{c=1}^{C} f_m A_{mtc} \qquad t = 1,\dots,T,\ m = 1,\dots,M \tag{30}$$

$$w_i Q_{it} = \sum_{c=1}^{C} X_{itc} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{31}$$

$$L_{itc} \le O_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ c = 1,\dots,C \tag{32}$$

$$L_{itc} \le U_{tc} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ c = 1,\dots,C \tag{33}$$

$$X_{itc} \le (\sum_{t'=t}^{T} w_i d_{it'}) L_{itc} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ c = 1,\dots,C \tag{34}$$

$$\sum_{c=1}^{C} L_{itc} \le 1 \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{35}$$

$$\sum_{i=1}^{I} X_{itc} \le \sum_{m=1}^{M} c_m A_{mtc} \qquad t = 1,\dots,T,\ c = 1,\dots,C \tag{36}$$

$$U_{tc} \le \sum_{m=1}^{M} A_{mtc} \qquad t = 1,\dots,T,\ c = 1,\dots,C \tag{37}$$

$$F_{it} \le \frac{w_i Q_{it}}{\max_{m=1}^{M} c_m} \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{38}$$

$$F_{it} \ge \frac{w_i Q_{it}}{\max_{m=1}^{M} c_m} - 1 + \epsilon \qquad t = 1,\dots,T,\ i = 1,\dots,I \tag{39}$$

$$\sum_{c=1}^{C}(\sum_{m=1}^{M} A_{mtc} - U_{tc}) - \sum_{i=1}^{I} F_{it} \le F \qquad t = 1,\dots,T \tag{40}$$

$$U_{t(c+1)} \le U_{tc} \qquad t = 1,\dots,T,\ c = 1,\dots,C-1 \tag{41}$$

$$L_{itc}, U_{tc} \in \{0,1\} \qquad t = 1,\dots,T,\ m = 1,\dots,M,\ c = 1,\dots,C \tag{42}$$

$$A_{mtc}, F_{it} \in \mathbf{N} \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M,\ c = 1,\dots,C \tag{43}$$

$$X_{itc} \ge 0 \qquad t = 1,\dots,T,\ i = 1,\dots,I,\ m = 1,\dots,M,\ c = 1,\dots,C \tag{44}$$

Constraints (30) compute the transportation costs (TC) in the objective function (1). Constraints (31) convert the quantity Q_{it} of item i ordered in period t to the consumption of the transportation capacity and enforce the acquisition of such required capacity. Constraints (32) avoid considering loading decisions of an item that is not ordered, while Constraints (33) forbid an item to be loaded on an unused chain c. Constraints (34) ensure that the binary variable modeling the use of chain c by item i is equal to 1 as soon as there is a strictly positive quantity loaded on chain c. Constraints (35) state that an item is loaded in at most one chain. Constraints (36) make sure that each chain c is allocated enough FTL capacity to transport the total quantity to ship in period t. Constraints (37) ensure that each selected chain has at least one FTL mode container. Constraints (38) and (39) compute for each item i and each period the number of mandatory fragmentations by using the floor function on the results of dividing the required capacity of the total quantity to be shipped in that period by the capacity of the largest container capacity. Constraints (40) impose a limit on the total number of fragmentations. For each chain, the number of fragmentations is computed as the sum of the number of used FTL mode containers minus the number of used chains. The upper bound is increased by the number of mandatory fragmentations. Constraints (41) are used to eliminate symmetries and reduce the search space. Constraints (42), (43) and (44) define the binary, integer, and non-negative variables, respectively.

In summary, four formulations of the problem studied in this paper are obtained by choosing one of the two formulations for each of its two subproblems. Table 2 provides the section number for each of the four formulations. The first part of each formulation name indicates the formulation for the lot-sizing problem: AGG stands for aggregate formulation while FAL stands for facility location formulation. The second part of the name of each formulation indicates the formulation for the variable cost and size bin packing problem: "S" stands for "straightforward" formulation, whereas "C" stands for the adaptation of the chain formulation. Note again that, independently of the selected formulation, the objective function (1) remains the same. Using realistic instances, the four formulations are compared in Section 7.2.

6. A relax-and-fix heuristic

The problem defined in this work combines two NP-hard problems: A multi-item lot-sizing problem and a variable cost and size bin packing problem with item fragmentation. Even with the modern fast computers and state-of-the-art optimization solvers, it is very unlikely that real-life instances of such a problem can be solved using exact methods. Therefore, this section introduces the relax-and-fix heuristic, denoted *RF*, to study its performance for this problem. The relax-and-fix heuristic is a commonly used heuristic to solve lot-sizing problems (Pochet and Wolsey, 2006 and Toledo et al., 2015) and mixed-integer linear programs in general (Joncour et al., 2023). A relax-and-fix heuristic builds an initial solution by sequentially solving several relaxed sub-problems and fixing integer variables. Before iterating over the sub-problems, this heuristic requires the partitioning of the integer variables into distinct subsets and the definition of the processing sequence. In multi-period optimization problems, time-based partitioning and a processing sequence from the first period to the last are natural choices. At each iteration, except for the current subset of integer variables and the integer variables already fixed in the previous iterations, the integrality constraints are relaxed for all the other integer variables. In the AGG-C formulation, the integer decision variables are: O_{it} (whether item i

is ordered in period t), U_{tk} (whether chain k is used in period t), L_{itk} (whether item i is loaded in chain k in period t), A_{mtk} (number of containers in FTL mode m used by chain k in period t) and F_{it} (number of mandatory fragmentations of item i in period t).

