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Treasury Bond Auctions: Repurchase Agreements and Bidding Behavior

Master Thesis

by
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ABSTRACT

We investigate bidding behavior and auction performance in Norwegian treasury bond auctions. We test theoretical predictions of bidding behavior introduced by Back and Zender (1993) and Kyle (1989). The models help explain many of the dynamics of bidding behavior we find, but the demand schedules submitted contributes to an overall rejection of the models. In addition, we introduce an explanatory variable that helps explain the interaction between bidding behavior and pre-auction inventory. We find that dealers with larger pre-auction repurchase agreements bid higher prices in the auction, implying that pre-auction inventory impacts the valuation of the auctioned asset, just as predicted in the simplified equilibrium model proposed by (Nyborg and Strebulaev (2004)).

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1 Introduction and motivation

In this thesis, we study Norwegian treasury auctions and bidding behavior. Governments issue debt to finance operations and deficits, and treasury auctions are the most common way for advanced economies to issue debt. However, there is a cost to this primary-market operation, and researchers have conducted countless studies on the most cost-efficient way of issuing government debt. The most common are discriminatory- and uniform price auctions. In both cases, the bidders submit demand schedules, and the securities are given out in descending order until supply is exhausted.[1] In a uniform auction format, the auction winners pay the stop-out price for all units, while in a discriminatory auction, the winners pay what they bid. Norway has utilized a uniform treasury auction system since 2000.

The Norwegian treasury market has a small number of bidders, a predetermined supply, and a liquid secondary market. In addition, Norway is in a unique position, primarily because there exists less demand to loan money for the budget deficit because of the petroleum oil fund. From 2006 onward, Norway has adopted a primary dealer system, where a selected number of banks are obligated to bid in the auction. Before the primary dealer system, any bank could attend the auction. In 2006, there were six primary dealers, while in our newest sample in 2021, there are only four primary dealers.

We study auction performance and bidding behavior in Norwegian treasury auctions from 2006-to 2021. Auction performance is an important metric, as even though the underpricing might be a small percentage, the large amount of quantity supplied can lead to billions in the cost of financing. We mainly investigate performance by analyzing how dealers submit their demand schedules compared to expected bidding behavior from theory, following (Keloharju

[1]Demand schedule is defined as all bids and quantity demanded for each bidder in an auction.

et al. (2005)). The theory of bidding behavior explains why dealers submit their respective demand schedules. A thorough understanding of bidding behavior is vital to maximize the value for both dealers and sellers. Most research is conducted on the US treasury market, but we have seen several studies on other economies in recent years, including the Scandinavian countries. In 2020 Robert B. Wilson and Paul R. Milgrom were awarded the Nobel prize for economics for their contribution to auction theory, solidifying the importance of the research.

We find the equilibrium models proposed by Wilson (1979), Back and Zender (1993) and Kyle (1989) help explain many characteristics found in the Norwegian market. Primary dealers submit steep demand schedules that are disadvantageous for the seller. Bidders respond to an increase in uncertainty by submitting deeper discounts and increase dispersion. Kurtosis and negative skewness increase with the number of bidders. However, the insignificance of important variables and the fact that dealers submit demand schedules above the secondary market price contribute to an overall rejection of the models.

In addition to our study of auction theory, we contribute to academic research by introducing pre-auction inventory as an explanatory variable. To our knowledge, this has not been done in the Norwegian treasury market thus far. Our research on pre-auction inventory is primarily based on (Nyborg and Strebulaev (2004)). They derive equilibrium models that show that in multi-unit auctions, the potential of a short-squeeze affects each bidder's behavior and valuation of the auctioned asset. Our secondary market interaction variable can provide a more complete picture of bidding behavior in the Norwegian market and contribute to academic literature. We determine the dealer's position in the secondary market through repurchase agreements. The primary dealer can borrow securities from the Norwegian government using repurchase agreements. We believe dealers will behave more aggressively in the auction if they have short exposure to the asset pre-auction. The thought process is that a dealer would want to cover their short position to avoid being short-squeezed. Therefore, our hypothesis would indicate that a dealer with a larger short position to the security would submit demand schedules with smaller discounts, demand more quantity, and increase the highest bid compared to a dealer with a long or smaller short position. Hence, our primary research question is the following:

Does the bidder's pre-auction position impact bidding behavior in Norwegian treasury bond auctions?

As a secondary research question, we control for auction theory and derive testable measures of bidding behavior to see how the models predict behavior. The research on testable measures follows the study conducted on Finnish treasury auctions by (Keloharju et al. (2005)).

Another proposition we test from Nyborg and Strebulaev (2004) states that the mean rate of bids is higher for dealers with higher short-exposure. The proposition follows the notion that dealers value the security differently dependent on pre-auction inventory. We find evidence that bidders with larger pre-auction repurchase agreements in Norwegian treasury auctions bid higher prices.

Section 2 of the thesis presents a detailed overview of existing literature and theories regarding bidding behavior and auction performance. Section 3 derives testable implications of bidding behavior from auction theory. We outline models put forward by Wilson (1979), Back and Zender (1993), and Kyle (1989). Which provides a detailed explanation of how different equilibrium strategies employed by bidders create underpricing in the auction. We also outline an equilibrium model by Nyborg and Strebulaev (2004) where pre-auction inventory varies amongst bidders.

Next, we explain our methodology and hypothesis. Followed by an institutional background of the Norwegian treasury market and presentation of the data. Section 6 presents univariate and multivariate analysis of the auctions. We present three univariate analyses that show different dynamics of treasury bond auctions. These are summary statistics for the auctions, reopening cycle, and individual bidding behavior. Section 6 also presents our main multivariate analysis and discusses the findings and theory. Finally, section 7 concludes the paper and recommends future research points.

2 Literature review

Early literature on bidding behavior and auction theory Wilson (1979) studies the price resulting from two auction pricing formats: unit and share auction. The study's conclusion is that share auctions can result in significantly lower prices, inducing loss in revenue for the seller. He describes the main features contributing to the underpricing. First, he argues that the seller might not obtain any advantages from the increased competition in the auction. Secondly, the variety of optimal strategies may result in the bidders adopting an unfavorable strategy for the seller. Further, he states that the conclusion might not be disadvantageous to the seller, as bidders might not adopt the optimal strategy. Wilson (1979) theory has contributed vastly to modern-day auction theory.

Back and Zender (1993) compare uniform-price auction with discriminatory price auction format, assuming the good is perfectly dividable between bidders. One of the article's main points revolves around collusive strategies and how they are self-enforcing in uniform-price auctions. They conclude that in equilibria, the underpricing in uniform auctions is more severe than in discriminatory auctions. However, a lot of newer research disagrees with their conclusion. Goldreich (2007) concludes that underpricing is cut in half by changing from discriminatory to uniform format in the US Treasury market. Back and Zender (1993) continues by suggesting a policy that might be effective for the auctioneer using a uniform format. Deciding on the quantity post-bidding can help remove inframarginal bids, so it might be able to eliminate the collusive equilibria described. This is an approach adopted by the Finnish Treasury, and newer studies by Keloharju et al. (2005), and McAdams (2007) suggest lower underpricing under this standard.

The empirical study by Goldreich (2007) shows that underpricing increases in the dispersion of signals, which can affect bidding behavior. Also, he finds that theory surrounding more underpricing in uniform auctions does not correspond to the results in the market being researched.

Existing theory suggests that uniform price auctions are subject to severe underpricing, resulting from market power that arises endogenously. Each participant is a monopsonist over the supply left after other bidders' demand has been filled. Kremer and Nyborg (2004) criticizes the theory put forward by Wilson (1979) and Back and Zender (1993) by presenting a framework that corresponds better to real-world market dynamics. They incorporate discrete bids in the model, meaning that the participants in the auctions submit price-quantity pairs rather than demand functions. Further, they argue that there is a minimum tick size for prices and quantity multiple. They show that this profoundly impacts the set of equilibria and that underpricing can be reduced or even eliminated. This is a result of discreteness, making the marginal residual supply large. This creates price competition for the marginal units and thus reduces the profits for bidders.

Also, altering how supply is allocated when there is excess demand shows how underpricing can be eliminated. Their suggestion is to give this inframarginal demand only partial preference rather than giving preference to demand above the stop-out price. This may lead to an increase in the stop-out price as it stimulates aggressive bidding on the margin.

We explore several non-linear equilibria in the thesis. The risk-neutral market power equilibrium is outlined by (Back and Zender (1993)). Wang and Zender (2002) and Kyle (1989) expand on the proposed equilibrium by introducing a risk-aversion coefficient. The main finding in all the models is that bidders exercise market power by submitting downward-sloping demand schedules.

Several research papers study bidding behavior, including extensive research in the Scandinavian countries. Nyborg et al. (2002) looks at bidding behavior in Swedish treasury auctions. They regress the endogenous bidding variables (dispersion, discount, and quantity) on volatility and auction size. Their main finding is that bidders respond with more caution when uncertainty increases. The bidders increase dispersion, reduce quantity demanded, and reduce price levels with uncertainty. The findings support the theory of bidders adjusting for the winner's curse. Bjønnes (2001) finds similar results in the Norwegian market regarding the winner's curse by testing how bidders respond to an increase in the number of bidders. However, this study was conducted when Norway practiced discriminatory auctions. He concludes that the Norwegian treasury could reduce underpricing by changing the auction format, which they did in October 2000.

Bjønnes and Rydqvist (2012) study bidder behavior and auction performance in the Norwegian treasury market. The market is unique because of its relatively small number of primary dealers, uniform-price auctions, and a treasury willing to commit to a pre-announced quantity for sale. They explore that bidders exercise market power by submitting steep demand schedules, resulting in less-than-optimal competition at the stop-out price, significant underpricing, and profit for bidders.

To get perspective, they compare the bidding behavior in the Norwegian market, where the treasury determines auction size before bidding, to that of the behavior in the Finnish market, where the supply is determined post-bidding. Bjønnes and Rydqvist (2012) shows that this slight difference in auction format leads to a dramatically different demand schedule. Keloharju et al. (2005) find that the average underpricing in Finland is only one-quarter of that in Norway.

Further, Bjønnes and Rydqvist (2012) finds that the change in auction format affects the bidding behavior. Around the time when the auction format changes from discriminatory to uniform, demand schedules shift outwards and become steeper. This means that they have smaller price discounts and more bid dispersion.

Our thesis's primary goal is to study pre-auction inventory's effect on bidding behavior. We introduce an explanatory variable that shows the effect of repurchase agreements on bidding behavior. We start our investigation by researching how market players position themselves before auctions.

Sigaux (2018) explains how bidders trade ahead of auctions. In the model, a risk-averse investor expects an uncertain increase in the supply of the risky asset. He explains that bidders face a trade-off between hedging the supply uncertainty by going long in the market and speculation with short positions. As the auction date approaches, uncertainty decreases because of information flowing in. Therefore, investors hedge less and speculate more through shorting, which results in a price decrease in the security trading in the market.

Nyborg and Strebulaev (2004) developed theories on how the possibility of a short squeeze in the secondary market can impact bidding behavior in multiunit auctions. They derive mixed-strategy equilibrium models. Their main finding is that different bidders have differing valuations of the auctioned asset depending on their pre-auction inventory. This is the basis for our analysis. They explain that short-bidders will act aggressively in the auction to prevent being short-squeezed. While long players could either act passively to free-ride on other longs attempts or bid aggressively to try and create a short squeeze. They show that short-players can implement a costless strategy to eliminate short-squeezing by acting aggressively in a uniform auction.

Chatterjea and Jarrow (1998) study the US treasury market and show how market manipulation can happen in an equilibrium model. The research is based on the Salomon Brothers short-squeeze of the treasury market in 1991, where they obtained 94% of a two-year treasury note in an auction. Followed by squeezing the short-players by charging a premium in the secondary market. Jegadeesh (1993) investigate the instance and find evidence of collusion and market manipulation. Chatterjea and Jarrow (1998) develop equilibrium models which show market manipulation in both uniform and discriminatory auction systems. However, they conclude that market manipulation does not happen in a long-term equilibrium in uniform auctions. They argue that this is because the uniform system encourages aggressive bidding.

