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Optimal Equity Share in the Norwegian Government Pension Fund Global in Terms of the Downside Risk

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ABSTRACT

Being a sovereign wealth fund (SWF), the Norwegian Government Pension Fund Global (NGPFG) operates in a political environment and must adapt to several considerations. We address to what extent the combination of equity share and payout policy influence the downside risk of the fund. Our results show that today's composition of the fund is sustainable in terms of the trade-off between maintaining both today's expected payouts and the real fund value and controlling the downside risk.

Acknowledgements

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1 Introduction

The discovery of oil on the Norwegian continental shelf in the late 1960's created the foundation of vast future wealth. As a result of the then governments quick thinking, laws and regulations was implemented to ensure that the profits of these natural resources would benefit both current and future generations of Norway's population. Eventually, the state's revenues from the oil sector had accumulated to such an extent that it was too much to spend, and a sovereign wealth fund (SWF) was setup into which the profits would be transferred and managed (NBIM, 2016). Today, the fund named, the National Government Pension Fund Global, is one of the largest SWFs in the world.

The aim of the fund is to ensure that Norway's oil wealth will benefit both current and future generations, implying that all generations to come should benefit equally from the fund. This means that the fund must seek returns as well as control the downside risk. The NGPFG is an important part of the state budget of Norway, providing means for 25 percent of the expected expenses in 2021 (Regjeringen, 2021).

The NGPFG was established after the discovery of oil in the North Sea with the purpose to protect and ensure long-term management of the oil wealth. Additionally, the fund could contribute to more flexibility during economic downturns and protect against fluctuations in the oil revenues. Surpluses in the state budget were transferred to the fund, while deficits were covered by payouts from the fund (NBIM, 2016).

The first deposit of money was done in 1996 and the fund is currently invested in 70 countries. In the beginning it was mainly invested in foreign government bonds, until 1997 when the government agreed to invest 40 percent in equities.

This share was increased to 60 percent in 2008 and later it was decided that the fund should also invest 5 percent in real estate to further diversify their portfolio. The latest change in the equity share happened in 2017 when the government decided to increase the equity share to 70 percent (NBIM, 2016).

The issue of downside risk matters because of the need for steady withdrawals from the fund. Without the yearly withdrawals from the government, it could have handled periods of low returns as the profits would even out over time. As of "Handlingsregelen", the government can only use an average of 3 percent of the fund each year. In times of recession or crises, the government can use more than 3 percent and likewise less than 3 percent in times of upswings (Finansdeparementet, *Hvor Mye Oljepenger Bruker Vi?* 2021). This means that it is in times of adversity that the fund is needed the most, which makes the downside risk even more important to consider.

Despite the significant increase in the size of the fund, there has only been one adjustment of the payout-policy; In 2017, the payout rate was lowered from 4 to 3 percent (Meld. St: 2 (2016-2017), p.11). The payout-rule must create sufficient room for adaption in times of need, but also ensure that the purchasing power of the fund is maintained. A higher equity share will cause a higher downside risk, but to what extent is what we want to investigate in this report. Therefore, we want to examine the impact of different payout rules, paired with a variation of asset allocations.

If unable to subsidize deficits in the national government budget with the national pension fund, the state will need to collect higher taxes or reduce welfare services. This will reduce the governments ability to smooth out fluctuations for its population by for instance big investments in infrastructure, subsidiaries to businesses and social benefits.

Based on this, we want to examine how much downside risk the NGPFG can take on before the potential downside of the strategy outweighs the potential profit. We want to look further into if the fund should rebalance their equity share to adjust the downside risk, and what the financial consequences of such a rebalancing would be in terms of expected returns and payouts. A way of reducing risk would be to increase the investments in fixed-income assets, that have a significantly lower volatility than the equity markets. Combined with the various equity shares, we will examine the consequences of variations in the fiscal rule.

2 Government Pension Fund Global

The fund's investment strategy is managed under The Ministry of Finance and any material changes to the fund's strategy are anchored in the Norwegian parliament. The operational management of the fund is delegated to Norges Bank (Regjeringen, 2021). Its strategy has evolved over the years and today they manage a global portfolio dominated by public market equity risk, although with an increasing share invested in other real assets.

For the fund to be able to achieve long-term profits they assess factors related to corporate governance and sustainability that may have an impact on the fund's return over time. In addition, their aim to act in a professional and transparent way is derived from their desire to build trust and legitimacy among the Norwegian people (NBIM, 2019).

2.0.1 Investment Strategy

The strategy of the fund is expressed through the management mandate that defines a benchmark index and puts certain constraints and requirements on the NBIM. The benchmark index includes indices from external providers based on mainly listed equities (NBIM, 2019). In addition, there are other investments available that increases the diversification of the portfolio which is not included in the benchmark. Consequently, they use a reference portfolio that includes certain emerging and frontier markets to calculate the exposure to these types of investments (NBIM, 2019).

Their goal is to generate risk premiums over time with a moderate level of risk. As a result, a key part of their investment strategy is diversification across asset classes, industries and countries. To ensure that the fund follows the guidelines form the mandate, they rebalance the equity share when needed. In addition, the fund includes responsible management, cost efficiency and transparency in their investment strategy (NBIM, 2019).

The fund is invested in three main asset classes including public equities, fixed income, and real assets. Moreover, the equity share at 70 percent is the greatest contributor to the risk and return of the fund. Due to the high exposure in the equity market, the fund may experience large fluctuations in value (NBIM, 2019). Said risk is a part of the investment strategy of the fund, as it is considered a prerequisite for being able to achieve sufficient returns.

The public equity investments constitute a significant part of the portfolio and consist of large, medium-sized and small listed companies. The fixed income portfolio is issued in more than 25 currencies and together with the public equity investments these makes up more than 95 percent of the fund. To better the diversification a minor part of the fund is invested in real assets such as office, retail, residential and logistics real estate. Lastly, a small portion of approximately 1 percent of the fund is invested in renewable energy infrastructure (NBIM, 2019).

The NBIM depends on sustainability, well-functioning markets, and good corporate governance to ensure long-term returns. Therefore, they try to influence the companies in the fund when possible. NBIM has published expectation documents that is shared with the companies they are invested in. These documents include human rights, children's rights, climate change, water management, ocean sustainability, tax and transparency and anti-corruption. Additionally, they engage in the companies included in the fund through voting at almost all shareholder meetings. They also believe that keeping a dialogue with the companies is a way of exercise their ownership control (NBIM, 2019).

The overall return of the fund from 1998 to September 2021 has been higher than the benchmark index that the fund's performance is measured against. The annual return on the fund has been 6.49 percent and after deductions for management costs and inflation the annual net real return has been 4.52 percent. This is 0.26 percentage points above the benchmark index (NBIM, 2019).

The management of the fund is divided into three strategies: asset management, security selection and fund allocation. The goal is to generate excess return over time and not necessarily all at once. These strategies have different time horizons, build on different analytical frameworks, and are expected to produce excess returns during different market conditions. In that case, the fund has generated annual excess return of 0.33 percentage point over the past five years (NBIM, 2019).

3 Literature Review

3.1 Payout policy

"Handlingsregelen" (the fiscal rule) was introduced in 2001 as a measure to avoid the spendings to depend too much on oil prices and to reduce the risk of high inflation (Hovland, 2021). The initial number of 4 percent (now 3 percent) was set to reflect the expected real return of the fund, to ensure that the fund stays solid for generations to come. Since 2001 the fund has grown substantially, and with only one adjustment of the fiscal rule along the way, the amount of money spent has increased significantly from NOK 32 billion in 2002 to NOK 395 billion in 2021 (Meld. St. 1 (2021-2022), p.61).

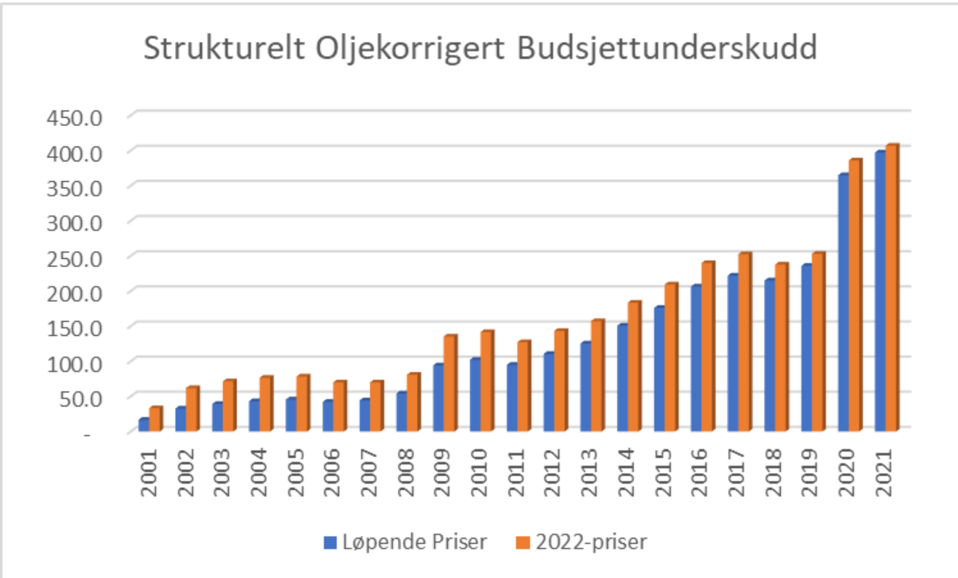

Figure 1: Structural Oil Adjusted Budget Deficits

The former governor of the Central Bank of Norway, Øystein Olsen, suggested that the fiscal rule should be reduced to 2 percent (Bjørnstad, 2021). The main argument was that he consider 2 percent as a more realistic level of expected real returns for the fund looking forward. He also argued that some of the factors why the fund had experienced its great returns, were a highly priced stock market and a weak Norwegian Krone. These are circumstances that can be subject to sudden changes, which can affect future returns of the fund.

Aase and Bjerksund (2021) argues that the diversification of the fund implies a level of risk aversion, and that risk averse investors' optimal spending rate is lower than the expected real rate of return. They argue that the current practice of spending the expected real return will be a sure way of draining the fund, rather than sustaining the fund for generations to come.

3.2 Payout rules

In his article on endowment spending in volatile markets (2010), Marshall Blume discusses the impact of short-term volatility in endowment funds and a variety of spending rules. The research assumes that an all-equity portfolio will have a higher return than a portfolio with both bonds and equities. Using 10,000 simulations over two 50-year periods, Blume analyzes three rules: the Ratchet Rule, the Flexible Rule, and the Average Rule. These rules vary in terms of how one responds to variations in fund value.

3.2.1 The Flexible Rule

The Flexible Rule simply follows the variations in value, and the payout is a fixed percentage regardless of the size of the fund. This would cause for big changes in withdrawals from one year to the next. The Flexible rule will not satisfy the intention of countercyclical spendings of the fund, but rather increase the impact of economic cycles due to the high payouts in good times and low payouts in poor times.

