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Introducing Subjective Logic in Portfolio Management

BY

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Abstract

In this research, we attempt to open a new window in finance by presenting and using the subjective logic. Recently, subjective logic, introduced by Prof. Audun Jøsang in UiO, is presented successfully in different fields of science, but it is not used in finance or economics up to now.

To open this new opportunity in finance, we choose to use subjective logic in portfolio management in a simple way to show that it is possible and valuable to employ subjective logic in finance and show how we can do that. Therefore, we choose the momentum strategy which is one of the famous trading strategies to reimplement it by subjective logic and compare their performances.

The results show that the subjective logic method can outperform the traditional momentum strategy in most cases, especially for the non-tech companies' stocks. The subjective logic method can generate relatively higher returns and bring fewer risks than the traditional momentum strategy. This can show that there is a great potential for research and implementation the subjective logic in the financial industry. We hope that the research in this new field continues in the future to employ all capacities of subjective logic in finance.

Keywords: Subjective logic, Belief mass, Opinion, Uncertainty, Portfolio management, Momentum Strategy

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And then, we would like to express our special appreciation and thanks to our family members for helping and supporting us during our study period.
Noor Hansen & Li Lin

Dear my beloved, Elham

I would like to express my special appreciation and gratitude for your support. You have been there for me every single day in any aspect of our life during my study period. I can’t thank you enough for encouraging me throughout this experience to complete this journey. Without your emotional support, I would not have been able to complete this study and research, and without all your supports in our daily life specially during the covid-19 pandemic I would not have made it through my master’s degree! Thank you for being there for me.

Yours truly, Noor (Mahdi)

Contents

1. Introduction

1.1 The Motivation of Our Research

One of the main topics in finance is portfolio management (Drake & Fabozzi, 2010; Aouni et al., 2014; Spronk & Hallerbach 1997) and strategy to rebalance the portfolio to maximize abnormal return and minimize the associated risk. Investors or portfolio managers' primary purpose is to find a profitable investing strategy which can yield excess returns. Most of the strategies and trading signals are using statistical parameters extracted from historical data and prices, because based on the efficient market hypothesis (Malkiel, 2003; Sewell, 2011; Malkiel, 2005) all public information, including prices and trading volumes should be reflected in current prices. But in reality, it may be not always true. Because there is overwhelming empirical evidence that even the simplest trend-following strategies can generate abnormal returns (Moskowitz et al., 2012).

In a general perspective, portfolio management is a process using historical data to extract desired information or signals to help us to make a decision regards our portfolio rebalancing among a massive uncertainty. One of the mathematical powerful tools that can be used in such problems is "**Subjective Logic**" which is introduced by Prof. Audun Jøsang (2016S) from the University of Oslo. Arguments in subjective logic are subjective *opinions* about state variables that can take values from a pre-determined domain. One of the very interesting properties of subjective logic is explicitly taking epistemic uncertainty and source trust into account. In general, subjective logic is suitable for modeling and analyzing situations involving uncertainty and relatively unreliable sources (Jøsang, 2016; Jøsang & Bhuiyan, 2008; Jøsang, 1997; Oren et al., 2007).

Based on Professor Jøsang's (2016) idea, there are some intrinsic problems in probability logic that are solved in subjective logic. Therefore, we believe that using the subjective logic technique in portfolio management can lead us to extract more precise information from historical data to reduce our decision-making errors in rebalancing our portfolio. Additionally, calculating the uncertainty quantitatively can help us to measure the reliability of our results in the portfolio management or risk management process and take it into account in our decision. Prof. Audun

Jøsang (2016S) shows that if our uncertainty goes to zero, the subjective logic tends and maps to probability logic. But in the real world it is impossible to find environment with zero uncertainty. In other words, probability logic (or zero uncertainty) is an ideal world and using probability logic in our calculations for our real world has this intrinsic issue.

One of the challenges in trading/rebalancing strategies is using qualitative parameters such as news impact or experts' judgment systematically. Subjective logic has the ability to combine this kind of information from different sources with different levels of uncertainty with our strategy systematically.

One of the challenges in trading strategies is combining different signals whose outputs are different. For example, combining volatility, trading volume, value signal and historical prices are challenging because they are values with different metrics. In subjective logic, all values are converted to opinion, then we combine opinions with each other, and we get rid of combining variables with different metrics. The same challenges can be solved by subjective logic when we want to combine different trading strategies.

In this report, we introduce the subjective logic as much as we need for replicating this research and we are not discussing about the proof of the formulas and definitions related to subjective logic. For more details and proof, please refer to the corresponding references.

Our scope in this research is to show that how we can employ the subjective logic in finance to enjoy its capacity and which question we must answer for. Therefore, we choose to use subjective logic in portfolio management and redesign a famous asset allocation strategy called momentum strategy. Momentum strategy help us to proof our idea in the simplest way and we need to use the simplest version of subjective logic to be valuable and more understandable in economic world.

1.2 Research Questions

1- Is it possible to use "Subjective Logic" in finance? If yes, is there any advantages for?

2- Does using the subjective logic technique in portfolio management can outperform the traditional momentum strategy method?

3- Does using the subjective logic technique in portfolio management can lead us to extract lower risks compared to using the traditional momentum strategy method?

The theoretical research will answer the following questions:

1- Which parameters from the market we can assign an opinion to?

2- How can we assign an opinion to our parameters?

3- Which kind of opinion is suitable: Binomial or Multinomial Opinion?

4- How should we combine opinions to construct our strategy?

5- Which information we must extract from opinions?

6- How can we use calculated opinions to predict the future of market properties?

7- How should we calculate the portfolio weights based on our opinion?

8- For which kind of market or asset, can we use this technique?

2. Literature Review

2.1 Portfolio Management

Portfolio management is very important to clients' and the companies' long-term objectives (Alexandrova, 2018). It needs to select and oversee a group of assets that can maximize investors' returns and at the same time to control the risk under an acceptable risk tolerance. In order to outperform the market index, active portfolio management entails systematically evaluating the assets, buying the undervalued assets, and short selling the overvalued assets (Grinold and Kahn, 2000). Opposite to active portfolio management, passive portfolio management aims to replicate market returns by simulating the composition of a specific index or indexes.

Portfolio management necessitates the capacity to assess advantages and disadvantages, as well as opportunities and risks, throughout the whole investment spectrum. There are a lot of trade-offs that need to be considered, such as should we invest more in bonds or more in equities, should we focus on domestic assets or foreign assets, or both? Should we choose growth companies or value companies, developing countries, or developed countries? All of these questions require the portfolio managers to have professional financial knowledge and verify each relevant investment. It also needs to consider the diversification of the whole portfolio and meet investors' financial objectives and risk tolerance.

Asset allocation is very important in the portfolio management process (Campbell et al., 2002). First, we need to select which assets we want to invest in, such as traditional assets: bonds and equities. Sometimes, we also choose alternative investments, such as derivatives, real estate (REITs), and commodities. Then we need to think about putting how many weights in each of these assets. In order to do a suitable strategic asset allocation, the portfolio managers need to know the characteristics of different types of assets, their risks, and returns. They also need to know how to combine them together to diversify portfolios and hedge the risks.

Diversification is very important to control the portfolios' risks and do not put all your eggs in one basket. The wise strategy is to put different types of assets together and it can help you to hedge some unnecessary risks (Abreu and Mendes, 2010). Diversification is achieved through investing in a variety of assets, including different industries, and different countries.

Additionally, portfolio management is a dynamic process and needs to rebalance the allocation and weight of assets at regularly, such as half-year or one year (Tokat and Wicas, 2007). Because the returns and the risks of each asset may change over time. To maintain investors' original objective and risk tolerance, portfolio managers need to rebalance the portfolio. The regular rebalancing procedure allows investors to take advantage of gains and at the same time to increase the chance to invest in high growth and high-potential industries. It also can help to maintain the portfolios' original risk and return objectives.

Why do some researchers and portfolio managers believe that active management can outperform the market index? Passive investing assumes that markets are completely efficient and that all the investors will behave rationally, and they can access the same information. While in reality, this is not true. Successful active portfolio investment depends on portfolio managers' experience and abilities. Good managers and their teams can research and analyze the stocks comprehensively and deeply. They can exploit the pricing anomalies and take advantage of them. They also need to follow the macroeconomic situation, policy changes, and big events in the relevant industries. They need to predict the market trends (Grinold and Kahn, 2000). All of these activities will require the portfolio managers to have higher competence and profound knowledge of financial markets.

However, active management also has some drawbacks. First, active investing may involve more risks than the market index. The reason is that active investors will look for stocks that can yield enormous profits. But these higher return stocks may also cause higher risks. In addition, this may result in overexposure to a specific position or industry. The less-diversified portfolios may bring higher risks than the market index. Furthermore, active management portfolios need to trade more frequently than index funds, and this will cause more transaction fees and taxes (Jones and Wermers, 2011). Last but not least, as we discussed, active management needs managers to put more effort and it will charge relatively higher management fees compared to passive portfolio management.

2.2 Momentum Strategy

The momentum strategy is one of the most frequently used traditional investing strategies, which takes advantage of the tendency for the stocks' historical returns. By using the historical performance to predict future returns. The objective of the momentum strategy is to generate excess returns by purchasing stocks with the best performance in the past, called winner portfolio. And selling the stocks with the worse returns, called loser portfolio, based on the historical record. This strategy was found by Jegadeesh and Titman (1993), they studied the stock market in the United States from 1965 to 1989 and found that buying stocks with higher returns over the past three to twelve months and at the same time selling out some stocks with lower returns over the same time period could generate around 1% profits per month for the next one year.

Another interesting finding is that the momentum strategy has a striking seasonality in January. Because the winner portfolios can also outperform the loser portfolios for all months except for January (Jegadeesh and Titman, 1993). Even though it will generate negative returns in January, the loss is much smaller than its profits in the other 11 months. Overall, a momentum strategy still can generate positive profits for investors. While momentum strategy has a higher requirement for the traders, they must have some experience and knowledge that when is the best time to enter into a position and need to estimate how long should we hold this position and when should we exit. Otherwise, you may miss the key trends and loss the profits because of entering or exiting the position too early or too late.

One criticism of the momentum strategy is that the profits are generated by data mining (Jegadeesh and Titman, 2001). To solve this problem, Jegadessh and Titman collected additional nine years of financial data to test it. The results of the out-of-sample tests showed that the momentum strategy still generates profits. The previous winner stocks still outperform the previous loser stocks by almost the same margin in the preceding period. After several years of study, Chan and Jegadeesh (1996) also found that this result will not be influenced by the companies' sizes or their book-to-market ratios. They retested the momentum strategy and found that this momentum would last for the subsequent 6 to 12 months.

A lot of researchers want to find the explanation for why the momentum strategy can generate profits. According to the behavioral models, abnormal returns are caused by the investors' delayed overreaction to the news and information (Jegadeesh and Titman, 2001). This will cause the price of the winner stocks above their long-run values and the price of the loser stocks to below their internal long-term value. After further research, Jegadeesh and Titman found that the behavioral models can at least partially explain the anomaly returns of the momentum strategy.