With time-based partitioning, it is common to name *window* the sequence of periods for which the related integer decision variables are the only variables on which the integrality constraints apply. The window size, denoted ws, corresponds to the number of periods in the window. After solving the relaxed sub-problem, it is possible to fix all the optimized integer variables, but also to only fix a subset and re-optimize the remaining variables in the next iterations. The overlap parameter, denoted ov, specifies the number of periods at the end of the window for which the decision variables are re-optimized in the next iterations. Contrary to the integer decision variables, the continuous variables are re-optimized in every iteration and only fixed in the last iteration of the heuristic. Finally, as sub-problems might be too time-consuming to solve, it is common to set up a time limit for the MILP solver in each iteration. In the implemented heuristic, the time limit for each iteration is determined by the window size (i.e., the number of periods) and the allocated computation time per period, denoted tl. In addition to the time limit, the solver is terminated when the solution is within the relative MILP optimality gap specified by a parameter denoted gap. In conclusion, the proposed relax-and-fix heuristic has four parameters: Window size (ws), overlap (ov), time limit per period (tl) and relative MILP gap (gap). The performance of this heuristic is evaluated in Section 7.4.

7. Computational analysis

This section aims at numerically exploring the computational complexity of the problem defined in this work. First, Section 7.1 describes the procedure used to generate the problem instances based on real data from a Scandinavian distribution company for fast-moving consumer goods. This section identifies the main parameters to be varied to analyze their impact on the problem resolution. Section 7.2 compares the four proposed problem formulations and Section 7.3 studies the impact of the parameters. Finally, Section 7.4 reports the impact of the size of the instances on the performance of the exact and heuristic approaches.

7.1. Design of experiments

To conduct the experiments, new instances are created to explore the efficiency of the formulations and characterize the impact of some of the problem attributes on the problem computational complexity and of the fragmentation constraints on the total cost. The input data related to decisions regarding the lot sizing and transportation mode selection are based on realistic data of a Scandinavian distribution company for fast-moving consumer goods. The other parameters that are closely related to fragmentation constraints are generated following a procedure inspired by Crainic et al. (2011) and Casazza (2019). The problem attributes are as follows (costs are expressed in NOK):

1. Combinations of the number of items and periods: $(I, T) \in \{(10, 10), (20, 10), (40, 20)\}$.
2. Number of available transportation modes $M \in \{2, 4\}$. The capacity and cost of the FTL modes are taken from the two lists $(33, 30, 25, 11)$ and $(4917, 4560, 4000, 2123)$, respectively. When $M = 2$, the characteristics of the available FTL modes correspond to the first elements of the two lists. The capacity is expressed in terms of the number of Euro-pallets that can fit in typical container sizes (Engebrethsen and Dauzère-Pérès, 2019).
3. Based on historical data from a Scandinavian company that distributes fast-moving consumer products, the data related to the items are generated as follows:
   - Given that the products from the practical setting belong to the same family, it is assumed that all items have a weight of $w_i = 1$.
   - The annual holding cost is equal to 20% of the product price.
   - Each item ordering cost is randomly generated according to a (discrete) uniform distribution in the set $\{1, 2, \ldots, 100\}$. This corresponds to a low time between orders (TBO) characterizing the practical setting.
   - The parameters of the normal distributions used to randomly generate the demand of the items are estimated based on the historical demand for 30 products. The demand is expressed in terms of the number of Euro-pallets. In the actual setting studied in this work, as highlighted by Heßler et al. (2021), the quantities are large enough so that pallets are not mixed, i.e., each pallet contains only one product. Based on the average weekly demand, the items are grouped into three categories: Small-sized items $S1$ with an average demand in the interval $[1, 7]$; Medium-sized items $S2$ with an average demand in the interval $[11, 21]$; and Big-sized items $S3$ with an average demand in the interval $[26, 33]$. The relative size of the items is determined based on the capacity of the largest FTL container, i.e., 33 pallets.
4. After grouping the items according to the proportion of the average demand relative to the largest FTL capacity, the item types are mixed in order to generate different demand configurations similarly to Crainic et al. (2011). Six demand configurations are generated:
   - Configuration $C1$ where 100% of items are in $S1$,
   - Configuration $C2$ where 100% of items are in $S2$,
   - Configuration $C3$ where 100% of items are in $S3$,
   - Configuration $C4$ where 50% of items are in $S1$ and 50% of items are in $S2$,
   - Configuration $C5$ where 50% of items are in $S1$ and 50% of items are in $S3$,
   - Configuration $C6$ where 10% of items are in $S1$, 70% of items are in $S2$ and 20% of items are in $S3$.
5. The allowed number of fragmentations for each configuration is $F = \lfloor fI \rfloor$, where f is a parameter in the set $\{0\%, 10\%, 20\%, 100\%\}$ and I is the number of items.

In general, the instances in this section and in Section 8 are characterized by the following elements: Number of items, number of periods, demand configuration, number and characteristics of the available FTL modes, and allowed number of fragmentations. The relax-and-fix heuristic is implemented in C++. The experiments have been conducted using Cplex 22.11 with default values for all parameters on a PC equipped with an Intel(R) Core(TM) i9-12900H CPU at 2.50 GHz with 32 GB of memory.

7.2. Comparison of the formulations

This section compares the four formulations: AGG-S, AGG-C, FAL-S and FAL-C using 240 instances with $I = 10$ and $T = 10$, generated following the procedure described in Section 7.1. Three maximum computational times are used: 60, 300, and 3600 seconds. Due to the difficulty of the problem instances, the optimality gap of each formulation on each instance is calculated with regards to the best lower bound denoted LB_{3600}. Furthermore, to tighten the lower bound, the same instances are solved after dropping all considerations related to fragmentation. Hence, the maximums of these bounds obtained after 3600 seconds are used for the values of LB_{3600}. Therefore, the optimality gap is calculated as $opt.\ gap = \frac{UB - LB_{3600}}{LB_{3600}}$, where UB is the

Table 3
Summary of experimental results for each of the four formulations over all instances.

	60 s					300 s				3600 s		
	nFS	nRS	nOSF	opt. gap (%)		nRS	nOSF	opt. gap (%)		nOSF	opt. gap (%)	
				avg	max			avg	max		avg	max
AGG-C	240	230	83	0.9	8.8	237	118	0.9	7.4	138	0.7	5.2
AGG-S	234	209	20	12.9	70.6	235	36	7.1	66.3	139	0.8	7.8
FAL-C	240	230	89	0.8	7.3	237	126	0.9	6.5	143	0.7	6.7
FAL-S	236	190	21	12.8	99.6	234	38	7.1	64.8	140	0.8	7.8

upper bound given by the solver in the allocated computational time. Table 3 reports the average and maximum optimality gaps for each formulation and for each maximum computation time.

