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Integrated planning of maintenance operations and workload allocation

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Abstract

Motivated by a practical problem, this paper investigates the integrated planning of maintenance operations and workload allocation on a set of machines in a workshop. Given quantities of products to be produced per period on a planning horizon must be processed on unrelated flexible machines. Moreover, each machine has to undergo one or more maintenance operations that must be planned within a given time window and impact products differently. The main goal is to find a feasible plan that satisfies the machine capacity by allocating the production quantities to machines and assigning maintenance operations as late as possible in their time windows. Various original mathematical models are presented. In particular, we propose models that allow maintenance operations and some production quantities to overlap two consecutive periods. Computational experiments based on industrial data show that allowing this overlapping helps the earliness of maintenance operations to be significantly reduced in the most difficult instances, going for example from a total of 14 periods to only 1 period, and by more than 35% on average.

Keywords: Maintenance, Workload allocation, Planning, mathematical modeling

1. Introduction

Following the development of concepts related to Industry 4.0, more and more machines are equipped with sensors, allowing their health statuses to be monitored with the objective of performing maintenance operations on machines as late as possible. This allows to reduce the maintenance costs and to use the machines for production as long as possible. However, it is also important to ensure that machines are available and not being maintained when they are required to process products, i.e. when production quantities are planned to be processed. Hence, depending on the future workload on the machines, it could be relevant to plan a Preventive Maintenance (PM) operation on a machine earlier than the latest time at which the PM operation should be performed to avoid a potential failure. Although the information is available, planned production quantities are usually not considered when planning PM operations. In practice, production and maintenance planning decisions are still often taken independently or se-

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quentially although, as pointed out for instance in Iravani and Duenyas (2002), integrating both planning decisions can lead to significant savings. This has led more and more researchers to study integrated maintenance and production problems, the main emphasis being on operational production control, as shown in the reviews of Hadidi et al. (2012), Budai et al. (2008) and de Jonge and Scarf (2020), whereas we are considering decisions on a horizon of several weeks.

In this paper, based on an industrial case, we are considering that a production plan in a workshop is given on a planning horizon discretized in periods, typically days, i.e. the production quantity of each product in each period is given. However, the workload allocation, i.e. the production quantities allocated to each machine, can still be optimized. An important characteristic of the problem is that machines are only qualified (also called eligible) to process a limited number of products. Machine qualifications make the workload allocation problem not trivial to solve and, to our knowledge, have never been considered in the literature on the integration of maintenance and production decisions. One or more PM operations should be performed on each machine in the planning horizon, and each PM operation has an earliest period before which it cannot be performed and a latest period before which it should be performed. Given the capacity of each machine in each period, the objective is to simultaneously decide a period for each PM operation and the workload allocation of the required quantities in the production plan on the machines in each period.

As mentioned by Budai et al. (2008) and Najib et al. (2009), the research on integrated production and maintenance problems can be classified into four categories: (1) Maintenance planning with inventory control, (2) Optimization of production and maintenance rates, (3) Economic manufacturing/production quantity (hereafter, EMQ or EPQ) with failure aspects, and (4) Aggregate production planning (AGG)/lot sizing models with PM or corrective maintenance (CM) decisions.

Van der Duyn Schouten and Vanneste (1995), Meller and Kim (1996), Kyriakidis and Dimitrakos (2006), Gharbi et al. (2007), or Kenné et al. (2007) can be given as examples of the first category. Van der Duyn Schouten and Vanneste (1995), Meller and Kim (1996) and Kyriakidis and Dimitrakos (2006) consider a two-machine production system, more precisely one deteriorating installation and one production unit processing the raw material transferred from the installation. Gharbi et al. (2007) and Kenné et al. (2007) consider systems producing a single item. The common feature of the research in this category is that a buffer is built up to guarantee the continuity of the production processes during the (CM or PM) maintenance operations. As these buffers alleviate the negative effects of any random failure on ongoing production, their levels also play a crucial role in answering the questions of whether a PM operation or production operations will be performed. In the second category, illustrated by the works of Gharbi and Kenné (2000) that consider multiple identical machines and a single item, Charlot et al. (2007), Song (2009), Nodem et al. (2011) and Kang and Subramaniam (2018) that consider a single machine and a single item, the main objective is to control the production and maintenance rates to minimize the sum of the related costs such as the inventory, backlog and maintenance costs. While Gharbi and Kenné (2000) and Charlot et al. (2007) find optimal PM/production rates policies based on the age of the machine, Song (2009) considers that the machine has multiple states and that failure rates are state dependent. Nodem et al. (2011) introduce the failure history of the machine to the problem, while Kang and Subramaniam (2018) evaluate different PM levels to find optimal production and PM rates.

Groenevelt et al. (1992a,b) address EMQ problem and investigate the effects of machine breakdowns and corrective maintenance on the EMQ decisions. While the repair time is considered negligible in their first work (Groenevelt et al. (1992b)), in their subsequent study Groenevelt et al. (1992a) aim to determine lot size and safety stock policies when repair times are large. In this stream of research, the studies of Groenevelt et al. (1992a,b), that deal with a single machine, can be referred as the pioneering works since many works such as Chung (1997), Kuhn (1997), Abboud (1997) and Kim and Hong (1997) have been following their footsteps. Different from these works, Cheung and Hausman (1997), Dohi et al. (2001), Giri and Dohi (2005) and El-Ferik (2008) incorporate PM and EQM decisions. Cheung and Hausman (1997) and Dohi et al. (2001) study on the joint optimization of the PM time and safety stocks for an EMQ model with stochastic machine breakdown for a production system with one machine and a single item. Giri and Dohi (2005) formulate an EMQ model for an unreliable manufacturing system that produces a single item with the assumption that the time to machine failure, that preventive and corrective repair times are random variables, and that the failure rate depends on the production rate. El-Ferik (2008) defines jointly EPQ and PM schedules for an unreliable machine under an imperfect age-based maintenance policy that minimizes the long-term average cost. Ben-Daya (2002), Chelbi et al. (2008) and Suliman and Jawad (2012) develop an integrated model that links quality, EMQ and preventive maintenance policies. While Ben-Daya (2002) considers periodical PM, Chelbi et al. (2008) include an age based PM policy in their joint model. Different from the study of Chelbi et al. (2008), Suliman and Jawad (2012) develop their model under the assumption that the failure may occur during the preparation period for its repair.

A significant number of studies have been conducted to address integrated lot sizing and PM planning. Aghezzaf et al. (2007) formulate a single-line production system, which is subject to random failures and cyclical PM, as a multi-item capacitated lot sizing problem and define the expected lost capacity due to the maintenance actions. Aghezzaf and Najid (2008) extend the study of Aghezzaf et al. (2007) to a multi-line production environment by considering cyclic and noncyclic maintenance policies, later, Yalaoui et al. (2014) propose a more compact formulation of the model studied by Aghezzaf and Najid (2008) which reduces the computational time and allows to cope with more complex problems. Najid et al. (2011) and Alaoui-Selsouli et al. (2012) propose to plan PM operations within defined time windows, and allow lost sales when the resource capacity is not sufficient to meet the demand. Nourelfath et al. (2010) study the integrated maintenance and production planning problem for multi-state systems with cyclic, and Fitouhi and Nourelfath (2012) extend their work by taking non cyclic PM into account. Yildirim and Nezami (2014) study a joint PM and production planning model for a multi-item, multi-period single-machine and capacitated lot-sizing problem by considering the energy consumption. Fakher et al. (2018) note that the results obtained by maximizing production or minimizing the maintenance cost can be different from the optimal solution of the joint model that maximizes the profit. Thus, they propose a multi-item capacitated lot sizing model that maximizes the total profit and integrates maintenance, production and quality decisions. The common feature of the studies in this research line is that the maintenance planning problem is integrated to variants of the lot sizing problem.

Let us discuss the main differences between our work and the studies discussed above. Production quantities in each period are decided at a higher decision level, as for example shown in Beraudy et al. (2022) or in Christ et al. (2018), where the total production cost is minimized. Hence, we want to allocate the production quantities, i.e. the workload, to machines and plan the maintenance operations on a horizon of several weeks while considering multiple unrelated machines (and thus machine qualifications) and product-based maintenance time windows. This allows to delay the maintenance operation of a machine by only assigning product quantities that can safely be processed on that machine. Hence, it allows machines to be used longer. Also, in most previous studies, a single machine is considered. To our knowledge, the combination of machine qualifications and product-based maintenance time windows when planning production and maintenance is not considered in the literature. Moritz et al. (2020) can be seen as a preliminary version of our work. However, in Moritz et al. (2020), (1) The workload allocation is already defined, (2) Product-based maintenance time windows are not considered, (3) The overlapping of maintenance operations is modeled in a simpler way and (4) Production quantities cannot overlap consecutive periods.

Let us summarize our contributions. First, a Mixed Integer Linear Program (MILP) (P_{init}) is proposed that optimizes the workload allocation on the set of machines and minimizes the earliness of the PM operations. Second, by considering two relevant assumptions, two mathematical models (P^{oper}_{shift} and P^{prod}_{shift}) are proposed to better use the available machine capacity. Third, computational experiments on instances generated from industrial are conducted, and managerial insights derived.