In this research, we consider the 11-2 momentum strategy which is calculating last 11 months momentum as a signal to classify the assets and leave next 2 months to pass the short reversal and then rebalance the portfolio.

2.3 Subjective Logic

To evaluate a proposition, we assign either TRUE or FALSE in a standard logic or a probability in the range [0, 1] in probabilistic logic. In this evaluation system (idealized world) we implicitly assume that we are 100% certain about the assigned values while in the real world almost never the evaluators can determine the probability of a proposition with absolute certainty. Uncertainty is one of the fundamental aspects of our real-world problems which are missing in the way standard logic and probabilistic logic capture our perception of reality.

To model situations with some uncertainty, belief calculus is suitable and can express the uncertainty by a belief mass assignment (BMA) (Shafer, 1976). To abandon the additivity principle of probability theory (only between the truth of the arguments values), is the main idea of belief theory. It means the sum of probabilities on all pairwise disjoint states add up to one in probability theory while in belief theory it can be less than one as much as we have uncertainty.

An extended version of belief calculus is presented as "*Subjective Logic*" by Audun Jøsang (2016S). Arguments in subjective logic are called "Subjective Opinions" or "**Opinions**" in short. Opinion can contain degrees of uncertainty regards the truth of the argument. Uncertainty explains the ignorance about the truth of the argument. The main advantage of this approach is explicitly and quantitatively extracting our uncertainty about the arguments existing in our observations and assigning an absolute belief to each argument which helps us to reduce our error during the decision-making process based on our observations. Criteria for decision-making can be articulated in terms of uncertainty in addition to expected utility (Jøsang, 2016d).

In subjective logic, we refer to the sets of all possible states (arguments or propositions) as "**Frame**" whose members called "**Element**". *An assigned belief mass to each element shows our probability estimation regards the truth of the related element*. For example, if our frame is $X = \{x_1, x_2, x_3, \ldots, x_n\}$ then our belief mass distribution is $\{b_1, b_2, b_3, \ldots, b_n\}$ and b_i, *shows our belief regards the truth of* x_i. The uncertainty related to the frame X is illustrated by u_X, and we have:

$$b_1 + b_2 + b_3 + \ldots + b_n + u_X = 1 \quad (1)$$

If the frame X consists of only two elements $\{x_1, x_2\}$ such that they are complement of each other (or $x_1 = \overline{x_2}$), the related opinion is called *binomial opinions* and the opinion related to the larger frame called *multinomial opinion*. In fact, any frame can be converted to a binomial frame by dividing its arguments into two disjoint groups $\{x, \bar{x}\}$. For simplicity, we use the binomial frame in this research, therefore in the rest of the report both terms "Binomial Opinion" and "Opinion" are used to refer to the binomial opinion unless specified the type of opinion.

2.4 Binomial Opinion

Definition 1: Let $X = \{x, \bar{x}\}$ be either a binary frame or a binary partitioning of an n-array frame. A binomial opinion about the truth of state X is the ordered quadruple $\omega_X^A = \{b, d, u, a\}$ where (Jøsang, 2016S):

- $\omega_{\boldsymbol{X}}^{\boldsymbol{A}}$: is used to show the subjective opinion about frame *X* while the opinion owner is A. We simply called "Opinion"
- ***b***: the belief mass of A about the truth of state *x* and called "*belief*"
- ***d***: the belief mass of A about state *x* to be false which is equivalent of the belief mass about the truth of state $\bar{x}$. The parameter *d* called "*disbelief*"
- ***u***: the amount of uncommitted belief mass and called "*uncertainty*"
- ***a***: called "*base rate*" which is a type of prior probability for the truth of state x. This parameter implement the norm of population belief regards the truth of state *x*. In binomial opinion, if we have no information or idea about this norm or the prior probability, we can consider an equal probability for both states. Therefore, a trivial value for this parameter is 0.5 which is considered in our research and implementation. We don't go into details about the base rate but for more information please refer to (Jøsang, 2016S).

The main relationship between the opinion parameters presented in equation (1) is also satisfied by binomial opinion as follow:

$$b + d + u = 1 \text{ and } b, d, u \in [0, 1] \tag{2}$$

Some special cases of binomial opinion are as follow (Jøsang, 2016s):

- where $b = 1$, the opinion is equivalent to binary logic TRUE
- where $d = 1$, the opinion is equivalent to binary logic FALSE
- where $b + d = 1$, the opinion is equivalent to a traditional probability
- where $b + d < 1$, the opinion expresses degrees of uncertainty
- where $b + d = 0$, the opinion expresses total uncertainty situation

We can use an equilateral triangle to show binomial opinions. As illustrated in figure 1, The belief, disbelief, and uncertainty-axes run from one edge (which is its starting point and corresponds to 0) to the opposite vertex (which is its ending point and corresponds to 1) indicated by the b_x axis, d_x axis and u_x axis labels. An opinion is a point inside the mentioned triangle corresponding to triple (b, d, u) with respect to the defined axes (Jøsang, 2016S). For example, a strong positive opinion or ω = (1,0,0) is represented by a point towards the bottom right belief vertex. The base rate (a) is a point on the baseline while its distance from the left disbelief vertex is equal to the base rate. As an example, the opinion ω_x= (0.2, 0.5, 0.3, 0.6) is illustrated in figure 1 (the blue point).

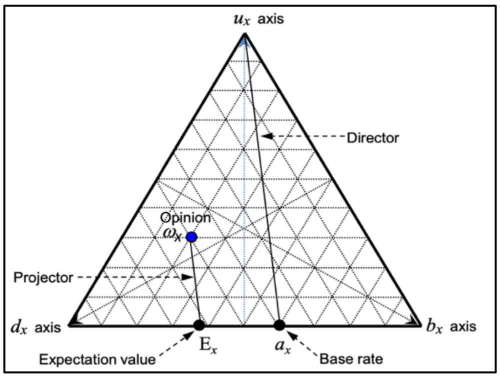


Figure 1: Opinion triangle with example opinion (Jøsang, 2016S)

2.5 Mapping Subjective Logic to Probability Logic

Each opinion can be represented in traditional probability logic by using Beta pdf (Beta distribution, n. d.). Each opinion is corresponding to exactly one beta probability distribution function denoted by Beta(p| α, β) where α and β are its two evidence parameters (Jøsang, 2016S). The corresponding beta pdf shows the probability distribution of the corresponding state or proposition x.

Figure 2 shows the beta probability density function for different parameters.

Let r and s denote the number of observations of x and $\bar{x}$ in our data sample respectively. Therefore, in (Jøsang, 2016s) and (Jøsang & Ismail, 2002) it is shown that the parameters α and β are calculated as follow:

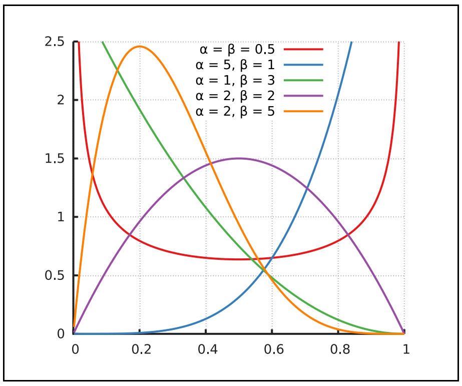


Figure 2: Beta pdf

$$\begin{aligned} \alpha &= r+2a, \\ \beta &= s+2(1-a) \end{aligned} \tag{3}$$

where a is base rate and for this research a is equal to $\frac{1}{2}$, then we can rewrite (3):

$$\begin{aligned} \alpha &= r+1 \\ \beta &= s+1 \end{aligned} \tag{4}$$

To find the corresponding opinion, Jøsang shows that we can use the following formula (2016a):

$$b = r/(r+s+2),$$
$$d = s/(r+s+2), \qquad (5)$$
$$u = 2/(r+s+2)$$

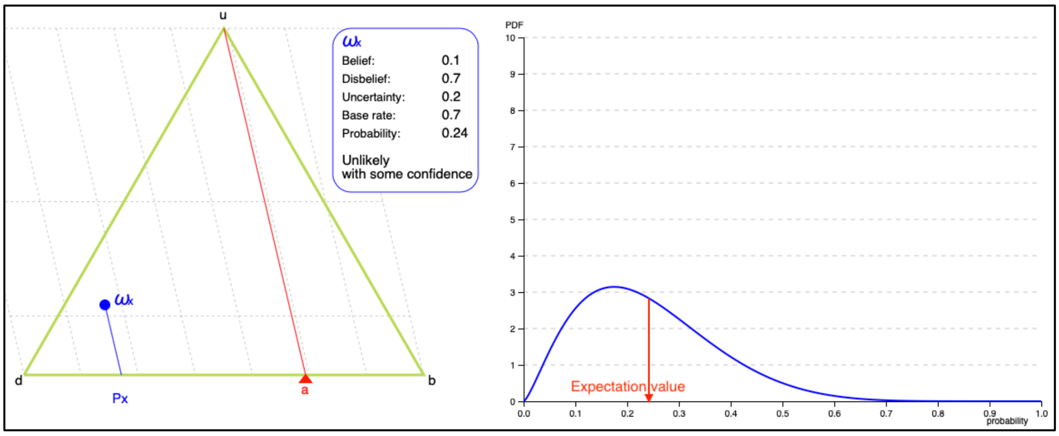


Figure 3: Representation of opinion ω_x and its corresponding beta pdf (Opinion Visualization Demo, n. d.)

Figure 3 illustrates the opinion ω_x = (0.1, 0.7, 0.2, 0.7) as example in the triangle and its corresponding beta pdf. By using the corresponding beta pdf, we can find the expectation value of the state x which is equal to $E[x] = \alpha/(\alpha+\beta)$. For our example, the expected value is equal to 0.24 which can be calculated directly from the opinion elements without mapping to beta pdf by using formula (6).

$$E[x] = b + u*a \qquad (6)$$

Table 1 shows example values of r and s (evidence notation) and corresponding opinions and traditional probability representation with their interpretations. The elements of probabilistic notation (E, c, a) are probability expectation value, the certainty function (c =1-u), and the base rate respectively.