It might be difficult to find a feasible solution or a feasible solution with a "reasonable" optimality gap when allowing short computational times. As this is relevant only when allowing a computational time of 60 seconds, Table 3 reports in Column *nFS* the number of instances for which the formulation found a feasible solution. The "reasonable" optimality gap filters out outliers when computing the average and maximum optimality gaps. The results regarding the number of instances for which a "reasonable" feasible solution was found are reported in Columns *nRS* using a threshold of 100%. As highlighted above, Table 3 also reports the average and maximum optimality gaps. However, to conduct a fair comparison, these statistics are computed only over the instances for which all the formulations found "reasonable" feasible solutions in the allocated computational time. Over 240 instances, the number of instances to compute the statistics related to the optimality gap is 190 when allowing 60 seconds, 234 with 300 seconds, and 240 with 3600 seconds. Finally, Table 3 also reports in Columns *nOSF* the number of instances for which each formulation determined an optimal solution in the allocated computational time. Note that, because LB_{3600} is used instead of the lower bound obtained by executing the solver, a solution does not need to be proven optimal to be considered as such and taken into account when calculating *nOSF*.

Table 3 shows that, when allowing a maximum computational time of 60 seconds, FAL-C significantly outperforms the other formulations. The role of the chain formulation for the variable cost and size bin packing problem in improving the solution efficiency seems to be significant, considering that AGG-C is the second-best performing model. This ranking also holds when analyzing the results obtained after 300 seconds. The impact of the choice of formulation for the lot-sizing problem is less clear. FAL-C outperforms AGG-C when the computational time is 60 or 300 seconds, but the ranking of AGG-S and FAL-S depends on the performance indicator. When comparing the maximum optimality gap between the results found after 60 and 300 seconds, it is important to remember that the number of instances used to compute these statistics is different (83 and 208, respectively).

The results obtained when allowing a computational time of 3600 seconds show that the difference between the four formulations is less significant, especially regarding the average optimality gap. The impact of the chain formulation is most significant when analyzing the worst-case performance (*max*) as AGG-C and FAL-C outperform the two other formulations. The analysis of the results in Columns *nOS* highlights the difficulty of reaching optimal solutions in reasonable computational times. To further analyze these results, Table 4 reports the deviation of the lower bounds obtained by each formulation to the best lower bounds LB_{3600}, i.e., $LB\ deviation = \frac{LB_{3600}-LB}{LB_{3600}}$ where LB denotes the lower bound obtained in the allocated computational time.

Table 4 provides the average (*avg*) and the maximum (*max*) deviations over all instances with 300 and 3600 seconds. The table includes two additional columns with the number of optimal solutions: *nOSP* and *nOSF*. Column *nOSP* reports the number of instances for which the solver proves the optimality of the solution. Similarly to Table 3, Column *nOSF* reports the number of instances for which the solver found a solution proven to be optimal, either by using the lower bound provided by the solver or by using LB_{3600}. The results obtained when allowing 300s show that the chain formulation helps to tighten the lower bounds quickly. Except for *nOSP*, the chain formulation also helps to get slightly better lower bounds when allowing 3600s. Despite the best performance in terms of the quality of solutions (Table 3) and lower bounds (Table 4), AGG-C and FAL-C struggle to prove the optimality as they have significantly a lower number of proven optimal solutions (*nOSP*) compared to AGG-S and FAL-S. Comparing the results obtained after 300s and 3600s, the quality of the lower bounds seems to stagnate for AGG-C and FAL-C.

Table 4
Quality of the lower bounds of the four formulations over all instances.

	300 s				3600 s			
	nOSP	nOSF	LB dev. (%)		nOSP	nOSF	LB dev. (%)	
			avg	max			avg	max
AGG-C	67	120	0.7	2.6	84	138	0.6	2.2
AGG-S	22	36	2.6	17.3	108	139	0.6	4.4
FAL-C	63	130	0.8	3.1	77	143	0.7	2.7
FAL-S	18	38	2.8	14.9	108	140	0.7	4.8

In conclusion, this section illustrates the difficulty of solving even small-sized instances of the problem studied in this work ($I = 10$, $T = 10$). By comparing the four formulations, the computational results show a significant contribution of the chain formulation to finding high-quality solutions quickly. On the contrary, the contribution of the facility location formulation for the lot-sizing problem is not consistent over the different performance indicators and computational times. Except for the number of proven optimal solutions, AGG-C and FAL-C outperform AGG-S and FAL-C on all other performance indicators. Choosing between AGG-C and FAL-C is possible only by favoring one of the performance indicators. For the remainder of this article, we use AGG-C considering that it significantly outperforms all the other formulations regarding the worst performance in terms of the quality of solutions (*max* in Table 3) and lower bounds (*max* in Table 4) when allowing 3600s.

7.3. Impact of problem parameters on computational complexity

The solution quality depends not only on the problem formulation but also on the problem characteristics. To illustrate this, Fig. 3 shows the average optimality gap for each problem type and each number of allowed fragmentations per period. The bar chart is plotted using the results obtained by the AGG-C formulations when solving the 240 instances for 3600 seconds. When analyzing Fig. 3, all the parameters that are varied to generate the instances seem to determine the complexity of the problem instances: The demand configuration, the number of available FTL modes, and the number of allowed fragmentations in each period.