The remainder of the paper is organized as follows. In Section 2, the problem and the related assumptions are presented, and Mixed Integer Linear Programming (MILP) models for our integrated production and PM planning problems are presented in Section 3. In Section 4, the proposed mathematical models are tested on industrial data obtained from a semiconductor manufacturing company. Section 5 then provides some practical implications of our work and managerial insights, before Section 6 concludes the paper by summarizing the contributions and proposing some perspectives.

2. Problem description

As discussed in the introduction, the problem introduced in this section is inspired and extended from Moritz et al. (2020). Our problem setting is as follows. A set of products ($p = 1, \dots, P$) is processed on a set of non-identical parallel machines ($p = 1, \dots, M$). Each machine has a given capacity in number of hours in each period. The products can be processed on a single machine or on multiple machines (Figure 1). Preventive Maintenance (PM) operations are required to ensure that the machine remains in operating conditions. Each PM operation o is unique and associated to a single machine m_o (Figure 1), and can only be performed in an interval defined by an earliest period e_o and a latest period l_o, i.e. the maintenance operation cannot be planned on machine m_o before period e_o and after period l_o (Figure 2). The PM operations to perform and their intervals are given by the maintenance department based on the status of the machines and the type of maintenance operations. Products cannot be processed on machine m_o after period l_o if PM operation o has not been performed. Additionally, some products are more critical than others, and the latest period $l_{o,p}$ in which operation o can be planned for a critical product p might be earlier than l_o. More precisely, product p cannot be produced on machine m_o after period $l_{o,p} + 1$ if maintenance operation o has not been performed. The duration of each PM operation o is denoted by d_o. The capacity of machine m in period t is denoted by $c_{m,t}$, and an operation o can only be planned in period t if the production capacity of machine m_o is not exceeded, i.e. $d_o \leq c_{m_o,t}$.

The mathematical models developed in this study aim to determine the optimal workload on a set of machines ($X_{p,m,t}$) and to optimally plan PM operations over a planning horizon (T) by minimizing their earliness. For instance, in Figure 2, Operation 1, which is related to the first machine (M1) should be performed between the first (e_o) period and the seventh (l_o) period. However, to be able to keep the production of product B on Machine 1 (M1), Operation 1 must be performed before the third period ($l_{o,p} = 3$), shown by an exclamation mark. If Operation 1 is not performed before the fifth period, product A cannot be processed on Machine 1 as well until Operation 1 is performed. In our illustration, since PM Operation 1 is performed in the seventh period, note that products A and B are not produced on Machine

1 in periods 4 and 6 respectively, since Operation 1 has not been performed yet. The other machines and the operations in Figure 2 can be discussed in the same way.

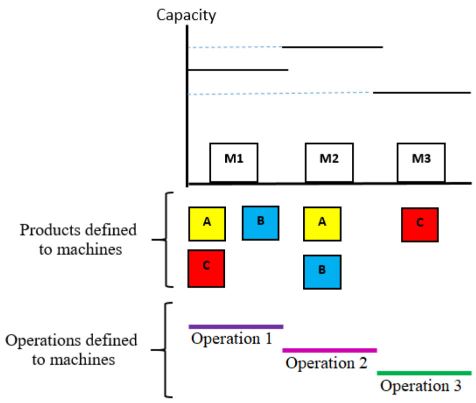

Figure 1: Machine with different qualifications and capacity.
Alt Text: An example with multiple machines with different capacity, products qualified on machines and maintenance operations to assign to machines.

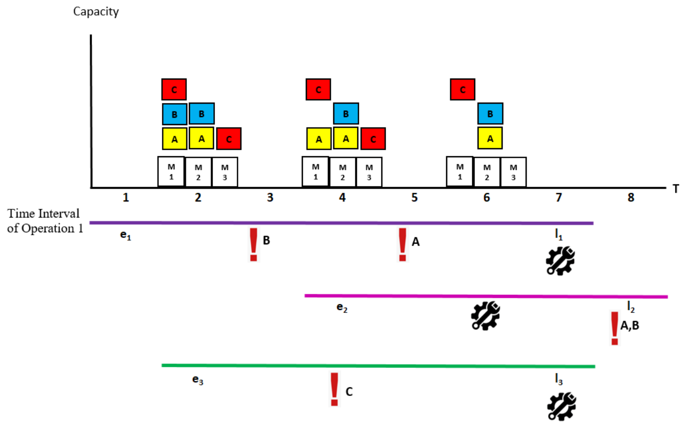

Figure 2: A production plan with maintenance operations.
Alt Text: An example of an assignment of products to machines over time with the periods in which maintenance operations are performed.

3. Mathematical modeling

After introducing the main notations in Section 3.1, the initial model P_{init} is presented in Section 3.2. Then, P_{init} is extended as follows:

- Model P^{oper}_{shift} in Section 3.3 allows maintenance operations to overlap two consecutive periods. This is more realistic as it is usually possible in practice to start a maintenance operation in a period and to complete in the next period.
- Model P^{prod}_{shift} in Section 3.4 allows the required production quantities to be partially shifted. More precisely, the operational production plan is provided with some flexibility by specifying a percentage of the production quantities that could actually be processed in the period preceding the period to which they are assigned, and a percentage of the production quantities that could actually be processed in the period following the period to which they are assigned. This is also more realistic as, although the planning horizon is discretized, production quantities are often could actually be assigned to one of two consecutive periods.
- Models P^{oper}_{shift} and P^{prod}_{shift} are combined in Section 3.5 to consider the flexibility offered by both models.

Note that the additional parameters and decision variables for Models (P^{oper}_{shift} and P^{prod}_{shift}) are introduced in the corresponding sections 3.3 and 3.4.

3.1. Notations

The following parameters are considered:

T: Number of periods in the planning horizon,
P: Number of products,
M: Number of machines,
O: Number of maintenance operations,
$a_{p,m}$: Process time (defined in time units per unit of product) of product p on machine m,
$q_{p,t}$: Required quantity of product p to be produced in period t,
$c_{m,t}$: Production capacity of machine m in period t,
m_o: Machine on which maintenance operation o should be performed,
d_o: Processing time of maintenance operation o,
e_o: Earliest period in which maintenance operation o can be planned,
l_o: Latest period in which maintenance operation o can be planned,
ω_o: Weight of operation o,
$l_{o,p}$ ($\leq l_o$): Latest period in which operation o can be planned for product p.

The following decision variables are used:

$X_{p,m,t}$: Quantity of product p assigned to machine m in period t,
$S_{o,t} \in \{0,1\}$: Equal to 1 if operation o is performed in period t, and to 0 otherwise,
$I_o \in \{0,1\}$: Equal to 1 if operation o is not planned, and to 0 otherwise.

3.2. Initial model (P_{init})

Based on the defined objective and the assumptions related to the production and maintenance operations, the initial model P_{init} is formalized below:

$$\min \sum_{o=1}^{O} \left((l_o - e_o + 1)^2 \omega_o I_o + \sum_{t=e_o}^{l_o} (l_o - t)^2 S_{o,t} \right) \tag{1}$$

$$\sum_{m=1;\ a_{p,m}>0}^{M} X_{p,m,t} = q_{p,t} \quad \forall p \in \{1,\dots,P\}, \forall t \in \{1,\dots,T\} \tag{2}$$

$$\sum_{o=1;\ m_o=m}^{O} d_o S_{o,t} \leq c_{m,t} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m} X_{p,m,t} \quad \forall t \in \{1,\dots,T\}, \forall m \in \{1,\dots,M\} \tag{3}$$

$$\sum_{t=e_o}^{l_o} S_{o,t} + I_o = 1 \quad \forall o \in \{1,\dots,O\} \tag{4}$$

$$X_{p,m_o,t} \leq \min(q_{p,t}, \frac{c_{m_o,t}}{a_{p,m_o}}) \sum_{l=e_o}^{\min(t,l_o)} S_{o,l} \quad \forall p \in \{1,\dots,p\}, \forall o \in \{1,\dots,O\} \text{ s.t. } a_{p,m_o} > 0, \forall t \in \{l_{o,p}+1,\dots,T\} \tag{5}$$

$$X_{p,m,t} \geq 0 \quad \forall p \in \{1,\ldots,P\}, m \in \{1,\ldots,M\}, \forall t \in \{1,\ldots,T\} \tag{6}$$

$$S_{o,t} \in \{0,1\} \quad \forall o \in \{1,\ldots,O\}, \forall t \in \{1,\ldots,T\} \tag{7}$$

$$I_o \in \{0,1\} \quad \forall o \in \{1,\ldots,O\} \tag{8}$$

Objective function (1) aims to minimize the number of unplanned maintenance operations ($\sum_{o=1}^{O}(l_o - e_o + 1)^2 \omega_o I_o$ and to minimize the earliness of the maintenance operations ($\sum_{o=1}^{O}\sum_{t=e_o}^{l_o}(l_o - t)^2 S_{o,t}$), i.e. to plan the maintenance operations as late as possible in the interval $[e_o, l_o]$. The earliness is weighted by the quadratic difference between the latest period in which PM operation o can be planned (l_o) and the period in which o is performed. A linear penalty would make equivalent an earliness of 1 period for two maintenance operations and an earliness of 2 periods for only one maintenance operation, whereas the first solution is preferable. Constraints (2) guarantee that the production quantities are assigned to the machines. Constraints (3) are the capacity constraints that impose that the production and maintenance activities assigned to a machine in a period cannot last more than the machine capacity in that period. Constraints (4) force maintenance operation o to be planned within the interval defined by periods e_o and l_o or not planned at all. Constraints (5) ensure that product p cannot be produced on machine m_o after period $l_{o,p}$ until PM operation o is planned. Constraints (6)-(8) are the non-negativity and integrality constraints.

