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Forecasting Energy Commodity Prices: A Large Global Dataset Sparse Approach*

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Abstract

This paper focuses on forecasting quarterly nominal global energy prices of commodities, such as oil, gas and coal, using the Global VAR dataset proposed by Mohaddes and Raissi (2018). This dataset includes a number of potentially informative quarterly macroeconomic variables for the 33 largest economies, overall accounting for more than 80% of the global GDP. To deal with the information on this large database, we apply dynamic factor models based on a penalized maximum likelihood approach that allows to shrink parameters to zero and to estimate sparse factor loadings. The estimated latent factors show considerable sparsity and heterogeneity in the selected loadings across variables. When the model is extended to predict energy commodity prices up to four periods ahead, results indicate larger predictability relative to the benchmark random walk model for 1-quarter ahead for all energy commodities and up to 4 quarters ahead for gas prices. Our model also provides superior forecasts than machine learning techniques, such as elastic net, LASSO and random forest, applied to the same database.

Keywords: Energy Prices, Forecasting, Dynamic Factor model, Sparse Estimation, Penalized Maximum Likelihood

JEL Codes: C1, C5, C8, E3, Q4

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1 Introduction

Global energy commodity prices play an important role in individual country-level economies. They are known to affect several macroeconomic variables including inflation, gross domestic product, exchange rate, stock market prices and interest rates as well as other commodity prices such as raw materials, metals, minerals and agricultural products whose cost depends on extraction and transportation. We focus on global nominal energy prices as reported by the World Bank, since these prices are key determinants of domestic inflation, and production costs worldwide. Global energy commodities (such as oil, gas and coal) are important inputs for many industries, such as transport, agriculture, metal and mineral, construction, manufacturing, and chemical, amongst others. Consequently, a reliable forecast of energy prices is critical for central banks, treasures, investors and international organisations such as the International Monetary Fund (IMF) and the World Bank (WB) for implementing policy responses and advising about energy commodity price fluctuations.[1]

Figure 1 shows a strong co-movement of oil, gas and coal, across three distinctive economic periods: the great moderation from the middle 1980s to early 2002, the global financial crisis and the drop and the following (partial) recover in the period 2015-2017.

The main focus of this paper is to forecast global energy commodity prices (oil, gas and coal) based on a large global database using a latent factor model. The approach to factor construction proposed in this paper is global in nature: it is not constrained within a dominant geographical region and instead relies on exploiting the information from a large number of observable variables representing the entire global economy. Moreover, different factors can be associated with specific variables, such as inflation, equity prices, exchange rates and interest rates, supporting the construction of global factors of macroeconomic variables. Commodity energy prices are obtained from the World Bank dataset. This dataset reports nominal average prices measured in US dollars for oil, coal and gas (see details in data section).

Our empirical findings support the presence of pronounced sparsity and heterogeneity in the estimates for the loading matrix, thus showing that each latent factor is likely to depend on a subset of the original variables in the Global VAR data set. Compared to the benchmark random walk (RW) model, autoregressive and vector autoregressive models, and four different types of machine learning methods, precisely stepwise regression, random forest, elastic net and LASSO, the proposed models based on sparse factors improve the forecast accuracy for all the energy

[1] The energy commodity prices forecasted in this study are nominal prices in US dollars as reported by the World Bank (WB). Nominal energy commodity prices are critical to understand the pass-through effect on domestic economies, including on domestic prices and/or other commodities, see, for example, Leibtag (2009) or Baffes (2007).

commodity prices. The largest accuracy gains occur when predicting gas prices, with the lowest mean square forecasting error (MSFE) corresponding to 1- to 4-quarters ahead.[2] Other machine learning methods in almost all cases give larger MSFEs than the RW and only the random forest method gives forecasting gains when predicting gas prices at longer horizons. This suggests that the increased prediction accuracy is not only due to the use of a larger database, but also by taking into account the sparsity of the latent factor structure behind these data.

The literature has applied, often separately, several macroeconomic and financial variables to predict energy and commodity prices. Different measures of economic activity have been largely used to predict energy commodity prices; e.g., see Caldara et al. (2019), Aastveit et al. (2015) and Bjørnland et al. (2018). Alquist et al. (2019) develop a factor-based identification strategy to link fluctuations in commodity prices and global economic activity. Baumeister et al. (2020) apply alternative indicators of global economic activity and other market fundamentals to predict oil prices and global petroleum consumption. Čeperić et al. (2017) forecast short term natural gas using both classical time series models and machine learning methods. They find that machine learning models without the pre-selection of variables are often inferior to time-series models in forecasting spot prices. Focusing on the impact of the shale revolution, Gao et al. (2020) forecast monthly gas prices using a time-varying parameters model with stochastic volatility, a Markov switching model and hybrid model. Their results suggest that time-varying parameters model perform better than static models. Matyjaszek et al. (2019) examine the performance in forecasting coking coal prices using ARIMA models and neural networks. They find that the best forecasts are achieved by combining transgenic time series with ARIMA models. Alameer et al. (2020) consider multistep-ahead forecasts of coal prices using a hybrid deep learning model. Their experimental results suggest that a hybrid model may be a promising technique for long-term prediction of coal prices.

Focusing on forecasting the U.S. real oil prices, Baumeister and Kilian (2012) present backcasting and nowcasting techniques to fill gaps in the real-time data. The main findings are that real-time forecasts of the real price of oil may be more accurate than the no-change forecast at horizons up to one year. Also with a narrower focus on the U.S real oil prices, Baumeister and Kilian (2015) evaluate six alternative VAR models using real-time forecast combinations. Alternative specifications are based on different weight combinations. The main results of this paper are that real-time forecast combinations of the U.S real oil prices lead to a considerable reduction in MSPE relative to the no-change forecast

Building on this evidence, we collect and update up to 2018Q4 the global VAR data set of Mohaddes and Raissi (2018). This large dataset contains around 200

[2]Note that if two models are to be compared, the one with the lower MSPE over the $n-q$ out-of-sample data points is regarded as the model with the larger prediction power

series for the 33 largest world economies. The data frequency is quarterly, it spans the sample period from 1979Q1 to 2018Q4, and it includes measures of economic activity (GDP and industrial production), inflation, short- and long-term interest rates, nominal exchange rates and stock market indices. Then, a sparse dynamic factor model is applied to these extended data to extract information simultaneously from all the available variables.

The large dimension of our data relative to the time-series length poses nontrivial challenges to standard dimension reduction techniques such as factor analysis. Particularly, when the sample size is small compared to the number of variables, the estimates from the standard factor model are very imprecise. In the most extreme case as in our data, where the sample size is smaller than the number of variables, the maximum likelihood estimates are meaningless without additional assumptions on the model structure. Motivated by these issues, we extend the recent literature on forecasting energy prices with factor models in which the sample size is larger than the number of variables (e.g., see Baumeister et al. (2020)). Our approach assumes sparse loadings meaning that a relatively large number of elements in the factor loading matrix is exactly zero; estimation is carried out by a penalized maximum likelihood approach that allows to shrink estimated parameters towards zero.

In standard factor analysis, all the estimated loadings are different from zero, meaning that all the observed variables from all countries in principle contribute to the latent factors. In reality, this may not be the case since certain observed variables may play a role in explaining the latent factors whilst others do not. Therefore, we apply a factor augmented autoregressive model (Boivin and Ng, 2005) in which factors are included as predictors in an autoregressive model to forecast energy commodity prices from 1 quarter to 4 quarters ahead, and a factor augmented vector-autoregressive model in which factors are included in a vector-autoregressive model.

The remainder of the paper is organized as follows. Section 2 describes the updated VAR global dataset. Section 3 outlines the proposed model and the sparse factor analysis methodology used for estimation. Section 4 summarizes our empirical findings. Section 5 concludes and provides final remarks.