3.2.2 The Ratchet Rule

The Ratchet Rule says that you never decrease your spendings, regardless of the return or size of the fund. This is a quite realistic approach, as it will be difficult for the government to lower the withdrawal from one year to the next. By increasing the spendings in times of high returns, but not adjust accordingly in poorer times, the average payout is expected to be high in comparison with the other rules. Furthermore, it is also expected to increase the probability of failure of the fund due to the high payouts.

3.2.3 The Average Rule

Lastly, the Average Rule creates a scenario in which one considers an average of the fund value of a given period back in time. Including a few years back in time, the effect of short-term changes is reduced. This reduces the volatility of payouts compared to the Flexible Rule and is also able to consider decreases in the fund in contrast to the Ratchet Rule.

Blume argues that after deciding to what extent an institution is willing to reduce its spendings in times of decreases in the value of the fund (what is their optimal spending rule), it must decide its optimal investment strategy.

3.3 Asset Allocation

What equity share that is set to be held in the NGPFG is said to be the single most important decision in terms of the risk and return of the fund (Olsen, 2016). The first investment in stocks from the fund was made in 1998, after a decision to invest 40 percent in stocks. Since then, two increases of the equity share have been made - one in 2007 to 60 percent and one in 2017 to 70 percent. We will look further into the reasoning of the last adjustment.

In 2015, the Norwegian Department of Finance, decided to put together a committee to assess the equity share of the fund (NOU 2016: 20). The report was handed over in October 2016 and consisted of a thorough analysis of the impact of changes in the asset allocation. The advice provided by the majority of the committee was to increase the equity share from 60 to 70 percent. The committee leader, Knut Anton Mork, was the only one advising to reduce the equity share to 50 percent, as he was concerned with the stability of the payouts. Norges Bank also recommended to increase the equity share of the NGPFG even further from 60 to 75 percent (Norges Bank, 2016).

One reason provided for the recommendation of an increase in the share of stocks was the relative size of the fund. At the time, the fund was at the size of two thirds of the state's petroleum wealth (the fund + estimated value of remaining oil- and gas reserves), which is a significant increase since 2006 when it was only one third. The risk of the natural resources is assumed to be higher than the diversified portfolio of the fund, which means that the total risk of the petroleum wealth is reduced because of the bigger slice of the pie of the fund. Assuming that the total risk of the resources should be constant, this is used as an argument to why the equity share, and thereby the risk, of the fund can be increased.

Expected return, and consequently the contribution to The National Budget, is assumed to be higher with an increase in the equity share. Mork's committee (except Mork) considers this upside to outweigh the added risk, as long as there is political agreement to adjust the policies accordingly to the new risk level (NOU 2016:20).

Norges Bank also mentioned in their recommendation that the correlation between bonds and stocks have turned from positive to negative, which lowers the required level of bonds to reduce the total risk of the fund, as they do not only reduce the volatility but also directly reduces the fluctuations by moving opposite of the equity.

Based on these recommendations, it was decided to change the asset allocation to the current share of stocks, to 70 percent in 2017.

4 Methodology

In accordance with Olsen and Grande, the equity share is said to be the single most important decision in terms of the risk and return of the fund (Olsen and Grande, 2016). We will therefore investigate the hypothesis that a reduction in today's equity share will be optimal, in order to provide a lower downside risk of the NGPFG under the assumption that today's expected level of payouts must be maintained.

In order to investigate the downside risk of the fund, we will perform an empirical study from simulated future portfolios based on a historical simulation approach. We make these simulations to assess the long-term impact of today's payout policy and equity allocation on the downside risk of the fund. We will also test portfolios with lower equity shares and both a higher and a lower payout policy. This will give us a basis of comparison and we will observe which portfolios who perform the best in terms of the downside risk.

4.1 Asset allocation

Careful considerations have been made to decide the asset allocations of the fund, so we will only be testing the equity shares that is and has been in use since the first introduction of stocks to the fund. These are 40, 60 and 70 percent equities.

4.2 Payout-policies

Based on the given pay out-policy, a withdrawal is made in the beginning of each year to cover the funds share of yearly government expenses. The Norwegian government makes a withdrawal from the fund to cover the oil-adjusted deficit in the national budget which varies in line with the needs of the socioeconomics. Due to this it is difficult to estimate an exact amount that will be withdrawn from the fund every year. Hence, the actual compliance of the payout rule of the NGPFG is hard to model in an exact matter, which is why we will test three different rules to cover the impact of the equity share and payout policy in more aspects. These rules are inspired by Blume's literature on endowment spending.

Additionally, we will test three different percentages of yearly withdrawals. To investigate what level of payout policy is required in order to maintain today's expected level of payouts, and simultaneously manage to maintain the real value of the fund. We will also be able to get a better understanding of the impact the level of withdrawals have on the downside risk of the fund. The chosen percentages are the current and previous policies of 3 and 4 percent. Additionally, we have included 2 percent which is a recommendation from the past governor of the fund. Also, if the fund keeps growing at the past rate, it might be sustainable to reduce the policy and maintain the payouts due to the size of the fund.

4.2.1 Flexible rule

The flexible rule is tested by simply withdrawing a given percentage of the starting value of each year. The withdrawal will fluctuate yearly and reflect the volatility of bond and equity returns. Whether the change in fund value from one year to the next is positive or negative, the payouts are adjusted proportionally. As a result, this rule will cause substantial variations of the payouts from one year to the next, which are incompatible with the national budget which will not vary accordingly. However, we believe the flexible rule will provide a reliable prediction of the withdrawals from the fund in the long run.

4.2.2 Ratchet rule

To test the Ratchet Rule, we made the withdrawal in year one the fixed percentage subject to testing. In the following years, the payout is coded to be the maximum of the given percentage of the beginning of year-value and the pay out of the previous year. The Ratchet rule provides realism in terms of its lack of decrease in spendings. For the government to reduce the national budget from one year to the next might be rather unlikely in most years. Consequently, we include the Ratchet rule in our simulations to test what asset allocations and payout policies that are able to maintain both payouts and fund value without adjusting for downturns of the fund value.

This rule ensures that the payouts will be adjusted accordingly when the fund generate positive returns but will guarantee stability as the spendings never decrease. As a result, payouts will be stable and will not require an adjustment in the national budget due to lower payments from the fund.

4.2.3 Average rule

The average rule ensures less variations in the spendings caused by the volatility of the fund value. As we use a five year-average, the return on the fund will only impact the spendings with one fifth of its impact for the following year. Consequently, when the fund value declines, the adjustment in pay-outs will happen smoothly. Like the ratchet rule, this ensures more stability of the pay-outs than with the flexible rule.

To find the appropriate withdrawal with the Average Rule, we found the mean starting value of the past five years and multiplied with the respective level of payout. In the first four years, year one was repeated as many times as required to have five observations.

4.3 Forecasting Horizon

As mentioned, we want to protect the real value of the fund so that current and future generations will benefit equally from the fund. Consequently, we will analyse whether the real value is maintained for the next generation. Additionally, our simulations will suffer increased uncertainty for each year we simulate, which creates a trade-off between wanting to see long term effects and the reliability of our results. We have therefore based our simulation on a 30-year horizon, which represents approximately one generation ("Generasjon",2021).

4.4 Rebalancing Cost

For the fund to stay in line with its mandate in terms of asset allocation and distribution within countries and regions, it needs to adjust its holdings every year. Both buying and selling assets are subject to transaction costs, in addition to the commission fees. This cost is found by taking a given percentage of 10 basis points, which is multiplied with the sum of absolute value of the change of each account.

4.5 Inflation

We have used monthly inflation to calculate real portfolio values. We downloaded the monthly Consumer Price Index from Statistics Norway, from 1970 to 2021, and calculated inflation based on monthly changes in the CPI.

$$Inflation_i = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} \tag{1}$$

$$Inflation\ index\ = \prod_{t=1}^{T}(1 + inflation_t) \tag{2}$$

$$Yearly\ inflation = Inflation\ index_{t=12,24,36...} \tag{3}$$

4.6 Tests

To assess the downside risk of the fund, we will analyze how many times out of 10,000 the fund will either decrease in real value or be less than half of the starting value per capita. Adjusting the end of period value for inflation is done by dividing the end-value by one plus each of the simulated monthly inflation values. To take population growth into account, we have divided the average end value by the estimated population of 2050.

We will also investigate the average pay out per capita and the volatility of withdrawals of each combination of asset allocations and spending-rules. The volatility is measured in terms of standard deviation, and the distribution is measured in terms of skewness.

5 Empirical Methodology

5.1 Historical simulations

To make a forecast of future returns and variations of the fund, we have used historical simulations. A historical simulation is made on the basis that the best guess for the future is the past. Simulating future returns is done by randomly sampling a row of past returns for each month in our model. All rows in the dataset have the same probability of being sampled, regardless of size or time of observation. Using this approach, we assume that past returns and patterns may repeat itself, which we have observed in history.

As we have monthly data for 50 years, we consider the dataset to be sufficiently big to be a fair representation of the future. However, the approach has received some criticism because it assigns each observation, regardless of how far back it goes, the same weight and probability of future observations (Pritsker, 2006). This implies that if there are fundamental changes in the market, these will not be taken into account. Therefore, a bigger dataset might not always provide better forecasts for the future. Despite the mixed reviews of the method, historical simulations are used because it is a simple and effective method.

5.2 With and Without Replacement

We have performed each test both with and without replacement. *With replacement* is a method in which each sampled observation is put back to the population and has an equal probability of being sampled again for the next observation.

$$Prob\,Obs\,1^{w/Replacement} = Prob/Obs/2^{w/Replacement} \;= ... = \frac{1}{N} \quad (4)$$

Without replacement means that for each sampled observation the probability of the remaining observations increases.

$$Prob\ Obs\ 1^{wo/Replacement} = \frac{1}{N} \neq Prob\,Obs\,2^{wo/Replacement} \;= ... = \frac{1}{N-1} \quad (5)$$

Our analyses focus mainly on the results of the simulations without replacement. Also, the samples with replacement will suffer more random drawings, in which the samples can affect the results as much as the asset allocation or payout policy subject to testing (Blume, 2010). The results from the simulations with replacement are considered to validate our results.

5.3 Model

To construct our model, we used Matlab R2022a. Our data input consists of historical monthly return for equity indices for 16 countries and three indices for bond returns. In total we have data from 1970 until 2022, giving 624 monthly observations paired with the respective monthly inflations. The value of the fund as of 31.12.2020 at 10,914 billion NOK is used as the starting value.

