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Saving Towards Homeownership in Norway: Including the BSU in Portfolio Choice

GRA 1970 - Master Thesis

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Abstract

We research the effect of housing and the Boligsparing for Unge (BSU) savings account on portfolio allocation and performance. By applying the mean-variance optimality framework to form portfolios, we use the Sharpe Ratio and tests of differences in return variance and mean to investigate performance. Graphing the efficient frontier, we find higher Sharpe Ratios for optimal portfolios with the BSU, which decreases when increasing housing. Without the BSU, portfolios with housing tend to use available cash towards reducing leverage. We conclude that portfolios with the BSU are less volatile, but do not offer significantly higher returns compared to the housing market. Still, the BSU is Norway's best savings scheme in the pursuit of achieving homeownership.

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1. Introduction and Motivation

With a keen interest in providing support for Norwegian investors in optimizing their portfolio choices, this thesis explores how such portfolios are allocated between the asset classes of stocks, bonds, housing, mortgage, and the Boligsparing for Unge (BSU) savings account.

Owner-occupied housing is the single most important asset in many investors' portfolios (Tracy, Schneider, & Chan, 1999). This is especially relevant in Norway, where the homeownership rate for individuals totaled 80.8% in 2020 (Eurostat, 2022). Norwegians on average own real estate by the age of 27.8, while the European first-time homeowner is on average 31 years old (NEF, 2021; Swiss Life, 2018). With the goal of homeownership being integral in the Norwegian government's housing policy, the BSU account is part of an incentive program to support equity build-up among young households. Industry experts, such as financial advisor Hallgeir Kvadsheim, often praise the savings account, deeming it "Norway's best savings scheme" (Kvadsheim, 2021). Kvadsheim claims that investing in the BSU could return up to 22% during the first year, with the savings account often offering higher interest rates than mortgages, implying that it is more profitable to save in the BSU than to repay mortgage loans.

Accordingly, section 3 in this thesis is dedicated to diving deeper into Norwegian homeownership rates and the BSU account to get a better grasp of optimal portfolio choices in the mean-variance optimization framework including these typical Norwegian asset classes.

Thus, our primary research question is:

What effect does the Boligsparing for Unge (BSU) savings account have on mean-variance optimal portfolio allocation?

Additionally, we analyze the following sub-questions:

- a. Does the BSU increase portfolio Sharpe Ratios for investors without housing?*
- b. Can the BSU account assist savers in obtaining homeownership?*

To conduct tests on optimal portfolios, we use the mean-variance optimality framework in portfolio formation, maximizing expected return for a given level of risk by choosing allocations in different asset classes. While classical portfolio optimization models focus on stocks and bonds, our model follows Flavin and Yamashita (2002) in

including housing and mortgage. Beyond this, we further extend the asset allocation choice with the BSU savings account. Throughout the thesis, we will analyze the risk and return trade-off for these portfolios, compare them to portfolios that do not have the possibility of saving in the BSU, and compare the portfolios' performance relative to that of the housing market. In this way, we examine whether investors increase their wealth by saving in the BSU, potentially gaining better chances of purchasing a house.

Finding that only 32.6% of the Norwegian population between the age of 17 to 33 saved in the BSU in 2020, we hope that our research can assist in gaining a better understanding of the savings account and contribute to this age group taking reasoned choices when it comes to portfolio allocation (SSB, 2021a). After all, given the high homeownership rate in Norway, it seems plausible that the majority of those without the account also will purchase a housing unit. Thus, for those 67.4% of 17- to 33-year-old Norwegians that do not have a BSU account, we wish to present a factual argumentation for opening a BSU account, although we stress that certain benefits associated with the savings account might not be fully applicable to everyone. For instance, households earning less than NOK 86,650 will not reap the full tax benefit of the BSU. Moreover, we disregarded the fact that saving in the BSU is highly illiquid, meaning that households forego immediate consumption by locking their savings in the savings account. Investigating such cases more closely is certainly interesting and relevant, however, in this thesis we focus on whether a fully utilized BSU could offer households added value when saving towards home purchases. Hence, our research examines the case closest to the borderline of homeownership.

Lastly, we define housing as private real estate that is owner-occupied and are not analyzing other real estate investments such as commercial properties or real estate funds. We would also like to point out to the reader that we use the expressions "households", "savers" and "investors" interchangeably, hoping readers can bear with us on this and enjoy our thesis.

2. Literature Review

Portfolio optimization through a mean-variance framework was first outlined by Harry Markowitz (1952), who laid the foundation for modern portfolio theory (MTP). In his article, Markowitz describes the impact of covariance relationships between assets in a portfolio and their diversification benefits. Combined with William F. Sharpe's work on the Capital Asset Pricing Model (Sharpe, 1964) and the Sharpe Ratio (1966; 1994) the literature provides us with an investment framework that is based on maximizing expected returns while minimizing risk in a portfolio setting of investment choices, i.e., finding the Sharpe optimal portfolio.

Our thesis closely follows Flavin and Yamashita (2002) in including housing and mortgage as assets when constructing an optimal portfolio using the mean-variance framework. Utilizing quantitative methods and quadratic programming of the covariance matrix and vector of expected returns, Flavin and Yamashita (2002) solve a partial equilibrium model, i.e., there is no aggregate market-clearing condition, in which they find the optimal portfolios for a hypothetical household. Furthermore, they show that the "housing constraint" (or the household's ratio of house value to wealth, h) has a major effect on the portfolio's risk and return. As an example, the paper finds that young households tend to have high h , i.e., housing is a major component of the household's wealth. With it, housing introduces high leverage, leaving young households in a situation of high risk (as well as return), which in turn results in a willingness to reduce their portfolio's risk by de-leveraging, or buying bonds instead of stocks. Naturally, as a household matures, h decreases while the percentage of financial assets in stocks is inversely related to h , meaning stockholdings increase with age. Thus, Flavin and Yamashita (2002) find that the efficient frontier faced by households is not static but changes significantly over the course of a lifetime. Lastly, the paper finds conflicting evidence of age impacting asset prices in existing studies but concludes that demographic changes could potentially explain effects on asset prices, e.g., the baby boom generation's relatively high savings rate and large fraction of the population is pushing up stock prices.

As an extension, Yamashita (2003) tests their 2002 model by analyzing the link between house-to-net-worth ratio (h) and stock investments (s) with empirical data. Here, he stresses once more the importance of housing's dual characteristic of being

both an investment and consumption good, referencing Brueckner (1997). At elevated levels of h , he finds overinvestment in housing due to high demand for consumption, leaving households in an exceedingly leveraged position. These households attempt to mitigate portfolio risk by lowering their stock market participation. However, when housing investments are constrained from the consumption demand for housing (i.e., controlling for h by holding the variable constant), part of the cross-sectional heterogeneity in stockholdings disappears. In doing so, the state of high leverage in the form of mortgage (or not) is a determinant for the relationship between h and s (negative with mortgage, and positive without mortgage). Hence, the paper finds that leverage is the major factor in crowding out stockholdings. While many aspects of the simulation fit the empirical data, Yamashita is clear in that a portfolio model of housing constraint alone is not sufficient in explaining the crowding out effect. This is due to many simplifying assumptions in the model. Among others, issues of endogeneity and sample selection bias must be addressed. We are noting that housing is a complex variable to include, as each homeowner exhibits varying risks and return features depending on their age, labor income, location of living, and many other factors, most of which we will have to abstract from.

Solving a model of optimal portfolio and consumption decisions for investors, Cocco (2005) finds that the inclusion of housing in the portfolio leads to crowding out of stock market participation, especially for younger and poorer investors. Furthermore, the model incorporates human capital, defined as the generation of dividends in the form of labor income. Here, Cocco argues that the human capital's resemblance to T-bills drives the individual's investments towards increasing levels of stockholdings in the portfolio, partly explaining the positive correlation between leverage and investments in risky assets, contrasting Yamashita's findings (2003). Cocco's reasoning is complemented by an analysis of the consumption dimension of housing, indicating that higher levels of human capital (particularly for young investors) lead to purchasing more expensive houses, inducing increased levels of debt. Hence, the level of human capital has great implications on purchasing housing, the level of leverage, and share of stocks in the financial portfolio.

Lastly, the empirical evidence listed in the paper highlights that the portfolio's share in stocks increases over a lifetime, partly for the reason mentioned above, but

also since the importance of liquid assets on future consumption is less than that of housing or human capital. As the investor matures, her future consumption is less correlated with the return on the portfolio of liquid assets, implying that older people are more willing to accept risk in this portfolio. Hence, housing not only leads to a crowding out effect from stock markets for young and lower-wealth investors, but it can also counter declines in the share of stocks in their financial portfolio.

Like Flavin and Yamashita (2002), Fagereng, Gottlieb and Guiso (2017) estimate a life-cycle portfolio model, but on a large data sample of Norwegian households. Parameterizing the model using a comparably high value of risk aversion (around 11), a per-period cost of market participation, and a probability for a large stock market tail event (like that of stock market crashes on the Oslo Børs), the authors find comparable patterns to that of the data in their model. One of the paper's major discoveries is that households rebalance their portfolios by reducing the share of stocks as they approach retirement, exiting the stock market around the retirement date. Interestingly, the estimated life-cycle profile shows a hump-shaped pattern in the risky asset market participation across the age-dimension. The paper also features a cohort effect where investors' participation rate in the risky asset markets is likely dependent on the stock market returns in the initial, formative years. Although the paper is highly relevant as it shows Norwegian portfolio allocations over the life-cycle, the paper does not include housing as an asset class.

Pelizzon and Weber (2008) build upon the work of Flavin and Yamashita (2002), differing from the paper in that housing and financial returns are considered correlated, and by abandoning the non-negativity constraints in constructing optimal portfolios. In addition, the research of Pelizzon and Weber (2008) is also linked to Cocco (2005), who solves an intertemporal optimization problem numerically and shows the crowding out effect housing has on stock market participation. However, rather than advancing the analysis on the intertemporal optimization problem, the authors derive the cross-sectional optimal portfolio for a given housing wealth, as well as empirically showing how the households' portfolios in Italy differ from optimality when faced with house price risk. The paper finds that housing returns in Italy are correlated with returns on financial assets (such as stocks and bonds), thus strengthening the need of including a hedging term in the portfolio optimization problem. Hence, the optimal investment

decision is affected by the need to hedge the risks arising from the illiquid position, which would not be necessary should the returns have zero correlation.

Furthermore, deriving the tangency portfolio without including housing shows that most portfolios (collected from SHIW for 1998) are efficient, although the findings reveal issues such as that of the optimal portfolio not holding stocks, as well as the limited precision in the test due to a small number of observations. However, including housing, very few portfolios are considered efficient, although treating housing as a constrained, rather than unconstrained asset increases the number of portfolios that are efficient. Drawing upon the results, the paper highlights that the standard tests of portfolio efficiency are considered biased as they neglect illiquid wealth. Hence, optimal portfolios in periods of no adjustments of the housing stock are affected by housing price risk.

Lastly, Pelizzon and Weber (2008) identifies the issue of housing as a consumption good, showing that the housing liability, i.e., the need to live somewhere, affects the optimal portfolio. By making adjustments through the additional variables “net housing wealth” and “fraction of housing services currently enjoyed”, the results show an increased number of conditionally efficient fully diversified portfolios.

The authors conclude the paper by highlighting strong evidence towards Italian households holding well-diversified portfolios failing to fully exploit hedging opportunities, thus not holding completely efficient portfolios.

Hu (2005) adds to the research review the analysis of households transitioning from renting to mortgage-financed housing. By modeling the transitioning decision from renting to owning a home, Hu can support housing’s substitutional effect regarding stocks, while bonds are distinguished as sources of liquidity in a portfolio. Interestingly, with the goal of becoming a homeowner, self-selected renters choose a suboptimal asset allocation. More specifically, they hold fewer stocks and more bonds than predicted by traditional models.

Yao and Zhang (2005) create an economic model and use data from PSID between 1984 and 2001 in analyzing the investor’s portfolio decisions when gaining housing services either through renting or owning. The paper finds results attributing a higher life-time wealth to those who own housing rather than those who rent and finds a substitution effect of home equity for risky stocks. This shows a diversification effect

of holding less stocks in proportion to net wealth while holding a higher proportion of stocks in their liquid financial portfolio. In addition, the paper argues that studies not considering the house rental market might be subject to a distortion in housing choices, especially for older investors or investors with high liquidity constraints.

The authors also show a hedging demand for holding stocks when stock market and housing market returns are correlated showing how renters often hold a substantial proportion of stocks in their portfolios compared to that of house owners.

Also consistent with other studies, Yao and Zhang (2005) find a hump-shaped pattern in stockholdings over age. However, even when incorporating moving shocks in their model, the authors find that the ownership percentage in housing is lower in the observed data than in the model, implying that there are significant economic frictions conscribed homeownership.

Lastly, investors that hold more wealth in liquid form, i.e., have a higher mortgage but use this to hold more liquid assets compared to illiquid housing, are usually more robust in the case of a shock with large negative returns in stocks. Hence, holding a proportionally larger mortgage to invest more in liquid asset classes is described in the paper as highly beneficial.

In summary, existing literature on portfolio choice in the presence of housing explores the relationship between stockholdings, housing, and leverage. Here, we find conflicting evidence on the impact of housing on stocks in the financial portfolio and the crowding out effect. Our research is novel by incorporating the BSU in the mean-variance optimality framework in Norway. We contribute to the literature the effect of adding and removing the BSU on optimal portfolios with housing.

3. Homeownership and Boligsparing for Unge – Investor’s Considerations

In this section, we describe the importance of homeownership in Norway and present stylized facts and figures about the BSU savings account. At its root, portfolio selection in the context of homeownership is influenced by domestic and local housing policies, incentivizing certain investment choices while deterring others, which we will now explore in detail.

3.1. The Importance of Homeownership

The importance of homeownership in Norway can be traced back to the nation’s egalitarian identity and historically low degree of urbanization before the discovery of oil on the Norwegian Continental Shelf accelerated wealth creation in the 1960s. Since World War II, the housing- and credit market had been thoroughly regulated by the state, ensuring that mortgages and housing were affordable for the average Norwegian (Sørvoll, 2007). While the government consistently supported high homeownership rates, the 1980s deregulation of both the housing- and credit markets unlocked possibilities for investors to further capitalize on price gains and use real estate as an investment vehicle. At that time, the government also reduced its social housing support, retreating its influence on the real estate market (Stamsø, 2012). Instead, tax incentives, such as mortgage interest deductions, a moderate wealth tax, and BSU were introduced, shouldering the responsibility of securing housing, first and foremost, on the individual. In terms of tax incentives today (in 2022), owner-occupied housing is the only investment vehicle that is exempt from capital gains tax at sale, under the premise that the housing unit was owned and occupied for at least 12 months out of the 24 months preceding the sale (Pihl, 2021). On the cost side of homeownership, there are typically maintenance costs, depreciation, and potential property taxes. There is also a wealth tax on housing (conditional on the owner having positive taxable net wealth), although owners enjoy a 75% valuation discount, giving them a wealth-tax shield on primary dwellings (Skatteetaten, 2022b).