2 Data

The data used in this paper is from the global VAR dataset from Mohaddes and Raissi (2018) for the period 1979Q1 to 2018Q4 (the most recent available data). The Global VAR dataset includes the most relevant quarterly macroeconomic variables for the 33 largest economies, accounting overall for more than 90% of the global economy as of 2018. For each country, this dataset includes the following variables: real gross domestic product, inflation rate, short-term interest rate, long-term interest rate, deflated exchange rate, and real equity prices. We also use quarterly energy commodity prices data from WB for the natural logarithm of the nominal price of oil, gas and coal. We use the global indices series as reported by the WB. The global gas prices indexes are constructed using Laspeyres indexes. Weights are based on five-year consumption volumes for Europe, U.S. and Japan (LNG prices), weights are updated by the WB every five years. Coal prices are the average prices of Australian and South African coal prices. Global crude oil prices are equally weighted average price of Brent, Dubai Fateh, and West Texas Intermediate. All the original series are denominated in nominal US dollars. All the energy commodity prices original indices are denominated in nominal US dollars (2010=100.[3] The energy commodity prices raw data are shown in Figure 1. Collectively the series followed similar trends and experienced strong growth from the beginning of 2000 to the US financial crisis when they dramatically dropped; they resurged from 2010 with another drop in 2014, and a final resurge in 2017-2018. There is, however, large heterogeneity across the individual series with gas, for example, being more volatile from 2000 to 2009. Therefore, the considered out-of-sample period from 2000 to 2018 corresponds to very volatile prices and overperforming a simple random walk benchmark might be not straightforward.

The country-level variables included in our model are: the natural logarithm of real GDP (RGDP), the rate of inflation calculated by taking the difference of the natural logarithm of the consumer price index (CPI), the natural logarithm of the nominal equity price index deflated by CPI (nomEQ), the natural logarithm of the exchange rate of country i at time t expressed in US dollars deflated by country i's CPI (Fxdol), the nominal short-term percent interest rate per quarter (Rshort) and the nominal long-term percent interest rate per quarter (Rlong). The main data sources are Haver Analytics, the International Monetary Funds International Financial Statistics (IFS) database, and Bloomberg. When data were not available from these sources, we employed the other national datasets described in Mohaddes and Raissi (2018). Finally, as in the original database from Mohaddes and Raissi (2018), we use final vintage data.

3 Model

Direct forecasting of multivariate commodities based on a large number of global macroeconomic potential predictors is challenging from the estimation point-of-view due to model over-parametrization, which often requires to introduce a dimension reduction of the predictor set. To avoid this issue, the approach proposed in this paper achieves forecasting of multiple commodity prices through two main modeling

[3]See for more details on the data: `https://www.worldbank.org/en/research/commodity-markets`. Therefore, we take a different approach from most of literature that focuses on a single series (e.g. oil), a single definition for it (e.g. WTI), and its real measure by deflating for national inflation. Our work provides a more global approach and evidence is valid for a large set of (global) operators.

components. The first component is a latent factor model used to achieve dimension reduction starting from a large number of potentially useful predictors. The second component is a simple bridge linear equation allowing us to use the sparse factors to forecast commodity prices.

Factor analysis and latent factor models are appealing due to their ability to summarize a large number of correlated macroeconomic predictors in terms of a few common latent factors, which capture most of the meaningful information in the data; e.g., see Bai and Wang (2016) for a general overview of theory and applications of large factor models in economics. Despite their utility, however, classical estimators for factor analysis models, such as the maximum likelihood estimator, are not generally trustworthy when the number of variables is large compared to the number observations, as it is the case for the global VAR data in this paper. Hence, we propose to use a penalized likelihood estimation approach which enables us to achieve dimension reduction reliably even in the presence of relatively few time points (Choi et al., 2010; Hirose and Yamamoto, 2015).

Let $x_t = (x_{1,t}, \ldots, x_{n,t})'$ be a vector of monthly observations at times $t = 1, \ldots, T$ included in our global dataset which have been standardized to have a mean equal to zero and variance equal to one. A factor model is then given by the following equation:

$$x_t = \Lambda f_t + \epsilon_t, \qquad \epsilon_t \sim N(0, \Psi), \tag{1}$$

where Λ is a fixed $(n \times r)$ matrix of factor loadings, $f_t = (f_{1t}, \ldots, f_{rt})'$ is a $(r \times 1)$ vector of static common factors and $\epsilon_t = (\epsilon_{1t}, \ldots, \epsilon_{nt})'$ is the idiosyncratic component with zero expectation and $n \times n$ covariance matrix Ψ. We assume independent idiosyncratic errors with Ψ being diagonal. Following Doz et al. (2012), we assume that the common factors f_t follow a stationary process independent of the idiosyncratic component ϵ_t. Throughout the paper, we take f_t as a stationary VAR process:

$$f_t = A f_{t-1} + u_t, \tag{2}$$

where $u_t \sim N(0, Q)$, and Q is a $(r \times r)$ full-rank matrix, A is a $(r \times r)$ matrix where all roots of $\det(I_r - Az)$ lie outside the unit circle. The idiosyncratic and VAR residuals are assumed to be independent and identically distributed as follows:

$$\begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix} \sim iid\ N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Psi & 0 \\ 0 & Q \end{bmatrix} \right). \tag{3}$$

Differently from Doz et al. (2012), we do not restrict the matrix Q to be the identity matrix. In our empirical analysis, we mainly consider an autoregressive lag structure for the VAR model equal to 1. We also investigated the usefulness of higher order lags, but results were qualitatively similar. We also note that factors are demeaned.

The h-step ahead predictions of each of the S commodity prices, $y_{s,t+h}$, $s =$

$1, \dots, S$, are obtained by using a factor augmented autoregressive model where commodity prices ($y_{s,t}$) are expressed as a linear function of the (lagged) expected common factors and lags of y_{t+h}, as described by the following bridge equation:

$$y_{s,t+h} = \alpha_s + \beta_s y_{s,t+h-1} + \zeta_s' f_{t+h-1} + e_{s,t+h}, \qquad e_{s,t+h} \sim N(0, \sigma_{s,e}) \tag{4}$$

where ζ_s is an $r \times 1$ vector of parameters.[4] Accordingly, predictions of commodity prices ($y_{s,t+h}$) are constructed from equation (4), conditional on the estimated parameters and the factor forecasts. Model (4) will be referred to as DFM-AR. Note, that equation (4) implies that h-step ahead forecasts are computed as iterative forecasts. An alternative approach, not considered in this paper, is the direct multi-step ahead forecasting suggested by Marcellino et al. (2006).

We also investigate the use a VAR model to predict jointly the full vector of commodity prices $y_{t+h} = (y_{1,t+h}, \cdots, y_{S,t+h})$ by the multivariate model

$$y_{t+h} = C + \Phi y_{t+h-1} + B' F_{t+h-1} + e_{t+h}, \qquad e_{t+h} \sim N(0, \Sigma_e). \tag{5}$$

Model (5) will be referred to as DFM-VAR in the rest of the paper.

Equations (1) and (2) are commonly estimated following a two-step procedure; e.g., see Giannone et al. (2008). In the first step, principal component analysis or factor analysis are used to estimate the latent factors, while the second step entails finding maximum likelihood estimates for the latent VAR process. One issue with this approach is that estimation becomes very imprecise when the number of variables n is relatively large compared to the number of observations. In the most extreme case where the number of observations T is much smaller than n, maximum likelihood estimates for the standard factor analysis model do not exist. An additional problem with the traditional approaches is that the estimated matrix of loadings Λ is not sparse, meaning that all its elements are different from zero even when in reality certain elements of Λ might be exactly zero, i.e. some variables may play no role in forming certain latent factors. This means that the additional statistical errors deriving from the estimation of the irrelevant loadings would result in larger forecasting errors.

