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Equilibrium Asset Pricing in Directed Networks

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Abstract

Directed links in cash flow networks affect the cross-section of risk premia through three channels. In a tractable consumption-based equilibrium asset pricing model, we obtain closed-form solutions that disentangle these channels for arbitrary directed networks. First, shocks that can propagate through the economy command a higher market price of risk. Second, shock-receiving assets earn an extra premium since their valuation ratios drop upon shocks in connected assets. Third, a hedge effect pushes risk premia down: when a shock propagates through the economy, an asset that is unconnected becomes relatively more attractive and its valuation ratio increases.

Keywords: Directed cash flow networks, directed shocks, mutually exciting processes, recursive preferences

JEL: G12, D85

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1. Introduction

The existence of network linkages between firms, industries, or countries can turn microeconomic shocks into aggregate fluctuations. Taking this as given, the question arises what such a potential amplification implies for asset prices. In this paper, we develop a tractable consumption-based equilibrium asset pricing model that allows us to trace risk premia back to the core input of any network model, namely the individual entries of the connectivity matrix. The fact that links in a network usually have a direction, i.e., it makes a difference whether a link goes from node i to node j or the other way around, turns out to be of first-order importance for expected excess returns.

We propose an equilibrium asset pricing model, in which negative cash flow shocks in some assets can increase the probability of *subsequent* cash flow shocks in other assets.[1] The direction and the magnitude of this "timing of shocks" characterize the network in our model. Based on a series expansion of the closed-form solution of our model, we prove for arbitrary networks that directed links between cash flows affect the cross-section of risk premia through three channels: (i) Shocks that can propagate through the economy command a higher market price of risk ("spreading channel"). (ii) Shock-receiving assets earn an extra premium for spillover risk because their valuation ratios drop upon shocks in connected assets ("receiving channel"). (iii) A hedge effect pushes risk premia down: when a shock propagates through the economy, an asset that is unconnected becomes relatively more attractive and its valuation ratio increases ("hedging channel").[2] The first two channels increase risk premia, while the third one pushes them down, so that the overall impact of directed shock propagation on risk premia depends on the tradeoff between these channels.

We disentangle the three channels for arbitrary directed networks. However, the intuition behind them is more easily understood by a stylized example. Suppose there are three assets in the economy. Shocks can propagate from asset 1 to asset 2, but there are no other links in the economy. In particular, asset 3 is unconnected to the rest of the economy. The intuition behind the spreading channel is as follows. Shocks to the cash flow of asset 1 increase cash flow risk in the rest of the economy (here: asset 2). Hence, they are more systematic and carry a higher market price of risk than the cash flow shocks of assets 2 and 3 which cannot propagate. The more an asset loads on cash flow risk of asset 1, the higher

[1]We will use the term "asset" to refer to a node in the network throughout the paper. Of course, nodes can represent industries, countries, or any other economic unit.

[2]As we explain below, the hedging channel is different from the market clearing channel discussed, e.g., by Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2013), which generally increases risk premia of unconnected assets.

is thus its risk premium.

The other two channels build on the general intuition that in equilibrium all price-to-cash flow ratios in the economy respond to any cash flow shock that has the potential to propagate. In the example, the valuation ratio of asset 2 decreases upon a shock to the cash flow of asset 1 because the cash flow of asset 2 becomes riskier upon this shock. Shock-receiving assets (here: asset 2) thus load on systematic risk factors (here: cash flow risk of asset 1) and this constitutes the receiving channel outlined above.

The hedging channel can be illustrated through the unconnected asset 3. Upon a cash flow shock of asset 1, the price-dividend ratio of asset 3 *increases*. This is because the entire economy becomes riskier, but asset 3 is unaffected and therefore, in relative terms, less risky as compared to the economy as a whole. Put differently, asset 3 is the best hedging device against the propagation of shocks (here: shocks to the cash flow of asset 1). The positive price reaction upon cash flow shocks pushes risk premia down for hedge assets (here: asset 3).

As our main theorem for arbitrary directed networks reveals, each asset's risk premium is affected by all three channels in general networks. The spreading and the receiving channel are determined only by the direct linkages from and to a particular asset. In contrast, the hedging channel is driven by *all other* linkages in the network. This has three important implications. First, in a given network, the hedging channel operates through all assets except the ones which cannot spread their shocks anywhere. Second, it is not possible to construct a network in which the hedging channel is shut down completely. Third, the risk premium of an asset also depends on the existence of cash flow linkages in very remote or unconnected parts of the economy.

Our consumption-based equilibrium asset pricing model features an arbitrary number of assets whose cash flows are linked via self and mutually exciting jump processes, and a representative investor with recursive preferences. An initial negative cash flow shock of asset i increases the *probability* of future cash flow shocks to connected assets $j \neq i$ (and potentially also to i itself), but it is unknown when (and if at all) these shocks will *materialize*. The network thus manifests itself only indirectly via the dynamics of jump intensities as state variables, but not directly through contemporaneous shocks to the levels of several cash flows. Aggregate consumption is driven by all individual jumps, while a given jump affects the cash flow of only one asset at a time. The representative investor cares about the risk associated with future values of the state variables. Hence, the price-to-cash flow ratios of all assets will react to a jump in any individual cash flow that has the potential to propagate, and it is the structure of the network which determines the sign and the magnitude of these reactions.

Our shock propagation setup is motivated by the theoretical work of Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), and Herskovic (2018) who show how intersectoral input-output linkages may turn microeconomic shocks into aggregate fluctuations and macroeconomic tail risks. Moreover, the crucial feature of our model that cash flow shocks to one node in the network affect other nodes only with a certain time lag has also been documented empirically. In a recent paper, Carvalho, Nirei, Saito, and Tahbaz-Salehi (2020) provide rich empirical evidence for such a delayed propagation of cash flow shocks at the firm level in a natural experiment setting around the nuclear incident of Fukushima in 2011. They summarize the intuition behind their result as follows: "When faced with a supply-chain disruption, individual firms are unable to find suitable alternatives in order to completely insulate themselves from the shock (at least in the short run). This is consistent with an emerging literature [...] that emphasizes the importance of search frictions and relation-specific investments along supply chains." (p. 34). However, even though the cash flow shocks propagate with a time lag, equilibrium prices react immediately to any shock in the economy since markets are efficient. It is precisely this instantaneous reaction of prices to cash flow shocks propagating slowly over time that is key for the risk premia in our equilibrium model.

By combining the exponentially affine equilibrium framework of Eraker and Shaliastovich (2008) with mutually exciting processes introduced into finance by Aït-Sahalia, Cacho-Diaz, and Laeven (2015), our model inherits the following two important properties. First, mutually exciting processes naturally feature directed links, with a shock going from i to j, but not necessarily vice versa. Second, the model belongs to the exponentially affine class for which there is a well-developed solution theory, and thus it remains tractable with at least semi-closed form expressions for all equilibrium quantities. A series expansion allows us to rewrite the market prices of jump risk, jump exposures, and expected excess returns as functions of row and column sums of the connectivity matrix for *arbitrary directed networks*.

We contribute to the literature in various other ways. Most importantly, we are the first to trace risk premia back to individual entries of the connectivity matrix in an equilibrium setup. Moreover, we dissect the three channels. In particular, the hedging channel is novel. Through this channel network linkages can have a *negative* effect on expected excess returns, and this effect is not limited to unconnected assets, but also prevalent for connected assets. Finally, the model we propose is as complex as necessary, but as simple as possible to tease out the three channels, while still allowing for closed-form solutions for arbitrary directed networks. In particular, our three channels change signs or do not even exist in simpler models. A model with contemporaneous jumps instead of sequential jumps lacks the spreading channel and the hedging channel described above. In contrast, in our model with mutually exciting jumps, all risk premia depend on all entries of the connectivity matrix. Replacing recursive preferences by CRRA utility shuts down the spreading channel and switches the sign of the receiving channel.

Without specifically considering network linkages, Martin (2013) also presents general closed-form solutions for consumption-based asset pricing models with multiple risky assets.[3] His focus is on the market clearing channel arising from the assumption that aggregate consumption equals the sum of all individual cash flows. The market clearing channel implies that upon a negative shock to the cash flow of one asset, the price-dividend ratios of all other assets *decrease*. Relaxing the market clearing assumption allows us to highlight that the hedging channel we uncover is different from the market clearing channel in his paper. Moreover, it facilitates tracing equilibrium quantities back to the individual entries of the connectivity matrix. In our model, the price responses work through time variation in the probability of future cash flow shocks which depends on these individual entries of the connectivity matrix. In particular, the hedging channel leads to *positive* price reactions.

Several papers study the asset pricing implications of networks at the production level. Herskovic (2018) highlights the role of sparsity and concentration of an entire network for capturing aggregate risk. Gofman, Segal, and Wu (2018) determine a firm's vertical position in the supply chain and calculate a top-minus-bottom spread which they explain in a production economy with layer-specific capital. In an international context, Richmond (2019) relies on Katz centrality and finds that more central countries have lower interest rates and currency risk premia. The purely empirical papers by Ahern (2013) and Aobdia, Caskey, and Ozel (2014) link equity returns to trade flows between industries. However, none of these papers focus explicitly on the impact of directedness and thus our findings are novel to this literature.[4] Besides our paper, Buraschi and Tebaldi (2019) are the only ones who model cash flow networks. However, their focus is on systemic risk in banking networks and multiple equilibria.

Another strand of literature analyzes networks estimated from return data. An example

[3]This strand of literature started with the seminal papers by Dumas (1992) and Cochrane, Longstaff, and Santa-Clara (2008).

[4]Concerning production or supply chain networks, there is also a large strand of literature in economics, in which the authors do not focus on the asset pricing implications of network structures. Examples include, among others, Long and Plosser (1983), Gabaix (2011), Carvalho and Voigtländer (2015), Wu (2015), Acemoglu, Akcigit, and Kerr (2016), Barrot and Sauvagnat (2016), Wu (2016), Ozdagli and Weber (2019), and Tascherau-Dumouchel (2018). Carvalho (2014) provides an excellent review of this literature.

for such papers is Diebold and Yilmaz (2014). Many papers dealing with the measurement of systemic risk also follow this route, e.g., Billio, Getmansky, Lo, and Pelizzon (2012) and Demirer, Diebold, Liu, and Yilmaz (2017). The main difference between these papers and ours is that we model the underlying fundamentals, i.e., cash flows, and prices and returns are then endogenously determined in equilibrium.

Finally, Aït-Sahalia, Cacho-Diaz, and Laeven (2015) are the first to discuss the role of mutually exciting jumps in finance applications. The methodological framework of our equilibrium model goes back to the paper by Eraker and Shaliastovich (2008). Besides, there is an increasing literature about consumption-based asset pricing models with stochastic jump intensities in the endowment process. For instance, Wachter (2013) and Gabaix (2012) analyze the equity premium puzzle and the excess volatility puzzle in economies with stochastic intensities for rare consumption disasters, but do so in models with only one endowment stream, which obviously does not lend itself to any network applications.[5]

The paper is structured as follows. In Section 2, we present the model, explain the solution, and summarize our theoretical results for arbitrary directed network structures. In Section 3, we dissect the three channels in a stylized three asset economy and in an n-asset star network. We discuss the key assumptions underlying our model in Section 4. In Section 5, we illustrate that our model features a centrality premium. Section 6 concludes.

2. Model

2.1. Fundamental Dynamics

We assume a Lucas endowment economy. Log aggregate consumption $y_t \equiv \ln Y_t$ follows

$$dy_t \quad = \quad \mu\, dt + \sum_{j=1}^{n} K\, dN_{j,t},$$

[5]This framework is extended to a two-sector economy with jump intensities driven by correlated Brownian motions in Tsai and Wachter (2016) and towards CDS pricing in Seo and Wachter (2018). Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015) analyze defaultable bonds subject to contagion risk in a general equilibrium model. Nowotny (2011) investigates a one-sector economy with consumption following a self exciting process. Branger, Kraft, and Meinerding (2014) show that self exciting processes can endogenously evolve in a framework with learning about latent disaster intensities. A comprehensive summary of the disaster risk literature is provided by Tsai and Wachter (2015).

where μ is the constant drift rate and the $N_{j,t}$ $(j = 1, \dots, n)$ are self and mutually exciting jump processes with constant jump sizes $K < 0$.[6,7] Their stochastic jump intensities $\ell_{j,t}$ have dynamics

$$d\ell_{j,t} \;=\; \kappa \left(\bar{\ell}_j - \ell_{j,t}\right) dt + \sum_{i=1}^{n} \beta_{j,i}\, dN_{i,t}. \tag{1}$$

The coefficients $\beta_{j,i}$ represent discrete changes in $\ell_{j,t}$ induced by a jump in $N_{i,t}$. The parameters $\beta_{j,i}$, collected in what we call the "beta matrix" or the connectivity matrix, completely determine the structure of a given network.[8] To preclude negative intensities we assume $\beta_{j,i} \geq 0$ for all pairs (j, i).

There are n industries in the economy, indexed by i, with the following dynamics for log cash flows $y_{i,t}$:

$$dy_{i,t} \;=\; \mu_i\, dt + L\, dN_{i,t} \qquad (i = 1, \dots, n). \tag{2}$$

Thus, the level of industry i's cash flow is affected by the jump process N_i only.

Equations (1) and (2) formalize how the beta matrix gives rise to a dynamic shock propagation mechanism by which negative shocks to one cash flow stream can spread across the economy. With $\beta_{j,i} > 0$, a downward jump in cash flow i immediately increases the jump intensity of cash flow j by the amount $\beta_{j,i}$. Once the increased intensity $\ell_{j,t}$ indeed leads to a jump in cash flow j and there is a nonzero coefficient $\beta_{k,j}$, the initial shock is passed on to asset k and can in this way be propagated through the whole network. Note that our specification is general in the sense that it also allows for "feedback loops", i.e., depending on the structure of the network, an initial shock to node i can, after a number of intermediate steps, eventually reach node i itself again. Nevertheless, each jump only affects one cash flow directly, so that network connectivity is captured exclusively via linkages in the dynamics of

[6] We do not include diffusion terms in the dynamics of aggregate consumption for parsimony. One could of course generalize the model to incorporate *additional* types of diffusive risk premia, which are *unrelated* to the network structure, as long as the framework remains affine.

[7] In principle, cash flows could also be subject to positive jumps. Our general model solution in Appendix A and the approximations in Appendix B are valid irrespective of the sign of K or L. The sign restrictions matter only for the theorems and corollaries derived from there. In Appendix C, we briefly summarize the impact of positive jumps on the three channels through which directed links in cash flow networks affect the cross-section of risk premia.

[8] Our network is weighted in the sense that the links between nodes are represented by (positive) real numbers, not just by the binary 0-1 information whether two nodes are linked or not.

the state variables, not at the cash flow level itself.

In the model, nodes in the network represent industries or groups of firms whose earnings are described by the cash flow processes. Thus, we refer to nodes as assets throughout the paper. The cash flow jumps may be interpreted as natural disasters or policy shocks, whose timing is uncertain, but to which investors assign a non-zero probability. The empirical literature documents that such shocks can spread through the economy along supply chains (see, e.g., Carvalho, Nirei, Saito, and Tahbaz-Salehi, 2020), and our network linkages can be interpreted as arising from customer-supplier relations. Importantly, cash flow shocks can propagate in customer-supplier networks with a certain delay. For instance, using a broad data set of natural disasters, Barrot and Sauvagnat (2016) find that the sales of firms linked to other firms affected by a disaster also decrease, but that this decrease materializes on average between one and four quarters after the initial shock.

Therefore, the main ingredient of our asset pricing model are sequential cash flow shocks. Suppose industry A experiences a negative cash flow shock, e.g., because its sales go down, or because it faces an unexpected rise in production costs following a natural disaster or a policy shock. How does this shock to industry A affect suppliers or customers in industries B and C related to A via contractual obligations? Firms in industry A may default on trade credit or delay the payment of bills (see Jacobson and von Schedvin, 2015; Murfin and Njoroge, 2015). This can lead to cash flow shocks in the linked industries B and C, possibly with a non-negligible delay (see Albuquerque, Ramadorai, and Watugala, 2015). Potential reasons for such a delay are search frictions as emphasized by Carvalho, Nirei, Saito, and Tahbaz-Salehi (2020), or that corporate restructuring takes time and does not necessarily affect the cash flow immediately.

Following this line of reasoning, the beta coefficients and the modeling of the network, exclusively through the dynamics of state variables, can be interpreted as follows. The larger $\beta_{j,i}$, the more likely a cash flow shock in industry i is followed by a cash flow shock in industry j. The amount of the increase in this likelihood is given exogenously. Thinking of the assets as industries, the beta coefficients might represent industry characteristics that explain the extent of shock propagation risk, e.g., the number of individual customer-supplier linkages of firms across these industries or the strengths of such linkages. All entries in the beta matrix are assumed to be non-negative values and not normalized, so that a change in one coefficient does not require an adjustment in any other coefficient in the corresponding row or column. Coefficients equal to zero indicate no direct dependence between the cash flows of the two respective industries, e.g., because there is no customer-supplier relation between them.