Rydqvist and Wu (2016) research pre-auction inventory and bidding behavior in Canadian treasury auctions. Canada utilizes a discriminatory treasury auction system. They find that bidders with short and long pre-auction inventory bid more aggressively in comparison with neutral bidders and that auction prices of treasury securities vary with the ownership structure of the asset. However, they conclude their research by stating that the occurrence of short-squeezes is unlikely to explain behavior in Canadian treasury auctions.

3 Testable implications of bidding behavior

This section serves several purposes. First, we will explain central theories regarding uniform auctions and introduce auction theory and underpricing equilibria. Finally, we present equilibrium models where bidders have different pre-auction inventory.

3.1 Theory of bidding behavior in treasury bond auctions

The general underpricing equilibria we explore is based on Wilson (1979) and Back and Zender (1993). They show equilibrium in uniform price auctions where all bidders submit downward-sloping demand functions. The decreasing demand functions lead to underpricing. That is, the secondary market price is higher than the stop-out price in the auction. The models are based on Nash equilibrium. In a Nash Equilibrium, when all bidders submit the same demand schedule, it is optimal for the last bidder to submit the same demand schedule. There is no other strategy that can give the player a better payoff. An agent could increase the quantity acquired by submitting a higher demand schedule. However, this would reduce the underpricing in the auction. Leading to less profit for the bidders. Therefore, each bidder is in an "agreement" to submit steep demand schedules to give each other market power and create underpricing. The model outlined by Wilson (1979) and Back and Zender (1993) investigate risk-averse bidders. We will also explore a framework outlined by Kyle (1989) and Wang and Zender (2002) that expands equilibrium to bidders with a risk-aversion coefficient. Later we will use these models to compare the predicted theoretical statistics to our findings in the Norwegian treasury auctions and check if the statistics respond to exogenous variables as the theory predict.

The second part of our theoretical section will explore the theory of pre-auction inventory and bidding behavior. The theory states that bidding behavior will vary with pre-auction trading and ownership structure. Nyborg and Strebulaev (2004) study the impact of potential short-squeezes on equilibrium bidding strategies. A short squeeze occurs when a player with negative post-auction inventory must cover their short position through a long-player in the secondary market. We outline some of Nyborg and Strebulaev (2004) theory and propositions. In a later section, we seek to test the predictions in the Norwegian treasury market through regression analysis.

3.1.1 Risk neutral market power

Our outline of the theory follows Kremer and Nyborg (2004). In this model, there are N identical bidders, Q is the auction size, v is the value of the security (which is known by all players in the auction), p_0 is the stop-out price, and $x\left(p\right)$ is the demand schedule submitted by each bidder. As mentioned earlier, the following model is a symmetric equilibrium, meaning that each bidder will submit the same demand schedule. Academic papers have found several equilibria that result in $p_0 < v$, leading to an underpriced auction. The theory is that bidder I maximize profit when submitting a steep demand schedule, and aggregate demand equals supply at the stop-out price. The problem is to determine a demand schedule that maximizes the following problem.

$$\max_{y}[v - p_0(y)]y(p_0) \tag{1}$$

With the condition that aggregate demand equal supply:

$$\left(N-1\right)x\left(p_0\right) + y(p_0) = Q \tag{2}$$

In a symmetric equilibrium $x = y$, hence Equation (2) can be written as:

$$Nx\left(p_0\right) = Q \tag{3}$$

The optimization problem can be modified and simplified to profit-maximizing the stop-out price (see Kremer and Nyborg (2004)). The bidders problem can then be written as:

$$\max_{p_0}[v - p_0][Q - (N-1)x(p_0)] \tag{4}$$

The first term is the profit per unit and the second term is the residual supply. The dealer can increase the stop-out price, but the residual supply will decrease and vice versa. The first-order condition is given by

$$(N-1)\,(v - p_0)\,x'(p_0) + Q - (N-1)x(p_0) = 0 \tag{5}$$

Substituting Equation (2) into the first-order condition gives a multitude of solutions. We examine non-symmetric equilibria for both risk-averse and risk-neutral bidders.

3.1.2 Non-linear equilibria

The non-linear equilibrium is derived by setting the first-order condition to hold at the set of all possible stop-out prices. This is particularly interesting for auctions with uncertainty about the stop-out price. The equilibrium is given by:

$$x\,(p) = a\left(1 - \frac{p}{v}\right)^{\frac{1}{N-1}} \tag{6}$$

Where a is an unknown paramater that satisfies $a \geq Q/N$. The inverse demand schedule is:

$$p = \left(1 - \left(\frac{q}{a}\right)^{N-1}\right)v \tag{7}$$

The demand schedule inhibits concavity when $N > 3$. Meaning that the slope is more negative for each additional unit demanded. If a player seeks to gain a more significant share of the auction, it must bid more aggressively. However,

Table 1: Testable measures of bidder behavior

	Formula	N = 4	N = 6
Standardized discount	$\frac{\sqrt{2N-1}}{N-1}$	0.882	0.663
Skewness	$-\frac{2(N-2)\sqrt{2N-1}}{3N-2}$	-1.058	-1.658
Kurtosis	$\frac{3(2N-1)(6-5N+2N^2)}{(4N-3)(3N-2)}$	2.908	4.714

The table shows testable measures of bidding behavior (standardized discount, skewness, and kurtosis) in the risk-averse model by Back and Zender (1993) for $N = 4$ and 6.

the player is offset by doing so because of the considerable price increase of the security.

Keloharju et al. (2005) derive testable measures of bidder behavior in the model. In Table 1 we present the variables that are independent of the reservation price (r) and unknown positive constant (a). The fact that they cancel out is a surprising result, seeing that a and r will likely vary from auction to auction. Nevertheless, this allows comparison in a cross-section of auctions. The variables are standardized discount, skewness, and kurtosis. The model studied is a risk-neutral model. However, they also provide measures for risk-averse models, which will be explored later (see Keloharju et al. (2005) for full exposition). We will compare the two different periods when we have four and six primary dealers.

As we can see, kurtosis and negative skewness increase with N. Keloharju et al. (2005) shows that the model exhibits a concave demand function that explains the negative skewness in the bid distribution. In this model by Back and Zender (1993) the predictions are constant only depending on the number of bidders in the auction. This model assumes that players are risk-neutral, and from Table 4, we find some evidence of risk-averse bidders in Norwegian treasury auctions because of dispersion of bids when uncertainty increase. It is important to note that there could be other reasons for the dispersion of bids we find, like bidders adjusting for the winner's curse. Kyle (1989) and Wang and Zender (2002) present symmetric non-linear equilibria that incorporates a risk-aversion coefficient. Wang and Zender (2002) finds the following inverse demand schedule:

$$p = \left[1 - \left(\frac{q}{a}\right)^{N-1}\right] v - \left[1 - \left(\frac{q}{a}\right)^{N-2}\right] \left(\frac{N-1}{N-2}\right) \rho\sigma^2 q \tag{8}$$

In this model, the bidders have CARA utility with risk-aversion coefficient ρ and the standard deviation of the secondary market σ^2. An attractive property of this model is that with a risk-aversion coefficient of 0, the model is only left with the first term on the right-hand side, which is the exact inverse demand schedule as Back and Zender (1993) equilibria. The second term is considered a risk premium. In the risk-averse model, the three bidder behavior measures are dependent on the unknown positive constant and reservation price, except for when a is equal to:

$$a = \frac{N-2}{N-1} \cdot \frac{1}{\rho\sigma^2} \tag{9}$$

In this case, the non-linear equilibrium presented transforms into Kyle (1989) linear equilibrium. This equilibrium can be viewed as Wang and Zender equilibrium with a risk-aversion coefficient of 0.5. The three bidder behavior statistics we are comparing have the surprising result of being constant because of the linearity of the model. Surprisingly, volatility does not affect the bidding variables, even though bidders are risk-averse. Keloharju et al. (2005) show that the volatility is precisely offset by the reduction in quantity demanded when uncertainty increase. The predicted bidder behavior measures that does not depend on a and r in Kyle (1989) equilibrium is in Table 2:

Keloharju et al. (2005) derive expressions for testable bidder measures in Wang and Zender (2002) equilibrium model. They are extremely complicated and were not reported in the paper. Nevertheless, we corresponded with professor Kristian Rydqvist and obtained the derivations. However, from advice given by Rydqvist, we drop these measures by the principle of Occam's razor.

Table 2: Prediction bidder behavior from Kyle

Variable	Prediction
Standardized discount	$\sqrt{3} = 1.732$
Skewness	0
Kurtosis	1.8

The table shows testable measures of bidder behavior (standardized discount, skewness, and kurtosis) in the risk-averse model by Kyle. The predictions put forward are constants.

3.2 Theory of pre-auction inventory impact on bidding behavior

In this section, we explore a theory of bidding behavior proposed by Nyborg and Strebulaev (2004). The theory is based on the notion that bidders will act differently depending on the pre-auction inventory. Most auctions are reopening of securities already traded in the secondary market. Therefore, bidders could have long, short, or neutral exposure to the security being auctioned. Nyborg and Strebulaev (2004) explain the difference in valuation between the bidders through the possibility of a short squeeze occurring post-auction. A short squeeze in this context is when a bidder with negative post-auction inventory needs to cover their short position by buying from a single long bidder. Intuitively a bidder with a short position would bid more aggressively in the auction to avoid this scenario. Furthermore, a bidder with a long position could also bid more aggressively to try and create a short squeeze. If this is the case, this would increase competition and benefit the auctioneer. The most important conclusion from Nyborg and Strebulaev (2004) theory is that players with different pre-auction inventory exposure have different valuations of the auctioned asset.

Suppose there are two bidders, one with long and short pre-auction exposure. However, in this model, all players do not value the auction at v_0. The short bidder needs to cover Z units to avoid being squeezed. Hence, the bidder will value these Z units at a higher price (v_h). In this equilibrium, the short submit bids for $Q - Z$ units at v_0 and z units at a very high price (v_h) to avoid being squeezed. The player will win the z units at v_h. The long bidder submits a bid for Q units at v_0. P_0 is the stop-out price, and in this equilibrium, $P_0 = v_0$. There will be no short squeeze in the secondary market, and the participants value the security differently.

The explanation is that a long bidder must submit bids above v_h for $Q - Z + 1$ units to squeeze the short. Nyborg and Strebulaev (2004) derive proof that the payoff for this strategy is worse than remaining passive, on the condition that the short bidder submits the respective demand schedule.[2] Another way to invoke a short-squeeze would be for the long bidder to corner the market and exhaust the entire supply.[3] However, Nyborg and Strebulaev (2004) shows that the payoff a long bidder would receive by winning the whole supply is lower than v_h.

$$Payoff = \frac{(Q - Z)V_0 + ZV_h}{Q} < V_h \tag{10}$$

In the model, the short takes advantage of the uniform auction format. He bids aggressively on the units needed to cover his short. The best cause of action for the long-bidder is to remain at his valuation so that he does not pay a premium for the security. All players pay the stop-out price for all units awarded in a uniform auction. So, the short player can adopt a costless strategy to remove the possibility of a short squeeze by submitting an aggressive demand schedule. The model suggests that the mean rate of bids submitted by the short is higher than any bids submitted by a long. However, this is under the assumption that the long-bidder only submits one bid at v_0. This is not the case for Norwegian treasury auctions. Nevertheless, the theory suggests that short bidders will behave more aggressively. We test this theorem in Section 6.3.