As a positive externality, high homeownership was supposed to keep the country’s real estate in good standard, induce Norwegians to save intelligently, and, through high employment and productivity, build up wealth among the general population (Lundesgaarden, 2018). While there is scarcely any academic literature to prove the

direct causality of these effects in Norway, studies in the US find that homeownership has had a significant impact on the wealth accumulation of households (Turner & Luea, 2009).¹ Proving these incentives successful in the European context, Norway exhibits one of the highest homeownership rates, with 80.8% of the population owning their home in 2020, as shown in Figure 1 below. By looking at data from 2016, we analyze the age groups that comprise this high rate and find that especially young Norwegians between 25 and 29 years enter the real estate market in large quantities, relative to other countries' populations (SSB, 2019). Historically, the Norwegian homeownership rate has fluctuated between 80.3% and 86.1% with a standard deviation of 1.4%, exhibiting a slight downward trend in the time series (Trading Economics, 2021).

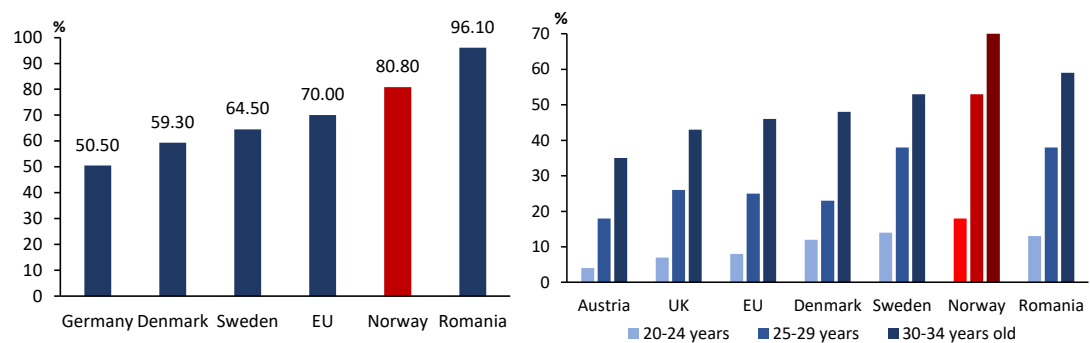


Figure 1. Homeownership rates in selected European countries in 2020; Homeownership rates among young adults in Europe in 2016 (by age groups). Based on (Eurostat 2022; SSB 2017).

At the same time, high homeownership in Norway has led to steady price increases in the housing market, with the capital of Oslo seemingly being pressured by a supply-demand imbalance driving up prices. In 2020, average square meter prices in Oslo were at a 122% premium compared to the rest of the country.² It seems housing can be seen as a double-edged sword, providing high asset returns for participants, but inferring high debt burdens, especially among young Norwegians lacking support from family or government. Hence, entering the housing market requires the young households to be highly dedicated and smart in their savings, or incentivized, to build up sufficient equity for an early home purchase.

¹ Investigating the cause and effect of homeownership is challenging due to the many potential sources of endogeneity, unobserved heterogeneity, as well as due to housing having both a consumption and investment component. Here, the renting vs. owning choice adds another layer of possible research that is significantly challenged by the self-selection bias.

² In 2020, the average m² price in Oslo for both new and used detached houses was NOK 66,929 while the rest of Norway (excluding Oslo) averaged a m² price of NOK 30,119 (SSB, 2021d).

Furthermore, certain lending regulations need to be considered in the context of homeownership in Norway, namely the Norwegian lending regulation “*Boliglånsforskriften*”. First adopted in 2011, the regulation was designed to set requirements for new residential mortgage loans and has since been extended to include consumer loans. The current regulations came into effect in January 2021 and will hold until the end of 2024, covering the following elements for mortgage lending:

Table 1. Norwegian mortgage lending regulation (2021-2024). Based on (Finanstilsynet 2021).

Item	Requirement
(1) The customer’s debt-serving ability	Stress testing the customer’s tolerance of a 5% interest rate increase
(2) The customer’s debt-to-income (DTI) ratio	Maximum DTI ratio of 500%
(3) The mortgage size in relation to property value (loan-to-value ratio (LTV ratio))	Maximum LTV ratio of 85%
(4) Required principal payments for mortgages with a high LTV ratio	For loans with LTV above 60%

However, it is worth noting that banks enjoy a flexibility quota when making customer-specific assessments where 8% to 10% of loans given may exceed the regulatory requirements each quarter. Lastly, refinancing is permitted if the new loan does not exceed the existing loan, has the same property as collateral, its duration does not exceed the remaining duration of the existing loan, and principal payment requirements are either the same or stricter than previously (for the legal text see: (Finansforetaksloven, 2015, §1-7)).

3.2. *Boligsparing for Ungdom (BSU)*

3.2.1. *Stylized Facts, Figures and Developments of the BSU Account*

The housing savings account BSU is a unique capital allocation choice for young adults under 34 years in Norway. Originated in 1992 as part of a federal tax reform and the governments housing policy, the BSU is a savings incentive program to support young people’s establishment in the Norwegian housing market (Gulbrandsen, 2000). With this, the BSU functions as a main pillar in Norwegian’s pursuit of owning their own home at a young age.

At its inception, the BSU covered a maximum annual savings amount of NOK 10,000, with a total maximum savings amount (before the age of 34) of NOK 60,000.

The prescribed savings amounts have regularly been increased over time, reaching NOK 27,500 and NOK 300,000 for the annual and total maximum savings amounts in 2022, respectively (see Table 7 in the Appendix for a summary of the BSU regulation from 1992 to 2021) (Finansdepartementet, 2021).³ Moreover, the annual savings amount is non-binding and can be freely chosen each year, as long as it is below the prescribed quota, thus providing some flexibility in the saver's yearly deposits. However, at the end of the year, any deposits made are frozen in the BSU and cannot be withdrawn without closing the account. We also note that, at current levels, it takes about 11 years to fill the BSU. Furthermore, the full account of NOK 300,000 could purchase a housing unit equivalent to NOK 2,000,000, meaning a unit of approximately 30m² in size using the average price in Oslo (SSB, 2021d).

Nevertheless, there are two main arguments for why 17 to 33-year-olds should save in the BSU account: (1) there is no other savings account offering higher interest rates, and (2) the BSU provides a unique tax deduction of 20% on the saver's taxable income.

In terms of interest rates, the BSU has historically given savers an average "premium" of 2.5% over the banks' deposit rate (see section 5.5 for a detailed description of BSU rates). Accounts are facilitated by commercial banks in Norway, where rates are market-driven, i.e., each bank is free to set its own interest rate on the BSU. Moreover, there are no restrictions regarding the maximum or minimum level of interest rate on the BSU and the interest rate is free to fluctuate over time as banks are following closely the developments of competitors, as well as the Norwegian Central Bank's policy rate. This means that customers can freely choose their bank and transfer the money in the BSU account (in most cases cost-free) to receive the highest possible interest rate at that time.⁴

While rates have been varying over time, the tax deduction (in percent of yearly savings amount) has remained the same since the inception of the BSU 30 years ago. Nevertheless, since the annual savings amount has increased over time, the maximum annual tax deduction (in absolute terms) also grew by the same magnitude, that is by

³ Note that interest is not included in the maximum savings amount.

⁴ Banks must operate within the regulatory framework of the BSU set by the government authorities. As part of this, they are obliged to offer mortgages to their BSU customers (Skatteloven, 1999/2021, §16-10). Interest rates, however, are not regulated. We presume that banks historically maintained the mentioned 2.5% premium on the BSU to recruit young savers, breaking even on the customer with additional products and services.

175%. Although an investor can keep saving in the BSU after purchasing a home (by using other financing sources), from 2021 onwards this investor may not enjoy the 20% tax deduction any longer.⁵

Figure 3 below presents the tax benefit of BSU-savings in three scenarios for the full year 2022. To achieve the maximum tax deduction of NOK 5,500 the investor needs to save the maximum prescribed amount of NOK 27,500 and have a yearly taxable income of at least NOK 86,650. If the user earns less than the NOK 86,650, she will also be taxed less and consequently miss out on a share of the BSU tax break. Note that in 2022 the minimum taxable income is NOK 64,650, implying that the income group below this threshold is entirely missing out on the BSU tax benefit. Driven by below-maximum investment amounts, the yellow and blue line exemplify scenarios where tax breaks of NOK 3,667 and NOK 1,833 were achieved. Naturally, we observe a marginally decreasing effect of the tax break (relative to income) with increasing taxable income for all scenarios due to the non-progressive annual BSU investment cap. The figure's x-axis zooms in on incomes between NOK 42,650 and NOK 193,507 covering the key tax implications of the BSU, presented by the kinks described above. Income levels outside this range are following the patterns presented at the edges of the graph.

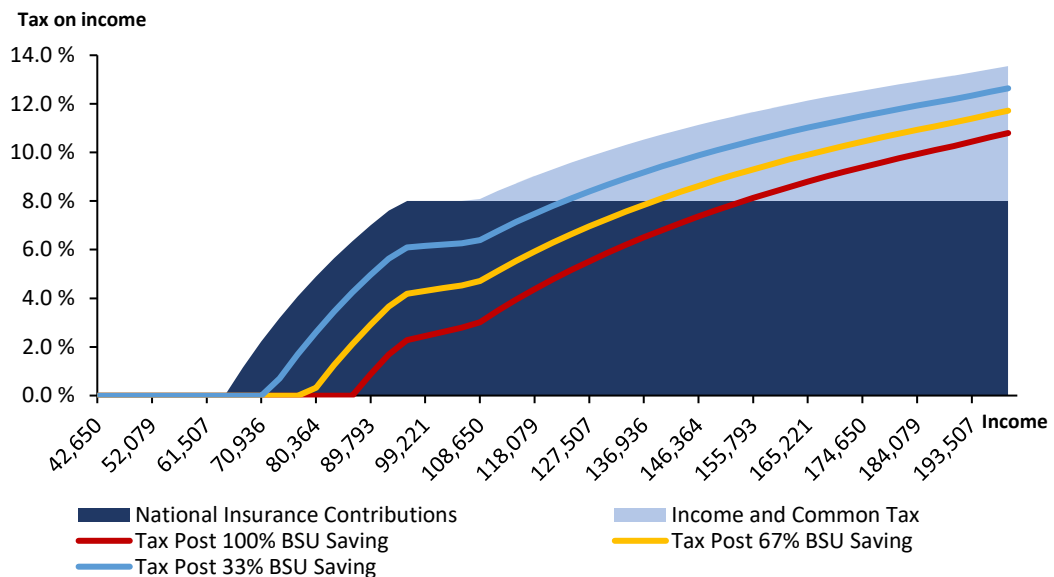


Figure 2. Tax implications for different levels of BSU saving in 2022. Based on (Skatteetaten 2022a).

⁵ Only since the new BSU regulation in 2021, home owning BSU savers cannot get a 20% tax deduction on their savings. Later in the analysis, we will see that this change induced a significant reduction in BSU balances.

Being a risk-free savings account with (historically) higher interest rates than comparable bank savings, the BSU has certain restrictions. Firstly, the saver may only use BSU savings on their first home, meaning a home purchase or home-related expenses. Such expenses could include payments on the mortgage or general upkeep of the housing unit. Failing to comply (i.e., breaching the contract) will result in repaying all tax benefit received during the entirety of the saving period. Once closed, that is, either used as equity for a home purchase or withdrawing more than the year's deposit, the BSU account may not be opened anew. Additionally, it is not possible to create multiple BSU accounts or share an account with a partner (although most banks offer a BSU 2.0 account with the same interest rate, but without tax benefits).

Having outlined the beneficial tax breaks on the BSU savings, we also find it of utmost importance to highlight the actual return on the investment. As Harald S. Olsen explains, the actual return is a function of the investment and interest earned, both after taxes (Olsen, 2015). Hence, we can set up the return on deposits in the BSU account as a present value formula:

$$PV = -I_0(1 - \tau_{BSU}) - \sum_{t=1}^T \frac{I_t(1 - \tau_{BSU}) + i_t \times \tau_p}{(1 + r_n)^t} + \frac{FV}{(1 + r_n)^T} = 0$$

where,

PV : Present value (set to zero as the formula is solved for the after-tax return),

I_0 : Initial savings amount,

I_t : Savings amount at time t ,

τ_{BSU} : Tax savings on BSU (20.0%),

i_t : Interest return in nominal terms at time t ,

τ_p : Income tax rate (22.0%),

FV : Future value after time T ,

r_n : Nominal after-tax return on total savings.

From the formula above, we can interpret the after-tax annual returns as the internal rate of return on the investment, which, because of the tax break, is only 80% of the initial deposit. As an example, we can assume a fixed yearly interest rate of 3.2% (close to recent rates as of June 2022), a tax rate of 22.0%, and full utilization of the annual savings amount, arriving at Figure 3 below. Graphing the investment horizon on the x-axis, the figure presents a high after-tax return of 28.1% should the investor only save

for one year, while savings over longer periods show marginally decreasing annual returns, exemplified by the 6.4% annual return received over a 10-year period. Nonetheless, this is an excellent after-tax return far outpacing interest on other bank deposits.

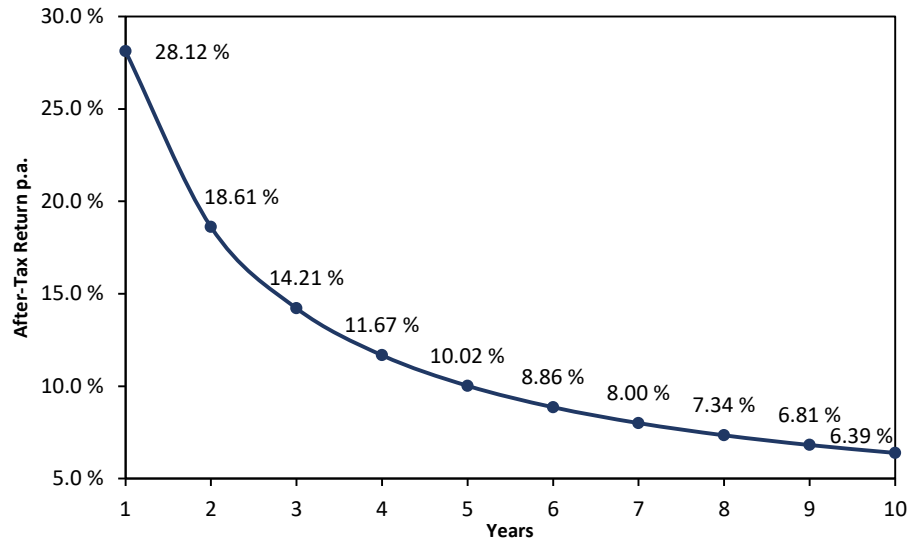


Figure 3. After-tax return p.a. on fully utilized BSU savings. Based on (Olsen 2015).