Table 1: **Extract of Returns**

Country	**30.01.70**	**27.02.70**	**31.03.70**
Aus	3.08 %	-1.10 %	-0.29%
Bel	1.07%	0.20%	0.68%
Can	2.43%	4.29%	1.05%
Den	0.68%	-1.43%	-2.34%
Fra	3.94%	-2.66%	-1.03%
Ger	-4.77%	-2.86%	0.95%
HKG	5.59%	7.13%	4.12%
Ita	4.18%	-2.64%	3.62%
Jap	-1.51%	1.30%	4.13%
Net	-5.36%	0.94%	1.44%
Sin	-1.02%	1.61%	-2.51%
Spa	3.94%	4.91%	-3.03%
Swe	-3.77%	-6.23%	2.94%
Swi	-2.50%	-1.66%	-0.03%
UK	-0.62%	-5.23%	2.58%
USA	-7.11%	5.65%	0.61%
Region	**30.01.70**	**27.02.70**	**31.03.70**
AsiaOZ	0.46 %	0.17 %	-2.67%
EU	0.62%	0.62%	0.23%
US	2.57%	2.80%	1.88%

5.3.1 Weights

Country and region weights are included as constants. The country weights were found by taking the amount invested in equity for each country divided with the total value of the replicated equity-portfolio. The same was used for region-weights, which were found by finding the relative sizes of the investments in each region per 31.12.2020.

5.3.2 Simulation

The first step in our simulation, is to find the initial value of each country and region for both equities and bonds. This is done by multiplying the initial value of the fund by the equity and bond share subject to testing. Thereafter, the resulting equity- and bond values are multiplied with the respective country and region weights of each asset class. We now have our starting portfolio, which our further simulations will be based on. This is done for each portfolio we are testing. We assume that the expenses of year 1 is already covered from the year before our simulation, which is why the first withdrawal is not until after a full year.

In the next step, our simulation in Matlab samples a random row with both equity and bond returns, and the corresponding inflation from our dataset. The initial investment in each country and region is then multiplied by 1 plus the simulated return of the respective market. This provides the starting value of the next month and is repeated 11 more times until we reach year end.

$$Month\ 1 : Starting\,Value * (1 + r_{simulated}) \tag{6}$$

$$Month\ 2 - 12 : (End\,Month\,t - 1) * (1 + r^{simulated}),\ t = [2 - 12] \tag{7}$$

In the case of a Flexible rule, the payouts are calculated as the given withdrawal rate times the sum of the fund at the beginning of the year.

$$Withdrawal\,Y_t^{Flex} = (Beginning\ of\ year_t * payout\ rule) \tag{8}$$

With a Ratchet rule, the payouts are coded to be the maximum of the beginning-of-year fund value multiplied with the withdrawal rate, and the payout of last year.

$$Withdrawal\, Y_t^{Ratch} = Max(Beginning\, of\, year_t * payout\, rule,\ Pay_{t-1}) \quad (9)$$

Finding the payout with the Average rule is done similarly to the process with the Flexible rule, with the difference being that the starting value is an average of the last five years, rather than the starting value of the current year. The example below is from year 3, in which year 1 is included three times.

$$Mean\ Starting\, Value\, Year\ 3 = \frac{Y3 + Y2 + Y1 + Y1 + Y1}{5} \quad (10)$$

Both equity/bond and country/region weights are reconsidered after the withdrawals of each year, and a rebalancing is performed. To do this, the reference weights are divided by the new weights which gives us a rebalancing coefficient for all weights. New weights are found by taking the value in each country and divide by the total sum of equity, and the value of each region divided by the total sum of bond of the respective year.

The equity/bond share is found by taking the sum of equity and divide by the total sum of the fund, and similarly the sum of bonds is used to find the bond share.

$$Coeff_{Country/Region} = \frac{Reference\, Country/Region\, Weight}{Simulated\, End\, of\, Year\, C/R\, Weight} \quad (11)$$

$$Coeff_{Equity/Bond} = \frac{Equity/Bond\,Share\,Subject\,to\,Testing}{Simulated\,End\,of\,Year\,Equity/\,Bond\,Share} \tag{12}$$

$$Coeff_k = Coeff_{Country/Region} * Coeff_{Equity/Bond} \tag{13}$$

Each country/region amount is then multiplied with both the asset allocation and country/region coefficients to find the end of year-value of each market balance. The rebalancing is subject to a cost that is subtracted after the rebalancing. Rebalancing cost is found by finding the difference between the value of each country and region before and after the rebalancing, which is multiplied by the set level of cost at 10 basis points. This marks the end of year one.

$$Rebalancing\,Cost = 0.1\% * [\sum_k w_k * Fund_t * \mid Coeff_k - 1 \mid] \tag{14}$$

$$Rebalancing\,Cost = 0.1\% * [\sum_k \mid Fund_{t,k} * Coeff_k - Fund_{t,k} \mid] \tag{15}$$

The end of year-value of the fund is found by taking the value of each market after 12 months of simulated returns and subtract the correct payout. After the withdrawal, the rebalancing is performed by multiplying each investment with the corresponding coefficient. Lastly, the rebalancing cost is subtracted, and we have the end of year 1. Every 12th row will then be

$$YearEnd_k = (Fund_{k,t} - w_k * Withdrawal) * Coeff_k - w_k * Rebal.cost \quad (16)$$

$$t = [12, 24, 36...] \quad (17)$$

$$Fund\,Value\,Year\;End = \sum_{k=1}^{19} YearEnd_k \quad (18)$$

This is the full simulation of one year, which is then repeated 29 times to find 30 years' worth of development for the fund.

To ensure that all portfolios are tested with the same simulated set of returns, we created a loop in which all 27 portfolios with 30 years of development each were included simultaneously. This means that we first put in all 30 years with a 40-60-2 portfolio with a Flexible rule, then followed by 30 years, starting from time zero again, of a 60-40-2 portfolio etc. After 9 portfolios with the Flexible rule, 9 portfolios with the Ratchet rule followed, and thereafter 9 portfolios with the Average rule. The loop was repeated 10,000 times, which helps us get a representative sample of the results.

We calculated the average payout per capita for each year. Then, the average payouts provided in our results was found by taking the average of these averages of each year. Lastly, we included codes to obtain the average end value per capita, skewness and standard deviations of payouts and end values of all 27 portfolios in order to analyze our results.

After 10,000 simulations we performed our tests to investigate how many of the end values of the 10,000 portfolios that are below the initial value in real terms, and below half of the initial value. This was done by making two logical tests, and then we simply summed how many times the logical test was true. To find the end value to compare with the starting value, we divided the simulated end value of each portfolio with the corresponding path of monthly inflation. Additionally, we divided all end values with the same projected population of 2050, which is 6.0 million people.

$$Inflation\ Adjusted\ EndValuePC = \frac{EndValue\ per\ Capita}{\prod_{t=1}^{T}(1 + Inflation_t)} \tag{19}$$

$$Logical\ test\ Prob\ Not\ Maintain = Real\ EndValuePC < Start\ Value\ PC \tag{20}$$

$$Logical\ test\ Prob\ Fail = Real\ EndValue\ PC < \frac{StartValue\ PC}{2} \tag{21}$$

To test the level of average payouts against our basis year, each yearly withdrawal is discounted by the corresponding yearly inflation to make sure we compare the payouts in real terms. The yearly inflation index is made by first making an inflation index of every month's inflation, end then pick out row 12, 24,..., to get the correct yearly inflation. This enables us to discount the payouts for year 1 with the cumulative inflation of the first 12 months, the payouts of year 2 with 24 months etc.

5.4 Criteria and Evaluation

In order to investigate the downside risk of the fund in combination with its ability to maintain real fund value and the current expected payouts, we have established the following evaluation criteria.

Firstly, the average end value of the fund is considered. It is important to investigate how the payouts and the asset allocation affect the size of the fund long term. In 2020 the fund was worth 2 033 275 NOK per capita. When the government is deciding the level of payouts and asset allocation of the fund, maintaining the real fund value is one of the most important requirements. We also want to include the expected growth in the population of Norway, to see the purchasing power of the fund per person. Our criterion is therefore that the fund value must strictly maintain the fund value of 2020 per projected capita in inflation-adjusted terms.

Our second measure is the probability that the fund will maintain its real value over time. A higher equity share may lead to a wider dispersion of future real value, and a greater likelihood of a lower future real fund value. The given payout rule will also impact the probability of maintaining the real value. A higher payout policy will increase the withdrawals and therefore reduce the fund value further. To observe this criterion, we have examined its real value after 30 years per capita for all 10 000 portfolios. Further, we have calculated how many times this value was below the initial fund value. To analyze the end value one step further in terms of downside risk, we look into how many of our portfolios that will remain within our criterion if they end up one standard deviation below the average end value. The reason for doing this is because the standard deviation is the expected difference from the average, and one could expect the end value to be within one standard deviation from the average.

There is still a possibility that the fund value will increase above the initial fund value over time, and we will therefore look at the fund's probability of failure as a criterion. We have chosen to consider a halving of the starting value in real terms to be a failure. For the fund to be able to return to its initial value it would have to generate 100 percent returns. This probability will provide further insight to the downside risk, as it measures how often the fund is severely drained and would require a significant amount of time to recover.

The next measure we will consider is the average real payouts per capita. From our first criteria we may get an optimal payout rule and asset allocation that results in withdrawals that are too low and unrealistic. From the current payout policy and the value of the fund in 2020, the payout per capita in 2020 would equal 60 998 NOK.

$$Expected\ Payouts\ TodayPC = \frac{(NOK 10\,913\,768\,061\,832) * 3\%}{5\,367\,580} \tag{22}$$

$$Required\ Payouts\ PC = Avg[\sum_{t=1}^{30} ExpPO\ Today * (1 + 1.97\%)^t] \tag{23}$$

The fund has on average increased its payouts with 1.97 percent the last three years before 2020. Our criterion is therefore that the fund must generate payouts of at least 60 998 NOK per capita in real terms with a 1.97 percent increase every year. Based on this, we find the required level of payouts for the 30 next years and calculate the average. Resultingly, we will require average payouts of at least 82 100 NOK. We will also consider the standard deviation of payouts. The standard deviation tells us how much the average observation varies from the mean value and is therefore a good measurement of the volatility.

Our next criterion is the skewness of average real payouts per capita. The skewness tells us how the payouts are distributed, relative to a normal distribution. If we observe positive skewness, we know that the payouts are right skewed and that the curve has a long tail on the right side. This means that the median of the payouts is below the average, which also implies that more than half of the payouts are below the mean. The skewness indicates how reliable the resulting average payouts are, in terms of how the observations are distributed around the mean.

These criteria are not subject to decision-making individually, but tradeoffs between them must be considered. We are therefore mostly interested in the extent of relative changes in our criteria, resulting from changes in either the payout policy or the asset allocation.

5.5 Data

To be able to forecast the future performance of the fund we have collected data that represents the fund's portfolio at the end of 2020. At this time the fund was worth 10,914 billions. Our data includes 16 of the countries with the largest investments in equity, and bond returns for three regions. We have also used today's asset allocation to calculate the country weights for the equity portfolio and region weights for the fixed income portfolio. Relative weights of each country and region will remain constant within its respective asset class, but the relative weights to the total fund value will vary across the different asset allocations.