[2]Full derivation in Nyborg and Strebulaev (2004) p.28

[3]Chatterjea and Jarrow (1998) derive equilibrium where the best response is not to corner the market

4 Institutional background

In this section, we discuss the mechanics of the Norwegian treasury market. We study bidder behavior from January 2006 to October 2021. Norway has had budget surpluses in the entire period we investigate, except for 2020 (which was an abnormal year because of the corona pandemic). The budget surplus ranges from 4% - 19% in percent of the GDP. In addition, Norway is backed by the world's largest sovereign wealth fund. Regardless, they still regularly issue government debt; in 2021, the debt was approximately 522B NOK. Norges Bank issues debt by selling treasury bonds and treasury bills. The debt is divided between 466B NOK in treasury bonds and 56B NOK in treasury bills.[4] Treasury bills are no-coupon securities that pay face value at maturity with a maturity of up to one year. Treasury bonds are non-callable bullet bonds that pay annual coupons with a maturity of over one year. The Norwegian Central Bank (Norges Bank) manages government debt. They issue debt for several reasons. It is an instrument used to cover deficits, and payments, generate yield curve information, and occasionally regulate the liquidity of the government budget. The debt also ensures a liquid and stable secondary market in Norway.

In the period we study (2006-2021), Norway issued government debt through a uniform auction system and syndication. They adopted a uniform treasury auction system in September 2000, which had previously been discriminatory for both T-bills and T-bonds. Changing to a uniform system was probably motivated by research showing that the dealer could reduce underpricing by changing the system. Goldreich (2007) finds that underpricing in US Treasury auctions is cut in half by changing from discriminatory to uniform auctions. Bjønnes (2001) conducts research on Norwegian treasury auctions under the discriminatory system and concludes that changing to uniform price auctions will lead to better loan terms for the Norwegian government. However, there

[4]Norges Bank "Årsrapport 2021", p.5.

are conflicting views on this in academics. Back and Zender (1993) conclude that uniform auctions lead to more severe underpricing in equilibria.

There have been several proposed solutions that could reduce the underpricing phenomenon. Keloharju et al. (2005) and McAdams (2007) show that determining supply after bidding is favorable for the seller. The Norwegian auction system currently has a predetermined supply. They have always had the right to cancel the auction if the bids are insufficient. However, this has never occurred. Nevertheless, as of June 2021, in response to a low-performing auction only a few days earlier, the auction terms were changed. There were substantial considerations to cancel the auction, but the proposal was rejected for the potential negative consequences for the secondary market and future auctions. Instead, Norges Bank now has the right to issue a lower volume than announced pre-auction on special occasions. The auction volume is still announced before the auction. However, the predetermined supply could change if the volume of accepted bids is lower than the announced sales volume.[5]

Starting in 2018, all new issues of T-bonds are issued through syndication. In our analysis, the focus is on uniform treasury bond auctions. There are only four syndications so far. Therefore, the current sample size of syndicated issues is too small to make meaningful comparisons. However, in finance, syndication is a common way to issue debt. A group of bookrunners is hired to issue the debt to other investors. In the Norwegian auctions, the primary dealers have acted as bookrunners. The primary dealers are compensated for the process, but the amount is not specified.[6] The most significant difference between auction and syndication is that the price of the syndicated asset is determined through dialogue between the investor and issuer. If the banks fail to issue the debt, they must take on the remaining supply, creating stability and certainty for Norges Bank. The reason for the implementation is to try

[5] Norges Bank "Årsrapport 2021", p.13.
[6] Norges Bank "Årsrapport 2021", p.9.

and decrease the costs of borrowing. There has been a higher demand for the new issues after 2018, but it is still early to tell if the syndication process has led to decreased costs.

The Bank of Norway has utilized a primary dealer system since 1995 for T-bonds and 2003 for T-bills. There are several reasons for this system. Primarily, it contributes to the primary goal of keeping the borrowing costs as small as possible. It also gives a certain degree of security that they can issue the desired amount of government debt. The primary dealer system obliges the dealer to attend the auctions, and only primary dealers are allowed to make bids. The dealers can buy the securities for investment or liquidity purposes and are obligated to trade them in the secondary market. Essentially making the dealers act as market-makers for Norges Bank. They are obligated to post a bid-ask quote and maximum spread, which varies depending on time to maturity and market conditions. In addition, they can place bids in the auction for other investors that want to attend. Investors could include retail funds, private investors, pension funds, and so on. We have no information about whether the bids submitted are intended for the banks or other investors. Therefore, investigating other investors' demands will not be a part of our analysis. The primary dealers include the biggest banks in Norway, which have been reduced from eight to four from 2000 to 2021. As of 2021 the four banks are DNB, Danske Bank, Nordea, and SEB (Skandinaviske Eskilda Banken).

Before 2016 the banks did not receive any compensation for their attendance in the primary dealership system. As of 2016, there was introduced a remuneration scheme. It is a performance-based incentive fee. The dealers are evaluated on their ability to attend the primary and secondary market, promote activity for government securities, and how they comply with the requirements set.

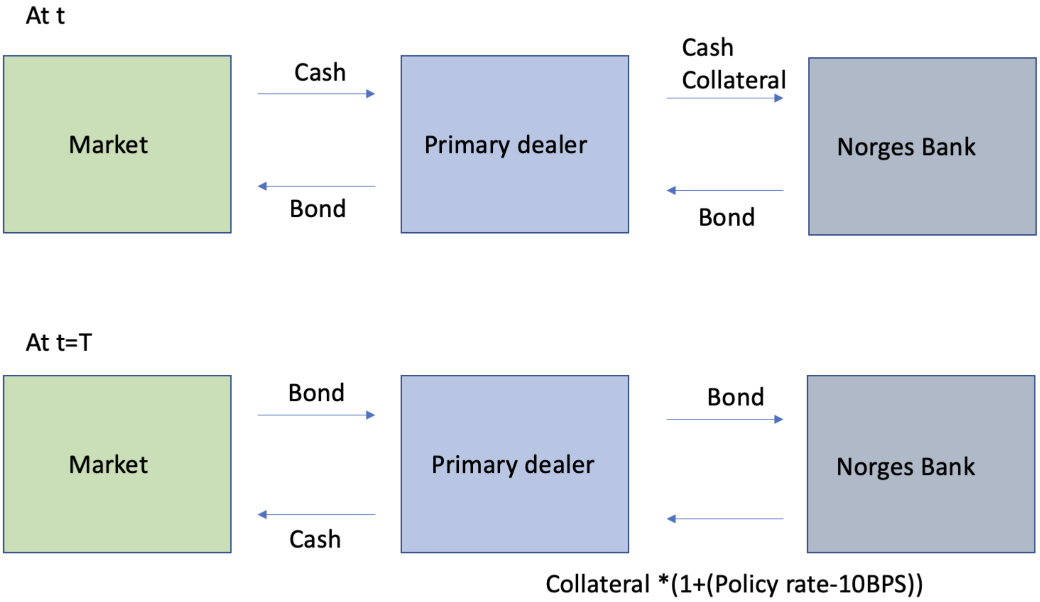
In 2021 Norges Bank paid collectively 5 MN NOK in reimbursements.[7] In addition, there are several other advantages for the banks. First, the dealers earn the bid-ask spread by selling the securities in the secondary market. Furthermore, they can enter repurchase agreements with Norges Bank. Therefore, primary dealers can borrow securities to meet demand in the secondary market. The agreement is a short-term loan with a maximum maturity of one week. The repurchase agreement is essential in our research, considering that borrowing securities impact the pre-auction inventory of the dealers. The possibility of borrowing securities helps the dealers always provide liquidity in the secondary market without having many outstanding securities themselves. The system was introduced for the dealers to overcome the short-squeeze problem.[8] The implementation could help the predictability of the bidding behavior. Figure 1 illustrates how the repurchase agreement works between the dealers and Norges Bank.

Figure 2 shows certain summary statistics of the auctions over the 15-year period. The average number of treasury bond auctions per year increases while the average auction size decreases. It decreases by over 30% from the start of 2006 to 2021. The average auction size is approximately 3 billion NOK. This is similar to other Scandinavian countries like Sweden (Nyborg et al. (2002)) and Finland (Keloharju et al. (2005)), but only about 5% of an average US auction (Goldreich (2007)). The number of primary dealers is relatively constant. Before 2011 there were six, and after 2012 there were four. The black line shows the primary dealers at the end of the year, while the black and grey display the total amount of primary dealers per year. The average demand in the auctions is 7.7 Billion NOK, and the average bid-to-cover ratio is 2.65. There are no undersubscribed auctions. This shows that there exists sufficient demand for the securities. We have plotted the average usage of

[7]Norges Bank "Årsrapport 2021", p.8.

[8]Nyborg and Strebulaev (2004) provide a detailed explanation of the short-squeeze problem in treasury auctions.

Figure 1: Repurchase agreement illustration

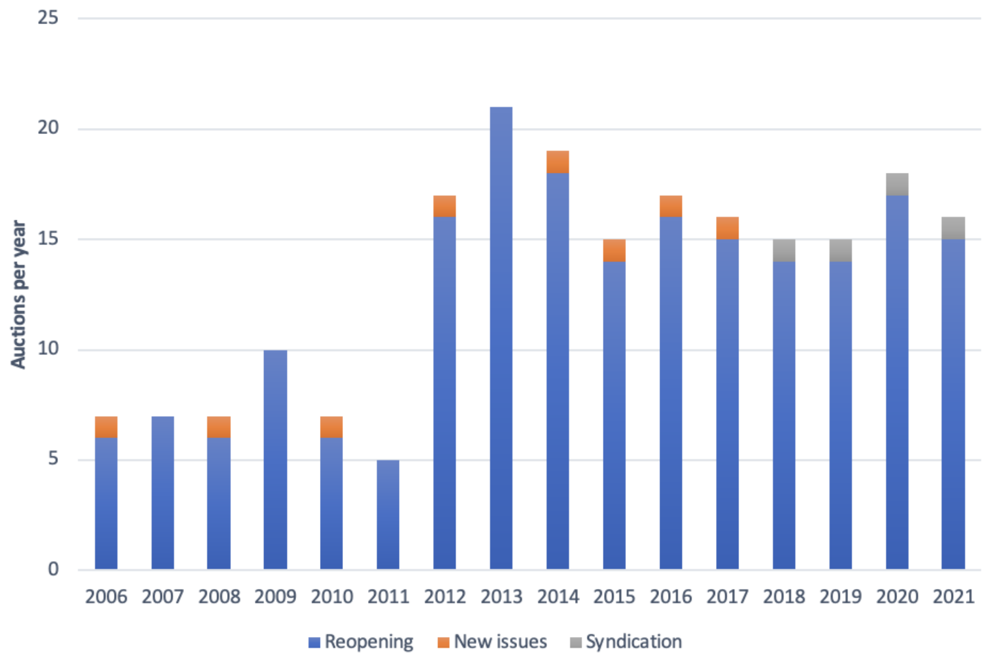


The figure displays how repurchase agreements work. At t: The primary dealer lends a bond to meet demand in the secondary market and gives cash collateral to Norges Bank. At t=T: the primary dealer delivers back the bond and receives the cash collateral with an interest of ten basis points below the policy rate.

Figure 2: Institutional background

The figure displays different summary statistics of Norwegian treasury bond auctions between 2006 and 2021. Top left: auctions per year, top right: auction size in MN NOK, bottom left: dark grey show primary dealers at the end of the year while light grey displays dealers during the year, and bottom right: shows the average number of repurchase agreements each year traded in the same month as the auction for the issued security.

Figure 3: Reopening frequency



The figure shows a detailed illustration of auctions per year from 2006-to 2021. The blue bar show reopenings, the orange shows new issue auctions, and the grey shows new issue syndication.

repurchase agreements on the auctioned securities in the bottom right. The data shows that dealers are utilizing it less as time passes.[9]

In our data set, there are 208 total treasury bond auctions.[10] There are 12 new issues of T-bonds.[11] The remaining auctions are reopenings of existing securities. A detailed overview of the distribution of reopenings, syndications, and new securities is plotted in Figure 3. From 2006 to 2014, the reopening frequency of T-bonds was once every second year and yearly after 2014. The reopening frequency is 94.33%. The primary purpose of reopening securities is to maintain liquidity in the secondary market while keeping the number of outstanding securities low for an accurate yield curve.