To summarize, the BSU savings account has strong tax benefits, as well as a high return on invested capital. Supported by regulators, steady increases in savings limits are pointing towards an expansion of BSU balances over time, which we will investigate in the upcoming section.

3.2.2. Actual Utilization of the BSU Account

Having set the stage with the theoretical framework of the BSU, we will now analyze the actual utilization of this savings form among the Norwegian population.

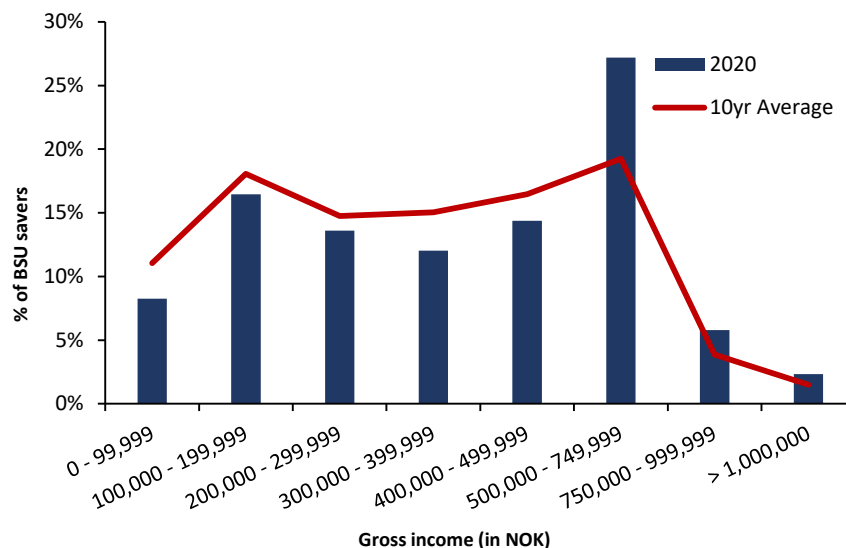


Figure 4. Income brackets of BSU savers. A comparison of the 10-year average (2011-2020) and 2020. Based on (SSB 2021a).

Data from Statistics Norway (SSB) describes that 43% of BSU users are in the 17 to 24-year-old age group and 57% are 25 to 33 years of age.⁶ Figure 4 further distinguishes BSU savers in different income brackets. Looking at the 10-year average (2011-2020) we observe a relatively flat distribution of the BSU population with gross income between NOK 100,000 and NOK 749,999. The latest data from 2020, on the other hand, allocates most BSU savers to the NOK 500,000 to NOK 749,999 income bracket. In the time-series, we confirm that there has been a transition from lower to higher income brackets. The most prominent explanation for this development is that the data is not adjusted for nominal wage growth during the 10-year period. Further, increased housing prices may have forced individuals to save longer (i.e., until later in their career), needing comparatively more equity for a home purchase.

⁶ These numbers are averaged over a 10-year period from 2011-2020. Note that about 0.5% of BSU savers are below 17 or above 34 years of age.

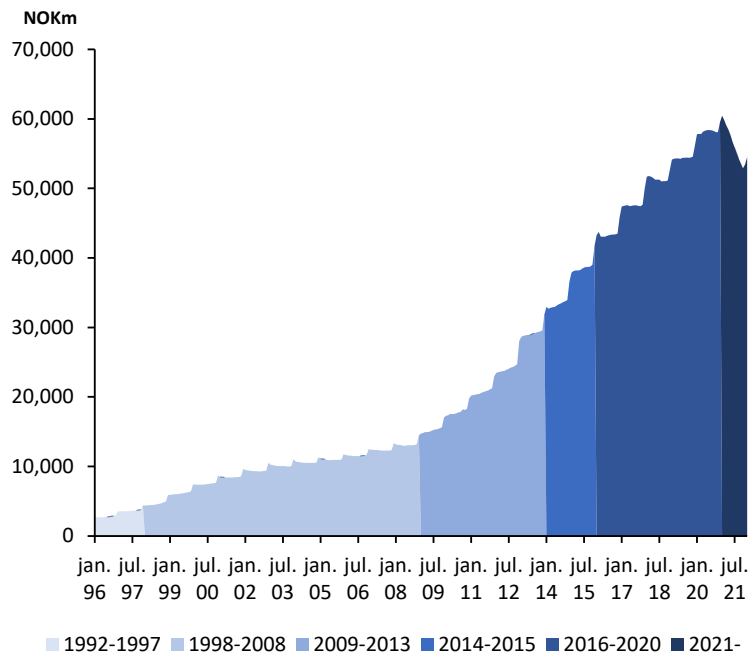


Figure 5. Total BSU account balance (in NOKm) for the various regulation periods. Based on (SSB 2022b).

Figure 5 above shows the total BSU account balance of all savers from 1996 to 2021, the blue areas highlight the various regulation periods. Since the data is on the aggregate level, it is important to note that various constituents affect the BSU balance, such as the number of savers, deposits, withdrawals, and interest on savings.⁷ While we do not have data available to analyze the effects of these variables in isolation, we suggest the following main takeaways: (1) BSU savings patterns are highly influenced by regulations, and (2) we observe a sharp increase in the total BSU balance during the last month of each year (presented by “step-wise” kinks).

With each regulation period having its own characteristic pattern we deduce that regulatory changes have been entirely beneficial for savers up until 2021. As noted earlier, from 2021 onwards, savers who already are homeowners may not enjoy the BSU tax deduction, which induced a significant sell-off for this investor group. Furthermore, the sharp increase in total BSU balance during the last month stems, first and foremost, from the interest payments on BSU investments, which are received on the last day of each year. A second contributor to the kink might be that investors fill

⁷ Interest on BSU savings is further dependent on the annual deposit amount, timing of the deposit, and investment horizon of savers.

up their annual deposit quota towards the end of the year such that they may reap the maximum tax break depending on their income (a possible “December Effect”).⁸

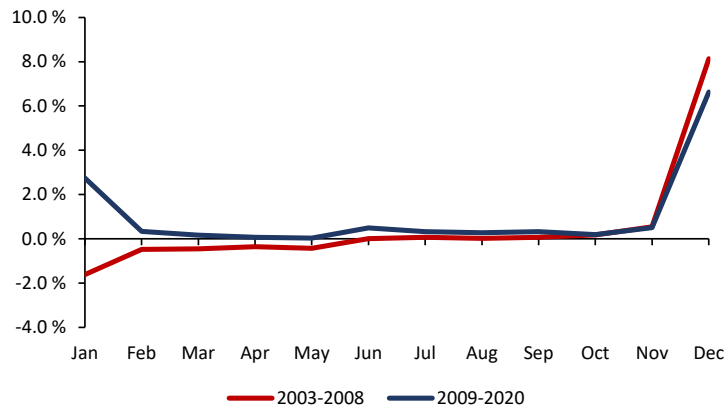


Figure 6. Average monthly change in the total BSU account balance for all BSU savers. Based on (SSB 2022b).

Figure 6 above extends the analysis of the BSU balance by contrasting its average monthly changes in 2003 to 2008 and 2009 to 2020. Focusing on end-of-year changes, we confirm that the kinks observed in Figure 6 arise from the discussed December Effect. Further, we corroborate that the magnitude of change cannot entirely be explained by the interest-effect but is most likely also affected by late deposits.⁹ A simplified explanation is that the average month’s nominal returns p.a. are not exhausting the percentage increase of balances during December. For the years 2003 to 2008 we compare an average month’s nominal returns p.a. of 5.0% with an average December balance change of 8.1% (with 3.1% being unexplained), and for 2009 to 2020 we are respectively comparing 3.8% with 6.6% (resulting in 2.8% being unexplained).

In the next step, we look closer at the BSU utilization of an average saver. In Figure 7 we see that in 2003 around 212,000 Norwegians between the ages 17 to 33 saved in the BSU, equivalent to about 20.8% of the population in this age group. This number increased steadily to around 375,000 savers in 2014 (or 32.5% of the population),

⁸ There are many potential reasons why investors choose to postpone filling up the annual BSU quota in December. E.g.: (1) depending on the investor’s risk-aversion and skill she might prioritize investment vehicles that promise higher returns throughout the year. (2) A young investor might have variable income, choosing not to lock up money early during the year (e.g., she might be operating on an annual budget). (3) The investor might be limitedly financial literate, unaware of the loss in interest return if she chooses to hold cash instead.

⁹ Analyzing the balance pattern could be an extension of our study. Referring to Figure 6, we suggest an investigation of both the January and December Effect, contrasting the 2003-2008 and 2009-2020 regulation periods.

before stagnating at this level. Recently, in 2019 and 2020, the number of savers increased again up to just shy of 400,000 (equivalent to 32.6% of the population). Taking the annual balance of all BSU savers from Figure 7 and averaging it down to the individual level presents us naturally with similar challenges when interpreting the data output. Nonetheless, we may deduce that annual BSU account balances on the individual level are, on average, not fully utilized regarding the maximum savings quota. This stands in contrast to the deposit pattern of BSU savers with taxable income. In reflection of regulatory changes, the latter type of BSU investor tries to allocate her capital close to the prescribed quotas, with the goal of maximizing the tax deduction.

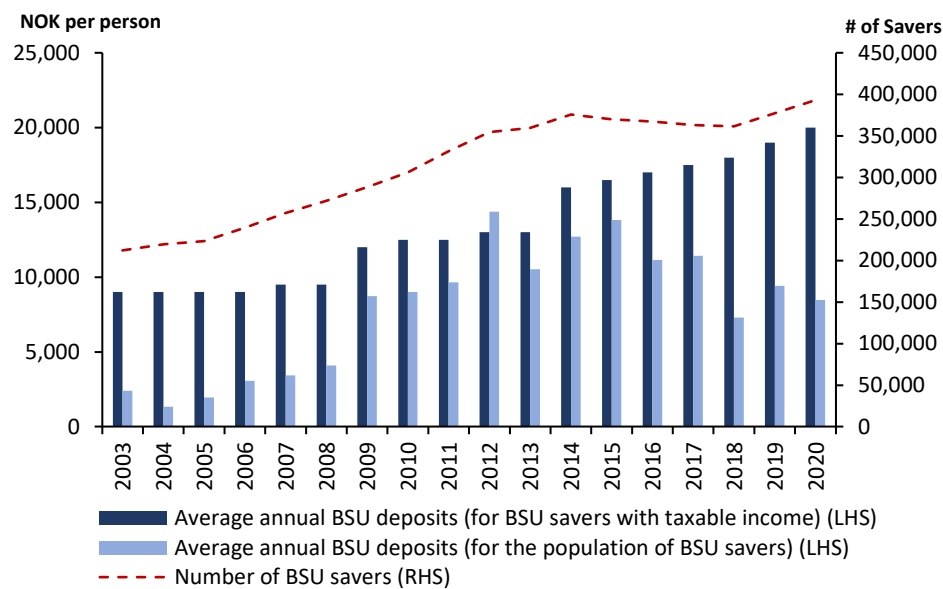


Figure 7. Average annual deposits (net deposits) in representative BSU accounts. Based on (SSB 2021a; SSB 2022a; SSB 2022b).

Moving on to the analysis of the BSU tax benefit, we present data on the utilization of the maximum possible tax break in Figure 8 below. We see that BSU-saving taxpayers between 2003 and 2015 achieve on average 62% of the maximum possible tax deduction. It is only in 2017 that we can observe a utilization rate of 70%, reaching 80% in 2020. There is a variety of factors that could explain why BSU savers on average are missing out on one-third of possible tax savings. While not exhaustive, we hypothesize the following scenarios: (1) A sizeable share of BSU savers does not earn the minimum taxable income to achieve the maximum BSU tax break, (2) the general BSU savings rate is below the annual prescribed maximum amount with savers choosing to allocate capital elsewhere or consume due to liquidity constraints, and (3) a breach of contract or lack of financial literacy.

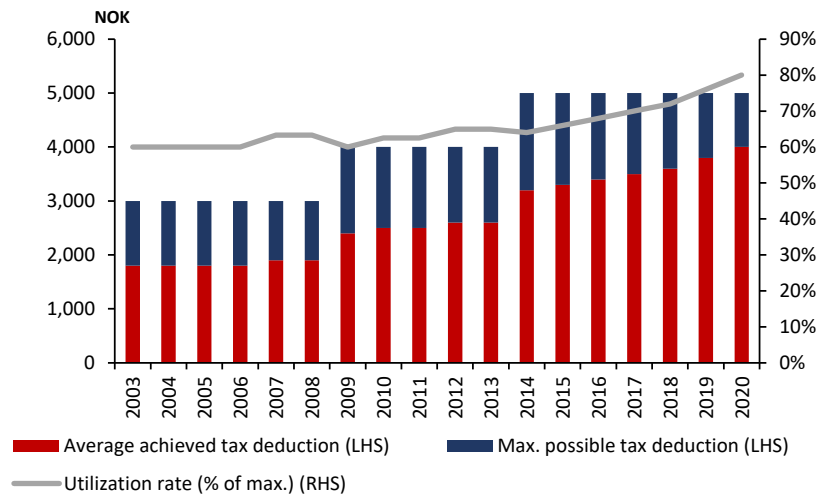


Figure 8. Utilization of the BSU tax deduction among savers Based on (SSB 2021a).

All three reasons could be prevalent but challenging to pinpoint. From Figure 4 we know that only 8.2% of BSU savers in 2020 had a gross income of less than NOK 100,000, weakening argument (1) from above, as most savers had high enough income to achieve the prescribed tax break. Additionally, the utilization rate has remained very stable in the years 2003 to 2014 before increasing, possibly due to savers gaining higher incomes and saving more. However, when facing the choice of where to save, the illiquidity of the BSU account must be considered. Although savers are free to withdraw any deposits done during the year, at year-end the retained amount is frozen unless the individual wants to close the account. Hence, by saving in the BSU, investors are foregoing liquidity and consumption in the presence of potentially gaining higher consumption of housing in the future. Lastly, as mentioned above, less than one-third of the population aged 17 to 33 has a BSU account, which could either imply that a large share already bought a house and closed their BSU, does not know of the savings account’s benefits, or both.

We cannot categorically say whether it is optimal for everyone to save in the BSU. For example, households might derive higher utility from consumption or from having cash available rather than tying up their money. Moreover, other households might not earn enough to take full advantage of the tax benefit. However, should it be the case that people are unaware of the benefits and costs of the BSU, we can conclude that the savings account offers substantial compensation in the form of high returns and tax deductions, but with the downside of locking the money until purchasing a house. Nevertheless, the high homeownership rate of above 80% in Norway implies that most

Norwegian residents, at some point, will face a home purchasing decision, meaning that the two-thirds of 17 to 33 year-olds without a BSU account should reexamine the benefits and costs of the BSU.

