Supply and Demand in a Two-Sector Matching Model

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I examine the distributional consequences of technological change in a framework that nests Roy's (1951) and Becker's (1973) classical models: workers self-select into two sectors and then match with heterogeneous firms within each sector. In this model, technological change can be decomposed into changes in (i) the degree to which sectors covet the same skill sets and (ii) the extent to which output varies with skill in each sector. By deriving monotone comparative statics results for each of these two types of changes, I am able to provide a comprehensive account of the distributional consequences of technological change.

I. Introduction

Changes in technology make some skill sets more valuable while rendering others obsolete, which can have dramatic effects on the distribution

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of wages. The advent of computers, for example, first increased the return to cognitive skill (Bound and Johnson 1992; Katz and Murphy 1992; Juhn, Murphy, and Pierce 1993), then led to the replacement of labor in routine manual tasks (Autor, Levy, and Murnane 2003; Autor, Katz, and Kearney 2006; Acemoglu and Autor 2011), and most recently caused an increase in return to those skills that computers find difficult to emulate: noncognitive skills in general and social skills in particular (Deming 2017; Edin et al. 2017). Over the same time span, the distribution of wages changed significantly: first, upper-tail wage inequality increased rapidly; later—in the 1990s—this increase was accompanied by a fall in lower-tail inequality, leading to a rise in wage polarization.

The overall distribution of wages was affected by changes to both the between- and the within-sector (or between- and within-occupation) distributions (Mouw and Kalleberg 2010; Akerman et al. 2013; Helpman et al. 2016).[1] The literature attributes changes in between-sector distributions to changes in the relative productivity of sectors or occupations (task-biased technological change). The widespread changes in within-sector distributions are interpreted as evidence of increases in the return to cognitive skill across all sectors (skill-biased technological change).

While this interpretation is natural, it ignores the possibility that sector-specific technological change can have ripple-through effects and can affect within-sector distributions in the rest of the economy. In particular, if a change in technology increases the return to cognitive skill in some but not all sectors, it must affect which workers self-select into which sectors. As workers possessing different skill sets tend to be imperfect substitutes (see Katz and Murphy 1992), these changes to workers' self-selection will affect the relative wages of workers with high and low cognitive skill levels across all sectors. To investigate such effects, one requires a model with multivariate skills, multiple sectors, and imperfect substitution of workers within sectors; however, no comparative statics results are available for this class of models in the existing literature.

In this paper, I derive monotone comparative statics results with respect to changes in technology. My framework nests the classical models of Roy (1951) and Becker (1973) by introducing within-sector assignment of workers to firms into a model of workers' self-selection into sectors. Firms differ in their univariate productivity and operate in one of two sectors—manufacturing or services; workers differ in their multivariate skills. The output produced by a worker-firm match depends on the firm's productivity and sector and the worker's skill. In the competitive equilibrium, both workers and firms take wages as given, each worker self-selects into the sector that pays a higher wage for her skill, and each firm hires at most one worker to maximize profits.

[1] Throughout the paper, I use the term "sectors" for the two broad types of jobs in my model, but they could equally well be interpreted as occupations.

I assume that firms from the same sector agree on how to rank workers with respect to the output they produce and that highly ranked workers and highly productive firms are complements. As a consequence, highly productive firms hire highly ranked workers in equilibrium, as in the model of Becker (1973), and thus—given workers' sectoral choices—the assignment of workers to firms within a sector can be determined without knowing the wage functions. As in the models of Sattinger (1979) and Tinbergen (1956), the wage function's slope is then equal to the rate of change of output with respect to the worker's skill (keeping constant the firm matched with that worker), which is entirely characterized by her rank. The wage received by each worker after joining a sector can thus be written as a function of only the worker's rank within that sector, ensuring that—for given wage functions—workers' self-selection into sectors is determined exactly as in the model of Roy (1951). Finally, the baseline comparative statics results assume that the sum of the measures of manufacturing firms and services firms is equal to the measure of workers.

In my model, technology is reflected by the mapping from a worker's skill and a firm's productivity (and sector) into the output they jointly produce. Crucially, any change to this mapping can be decomposed into (i) a change in how similarly the two sectors rank workers and (ii) a change in the degree to which output varies with rank in each sector.[2] By deriving results concerning changes to both of these aspects of technology, I am able to provide a full account of the distributional implications of technological change. The more significant contribution is the derivation of the results regarding the similarity of rankings, as this aspect of technology has thus far received little attention in the literature. However, my model's predictions differ from the predictions of single-sector assignment models and standard selection models in regard to each of the two types of changes. Many of these differences derive from the fact that workers are perfect substitutes within sectors in the standard selection models (e.g., Heckman and Sedlacek 1985; Heckman and Honoré 1990; Gould 2002; Böhm 2020). In my model, workers with very different ranks are imperfect substitutes within sectors: while each firm is nearly indifferent between the worker matched to that firm and workers with close rankings, hiring a worker with a very different ranking would cause the firm to have significant losses.

To understand what happens to output and the distribution of wages if sectors start ranking workers more similarly, suppose that it has become more likely that a high-ranked manufacturing worker is ranked highly in services as well. This change on its own is akin to a fall in the sum of skill supplies in the two sectors: initially, when the two sectors ranked workers

[2] While the output produced by the lowest-ranked worker may change as well, this would result only in a shift of the distribution of wages by a constant.

differently, there were plenty of high-ranked workers to go around, and each sector was able to hire many workers that it regarded highly; after the change, the two sectors have to settle (in total) for more of the low-ranked workers. As far as the distribution of wages is concerned, this change has a composition effect and a wage effect. The composition effect holds constant the wage paid to workers of each rank. The literature has established that if the two sectors start valuing the same skill sets in a symmetric self-selection model with lognormally distributed skills, then the composition effect increases the variance of log wages (Owen and Steck 1962; Heckman and Honoré 1990; Gould 2002).[3] However, by focusing on the variance, these papers have missed the fact that the composition effect has a different impact on lower-tail than on upper-tail inequality: as the sectors hire more workers of low rank, the median-earning worker is of lower rank than previously, which decreases the median wage compared with the lowest and highest wages in the economy. As a result, lower-tail wage inequality falls, whereas upper-tail inequality increases.

The wage effect varies the wage paid to workers of a given rank and has not previously been studied in this context. To better analyze the change in within-sector distributions, I hold the between-sector distribution constant by focusing on the case where wages in the two sectors change by the same amount for marginal workers (i.e., those indifferent between the sectors). In that case, an increase in the similarity of rankings implies that firms in the two sectors increasingly compete for the same workers, which—because of the imperfect substitution of workers within sectors—increases both upper- and lower-tail inequality: the wages of the highest earners increase more than those of medium earners, and the wages of high and medium earners increase by a greater proportion than those of the lowest earners. The latter in particular is in stark contrast to the standard self-selection models, in which wages would increase by the same proportion for workers of all ranks. Overall, upper-tail wage inequality increases, as the composition and wage effects work in the same direction, whereas lower-tail wage inequality falls because the (first-order) composition effect dominates the (second-order) wage effect.

My model's predictions also differ from the predictions of standard models in regard to changes in the extent to which output varies with rank. To see why, suppose that the difference in the output produced by workers of any two ranks has increased in services. In a single-sector assignment model, the only available margin of adjustment would be a change to wages—and the difference in wages between workers of any two ranks would increase (Costrell and Loury 2004; Gabaix and Landier

[3] Owen and Steck (1962) derive order statistics for normally distributed variables with no reference to technology or wages. Heckman and Honoré (1990) are interested in wages, but they do not relate their theorem 2 to changes in technology. Gould (2002) provides the technology interpretation for the results of Heckman and Honoré (1990).

2008; Tervio 2008). With two sectors, however, self-selection changes as well: services draws in marginal high-ranked workers from manufacturing, whereas marginal low-ranked workers leave services. As a result, wage inequality increases in both sectors; however, the increase in services is smaller than in a single-sector model. There is also a change in the between-sector distribution of wages, as the wages of low-ranked workers increase in manufacturing relative to services. In particular, if the lowest-paid workers were concentrated in manufacturing initially, lower-tail between-sector inequality decreases. While standard self-selection models are able to produce qualitatively similar changes in the between-sector and within-services wage distributions, their predictions concerning the within-manufacturing distribution will differ from mine in at least one respect: in standard models, manufacturing wages would increase by the same proportion for workers of all ranks. In my model, the wages of high-ranked workers change by a different proportion than those of low-ranked workers, because of the imperfect substitution of workers within sectors.

Overall, my findings imply that the difference in outcomes of sector- (or task-) and skill-biased technological change is much less clear-cut than the literature suggests. The distribution of wages is determined, among other things, by how similar a worker's ranking in services relative to that in manufacturing is, which can be affected just as well by a technological change originating in one sector as by a change originating in both sectors. Specifically, a services-specific technological change may create demand in services for skill sets that initially were valued only in manufacturing, which can easily increase wage inequality to a similar (or greater) degree in manufacturing as in services. Thus, the fact that wage inequality increased by more in one sector than in another provides little evidence as to which sector was more affected by technological change, and the fact that wage inequality increased across all sectors does not prove that skill-biased technological change affected production in all sectors.[4] Similarly, skill-biased technological change makes all sectors value the same skill sets, which can decrease lower-tail wage inequality. This finding is in contrast to the standard view in the literature (see, e.g., Acemoglu and Autor 2011; Lindenlaub 2017) that one needs task-biased technological change (i.e., a change in the productivity of low-paying occupations relative to medium-paying occupations) to account for decreases in lower-tail wage inequality.

The rest of the paper is organized as follows. Section II further discusses the related literature. Section III develops the model and characterizes the unique equilibrium. Section IV derives comparative statics results concerning changes in ranking similarity and the degree to which output

[4] Both assumptions are widespread in the empirical literature on wage inequality. In some cases they are stated explicitly (e.g., Kaplan and Rauh 2013), but in most cases they are implicit (e.g., Böhm, Metzger, and Strömberg 2018).

varies with rank. Section V uses a special case of the model to demonstrate how changes to the surplus function (the mapping from a worker's skill and a firm's productivity into output) can be decomposed into the two types of changes considered in section IV and then uses the results from section IV to derive the distributional impact of sector- and skill-biased technological change. Section VI discusses what happens if (*a*) the measures of firms and workers are not equal or (*b*) firm entry is endogenous. Section VII concludes, and appendix A provides proofs of the comparative statics results.

II. Related Literature

A. Roy-Like Assignment Models

In addition to the standard selection models, which I discussed at length, the literature on self-selection includes the so-called Roy-like assignment models (e.g., Teulings 1995, 2005; Costinot and Vogel 2010; Acemoglu and Autor 2011), in which there is a single ranking of workers and the assignment of workers to a continuum of sectors is determined by comparative advantage. While, technically speaking, workers are perfect substitutes within sectors in these models, Costinot and Vogel (2010) point out that with a continuum of sectors, broadly defined real-world sectors (such as manufacturing) should be mapped into more than one sector in the model. Under this interpretation, workers would be imperfect substitutes within manufacturing, similarly to my model. However, the Roy-like assignment models lack the main mechanism driving my results (changes in ranking similarity), and the comparative statics results available for these models either rely on strong functional form restrictions (Teulings 1995, 2005) or ignore composition effects (Costinot and Vogel 2010).

B. Differential Rents Assignment Models

My baseline model provides a two-sector version of the Becker (1973) and Sattinger (1979) assignment models, while the endogenous entry extension provides a two-sector version of the Costrell and Loury (2004) model. In addition to the body of work that assumes one-dimensional skills, which I have already discussed, Tinbergen (1956) and Lindenlaub (2017) study the determinants of the distribution of wages in a single-sector, multivariate differential rents assignment model. The paper by Lindenlaub (2017) is particularly relevant, as it investigates the impact of skill- and task-biased technological change on the distribution of wages. Lindenlaub (2017) and this paper make different trade-offs to ensure tractability: Lindenlaub's model could be reinterpreted as a model where firms are divided into a continuum of sectors, but her model is solved—and comparative statics are provided—only for the very special Gaussian-quadratic case.[5] For example, the Gaussian-quadratic specification does not allow for skill-biased technological change that has any impact on sorting, which leads to the conclusion that technological change must be task biased to increase wage polarization.

C. "Becker Meets Roy" Models

My paper is not the first to combine assignment and self-selection in order to consider changes to both between- and within-group distributions. Grossman, Helpman, and Kircher (2017) study the impact of trade liberalization on within-industry and within-occupation distributions in a model with assignment between workers and managers and endogenous choice of firm size. However, they focus on trade rather than technology, and their model assumes a single ranking of both workers and managers, so that the key mechanism driving my results is absent. Dupuy (2015), McCann et al. (2015), and Mak and Siow (2017) also develop models in which workers self-select into sectors and occupations and match with firms. While the models in Dupuy (2015) and Mak and Siow (2017) share many features with my framework, none of those three papers provides any comparative statics results.[6]

III. The Model

In this section, I set up the model, characterize the equilibrium, prove its uniqueness, and compare it to the equilibria of the models in Sattinger (1979) and Roy (1951).

A. Setup

There are two sectors (manufacturing and services) and two populations (workers and firms).

Workers.—There is a unit measure of workers, each endowed with a vector of basic skills $\mathbf{x} = (x_1, x_2, \dots, x_N) \in I_{\mathbf{x}} \subset \mathbf{R}^N$. Denote the distribution of $\mathbf{x}$ by F. Workers can either work for a firm and receive a market wage or remain unemployed and receive a reservation wage (normalized to zero).

[5] The quadratic surplus function coupled with the normal skill distribution implies a surplus function that is not monotonic in workers' skills, even after the addition of noninteraction skill terms. For example, the estimated surplus function—see table 10 in Lindenlaub (2017)—is nonmonotonic in manual skill.

[6] Dupuy (2015) investigates the impact of multiplicative changes to the production function on self-selection and wage inequality. However, unlike this paper, Dupuy (2015) does not show the equilibrium effect of such changes, providing only a first-order result. That is, the results in Dupuy (2015) consider only the first step in the following equilibrium adjustment process: the change in production function affects wages, which affects sorting, which affects wages, etc., until a new equilibrium is reached.