Model P_{init} forces the maintenance operations to be started and completed in a single period, which could potentially lead to infeasible solutions (not enough capacity to produce in a period), to unplanned maintenance operations or to large earliness values. In particular, the combination of Constraints (2), (3) and (5) can lead to infeasible solutions. This is because Constraints (5) might prevent product p to be processed on machine m_o after period $l_{o,p}$ if maintenance operation o is not performed before period $l_{o,p}$. Also, Constraints (2) force the production quantity $q_{p,t}$ of product p in period t to be completed. Finally, the capacity of the remaining machines might not be enough to complete the required production quantities $q_{p,t}$.

The complexity of P_{init} can be analyzed by reducing the problem to a knapsack problem which is NP-complete (see for example Garey and Johnson (1979)). This can be done by fixing variables $X_{p,m,t}$ so that all periods are fully used except a single period t and by allowing all maintenance operations to be performed in period t. Then, the problem is equivalent to minimizing the total cost of not planning the maintenance operations in a single period (the knapsack), where each maintenance operation o (object to be added in the knapsack) with processing time d_o (size of object in knapsack) has a cost $(l_o - e_o + 1)^2\omega_o - \sum_{t=e_o}^{l_o}(l_o - t)^2$ if it is planned in t (or equivalently a profit of $\sum_{t=e_o}^{l_o}(l_o - t)^2 - (l_o - e_o + 1)^2\omega_o$ if the object is added in the knapsack). Hence, our problem is at least as hard as the knapsack problem.

In practice, maintenance operations might start in a period and be completed in the next period. This possibility is modeled in Section 3.3 (Model P_{shift}^{oper}) as an extension of Model P_{init}. Moreover, some of the production quantities planned in the workshop in a period might be advanced to the previous period or postponed to the following period without significantly impacting the overall production plan. This possibility is modeled in Section 3.4 (Model P_{shift}^{prod}) as an extension of Model P_{init}.

3.3. *Shifting maintenance operations* (P_{shift}^{oper})

Following the perspectives of Moritz et al. (2020), each maintenance operation can be distributed on two consecutive periods. To do so, the following decision variables are introduced:

$P_{o,t} \in [0,1]$: Percentage of maintenance operation o performed in period t.

Thus, if a PM operation starts on a machine in period t, it can be completed in period $t+1$. This is quite common in real life, in particular to carry out long PM operations that take more than half a day. Accordingly, Model P^{oper}_{shift} below is proposed:

$$\min \sum_{o=1}^{O} \left((l_o - e_o + 1)^2 \omega_o I_o + \sum_{t=e_o}^{l_o} (l_o - t)^2 S_{o,t} \right) \tag{9}$$

subject to:
Constraints (2), (4), (6)-(8)

$$\sum_{o=1;\ m_o=m}^{O} d_o P_{o,t} \leq c_{m,t} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m} X_{p,m,t} \quad \forall t \in \{1,\ldots,T\}, \forall m \in \{1,\ldots,M\} \tag{10}$$

$$X_{p,m_o,t} \leq \min(q_{p,t}, \frac{c_{m_o,t}}{a_{p,m_o}}) \sum_{l=e_o}^{\min(t,l_o)} P_{o,l} \quad \forall p \in \{1,\ldots,p\}, \forall o \in \{1,\ldots,O\} \text{ s.t. } a_{p,m_o} > 0, \quad \forall t \in \{l_{o,p}+1,\ldots,T\} \tag{11}$$

$$\sum_{t=e_o}^{l_o} P_{o,t} = \sum_{t=e_o}^{l_o} S_{o,t} \quad \forall o \in \{1,\ldots,O\} \tag{12}$$

$$P_{o,t} \leq S_{o,t-1} + S_{o,t} \quad \forall o \in \{1,\ldots,O\}, \forall t \in \{e_o+1,\ldots,l_o\}, \tag{13}$$

$$P_{o,e_o} \leq S_{o,e_o} \quad \forall o \in \{1,\ldots,O\}, \tag{14}$$

Constraints (10) replace Constraints (3) in P_{init} as the capacity constraints except that maintenance operation o can be partially performed on machine m_o in period t, i.e. $P_{o,t}$, instead of only a full maintenance operation. Hence, the completion rate of maintenance operations is taken into account when calculating the total capacity consumption. Constraints (11) is equivalent to Constraints (5) in P_{init}, where machine m_o is available to process product p even if PM operation o is partially completed. Constraints (12) ensure that PM operation o is completed within its time window $[e_o, l_o]$ even if o is not completed in its starting period e_o. Constraints (13) guarantee that a maintenance operation is performed in at most two consecutive periods, while Constraints (14) guarantee that the first period in which maintenance operation o cannot partially start before the first period e_o of the interval $[e_o, l_o]$.

3.4. Shifting production (P^{prod}_{shift})

Instead of allowing maintenance operations to overlap two consecutive periods, we now allow some of the production quantity $q_{p,t}$ to be processed one period earlier, i.e. in period $t-1$, and some of the production quantity $q_{p,t}$ to be processed one period later, i.e. in period $t+1$. A critical point is that a limited amount of the production quantities can be advanced or postponed to avoid disturbing the production plan of the whole factory. This is why the two following parameters are introduced:

$r^-_{p,t} \in [0,1]$: Allowed ratio of product p that can be produced for period t in period $t-1$,
$r^+_{p,t} \in [0,1]$: Allowed ratio of product p that can be produced for period t in period $t+1$,

and also the two following decision variables:

$X^-_{p,m,t}$: Production quantity of product p produced for period t in period $t-1$,
$X^+_{p,m,t}$: Production quantity of product p produced for period t in period $t+1$.

Then, Model P^{prod}_{shift} below can be written:

$$\min \sum_{o=1}^{O} \left((l_o - e_o + 1)^2 \omega_o I_o + \sum_{t=e_o}^{l_o} (l_o - t)^2 S_{o,t} \right) \tag{15}$$

subject to:
Constraints (4), (7), (8)

$$\sum_{m=1,\ a_{p,m}>0}^{M} (X_{p,m,t} + X^-_{p,m,t} + X^+_{p,m,t}) = q_{p,t} \quad \forall p \in \{1,\dots,P\}, \forall t \in \{1,\dots,T\} \tag{16}$$

$$\sum_{m=1,\ a_{p,m}>0}^{M} X^-_{p,m,t} \le r^-_{p,t}\ q_{p,t} \quad \forall p \in \{1,\dots,P\}, \forall t \in \{1,\dots,T\} \tag{17}$$

$$\sum_{m=1,\ a_{p,m}>0}^{M} X^+_{p,m,t} \le r^+_{p,t}\ q_{p,t} \quad \forall p \in \{1,\dots,P\}, \forall t \in \{1,\dots,T\} \tag{18}$$

$$\begin{aligned} &\sum_{o=1;\ m_o=m}^{O} d_o S_{o,t} \le c_{m,t} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m}(X_{p,m,t} + X^-_{p,m,t+1} + X^+_{p,m,t-1}) \\ &\forall t \in \{2,\dots,T-1\}, \forall m \in \{1,\dots,M\} \end{aligned} \tag{19}$$

$$\sum_{o=1;\ m_o=m}^{O} d_o S_{o,1} \le c_{m,1} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m}(X_{p,m,1} + X^-_{p,m,2}) \quad \forall m \in \{1,\dots,M\} \tag{20}$$

$$\sum_{o=1;\ m_o=m}^{O} d_o S_{o,T} \le c_{m,T} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m}(X_{p,m,T} + X^+_{p,m,T-1}) \quad \forall m \in \{1,\dots,M\} \tag{21}$$

$$\begin{aligned} &X_{p,m_o,t} + X^-_{p,m_o,t+1} + X^+_{p,m_o,t-1} \le \min((q_{p,t} + r^-_{p,t+1} q_{p,t+1} + r^+_{p,t-1} q_{p,t-1}), \frac{c_{m_o,t}}{a_{p,m_o}}) \sum_{l=e_o}^{\min(t,l_o)} S_{o,l} \\ &\forall p \in \{1,\dots,p\}, \forall o \in \{1,\dots,O\} \text{ s.t. } a_{p,m_o} > 0, \forall t \in \{l_{o,p}+1,\dots,T-1\} \end{aligned} \tag{22}$$

$$X_{p,m_o,T} + X^{+}_{p,m_o,T-1} \leq \min((q_{p,T} + r^{+}_{p,T-1} q_{p,T-1}), \frac{c_{m_o,T}}{a_{p,m_o}}) \sum_{l=e_o}^{T} S_{o,l}$$
$$\forall p \in \{1,\ldots,p\}, \forall o \in \{1,\ldots,O\} \text{ s.t. } a_{p,m_o} > 0 \tag{23}$$

$$X_{p,m,t},\, X^{-}_{p,m,t},\, X^{+}_{p,m,t} \geq 0 \quad \forall p \in \{1,\ldots,P\}, m \in \{1,\ldots,M\}, \forall t \in \{1,\ldots,T\} \tag{24}$$