Table 1: Equivalence of opinion, evidence, and prob. notation (Jøsang, 2016S)

Evidence (r, s, a)	Belief (b, d, u, a)	Probabilistic (E, c, a)	Equivalent interpretation in binary logic and/or as probability value
(∞,0,a)	(1,0,0,a)	(1,1,a)	Binary logic TRUE, and probability p = 1
(0,∞,a)	(0,1,0,a)	(0,1,a)	Binary logic FALSE, and probability p = 0
(0,0,a)	(0,0,1,a)	(a,0,a)	Vacuous opinion, Beta density with prior a
(∞,∞,a)	(½,½,0,a)	(½,1,a)	Dogmatic opinion, probability p =1/2, Dirac delta function with (irrelevant) prior a
(0,0,½)	(0,0,1,½)	(½,0,½)	Vacuous opinion, uniform Beta distribution over the binary frame
(1,1,½)	(¼,¼,½,½)	(½,½,½)	Symmetric Beta density after 1 positive and 1 negative observation, binary frame

2.6 Binomial Opinion Operators

When the arguments opinions are equivalent to binary or traditional probabilistic logic (zero uncertainty), the result of any subjective logic operator is always equal to the corresponding operator in that logic. But when the uncertainty is not zero, only the expectation value of arguments is the same in corresponding logic. We consider that subjective logic is a generalization of binary logic and probability calculus.

A completed list of subjective operators' definition is presented in (Jøsang, 2016S) and (Jøsang, 2001). Based on the definitions, all operators are computationally very simple and easy to combine the opinions which is one of the strong advantages of subjective logic. Using subjective logic can reduce the computational complexity efficiently. For example, multiplication of two beta pdf in probabilistic logic is very hard and complex while we can do it very simply by multiplying their corresponding opinions.

For the formula of all operators in subjective logic, please refer to (Jøsang, 2001). The only operator that is important in our research is called "*the Cumulative Fusion Operator*" which is presented in Jøsang's (2016S). This operator can be used to combine two opinions from two different agents about the same frame as well as combining two different opinions from one agent in two disjoint time about the same frame. A simplified definition of this operator is presented in definition 2.

Definition 2 (Cumulative Fusion Operator)

Let $\omega^A = \{b^A, d^A, u^A, a^A\}$ and $\omega^B = \{b^B, d^B, u^B, a^B\}$ be opinions respectively held by agents A and B (or two opinions from one agent in two disjoint times A and B) about the same frame X. The cumulative fusion combination of ω^A and ω^B is presented by $\omega^{A\diamond B}$ and formulated as follows (Jøsang, 2002):

Case I: For $u^A \neq 0 \ \vee \ u^B \neq 0$:

$$\begin{cases} b^{A\diamond B}(x_i) & = \frac{b^A(x_i)u^B + b^B(x_i)u^A}{u^A + u^B - u^A u^B} \\ u^{A\diamond B} & = \frac{u^A u^B}{u^A + u^B - u^A u^B} \end{cases} \tag{7}$$

Case II: For $u^A = 0 \;\wedge\; u^B = 0$:

$$\begin{cases} b^{A\diamond B}(x_i) & = \gamma^A\, b^A(x_i) + \gamma^B b^B(x_i) \\ u^{A\diamond B} & = 0 \end{cases} \quad \textit{where} \quad \begin{cases} \gamma^A = \lim\limits_{\substack{u^A \to 0 \\ u^B \to 0}} \dfrac{u^B}{u^A+u^B} \\ \gamma^B = \lim\limits_{\substack{u^A \to 0 \\ u^B \to 0}} \dfrac{u^A}{u^A+u^B} \end{cases} \tag{8}$$

Then $\omega^{A\diamond B}$ is called the cumulatively fused of ω^A and ω^B, representing the combination of independent opinions of A and B. By using the symbol '$\oplus$' to designate this belief operator, we define $\omega^{A\diamond B} \equiv \omega^A \oplus \omega^B$.

2.7 Subjective Logic in Finance for the First Time

The concept of belief mass is not a new idea, but during the last decade, the concept of subjective logic has become more popular and used in different fields of science. Unfortunately, this concept is still not presented in finance and this research is the first attempt to employ subjective logic in finance to open this new window for future research and lock up more capacity of this method. We believe that there are many opportunities to use subjective logic in finance.

Generally, subjective logic can be used in any decision-making situation under any degree of uncertainty that comes from observations. One situation in finance that we can use subjective logic is asset allocation which is a decision-making situation based on the estimation of future assets' return which is consisting of uncertainty.

3. The Research Methodology and Design in Subjective Logic

To show how we can employ subjective logic in finance, we attempt to use it to reimplement the momentum strategy. Then we will compare the performance of our implementation with the traditional momentum strategy to show the advantage of this new method. The same approach can be used to design new strategy or modify existing strategy by using subjective logic. Diagrams 1 and 2 illustrate the traditional 11-2 momentum strategy, we call **TMS**, and the 11-2 moment strategy based on subjective logic, we call **SLMS,** method respectively.

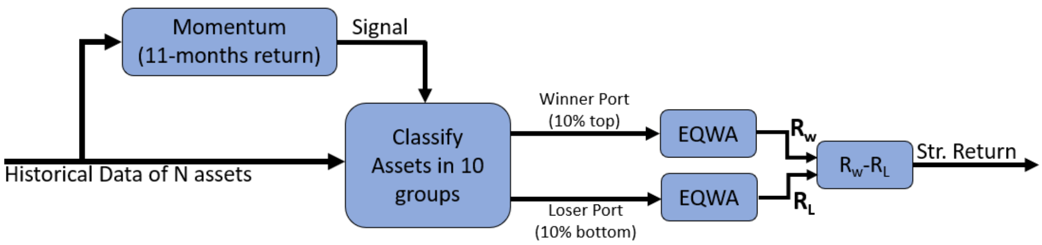


Diagram 1: Traditional 11-2 Momentum Strategy

In both methods, all assets will be ranked into 10 groups based on a signal which extracted from historical data. When the signal is produced in each period, before ranking the assets, we pass over the next two months to prevent the short reversal of return. The top 10% group called the winner and the lowest 10% group called loser. Both methods make an equal weighted average portfolio from winner and loser. Finally, we buy the winner and sell the loser to construct our portfolio and rebalancing in each period.

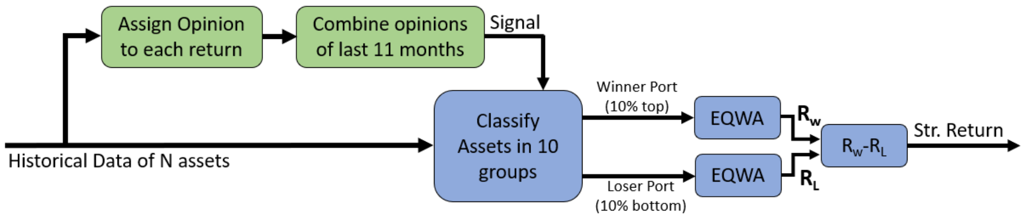


Diagram 2: Momentum Strategy based on Subjective logic method

The only difference between these two methods is the process and information to produce the signal to classify the assets. The traditional momentum strategy used previous 11 months returns as the signal, while the subjective logic method uses the combination of the last 11 months' opinions assigned to the returns. To complete our strategy based on subjective logic, we must answer the following questions:

1- How to define a suitable frame for each asset (or return)?
2- How to assign the appropriate opinion to each asset (or return)?
3- How to update or combine our opinion with other/new observations?
4- How to calculate the weight of assets based on our opinion about their return?
5- Which class of assets is suitable to apply our final strategy?

In the following, we answer the mentioned questions.

3.1 How to define a suitable frame for each asset's return?

For simplicity, we decided to work with binomial opinion and leave the usage of multinomial or hyper opinion for future research. Therefore, we must define a binomial frame $X = \{x, \bar{x}\}$. The main question to design an asset allocation strategy is "Will the return of asset i^{th} be positive or negative?". To simplify this question, we can use momentum based on this idea that the current situation will persist in the next few months. Therefore, we chose our binomial proposition (or state x) as follow:

"Will the asset j continue its upward trend?"

The answer to our proposition is either True or False but with a degree of uncertainty.

3.2 How to assign the appropriate opinion to each asset's return?

Solving this challenge is the most important and hardest part of this method. Generally, to assign an opinion to asset's return, the first and trivial idea is using formula (5) to find the opinion elements (*b, d, u, a*) by setting the parameters *r* and *s* to the number of observations (or periods) that the corresponding asset had positive or negative returns respectively. In this idea, we explicitly supposed that the sign of next period return is similar to the sign of current return (or few last returns).

We know that the historical return has no strong explanatory power about the future (next period) return. And also, an asset with higher volatility has more chance to flip their return sign.

Therefore, we need an idea for setting the value of *r* and *s* such that it considers the following facts:

a) If an asset with lower volatility experiences a higher current return, its next period return has a bigger chance to have the same sign as the current return. Because it shows a strong movement or trend in the market price.
b) Some consecutive periods with the same positive (negative) return mean the asset is in an upward (downward) trend. The longer trend means bigger mispricing and more chance to return the price to its true value which means a bigger chance to flip the trend direction or return sign.

c) The total returns from the beginning of the current trend are important and its effect is similar to the fact b)

d) Assets with higher volatility will have a bigger jump to up or down. Therefore, they have more chances to flip their return sign.

e) Assets with higher volatility will have a smaller chance to have a longer trend period and vice versa.

Let R_t, V, T, and R_T denote the current return, volatility, length of the current trend, and the total return in the current trend respectively. Based on the mentioned facts, for our belief mass we can say:

$$b \propto R_t, b \propto V^{-1}, b \propto T^{-1} \ and \ b \propto R_T^{-1} \tag{9}$$

In other words, we have $b = f_b(R_t, V^{-1}, T^{-1}, R_T^{-1})$. By the same method we must find a function, called $f_d(.)$ to calculate the disbelief. Now we must find the functions $f_b(.)$ and $f_d(.)$ which are the most important part and hardest part in designing a trading strategy based on subjective logic. Based on the chosen strategy, any other variables or signal can be considered as the input of the functions. The assigned opinion is:

$$\omega = \{ \boldsymbol{b}=f_b(.),\ \boldsymbol{d}=f_d(.),\ \boldsymbol{u}=1-f_b(.)-f_d(.),\ \boldsymbol{a}=0.5 \}$$

As mentioned, the 11-2 momentum strategy is the focus of this research, therefore the functions $f_b(.)$ and $f_d(.)$ are only dependents on the last 11 months return of the asset.

$$b = f_b(R_t, R_{t-1}, \dots, R_{t-10}) \ and \ \ d = f_d(R_t, R_{t-1}, \dots, R_{t-10}) \tag{10}$$

The functions $f_b(.)$ and $f_d(.)$ which are designed and used in this research can be explained as follow:

1) As returns normally are small numbers, we need to scale up all returns by a constant number called "*Scale*" to be able to use in formula (5) to assign belief and disbelief to each return. Our recommendation is *Scale=1000* what we use in our simulations, but the best value can be found in training our algorithm over the in-sample or historical data.

2) We consider a threshold called "*Threshold*" to eliminate return around the zero. The best value for Threshold is differ based on the asset environment and can be set by training such as *Scale*.