The above interpretation is consistent with two other model assumptions. First, all cash flow and consumption jumps in our model are negative, because the propagation of cash flow shocks is usually considered more relevant for negative than for positive shocks. For instance, think of increased default risk after a natural disaster. The literature on financial frictions, contagion, and systemic risk has largely documented that the amplification and propagation of negative shocks during economic busts is very different from the gradual build-up during economic booms (see, e.g., Bernanke, Gertler, and Gilchrist, 1996; Fostel and Geanakoplos, 2008; Brunnermeier and Sannikov, 2014). Second, the interpretation above implies that each asset represents a substantial fraction of the economy. Therefore, it is natural to assume that each cash flow jump is accompanied by a jump in aggregate consumption at the same time. In this sense, the ratio L/K can be viewed as a rough proxy for the relative size of an industry.

Mutually exciting jumps provide certainly not the only, but a very lean and reduced-form modeling tool to capture exactly this propagation mechanism. An initial cash flow shock in industry i increases the *probability* of future cash flow shocks to a connected industry $j \neq i$ (and potentially also industry i itself), but it is unknown when (and if at all) these shocks will *materialize*. Stated differently, a cash flow shock of one industry changes the conditional distribution of future cash flows of other industries, but does not affect the level of these cash flows instantaneously. The structure of the jump processes in our model thus differs in a time series and in a cross-sectional dimension from, for instance, contemporaneous jumps in many assets. As an alternative to continuous-time mutually exciting processes, the time dimension of shock propagation could also be represented by, e.g., a discrete-time vector autoregressive model. However, this would lead to the problem that the sum of state variables following an AR(1) processes does not necessarily follow an AR(1) process itself (see Granger and Morris, 1976), so that the standard affine solution machinery cannot be applied.

Our specification ensures that the vector $X_t = (y_t, \ell_{1,t}, \ldots, \ell_{n,t}, y_{1,t}, \ldots, y_{n,t})'$ follows an affine jump process.[9] The joint process (N_t, ℓ_t) is Markov. In all applications of the model, we assume $\kappa > \beta_{i,i}$ for $i = 1, \ldots, n$, so that the vector of intensities ℓ is stationary.[10]

Remark 1. *We do not link aggregate consumption to the sum of cash flows, but model cash flows as claims on the risk factors in the consumption process. The difference between aggregate consumption and the sum of all individual cash flows can be thought of, e.g., as the investor's implicit labor income. This specification is consistent with empirical data, e.g., Santos and Veronesi (2006) point out that the sum of cash flows is only a fraction of*

[9]See Appendix A for details.

[10]See, e.g., Aït-Sahalia, Cacho-Diaz, and Laeven (2015) for details about mutually exciting processes, in particular, concerning conditions for stationarity.

aggregate consumption. This assumption also underlies asset pricing models like Campbell and Cochrane (1999), Longstaff and Piazzesi (2004), Bansal and Yaron (2004), Backus, Chernov, and Martin (2011), or Barberis, Greenwood, Jin, and Shleifer (2015).

An alternative approach to modeling economies with multiple Lucas trees has been pioneered by Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2013). These authors assume that aggregate consumption equals the sum of all cash flows. The shares of each cash flow in aggregate consumption become state variables and affect all equilibrium prices. These state variables follow highly nonlinear dynamics, so that such a model cannot be solved using the affine machinery applied in this paper.

We follow the former approach because only this framework, in conjunction with the affine machinery, allows us to trace risk premia back to the core input of any network model, namely the individual entries of the connectivity matrix, and to isolate the three channels affecting risk premia. If one were to solve a general model in which the market clearing channel and the three channels of our paper coexist and complement each other, i.e., a model with multiple risky assets, market clearing and directed cash flow networks, one would have to rely on numerical solutions. Such a numerical solution would only be valid for a given network structure. In contrast, the series expansion of our closed-form solution allows us to derive statements that hold for arbitrary directed network structures.

Our solution approach using first-order approximations which will be explained below highlights two straightforward and very basic measures for the directedness of cash flow shocks. The *spreading capacity*, denoted by $spread_i$, of asset i is defined as the respective column sum of the beta matrix:

$$spread_i = \sum_{j=1}^{n} \beta_{j,i}. \tag{3}$$

Its *receiving capacity*, denoted by $receive_i$, is defined as the respective row sum:

$$receive_i = \sum_{j=1}^{n} \beta_{i,j}. \tag{4}$$

These measures have also been proposed by, e.g., Jackson (2008) and Diebold and Yilmaz (2014) and represent the total strength of the network links going from node i to all other nodes or vice versa. In the framework of our model, the higher $spread_i$, the more a shock to cash flows of asset i increases the jump intensities of other nodes. The higher $receive_i$, the more asset i is affected by other cash flow jumps somewhere in the economy.[11]

2.2. Model Solution

Our economy is populated by a representative agent with an infinite planning horizon. We assume that the agent has recursive preferences so that the risk generated by state variables (in this case the intensities $\ell_{i,t}$) will be priced in equilibrium.

The derivation of the model solution closely follows Eraker and Shaliastovich (2008).[12] They show that the continuous-time dynamics of the pricing kernel M_t can be written as

$$d \ln M_t \;=\; -\delta\,\theta\,dt - (1-\theta)\;d\ln R_t - \frac{\theta}{\psi}\,dy_t,$$

where δ is the subjective time preference rate, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. We assume $\gamma > 1$ and $\psi > 1$, so that $\theta < 0$. In particular, with $\gamma > \frac{1}{\psi}$, the representative agent has a preference for early resolution of uncertainty.

The return on the consumption claim R_t satisfies the following continuous-time version of the Euler equation

$$0 \;=\; \frac{1}{dt}\,\mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}}\right].$$

R_t depends on the dynamics of the log wealth-consumption ratio v and aggregate consumption. To compute R_t, we use the Campbell-Shiller log-linear approximation $d\ln R_t = k_{v,0}\,dt + k_{v,1}\,dv_t - (1-k_{v,1})\;v_t\,dt + dy_t$ with linearizing constants $k_{v,0}$ and $0 \ll k_{v,1} < 1$. Employing the usual affine guess for the log wealth-consumption ratio v_t, i.e., assuming $v_t = A + B'\,\ell_t$ with $B = (B_1, \ldots, B_n)'$ and $\ell_t = (\ell_{1,t}, \ldots, \ell_{n,t})'$, we can solve numerically for the coefficients A and B as well as for the linearizing constants (see Appendix A).

The dynamics of the pricing kernel are

$$\frac{dM_t}{M_t} \;=\; -r_t\,dt - \sum_{i=1}^{n} \mathrm{MPJR}_i\,(dN_{i,t} - \ell_{i,t}dt), \tag{5}$$

[11] Although we call *spread* and *receive* measure of *directedness*, they can of course also be applied in an *undirected* network, i.e., in a network where the connectivity matrix is symmetric. In this case, we have $\text{spread}_i = \text{receive}_i$ for all i.

[12] Details are presented in Appendix A.

where r_t is the equilibrium risk-free rate and MPJR_i is the market price of risk for the jump process N_i. These in general negative market prices of jump risk are given as

$$\mathrm{MPJR}_i \;=\; 1 - \exp\left\{-\gamma\, K + k_{v,1}\,(\theta - 1) \sum_{j=1}^{n} B_j\, \beta_{j,i}\right\}, \tag{6}$$

with $k_{v,1} = \frac{e^{\bar{v}}}{1+e^{\bar{v}}}$, where $e^{\bar{v}}$ is the steady-state wealth-consumption ratio. The exponential term is a product of two factors. The first factor, $\exp\{-\gamma\, K\}$, represents the compensation for the immediate shock caused by the jump in cash flow i. Since $K < 0$ these market prices of jump risk are in general negative. The second factor with the remaining exponents is the compensation for the risk caused by variations in the state variables and is one of the key features of our model. It depends on the impact of the intensities ℓ_i on the equilibrium wealth-consumption ratio, represented by the components of the vector B.

In analogy to the return on the consumption claim, the returns $R_{i,t}$ on the individual cash flow claims satisfy the continuous-time Euler equations

$$0 \;=\; \frac{1}{dt}\,\mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_{i,t}}\right)}{e^{\ln M_t + \ln R_{i,t}}}\right].$$

The local expected excess return of asset i can be written as

$$\mathrm{EER}_i \;=\; \frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] - r_t \;=\; \sum_{j=1}^{n} \ell_{j,t}\,\mathrm{MPJR}_j\,\mathrm{JEXP}_{i,j}, \tag{7}$$

i.e., the risk premium of asset i is given by the sum of the products of jump intensity, market price of risk, and jump exposures denoted by $\mathrm{JEXP}_{i,j}$.

These jump exposures are defined as the response of the return on asset i to a shock in the cash flow of asset j, i.e.,

$$dR_{i,t} \;=\; \ldots dt + \sum_{j=1}^{n} \mathrm{JEXP}_{i,j}\, dN_{j,t}.$$

To compute these, we proceed as in the case of the consumption claim, i.e., we employ an affine guess for the log price-to-cash flow ratio of asset i, $v_{i,t} = A_i + C_i'\ell_t$ with $C_i = (C_{i,1}, \ldots, C_{i,n})'$, and use the Campbell-Shiller approximation $d\ln R_{i,t} = k_{i,0}\,dt + k_{i,1}\,dv_{i,t} - (1 - k_{i,1})\; v_{i,t}\,dt + dy_{i,t}$ with linearization constants $k_{i,0}$ and $0 \ll k_{i,1} < 1$. Again, we solve for the coefficients A_i and $C_{i,j}$ $(j = 1, \ldots, n)$ as well as for the linearization constants $k_{i,0}$ and $k_{i,1}$ numerically (see Appendix A).

Finally, the jump exposure of asset i to shocks in the cash flow of asset j is given by

$$\mathrm{JEXP}_{i,j} = \begin{cases} \exp\left(L + k_{i,1} \sum_{k=1}^{n} C_{i,k}\, \beta_{k,i}\right) - 1 & \text{for } j = i \\ \exp\left(k_{i,1} \sum_{k=1}^{n} C_{i,k}\, \beta_{k,j}\right) - 1 & \text{for } j \neq i. \end{cases} \tag{8}$$

The exponential term in the exposure of asset i to jumps in its own cash flow, $\mathrm{JEXP}_{i,i}$, has two components. First, there is the price change due to the immediate cash flow shock, represented via the jump size L. By assumption this component is only present in the exposure of asset i to jumps in its own cash flow i because N_i exclusively affects y_i, i.e., jumps in other assets do not have a direct impact on the cash flow y_i. The second term is a special feature of models with recursive utility and captures the effect of a shock in cash flow j on asset i's price-to-cash flow ratio. For $j \neq i$, the exposure $\mathrm{JEXP}_{i,j}$ only consists of this valuation ratio effect.

Finally, the risk-free rate is given as

$$r_t = \Phi_0 + \Phi_1'\, \ell_t,$$

with Φ_0 and Φ_1 given in Appendix A.

2.3. First-Order Approximations

The coefficients B and C_i in Equations (6) and (8) are the solutions of ordinary differential equations given in Appendix A. They are non-linear functions of all the coefficients in the network connectivity matrix β and cannot be given in closed-form. Therefore, there is also no closed-form solution for the market prices of risk or the jump exposures as functions of entries of the connectivity matrix just from Equation (6) or (8). However, motivated by Carvalho, Nirei, Saito, and Tahbaz-Salehi (2020) and Walden (2019), we can derive theorems through a first-order approximation. This implies that we summarize polynomial terms of order 2 or higher in the network coefficients using the notation $O(\beta^2)$. Throughout the paper, quantities in which $O(\beta^2)$ terms have been omitted are denoted by the superscript **. In this section, we impose some mild parameter restrictions to dissect the three channels outlined in the introduction.

Assumptions.

(A1) $0 < \kappa < 1$

(A2) $-\ln(2) < K$

(A3) $L < K < 0$

(A4) $\ell_{1,t} = \ldots = \ell_{n,t} = \ell_t$

(A1) is an assumption for convenience which allows us to shorten the proofs in the appendix. (A2) puts a mild bound on the consumption jump size which is in line with the estimates of Barro (2006). (A3) states that the cash flow jump size is more negative than the consumption jump size. This is in line with the concept of levered consumption dating back to Abel (1999). (A4) makes sure that cross-sectional differences can arise through network linkages only.

We obtain the following theorem for the market prices of jump risk.

Theorem 1. *The first-order approximation of asset i's market price of jump risk is given by*

$$\begin{aligned} MPJR_i^{**} &= 1 - \exp\left\{\mathcal{A} + \mathcal{B}\sum_{j=1}^{n}\beta_{j,i}\right\} = 1 - \exp\left\{\mathcal{A} + \mathcal{B}\, spread_i\right\} \qquad (9) \\ \text{with} \qquad \mathcal{A} &= -\gamma\, K \\ \mathcal{B} &= \frac{(\theta-1)\,\left(1-\exp\left\{K\,(1-\gamma)\right\}\right)}{\theta\left[(1-\kappa)-\frac{1}{k_{v,1}}\right]}. \end{aligned}$$

Assuming (A1), we obtain the following results:

(1) $\mathcal{A} > 0$ *and* $\mathcal{B} > 0$.

(2) If $spread_i > spread_j$*, then* $|MPJR_i^{**}| > \left|MPJR_j^{**}\right|$.

Proof: See Appendix B.1.

The second exponential factor on the right-hand side of (9) is one of the key features of our model. The spreading capacity of an asset is the main driver of cross-sectional differences in the equilibrium market prices of risk. The theorem states that the market prices of risk for jumps associated with high *spread* assets are larger (in absolute terms) than those of low *spread* assets (note that $\mathcal{A}$ and $\mathcal{B}$ do not depend on i). Throughout the paper, we will refer to the spreading channel whenever equilibrium quantities involve $\mathcal{B}$.

The economic intuition behind this key result is the following. By definition, high *spread* industries have more links or stronger links to other industries, relative to their low *spread* counterparts. Hence, cash flow shocks originating from a high *spread* industry have a more pronounced impact on the rest of the economy, i.e., they increase the aggregate risk of subsequent shocks by a larger amount. In models with stochastic cash flow jump intensities and recursive preferences, the wealth-consumption ratio is generally decreasing in the aggregate jump risk.[13] The wealth-consumption ratio in our economy thus reacts more negatively to cash flow shocks of high *spread* assets. These shocks are thus more systematic and carry a higher (i.e., more negative) market price of risk in equilibrium.

The theorem explicates that a necessary condition for this key result is that $\mathcal{B} > 0$. This condition is satisfied under some mild preference parameter restrictions like $\theta < 0$, which implies $\psi > 1$ (if $\gamma > 1$). In this situation, the intertemporal substitution effect dominates the income effect, so that the investor wants to consume more and save less in bad times with high jump intensities. Moreover, MPJR_i is the larger, the larger the impact of jumps in asset i on aggregate consumption, as measured by K.

For the jump exposures, we can formulate the following theorem.

Theorem 2. *The first-order approximation of the jump exposures of asset i against shocks to cash flow j is given by*

$$\begin{aligned}
JEXP_{i,j}^{**} &:= \begin{cases} \exp\left\{\mathcal{D}_i \cdot \beta_{i,j} + \mathcal{C}_i \cdot (spread_j - \beta_{i,j})\right\} - 1 & \text{for } j \neq i \\ \exp\left\{L + \mathcal{D}_i \cdot \beta_{i,i} + \mathcal{C}_i \cdot (spread_i - \beta_{i,i})\right\} - 1 & \text{for } j = i \end{cases} \qquad (10) \\
\text{with} \qquad \mathcal{C}_i &= \frac{1 - \exp\{-K\,\gamma\} - \frac{\theta-1}{\theta}\left[1 - \exp\{K\,(1-\gamma)\}\right]}{1 - \kappa - \frac{1}{k_{i,1}}} \\
\mathcal{D}_i &= \frac{1 - \exp\{L - K\,\gamma\} - \frac{\theta-1}{\theta}\left[1 - \exp\{K\,(1-\gamma)\}\right]}{1 - \kappa - \frac{1}{k_{i,1}}}.
\end{aligned}$$

Assuming (A1), (A2), and (A3) we obtain

(1) $\mathcal{C}_i > 0$ for all i and for $\gamma > 2\psi - 1$.

(2) $\mathcal{D}_i < 0$ for all i.

(3) If $JEXP_{i,i}^{**}$, $JEXP_{j,j}^{**} < 0$, $k_{i,1} = k_{j,1}$, $\beta_{i,i} = \beta_{j,j}$, and $spread_i > spread_j$, then $\left|JEXP_{i,i}^{**}\right| < \left|JEXP_{j,j}^{**}\right|$.

Proof: See Appendix B.2.