Constraints (16) ensure that the required production quantity $q_{p,t}$ of product p in period t can be satisfied by some of the quantity produced in period $t-1$ and by some of the quantity produced in period $t+1$. Constraints (17), resp. (18), impose that the production quantities advanced in period $t-1$, resp. in period $t+1$, are limited by the allowed ratio $r^{-}_{p,t}$, resp. allowed ratio $r^{+}_{p,t}$. Constraints (19) are equivalent to Constraints (3) by taking the quantities produced in period t into account to satisfy the required production quantities in periods $t-1$ and $t+1$. Constraints (20), resp. (21), are the capacity constraints for the first period, resp. last period, both periods being excluded from Constraints (19). Constraints (22) and (23) replace Constraints (4) that consider early, resp. late, satisfied production quantities for all periods, resp. the last period. Constraints (24) guarantee the non-negativity of the production quantities.

3.5. Shifting both maintenance operations (P^{oper}_{shift}) and production (P^{prod}_{shift})

In the last model, the two models in the previous sections are combined a follows:

$$\min \sum_{o=1}^{O} \left((l_o - e_o + 1)^2 \omega_o I_o + \sum_{t=e_o}^{l_o} (l_o - t)^2 S_{o,t} \right) \tag{25}$$

subject to:
Constraints (12), (13), (14), (16), (17), (18), (24)

$$\sum_{o=1;\ m_o=m}^{O} d_o P_{o,t} \leq c_{m,t} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m}(X_{p,m,t} + X^{-}_{p,m,t+1} + X^{+}_{p,m,t-1})$$
$$\forall t \in \{2,\ldots,T-1\}, \forall m \in \{1,\ldots,M\} \tag{26}$$

$$\sum_{o=1;\ m_o=m}^{O} d_o P_{o,1} \leq c_{m,1} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m}(X_{p,m,1} + X^{-}_{p,m,2}) \quad \forall m \in \{1,\ldots,M\} \tag{27}$$

$$\sum_{o=1;\ m_o=m}^{O} d_o P_{o,T} \leq c_{m,T} - \sum_{p=1;\ a_{p,m}>0}^{P} a_{p,m}(X_{p,m,T} + X^{+}_{p,m,T-1}) \quad \forall m \in \{1,\ldots,M\} \tag{28}$$

$$X_{p,m_o,t} + X^{-}_{p,m_o,t+1} + X^{+}_{p,m_o,t-1} \leq \min((q_{p,t} + r^{-}_{p,t+1} q_{p,t+1} + r^{+}_{p,t-1} q_{p,t-1}), \frac{c_{m_o,t}}{a_{p,m_o}}) \sum_{l=e_o}^{\min(t,l_o)} P_{o,l}$$
$$\forall p \in \{1,\ldots,p\}, \forall o \in \{1,\ldots,O\} \text{ s.t. } a_{p,m_o} > 0, \forall t \in \{l_{o,p}+1,\ldots,T-1\} \tag{29}$$

$$X_{p,m_o,T} + X^+_{p,m_o,T-1} \leq \min((q_{p,T} + r^+_{p,T-1} q_{p,T-1}), \frac{c_{m_o,T}}{a_{p,m_o}}) \sum_{l=e_o}^{T} P_{o,l}$$

$$\forall p \in \{1,\ldots,p\}, \forall o \in \{1,\ldots,O\} \text{ s.t. } a_{p,m_o} > 0 \tag{30}$$

Constraints (26)-(30) are equivalent to Constraints (19)-(23) that take into account the completion ratio $P_{o,t}$ of PM operation o in period t. The remaining constraints are directly inherited from Models P^{prod}_{shift} and P^{oper}_{shift}.

4. Computational experiments

In this section, the models proposed in Section 3 are tested and compared on instances that are randomly generated from industrial data. Section 4.1 presents how the instances are created. Some instances cannot be solved optimally in one hour of computational time. In this work, we only present the instances for which optimal solutions are obtained to be able to compare and discuss the different models. Section 4.2 first analyzes the case where both critical and non-critical products are considered. Note that a non-critical product corresponds to a product whose latest period l_{op} to perform PM operation o for operation p is equal to l_o. Non-critical products can thus be produced longer on machine m_o than critical products if o is not performed. In our experiments, we assume 50% of non-critical products. Finally, the analysis in Section 4.3 is conducted on the case where all the products can be critical, i.e. l_{op} can be strictly smaller than l_o for any product p.

All computational experiments have been conducted on a computer with a processor Intel Core i5 with 1.60GHz and 16 Gigabyte of RAM Memory, and the integer linear programming models have been solved using Version 12.9 of IBM ILOG CPLEX with the default settings. The models comprise about 336,000 constraints and 114,000 variables, including about 4,700 binary variables, for the smallest instances, to up to about 853,000 constraints and 231,000 variables, including about 6,000 binary variables, for the largest instances.

4.1. Design of experiments

The instances were generated from the industrial data of a semiconductor manufacturing facility in France. The considered workshop includes 20 machines. The characteristics of the instances can be found in Table 1. Only industrial data are in italic. They correspond to critical parameters, in particular: (1) The number of PM operations, (2) For each PM operation, its duration, its earliest and latest periods and the machine on which it should be performed, and (3) The allowed production ratios ($r^-_{p,t}$, $r^+_{p,t}$) that can be shifted to previous or following periods. Although they are randomly generated, the other data are inspired from the industrial data, such as the ranges for the production quantities, the largest and smallest possible unit processing times, and the number of qualified machines for each product that varies between 2 and 7. The production horizon includes 60 periods of 24 hours. In terms of production quantity, two groups of five instances are considered. In the first group (G1), the production quantities vary in the range [100-200] whereas, in the second group (G2), they vary in the range [180-300]. The goal of these two groups of instances is to analyze the impact of larger production quantities, which require more production capacity, on the maintenance plan.

The numerical results are displayed in Table 2 through Table 7. The data set is divided into six groups based on the number of PM operations. Each group of instances is tested by considering different numbers of products (400, 500, 600) and different production quantities represented by G1 and G2. As a

Table 1: Characteristics of the instances

Parameters	**Values**
Number of PM operations	77, 80, 84, 88, 94, 97
Number of products	400, 500, 600
Production quantity (G1)	[100 - 200]
Production quantity (G2)	[180 - 300]
Unit processing time (sec.)	[15 - 50]
$r^{-}_{p,t}$ *(%)*	[10 - 50]
$r^{+}_{p,t}$ *(%)*	[10 - 20]
d_o *(min.)*	[30 - 1440]

result of the experiments, the number of unplanned PM operations, the total earliness of the planned PM operations and the computational time are presented. When it is possible to plan all PM operations in the latest period of their interval using P_{init}, i.e. *earliness=0*, Models P^{oper}_{shift} and P^{prod}_{shift} have not been used to solve these instances. The ratios in Columns *Backward Shift* and *Backward Shift* are the ratios of the required production quantities that are shifted to the previous or following periods. When Model P^{oper}_{shift} is used, since PM operations that start in period t can be completed in $t+1$, the *Earliness* values shown in the tables are counted using the periods in which maintenance operations are completed. For instance, if the latest period l_o to perform maintenance operation o is period 15 and 60% of the maintenance operation is performed in period 14 and 40% in period 15, the earliness is assumed to be equal to 0.

4.2. Critical and non-critical products

The numerical results, when non-critical and critical products are considered, show that it is often possible to plan the required production quantities by performing the PM operations in the last periods of their intervals. Note that the results differ for the first two groups of instances with 77 and 80 operations, since they are close to the results of the case with only critical products (Table 5). A tentative explanation of this behavior is provided in Section 4.3. When there are both non-critical products and critical products, machines can be used for longer periods for the non-critical products, and thus the available capacity of the other machines can be used to process other products and to perform PM operations. This flexibility seems to reduce the complexity of the problem, since the instances are solved in at most 6 minutes, even with large required production quantities (G2).

Since the earliness values are already small with P_{init}, solving Models P^{oper}_{shift} and P^{prod}_{shift} do not lead to significant improvements in most cases. However, when looking at Instance 97-400-G2-4, P_{init} reaches an optimal solution with PM operations being performed 3 periods earlier in total, which is the maximum total earliness of the instances considered in this section, whereas Models P^{oper}_{shift} and P^{prod}_{shift} find solutions where PM operations are performed in later periods and the earliness decreases to 1 period. The use of Models P^{oper}_{shift} and P^{prod}_{shift} also impacts the solutions for Instances 94-500-G1-2, 94-500-G1-4, 97-500-G2-2, 97-500-G2-3, 97-600-G2-1, 97-600-G2-2 and 97-600-G2-4, although the earliness only slightly decreases. Since most of these instances mainly belong to group G2, we can conclude that Models P^{oper}_{shift} and P^{prod}_{shift} are more effective with large required production quantities when there are both non-critical and critical products.