Motivated by these issues, we propose to use a penalized likelihood approach to extract the latent factors. Penalized likelihood estimation, such as Lasso-type penalization, has been successfully applied in several domains in statistics and econometrics to deal with complex likelihood functions with a large number of parameters. In this paper, we follow the penalized factor analysis estimation methodology first considered by Choi et al. (2010). Hirose and Yamamoto (2015) developed al-

[4]As for the VAR model in (2) we tried to include more lags of $y_{s,t}$ in (4), but forecast accuracy did not improve. Moreover, we also investigated to regress $y_{s,t+h}$ on the contemporaneous factors f_{t+h}. This would mean that commodity prices do not affect macro variables contemporaneously. We believe this is not a realistic assumption. Unreported results show that prediction accuracy decreases when using factors contemporaneously, probably because equation (4) suffers from a simultaneity bias.

gorithms to compute the entire solution path, permitting the application of a wide variety of convex and nonconvex penalties.

3.1 Sparse penalized quasi-likelihood estimation

Our goal is to extract the common factors from the observations by using estimates of the model. To this end, we consider a two-step estimation approach. In the first step, loadings and approximate factors are computed using a penalized quasi-maximum likelihood approach. In the second step, we recover the factor dynamics using approximate factors.

Following Doz et al. (2012), a quasi log-likelihood objective function is obtained by noting that the observable vector x_t follows approximately a multivariate normal distribution with variance-covariance matrix $\Sigma = \Lambda\Lambda' + \Psi$. This quasi log-likelihood function is, up to an additive constant not depending on parameters

$$\ell(\Lambda, \Psi) = -\frac{T}{2}\left[\log|\Lambda\Lambda' + \Psi| + \mathrm{tr}\{(\Lambda\Lambda' + \Psi)^{-1}\widehat{\Sigma}\}\right], \tag{6}$$

where $\widehat{\Sigma}$ is the sample variance-covariance matrix. Then estimates of Λ and Ψ are found by solving $\partial\ell(\Lambda, \Psi)/\partial\Lambda = 0$ and $\partial\ell(\Lambda, \Psi)/\partial\Lambda = 0$. Unfortunately, this cannot be done without further restrictions since the sample size T is smaller than the number of parameters. Thus, following Hirose and Yamamoto (2015), the estimates $\widehat{\Lambda}_{\rho,\gamma}$ and $\widehat{\Psi}_{\rho,\gamma}$ are obtained by maximizing the penalized quasi log-likelihood function

$$\ell_{\rho,\gamma}(\Lambda, \Psi) = \ell(\Lambda, \Psi) - T\sum_{i=1}^{n}\sum_{j=1}^{r} \mathrm{pen}_{\rho,\gamma}(|\Lambda_{ij}|), \tag{7}$$

where $\rho > 0$ and $\gamma > 0$ are regularization parameter. Here pen$(\cdot)$ is a penalty with amount of shrinkage controlled by ρ and γ. Particularly, for smaller values of ρ, we have greater shrinkage, meaning that more elements of the loading matrix Λ become zero.

The estimator of the common factors is given by

$$\widehat{f}_t = (\widehat{\Lambda}_{\rho,\gamma}\widehat{\Psi}^{-1}_{\rho,\gamma}\widehat{\Lambda}_{\rho,\gamma})^{-1}\widehat{\Lambda}_{\rho,\gamma}\widehat{\Psi}^{-1}_{\rho,\gamma}x_t, \quad t = 1, \ldots, T. \tag{8}$$

The VAR model in (2) and the bridge equation (5) are then subsequently estimated using the approximated factors $\widehat{f}_1, \ldots, \widehat{f}_T$.

In this paper, we focus on the following popular nonconvex penalties:

- The smoothly clipped absolute deviation (SCAD) penalty defined by

$$\mathrm{pen}_{\rho,\gamma}(z) = \begin{cases} \rho z, & z \leq \rho \\ \dfrac{\gamma\rho z - (z^2 + \rho^2)^2}{\gamma - 1}, & \rho \leq z \leq, \gamma\rho \\ \dfrac{\gamma^2(\gamma^2 - 1)}{2(\gamma - 1)}, & z > \gamma\rho. \end{cases}$$

- The minimax convex penalty (MC+) defined by

$$\text{pen}_{\rho,\gamma}(z) = \begin{cases} \rho z - \dfrac{z^2}{2\gamma}, & z \leq \rho\gamma, \\ \dfrac{1}{2}\gamma\rho^2, & z > \gamma\lambda. \end{cases}$$

  For each value of $\rho > 0$, $\gamma \to \infty$ yields the soft threshold operator (i.e., the Lasso penalty), whilst $\gamma \to 1^+$ produces hard threshold operator.

The maximization of the penalized likelihood is computed efficiently through a generalized Expectation-Maximization (EM) algorithm. Given the number of latent factors $r \geq 1$, and a grid of choices for the tuning parameters γ and ρ, estimated loading matrices are computed in the maximization step through the coordinate-descent approach described in Hirose and Yamamoto (2015). Estimates in this paper are obtained using the R package `fanc` (Hirose et al., 2016).

For our empirical analyses, it is important to select the appropriate value of the regularization parameters ρ and γ, as well as the unknown number of latent factors r. The selection of the triple (ρ, γ, r) can be viewed as a model selection. To this end, we use the Bayesian information criterion (BIC)

$$BIC(\rho, \lambda, r) = -2\ell_{\rho,\gamma}(\widehat{\Lambda}_{\rho,\gamma}, \widehat{\Psi}_{\rho,\gamma}) + \log(T) \times \text{df}(r, n), \tag{9}$$

where $\text{df}(r, n) = rn + r(r + 1)/2$ is an estimate of the degrees of freedom, i.e. the number of effective parameters. Note that our estimate for the degrees of freedom is naive in the sense that we are ignoring the constraints on certain loadings to be zero; thus, we have no guarantee that $\text{df}(r, n)$ results in model selection consistency. On the other hand, estimation of degrees of freedom in high-dimensional factor analysis is an ongoing research topic and more theory would be needed in this context. In other works involving penalized likelihood approaches, the number of effective parameters is often approximated by taking the number of nonzero elements in $\widehat{\Lambda}_{\rho,\gamma}$. By doing this, however, we would under-estimate the effective degrees of freedom of our model since we would treat the selected model as the true model, thus ignoring completely the uncertainty related to the model selection process. Following the Generalized information criterion approach of Konishi and Kitagawa (1996), we also estimated directly the model-selection bias by non-parametric bootstrap for the log-likelihood $\ell_{\rho,\gamma}(\widehat{\Lambda}_{\rho,\gamma}, \widehat{\Psi}_{\rho,\gamma})$, finding models with complexity similar to those selected by our naive BIC approach.

4 Empirical analysis

This section provides the results of the application of the proposed dynamic factor models to predict energy commodity prices. In the first subsection, we focus on in-sample results, in particular evaluating factor loadings, sparsity and factor dynamics. The second subsection provides details of the forecast exercises and results.

4.1 In-sample results

Our in-sample analysis focuses on the estimation via penalized maximum likelihood of factors f_t and factor loadings Λ in equation (1) when applied to the updated global VAR dataset.

The first step is the estimation of the number of factors. We apply the BIC selection in equation (3.1) resulting in $r = 8$ factors. In particular, the BIC value steadily declines up to 8 factors, then it stabilizes before increasing again when $r > 10$. The selected number of factors partly confirms previous findings. Based on US data, Ludvigson and Ng (2009), Stock and Watson (2012) and Casarin et al. (2015) find optimal number of factors equal to 7, 5 and 7, respectively.