- **Demand configuration.** The results show that the problem complexity measured through the optimality gap is negatively correlated with the proportion of small and positively correlated with the proportion of big items. All the instances with demand configuration $C1$ (i.e., 100% of the items are small-sized) are solved optimally. The second easier subset of instances to solve

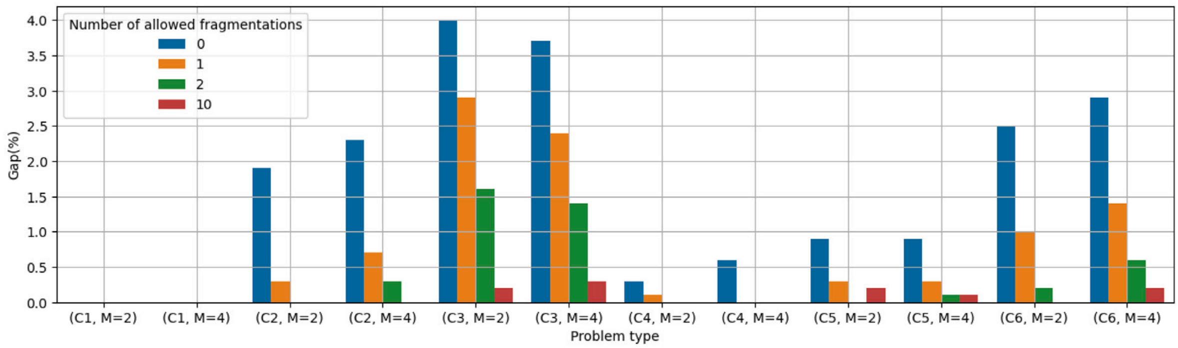


Fig. 3. Optimality gap (%) by problem type and number of allowed fragmentations per period.

are those with demand configuration $C4$ (i.e., half of the items are small-sized and the other half are medium-sized). The instances with the largest average optimality gap are those with demand configuration $C3$ (i.e., 100% of the items are big-sized). These results are in line with the findings of Heßler et al. (2021), showing that more difficult instances are obtained by increasing the proportion of the size of the items to pack in the containers. In addition to instances with demand configuration $C3$, such difficulty can be observed by the large optimality gap when solving instances with demand configuration $C6$ mixing 20% of big-sized items and 70% of medium-sized items.

- **Number of allowed fragmentations.** Overall, Fig. 3 shows that tightening the fragmentation constraints increases the complexity of the problem, especially when only two FTL modes are available.
- **Number of available FTL modes.** Increasing the number of FTL modes from $M = 2$ to $M = 4$ makes the problem more challenging due to the significant increase in the number of integer decision variables. The instances with demand configuration $C3$ are the only ones that deviate from this general pattern. With tight limits on the number of allowed fragmentation, the optimality gaps are lower when using $M = 4$ than $M = 2$. As highlighted above, the bin packing problem is the most difficult when all items are big-sized. It is possible that increasing the number of FTL modes, despite leading to a significant increase in the number of integer decision variables, provides more options to pack the big items, especially in the presence of tight fragmentation constraints.

7.4. Performance of exact and heuristic approaches on larger instances

In the previous sections, only instances with $I = 10$ and $T = 10$ are used in the computational experiments. This section aims to computationally evaluate the difficulty of the problem when solving larger instances. To do so, we report results on the optimality gap when solving instances with $(I, T) \in \{(10, 10), (20, 10), (40, 20)\}$. The instances are restricted to the most realistic demand configuration $C6$. Three limits on the maximum number of fragmentations are considered when considering $f \in \{0\%, 10\%, 20\%\}$. Five instances are generated for each instance size (I, T) and maximum number of fragmentations $F = fI$. In total, 45 instances are used in this section. Note that instances with $(10, 10)$ are a subset of the instances used in the previous sections included here to study the impact of the problem size increase on its difficulty. All the considered instances are solved using AGG-C formulation. This section also aims at analyzing the performance of the relax-and-fix heuristic. To tune its parameters, preliminary experiments on $(10, 10)$ instances are conducted to select the best combination among the ones initially studied. The results reported below are obtained using the following parameters: Window size ($ws = 2$), overlap ($ov = 1$), time limit per period ($tl = 90s$), and relative MILP gap ($gap = 0.2\%$).

Table 5
Results of AGG-C and the relax-and-fix heuristic (RF) heuristic on three instance sizes.

Instance size	Gap(%)				Computational time (s)			
	Average		Max		Average		Max	
	AGG-C	RF	AGG-C	RF	AGG-C	RF	AGG-C	RF
(I = 10, T = 10)	**1.9**	2.0	5.5	**5.2**	3651	866	3723	1702
(I = 20, T = 20)	5.7	**4.0**	11.0	**6.3**	3633	3171	3658	3666
(I = 40, T = 20)	9.2	**8.9**	14.7	**14.4**	3662	5171	3687	5319
All	5.6	**5.0**	14.7	**14.4**	3648	3069	3723	5319

Table 5 reports the average and maximum optimality gaps and computational times. The optimality gap is based on the lower bounds obtained by the AGG-C formulation. Focusing only on AGG-C, the results, either on average or considering the worst case, show that the optimality gap increases significantly when solving larger instances. For example, the average gap goes from 1.9% when solving (10.10) instances to 9.2% for (40.20) instances. We can observe the same trend when analyzing the results of the relax-and-fix heuristic. Considering the overall performance across all instances, the relax-and-fix heuristic provides slightly better average and maximum optimality gaps in a shorter computational time than AGG-C. However, the conclusions of the comparison between the two approaches vary depending on the size of the instance. Both approaches have similar optimality gap indicators on $(10, 10)$ instances, but RF has shorter computation times. On (20.20) instances, RF outperforms AGG-C on the optimality gap and computation time indicators. Both approaches again have similar optimality gap indicators on (40.20) instances, but RF, this time, has longer computation times. All the results of this section confirm the problem's difficulty, already highlighted in the previous sections, and suggest the need to develop tailored exact or heuristic approaches to solve real-life instances.

8. Model exploitation for managerial insights

This section further investigates the proposed optimization model to derive some managerial insights. Section 8.1 compares the formulated integrated model with the sequential approach where the decisions related to transportation are taken in a second stage using the lot sizes determined in the first stage, ignoring the transportation costs and capacities. Such an approach is often used in practice. The comparison helps to identify the cases where using the integrated model may lead to significant cost savings for companies. Also, the analysis in Section 8.2 suggests that the potential cost savings from using the integrated model can cover the increase in the total cost induced by reasonable limits on the allowed number of fragmentations. Section 8.2 also reports the results allowing to measure the impact of the fragmentation constraint tightness on the total cost. Finally, Section 8.3 explores the interplay between decision integration and flexibility of FTL modes and their marginal effect on the total cost.