3) Assign an opinion to each return as follow:
   a) scale up all returns by a constant number called "*Scale*"
   b) if (*Scale**R_t) > *Threshold*, then r = (*Scale**R_t) and s = 0
   c) if (*Scale**R_t) < (-*Threshold*), then s=(*Scale**R_t) and r=0
   d) otherwise, $r = s = 0$
   e) Use formula (5) and parameters r and s to calculate belief b_t and disbelief d_t corresponding to return R_t
   f) Set u_t =1- b_t - d_t or use formula (5). And also, set a_t =0.5
   g) Now we can assign opinion ω(R_t)=(b_t, d_t, u_t, a_t) to return R_t
   h) Use formula (7) and (8) to find the combination of the opinion assigned to the last 11 months return.

Therefore, for our strategy (and in our implementation) the function *f*(.) is defined as follow:

$$\omega(R_t, R_{t-1}, \dots, R_{t-10}) = \omega(R_t) \oplus \omega(R_{t-1}) \oplus \dots \oplus \omega(R_{t-10}) \tag{11}$$

3.3 How to update our opinion with new observations?

The method to update our opinion with new observations depends on the source of new observations and their relationship with our original data history. In this research, we attempt to update our opinion with the information extracted from VIX when we work with the stocks of technical companies. First of all, we have to find the effect of VIX on our decision-making process in our trading strategy. VIX generates a 30-day forward projection of volatility. When the VIX is high (low) volatility is high (low), which is usually accompanied by market fear and traders will go long when the VIX is high and short when VIX is low. In other words:

- When VIX<20 then a major sell-off has taken place shortly after
- When VIX>30 then a major buy has taken place shortly after

Therefore, we assign an opinion to each monthly VIX value such that:

- ω(VIX_t <= *Lbound*) = {b_t =0, d_t =1, u_t =0, a_t =0.5}
- ω(VIX_t ≈ *Mean*) = {b_t =0, d_t =0, u_t =1, a_t =0.5}
- ω(VIX_t >= *Hbound*) = {b_t =1, d_t =0, u_t =0, a_t =0.5}
- ω(VIX_t >*Lbound* & VIX_t <*Mean*)= {b_t =0, d_t = -2/(1+exp(Mean-VIX_t))-1, u_t =1-d_t , a_t =0.5}
- ω(VIX_t < *Hbound* & VIX_t > *Mean*) = {b_t =2/(1+exp(Mean-VIX_t))-1, d_t =0, u_t =1-b_t , a_t =0.5}

Mean and σ_{VIX} are the average value and standard deviation of VIX in the past, *Lbound* is equal to (*Mean*-0.99*σ_{VIX}) and *Hbound*=(*Mean*+0.99*σ_{VIX}). Finally, we update the opinion or formula (11) again by using cumulative fusion as follow:

$$\omega(R_t, R_{t-1}, \ldots, R_{t-10}, VIX_t) \\ = \omega(R_t) \oplus \omega(R_{t-1}) \oplus \ldots \oplus \omega(R_{t-10}) \oplus \omega(VIX_t) \tag{12}$$

Later we will show that inserting VIX information can help to improve the performance of our result but not so much. It can be due to the weakness of our method to assign opinion to VIX or the weakness of VIX to estimate the future volatility. The former is more probable, so designing a better method to assign opinion to VIX may improve our result a lot and we leave it for future research. For example, there is another operator in subjective logic to combine two opinions which is called "the consensus operator" and presented in (Jøsang, 2002). The cumulative fusion operator is aggregating the disjoint opinions about the same frame, but the consensus operator calculates the general agreement between two disjoint opinions about the same frame. Using the consensus operator instead of cumulative fusion operator to combine ω(VIX$_t$) with other opinion may can improve our result.

3.4 How to calculate the weight of assets based on our opinion about their return?

In our final implementation, we decided to use an equal weighted average sum to calculate the portfolio return for both winner and loser to be consistent with the normal momentum strategy we compare our result with. But it is possible to consider different weights for assets when we combine them in the winner and loser portfolios based on their belief mass (b_t) or the level of their uncertainty (u_t) when we calculate the opinion $\omega(R_t, R_{t-1}, \ldots, R_{t-10})$. As bigger belief mass shows stronger positive return in winner portfolio, the weight of the corresponding asset also must be bigger. For loser portfolio, the smaller belief mass or bigger disbelief mass (d_t) correspond to the more negative return which must have bigger weight to short more of the corresponding asset. Another idea is considering the bigger weight for the asset with smaller uncertainty. As it is out of scope of this research, we leave this topic for future research.

3.5 Which class of assets is suitable to apply our final strategy?

We started by using equities to apply our strategy. As we will present later, our result on the stocks of non-technical companies is better than the normal momentum strategy in terms of both return and risk parameters. But our result on the stocks of technical companies is not better than the normal momentum strategy. The reason is presented later in the result session. Due to the limitation of the time, we leave the research on the performance of this new method on the different asset classes for future research.

4. Data Collection and Analyses

In this thesis, we will research the possibility of using subjective logic in finance especially in portfolio management and introduce a new quantitative approach. We consider the momentum strategy results, presented in Moskowitz et al.'s research (2012), as a pilot to compare the performance of our introduced strategy.

4.1 Data Selection

When it comes to data selection, we used historical investment and stock market data. First, we will use some US famous stocks' monthly returns to test our subjective logic strategy and make a comparison to the original strategies, such as momentum strategy. Our data is downloaded from the Bloomberg terminal. All stocks are traded on the US market and the time period is from 31st January 2000 to 31st May 2022. Because we try to include the financial crisis and COVID-19 period and want to test whether the Subject Logic strategy can perform well during these special crisis periods. Risk management should consider both good times and bad times.

We want our dataset to cover a long-time span and a lot of stocks, it will make our results more convincing, and it could be applied at any time in the financial markets. Therefore, we also applied our method on the 810 German stocks (monthly return from January 1991 to December 2004) which was available from "Asset Management" course 2022 to test our subjective logic strategy's performance in the different countries' stocks. In addition, we choose the monthly data instead of daily or yearly, because most of previous studies related to the momentum strategy was used monthly returns. In this way, it will be making it easier to compare our results with previous studies.

4.2 Data Cleaning

We excluded all stocks which lack information during these periods. When we downloaded data from the Bloomberg Terminal, if there is some missing information within our required time period. We deleted these stocks, even for some big and famous companies' stocks. Because we want to make a reliable calculation and comparison between these stocks during the same time period. Furthermore, when we downloaded data, we already excluded the preference shares, warrants, REITS or any other special stocks. We make sure to only included the ordinary shares. This is consistent with prior momentum studies' selection of stocks.

After data selecting and cleaning, we got three groups of datasets. The first one is a big dataset which includes 810 German stocks including all industries. The second group is 50 technical companies' stocks (in USD) and the third group is 50 non-tech companies' stocks (in USD), which are from pharma, energy, financial, food and clothing industries. The reason is, we want to test that is there any difference in results between Technology companies' stocks and non-tech stocks. We downloaded the monthly last price (in USD) and calculated the monthly returns for these stocks.

5. Legal and Ethical Regulations-NSD

Our research and data collection method will comply with legal and ethical regulations. We also strictly follow the guidelines of the National Committee for Research Ethics in the Social Science and the Humanities (NESH). They are the most important guidelines related to research ethics and they can help researchers put theories into practice in a more scientific and ethical way. NESH belongs to the Norwegian National Research Ethics Committees and all research activities at BI Norwegian Business School must be conducted in accordance with guidelines that were formulated by NESH. In addition, our research did not use any personal information. We downloaded the data from published Bloomberg Terminal, and we did not collect data from employees or students. Therefore, we no need to report the personal information and also no need to apply to the coordination group for permission.

6. Implementation

Throughout the entire investigation, we mainly used the R programming language to analyze the data. For the traditional 11-2 momentum strategy, we used the code given by "Asset Management" course, but the other parts related to the subjective logic is coded by ourselves.

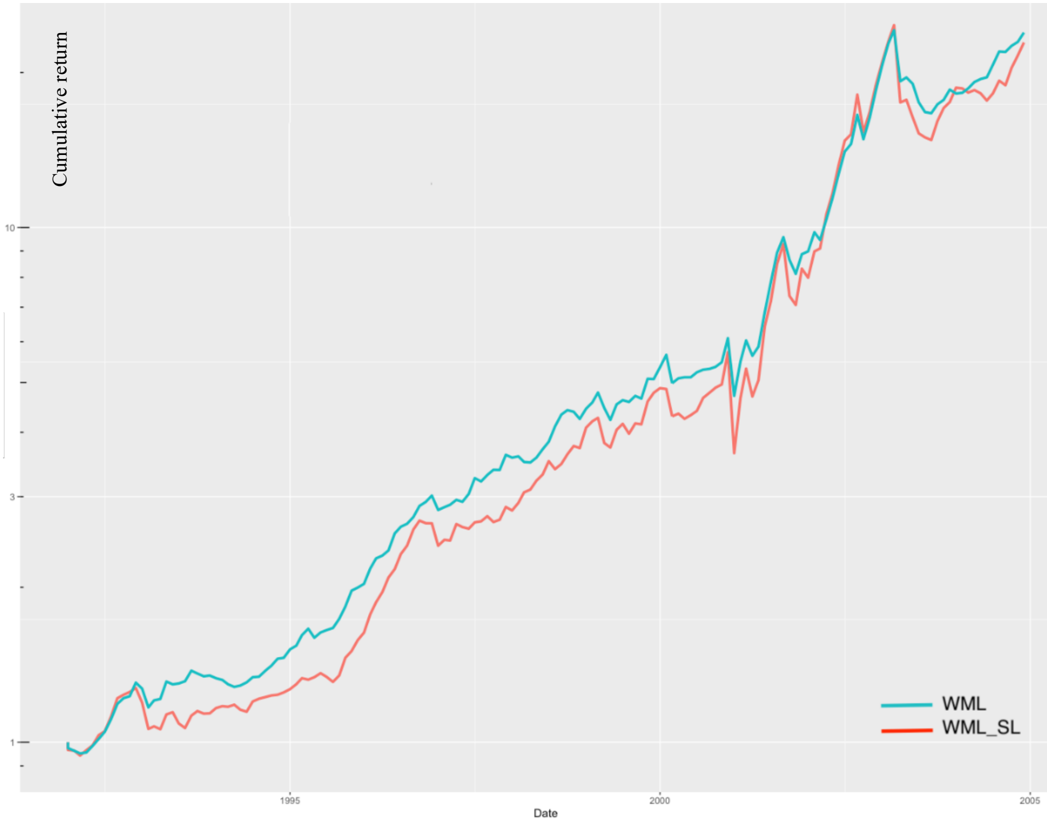
For implementing our design, we use the algorithms presented in subsection 3.2 and the main formula is formula (11). For including the VIX information in our strategy, we implemented formula (12) which is explained in subsection 3.3. The detailed data construction, calculation and analysis could be found in the Appendix.

7. Results and Interpretation

We compare the result of our strategies' performance with the performance of the traditional momentum strategy over the same datasets. We use the accumulate returns and Sharpe ratio as the performance measures and we use the Value at Risk (VaR), Expected Shortfall (ES) and Maximum Drawdown (MDD) as the risk measures.