[9]See section 4.1.Data for an explanation

[10]One auction (NST 476, 5/13/2014) is missing because of lacking data

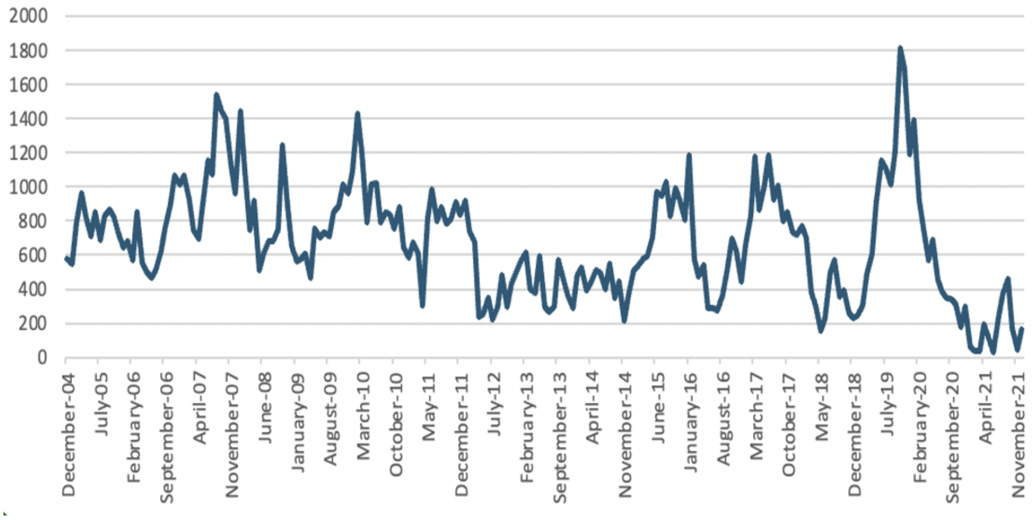
[11]Four are removed in our analysis because they are issued by syndication

4.1 Data

We obtained auction data from Norges Bank for the period 2006-2021. It includes both bids and trades for T-bonds. The bid data is comprised of all price-quantity pairs, yield, and individual bidder identity. It gives us a complete overview of each individual demand schedule. A unique aspect of the dataset is that each bidder is identified with a mark. The mark allows us to follow the individual bidder over time. The bidders are anonymized, so we know the whole history of each bidder but not who they are. Since other market participants can bid through the primary dealers, we cannot identify whether the bid is requested by another investor or the primary dealer itself. In addition, we have the trades of the auction. Here we find the information about which bid was awarded volume.

Oslo Stock Exchange provides secondary market data. The securities are traded over the counter and on the Norwegian stock exchange. Dealers must report the price, volume, and time of all transactions to the Oslo Stock Exchange. We obtain the data from a Bloomberg terminal. We have extracted the closing bid and ask prices and used the closing midpoint as a benchmark for the secondary market price to calculate bid discount and auction performance. Bjønnes (2001) argue that the midpoint is the best proxy for the secondary market price as transactions are conducted on both closing bid and ask. He continues by stating that practitioners also recommended this. It is important to note that when we compare auction performance with other studies they use different benchmarks. Keloharju et al. (2005) use the bid quote subtracted with the dealer's markup. In markets with a when-issued market for treasuries, the when-issued price is most commonly used (Goldreich (2007), amongst others). However, this is not an option in studies of Scandinavian treasury auctions.

Figure 4: Outstanding repurchase agreements



The figure shows monthly average daily outstanding repurchase agreements in MN NOK. The data includes both repurchase agreements for treasury bills and bonds.

When the security auctioned is a new issue, the secondary market does not open until 2-3 days after the auction. Therefore, the new issues observations are removed in the primary analysis instead of inferring secondary market yields from the data. We adjust the secondary market data using yield mimicking in the reopening cycle analysis.

We have obtained repurchase agreement data in the corresponding period from Norges Bank. It contains all repurchase agreements by the primary dealers with an anonymous mark consistent with the auction data. The repo data contain cumulative net inventory, net inventory, and inventory in and -out. Cumulative net inventory is the outstanding repurchase agreements Norges Bank has with the primary dealer, and the net inventory represents the change in cumulative net inventory. The data is only reported on dates with changes in outstanding repurchase agreements or where outstanding volume is larger than zero.

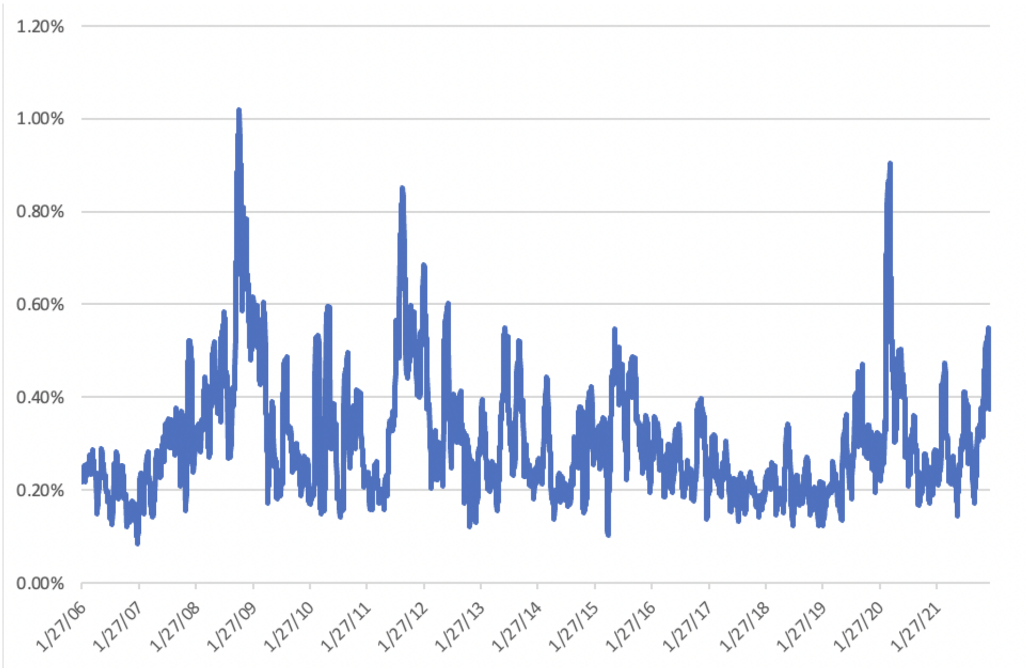
Figure 4 shows how much the primary dealers use the repurchase agreement. It is the monthly average daily outstanding repurchase agreements in MN NOK. We can see a sharp decline in 2021. At the start of 2021, the conditions of the agreement changed. Previously, the dealers posted cash collateral gained an interest of 5bps below the policy rate. The updated terms are that the cash collateral of the first 2BN NOK is given 10bps below the policy rate. After that, they can borrow another 2BN NOK of securities, but their cash collateral is then given a rate of 100bps below the policy rate. Norges Bank states that the change is to incentivize the dealer to obtain securities in the secondary market and reduce the cost for the government.[12] It is working, as repurchase agreements between primary dealers have significantly increased in the last year.[13] This is important to keep in mind. We have no information about the pre-auction inventory gained from other dealers in the secondary market. Before 2021, most primary dealer's repurchase agreements came from Norges Bank.

We obtain the return on constant duration bond indices to model volatility. We use seven different indices. In 2021, DNB decided to discontinue its treasury indices (STX) and use independent indices from Nordic Bond Pricing (NBP) as a replacement. However, NBP indices only go back to 2015. Therefore, we obtain STX indices from Bloomberg until 2015 and combine the conditional volatility obtained from GARCH(1,1) with the volatility obtained from the NBP indices from 2015 until 2021, obtained directly from Nordic Bond Pricing. So, from that methodology, we have obtained 1-year, 3-year, and 5-year constant duration bond indices. In addition, we extract one 10-year index from Refintiv Eikon

The historic time-series volatility of bond returns from a 10-year constant duration index is plotted in figure 5. We calculate the daily standard deviation with a 20-day moving average from the return on a 10-year bond index with constant duration. There are clear volatility spikes around when financial markets are in distress. This includes the financial crisis in 2008, black monday

[12]Norges Bank "Årsrapport 2021", p.19.
[13]Norges Bank "Årsrapport 2021", p.20.

Figure 5: 10-year bond index volatility



The figure shows the rolling 20-day standard deviation of returns on a 10-year constant duration index.

and the eurozone crisis in 2011, and a volatility spike around the corona pandemic in 2020. More importantly, we perform an augmented dickey fuller and KPPS test to test for a unit root in the time series and conclude that the return series is stationary.

5 Hypothesis and Methodology

The methodology is crucial to answer the research question. The field of auction theory is full of comprehensive and reliable literature from respected authors. We will replicate methodologies done in previous research and expand on them by introducing a new explanatory variable in the Norwegian market. By doing so, we are ensuring that our methodology is valid and thorough.

The thesis is a quantitative study where we start by performing a univariate analysis of our variables, where we gain insight, determine performance, behavior, and find patterns. We do this by presenting descriptive statistics and discussing the results. Calculations of variables and statistics closely follow previous academic studies. This analysis is based on research done by Bjønnes (2001), Keloharju et al. (2005) and Nyborg et al. (2002). Next, we perform a multivariate analysis with ordinary least squares regressions as our primary econometric model to test hypotheses and bidding behavior. We test all variables for heteroskedasticity (Whites test) and autocorrelation (Breusch-Godfrey test). We use Newey-West heteroskedasticity and autocorrelation corrected standard errors to adjust the variables. We determine the statistical significance and impact of our explanatory variables through regression.

The primary purpose of this paper is to contribute to existing research by determining whether pre-auction inventory impacts bidding behavior in Norwegian treasury bond auctions. As stated earlier, we use repurchase agreements between the seller and bidders as a proxy for the short position. Hence, our first explanatory variable is REPO. Our regression assumes a linear relationship between the net inventory of repurchase agreements and bidding behavior. We duration weight the repo variable when we analyze variables that include prices following Rydqvist and Wu (2016). In addition, we include explanatory variables that have been used in other Scandinavian literature to help explain each bidder's demand schedule.[14] The first explanatory variable will be volatility. We estimate volatility using a GARCH (1,1) model.[15] This is the methodology used in Bjønnes (2001). Auction size is the final explanatory variable in our main analysis, defined as the quantity supplied in the auction. The explanatory variables will be regressed on the individual bidding variables (bidder dispersion, discount, profit, number of bids, number of bidders, quantity demanded, bid-to-cover ratio, high minus market, and high minus low). Section 6.2 investigates the testable predictions of the outlined models and introduces the number of bidders as an explanatory variable. As number of bidders is an important part of models by Back and Zender (1993), Kyle (1989) and Wang and Zender (2002).

Auction performance variables require secondary market quotes to compare auction prices. When a new security is auctioned, the secondary market does not open until 2-3 business days after the auction.[16] In our primary univariate and multivariate analysis, we drop these observations instead of trying to infer secondary market quotes. However, when we research the reopening cycle, we compare new issues with reopenings. Hence, we need secondary market quotes for the new issues. Therefore, we adjust the new issue secondary price for market changes and time elapsed. First, we find the bond that mimics the change in yield of the auctioned assets the closest. Afterward, we use the yield from the first trading day for the auctioned security and multiply it with the reference bond yield ratio as a correction factor.[17] Subsequently, we find the estimated auction midpoint price by calculating the security price from the yield. This methodology follows Hamao and Jegadeesh (1998).

[14] Bjønnes and Rydqvist (2012), Keloharju et al. (2005) and Nyborg et al. (2002), amongst others.