The selected tuning parameters are $\rho = 0.14$ and $\gamma = 1.01$ essentially corresponding to the hard threshold operator and indicate a high level of sparsity in the factor loadings. Indeed, we recall that Λ is a (174×8) matrix and therefore it requires the estimation of 1392 parameters. Thus, estimation would be imprecise without imposing sparsity. Moreover, it could not be realistic imposing that all variables from all 33 countries contribute to all the factors. Tables 1 and 4 summarize the composition of the estimated factor loadings. In particular, Table 1 reports the proportion of times a certain variable is included in constructing factors. A variable i has a significant contribution to factor j if the corresponding factor loading estimate $\widehat{\Lambda}_{ij}$ is different from zero, where $\widehat{\Lambda}$ is the estimated loading matrix. A value of 1 means that all variables associated with a specific group are included; a value of 0 indicates not variables are included. Table 4 provides detailed results. Table 1 organizes the results in two groups: by variables and by geographical regions.

When grouping across all variables (column All), we find that from 30% to 40% of the estimated loadings were exactly zero. This is a substantial sparsity level, indicating that not all variables are useful for all factors. On the other hand, we found a large heterogeneity across factors. All 8 factors load on most of the real GDP (RGDP) variables, with a minimum of 7% of the real variables restricted to zero for the eight factor and a maximum of 21% zero restrictions for the seventh factor. CPI variables are important for the third and, above all, the sixth factors. The seventh factor is an equity price factor (nomEQ) with no sparsity, whether the second factor loads mainly on the nominal exchange rate (Fxdol). Factor three loads with no sparsity on short interest rates (Rshort), but this variable is also relevant for the first, the fifth and the sixth factors, supporting the connections of interest rates to many variables, in particular CPI (sixth factors). Finally, all long interest rates (Rlong) variables are included to compute the eight factor, and most of them in factors one, three, four, five and six.

As second evidence we classify results in geographical regions. Following Bjørnland et al. (2017), we divide countries into four main continents: North America, South America, Europe and Asia.[5] The heterogeneity is very large, with only the third factor loading on all South American variables, and from 40% to 25% of factor estimates are in most cases restricted to zero. Therefore, our evidence indicates that factors are strongly connected to specific variables and not geographical regions, providing supportive evidence of the importance of the global database.

Figure 2 shows the estimated eight factors. We note specific patterns of the factors that can be explained by the factor loadings. For example, the sixth factor is the one that resembles a global inflation factor. It shows large values in the early part of the sample, then it decreases over the '80s, stabilizes in the '90s and beginning of 2000 before becoming negative starting from the US financial crises. The seventh factor captures equity collapses in October 1987, for the Gulf War (1990-1991), around the Asian financial crisis (1997-1998), associated to the burst of the internet bubble (2000), and the US financial crisis (2008). It is also the more volatile of all factors. Factor three proxies the declines of the short interest rates over the sample. Also, there is not a clear output factor because most factors upload on RGDP variables and output growth has been quite heterogeneous across countries in our sample. Finally, Table 2 provides the estimate of the matrix Q in equation (2), supporting the assumption of a full variance-covariance residual matrix.

4.2 Forecasting results

4.2.1 Alternative time-series models

We compare the dynamic factor augmented AR and VAR models, DFM-AR in (1)-(2)-(4) and DFM-VAR in (1)-(2)-(5) to three standard models usually applied when investigating energy commodity price predictability, see Alquist et al. (2013).

The benchmark model is the random walk model:

$$y_{t+h,s} = y_{t,s} + e_{t+h,s}. \tag{10}$$

The model assumes that the most accurate prediction for future values of variables $y_{t+h,s}$ is the last available price $y_{t,s}$.

The second model is the (parsimonious) univariate asutoregressive (AR) specification, whereas the errors are assumed to be normally distributed with zero mean and σ_h^2 variance. The autoregressive model can be written as follows

$$y_{t,s} = \sum_{l=1}^{p} \phi_l y_{t-l,s} + e_{t,s}. \tag{11}$$

[5] Australia, New Zealand and South Africa are grouped in the Asia continent to avoid groups with few variables. We also note that not all variables are available for all countries, in particular, equity premium and long interest rates are missing for some of the Asian and South American countries.

To account for the long autoregressive lag properties of energy commodity prices, see Kilian (2009), we fix p=8 (two years) and select with the AIC criterion the optimal p lag for each series and for each vintage.

The third model is the vector autoregressive (VAR) model. Let $y_{t+h} = (y_{t+h,1}, \ldots, y_{t+h,3})'$ denote the (3×1) vector of energy commodity prices. The VAR model of order p is formulated as follows:

$$y_{t+h} = \sum_{l=1}^{p} \Phi_l y_{t+h-l} + e_t, \tag{12}$$

where Φ_l is the (3×3) matrix of autoregressive coefficients. The vector of errors e_t is assumed to be serially uncorrelated and normally distributed with zero mean and a full covariance matrix Σ. As for the AR model, we choose optimal p via AIC for each vintage.

4.2.2 Machine learning methods

Alternative methods to our factor augmented sparse autoregressive model when the number of regressors is larger than the number of observations rely on reducing the number of estimated parameters in the model by penalizing the objective function to obtain more frugal characteristics. We consider four different methods: stepwise regression, random forest, elastic net and LASSO.

Stepwise regression The stepwise regression (SR) is a method of fitting regression models in which the choice of predictive variables is carried out by an automatic procedure. In each step, a variable is considered for addition to or subtraction from the set of explanatory variables based on some pre-specified criterion. We apply the Akaike information criterion. We start with no predictors and let the algorithm searching for the best set of predictors.

Random forests Random forests (RF) or random decision forests are an ensemble learning method that operates by constructing a multitude of decision trees at training time and outputting the class that is the mode of the classes or mean prediction (regression) of the individual trees. It is a variation of a procedure known as bootstrap aggregation that involves taking random samples from the dataset against which to evaluate the model. A sequence of steps is generated and, at each step, the residuals from the preceding step are sorted into different "branches" based on one or a set of the predictor variables. This sequential branching slices the space of predictors into rectangular partitions and approximates the unknown function with the average value of the outcome variable within each partition. Then, it combines forecasts from many different trees.

Elastic net and LASSO multiple regression Different penalty functions of equation 7 include elastic net (EN) and LASSO penalties. The general form is specified as:

$$\text{pen}_{\lambda,\rho}(\theta) = \lambda(1-\rho)\sum_{j=1}^{P}|\theta_j| + \frac{1}{2}\lambda\rho\sum_{j=1}^{P}\theta_j^2 \tag{13}$$

where $\theta = (\theta_1, \cdot, \theta_P)'$ is the set of parameters in the regression models. If there is no penalty term, the process is ordinary least squares. If $\rho = 0$, it is called the LASSO, or "Least Absolute Shrinkage and Selection Operator". If $0 < \rho < 1$ is estimated, the model is labelled elastic net. If $\rho = 1$, there is only the quadratic constraint and the process corresponds to the Ridge regression. We apply elastic net and LASSO regressions.[6]

Elastic net starts with the sum of squared residuals and combines the LASSO regression Penalty with the ridge regression penalty. Each penalty gets its own λ. Cross-validation on different combinations of the two λ's is tied to find the best values. Elastic Net penalty is especially good at dealing with situations in which there are correlations between the parameters. The elastic net groups and shrinks the parameters associated with the correlated variables and leaves them in the equation or removes them all at once.

LASSO regression is considered to work better when the model contains variables that might be considered as non-meaningful. The penalty function will eliminate them setting coefficients equal to zero creating a simpler model. For this reason, LASSO can also be considered as a variable selection method.

4.2.3 Forecast evaluation

We compare the different models in predicting the three energy commodity (logarithmic) prices for four horizons, h =1, 2, 3 and 4 quarters ahead. We assess the goodness of our forecasts using the mean square forecasting errors (MSFEs) for each forecast horizon h. The MSFE for $h = 1, \ldots, 4$, model i= RW, AR, VAR, DFM-AR, DFM-VAR, $s = \{\text{coal}, \text{gas}, \text{oil}\}$ is computed as

$$\text{MSFE}_{h,i,s} = \frac{1}{T-R}\sum_{t=R}^{T-1}\left(\widehat{y}_{t+h,i,s} - y_{t+h,s}\right)^2, \tag{14}$$

where T is the number of observations, R is the initial in-sample period to compute the first out-of-sample forecasts, $y_{t+h,s}$ is the realized value of the variable s at time $t+h$, and $\widehat{y}_{t+h,i,s}$ is the model i forecast of $y_{t+h,s}$ made at time t.