3.3. Life-Cycle Considerations

Much of the reviewed literature points towards the importance of life-cycle consideration in the portfolio composition with housing. Norwegian data matches the statement from Flavin and Yamashita (2002) that the housing constraint generally decreases with age of the homeowner. That is, young homeowners are often burdened with substantial levels of mortgage debt, such that they may hold higher levels of real estate, relative to their net wealth (Solheim & Vatne, 2018).

While it may certainly be relevant to model Norwegians' portfolio choice in the context of a typical lifecycle, we chose to largely abstract from this research angle. However, due to our focus on the savings implications of the BSU, which is restricted to the age group 17 to 33, we are naturally moving in the early life-cycle stage. In the main body of our analysis, we will thus pay special attention to portfolios that are endogenously appointed with a high housing constraint. The beforementioned data from Norges Bank (Solheim & Vatne) allocates 80% LTV to the average homeowner after her first home purchase (in 2015). This equates to a housing constraint of $h_t = 5.00$, while, for reference, the maximum LTV of 85% corresponds to $h_t = 6.66$ (h_t will be closely defined in the upcoming section). This gives reason to believe, that young homeowners are relatively more constrained in their portfolio choice and further consumption of goods and services. In this thesis, we will mostly abstract from life-cycle consideration of consumption by taking the "balance sheet" point-of-view, i.e., regarding the financial portfolio at a given point in time. While disregarding consumption smoothing throughout a life-cycle, our focus lies on modeling how portfolios are formed and their performance in the following years. As Fagereng, Gottlieb, and Guisso (2017) point out, the share of risky assets in a portfolio varies throughout the life-cycle, taking on the shape of a hump across the age-dimension. Due to the lack of portfolio data including housing at the individual level, we will instead investigate how the allocation of risky assets behaves across the varying levels of housing constraints.

4. Methodology

4.1. Housing and BSU in a Mean-Variance Efficiency Framework

Developed by Harry Markowitz in 1954, the mean-variance efficient portfolio framework is used to characterize an investment opportunity set of assets. Here, it is assumed that the investor rationally chooses her portfolio either to maximize expected return given a target level of risk or minimize risk given a target level of expected return. The investment opportunity set, or the efficient frontier, is constructed by changing portfolio compositions, effectively altering risk and return such that the investor can choose the portfolio suiting her preferences. We use the mean-variance efficiency framework to investigate optimal portfolios in the Norwegian setting, with the inclusion of the asset classes housing and BSU.

Guided by Flavin's and Yamashita's (2002) paper, we compute the mean-variance efficient frontier with the inclusion of both housing and mortgage in addition to the BSU account. The household's total wealth is regarded in relative terms, i.e., all weights of assets in the total portfolio must sum to one:

$$1 = h_t + x_t$$

where h_t is the amount of housing relative to net wealth, denoted as the "housing constraint". Effectively, h_t can be thought of as

$$\frac{P_t H_t}{W_t}$$

where P_t can be the price per square meter, and H_t is the size of the housing unit, measured in square meters. W_t is then the households net worth, and taking an example, with a net wealth of NOK 1,000,000 an h_t of 3.50 translates to a house worth NOK 3,500,000.¹⁰

Similarly, x_t is a $(1 \times n)$ vector of the relative weight of the n risky assets held in the portfolio: the OSEBX stock market index, the MSCI World Index, the 10yr Norwegian Government Bond, the BSU account, and the mortgage (also denoted x_n). Furthermore, in line with the Norwegian Mortgage Lending Regulation, we assume that the mortgage cannot exceed the limit of 85% LTV (as explained in section 3.2.).

¹⁰ This is possible through leverage. In this example, the household's wealth of NOK 1,000,000 could be used as equity in a home purchase with the remaining NOK 2,500,000 being financed by a mortgage.

Hence, a household with an h_t of 6.66 contributes all her initial net wealth towards the 15% equity requirement. The housing constraint, h_t , mimics somewhat a household's life-cycle pattern in that, higher levels of h_t are most prevalent for younger households who spend most (if not all) of their wealth on housing, while older households tend to have more wealth, already having repaid larger portions of their mortgages. In this thesis, we think of housing and asset allocations in relative terms, abstracting from the physical size of the real estate. This implies that households with different levels of wealth could have the same house by varying h_t , i.e., a house of value NOK 4,000,000 could be owned by both investors with h_t of 6.66 (net wealth of NOK 600,601) and 3.50 (net wealth of NOK 1,142,857).¹¹ By keeping the analysis in relative terms, we can better compare returns of the portfolios. Lastly, our mean-variance optimal portfolios restrict investors from shorting risky assets in our model.

The mean-variance framework utilizes return data on the assets (the data is presented in further detail in section 5), of which we compute average monthly after-tax, real returns and construct a matrix of the specific monthly return's deviation from the mean. The deviations highlight the stochastic component of the after-tax, real returns for each asset class:

$$R_{i,t} = \mu_i + \epsilon_{i,t}$$

where,

$R_{i,t}$ is the after-tax, real return for asset i at month t ,

μ_i is the average monthly return for asset i ,

$\epsilon_{i,t}$ is the stochastic component measured as the deviations from the mean.

The $(n \times T)$ matrix of return deviations is then used to compute the variance-covariance matrix, denoted Ω , using matrix multiplication:

$$\frac{\epsilon\epsilon^{-1}}{T}$$

where, ϵ is the $(n \times T)$ matrix of after-tax, real return deviations from the mean for all n (six) assets and T (114) months. The variance-covariance matrix is essential in calculating portfolio standard deviation which is also performed using matrix multiplication:

¹¹ We also disregard from the mortgage restriction of five times income.

$$\sqrt{[x_t, h_t]\Omega[x_t, h_t]^{-1}}$$

As shown in Flavin & Yamashita (2002) the mean-variance efficient framework can be used in combination with a model of a household's utility. The expected return, or mean return, of the assets and the variance-covariance matrix are then the basis for the household's utility function. Here, higher portfolio expected returns increase utility, while higher standard deviation (or risk) multiplied by a factor capturing the household's risk aversion decreases utility.

We model the household as maximizing their utility where the optimal portfolios are considered based on the return and risk profile of the assets while considering the set of constraints mentioned above. We can write the problem formally as:

$$\max_{x_t} \left\{ (x_t\mu + h_t\mu_H) - \frac{A}{2} [x_t, h_t]\Omega[x_t, h_t]^{-1} \right\}$$

subject to:

$$1 = h_t + x_t$$

(the sum of the weights of portfolio assets cannot exceed one:
total portfolio constraint)

$$-0.85 \times h_t \leq x_{n,t} \leq 0$$

(the mortgage (x_n) is represented as a negative value and
cannot exceed 85% of house value: mortgage constraint)

$$0 \leq x_{i,t} \quad \text{for } i = 1 \text{ to } n - 1$$

(all assets, except the mortgage, need to have positive weights:
no shorting constraint).

Furthermore, we take the housing constraint, h_t , and the investor's risk aversion coefficient, A , as given and solve the maximization problem using quadratic programming. The portfolio optimization is performed for six different levels of h_t (0.00, 0.65, 1.50, 2.40, 3.50 and 6.66), consistent with the life-cycle pattern of asset holdings from Flavin and Yamashita (2002). In extension to their paper, we added $h_t = 6.66$ as this constraint translates to a household fully invested in housing, i.e., the household maximizes its mortgage LTV of 85%. Inspired by Fagereng, Gottlieb and Guiso (2017) who find a relatively high coefficient of risk aversion between 11 and 14 for Norwegian households, we choose to focus our analysis on the levels 2, 6, 8, 10, 12

and 14 for A . This showcases optimal portfolios for the most common risk aversion levels in Norway, as well as one extreme outlier, providing perspective to the analysis.

After computing the efficient frontiers for all six housing constraints, we compare Sharpe Ratios of the optimal portfolios measured as the expected return per unit of risk:

$$SR = \frac{\mu_P}{\sigma_P}$$

Solving the optimization problem and finding the six portfolios with the highest Sharpe Ratio will yield valuable information regarding portfolio composition and the role of BSU in the mean-variance efficiency framework. We will perform two portfolio formations, one with the choice of saving in the BSU and one without the BSU and investigate how portfolio compositions and performances change. The resulting portfolios could assist in interpreting the importance of the BSU by seeing how much portfolio composition and expected return change without the BSU.

Furthermore, the data set of returns spanning 228 months will be split evenly in two at August 2012. Hence, the formation of optimal portfolios (described above) will be conducted on in-sample data from January 2003 to June 2012, while out-of-sample data from August 2012 to January 2022 will be used to examine each optimal portfolio's actual performance.

We then compare the two scenarios when investors either have or do not have the possibility of saving in the BSU account, investigating the effect of BSU on portfolio composition and performance, comparing Sharpe Ratios and annual returns.¹²

Lastly, we determine if there are significant differences in the mean returns of the two comparable optimal portfolios with and without BSU (i.e., portfolios with the same risk aversion), as well as BSU versus the Housing Index. To test differences in the returns between the two samples (the portfolios with and without BSU, as well as the Housing Index), we perform a one-sided t -test.¹³ First, we need to run an F -test of the differences in variance between the two returns. There are three different cases where the t -distribution is used, all with differing degrees of freedom. The null hypothesis of

¹² It is unwise to look at the Sharpe Ratio and return in isolation since two portfolios can have the same realized, or expected, return but in most cases, investors will prefer the one with higher Sharpe Ratio. The opposite is also true, as investors will maximize their wealth by choosing higher returns among portfolios with the same Sharpe Ratio.

¹³ We choose a one-sided test to see whether the portfolios with BSU yield higher returns.

the F -test on variances is that the two samples have equal variances. We test this using the F -statistic formulated below

$$F\text{-statistic} = \frac{\sigma_1^2}{\sigma_2^2}$$

where the ratio of variances is compared to a $F^{-1}(n_1 - 1, n_2 - 1)$ distribution at the 0.05 level of significance.¹⁴ With both samples of returns having the same number of observations (114 months), the critical value must lie in the interval 0.6903 to 1.4486 to fail to reject the null hypothesis. If the F -statistic lies outside the interval, the two variances are unequal.

Having confirmed whether there are differences in variance between the two samples of returns, we use the appropriate t -distribution to test differences in the mean of the return samples. The null hypothesis is that the two mean returns are equal (i.e., $\mu_1 - \mu_2 = 0$), and if the variances are equal, the test statistic is given as

$$t\text{-statistic} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{1,2} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where, the \bar{x} signifies the mean returns, and $\sigma_{1,2} = \frac{(n_1-1)\sigma_1^2 + (n_2-1)\sigma_2^2}{(n_1-1) + (n_2-1)}$ which is an estimate of the two sample's common variance. This t -statistic is compared to a critical value with a $t^{-1}(n_1 + n_2 - 2)$ distribution, where if the test statistic is larger or equal to the critical value, we reject the null hypothesis stating that the two means are equal.

However, if the variances are unequal, we use the following test statistic

$$t\text{-statistic} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\left(\frac{\sigma_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{\sigma_2^2}{n_2}\right)^2}{n_2 - 1}}}$$

with a $t^{-1}(df)$ distribution where the degrees of freedom are computed as

$$df = \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{\left(\frac{\sigma_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{\sigma_2^2}{n_2}\right)^2}{n_2 - 1}}$$

¹⁴ Degrees of freedom (df) are given in the parentheses.

As outlined above, we perform two sets of the tests. Keeping the portfolios that include the BSU as a baseline, we test these returns first against returns of portfolios without the possibility of saving in the BSU, and then against the returns of the Housing Index. We can interpret a rejection of the null hypothesis as portfolios including the BSU yielding higher returns than portfolios without the BSU and the Housing Index. On the other hand, failing to reject the null hypothesis indicates that the returns either are very similar, or the portfolios with BSU offer lower returns. Lastly, we will consider the results on the 0.10, 0.05 and 0.01 levels of significance.

It should be noted that the test of variances could result in unequal variances, which, if the results from the t -test of differences in means are insignificant, could still imply that the portfolios offer better risk-adjusted returns.¹⁵ Hence, the results from the tests should be viewed together with the analysis of Sharpe Ratios and returns. Moreover, if the tests of difference of means are significant, this could imply an added benefit of saving in the BSU, and potentially increasing the possibility of acquiring a house.

In summary, we use the mean-variance optimization framework to form portfolios consisting of stocks, bonds, housing, mortgage, and BSU that maximizes a household's utility. Moreover, we restrict investments in BSU to test differences in variance and mean returns between portfolios with and without the savings account. Lastly, the same is done with portfolios including BSU versus the Housing Index. Based on the results, we can draw conclusions of whether the BSU assists in achieving homeownership.

¹⁵ As the Sharpe Ratio is a function of return and risk, out of two portfolios with the same return, the one offering lower risk has a higher Sharpe Ratio.

5. Data Description

When analyzing a representative investors' portfolio, we carry out the computations using return data on housing, mortgage rates, stock market indices, bonds, and Norges Bank's deposit rates. In today's environment, the data we use is publicly available, hence providing a low barrier for investors to analyze their portfolios. Furthermore, we adjust the data for inflation, as well as taxations to provide real after-tax return data. The adjustment will in most cases look like this:

$$\frac{1 + r_t \times (1 - \tau)}{1 + \pi_t} - 1$$

where,

r_t is the month's nominal return,

τ_t is the month's tax rate,

π_t is the period's CPI rate.

It is important to note that the tax rate has been gradually lowered to the current 22%, down from 28% in 2003. To get comparable return data, we assume the investor holds the asset at the beginning of the month, sells it at month-end, and buys it back again at the beginning of the next month, i.e., fully experiencing the monthly variations.

Lastly, it is important to note that our data set of returns spans 228 months split in half so to perform an in-sample/out-of-sample analysis as described in the methodology section above. Thus, our data analysis is based on the returns counting January 2003 until June 2012.

5.1. Housing

As described earlier, Norway enjoys one of the highest homeownership rates in the world signaling the importance of this asset in a household's portfolio. We define housing as private real estate and are not analyzing other real estate investments such as commercial properties or real estate funds. The primary vehicle for this part of our analysis is data on Norwegian homeowner's prices, which we collect from Eiendom Norge's house price index. The price index is based on sales reported by realtors or announced on the platform Finn.no, which we believe to be of relatively high accuracy (Eiendom Norge, 2022).

As housing returns have a considerable idiosyncratic component (Flavin & Yamashita, 2002), this factor will not be properly reflected through the aggregated index. Hence, our analysis can be useful seen through the lenses of the average investor, but it is weak on the individual level. Taking the raw data from Eiendom Norge, we find yearly returns of 6.20% with a standard deviation of 4.43% for the whole twenty-year period. Dividing the yearly return by its standard deviation gives us 1.40 which is a considerably higher value than the overall data in Flavin and Yamashita (2002) of 0.46, although it is close to the value of individual cities such as Dallas (1.48) and San Francisco (1.50).