Table 6
Cost saving (%) by using the integrated model compared to sequential approach.

		Demand configuration						avg	max
		1	2	3	4	5	6		
# FTL modes	1	13.7	4.2	2.5	5.0	5.0	4.2	5.8	22.7
	2	10.9	1.5	0.3	2.9	2.0	1.6	3.2	17.8
	4	1.8	0.2	0.1	0.3	0.3	0.2	0.5	4.0
avg		8.8	2.0	1.0	2.7	2.5	2.0	3.2	
max		22.7	9.0	4.7	11.4	10.5	9.0		22.7

8.1. Benefits of integrating decisions on transportation modes in lot-sizing model

This section explores the benefits of explicitly modeling the transportation modes in a lot-sizing model. To do so, the integrated model is compared to the practical approach, where the decisions are taken sequentially:

1. Optimize the lot-sizing decisions considering only the traditional inventory and ordering costs,
2. Select the optimal transportation modes to transport the optimized quantities in each period.

The comparison uses the optimal solutions for both models on 360 instances. To evaluate the benefits of using the integrated model, the constraints on the number of fragmentations are relaxed in this section. To ensure reasonable computational times, all the instances are characterized by a number of items $I = 10$ and a number of periods $T = 10$. The demand of items in each instance is generated according to one of the six demand configurations described in Section 7.1. For each demand configuration, 20 instances are generated. The other characteristics of items are the same as in Section 7.1. Each of the 120 instances results in three complete instances by choosing the number of available FTL modes, chosen to be one of the three alternatives $M \in \{1, 2, 4\}$. Similarly to Section 7.1, the capacity and cost of the FTL modes are taken from the two lists $(33, 30, 25, 11)$ and $(4917, 4560, 4000, 2123)$, respectively. The characteristics of the available FTL modes correspond to the first M elements of the two lists. By dropping fragmentation considerations, it is no longer necessary to explicitly model the assignment of the orders of items to the containers. It is only required to ensure enough aggregated capacity to transport all the shipments in the period. By doing so, either with the integrated approach or the sequential approach, it takes, on average, a few seconds to optimally solve each of the 360 instances.

Table 6 reports the cost saving as a percentage achieved by using the integrated model when compared to the sequential approach. The overall saving is about 3.2% across 360 problems, with maximum savings up to 22.7% of the total costs for some cases. When analyzing the saving based on the demand configuration, note that the saving is positively correlated with the proportion of small-sized items and negatively correlated with the proportion of big-sized items. The savings are the largest for the instances where 100% of the items are small-sized ($C1$), and the savings are the lowest for instances where 100% are big-sized ($C3$). The demand configuration presenting the second-largest average savings is $C4$, where half of the items are small-sized and the other half medium-sized. To facilitate the interpretation of the results, Fig. 4 plots the average savings according to the demand configuration and the number of available FTL modes. The error bars show the savings between the first and third quartiles.

To understand these results, it is essential to remember that the classification of the items based on their size uses the capacity of the largest container as a reference. Also, when analyzing the optimal solutions, the results show that those produced by the first stage of the sequential approach follow the lot-for-lot policy, which is consistent with the context studied with low time between orders (high inventory costs and low ordering costs). Therefore, the integrated model does not have much room for improvements for instances with mainly big-sized items as their lots in each period found in the first stage of the sequential approach already utilize most of the container capacities. Conversely, for instances with mainly small-sized items, the sequential approach results in underutilized transportation capacity, which is better used when using the integrated model by consolidating the demand of the products with the lowest inventory holding cost over several periods. However, the large saving for instances with all the items being small-sized should be carefully considered. The results show significant savings for all demand configurations, especially when there are few FTL modes. However, the large savings when all the items are small-sized are primarily due to the poor selection of the FTL modes.

If the results in Table 6 are to be analyzed in light of the number of available transportation modes, there is a decrease in the savings when there are more FTL modes. On average, the savings decrease from 5.8% when only one FTL mode is available to 0.5% when 4 FTL modes are used instead. Intuitively, the additional flexibility makes it possible for the second stage in the sequential approach to correct the uninformed lot-sizing decisions taken in the first stage, especially when a high proportion of items are small-sized. However, not every additional flexibility can make the sequential approach relevant in practice, given that the potential for improvement of a more complex integrated model

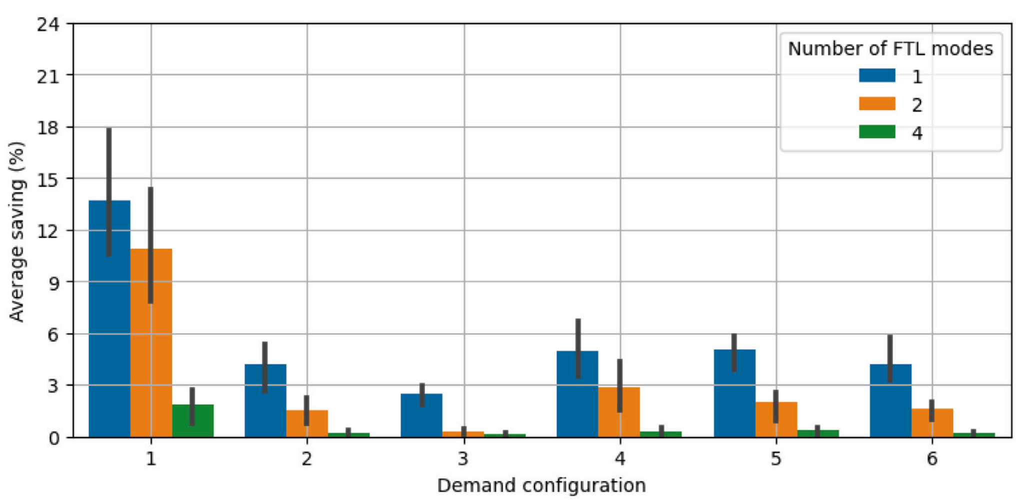


Fig. 4. Cost saving (%) by using the integrated model compared to sequential approach.

is low. Two additional characteristics of the instances used to conduct the experiments should be considered to explain these results. First, the scenarios with 4 FTL modes include two additional modes with lower capacities (11 and 25) than the two others (30 and 33) used in all the scenarios. Also, the difference between unit transportation costs for each fully utilized mode is relatively small: Cost per pallet per mode $(11, 193), (25, 160), (30, 152), (33, 149)$. The additional flexibility would have less impact on potential savings when using the integrated model instead of the sequential approach if transport unit costs were higher or the difference between capacities was lower.