Table 2: Critical and non-critical products: Instances with 77, 80 and 84 PM operations

			P_{init}			P^{oper}_{shift}			P^{prod}_{shift}				
Nb, Oper.	Nb. Prod.	Instance	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	Backward Shift(%)	Forward Shift(%)	CPU(s)
77	400	G1-1	0	**4**	34	0	**2**	43	0	**4**	3,16	1,71	66
		G1-2	0	**4**	45	0	**2**	45	0	**4**	3,25	1,69	62
		G1-3	0	**3**	40	0	**2**	45	0	**3**	3,25	1,63	62
		G1-4	0	**11**	35	0	**9**	54	0	**11**	3,66	1,88	86
		G1-5	0	**4**	36	0	**1**	59	0	**4**	3,54	1,92	79
	500	G1-1	0	**15**	50	0	**12**	89	0	**14**	3,72	1,90	171
		G1-2	0	**4**	43	0	**1**	81	0	**3**	3,77	1,97	114
		G1-3	0	**2**	33	0	**0**	65	0	**2**	3,36	1,73	83
		G1-4	0	**9**	37	0	**5**	86	0	**7**	3,82	1,99	123
		G1-5	0	**18**	46	0	**14**	169	0	**16**	3,81	2,05	126
	600	G1-1	0	**3**	99	0	**0**	115	0	**2**	4,38	2,25	213
		G1-2	0	**5**	59	0	**3**	100	0	**5**	4,19	2,24	175
		G1-3	0	**5**	55	0	**5**	82	0	**5**	3,85	2,04	156
		G1-4	0	**14**	54	0	**12**	84	0	**14**	4,2	2,22	193
		G1-5			**INF**	**0**	**7**	112		**INF**			
80	400	G1-1	0	**12**	29	0	**9**	72	0	**12**	3,27	1,66	63
		G1-2	0	**8**	28	0	**5**	81	0	**8**	3,14	1,61	62
		G1-3	0	**7**	28	0	**5**	60	0	**7**	3,18	1,63	60
		G1-4	0	**6**	29	0	**3**	53	0	**6**	3,24	1,66	61
		G1-5	0	**3**	27	0	**0**	48	0	**3**	3,24	1,67	60
	500	G1-1	0	**12**	108	0	**7**	328	0	**9**	3,50	1,87	131
		G1-2	0	**15**	53	0	**7**	258	0	**12**	3,58	1,92	129
		G1-3	0	**11**	67	0	**7**	136	0	**8**	3,71	1,91	101
		G1-4	0	**11**	161	0	**5**	85	0	**8**	3,46	1,91	108
		G1-5	0	**9**	93	0	**3**	119	0	**6**	3,77	1,99	111
	600	G1-1	0	**14**	55	0	**11**	119	0	**14**	3,55	1,88	159
		G1-2	0	**10**	74	0	**6**	105	0	**8**	3,75	1,96	144
		G1-3	0	**7**	94	0	**4**	155	0	**5**	3,69	1,91	128
		G1-4	0	**15**	80	0	**10**	281	0	**13**	3,61	1,88	148
		G1-5			**INF**	0	**15**	334		**INF**			
84	400	G1-1	0	**7**	26	0	**7**	48	0	**7**	3,28	1,54	78
		G1-2	0	**2**	27	0	**2**	39	0	**2**	3,18	1,63	80
		G1-3	0	**0**	19								
		G1-4	0	**0**	19								
		G1-5	0	**0**	19								
	500	G1-1	0	**0**	27								
		G1-2	0	**0**	30								
		G1-3	0	**0**	26								
		G1-4	0	**0**	26								
		G1-5	0	**0**	26								
		G2-1	0	**0**	26								
		G2-2	0	**2**	78	0	**1**	91	0	**1**	4,36	2,24	197
		G2-3	0	**0**	26								
		G2-4	0	**0**	27								
		G2-5	0	**0**	27								
	600	G1-1	0	**3**	86	0	**3**	91	0	**3**	3,56	1,82	220
		G1-2	0	**0**	37								
		G1-3	0	**1**	88	0	**1**	93	0	**1**	3,7	1,83	260
		G1-4	0	**0**	35								
		G1-5	0	**0**	37								
		G2-1	0	**0**	36								
		G2-2	0	**0**	37								
		G2-3	0	**0**	39								
		G2-4	0	**2**	102	0	**2**	90	0	**2**	4,30	2,12	258
		G2-5	0	**0**	35								

Table 3: Critical and non-critical products: Instances with 88 and 94 PM operations

			P_{init}			P_{shift}^{oper}			P_{shift}^{prod}				
Nb, Oper.	Nb. Prod.	Instance	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	Backward Shift(%)	Forward Shift(%)	CPU(s)
88		G1-1	0	**0**	24								
		G1-2	0	**0**	23								
		G1-3	0	**0**	24								
		G1-4	0	**0**	24								
		G1-5	0	**0**	23								
	400	G2-1	0	**0**	23								
		G2-2	0	**0**	23								
		G2-3	0	**0**	23								
		G2-4	0	**0**	23								
		G2-5	0	**0**	29								
		G1-1	0	**0**	56								
		G1-2	0	**1**	130	0	**1**	73	0	**1**	3,44	1,73	207
		G1-3	0	**0**	38								
		G1-4	0	**0**	37								
		G1-5	0	**0**	39								
	500	G2-1	0	**0**	39								
		G2-2	0	**0**	39								
		G2-3	0	**0**	40								
		G2-4	0	**0**	39								
		G2-5	0	**0**	40								
		G1-1	0	**0**	54								
		G1-2	0	**0**	54								
		G1-3	0	**1**	161	0	**1**	96	0	**1**	3,60	1,77	276
		G1-4	0	**0**	43								
		G1-5	0	**0**	41								
	600	G2-1	0	**1**	98	0	**1**	121	0	**1**	4,16	2,14	267
		G2-2	0	**1**	103	0	**1**	109	0	**1**	4,09	2,12	303
		G2-3	0	**2**	103	0	**2**	111	0	**2**	4,01	2,08	261
		G2-4	0	**0**	42								
		G2-5	0	**0**	42								
94		G1-1	0	**1**	92	0	**1**	80	0	**1**	3,25	1,67	165
		G1-2	0	**0**	32								
		G1-3	0	**0**	29								
		G1-4	0	**0**	31								
		G1-5	0	**0**	27								
	400	G2-1	0	**1**	67	0	**1**	82	0	**1**	4,31	2,11	155
		G2-2	0	**0**	26								
		G2-3	0	**0**	27								
		G2-4	0	**0**	32								
		G2-5	0	**0**	37								
		G1-1	0	**1**	115	0	**1**	77	0	**1**	3,41	1,81	190
		G1-2	0	**0**	50								
		G1-3	0	**0**	38								
		G1-4	0	**1**	87	0	**1**	96	0	**1**	3,43	1,73	209
		G1-5	0	**0**	40								
	500	G2-1	0	**1**	146	0	**1**	116	0	**1**	4,15	2,12	205
		G2-2	0	**0**	53								
		G2-3	0	**0**	56								
		G2-4	0	**0**	54								
		G2-5	0	**0**	55								
		G1-1	0	**0**	70								
		G1-2	0	**1**	191	0	**0**	96	0	**1**	3,49	1,77	282
		G1-3	0	**0**	75								
		G1-4	0	**0**	79								
		G1-5	0	**1**	182	0	**0**	114	0	**1**	3,64	1,91	271
	600	G2-1	0	**0**	77								
		G2-2	0	**0**	79								
		G2-3	0	**0**	79								
		G2-4	0	**1**	187	0	**1**	139	0	**1**	4,16	2,14	321
		G2-5	0	**0**	75								

Table 4: Critical and non-critical products: Instances with 97 PM operations

			P_{init}			P^{oper}_{shift}			P^{prod}_{shift}				
Nb, Oper.	Nb. Prod.	Instance	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	Backward Shift(%)	Forward Shift(%)	CPU(s)
97		G1-1	1	**0**	122								
		G1-2	1	**0**	111								
		G1-3	1	**0**	105								
		G1-4	1	**1**	136	1	**1**	50	1	**1**	3,46	1,73	174
		G1-5	1	**0**	91								
	400	G2-1	1	**1**	147	1	**0**	67	1	**0**	4,25	2,18	165
		G2-2	1	**0**	128								
		G2-3	1	**0**	129								
		G2-4	1	**3**	127	1	**1**	102	1	**1**	4,50	2,24	188
		G2-5	1	**2**	100	1	**1**	73		**INF**			
		G1-1	1	**0**	123								
		G1-2	1	**0**	128								
		G1-3	1	**0**	137								
		G1-4	1	**0**	175								
		G1-5	1	**0**	182								
	500	G2-1	1	**0**	181								
		G2-2	1	**1**	152	1	**0**	142	1	**0**	4,30	2,10	262
		G2-3	1	**1**	163	1	**0**	80	1	**0**	4,36	2,21	270
		G2-4	1	**0**	198								
		G2-5	1	**0**	143								
		G1-1	1	**0**	312								
		G1-2	1	**0**	206								
		G1-3	1	**1**	244	1	**1**	92	1	**1**	3,95	1,98	360
		G1-4	1	**0**	302								
		G1-5	1	**0**	268								
	600	G2-1	1	**1**	296	1	**0**	105	1	**0**	4,44	2,25	328
		G2-2	1	**1**	247	1	**0**	137	1	**0**	4,38	2,13	284
		G2-3	1	**0**	272								
		G2-4	1	**2**	296	1	**0**	112	1	**0**	4,26	2,15	397
		G2-5	1	**0**	297								