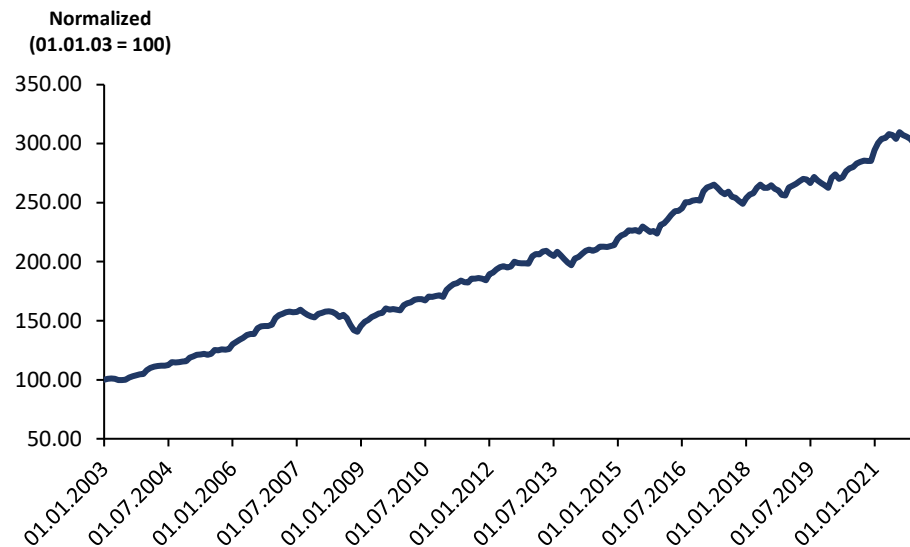


Figure 9. Cumulative returns of the Norwegian house price index. Based on (Eiendom Norge 2022).

5.2. Mortgage Rate

Mortgages are the most common form of financing when purchasing a house. We find data on historical mortgage rates from SSB and use the rate on existing amortizable loans with housing as collateral (SSB, 2022c). A weak point in our data is the lack of available data points before 2013 as SSB reports monthly rates only after 2013. Thus, we use average quarterly data for the period 2003 to 2013, setting the rate for all three months within the specific quarter.

Historically, the interest rate on Norwegian mortgages has declined from 6.75% in 2003 to 2.07% in January 2022, with sporadic increases during the period such as in

the financial crisis of 2008. We assume investors hold variable-rate mortgages¹⁶ and introduce a monthly holding period of the mortgage which is then adjusted for taxes and inflation:

$$\frac{1 + ((1 + r_t)^{\frac{1}{12}} - 1) \times (1 - \tau)}{1 + \pi_t} - 1$$

where r_t is the yearly mortgage rate in month t , τ is the tax rate, and π_t is the inflation during the period. Unsurprisingly, the mortgage rate is highly correlated with the 10yr Norwegian Government Bond, as well as the banks' BSU rate, which we will investigate further below.

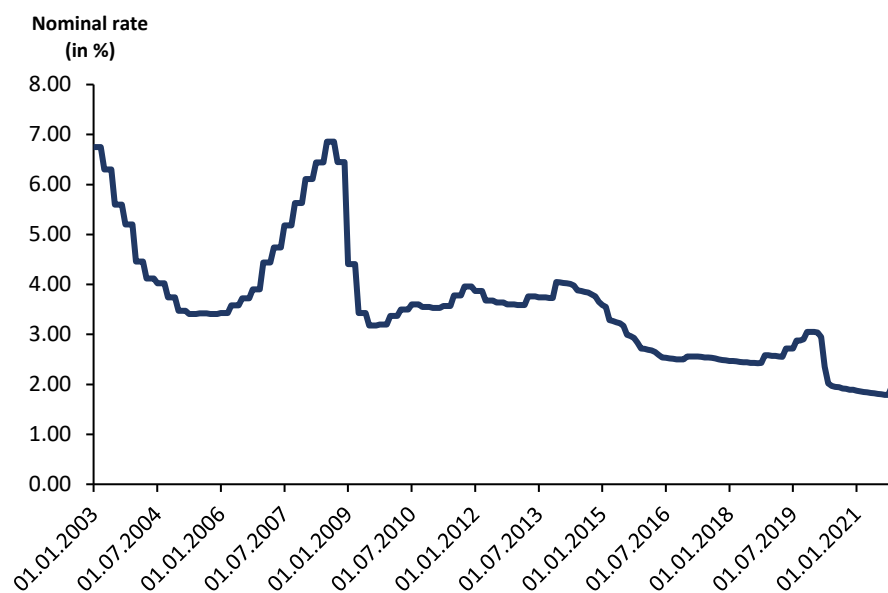


Figure 10. Norwegian yearly floating interest rate on mortgages. Based on (SSB 2022c).

5.3. Stocks – The OSEBX & MSCI World Indices

When it comes to stock holdings, we assume the Norwegian household to be predominantly invested in the home index, i.e., the OSEBX. Furthermore, we will include the MSCI World Index in our analysis, as Fagereng, Gottlieb and Guiso (2017) show the average Norwegian household holds 80% OSEBX and 20% MSCI World Index in their portfolios.

Since the beginning of 2003, a buy-and-hold investment of NOK 100 in the OSEBX would have grown to NOK 1,105.11 by year-end 2021 (Euronext, 2022). Historically,

¹⁶ As of Q4 2021, 93.6% of Norwegians held variable-rate mortgages (SSB, 2022d).

the index has yielded an arithmetic average annual return of 14.5% with a standard deviation of 18.7%. In comparison, a NOK 100 investment in the MSCI at the beginning of 2003 would have evolved into NOK 617.32, resulting in the World index returning 10.5% per year with a 14.9% standard deviation (MSCI, 2022). Hence, the OSEBX has provided investors with higher risk-adjusted returns of 0.77 compared to 0.71 for the MSCI World. There are many possible explanations for why OSEBX has yielded higher risk-adjusted returns like the political stability and transparency in Norway, the index's heavy tilt towards the energy and industrial sectors, or the country's general economic prosperity.¹⁷ As of March 2022, the OSEBX holds 69 companies where the top five sectors are energy (28.7%), financial (18.2%), consumer staples (14.0%), basic materials (12.4%) and industrials (11.3%). In comparison, the MSCI World Index has more than 1,600 constituents across 23 countries in five different regions (as of year-end 2018) (MSCI, 2019). The top five sectors in the index are financials (16.2%), information technology (14.9%), health care (13.4%), industrials (10.9%) and consumer discretionary (10.4%).

We collected data on OSEBX from Euronext, while the historical performance of the MSCI World Index was gathered directly from MSCI.

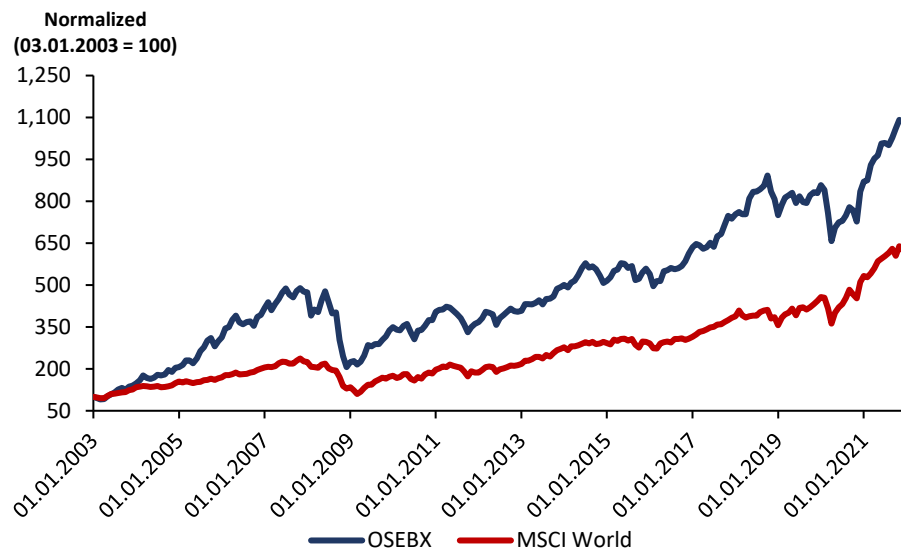


Figure 11. Cumulative returns of the OSEBX (blue) and MSCI World Index (red). Based on (Euronext 2022; MSCI 2022).

¹⁷ This thesis will not dive into the reasons why the Norwegian stock market is seemingly outperforming the world, although this could be an interesting analysis.

5.4. Bonds – The 10yr Norwegian Government Bond

Following Fagereng, Gottlieb and Guiso (2017), we set the risk-free financial asset as long-term government bonds. According to the authors, the average Norwegian household holds 63% of its portfolio in such “safer” assets (e.g., bonds and cash) and are typically risk-averse. Furthermore, Norway can be seen as one of the safest countries with a low probability of sovereign default. In 2020, the country’s government debt was 53% of its GDP, compared to 94% for the OECD average and 238% for the debt-haunted Greece (OECD, 2022). In addition, apart from 2020, Norway has had a positive trade balance since 1988. The period 2015 to 2020 saw an average balance of trade of NOK 80,930m, while oil and gas prices increased in 2021, contributing to a record trade balance of NOK 516,208m (SSB, 2022e). Hence, we deem the 10-year Norwegian government bond to be the Norwegian household’s safe asset.¹⁸

During the entire span of our data, the 10yr government bond reached its highest level of 5.68% at the beginning of 2003 and has steadily decreased over the years (Norges Bank, 2022b). Its lowest point happened in May 2021 when the 10yr was at 0.47% before reaching 1.56% at the end of 2021. As mentioned above, the mortgage rate and BSU are both highly correlated with the 10yr government bond, as is Norges Bank’s deposit rate. Regressing the policy rate on the 10yr government bond gives an R^2 of 0.66 (correlation of 0.81), as well as a highly statistically significant intercept and slope coefficient of 1.49 and 0.75, respectively (measured in percent). Hence, a deposit rate of zero percent has given a bond rate of 1.49% on average, whereas a 1% rise in the deposit rate has (on average) increased the bond yield by 0.75%, possibly implying that the bond yields are responding tentatively to changes in the Central Bank’s deposit rate.

¹⁸ The 10yr govt. bond also provided a longer and more consistent data series than the Norwegian cash deposit rate, in addition to the two being highly correlated so to include both would be unnecessary.

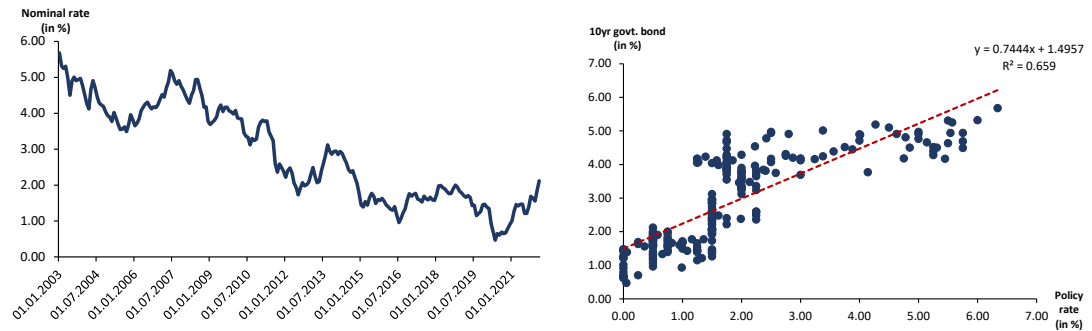


Figure 12. Interest rate on 10yr Norwegian Government bonds and scatterplot of Norges Bank’s policy rate vs 10yr government bond returns. Based on (Norges Bank 2022b).

5.5. BSU – Boligsparing for Unge

As mentioned in section 3.2, the BSU was created as a tax-incentivized instrument for 17 to 33-year-old Norwegian residents to save for housing. Using the BSU, investors in 2022 face a savings constraint of a yearly maximum amount of NOK 27,500 and a total maximum of NOK 300,000. In addition, BSU savers enjoy an annual tax benefit of 20% on the year’s deposit, meaning a total tax savings of NOK 5,500 (given that the investor has at least NOK 86,650 in income) (Skatteetaten, 2021a). To include the actual after-tax BSU return in our model, we modify the formula by Harald S. Olsen (2015) using relative numbers and assumptions. For one, we assume a holding period of five years, which can be reasonable as data presented in section 3.2 show a homeownership rate of more than 50% for young adults between 25 and 29 years of age.

Furthermore, as the interest rate on the BSU account varies between banks, we use the cross-sectional average of three of the largest banks in Norway for each month, gathered from Finansportalen (2022). However, data is not available before 2018, hence we use the bank deposit rate for the years 2003 to 2017 (gathered from SSB (2022f) and add a “BSU premium” of 2.5% which is the average BSU rate above the bank deposit rate for the years 2008 to 2022, assuming banks had the incentive to yield the same rate premiums for the two periods in question. It is also worth noting that the BSU can be considered a risk-free asset as the Norwegian Bank’s Guarantee Fund covers deposits up to NOK 2m per depositor per member bank (Norges Bank, 2022a). Hence, there is a minimal risk of loss on the BSU.

As the BSU rate has a variable return depending on the savings horizon, yearly savings amount, and utilization of tax benefit (mentioned in section 3.2), we compute the monthly after-tax real return as follows:

$$\frac{1 + \left((-1 \times (1 - \tau_{BSU}) \times ((1 + r_t)^5 - 1) \times \tau_t) + \frac{r_t + (\sum_{t=1}^5 (1 + r_t)^t - 1)}{5} \right)}{1 + \pi_t} - 1$$

The formula above gives a close approximation to Olsen's formula from section 3.2.1, but we alter it to have altered it to input relative terms such as the return on the BSU account rather than absolute figures. We find the formula tends to give a slightly lower return than the actual r_t using the original formula.

Lastly, we assume the investor fully utilizes the yearly savings and gains the full 20% tax break on income. However, we adjust the BSU returns for portfolios with housing, as the household loses the tax benefit after a home purchase. Hence, for portfolios with housing the after-tax, real return is computed like the mortgage rate as:

$$\frac{1 + ((1 + r_t)^{\frac{1}{12}} - 1) \times (1 - \tau)}{1 + \pi_t} - 1$$

where, r_t is the BSU account's annual return in month t , adjusted for tax paid on interest and the month's inflation rate.

As with the 10yr government bonds, the BSU is highly correlated with the Central Bank's policy rate (correlation coefficient of 0.90) and has a positive slope of 0.74 in the univariate regression of the two variables (statistically significant at the 1%-level). Furthermore, the regression line intercepts the y-axis at 2.86% (also statistically significant), suggesting a solid BSU premium. However, a slope coefficient below one can indicate that banks are slow in changing the rates for the BSU (which could be either beneficial or disadvantageous for the investors depending on the direction of change), or there is a different pass-through at the different rates (i.e., varying rate adjustments depending on what state the economy is in).

