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The low volatility effect in the Norwegian stock market

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Abstract

In this paper we seek to investigate whether the low volatility puzzle is present in Norwegian stock market during the period January 1980 - December 2019. By examining the relationship of total volatility and returns and idiosyncratic volatility and returns, we conclude that low volatility securities do not outperform high volatility securities. Contrary, we find that investors generally are being compensated for holding more volatile (risker) securities in line with traditional finance theory. We arrive at this conclusion by sorting available securities into portfolios based on either total volatility or idiosyncratic volatility using a rolling window approach. The return performances of these portfolios are then analyzed. We explore different variations of rolling windows and holding periods, as well as different filtration techniques. Furthermore, the portfolios' performances are analyzed and controlled for different factors such as High-minus-Low (HML), Small-minus-Big (SMB), Up-minus-Down (UMD) and Liquidity (LQD). Lastly, we explore the industry exposure in the different portfolios and the effect this has on the respective portfolios.

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1. Introduction

This paper seeks to obtain and gather information to answer the research question "*Is there a low volatility effect in the Norwegian stock market? Can controlling for value, momentum and mean reversal explain this anomaly, and does the anomaly persist after transaction cost is accounted for?"*. To do so, we have conducted a literature review of recent findings within the field of study, then conducted a methodical investigation and lastly presented the relevant findings in this paper.

The persistence of the low-volatility anomaly is one of the biggest mysteries in modern finance. In the capital asset pricing model (CAPM) theory an investor is expected to earn returns based on the level of market risk (beta) the investor bears. The beta is an expression of the asset's level of covariance with the market. However, this expectation is not in line with what historical data suggests (Black, 1972). Leading studies have shown that there exists a low volatility anomaly where less risky stocks outperform riskier stocks (Haugan & Heins, 1975; Ang et al. 2009; Baker et al. 2001; Frazzini & Pedersen, 2014), contrary to the theory. Therefore, sorting stocks into portfolios based on their market risk/beta would lead to the portfolios with a lower beta outperforming the portfolios with higher beta. Such trading strategies would therefore generate alpha and consequently be seen as an anomaly in the CAPM.

In this paper, we seek to test whether there is evidence that stocks with low volatility outperform stocks with a high volatility in the Norwegian stock market, as most previous studies have focused on American and international markets. Previous studies on the Norwegian stock market have yielded mixed results. Hafskjær & Østnes (2013) concluded that there were no "Low Volatility Puzzle" in the Norwegian stock market, contrary to Høgenhaug & Nydal (2018) which found opposite results pointing to the existence of the anomaly. Furthermore, a recent study of Koskela (2019) shows there is overall strong evidence of low beta stock outperformance among Nordic countries, after controlling for size and quality factor. By examining possible factors and filtering the return with the value factor, this paper seeks evidence of the low volatility as an independent factor.

The primary dataset is gathered from Oslo Børs Information (OBI) from January 1980 until December 2019 and consists of daily prices, as well as other financial information. The data is then processed by applying certain filters. The return data is winsorized to reduce the impact of extreme outliers. Then, we eliminate the companies with the five percent lowest market capitalization, in order to prevent a biased estimate of the intrinsic volatility. Both the idiosyncratic volatility and the total volatility is then used to construct the portfolios from securities using a rolling window approach. Lastly, the performances of these portfolios are then compared to study whether there is a significant difference between the low-volatility- and high-volatility- portfolios and whether the low-volatility puzzle exists in the Norwegian stock market.

The report is structured as follows: Literature Review, a quick review of relevant, recent literature on the subject. Methodology, description and justification of the appropriate methods and choices made in the thesis. Data, insight into the data utilized in the paper. Results, containing the findings and discoveries made in the thesis. Lastly, conclusion, where a final conclusion is presented.

2. Literature Review

The literature review is a crucial part in order to be able to answer the research question. Reviewing recent studies on the subject provides insight into methodologies, data collection and their possible pitfalls. The literature review also shines light upon which areas might require more research and helps to pose the interesting questions.