Firms.—There is a measure R of firms, each endowed with a vector $(z, i) \in I_z \times \{M, S\}$, where z denotes the firm's basic productivity, $i \in \{M, S\}$ denotes the sector in which the firm operates (manufacturing or services), and $I_z \subset \mathbf{R}$. The (exogenous) distribution of (z, i) is denoted by H_Z, and the measure of firms in sector i is denoted by $R_i > 0$, with $R_M + R_S = R$. Each firm hires at most one worker. A worker-firm pair produces surplus according to the basic surplus function $\Pi : I_{\mathbf{x}} \times I_z \times \{M, S\} \to \mathbf{R}_{\geq 0}$. For example, if a manufacturing firm with basic productivity z hires a worker with basic skill $\mathbf{x}$, they produce surplus $\Pi(\mathbf{x}, z, M)$. The fact that the basic surplus function depends on the sector means that workers' basic skill and firms' basic productivity might be used differently in each sector. If a firm does not hire a worker, it receives a reservation profit normalized to zero.

1. Assumptions

I assume that within each sector all firms rank workers in the same way and all workers rank firms in the same way.[7] As sectors are traditionally defined with respect to the goods produced and firms producing similar goods are likely to highly value the same types of workers, this appears to be a reasonable simplifying assumption.[8] Formally, firm (z, i) ranks workers according to the function $v_i(\mathbf{x}, z) \equiv \Pr(\Pi(\mathbf{X}, z, i) \leq \Pi(\mathbf{x}, z, i))$, and worker $\mathbf{x}$ ranks firms in sector i according to the function $h_i(\mathbf{x}, z) \equiv \Pr(\Pi(\mathbf{x}, Z, i) < \Pi(\mathbf{x}, z, i)) + \Pr(\Pi(\mathbf{x}, Z, i) = \Pi(\mathbf{x}, z, i), Z \leq z)$.[9]

Assumption 1 (Common rankings). The rank $v_i(\mathbf{x})$ of worker $\mathbf{x}$ depends on the firm's sector but not on its productivity ($v_i(\mathbf{x}) \equiv v_i(\mathbf{x}, z) = v_i(\mathbf{x}, z')$ for any $z, z' \in I_z$), and the rank $h_i(z)$ of firm (z, i) does not depend on the worker ($h_i(z) \equiv h_i(\mathbf{x}, z) = h_i(\mathbf{x}', z)$ for any $\mathbf{x}, \mathbf{x}' \in I_x$).

It follows that a worker's ranks v_M, v_S and a firm's rank h_i contain all relevant information about $\mathbf{x}$ and z, respectively; henceforth, I will refer to them as skill and productivity.[10] We can now define the surplus function $\pi_i(v_i(\mathbf{x}), h_i(z)) = \Pi(\mathbf{x}, z, i)$; note that π_i is increasing in both the worker's skill and the firm's productivity by the definitions of v_i, h_i. Additionally, I require π_i to be twice continuously differentiable and supermodular.

Assumption 2 (Properties of surplus). The surplus function $\pi_i : [0, 1]^2 \to \mathbf{R}_{\geq 0}$ satisfies

[7] Chiappori, Oreffice, and Quintana-Domeque (2012) refer to this assumption as the separability of surplus.

[8] Alternatively, this assumption is equivalent to defining a sector as a collection of firms that rank (and are ranked by) workers in the same way and then assuming that there are only two sectors. All results from sec. IV.A easily generalize to the case on N sectors, but the problem studied in sec. IV.B is intractable with more than two sectors.

[9] The second term in $h_i(\mathbf{x}, z)$ is a tiebreaker needed to allow for surplus to increase weakly in h_i.

[10] Note that v_i bears a close resemblance to the "task" t_i in Heckman and Sedlacek (1985), in that both are a univariate sufficient statistics for the worker's multivariate skill.

i. differentiability (π_i is twice continuously differentiable),
ii. increasingness ($((\partial/\partial v_i)\pi_i > 0, (\partial/\partial h_i)\pi_i \geq 0)$, and
iii. supermodularity $((\partial^2/(\partial v_i \partial h_i))\pi_i \geq 0)$.[11]

Supermodularity ensures that highly productive firms benefit more from hiring high-skilled workers, and thus within-sector matching is positive and assortative. This brings tractability. However, as workers employed by similar firms must be close substitutes in production, the monotonicity of the matching function implies that workers of similar rank are closer substitutes than workers of very different rank: in equilibrium, each firm's profit is unimodal in the worker's skill (see app. OA.1 for a proof; apps. OA.1–OA.6 are available online). This pattern of substitution drives most of my results; for example, changes in skill supply can affect the relative wages of the highest- and lowest-ranked workers only if they are imperfect substitutes.

Assumption 3 (Properties of the copula). The distributions F and H_Z and the surplus Π are such that the joint distribution C of $(v_M, v_S) \in [0, 1]^2$ and the sectoral distributions H_i of $h_i \in [0, 1]$ satisfy

i. differentiability (are twice continuously differentiable) and
ii. full support (have strictly positive, finite density on their respective supports).[12]

The full support assumption implies that the marginal distributions of ranks v_M, v_S and h_M, h_S are standard uniform. Thus, C is a joint distribution function with standard uniform marginals, which is—effectively—the definition of a copula (Sklar 1959). Full support precludes perfect positive and negative dependence between skill: the copula is given by $\min\{v_M, v_S\}$ in the former case and by $\max\{v_M + v_S - 1, 0\}$ in the latter, and thus the density is concentrated only on the diagonal ($v_M = v_S$) and the antidiagonal ($v_M = 1 - v_S$). However, assumption 3 otherwise allows for very general dependence structures. For example, for C belonging to the family of Gaussian copulas, it allows for any correlation parameter $\rho \in (-1, 1)$.

I will refer to the formulation of the model in terms of v_i, h_i, and π_i as the canonical formulation, because it defines equivalence classes: any two models with the same canonical formulation will give rise to the same wage and output distributions.

Assumption 4 (Nondegenerate solutions). For $i, j \in \{M, S\}$ with $j \neq i$, either $R_i < 1$ or $\pi_i(0, 1 - 1/R^i) < \pi_j(1, 1)$.

[11] Common rankings ensure that π_i is strictly increasing in v_i, but $(\partial/\partial v_i)/\pi_i > 0$ is a stronger requirement.

[12] In the case of C, it suffices for both conditions to hold on $(0, 1)^2$. In particular, all results hold for Gaussian copulas.

This assumption is necessary and sufficient for all equilibria of this model to be nondegenerate, so that a positive measure of workers is employed in each sector.

2. Supply, Demand, and Equilibrium

Supply of skills.—A worker with skill (v_M, v_S) who joins sector i receives the competitive wage $w_i(v_i)$, where $w_i:[0,1]\rightarrow\mathbf{R}$. Workers sort into the sector that maximizes their wages. A worker with skill (v_M, v_S) joins manufacturing if $w_M(v_M)\geq\max\{w_S(v_S),0\}$, joins services if $w_S(v_S)>\max\{w_M(v_M),0\}$, and remains unemployed otherwise.[13]

The supply of skill of rank v_i in sector i, $S_i(v_i)$, is defined cumulatively as the measure of workers ranked higher than v_i who join sector i for given wage functions w_M, w_S:[14]

$$S_M(v_M) = \Pr(V_M\geq v_M, w_M(V_M)\geq w_S(V_S), w_M(V_M)\geq 0), \tag{1}$$

$$S_S(v_S) = \Pr(V_S\geq v_S, w_M(V_M)< w_S(V_S), w_S(V_S)> 0). \tag{2}$$

Note that $S_i(0)$ gives the total measure of workers who joined sector i.

Demand for skills.—The demand for skills in each sector is determined by the firms' hiring decisions, which in turn are driven by profit maximization, with firms taking the wage function as given. Firm h_i earns profit $r_i(h_i)$ and hires worker $v_i^*(h_i)$, where $r_i:[0,1]\rightarrow\mathbf{R}$ and $v_i^*:[0,1]\rightarrow[0,1]$, with

$$r_i(h_i) = \max_{v\in[0,1]}\pi_i(v,h_i) - w_i(v), \tag{3}$$

$$v_i^*(h_i)\in\arg\max_{v\in[0,1]}\pi_i(v,h_i) - w_i(v). \tag{4}$$

Demand for skills is defined analogously to skill supply. The demand for skill of rank v_i in sector i, $D_i(v_i)$, is equal to the measure of sector i firms that hire workers ranked higher than v_i for a given wage function w_i:

$$D_i(v_i) = R_i\Pr(v_i^*(H_i)\geq v_i, r_i(H_i)\geq 0). \tag{5}$$

This definition assumes that profits are strictly increasing in productivity, which is the case as long as $(\partial/\partial h_i)\pi_i>0$. A more general definition,

[13] If $w_S(v_S) = w_M(v_M)$, the worker could join either sector; however, as the set of all such workers is of measure zero in equilibrium, we can assign all of them to manufacturing without loss of generality.

[14] Throughout the paper, uppercase letters refer to random variables and lowercase letters refer to their realizations.

which holds even when surplus does not depend on productivity, is provided in appendix OA.1.[15]

The competitive equilibrium.—I focus on the competitive equilibrium, which is defined as follows.

DEFINITION 1 (Equilibrium). An equilibrium is characterized by

i. two skill supply functions $S_i:[0,1]\to[0,1]$, consistent with workers' sorting decisions and given by equations (1) and (2);
ii. two skill demand functions $D_i:[0,1]\to[0,1]$, consistent with firms' profit maximization and given by equation (5); and
iii. two wage functions $w_i:[0,1]\to\mathbf{R}$, which clear the markets: $S_i(v_i) = D_i(v_i)$ for $i\in\{M,S\}$ and all $v_i\in[0,1]$.

It is worth noting that because this model is an assignment game, the competitive equilibrium coincides with the core (Gretsky, Ostroy, and Zame 1992).

B. *Equilibrium Characterization*

To characterize the competitive equilibrium, I employ a two-step strategy.

First step.— In the first step, I treat the sectoral supply functions as given and find the wage functions for which demand equals supply. Let the critical skill v_i^c be the skill of the least skilled worker who joins sector i:

$$v_i^c = \sup\{v\in[0,1]: S_i(v) = S_i(0)\}\,. \tag{6}$$

In equilibrium, $S_i(0)$ cannot be greater than $R_i \geq D_i(0)$, as otherwise the market would never clear; I will restrict attention to supply functions that meet this condition.

When the supply of skill is fixed, the model becomes equivalent to two independent single-sector assignment models. It is well known that in a single-sector model supermodularity of surplus implies that workers and firms match positively and assortatively (Becker 1973; Sattinger 1979); that is, the most productive firm matches with the worker of highest skill, the second-most-productive firm matches with the second-most-skilled worker, and so on. The first-order condition of the firm's hiring decision is then

[15] Additionally, my definition implies that v_i^* is a function, which excludes the possibility of impure assignments. This greatly simplifies notation and is without loss of generality, because there will always exist a pure equilibrium assignment, and—by proposition 3 in Chiappori, McCann, and Nesheim (2010)—in my model the wage function is identical for all equilibrium assignments. Legros and Newman (2002) make a similar point for (supermodular) one-sided matching markets.

$$\frac{\partial}{\partial v_i} w_i(v_i^*(h_i)) = \frac{\partial}{\partial v_i} \pi_i \left(v_i^*(h_i), 1 - \frac{S_i(v_i^*(h_i))}{R_i} \right). \tag{7}$$

The difference in wages paid to workers of marginally different skill is equal to the difference in the surplus they produce, evaluated for the firm that is the optimal match for one of them. The optimal match depends in turn on the supply of skills in that sector: the fewer high-skilled workers available, the better the match that can be secured by any worker. Overall, the wage earned by a worker of skill v_i can be found by integrating equation (7) from v_i^c to v_i:[16]

$$w_i(v_i) = \int_{v_i^c}^{v_i} \frac{\partial}{\partial v_i} \pi_i \left(v, 1 - \frac{S_i(v)}{R_i} \right) \mathrm{d}v + w_i(v_i^c). \tag{8}$$

The wage paid to the worker with critical skill v_i^c is equal to the worker's share in the surplus produced by the worst match, and $w_i(v_i^c) \in [0, \pi_i(v_i^c, 1 - S_i(0)/R_i)]$. In particular, if workers are in short supply $(S_i(0) < R_i)$, then competition drives the profits of the least productive matched firm to zero and $w_i(v_i^c) = \pi_i(v_i^c, 1 - S_i(0)/R_i)$.

Second step.—In the second step, I treat the wage functions w_M, w_S as given and derive the corresponding sectoral supply functions, starting with the critical skill v_M^c, v_S^c. Note that a worker with a critical services skill prefers to join services over joining manufacturing or remaining unemployed and thus earns a wage that is greater than the reservation wage (zero) and the lowest offered wage in manufacturing $(w_M(0))$.[17] Furthermore, she must be the least skilled worker who can earn a wage greater than $\max\{w_M(0), 0\}$ in services, as otherwise, by continuity of $w_S(\cdot)$, a positive measure of workers with skill lower than v_S^c would also join services. This reasoning leads to the following lemma (a formal proof of this and other results from this section is provided in app. OA.1).

[16] Technically, for v_i such that $(\partial/\partial v_i)S_i(v_i) = 0$ market clearing requires only that $w_i(v_i)$ is weakly greater than the right-hand side. The assumption that $w_i(v_i)$ is equal to the right-hand side facilitates the derivation of more intuitive expressions and is without loss of generality: as $w_i(v_i)$ can be strictly greater than the right-hand side only for such v_i for which no workers join sector i, any lower wage (including the one equal to the right-hand side) would result in the same sorting. Therefore, for any equilibrium in which $w_i(v_i)$ is greater than the right-hand side for some v_i, there must exist an equilibrium with identical supply and demand functions and a wage function for which $w_i(v_i)$ is equal to the right-hand side for all v_i.

[17] Note that the lowest paid wage in manufacturing is $w_M(v_M^c) \geq w_M(0)$. This is because workers with $v_M < v_M^c$ choose not to join manufacturing; however, even if they joined manufacturing, the lowest wage they could be offered is $w_M(0)$.