[15] See appendix B for a full explanation of our GARCH model

[16] From 2014, some auctions post market quotes on the auction day

[17] See Appendix A for complete derivation

6 Bidder Behavior in Treasury Bond Auctions

The main goal of this paper is to study bidding behavior and how it corresponds to auction theory and changes in pre-auction inventory. We test the models examined earlier and check if our results are consistent with the theoretical predictions. In subsection 5.1, we thoroughly introduce the variables included in our analysis, followed by a univariate investigation of the bidder variables. In subsection 5.2, we perform a multivariate analysis via regression to see how the intrabidder statistics are influenced by our explanatory variables (volatility, size, and repo). In subsection 6.3, we discuss our results and compare them with predictions from theory. In 6.4, we do a similar univariate analysis on how bidder behavior responds to the reopening cycle. Finally, in 6.5, we study individual bidding behavior.

6.1 Variable definition

Keloharju et al. (2005), and Bjønnes and Rydqvist (2012) derive descriptive statistics that measure the properties of the bidder's demand schedule in their empirical analysis of bidding behavior. We follow their methodology and calculations when we define variables. We use subindex j for auction and i for the bidder. First, we define the quantity-weighted average bid of each bidder's demand schedule.

$$\mu_{i,j} = \sum w_{i,jk} \cdot p_{i,jk}, \; where \; w_{i,jk} = \frac{q_{i,jk}}{\sum q_{i,jk}} \tag{11}$$

In this case each bidder submits a set of bids at price p and quantity q $\{(p_{i,jk}, q_{i,jk})\}_{k=1}^{m}$, where m represents the number of bids submitted. $w_{i,jk}$ represents the quantity demanded as a fraction of the total quantity demanded in the auction by the bidder. The discount, which shows us the bid shading of each bidder, is the difference between the secondary market price at the time of the auction and the quantity weighted average bid.

$$Discount_{i,j} \;=\; v_j \;-\; \mu_{i,j} \tag{12}$$

The profit in the auction is calculated as the secondary market price minus the stop-out (p_0). We follow the methodology of Keloharju et al. (2005) and use the stop-out to calculate performance, which is the price the dealers ultimately pay for the auctioned security. Other studies, like Bjønnes (2001) and Nyborg et al. (2002), use the award-winning bid price instead of the stop-out. For all reopening of securities (196 auctions), the secondary market price is available on the auction date and can be used directly to calculate profit. For most of the auction dates we study, the secondary market quote is unavailable on the same day for new issues. The first trading day takes place two to three business days after the auction. This is problematic as market events and time elapsed could greatly impact the results. Many academic studies calculate an implied secondary market price. However, we choose to drop these observations instead of inferring prices from market data, following Bjønnes and Rydqvist (2012). [18] Profit is defined as:

$$Profit \;=\; v_j - p_0 \tag{13}$$

We are also interested in how the dispersion of each bidder's demand schedule reacts to changes in pre-auction inventory. Previous research shows that if uncertainty or disagreement exists about the actual value of the security auctioned, the dispersion will increase. This is intuitive, as bidders will be worried about overvaluing the security and instead disperse their bids to avoid buying at a premium. Seeing that all players pay the same stop-out price, the bidders exercise their market power by dispersing bids. However, bidder behavior could respond differently to uncertainty if there is a potential for a short

[18] New issue observations are present in the variables that do not incorporate secondary market prices.

squeeze in the market. We choose to look at intra-bidder dispersion instead of auction dispersion, as individual bidder behavior is what we are interested in. The variables for bidder dispersion are defined in the following manner:

$$STD = \sigma_{i,j} = \sqrt{\sum w_{i,jk}\left(p_{i,jk} - \mu_{i,j}\right)^2} \tag{14}$$

$$Skewness = \frac{1}{\sigma_{i,j}^3}\left[\sum w_{i,jk}\left(p_{i,jk} - \mu_{i,j}\right)^3\right] \tag{15}$$

$$Kurtosis = \frac{1}{\sigma_{i,j}^4}\left[\sum w_{i,jk}\left(p_{i,jk} - \mu_{i,j}\right)^4\right] \tag{16}$$

The central concept of our research is how the bidders' demand varies with pre-auction inventory. The quantity demanded is defined as the demand by a bidder divided by the auction size (Q_j):

$$QD_{i,j} = \frac{q_{i,j}}{Q_j} \tag{17}$$

The second quantity bidding variable is quantity awarded, which is the respective bidders volume awarded $\left(Q_{i,j}^*\right)$ divided by the auction size:

$$QA = \frac{Q_{i,j}^*}{Q_j} \tag{18}$$

Lastly, we are interested in both the spread between the highest bid and secondary market price and the spread between the lowest and highest bid within each demand schedule:

$$HMM = p_{max} - v_j \tag{19}$$

$$HML = p_{max} - P_{min} \tag{20}$$

We follow the methodology of Rydqvist and Wu (2016) scaling the explanatory variable (REPO) by auction size and multiplying it with -1 for interpretation purposes. It is defined as the cumulative net inventory of each bidder at auction day (y_{ij}) divided by the auction size (Q_j)

$$REPO = -\frac{y_{i,j}}{Q_j} \tag{21}$$

In addition, following Rydqvist and Wu (2016), the regressions containing prices and bid dispersion variables are controlled for duration. [19] The adjustment results from duration being the primary effect on a bond's percentage price change.

6.1.1 Descriptive statistics

This section reports the statistics for the outlined variables and conducts a univariate analysis. We have chosen to separate the data into two periods based on the number of primary dealers. This is being done to keep the bidding environment consistent in our analysis, as some variables might be impacted by the change in dealers. In 2012 the number of primary dealers was reduced from six to four. For our analysis, we compute the equally-weighted average across the primary dealer and take the auction average as the observation. Hence, we treat each auction average as one observation. The descriptive statistics are reported in Table 3.

In section A, we report the exogenous variables. Duration and volatility are approximately constant and not impacted by the bidding environment. For the entire period, their averages are respectively 6.35 and 0.22%. The average auction size has reduced while the number of auctions per year has increased. There are approximately ten more auctions per year in the second period. This is consistent with the reopening frequency increasing over time. Discount and profit decrease when primary dealers are reduced to four. The discount shows that the average bid submitted is below the secondary market price. Underpricing for the entire period is about 25 times larger than in the United States (Goldreich (2007)) and triple that in Finland (Keloharju et al. (2005)).

[19] To control for duration we multiply the duration of the bond in the REPO equation.

Table 3: Univariate analysis of treasury bond auctions

	2006-2011	2012-2021	Entire period
A: Exogenous			
Duration	6.17	6.40	6.35
Volatility(%)	0.256	0.210	0.22
Auction size	3420	2870	3000
# of auctions	43	165	208
B: Location and Performance			
Discount (%)	0.377	0.194	0.232
Profit (%)	0.275	0.11	0.144
Bidders at stop-out	1.791	1.552	1.601
Bidders	5.95	4	4.41
Bid-to-cover	2.73	2.65	2.67
High minus Low (%)	1.42	1.52	1.51
High minus Market (%)	0.15	0.35	0.30
C: Dispersion			
Standard deviation (%)	0.357	0.395	0.387
Skewness	-0.611	-0.398	-0.442
Kurtosis	4.551	3.610	3.804
D: Quantity			
Quantity demanded	0.458	0.663	0.620
E: Repo			
Total repo ratio	0.343	0.236	0.258

The table shows descriptive statistics for Norwegian treasury bond auctions. Panel A is equally weighted auction averages. Panel B, C, D, and E are the equally-weighted average across dealers and then across auctions. Volatility is calculated using a GARCH (1,1) model. The complete derivation is found in Appendix B. Auction size is in MN NOK. Bidders at stop-out is the number of bidders submitting the stop-out price as a bid. Bidders indicate, on average, how many dealers attended the auction. Bid-to-cover is the quantity demanded divided by the quantity awarded for each auction. The total repo ratio shows the total amount of outstanding repurchase agreements for each auction divided by auction size. Discount is defined as the secondary market price (average of bid-ask at closing) minus the stop-out in the auction. Profit is defined as the secondary market price minus the quantity weighted winning bid price. All other variables are defined in subsection 6.1.

[20] Bidders at stop-out and the average bid-to-cover ratio are approximately constant over the whole dataset, signaling that the treasury auctions are attractive to attend. On average, each demand schedule submits the highest bid above the secondary market price (high minus market), and 1.51% is the average spread between the highest and lowest bids. The total repo ratio shows the total repurchase agreements with Norges Bank adjusted for auction size. The total repo ratio has decreased over time. This could be a consequence of either auction size increasing, repurchase agreements being used less by the primary dealers, or a combination. However, auction size is also decreasing over time. Hence, repurchase agreements have been used less by the primary dealers in later years. All of the reported variables are significantly different from 0.

6.2 Multivariate analysis: bidding behavior

In this section, we perform regression analysis on bidding variables to determine statistical significance and explain bidding behavior. We regress the performance and bidding variables on volatility (VOLAT), size (SIZE), and cumulative net inventory of repurchase agreements (REPO). The exogenous variable REPO is a proxy for short exposure to the auctioned asset in our regression. In addition, the REPO variable is controlled for duration in regression containing prices, following Rydqvist and Wu (2016).

$$y_j = \beta_1 + \beta_2 \, \mathrm{VOLAT}_j + \beta_3 \, \mathrm{SIZE}_j + \beta_4 \, \mathrm{REPO}_j + \epsilon_j \qquad (22)$$

Following Nyborg et al. (2002) amongst others, we regress each variable separately instead of in a joint system. Variables that are linked to prices show signs of heteroskedasticity and autocorrelation. The dispersion variables (stan-

[20] The underpricing is captured with the variable: profit. Goldreich (2007) use the when-issued price as a benchmark for the secondary market price, our study uses the average of bid and ask at the close, and Keloharju et al. (2005) use the bid quote minus the dealer's markup.

dard deviation, skewness, and kurtosis), discount, high minus low, and high minus market have been regressed with the Newey-West heteroskedasticity and autocorrelation consistent (HAC) standard errors. The remaining regressions are regressed using ordinary least squares. [21] We treat each auction as a single observation using the auction day average. Discount, profit, high minus market, and high minus low are reported as a percentage of face value. The quantity demanded is adjusted for auction size. Finally, the regression with profit, discount, and high minus market has fewer observations. In these regressions, we remove new issues of securities because the secondary market price is not available on auction day.

We start by analyzing the results for profit and discount. Non-surprisingly the regressions react very similarly to volatility. Both increase significantly with volatility. A one standard deviation increase in volatility implies an increase in profit and discount by 0.132% and 0.196% respectively of face value. Hence, the results are not only statistically significant but also economically significant. This result of uncertainty is similar to what is observed in other treasury markets. Bjønnes (2001) and Nyborg et al. (2002) present the hypothesis that this is a way for bidders to adjust for the winner's curse. These studies are conducted on discriminatory auctions. Another explanation could be that the bidders are risk-averse or imperfect competition as Bjønnes and Rydqvist (2012) suspect. Profit increase significantly with the auction size, and repo is statistically insignificant. Auction size and repo are statistically insignificant for discount. As for the rest of the performance variables. Bidders at stop-out increase with size, and bid-to-cover decrease with size. The two other explanatory variables are not statistically significant.