In the paper "Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly", Baker et al. (2011) argues that the low-volatility anomaly is in part a result of institutional investors being discouraged from arbitrage activity due to their mandate to beat their fixed benchmark. Further, they argue that there is a behavioral preference for high volatility stocks, which causes the low volatility stocks to outperform. This preference is derived from investors' preference for lotteries ("Loss aversion"), representativeness and overconfidence. Preference for lotteries indicates that investors are more likely to seek positive skewness and therefore control volatility. Representativeness explains that there is no clear way to distinguish a winner from all the losers and that investors are ignoring the fact and therefore overpaying for volatile stocks. Lastly, overconfidence is a pervasive bias where analysts see past each other's opinions and therefore cause high volatility stocks to remain volatile.

"Betting Against Beta" by Franzzini and Pedersen (2014) explores the betting-against-beta (BAB) factor, a factor composed of long leveraged low beta assets and short high beta assets. Their findings show that this factor generates noteworthy positive risk adjusted returns. A reason for this is that constrained investors push up higher beta assets and decrease these assets' alpha. Frazzini and Pedersen find that this holds not only for the American equity market, but also for 20 other international equity markets, American treasury bonds, corporate bonds, and the future market. Furthermore, the returns of the BAB factor increases when investors experience funding constraints.

Investors with larger funding constraints tend to hold higher beta assets to compensate for this.

Baker et al. (2014) examines the low-risk anomaly by decomposing it into micro and macro effects. The micro effects result from the choice of low beta stocks, while the macro effects are results of the selection of low beta industries and/or countries. Their findings indicate that both the micro and macro effects contribute to the anomaly. Choice of stock impacts the alpha though a combination of noteworthy risk reduction and small return improvements. Whereas country selection impacts alpha, oppositely, through a combination of small risk reduction and noteworthy return improvements. The effect from industry selection is not statistically distinguishable from zero.

"Low-Volatility Cycles: The Influence of Valuation and Momentum on Low-Volatility Portfolios" by Garcia-Feijóo et al. (2015) explores the time-varying aspect of the performance of low-risk investing. Their findings indicate that this is indeed the case, and that momentum, value and size dynamically explains the low-risk strategy over time, which in turn appears to be influenced by the general economy. Furthermore, the performance of this strategy depends upon current valuation. Put in another way, the low-risk strategy is more likely to outperform a high-risk strategy whenever the current conditions are favorable. We also point out that during the last 85 years of financial history, there have been multiple occasions where a high-risk strategy would cumulatively outperform a low-risk strategy.

Li et al. (2016) seeks to offer important insight into the low-risk effect. In "The Low-Volatility Anomaly: Market Evidence on Systematic Risk vs. Mispricing" they argue that the excess returns experienced in low-volatility stocks are driven by systematic mispricing attributed to volatility as a stock feature. The reason being that the returns of the low-volatility portfolios are set on common variations connected to the idiosyncratic volatility and not factor loadings. As a result of that, investors seem to prefer stocks with higher volatility to stocks with lower volatility. They therefore provide supplementary support to other studies which surmise that the low-volatility anomaly originates from behavioral biases, such as the previously mentioned paper “Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly” by Baker et al. (2014).

In seeking explanation for the low volatility anomaly, Stambaugh et al. (2015)'s finding shows the negative relation of idiosyncratic volatility and stock return is stronger within firms that have low institutional ownership (IO). Short selling stocks with low IO tend to be more expensive due to the smaller size of stock loan supply (Stambaugh, 2015) and higher lending fee (D'Avolio, 2002). Limits in arbitrage might add up to the mispricing of these stocks.

Beijer (2015) found a significant relationship between low volatility and strong operating performance, in the way that operating performance partially explains the low volatility effect. The logic behind this relationship is that a firm with stable, predictable operating performance is more capable of obtaining capital, which is used to finance profitable projects for the firm. When the high returns from the projects are realized or when the pay-off becomes more certain as investment risk decreases, the market will bid up the stock price.

In “Implied volatility index for the Norwegian equity market”, Bugge et al. (2016) first introduced and then evaluated an implied volatility index for the Norwegian stock index OBX, called NOVIX. The NOVIX is constructed using the same methodology as the American Cboe Volatility Index (VIX), using options on the OBX Total Return Index. Furthermore, when evaluating the NOVIX, comparisons show that the NOXIV exhibits similar properties to that of VIX and the german VDAX-NEW. Nonetheless, the

NOVIX excerpts less of an improvement in volatility forecasting than its American and German equivalents. This is an indication that the forward-looking predictability is largely dependent on larger and more liquid options markets.