To conclude this section, the experimental results using the instances defined in 7.1 show the benefits in terms of cost savings when explicitly modeling the transportation capacities and costs in a lot-sizing model. Such integration is more critical when the actual flexibility in the transportation modes to use is limited, the actual flexibility being the situation with several types of transportation modes with a large range of capacity and low range of unit costs. Using the sequential approach might be relevant in the presence of high actual flexibility. However, in a situation where only one transportation mode is available, it is highly improbable that the lot-sizing decisions taken at the first stage will result in high utilization of the transportation capacities for the different demand configurations. This is good news from a computational cost perspective, as the integrated model can be useful in practice when its complexity is the lowest. Considering the realistic instances with demand configuration $C6$, the managers should be aware that the total costs might be reduced by up to 9% with the integrated model compared to the sequential approach, particularly for contexts with few alternative transportation modes and low flexibility of the modes.

8.2. Impact of item fragmentation on total cost

This section aims at evaluating the impact of the fragmentation constraint tightness on the total cost, which is a challenge given the difficulty of generic solvers in finding optimal solutions, as illustrated in Section 7. 180 new instances with $I = 5$ and $T = 5$ are generated to make sure that optimal solutions are obtained in reasonable computational times. 20 instances are first obtained by generating the demand of items for each instance by only considering $C6$ as demand configuration mixing 20% of big-sized items, 70% of medium-sized items and 10% of small-sized items. Each of the twenty instances is allowed to use one of the three alternatives $M \in \{1, 2, 4\}$. Similarly to Section 7.1, the capacity and cost of the FTL modes are taken from the two lists $(33, 30, 25, 11)$ and $(4917, 4560, 4000, 2123)$, respectively. The characteristics of the available FTL modes correspond to the first M elements of the two lists.

To fully define the 180 instances starting from the 60 generated instances, as explained above, different values for the number of fragmentations per period should be selected. The sequential approach is first used to find the threshold beyond which the fragmentation constraints are ignored for all the instances. As the periods are independent of each other after fixing the replenishment quantities and the FTL mode selection decisions, minimizing the total number of fragmentations also minimizes the number of fragmentations in each period. Several valuable pieces of information can be extracted from the analysis of the results of this sequential approach:

- The total cost of the replenishment plans obtained by the first phase is the lowest cost independent of the fragmentation constraints. Therefore, this cost can be used as a reference to measure the impact of the fragmentation constraints.
- The sequential approach reproduces the practice where fragmentation is dealt with only when solving the operational problem of loading the containers in each period as illustrated by Heßler et al. (2021).
- As mentioned above, the sequential approach results help to identify the different tightness levels of the fragmentation constraints.

Table 7 provides the results of the comparison between the sequential model and the integrated model. Columns *Max* and *Sum* are related to the sequential approach and refer to the maximum number of fragmentations and the total number of fragmentations in the planning horizon, respectively. As the maximum number of fragmentations, independently of the number of FTL modes, is at most 2, the fragmentation constraint might increase the cost only for limits strictly below 2. Therefore, 180 instances are solved by the integrated model using three values of the allowed number of fragmentations in each period: $F \in \{2, 1, 0\}$. After solving these instances to optimality, the relative deviations of the total costs compared to the costs obtained by the first phase of the sequential approach are computed and reported in Table 7 under *Total cost increase (%)*. Note that there is no need to report the results when $F = 2$ as the fragmentation constraints have no impact on the cost in this case.

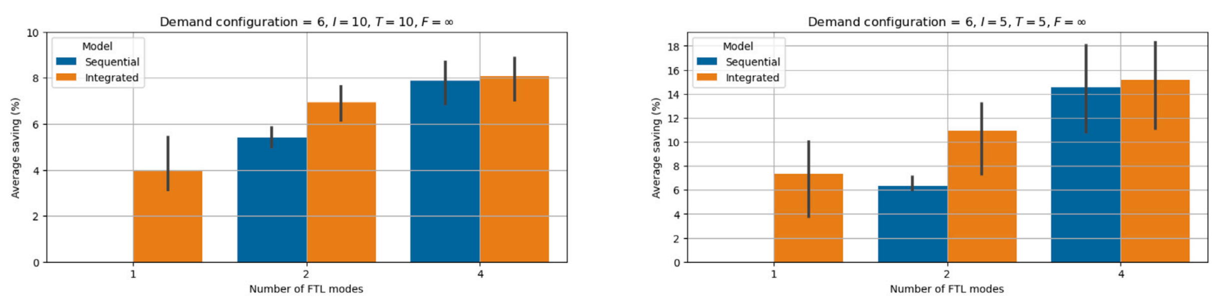
Table 7 shows that forbidding fragmentation ($F = 0$) leads to a significant increase in the total cost, around 3% on average. In the absence of a clear approach to evaluating the impact of fragmentation on the handling operations, forbidding fragmentation in the whole planning horizon might be a radical approach. Setting reasonable limits on the number of fragmentations, such as $F = 1$ in the case of the used instances, might be beneficial for handling operations without a significant increase in the total cost ($\leq 0.2\%$, on average). Table 6, reporting the cost saving by integrating the transportation modes in the lot-sizing models, shows that the savings are on average 2% when solving the instances with $C6$ as demand configuration. Making the reasonable assumption that the same saving estimation holds in the case of the smaller instances solved in this section, the integrated model proposed in this work both allows saving in the total cost and ensures more efficient handling operations. If the results in Table 7 are to be analyzed based on the number of available transportation modes, the solution quality indicators (*Sum* and *Max* for the sequential approach, and *Total cost increase (%)* for the integrated approach) deteriorate when there are more FTL modes. This can be explained by the trade-off between the efficiency of transportation operations and the efficiency of handling operations under the prism of fragmentation. The main advantage of increasing the diversity of the FTL modes is to minimize the proportion of unused transportation capacity. However, such unused capacity is a buffer that lessens the effect of the constraints on the number of fragmentations. A more detailed analysis of these results is provided in the next section.