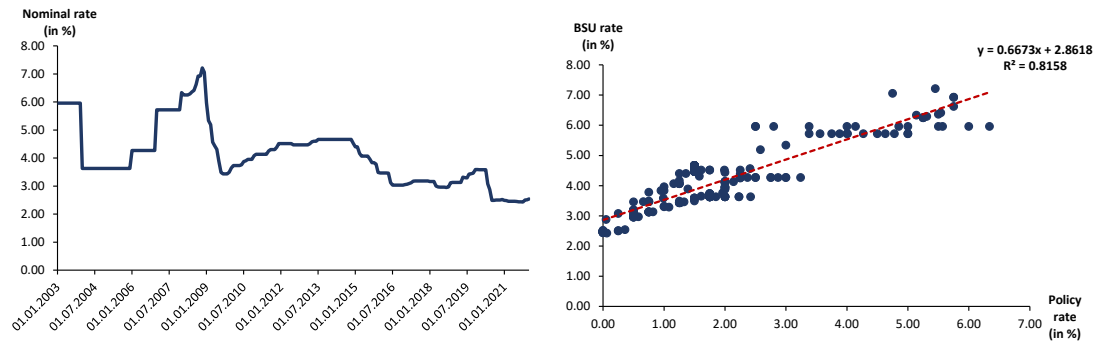


Figure 13. Yearly nominal interest rate on BSU and scatterplot of Norges Bank’s policy rate vs. BSU interest rates. Based on (SSB 2022f; Norges Bank 2022b).

5.6. Inflation

In recent years, Norges Bank has set its policy rate with the goal of securing low and stable inflation at 2%. However, supply chain disruptions and rising commodity and energy prices have recently led to higher fluctuations in inflation rates. Following Flavin and Yamashita (2002) we adjust nominal returns to reflect real price changes. We intend to use the KPI, a measure of the core inflation for private Norwegian households’ CPI, as reported by SSB KPI (core inflation for private households) instead of the KPI-JAE which is inflation-adjusted for tax changes and excluding energy products (SSB, 2022g). The KPI tends to be more volatile than the KPI-JAE as it takes into consideration housing-related costs, which are typically weighted around 24% of the index. Hence, we argue that the KPI is the best index to highlight the overall price changes given that housing also has a consumption dimension.

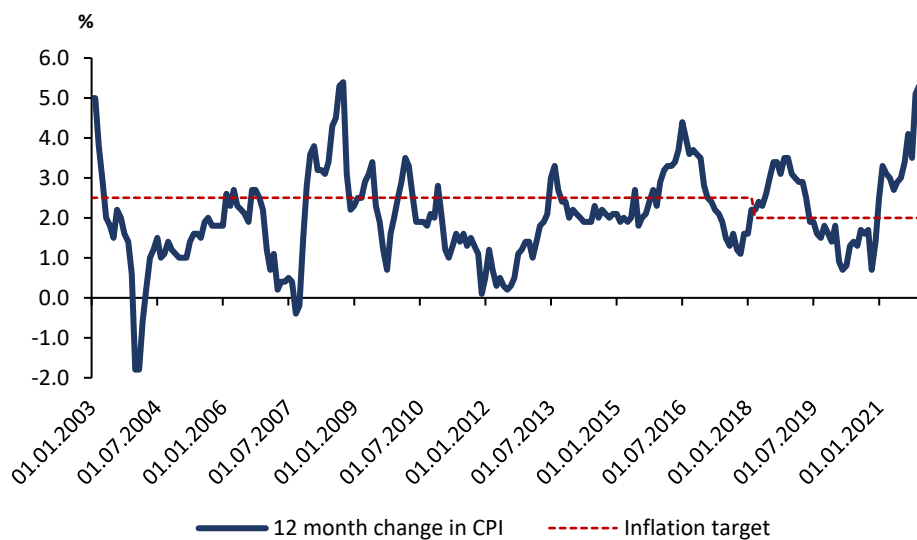


Figure 14. Norwegian CPI rate with yearly change and inflation target. Based on (SSB 2022g).

The Norwegian CPI has averaged 2.1% over the period 2003 to 2022, ranging between negative 1.8% and positive 5.4% during that time. In March 2018, the Norwegian Central Bank decreased its inflation target from 2.5% to 2.0%, although the inflation rate has been higher after this, averaging 2.5% versus 2.0% for the period 2003 to 2018, most likely due to the high inflation we are currently experiencing in 2021 and 2022.

5.7. Descriptive Statistics

The arithmetic mean returns and standard deviations for the six assets, as well as the covariance and correlation matrices, are presented in Table 2 below. As mentioned earlier, housing and the BSU are the only asset class in our selection with higher returns relative to their standard deviations. For housing, this primarily stems from the returns being gathered from an aggregate index which limits the idiosyncratic component of the asset's riskiness. Well ahead of any asset class is the BSU, measured on the basis of risk-adjusted returns, yielding an average yearly return of 7.14% with a standard deviation of 1.73%, albeit practically being a risk-free asset. Even without the tax benefit, the BSU has returned an average of 1.62% p.a. (see the Appendix for descriptive statistics where we take out the BSU tax benefit). Housing scores second-highest on risk-adjusted returns, while bonds, on the other hand, performed poorest with a yearly after-tax real return of negative 0.43% and a standard deviation of 1.61%. The mortgage rate has outperformed the bonds and, lastly, the stock markets have by far the highest yearly returns with the OSEBX yielding more than 12% after adjusting

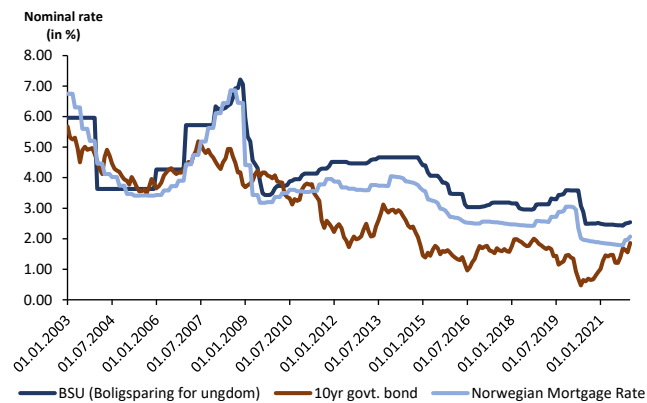


Figure 15. Nominal interest rates for BSU, bond and mortgage.

for taxes and inflation.

Unsurprisingly, the three rates (mortgage, bonds, and BSU) are all near perfectly correlated as they all follow closely the Norges Bank deposit rate (as seen in Figure 15), while the two stock market indices are also strongly correlated. Interestingly, the

housing index is also moderately correlated with all the other asset classes.

Table 2. Expected return, standard deviation and covariance matrix of the six asset classes.

	Housing	Mortgage	OSEBX	MSCI World	Bonds	BSU
Mean return (arithmetic)	0.0553	0.0140	0.1221	0.0623	-0.0043	0.0714
Standard deviation	0.0495	0.0163	0.1368	0.1097	0.0161	0.0173
Covariance Matrix						
Housing	0.00243					
Mortgage	0.00037	0.00032				
OSEBX	0.00330	0.00043	0.02726			
MSCI World	0.00211	0.00015	0.01542	0.01413		
Bonds	0.00040	0.00032	0.00051	0.00021	0.00032	
BSU	0.00035	0.00033	0.00028	0.00005	0.00032	0.00036
Correlation Matrix						
Housing	1.0000					
Mortgage	0.4227 (4.9354)	1.0000				
OSEBX	0.4050 (4.6879)	0.1435 (1.5343)	1.0000			
MSCI World	0.3602 (4.0856)	0.0686 (0.728)	0.7856 (13.4353)	1.0000		
Bonds	0.4566 (5.4314)	0.9936 (92.9696)	0.1744 (1.8744)	0.0988 (1.0502)	1.0000	
BSU	0.3729 (4.2536)	0.9788 (50.5613)	0.0892 (0.948)	0.0236 (0.2498)	0.9546 (33.9012)	1.0000

Note: t-values are in paranthesis

6. Results & Discussion

6.1. In-sample Mean-Variance Portfolio Optimization with Housing Constraint

By including housing and BSU in the mean-variance efficiency framework, investors gain two diversification options beyond the typical allocation choices in stocks and bonds, as well as the possibility of leveraging their portfolios relatively cheaply through a mortgage.¹⁹ Figure 16 below depicts the mean-variance efficient frontiers for the six levels of housing constraints (0.00, 0.65, 1.50, 2.40, 3.50 and 6.66), in addition to an overall efficient frontier (red dotted line) that is tangent to the best sets of portfolios in our investment universe. Further, investors with different levels of risk-aversion are placed along the efficient frontiers respective to their housing constraints. As defined by the model, risk-tolerant investors allocate towards the upper end of their efficient frontier, while risk-averse investors get the most utility from choosing a portfolio that is defined on the lower end of the frontier.

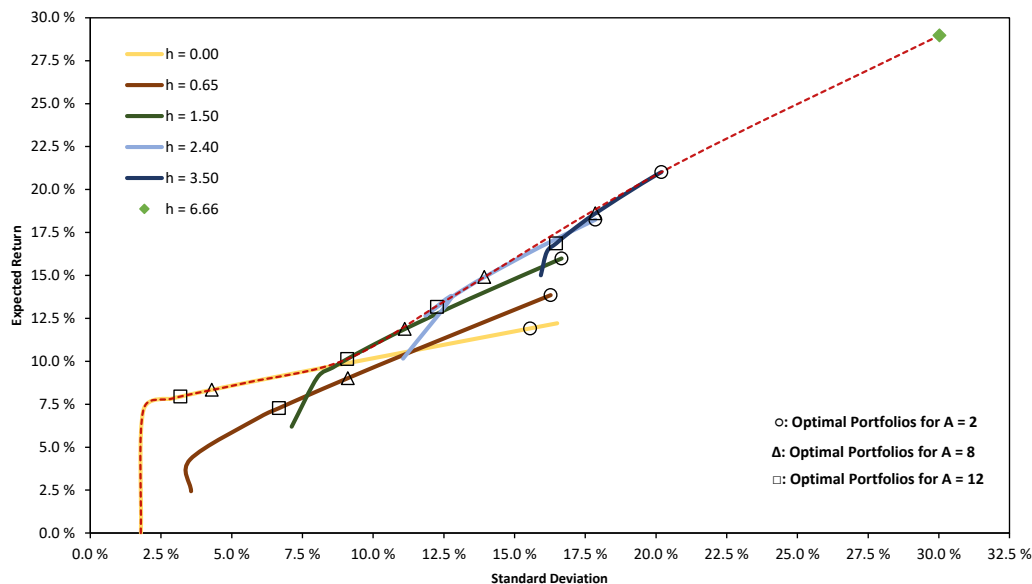


Figure 16. Mean-variance efficient frontiers with varying housing constraints and optimal portfolios for three levels of risk aversion.

As a first observation, we note the perpendicularity of the efficient frontier for investors without housing, stemming from the BSU tax benefit, that outperforms the other assets choices on a Sharpe Ratio basis. Here, even highly risk-averse,

¹⁹ Cheapness is relative, but compared to the real, after-tax returns of the other asset classes, a mortgage in Norway offers a low interest rate cost (excluding transaction costs).

undiversified investors are offered a formidable return of up to 7.1% (100% allocation in the BSU). Beyond this, diversification in riskier assets is needed to push returns further up, although this induces a higher marginal increase of risk compared to return.

The overall tangency portfolio (red dotted line) is kinked a second time at the intercept between the efficient frontier of no housing (yellow line) and the efficient frontier for $h = 1.50$ (the green line). The kink stems from the BSU account utilizing its tax benefit (only available for investors without housing), yielding significant return at very low levels of risk. The efficient set of portfolios for $h_t = 0$ is thus defining the overall tangency portfolio, outperforming the $h_t = 0.65$ and $h_t = 1.50$ frontiers up until this point. Additionally, we observe that the efficient frontiers become shorter for higher levels of housing constraints. We reason that when the household constraints its portfolio by holding elevated positions in housing, it limits itself from holding other asset classes.

Hence, it is evident that the efficient frontier at $h_t = 6.66$ (in the top, right corner) only consists of one point with the investor's initial wealth being entirely spent on the 15% equity requirement for the housing unit, meaning there is no cash left to allocate to other asset classes. This portfolio forfeits diversification opportunities for higher returns, yielding an expected return of 29.0% with a standard deviation of 30.0%. The resulting Sharpe Ratio of 0.97 is the lowest of the optimal portfolios for the six different levels of h_t .

Performing the best on a risk-adjusted return basis is the optimal portfolio with no housing, which holds 98.2% of assets in the BSU with the rest invested in the stock market. Moreover, referring to the kink in Figure 16 above, when decreasing the household's level of risk-aversion (i.e., taking more risk), the portfolio re-allocates its holdings towards more in the stock market index and fewer savings in the BSU. By investing more in the stock market and less in the BSU, the portfolio experiences a higher increase in standard deviation relative to the expected return, translating to a significant drop in Sharpe Ratio.

As presented in Figure 17, a pattern in the portfolios' Sharpe Ratio emerges when

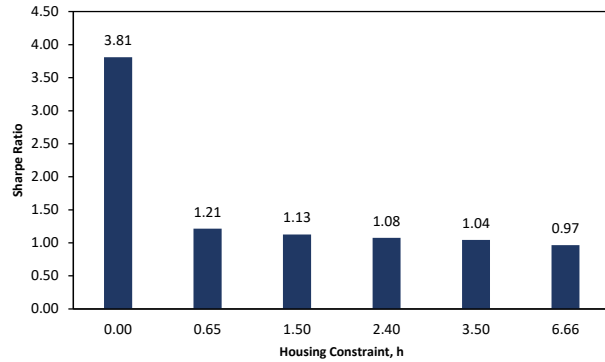


Figure 17. Sharpe Ratios for optimal portfolios at different housing constraints.

varying the housing constraint, where it seems that increased levels of h_t decrease the efficient frontier's optimal portfolio Sharpe Ratio. A lower level of h_t usually implies the household use less leverage which results in lowering the portfolio's expected return but

yields a higher Sharpe Ratio. Thus, optimal portfolio's for $h_t = 0.65, 1.50, 2.40, 3.50$ and 6.66 yield Sharpe Ratios of $1.21, 1.11, 1.06, 1.04$ and 0.97 , and expected returns of $4.3\%, 9.1\%, 13.7\%, 19.6$ and 29.0% , respectively. In comparison, the portfolio without housing has a Sharpe Ratio of 3.81 , yielding 7.1% in expected return.

In extension to Figure 17, Table 3 presents the various optimal portfolios at different degrees of risk aversion and housing constraints. A first observation is that for the entire sample, no portfolios seem to allocate capital to the MSCI World index or the Norwegian 10yr government bond. Instead, all portfolios at the different levels of h_t , take full advantage of leverage (mortgaging 85% LTV) and invest to varying degrees in the OSEBX and the BSU. For $h_t = 0.00$ (no housing), lower levels of risk-aversion, A , invest more in the OSEBX, while still holding some portion in the BSU which increases its share rapidly for higher levels of A . On the other extreme, for $h_t = 6.66$, the entire mortgage is needed to finance the housing unit, leaving no cash to be invested in other asset classes. While this is true for all levels of A , we do observe a dominant pattern in the allocation choice of the most risk-tolerant investors ($A = 2$), where investors with housing hold all available cash in the OSEBX rather than investing in the BSU or other assets.