LEMMA 1. The critical skill in sector i depends on the wage functions w_M, w_S as follows:

$$v_i^c = \min\{v_i \in [0,1] : w_i(v_i) \geq \max\{w_j(0), 0\}\}, \tag{9}$$

with $i, j \in \{M, S\}$ and $i \neq j$. Further, $w_M(v_M^c) = w_S(v_S^c)$.

By equation (8), wages are strictly increasing in each sector. This has two important implications for sorting. First, any worker with services skill $v_S > v_S^c$ can earn a strictly positive wage and will never choose to remain unemployed. Second, for any such worker there will exist a cutoff value $\psi(v_S)$ of the manufacturing skill such that she will strictly prefer to join services if $v_M < \psi(v_S)$ and strictly prefer to join manufacturing if $v_M > \psi(v_S)$. Therefore, the sorting of workers to sectors can be expressed by the means of the critical skills v_M^c, v_S^c and the separation function $\psi : [v_S^c, 1] \to [v_M^c, 1]$, which takes the services skill as an argument and returns the corresponding cutoff value of the manufacturing skill. Formally, the separation function depends on wages as follows:

$$\psi(v_S) = \max\{v_M \in [v_M^c, 1] : w_M(v_M) \leq w_S(v_S)\}. \tag{10}$$

Note that for values of v_S such that $w_S(v_S) \leq w_M(1)$, this implies that

$$w_S(v_S) = w_M(\psi(v_S)). \tag{11}$$

Figure 1 shows how the critical skills and separation function determine the sorting of workers to sectors. Services are populated by workers with $v_S \geq v_S^c$ and $v_M < \psi(v_S)$. Manufacturing is populated by workers with $v_M \geq v_M^c$ and $v_M \geq \psi(v_S)$. The remaining workers are unmatched, and their number is equal to $C(v_M^c, v_S^c) = 1 - S_S(0) - S_M(0)$. The number of workers in each sector depends on the position of the separation function: if ψ shifts up, then the number of workers in services (manufacturing) increases (decreases). The exact relation between ψ and the supply of skill is specified in the following lemma.

LEMMA 2. Given the critical skills v_M, v_S and the separation function ψ, the supply of skill in manufacturing and services is respectively

$$S_M(v) = \begin{cases} \int_v^1 \frac{\partial}{\partial v_M} C(r, \phi(r))\, \mathrm{d}r, & v \geq v_M^c \\ S_M(v_M^c), & v < v_M^c \end{cases}, \quad S_S(v) = \begin{cases} \int_v^1 \frac{\partial}{\partial v_S} C(\psi(r), r)\, \mathrm{d}r, & v \geq v_S^c \\ S_S(v_S^c), & v < v_S^c \end{cases}, \tag{12}$$

where $\phi : [v_M^c, 1] \to [v_S^c, 1]$ depends on ψ as follows:

$$\phi(v_M) = \sup\{v_S \in [v_S^c, 1] : \psi(v_S) < v_M\}.$$

In equilibrium, the separation function and critical values determine the supply of skill in each sector, the supply of skill determines wages, and wages determine the separation function and critical values. Thus,

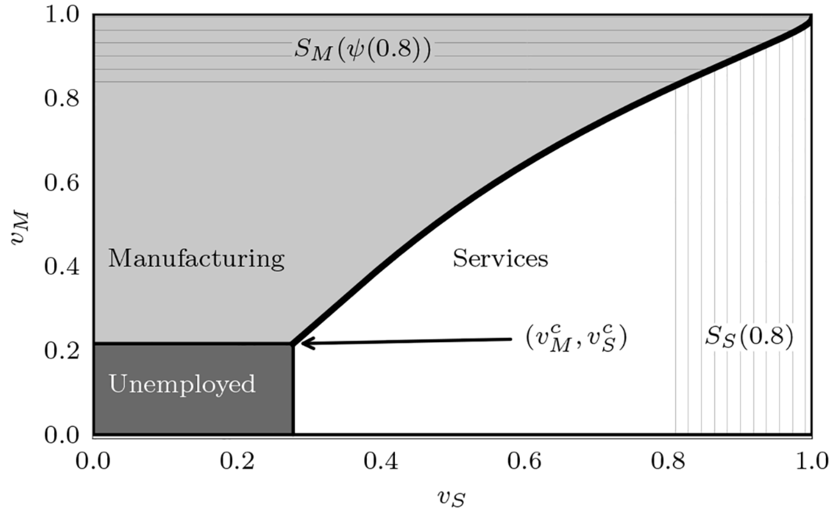

FIG. 1.—Separation function and self-selection. The solid black line depicts the separation function ψ. The white (light gray) area below (above) ψ depicts the space of workers joining services (manufacturing), and the gray area in the lower left corner depicts the space of workers who remain unemployed. The vertically (horizontally) hatched area represents the space of workers with skill $v_S \geq 0.8$ ($v_M \geq \psi(0.8)$) who join services (manufacturing); their supply depends on how many workers reside in this space, with $S_S(v_S) + S_M(\psi(v_S)) = 1 - C(\psi(v_S), v_S)$.

the equilibrium separation function can be found by substituting the supply functions from equation (12) into equation (8) and then substituting the resulting wage functions into equations (9) and (10).

THEOREM 1. An equilibrium exists. The equilibrium supply and demand functions are unique.

The proof (see app. OA.1) entails constructing a map, the fixed point of which solves equation (10), and finding a norm for which this map is a contraction mapping.[18] This proves that $\psi(\cdot)$ is unique given (v_M^c, v_S^c)—and that it is continuous in both v_M^c and v_S^c. To show that the equilibrium supply and demand functions are unique, it suffices to prove that equation (9) has a unique solution given $\psi(\cdot; v_M^c, v_S^c)$. However, if $R_M + R_S = 1$, then the wage functions are uniquely determined only up to the lowest wage $w_i(v_i^c)$.

C. *Sattinger and Roy*

The first step in my characterization strategy is very similar to Sattinger (1979), and the second step is very similar to Roy (1951). This is not a coincidence: the model nests both one-sector assignment models and Roy's

[18] The norm I use is Bielecki's norm for a sufficiently high parameter λ.

model of self-selection.[19] In this section, I compare the wage functions—appendix OA.1.1 provides the comparison of production functions.

Sattinger (1979).—The model reduces to the single-sector assignment model if one sector does not employ any workers. This can happen if either assumption 4 is not satisfied or there exist no firms in services ($R_S = 0$). In either case, wages in manufacturing are represented by

$$w_M^{\mathrm{SAT}}(v_M) = \int_{v_M^c}^{v_M} \frac{\partial}{\partial v_M} \pi_M \left(v, 1 - \frac{1-v}{R_M} \right) \mathrm{d}v + w_M(v_M^c), \tag{13}$$

where $v_M^c = \max\{0, 1 - R_M\}$, with $w_M(v_M^c) = 0$ if $R_M < 1$ and $w_M(v_M^c) = \pi_M(0, 1 - 1/R_M)$ if $R_M > 1$.

Roy (1951).—The model reduces to Roy's model if firms are identical within each sector, so that the surplus produced by any match depends only on the worker's skill (i.e., $(\partial/\partial h_i)\pi_i = 0$), and additionally there is an abundance of firms in each sector ($R_i > 1$). Under these assumptions, firms earn no rents and workers receive the entire surplus, so that $w_i(v_i) = \pi_i(v_i, 0)$. This is exactly as in Roy's model. In other words, Roy-like models can be seen as two-sector matching models in which all firms from the same sector are homogeneous.[20]

IV. Comparative Statics

In this section, I provide comparative statics results for changes in the interdependence of workers' skills (sec. IV.A) and for sector-specific changes in the surplus function (sec. IV.B). In all comparative statics exercises in this paper, I compare the equilibria of two specifications of the model: the old and the new. The old specification is denoted by θ_1 and the new one by θ_2. For example, $C(\theta_1)$ is the old copula of skills, and $C(\theta_2)$ is the new one. Most of the time, I will focus on small changes to the copula and surplus, using the following families of functions: $C(\theta) = \theta C(\theta_2) + (1-\theta)C(\theta_1)$ and $\pi_i(\theta) = \theta\pi_i(\theta_2) + (1-\theta)\pi_i(\theta_1)$.

A. *Interdependence of Skill*

The interdependence of v_M, v_S captures how similarly the two sectors rank workers. In section IV.A.1, I derive the relationship between skill

[19] Formally, it does not fully nest the actual models by Sattinger and Roy. Sattinger allows for cases in which there is unemployment even though some firms remain unmatched, whereas my assumption of positive surpluses implies that unemployment is possible only if all firms are matched. Roy uses bivariate lognormal distribution of skills, which violates the differentiability assumption at $v_i \in \{0, 1\}$; however, Roy's model can be approximated arbitrarily well by using bivariate lognormal distribution, truncated arbitrarily high and arbitrarily close to zero. This is done in sec. V and app. OA.3.

[20] The Cobb-Douglas lognormal (CDL) specification from sec. V with $R_i > 1$ and $\alpha_{iF} = 0$ provides a good example.

interdependence and the sum of equilibrium skill supplies in the two sectors, and in section IV.A.2 I explore the implications of this relationship for the distribution of wages. Note that since the marginal distributions of skill are standard uniform by construction, they are held constant throughout.

1. Concordance and the Supply of Skill

I will use the concordance ordering as the notion of interdependence.

Definition 2 (Scarsini 1984). Copula $C(\theta_2)$ is more concordant than copula $C(\theta_1)$ if $C(v_M, v_S, \theta_2) \geq C(v_M, v_S, \theta_1)$ for all $(v_M, v_S) \in [0, 1]^2$.

Concordance is a natural nonparametric partial ordering of interdependence. An increase in the concordance of skills captures the fact that there are fewer workers who are highly ranked in manufacturing but have low rank in services; equivalently, there are more workers who have high (or low) rank in both sectors:

$$\Pr(V_M \geq v_M, V_S \leq v_S) = v_M - C(v_M, v_S),$$

$$\Pr(V_M \leq v_M, V_S \leq v_S) = C(v_M, v_S),$$

$$\Pr(V_M \geq v_M, V_S \geq v_S) = C(v_M, v_S) + 1 - v_M - v_S.$$

For multivariate normal variables, an increase in the copula's concordance is equivalent to an increase in the correlation coefficient ρ (Joe 1997).[21] In general, an increase in concordance of the copula of a multivariate distribution implies an increase in all the standard measures of interdependence: Kendall's τ, Spearman's ρ (Nelsen 1999), the Gini coefficient of association (Nelsen 1998), and linear correlation (keeping marginals fixed). Figure 2 depicts the contour sets for Gaussian copulas with different correlation values and demonstrates how the same value of C is attained by lower pairs (v_M, v_S) as correlation increases.

A fall in the concordance of skill increases the sum $S_M(\psi(v_S; \theta_1)) + S_S(v_S)$ of equilibrium skill supplies in the two sectors; it follows from lemma 2 that

$$\frac{\partial}{\partial\theta}(S_M(\psi(v_S; \theta); \theta) + S_S(v_S; \theta)) = -\frac{\partial}{\partial\theta} C(\psi(v_S; \theta), v_S; \theta). \qquad (14)$$

Intuitively, if the concordance of skill is very high, then a select few workers are good at everything, while the rest are good at nothing. In contrast, if the concordance of skill is very low, then most workers are good at one type of job but are bad at the other. Since each worker can hold only one job, the latter case results in a larger number of workers performing a job at which they are good.

[21] By Sklar's (1959) theorem, the copula C of any continuous bivariate distribution function H with marginals H_1, H_2 is defined uniquely by $C(H_1(x_1), H_2(x_2)) = H(x_1, x_2)$.

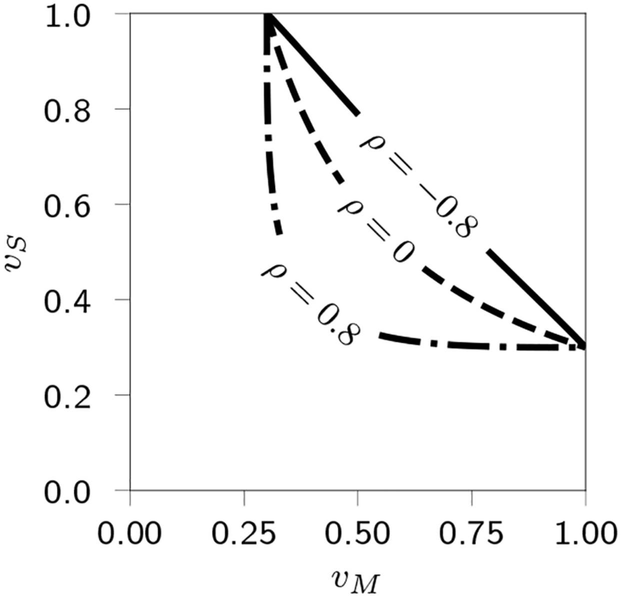

FIG. 2.—Concordance: a contour plot. Contour plots for $C(v_M, v_S) = 0.3$ for three bivariate Gaussian copulas with correlation coefficient ρ equal to (from left to right) 0.8, 0, and −0.8.

As a change in concordance affects the sum of skill supplies, it must also affect the total surplus produced in the economy. The total surplus is equal to the sum of the average wage and the (appropriately weighted) average profit earned in each sector:

$$T(C) \equiv E_C(w(V_M, V_S)) + \sum_{i \in \{M,S\}} R_i E_{U[0,1]}(\max\{r_i(H_i), 0\}),$$

where the subscript on E denotes the cumulative distribution function (CDF) with respect to which the expectation is taken and $w(V_M, V_S) = \max\{w_M(V_M), w(V_S), 0\}$.

Let us first consider the effect that a change in concordance has on the total surplus in a standard Roy's model—that is, when firms are abundant and homogeneous within sectors (see sec. III.C). In such a case, wage functions are exogenous and there are no profits, implying that the total surplus is equal to the average wage $E_C(w(V_M, V_S))$. To see the impact of a change in concordance on the average wage, consider an arbitrary wage level $x \geq 0$. Self-selection implies that only workers with skill such that $w_M(v_M), w_S(v_S) \leq x$ will earn less than x. Therefore, the economy-wide distribution of wages is simply

$$F_W(x) = \Pr(w(V_M, V_S) \leq x) = C(w_M^{-1}(x), w_S^{-1}(x)). \quad (15)$$

Therefore, in a Roy's model, a fall in concordance causes a first-order stochastic dominance improvement in the distribution of wages, which increases the average wage. Intuitively, if wages in manufacturing and services are perfectly positively correlated, then effectively each worker receives a single draw from the distribution of wages. If instead concordance falls and wages in services become independent of wages in manufacturing, then each worker gets two chances to draw a wage—and, because of self-selection, keeps the higher one. Clearly, expected wages must always be greater in that case.