8.3. Decision integration vs. Mode flexibility

In the previous two sections, the different analyzes involve the comparison of optimal solutions obtained by the sequential and integrated approaches. When comparing these approaches, the same problem instance (demand and costs of items, available FTL modes, and maximum allowed number of fragmentation) is used to compute the cost saving or increase. The purpose of this section is to explore the interaction between the integration of the decisions and the flexibility of FTL modes and their effect on the total cost. To do so, a problem instance is characterized by the demand and costs of items, and the maximum allowed number of fragmentation. Each instance is solved after choosing the number of FTL modes ($M \in \{1, 2, 4\}$) and the solution approach (integrated or sequential). Each instance has, therefore, 6 possible outcomes. To study the interaction between decision integration and mode flexibility, a baseline situation should be chosen relative to which the cost saving or increase is computed. We made a choice to consider a baseline approach that we believe is the most commonly applied in the industry: The sequential approach with one FTL mode. In the baseline approach, lot-sizing decisions are taken in the first stage without considering the transportation requirements.

Table 7
Cost increase for various fragmentation constraints for instances with demand configuration $C6$ and two FTL modes.

Instance	M = 1				M = 2				M = 4			
	Sum	Max	Cost increase (%)		Sum	Max	Cost increase (%)		Sum	Max	Cost increase (%)	
			$F = 0$	$F = 1$			$F = 0$	$F = 1$			$F = 0$	$F = 1$
1	3	1	1.6	0	4	2	1.2	0.3	6	2	3.8	0.3
2	4	2	9.6	0.8	6	2	11.4	0.8	6	2	7.7	0.8
3	1	1	0.3	0	0	0	0	0	2	1	2.3	0
4	0	0	0	0	1	1	0.7	0	4	1	3.2	0
5	3	1	1.2	0	4	2	1.7	0.5	6	2	1.2	0.5
6	1	1	0.9	0	1	1	1	0	2	1	1	0
7	1	1	0.1	0	0	0	0	0	0	0	0	0
8	3	2	3.3	0.3	5	2	3.7	0.4	6	2	5.9	0.4
9	3	2	1.3	0.1	4	1	1.7	0	7	2	4.2	1.9
10	0	0	0	0	0	0	0	0	1	1	2.1	0
11	2	1	7.2	0	1	1	6	0	2	1	3.9	0
12	0	0	0	0	2	1	1.7	0	3	1	1.9	0
13	2	1	4.1	0	2	1	3.1	0	3	1	0.9	0
14	0	0	0	0	0	0	0	0	1	1	3.1	0
15	2	1	4.4	0	3	1	4.5	0	3	1	3.4	0
16	3	2	3.6	1.1	4	2	4.2	1.1	4	2	2.8	0.5
17	2	1	1.2	0	2	1	1.2	0	4	2	1.6	0.2
18	1	1	4	0	1	1	4.2	0	2	1	6.2	0
19	2	1	9.1	0	2	1	7.4	0	3	1	4.2	0
20	3	1	3.3	0	3	1	4.4	0	4	1	4	0
avg	1.8	1	2.8	0.1	2.3	1.1	2.9	0.2	3.5	1.3	3.2	0.2
med	2	1	1.5	0	2	1	1.7	0	3	1	3.2	0
max	4	2	9.6	1.1	6	2	11.4	1.1	7	2	7.7	1.9



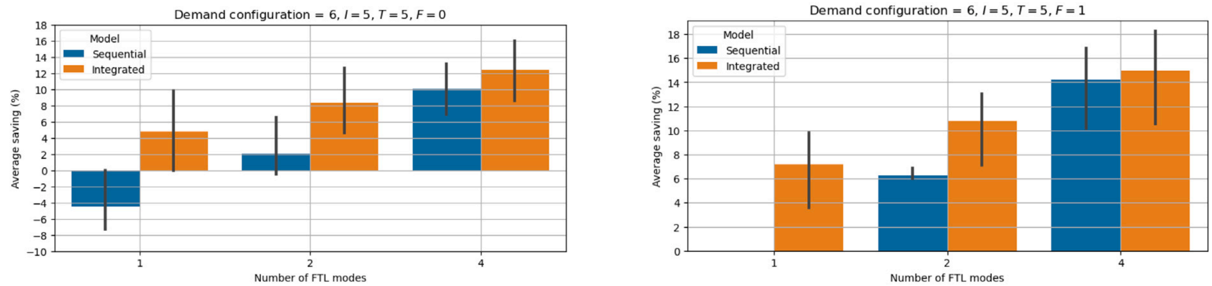
(a) $I = 10$, $T = 10$, $F = \infty$ (b) $I = 5$, $T = 5$, $F = \infty$

Fig. 5. Decision integration vs. Mode flexibility: Without fragmentation constraints.

In the second stage, the optimal loading of the quantities determined in the first stage is defined using only one transportation mode. To conduct this study, the instances in Sections 8.1 and 8.2 are used.