The recent paper “Low-Risk Anomalies?” by Schneider et al. (2020) shows that the low-volatility anomaly arises when investors demand compensation for coskewness risk and that the returns of low volatility anomalies can be explained by stock returns’ skewness. Followingly, Schneider et al. also conclude that betting-against-beta (BAB) and betting-against-volatility (BAV)’s alphas are associated with negative residual coskewness. Furthermore, after the BAB and BAV factors have been controlled for shrewdness, both prove insignificant, and the low-volatility anomalies vanish.

In “Do the Rich Gamble in the Stock Market? Low Risk Anomalies and Wealthy Households”, Bali et al. (2020) provide direct evidence that high-volatility stocks are subject to overpricing due to individual retail investors’ demand. As a result of this, this leads to short term overpricing and abnormal negative future returns. Further, they utilize a large individual level dataset from Sweden to understand how individual investors contribute to the anomaly. Their findings show that the anomaly is confined to stocks held by rich households and further test suggests that this is an effect of differences in skewness preference.

Joshipura & Joshipura (2020) conducted an analysis of the top 500 largest stocks in the Indian stock market, seeking to answer several questions about the anomaly. In their paper “Low-risk effect: evidence, explanations and approaches to enhancing the performance of low-risk investment strategies”, they find strong evidence that the low-volatility anomaly exists with a flat/negative risk-return relation formed from the average returns. Followingly, they conclude that the low-volatility effect is independent of the momentum, value and size effects. Moreover, they find that the low-volatility effect is a combination of stock- and sector level effects. The effect cannot be entirely captured by specific sector exposure. Furthermore, their paper explores how the low-volatility trading strategy could be improved upon. It turns out that by incorporating the momentum effect, the performance can be improved in terms of risk-adjusted returns as well as absolute.

3. Methodology

This chapter contains description and justification for the appropriate methods and choices made in this paper.

3.1 Idiosyncratic Volatility

This paper looks at two measures of volatility: Idiosyncratic volatility and total volatility. In accordance with Ang et al. (2006), idiosyncratic volatility (IVOL) is the variance of the residual $\varepsilon_{i,t}$ from the regression of stocks excess returns on Fama French 3 factors in formula one.

$$r_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{i,t} * (r_{m,t} - r_{i,t}) + s_{i,t} * SMB_t + h_{i,t} * HML_t + \varepsilon_{i,t} \tag{1}$$

Where $r_{i,t} - r_{f,t}$ is the excess return of a stock at time t; SMB_t is referred to as the size effect, which is computed as the return of a portfolio of small cap companies in excess of that of big cap companies; HML_t is the value factor, equal to the returns of a high book-to-market portfolio of stocks minus the returns of a low book-to-market portfolio of stocks. $\beta_{i,t}$ is the standard market return factor.

3.2 Total Volatility

Total volatility represents both stocks' idiosyncratic volatility and the systematic volatility, which is computed as the standard deviation of stocks' return from its average return in each time period. The formula is shown in formula two.

$$TVOL = \sqrt{\sum_{i=1}^{N-1} \frac{R_i - \underline{R}}{n-1}} \tag{2}$$

3.4 Filtration and Data Manipulation

Before constructing the portfolios that are used to test whether the low volatility puzzle exists, we first apply a set of filters to reduce the impact outliers cause the analyses.

3.4.1 Delisted Stocks

To implement a realistic trading strategy, we exclude the securities that are delisted during the holding period. This means that any security that is either delisted from Oslo Stock Exchange by going private, merger or acquisitions would be excluded from the sample as of the start of that year. By going private, we do no longer have access to the daily fluctuations in returns and are therefore exempt. Furthermore, as mergers and acquisitions happen, the characteristics of the companies change greatly and no longer reflect the old attributes. In order to preserve continuity, these securities are omitted. If a company is bought by another traded company, the returns data for the purchaser is kept.