Next, the bidding variables in section B. Dispersion increase with volatility: dealers increase the number of bids, spread between their bids (high minus

[21]The results of using weighted least squares with volatility as weight on heteroscedastic variables, as done in Keloharju et al. (2005), show little difference from our regressions

Table 4: Multivariate analysis

	Intercept	Volat	Size	Repo	R^2	Obs.
A: Performance						
Profit	−0.118 (−1.59)	0.553 (3.66)***	0.047 (2.035)**	−0.019 (−0.566)	0.0912	200
Bidders at stop-out	1.343 (6.54)***	−0.678 (−1.41)	0.133 (2.27)**	0.0979 (0.143)	0.0295	208
Bidders	3.63 (14.158)***	0.808 (1.19)	0.191 (2.03)**	0.51 (0.64)	0.0674	208
Bid-to-cover	3.259 (15.137)***	−0.135 (−0.27)	−0.20 (−3.27)***	0.66 (0.91)	0.0652	208
B: Bidding variables						
Discount	−0.026 (−0.23)	0.729 (3.03)***	0.035 (1.10)	−0.023 (−0.77)	0.1029	200
High minus market	−0.012 (−0.153)	0.87 (3.06)***	0.040 (1.847)*	−0.028 (−0.935)	0.124	200
High minus low	−0.012 (−3.49)***	0.050 (8.03)***	0.006 (4.90)***	−0.002 (−2.36)**	0.457	208
Number of bids	1.77 (1.03)	11.74 (3.60)***	3.40 (5.87)***	−1.68 (−4.76)*	0.444	208
Standard deviation	−0.073 (−1.22)	1.151 (7.03)***	0.0758 (4.22)***	−0.042 (−1.85)*	0.383	208
Skewness	−0.114 (−1.01)	−0.333 (−1.04)	−0.10 (−3.38)***	0.1291 (1.70)*	0.056	208
Kurtosis	0.538 (0.61)	1.88 (0.97)	1.00 (2.76)***	−0.378 (−2.24)**	0.161	208
Quantity demanded	0.84 (9.88)***	−0.124 (−0.89)	−0.066 (−2.71)***	0.128 (0.844)	0.1091	208

The table shows the performance and bidding variables regressed on volatility, auction size, and repurchase agreements. In all regressions with variables that include prices (profit, discount, dispersion, high minus market, and high minus low) the repo variable is duration weighted. Discount, profit, dispersion, high minus market, and high minus low are measured as a percent of face value. The quantity demanded is adjusted for auction size. Bidders are the number of bidders in each auction. Bidders at stop-out is the number of bidders submitting the stop-out price as a bid. All bidding variables except quantity demanded are regressed using HAC robust standard errors, and the rest is regressed using ordinary least squares. T-stat is displayed in parenthesis. The symbols *,** and *** indicate respectively 10%,5% and 1% statistical significance.

low), and standard deviation. This is consistent with results from theory and other treasury markets.[22] Wang and Zender (2002) explain that risk-averse bidders disperse bids more when uncertainty increase. Intuitively, risk-neutral bidders would not react to increased uncertainty by dispersing bids. Therefore, we believe that bidders in Norwegian treasury bond auctions are risk-averse. Auction size also increases dispersion (standard deviation, high minus low, and number of bids), just less economically impactful, as the coefficients are much smaller. A 1 billion kroner auction increase will typically increase the spread between the highest and lowest bid by 0.6%. In addition, when auction size increases, the dealers submit schedules with more negative skewness and fatter tails. The quantity demanded per bidder as a fraction of auction size decrease with auction size. A 1 billion kroner increase in auction size decreases the demand by a significant 6.6%. However, the quantity demanded is scaled by auction size. The results are different if we look at total demand. The REPO variable reveals less dispersing of bids (high minus low, number of bids and standard deviation) when pre-auction inventory increases. Which could indicate that short bidders are more interested that the quantity demanded is fulfilled. However, our result from the REPO variable is not entirely consistent with the results we expect from the model outlined by (Nyborg and Strebulaev (2004)). Our main prediction was that the quantity demanded increases when the dealers increase their repurchase agreement position. The coefficient sign supports our theory, but the variable is statistically insignificant.

We test the robustness of the result by running regressions on selected variables, excluding the explanatory variable REPO, and compare them with results from other studies. We remove the repo variable to see how the variables respond and to compare the regressions with other studies that do not have this explanatory variable. We see no major change in the economic or statistical significance of the coefficients being tested. Therefore, we conclude with

[22]Keloharju et al. (2005) and Nyborg et al. (2002)

them being rather stable. We regress profit, standard deviation, skewness, and kurtosis on volatility and size. Thereafter, we compare them with results from Bjønnes and Rydqvist (2012) and Keloharju et al. (2005). The results can be found in Table 5.

Bjønnes and Rydqvist (2012) include two explanatory variables in their regression that we omit. They are EXCH and PRIM that represent respectively which exchange system is being utilized (Uniform/Discriminatory) and if there is a primary dealer system.[23] Keloharju et al. (2005) include the number of bidders as an explanatory variable, which is excluded from our regression. It is defined as how many bidders participate in an auction. [24] Hence, we only compare the explanatory variables all three studies include.

We start by comparing the profit regressions. In all studies, volatility has a positive relationship and is significant at the 1% level. In the size regression, we find a similar result to Bjønnes and Rydqvist (2012) with the same sign on the coefficient and significance at the 5% level. While Keloharju et al. (2005) has s negative sign on the coefficient and is statistically insignificant. We theorize that this could be related to an alternative measurement of the explanatory variable size. In the Finnish treasury auction, the supply is determined after the auction. Therefore, Keloharju et al. (2005) use expected size to better relate to the position of bidders. Volatility is significant with respect to standard deviation across the studies. Keloharju et al. (2005) find that size positively impacts kurtosis and skewness. Our study and Bjønnes and Rydqvist (2012) find the opposite sign in the skewness regression.

[23]This is not included in any of our analysis because the auction system is uniform and utilize primary dealers throughout the entire period we study.

[24]This is not included in our primary multivariate analysis as the Norwegian treasury auctions have a fixed number of participants. However, the number of bidders is included as an explanatory variable in section 6.3.1

Table 5: Robustness check

	Volat	Size	R^2	Obs
Panel A: Profit				
Our paper	0.542 (3.62)***	0.049 (2.15)**	0.09	200
Bjønnes and Rydqvist (2012)	0.436 (3.0)***	0.062 (2.9)***	0.231	96
Keloharju et al. (2005)	0.215 (3.8)***	−0.009 (−0.6)	0.054	156
Panel B: Standard deviation				
Our paper	1.129 (7.24)***	0.081 (3.45)***	0.3774	208
Bjønnes and Rydqvist (2012)	0.209 (2.1)**	0.053 (3.9)***	0.249	92
Keloharju et al. (2005)	0.161 (5.9)***	−0.019 (−0.4)	0.222	175
Panel C: Skewness				
Our paper	−0.26 (−0.86)	−0.11 (−3.32)***	0.044	208
Bjønnes and Rydqvist (2012)	0.191 (0.6)	−0.027 (−0.6)	0.238	92
Keloharju et al. (2005)	−0.005 (−0.0)	0.101 (2.0)**	0.172	175
Panel D: Kurtosis				
Our paper	1.68 (0.88)	1.04 (2.93)***	0.156	208
Bjønnes and Rydqvist (2012)	−0.077 (−0.1)	0.156 (0.9)	0.083	92
Keloharju et al. (2005)	0.436 (0.4)	0.543 (2.7)***	0.057	175

The table shows profit, standard deviation, skewness, and kurtosis regressed on volatility and auction size. We compare our regressions with Bjønnes and Rydqvist (2012) and Keloharju et al. (2005) for a robustness check. In our study, all variables except profit are regressed using HAC robust standard errors. Profit is regressed using ordinary least squares. Bjønnes and Rydqvist (2012) estimate all regressions using ordinary least squares. Keloharju et al. (2005) estimate all regression using weighted least squares using volatility as weight. T-stat is displayed in parenthesis. The symbols *,** and *** indicate respectively 10%, 5% and 1% statistical significance.

6.3 Analysis: Discussion of results and theory

This section test and discuss qualitative predictions from section 3. First, we start by testing the non-linear equilibrium predictions from Wang and Zender (2002), Kyle (1989) and Back and Zender (1993). We use the result from (4) and perform new regression where we introduce one new bidding variable (standardized discount) and an explanatory variable (number of bidders). We define standardized discount as the discount divided by the standard deviation. Secondly, we look at predictions about pre-auction inventory made by Nyborg and Strebulaev (2004) and discuss the results found in the multivariate regressions.

6.3.1 Non-linear equilibria

Standardized discount is regressed with ordinary least squares, kurtosis and skewness is regressed with HAC robust standard errors. The regressions are given by:

$$y_j \;=\; \beta_1 + \beta_2\,\mathrm{VOLAT}_j + \beta_3\,\mathrm{SIZE}_j + \beta_4\,\mathrm{BIDDERS}_j + \epsilon_j \qquad (23)$$

The results from the regressions are in Table 6. First, we see that standardized discount is only statistically dependent on the number of bidders. This is inconsistent with Kyle, who states that the variable is a constant. It is also inconsistent with Back and Zender. They predict that standardized discount decrease when the number of bidders increases. We get the opposite result. Interestingly, this could mean that competition has increased amongst the bidders when primary dealers were reduced from six to four. The remuneration system or different market conditions could be a factor.

Keloharju et al. (2005), and Bjønnes and Rydqvist (2012) explore the market power theory and show that negative skewness is an integral part of it. The

Table 6: Multivariate analysis: testable measures of bidding behavior

	Intercept	Volat	Size	Bidders	Rˆ2	Obs.
A: Performance						
Standardized discount	−0.343 (−1.31)	−0.228 (−0.61)	−0.036 (−0.618)	0.25 (4.65)***	0.099	200
Skewness	0.407 (1.66)*	−0.319 (−0.91)	−0.147 (−2.68)***	−0.08 (−1.60)	0.0674	208
Kurtosis	−0.65 (−0.61)	1.44 (0.79)	1.06 (2.10)**	0.224 (0.89)	0.129	208

The table shows the three testable measures outlined in section 3 (standardized discount, skewness, and kurtosis) regressed on volatility, auction size, and the number of bidders. Standardized discount is defined as the discount divided by standard deviation. Bidders are the number of bidders that attended the auction. Standardized discount is regressed using ordinary least squares, and skewness and kurtosis are regressed using HAC robust standard errors. Repo is not included as the effect of repurchase agreements is not a part of the models we investigate. The standardized discount is 200 observations as we remove new issue auctions. T-stat is displayed in parenthesis. The symbols *,** and *** indicate respectively 10%, 5% and 1% statistical significance.

dealers submit demand schedules with small prices because the lower portion of schedules, in some cases, helps determine the stop-out price. According to the models, negative skewness should increase with the number of bidders. We find that skewness depends on the auction size, and the number of bidders is barely non-significant. However, the sign of the coefficient is the same as the prediction by Back and Zender (1993) and Wang and Zender (2002). Negative skewness increase when there are more bidders in the auction. Finally, kurtosis is affected by the auction size, and the number of bidders is insignificant. Nevertheless, the sign of the coefficient is the same as predicted.

Next, we perform a univariate analysis and see if any predictions align with what we see in the Norwegian market. The results is presented in Table 7.

The risk-neutral model of Back and Zender (1993) predicts that negative skewness and kurtosis increase when the number of bidders increases. Consistent with what we find in the observed values. However, the multivariate analysis shows that the change is not statistically significant. We find that every observed value, except kurtosis and standardized discount when $N = 4$, is between the risk-averse model and Kyles model with a risk-aversion coefficient

Table 7: Comparison of predictions and results

Variable	Back and Zender	Kyle	Observed
N = 4			
Standardized discount	0.882	1.732	0.504
Skewness	-1.058	0	-0.388
Kurtosis	2.908	1.8	3.508
N = 6			
Standardized discount	0.663	1.732	0.97
Skewness	-1.658	0	-0.61
Kurtosis	4.714	1.8	4.47

The table shows observed values and predictions (standardized discount, skewness and kurtosis) from the risk-neutral model by Back and Zender (1993) and the risk-averse model by Kyle (1989) for $N = 4$ and 6. The observed values are the equally-weighted average across bidders followed by the average of the auctions.

of 0.5. Therefore, the Norwegian primary dealers likely have a risk-aversion coefficient somewhere in between.

As shown above, some of the predictions made by the model help explain bidding behavior. However, overall we reject the models. One key factor missing is the statistical significance of the number of bidders in our multivariate analysis. The number of bidders is an integral part of the models explaining imperfect competition in auctions. In addition, as Bjønnes and Rydqvist (2012) report, we find that, on average, the bidders submit demand schedules above the secondary market price (high minus market), which is not the case in the non-linear equilibrium models.