For $j \neq i$, the expression for $\mathrm{JEXP}_{i,j}^{**}$ comprises two terms. The first one represents the impact of the jump in cash flow j on the jump intensity of asset i, whereas the second term captures the effect on the jump intensities of the other assets in the economy.

The first quantity, $\mathcal{D}_i\,\beta_{i,j}$, describes a price effect through direct spillover of shocks from j to i. A jump in asset j increases the jump intensity of asset i by $\beta_{i,j}$. The reaction

[13] This has been shown, e.g., by Wachter (2013).

of the price-dividend ratio of i due to this direct effect, $\exp\{\mathcal{D}_i\,\beta_{i,j}\}-1$, is negative since $\mathcal{D}_i<0$. Throughout the paper, we will refer to this effect as the receiving channel.

In contrast, the second term, $\mathcal{C}_i\cdot(spread_j-\beta_{i,j})=\mathcal{C}_i\cdot\sum_{k=1,k\neq i}^{n}\beta_{k,j}$, is positive, since $\mathcal{C}_i>0$. This term represents what we label as hedging channel throughout the paper, namely the effect of a jump in cash flow j on the price of asset i that arises through the propagation of shocks to *other* parts of the economy. The economic intuition is as follows. In equilibrium, the jump in asset j's cash flow makes asset i *relatively* more attractive if the jump in j spreads widely through the rest of the economy. Hence, asset i can be viewed as a hedging device against shock propagation risk: its price-dividend ratio increases upon a shock in j through this channel. I.e., if a jump in asset j's cash flow makes the entire economy riskier, the drop in the price-dividend ratio of asset i coming from the receiving channel is dampened by this second term. The hedging channel is more pronounced for shocks originating from high *spread* assets than from low *spread* assets. The hedging term is always positive, as long as at least one $\beta_{k,j}\neq 0$ (for $k\neq i$) and for mild restrictions on the preference parameters.

The two previous effects describe the response of the price-dividend ratio to shocks. The ultimate sign of $\mathrm{JEXP}_{i,j}^{**}$ depends on the trade-off between the positive hedging term, $\mathcal{C}_i\cdot(spread_j-\beta_{i,j})$, and the negative receiving channel term, $\mathcal{D}_i\cdot\beta_{i,j}$, and thus on the network structure. For $j=i$, there is an additional term, the negative cash flow effect of a jump in i on the price of asset i itself, represented by $\exp\{L\}-1$. If L is chosen strongly negative, then $\mathrm{JEXP}_{i,i}^{**}$ will be negative. Since the hedging channel is more pronounced for a high *spread* asset than for a low *spread* asset, we have that $\left|\mathrm{JEXP}_{i,i}^{**}\right|<\left|\mathrm{JEXP}_{j,j}^{**}\right|$ for $spread_i>spread_j$.

Remark 2. *Formally, the notion of an asset's relative attractiveness to which we refer above is linked to the quantity* $\frac{\ell_{i,t}}{\sum_j \ell_{j,t}}$*, i.e., the ratio of the jump intensities of cash flow i and of aggregate consumption. In a pure jump model like ours, this ratio drives all asset prices and risk premia because* $\frac{Cov_t(M_t,y_{i,t})}{Var_t(M_t)}$ *is a function of it. To see this, remember that according to Equation (5) the pricing kernel* M_t *reacts to all jumps* N_j $(j=1,\ldots,N)$*, whereas the cash flow* $y_{i,t}$ *reacts to the jump* N_i *only, i.e., the covariance term is a function of* $\ell_{i,t}$*. By the same reasoning, the variance of* M_t *is a function of* $\sum_j \ell_{j,t}$*. In case there is a jump in one of the cash flows in the economy,* $\sum_j \ell_{j,t}$ *goes up if the jump spreads throughout the economy and this increase is the larger the more widely the jump spreads. Suppose asset i is not affected by this jump, i.e.,* $\ell_{i,t}$ *remains unchanged. Then, the ratio* $\frac{\ell_{i,t}}{\sum_j \ell_{j,t}}$ *goes down, asset i becomes relatively more attractive, and asset i's price-dividend ratio goes up.*

Using the closed-form solutions derived for the market prices of jump risk and the jump exposures, the proof of the following theorem for the expected excess return is straightforward.

Theorem 3. *Under the assumptions (A1) to (A4), the first-order approximation of asset i's expected excess return is given by*

$$
\begin{aligned}
EER^{**}_{i,t} &= -\ell_t\,\mathcal{A}\,L \underbrace{-\ell_t\,\mathcal{B}\,L\sum_{j=1}^{n}\beta_{j,i}}_{\text{spreading channel}} \underbrace{-\ell_t\,\mathcal{A}\,\mathcal{D}_i\sum_{j=1}^{n}\beta_{i,j}}_{\text{receiving channel}} \underbrace{-\ell_t\,\mathcal{A}\,\mathcal{C}_i\sum_{j=1}^{n}\sum_{k=1,\,k\neq i}^{n}\beta_{k,j}}_{\text{hedging channel}} \qquad (11)\\
&= -\ell_t\,\mathcal{A}\,L - \ell_t\,\mathcal{B}\,L\,spread_i - \ell_t\,\mathcal{A}\,\mathcal{D}_i\,receive_i - \ell_t\,\mathcal{A}\,\mathcal{C}_i\sum_{j=1}^{n}\left(spread_j - \beta_{i,j}\right)\\
&= -\ell_t\,\mathcal{A}\,L - \ell_t\,\mathcal{B}\,L\,spread_i - \ell_t\,\mathcal{A}\,\mathcal{D}_i\,receive_i - \ell_t\,\mathcal{A}\,\mathcal{C}_i\sum_{k=1,\,k\neq i}^{n} receive_k.
\end{aligned}
$$

The expected excess return comprises four terms. The first one, $-\ell_t\mathcal{A}L = \ell_t\gamma KL$, is positive and represents the well-known risk premium for jumps in cash flows disregarding any network features.

The second term, $-\ell_t\,\mathcal{B}\,L\,spread_i$, is positive. It captures the extra risk premium for asset i arising through the spreading channel. If shocks to asset i can spread through the economy, these shocks earn a higher market price of risk, as outlined above. This term is the larger for high *spread* assets relative to low *spread* assets.

Also the third term, $-\ell_t\,\mathcal{A}\,\mathcal{D}_i\,receive_i$, is positive. It captures the additional risk premium arising through the receiving channel. If there is a link from some other assets j to asset i, then jumps in j make i riskier, and this commands an extra premium in equilibrium for holding asset i.

The last term, $-\ell_t\,\mathcal{A}\,\mathcal{C}_i\sum_{j=1}^{n}\left(spread_j - \beta_{i,j}\right) = -\ell_t\,\mathcal{A}\,\mathcal{C}_i\sum_{k=1,\,k\neq i}^{n} receive_k$, is negative and represents the hedging channel. If a jump spreads widely throughout the economy, this "diversification" makes the price-dividend ratio of asset i react less to the jump. In equilibrium, this lowers asset i's total risk premium.

Overall, we see that the risk premium of asset i depends on *spread* and *receive* of all assets in the economy. The dependence on its own spreading capacity comes through the spreading channel in the second term and the hedging channel in the fourth term. The dependence on the spreading capacity of all the *other* assets in the economy is captured by the fourth term. In particular, $spread_i$ enters the above formula with two opposing signs: high *spread* assets earn an extra risk premium through the spreading channel, but at the same time a strong hedging channel lowers their risk premium. Therefore, we cannot make a general statement about the impact of *spread* on the cross-section of risk premia. However, we derive the following corollary.

Corollary 1. *If assumptions (A1) to (A4), and $k_{i,1} = k_{i',1}$ hold, we have $\mathcal{C}_i = \mathcal{C}_{i'}$ and $\mathcal{D}_i = \mathcal{D}_{i'}$ and the difference between the expected excess returns of two assets i and i' is given by*

$$\begin{aligned} EER^{**}_{i,t} - EER^{**}_{i',t} &= -\ell_t \, \mathcal{B} \, L \, (spread_i - spread_{i'}) - \ell_t \, \mathcal{A} \, \mathcal{D}_i \, (receive_i - receive_{i'}) \\ &\quad + \ell_t \, \mathcal{A} \, \mathcal{C}_i \, (receive_i - receive_{i'}) \, . \end{aligned} \tag{12}$$

If $spread_i > spread_{i'}$ and $receive_i > receive_{i'}$, then $EER^{**}_{i,t} > EER^{**}_{i',t}$.

All three terms in Equation (12) are positive. First, risk premia for high *spread* assets are higher than risk premia of low *spread* assets through the spreading channel. Second, if $receive_i$ is larger than $receive_{i'}$, the receiving channel is larger for asset i than for i'. Third, for the same reason, the hedging channel pushing risk premia down is less pronounced for i than for i'.

Finally, the theorem again shows that our two measures for the directedness of cash-flow shocks, *spread* and *receive*, are consistent with the first-order approximation. Stated differently, we have chosen them because the key building blocks of expected excess returns are determined by row and column sums.

Remark 3. *The hedging channel uncovered here is related to, but different from the market clearing channel put forward by Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2013). The key quantity in a model in which the market for the consumption good has to clear, is the dividend share $\frac{y_{i,t}}{\sum_j y_{j,t}}$. A negative shock to cash flow i reduces the dividend share of asset i, whereas it mechanically increases the dividend shares of all other assets $j \neq i$ (assuming that all cash flow processes are locally uncorrelated). The cash flow risk of asset i is then less systematic, while the risks of the other assets become more systematic. Thus, asset i's price-dividend ratio increases and the price-dividend ratios of all other assets decrease upon this shock. Hence, the price reactions of all assets in the rest of the economy have a negative sign.*

In contrast, in our model, it is not the sum of the individual cash flows, but the aggregate jump intensity which matters. The equilibrium price of asset i depends on $\frac{\ell_{i,t}}{\sum_j \ell_{j,t}}$, i.e., the ratio of the jump intensities of cash flow i and of aggregate consumption. We then distinguish between hedge and non-hedge assets, where hedge assets are those for which $\frac{\ell_{i,t}}{\sum_j \ell_{j,t}}$ decreases upon a negative cash flow shock in the economy. Upon such a shock, the hedge assets become relatively more attractive and their price-dividend ratios increase, while the non-hedge assets become relatively less attractive and their price-dividend ratios decrease. Hence, the price *reactions of hedge assets have a positive sign, while those of non-hedge assets have a negative sign.*

For the sake of completeness, we also derive the following theorem for the risk-free rate.

Theorem 4. *The first-order approximation of the risk-free rate is given by*

$$\begin{aligned} r_t^{**} &= \mathcal{F} + \mathcal{G} \sum_{j=1}^{n} \sum_{i=1}^{n} \beta_{i,j} + \mathcal{H} \sum_{j=1}^{n} \ell_{j,t} + \mathcal{I} \sum_{j=1}^{n} \ell_{j,t} \sum_{i=1}^{n} \beta_{i,j} \\ &= \mathcal{F} + \mathcal{G} \sum_{j=1}^{n} spread_j + \mathcal{H} \sum_{j=1}^{n} \ell_{j,t} + \mathcal{I} \sum_{j=1}^{n} \ell_{j,t}\, spread_j \\ &= \mathcal{F} + \mathcal{G} \sum_{i=1}^{n} receive_i + \mathcal{H} \sum_{j=1}^{n} \ell_{j,t} + \mathcal{I} \sum_{j=1}^{n} \ell_{j,t}\, spread_j \end{aligned}$$

where the coefficients $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{I}$ are given in Appendix B.3. Assuming (A1), we obtain

(1) $\mathcal{G} < 0$

(2) $\mathcal{I} > 0$.

Proof: See Appendix B.3.

The risk-free rate is the negative of the conditional expectation of the pricing kernel. Thus, all the channels affecting equilibrium valuation ratios show up in the risk-free rate as well. In particular, every single network coefficient $\beta_{i,j}$ enters the formula with a positive and with a negative sign ($\mathcal{I} > 0$ and $\mathcal{G} < 0$) so that we cannot make general statements about the impact of the network structure on the risk-free rate.

3. Dissecting the Three Channels

The general theorems derived above hold for arbitrary directed networks, i.e., for any possible network structure and any number of assets. In principle, each asset's expected excess return is affected by all three channels. The channels we derived and discussed in the previous section are present in any directed network, but it is instructive to study them in more stylized networks in which they can be dissected more easily. To achieve this, we frequently assume $\beta_{i,j} > 0$ and $\beta_{j,i} = 0$.

3.1. A Stylized Three Asset Economy

In this section, we set $n = 3$ and choose the following beta matrix:

$$\beta = \begin{pmatrix} 0 & 0 & 0 \\ \beta_{2,1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Shocks can propagate from asset 1 to asset 2, but there are no other links in the economy, and there is no self-excitation, i.e., the diagonal of β is set to 0. Asset 3 is separated from the rest of the economy, i.e., it can neither spread shocks to other assets nor does it receive any shocks from them. The network is shown graphically in Figure 1.

We choose this network because it is the smallest possible network that has the following two features: (i) There exists at least one link between two assets. (ii) There exists at least one additional asset that has no ties to the link in (i). For simplicity, we assume that this additional asset (asset 3) has no link at all to the rest of the economy (assets 1 and 2). That implies that we can study the general equilibrium pricing effects of shock propagation on an asset that has no fundamental link to the assets sending out or receiving the shock.

Table 1 summarizes the expected excess returns of the three assets and highlights the contributions of the three channels to these risk premia. For the sake of illustration, we start our explanations with asset 2 which receives shocks (from asset 1) in this economy, but does not spread any shocks. We arrive at the following corollary.

Corollary 2. *Under the assumptions (A1) to (A3), the first-order approximation of asset 2's expected excess return is given by*

$$EER_{2,t}^{**} = -\ell_{2,t}\,\mathcal{A}\,L \underbrace{-\,\ell_{1,t}\,\mathcal{A}\,\mathcal{D}_2\,\beta_{2,1}}_{\textit{receiving channel}}$$

which results from the following first-order approximation of the jump exposures and the market prices of jump risk

$$\begin{array}{lll} JEXP_{2,1}^{**} = \exp\{\mathcal{D}_2\,\beta_{2,1}\} - 1 & JEXP_{2,2}^{**} = \exp\{L\} - 1 & JEXP_{2,3}^{**} = 0 \\ MPJR_1^{**} = 1 - \exp\{\mathcal{A} + \mathcal{B}\,\beta_{2,1}\} & MPJR_2^{**} = 1 - \exp\{\mathcal{A}\} & MPJR_3^{**} = 1 - \exp\{\mathcal{A}\} \end{array}$$

where $\mathcal{A} > 0$, $\mathcal{B} > 0$, and $\mathcal{D}_2 < 0$ are given in Theorems 1 and 2.

The expected excess return is given by $\text{EER}_2 = \sum_{j=1}^{3} \ell_{j,t}\,\text{MPJR}_j\,\text{JEXP}_{2,j}$. Given the sparse beta matrix, this sum collapses to the expression given in the corollary.

As explained in the general framework, the first term represents the standard risk premium for jumps in cash flows, $-\ell_{2,t}\,\mathcal{A}\,L = \ell_{2,t}\,\gamma\,K\,L$. Jumps in the cash flow of asset 2 do not spread, i.e., do not change the distribution of any other cash flow throughout the economy. Hence, the risk premium that asset 2 commands for loading on these cash flow jumps, consists only of this standard term. This part of the premium is positive and arises irrespective of any network linkages. Neither the exposure, $\mathrm{JEXP}^{**}_{2,2}$, nor the market price of risk for jumps in cash flow 2, MPJR^{**}_2, depend on the structure of the network.

The second term is caused by the receiving channel, $-\ell_{1,t}\,\mathcal{A}\,\mathcal{D}_2\,\beta_{2,1}$. This positive term reflects the additional risk premium which asset 2 commands. A shock to the cash flow of asset 1 propagates to asset 2, i.e., it increases the intensity of subsequent cash flow shocks in asset 2. Hence, the price-dividend ratio of asset 2 reacts negatively to shocks in the cash flow of asset 1, i.e., $\mathrm{JEXP}^{**}_{2,1} < 0$. The market price of jump risk MPJR^{**}_1 is negative (like all market prices of jump risk in our economy), and so this receiving premium is positive.

Finally, since asset 2's price-dividend ratio does not react to shocks in asset 3, $\mathrm{JEXP}^{**}_{2,3} = 0$, there is no additional premium to be earned for these shocks. To sum up, an asset which is purely shock receiving, but does not spread its own shocks throughout the economy, earns an additional positive risk premium in equilibrium.

Next, we turn to asset 3 which neither receives nor spreads any shocks.