3.4.2 Small and Illiquid Stocks

We recognize that many of the smaller companies traded at Oslo Stock Exchange are illiquid and therefore suffer from large bid-ask spreads. In line with previous studies, an elimination of the lowest market capitalization stocks from the sample in order to prevent a biased estimate of the intrinsic volatility. However, due to the size of the Norwegian stock market, eliminating too many small capitalization stocks might cause the stock sample to become impractical due to portfolio creation based on separate volatility levels. Both Baker et al. (2011) and Ang et al. (2009) implement a cut-off where the five percent lowest market capitalization securities are removed from the sample. Furthermore, to counteract low liquidity securities from affecting the analysis, we implement a filtration where securities with less than 125 trading days per year (or

10 trading days per month during certain analysis) are omitted from the sample. This filtration functions on a rolling window basis and only comes into effect if the security is illiquid during this period.

3.4.3 Data Outliers and Winsorization of Return Data

To counteract and reduce the effect of spurious outliers in the daily returns, we apply two techniques. Firstly, we apply a cut-off point to the returns. Meaning that any daily return above or below this amount will be reduced to that of this amount. In line with previous studies, this return threshold has been set to + 100 percent and - 50 percent (Hafskjær & Østnes, 2013). I.e., a security with a daily return exceeding + 100 percent, will be reduced to + 100 percent for that single day.

Further, we apply winsorization to the daily returns. The daily returns have been consolidated to between the 1st percentile and the 99th percentile. This means that any value outside of this boundary is reduced/increased to 1st/99th percentile value, effectively giving the outliers less weight (Dixon, 1960). The winsorization is applied to each rolling window or holding period sample individually. An example of this is that the winsorization of a rolling window of 24 months would reduce the highest 1st percentile within the 24 months to the 1st percentile value within the 24 months, and not within the entirety of the returns' dataset.

3.4 Portfolio construction

To test whether the low volatility puzzle exists in the Norwegian stock market, the performance of low-volatility stocks needs to be statistically better than the high-volatility stocks. This is tested by sorting the stocks into portfolios based on their level of idiosyncratic volatility. To get a good understanding of the span and to what degree certain securities are outperforming, we have chosen to analyze the market by dividing it into three, four, five and ten portfolios. By doing this, we can capture if the anomaly exists in a broader or smaller fraction of the market. This can also shine some light upon whether the anomaly already is exploited. E.g., if the anomaly exists in the earlier periods, but ceases to exist later on.

To construct the portfolios, we utilize a rolling window approach in order to capture the relevant return behaviour. Previous studies, such as that of Baker & Haugen (2012) and Bali & Cakici (2006), have utilized a rolling window of 24-months. However, we also look at smaller and larger windows (12 and 36-month) in order to see if these approaches better capture the anomaly.

Based on the volatility, the securities are arranged from least to most volatile, using either the total volatility or the idiosyncratic volatility, respectively. The securities are then sorted into three, four, five or ten portfolios based on this arrangement. E.g., if the securities are sorted into ten different portfolios using a 24-month rolling window, portfolio one will contain the ten percent least volatile securities during the 24-month period.

3.5 Evaluating portfolios

To evaluate the performance of the portfolios, each of the portfolio's monthly and yearly return, standard deviation, and Sharpe ratio is calculated. The portfolios have been constructed using both equal weighted and value weighted approaches. We regress portfolios' excess returns on the Fama French (1992) three-factor model including market return, small minus big factor (SMB) and high minus low factor (HML). Additionally, to a liquidity factor (LQD) and a momentum factor (UMD) to assess the persistence of the low volatility factor in the portfolios after controlling for common return explaining factors.

While previous studies by Baker & Haugen (2012) and Bali & Cakici (2006) have utilized 12-months periods of which the securities have been held in the portfolio, this paper utilizes both 12- and 24 month holding periods. Looking at multiple lengths of holding periods gives us the opportunity to see whether the attributes associated with the low volatility puzzle are short- or long term.

4. Data

The data chapter provides insight into the data utilized in the paper. Furthermore, it also provides awareness to which challenges are associated with the data and its sources. Any changes to the data sample are explained in chapter 3.4 Filtration and Data Manipulation.