In addition, to provide a rough gauge of whether the differences in forecast accuracy based on square forecasting errors are significant, we apply the Model Confidence Set procedure of Hansen et al. (2011) across models for a fixed variable s and a fixed horizon h to jointly compare their predictive power. We implemented the MCS procedure with $T_{max,\mathcal{M}}$ test (Hansen et al., 2011, p. 465) at the $\alpha = 0.15$ significance level, using the `R` function `MCSprocedure` within the package `MCS` in

[6] We have also tried ridge regression, but results were inferior to elastic net and LASSO.

Bernardi and Catania (2018).

4.2.4 Results

We split the full quarterly sample 1979Q1-2018Q4 into two periods: an initial in-sample period 1979Q1-1999Q4 and an out-of-sample (OOS) period 2000Q1-2018Q4. We use an expanding recursive window to estimate the models and produce 76 forecasts (2000Q1-2018Q4) for each of the energy commodity prices by extending the in-sample period with a new observation at each step. For each of the 76 OOS values, we produce from 1- to 4-step ahead forecasts using several different models described in the previous section. Models are re-estimated for each one of the 76 vintages.

Table 3 reports the OOS forecasting results for the different individual models when predicting the three energy commodity prices at the four horizons we consider. In the second row, the MSFE for the benchmark RW model is reported; on the contrary ratios between the MSFEs of the AR, VAR, SR, RF, EL, LASSO, FDM-AR and DFM-VAR models and the MSFE of the RW model are reported in the other rows. Values less than 1 indicates that forecasts from the current model are more accurate than forecasts of the RW model.

We find three clear results. The first one is that the benchmark model RW is difficult to beat, as documented in many studies on oil price predictability, see e.g. Alquist et al. (2013) and Baumesteir and Kilian (2015). We generalize the finding to other energy (global) commodity prices.

Second, both our models offer larger predictability relative to the RW. The DFM-AR provides the lowest MSFE at 1-quarter ahead horizon for coal and gas prices, the DFM-VAR at 1-quarter ahead horizon for oil prices. Moreover, both models also do very well when predicting gas prices at longer horizons with MSFE reductions larger than 20% relative to the benchmark when using the DFM-VAR model. Also, the DFM-AR and the DFM-VAR are the only models always included in the model confidence set in all twelve forecasting exercises we consider. Studying predictability over time, we notice that on average our model class performs well both in periods of price increase such as the first part of 2000 and in the price drop related to the US financial crisis. It is a bit less accurate in the recovery phase, but it provides new gains in the last part of the sample, associated with a new period of high volatility, see Figure 1. When comparing the two models, the DFM-VAR does better for gas prices and DFM-AR when predicting coal and oil. In the letter case, it is not included in the model confidence set at any horizon. This results in hints to the different behaviour of gas prices that respond both to gas-market fundamentals and too other commodity energy markets. Recent evidence (Hulshof et al., 2016) show that supply and demand fundamentals are important for gas price determination. However, gas markets are less liquid than oil and coal markets, frictions exist and these frictions increase predictability. Our global factors and their jointly modelling capture movements in several specific variables, and they help to calculate these frictions.

Finally, the other four different types of machine learning methods, stepwise regression, random forest, elastic net and LASSO, do not offer similar gains. In almost all the twelve cases we study, none of them provides a smaller MSFE than the RW. Only random forest is included in the model set for longer horizons. We interpret this evidence as an indication that the predictability is driven not only by the use of a large database but mainly by the factor structure when accounting for sparsity. The VAR model also performs very poorly.

5 Conclusion

This paper proposes the use of dynamic factor augmented models based on a penalized maximum likelihood approach to forecast oil, gas and coal commodity prices. The nature of our approach is global since our model is constructed using an updated version of the Global VAR dataset proposed by Mohaddes and Raissi (2018), which includes important information from the largest 33 economies globally accounting for more than 80% of the global economy. Specifically, we consider measures of output, inflation, interest rates, equity markets and exchange rates of these countries as well as data for other non-energy commodities (metal and agricultural commodity prices). We estimate a factor model using a penalized likelihood approach that enables shrinking the elements of the loading parameters to zero, thus resulting in sparse factor loadings. The sparse estimates are shown to improve forecasts compared to standard methods and enhance the interpretation of the role played by observable variables in forming latent factors. Interestingly, each factor mostly captures separate groups of economic information. Among the estimated factors, we identify global inflation, interest rates and equity premium factors.

For forecasting, we introduce factor augmented AR and VAR equations relating the latent factors to energy commodity prices. We carried out out-of-sample testing of the accuracy of our forecasting by splitting the original sample into an initial sample period (1979Q1-199Q4) and out-of-sample period (2000Q1-2018Q4). Using a moving forecasting window approach, we estimated the models and produced 76 forecasts (1 to 4 periods ahead) for each energy commodity prices (oil, gas and coal). Our main results indicate that the models which provide the largest predictability gains relative to the RW are our dynamic factor models. Precisely, our models provide more accurate forecasts than the benchmark RW model for 1-quarter ahead for all energy commodities. When predicting gas prices, the factor augmented VAR model gives the lowest MSFE values for 1 to 4 quarters ahead. Why predictability is higher for gas prices than other energy commodities? A possible answer to this question may be related to higher inefficiencies in the gas market, compared to the most efficient and liquid global oil and coal markets. However, this issue is beyond the scope of this paper and further research is needed. Other four different types of machine learning methods, precisely stepwise regression, random forest, elastic net and LASSO, do not offer similar gains. Therefore, the increased predictability is not only driven by the use of an extended database with more variables but also by the estimation of a parsimonious model by taking into account the sparse structure of the latent factors.

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Table 1: Factor loadings summary

Factors	1	2	3	4	5	6	7	8
				Variable groups				
All	0.644	0.592	0.713	0.632	0.707	0.678	0.672	0.609
RGDP	0.909	0.818	0.848	0.909	0.879	0.848	0.788	0.939
CPI	0.606	0.364	0.727	0.424	0.606	0.848	0.424	0.485
nomEQ	0.192	0.269	0.192	0.423	0.462	0.385	1.000	0.154
Fxdol	0.406	0.875	0.563	0.531	0.594	0.250	0.656	0.531
Rshort	0.875	0.531	1.000	0.688	0.844	0.844	0.625	0.625
Rlong	0.889	0.667	0.944	0.889	0.889	0.944	0.556	1.000
				Geographical regions				
North America	0.727	0.455	0.636	0.636	0.546	0.546	0.818	0.727
South America	0.810	0.524	1.000	0.524	0.714	0.381	0.524	0.619
Europe	0.662	0.662	0.676	0.662	0.732	0.775	0.747	0.648
Asia	0.563	0.563	0.676	0.634	0.704	0.690	0.620	0.549

Notes: Percentage of significant variables included in constructing factors. A variable (RDGP, CPI, etc.) has a significant contribution to a given factor j, with $j = 1, \ldots, 8$, if $\widehat{\Lambda}_{ij} \neq 0$ where $\widehat{\Lambda}_{ij}$ is the (ij)th element of the estimated loading matrix $\widehat{\Lambda}$. A value of 1 indicates all variables associated to a specific group are included; a value of 0 indicates not variables are included.