6.3.2 Pre-auction inventory

The multivariate analysis in Table 4 is inconsistent with our research question's predictions. We find no statistically significant evidence that pre-auction inventory impacts quantity demanded. The statistically significant variables that REPO influences are dispersion and high minus low. Hence, we see little to no sign of more aggressive bidding from the auction day averages. This supports the equilibrium model proposed by Chatterjea and Jarrow (1998), where short-squeezes are not present in a long-term equilibrium in uniform auctions. Because the uniform auction system already promotes aggressive bidding. The dilemma is a case of a Nash Equilibrium where all dealers have the incentive to bid at the same valuation. Enforcing another strategy would lead to a worse outcome for the dealer.[25]

The results could also be a consequence of lacking data. As Figure 4 and the total repo ratio variable from Table 3 show, the use of repurchase agreements with Norges Bank has taken a downturn the last year. We suspect that the primary dealers have dramatically increased the use of repurchase agreements amongst each other to meet demand, which means that the dealers could have different pre-auction inventory than present in our dataset.

We finish the empirical investigation of pre-auction inventory by testing a prediction made by Nyborg and Strebulaev (2004).[26] They state that the mean-rate of bids is larger for someone with short pre-auction inventory than any bids from a dealer with long pre-auction inventory.

We test this hypothesis by conducting a regression of the price of all bids submitted in every auction with our explanatory variables (VOLAT, SIZE, and REPO).[27] We use HAC robust standard errors in the regression to adjust for heteroscedasticity and autocorrelation. The results is found in Table 8.

All variables are statistically significant. We find that the bidders submit lower bids as uncertainty and size increase. However, we can see that other important variables are omitted from the regression that impacts the price, as the regression shows a very low R^2.

[25]Chatterjea and Jarrow (1998), p.271. for a full exposition of the dilemma
[26]Proposition 4 by Nyborg and Strebulaev (2004), p. 18.
[27]Every bid submitted equals 12877 observations.

Table 8: Multivariate analysis: price and pre-auction inventory

	Intercept	Volat	Size	Repo	Rˆ2	Obs.
			A: Performance			
Price	1.06 (206.15)***	-0.065 (-4.03)***	$-2.5e-06$ (-1.69)*	0.003 (2.06)**	0.025	12877

The table shows the bid price regressed on volatility, size, and repo. The repo variable is duration weighted. The regression use HAC standard errors. T-stat is displayed in parenthesis. The symbols *,** and *** indicate respectively 10%, 5% and 1% statistical significance.

Nevertheless, when a bidder has a larger short position, the bid price increases. The results are both statistically and economically significant. A dealer with a higher duration weighted net inventory scaled by auction size submits larger bids than a neutral bidder. The results support what Nyborg and Strebulaev (2004) model explain and Rydqvist and Wu (2016) find in the Canadian treasury market. It is important to note that dealers could have larger short or long positions in addition to the repurchase agreements with Norges Bank. Hence, we cannot conclude that the model and proposition hold. Regardless, the evidence supports the notion that pre-auction inventory impact bidders valuation of the security.

6.4 Reopening cycle

To illustrate how uncertainty affects the bidder's behavior, we analyze the same descriptive statistics as before and separate them into issuance orders. Nyborg et al. (2002) shows that bidders react to uncertainty by dispersing their bids more, reducing the quantity demanded, and increasing underpricing. There is more uncertainty surrounding auctions of new-issue bonds. Reopenings already exist in the secondary market. Hence, price and demand are more predictable for the dealers. The first four reopenings in our sample occur approximately one month after the new issue, while two weeks elapse between new issuance and reopenings for the last four issues. Table 9 reports the descriptive statistics for the new issue, first reopening, and all subsequent reopenings. We saw no difference in trend between first and later reopenings. Therefore we bundled all subsequent reopenings together. We find that bidders respond to uncertainty as theory would suggest. The auction size is almost double the size in the first auction, and the bidders disperse their bids more in new issues. Standard deviation, high minus low, and high minus market decrease over the reopening cycle. When new securities are auctioned, the bidders separate their highest and lowest bid by more than four percent. Furthermore, auction profit and discount decrease over the reopening cycle. The quantity demanded increases as we progress in the reopening cycle. We have omitted all repurchase agreement variables. The dealers will increase repurchase agreements on securities with the reopening cycle. For new issue auctions, the repurchase agreements will always be zero.

6.5 Individual bidder strategies

In this section, we look at descriptive statistics for each bidder. A similar analysis of individual bidding behavior is done by Bjønnes and Rydqvist (2012). Most auction theory assumes symmetric bidders. However, our unique data lets us study each individual bidder. Bjønnes (2001) and Umlauf (1993) find that bidders of different sizes submit strikingly different demand schedules. Umlauf (1993) show that in Mexican treasury bill auctions, the six most prominent bidders earn 80% of the auction profit. He suggests that the reason is because of information asymmetry and collusion.

In our data, two bidders are only present in 42 and 43 of the auctions. However, we find the same pattern displayed if we only include the auctions where all 6 participants are present, with few exceptions.

The individual bidding behavior of the Norwegian treasury bond auctions is outlined in Table 10. The bidders are sorted from smallest (Bidder 1) to

Table 9: Reopening cycle

	New issue	First reopening	Subsequent reopenings
A: Exogenous			
Duration	9.12	9.05	6.06
Volatility(%)	0.332	0.335	0.20
Auction size	5875	3500	2850
# of auctions	8	8	188
B: Location and Performance			
Discount (%)	*0.23*	0.359	0.155
Profit (%)	*0.340*	0.252	0.064
Bidders at stop-out	1.75	2.125	1.585
Bid-to-cover	2.11	2.72	2.68
High minus Low (%)	4.519	2.230	1.356
High minus Market (%)	*1.11*	0.441	0.332
C: Dispersion			
Standard deviation (%)	0.803	0.487	0.366
Skewness	-0.554	-0.546	-0.436
Kurtosis	6.591	4.705	3.678
D: Quantity			
Quantity demanded	0.478	0.589	0.624

The table reports descriptive statistics over the reopening cycle. There are 208 auctions in the dataset with eight new securities, eight first reopenings, and 188 subsequent reopenings. The remaining four auctions are syndications and are excluded from our analysis. The exogenous variables are equal weight averages across auctions. The remaining variables are equal weight averages for each primary dealer, followed by equal-weighted averages across auctions. Discount, profit, and high minus market in new issues are italicized, as the new security secondary market price is computed differently. The methodology follows Hamao and Jegadeesh (1998) and is outlined in appendix A largest (Bidder 6). Bidder size is measured as quantity demand relative to auctions size. The four biggest dealers are much larger than the two smallest. Immediately it is apparent that the bidder's demand increase with bidder size. The larger bidders demand and are awarded remarkably more quantity than smaller bidders. The average quantity awarded by the three most prominent bidders is higher than the maximum allowed quantity awarded in Canadian treasury auctions (Rydqvist and Wu (2016)). Nevertheless, the award ratio shows that small bidders can expect to receive the same fraction of quantity demanded as larger bidders. The appearance at stop-out also increases with bidder size. We can expect that bidder six determines the stop-out price approximately every second auction.

Studying the bidding variables, we observe the same pattern as in Bjønnes (2001) and Umlauf (1993); the bidders submit very different demand schedules. The larger bidders disperse their bids more. [28] On average, the largest bidder submits a ten times larger spread between the highest and lowest bid and ten times as many bids as the smallest bidder. The smaller bidders are therefore forced to submit higher bids to obtain quantity. Hence, we should observe deeper discounts from larger bidders. Discount is one of the only variables where the pattern is affected by the auction change from six to four bidders. Looking only at the 42 auctions with all participants, we see a clear pattern where the biggest bidder, on average, has a five times deeper discount than the smallest. While in the full sample, bidder four submit the lowest discount on average. We test if the mean discount is statistically different for bidder four when the auction format goes from six to four bidders. We reject the null hypothesis at 5% significance level and find that the mean discount change after primary dealers reduces from six to four. Five of the bidders submit demand schedules with negative skewness and fat tails. Five bidders also, on average, submit the highest bid over the secondary market price (high minus market).

[28] This is apparent from high minus low, the number of bids, and the standard deviation.

Table 10: Individual bidding behavior

Bidder	1	2	3	4	5	6
A: Market Share						
Auctions attended	42	43	208	208	208	208
Quantity demanded	0.095	0.215	0.451	0.519	0.585	1.050
Quantity awarded	0.049	0.087	0.143	0.292	0.261	0.277
Award ratio	0.518	0.404	0.316	0.562	0.447	0.263
Apperance at stop-out	0.095	0.209	0.337	0.317	0.370	0.514
B: Bidding variables						
Standard deviation (%)	0.069	0.325	0.334	0.451	0.356	0.472
Skewness	-0.999	0.053	-0.457	-0.270	-0.356	-0.778
Kurtosis	6.220	2.283	3.943	3.163	3.370	5.033
High Minus Low (%)	0.21	0.96	1.29	1.67	1.29	2.16
High Minus Market (%)	-0.08	0.08	0.14	0.52	0.30	0.19
Discount (%)	0.154	0.279	0.289	0.044	0.138	0.447
# Bids	2	6	12.5	14.45	13	20.25
C: Repo agreements						
REPO	0.036	0.049	0.079	0.046	0.054	0.062

The table reports individual bidding strategies. The bidders are sorted from smallest (Bidder 1) to largest (Bidder 6). Bidders 1 and 2 are only present in auctions until 2012. All variables have 208 observations, except for high minus market and discount (with 200 observations), since new issue observations is excluded. The award ratio is the quantity demanded divided by the quantity awarded. Appearance at stop-out show if the bidder submitted a bid at the stop-out price. All other variables are defined in section 6.1. The variables are equally weighted across auctions, followed by equally weighted across dealers.

Finally, we do not find any pattern between bidder size and cumulative net inventory.

7 Conclusion

This thesis studies bidding behavior and auction performance in Norwegian treasury auctions from 2006 to 2021. We derive theory and testable measures of bidding behavior and qualitatively test the predictions. In addition, the unique dataset lets us introduce an explanatory variable that captures the primary dealer's pre-auction short exposure using repurchase agreements.

First, we find that pre-auction repurchase agreements impact the bidding behavior of the primary dealers. Primarily by affecting the bid price, which would support the proposition that bidders with different pre-auction inventory positions value securities differently, as Nyborg and Strebulaev (2004) predict. The implications of the findings help amplify the current knowledge of bidding behavior in Norwegian treasury auctions. Nevertheless, we reject other parts of our hypothesis. For example, we find no evidence that bidders with larger repurchase agreement positions submit demand schedules with smaller discounts or demand more quantity. Hence, we do not have enough evidence to reject nor support the hypothesis that short dealers bid more aggressively.

Rydqvist and Wu (2016) find evidence of smaller discounts and an increase in demand with short exposure in the Canadian treasury market. Similar to our expectations. Canadian treasury auctions have different institutional properties with the discriminatory format, maximum number of bids, and maximum awarded quantity. In addition, Rydqvist and Wu (2016) has data on both long and short positions, while we only investigate repurchase agreements as a proxy for pre-auction short positions. Therefore, we are not surprised by the contrasting results. However, the Norwegian treasury auctions result could support the theory proposed by Chatterjea and Jarrow (1998), which concludes that market manipulation does not occur in long-term uniform equilibrium, lowering the possibility of a short-squeeze. He shows that every bidder's best action in this Nash equilibrium is to submit aggressive demand schedules.

Secondly, auction theory help explain some characteristics of the bidding behavior in Norwegian treasury auctions. Bidders react to uncertainty by increasing dispersion and submitting demand schedules with deeper discounts. Bidders increase dispersion with auction size. However, auction size has overall less economic impact than volatility. We also find that the primary dealers submit demand schedules which are disadvantageous for the seller like Wilson (1979) predict, inducing profit for the primary dealers.

We also investigate non-linear equilibrium models proposed by Back and Zender (1993), Wang and Zender (2002) and Kyle (1989). The models emphasize that negative skewness increase with the number of bidders. We find the same, but our result is insignificant. In our univariate analysis, none of the risk-aversion coefficients we test correctly predict the observed values in Norwegian treasury auctions. We theorize that the primary dealers have a risk-aversion coefficient between Back and Zender (1993) risk-neutral model and Kyle (1989) risk-averse model. The models explain many characteristics of the demand schedules, like kurtosis and negative skewness that increase with the number of bidders, dispersion of bids with uncertainty, imperfect competition, and dealers' risk-averse nature. Nevertheless, we reject the models because of the statistical insignificance of the explanatory variable number of bidders.