Corollary 3. *Under the assumptions (A1) to (A3), the first-order approximation of asset 3's expected excess return is given by*

$$EER^{**}_{3,t} \;=\; -\ell_{3,t}\,\mathcal{A}\,L \;\underbrace{-\,\ell_{1,t}\,\mathcal{A}\,\mathcal{C}_3\,\beta_{2,1}}_{\textit{hedging channel}}$$

which results from the following first-order approximation of the jump exposures and the market prices of jump risk

$$\begin{array}{lll} JEXP^{**}_{3,1} = \exp\{\mathcal{C}_3\,\beta_{2,1}\} - 1 & JEXP^{**}_{3,2} = 0 & JEXP^{**}_{3,3} = \exp\{L\} - 1 \\ MPJR^{**}_1 = 1 - \exp\{\mathcal{A} + \mathcal{B}\,\beta_{2,1}\} & MPJR^{**}_2 = 1 - \exp\{\mathcal{A}\} & MPJR^{**}_3 = 1 - \exp\{\mathcal{A}\} \end{array}$$

where $\mathcal{A} > 0$, $\mathcal{B} > 0$, and $\mathcal{C}_3 > 0$ are given in Theorems 1 and 2.

The corollary illustrates one of the key results from our equilibrium model: Although asset 3 is unconnected to the rest of the economy, its risk premium contains a term reflecting the link from asset 1 to 2, $-\ell_{1,t}\,\mathcal{A}\,\mathcal{C}_3\,\beta_{2,1}$.

This term is negative and can be explained through the hedging channel. Upon a shock to asset 1, the cash flow of asset 2 and thus also aggregate consumption becomes riskier, while the distribution of asset 3's cash flows remains unchanged. In relative terms, the cash flow risk of asset 3 then represents a smaller fraction of the overall cash flow risk in the economy. Consequently, through the equilibrium pricing mechanism, the price-dividend ratio of asset 3 increases, $\mathrm{JEXP}^{**}_{3,1} > 0$. Because of this positive exposure, asset 3 can be considered as a hedging device against the risk of losses due to shock propagation from asset 1 to asset 2. This makes asset 3 more attractive and lowers its expected return. Put differently, its risk premium, $\mathrm{EER}^{**}_{3,t}$, is lower than the (hypothetical) jump risk premium which would arise in an economy without any network linkages and cash-flow shocks only, $-\ell_{3,t}\,\mathcal{A}\,L$.

Finally, we present the results for asset 1.

Corollary 4. *Under the assumptions (A1) to (A3), the first-order approximation of asset 1's expected excess return is given by*

$$EER^{**}_{1,t} \;=\; -\ell_{1,t}\,\mathcal{A}\,L \;\underbrace{-\,\ell_{1,t}\,\mathcal{B}\,L\,\beta_{2,1}}_{\textit{spreading channel}}\;\underbrace{-\,\ell_{1,t}\,\mathcal{A}\,\mathcal{C}_1\,\beta_{2,1}}_{\textit{hedging channel}}$$

which results from the following first-order approximation of the jump exposures and the market prices of jump risk

$$\begin{array}{lll} JEXP^{**}_{1,1} = \exp\{L + \mathcal{C}_1\,\beta_{2,1}\} - 1 & JEXP^{**}_{1,2} = 0 & JEXP^{**}_{1,3} = 0 \\ MPJR^{**}_{1} = 1 - \exp\{\mathcal{A} + \mathcal{B}\,\beta_{2,1}\} & MPJR^{**}_{2} = 1 - \exp\{\mathcal{A}\} & MPJR^{**}_{3} = 1 - \exp\{\mathcal{A}\} \end{array}$$

where $\mathcal{A} > 0$, $\mathcal{B} > 0$, and $\mathcal{C}_1 > 0$ are given in Theorems 1 and 2.

Asset 1 is the shock-spreading asset in this economy. Besides the risk premium for cash-flow jumps, its expected excess return contains two network terms.

First, asset 1 earns a positive extra risk premium $-\ell_{1,t}\,\mathcal{B}\,L\,\beta_{2,1}$ which arises through the spreading channel. Out of the three possible cash flow jumps in this economy, only the shocks to the cash flow of asset 1 have the potential to change the future distribution of cash flows in the economy. Hence, the market price of risk for these shocks contains an additional network component, $\mathcal{B}\,\beta_{2,1}$, which implies an extra risk premium on the cash flow jump size L of asset 1. Overall, we thus see that shock-spreading assets command an extra risk premium for the additional systematic risk they induce.

The second component, $-\ell_{1,t}\,\mathcal{A}\,\mathcal{C}_1\,\beta_{2,1}$, reflects the hedging channel we just discussed for asset 3. Such a term also appears for asset 1 because the same equilibrium pricing mechanism is at work here. When asset 1 is hit by a cash flow shock, the intensity of subsequent cash flow shocks of asset 2 increases. Consequently, in relative terms the cash flow risk of asset

1 then represents a smaller fraction of the overall cash flow risk in the economy. Hence, the price-dividend ratio of asset 1 *increases* after a shock to its own cash flow. As already explained for asset 3 above, the additional risk premium arising through the hedging channel is thus negative.

This stylized example reveals another striking fact: The hedging channel is effective in any network, even in those that exhibit only one link. Even if there would be no unconnected third asset, the hedging channel would still show up in the expected excess return of asset 1, i.e., it can never be shut down completely.

Comparing all three assets with each other, we see that asset 3 has the lowest expected excess return in the economy because the hedging channel pushes its risk premium down, i.e., $\mathrm{EER}_{1,t}^{**} > \mathrm{EER}_{3,t}^{**}$ and $\mathrm{EER}_{2,t}^{**} > \mathrm{EER}_{3,t}^{**}$. However, we cannot rank assets 1 and 2 relative to each other. Both assets command additional positive risk premia due to network connectivity, but from distinct channels, and the sizes of these risk premia depend on the parametrization of the model.

Finally, we acknowledge that we are discussing first-order approximations here, thereby omitting higher order terms involving $\beta_{2,1}^2$, $\beta_{2,1}^3$ and so forth. It is this approximation which allows us to uncover the structure of expected excess returns in such a comprehensive way. Without it, we would have numerous additional interaction terms, in which the four key channels would be intertwined and reinforce or weaken each other. However, the building blocks are the ones we outlined above.

3.2. An n-Asset Star Network

The 3-asset network above allowed us to separate the four key channels driving expected excess return. Such a clear separation is not possible in an arbitrary network with n assets. For instance, the hedging channel, $-\ell_t \, \mathcal{A} \, \mathcal{C}_i \sum_{j=1}^n \sum_{k=1,k\neq i}^n \beta_{k,j}$, involves all entries of the beta matrix except for the i-th row. Hence, it is difficult to isolate.

However, Equation (11) allows us to derive conditions for a sparse network with n assets in which at least the spreading and the receiving channel can be disentangled. The spreading channel, $-\ell_t \, \mathcal{B} \, L \sum_{j=1}^n \beta_{j,i}$, appears in the risk premium of asset i whenever the i-th column of the beta matrix has nonzero entries, whereas the receiving channel, $-\ell_t \, \mathcal{A} \, \mathcal{D}_i \sum_{j=1}^n \beta_{i,j}$, shows up whenever there are nonzero entries in the i-th row. Both channels can thus be separated in so-called star networks or core-periphery networks, where only one row or one column has nonzero entries.

For ease of exposition, we study the two following beta matrices:

$$\beta^{OS} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \beta^* & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta^* & 0 & \dots & 0 \end{pmatrix} \qquad \beta^{IS} = \begin{pmatrix} 0 & \beta^* & \dots & \beta^* \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

In the outward star network (OS), shocks can only propagate from the core asset 1 to the periphery assets 2, …, n. In the inward star network (IS), the core asset receives shocks from the periphery assets. The networks are shown graphically in Figure 2.

Corollary 5. *Under the assumptions (A1) to (A4), the first-order approximations for expected excess returns in the inward (IS) and outward (OS) star networks are given by*

$$\begin{aligned}
EER^{OS,**}_{core,t} &= -\ell_t\,\mathcal{A}\,L \quad - \ell_t\,\mathcal{B}\,L\,(n-1)\,\beta^* && && - \ell_t\,\mathcal{A}\,\mathcal{C}_{core}\,(n-1)\,\beta^* \\
EER^{OS,**}_{per,t} &= -\ell_t\,\mathcal{A}\,L && - \ell_t\,\mathcal{A}\,\mathcal{D}_{per}\,\beta^* && - \ell_t\,\mathcal{A}\,\mathcal{C}_{per}\,(n-2)\,\beta^*
\end{aligned}$$

$$\begin{aligned}
EER^{IS,**}_{core,t} &= -\ell_t\,\mathcal{A}\,L && - \ell_t\,\mathcal{A}\,\mathcal{D}_{core}\,(n-1)\,\beta^* && \\
EER^{IS,**}_{per,t} &= -\ell_t\,\mathcal{A}\,L \quad - \ell_t\,\mathcal{B}\,L\beta^* && && - \ell_t\,\mathcal{A}\,\mathcal{C}_{per}\,(n-1)\,\beta^*.
\end{aligned}$$

As explained previously, we have $\mathcal{A} > 0$, $\mathcal{B} > 0$, $\mathcal{C}_i > 0$, and $\mathcal{D}_i < 0$.

In the outward star network, the core asset earns an extra risk premium because it is the only asset in the economy which can spread shocks. The periphery assets earn an additional risk premium, $-\ell_t\,\mathcal{A}\,\mathcal{D}_{per}\,\beta^*$, through the receiving channel. For both core and periphery assets the risk premium is reduced through the hedging channel. While this is rather obvious for the core asset, the hedge term for the periphery asset, $-\ell_t\,\mathcal{A}\,\mathcal{C}_{per}\,(n-2)\,\beta^*$, reflects the fact that a jump in the cash flow of the core asset affects all periphery assets at the same time. For instance, upon a cash flow shock of asset 1, the price-dividend ratio of asset 2 increases because the remaining assets $3, \dots, n$ also become riskier. That is why the hedge term is multiplied by $n-2$ in the above formula.

In the inward star network, the hedging channel does not affect the core asset because it is the only asset in the economy that receives shocks. Instead, the receiving channel is very pronounced here (multiplied by n-1). For the periphery assets, we see the spreading channel and the hedging channel at work. Here, the hedging channel is actually the sum of two terms: (i) $-\ell_t\,\mathcal{A}\,\mathcal{C}_{per}\,\beta^*$, which describes the reaction of a periphery asset to its own shocks, and (ii) $-\ell_t\,\mathcal{A}\,\mathcal{C}_{per}\,(n-2)\,\beta^*$, capturing the reaction to the shocks from the other $n-2$ periphery assets.

4. Discussing Key Assumptions of the Model

Our results depend crucially on combining two key features, namely mutually exciting processes for cash flows and recursive preferences. While the former capture the way how shocks propagate through the economy, the latter relate to the pricing of these shocks. In the following, we document that both features are crucial to obtain our results.

4.1. Mutually Exciting Jumps versus Contemporaneous Jumps

In our equilibrium asset pricing model negative cash flow shocks in some assets can increase the probability of *subsequent* cash flow shocks in other assets. The direction and the magnitude of this timing of shocks characterize the network in our model. We now compare our results to the ones in a model in which this time dimension is disregarded.

Switching off the time dimension of shock propagation in our model translates into the beta matrix being zero. The only way to then have some form of shock propagation is through contemporaneous jumps. Assume that log aggregate consumption follows

$$dy_t \;=\; \mu\, dt + \sum_{j=1}^{n} K_j\, dN_{j,t},$$

but the dynamics for the log cash flows are now given by

$$dy_{i,t} \;=\; \mu_i\, dt + \sum_{j=1}^{n} L_{i,j}\, dN_{j,t} \qquad (i = 1, \dots, n)$$

with constant jump intensities $\bar{\ell}_j$. A jump in cash flow j affects the level of all cash flows for which $L_{i,j} \neq 0$. We collect the jump sizes $L_{i,j}$ in a matrix, which can then be viewed as the connectivity matrix. In Appendix D, we show that the solution of this model is nested in our framework.

The main quantities of interest can be summarized as follows:

$$\begin{aligned}
\text{MPJR}_i &= 1 - \exp\{-\gamma\, K_i\} \\
\text{JEXP}_{i,j} &= \exp\{L_{i,j}\} - 1 \\
\text{EER}_i &= \sum_{j=1}^{n} \ell_j \,[1 - \exp\{-\gamma\, K_j\}]\; [\exp\{L_{i,j}\} - 1] \;\approx\; \gamma \sum_{j=1}^{n} \ell_j\, K_j\, L_{i,j}
\end{aligned}$$

First of all, the market prices of risk do not depend on the structure of the network, hence there is no room for the spreading channel outlined previously. Second, jump exposures depend on the network structure only through the exogenously assumed $L_{i,j}$, which represent the size of the co-jumps of cash flow i upon a jump in cash flow j. Although this extra exposure to shocks originating from other assets does not involve any state variables, it can be regarded as an analogue to the receiving channel described previously. However, the jump exposures do not give rise to the hedging channel that we uncovered in the previous section. The reason is that in this model valuation ratios are constant over time and, in particular, not reflecting the dynamic shock propagation mechanism.

Structurally, risk premia in a model with contemporaneous jumps are similar to the standard cash flow jump risk premia in a model *without* network linkages, i.e., the first term in Equation (11). In particular, assets that are unconnected to the rest of the economy, like asset 3 in the example in Section 3.1, do not command any additional risk premium for shock propagation risk, neither positive nor negative because the hedging channel does not exist. In contrast, in our proposed model with mutually exciting jumps, all risk premia depend on all entries of the connectivity matrix.

4.2. Recursive Preferences versus CRRA Utility

The second key feature of our model is a representative investor with recursive preferences. If we restrict the utility specification to constant relative risk aversion (CRRA), i.e., $\theta = 1$, then all formulas derived in Section 2 still hold. However the sign of $\mathcal{D}$ changes and we get $\mathcal{B} = 0$.

For the market price of jump risk described in Theorem 1 this implies $\text{MPJR}_i^{**} \equiv 1 - \exp\{\mathcal{A}\}$. Hence, there is no effect of shock propagation through the spreading channel. In Theorem 2, $\mathcal{D}_i$ becomes positive if $L > K\,\gamma$. Hence, the receiving channel switches sign from negative to positive. This switch goes back to a well-known flaw of CRRA utility: valuation ratios increase if the economy as a whole becomes riskier.[14] Put differently, if there is a shock to the cash flow of asset i and this shock is propagated to asset j, then asset j's price-dividend ratio *increases* with CRRA utility, while it decreases with recursive preferences. Finally, the coefficient $\mathcal{C}$ does not change sign. It depends on the riskiness of a cash flow relative to all other cash flows in the economy, and such cross-sectional relations between valuation ratios are not affected by CRRA utility.

To sum up, in Theorem 3, all terms related to shock propagation are either zero or

[14]The reason is that the income effect dominates the substitution effect in this case. This has already been pointed out, e.g., by Bansal and Yaron (2004).

negative. Shock propagation thus always *decreases* expected excess returns in an economy with CRRA utility.

5. Centrality Premium

The empirical asset pricing literature on networks documents a relation between the cross-section of expected excess returns and the degree of connectivity of an asset within the network, referred to in the literature as *network centrality*. In his study, Ahern (2013) finds that more central assets earn higher average returns. He focuses on undirected networks characterized by symmetric beta matrices, and so we also restrict the analysis in this section to this type of networks.

Our model provides an explanation for the main finding of Ahern (2013). Since the theorems obtained from our first-order approximation do not allow us to rank assets according to their risk premia, we illustrate this with a meaningful example. Specifically, we analyze the cross-section of expected excess returns in an economy in which assets differ in their eigenvector centrality. Besides Ahern (2013), this centrality measure has been suggested by, among others, Ahern and Harford (2014) and Ozsoylev, Walden, Yavuz, and Bildik (2014). The general idea behind the concept of eigenvector centrality is that the centrality of a given node depends on the centrality of its neighbors, so that a node is supposed to be central when it has many neighbors, important neighbors, or both.

Eigenvector centrality is related to the eigenvalues and eigenvectors of the beta matrix characterizing the network. Formally, let $\varphi_1, \ldots, \varphi_n$ denote the eigenvectors of β, sorted in descending order by their absolute values, and $\alpha \in \mathbb{R}^{n \times n}$ (with generic element $\alpha_{i,j}$) the so-called centrality matrix containing the associated eigenvectors as columns. Then the eigenvector centralities of the network nodes are given by the eigenvector associated with the principal eigenvalue φ_1, i.e., by the first column of α with elements $\alpha_{i,1}$ ($i = 1, \ldots, n$).