In the general model (i.e., when firms are heterogeneous), the analysis becomes only marginally more complicated. Of course, now a change in concordance affects the wage functions as well and therefore also affects firms' profits. However, because the total surplus can be written as the solution of a minimization problem with respect to the wage functions (by the Monge-Kantorovich theorem), the envelope theorem implies that the new effects are of second order.[22] Formally, we can write

$$\begin{aligned} T(C) \equiv \inf_{\tilde{w}_M, \tilde{w}_S} & E_C(\tilde{w}(V_M, V_S)) \\ & + \sum_{i \in \{M,S\}} R_i E_{U[0,1]}\left(\max\left\{0, \max_{v_i \in [0,1]} \pi_i(v_i, H_i) - \tilde{w}_i(v_i)\right\}\right), \end{aligned}$$

where the infimum is over integrable functions $\tilde{w}_M, \tilde{w}_S : [0,1] \to \mathbf{R}$, with $\tilde{w}(V_M, V_S) = \max\{\tilde{w}_M(V_M), \tilde{w}(V_S), 0\}$. We can apply theorem 2 in Milgrom and Segal (2002) to get

$$T(C(\theta_2) - T(C(\theta_1)) = \int_0^1 \frac{\partial}{\partial \theta} E_{C(r)}(w(V_M, V_S; r))\, \mathrm{d}r, \quad (16)$$

where $w(\cdot, \cdot; r)$ denotes the equilibrium wage function for copula $rC(\theta_2) + (1 - r)C(\theta_1)$.

It therefore follows from equation (15) that a fall in concordance of the joint distribution of skills increases total surplus in every specification of the model. In appendix OA.2, I show that this implication goes in the other direction as well: total surplus increases in every specification of the model only if concordance falls.

2. Wage Polarization

I will now address the impact that an increase in skill concordance has on wage polarization. To the best of my knowledge, there does not exist a

[22] For an introduction to the Monge-Kantorovich theorem, see chap. 2 of Galichon (2016). In my case, the formulation of the theorem from Chiappori, McCann, and Nesheim (2010, theorem 1) can be applied directly, as Chiappori, McCann, and Nesheim (2010) explicitly allow for unmatched agents.

standard, formal definition of wage polarization; hence, I will provide my own definition. In doing so, it will be convenient to focus on the inverse wage distribution $W(t) = \min\{x \geq 0 : F_W(x) \geq t\}$, which returns the tth quantile of the wage distribution (e.g., $W(0.5)$ is the median wage).

DEFINITION 3 (Wage polarization). Wage polarization increases in absolute terms if (i) there exists $\bar{t} \in (0, 1)$ such that the difference $W(t) - W(0)$ falls for all $t \in (0, \bar{t})$ and (ii) the difference $W(1) - W(0)$ increases. Wage polarization increases in relative terms if conditions i and ii hold for log $W(t)$.

Definition 3 captures the most salient empirical facts about wage polarization: the decrease in lower-tail wage inequality and the increase in overall and upper-tail inequality (see fig. 9 in Acemoglu and Autor [2011] and the accompanying discussion).[23]

I will decompose the change in the inverse wage distribution into the (first-order) *composition effect* and the (second-order) *wage effect*. This is the same in spirit as the empirical decomposition proposed by Machado and Mata (2005), with the crucial difference that the notion of skill Machado and Mata (2005) use in their application is one-dimensional and thus the driving force is changes in the marginal distribution rather than concordance. For $t \geq C(v_M^c, v_S^c)$, it is the case that $F_W(W(t)) = t$; differentiating this with respect to θ and rearranging yields

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\theta} W(t) = & \underbrace{- W'(t) \frac{\partial}{\partial\theta} C(v_M(t), v_S(t); \theta)}_{\text{composition effect}} \\ & + \underbrace{p_M(t) \frac{\partial}{\partial\theta} w_M(v_M(t)) + p_S(t) \frac{\partial}{\partial\theta} w_S(v_S(t))}_{\text{wage effect}},\end{aligned} \tag{17}$$

where $v_i(t) = w_i^{-1}(W(t))$ denotes the skill in sector i of a worker occupying quantile t and $p_i(t)$ denotes the probability that a worker at the tth

[23] First, note that my definition does not require that $W(0.5) - W(0)$ decreases, which did happen in the 1990s in the United States. This will happen in my model for plausible parameterizations (see fig. 3*A*), but this outcome is not universal. Second, notice that I focus on wage rather than job polarization, i.e., the fall in employment in medium-paid occupations (Goos and Manning 2007). These two are related but not the same. For example, in my model there are just two sectors/occupations, so there can be no job polarization, and yet there will be wage polarization. Empirically, Goos and Manning (2007) provide evidence of increased job polarization in the United Kingdom but find no evidence of increased wage polarization. Finally, observe that my definition of a fall in relative lower-tail inequality implies that the least earning workers earn a higher proportion of the income earned in the lower tail of the wage distribution. To see this, consider a conditional Lorenz curve $CL(t; \bar{t}) = (\int_0^t W(s)\,\mathrm{d}s)(\int_0^{\bar{t}} W(s)\,\mathrm{d}s)^{-1}$, which measures how much income earned by workers of rank $\bar{t}$ or lower accrues to workers of rank t or lower. Notice that $(\partial/\partial t)CL(t; \bar{t}) = W(t)(\int_0^{\bar{t}} W(s)\,\mathrm{d}s)^{-1}$; a fall in $\ln W(t) - \ln W(0)$ for all $t \leq \bar{t}$ implies that $W(t)/W(0)$ falls as well, from which it follows that $(\partial/\partial\theta)(\partial/\partial t)CL(0; \bar{t}) > 0$.

percentile of the wage distribution works in sector i. A detailed derivation can be found in appendix OA.2.

Composition effect.—This effect holds constant the wage functions $w_i(v_i)$ and captures the fact that as concordance increases, the two sectors end up hiring (in total) fewer workers of high skill, which places a worker of given skill (v_M, v_S) at a higher percentile in the distribution of wages. Therefore, the median-earning worker is of lower skill than previously, which decreases the median wage compared with the lowest and highest wages in the economy. As a result, lower-tail wage inequality falls, whereas upper-tail inequality increases. Note that in Roy's (1951) model, only the composition effect is present.

Wage effect.—This effect captures the impact of the changes in the wage functions $w_M(v_M)$, $w_S(v_S)$ induced by the change in the copula. It is equal to the sum of the changes to $w_M(v_M)$, $w_S(v_S)$ weighted by the probability that a worker who occupies rank t works in sector i.

The impact that the wage effect has on wage inequality can be further decomposed into the *within- and between-sector* components. The within-sector component measures how much wage inequality is affected by changes to the gradients of the within-sector wage functions. Specifically, it captures by how much the gradient of the wage effect at rank t would increase if wage functions changed symmetrically across sectors and the change to their gradient was equal to the average of the actual changes (weighted by the probability that a worker of rank t works in a given sector). The between-sector component measures how much wage inequality is affected by the fact that wage functions change asymmetrically across the two sectors. In particular, it can be interpreted as the change in the gradient of the wage effect at rank t if wages in both sectors changed by a constant and this constant was different for manufacturing than for services. Formally:

$$\underbrace{\frac{\partial}{\partial t}\sum_{i\in\{M,S\}} p_i(t)\frac{\partial}{\partial\theta}w_i(v_i(t))}_{\text{wage effect}} = \underbrace{\sum_{i\in\{M,S\}} p_i(t)\frac{\partial}{\partial t}v_i(t)\frac{\partial^2}{\partial\theta\partial v_M}w_M(v_i(t))}_{\text{(1): within-sector component}} + \underbrace{\frac{\partial}{\partial t}p_M(t)\frac{\partial}{\partial\theta}\left(w_M(\psi(v_S(t))) - w_S(v_S(t))\right)}_{\text{(2): between-sector component}}. \tag{18}$$

Suppose that the change in concordance has no effect on between-sector inequality, which is ensured if wages in both sectors change by the same amount along the separation line:

$$\frac{\partial}{\partial\theta}\left(w_M(\psi(v_S)) - w_S(v_S)\right) = 0 \tag{19}$$

for all $v_S \geq v_S^c(\theta)$ and all $\theta \in [0, 1]$. This condition is met, for example, if the two sectors are symmetric—that is, if $R_M = R_S$, $\pi_M = \pi_S$, and $C(x,y;\theta_i) = C(y, x; \theta_i)$. If equation (19) is satisfied, the change in skill supply must be of the same sign in both sectors, as otherwise wage gradients would change by different amounts (by eq. [8]). Thus, the supply of skill falls in both sectors by equation (14), and the gradient of the wage function increases for all skill levels. It follows that the wage effect can increase wage polarization only if it changes the between-sector distribution of wages, typically by increasing wages by more in the sector that employs most of the lowest and highest earners (see app. OA.2 for a numerical example).

Overall effect.—In the existing literature, an increase in wage polarization is always caused by the between-sector component of the wage effect (see, e.g., Costinot and Vogel 2010; Acemoglu and Autor 2011; Böhm 2020). However, I have shown that if the change in the distribution of wages is driven by an increase in skill concordance, then the composition effect also contributes to the increase in polarization. In fact, it is precisely the case in which the between-sector component of the wage effect does not change for which it can be shown that—under fairly mild conditions—wage polarization must increase overall. This is not obvious, because in this very case the increase in polarization caused by the composition effect is counteracted by the increase in lower-tail inequality coming from the within-sector component of the wage effect.

To simplify the exposition and highlight the key mechanisms, I will focus on the case where the total measure of firms is equal to the measure of workers, which is a standard assumption in the literature on worker-firm assignment (see, e.g., Mailath, Postlewaite, and Samuelson 2013, 2017; Lindenlaub 2017). To avoid the problem of indeterminate lowest wage, I will restrict attention to the equilibria in which workers have all the bargaining power. These assumptions are relaxed in section VI.

Assumption 5. $R_M + R_S = 1$, and $w_i(v_i^c) = \min_{j \in \{M,S\}} \pi_j(v_j^c, 0)$.

Additionally, to show that $W(t) - W(0)$ decreases for all $t \leq \bar{t}$ (rather than only for some), I impose a mild regularity condition: I say that an increase in interdependence is regular if there exists $\bar{v} \in (v_S^c, 1)$ such that $C(\psi(v_S; \theta_1), v_S; \theta_2) - C(\psi(v_S; \theta_1), v_S; \theta_1)$ is weakly increasing on $v \in (v_S^c, \bar{v})$ and is strictly increasing on a subinterval of $v \in (v_S^c, \bar{v})$. This excludes pathological cases, where the difference in derivatives oscillates between positive and negative values in any neighborhood of zero.[24]

[24] This condition is satisfied by all commonly used copulas, including all of the one-parameter families listed in chap. 4 of Joe (1997) that satisfy assumption 3—i.e., families B1–B8 and B10. It is possible that the condition is not needed at all, but all my attempts to prove this have failed.

Proposition 1 (Wage polarization). Suppose that assumption 5 is satisfied, the concordance of the skill distribution increases regularly, and equation (19) holds. Then (i) wage polarization increases in both absolute and relative terms, with both $W(t) - W(0)$ and $\log W(t) - \log W(0)$ strictly falling for some $t \in (0, \bar{t})$. If, in addition, $(\partial^2/(\partial v_i \partial h))\pi_i > 0$ for $i \in \{M, S\}$, then (ii) $W(1) - W(0)$ and $\log W(1) - \log W(0)$ strictly increase.

The impact of an increase in concordance on the inverse distribution of log wages is depicted in figure 3*A*. If there is no change in between-sector inequality, then the competition for marginal workers (i.e., those whose skills lie on the separation line) must change symmetrically across sectors. In particular, as the sectors start agreeing to a great extent on which workers are most skilled, the competition for these workers must increase. As high- and low-ranked workers are imperfect substitutes, this increases the wages of high earners by a greater proportion than those of the lowest earners; the lowest wage in particular continues to equal $\pi_i(v_i^c(\theta_1), 0)$. Lower-tail inequality falls because the increase in concordance affects the wage function through the wage gradient and thus has only an indirect impact on wage levels (i.e., the wage effect is of second order), whereas the composition effect is a direct consequence of the increase in concordance (and thus is of first order). This intuition can be glanced from equation (17), which for t close to zero implies that[25]

$$\frac{\mathrm{d}}{\mathrm{d}\theta} W(t) \leq \underbrace{\frac{\partial}{\partial \theta} C(v_S(t), v_S(t))}_{>0} \left[\underbrace{\int_{v_S^c}^{v_S(t)} \frac{1}{R_S} \frac{\partial^2}{\partial v_S \partial h_S} \pi_S \left(s, 1 - \frac{S_S(s)}{R_S} \right) \mathrm{d}s}_{(1)\approx 0} \underbrace{- W'(t)}_{(2)<0} \right] < 0. \quad (20)$$

The increase in concordance changes the matching between workers and firms, which in turn affects the gradient of the wage function. However, the change in the gradient accumulates only gradually over skill levels (term [1]). Conversely, the composition effect is always finite (term [2]), and thus for ranks arbitrarily close to zero the wage effect is arbitrarily small compared with the composition effect (see fig. 3*B*).

The conclusions of proposition 1 are unique to models that combine assignment and self-selection. In standard self-selection models, workers are perfect substitutes within each sector, and so the range of log wages $(\log W(1) - \log W(0))$ would remain unchanged, which is inconsistent with the empirical trends in the last decades. And while it is true that a fall in the supply of skill would increase polarization and inequality in a single-sector assignment model (which has not been pointed out before), workers

[25] Equation (20) follows from substituting eq. (19) into eq. (17), using eqq. (14) and (19) to bound $(\partial/\partial\theta)(\partial/\partial v_S) w_S(v_S)$, and finally using the regularity condition to bound the entire expression. A detailed derivation is provided in the proof of proposition 1 in app. A.