5.5.1 Data source and treatment

Firstly, we found the Financial Times Stock Exchange (FTSE) total return-indices for said countries in Eikon Refinitiv, which dated back to 1994. As this is the benchmark-index of the fund, we included all the FTSE data. To have a larger dataset, we spliced the FTSE indices with the Morgan Stanley Capital International (MSCI) indices from 1970 to 1994.

The equity returns are downloaded as return indices in American dollars. Return indices consist of capital gains including dividends and interest rates. This means that dividends are reinvested in the indices. We adjusted the indices to NOK with the corresponding MSCI exchange rate between US dollars and NOK. Thereafter, we found the monthly return in percent by taking the difference between two consecutive months and divided it by the first one.

Table 2: **Equity indices**

Equity 1970 - 1994		**Equity 1994 - 2021**	
Tot Ret Ind	**Refinitiv Code**	**Tot Ret Ind**	**Refinitiv Code**
MSCI AUS	MSAUST$	FTSE AUS	WIAUST$
MSCI BEL	MSBELG$	FTSE BEL/LUX	WIBELGL$
MSCI CAN	MSCNDA$	FTSE CAN	WICANDA$
MSCI DEN	MSCDNMK$	FTSE DEN	WIDNMK$
MSCI FRA	MSFRNCL$	FTSE FRA	WIFRNC$
MSCI GER	MSGERM$	FTSE GERY	WIWFRM$
MSCI HKG	MSHGKG$	FTSE HKG	WIHGK$
MSCI ITA	MSITAL$	FTSE ITA	WIITAL$
MSCI JAP	MSJPAN$	FTSE JAP	WIJPAN$
MSCI NET	MSNETHL$	FTSE NET	WINETH$
MSCI SPA	MSSPAN$	FTSE SPA	WISPAN$
MSCI SWE	MSSWDNL$	FTSE SWE	WISWDNL$
MSCI SWI	MSSWIT$	FTSE SWI	WISWITL$
MSCI UK	MSUTDK$	FTSE UK	WIUTDL$
MSCI USA	MSUSAM$	FTSE USA	WIUSAM$
MSCI SIN	MSSING$	FTSE W SIN	WISNGP$

For bond returns we used Bloomberg Fixed Income Indices (BBG FI Index) for the three following regions: U.S. Aggregate, Asian-Pacific Aggregate and Euro Aggregate. Because this data only goes back to 2000, we calculated a proxy-index for fixed income-returns per country and combined these with equal weights to their respective region-index. We then spliced the monthly returns for both the Bloomberg and the calculated indices to get a dataset from 1970 to 2021.

Table 3: **Fixed Income Indices**

Fixed Income 1970 - 2000		**Fixed Income 2000 - 2021**	
GVT Bond Yield	Refinitiv Code	BBG FI Index	BBG Code
Aus	AUGBOND	Asian-Pacific Agg.	I00163JP
Jap	JPGBOND		
Bel	BGGBOND	Pan-Europe Agg.	LP06TREU
Fra	FRGBOND		
Ger	BDGBOND		
Net	NLGBOND		
Swe	SDGBOND		
Swi	SWGBOND		
UK	UKGBOND		
Can	CNGBOND	U.S. Agg.	LBUSTRUU
US	USGBOND		

These calculations were performed by downloading long term bond yields for government bonds per country. Then the returns were calculated by assuming that the bond is bought in period one and sold in the following month. Face value is assumed to be 1,000 in the local currency for all bonds. The closest quarterly percentage in yield was used as a proxy for the coupon rate. In the U.S., Canada and Japan semi-annual coupons are used. After selling the bond in the following month, a new, identical bond is bought, priced by the then current yield. To find the price of the bond, we used the following equation:

$$Bond\ Price_i = \sum_{t=1}^{T} \frac{C_t}{(1+Yield)^t} + \frac{Face\,Value}{(1+Yield)^T} \tag{24}$$

After finding the prices, we calculated the monthly returns from holding the bond and reselling it after one month, and created an index based on this. The price in period 2 of the bond bought in period 1, was found with the same formula, but discounted with only T-1 months and the yield of period 2.

To make the data more comparable, we then used the MSCI historical exchange rates to convert the indices from local currencies into US dollars, and further from dollars to Norwegian kroner.

Table 4: **MSCI FX rates**

Refinitiv Code	**Currency**	**Refinitiv Code**	**Currency**
MSERAUD	Australian Dollar	MSERITL	Italian Lire
MSERCAD	Canadian Dollar	MSERGBP	UK Pound
MSERJPY	Japanese Yen	MSERSGD	Singapore Dollar
MSERCHF	Swiss Franc	MSERFRF	French Franc
MSERDKK	Danish Krone	MSERHKD	Hong Kong Dollar
MSEREUR	Euro	MSERNLG	Netherlands Guilder
MSERESP	Spanish Peseta	MSERSEK	Swedish Krona
MSERDEM	German Mark	MSERNOK	Norwegian Krone
MSERBF	Belgian Franc		

From the index in NOK, we found monthly, continuously compounded returns by using the logarithmic model:

$$Monthly\, return_i = ln[\frac{l_t^{NOK}}{l_{t-1}^{NOK}}] \tag{25}$$

For inflation, we downloaded the monthly Consumer Price Index from Statistics Norway, from 1970 to 2021, and calculated inflation based on monthly changes in the CPI.

$$Inflation_i = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} \tag{26}$$

In order to provide an estimate of the future population of Norway, we have used Statistics Norway's National population projections, updated June 3rd, 2020. These projections are divided into a main, low, and high alternative, but we have used the main one, as it is considered to be the most likely scenario. We have used the population estimates for 2021-2050. The starting value of the fund was divided by the actual population of 2020 to find the per capita value.

5.6 Customizing Weights

Table 5: **Equity Weights**

Country	Value 2020	Equity Weight
Australia	18 491 233 960	2.35%
Belgium	4 739 087 349	0.60%
Canada	180 082 385 028	2.30%
Denmark	11 121 341 438	1.41%
France	44 860 901 335	5.70%
Germany	41 766 033 306	5.30%
Hong Kong	9 957 496 544	1.26%
Italy	10 935 780 889	1.39%
Japan	76 184 517 810	9.68%
Netherlands	18 113 877 057	2.30%
Singapore	3 917 088 533	0.50%
Spain	13 234 240 750	1.68%
Sweden	17 839 742 466	2.27%
Switzerland	41 887 827 587	5.32%
UK	66 656 141 437	8.47%
USA	389 641 136 303	49.48%
SUM	787 428 831 792	100.00%

Table 6: **Bond Weights**

Country	**Value 2020**	**FI Weight**
AsiaOz	61 163 670 635	19%
US	150 444 482 581	47%
EU	108 607 900 787	34%
SUM	320 216 054 003.00	100.00%

Additionally, we included parameters for the equity and bond share. To calculate the starting portfolio, we multiplied the constant starting value with the equity and bond share subject to testing before the resulting values was divided into the given country and region weights. These numbers are obtained from NBIMs report of historic investments per 31/12-2020.

5.7 Descriptive Statistics

In our dataset, the average monthly return of equity is 0.96 percent. Average monthly return of bonds is 0.65 percent and average inflation 0.37 percent.

Table 7: **Descriptive Statistics**

	Monthly Return	**Standard Deviation**
Equity	0.96%	6.63%
Bonds	0.65%	2.90%
Inflation	0.37%	0.57%
SUM	320 216 054 003.00	100.00%

This results in expected annual real returns of 4.91 percent with a 40 percent-equity share, 5.71 percent with 60 percent equities and 6.11 percent with 70 percent equities.

Table 8: **Expected Returns**

Asset Allocation	**Exp.Monthly Return**	**Exp.Annual Return**
40% Equity 60% Bonds	0.40%	4.91%
60% Equity 40% Bonds	0.46%	5.71%
70% Equity 30% Bonds	0.50%	6.11%

5.8 Limitations of the model

There are some limitations in our model that has an impact on the results. Parts of our bond returns are calculated to a proxy based on bond yields. These are an estimate and may differ from the actual bond returns in the past.

Secondly, every time we run a simulation, we will get different results as the criteria are tested on differently sampled portfolios. However, as we have performed the tests of each asset allocation and payout policy on the same simulations, we consider these differences to be very small and the results of the relative impacts to be reliable.

In addition, in the model, there are no oil-income taken into consideration, which there will be for several years to come. However, as oil is a limited resource, the oil-income will not last forever, meaning that the model will be even more relevant when these revenues stop coming.

The payout rules we have tested are not perfectly representable of how the payouts are implemented in real life. Rather than downsizing payouts in times of low returns, the opposite could be a realistic expectation for the government to work against economic oscillations. What causes the payouts is the deficit of the national budget, and not the funds performance.

Our results are based on past returns, which are not an accurate representation of future returns. We know that for instance Norges Bank expects the future returns to be significantly lower in the coming years, than for the past year. This will make an impact on the effect of a higher equity share.

6 Empirical results

6.1 Without Replacement 30-Year Forecast

6.1.1 Flexible rule

Table 9: **Results End Value Flexible Rule**

		End Value	Std End Value	% Not Maintain	% Fail
2%	**40/60**	4 016 252	1 348 224	2.64%	0%
2%	**60/40**	4 685 830	2 002 884	3.27%	0%
2%	**70/30**	5 013 601	2 439 544	3.97%	0%
3%	**40/60**	3 025 069	1 024 507	15.08%	0%
3%	**60/40**	3 532 897	1 524 125	12.81%	0.18%
3%	**70/30**	3 781 274	1 857 540	13.44%	0.37%
4%	**40/60**	2 272 290	776 490	43.05%	1.06%
4%	**60/40**	2 656 416	1 156 814	33.19%	1.62%
4%	**70/30**	2 844 124	1 410 750	31.41%	2.44%

Average real end values per capita decrease with a higher rate of withdrawals, giving almost a halving in expected end value by increasing the policy from 2 to 4 percent within each asset allocation. When changing the asset allocation from 40 to 70 percent, we observe an increase in average end value of almost exactly 25 percent within all payout policies. In line with this, we see that the highest end value is in the portfolio with the lowest payout policy and highest equity share, and oppositely the lowest end value is seen in the portfolio with the highest payouts and lowest equity share.

The probability of not maintaining the fund value with a 2 percent payout policy increases with a higher equity share. With 3 and 4 percent payouts the opposite pattern occurs. The probability of not maintaining the fund value decreases from 15 to 13 percent when the equity share goes from 40 to 60 percent. Increasing the equity share from 60 to 70 percent provides only a small difference in the probability of not maintaining the real fund value per capita. 4 percent payouts and 70 percent equities provide a probability of not maintaining fund value of 31 percent, whereas 40 percent equities have a 43 percent probability.

The fund will not fail with 2 percent and it is also very unlikely with a 3 percent payout policy for all asset allocations according to our model. Increasing the payout policy to 4 percent results in the highest probabilities of failure of the fund. Within the 4 percent, increases in the equity share provide larger probabilities of failure, going from 1.06 percent with the lowest share of equities, to 2.44 percent with the largest equity share.