7.1 Cumulate Returns

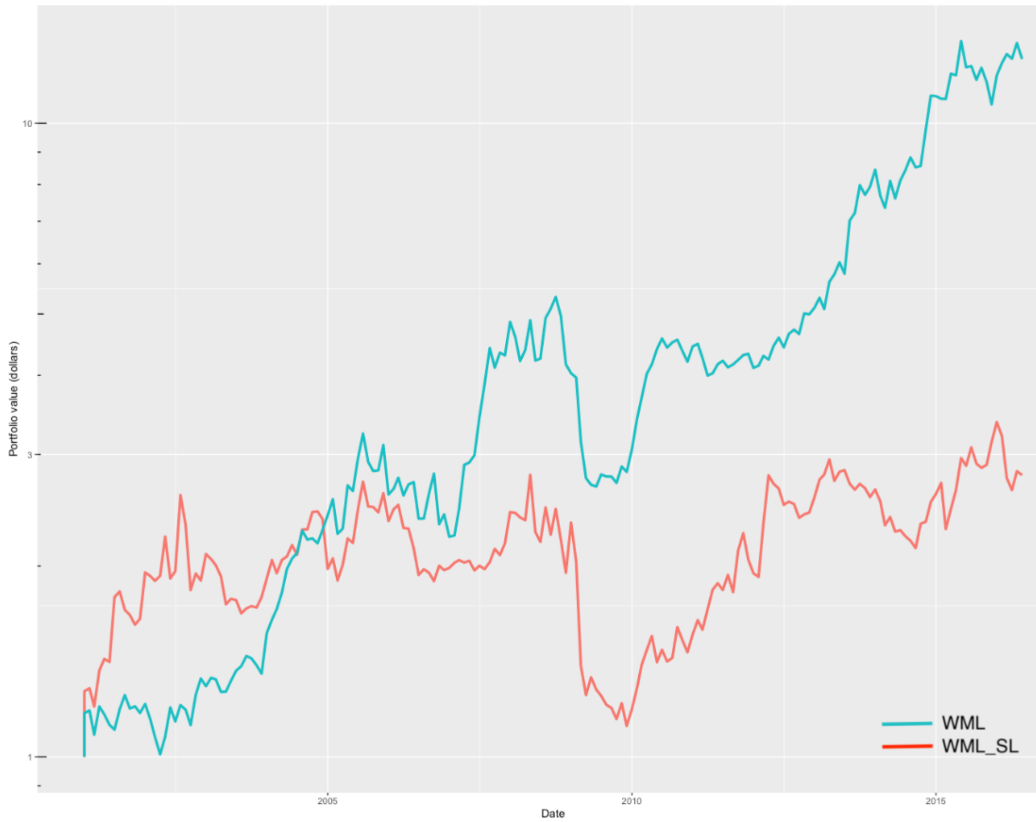
We calculated the monthly Cumulative returns over the holding time period for four portfolios: 810 German stocks, 50 US Non-Tech Stocks, 50 US Tech Stocks without using VIX and 50 US Tech Stocks using VIX. In order to visualize the performance of two different strategies, we drew the following graphs for each portfolio. It will be easier for us to make a comparison between the traditional momentum strategy, which is called WML (the red line) in the following graphs and the momentum strategy based on subjective logic method, which is called WML_SL (the blue line) in the following graphs.



Graph 1: Performance over 810 German stocks

From graph 1, we tested two different strategies based on 810 Non-Tech German stocks. As we can see from the two-color lines, most of the time, the blue line (subjective logic method) will outperform the red line (momentum strategy), especially from the year 1993 to 2002. Sometimes, these two strategies will also have some overlaps, such as at the beginning from 1992 to 1993 and from 2002 to 2004. Overall, we can see that more than 95% of the time, the subjective logic method can generate higher returns than the traditional momentum strategies for these 810 Non-Tech German stocks portfolio.

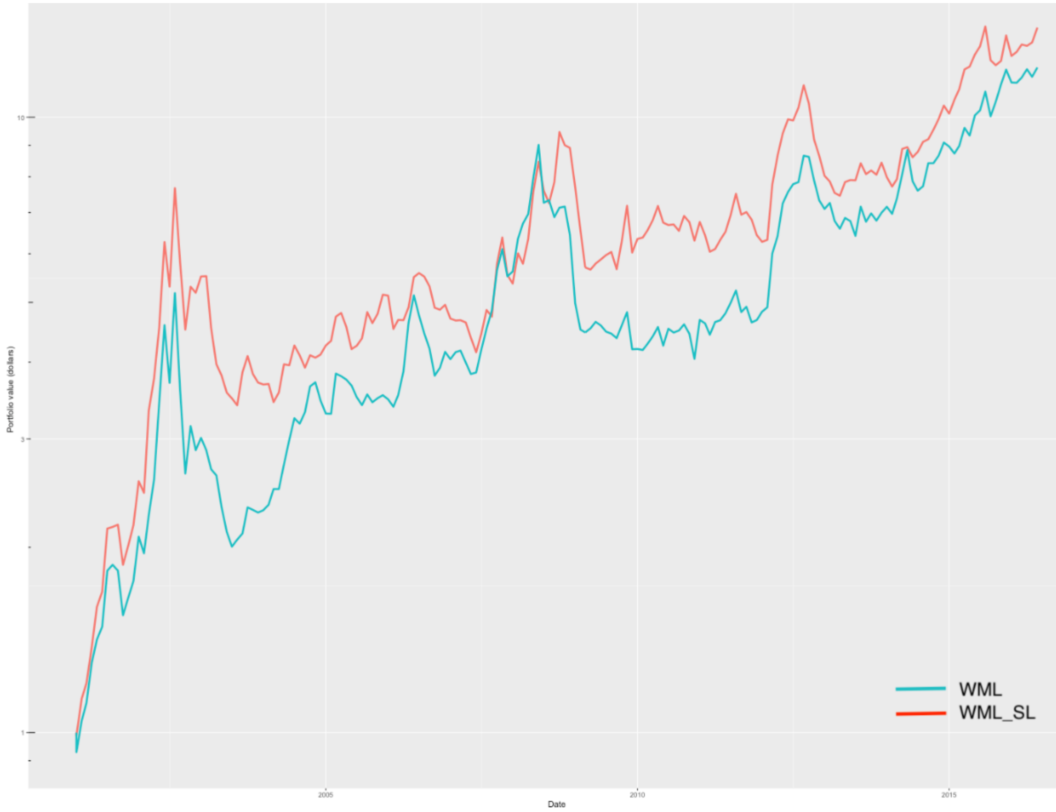
In order to check whether the result will be consistence in other markets, we also applied these two strategies on the 50 US Non-Tech companies' stocks. The results are illustrated in graph 2. It shows that our strategy based on subjective logic outperform the traditional momentum strategy very well.



Graph 2: Performance over 50 US Non-Tech Companies' Stocks

The reason is that the subjective logic method can detect and extract the noises from the historical data as uncertainty and make decision based on the pure information extracted from historical data as belief mass. In graph 1, the two lines are always upward. When the economic situation is boosted, the normal momentum strategy and subjective logic method are overlapped, and it means the data is less noisy.

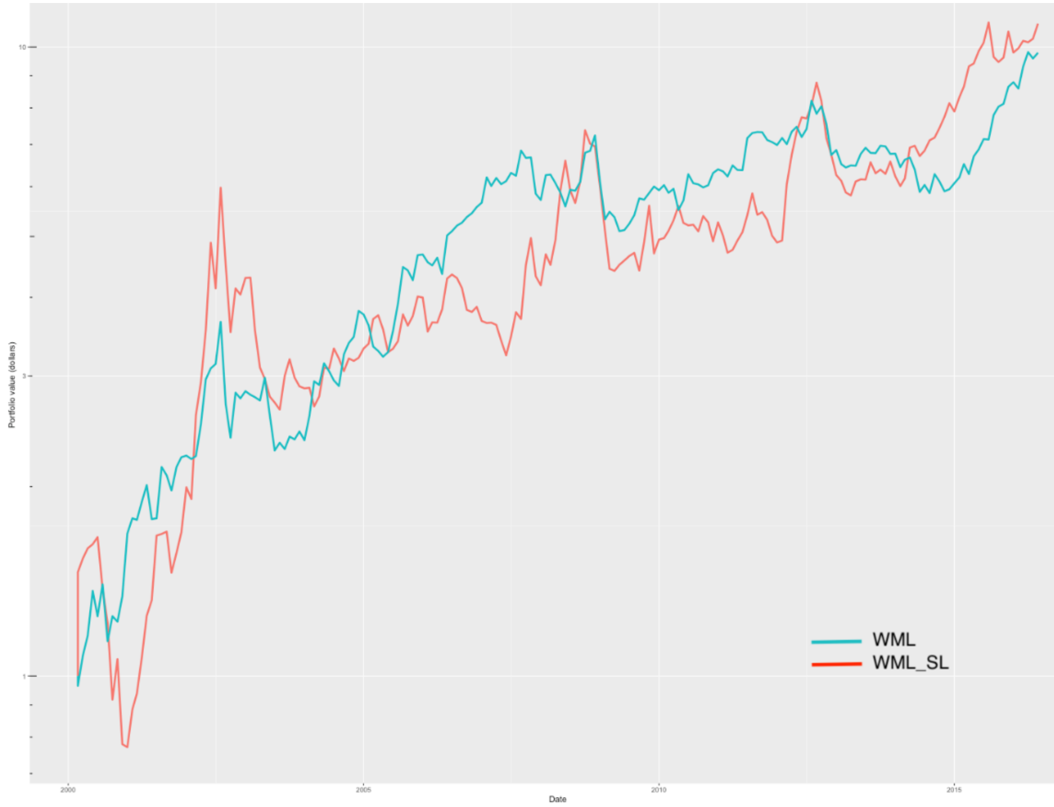
While for graph 2, this holding period includes the 2008 financial crisis, and this will make these two strategies perform differently. From 2001 to 2005, the traditional momentum strategy outperformed the subjective logic method. While after 2005, the subjective logic method can generate higher returns than the momentum strategy. As time goes by, we can see that the blue line (WML_SL) performed much better than the red line (WML). The most interesting thing happened after the 2008 financial crisis. The traditional momentum strategy performed worst and reached its lowest point around 2010. While at the same time point, the subjective logic method can be performed very well. After 2010, the blue line even began to increase dramatically in the following 10 years. This can show that the subjective logic method can perform very well after the financial crisis compared to the traditional momentum strategy.



Graph 3: Performance over 50 US Tech-Companies' Stocks without using VIX

We also test that is there any difference in the subjective logic's performance between the technology and non-technology stocks. So, we applied these two strategies into 50 US Tech Companies' stocks without using VIX. The results are opposite to the non-tech stocks. As we can see in graph 3, these two lines follow similar trends and the red line (the traditional momentum strategy) can outperform the blue line (the subjective logic method) the most of time, even after the financial crisis.