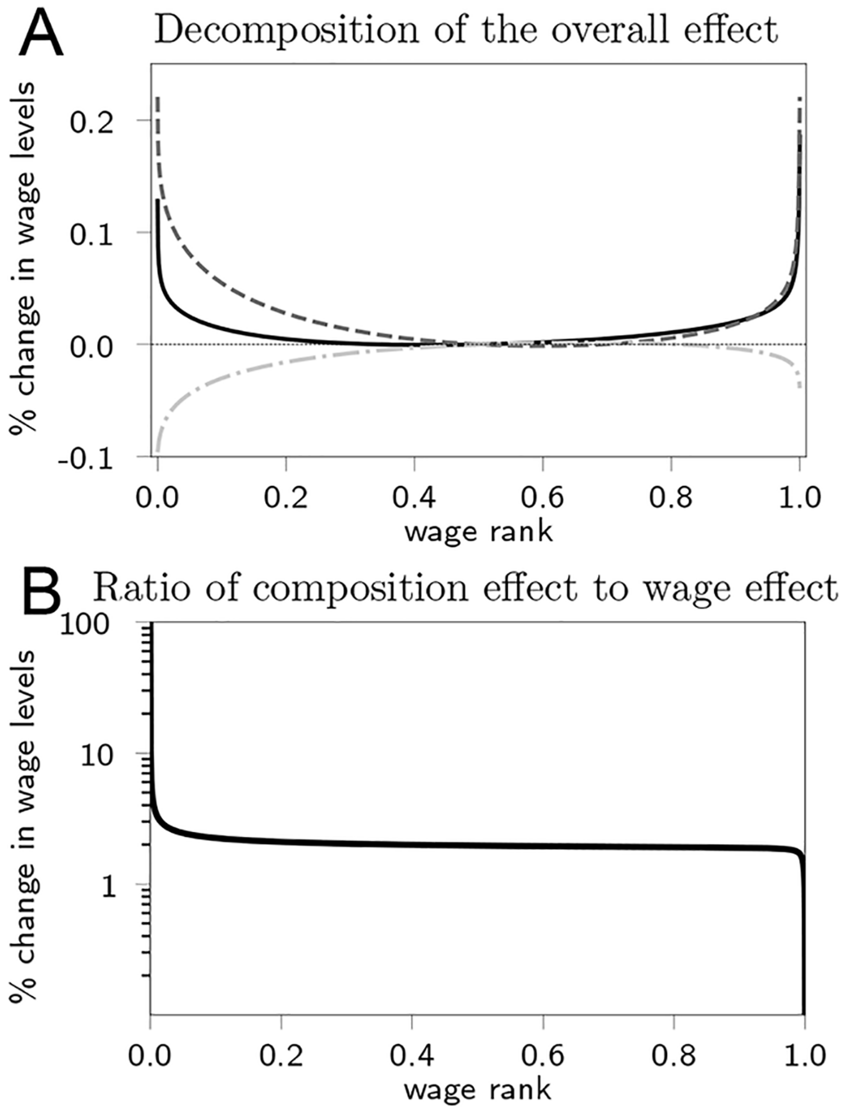

FIG. 3.—Concordance and wage polarization. *A*, Effect of an increase in concordance on the inverse distribution of log wages, relative to the median (i.e., $\ln W(t) - \ln W(0.5)$). The solid black line depicts the overall effect, the dashed gray line depicts the composition effect, and the dashed-dotted light gray line depicts the wage effect. *B*, Absolute value of the ratio of the composition effect to the wage effect ($|-W'(t)/\int_{v_S^c}^{v_S(t)}(1/R_S)(\partial^2/(\partial v_S \partial h_S))\pi_S(s, 1 - S_S(s)/R_S)\,\mathrm{d}s|$). The specification used to create this figure is reported in appendix OA.4.

must self-select into particular sectors or tasks for the increase in concordance to cause a fall in sectoral skill supplies in the first place.[26]

B. *Changes to the Reduced Surplus Function*

In this section, I investigate the effects of sector-specific changes to the surplus function. Under assumption 5, which will be maintained in this section, the crucial feature of the surplus function that determines workers' self-selection is their vertical differentiation.

Definition 4 (Vertical differentiation). Workers in services become (strictly) more vertically differentiated if, for any $h_S \in [0, 1]$ and any $v_S \in [0, 1]$,

$$\frac{\partial}{\partial v_S}\pi_S(v_S, h_S; \theta_2) \geq (>) \frac{\partial}{\partial v_S}\pi_S(v_S, h_S; \theta_1).$$

Workers become more vertically differentiated in services if the difference in the surplus they produce increases for all levels of skill and all firms. An increase in vertical differentiation can be caused by a number of changes to the fundamentals: a change to the actual production process (see sec. V), a fall in the price of an input that is a complement to skill, or an increase in the price of output produced in services. The distribution of skill v_i in sector i will be denoted by $G_i(v_i)$, with

$$G_i(v_i) = \Pr(V_i \leq v_i | w_i(V_i) \geq w_j(V_j), w_S(V_i) \geq 0),$$

where $j \neq i$. Under assumption 5 all workers are employed, and hence a worker of skill v_i is matched with a firm of productivity $G_i(v_i) = 1 - S_i(v_i)/R_i$. The within-sector distribution of wages is denoted by $F_{Wi}(x) = G_i(w_i^{-1}(x))$, and the inverse distribution of wages in sector i is denoted by $W_i(t) = w_i(G_i^{-1}(t))$.

Proposition 2. Suppose that assumption 5 is satisfied and workers in services become more vertically differentiated. Then (i) $G_S(v_S)$ falls for all v_S and (ii) $G_M(v_M)$ increases for all v_M. As a consequence, (iii) $(\partial/\partial v_M)w_M(v_M)$ increases for all $v_M \geq v_M^c(\theta_1)$ and (iv) $W_M(1) - W_M(0)$ increases. In services, (v) $W_S(1) - W_S(0)$ increases by more than $W_M(1) - W_M(0)$. Furthermore, if $v_S^c(\theta_1) > 0$ and the increase in differentiation is strict, then (vi) there exists a $\bar{t} > 0$ such that $W(t) - W(0)$ falls for all $t \in (0, \bar{t})$, so that wage polarization increases in absolute terms. Finally, if $\pi_S(v_S^c(\theta_1), 0; \theta_2) \geq \pi_S(v_S^c(\theta_1), 0; \theta_1) \geq \pi_M(v_M^c(\theta_1), 0)$, then conditions iv–vi hold for log $W(t)$ as well.

As workers in services become more vertically differentiated, the distribution of skill improves in services and deteriorates in manufacturing,

[26] The self-selection could take place within firms. In fact, an assignment model with within-firm self-selection is mathematically equivalent to the symmetric case of this model.

both in the sense of first-order stochastic dominance. This result can be best understood by focusing on the impact on the demand for skills. How do services firms' hiring decisions change after an increase in vertical differentiation but before the wage functions have time to adjust?[27] Under assumption 5, all firms want to hire some worker, and hence their hiring decisions depend on the differences in surplus only; the levels play no role at all. And because the difference in surplus produced by different workers has increased but the difference in wages has not, all firms want to hire a worker of higher skill than in the old equilibrium: the demand for skill shifts upward. This is depicted in figure 4*A*. The shift in skill demand draws in marginal high-skilled workers into services and causes marginal low-skilled workers to leave for manufacturing, which is depicted in the right panel of figure 4*B*. Naturally, wages increase more (decrease less) for services workers of highest skill than those of lowest skill, as the former are now in higher demand; accordingly, the wage range increases. In manufacturing, the supply of skill falls, and thus workers of all skill levels are matched with more productive firms than previously, which—by the supermodularity of surplus—increases the difference in wages earned by workers of any two skill ranks. It follows that the highest wages increase more than the lowest wages, in manufacturing and overall.

The change in overall wage inequality can be decomposed into the within- and between-sector components (see eq. [18]). The fall in lower-tail inequality is because of the latter component. The condition $v_S^c(\theta_1) > 0$ implies that the least earning workers are all employed in manufacturing ($p_S(0) = 0$), while workers who earn slightly higher wages can be found in both sectors. Therefore, the average wage among low-earning workers (i.e., those below the $\bar{t}$th percentile) is lower in manufacturing than in services. Recall that marginal low-skilled workers leave services for manufacturing; hence, the wages of low earners increase more in manufacturing, and between-sector lower-tail inequality falls. This causes an overall fall in lower-tail wage inequality, because the change in within-sector inequality is very small for low-earning workers, as (*a*) the change in wage inequality in services enters with a very low weight ($p_S(0) = 0$) and (*b*) the increase in the wage gradient in manufacturing is very small for low-skilled workers $((\partial^2/(\partial\theta\partial v_M))w_M(v_M(0)) = 0)$.

Finally, the impact of a change in surplus on the difference in log wages $(\log W(t) - \log W(0))$ depends on the change in $W(t) - W(0)$ and the change in the lowest wage. The condition $\pi_S(v_S^c(\theta_1), 0; \theta_2) \geq \pi_S(v_S^c(\theta_1), 0; \theta_1) \geq \pi_M(v_M^c(\theta_1), 0)$ ensures that the lowest wage is given by the surplus produced by the least skilled manufacturing worker and hence either falls

[27] That is, if firms still have to pay wage $w_S(v_S; \theta_1)$ for skill v_S.

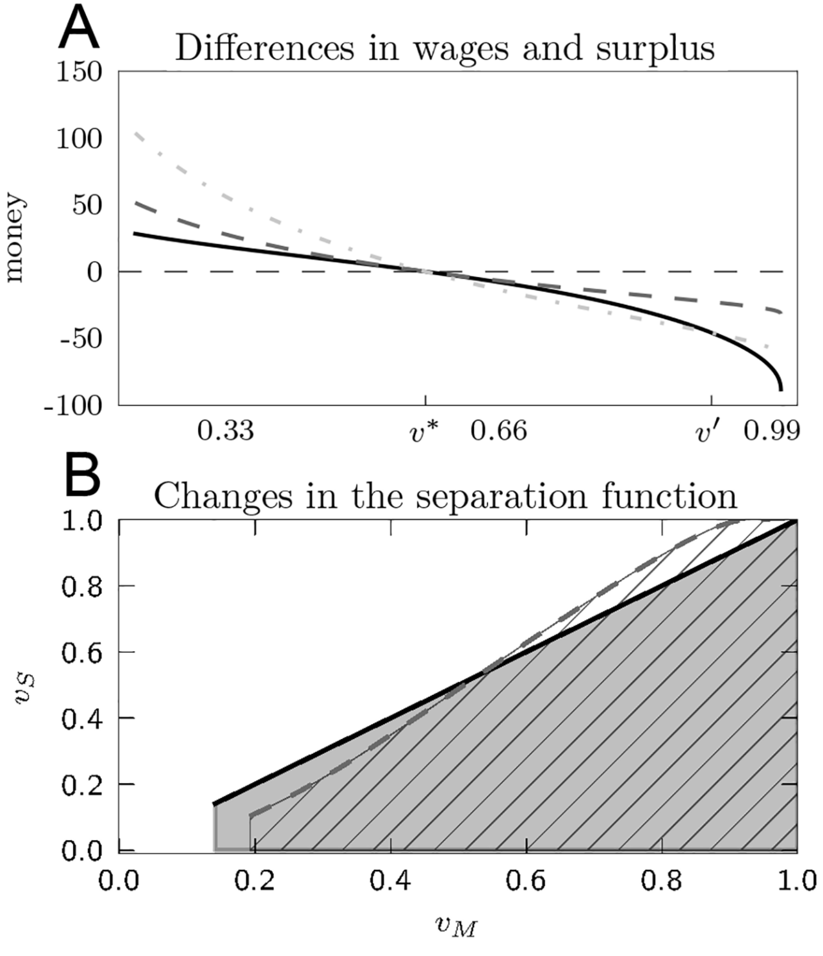

FIG. 4.—Changes in hiring decisions and the separation function. In *A*, the solid black line depicts $w_S(v^*(h), \theta_1) - w_S(v, \theta_1)$, the dashed gray line depicts $\pi_S(v^*(h), h, \theta_1) - \pi_S(v, h, \theta_1)$, and the dashed-dotted light gray line depicts $\pi_S(v^*(h), h, \theta_2) - \pi_S(v, h, \theta_2)$. The skill of the worker hired by firm h in the old equilibrium is denoted by v^*: the firm prefers all workers with skill $v \in (v^*, v')$ over worker v^* after the increase in differentiation but before wage functions have time to adjust. *B*, Change in the equilibrium separation function: the solid black line depicts $\psi(\cdot; \theta_1)$, and the dashed gray line depicts $\psi(\cdot; \theta_2)$.

(if $v_M^c > 0$) or remains constant. Thus, the results regarding changes in wage range and lower-tail inequality also hold in logs. The impact of an increase in vertical differentiation on the inverse distributions of log wages in the economy and each sector is depicted by the dashed-dotted line in figure 5*A*.

The conclusions of proposition 2 differ from what would happen in either the standard Roy's model or a single-sector assignment model. Single-sector assignment models are silent on the spillover effect in the other sector. In particular, they cannot produce a fall in (absolute) lower-tail wage inequality in response to an increase in the gradient of the surplus function. In the standard Roy's model, firms earn no rents and workers always receive the entire surplus; as a result, sorting depends only on the level of surplus, not on its gradient. In particular, an increase in the level of surplus in services would raise wages in that sector and the least skilled manufacturing workers would leave for services $((\partial/\partial\theta)v_M^c \geq 0)$; as a result, overall lower-tail wage inequality would increase and the wage range in manufacturing would fall (see proposition OA.3; propositions OA.1–OA.13 are available online). In the self-selection models with endogenous wages and perfect substitution of labor within sectors, the change in sorting would further depend on the concavity of the aggregate production function in each sector; however, perfect substitution of labor ensures that changes in surplus in services would have no effect on the gradient of log wages in manufacturing. Table OA.1 (available online) summarizes the predictions of my model, a single-sector assignment model and Roy's self-selection model for the two comparative statics exercises conducted in this section.

V. Sector- and Skill-Biased Technological Change

In this section, I will use the results from section IV to explain how a change to the basic surplus function Π in one or both sectors affects the distribution of wages in a simple CDL specification of the model.