We construct a beta matrix such that it represents an economy where all assets in the network differ with respect to their eigenvector centrality. We can represent this matrix $\beta = \alpha \, \varphi \, \alpha^{-1}$ with φ as the diagonal matrix containing the eigenvalues of β. We choose $\varphi_1 = 0.4$ and $\varphi_j = 0$ for $j = 2, \ldots, 10$. For the centrality vector, i.e., the principal eigenvector $\alpha_1 = (\alpha_{1,1}, \ldots, \alpha_{10,1})$ corresponding to the eigenvalue φ_1, we choose the components as $\alpha_{10,1} = 0.25$ and then with step size s, $\alpha_{i,1} = \alpha_{i+1,1} + s$ for i from 9 down to 2. Lastly, $\alpha_{1,1}$ is chosen such that the vector has unit length. With our benchmark step size of $s = 0.014$, this results in $\alpha_1 = (0.382, 0.362, 0.348, \ldots, 0.264, 0.25)'$. The remaining eigenvectors are chosen such that the beta matrix is symmetric and the network is undirected.[15] The top-left graph of Figure 3 depicts the corresponding network graphically.

For the remaining parameters, we choose the following values: $\gamma = 10$, $\psi = 1.5$, $\mu = 0.05$, $K = -0.01$, $\mu_i = 0.05$, $L = -0.10$, $\kappa = 0.8$, $\bar{\ell} = 0.10$, and we evaluate the model at $\ell_{1,t}, \ldots, \ell_{n,t} = \bar{\ell}$. Figure 3 shows in the top-right plot the expected excess returns computed using the first-order approximations from Section 2.3, denoted by EER**, and in the bottom-left plot, the expected excess returns obtained from the numerical solution described in Appendix A, denoted by EER. We see that the relation between eigenvector centrality and expected excess returns is basically perfectly linear, both for the linear approximation and for the numerical solution.[16] This shows that our model can produce a centrality premium in the spirit of Ahern (2013). Moreover, the bottom-right plot shows that the approximation produces the same sorting as the numerical solution.[17] This is consistent with the results we obtain from regressing EER** on EER which yields the following parameter estimates, t-stats (in parentheses), R^2, and correlations:

$$\begin{array}{ccccccc} \mathrm{EER}_i^{**} & = & \underset{(437.5)}{0.0011} & + & \underset{(188.1)}{0.0812} & \mathrm{EER}_i & + \quad u_i, \end{array} \qquad R^2 = 0.9998, \quad \mathrm{Corr} = 0.9999.$$

The approximated risk premia are an affine function of the exact risk premia. However, the levels differ because there are no higher-order terms in the first-order approximation.[18]

6. Conclusion

We develop a tractable consumption-based equilibrium asset pricing model that allows us to trace risk premia back to the core input of any network model, namely the individual entries of the connectivity matrix. Based on a series expansion of the closed-form solution of our model, we prove for arbitrary networks that directed links between cash flows affect the cross-section of risk premia through three channels: a spreading channel, a receiving channel

[15] A sufficient condition for a symmetric beta matrix is that its eigenvectors form an orthonormal basis of $\mathbb{R}^n$, i.e., the eigenvector matrix is an orthogonal matrix. Further details are given in Appendix E.

[16] When we vary μ, K, μ_i, L, κ, $\bar{\ell}$, φ, s, and $\alpha_{10,1}$, we find that this linear pattern is robust with respect to the choice of these parameters.

[17] The approximation quality depends on the degree of precision with which the Leontief inverse of the beta matrix can be approximated by an affine function of this matrix itself, see Appendices B.1.2 and B.2.2.

[18] In the Online Appendix, we document that this high approximation quality carries over to the market prices of risk and the jump exposures.

and a hedging channel. The first two increase risk premia, while the third one pushes them down. The overall impact of directed shock propagation on risk premia depends on the tradeoff between these channels.

We are the first to trace risk premia back to column sums and row sums of the connectivity matrix in an equilibrium setup. In particular, the hedging channel is novel to the literature. When a shock propagates through the economy, parts of the economy and aggregate consumption become riskier for a prolonged time period. At the same time, an asset that is unconnected and immune to this propagation becomes relatively more attractive. Hence, its valuation ratio increases in equilibrium, so that network linkages have a *negative* effect on its expected excess return.

The three channels that we uncover are the key building blocks of equilibrium risk premia in any directed cash flow network. More broadly, our results highlight that the price of an asset i can react to cash flow shocks of another asset j even though there is no fundamental link between these two assets.

This key insight has consequences for empirical research. The risk premium of an asset does not only depend on its own characteristics (e.g., its position in the network), but also on the characteristics of *all* other assets, irrespective of whether they are connected or unconnected. Hence, the results using an isolated approach, focussing only on the influence of direct links between two assets, may be confounded by equilibrium effects such as the hedging channel outlined here, when the rest of the economy is disregarded. In this, our findings also touch upon the literature on banking networks, input-output networks or international trade networks. Researchers who study return relations between countries, industries, firms, or banks should make sure not to overlook the influence of links among the rest of the economy.

Data availability

No new data were generated or analysed in support of this research.

Appendix

A. Model Solution

We derive the solution for a slightly more general version of the model presented in Section 2:

$$\begin{aligned} dy_t &= \mu\, dt + \sum_{j=1}^{n} K_j\, dN_{j,t}, \\ d\ell_{j,t} &= \kappa_j\, \left(\bar{\ell}_j - \ell_{j,t}\right)\, dt + \sum_{i=1}^{n} \beta_{j,i}\, dN_{i,t}, \\ dy_{i,t} &= \mu_i\, dt + L_i\, dN_{i,t} \qquad (i = 1, \ldots, n). \end{aligned}$$

To solve for the equilibrium we apply the approach proposed in Eraker and Shaliastovich (2008). The vector $X = (y, \ell_1, \ldots, \ell_n, y_1, \ldots, y_n)'$ follows the affine jump process

$$dX_t \;=\; \mu\left(X_t\right)\, dt + \xi_t\, dN_t,$$

where we use the following notation:

- $\mu(X_t) = \mathcal{M} + \mathcal{K}\, X_t$
  with $\mathcal{M} = \begin{pmatrix} \mu \\ \kappa_1\, \bar{\ell}_1 \\ \vdots \\ \kappa_n\, \bar{\ell}_n \\ \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}$ and $\mathcal{K} = \begin{pmatrix} 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & -\kappa_1 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & -\kappa_n & \ldots & 0 \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0 \end{pmatrix}$,
- $\ell_t = l_0 + l_1\, X_t$
  with $l_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ and $l_1 = \begin{pmatrix} 0 & 1 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 & 0 & \ldots & 0 \end{pmatrix}$,
- $\xi_t = \begin{pmatrix} \xi_{1,t}, & \ldots, & \xi_{n,t} \end{pmatrix} = \begin{pmatrix} K_1 & \ldots & K_n \\ \beta_{1,1} & \ldots & \beta_{1,n} \\ \vdots & \ddots & \vdots \\ \beta_{n,1} & \ldots & \beta_{n,n} \\ L_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & L_n \end{pmatrix}$.

The jump transform $\varrho(u) = \mathbb{E}\left[\left(e^{u'\xi_{1,t}}, \ldots, e^{u'\xi_{n,t}}\right)\right]'$ is in our case simply equal to $\left(e^{u'\xi_{1,t}}, \ldots, e^{u'\xi_{n,t}}\right)'$, since the jump sizes are all constant. We define the selection vectors δ_y, $\delta_{\ell_{i,t}}$ $(i = 1, \ldots, n)$, and $\delta_{y,i}$ $(i = 1, \ldots, n)$ implicitly via $dy_t = \delta_y'\, dX_t$, $d\ell_{i,t} = \delta_{\ell,i}'\, dX_t$, and $dy_{i,t} = \delta_{y,i}'\, dX_t$.

The continuous-time version of the Euler equation can be written as

$$0 = \frac{1}{dt}\,\mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}}\right], \tag{A.1}$$

where R is the return on the claim to aggregate consumption. The logarithm of the pricing kernel has the dynamics

$$d\ln M_t = -\delta\,\theta\,dt - (1-\theta)\, d\ln R_t - \frac{\theta}{\psi}\, dy_t.$$

We apply the usual affine conjecture for the log wealth-consumption ratio

$$\begin{aligned} v_t &= A + (0, B_1, \ldots, B_n, 0, \ldots, 0)\; X_t \\ &= A + (B_1, \ldots, B_n)\, \ell_t, \end{aligned}$$

and use the Campbell-Shiller approximation for the return on the consumption claim

$$d\ln R_t = k_{v,0}\, dt + k_{v,1}\, dv_t - (1 - k_{v,1})\; v_t\, dt + dy_t.$$

Combining the Campbell-Shiller approximation, the affine guess for v_t, and the dynamics of the log pricing kernel, we get

$$\begin{aligned} \frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}} &= \left\{-\delta\,\theta + \theta\, k_{v,0} - \theta\,(1-k_{v,1})\,\left(A + B'\,X_t\right) + \chi_y'\,(\mathcal{M} + \mathcal{K}\,X_t)\right\}\, dt \\ &\quad + \left\{e^{\chi_y'\,\xi_t} - \mathbb{1}\right\}\, dN_t, \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} \chi_y &= \theta\left[\left(1 - \frac{1}{\psi}\right)\,\delta_y + k_{v,1}\, B\right] \\ &= \left(-\theta\left(\frac{1}{\psi} - 1\right), \theta\, k_{v,1}\, B_1, \ldots, \theta\, k_{v,1}\, B_n, 0, \ldots, 0\right)', \end{aligned}$$

and where $\mathbb{1}$ is a vector of ones with length n. We plug expression (A.2) into the Euler equation (A.1) to get a system of equations for A and B:

$$0 = \theta\,[-\delta + k_{v,0} - (1-k_{v,1})\; A] + \mathcal{M}'\,\chi_y + l_0'\;[\varrho(\chi_y) - \mathbb{1}] \tag{A.3}$$

$$0 = \mathcal{K}'\,\chi_y - \theta\,(1-k_{v,1})\; B + l_1'\;[\varrho(\chi_y) - \mathbb{1}]\,. \tag{A.4}$$

We have two additional equations for the loglinearization constants $k_{v,0}$ and $k_{v,1}$:

$$0 = -k_{v,0} - \ln k_{v,1} + (1 - k_{v,1}) \left[A + B' \mu_X\right] \quad \text{(A.5)}$$

$$0 = A + B' \mu_X - \ln (k_{v,1}) + \ln (1 - k_{v,1}) , \quad \text{(A.6)}$$

where μ_X is a vector with i-th component $\mathbb{E}[X_i]$ if that expectation is finite and 0 otherwise. Due to the presence of the mutually exciting jump terms, the long-run means $\bar{\bar{\ell}}_i$, i.e., the unconditional expectations, are not equal to the respective mean reversion levels $\bar{\ell}_i$, as it would be the case, e.g., for a standard square-root process. According to Aït-Sahalia, Cacho-Diaz, and Laeven (2015), the $\bar{\bar{\ell}}_i$ are the solution to the following system of equations:

$$\bar{\bar{\ell}}_i = \frac{\kappa_i \, \bar{\ell}_i + \sum_{j \neq i} \beta_{i,j} \, \bar{\bar{\ell}}_j}{\kappa_i - \beta_{i,i}} \qquad (i = 1, \ldots, n). \quad \text{(A.7)}$$

We assume $\kappa_i > \beta_{i,i}$ for $i = 1, \ldots, n$ to ensure that all the $\bar{\bar{\ell}}_i$ are positive.

We solve the four equations (A.3), (A.4), (A.5), and (A.6) via an iterative procedure. We initialize $k_{v,1}$ by setting it equal to δ, then compute $k_{v,0}$, A, and B. Given these we then compute $k_{v,1}$ again and iterate forward until the system converges.

The pricing kernel has dynamics

$$\begin{aligned} \frac{dM_t}{M_t} &= -r_t \, dt - \left[\mathbb{1} - \varrho(-\lambda)\right]' (dN_t - \ell_t dt) \\ \text{with} \qquad \lambda &= \gamma \, \delta_y + (1 - \theta) \; k_{v,1} \, B \\ &= \left(\gamma, (1 - \theta) \; k_{v,1} \, B_1, \ldots, (1 - \theta) \; k_{v,1} \, B_n, 0, \ldots, 0\right)' , \end{aligned}$$

so that we can immediately read off the risk-free rate and the market prices of risk. The risk-free rate is given as

$$\begin{aligned} r_t &= \Phi_0 + \Phi_1' \, X_t \\ \text{with} \qquad \Phi_0 &= \theta \, \delta + (\theta - 1) \left[\ln k_{v,1} + (k_{v,1} - 1) \; B' \mu_X\right] + \mathcal{M}' \lambda - l_0' \left[\varrho(-\lambda) - \mathbb{1}\right] \\ \Phi_1 &= (1 - \theta) \; (k_{v,1} - 1) \; B + \mathcal{K}' \lambda - l_1' \left[\varrho(-\lambda) - \mathbb{1}\right] . \end{aligned}$$

The market prices of jump risk are given as

$$\begin{pmatrix} \mathrm{MPJR}_1 \\ \vdots \\ \mathrm{MPJR}_n \end{pmatrix} = [\mathbb{1} - \varrho(-\lambda)]$$
$$= \begin{pmatrix} 1 - \exp\left(-\gamma K_1 + k_{v,1}\ (\theta - 1)\ [B_1\, \beta_{1,1} + \ldots + B_n\, \beta_{n,1}]\right) \\ \vdots \\ 1 - \exp\left(-\gamma K_n + k_{v,1}\ (\theta - 1)\ [B_1\, \beta_{1,n} + \ldots + B_n\, \beta_{n,n}]\right) \end{pmatrix}.$$

The return on the consumption claim is given by

$$dR_t = \{\ldots\}\ dt + \{\varrho(\delta_y + k_{v,1}\, B) - \mathbb{1}\}\ dN_t$$

with jump exposures

$$\begin{pmatrix} \mathrm{JEXP}_{y,1} \\ \vdots \\ \mathrm{JEXP}_{y,n} \end{pmatrix} = \varrho(\delta_y + k_{v,1}\, B) - \mathbb{1},$$

where

$$\mathrm{JEXP}_{y,i} = \exp\left[K_i + k_{v,1}\ (B_1\, \beta_{1,i} + \ldots + B_n\, \beta_{n,i})\right] - 1$$

for $i = 1, \ldots, n$.

To obtain the expected excess returns on the cash flow claims, we follow the same approach as for the consumption claim. The continuous-time Euler equation again reads

$$0 = \frac{1}{dt}\, \mathbb{E}_t \left[\frac{d\left(e^{\ln M_t + \ln R_{i,t}}\right)}{e^{\ln M_t + \ln R_{i,t}}} \right].$$

Applying the Campbell-Shiller approximation

$$d \ln R_{i,t} = k_{i,0}\, dt + k_{i,1}\, dv_{i,t} - (1 - k_{i,1})\ v_{i,t}\, dt + dy_{i,t}$$

and the usual affine guess for the log price-to cash flow ratio

$$\begin{aligned} v_{i,t} &= A_i + (0, C_{i,1}, \ldots, C_{i,n}, 0, \ldots, 0)\ X_t \\ &= A_i + (C_{i,1}, \ldots, C_{i,n})\ \ell_t, \end{aligned}$$

we arrive at

$$\begin{aligned}
\frac{d\left(e^{\ln M_t + \ln R_{i,t}}\right)}{e^{\ln M_t + \ln R_{i,t}}} &= \Big\{-\delta\,\theta - (1-\theta)\,\left[k_{v,0} - (1-k_{v,1})\,\left(A + B'\,X_t\right)\right] + k_{i,0} \\
&\quad - (1-k_{i,1})\,\left[A_i + C_i'\,X_t\right] + \chi_{y,i}'\,\left(\mathcal{M} + \mathcal{K}\,X_t\right)\Big\}\,dt \\
&\quad + \left\{e^{\chi_{y,i}'\,\xi_t} - \mathbb{1}\right\}\,dN_t, \qquad \text{(A.8)}
\end{aligned}$$

where $\chi_{y,i} = k_{i,1}\,C_i + \delta_{y,i} - \lambda$. Plugging (A.8) into the Euler equation yields a system of equations for the coefficients A_i and C_i:

$$\begin{aligned}
0 &= -\theta\,\delta + (1-\theta)\,\left[\ln k_{v,1} - (1-k_{v,1})\,B'\,\mu_X\right] - \ln k_{i,1} + (1-k_{i,1})\,C_i'\,\mu_X \\
&\quad + \mathcal{M}'\,\chi_{y,i} + l_0'\,\left[\varrho\left(\chi_{y,i}\right) - \mathbb{1}\right] \qquad \text{(A.9)} \\
0 &= \mathcal{K}'\,\chi_{y,i} + (1-\theta)\,(1-k_{v,1})\,B - (1-k_{i,1})\,C_i + l_1'\,\left[\varrho\left(\chi_{y,i}\right) - \mathbb{1}\right]. \qquad \text{(A.10)}
\end{aligned}$$

The two additional equations for the log-linearization constants $k_{i,0}$ and $k_{i,1}$ are

$$\begin{aligned}
0 &= -k_{i,0} - \ln k_{i,1} + (1-k_{i,1})\,\left(A_i + C_i'\,\mu_X\right) \qquad \text{(A.11)} \\
0 &= A_i + C_i'\,\mu_X - \ln k_{i,1} + \ln\left(1-k_{i,1}\right). \qquad \text{(A.12)}
\end{aligned}$$