4.1 Returns and Market Capitalization

The primary dataset is obtained from the Oslo Stock Exchange, through Oslo Børs Information (OBI) and consists of the daily price and the total return of all securities listed on the exchange from 03.01.1980 - 30.12.2019, a time period of 40 years. The returns are adjusted for dividends, stock dividends, stock splits and other corporate events. In total, the dataset incorporates more than 6,500 individual, public securities, including stocks, warrants and exchange traded funds (ETFs). Due to the sheer size of the data sample, a complete securities list will not be provided. However, we refer to Table 6 and 7 for an overview of the number of securities each year and the return characteristics.

The market capitalization of the companies has been computed based on the daily prices of the securities multiplied with the number of outstanding shares. For the daily price of the securities, the closing price has been utilized. In the case a security has not been traded that day, which is a more prominent case earlier in the data sample, the previous closing price has been utilized.

4.2 Factor data

The value weighted Norwegian stock market return and the High-minus-low (HML), Small-minus-big (SMB), Up-minus-down (UMD), Liquidity (LQD), PR1YR factor returns are gathered from Bernt Arne Ødegaars's data library. Ødegaars's method of creating and obtaining the factors is similar to that of Fama French (1992).

The HML factor along with SMB are two of the Fama and French Three Factor Model factors. The HML factor represents the relative performance of returns between securities with high book to market ratios and low book to market ratios. A positive coefficient would indicate exposure to value securities. Further, the SMB factor represents the performance relative to the size of the securities. A positive SMB coefficient would indicate larger exposure towards smaller companies.

LQD factor captures the ease of a trading execution, given the rationale that illiquid securities should require a premium. Portfolios with a high LQD coefficient are exposed to more liquid securities. Furthermore, the UMD factor, also known as the momentum factor, captures the performance of trending price action. It is calculated by taking the average return of the top 30 percent, minus the average return of the bottom 30 percent, ranked by return measured over the last 12 months. Additionally, we have included another factor capturing the momentum effect. The PR1Y factor is essentially constructed equally, however it uses the last 13 months of observations and omits the last month to account for a bid-ask bounce.

4.3 Industry data

To determine the effects of the portfolios' composition, we have utilized return data for separate industries. This data has been gathered from Bernt Arne Ødegaard’s data library and contains daily return data for each industry sector using an equal weighted approach. The industries included are Energy, Material, Industry, Consumer Discretionary, Consumer Staples, Health, Finance, IT, Telecommunications and Utilities. The data span the time period 31.01.1980 - 30.11.2020. However, only until 31.12.2019 have been utilized. An overview of the industry data can be viewed in Appendix 6.

4.4 Risk-free rate

Finding a suitable proxy for the risk-free rate in Norway is more challenging than in other countries due to the unusual position of the Norwegian government and its balance. In many other countries a suitable solution would be to utilize the country’s government’s treasury bonds, but since Norway has a positive balance and does not issue treasury bonds and bills in the same magnitude as other sovereign states, there might be irregularities affecting the characteristics of these. This makes the treasury bonds, in turn, impractical to utilize as the effects of low supply would have to be accounted for, among other things. Therefore, we utilize Bernt Arne Ødegaard’s estimate of the risk-free rate. The data has been collected from his homepage and have been used for the period from January 1980 to December 2019. These rates are in daily frequency and therefore consistent with the stock data.

5. Results

In this chapter, the findings and results of the analysis conducted in this paper are presented and discussed. First the results of the two different portfolio construction approaches are outlined, followed by common observations.

5.1 The Total Volatility Approach

Table 1 reports performance of portfolios sorted by the total volatility approach. The portfolios have been constructed with one month looking back (rolling window) and one month holding. Panel A shows returns of equally weighted portfolios and panel B presents value weighted portfolios returns. In both panels, the returns are monotonically increasing from portfolio 1 (lowest volatility) to portfolio 5 (highest volatility). However, the patterns seen on the other performance measurements are notably contrast between Table 1, Panel A and Panel B. Sharpe Ratio and alphas are monotonically decreasing from portfolio 1-5 for equally weighted portfolios.