Table 10: **Result Payouts Flexible Rule**

		Avg Payout	**StdDev Payout**	**Skewness**
2%	**40/60**	51 830	12 873	0.8156
2%	**60/40**	60 977	18 102	1.0680
2%	**70/30**	61 360	21 003	1.2226
3%	**40/60**	71 851	16 328	0.8047
3%	**60/40**	78 540	22 907	1.0571
3%	**70/30**	81 881	27 256	1.2036
4%	**40/60**	82 939	18 481	0.7930
4%	**60/40**	90 384	25 864	1.0452
4%	**70/30**	94 103	30 744	1.1916

Average real payouts per capita vary across the asset allocations and payout policices. An increase in the asset allocation increases the average payouts for all payout policies. We also observe higher payouts with an increase in the payout policy. Consequently, the highest average payouts are in the portfolio with 70 percent shares and a 4 percent payout policy. This portfolio generates almost twice as high payouts as the portfolio with payouts of 2 percent and 40 percent equities. We notice that the portfolios with 3 percent payouts and 60 and 70 percent equities provide almost the same payouts as the portfolio with a 4 percent payout policy and 40 percent equities.

The standard deviations of payouts also increase with a higher asset allocation for all payout policices. It is a significant increase in the standard deviation of payouts when increasing the payout policy for almost all asset allocation.

The skewness of the payouts is higher with a higher equity share. Within each asset allocation, the skewness is marginally lower with a higher payout policy. The differences are significantly larger between different asset allocations than between different payout policies. See figures below for distribution of average payouts per capita within the 4 percent policy across asset allocations. The average payout is marked with the vertical line in all the figures. This pattern also applies to 2 and 3 percent, see appendix A for all histograms.

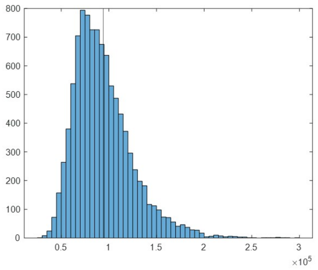

Figure 2: 40% Equities

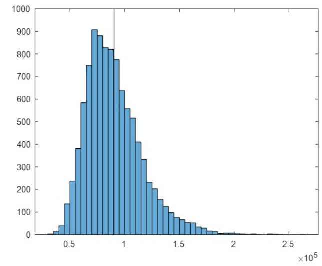

Figure 3: 60% Equities

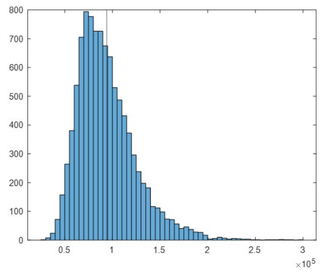

Figure 4: 70% Equities

Figure 5: Histograms 4% Payouts Flexible Rule

6.1.2 Ratchet rule

Table 11: **Results End Value Ratchet Rule**

		End Value	Std End Value	% Not Maintain	% Fail
2%	**40/60**	3 975 659	1 348 856	3.11%	0%
2%	**60/40**	4 603 438	1 999 548	4.02%	0.04%
2%	**70/30**	4 895 623	2 430 978	5.08%	0.11%
3%	**40/60**	2 960 608	1 026 951	17.37%	0.11%
3%	**60/40**	3 407 969	1 522 304	15.95&	0.53%
3%	**70/30**	3 605 781	1 849 734	17.23%	1.07%
4%	**40/60**	2 179 069	783 755	48.23%	2.67%
4%	**60/40**	2 483 851	1 162 171	40.18%	4.57%
4%	**70/30**	2 606 161	1 411 888	39.83%	6.64%

The average real end value per capita increases with an increase in the equity share, but with a higher rate of withdrawal we observe a reduction in the end value. When increasing the equity share from 40 to 60 percent the average end value increases with approximately 14 percent for all payout policies. When increasing the equity share further to 70 percent the end value increase by 6 percent. This pattern is repeated for all payout policies.

The probability of not maintaining the real fund value increases with a higher payout policy. With a 2 percent payout policy, a higher equity share increases the probability of not satisfying this criterion. When we look at the 3 percent portfolio, we see that the 60 percent equity portfolio has a lower probability of not maintaining the real fund value than both the higher and the lower equity shares. The policy to be the most sensitive to changes in asset allocation is the one with 4 percent payouts, in which 48 percent of the 40-percent equity-portfolios suffers loss of real value, relative to 40 percent of the ones with 70 percent equities.

The fund will not fail in our model with the ratchet rule given a 2 percent payout policy for all asset allocations. Also with a 3 percent payout policy the risk of failure is marginal. The probability of failure increases significantly when we increase the payouts further to 4 percent. With this payout policy we also observe that the probability increases with approximately 2 percentage points for each increase in the equity share.

Table 12: **Results payouts Ratchet Rule**

		Avg Payout	**StdDev Payout**	**Skewness**
2%	**40/60**	56 308	12 965	0.8298
2%	**60/40**	62 277	18 376	1.0858
2%	**70/30**	65 461	22 028	1.2377
3%	**40/60**	72 857	16 486	0.8236
3%	**60/40**	80 349	23 346	1.0824
3%	**70/30**	84 342	27 976	1.2370
4%	**40/60**	84 270	18 729	0.8203
4%	**60/40**	92 654	26 485	1.0830
4%	**70/30**	97 128	31 726	1.2403

Average real payouts per capita increase with an increase in the payout policy and asset allocation. For all payout policies we observe that there is an increase of 10 percent in payouts when increasing the equity share from 40 to 60 percent. When increasing the equity share further to 70 percent there is a 5 percent difference.

An increase in the equity share provides higher standard deviations. The standard deviations of payouts relative to the average, is rather constant in percentage terms within each asset allocation, being approximately 26 percent with 40 percent equities, 32 percentt with 60 percent equities and 37 percent with 70 percent equities. The absolute level of the standard deviation varies with the payout policies.

We observe a higher skewness with a higher equity share for all payout policies. Within each asset allocation, the skewness is marginally higher with higher withdrawals. We obtain larger differences between asset allocations than between different payout policies.

6.1.3 Average rule

Table 13: **Results End Value Average Rule**

		End Value	Std End Value	% Not Maintain	% Fail
2%	**40/60**	4 296 778	1 482 104	1.75%	0%
2%	**60/40**	5 029 303	2 090 986	2.51%	0.03%
2%	**70/30**	5 387 132	2 696 044	3.30%	0.05%
3%	**40/60**	3 312 376	1 171 235	10.64%	0.02%
3%	**60/40**	3 888 323	1 752 780	9.87%	0.14%
3%	**70/30**	4 169 040	2 141 584	10.60%	0.32%
4%	**40/60**	2 525 377	917 254	32.67%	0.69%
4%	**60/40**	2 973 836	1 378 096	25.69%	1.26%
4%	**70/30**	3 191 861	1 686 604	25.24%	1.99%

The average real end value per capita increase with an increase in the equity share. A lower rate of withdrawal also provides a higher average real end value per capita. The portfolio with a 2 percent rate of withdrawal and a 70 percent equity share provides the largest real end value per capita.

Higher payout policies increase the probabilities of not maintaining the fund value per capita with the average rule. With a 2 percent payout policy, the probability also increases with a higher equity share. In the scenario with 3 percent of withdrawals the probability stays approximately the same with an increase in the equity share and with a 4 percent policy the probability decreases with an increase in the equity share.

The probability of failure is close to zero for all asset allocations within a 2 and 3 percent rate of withdrawal. When increasing the payout policy to 4 percent the probability increases, but the increase is small with a maximum of only 2 percent. The max is found in the portfolio with the highest equity share.

Table 14: **Results payouts Average Rule**

		Avg Payout	**StdDev Payout**	**Skewness**
2%	**40/60**	50 398	11 328	0.8191
2%	**60/40**	54 885	15 867	1.0739
2%	**70/30**	57 129	18 871	1.2235
3%	**40/60**	66 828	14 922	0.8198
3%	**60/40**	72 648	20 893	1.0776
3%	**70/30**	75 559	24 847	1.2292
4%	**40/60**	78 839	17 465	0.8199
4%	**60/40**	85 537	24 440	1.0807
4%	**70/30**	88 886	29 062	1.2344

Average real payouts per capita increase with a higher equity share and payout policy. The portfolio with 70 percent equities and a 4 percent payout policy provides the highest average payouts. These payouts are 75 percent larger than the portfolio with the lowest average payouts. The standard deviations of the payouts are higher with both a higher equity share and a higher payout policy.

The skewness varies mostly across the three asset allocations, with the highest skewness found within the highest equity shares.

6.2 Interpretation of Results

6.2.1 Average Real End Value

Table 15: **Average Real End Value Per Capita WO Repl.**

		Flexible	Ratchet	Average
2%	**40/60**	4 016 252	3 975 659	4 296 778
2%	**60/40**	4 685 830	4 603 438	5 387 132
2%	**70/30**	5 013 601	4 895 623	5 387 132
3%	**40/60**	3 025 069	2 960 608	3 312 376
3%	**60/40**	3 532 897	3 407 969	3 888 323
3%	**70/30**	3 781 274	3 605 781	4 169 040
4%	**40/60**	2 272 290	2 179 069	2 525 377
4%	**60/40**	2 6565 416	2 483 851	2 973 836
4%	**70/30**	2 844 124	2 606 161	3 194 861

Our criterion is that the expected real end value per capita of the fund must equal at least 2 033 275 NOK per capita. As we can see from our results, all portfolios expected real end value satisfy this criterion.

The average end value decreases with a higher payout policy, which is expected as the withdrawals are higher while the expected return is constant within each asset allocation. Within each policy, a higher equity share provides higher average end values. From our results, we gather that a reduction in the payout policy has a larger impact on the average end value than the added expected return of an increase in the equity share.

Table 16: **1 STD Average Real End Value Per Capita WO Repl.**

		Flexible	Ratchet	Average
2%	**40/60**	2 668 028	2 626 802	2 814 673
2%	**60/40**	2 682 947	2 603 890	2 819 317
2%	**70/30**	2 574 056	2 464 645	2 691 088
3%	**40/60**	2 000 562	1 933 657	2 141 141
3%	**60/40**	2 008 772	1 885 665	2 135 543
3%	**70/30**	1 923 734	1 756 048	2 027 455
4%	**40/60**	1 495 800	1 395 315	1 608 123
4%	**60/40**	1 499 602	1 321 679	1 595 739
4%	**70/30**	1 433 374	1 194 273	1 505 257

Table 17: **Real End value - Standard Deviation WO Repl.**

		Flexible Rule	Ratchet Rule	Average Rule
2%	**40/60**	1 384 224	1 384 856	1 482 104
2%	**60/40**	2 002 884	1 999 548	2 209 986
2%	**70/30**	2 439 544	2 430 978	2 696 044
3%	**40/60**	1 024 507	1 026 951	1 171 235
3%	**60/40**	1 524 125	1 522 304	1 752 780
3%	**70/30**	1 857 540	1 849 743	2 141 584
4%	**40/60**	776 490	783 755	917 254
4%	**60/40**	1 156 814	1 162 171	1 378 096
4%	**70/30**	1 410 750	1 411 888	1 686 604

When looking at the average end value subtracted by one standard deviation, none of the portfolios with 4 percent payouts remain within the criterion. All the portfolios with 2 percent payouts are still satisfying the required end value. Within the 3 percent payouts, only the average rule with 40 and 60 percent equities remain within the requirement. This balances out the strictly higher average end value with a higher equity share, as we now see that the added risk of more equities does create a trade-off between a highest possible expected end value, and what certainty one can expect in terms of maintaining the real fund value.