Cobb-Douglas lognormal.—The basic surplus in each sector is a (shifted) Cobb-Douglas function of cognitive (x_C) and noncognitive (x_N) basic skills, as well as the firm's basic productivity (z): $\Pi(\mathbf{x}, z, i) = x_C^{\alpha_{iC}} x_N^{\alpha_{iN}} z^{\alpha_{iF}} + A_i$, where $\alpha_{ij}, A_i \geq 0$. The two basic skills are jointly lognormally distributed, with $\log \mathbf{x} \sim \mathbf{N}(\boldsymbol{\mu}_x, \mathbf{I})$; basic productivity z, conditional on the firm operating in sector i, is univariate lognormally distributed, with $\log z \sim N(\mu_{iF}, 1)$.

Denote the Cobb-Douglas skill composite used in sector i by $v_i'(\mathbf{x}) \equiv x_C^{\alpha_{iC}} x_N^{\alpha_{iN}}$. The vector of indexes $\mathbf{v}' \equiv (\log v_M', \log v_S')$ is jointly normally distributed, with mean $\mu_i = \alpha_{iC}\mu_{xC} + \alpha_{iN}\mu_{xN}$, standard deviation $\sigma_i = \sqrt{\alpha_{iC}^2 + \alpha_{iN}^2}$, and correlation $\rho = (\alpha_{MC}\alpha_{SC} + \alpha_{MN}\alpha_{SN})(1/\sigma_M\sigma_S)$. Normalizing v_i' and $z|i$ so that their marginal distributions are standard uniform yields the following canonical formulation of the model:

$$\begin{aligned} \pi_i(v_i, h_i) &= e^{\Phi^{-1}(v_i)\sigma_i + \mu_i + \alpha_{iF}(\Phi^{-1}(h_i) + \mu_{iF})} + A_i, \\ C(v_M, v_S) &= \boldsymbol{\Phi}_\rho(\Phi^{-1}(v_M), \Phi^{-1}(v_S)), \end{aligned}$$

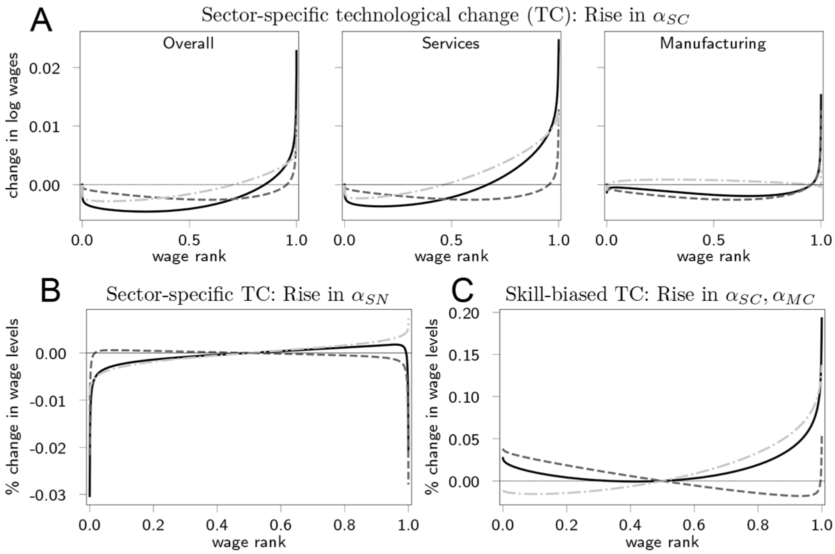

FIG. 5.—Impact of changes in skill intensity. *A*, Effect of an increase in α_{SC} on the inverse distribution of log wages (i.e., $W(t)$). *B*, *C*, Effects of increases in α_{SN} (*B*) and α_{SC}, α_{MC} (*C*) on the inverse distribution of log wages relative to the median (i.e., $\ln W(t) - \ln W(0.5)$). The solid black line represents the overall effect, the dashed gray line represents the effect of the change in ρ, and the dashed-dotted light gray line represents the effect of the change in π_i. The specifications for which these figures were created are reported in appendix OA.4.

where Φ_ρ represents the CDF of a standardized bivariate normal distribution with correlation ρ and Φ represents the CDF of the univariate standard normal distribution.[28]

In what follows, I assume that assumption 5 is satisfied and that the model is initially symmetric. The latter ensures that any increase in concordance is without effect on the between-sector distribution (needed for proposition 1) and that services hire no more low-income workers than manufacturing (needed for proposition 2, result vi). Services are labeled as the relative noncognitive-skill-intensive sector, so that $\alpha_{SN}/\alpha_{MN} \geq \alpha_{SC}/\alpha_{MC}$.

Sector-specific technological change.—Suppose that services become more cognitive skill intensive—that is, α_{SC} increases slightly. Such a change affects the surplus function through μ_i and σ_i and the copula of skills through ρ. As services are relative noncognitive skill intensive, the concordance of skills increases:

[28] The Gaussian-exponential specification does not satisfy assumption 2, as the surplus function is not defined for $v_i = 1$ and not differentiable for $v_i \in \{0, 1\}$. Formally, I solve this problem by working with a surplus function with an additional truncation parameter a_i, which approaches π_i as $a_i \rightarrow 0$ (see app. OA.3 for details). Then, in simulations I simply set a_i close to zero.

$$\frac{\mathrm{d}}{\mathrm{d}\alpha_{SC}}\rho = \frac{\alpha_{SN}(\alpha_{MC}\alpha_{SN} - \alpha_{MN}\alpha_{SC})}{\sigma_S^3\sigma_M^{1/2}} > 0. \tag{21}$$

The gradient of the surplus function in services increases for $v_S \geq \Phi(-((\sigma_S/\alpha_{SC})\mu_{xC} + (1/\sigma_S)))$ and falls otherwise:

$$\frac{\mathrm{d}}{\mathrm{d}\alpha_{SC}}\frac{\partial}{\partial v_S}\pi_S(v_S, h_S) = \frac{\pi_S(v_S, h_S) - A_S}{\Phi'(\Phi^{-1}(v_S))}\left(\frac{\alpha_{SC}}{\sigma_S} + \alpha_{SC}\Phi^{-1}(v_S) + \sigma_S\mu_{xC}\right). \tag{22}$$

To ensure that the gradient increases for all v_S and thus that workers become more vertically differentiated in services, I further assume that $\mu_{xC} > 0$ and $\alpha_{SC} = 0$ initially.[29]

An increase in α_{SC} raises the return to skill and causes sectors to rank workers more similarly. Therefore, it follows from propositions 1 and 2 that, as shown in figure 5*A*, the change in concordance (dashed gray line) and the change in the surplus function (dashed-dotted light gray line) have the same effect on the overall distribution of wages (solid black line): they raise overall wage polarization and increase the wage range in both sectors. Notice that while the change in the surplus function increases the wage range in services more than in manufacturing, the change in concordance increases the wage range symmetrically across sectors. In the parametrization used to create figure 5*A*, most of the increase in wage range in services is caused by the increase in concordance; as a result, the wage range increases by the same order of magnitude in both sectors. Indeed, there exist asymmetric parameterizations in which wages of highest earners increase more in manufacturing than in services (see fig. OA.1, available online). This suggests that empirical studies attempting to infer the causes of increased wage inequality from cross-sector comparisons (e.g., Kaplan and Rauh 2013; Böhm, Metzger, and Strömberg 2018) should exercise a certain degree of caution: the fact that inequality rose more in financial services than in other sectors does not imply that the underlying technological change has its origin in that sector.

Suppose instead that services become even more noncognitive skill intensive—that is, α_{SN} increases slightly. As a result, the return to skill increases (eq. [22]) but sectors rank workers less similarly (eq. [21]). In such a case, the changes in the surplus function and concordance work in opposite directions, and the impact on wage polarization and inequality is ambiguous. In particular, figure 5*B* depicts a parametrization of the model for which the effect of the change in concordance dominates and both wage polarization and upper-tail wage inequality fall. This case matches well the recent findings that noncognitive skills became more

[29] While this assumption is necessary to ensure an increase in vertical differentiation, it can easily be shown that the conclusions of proposition 2 hold for any sufficiently large increase in α_{SC}.

important in the 2000s and 2010s in the United States and Sweden. On one hand, these changes increased wages in high-paying occupations other than science, technology, engineering, and mathematics (Deming 2017), which should have increased upper-tail wage inequality. On the other hand, the increase in the return to noncognitive skills was particularly pronounced in occupations that were already noncognitive skill intensive (table V in Deming 2017; table 4 in Edin et al. 2017), which would decrease skill concordance and thus also upper-tail wage inequality. Figure 2 in Edin et al. (2017) shows that wage polarization and upper-tail inequality in Sweden have indeed fallen in the 2000s and 2010s.

Skill-biased technological change.—Finally, consider technological change that makes both sectors more cognitive skill intensive but is not biased toward either sector. Specifically, suppose that α_{MC}, α_{SC} increase in such a way that $\pi_M = \pi_S$ after the change; this is the case if and only if $\mu_{xC} = 0$ and α_{MC} as a function of α_{SC} satisfies $d\alpha_{MC}/d\alpha_{SC} = \alpha_{SC}/\alpha_{MC}$. Of course, as a result, the gradient of surplus falls for low-skilled workers and increases for high-skilled workers in both sectors (by eq. [22]), which leaves sorting unaffected. Further, the concordance of skill increases, as $d\rho/d\alpha_{SC} = (\alpha_{MC}/\sigma_S^2)(\alpha_{MC}\alpha_{SN} - \alpha_{MN}\alpha_{SC})^2 > 0$.

Therefore, skill-biased technological change increases wage polarization unambiguously, as both the change in concordance (by proposition 1) and the (symmetric) change in surplus (by eq. [22]) cause an increase in polarization. This is depicted in figure 5*C*; note that while the change in surplus produces a small fall in lower-tail inequality on its own, one needs the increase in concordance of skill to match the empirical fact that the increase in wages was smallest for wages close to the median.[30] Crucially, skill concordance is certain to increase precisely because skill-biased technological change is not biased toward any sector, which is possible only if the increase in cognitive skill intensity is weaker in the relatively cognitive-skill-intensive sector. Therefore, the fact that the rise in the return to cognitive skills took place across all sectors (Bound and Johnson 1992; Juhn, Murphy, and Pierce 1993) makes it more, rather than less, likely that skill-biased technological change contributed to the increase in wage polarization in the 1990s.

VI. Extensions and Generalizations

In this section, I briefly discuss the consequences of relaxing some of my assumptions. The results on which this discussion is based can be found in appendix OA.5, which also includes a discussion of a dynamic version of the model.

[30] This is not an artifact of the parametrization used to create fig. 5*C* but holds true in general. The reason is that $d\pi_S(0.5, h_i)/d\alpha_{SC} > 0$ and the skill of the median-earning worker is greater than 0.5.

A. *Scarce and Abundant Jobs*

I will first relax assumption 5 and allow for jobs being either scarce ($R_M + R_S < 1$) or abundant ($R_M + R_S > 1$).

1. Scarce Jobs

In the scarce jobs case, the number of workers in each sector is fixed (as in the baseline), but the wage of the lowest-ranked employed worker is equal to the reservation wage. All results concerning changes in the distribution of skill and upper-tail wage inequality—that is, proposition 1(ii) and proposition 2(i–v)—carry through unchanged. The results concerning lower-tail wage inequality are ambiguous: intuitively, if there are few firms, then most workers receive the reservation wage and the matched workers are effectively in the upper tail of the overall wage distribution, and hence wage inequality either remains unchanged (for those earning the reservation wage) or increases (for everyone else).

2. Abundant Jobs

In the abundant jobs case, the lowest wage is still equal to $\min_{i \in \{M,S\}} \pi_i(v_i^c, 1 - S_i(0)/R_i)$, but the number of workers in each sector is endogenous. The change in wages can be decomposed into the standard *baseline effect* and the new *size effect.* Formally, consider an intermediate specification of the model (θ_3) in which the initial surplus function in services is shifted upward by a constant ($\pi_M(v_M, h_M; \theta_3) = \pi_M(v_M, h_M; \theta_1) + A$) in such a way that the measure of workers in each sector is the same as in the final specification ($S_i(0; \theta_3) = S_i(0; \theta_2)$). We can then decompose the gradient of the inverse wage distribution:

$$W'(t; \theta_2) - W'(t; \theta_1) = \underbrace{W'(t; \theta_2) - W'(t; \theta_3)}_{\text{baseline effect}} + \underbrace{W'(t; \theta_3) - W'(t; \theta_1)}_{\text{size effect}}. \quad (23)$$

Skill interdependence.—If equation (19) is satisfied, then after an increase in the concordance of the skill distribution, either there is no effect on the size of either sector or the model behaves like Roy's model. Proposition 1 carries through unchanged in either case.

Changes in surplus function.—As explained in section IV.B, the intensive margin of the demand for skill depends on the vertical differentiation of workers. The extensive margin (how many firms want to hire a worker), however, depends on the level of surplus as well. In general, if workers become more vertically differentiated and the surplus produced by the lowest-ranked worker ($\pi_S(v_S^c(\theta_1), 1 - (S_S(0; \theta_1)/R_S))$) increases, then the supply of skill ($S_S(\cdot)$) increases in services for all skill ranks. In such a case, the baseline effect is exactly as in proposition 2.

The size effect increases wage levels in both sectors but more so in services, in the sense that the separation function ψ shifts upward. Furthermore, it increases the wage gradient $((\partial/\partial v_i)w_i)$ in manufacturing and decreases it in services. Therefore, if $v_S^c > 0$, then both components of equation (18) become positive for small t and the impact of the size effect on lower-tail wage inequality is positive (proposition OA.8). Thus, the overall effect depends on whether the baseline or the size effect dominates, which in turn depends on the increase in workers' vertical differentiation relative to the increase in surplus levels.