We recommend that future studies help expand our work by introducing long positions and repurchase agreements between the primary dealers. In the end, we believe the results could provide more explanatory power if these are introduced. We did not have data that captured the entire ownership structure of the primary dealer's pre-auction inventory. We would also like to see a study of performance and bidding behavior in the new syndication auctions. Norges Bank has changed the system to reduce the cost of borrowing, and it would be interesting to see how they perform.

REFERENCES

Back, K., and J. F. Zender. 1993. Auctions of Divisible Goods: On the Rationale for the Treasury Experiment. *The Review of Financial Studies* 6:733–764. doi:10.1093/rfs/6.4.733.

Bjønnes, G. H. 2001. Winners Curse in Discriminatory Price Auctions: Evidence from Norwegian Treasury Bill Auctions. *Institute for Financial Research* doi:10.2139/ssrn.290743.

Bjønnes, G. H., and K. Rydqvist. 2012. Imperfect Competition in Uniform-Price Auctions: Evidence from Norwegian Treasury Auctions. *Working Paper* .

Chatterjea, A., and R. A. Jarrow. 1998. Market Manipulation, Price Bubbles, and a Model of the U.S. Treasury Securities Auction Market. *Journal of Financial and Quantitative Analysis* 33:255–289. doi:10.2307/2331310.

Goldreich, D. 2007. Underpricing in Discriminatory and Uniform-Price Treasury Auctions. *Journal of Financial and Quantitative Analysis* 42:443–466. doi:10.1017/S0022109000003343.

Hamao, Y., and N. Jegadeesh. 1998. An Analysis of Bidding in the Japanese Government Bond Auctions. *The Journal of Finance (New York)* 53:755–772. doi:10.1111/0022-1082.305342.

Jegadeesh, N. 1993. Treasury Auction Bids and the Salomon Squeeze. *The Journal of Finance* 48:1403–1419. doi:10.1111/j.1540-6261.1993.tb04759.x.

Keloharju, M., K. G. Nyborg, and K. Rydqvist. 2005. Strategic Behavior and Underpricing in Uniform Price Auctions: Evidence from Finnish Treasury Auctions. *Journal of Finance* 60:1865–1902. doi:10.1111/j.1540-6261.2005.00782.x.

Kremer, I., and K. G. Nyborg. 2004. Underpricing and Market Power in Uniform Price Auctions. *The Review of Financial Studies* 17:849–877. doi:10.2139/ssrn.305373.

Kyle, A. 1989. Informed Speculation with Imperfect Competition. *Review of Economic Studies* 56:317–55. doi:10.2307/2297551.

McAdams, D. 2007. Adjustable Supply in Uniform Price Auctions: Non-commitment as a Strategic Tool. *Economics Letters* 95:48–53. doi:10.1016/j.econlet.2006.08.035.

Nyborg, K., K. Rydqvist, and S. Sundaresan. 2002. Bidder Behavior in Multiple Unit Auctions – Evidence from Swedish Treasury Auctions. *Journal of Political Economy* 110:394–424. doi:10.1086/338748.

Nyborg, K. G., and I. A. Strebulaev. 2004. Multiple Unit Auctions and Short Squeezes. *The Review of Financial Studies* 17:545–580. doi:10.1093/rfs/hhg038.

Rydqvist, K., and M. Wu. 2016. Pre-auction inventory and bidding behavior: Evidence from Canadian Treasury auctions. *Journal of Financial Markets* 30:78–102. doi:10.1016/j.finmar.2016.07.001.

Sigaux, J. D. 2018. Trading ahead of treasury auctions. *Working Paper Series* doi:10.2139/ssrn.2789988.

Umlauf, S. R. 1993. An empirical study of the Mexican Treasury bill auction. *Journal of Financial Economics* 33:313–340. doi:10.1016/0304-405X(93)90010-9.

Wang, J. J., and J. F. Zender. 2002. Auctioning Divisible Goods. *Economic Theory* 19:673–705. doi:10.1007/s001990100191.

Wilson, R. 1979. Auctions of Shares. *The Quarterly Journal of Economics* 93:675–689. doi:10.2307/1884475.

APPENDIX

A Yield Change Mimicking

We use the following equation to estimate secondary market yields for a newly issued security where the secondary market price is unavailable:

$$y_t = \frac{y_t^r}{y_{t+\tau}^r} * y_{t+\tau} \tag{A.1}$$

Where y_t is the estimated yield on the day of the auction and $y_{t+\tau}$, is the observed secondary market yield on the first day of active trading, y_t^r and $y_{t+\tau}^r$ is the yield of the reference bond on the day of the auction and the first trading days of issued bond respectively.

We obtain the reference bond through an empirical analysis, checking the 10 first active trading days of the new bond and 5 bonds with the closest maturity already trading in the market. The analysis aims to reveal how well bonds trading in the secondary market mimics the changes in the yield of the newly auctioned bond.

We begin by reindexing the auctioned bond to 100 and the other bonds in the analysis relative to this. For example, if the yield of the auctioned security is 1%, and a bond trading in the market is 0.6%, they would be listed as 100 and 60, respectively. This process would be repeated over the ten first trading days, and if the newly issued bond experiences a change in the yield, the bond being compared must experience an equal change to keep the ratio constant. Finally, we choose the bond with the lowest variation in the relationship with the issued bonds yield change.

Figure 6 displays plots for all newly issued securities over the 10 first trading days. NST476 and - 479 are omitted as they are traded on the secondary market on auction day. The issued bond is constant at 100 for the whole period, and the relevant bonds vary in relation to the issued bond.

From Figure 6, we see that at least one bond mimics the yield change relatively well. For all bond bonds except NST474 and NST475, the bond that mimics the yield change best is the bond with the closest maturity. For NST474 and NST475, NST472 has the most constant relation.

Finally, we follow Bjønnes (2001) and convert yields into prices using the following equation:

$$P = (100 + R) * \left[\frac{1}{1+i}\right]^{\frac{N}{T}} - \frac{A * C * 100}{T} \tag{A.2}$$

Where

$$R = \frac{A+N}{T} * C * 100 \tag{A.3}$$

P is the price of the security, R is the interest paid at maturity, T is the days from issued until maturity, N is time to maturity, A is the time since issue, and C is the coupon rate.

B GARCH

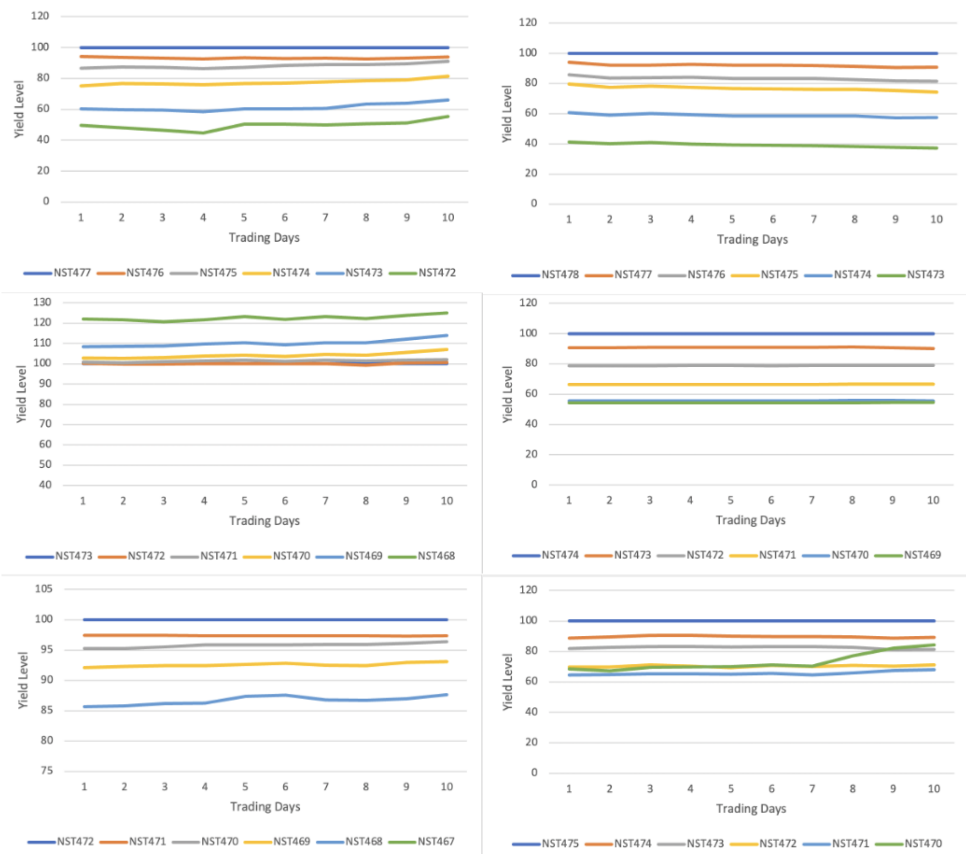
We use a GARCH(1,1) model to estimate the conditional volatility, following Bjønnes (2001). We estimate the return for 7 bond indices with a constant duration of 1, 3, 5, and 10 years respectively. Let R_t denote the logarithmic return of the index at time t. First, we estimate the return on the price index

$$R_t = \mu + \varphi_t R_{t-1} + u_t \tag{B.1}$$

Secondly, we estimate the conditional variance for each index.

$$\sigma_t^2 = \alpha_o + \alpha_1 \mu_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{B.2}$$

Figure 6: Yield mimicking



The figure displays the newly issued bonds and the 5 bonds with the closest maturity. The leftmost legend in each plot represents the new issue. It is reindexed to a constant of 100, and the other bonds are calculated relative to this.

Table 11: GARCH

Index	Duration (years)	α_0	α_1	β
ST3X	1	$5.8e-06$ (0.066)	0.166 (816)	0.884 (123.8)
ST4X	3	$6.6e-05$ ($1.7e-05$)	0.063 (0.007)	0.932 (0.007)
ST5X	5	$3.7e-04$ ($9.6e-05$)	0.061 (0.008)	0.930 (0.010)
NOGOVD1	1	$3.7e-05$ ($5.0e-06$)	0.201 (0.027)	0.717 (0.028)
NOGOVD3	3	$3.7e-05$ ($5.0e-06$)	0.201 (0.027)	0.717 (0.028)
NOGOVD5	5	$3.7e-05$ ($5.0e-06$)	0.201 (0.027)	0.717 (0.028)
10 year GOVT.INDEX	10	$2.4e-03$ ($6.0e-04$)	0.068 (0.009)	0.911 (0.013)

The table show the intercept, alpha and beta of each conditional variance. The standard errors are displayed in parenthesis

Where σ^2_{t-1} is the lagged forecast variance and μ^2_{t-1} is the squared residual from the previous period. We use data from 2006-2020 for index ST3X, ST4X, ST5X, 2006-2021 for the 10year index and 2015-2021 for NOGOVD1, NOGOVD3 and NOGOVD5.

The volatility of the auctioned bond is calculated as the duration-weighted average of the conditional volatility for the two bond indices with the closest maturity. We set a targeted duration corresponding to the auctioned bond for two assets and solve for the weights. These weights are then used on the volatilities of the two bond indices to obtain the volatility of the auction.

For example, if the auctioned security has a duration of 7.5 years, the weights are calculated to 50% of the 5-year index and 50% of the 10-year index. These weights are then used on the auction day's conditional volatility from the 5- and 10-year index. If a bond has a shorter duration than 1 year in the auction, the conditional volatility of the index with 1 year is used.

The STX indices were retired in January 2021 and replaced by indices supplied from Nordic Bond Pricing. However, the data is only available from 2015;

therefore, combining the two indices is necessary to cover the entire period studied.