4.3. Only critical products

Contrary to the results obtained in the previous section, shifting maintenance operations or production quantities are more efficient to perform PM operations in later periods when only critical products are considered. Table 5 shows that shifting maintenance operations leads to better results in terms of earliness for the first two groups of instances with 77 and 80 PM operations. However, this is not the case when observing the results of the instances with more PM operations. In the instances with 88, 94 and 97 operations with larger required production quantities (G2), the earliness significantly drops when production quantities are shifted instead of maintenance operations (see Tables 6 and 7). When these contradictory results are analyzed in more details, we can observe that there are differences between the instances. The PM operations in the instances with 77 and 80 PM operations have longer processing times than the instances with 88, 94 and 97 PM operations. Shorter PM operations provide more flexibility to shift production quantities, whereas longer PM operations prevent the machine capacity in a period to be used for production for earlier or later periods. Hence, based on this analysis, we can conclude that Model P_{shift}^{prod} is more effective with short PM operations, whereas Model P_{shift}^{oper} is more effective with long PM operations. In addition, shifting production quantities (Model P_{shift}^{prod}) is efficient if the remaining machine capacity, after PM operations are planned, allows about 4.5% of the product quantities to be shifted (see Tables 6 and 7). In terms of computational complexity, shifting maintenance operations (Model P_{shift}^{oper}) requires longer computational times than shifting production quantities (Model P_{shift}^{prod}). Consider in particular Instances 94-600-G2-1 and 94-600-G2-2 in Table 7, where Model P_{shift}^{prod} provides much better results than Model P_{shift}^{oper} in terms of earliness and with shorter computational times. The same pattern can be observed in Instances 97-600-G2-1 and 97-600-G2-2. In Instance 97-600-G2-2, although shifting maintenance operations enables PM operations to be planned 5 periods later in total (earliness from 17 to 12) with a computational time of about one hour (3 450 sec.), PM operations can be planned in later periods (earliness equal to 4) by shifting production quantities with a computational time of about 10 minutes (627 sec.).

When the case with critical and non-critical products (Section 4.2) and the case with only critical products (this section) are compared, it can first be noted that the earliness is smaller in the first case as expected. This can be observed in particular in the instances with 88, 94 and 97 operations with larger production quantities (G2). In the first case, since $l_{op} = l_o$ for half of the products, solving Model P_{init} is faster than in the second case as less Constraints (5) are required. In both cases, Tables 2 and 5 show that Model P_{init} cannot find a feasible solution for Instances 77-600-G1-5 and 80-600-G1-5, whereas these instances are optimally solved by Model P_{shift}^{oper}. Note also that Model P_{shift}^{prod} cannot find a feasible solution for Instances 77-600-G1-5 and 80-600-G1-5. This illustrates again that shifting maintenance operations is more relevant than shifting production quantities when maintenance operations have long processing times.

4.4. Combining Models P_{shift}^{oper} and P_{shift}^{oper}

The motivation behind combining Models P_{shift}^{oper} and P_{shift}^{oper} is to better use machine capacity to plan PM operations. The experiments were only conducted on instances with relatively large earliness that could not be significantly improved by using Models P_{shift}^{oper} and P_{shift}^{oper} separately. The results can be found in Table 8. As it can be observed, the earliness can again be reduced, for example in Instances 77-500-G1-2 and 77-600-G1-4 for only critical products, where the earliness decreases from 13 to 6 and from 10 to 4. Note also that the computational times most often increase, sometimes significantly such as again

Table 5: Only critical products: Instances with 77 and 80 PM operations

			P_{init}			P^{oper}_{shift}			P^{prod}_{shift}				
Nb. Oper.	Nb. Prod.	Instance	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	Backward Shift(%)	Forward Shift(%)	CPU(s)
77	400	G1-1	0	**4**	40	0	**3**	38	0	**4**	3,26	1,73	73
		G1-2	0	**7**	42	0	**6**	40	0	**7**	3,30	1,70	90
		G1-3	0	**8**	41	0	**6**	43	0	**8**	3,33	1,64	120
		G1-4	0	**11**	80	0	**10**	70	0	**11**	3,73	1,93	111
		G1-5	0	**7**	52	0	**4**	63	0	**7**	3,76	2,00	106
	500	G1-1	0	**16**	673	0	**9**	241	0	**9**	3,85	2,00	306
		G1-2	0	**22**	615	0	**13**	1057	0	**15**	4,35	2,36	274
		G1-3	0	**2**	48	0	**2**	62	0	**2**	3,41	1,78	87
		G1-4	0	**22**	1586	0	**10**	421	0	**10**	3,79	1,96	153
		G1-5	0	**21**	238	0	**13**	727	0	**15**	3,80	2,05	154
	600	G1-1	0	**11**	1330	0	**1**	543	0	**3**	4,56	2,35	405
		G1-2	0	**16**	449	0	**7**	797	0	**9**	4,07	2,09	442
		G1-3	0	**19**	103	0	**16**	311	0	**19**	3,98	2,14	442
		G1-4	0	**13**	98	0	**10**	125	0	**12**	3,98	2,10	279
		G1-5	0	**INF**			**11**	280		**INF**			
80	400	G1-1	0	**11**	40	0	**9**	54	0	**11**	3,32	1,69	65
		G1-2	0	**5**	44	0	**2**	46	0	**5**	3,22	1,62	62
		G1-3	0	**7**	30	0	**3**	59	0	**7**	3,25	1,7	80
		G1-4	0	**9**	28	0	**6**	50	0	**9**	3,32	1,7	61
		G1-5	0	**6**	29	0	**3**	56	0	**6**	3,30	1,66	62
	500	G1-1	0	**17**	125	0	**10**	345	0	**10**	3,73	1,96	133
		G1-2	0	**24**	268	0	**14**	481	0	**17**	3,7	2	204
		G1-3	0	**23**	575	0	**12**	661	0	**13**	3,84	2	219
		G1-4	0	**22**	292	0	**14**	258,	0	**18**	3,74	2,00	197
		G1-5	0	**29**	831	0	**15**	603	0	**16**	3,77	2,00	257
	600	G1-1	0	**21**	374		**14**	479		**16**	3,78	1,95	249
		G1-2	0	**22**	364		**14**	392		**16**	3,91	2,02	244
		G1-3	0	**17**	317		**9**	988		**11**	3,83	1,94	298
		G1-4	0	**22**	86		**17**	360		**18**	3,67	1,98	285
		G1-5	0	**INF**			**22**	564		**INF**			