B. Endogenous Entry

To model firm entry, I build on the approaches of Hopenhayn (1992) and Melitz (2003). There is an unlimited supply of potential firms that are ex ante identical. If a potential firm decides to enter sector i, it pays cost $c_i > 0$ and draws productivity h_i from a standard uniform distribution. Firms enter the sector that maximizes their expected profits net of entry cost (if any), where the expected profit in sector i is defined as

$$\bar{r}_i \equiv \int_0^1 \max\{r_i(h), 0\}\, \mathrm{d}h.$$

Compared with definition 1, the equilibrium must satisfy one additional condition: if entry is positive in sector i ($R_i > 0$), the expected profit must equal the cost of entry: $\bar{r}_i = c_i$.

The equilibrium exists and is unique (theorem OA.2, available online) and efficient (proposition OA.9). In contrast to the baseline, wages are uniquely determined even if $R_M + R_S = 1$ in equilibrium, because constant average profits pinpoint the split of surplus in the least productive match. To derive comparative statics results, I assume—analogously to assumption 5—that the equilibrium measure of firms across the two sectors is equal to the measure of workers. This is ensured if, for example, $\pi_i(1,1) - \pi_i(1,0) \le c_i$ for both $i \in \{M, S\}$ and $\pi_i(0,0) > c_i$ for some $i \in \{M, S\}$. Notably, under this assumption the model nests the Costrell and Loury (2004) assignment model (app. OA.5.3).

The change in wages can again be decomposed, this time into the baseline effect and the *entry effect*. The intermediate specification still shifts the initial surplus function in services by a constant, this time in such a way that entry is the same as in the final specification ($R_i(\theta_3) = R_i(\theta_2)$). Then the wage gradient can be written as

$$\begin{aligned}\frac{\partial}{\partial v_i} w_i(v_i;\theta_2) - \frac{\partial}{\partial v_i} w_i(v_i;\theta_1) &= \underbrace{\frac{\partial}{\partial v_i} w_i(v_i;\theta_2) - \frac{\partial}{\partial v_i} w_i(v_i;\theta_3)}_{\text{baseline effect}} \\ &\quad + \underbrace{\frac{\partial}{\partial v_i} w_i(v_i;\theta_3) - \frac{\partial}{\partial v_i} w_i(v_i;\theta_1)}_{\text{entry effect}}.\end{aligned}$$

Skill interdependence.—If surplus is additively separable $((\partial^2/\partial v_i h_i)\pi_i = 0)$, then the endogenous entry extension is equivalent to Roy's model and proposition 1(i) holds unchanged. If surplus is strictly supermodular $((\partial^2/\partial v_i h_i)\pi_i > 0)$, then equation (19) ensures that an increase in the concordance of the skill distribution has no effect on firm entry. Thus, the change in the wage gradient is as in the baseline, and the results about the impact of concordance on absolute wage polarization are unchanged compared with the conclusions of proposition 1. The results regarding relative wage polarization differ: with endogenous entry, high- and low-ranked workers are complements rather than imperfect substitutes (proposition 2 in Costrell and Loury 2004), so that a worsening of the skill distribution in a sector increases the wages of high-ranked workers but decreases the wages of low-ranked workers. Thus, lowest wages increase as a result of higher concordance, which contributes to a decrease in relative lower-tail wage inequality. In the symmetric case, however, it can be shown that if the lowest wage was sufficiently high initially or the supermodularity of surplus is sufficiently weak, then lower-tail inequality falls in relative terms as well (proposition OA.10).

Changes in surplus functions.—With endogenous entry, the equilibrium supply of skill in each sector depends not only on the gradient of surplus with respect to a worker's rank and the level of surplus but also on the gradient of surplus with respect to a firm's productivity. In particular, if the level of surplus increases and both workers and firms become more vertically differentiated in services (both $(\partial/\partial v_S)\pi_S$ and $(\partial/\partial h_S)\pi_S$ increase for all $(v, h) \in [0, 1]^2$), then the supply of skill and the measure of firms must increase in services and fall in manufacturing (proposition OA.11). In such a case, the baseline effect's impact on wage range and gradients is as in proposition 2. The entry effect is identical to the effect of increasing entry in services in the baseline while also decreasing it in manufacturing by the same amount. As entry changes but the gradient of surplus does not, the wage must go up for some workers and down for others; otherwise, firm profits would not be equal to the cost of entry. The impact that the entry effect has on the wage gradient is ambiguous: in particular, if the changes in entry are large enough, then the entry effect must reduce wage inequality in manufacturing and thus counteract the increase caused by the baseline effect (proposition OA.12).

VII. Concluding Remarks

This paper derived monotone comparative statics results with respect to changes in technology for a selection model with imperfect substitution of workers within sectors. The main message is that sector- and skill-biased technological change can have similar impact on the distribution of wages once we take into account that technological change affects the degree to which sectors value the same skill sets.

How could one then empirically distinguish between sector- and skill-biased technological change? I see two paths forward. The first is to make use of the testable implications of my model: while my results indicate that these two types of technological change can have a similar impact on the overall distribution of wages, they often differ in their impact on within- and between-sector distributions and/or in the mechanisms that drive the outcomes. For example, skill-biased technological change affects lower-tail wage inequality within each sector, whereas sector-biased technological change decreases lower-tail inequality by increasing wages in one sector relative to the other. Thus, to determine the causes of a change in lower-tail inequality, one needs to check whether it has fallen between sectors, within sectors, or both. Similarly, if within-sector wage inequality increases across sectors as a result of a services-specific technological change, then it will be accompanied by changes in the sorting of workers to sectors. Skill-biased technological change, however, could produce the same effect with minimal reallocation of workers. Thus, to evaluate the causes of a widespread increase in within-sector inequality, one should look at the magnitude of the accompanying changes in workers' self-selection.

Alternatively, estimating the model at two different points in time would also reveal the causes of the observed changes in wage distributions. It is well known that the standard self-selection model can be nonparametrically identified from multimarket data (Heckman and Honoré 1990), whereas a single-sector assignment model with random search can be nonparametrically identified from matched employer-employee data on wages and labor market transitions (Hagedorn, Law, and Manovskii 2017). While combining these two results is conceptually straightforward, doing so would require introducing within-sector random search into my model, which is left for future research. More immediately, the frictionless model from this paper can be estimated or calibrated parametrically. This is done in Burzyński and Gola (2020), where we calibrate a close cousin of the model from this paper and use it to quantify the impact of changes in migration costs on welfare in the sending and destination countries.

Appendix A

A. *Proofs for Section IV*

To simplify notation, I will assume that $(\partial/\partial\theta)\psi(\cdot;\theta)$ exists and is continuous in both v_S and θ; this is proved in appendix OA.6, lemma OA.13. In addition, appendix OA.5 provides proofs of more general results that do not rely on differentiability.

1. Proof of Proposition 1

i. To show that $(\mathrm{d}/\mathrm{d}\theta)(W(t) - W(0)) < 0$ for $t \approx 0$, it suffices to show that $(\mathrm{d}/\mathrm{d}\theta)W(0) = 0$ and to derive equation (20). First, notice that under assumption 5 all workers are matched, and thus $C(v_M^c, v_M^c) = 0$ and $\min\{v_M^c, v_M^c\} = 0$. Together with (19), this implies that $(\mathrm{d}/\mathrm{d}\theta)v_i^c = 0$ and thus

$(\mathrm{d}/\mathrm{d}\theta)W(0) = 0$. Second, using equations (11) and (19) we can rewrite equation (17) as $(\mathrm{d}/\mathrm{d}\theta)W(t) = -W'(t)(\partial/\partial\theta)C(v_M(t), v_S(t)) + (\partial/\partial\theta)w_S(v_S(t))$. Differentiating equation (19) yields $(\partial/\partial\theta)(\partial/\partial v_S)w_S(v_S) = (\partial/\partial v_S)\psi(v_S)(\partial/\partial\theta)(\partial/\partial v_M)w_M(\psi(v_S))$, which together with equation (14) implies that $0 \leq (\partial/\partial\theta)(\partial/\partial v_S)w_S(v_S)$ and

$$\frac{\partial}{\partial\theta}\frac{\partial}{\partial v_S}w_S(v_S) \leq \frac{\partial^2}{\partial v_S \partial h_S}\pi_S\left(v_S, 1 - \frac{S_i(v_i)}{R_i}\right)\bigg)\frac{\partial}{\partial\theta}\frac{C(\psi(v_S), v_S)}{R_S}.$$

The regularity condition implies that $((\partial/\partial\theta)C(v_M(s), v_S(s)))/((\partial/\partial\theta)C(v_M(t), v_S(t))) \leq 1$ for all $s < t \approx 0$ from which equation (20) follows. The result for log holds because

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}\theta}(\ln W(t) - \ln W(0))\\ &= \frac{1}{W(t)W(0)}\left(W(0)\frac{\mathrm{d}}{\mathrm{d}\theta}(W(t) - W(0)) - (W(t) - W(0))\frac{\mathrm{d}}{\mathrm{d}\theta}W(0)\right).\end{aligned} \tag{24}$$

ii. From i, $(\partial/\partial\theta)(1 - ((S_i(v_i))/R_i))) \geq 0$, and—by regularity—this inequality holds strictly for some v_S. Therefore, $(\mathrm{d}/\mathrm{d}\theta)(W(1) - W(0)) > 0$ follows by inspection of equation (8) and $(\mathrm{d}/\mathrm{d}\theta)(W(1) - W(0)) > 1$ from equation (24).

2. Proof of Proposition 2

i. Define $\bar{v}_S \equiv \sup\{v_S \in [0,1] : \psi(v_S) < 1\}$, $\bar{v}_M \equiv \psi(\bar{v}_S)$ and the sets $\Xi^0 \equiv \{v_S \in [v_S^c, \bar{v}_S] : (\partial/\partial\theta)G_S(v_S) \geq 0\}$ and $\Xi^1 \equiv \{v_S \in [v_S^c, \bar{v}_S] : (\partial/\partial\theta)\psi(v_S) > 0$ and $(\partial/\partial\theta)G_S(v_S) > 0\}$. Consider any $v_1 \in \Xi^1$, and define $\Xi^3 = \{v_S \in [v_1, \bar{v}_S] : v_S \notin \Xi^1\}$. Suppose that Ξ^3 is nonempty—then $\min \Xi^3$ exists by continuity of ψ and G_S; clearly, $\min \Xi^3 > v_1$ and $[v_1, \min \Xi^3] \subset \Xi^0$. Define $\kappa(v_S) \equiv (\partial/\partial\theta)\psi(v_S)(\partial/\partial v_M)w_M(\psi(v_S))$, and note that differentiating equation (11) yields that

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}v_S}\kappa(v_S) &= \frac{\partial^2}{\partial\theta\partial v_S}\pi_S(v_S, G_S(v_S)) + \frac{\partial}{\partial\theta}G_S(v_S)\frac{\partial^2}{\partial v_S\partial h_S}\pi_S(v_S, G_S(v_S))\\ &\quad - \frac{\partial}{\partial v_S}\psi(v_S)\frac{\partial}{\partial\theta}G_M(\psi(v_S))\frac{\partial^2}{v_M\partial h_M}\pi_M(\psi(v_S), G_M(\psi(v_S))) \geq 0\end{aligned} \tag{25}$$

for any $v_S \in \Xi^0$, because $(\partial/\partial\theta)G_M(\psi(v_S)) = -(R_S/R_M)(\partial/\partial\theta)G_S(v_S)$ by equation (14). As $\kappa(v_1) > 0$ by the definition of v_1, it follows that $(\partial/\partial\theta)\psi(v_S) > 0$ for all $v_S \in [v_1, \min \Xi^3]$. However, then $(\partial/\partial\theta)G_S(\min \Xi_3) = (\partial/\partial\theta)G_S(v_1) + \int_{\min v_1}^{\min \Xi_3}(\partial/\partial\theta)\psi(r)(\partial^2/(\partial v_M\partial v_S))C(\psi(r), r)\,\mathrm{d}r > 0$ and $\min \Xi_3 \in \Xi_1$, which is a contradiction. Thus, if $v_1 \in \Xi^1$, then all $[v_1, \bar{v}_s) \subset \Xi^1$, and hence

$$\frac{\partial}{\partial\theta}G_S(1) = \frac{\partial}{\partial\theta}G_S(v_1) + \int_{v_1}^{\bar{v}_S}\frac{\partial}{\partial\theta}\psi(r)\frac{\partial^2}{\partial v_M\partial v_S}C(\psi(r), r)\,\mathrm{d}r > 0, \tag{26}$$

which contradicts $(\partial/\partial\theta)G_S(1) = 0$. Thus, Ξ^1 must be empty. Suppose that $(\partial/\partial\theta)v_S^c < 0$. This implies (*a*) that $(\partial/\partial\theta)\psi(v_S^c) > 0$ (by differentiating $C(\psi(v_S^c), v_S^c) = 0$ with respect to θ) and (*b*) that $(\partial/\partial\theta)G_S(v_S^c) > 0$. Hence, $v_S^c \in \Xi^1$, which is a contradiction. Thus, $(\partial/\partial\theta)v_S^c \geq 0$, and $(\partial/\partial\theta)G_S(v_S^c) \leq 0$. Now suppose that the set $\Upsilon^1 = \{v_S \in [v_S^c, \bar{v}_S] : (\partial/\partial\theta) G_S(v_S) > 0\}$ is nonempty. Take some $v_2 \in \Upsilon^1$ and define $\Upsilon^2 = \{v_S \in [v_S^c, v_2] : (\partial/\partial\theta) G_S(v_S^c) \leq 0\}$. By continuity of $(\partial/\partial\theta) G_S(v_S)$, the point $v_3 = \max \Upsilon^2$ exists and is less than v_2. Therefore, for any $v_S \in (v_3, v_2]$ we have $(\partial/\partial\theta) G_S(v_S) > 0$. However, as

$$\frac{\partial}{\partial\theta} G_S(v_2) = \frac{\partial}{\partial\theta} G_S(v_3) + \int_{v_3}^{v_2} \frac{\partial}{\partial\theta} \psi(r) \frac{\partial^2}{\partial v_M \partial v_S} C(\psi(r), r) \mathrm{d}r,$$

this implies that there exists some $v_4 \in (v_3, v_2]$ such that $(\partial/\partial\theta)\psi(v_4) > 0$ and thus $v_4 \in \Xi^1$, which is a contradiction. Thus, $(\partial/\partial\theta) G_S(v_S) \leq 0$ for all $v_S \in [v_S^c, \bar{v}_S)$. Finally, as $G_S(v_S) = 1 - (1 - v_S)/R_S$ for $v_S > \bar{v}_S$, it follows that $(\partial/\partial\theta) G_S(v_S) \geq 0$ for $v_S > \bar{v}_S$, and thus by continuity, $(\partial/\partial\theta) G_S(v_S) \geq 0$ for all v_S, as required.