The findings are consistent with the findings from Baker and Haugen (2012) and Hafskjær & Østnes (2013). Both studies find a higher Sharpe Ratio for low volatility portfolios, but Baker and Haugen (2012) reports positive differentials between the returns of the two extremity portfolios. On the other hand, in Hafskjær & Østnes (2013) study, only high volatility earns high returns. Our findings are more similar to Hafskjær & Østnes (2013) results. In our case, we see no significant difference between returns of two extremity portfolios. Portfolio 1 has an average return of 1.27 percent and Portofolio 5 has an average return of 1.61 percent. Likewise, in Panel B - Value-weighted Portfolios, high volatility performs significantly better than low volatility. We can see a clear, monotonically increasing average returns and all measures of alphas from portfolio 1-5. The average monthly return of the first portfolio is 1.62 percent while that of the fifth portfolio is significantly larger at 2.5 percent. Value weighted portfolios also have overall higher volatility than equally weighted.

An explanation for this might be the high concentration in Norwegian stock market that gives significant weights to some stocks, which then greatly influence the portfolio performance. Ødegaard (2020) also found that the value-weighted approach yielded better results when examining OSE. Thereby further validating our findings.

In a one month, one month strategy (1/1 strategy), the highest volatility portfolio has 2.86 percent CAPM alpha while the lowest volatility returns 0.305 percent alpha. Both alphas are statistically significant. To sum up, the initial 1/1-month strategy does not give significant evidence of low volatility anomaly in Norwegian Stock Market.

Panel A - Equal-weighted						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factor alpha**
1	1.27%	4.87%	0.15	-0.0017	-0.0007	-0.0015
				[-0.962]	[0.325]	[0.286]
2	1.63%	6.19%	0.18	-0.0010	0.0000	-0.0001
				[-1.995]	[-0.573]	[-0.123]
3	1.31%	6.69%	0.11	-0.0050	-0.0043	-0.0026
				[-4.423]	[-4.087]	[-4.097]
4	1.36%	7.92%	0.10	-0.0061	-0.0073	-0.0062
				[-3.960]	[-4.619]	[-4.972]
5	1.61%	9.16%	0.10	-0.0041	-0.0076	-0.0055
				[-2.529]	[-3.715]	[-4.328]
Panel B - Value-weighted						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factor alpha**
1	1.62%	5.77%	0.22	0.0031	0.0066	0.0058
				[1.958]	[4.725]	[4.173]
2	1.73%	6.55%	0.24	0.0047	0.0096	0.0097
				[2.731]	[6.447]	[6.659]
3	1.66%	7.47%	0.22	0.0054	0.0089	0.0103
				[2.878]	[4.971]	[5.859]
4	2.35%	9.97%	0.28	0.0124	0.0151	0.0170
				[4.125]	[5.157]	[5.831]
5	2.58%	11.27%	0.32	0.0286	0.0288	0.0246
				[5.208]	[5.831]	[6.435]

Table 1: Table shows 1/1-month strategy data using total volatility method for portfolio construction. Panel A presents data from equal-weighted portfolios and Panel B presents value-weighted portfolios data. Test statistics are shown in square brackets.

5.1.1 The Effects of Rolling Window Length

In this section, we explore the influence of varying rolling window lengths on the performance of portfolios. A multitude of different window lengths has been utilized. However, for simplicity, only the results of the three-month are being directly disguised in this part. Appendices 1 and 2 respectively report equal-weighted portfolios constructed on a six-month and 24-month looking back period (rolling window) and one-month holding period. During these tests, the filtration parameters were accordingly tweaked so that the required trading days were 30 days, 60 days, and 240 days respectively.

In table 2 Panel A, we can see portfolio 5, containing the highest volatility securities, having the highest returns. Similar results can also be seen in Appendix 1 and 2. Additionally, portfolio 5 also has the highest alphas, which is contradictory to the results in Panel A in the previous section. Furthermore, if we exclude portfolio 5's observations, we can see there is seemingly a trend of diminishing performance from portfolio 1 to 4 even though the differences are not significant. On the contrary, in Table 2 Panel B, we see a very consistent outperformance of volatile securities. All performance measurements are increasing from portfolio 1-5. This implies that it is more profitable investing in volatile securities and therefore the lack of the low volatility anomaly in the Norwegian stock market. Lastly, it is also worth noting that the Sharpe Ratio for portfolios containing more volatile securities is higher when utilizing a value-weighted portfolio construction method. This indicates that larger market capitalization securities categorized as high volatility likely outperform smaller capitalization securities in the same category.