The return of the individual cash flow claim i is then given by

$$dR_{i,t} = \{\ldots\}\,dt + \left\{\varrho\left(\delta_{y,i} + k_{i,1}\,C_i\right) - \mathbb{1}\right\}\,dN_t$$

so that the jump exposure of the return is thus given by

$$\begin{aligned}
\begin{pmatrix} \mathrm{JEXP}_{i,1} \\ \vdots \\ \mathrm{JEXP}_{i,i} \\ \vdots \\ \mathrm{JEXP}_{i,n} \end{pmatrix} &= \left[\varrho\left(\delta_{y,i} + k_{i,1}\,C_i\right) - \mathbb{1}\right] \\
&= \begin{pmatrix} \exp\left(k_{i,1}\,\left[C_{i,1}\,\beta_{1,1} + \ldots + C_{i,n}\,\beta_{n,1}\right]\right) - 1 \\ \vdots \\ \exp\left(L_i + k_{i,1}\,\left[C_{i,1}\,\beta_{1,i} + \ldots + C_{i,n}\,\beta_{n,i}\right]\right) - 1 \\ \vdots \\ \exp\left(k_{i,1}\,\left[C_{i,1}\,\beta_{1,n} + \ldots + C_{i,n}\,\beta_{n,n}\right]\right) - 1 \end{pmatrix}.
\end{aligned}$$

The expected return on the claim to cash flow i can then be written as

$$\begin{aligned}\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] &= -\ln k_{i,1} + (1-k_{i,1})\;C_i'\;(\mu_X - X_t) + \left[\delta_i + k_{i,1}\,C_i\right]'\;(\mathcal{M} + \mathcal{K}\,X_t)\\ &\quad + \left[\varrho\left(\delta_{y,i} + k_{i,1}\,C_i\right) - \mathbb{1}\right]\;(l_0 + l_1\,X_t)\,.\end{aligned}$$

The expected excess return is given by

$$\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] - r_t \;=\; (l_0 + l_1\,X_t)'\;\left[\varrho\left(\chi_{y,i} + \lambda\right) + \varrho\left(-\lambda\right) - \varrho\left(\chi_{y,i}\right) - \mathbb{1}\right]$$

which can be represented as

$$\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] - r_t \;=\; \sum_{j=1}^{n} \ell_{j,t}\,\mathrm{MPJR}_j\,\mathrm{JEXP}_{i,j}.$$

B. Approximation for General Network Structures

B.1. Market Prices of Jump Risk

B.1.1. First Approximation Step

Rewriting Equation (A.4) for $\kappa_1 = \ldots = \kappa_n = \kappa$ and $K_1 = \ldots = K_n = K$ gives the following system of equations

$$\begin{aligned}0 &= B_1\,\theta\,\left[k_{v,1}\;(1-\kappa) - 1\right] + \exp\left\{K\;(1-\gamma) + \theta\,k_{v,1}\;(B_1\,\beta_{1,1} + \ldots + B_n\,\beta_{n,1})\right\} - 1\\ &\;\;\vdots\\ 0 &= B_n\,\theta\,\left[k_{v,1}\;(1-\kappa) - 1\right] + \exp\left\{K\;(1-\gamma) + \theta\,k_{v,1}\;(B_1\,\beta_{1,n} + \ldots + B_n\,\beta_{n,n})\right\} - 1\end{aligned}$$

and translating this into matrix notation yields

$$\mathbb{1} \;=\; \theta\,\left[k_{v,1}\;(1-\kappa) - 1\right]\;B + \exp\left\{K\;(1-\gamma)\right\}\,\exp\left\{\theta\,k_{v,1}\,\beta'\,B\right\},$$

where now and in the following, the "exp" operator, applied to a vector, stands for element-wise application of the "exp" operator to the vector.

Next, we apply the approximation $\exp(x) = 1 + x + O\left(x^2\right)$ and solve for B:

$$B \;=\; \left(I_{n\times n} + \frac{\exp\left\{K\;(1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)^{-1} \frac{1}{\theta\;\left[k_{v,1}\;(1-\kappa) - 1\right]}\;\left[\mathbb{1} - \exp\left\{K\;(1-\gamma)\right\}\right] + O\left(\beta^2\right) \quad \text{(B.1)}$$

where $I_{n\times n}$ denotes an $n\times n$ identity matrix and $\frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}} < 0$ since $\frac{1}{k_{v,1}} > 1-\kappa$ (due to $\frac{1}{k_{v,1}} = \frac{1+e^{\bar{v}}}{e^{\bar{v}}} > 1 > 1-\kappa$ for $0<\kappa<1$).

To conclude the first approximation step, we define

$$B^{*} \;=\; \left(I_{n\times n}+\frac{\exp\left\{K\ (1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)^{-1}\frac{1}{\theta\ [k_{v,1}\ (1-\kappa)-1]}\ [\mathbb{1}-\exp\left\{K\ (1-\gamma)\right\}]\,. \tag{B.2}$$

B.1.2. Second Approximation Step

Since the inverse term in Equation (B.1) has the structure of a Leontief inverse, $(I-A)^{-1} = I + A^1 + A^2 + \ldots$, we rewrite (B.1) as:

$$\begin{aligned}
B \;&=\; \left[I_{n\times n}-\frac{\exp\left\{K\ (1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'-\left(\frac{\exp\left\{K\ (1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)^{2}-\ldots\right]\frac{1}{\theta\ [k_{v,1}\ (1-\kappa)-1]}\\
&\quad\times [\mathbb{1}-\exp\left\{K\ (1-\gamma)\right\}]+O\left(\beta^2\right)\\
&=\; \left(I_{n\times n}-\frac{\exp\left\{K\ (1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)\frac{1}{\theta\ [k_{v,1}\ (1-\kappa)-1]}\ [\mathbb{1}-\exp\left\{K\ (1-\gamma)\right\}]\\
&\quad+O\left(\beta^2\right)
\end{aligned} \tag{B.3}$$

To conclude the second approximation step, we define

$$B^{**} \;=\; \left(I_{n\times n}-\frac{\exp\left\{K\ (1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)\frac{1}{\theta\ [k_{v,1}\ (1-\kappa)-1]}\ [\mathbb{1}-\exp\left\{K\ (1-\gamma)\right\}]\,. \tag{B.4}$$

Plugging (B.3) into the market price of risk from Equation (6) and rewriting this in matrix notation yields:

$$\begin{aligned}
\mathrm{MPJR} \;&=\; 1-\exp\left\{-\gamma\,K+\frac{k_{v,1}\ (\theta-1)}{\theta\ [k_{v,1}\ (1-\kappa)-1]}\ \left[\beta'\ [\mathbb{1}-\exp\left\{K\ (1-\gamma)\right\}]+O\left(\beta^2\right)\right]\right\}\\
&=\; 1-\exp\left\{-\gamma\,K+\frac{(\theta-1)\ (1-\exp\left\{K\ (1-\gamma)\right\})}{\theta\ \left[(1-\kappa)-\frac{1}{k_{v,1}}\right]}\,\mathit{spread}+O\left(\beta^2\right)\right\}\\
&=\; 1-\exp\left\{\mathcal{A}+\mathcal{B}\,\mathit{spread}+O\left(\beta^2\right)\right\}
\end{aligned}$$

with $\mathcal{A}$ and $\mathcal{B}$ given in Theorem 1. Thus we define

$$\mathrm{MPJR}^{**} \;=\; 1-\exp\left\{\mathcal{A}+\mathcal{B}\,\mathit{spread}\right\}. \tag{B.5}$$

For $\gamma > 1$, $\theta < 0$, $0 < \kappa < 1$, and $K < 0$, we have $\mathcal{A} > 0$ and $\mathcal{B} > 0$ since $\frac{1}{k_{v,1}} > 1-\kappa$.

B.2. Jump Exposures

B.2.1. First Approximation Step

Rewriting Equation (A.10) for $\kappa_1 = \ldots = \kappa_n = \kappa$ and $K_1 = \ldots = K_n = K$ gives a system of equations for each i, exemplified in the following for $i = 1$:

$$\begin{aligned}
0 &= B_1\ (k_{v,1} - 1)\ (\theta - 1) + C_{1,1}\ (k_{1,1} - 1) - \kappa\ [B_1\, k_{v,1}\ (\theta - 1) + C_{1,1}\, k_{1,1}] \\
&\quad + \exp\{L - K\,\gamma + \beta_{1,1}\ [B_1\, k_{v,1}\ (\theta - 1) + C_{1,1}\, k_{1,1}] + \ldots + \beta_{n,1}\ [B_n\, k_{v,1}\ (\theta - 1) + C_{1,n}\, k_{1,1}]\} - 1 \\
&\ \ \vdots \\
0 &= B_n\ (k_{v,1} - 1)\ (\theta - 1) + C_{1,n}\ (k_{1,1} - 1) - \kappa\ [B_n\, k_{v,1}\ (\theta - 1) + C_{1,n}\, k_{1,1}] \\
&\quad + \exp\{-K\,\gamma + \beta_{1,n}\ [B_1\, k_{v,1}\ (\theta - 1) + C_{1,1}\, k_{1,1}] + \ldots + \beta_{n,n}\ [B_n\, k_{v,1}\ (\theta - 1) + C_{1,n}\, k_{1,1}]\} - 1.
\end{aligned}$$

Collecting terms and introducing matrix notation yields the following system for each i:

$$\begin{aligned}
\mathbb{1} &= B\ (\theta - 1)\ [k_{v,1}\ (1 - \kappa) - 1] + C_i\ [k_{i,1}\ (1 - \kappa) - 1] \\
&\quad + [\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}] \bullet \exp\left\{k_{v,1}\ (\theta - 1)\ \beta'\, B + k_{i,1}\, \beta'\, C_i\right\},
\end{aligned}$$

where now and in the following, $\bullet$ represents element-wise multiplication of the vectors. $I_{n\times 1,i}$ is an $n \times 1$ vector with the i-th entry equal to 1 and zeros otherwise.

Again, we employ $\exp(x) = 1 + x + O\left(x^2\right)$ and solve for C_i:

$$\begin{aligned}
C_i \ &= \ \left(I + \frac{\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}}{1 - \kappa - \frac{1}{k_{i,1}}} \bullet \beta'\right)^{-1} \frac{1}{k_{i,1}\ (1 - \kappa) - 1} \\
&\quad \times \Bigg[\mathbb{1} - (\theta - 1)\ [k_{v,1}\ (1 - \kappa) - 1]\ B \\
&\quad - [\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}] \bullet k_{v,1}\ (\theta - 1)\ \beta'\, B \\
&\quad - \exp\{-K\,\gamma\}\ \mathbb{1} - (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}\Bigg] + O\left(\beta^2\right),
\end{aligned} \tag{B.6}$$

where $\frac{\exp\{-K\,\gamma\}\,\mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\, I_{n\times 1,i}}{1 - \kappa - \frac{1}{k_{i,1}}} < 0$ since $\frac{1}{k_{i,1}} > 1 - \kappa$ (due to $\frac{1}{k_{i,1}} = \frac{1 + e^{\bar{v}_i}}{e^{\bar{v}_i}} > 1 > 1 - \kappa$ for $0 < \kappa < 1$).

To conclude the first approximation step, we define

$$
\begin{aligned}
C_i^* \;=\;& \left(I + \frac{\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \right)^{-1} \frac{1}{k_{i,1}\;(1-\kappa)-1} \\
& \times \Bigg[\mathbb{1} - (\theta-1)\;\left[k_{v,1}\;(1-\kappa)-1\right]\;B \\
& \quad - \left[\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}\right] \bullet k_{v,1}\;(\theta-1)\;\beta'\,B \\
& \quad - \exp\{-K\,\gamma\}\;\mathbb{1} - \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i} \Bigg]. \qquad \text{(B.7)}
\end{aligned}
$$

B.2.2. Second Approximation Step

Again the inverse term in Equation (B.6) has the structure of a Leontief inverse, and we rewrite (B.6) as:

$$
\begin{aligned}
C_i \;=\;& \Bigg[I_{n\times n} - \frac{\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \\
& \quad - \left(\frac{\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \right)^2 - \ldots \Bigg] \frac{1}{k_{i,1}\;(1-\kappa)-1} \\
& \times \Bigg[\mathbb{1} - (\theta-1)\;\left[k_{v,1}\;(1-\kappa)-1\right]\;B \\
& \quad - \left[\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}\right] \bullet k_{v,1}\;(\theta-1)\;\beta'\,B \\
& \quad - \exp\{-K\,\gamma\}\;\mathbb{1} - \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i} \Bigg] + O\left(\beta^2\right) \\
=\;& \left(I_{n\times n} - \frac{\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \right) \frac{1}{k_{i,1}\;(1-\kappa)-1} \\
& \times \Bigg[\mathbb{1} - (\theta-1)\;\left[k_{v,1}\;(1-\kappa)-1\right]\;B \\
& \quad - \left[\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}\right] \bullet k_{v,1}\;(\theta-1)\;\beta'\,B \\
& \quad - \exp\{-K\,\gamma\}\;\mathbb{1} - \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i} \Bigg] + O\left(\beta^2\right). \qquad \text{(B.8)}
\end{aligned}
$$

To conclude the second approximation step, we define

$$
\begin{aligned}
C_i^{**} \;=\;& \left(I_{n\times n} - \frac{\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \right) \frac{1}{k_{i,1}\;(1-\kappa)-1} \\
& \times \Bigg[\mathbb{1} - (\theta-1)\;\left[k_{v,1}\;(1-\kappa)-1\right]\;B \\
& \quad - \left[\exp\{-K\,\gamma\}\;\mathbb{1} + \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i}\right] \bullet k_{v,1}\;(\theta-1)\;\beta'\,B \\
& \quad - \exp\{-K\,\gamma\}\;\mathbb{1} - \left(\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\}\right)\; I_{n\times 1,i} \Bigg] \qquad \text{(B.9)}
\end{aligned}
$$

Plugging (B.8) into the jump exposures from Equation (8) and rewriting them in matrix notation yields:

$$\begin{aligned}
\text{JEXP}_i &= \exp\left\{ L\, I_{n\times 1,i} + \frac{1-\frac{\theta-1}{\theta}\,\left(1-\exp\left\{K\,\left(1-\gamma\right)\right\}\right) - \exp\left\{-K\,\gamma\right\}}{1-\kappa-\frac{1}{k_{i,1}}}\,\beta'\,\mathbb{1} \right.\\
&\qquad \left. -\frac{\exp\left\{-K\,\gamma\right\}\,\left(\exp\left\{L\right\}-1\right)}{1-\kappa-\frac{1}{k_{i,1}}}\,\beta'\,I_{n\times 1,i} + O\left(\beta^2\right)\right\} - 1.
\end{aligned}$$

Breaking this expression down into the jump exposures $\text{JEXP}_{i,j}$ yields:

$$\begin{aligned}
\text{JEXP}_{i,j} &= \begin{cases} \exp\left\{\mathcal{D}_i \cdot \beta_{i,j} + \mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,j} + O\left(\beta^2\right)\right\} - 1 & \text{for } j \neq i \\ \exp\left\{L + \mathcal{D}_i \cdot \beta_{i,i} + \mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,i} + O\left(\beta^2\right)\right\} - 1 & \text{for } j = i \end{cases}\\
&= \begin{cases} \exp\left\{\mathcal{D}_i\,\beta_{i,j} + \mathcal{C}_i\,\left(\textit{spread}_j - \beta_{i,j}\right) + O\left(\beta^2\right)\right\} - 1 & \text{for } j \neq i \\ \exp\left\{L + \mathcal{D}_i\,\beta_{i,i} + \mathcal{C}_i\,\left(\textit{spread}_i - \beta_{i,i}\right) + O\left(\beta^2\right)\right\} - 1 & \text{for } j = i \end{cases}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{C}_i &= \frac{1-\frac{\theta-1}{\theta}\,\left[1-\exp\left\{K\,\left(1-\gamma\right)\right\}\right] - \exp\left\{-K\,\gamma\right\}}{1-\kappa-\frac{1}{k_{i,1}}}\\
\mathcal{D}_i &= \frac{1-\frac{\theta-1}{\theta}\,\left[1-\exp\left\{K\,\left(1-\gamma\right)\right\} - \exp\left\{L-K\,\gamma\right\}\right]}{1-\kappa-\frac{1}{k_{i,1}}}\\
\mathcal{D}_i - \mathcal{C}_i &= \frac{\exp\left\{-K\,\gamma\right\}\,\left(1-\exp\left\{L\right\}\right)}{1-\kappa-\frac{1}{k_{i,1}}}.
\end{aligned}$$

Note that $\frac{1}{k_{i,1}} > 1-\kappa$ (see above). For $\gamma > 1$, $0 < \kappa < 1$, and $-\log(2) < K < 0$, we have $\mathcal{C}_i > 0$. Additionally assuming $\theta < 0$, we obtain $\mathcal{D}_i < 0$.