The reason is not due to the worse performance of our method based on the subjective logic. If we compare the blue curve in graph 2 and 3, we can recognize the same behavior and performance for both tech and non-tech companies' stocks. In fact, the traditional momentum strategy treats differently by the tech and non-tech companies' stocks. It works worse for non-tech companies' stocks.



Graph 4: Performance over 50 US Tech-Companies' Stocks and using VIX in our method

In order to check whether using VIX can influence our results or not, we use the same data set, which is 50 US Tech-Companies' stocks and updated the opinion by the opinion assigned to the VIX by using formula (11). Adding VIX can help us to remove a small part of the noises in the historical data, but it cannot help us to improve the results a lot. By using VIX, the subjective logic method can outperform the momentum strategy most of the time, but it cannot generate much higher returns than the momentum strategy.

7.2 Sharpe Ratio

We calculate the Sharpe ratio to show that our method can outperform the momentum strategy in terms of risk adjusted return.

$$\text{Sharpe Ratio} = \frac{\text{Return} - \text{Risk Free Rate Return}}{\text{Standard Deviation}} \tag{13}$$

Sharpe ratio was employed to assist us to determine how well the investments will perform given their risks. The Sharpe ratio shows how much excess returns the investors will get to take one more unit of extra risk. The higher the ratio, the better for the investors. Table 2 shows the Sharpe ratio for both strategies applied over four different datasets. In most cases, the Subjective Logic (SL) method can generate higher Sharpe ratio to the investors.

Table 2: Sharpe Ratios

	810 Non-tech German eq.	50 Non-Tech US eq.	50 Tech US eq. (Without VIX)	50 Tech US eq. (With VIX)
Mom	0.882	0.310	0.623	0.623
SL	1.149	0.721	0.607	0.634

From Table 2, we can see that the subjective logic method has higher Sharpe ratio than the traditional momentum strategy, for Non-Tech 810, Non-Tech 50 and Tech 50 (with VIX). Only for the Tech 50 (without VIX) group, the momentum strategy has a little higher Sharpe ratio. We can see that subjective logic method can outperform the Momentum strategy in most of the cases. Especially for the non-tech companies' stocks, the Sharpe ratio of our sujective logic method will be much higher than Momentum strategy.

7.3 Risk Measurement

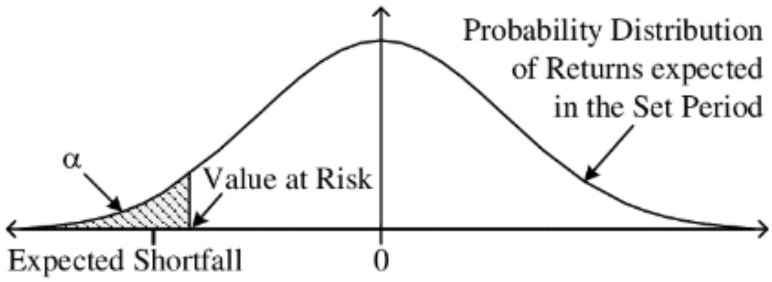
Risk is also a very important element when we are investing. We also investigate whether using the subjective logic technique in portfolio management can lead us the extract lower risks compared to using the traditional momentum strategy method or not. Therefore, we calculated the risk parameters, such as Value at Risk (VaR), Expected Shortfall (ES) and Maximum Drawdown (MDD) which are illustrated in Table 3.

Table 3: Risk Parameters

	810 Non-tech German eq.		50 Non-Tech US eq.		50 Tech US eq. (Without VIX)		50 Tech US eq. (With VIX)	
	Mom	**SL**	**Mom**	**SL**	**Mom**	**SL**	**Mom**	**SL**
VaR	-0.210	-0.122	-0.191	-0.167	-0.177	-0.228	-0.177	-0.158
ES	-0.294	-0.186	-0.240	-0.182	-0.216	-0.268	-0.216	-0.204
MDD	41.4%	31.8%	64.5%	50.5%	56.9%	62.3%	56.9%	39.2%

First, we look at the Value at Risk (VaR) and it could be used to help portfolio managers to foresee the biggest losses that could occur over a certain period of time. With 98% confidence, we can see the worst monthly loss for the subjective logic method is lower than the momentum strategy for the Non-Tech 810, Non-Tech 50, and Tech 50 (with VIX). Only for the Tech 50 (without VIX), the worst monthly loss for the subjective logic method is not exceeding 22.8%, while the worst monthly loss for momentum strategy will not exceed 17.7%. Except for this group, all other three groups showed that the subjective logic method will cause less expected losses than the momentum strategy.

Expected Shortfall (ES) got similar results as VaR. Since ES has a very close correlation with VaR (Graph 5) and it was used to measure the tail risk. As we can see that VaR has the drawback of providing no information regarding losses beyond the VaR level. Then we need to calculate the ES to know the expected average losses when it exceeds the VaR. Therefore, ES is also called conditional value at risk (CVaR).



Graph 5: Expected Shortall (ES) and Value at Risk (VaR)

From table 3, we can see that the Expected Shortfall for the Non-Tech 810 of Momentum strategy is -0.294. It means that 2% of the time our investment will return an average loss of 29.4% in one month, while the ES for our method is equal to -0.186. It means that 2% of the time our investment will return an average loss of 18.6%, which is lower than the momentum strategy. By using the subjective logic method, the average losses will also be lower for the Non-tech 50 and Tech 50 (with VIX) groups. The results showed that in most cases, the subjective logic method will bring fewer losses than the traditional momentum strategy.

Another risk parameter we measured is the Maximum Drawdown (MDD). It measures the maximum loss that could be observed between the portfolios' peaks and through before a new peak is reached. We can use MDD to measure the downside risks during a certain period.

$$\text{MDD} = \frac{\text{Trough Value} - \text{Peak Value}}{\text{Peak Value}} \tag{14}$$

When we looked at the MDD in table 3, we will find that the MDD of the subjective logic method is lower than the momentum strategy for Non-Tech 810, Non-Tech 50, and Tech 50 (with VIX). We can find similar results as VaR and ES, in most cases, our method has fewer risks than the momentum strategy.

8. Conclusion

From the above analysis, we can conclude that the subjective logic can outperform the momentum strategy most of the time. Subjective logic can generate higher returns and bring fewer risks than the traditional momentum strategies. When the economic situation is boosted, the normal momentum strategy and subjective logic method are overlapped and the performances of these two strategies will be similar. But during the financial crisis, the subjective logic method can perform very well compared to the traditional momentum strategy. This behavior can be explained from the noisy historical data point of view. Our core idea to use subjective logic is eliminating part of the noise as uncertainty and make decision based on our trusted belief mass. In general, there are many different sources of uncertainty among of our data and one of them is noise. Whenever our data has more noise and subjective logic can extract bigger part of noise as uncertainty, our method outperform the traditional strategy.

In addition, we also found that subjective logic can perform better for non-tech companies' stocks than for tech companies' stocks. We also found that our formula and method to consider VIX can help us to remove a small part of the noises in the historical data, but it cannot help us to improve the results a lot.

When it comes to risk management, we found that the subjective logic will cause less expected losses than the momentum strategy. This can show that there is a great potential for research and implementation the subjective logic in the financial industry. As we show a simplest method to use subjective logic in finance, we hope that the research in this new field continues in the future to employ and unlock all capacities of subjective logic in finance. In the following part, we will give some suggestions for the future research.

9. Future Research Topics

Based on our results and experience in this research, new questions come up which can be a topic for any interested researcher. It is our honor to share them with academic world for future research:

1) How can we expand our method to multinomial or hyper opinion? Is it valuable?
2) Applying our method on different asset classes and investigating its performance and the reason for each asset classes.
3) What is the best method to assign opinion to VIX to improve the portfolio return when we work with technical companies' equity?
4) How to apply transaction costs and other costs in our strategy based on subjective logic.
5) What is the best method to determine the weight of assets in the winner and loser portfolio based on the opinion calculated for each asset?
6) Can we improve our results by using "the consensus operator" instead of "the cumulative fusion operator"?
7) How can we assign opinion to the important news, expert judgment or any other qualitative parameter which has impact on the market prices? Which operator is suitable to combine their opinion with other opinions?
8) Reimplementation of other famous trading strategy based on subjective logic similar to what is done in this research for momentum strategy.
9) How can we use subjective logic in time-series trading strategy?
10) How can we implement the technical analysis by subjective logic? Are there any advantages for?