ii. Differentiating equation (14) with respect to θ yields $(\partial/\partial\theta) G_M(v_M) = -(\partial/\partial\theta)(R_S/R_M) G_S(\phi(v_M)) > 0$.

iii. Result iii follows immediately from ii by inspection of equation (8).

iv. $(\partial/\partial\theta) v_M^c \leq 0$ by ii and iv follows by inspection of equation (8).

v. Suppose that $(\partial/\partial\theta)\bar{v}_S > 0$. This implies that $\bar{v}_S < 1$ and hence $\psi(\bar{v}_S) = 1$. Differentiating this equation with respect to θ yields $(\partial/\partial\theta)\psi(\bar{v}_S) < 0$. By continuity, there then exists $\delta > 0$ such that $(\partial/\partial\theta)\psi(v_S) < 0$ for all $v_S \in (\bar{v}_S - \delta, \bar{v}_S)$. But this implies that $(\partial/\partial\theta) G_S(1) < 0$ by i and equation (26), which is a contradiction. It follows that $(\partial/\partial\theta)\bar{v}_S \leq 0$ and thus $(\partial/\partial\theta)\bar{v}_M \geq 0$. Differentiating equation (11) with respect to θ yields

$$\frac{\partial}{\partial\theta}(w_S(\bar{v}_S) - w_M(\bar{v}_M)) = \frac{\partial}{\partial\theta}\bar{v}_M \frac{\partial}{\partial v_M} w_M(\bar{v}_M) - \frac{\partial}{\partial\theta}\bar{v}_S \frac{\partial}{\partial v_S} w_S(\bar{v}_S) \geq 0.$$

Note that $(\partial/\partial\theta) G_i(v_i) = 0$ for $v_i > \bar{v}_i$. Hence, $(\partial/\partial\theta)(\partial/\partial v_i) w_i(\bar{v}_i) = (\partial/\partial\theta)(\partial/\partial v_i)\pi_i(v_i, G_i(v_i))$ for $v_i > \bar{v}_i$ and $(\partial/\partial\theta) w_S(1) \geq w_S(\bar{v}_S)$ while $(\partial/\partial\theta) w_M(1) = w_M(\bar{v}_M)$. As $W_i(0) = w_M(v_M^c) = w_S(v_S^c)$, it follows that $(\partial/\partial\theta)(W_S(1) - W_S(0)) \geq (\partial/\partial\theta)(W_M(1) - W_M(0))$.

vi. As I explained in the proof of proposition 1, $\min\{v_M^c, v_S^c\} = 0$ under assumption 5. From i, we have $(\partial/\partial\theta) v_S^c \geq 0$, which together with $v_S^c(\theta_1) > 0$ implies $v_M^c(\theta_2) = v_M^c(\theta_1) = 0$. First, this implies that

$$p_S(0) = \frac{(\partial/\partial v_S) C(v_M^c, v_S^c))}{(\partial/\partial v_S) w_S(v_S^c)))} W'(0) = 0.$$

Second, $(\mathrm{d}/\mathrm{d}\theta)(\partial/\partial v_M)\pi_M(0, G_M(0)) = 0$. Third, it implies that $p_M(0) = 1$ and thus $(\partial/\partial t) p_M(0) < 0$. A strict increase in differentiation implies that $(\partial/\partial\theta) v_S^c > 0$—suppose not. Then $(\partial/\partial\theta) G_i(v_i^c) = 0$ for both i, and thus equation (25) implies that $(\partial/\partial\theta)(\partial/\partial v_S)\psi(v_S) > 0$ and thus by continuity there exists some δ such that $(\partial/\partial\theta)\psi(v_S) > 0$ for all $v_S \in [v_S^c, v_S^c + \delta]$. It follows, therefore, that $(\partial/\partial\theta) G_S(v_S) > 0$ for all $v_S \in [v_S^c, v_S^c + \delta]$, which contradicts result i. By differentiating equation (11), we get $(\partial/\partial\theta)(w_M(v_M^c) - w_S(v_S^c)) = (\partial/\partial\theta) v_S^c(\partial/\partial v_S) w_S(v_S^c) - (\partial/\partial\theta) v_M^c(\partial/\partial v_M) w_M(v_M^c) > 0$. Thus, it follows by inspection of equation (18) that $(\partial/\partial\theta) W'(0) < 0$, which implies the result.

vii. As $\pi_S(v_S^C(\theta_1), 0; \theta_2) \geq \pi_S(v_S^C(\theta_1), 0; \theta_1) \geq \pi_M(v_M^C(\theta_1), 0)$ and $v_M^c(\theta_2) \leq v_M^c(\theta_1)$, $W_i(0; \theta_i) = \pi_M(v_M^C(\theta_i), 0)$. Thus, $W_i(0; \theta_2) \leq W_i(0; \theta_1)$, and iv and v follow from equation (24). However, if $v_S^c > 0$, then $v_M^c(\theta_2) = v_M^c(\theta_1) = 0$ and $W_i(0; \theta_2) = W_i(0; \theta_1)$. Thus, vi follows from equation (24) as well.

References

Acemoglu, Daron, and David Autor. 2011. “Skills, Tasks and Technologies: Implications for Employment and Earnings.” In *Handbook of Labor Economics*, vol. 4, edited by David Card and Orley Ashenfelter, 1043–171. Amsterdam: North-Holland.

Akerman, Anders, Elhanan Helpman, Oleg Itskhoki, Marc-Andreas Muendler, and Stephen Redding. 2013. “Sources of Wage Inequality.” *A.E.R.* 103 (3): 214–19.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. 2006. “The Polarization of the U.S. Labor Market.” *A.E.R.* 96 (2): 189–94.

Autor, David H., Frank Levy, and Richard J. Murnane. 2003. “The Skill Content of Recent Technological Change: An Empirical Exploration.” *Q.J.E.* 118 (4): 1279–333.

Becker, Gary S. 1973. “A Theory of Marriage: Part I.” *J.P.E.* 81 (4): 813–46.

Böhm, Michael J. 2020. “The Price of Polarization: Estimating Task Prices under Routine-Biased Technical Change.” *Quantitative Econ.* 11 (2): 761–99.

Böhm, Michael J., Daniel Metzger, and Per J. Strömberg. 2018. “‘Since You’re So Rich, You Must Be Really Smart’: Talent and the Finance Wage Premium.” CEPR Discussion Paper no. 12711 (February), Centre Econ. Policy Res., London.

Bound, John, and George Johnson. 1992. “Changes in the Structure of Wages in the 1980’s: An Evaluation of Alternative Explanations.” *A.E.R.* 82 (3): 371–92.

Burzyński, Michał, and Paweł Gola. 2020. “Mexican Migration to the United States: Selection, Assignment, and Welfare.” LISER Working Paper no. 2019-10 (July), Luxembourg Inst. Socio-Econ. Res.

Chiappori, Pierre-André, Robert J. McCann, and Lars P. Nesheim. 2010. “Hedonic Price Equilibria, Stable Matching, and Optimal Transport: Equivalence, Topology, and Uniqueness.” *Econ. Theory* 42 (2): 317–54.

Chiappori, Pierre-André, Sonia Oreffice, and Climent Quintana-Domeque. 2012. “Fatter Attraction: Anthropometric and Socioeconomic Matching on the Marriage Market.” *J.P.E.* 120 (4): 659–95.

Costinot, Arnaud, and Jonathan Vogel. 2010. “Matching and Inequality in the World Economy.” *J.P.E.* 118 (4): 747–86.

Costrell, Robert M., and Glenn C. Loury. 2004. “Distribution of Ability and Earnings in a Hierarchical Job Assignment Model.” *J.P.E.* 112 (6): 1322–63.

Deming, David J. 2017. “The Growing Importance of Social Skills in the Labor Market.” *Q.J.E.*132 (4): 1593–640.

Dupuy, Arnaud. 2015. “The Assignment of Workers to Tasks with Endogenous Supply of Skills.” *Economica* 82 (325): 24–45.

Edin, Per-Anders, Peter Fredriksson, Martin Nybom, and Björn Öckert. 2017. “The Rising Return to Non-cognitive Skill.” IZA Discussion Paper no. 10914 (July), Inst. Labor Econ., Bonn.

Gabaix, Xavier, and Augustin Landier. 2008. “Why Has CEO Pay Increased So Much?” *Q.J.E.* 123 (1): 49–100.

Galichon, Alfred. 2016. *Optimal Transport Methods in Economics*. Princeton, NJ: Princeton Univ. Press.

Goos, Maarten, and Alan Manning. 2007. “Lousy and Lovely Jobs: The Rising Polarization of Work in Britain.” *Rev. Econ. and Statis.* 89 (1): 118–33.

Gould, Eric D. 2002. “Rising Wage Inequality, Comparative Advantage, and the Growing Importance of General Skills in the United States.” *J. Labor Econ.* 20 (1): 105–47.

Gretsky, Neil E., Joseph M. Ostroy, and William R. Zame. 1992. “The Nonatomic Assignment Model.” *Econ. Theory* 2 (1): 103–27.

Grossman, Gene M., Elhanan Helpman, and Philipp Kircher. 2017. “Matching, Sorting, and the Distributional Effects of International Trade.” *J.P.E.* 125 (1): 224–64.

Hagedorn, Marcus, Tzuo Hann Law, and Iourii Manovskii. 2017. "Identifying Equilibrium Models of Labor Market Sorting." *Econometrica* 85 (1): 29–65.

Heckman, James J., and Guilherme Sedlacek. 1985. "Heterogeneity, Aggregation, and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market." *J.P.E.* 93 (6): 1077–125.

Heckman, James J., and Bo E. Honoré. 1990. "The Empirical Content of the Roy Model." *Econometrica* 58 (5): 1121–49.

Helpman, Elhanan, Oleg Itskhoki, Marc-Andreas Muendler, and Stephen J. Redding. 2016. "Trade and Inequality: From Theory to Estimation." *Rev. Econ. Studies* 84 (1): 357–405.

Hopenhayn, Hugo A. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica* 60 (5): 1127–50.

Joe, Harry. 1997. *Multivariate Models and Dependence Concepts*. Monographs on Statistics and Applied Probability 73. London: Chapman & Hall.

Juhn, Chinhui, Kevin M. Murphy, and Brooks Pierce. 1993. "Wage Inequality and the Rise in Returns to Skill." *J.P.E.* 101 (3): 410–42.

Kaplan, Steven N., and Joshua Rauh. 2013. "It's the Market: The Broad-Based Rise in the Return to Top Talent." *J. Econ. Perspectives* 27 (3): 35–56.

Katz, Lawrence F, and Kevin M. Murphy. 1992. "Changes in Relative Wages, 1963–1987: Supply and Demand Factors." *Q.J.E.* 107 (1): 35–78.

Legros, Patrick, and Andrew F. Newman. 2002. "Monotone Matching in Perfect and Imperfect Worlds." *Rev. Econ. Studies* 69 (4): 925–42.

Lindenlaub, Ilse. 2017. "Sorting Multidimensional Types: Theory and Application." *Rev. Econ. Studies* 84 (2): 718–89.

Machado, José A. F., and José Mata. 2005. "Counterfactual Decomposition of Changes in Wage Distributions Using Quantile Regression." *J. Appl. Econometrics* 20 (4): 445–65.

Mailath, George J., Andrew Postlewaite, and Larry Samuelson. 2013. "Pricing and Investments in Matching Markets." *Theoretical Econ.* 8 (2): 535–90.

———. 2017. "Premuneration Values and Investments in Matching Markets." *Econ. J.* 127 (604): 2041–65.

Mak, Eric, and Aloysius Siow. 2017. "Occupational Choice and Matching in the Labor Market." IZA Discussion Paper no. 10584 (February), Inst. Labor Econ., Bonn.

McCann, Robert J., Xianwen Shi, Aloysius Siow, and Ronald Wolthoff. 2015. "Becker Meets Ricardo: Multisector Matching with Communication and Cognitive Skills." *J. Law Econ. and Org.* 31 (4): 690–720.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71 (6): 1695–725.

Milgrom, Paul, and Ilya Segal. 2002. "Envelope Theorems for Arbitrary Choice Sets." *Econometrica* 70 (2): 583–601.

Mouw, Ted, and Arne L. Kalleberg. 2010. "Occupations and the Structure of Wage Inequality in the United States, 1980s to 2000s." *American Soc. Rev.* 75 (3): 402–31.

Nelsen, Roger B. 1998. "Concordance and Gini's Measure of Association." *J. Nonparametric Statis.* 9 (3): 227–38.

———. 1999. *An Introduction to Copulas*. New York: Springer.

Owen, D. B., and G. P. Steck. 1962. "Moments of Order Statistics from the Equicorrelated Multivariate Normal Distribution." *Annals Math. Statis.* 33 (4): 1286–91.

Roy, A. D. 1951. "Some Thoughts on the Distribution of Earnings." *Oxford Econ. Papers* 3 (2): 135–46.

Sattinger, Michael. 1979. “Differential Rents and the Distribution of Earnings.” *Oxford Econ. Papers* 31 (1): 60–71.

Scarsini, Marco. 1984. “On Measures of Concordance.” *Stochastica* 8 (3): 201–18.

Sklar, Abe. 1959. “Fonctions de Répartition à *n* Dimensions et Leurs Marges.” *Publications Inst. Statis. Univ. Paris* 8:229–31.

Tervio, Marko. 2008. “The Difference That CEOs Make: An Assignment Model Approach.” *A.E.R.* 98 (3): 642–68.

Teulings, Coen N. 1995. “The Wage Distribution in a Model of the Assignment of Skills to Jobs.” *J.P.E.* 103 (2): 280–315.

———. 2005. “Comparative Advantage, Relative Wages, and the Accumulation of Human Capital.” *J.P.E.* 113 (2): 425–61.

Tinbergen, Jan. 1956. “On the Theory of Income Distribution.” *Weltwirtschaftliches Archiv* 77:155–75.