Proof that $\mathcal{C}_\mathbf{i} > \mathbf{0}$: We rewrite $\mathcal{C}_i$ as follows:

$$\begin{aligned}
\mathcal{C}_i &= \frac{1-\frac{\theta-1}{\theta}\,\left[1-\exp\left\{K\,\left(1-\gamma\right)\right\}\right] - \exp\left\{-K\,\gamma\right\}}{1-\kappa-\frac{1}{k_{i,1}}}\\
&= \frac{\exp\left\{-K\,\gamma\right\}\,\left[\frac{1}{\theta}\,\left(\exp\left\{K\,\gamma\right\}-1\right) + \exp\left\{K\right\} - 1\right]}{1-\kappa-\frac{1}{k_{i,1}}}
\end{aligned}$$

Here, we have $1-\kappa-\frac{1}{k_{i,1}} < 0$ by assumption (since $0 < \kappa < 1$). Moreover, we have $\exp\left\{-K\,\gamma\right\} > 0$ and $\frac{1}{\theta}\,\left(\exp\left\{K\,\gamma\right\}-1\right) + \exp\left\{K\right\} - 1 < 0$.

To see the last inequality, define

$$\begin{aligned}
f\left(K\right) &= \exp\left\{K\,\gamma\right\} - 1 - \left(\exp\left\{K\right\} + \gamma\right)\,\left(\exp\left\{K\right\} - 1\right)\\
&= \exp\left\{K\,\gamma\right\} - 1 - \exp\left\{2\,K\right\} - \gamma\,\exp\left\{K\right\} + \exp\left\{K\right\} + \gamma
\end{aligned}$$

Then $f(0) = 0$ and

$$\begin{aligned} f'(K) &= \gamma \exp\{K\,\gamma\} - 2\exp\{2\,K\} - \gamma\exp\{K\} + \exp\{K\} \\ &= \gamma\left(\exp\{K\,\gamma\} - \exp\{K\}\right) + \exp\{K\} - 2\exp\{2\,K\} \end{aligned}$$

If $\gamma > 1$ and $-\ln(2) < K < 0$, then $f'(K) < 0$ which implies $f(K) > 0$. In particular,

$$\frac{\exp\{K\,\gamma\} - 1}{\exp\{K\} - 1} \;<\; \exp\{K\} + \gamma < -\theta.$$

The latter inequality is satisfied if $\gamma > 2\psi - 1$. The statement then follows. Altogether, we thus get $\mathcal{C}_i > 0$.

Proof that $\mathcal{D}_\mathbf{i} < \mathbf{0}$: We rewrite $\mathcal{D}_i$ as follows:

$$\begin{aligned} \mathcal{D}_i &= \frac{1 - \frac{\theta-1}{\theta}\left[1 - \exp\{K\,(1-\gamma)\}\right] - \exp\{L - K\,\gamma\}}{1 - \kappa - \frac{1}{k_{i,1}}} \\ &= \frac{\frac{1}{\theta} + \exp\{-K\,\gamma\}\left[\left(1 - \frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right]}{1 - \kappa - \frac{1}{k_{i,1}}} \end{aligned}$$

Again, we have $1 - \kappa - \frac{1}{k_{i,1}} < 0$. Moreover, we have

$$\begin{aligned} & \frac{1}{\theta} + \exp\{-K\,\gamma\}\left[\left(1 - \frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right] &>& \; 0 \\ \Leftrightarrow \quad & \exp\{-K\,\gamma\}\left[\left(1 - \frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right] &>& \; -\frac{1}{\theta} \\ \Leftrightarrow \quad & \left(1 - \frac{1}{\theta}\right)\exp\{K\} + \frac{1}{\theta}\exp\{K\,\gamma\} - \exp\{L\} &>& \; 0 \\ \Leftrightarrow \quad & \left(\exp\{K\} - \exp\{L\}\right) + \frac{1}{\theta}\left(\exp\{K\,\gamma\} - \exp\{K\}\right) &>& \; 0 \end{aligned}$$

which is true if $L < K$, $\gamma > 1$ and $\theta < 0$. This completes the proof.

B.3. Risk-free rate

The risk-free rate is given by

$$\begin{aligned} r_t &= \Phi_0 + \Phi_1'\,\ell_t \\ \text{with} \qquad \Phi_0 &= \theta\,\delta + (\theta - 1)\left[\ln k_{v,1} + (k_{v,1} - 1)\,B'\,\mu_X\right] + \mathcal{M}'\,\lambda - l_0'\left[\varrho(-\lambda) - \mathbb{1}\right] \\ \Phi_1 &= (1 - \theta)\,(k_{v,1} - 1)\,B + \mathcal{K}'\,\lambda - l_1'\left[\varrho(-\lambda) - \mathbb{1}\right]. \end{aligned}$$

Rewriting the expressions for Φ_0 and Φ_1 yields:

$$\begin{aligned}
\Phi_0 &= \theta\,\delta + \gamma\,\mu + (\theta-1)\,\ln k_{v_1} + (\theta-1)\,\kappa\,\left[\bar{\ell}\,\mathbb{1} - (k_{v_1}-1)\,\ell_t\right]'\,B \\
\Phi_1 &= (1-\theta)\,\left[k_{v_1}\,(1-\kappa)-1\right]\,B - \exp\left\{-K\,\gamma + (\theta-1)\,k_{v_1}\,\beta'\,B\right\} + \mathbb{1}.
\end{aligned}$$

After substituting B by Equation (B.3), we receive:

$$\begin{aligned}
\Phi_0 &= \theta\,\delta + \gamma\,\mu + (\theta-1)\,\ln k_{v_1} + \frac{(\theta-1)\,\kappa}{\theta\,\left[k_{v,1}\,(1-\kappa)-1\right]}\,\left[\bar{\ell}\,\mathbb{1} - (k_{v_1}-1)\,\ell_t\right]' \\
&\quad \times \left(I_{n\times n} - \frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)\,\left[\mathbb{1} - \exp\{K\,(1-\gamma)\}\right] + O\left(\beta^2\right) \\
\Phi_1 &= \frac{(1-\theta)\,\left[k_{v_1}\,(1-\kappa)-1\right]}{\theta\,\left[k_{v,1}\,(1-\kappa)-1\right]}\,\left(I_{n\times n} - \frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)\,\left[\mathbb{1} - \exp\{K\,(1-\gamma)\}\right] \\
&\quad - \exp\left\{-K\,\gamma + \frac{(\theta-1)\,k_{v_1}}{\theta\,\left[k_{v,1}\,(1-\kappa)-1\right]}\,\beta'\,\left(I_{n\times n} - \frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)\,\left[\mathbb{1} - \exp\{K\,(1-\gamma)\}\right]\right\} \\
&\quad + \mathbb{1} + O\left(\beta^2\right)
\end{aligned}$$

Rewriting the resulting expression and subsuming higher-order terms of β using $O\left(\beta^2\right)$:

$$\begin{aligned}
\Phi_0 &= \theta\,\delta + \gamma\,\mu + (\theta-1)\,\ln k_{v_1} + \mathcal{B}\,\left(1-\frac{1}{k_{v_1}}\right)\,\kappa\,\left[\bar{\ell}\,\mathbb{1} - (k_{v_1}-1)\,\ell_t\right]'\,\left[\mathbb{1} - \frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\mathit{spread}\right] \\
&\quad + O\left(\beta^2\right) \\
\Phi_1 &= \frac{1-\theta}{\theta}\,\left[\mathbb{1} - \exp\{K\,(1-\gamma)\}\right] + \mathcal{B}\,\exp\{K\,(1-\gamma)\}\,\mathit{spread} + 1 - \exp\{\mathcal{A} + \mathcal{B}\,\mathit{spread}\} + O\left(\beta^2\right).
\end{aligned}$$

Finally, applying the approximation $\exp\{x\} = 1 + x + O\left(x^2\right)$ allows us to rewrite Φ_1 as follows:

$$\Phi_1 = \frac{1-\theta}{\theta}\,\left[\mathbb{1} - \exp\{K\,(1-\gamma)\}\right] + \mathcal{B}\,\exp\{K\,(1-\gamma)\}\,\mathit{spread} - \mathcal{A} - \mathcal{B}\,\mathit{spread} + O\left(\beta^2\right).$$

Thus, the first-order approximation of the risk-free rate is given by

$$\begin{aligned}
r_t &= \theta\,\delta + \gamma\,\mu + (\theta-1)\,\ln k_{v_1} + \mathcal{B}\,\left(1-\frac{1}{k_{v_1}}\right)\,\kappa\,\bar{\ell}\,\mathbb{1}'\,\mathbb{1} - \mathcal{B}\,\frac{\left(1-\frac{1}{k_{v_1}}\right)\,\kappa\,\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\bar{\ell}\,\mathbb{1}'\,\mathit{spread} \\
&\quad + \mathcal{B}\,\left(1-\frac{1}{k_{v_1}}\right)\,\kappa\,(1-k_{v_1})\,\ell_t'\,\mathbb{1} + \mathcal{B}\,\frac{\left(1-\frac{1}{k_{v_1}}\right)\,\kappa\,(k_{v_1}-1)\,\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\ell_t'\,\mathit{spread} \\
&\quad + \left[\frac{1-\theta}{\theta}\,\left[\mathbb{1} - \exp\{K\,(1-\gamma)\}\right] - \mathcal{A}\right]'\,\ell_t + \mathcal{B}\,\left[\exp\{K\,(1-\gamma)\} - 1\right]\,\mathit{spread}'\,\ell_t.
\end{aligned}$$

Sorting terms, we can rewrite this as follows:

$$
\begin{aligned}
r_t^{**} &= \mathcal{F} + \mathcal{G} \sum_{j=1}^{n} \sum_{i=1}^{n} \beta_{i,j} + \mathcal{H} \sum_{j=1}^{n} \ell_{j,t} + \mathcal{I} \sum_{j=1}^{n} \ell_{j,t} \sum_{i=1}^{n} \beta_{i,j} \\
&= \mathcal{F} + \mathcal{G} \sum_{j=1}^{n} spread_j + \mathcal{H} \sum_{j=1}^{n} \ell_{j,t} + \mathcal{I} \sum_{j=1}^{n} \ell_{j,t}\, spread_j \\
&= \mathcal{F} + \mathcal{G} \sum_{i=1}^{n} receive_i + \mathcal{H} \sum_{j=1}^{n} \ell_{j,t} + \mathcal{I} \sum_{j=1}^{n} \ell_{j,t}\, spread_j \\
\text{with} \quad \mathcal{F} &= \theta\,\delta + \gamma\,\mu + (\theta - 1) \ln k_{v_1} + \mathcal{B} \left(1 - \frac{1}{k_{v_1}}\right) \kappa\, \bar{\ell}\, n \\
\mathcal{G} &= -\mathcal{B} \frac{\left(1 - \frac{1}{k_{v_1}}\right) \kappa \exp\{K\ (1-\gamma)\}}{1 - \kappa - \frac{1}{k_{v,1}}} \bar{\ell} \\
\mathcal{H} &= \mathcal{B} \left(1 - \frac{1}{k_{v_1}}\right) (1 - k_{v_1})\, \kappa + \frac{1-\theta}{\theta} \left[1 - \exp\{K\ (1-\gamma)\}\right] - \mathcal{A} \\
\mathcal{I} &= \mathcal{B} \left[\frac{\left(1 - \frac{1}{k_{v_1}}\right) (k_{v_1} - 1)\, \kappa \exp\{K\ (1-\gamma)\}}{1 - \kappa - \frac{1}{k_{v,1}}} + \exp\{K\ (1-\gamma)\} - 1 \right].
\end{aligned}
$$

C. Implications of Positive Jumps for the Three Channels

For all statements regarding the signs of the channels, the assumption $K < 0$ is sufficient, but not necessary. Hence, the signs of the relevant coefficients would not necessarily flip when this assumption is violated. In the following, we briefly summarize the impact of positive jumps in consumption on the three channels from Theorem 3 through which directed links in cash flow networks affect the cross-section of risk premia.

The spreading channel is given by $-\ell_t \mathcal{B} L\, spread_i$. According to Theorem 1, K enters the coefficient $\mathcal{B}$. The assumption $K < 0$ is necessary and sufficient for $\mathcal{B} > 0$. If K is positive, then $\mathcal{B} < 0$. If we additionally assume $spread_i > spread_j$, then $\text{MPJR}_i^{**} > \text{MPJR}_j^{**}$, i.e., statement (2) in Theorem 1 is still valid. If K is positive and we additionally assume $L > 0$, the spreading channel thus still increases risk premia.

The receiving channel is defined as $-\ell_t\, \mathcal{A}\, \mathcal{D}_i\, receive_i$. As shown in Theorem 1, K enters the coefficient $\mathcal{A}$. The assumption $K < 0$ is necessary and sufficient for $\mathcal{A} > 0$. If K is positive, then $\mathcal{A} < 0$. Theorem 2 shows that the coefficient $\mathcal{D}_i$ depends on K and L. The proof on pages 44 and 45 documents that $\mathcal{D}_i$ is negative if K is negative and

$$
L \;<\; \ln\left(\frac{1}{\theta} e^{\gamma K} + \left(1 - \frac{1}{\theta}\right) e^{K}\right), \tag{C.1}
$$

which is satisfied for $L < K$ because $K < \ln\left(\frac{1}{\theta}e^{\gamma K} + \left(1 - \frac{1}{\theta}\right)e^{K}\right)$. If K is positive and

$$L \quad > \quad \ln\left(\frac{1}{\theta}e^{\gamma K} + \left(1 - \frac{1}{\theta}\right)e^{K}\right), \tag{C.2}$$

then $\mathcal{D}_i$ is positive. Thus, if L satisfies the conditions (C.1) or (C.2) above, then the receiving channel still increases risk premia. Note that conditions (C.1) or (C.2) are sufficient, but not necessary. In particular, we cannot compare the magnitude of the receiving channel for positive jumps to the one for negative jumps. Numerical examples indicate, however, that it is larger for negative jumps.

The hedging channel is given by $-\ell_t\,\mathcal{A}\,\mathcal{C}_i\,\sum_{j=1}^{n}\left(spread_j - \beta_{i,j}\right)$. If K is positive, then $\mathcal{A} < 0$. The sign of $\mathcal{C}_i$, however, is ambiguous. In particular, we cannot conclude that the hedging channel is always positive or always negative.

Loosely speaking, if positive jumps in consumption are accompanied by more pronounced positive cash flow jumps, then the spreading and the receiving channel still increase risk premia.

D. Contemporaneous Jumps in Many Assets

The vector $X = (y, \ell_1, \ldots, \ell_n, y_1, \ldots, y_n)'$ follows the affine jump process

$$dX_t \quad = \quad \mu\left(X_t\right)\,dt + \xi_t\,dN_t,$$

where

- $\mu(X_t) = \mathcal{M} + \mathcal{K}\,X_t$
  with $\mathcal{M} = \begin{pmatrix} \mu \\ 0 \\ \vdots \\ 0 \\ \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}$ and $\mathcal{K} = \begin{pmatrix} 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0 \end{pmatrix}$,
- $\ell_t = l_0 + l_1\,X_t$
  with $l_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ and $l_1 = \begin{pmatrix} 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \end{pmatrix}$,

- $\xi_t = \begin{pmatrix} \xi_{1,t}, & \dots, & \xi_{n,t} \end{pmatrix} = \begin{pmatrix} K_1 & \dots & K_n \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \\ L_{1,1} & \dots & L_{1,n} \\ \vdots & \ddots & \vdots \\ L_{n,1} & \dots & L_{n,n} \end{pmatrix}$.

The solution of such a model is nested in the general solution of our original model since only the matrix ξ_t and the jump transform $\varrho(\cdot)$ change. It is given by $\varrho(u) = \left(e^{u'\xi_{1,t}}, \dots, e^{u'\xi_{n,t}}\right)'$.

The pricing kernel has dynamics

$$\begin{aligned} \frac{dM_t}{M_t} &= -r_t\, dt - [\mathbb{1} - \varrho(-\lambda)]' \,(dN_t - \ell_t dt) \\ \text{with} \qquad \lambda &= \gamma\, \delta_y \;=\; (\gamma, 0, \dots, 0)'. \end{aligned}$$

Given the new jump transform above, we end up with the result $\mathrm{MPJR}_i = 1 - e^{-\gamma K_i}$.