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APPENDIX

```
################################################################################
# Final version of our implementation for Master Thesis 2022
# Master of Science in Quantitative Finance, BI
# Supervisor: Professor Paolo Giordani paolo.giordani@bi.no
# Students:  Noor Hansen: Noor.hansen@student.bi.no
#           Li Lin:      Li.Lin@student.bi.no
################################################################################

rm(list = ls())
dev.off()
## load the required packages
require(data.table)
require(ggplot2)
require(reshape2)
library(xlsx)
library(readxl)
library("writexl")
library(PerformanceAnalytics)
library(R.utils)
library(plyr)
library(tictoc)

################################################################################
################################################################################
sortPort <- function(x, Prefix="P", mdate = 0, P = 10) {
  # define our function for sorting portfolios
  # arguments are: x: a vector of returns (or characteristics),
  # P: the number of portfolios we want
  # we assign P=10 as a default value, so unless we want a different number of
  # portfolios than 10, we do not need to set a value for P
  # first, determine the breakpoints of the sorting variable
  x1=as.numeric(na.omit(x))
  if(length(unique(x1))<5){return(NA)}
  b <- unique(quantile(x1, 0:P/P, na.rm = FALSE))
  # then, assign a portfolio-number to each stock-observation in a particular month
  p <- cut(x, breaks = b, labels = FALSE, include.lowest = TRUE, right = TRUE)
  # return the portfolio-number and add a "P" in front
  return(paste0(Prefix,p))
}
################################################################################
################################################################################
Momentum11m <- function(ISIN_RET, mom_length=11){
  # this function calculate the momentum of the last "mom_length" periods
  ISIN_RET <- ISIN_RET[order(ISIN_RET$mdate),]
  for(t in 2:nrow(ISIN_RET)){   # calculate return of last 11 months
    StartIndex = t-mom_length+1
    if (StartIndex<1){StartIndex = 1}
    ISIN_RET[t,"RET11"] = prod(1+ISIN_RET[StartIndex:t,RET])-1
  }
  return(ISIN_RET)
}
################################################################################
################################################################################
Momentum_Strg <- function(d, mom_gap=2){
  # This function implemented the traditional 11-2 momentum strategy
  d = arrange(d,ISIN,mdate)

  # calculate last 11 months return
  d$RET11= d$RET
  result <- data.frame()
  for (isin in unique(d[,ISIN])){ result <- rbind(result, Momentum11m(d[ISIN==isin, ])) }
  d <- result
    # lag 11-month return by 2 months
  d[,RET11_lag2 := shift(RET11, n = mom_gap, type = "lag"), by = ISIN]

  # apply our sortPort function to each month (by=mdate) separately
  # here, we are using the lagged 11-month return as a sorting variable
  # only use months, where RET11_lag2 is not missing (!is.na)
```

```
  d[!is.na(RET11_lag2), momport := sortPort(RET11_lag2, Prefix="P"), by = mdate]

  # calculate (equal weighted) portfolio returns for each of the 10 portfolios in each month
  momret.long <- d[!is.na(RET11_lag2), list(port.return = mean(RET)), keyby = list(momport, mdate)]

  # convert from long to wide format
  momret.wide <- dcast.data.table(momret.long, formula="mdate~momport", value.var="port.return")

  # calculate long-short winner-loser portfolio (excess) return
  momret.wide[, WML := P10-P1]
  return(momret.wide)
}
################################################################################
################################################################################
SLmomentum <- function(Portfolio, mom_lag=2, FileName){
  # This function calculates the momentum return based on the subjective logic
  # Our mom strategy is 11-1 it means 11 months return is considered
  # and 1 month lag consider for short term reversal
  # Portfolio variable names must be consist of: mdate, ISIN, RET, momport
  # lag 11-month opinion by 2 months
  Portfolio[ ,Exp_lag := shift(Exp, n = mom_lag, type = "lag"), by = 'ISIN']
  # apply our sortPort function to each month (by=mdate) separately
  # here, we are using the lagged 11-month expectation as a sorting variable
  # only use months, where Exp_lag is not missing (!is.na)
  Portfolio[!is.na(Exp_lag) & !isZero(Exp_lag), SLmomport := sortPort(Exp_lag, Prefix="P", mdate), by =
'mdate']
  Portfolio <- Portfolio[!is.na(SLmomport) & (SLmomport != "PNA"), ]

  # calculate (equal weighted) portfolio returns for each of the 10 portfolios in each month
  SLmomret.long <- Portfolio[!is.na(Exp_lag), list(port.return = mean(RET)), keyby = list(SLmomport, mdate)]
  # convert from long to wide format
  SLmomret.wide <- dcast.data.table(SLmomret.long, formula="mdate~SLmomport", value.var="port.return")

  # calculate long-short winner-loser portfolio (excess) return
  SLmomret.wide[, WML_SL := P10-P1]
  return(SLmomret.wide)
}
################################################################################
################################################################################
PlotPortfolios<- function(portfolios, Rf, Plottype= "Momentum", myTitle="", mySubTitle=NULL,
myCaption=NULL){
  # plot cumulated returns
  portfolios[is.na(portfolios)] <- 0   # convert na to 0
  # convert portfolios returns to long format (ggplot requires this)
  plotdata <- melt.data.table(portfolios, id.vars="mdate", variable.name="portfolio", value.name="return")

  # merge each return with risk-free rate in that month
  plotdata <- merge(plotdata, Rf, by="mdate")

  # calculate 1 + portfolio return (not excess!)
  # so, since all of the factors are long-short portfolios,
  # we need to add the risk-free rate (that's what you earn on your collateral for the short-side)
  plotdata[portfolio %in% c("WML_SL","WML","MktRf","HML","SMB"), cumret := (1 + return + Rf), by =
portfolio]

  # if we want to plot the 10 past-return-sorted portfolios later (later, we do not do it, yet)
  # then we do not need to add the risk-free rate, because these are not excess returns!
  # so we filter by saying, whenever the portfolio is NOT in this list of portfolionames, by using "!"
  plotdata[!portfolio %in% c("WML_SL","WML","MktRf","HML","SMB"), cumret := (1 + return), by =
portfolio]

  # set 1 dollar for first date, i.e., end of February 1991 (your initial investment)
  StartDate = min(portfolios$mdate)
  initial.investment <- data.table(mdate = StartDate, portfolio = unique(plotdata[, portfolio]),
                                return = NA, Rf = NA, cumret = 1)
  plotdata <- rbind(initial.investment, plotdata)

  # calculate cumulated return (that's why we needed the 1+ earlier)
  # now we get 1*(1+return_1stmonth)*(1+return_2ndmonth)*...
```

```
  # so the cumulated amount of dollars of that portfolio
  plotdata[, cumret := cumprod(cumret), by = portfolio]

  # convert mdates to actual dates (ggplot then knows its a date and that 12 is the last month in a year, etc.)
  plotdata[, date := as.Date(paste0(substr(mdate,1,4), "-", substr(mdate, 5, 6), "-01"))]

  # create a ggplot that plots the cumulated returns (of just WML, Market, HML and SMB) with a log-scale
  # ggplot is a really powerful plotting package
  # documentation available at: http://ggplot2.tidyverse.org/reference/
  if (Plottype == "Momentum"){
    list = plotdata[portfolio%in%c("WML_SL","WML")]
    #list = plotdata[, c("WML_SL","WML")]
  }else if (Plottype == "Mom+Winner+Loser"){
     list = plotdata[portfolio%in%c("WML_SL","WML","P1.x","P10.x","P1.y","P10.y")]
    #list = plotdata[, c("WML_SL","WML","P1.x","P10.x","P1.y","P10.y")]
  }else if (Plottype == "Mom+FF"){
     list = plotdata[portfolio%in%c("WML_SL","WML","MktRf","HML","SMB")]
    #list = plotdata[, c("WML_SL","WML","MktRf","HML","SMB")]
  }else{
    list = plotdata
  }
  plt <- ggplot(list,
               aes(x=date, y=cumret, group=portfolio, colour=portfolio)) +
    geom_line(size=1.2) +
    ylab("Portfolio value (dollars)") +
    xlab("Date") +
    scale_y_log10() +
    annotation_logticks(sides = "lr") +
    labs(title = myTitle,
         subtitle = mySubTitle,
         caption = myCaption)
  print(plt)
  return(list)
}
############################################################################
############################################################################
AssignOpenion2VIX <- function(VIX_data_File_Name="VIXCLS-Monthly.xlsx"){
  ### assigne opinion to the current VIX
  ### This part assign an opinion to the VIX(t) -> VOp(t)={Vb(t), Vd(t), Vu(t), Va(t)}
  ### When the VIX is low, volatility is low. When the VIX is high volatility is high,
  ### which is usually accompanied by market fear
  ### Buying when the VIX is high and selling when it is low is a strategy
  ### When VIX<20 then a major sell-off has taken place shortly after
  ### When VIX>30 then a major buy      has taken place shortly after
  VIX_Data <- as.data.table(read_excel(paste('/Volumes/Developer/MS Thesis/
Data/',VIX_data_File_Name,sep="")))
  VIX_mean = mean(VIX_Data$VIX)
  VIX_std  = sd(VIX_Data$VIX)
  VIX_LT   = VIX_mean-VIX_std*0.99
  VIX_HT   = VIX_mean+VIX_std*0.99
  VIX_Treshold = 0.05
  VIX_Data[, 'b'] = rep(0, each = nrow(VIX_Data))
  VIX_Data[, 'd'] = rep(0, each = nrow(VIX_Data))
  VIX_Data[, 'u'] = rep(1, each = nrow(VIX_Data))
  VIX_Data[, 'a'] = rep(0.5, each = nrow(VIX_Data))

  VIX_Data$mass_fun = 2/(1+exp(-(VIX_Data$VIX-VIX_mean)))-1
  VIX_Data[mass_fun<-1,]$mass_fun= -1
  VIX_Data[mass_fun>1 ,]$mass_fun= +1

  VIX_Data[mass_fun<0-VIX_Treshold,]$d= -VIX_Data[mass_fun<0-VIX_Treshold,]$mass_fun
  VIX_Data[mass_fun>0+VIX_Treshold,]$b=  VIX_Data[mass_fun>0+VIX_Treshold,]$mass_fun

  VIX_Data[, 'u'] = 1-VIX_Data[, 'b']-VIX_Data[, 'd']

  return(VIX_Data[,c(1:6)])
}
############################################################################
############################################################################
```

```
AssignOpenion2Return <- function(Return, Threshold=0.001){