6.2.2 Probability of Not Maintaining Fund Value

Table 18: **Probability of not Maintaining Fund Value WO Repl.**

		Flexible Rule	Ratchet Rule	Average Rule
2%	**40/60**	2.64%	3.11%	1.75%
2%	**60/40**	3.27%	4.02%	2.51%
2%	**70/30**	3.97%	5.08%	3.30%
3%	**40/60**	15.08%	17.37%	10.64%
3%	**60/40**	12.81%	15.95%	9.87%
3%	**70/30**	13.44%	17.23%	10.60%
4%	**40/60**	43.05%	48.23%	32.67%
4%	**60/40**	33.19%	40.18%	25.69%
4%	**70/30**	31.41%	39.83%	25.24%

The probability of not maintaining the fund value increases significantly with a higher payout policy. Consequently, the highest probability of not maintaining the fund value is found within the 4 percent payouts, and lowest within the 2 percent.

If we use today's composition of the fund, 70-30-3 as a starting point, we see that an increase in the payout policy to 4 percent more than doubles the probability of not maintaining the fund value, regardless of which rule we consider. By also reducing the equity share, the probability of not maintaining fund value increases even further, especially if we go all the way down to 40 percent equities. Should we rather reduce the payout policy from today's 3 to 2 percent, the risk of not maintaining the fund value will decrease significantly.

From this, we see a clear correlation between a higher payout policy and the downside risk of the fund.

With only 2 percent payouts, a decrease in the equity share results in a decrease in the probability of not maintaining the fund value. At this level of payouts, the fund operates with withdrawals well lower than expected return, regardless of the asset allocation. Therefore, the benefit of increasing the expected return with a higher equity share is lower than in portfolios with 3 or 4 percent payouts. Following higher rate of withdrawals, the bond share does not provide sufficient returns, and the fund relies on equities to obtain the required level of returns over time. Consequently, a higher equity share is beneficial for the fund's probability of maintaining its real value per capita when the withdrawals are close to the expected level of return. If the probability of maintaining the fund value was the most important criterion, the optimal equity share would change with different payout policies.

The ratchet rule has the highest probability of not maintaining the fund value for all portfolios. This is in line with what we expected because one will never reduce the spendings from the fund. During economic downturns with lower returns of the fund, the withdrawals will not be adjusted accordingly with a ratchet rule - in contrast to the flexible and average rule. Oppositely, with the average rule, both up- and downturns are only taken into account by one fifth of the yearly impact, which reduces volatility and ensures stability. This results in lower average payouts, which in turn provides the lowest probabilities of not maintaining the real fund value compared to the flexible and the ratchet rule.

6.2.3 Probability of Failure

Table 19: **Probability of Failure WO Replacement**

		Flexible Rule	**Ratchet Rule**	**Average Rule**
2%	**40/60**	0%	0%	0%
2%	**60/40**	0.05%	0.04%	0.03%
2%	**70/30**	0.04%	0.11%	0.05%
3%	**40/60**	0.03%	0.11%	0.02%
3%	**60/40**	0.18%	0.53%	0.14%
3%	**70/30**	0.37%	1.07%	0.32%
4%	**40/60**	1.06%	2.67%	0.69%
4%	**60/40**	1.62%	4.57%	1.26%
4%	**70/30**	2.44%	6.64%	1.99%

We see that the highest level of payouts provides the highest probability of failure. When using a 4 percent payout policy, the fund obtains a significantly higher probability of failure than with a 2 percent policy. All payout policies produce higher probabilities of failure with a higher equity share. Due to the higher payouts, the Ratchet rule suffers approximately three times higher probabilities of failure than the Average and the Flexible rule.

Despite relatively big changes from increases in the payout policies, all probabilities are quite small, and it seems unlikely that the fund will suffer failure based on our results.

6.2.4 Average Real Payouts Per Capita

Table 20: **Average payouts Without Replacement**

		Flexible Rule	**Ratchet Rule**	**Average Rule**
2%	**40/60**	51 830	56 308	50 398
2%	**60/40**	60 977	62 277	54 885
2%	**70/30**	61 360	65 461	57 129
3%	**40/60**	71 851	72 857	66 828
3%	**60/40**	78 540	80 349	72 648
3%	**70/30**	81 881	84 342	75 559
4%	**40/60**	82 939	84 270	78 839
4%	**60/40**	90 384	92 654	85 537
4%	**70/30**	94 103	97 128	88 886

Table 21: **Standard Deviation of Payouts WO Repl.**

		Flexible	**%**	**Ratchet**	**%**	**Average**	**%**
2%	**40/60**	12 873	25%	12 965	23%	11 328	22%
2%	**40/60**	18 102	30%	18 376	30%	15 867	29%
2%	**40/60**	21 003	34%	22 028	34%	18 871	33%
2%	**40/60**	16 328	23%	16 486	23%	14 922	22%
2%	**40/60**	27 256	33%	27 976	33%	24 847	33%
2%	**40/60**	18 481	22%	18 729	22%	17 465	22%
2%	**40/60**	25 864	29%	26 485	29%	24 440	29%
2%	**40/60**	30 744	33%	31 726	33%	29 062	33%

Today's portfolio satisfies our criterion of real average payout of 82 099 NOK per capita only using the ratchet rule. The rest of the portfolios with 2 or 3 percent payouts do not provide sufficient payouts for our criterion. With 4 percent payouts, only the portfolio with 40 percent equities and the average rule fails to meet the required level of withdrawals.

Based on our results, we notice that the main driver of the level of average payouts is the set policy. An increase in the equity share will also increase the average payout. The equity share obtains higher payouts as they yield higher returns on average, and therefore ensure the set percentage of a larger fund.

The standard deviation of payouts varies in accordance with the payout policy, and thereby the average payouts. A higher equity share increases the standard deviation, both in absolute values and in percentages of the average payouts. The percentages stay rather constant within each asset allocation across payout policies. This pattern is true for all payout rules. The standard deviation of the equities is higher than the one of bonds, which is why a higher equity share causes a higher standard deviation of the portfolio. A higher standard deviation implies more volatile and less certain expected future payouts in our model.

6.2.5 Skewness of Average Payouts

Table 22: **Skewness of Avg Payouts WO Repl.**

		Flexible	**Ratchet**	**Average**
2%	**40/60**	0.8156	0.8298	0.8191
2%	**60/40**	1.0680	1.0858	1.0739
2%	**70/30**	1.2226	1.2377	1.2235
3%	**40/60**	0.8047	0.8236	0.8198
3%	**60/40**	1.0571	1.0824	1.0776
3%	**70/30**	1.2036	1.2370	1.2292
4%	**40/60**	0.7930	0.8203	0.8199
4%	**60/40**	1.0452	1.0830	1.0807
4%	**70/30**	1.1916	1.2403	1.2344

We observe that all portfolios have positive skewness-coefficients which means that they are all right skewed.

In terms of downside risk, we see that a higher equity share increases the skewness of inflation adjusted payouts per capita. This tells us that we have more observations below the mean at a higher equity share, which implies that despite the level of average payout being satisfactorily, more of the yearly payments will be expected below the average and therefore not be adequate to satisfy our requirement.

Within each asset allocation, the skewness is not affected by what payout policy or payout rule selected. The Ratchet rule obtains fewer of the lowest-end average payouts than the Average and the Flexible rule due to its refusal of reducing payouts. This is visualised in the histograms below, all with 70-30-4 portfolios, with the Flexible to the left, Ratchet in the middle and the Average rule to the right.

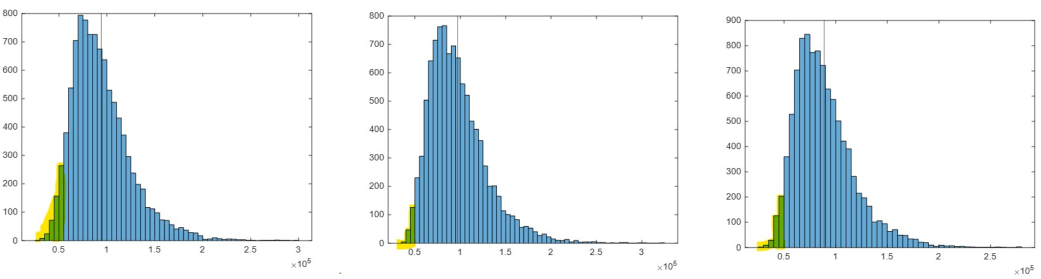

Figure 6: Lowest-end Average Payouts

6.3 Summary interpretation

All portfolios have an expected real end value per capita above today's value of 2 033 275 NOK. However, we see that the number of portfolios below the required level increases with a higher payout policy in both the probability of not maintaining real fund value and the probability of failure. Higher payout policies are also observed to require higher equity shares for the returns to keep up with the withdrawals and avoid drainage of the fund's value.

To satisfy the criteria of average payouts the withdrawal rate must be set to 3 percent with the ratchet rule or 4 percent for almost all rules. Today's portfolio of 70-30-3 is the one that has the highest expected average fund value among these portfolios and the lowest probability of not maintaining the fund value. Increasing the payout policy from today's portfolio will therefore increase the downside risk of the fund in terms of lower expected end value and a higher probability of not maintaining the fund value. This despite both 3 and 4 percent being significantly below the expected real return of our dataset, which is noteworthy due to the current argumentation that the payout policy should reflect the expected real return of the fund.

The Ratchet rule suffers the largest downside risk, in terms of probability of failure and expected ability to maintain its real fund value. Oppositely, the Average rule obtains the lowest chances of failure and to not maintain the fund value. All probabilities of failure are relatively small, except for the portfolios with the Ratchet rule and 4 percent payouts.

6.4 With Replacement 30-Year Forecast

6.4.1 Average Real End Value Per Capita

Table 23: **Average End Value With Replacement**

		Flexible Rule	Ratchet Rule	Average Rule
2%	**40/60**	4 335 503	4 294 826	4 654 576
2%	**60/40**	5 274 503	5 191 450	5 691 707
2%	**70/30**	5 816 110	5 191 450	6 291 769
3%	**40/60**	3 269 451	3 204 550	3 600 430
3%	**60/40**	3 984 262	3 857 901	4 423 730
3%	**70/30**	4 397 071	4 218 361	4 901 556
4%	**40/60**	2 458 889	2 364 131	2 755 504
4%	**60/40**	3 001 635	2 826 060	3 403 395
4%	**70/30**	3 315 470	3 072 204	3 780 693

All portfolios satisfy our criteria of an expected average end value of 2 033 275 NOK per capita. The expected values are higher in the simulation with replacement than without. However, we see that the same patterns apply. A higher payout policy provides lower average end values. Also, a higher equity share provides higher average end values.