Table 2: Factor residual correlations

Factors	1	2	3	4	5	6	7	8
1	1.140	-0.051	-0.153	-0.052	0.330	-0.051	0.126	0.184
2	-0.051	1.312	-0.121	-0.159	0.164	0.288	-0.283	0.059
3	-0.153	-0.121	1.071	0.016	-0.037	0.128	0.099	0.207
4	-0.052	-0.159	0.016	1.330	-0.618	0.157	-0.175	-0.228
5	0.330	0.164	-0.037	-0.618	1.524	0.025	0.127	-0.181
6	-0.051	0.288	0.128	0.157	0.025	1.169	-0.019	0.277
7	0.126	-0.283	0.099	-0.175	0.127	-0.019	1.257	0.163
8	0.184	0.059	0.207	-0.228	-0.181	0.277	0.163	1.354

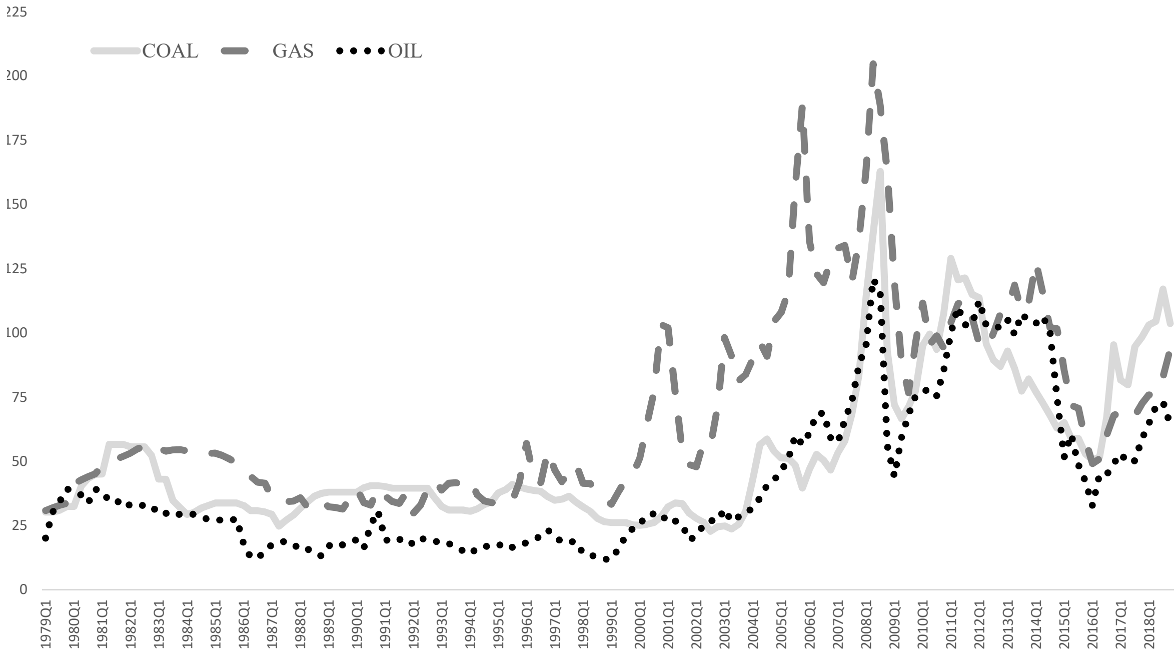
Notes: Estimates of the covariance matrix Q in equation (2).

Table 3: MSFE

	h=1	h=2	h=3	h=4
		COAL		
RW	0.020*	0.056*	0.093*	0.127*
AR	1.208*	1.245*	1.299*	1.345*
VAR	1.200	1.441	1.608	1.676
SR	1.584	2.944	1.837	1.030
RF	1.443	1.517*	1.307*	1.169*
EN	2.644	2.260	1.456	1.213
LASSO	2.761	2.260	1.456	1.205
DFM-AR	0.947*	1.044*	1.125*	1.165*
DFM-VAR	0.975*	1.101*	1.191*	1.227*
		GAS		
RW	0.022*	0.061	0.097*	0.123*
AR	1.239*	1.270	1.261*	1.346*
VAR	3.022	2.789	2.750	3.106
SR	1.355	2.196	1.870	1.501
RF	1.217	1.426	0.992*	0.947*
EN	2.474	2.650	1.753	1.444
LASSO	2.474	2.650	1.753	1.444
DFM-AR	0.945*	0.844*	0.846*	0.852*
DFM-VAR	0.922*	0.772*	0.801*	0.902*
		OIL		
RW	0.024*	0.057*	0.084*	0.106*
AR	1.355	1.473*	1.557*	1.665*
VAR	1.322	1.781*	2.024*	2.039*
SR	1.971	2.153	2.199	2.382
RF	1.515	1.640*	1.445*	1.419*
EN	2.719	1.707	1.523	1.628
LASSO	2.811	1.714	1.642	1.530
DFM-AR	0.956*	1.069*	1.099*	1.145*
DFM-VAR	0.996*	1.158*	1.213*	1.260*

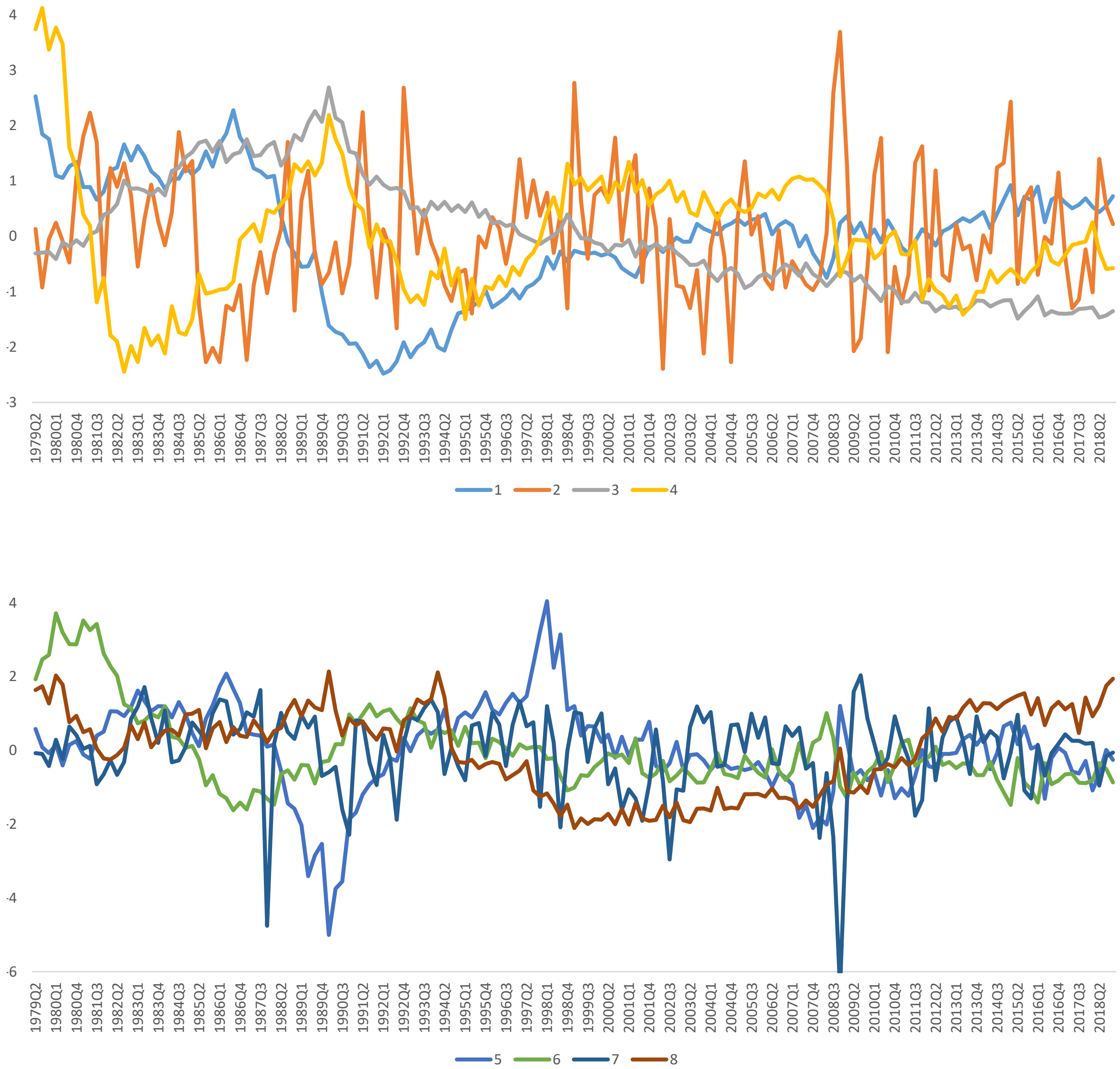
Notes: Notes: The Table reports mean square forecast error (MSFE) for different models and four different forecasting horizons, h=1, 2, 3 and 4 quarters ahead. For the RW baseline models, the Table reports MSFEs; for all other models, table reports ratio between MSFE of current model and MSFE of RW benchmark. Entries less than 1 indicate that forecasts from current model are more accurate than forecasts from corresponding baseline model. A * indicates the models that belong to the Superior Set of Models delivered by the Model Confidence Set procedure for each horizon h at confidence level 15%.