Starting with the situation where fragmentation considerations are ignored, Fig. 5 reports the average savings by using more FTL modes and/or integrated approaches compared to the baseline approach described above. Focusing on the results where the sequential approach is used but the diversity of FTL modes is increased, the results show that the cost saving increases as the diversity of FTL modes increases. It is important to keep in mind that the diversity of FTL modes is increased by adding additional modes with larger unit transportation costs. Despite the increase in the unit transportation cost, the additional flexibility allows significant savings compared to the situation where only one mode is used. Focusing now on the situation where only one FTL mode is used, as shown in the previous sections, the integrated model leads to significant cost savings. The most interesting insight from these results comes from the comparison of the sequential approach and the integrated approach for each fixed number of FTL modes. The difference between the savings when using the integrated model compared to using the sequential model represents the marginal contribution of integrating decisions. The results show diminishing returns of the marginal contribution of integrating decisions when increasing the diversity of the FTL modes. In other words, more diversity in the available transportation modes reduces the necessity of solving increasingly complex integrated models. As highlighted earlier, any increase in the FTL mode diversity is not necessarily beneficial. The diversity increase should reflect the additional flexibility in the choice of transportation modes: a large range of mode capacities and a low range of unit transportation costs.

The results of Section 8.2 on the impact of fragmentation on the total cost show that the cost increase is more severe when there are more FTL modes. One has to be careful not to conclude that fragmentation constraints are less costly with low diversity in transportation modes. To show the opposite, an analysis, similar to the one above, is performed while considering item fragmentation. To analyze the interaction between decision integration, mode flexibility and tightness of the fragmentation constraints, we use the same instances than in Section 8.2. We also consider the same baseline approach as above: Sequential approach, one FTL mode, and relaxed fragmentation constraints. Fig. 6 reports the average savings, relative to the baseline approach, obtained by varying the tightness of the fragmentation constraints ($F \in \{0, 1\}$) and the number of FTL modes ($M \in \{1, 2, 4\}$) and by choosing either the sequential model or the integrated model. Fig. 6(a) shows that the cost increase resulting from forbidding fragmentation is completely compensated by the savings obtained by solving the integrated model or by increasing the diversity



(a) $I = 5, T = 5, F = 0$

(b) $I = 5, T = 5, F = 1$

Fig. 6. Decision integration vs. Mode flexibility: With fragmentation constraints.

of the transportation modes. Either with $F = 0$ or $F = 1$, note that the same pattern as in Fig. 5 on the diminishing returns of the marginal contribution of integrating decisions when increasing the diversity of the FTL modes. However, such a decrease is slower when the constraints on fragmentation are tighter. This shows the benefit of solving the integrated model when there are tight limits on the allowed number of fragmentations, even when there is high mode flexibility.

9. Conclusion

In this paper, we proposed and analyzed a multi-item joint inventory and transportation planning problem with dynamic deterministic demand and multiple transportation modes, considering operational loading constraints in terms of fragmentation policies. We have proposed and tested several Mixed Integer Linear Programming formulations for the problem, which combines a multi-item lot-sizing problem and a variable cost and size bin packing problem with fragmentation. Based on real data from a Scandinavian distribution company for fast-moving consumer goods, we generated problem instances and identified the most efficient mathematical formulations in terms of computational time and number of solved instances using a standard solver. The chain-based models (AGG-C and FAL-C) outperform the other models, yielding a smaller duality gap. However, regardless of the formulation used, the experimental results show that it is unlikely to find high-quality solutions for practical problems relying only on standard solvers. The unsatisfactory performance of the relax-and-fix heuristic also illustrates the difficulty of the problem. These results are in line with the results of Casazza and Ceselli (2016), Casazza (2019), demonstrating the hardness of bin-packing problems with item fragmentation for generic solvers. The computational results are also analyzed to confirm the relevance of the parameters selected to be varied when generating the problem instances. In particular, the problem complexity measured through the optimality gap is positively correlated with the proportion of big items in the total demand, the tightness of the fragmentation constraints, and the number of FTL modes.

Given the novelty of the problem, we were also interested in drawing out some managerial insights that can help identify contexts where solving such a problem can be beneficial. Ignoring fragmentation considerations and solving the integrated model could result in 3.2% lower costs on average and up to 23% in some cases compared to the sequential planning approach where decisions related to transport are taken at the second stage after the lot sizes have been determined during the first stage. More importantly, the computational results emphasize that the integrated model is more relevant in the presence of the common setting where one or few FTL modes are used. Increasing the number of FTL modes leads to less cost savings due to decision integration. However, the increase in the number of FTL modes should reflect real flexibility by offering a wide range of capacity without significant differences in unit transportation costs among the modes. When integrating fragmentation considerations, the computational results show that forbidding fragmentation may lead to a significant increase in costs. However, with reasonable limits on the number of allowed fragmentations, the results show that the cost increase is significantly lower than the savings obtained through the decision integration. In other words, it is possible to contribute to more efficient handling operations by limiting the number of fragmentations without a significant impact on the total inventory, ordering and transportation costs.

The computational results show, at the same time, the hardness and relevance of investigating the proposed problem. However, it is important to consider the different insights carefully. First of all, it should be remembered that all instances of the problem are characterized by low TBO. Furthermore, given the difficulty of the problem, all conclusions are drawn from the analysis of optimal solutions for small instances. It is, therefore, interesting in the future to study the impact of increasing TBO and instance sizes on the various conclusions drawn in this work. To do this, and in general, to realize the benefits of solving the proposed problem, it is necessary to design efficient approaches capable of producing high-quality solutions for real instances. Several other directions for future research and model extension are worth exploring: .

- The proposed models can easily be modified to model the situation where the buyer imposes a limit on the number of fragmentations on the horizon instead of in each period. In general, it is also worth studying the relevance and impact of integrating other ways fragmentation impacts handling operations. For example, it may not be important whether the pallets of a fragmented item are dispersed over two or several containers. This modeling corresponds to a context described in Heßler et al. (2021), where the main effort of the buyer is due to the necessity of reserving an area where all the fragments of a product lot would be collected before starting the put-away operation. Instead of the number of fragmentations, it is more relevant in such a context to minimize or set up constraints on the number of fragmented items.
- We believe that relevant managerial insights can be drawn when allowing backlogging or transportation modes with other price schedules, such as parcels or LTL. One might consider other loading constraints in lot-sizing problems, in particular when dealing with other supply chain configurations. For example, with a single supplier and multiple customers, one might consider grouping constraints when solving the integrated model to make sure that the set of items to be unloaded at a given customer is not unnecessarily dispersed in several containers.

Data availability

Data will be made available on request.

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