Table 24: **1 STD Average Real End Value Per Capita W Repl.**

		Flexible	Ratchet	Average
2%	**40/60**	2 286 711	2 286 917	2 523 225
2%	**60/40**	3 605 582	3 597 738	4 002 388
2%	**70/30**	4 590 883	4 572 561	5 112 906
3%	**40/60**	1 740	1 742 392	2 001 856
3%	**60/40**	2 750 787	2 742 475	3 194 419
3%	**70/30**	3 507 197	3 485 661	4 094 383
4%	**40/60**	1 321 298	1 328 513	1 574 603
4%	**60/40**	2 093 360	2 090 503	2 529 099
4%	**70/30**	2 672 651	2 655 6	3 253 481

Table 25: **Real End value - Standard Deviation W Repl.**

		Flexible Rule	Ratchet Rule	Average Rule
2%	**40/60**	2 048 792	2 007 909	2 131 351
2%	**60/40**	1 668 922	1 593 712	1 689 318
2%	**70/30**	1 225 228	1 123 732	1 178 863
3%	**40/60**	1 528 992	1 462 158	1 598 574
3%	**60/40**	1 233 475	1 115 425	1 229 311
3%	**70/30**	889 874	732 700	807 173
4%	**40/60**	1 137 591	1 035 619	1 180 901
4%	**60/40**	908 275	735 557	874 296
4%	**70/30**	642 820	416 524	527 212

The standard deviations of average real end value per capita with replacement are larger in both absolute and relative terms than without replacement, which implies even fewer portfolios which satisfies our required level if they end up one standard deviation below the average.

6.4.2 Probability of Not Maintaining Fund Value

Table 26: **Probability of not Maintaining Fund Value w Repl.**

		Flexible Rule	Ratchet Rule	Average Rule
2%	**40/60**	10.34%	11.14%	8.84%
2%	**60/40**	11.40%	12.39%	10.16%
2%	**70/30**	12.45%	13.90%	11.37%
3%	**40/60**	24.59%	26.42%	20.50%
3%	**60/40**	22.40%	25.20%	19.21%
3%	**70/30**	22.97%	26.50%	20.09%
4%	**40/60**	44.96%	48.55%	38.29%
4%	**60/40**	38.52%	43.10%	33.36%
4%	**70/30**	37.06%	42.73%	32.98%

All probabilities are significantly larger in the simulation with replacement compared to without. Also with replacement, a higher payout policy obtains higher probabilities of not maintaining the fund's real value per capita. In terms of equity share, we observe the same pattern as without replacement, in which a higher payout policy experiences a larger benefit of an increase in equity share. In relative sizes, there is a smaller difference between the obtained results with or without replacement with a 4 percent policy than the two lower.

6.4.3 Probability of Failure

Table 27: **Probability of Failure with Replacement**

		Flexible Rule	Ratchet Rule	Average Rule
2%	**40/60**	0.50%	0.63%	0.41%
2%	**60/40**	1.06%	1.48%	1.01%
2%	**70/30**	1.72%	2.38%	1.61%
3%	**40/60**	2.06%	2.66%	1.73%
3%	**60/40**	3.11%	4.79%	2.81%
3%	**70/30**	4.57%	6.57%	4.01%
4%	**40/60**	7.08%	10.06%	5.73%
4%	**60/40**	8.36%	12.83%	7.27%
4%	**70/30**	9.76%	15.33%	8.93%

The simulation with replacement suffers significantly higher probabilities of failure relative to the simulation without. Both a higher equity share and a higher payout policy provides strictly higher probabilities of failure.

6.4.4 Average Payouts

Table 28: **Average Payout Per Capita w Repl.**

		Flexible Rule	Ratchet Rule	Average Rule
2%	**40/60**	34 771	57 989	51 748
2%	**60/40**	63 907	65 232	57 230
2%	**70/30**	65 207	69 402	60 243
3%	**40/60**	73 907	74 948	68 580
3%	**60/40**	82 147	84 015	75 697
3%	**70/30**	86 687	89 232	79 608
4%	**40/60**	85 194	86 609	80 855
4%	**60/40**	94 335	96 739	89 049
4%	**70/30**	99 366	102 568	93 554

Table 29: **Standard Deviation of Payouts with Replacement**

		Flex	%	Ratch	%	AvgR	%
2%	**40/60**	18 297	52.6%	18 287	31.5%	15 703	30.3%
2%	**60/40**	26 511	41.5%	26 605	40.8%	22 574	39.4%
2%	**70/30**	31 569	31.1%	32 499	30.6%	27 374	45.4%
3%	**40/60**	22 995	31.1%	22 967	30.6%	20 543	30.0%
3%	**60/40**	33 230	40.5%	33 325	39.7%	29 530	39.0%
3%	**70/30**	40 412	46.6%	40 652	45.6%	35 813	45.0%
4%	**40/60**	25 774	30.3%	25 689	29.7%	23 863	29.5%
4%	**60/40**	37 135	39.4%	37 163	38.4%	34 293	38.5%
4%	**70/30**	45 096	45.4%	45 270	44.1%	41 591	44.5%

Both a higher equity share and a higher payout policy results in higher average payouts. Also the payouts are larger with replacement than without. The standard deviation of payouts is also larger, not only in absolute terms but also in relative sizes of the average payouts.

Table 30: **Skewness of Avg Payouts W Repl.**

		Flexible	Ratchet	Average
2%	**40/60**	1.1158	1.1264	1.0933
2%	**60/40**	1.5060	1.5181	1.4626
2%	**70/30**	1.8037	1.7998	1.7223
3%	**40/60**	1.0997	1.1162	1.0947
3%	**60/40**	1.4848	1.5045	1.4660
3%	**70/30**	1.7621	1.7829	1.7277
4%	**40/60**	1.0821	1.1115	1.0950
4%	**60/40**	1.4611	1.4958	1.4682
4%	**70/30**	1.7334	1.7720	1.7318

Lastly, we see that the distribution of payouts is more skewed with than without replacement. In the histograms on the following page, we see that outliers in the 70 percent equity portfolio is significantly larger than the ones in the same portfolio without replacment.

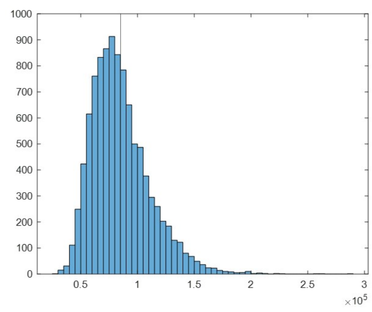

Figure 7: 40% Equities

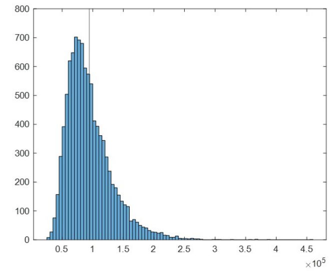

Figure 8: 60% Equities

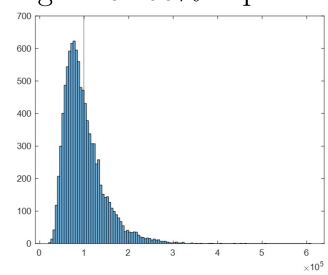

Figure 9: 70% Equities

Figure 10: Histograms 4% Payouts Flexible Rule With Replacement

6.4.5 Interpretation of results

Overall, all numbers provided by the simulation with replacement is substantially higher than the one without. This is because the simulation with replacement, unlike the simulation without, has almost infinitely many possible paths the fund can take, and the results will vary substantially more in both positive and negative direction. From this, there will be more portfolios unable to maintain real fund value and fail, but also more portfolios that obtains very high returns which causes the higher average end values and payouts. From this, more of the portfolios manage to meet the required level of average payouts from our criterion, but they obtain the same probability of not maintaining the real fund value and lower average end value than today's portfolio. Therefore, our interpretation does not change from the results without replacement.

We observe the same trends and patterns in terms of impact of both equity share and payout policy. Therefore, we conclude that our observations of previous results stay relevant.

7 Conclusion

We conclude that if we assume that the current level of payouts is desired to be maintained, the current equity share of 70 percent with a ratchet rule is sustainable in terms of the downside risk of the fund. If we lower the equity share today in order to reduce the downside risk in terms of volatility, the average payouts will not satisfy our criterion and is therefore not a plausible option based on our model. Therefore, we reject our hypothesis that it would be optimal to reduce the equity share to reduce the downside risk of the fund. Among all portfolios that can maintain the required level of payouts, today's portfolio with the ratchet rule obtains the lowest probability of not maintaining the real fund value and failure, in addition to the highest expected end value. We therefore conclude, given that the level of payouts is desired to be maintained, it's not beneficial in terms of the downside risk to neither increase the payout policy nor reduce the equity share.

Our conclusion is also confirmed from our simulation with replacement. The probability of maintaining the fund value does not decrease with a lower equity share given today's payout rule.

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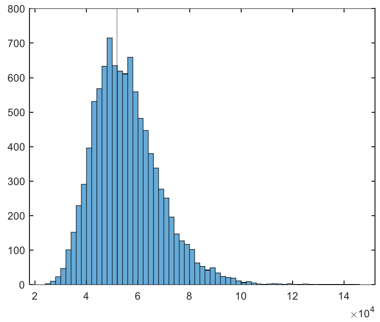
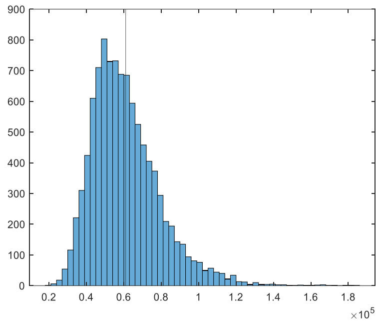
Appendix A

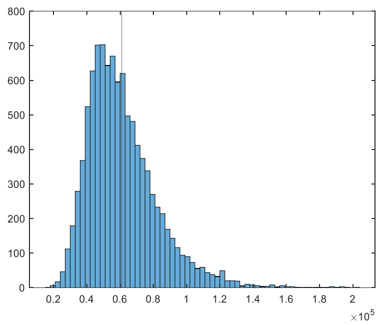
A. Flexible Rule, 40% Equities. 2% Payouts



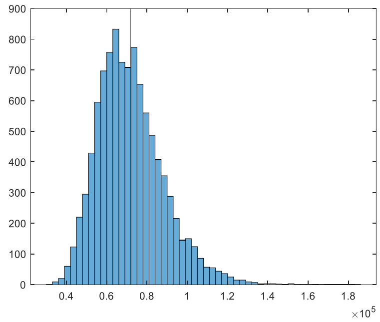
B. Flexible Rule, 60% Equities. 2% Payouts