Figure 1: Energy Commodity Prices



Notes: The Figure shows coal, gas and oil prices.

Figure 2: Factors



Notes: The Figure shows the first four estimated factors in panel a) and the last four estimated factors in panel b).

Appendix

This appendix reports details results for factor loadings, displaying results for each variable and each country.

Table 4: Factor loadings

		Factors							
Series	Countries	1	2	3	4	5	6	7	8
	'Argentina'	0.648	0.012	0.482		0.053	0.522	0.006	0.003
	'Australia'	0.734		-0.160	0.148	0.034	0.528	-0.026	0.000
	'Austria'	0.310			0.484	-0.079	0.599		-0.266
	'Belgium'	0.317	0.011	0.228	0.423	0.085	0.700		-0.151
	'Brazil'	0.553	0.013	0.639	-0.012	-0.029	0.414	-0.002	0.114
	'Canada'	0.711		-0.246	0.359	-0.027	0.345	-0.047	0.015
	'China '	0.035	-0.020	-0.704	-0.217	-0.332	-0.446	-0.058	0.194
	'Chile'	0.624	-0.002	0.293	0.027		0.625	0.001	0.083
	'Finland'	0.843	0.015			-0.489			-0.501
	'France'	0.467	-0.012	-0.086	0.395	0.052	0.675	-0.034	0.000
	'Germany'		0.018	0.387	0.617	0.039	0.329	-0.077	0.284
	'India'	0.605	-0.008	0.507	-0.006	-0.177	0.500	0.009	0.078
	'Indonesia'			0.711	0.088	0.252	0.382	0.051	0.185
	'Italy'	0.380		0.551	0.222	0.127	0.589	0.007	-0.001
	'Japan'	-0.375	-0.011	-0.655	0.001	-0.209	-0.495	-0.007	0.013
	'Korea'	-0.322	-0.030	-0.707	0.173	-0.101	-0.517	-0.011	0.172
RGDP	'Malaysia'	-0.308	-0.010	-0.816	-0.057	-0.109	-0.384	-0.014	0.055
	'Mexico'	0.503	-0.017	0.475	-0.011	0.079	0.610		0.059
	'Netherlands'	-0.281	0.044	-0.425	0.612			-0.058	-0.232
	'Norway'	0.617	-0.026		0.258	0.183	0.598		-0.138
	NewZealand'	0.733	-0.017		0.047	0.040	0.578		-0.003
	'Peru'	0.683	0.011	0.554	-0.026		0.396	-0.003	0.072
	Philippines'	0.543	0.002	0.370	0.004	0.007	0.592	-0.001	0.157
	SouthAfrica'	0.472	0.007	0.630	0.024	0.037	0.530	0.015	0.012
	SaudiArabia'	-0.075	-0.016	-0.880	-0.061	-0.274	-0.187	-0.038	0.055
	'Singapore'	-0.256		-0.766	-0.071	-0.172	-0.419	-0.014	-0.031
	'Spain'	0.516	0.002	0.375	0.219	0.018	0.647	-0.009	-0.013
	'Sweden'	0.820	0.001		0.116	-0.168	0.414	-0.047	0.126
	Switzerland'	0.529	0.008	-0.427		-0.387		-0.097	0.453
	'Thailand'		-0.189	-0.451	-0.466	-0.384		0.047	0.158
	'Turkey'	0.272	0.004	0.725	0.041	0.109	0.410	0.020	0.159
	'UK'	0.676	-0.005	0.329	0.341	-0.020	0.440	0.001	0.007
	'USA'	0.510	-0.032	-0.333	0.619				-0.028
	'Argentina'	0.258	0.045	0.546		-0.293			0.160
	'Australia'	0.520	-0.029	0.448		-0.152	0.333		-0.024
CPI	'Austria'					-0.152	0.474	0.162	
	'Belgium'	0.337		0.145	-0.238	-0.254	0.672	0.040	-0.152

Notes: The Table reports factor loading estimates Λ in equation (1). Empty cells indicate the corresponding λ_{ij} are shrink to zero.

Series	Countries	Factors 1	2	3	4	5	6	7	8
CPI	'Brazil'	-0.259		0.549	0.022	-0.041			0.331
	'Canada'	0.408		0.257	0.006	-0.212	0.582	0.059	-0.030
	'China '	-0.295		0.222			-0.147	0.014	0.163
	'Chile'	0.221		0.545	0.152		0.275	-0.027	0.261
	'Finland'	0.422	0.046	0.360		-0.222	0.564	0.018	
	'France'	0.447		0.222		-0.068	0.742	0.044	-0.040
	'Germany'					-0.190	0.619	0.092	
	'India'		-0.150				0.239		
	'Indonesia'				0.314	0.354			
	'Italy'	0.370	-0.015	0.345	0.012		0.733	0.008	
	'Japan'				0.193		0.383		0.282
	'Korea'			0.040	0.411	0.045	0.529	0.088	0.143
	'Malaysia'						0.462	0.105	
	'Mexico'	0.438		0.722	-0.171	0.074			
	'Netherlands'						0.603		-0.060
	'Norway'	0.457	0.067	0.353			0.378		
	'New Zealand'	0.547	-0.055	0.418	0.074		0.367		
	'Peru'			0.558		-0.372			0.155
	'Philippines'	0.165	0.080	0.343			0.289		
	'South Africa'	0.158	-0.039	0.596			0.208	-0.101	0.192
	'Saudi Arabia'			-0.245		-0.294	0.148		
	'Singapore'					-0.228	0.515		
	'Spain'	0.308	-0.051	0.348			0.555		
	'Sweden'	0.298		0.409		-0.249	0.503		0.037
	'Switzerland'			0.327		-0.217	0.463		
	'Thailand'		-0.047		0.316		0.512	0.192	
	'Turkey'	-0.414	0.008	0.433	0.420	0.548	0.096		0.199
	'UK'	0.204		0.184	0.298	-0.002	0.495		0.186
	'USA'	0.211		0.144	0.309	-0.080	0.532	0.162	
nomEQ	'Argentina'	0.199		0.386		-0.240		0.235	
	'Australia'				0.201			0.721	
	'Austria'		-0.094			-0.152		0.606	
	'Belgium'		-0.078		-0.002	0.049		0.711	
	'Canada'				0.108			0.610	-0.018
	'Chile'		-0.034	0.257	0.326			0.259	0.161
	'Finland'		0.030		0.000	0.190		0.543	
	'France'				0.096	0.013		0.775	
	'Germany'							0.510	
	'India'					-0.168		0.448	
	'Italy'				0.146	0.119	0.095	0.604	
	'Japan'	0.194						0.567	
	'Korea'						-0.151	0.402	
	'Malaysia'				0.206	-0.023		0.470	
	'Netherlands'	-0.035	0.079		-0.005		0.030	0.825	
	'Norway'				0.133	-0.025		0.724	
	'New Zealand'	0.057					0.112	0.521	
	'Philippines'			0.207			-0.250	0.409	

Notes: Continue Table 4.