Table 3. Portfolio weights for differing values of h and A .

Housing Constraint, h	Asset Classes in Portfolio	Degree of Risk Aversion, A					
		2	6	8	10	12	14
0.00	Mortgage	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	OSEBX	0.9411	0.3156	0.2374	0.1905	0.1592	0.1369
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BSU	0.0589	0.6844	0.7626	0.8095	0.8408	0.8631
0.65	Mortgage	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
	OSEBX	1.0000	0.6773	0.4944	0.3847	0.3115	0.2593
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BSU	0.0000	0.3227	0.5056	0.6153	0.6885	0.7407
1.50	Mortgage	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
	OSEBX	1.0000	0.7126	0.4996	0.3718	0.2866	0.2257
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BSU	0.0000	0.2874	0.5004	0.6282	0.7134	0.7743
2.40	Mortgage	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
	OSEBX	1.0000	0.7652	0.5072	0.3525	0.2493	0.1756
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BSU	0.0000	0.2348	0.4928	0.6475	0.7507	0.8244
3.50	Mortgage	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
	OSEBX	1.0000	0.8700	0.5225	0.3140	0.1750	0.0757
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BSU	0.0000	0.1300	0.4775	0.6860	0.8250	0.9243
6.66	Mortgage	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
	OSEBX	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BSU	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Note: Mortgage is stated in percent of the total mortgage available, i.e., a mortgage of -1.0000 implies full mortgage of 85% LTV on the housing unit. Weights of the other four assets are stated in terms of their invested relative weight, meaning left-over cash after housing and mortgage is invested in these four asset classes. Hence, the columns do not always sum to 1.

Interestingly, the BSU stays attractive even after buying a house, despite losing the 20% tax benefit on yearly savings. As mentioned in section 5.7, the BSU account's interest rate has yielded annual after-tax, real returns of 1.62%, which is higher than the mortgage rate of 1.40%. This could explain why investors with housing, even the most risk-averse ones, keep full leverage and instead save more in the BSU, which is also consistent with findings from Yao and Zhang (2005). Here, the authors find that utilizing the full amount of mortgage possible to invest in stocks has a highly beneficial effect on wealth.

Implementing this strategy implies that the household gains the spread between the BSU rate and mortgage rate, effectively using the BSU to finance the mortgage while receiving a return. However, this strategy must be viewed in relative terms (or marginal gains), i.e., for households contemplating whether to use any disposable cash for repaying their mortgage or saving in the BSU account. Here, households are better off saving excess cash in the BSU, as explained above. Similarly, the strategy must also be viewed considering liquidity constraints. In absolute terms, mortgages are often much larger than holdings in the BSU account (which are capped at NOK 300,000). Taking an example of the optimal portfolio at $h_t = 3.50$ and $A = 10$, we compute that the interest paid on the mortgage alone is 7.85 times larger than the interest gained from the BSU account (both measured as after-tax, real returns) at all levels of wealth. Hence, households in this example with a maximum loan-to-income ratio (five times income) pay 6.99% of their income on mortgage interest rates, excluding monthly amortizations which are common for most mortgages. By comparison, the BSU interest after taxes is equivalent to 0.89% of the annual income, which implies lower levels of liquidity, and constraint consumption, when saving in the BSU.

In summary, the optimal portfolios are, in all cases of housing, fully leveraged and allocate capital either in the OSEBX or BSU, where the latter yields the highest Sharpe Ratios in the scenario without housing due to the strong tax benefit. Moreover, households with housing gain the spread between the BSU rate and mortgage rate, which is highly beneficial when considering investing excess capital. At the same time, households are less liquid through the comparably higher interest payments (in absolute terms) of mortgages, thus also forfeiting consumption when saving in the BSU.

In the next sections, we will use the out-of-sample return data to investigate the performance of the optimal portfolios formed with the in-sample data. We answer the following questions: What is the effect of the BSU on optimal portfolios? Does saving in the BSU account contribute towards homeownership? Which of the portfolios without any initial housing will bring our investor closer to homeownership?

6.2. Mean-Variance Portfolio Optimization without BSU

To find the effect of BSU on optimal portfolios, we turn to the scenario where households do not have the possibility of saving in the BSU, i.e., we eliminate the asset class from the portfolio choice. Table 4 below highlights portfolio compositions at the six levels of housing and risk aversion.

Interestingly, we see that at low levels of risk-aversion ($A = 2$), portfolios consist of the maximum possible mortgage (85% LTV) and invest all available cash in the OSEBX. However, for higher levels of risk aversion, it seems more optimal to reduce leverage rather than invest in other asset classes. This underutilization of the maximum LTV is present at all levels of housing. Yamashita (2003) finds that increased leverage has a negative effect on stockholdings, i.e., a crowding out effect, and we detect similar results for portfolios without the BSU in that the optimal portfolios reduce leverage.

However, the de-leveraging could also be considered consistent with Yao and Zhang (2005) in that portfolio allocations are chosen based on the assets' after-tax, real returns. With the mortgage rate being significantly higher than that of the bond, and the MSCI World index is highly correlated with the OSEBX, decreasing leverage offers portfolios the greatest risk-adjusted returns. Hence, it is only when the mortgage is fully paid (as in the two most risk-averse portfolios at $h_t = 0.65$), that the optimal portfolio invests in the negative-yielding bond. However, even though portfolios holding bonds offer higher diversification, with bonds and stock markets being mostly uncorrelated, these portfolios yield the lowest returns and Sharpe Ratios.

Table 4. Portfolio weights for differing values of h and A without BSU.

Housing Constraint, h	Asset Classes in Portfolio	Degree of Risk Aversion, A					
		2	6	8	10	12	14
0.00	Mortgage	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	OSEBX	1.0000	0.7862	0.5878	0.4688	0.3894	0.3327
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.2138	0.4122	0.5312	0.6106	0.6673
0.65	Mortgage	-1.0000	-0.4982	-0.1931	-0.0100	0.0000	0.0000
	OSEBX	1.0000	1.0000	1.0000	1.0000	0.9902	0.8283
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0098	0.1717
1.50	Mortgage	-1.0000	-0.8361	-0.7039	-0.6246	-0.5717	-0.5339
	OSEBX	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.40	Mortgage	-1.0000	-0.9330	-0.8504	-0.8008	-0.7677	-0.7441
	OSEBX	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.50	Mortgage	-1.0000	-0.9838	-0.9271	-0.8931	-0.8704	-0.8542
	OSEBX	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6.66	Mortgage	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
	OSEBX	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	MSCI World	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	10yr govt. bond	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Note: Mortgage is stated in percent of the total mortgage available, i.e., a mortgage of -1.0000 implies full mortgage of 85% LTV on the housing unit. Weights of the other three assets are stated in terms of their invested relative weight, meaning left-over cash after housing and mortgage is invested in these three asset classes. Hence, the columns do not always sum to 1.

In an investment environment without BSU, it would be more profitable to reduce the mortgage loan rather than invest in bonds and stocks, the exception being portfolios without housing and with higher levels of risk aversion.

As mentioned, these portfolios yield the lowest Sharpe Ratio, as graphically depicted in Figure 18 below. Here, we also compare Sharpe Ratios for portfolios with BSU (shown on the left-hand side) and without BSU (on the right-hand side). From this, we can see portfolios that enjoy the maximum tax benefit of BSU (i.e., portfolios without housing) significantly outperform other portfolios, as measured by the Sharpe Ratio. Strikingly, both portfolio allocation scenarios showcase a quite similar “hump” shape around the housing constraint of 1.50, where Sharpe Ratios are only marginally different. We find that due to small differences in interest rates between the mortgage and the BSU (the average spread is approximately 0.22%), the optimal portfolios in the

scenarios with and without BSU behave consistently, that is, they are investing in the higher-yielding assets. Hence, the portfolios in both cases are holding close to the same weights in the OSEBX, the only difference being the asset allocation between saving in the BSU and paying down the mortgage.

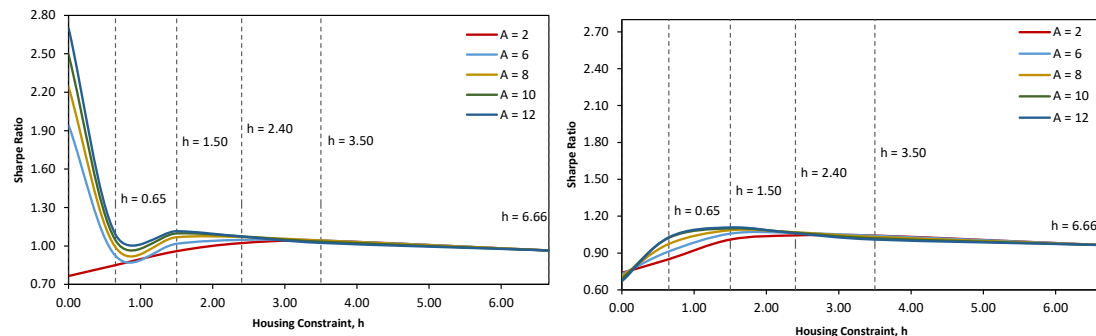


Figure 18. Comparison of Sharpe Ratio at differing h_t for optimal portfolios with (LHS) and without (RHS) BSU.

Concluding from the optimal portfolio compositions, we find the BSU to be an attractive asset class. According to the mean-variance optimization framework, it appears more beneficial to save in the BSU rather than to decrease leverage.

Next, we will test our optimal portfolios in out-of-sample data and investigate whether saving in the BSU will aid households in affording a house.

6.3. Out-of-Sample Tests of BSU Aiding in Homeownership

Having formed portfolios using the in-sample data from January 2003 to July 2012, we found a high prevalence of BSU holdings from our mean-variance optimization method. Using out-of-sample return data from August 2012 to January 2022, we compute and graph holding returns for five optimal portfolios without housing, comparing them to the Housing Index and five optimal portfolios without BSU, as seen in Figure 19 below.

Here, we index the portfolios' return at 100, simulating a buy-and-hold strategy over a time horizon of just over 10 years. From the graph below on the left-hand side, we can see that for portfolios without housing, the inclusion of the BSU as a portfolio choice generally tends to aid in portfolios outperforming the housing index. Moreover, we see that risk aversion levels of 6 to 12 are clustered closely together, while less risk-averse investors ($A = 2$) are rewarded with above-average returns, albeit enduring significant volatility. It is worth noting that this portfolio is invested 94.1% in the

OSEBX and 5.9% in the BSU. The remaining portfolios are more tilted towards the latter asset class and are characterized by steady returns throughout the 10 years. All portfolios end the out-of-sample period with higher cumulative returns than the Housing Index, as well as Sharpe ratios in the range of the low 0.92 for $A = 2$ and the high 2.65 for $A = 12$ (the Housing Index has a Sharpe Ratio of 1.15 for the same period). Thus, it seems the BSU assists in enabling medium-to-high risk-averse investors (who allocated more than 50% of their portfolio in the BSU) to increase their equity before a home purchase.

The right-hand side of Figure 19 below shows the investment scenario where the BSU account is excluded from the allocation choice for investors. Different from the portfolios with BSU, it appears that in the absence of the savings account, investors with higher risk aversion tend to hold portfolios that are underperforming respective to the housing market. This can be explained by the tendency to diversify risk by holding more of the portfolio in bonds, which have yielded negative after-tax, real returns. Still, more risk-averse investors are holding up to one-third of their portfolio in the OSEBX, which is not enough to yield high returns for low volatility. Hence, the more risk-averse portfolios also have lower Sharpe Ratios of 0.61 for $A = 12$ compared to 0.90 for $A = 2$.

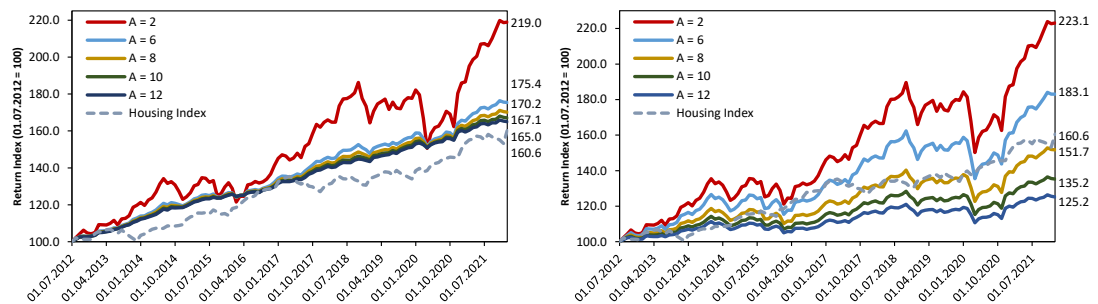


Figure 19. After-tax, real return for five optimal portfolios without housing versus the Housing Index in two scenarios: with BSU (LHS) and without BSU (RHS).

In total, the graph shows that portfolios at the three lowest levels of risk aversion outperform the Housing Index and could potentially assist the investor in purchasing a house. While the graph compares the cumulative returns of the portfolios, this type of analysis has its limitations. Most importantly, but not exhaustive, the investor is constrained by the maximum annual and total savings amounts, when allocating parts of her capital towards the BSU. Nonetheless, the analysis helps us to further investigate whether the BSU impacts investors' portfolios in outperforming the housing market,

such that savings may grow at a higher pace than the price inflation of potential future housing units.

6.4. Test of Monthly Mean Return Outperformance

Following the patterns found in Figure 19 above, we formally test whether portfolios with BSU yield higher monthly returns compared to portfolios without the BSU, and versus the housing market. Table 5 below shows a summary of the F - and t -test of difference in variance and mean for the five pairs of portfolios with and without BSU.

From the three rows above the stippled line, we see the variances of the two portfolios, as well as the test's F -statistic. Naturally, portfolios with higher risk aversion have lower variances for both pairs of portfolios. Moreover, the two portfolios with a risk aversion coefficient of 2 are very closely constructed (i.e., both are heavily invested in the OSEBX), thus being the only pair where we fail to reject the null hypothesis of same variances. For all portfolios with higher risk aversion, the inclusion of BSU results in significantly lower variances compared to portfolios without the BSU, at the 0.05 level of significance.

Turning to differences in monthly mean returns, we see that only the portfolio with risk aversion $A = 12$ yields significantly higher returns, at all levels of significance. The portfolio with risk aversion coefficient 10 is close to the 0.10 level of significance, but ultimately, we fail to reject the null hypothesis of higher monthly mean returns for the four remaining portfolios compared to their counterparties without BSU.