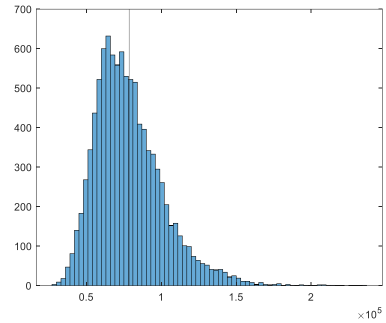
C. Flexible Rule, 70% Equities. 2% Payouts



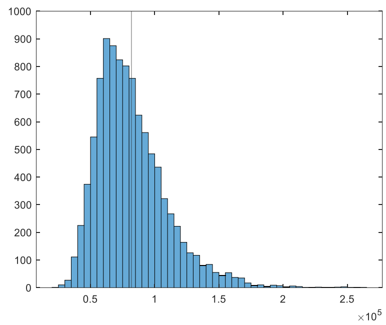
D. Flexible Rule, 40% Equities. 3% Payouts



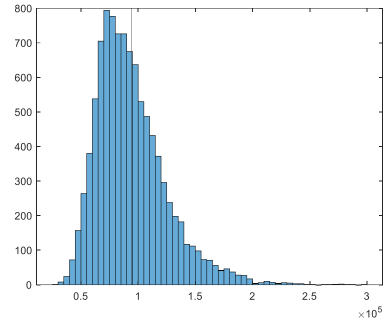
E. Flexible Rule, 60% Equities, 3% Payouts



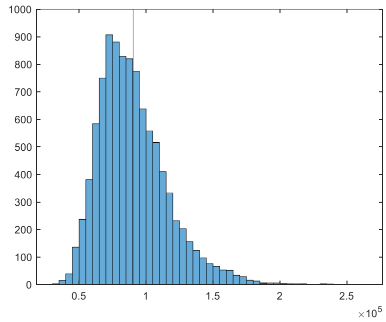
F. Flexible Rule, 70% Equities, 3% Payouts



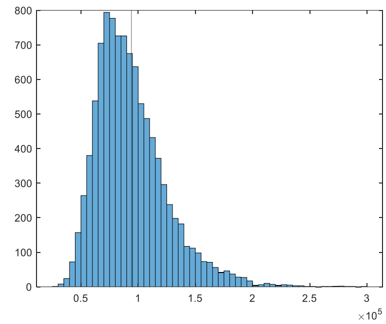
G. Flexible Rule, 40% Equities, 4% Payouts



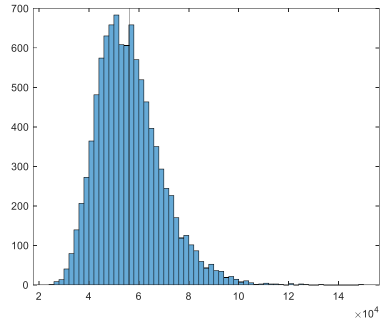
H. Flexible Rule, 60% Equities, 4% Payouts



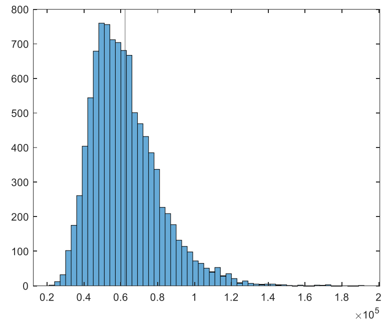
I. Flexible Rule, 70% Equities, 4% Payouts



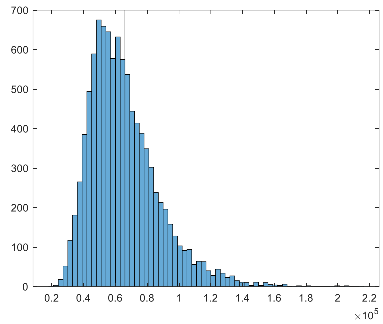
J. Ratchet Rule, 40% Equities, 2% Payouts



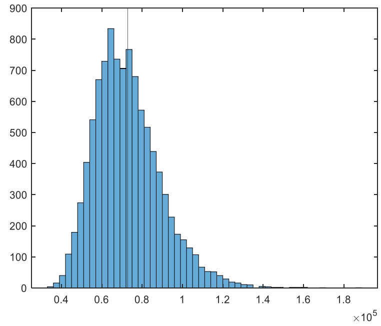
K. Ratchet Rule, 60% Equities, 2% Payouts



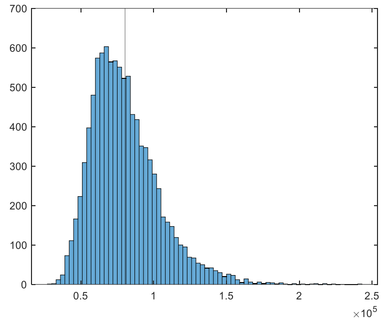
L. Ratchet Rule, 70% Equities, 2% Payouts



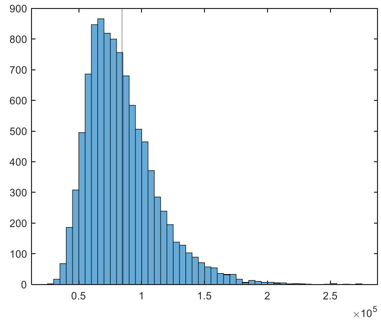
M. Ratchet Rule, 40% Equities, 3% Payouts



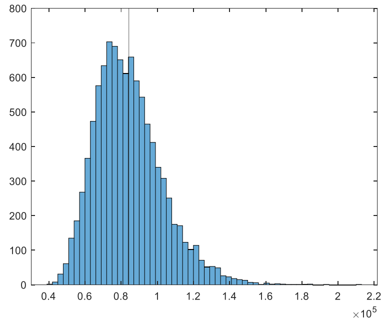
N. Ratchet Rule, 60% Equities, 3% Payouts



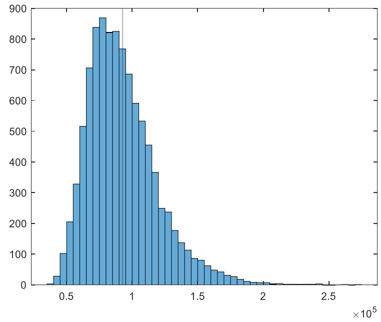
O. Ratchet Rule, 70% Equities, 3% Payouts



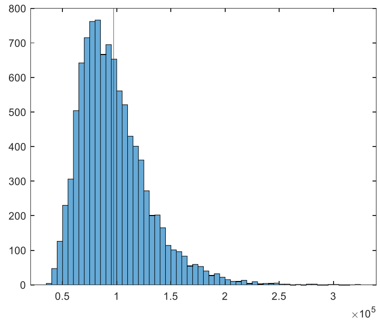
P. Ratchet Rule, 40% Equities, 4% Payouts



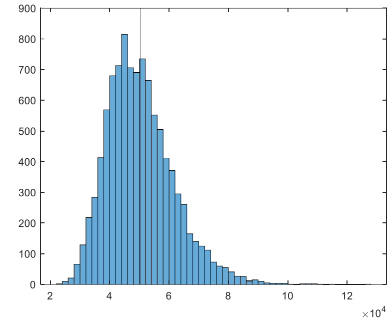
Q. Ratchet Rule, 60% Equities, 4% Payouts



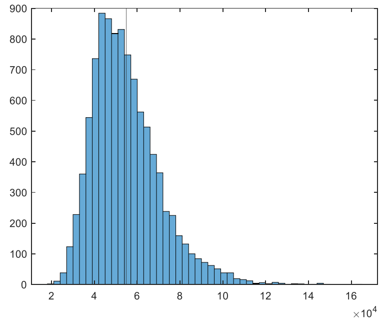
R. Ratchet Rule, 70% Equities, 4% Payouts



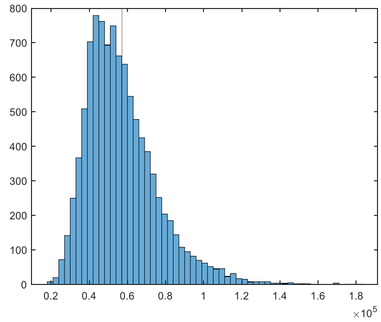
S. Average Rule, 40% Equities, 2% Payouts



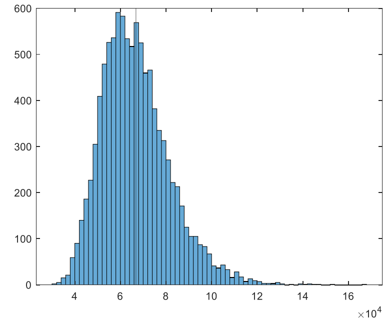
T. Average Rule, 60% Equities, 2% Payouts



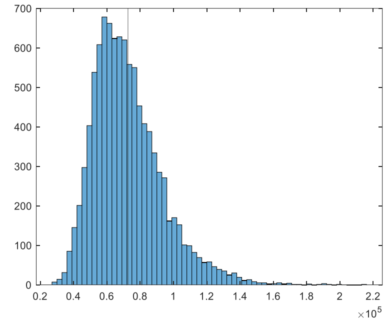
U. Average Rule, 70% Equities, 2% Payouts



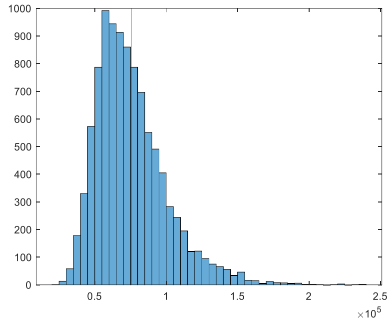
V. Average Rule, 40% Equities, 3% Payouts



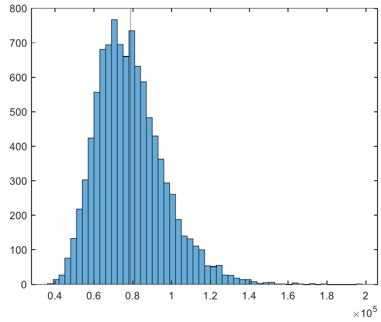
W. Average Rule, 60% Equities, 3% Payouts



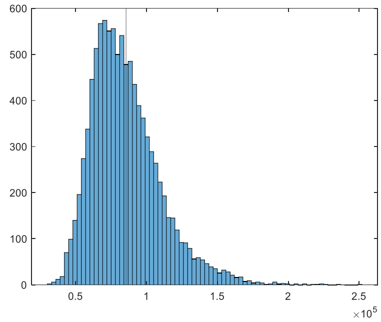
X. Average Rule, 70% Equities, 3% Payouts



Y. Average Rule, 40% Equities, 4% Payouts



Z. Average Rule, 60% Equities, 4% Payouts



AA. Average Rule, 70% Equities, 4% Payouts