  # This function assign an opinion to each period's return r(t) -> ROp(t)={Rb(t), Rd(t), Ru(t), Ra(t)}
  # if Rt>Threshold => b>0 and d=0,   if Rt<-Threshold => b=0 and d>0, else b=d=0, u=1
  # Threshold = the area (-Threshold, +Threshold) is our uncertain area
  if(Return>=Threshold){
    b = Return/(2 +Return)
    d = 0
  }else if(Return<=-Threshold){
    b = 0
    d = -Return/(2 -Return)
  }else{
    b = 0
    d = 0
  }
  u= 1-b-d
  a=1/2   # better to replace by the cummulative blief mass for last 3 years ???????
  ReturnOpinion = data.frame(b=b,d=d,u=u,a=a)
  return(ReturnOpinion)
}
###########################################################################
###########################################################################
Cumulative_Fusion <- function(Op1, Op2){
  # This operator is used for cumulative updating the opinions generated in disjoint time: for example calculating
  # the cumulative opinion of a trend which shows how much we are close to the breaking point
  # Assume two observers A and B who observe the outcomes of the process over two separate time periods.
  # The cumulative rule is commutative, associative and non-idempotent.
  # base rate for both opinion must be equal to each other
  if (Op1$u+Op2$u == 0){
    b = (Op1$b + Op2$b)/2
    d = (Op1$d + Op2$d)/2
  }else{
    b = (Op1$b*Op2$u + Op1$u*Op2$b)/(Op1$u + Op2$u - Op1$u*Op2$u)
    d = (Op1$d*Op2$u + Op1$u*Op2$d)/(Op1$u + Op2$u - Op1$u*Op2$u)
  }
  u= 1-b-d
  a= Op1$a
  COpinion = data.frame(b=b,d=d,u=u,a=a)
  return(COpinion)
}
###########################################################################
###########################################################################
Combine_VIX <- function(COp, VOp){
  # in this version we apply VIX if it is consistent with momentum
  VIX_weight = 1
  colnames(VOp)<-c('b', 'd', 'u', 'a')
  colnames(COp)<-c('b', 'd', 'u', 'a')
  FOp <- data.frame(b=0, d=0, u=1, a=1/2)
  # Combine VIX and Momentum
  VOp1 = VIX_weight * VOp
  VOp1['u'] = 1-VOp1['b']-VOp1['d']
  VOp1['a'] = 0.5
  FOp <- Cumulative_Fusion(COp, VOp1)
  # Update special cases
  if ((VOp['u']==0)&(VOp['b']*VOp['d']*COp['b']*COp['d']*COp['u']!=0)){VOp = c(0,0,1,0.5)}
  if(COp['b']==1){
    FOp[c('b', 'd')]= c(1-VOp['d']/6-VOp['u']/20, VOp['d']/6)
    if(VOp['u']==1){FOp<-COp}
    else if(VOp['d']*VOp['u']!=0){FOp<-COp}
  }

  else if(COp['d']==1){
    FOp[c('b', 'd')]= c(VOp['b']/6, 1-VOp['b']/6-VOp['u']/20)
    if(VOp['u']==1){FOp<-COp}
    else if(VOp['b']*VOp['u']!=0){FOp<-COp}
  }

  else if(COp['u']==1)   {FOp[c('b', 'd')]= c(VOp['b']/6, VOp['d']/6)}
  else if((COp['b']==0)&(VOp['b']*VOp['u']!=0)) {FOp<-COp}
  else if((COp['d']==0)&(VOp['d']*VOp['u']!=0)) {FOp<-COp}
```

```
  else if(COp['u']==0){
    if (VOp['d']*VOp['u']!=0){FOp<-COp}
    else if (VOp['b']*VOp['u']!=0){FOp<-COp}
    else if ((VOp['b']==1)|(VOp['d']==1)) {FOp <- Cumulative_Fusion(COp, VOp)}
  }

  if (FOp['b']>1) {FOp[c('b', 'd')]=c(1,0)}
  if (FOp['b']<0) {FOp['b']=0}
  if (FOp['d']>1) {FOp[c('b', 'd')]=c(0,1)}
  if (FOp['d']<0) {FOp['d']=0}
  FOp['u'] = 1-FOp['b']-FOp['d']
  FOp['a'] = 0.5

  return(FOp)

}
##############################################################################
##############################################################################
AssignOpenion <- function(DataF, Portfolio, scale=5, Threshold=0.001, ApplyVIX=FALSE){
  # scale = for re-scaling the Jumps to be suitable to calculate opinion, 10000 for converting to bps
  # Threshold = the area (-Threshold, +Threshold) is our uncertain area, Scale =10000 and Threshold=1 means
one bps
  # Portfolio is consisting the in DataF but in long format. DataF is in wide format
  ##### Initialization
  Portfolio_b <- data.frame(ISIN=character(), mdate=numeric(), RET=numeric(), Exp=numeric())

  mom_lenght  = 11   # Lenght of momentum strategy
  nrAssets = ncol(DataF)-1 # not counting the 'mdate' and 'VIX'

  ### Add VIX opinions to the data set
  VIX_Openion <- AssignOpenion2VIX()
  colnames(VIX_Openion)[c(3,4,5,6)] <- c("Vb","Vd","Vu","Va")
  DataF <- merge(DataF, VIX_Openion, by="mdate")
  VOp_columns <- c("Vb","Vd","Vu","Va")

  for (i in 2:(nrAssets+1)){   # loop over all assets
    AssetName = colnames(DataF)[i]
    print(paste('--> The asset number under process is: (',toString(i),') with ISIN/Name: ',AssetName,sep=""))

    # Opinion assigned to the current standardized return
    Rb =paste('Rb_',toString(i-1),sep="")
    Rd =paste('Rd_',toString(i-1),sep="")
    Ru =paste('Ru_',toString(i-1),sep="")
    Ra =paste('Ra_',toString(i-1),sep="")
    ROp_columns <- c(Rb, Rd, Ru, Ra)

    # assign opinion to the current asset Returns
    DataF$RET_Scaled  = DataF[,i]*scale
    DataF[,Rb] =  DataF$RET_Scaled/(2+DataF$RET_Scaled)
    DataF[,Rd] = -DataF$RET_Scaled/(2-DataF$RET_Scaled)
    DataF[(DataF$RET_Scaled)< Threshold, Rb] =0
    DataF[(DataF$RET_Scaled)>-Threshold, Rd] =0
    DataF[,Ru] = 1- DataF[,Rb]- DataF[,Rd]
    DataF[,Ra] = 0.5

    # Opinion assigned to the cummulative fusion of returns-opinion for last 11 months
    Cb =paste('Cb_',toString(i-1),sep="")
    Cd =paste('Cd_',toString(i-1),sep="")
    Cu =paste('Cu_',toString(i-1),sep="")
    Ca =paste('Ca_',toString(i-1),sep="")
    COp_columns <- c(Cb, Cd, Cu, Ca)
    DataF[, COp_columns] <- DataF[, ROp_columns]
    DataF[, c('Rbt', 'Rdt', 'Rut', 'Rat')] <- DataF[, ROp_columns]

    for (lag in 1:(mom_lenght-1)){
      DataF[, c('Rbt', 'Rdt', 'Rut')] = shift(DataF[, c('Rbt', 'Rdt', 'Rut')], n=1, fill=0, type = "lag")
      DataF[, c('b0','d0')] = list( (DataF[,Cb] + DataF[,'Rbt'])/2, (DataF[, Cd] + DataF[,'Rdt'])/2)
      DataF[, c(Cb)] = (DataF[,Cb]*DataF[,'Rut'] + DataF[,Cu]*DataF[,'Rbt'])/(DataF[,Cu]+DataF[,'Rut']-
DataF[,Cu]*DataF[,'Rut'])
```

```
    DataF[, c(Cd)] = (DataF[,Cd]*DataF[,'Rut'] + DataF[,Cu]*DataF[,'Rdt'])/(DataF[,Cu]+DataF[,'Rut']-
DataF[,Cu]*DataF[,'Rut'])
    DataF <- setDT(DataF)[DataF$Cu + DataF$Rut ==0, c(Cb,Cd):= list(b0,d0)]
    DataF <- as.data.frame(DataF)
    DataF[, c(Cu)] = 1-DataF[, c(Cb)]-DataF[, c(Cd)]
  }
  # Consensus combination of all opinion
  Fb =paste('Fb_',toString(i-1),sep="")
  Fd =paste('Fd_',toString(i-1),sep="")
  Fu =paste('Fu_',toString(i-1),sep="")
  Fa =paste('Fa_',toString(i-1),sep="")

  if (ApplyVIX) {
  #### Apply VIX to Cop: Only for tech-stocks
  for(t in 3:nrow(DataF)){   # Loop over all periods
    mdate_t = DataF[t,1]
    # calculate the cummulative opinion for last 11 months
    COp <- data.frame(b=0,d=0,u=1,a=1/2)
    COp <- DataF[t, COp_columns]

    VOp <- DataF[t, VOp_columns] # Coresponding VIX opinion
    FOp <- Combine_VIX(COp, VOp)
    DataF[, c(Fb, Fd, Fu, Fa)] <- FOp
  }
  }else{
    DataF[, c(Fb, Fd, Fu, Fa)] <- DataF[, COp_columns]
  }

  DataF[,"Expb"] = DataF[,Fb] +DataF[,Fa]*DataF[,Fu]
  DataF[,"ExpbMd"] = DataF[,Fb] - DataF[,Fd]
  temp = DataF[,c('mdate', Fb, 'Expb', 'ExpbMd')]
  temp <- merge(Portfolio[ISIN == AssetName,], temp, by=c('mdate'))

  colnames(temp)[4] <- "Exp"
  Portfolio_b = rbind(Portfolio_b,temp[, c('mdate','ISIN', 'RET', 'Exp')])

 } # end of loop for i = asset number

 temp<-NULL
 DataF[, c('RET_Scaled', "Expb", "ExpbMd", 'Rbt', 'Rdt', 'Rut', 'Rat', 'b0','d0')] <- NULL

 return(Portfolio_b)

}

################################################################################
#########################          MAIN         ############################
################################################################################
## automatic detection of current file location
setwd(dirname(rstudioapi::getActiveDocumentContext()$path))

# Load Fama-French factors and risk-free rate (Rf). Rf is needed for plot wealth
ff <- as.data.table(read.csv2("Data/DE_FF_Factors.csv"))
setkey(ff, mdate)

IsWideFormat=TRUE # choose FALSE if our dataset is in long format and TRUE if it is in wide format
if(IsWideFormat){
 # This part is for reading data for 810 German stocks
 d_long <- as.data.table (read.csv2("Data/DE_data_long.csv"))[,c("mdate", "ISIN", "RET")]
 setkey(d_long, ISIN, mdate)

 # changing the dataset from long to wide format (each column is a time series of each asset returens)
 d_wide <- d_long[,c("mdate", "ISIN", "RET")]
 d_wide = dcast(d_wide, mdate~ISIN, value.var ="RET")
 d_wide[is.na(d_wide)] <- 0
}else{
 # data from Bloomberg (US equity)
 d_wide <- as.data.table(read_excel("Data/Tech50_Return.xlsx"))
 d_wide[is.na(d_wide)] <- 0
```

```
  # changing the dataset from long to wide format (each column is a time series of each asset returens)
  d_long <- melt.data.table(d_wide, id.vars="mdate", variable.name="ISIN", value.name="RET")
  setkey(d_long, ISIN, mdate)
  d_wide <- as.data.frame(d_wide)
}
# apply normal momentum strategy
momret.wide <- setDT(Momentum_Strg(d_long))

# apply momentum strategy based on SL
Return <- setDT(AssignOpenion(d_wide, d_long[,c('mdate','ISIN', 'RET')], 1000, 42.5))   # assign opinion
SLmomret_b.wide <- SLmomentum(Return, mom_lag=2, "Momret_Long_SL_b_lag2.xlsx") #apply mom strg
based on SL

# merge returns of both Normal momentum and SLmomentum
MomReturns.wide <- merge(momret.wide,SLmomret_b.wide[,list(mdate, WML_SL, P1, P10)],by="mdate")
plotdata=PlotPortfolios(MomReturns.wide, ff[,list(mdate,Rf)],Plottype= "Momentum",
                    myTitle="belief(Tech50) VIX: S=1000, L=2, T=10") # plot returns

plotdata1 =  plotdata[portfolio=='WML',c('mdate', 'return')]
plotdata1 = plotdata1[!is.na(return),]
n = nrow(plotdata1)
AveR1 = mean(plotdata1$return)
stdR1 = sd(plotdata1$return)
SR1 = AveR1/stdR1*sqrt(12)
VaR1 = sort(plotdata1$return)[0.02*n]
esh1 = mean(sort(plotdata1$return)[0:(0.02*n)])
MDD1 = maxDrawdown(plotdata1$return)

plotdata2 =  plotdata[portfolio=='WML_SL',c('mdate', 'return')]
plotdata2 = plotdata2[!is.na(return),]
AveR2 = mean(plotdata2$return)
stdR2 = sd(plotdata2$return)
SR2 = AveR2/stdR2*sqrt(12)
VaR2   = sort(plotdata2$return)[0.02*n]
esh2 = mean(sort(plotdata2$return)[0:(0.02*n)])
MDD2 = maxDrawdown(plotdata2$return)
# make a table of the results
resl = data.frame(rbind(c(SR1,SR2),c(VaR1,VaR2),c(esh1,esh2),c(MDD1,MDD2)))
colnames(resl) = c('Mom','SL')            # define column names of result table
rownames(resl) = c('SR' ,'VaR','ES', 'Max DD') # define rownames of result table
resl
##############################################################################
```