Table 6: Only critical products: Instances with 84 and 88 PM operations

			P_{init}			P^{oper}_{shift}			P^{prod}_{shift}				
Nb. Oper.	Nb. Prod.	Instance	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	Backward Shift(%)	Forward Shift(%)	CPU(s)
84	400	G1-1	0	**16**	37	0	**16**	52	0	**16**	3,23	1,62	63
		G1-2	0	**12**	37	0	**12**	50	0	**12**	3,29	1,70	67
		G1-3	0	**0**	22	0							
		G1-4	0	**3**	54	0	**3**	54	0	**3**	3,14	1,56	81
		G1-5	0	**1**	63	0	**1**	47	0	**1**	3,3	1,66	82
	500	G1-1	0	**1**	86	0	**1**	60	0	**1**	3,48	1,82	132
		G1-2	0	**1**	117	0	**1**	75	0	**1**	3,75	2,00	161
		G1-3	0	**1**	107	0	**1**	56	0	**1**	3,58	1,81	121
		G1-4	0	**0**	38								
		G1-5	0	**2**	107	0	**2**	68	0	**2**	3,53	1,81	111
		G2-1	0	**7**	126	0	**5**	379	0	**2**	4,38	2,18	163
		G2-2	0	**13**	662	0	**7**	935	0	**1**	4,49	2,29	230
		G2-3	0	**11**	886	0	**3**	940	0	**1**	4,37	2,26	179
		G2-4	0	**9**	360	0	**4**	515	0	**0**	3,90	1,92	76
		G2-5	0	**10**	584	0	**3**	608	0	**0**	3,98	1,87	75
	600	G1-1	0	**6**	116	0	**6**	98	0	**6**	3,75	1,89	185
		G1-2	0	**0**	43								
		G1-3	0	**0**	50								
		G1-4	0	**1**	122	0	**1**	121	0	**1**	3,81	1,96	188
		G1-5	0	**0**	35								
		G2-1	0	**8**	157	0	**5**	163	0	**6**	4,33	2,20	304
		G2-2	0	**4**	227	0	**1**	143	0	**0**	3,77	1,90	110
		G2-3	0	**4**	411	0	**3**	177	0	**0**	3,73	1,89	126
		G2-4	0	**4**	153	0	**2**	152	0	**1**	4,45	2,21	280
		G2-5	0	**3**	192	0	**0**	139	0	**0**	3,84	1,95	102
88	400	G1-1	0	**0**	24								
		G1-2	0	**0**	26								
		G1-3	0	**0**	31								
		G1-4	0	**0**	36								
		G1-5	0	**0**	36								
		G2-1	0	**7**	386	0	**4**	693	0	**0**	3,66	2,00	66
		G2-2	0	**13**	317	0	**6**	835	0	**1**	4,18	2,28	146
		G2-3	0	**10**	421	0	**5**	580	0	**1**	4,18	2,36	136
		G2-4	0	**14**	426	0	**7**	535	0	**4**	4,28	2,22	233
		G2-5	0	**11**	412	0	**6**	761	0	**0**	3,75	2,03	75
	500	G1-1	0	**0**	38								
		G1-2	0	**0**	42								
		G1-3	0	**0**	45								
		G1-4	0	**0**	42								
		G1-5	0	**0**	45								
		G2-1	0	**8**	510	0	**4**	859	0	**1**	4,11	2,31	158
		G2-2	0	**4**	159	0	**3**	225	0	**0**	3,72	1,89	85
		G2-3	0	**8**	364	0	**8**	497	0	**1**	4,26	2,28	158
		G2-4	0	**4**	269	0	**1**	1014	0	**0**	3,72	1,89	86
		G2-5	0	**5**	186	0	**2**	338	0	**0**	3,71	1,92	86
	600	G1-1	0	**1**	157	0	**1**	101	0	**1**	3,73	1,87	198
		G1-2	0	**0**	44								
		G1-3	0	**0**	43								
		G1-4	0	**2**	124	0	**2**	106	0	**2**	3,56	1,87	189
		G1-5	0	**0**	42								
		G2-1	0	**11**	954	0	**6**	1177	0	**1**	4,33	2,25	237
		G2-2	0	**6**	284	0	**2**	452	0	**0**	3,82	2,00	110
		G2-3	0	**5**	219	0	**3**	249	0	**0**	3,70	1,93	109
		G2-4	0	**6**	150	0	**5**	307	0	**2**	4,20	2,13	205
		G2-5	0	**8**	451	0	**3**	487	0	**0**	3,76	1,96	109

Table 7: Only critical products: Instances with 94 and 97 PM operations

			P_{init}			P_{shift}^{oper}			P_{shift}^{prod}				
Nb. Oper.	Nb. Prod.	Instance	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	CPU(s)	Unplan. Oper.	**Earliness**	Backward Shift(%)	Forward Shift(%)	CPU(s)
94		G1-1	0	**1**	64	0	**1**	68	0	**1**	3,30	1,73	109
		G1-2	0	**1**	58	0	**1**	51	0	**1**	3,26	1,64	112
		G1-3	0	**4**	81	0	**4**	56	0	**4**	3,29	1,62	116
		G1-4	0	**0**	26								
		G1-5	0	**0**	33								
	400	G2-1	0	**8**	363	0	**4**	330	0	**1**	4,42	2,26	129
		G2-2	0	**6**	176	0	**5**	233	0	**1**	4,10	2,16	125
		G2-3	0	**7**	99	0	**7**	118	0	**4**	4,12	2,00	163
		G2-4	0	**6**	238	0	**5**	341	0	**0**	3,73	1,88	79
		G2-5	0	**3**	102	0	**3**	130	0	**0**	3,89	1,85	77
		G1-1	0	**1**	120	0	**1**	69	0	**1**	3,50	1,87	140
		G1-2	0	**0**	37								
		G1-3	0	**0**	37								
		G1-4	0	**1**	108	0	**1**	78	0	**1**	3,41	1,81	166
		G1-5	0	**1**	107	0	**1**	78	0	**1**	3,66	1,84	147
	500	G2-1	0	**11**	377	0	**5**	404	0	**1**	4,33	2,19	160
		G2-2	0	**9**	701	0	**6**	965	0	**0**	4,11	1,96	104
		G2-3	0	**11**	346	0	**6**	477	0	**0**	3,98	1,98	104
		G2-4	0	**12**	421	0	**9**	946	0	**2**	4,25	2,15	301
		G2-5	0	**5**	221	0	**4**	260	0	**1**	4,52	2,2	179
		G1-1	0	**1**	122	0	**1**	102	0	**1**	3,58	1,90	216
		G1-2	0	**1**	137	0	**1**	106	0	**1**	3,58	1,86	271
		G1-3	0	**2**	128	0	**2**	107	0	**2**	3,82	1,84	226
		G1-4	0	**1**	129	0	**1**	111	0	**1**	3,91	1,88	236
		G1-5	0	**1**	166	0	**1**	113	0	**1**	3,83	1,98	265
	600	G2-1	0	**13**	658	0	**8**	2274	0	**2**	4,20	2,10	390
		G2-2	0	**14**	877	0	**8**	2530	0	**1**	4,26	2,12	242
		G2-3	0	**11**	390	0	**6**	600	0	**2**	4,22	2,16	308
		G2-4	0	**11**	419	0	**11**	721	0	**1**	4,30	2,24	242
		G2-5	0	**14**	890	0	**6**	1489	0	**1**	4,46	2,24	248
97		G1-1	1	**3**	85	1	**3**	67	1	**3**	3,38	1,74	159
		G1-2	1	**0**	86	1	**0**	67	1	**0**	3,45	1,68	168
		G1-3	1	**0**	86	1	**0**	49	1	**0**	3,36	1,74	175
		G1-4	1	**3**	81	1	**3**	50	1	**3**	3,50	1,76	188
		G1-5	1	**2**	75	1	**2**	50	1	**2**	3,48	1,79	178
	400	G2-1	1	**10**	276	1	**5**	366	1	**3**	4,21	2,33	212
		G2-2	1	**5**	461	1	**3**	440	1	**0**	4,26	2,27	203
		G2-3	1	**6**	389	1	**0**	364	1	**0**	4,26	2,26	202
		G2-4	1	**9**	417	1	**3**	579	1	**4**	4,61	2,28	356
		G2-5	1	**13**	559	1	**6**	396	1	**4**	4,17	2,33	288
		G1-1	1	**1**	124	1	**1**	94	1	**1**	3,74	1,98	255
		G1-2	1	**3**	126	1	**3**	74	1	**3**	3,64	1,87	256
		G1-3	1	**0**	99	1	**0**	77	1	**0**	3,83	1,91	282
		G1-4	1	**1**	143	1	**1**	77	1	**1**	3,83	1,96	282
		G1-5	1	**2**	118	1	**2**	79	1	**2**	3,95	1,95	282
	500	G2-1	1	**8**	260	1	**2**	454	1	**1**	4,49	2,32	300
		G2-2	1	**11**	566	1	**5**	480	1	**4**	4,28	2,19	363
		G2-3	1	**14**	716	1	**8**	1459	1	**3**	4,31	2,39	628
		G2-4	1	**8**	571	1	**4**	508	1	**1**	4,22	2,33	387
		G2-5	1	**7**	258	1	**2**	356	1	**2**	4,52	2,29	277
		G1-1	1	**1**	184	1	**1**	107	1	**1**	4,00	2,00	273
		G1-2	1	**0**	189	1	**0**	117	1	**0**	4,00	2,00	274
		G1-3	1	**1**	196	1	**1**	127	1	**1**	4,05	2,04	278
		G1-4	1	**0**	200	1	**0**	130	1	**0**	4,00	1,99	278
		G1-5	1	**2**	196	1	**2**	119	1	**2**	3,94	1,97	274
	600	G2-1	1	**15**	1271	1	**11**	2249	1	**3**	4,40	2,36	519
		G2-2	1	**17**	1279	1	**12**	3450	1	**4**	4,28	2,33	627
		G2-3	1	**7**	675	1	**3**	617	1	**1**	4,29	2,26	303
		G2-4	1	**7**	720	1	**3**	343	1	**0**	4,31	2,28	261
		G2-5	1	**5**	315	1	**3**	433	1	**2**	4,20	2,16	354

for Instance 77-500-G1-2 for only critical products, where the computational time increases from at most 1057 seconds for Model P^{oper}_{shift} to 1580 seconds when Models P^{oper}_{shift} and P^{prod}_{shift} are combined.