		Factors							
Series	Countries	1	2	3	4	5	6	7	8
	'South Africa'				0.157			0.584	
	'Singapore'				0.219	-0.032	0.005	0.680	
	'Spain'			0.076		0.016	-0.020	0.682	-0.045
nomEQ	'Sweden'		0.095				0.016	0.749	-0.014
	'Switzerland'	-0.143						0.782	
	'Thailand'					-0.179	-0.147	0.453	
	'UK'		0.010	0.007			0.075	0.774	
	'USA'							0.550	
	'Argentina'	0.329	0.107	0.504	-0.163	-0.327			
	'Australia'		0.271			0.193		-0.344	0.188
	'Austria'	-0.012	0.941	-0.032	0.043	-0.003	-0.073	0.327	-0.006
	'Belgium'	0.040	0.921	-0.016	0.010	-0.001		0.307	-0.023
	'Brazil'	-0.301	0.013	0.530				-0.011	0.356
	'Canada'	-0.053	0.256			0.250		-0.365	0.252
	'China '			0.240	-0.029				0.156
	'Chile'	0.261	0.202	0.368	-0.221			-0.287	0.000
	'Finland'	-0.038	0.854				-0.029	0.260	0.009
	'France'	0.055	0.920	0.001				0.300	-0.019
	'Germany'	-0.007	0.939	-0.034	0.054		-0.062	0.324	-0.001
	'India'	-0.146	0.236	0.177	-0.119			-0.180	0.158
	'Indonesia'		0.069			0.171			
	'Italy'		0.872	0.018	-0.003		0.026	0.262	0.008
	'Japan'		0.341	-0.148	0.000			0.228	
Fxdol	'Korea'		0.000			0.113	0.204	-0.247	
	'Malaysia'		0.258			0.189		0.000	
	'Mexico'	0.348	-0.029	0.475	-0.287	0.106		-0.255	
	'Netherlands'		0.938	-0.033	0.059	0.005	-0.056	0.323	
	'Norway'		0.596			0.112			0.107
	'New Zealand'		0.388			0.026		-0.169	0.100
	'Peru'		0.009	0.476		-0.308		0.000	0.112
	'Philippines'			0.183	-0.004	0.234		-0.154	
	'South Africa'		0.347	0.057	-0.165			-0.162	
	'Saudi Arabia'	0.326		0.311	-0.218				
	'Singapore'		0.493			0.232	-0.150		
	'Spain'		0.848		-0.025	0.043	0.048	0.282	0.003
	'Sweden'		0.601		-0.053	0.030			0.054
	'Switzerland'		0.672					0.269	
	'Thailand'		0.235			0.155			
	'Turkey'	-0.263	0.237	0.223	0.294	0.462		-0.033	0.278
	'UK'		0.535		-0.132				
	'Argentina'			0.154		-0.144			
Rshort	'Australia'	0.338		0.872	-0.054	-0.224	0.169		-0.051
	'Austria'	-0.043	0.007	0.639	0.037	-0.053	0.585	-0.067	-0.036
	'Belgium'	0.093	-0.001	0.634	-0.013		0.599	-0.051	0.008
	'Brazil'	-0.010		0.234		-0.229			

Notes: Continue Table 4.

Series	Countries	Factors 1	2	3	4	5	6	7	8
Rshort	'Canada'	0.326	0.019	0.600		-0.051	0.594	-0.031	0.000
	'China '	-0.371	-0.002	0.654		0.157	0.101	0.072	0.232
	'Chile'	0.234		0.548			0.516	-0.054	0.122
	'Finland'	0.187		0.812	-0.086	-0.114	0.378	-0.040	0.019
	'France'	0.296		0.640	-0.142	0.073	0.621	-0.026	-0.125
	'Germany'	-0.044	0.017	0.585	0.029	-0.065	0.653	-0.083	-0.071
	'India'	-0.412	-0.004	0.501	-0.029	0.420	0.138	0.061	0.386
	'Indonesia'	-0.166	0.047	0.253	0.284	0.512			
	'Italy'	0.214		0.710	-0.169	0.007	0.542	-0.016	0.001
	'Japan'	0.258		0.539	-0.019	-0.116	0.584	-0.005	0.170
	'Korea'	0.136	0.001	0.419	0.393	0.150	0.556	0.002	0.158
	'Malaysia'	0.239		0.691	-0.045	0.304	0.408	0.018	-0.028
	'Mexico'	0.379		0.845	-0.100	0.108	-0.090	-0.004	0.052
	'Netherlands'	0.000	-0.011	0.620	0.136		0.588	-0.077	
	'Norway'	0.248	0.036	0.867	0.000		0.235	-0.110	
	'New Zealand'	0.331	-0.029	0.868	0.077		0.065		
	'Peru'			0.313		-0.362			
	'Philippines'		0.030	0.798		0.067	0.172		
	'South Africa'	-0.248	0.060	0.699	-0.130	0.007	0.000		-0.055
	'Singapore'	0.225	0.020	0.495	0.000	0.155	0.660		
	'Spain'	0.146	0.019	0.758	-0.022	0.009	0.428		0.066
	'Sweden'	0.077		0.810	-0.107	-0.047	0.408	-0.023	0.000
	'Switzerland'	-0.395	-0.002	0.643	0.019	-0.197	0.186	-0.003	0.116
	'Thailand'	-0.010		0.587	0.000	0.277	0.518	0.020	0.040
	'Turkey'	-0.552		0.494	0.031	0.463	-0.013	0.000	-0.165
	'UK'	0.170		0.747	0.212	-0.009	0.421		
	'USA'	0.324	0.063	0.596	0.068	0.006	0.572	0.018	-0.102
Rlong	'Australia'	0.308	-0.006	0.860	-0.148	-0.102	0.373	0.006	-0.096
	'Austria'	0.057	-0.017	0.748	-0.032	0.033	0.571	-0.018	-0.191
	'Belgium'	0.222	-0.009	0.725	-0.145	-0.012	0.601	-0.016	-0.149
	'Canada'								0.143
	'France'	0.297		0.702	-0.158	-0.030	0.616	-0.021	-0.150
	'Germany'			0.732	-0.009	0.033	0.579	-0.002	-0.210
	'Italy'	0.167	-0.018	0.652	-0.209	-0.012	0.610	-0.005	-0.001
	'Japan'	0.262		0.621	-0.035	-0.087	0.618		0.040
	'Korea'	0.058		0.520	0.260	0.206	0.605		0.096
	'Netherlands'	0.071	-0.005	0.686	-0.012	0.018	0.644	-0.014	-0.185
	'Norway'	0.293		0.903	-0.076	-0.011	0.288	-0.021	-0.085
	'New Zealand'	0.395	-0.015	0.874	0.043	-0.011	0.203		-0.039
	'South Africa'	-0.233	0.040	0.831	0.002	0.233	-0.027		0.007
	'Spain'	0.184	-0.003	0.730	-0.129	-0.014	0.502		0.032
	'Sweden'	0.121	-0.021	0.828	-0.051	0.025	0.423	0.001	-0.052
	'Switzerland'	-0.152	-0.024	0.784		-0.024	0.436		-0.208
	'UK'	0.188	-0.007	0.726	0.036	0.027	0.549		-0.047
	'USA'	0.324	0.047	0.704	-0.092		0.562	0.007	-0.138

Notes: Continue Table 4.