Table 5. Test of difference in variance and returns being greater for portfolios with BSU versus without. We use the 0.05 level of significance for the F-tests where * signals rejection of the null hypothesis of equal variance. *, **, and *** show rejection of the null hypothesis of equal means for the t-test at significance levels 0.10, 0.05, and 0.01, respectively. Degrees of freedom (df) are rounded to the nearest two decimals.

	Degree of Risk Aversion, A				
	12	10	8	6	2
σ_{BSU}^2	0.0004	0.0005	0.0007	0.0011	0.0089
$\sigma_{No\ BSU}^2$	0.0016	0.0023	0.0035	0.0062	0.0101
F-statistic	0.2500*	0.2196*	0.1945*	0.1754*	0.8855
μ_{BSU}	0.0044	0.0045	0.0047	0.0050	0.0073
$\mu_{No\ BSU}$	0.0020	0.0027	0.0038	0.0056	0.0075
t-statistic	1.9647***	1.2545	0.5112	-0.2541	-0.0576
df	166.18	160.35	155.36	151.45	1225.17

Even though the t -test of difference in means did not prove significantly higher returns for portfolios with BSU, the considerably low variances point towards higher risk-adjusted returns (Sharpe Ratio) compared to portfolios without the possibility of saving in the BSU. However, it is also worth noting that performing a t -test between the two portfolios with highest and lowest risk aversion (both with BSU) yields a t -statistic of 1.0923, hence also failing to reject the null hypothesis of greater monthly mean returns for the seemingly higher-yielding portfolio with $A = 2$. Thus, investors holding most of their savings in the BSU account are certainly not statistically significantly worse off than investors tilted more towards the stock market. Moreover, it seems the inclusion of the BSU account contributes to comparable portfolio returns to those without the savings account and even the stock market, but at a lower risk, which is arguably more attractive for any investor.

Next, we focus on the portfolios with BSU versus the Housing Index. Although the Housing Index has delivered extraordinarily low variance during the out-of-sample period (the index's yearly standard deviation is only 4.4%), the four portfolios with the highest risk aversion have experienced even lower variances. On the other hand, the portfolio with $A = 2$ had higher volatility compared to the Housing Index. Hence, all

five portfolios reject the null hypothesis of equal variances, at the 0.05 level of significance.

Remarkably, all five portfolios fail to reject the null hypothesis of greater mean returns compared to the Housing Index. Most noticeably, recording a p -value of 0.1425, even the portfolio with $A = 2$ is not yielding statistically significant higher monthly returns compared to the housing market. A test of differences in mean monthly returns for the Housing Index and the OSEBX also fails to reject the null hypothesis, implying that the two assets have yielded close to similar returns for the out-of-sample period. Nevertheless, given the significantly lower variances, we can still infer from the tests that portfolios with higher allocations towards the BSU outperform the Housing Index on a risk-adjusted return basis, although these portfolios do not yield significantly higher returns.

*Table 6. Test of difference in variance and returns being greater for portfolios with BSU versus Housing Index. We use the 0.05 level of significance for the F-tests where * signals rejection of the null hypothesis of equal variance. *, **, and *** show rejection of the null hypothesis of equal means for the t-test at significance levels 0.10, 0.05, and 0.01, respectively. Degrees of freedom (df) are rounded to the nearest two decimals.*

	Degree of Risk Aversion, A				
	12	10	8	6	2
σ_{BSU}^2	0.0004	0.0005	0.0007	0.0011	0.0089
$\sigma_{Housing}^2$	0.0020	0.0020	0.0020	0.0020	0.0020
F-statistic	0.2041*	0.2539*	0.3478*	0.5557*	4.5286*
μ_{BSU}	0.0044	0.0045	0.0047	0.0050	0.0073
$\mu_{Housing}$	0.0042	0.0042	0.0042	0.0042	0.0042
t-statistic	0.1328	0.2150	0.3302	0.4978	1.0726
df	157.2779	166.9020	183.1238	208.9481	160.5851

From Figure 19, it seems that portfolios with BSU outperform the Housing Index. However, the formal tests of the mean monthly returns do not find evidence for this. Apart from the least risk-averse optimal portfolio, we can see the Housing Index being more volatile, realizing a period of strong real estate appreciation in 2016 before weaker markets in 2017. The more risk-averse portfolios (with $A \geq 6$) on the other

hand, offer investors significant respite from tail-risks. In fact, over the entire out-of-sample period (noticeably a strong bull market), the maximum drawdown of the four most risk-averse optimal portfolios with BSU was 7.5%, with the portfolio of $A = 12$ being most stable, losing a maximum of 2.6% over the holding period. By comparison, the Housing Index experienced a modest drawdown of 8.2%, while the optimal portfolio for $A = 2$ had a maximum drawdown of 35.7% over the same duration.

Thus, although the inclusion of the BSU in asset allocation might not yield significantly higher returns and increase the possibility of acquiring a house, it is at least not underperforming the housing market either. Moreover, by allocating capital towards the savings account, young investors eager to enter the housing market will still reap comparable returns but with a much lower downside, potentially gaining ground to the housing market in tumultuous times.

7. Limitations & Possible Extensions

Before concluding on our research questions, we briefly summarize the main limitations and simplifying assumptions within our thesis, to reflect on possible influences of biases which might have affected our research outcome.

Firstly, the mean-variance optimization framework's inability to capture dynamic components of portfolio choices, especially within the context of the life-cycle, limits the scope of analysis of the BSU. One of the main findings in the thesis is the high returns on investment in the BSU taking full advantage of the tax break (as shown in section 3.2.). However, the model we use to form optimal portfolios is static using historic data, where past returns may not be indicative of future performance. In confirming this, the optimal portfolios formed using in-sample data often have lower Sharpe Ratios in the out-of-sample data (see Figure 20 in the Appendix).

Secondly, we have chosen to utilize monthly data, which usually makes sense in asset allocation analyses (many portfolios use monthly rebalancing). However, this frequency does not match the allocation choice for housing. Most homeowners hold their housing unit for multiple years before making a new purchase decision, meaning that price changes are not as important throughout this period where housing is held as an illiquid asset. Similarly, the model does not account for the illiquidity of the BSU either. This is especially relevant for the early life-cycle stage, where investors are most constrained in their disposable income and may have to significantly cut down on consumption - an opportunity cost that we did not consider.

While we have not come across another portfolio allocation model that better fits for portfolio choice with housing, we are suggesting that a cash flow model could be of value when trying to mitigate some of the limitations of the Markowitz model. Resembling a Discounted Cash Flow (DCF) model, focusing on a representative agent's monthly income from labor, as well as return from investments in assets and payments of mortgages, such a cash flow analysis could better incorporate an investor's liquidity and consumption considerations. However, such a model relies heavily on assumptions and should be accompanied by sensitivity analyses of input variables.

Lastly, we are reminded of potential sources of biases that might be baked into the real returns of some of our asset allocation choices. For the returns on the OSEBX, we are not including benefits and costs of trading. On the upside, we are disregarding the

capital gains tax break achieved by saving in broker accounts such as the “Aksjesparekonto” (ASK), and the valuation discount on wealth tax of 45% on stocks and bonds (Skatteetaten, 2021b). On the downside, transaction- and platform costs, as well as management fees are not included.

For the housing variable, our return data was gathered from an index of national price levels. A more relevant approach could be to capture housing returns as returning dividends, where housing service flows and depreciation are included in addition to price changes. For the latter, it would also be preferable to obtain repeated-sales data (i.e., follow the price development of specific housing units over time) instead of using a housing index.²⁰ This approach would better highlight the idiosyncratic risks of housing, as our results imply a very stable market with strong returns. In addition, such data could extend the research to highlight regional differences in housing returns.

²⁰ Here, service flows would include in-and-outflows from maintenance, property taxes, wealth tax, transaction costs, etc.

8. Conclusion

Throughout this thesis, we have introduced the importance of homeownership in Norway and the BSU savings account designed to assist young households in entering the housing market. From section 3, it seems the BSU savings account is not being fully utilized (perhaps because of liquidity considerations or lack of knowledge), even though our findings point towards the great benefits of such savings.

Answering our research question, we conclude that including the BSU savings account significantly improves the mean-variance efficient frontier by offering investors higher returns at lower risks. Furthermore, the BSU increases portfolio Sharpe Ratios, especially in the absence of housing and for portfolios with higher risk aversion. However, from an investment point of view, we cannot deduce that including the BSU in optimal portfolios necessarily outperforms the housing market, although these portfolios yield higher risk-adjusted returns (apart from at the lowest level of risk aversion). Interestingly, when considering additional investments for investors holding housing and mortgage, our results point towards saving in the BSU rather than repaying the loan, due to the comparably higher after-tax, real returns of the savings account.

These results are especially relevant for individuals with the goal of owning a housing unit, but who have not yet opened a BSU account. Such investors should start saving in the BSU. Furthermore, our research points toward the limitations of current policies in that the limitations of the BSU do not sufficiently assist new homeowners from entering the housing market. Although the BSU offers a very good interest rate, the savings limit of NOK 27,500 per year means it takes about 11 years to fill the BSU (to NOK 300,000), which, using it as 15% equity, equates to a housing unit worth NOK 2,000,000, equivalent to a unit of 30m² in size using the latest price level in Oslo.

Our research is novel in that it applies the BSU to the framework of mean-variance optimization. Hence, our findings add to the literature in providing an investment context for housing and the BSU. It would be interesting for further research to develop a model of households' life-cycle choices of asset allocations regarding income, liquidity, consumption, and other constraints. Investigating such dynamics could foster a better understanding of the implications of saving in the BSU and whether it assists in achieving homeownership.

9. Appendix

Table 7. Developments in the BSU regulation 1992-2021. Based on (Finansdepartementet, 2021).

Years in effect	Max. deposit p.a.	Total max. deposit	Max. tax deduction p.a.
1992 – 1997	NOK 10,000	NOK 60,000	NOK 2,000
1998 – 2008	NOK 15,000	NOK 100,000	NOK 3,000
2009 – 2013	NOK 20,000	NOK 150,000	NOK 4,000
2014 – 2015	NOK 25,000	NOK 200,000	NOK 5,000
2016 – 2020	NOK 25,000	NOK 300,000	NOK 5,000
2021 –	NOK 27,500	NOK 300,000	NOK 5,500*

*From 2021 onwards, a BSU saver may not receive the tax benefit if she is already a homeowner.

Slope of the Household's Indifference Curve:

$$U = \bar{\mu} - \frac{A}{2}\sigma^2$$

where

$$\begin{aligned}\bar{\mu} &= x_t\mu + h_t\mu_H \\ \sigma^2 &= [x_t, h_t]\Omega[x_t, h_t]^{-1}.\end{aligned}$$

We assume portfolio returns are normally distributed with mean μ and variance σ^2 and that the investor's expected utility is maximized. Hence the indifference curves of the utility function can be represented by a local approximation given as:

$$\mu = U + \frac{A}{2}\sigma^2$$

where the return's marginal rate of substitution with respect to the standard deviation (or risk) is given as:

$$\frac{\delta\mu}{\delta\sigma} = A\sigma$$

This is then the slope of the indifference curve used in our analysis.

Table 8. Return, standard deviation and covariance matrix with BSU without tax benefit.

	Housing	Mortgage	OSEBX	MSCI World	Bonds	BSU
Mean return (arithmetic)	0.0553	0.0140	0.1221	0.0623	-0.0043	0.0162
Standard deviation	0.0495	0.0163	0.1368	0.1097	0.0161	0.0163

	Housing	Mortgage	OSEBX	MSCI World	Bonds	BSU
Housing	0.00243					
Mortgage	0.00037	0.00032				
OSEBX	0.00330	0.00043	0.02726			
MSCI World	0.00211	0.00015	0.01542	0.01413		
Bonds	0.00040	0.00032	0.00051	0.00021	0.00032	
BSU	0.00038	0.00032	0.00043	0.00015	0.00032	0.00032

	Housing	Mortgage	OSEBX	MSCI World	Bonds	BSU
Housing	1.0000					
Mortgage	0.4227 (4.9354)	1.0000				
OSEBX	0.4050 (4.6879)	0.1435 (1.5343)	1.0000			
MSCI World	0.3602 (4.0856)	0.0686 (0.728)	0.7856 (13.4353)	1.0000		
Bonds	0.4566 (5.4314)	0.9936 (92.9696)	0.1744 (1.8744)	0.0988 (1.0502)	1.0000	
BSU	0.4311 (5.0568)	0.9986 (196.8383)	0.1438 (1.5378)	0.0709 (0.7523)	0.9940 (96.2419)	1.0000

Note: t-values are in paranthesis

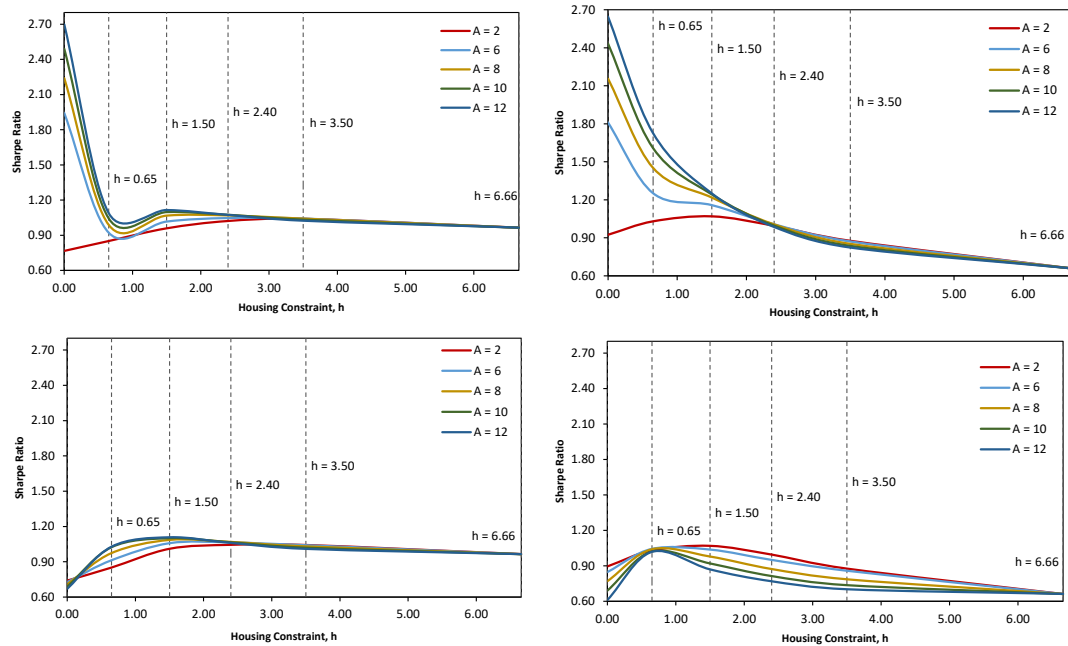


Figure 20. In-sample (LHS) versus out-of-sample Sharpe Ratios for optimal portfolios with BSU (above) and without BSU (below).

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