Table 8: Combining Models P^{oper}_{shift} and P^{prod}_{shift}: Some challenging instances with 77 and 80 operations

				P^{oper}_{shift}.		P^{prod}_{shift}		P^{prod}_{shift}+P^{oper}_{shift}	
Case	Nb, Oper.	Nb. Prod.	Instance	**Earliness**	CPU(s)	**Earliness**	CPU(s)	**Earliness**	CPU(s)
Crit./Non crit.	77	400	G1-4	**9**	54	**11**	86	**6**	129
Crit./Non crit.	77	500	G1-1	**12**	89	**14**	171	**7**	352
Crit./Non crit.	77	500	G1-5	**14**	169	**16**	126	**10**	272
Crit./Non crit.	77	600	G1-4	**12**	84	**14**	193	**6**	303
Crit./Non crit.	77	600	G1-5	**7**	112	**INF**		**6**	273
Crit./Non crit.	80	400	G1-1	**9**	72	**12**	63	**7**	103
Crit./Non crit.	80	600	G1-1	**11**	119	**14**	159	**7**	280
Crit./Non crit.	80	600	G1-4	**10**	281	**13**	148	**6**	399
Crit./Non crit.	80	600	G1-5	**15**	334	**INF**		**5**	575
Only crit.	77	400	G1-4	**10**	70	**11**	111	**6**	229
Only crit.	77	500	G1-2	**13**	1057	**15**	274	**6**	1580
Only crit.	77	500	G1-5	**13**	727	**15**	154	**8**	694
Only crit.	77	600	G1-3	**16**	311	**19**	442	**10**	1132
Only crit.	77	600	G1-4	**10**	125	**12**	279	**4**	548
Only crit.	77	600	G1-5	**11**	280	**INF**		**2**	427
Only crit.	80	500	G1-1	**10**	345	**10**	133	**5**	844
Only crit.	80	500	G1-2	**14**	481	**17**	204	**10**	970
Only crit.	80	500	G1-3	**12**	661	**13**	219	**5**	501
Only crit.	80	500	G1-4	**14**	258	**18**	197	**10**	587
Only crit.	80	500	G1-5	**15**	603	**16**	257	**9**	799
Only crit.	80	600	G1-1	**14**	479	**16**	249	**10**	296
Only crit.	80	600	G1-2	**14**	392	**16**	244	**10**	801
Only crit.	80	600	G1-3	**9**	988	**11**	298	**7**	621
Only crit.	80	600	G1-4	**17**	360	**18**	285	**11**	341
Only crit.	80	600	G1-5	**22**	564	**INF**		**15**	1002

5. Managerial insights

Our numerical results show that planning all maintenance operations might not always possible if the capacity is too tight or if there are too long maintenance operations. Indeed, instances with 77 and 80 operations are more difficult to solve than instances with 84, 88, 94 and 97 operations which include shortest maintenance operations. Moreover, the earliness is also significantly larger for instances with 77 and 80 operations as shown for example in Table 2. Another interesting observation is that an optimization approach is required as CPU times of more 60 seconds are nearly always required, which increase up to more than 1,200 seconds for the initial model P_{init}, which means that an optimal solution is not easily found by IBM ILOG CPLEX.

An important managerial insight is that allowing flexibility on the planning of the maintenance operations in two consecutive periods, and on the partial shifting of production quantities in the previous period or the following period, may help to significantly reduce the earliness, and even to find feasible solutions. Moreover, as shown in Table 8, combining both types of flexibility is also very interesting, as it may leads to again significant earliness reductions compared to considering each type of flexibility separately. Hence, it is important for production planners to inform on the allowed ratio $r^-_{p,t}$, resp. $r^+_{p,t}$, of product p that can be produced in period $t-1$, resp. in period $t+1$, for period t.

In the remainder of this section, we perform a sensitivity analysis. As pointed out in Section 3.2, the combination of Constraints (2), (3) and (5) can lead to infeasibility and increase the difficulty of the problem. As the production quantities are known, and the maintenance operations must be performed within time windows that are also known, we are interested in investigating the impact of the required capacity on key performance indicators. Hence, it makes more sense to perform the sensitivity analysis on Constraints (3) than on Constraints (2) and (5). We propose two strategies and three configurations for each strategy:

- In Strategy 1, the processing times of maintenance operations are reduced by 10%, 20% and 50%, respectively, i.e. Configurations 1, 2 and 3. In practice, reducing these processing times, which are typically overestimated, can be done by rationalizing the maintenance process or accelerating it by increasing the maintenance manpower, although 50% is of course extreme but interesting for our analysis.

- In Strategy 2, the speeds of the machines are increased, and thus the process times of the products are decreased, by 10%, 20% and 50%, respectively, i.e. Configurations 1, 2 and 3. In practice, significantly increasing the speeds of machines might be difficult, but 50% is also interesting for our analysis.

The common feature of both strategies is that some additional costs are usually associated to significant changes, i.e. when additional maintenance workforce are required or when the speeds of machines are increased (typically more energy consumption). The sensitivity analysis shows whether the improvement of the earliness values is worth implementing new strategies.

Let us consider instances 77_500_G1-5 and 80_500_G1-5, in which the maintenance operations are planned relatively early by the initial model (P_{init}), 21 and 29 respectively (see Table 5). In Strategy 1, resp. Strategy 2, the processing times of the maintenance operations, resp. of the products on their eligible machines, have been reduced by the defined ratio.

Table 9: Sensitivity Analysis: Strategy 1

Instance	Original	Config. 1		Config. 2		Config. 3	
77_500-G1-5	Earliness	Earliness	Gap	Earliness	Gap	Earliness	Gap
P_{init}	**21**	17	-4	16	-5	13	-8
P^{oper}_{shift}	**13**	13	0	13	0	13	0
P^{prod}_{shift}	**15**	15	0	15	0	13	-2
80_500-G1-5							
P_{init}	**29**	21	-8	19	-10	14	-15
P^{oper}_{shift}	**15**	14	-1	13	-2	13	-2
P^{prod}_{shift}	**16**	16	0	15	-1	13	-3

Table 9 presents the results obtained for Strategy 1. For instance 77_500-G1-5, with Model P_{init} and when the processing times of maintenance operations are decreased by 10% and 20%, the earliness value decreases to 17 and 16 respectively. These gains show that adjusting the processing times of maintenance operations can help to significantly reduce the earliness values. The improvement is even more significant in Configuration 3, although a 50% decrease should be very costly to implement. The results for instance 80_500-G1-5 show an even larger improvement, with an already significant gain in Configuration 1 (10%). It is interesting to note that, when Models P^{oper}_{shift} or P^{prod}_{shift} can be used, there is little or no gain induced by reducing the processing times of maintenance operations. These results show that it is more important to prioritize a better use of the remaining capacity on machines than to reduce the processing times of maintenance operations.

Table 10: Sensitivity Analysis: Strategy 2

Instance	Original	Config. 1		Config. 2		Config. 3	
77_500-G1-5	Earliness	Earliness	Gap	Earliness	Gap	Earliness	Gap
P_{init}	**21**	15	-6	15	-6	15	-6
P^{oper}_{shift}	**13**	13	0	13	0	13	0
P^{prod}_{shift}	**15**	15	0	15	0	15	0
80_500-G1-5							
P_{init}	**29**	16	-13	16	-13	16	-13
P^{oper}_{shift}	**15**	14	-1	14	-1	14	-1
P^{prod}_{shift}	**16**	16	0	16	0	16	0

Looking at Table 10, which presents the results of Strategy 2, the first noteworthy result is that decreasing the process times of products leads to larger improvements than with Strategy 1 except for Configuration 3 (50%). Even more important, a decrease of 10% (Configuration 1) is enough to get the best results. The results for Models P^{oper}_{shift} or P^{prod}_{shift} are in line with the ones in Table 9, i.e. little or no gain induced by reducing the process times of products. This emphasizes again the importance of optimizing the use of the remaining capacity on machines to perform maintenance operations.

6. Conclusions and perspectives

In this paper, inspired by practical settings, we proposed new mathematical models for the integrated planning of maintenance operations and workload allocation on a set of machines. To provide guidelines to practitioners, we tested the proposed models on industrial data and showed that the production and PM operations can be planned in different ways to minimize the earliness of maintenance operations. The results show that providing more flexibility in the planning of maintenance operations and of production quantities, by shifting them to several periods, can lead to significant earliness savings, and thus to a better use of the machine capacity.

Only instances that can be solved optimally with an integer linear programming solver are presented and discussed in this paper. However, some instances require very large CPU times and other instances of larger sizes cannot be solved optimally in reasonable CPU times. Hence, we are investigating dedicated solution approaches such as a Lagrangian Relaxation heuristic or a Column Generation approach.

In addition, the models in this paper assume that product p can no longer be produced if maintenance period o is not performed before period $l_{o,p}$. However, it could be relevant to allow p to be produced after $l_{o,p}$ even though it induces some risk on product quality. Hence, we are developing another mathematical model, which allows products to be processed on a machine even if their maintenance operation has not been performed by minimizing the number of products at risk. Another interesting and related perspective is to integrate the health index of the machines, and to provide a decision making tool to practitioners.

Finally, a more ambitious work would consist in integrating production planning decisions, i.e. the quantities to produce in each period, and maintenance planning decisions at a higher decision level. A cost minimization or a profit maximization objective function would have to be considered.

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Data Availability Statement

The data that support the findings of this study are available on request from the corresponding author. The industrial data are not publicly available due to their confidentiality.

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