Similarly, we can compute the jump exposures of asset i to the various jumps in the economy as

$$\begin{pmatrix} \mathrm{JEXP}_{i,1} \\ \vdots \\ \mathrm{JEXP}_{i,n} \end{pmatrix} = [\varrho(\delta_{y,i} + k_{i,1}\, C_i) - \mathbb{1}] = \begin{pmatrix} \exp(L_{i,1}) - 1 \\ \vdots \\ \exp(L_{i,n}) - 1 \end{pmatrix}.$$

Expected excess returns follow from the formula

$$\mathrm{EER}_i = \sum_{j=1}^{n} \ell_j\, \mathrm{MPJR}_j\, \mathrm{JEXP}_{i,j}$$

with constant jump intensities. Plugging in, we get

$$\mathrm{EER}_i = \sum_{j=1}^{n} \ell_j \left(1 - e^{-\gamma K_j}\right)\left(e^{L_{i,j}} - 1\right) \approx \gamma \sum_{j=1}^{n} \ell_j\, K_j\, L_{i,j}.$$

E. Setting up the Beta Matrix

In the following, we describe how we operationalize the concept of eigenvector centrality to determine the beta matrix

$$\beta \;=\; \begin{pmatrix} \beta_{1,1} & \dots & \beta_{1,n} \\ \vdots & \ddots & \vdots \\ \beta_{n,1} & \dots & \beta_{n,n} \end{pmatrix}.$$

W.l.o.g., we assume that the eigenvalues $\varphi_1, \dots, \varphi_n$ of β are sorted according to their absolute size, i.e., φ_1 is the principal eigenvalue. The eigenvectors for these eigenvalues are collected in the matrix

$$\alpha \;=\; \begin{pmatrix} \alpha_{1,1} & \dots & \alpha_{1,n} \\ \vdots & \ddots & \vdots \\ \alpha_{n,1} & \dots & \alpha_{n,n} \end{pmatrix}$$

so that we have the usual diagonalization

$$\beta \;=\; \alpha \begin{pmatrix} \varphi_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \varphi_n \end{pmatrix} \alpha^{-1}.$$

W.l.o.g. we assume that the eigenvectors in α are normalized to have length 1.

The β matrix is supposed to be non-negative because there is no economic interpretation for a negative β_{ij} in our model. The Perron-Frobenius theorem then says that there exists an eigenvector with only non-negative components. The Perron-Frobenius theorem also says that the non-negative eigenvector is associated with the absolutely largest eigenvalue (called the spectral radius) which is also non-negative. The eigenvectors related to all other eigenvalues must contain negative entries.[19]

Besides, the beta matrix is supposed to be symmetric in the benchmark case. Simple linear algebra implies that the matrix β is symmetric if the eigenvector matrix is an orthogonal matrix (i.e. $\alpha\alpha' = \mathbf{1}$ or $\alpha' = \alpha^{-1}$):

$$\left(\alpha \cdot \mathrm{diag}(\varphi_1, \dots, \varphi_n) \cdot \alpha^{-1}\right)' \;=\; \left(\alpha^{-1}\right)' \cdot \mathrm{diag}(\varphi_1, \dots, \varphi_n)' \cdot \alpha' \;=\; \alpha \cdot \mathrm{diag}(\varphi_1, \dots, \varphi_n) \cdot \alpha^{-1}.$$

Taking these two facts into account, we construct the beta matrix using the following al-

[19]Moreover, the Perron-Frobenius theorem says that the principle eigenvalue is simple if the matrix is irreducible and that there is no other eigenvalue with the same absolute value if the matrix is aperiodic. The matrix constructed below is irreducible and aperiodic.

gorithm. We assume that $\varphi_2 = \ldots = \varphi_n$, i.e., the eigenvalues other than φ_1 are equal and then construct the eigenvectors as follows. We assume that $\alpha_{1,1}^2 + \ldots + \alpha_{n,1}^2 = 1$. This implies that the first eigenvector is normalized to length 1. Moreover, we assume that all entries $\alpha_{i,1}$ in the first column are positive because of the Perron-Frobenius theorem. The remaining elements of the matrix α are then chosen such that the matrix becomes an orthogonal matrix, i.e. the columns of the matrix are mutually orthogonal and normalized to length 1. In a first step, we choose the vectors such that they are all mutually orthogonal:

$$\begin{pmatrix} \alpha_{1,1} & 1 & 1 & 1 & \ldots & 1 \\ \alpha_{2,1} & -\frac{\alpha_{1,1}}{\alpha_{2,1}} & -\frac{1}{\alpha_{2,2}} & -\frac{1}{\alpha_{2,2}} & \ldots & -\frac{1}{\alpha_{2,2}} \\ \alpha_{3,1} & 0 & -\frac{\alpha_{1,1}+\alpha_{2,1}\alpha_{2,3}}{\alpha_{3,1}} & -\frac{1+\alpha_{2,4}^2}{\alpha_{3,3}} & \ldots & -\frac{1+\alpha_{2,n}^2}{\alpha_{3,3}} \\ \alpha_{4,1} & 0 & 0 & -\frac{\alpha_{1,1}+\alpha_{2,1}\alpha_{2,4}+\alpha_{3,1}\alpha_{3,4}}{\alpha_{4,1}} & \ldots & -\frac{1+\alpha_{2,n}^2+\alpha_{3,n}^2}{\alpha_{4,4}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{n,1} & 0 & 0 & 0 & \ldots & -\frac{\alpha_{1,1}+\alpha_{2,1}\alpha_{2,n}+\alpha_{3,1}\alpha_{3,n}+\ldots+\alpha_{n-1,1}\alpha_{n-1,n}}{\alpha_{n,1}} \end{pmatrix}$$

In a second step, we scale every column by its norm so that all eigenvectors have length 1. The eigenvalues and eigenvectors uniquely determine the beta matrix. We have thus reduced the choice of the beta matrix to the choice of two eigenvalues φ_1 and φ_2 and one eigenvector which contains the eigenvector centrality of each node.

In Section 5, we choose the eigenvalues $\varphi_1 = 0.4$, $\varphi_2 = \ldots = \varphi_{10} = 0$, and the entries of the principal eigenvector are $\alpha_{1,1} = 0.3824$, $\alpha_{2,1} = 0.3620$, $\ldots$, $\alpha_{10,1} = 0.2500$. This leads to the following beta matrix:

$$\beta = \begin{pmatrix} 0.0584 & 0.0553 & \ldots & 0.0404 & 0.0382 \\ 0.0553 & 0.0524 & \ldots & 0.0382 & 0.0362 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.0404 & 0.0382 & \ldots & 0.0279 & 0.0264 \\ 0.0382 & 0.0362 & \ldots & 0.0264 & 0.0250 \end{pmatrix}.$$

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Expected excess return of...	Risk premium for jumps in cash flows	Risk premium for time variation in price-dividend ratios		
		spreading channel	receiving channel	hedging channel
Shock-spreading asset 1	$-\ell_{1,t}\,\mathcal{A}\,L$	$-\ell_{1,t}\,\mathcal{B}\,L\,\beta_{2,1}$		$-\ell_{1,t}\,\mathcal{A}\,\mathcal{C}_1\,\beta_{2,1}$
Shock-receiving asset 2	$-\ell_{2,t}\,\mathcal{A}\,L$		$-\ell_{1,t}\,\mathcal{A}\,\mathcal{D}_2\,\beta_{2,1}$	
Unconnected asset 3	$-\ell_{3,t}\,\mathcal{A}\,L$			$-\ell_{1,t}\,\mathcal{A}\,\mathcal{C}_3\,\beta_{2,1}$
	> 0	> 0	> 0	< 0

Table 1
Decomposition of risk premia

For the stylized three asset network discussed in Section 3.1, this table decomposes the expected excess returns of the assets and provides the contribution of each channel to these risk premia. The formulas are given in Corollaries 2 to 4.

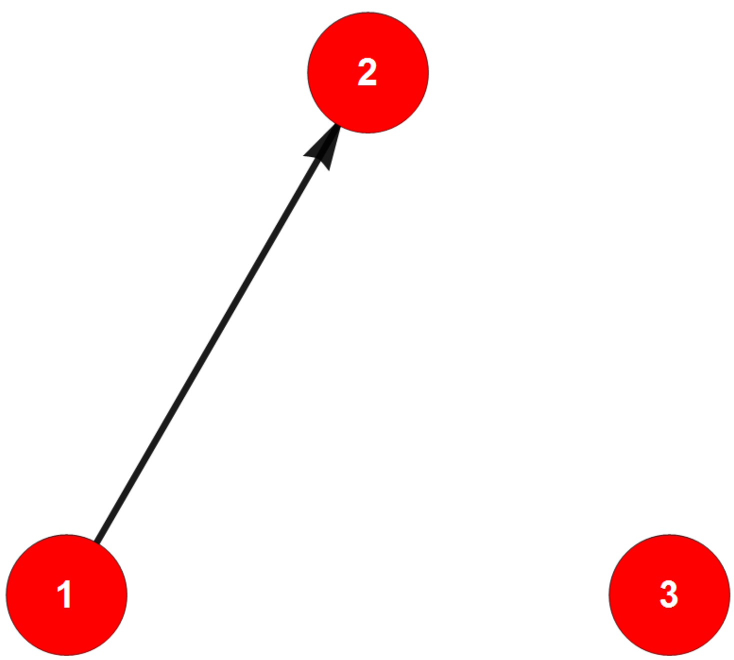

Figure 1
Stylized network

The figure shows the stylized three asset network discussed in Section 3.1.

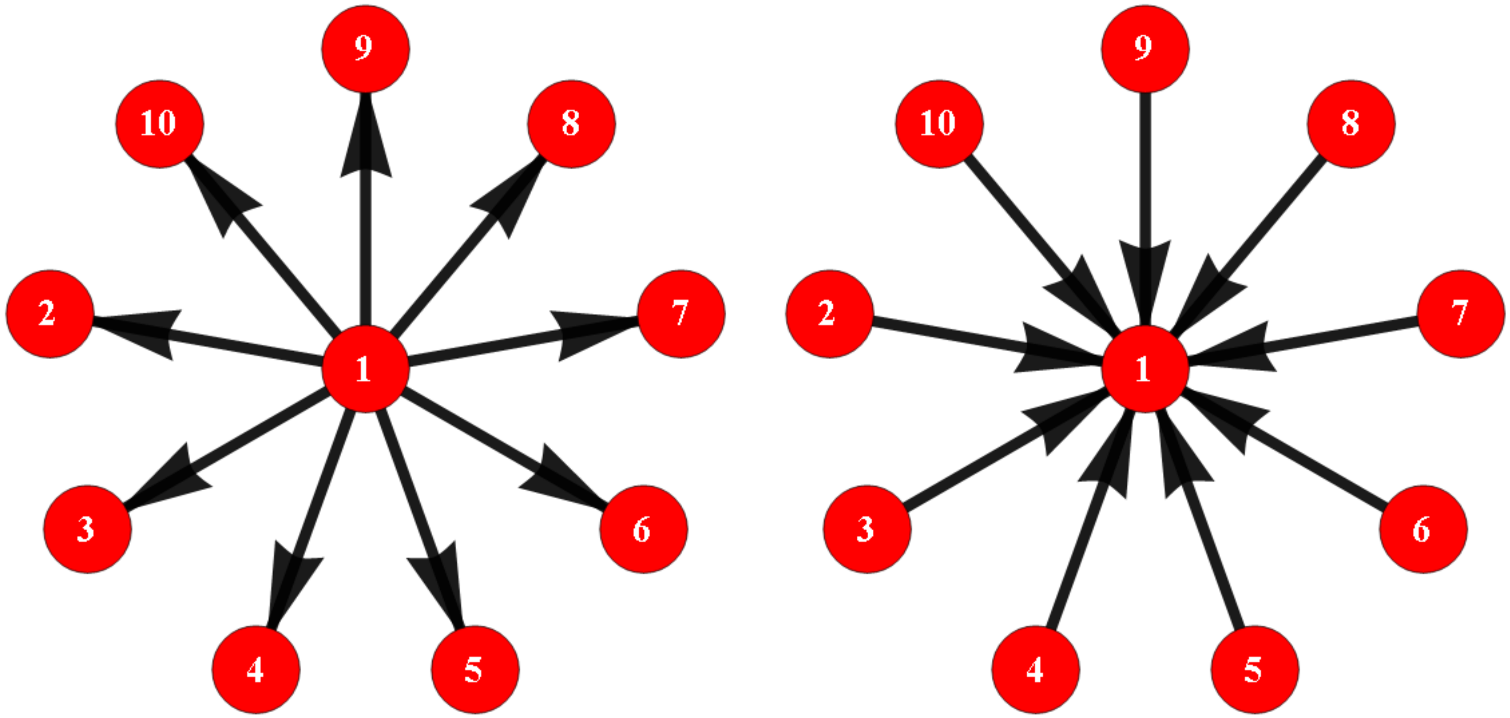

Figure 2
Outward and inward star network

The pictures show the outward (left) and the inward star network (right) discussed in Section 3.2 for $n = 10$. An arrow from node i to node j implies that a jump in the cash flow of industry i increases the jump intensity of industry j, i.e., $\beta_{j,i} > 0$.

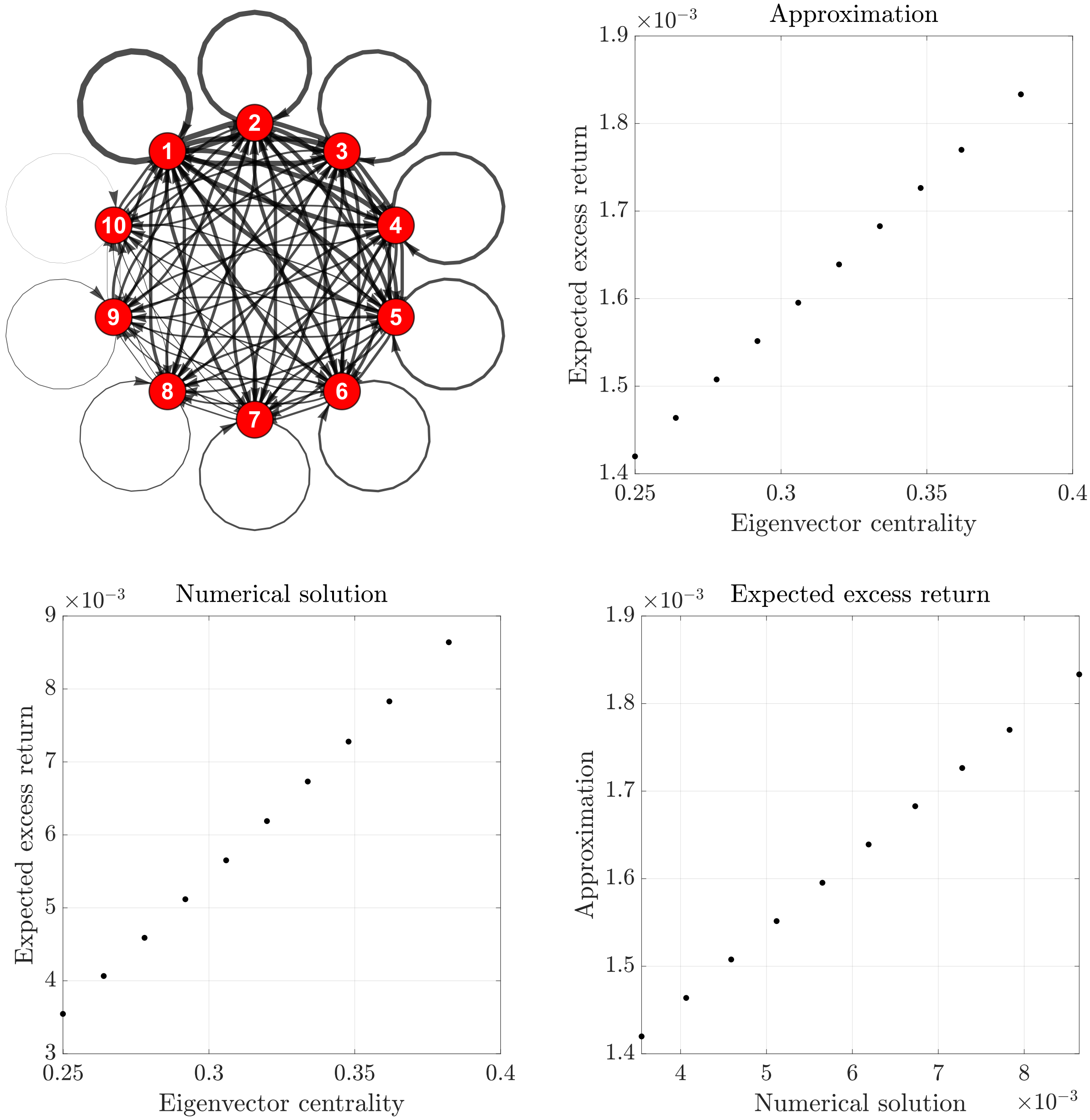

Figure 3
Centrality premium

The top-left graph shows the network used in Section 5. The top-right plot shows the expected excess return (computed using the first-order approximation from Section 2.3) as a function of each asset's eigenvector centrality. The bottom-left plot shows the expected excess return (obtained from the numerical solution described in Appendix A) as a function of each asset's eigenvector centrality. The bottom-right plot shows the expected excess return from the first-order approximation as a function of the expected excess return from the numerical solution.