Panel A - Equal-weighted 3/1-Strategy						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factors alpha**
1	1.29%	4.54%	0.16	0.0064	0.0044	0.0038
				[3.036]	[2.010]	[1.687]
2	1.58%	5.87%	0.18	0.0092	0.0071	0.0063
				[3.348]	[2.488]	[2.164]
3	1.22%	6.35%	0.10	0.0052	0.0021	0.0013
				[1.800]	[0.670]	[0.405]
4	1.22%	7.32%	0.09	0.0051	0.0015	0.0013
				[1.496]	[0.427]	[0.353]
5	2.25%	17.19%	0.10	0.0137	0.0092	0.0077
				[1.714]	[1.105]	[0.902]
Panel B - Value-weighted 3/1-Strategy						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factors alpha**
1	1.85%	5.59%	0.23	0.0123	0.0111	0.0106
				[3.036]	[2.010]	[1.687]
2	1.81%	6.58%	0.22	0.0167	0.0152	0.0146
				[3.348]	[2.488]	[2.164]
3	1.95%	7.14%	0.24	0.0167	0.0141	0.0132
				[1.800]	[0.670]	[0.405]
4	2.23%	9.61%	0.26	0.0292	0.0248	0.0246
				[1.496]	[0.427]	[0.353]
5	2.58%	19.39%	0.34	0.0337	0.0330	0.0311
				[1.714]	[1.105]	[0.902]

Table 2: Table shows 3/1-month strategy data using total volatility method for portfolio construction. Panel A presents data from equal-weighted portfolios and Panel B presents value-weighted portfolios data. Test statistics are shown in square brackets.

5.2 The Idiosyncratic Volatility Approach

In this section, we explore whether a different approach to risk measure may change the patterns seen in the previous approach. Table 3 reports performance of portfolios constructed on their idiosyncratic risk. Both equal-weighted and value-weighted are presented. To be consistent with the previous section and to better show similarities/differences, we continue to utilize the one month looking back and one month holding strategy.

Table 3 Panel B shows a similar pattern as the total volatility portfolios strategy above. We can see a drastic increase in average returns, Sharpe Ratio and alphas from portfolio 1 to portfolio 5. Portfolio 1 results in 0.83 percent 3FF alpha and portfolio 5 results in an impressive 3FF alpha of 2.579 percent, both being statistically significant. The witnessed patterns from Panel B are consistent with the findings of Hafskjær & Østnes (2013), for the same portfolio construction method.

On the other hand, we find a contrasting pattern in the performance of equally weighted portfolios constructed from idiosyncratic volatility, reported in Table 3 Panel A. Based on all performance criteria, the low volatility portfolios achieve better than the high volatility counterparts. The alphas of all five portfolios are negative and monotonically decreasing from portfolio 1 - 5. In contrast to the results in Panel B, we can see that there is not a stark difference between the two extremity portfolios. To test whether the returns are notably different, we conduct a simple t-test on the difference in mean of monthly returns of portfolio 1 and 5. The test returns a T-stat of 0.4750 and a P-value of 0.6350. Based on this, we do not reject the null hypothesis that there is no difference between the mean of two portfolio returns. In other words, we do not see a significant outperformance of low volatility in comparison with high volatility.

Panel A - Equal-weighted						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factors alpha**
1	1.39%	5.16%	0.17	-0.0006	0.0000	-0.0005
				[-0.632]	[0.040]	[-0.530]
2	1.48%	6.39%	0.15	-0.0022	-0.0019	-0.0014
				[-2.020]	[-1.754]	[-1.293]
3	1.17%	7.05%	0.09	-0.0061	-0.0060	-0.0050
				[-4.306]	[-4.183]	[-3.482]
4	1.05%	7.61%	0.07	-0.0077	-0.0089	-0.0075
				[-4.270]	[-4.973]	[-4.157]
5	1.24%	9.06%	0.08	-0.0066	-0.0085	-0.0070
				[-2.417]	[-3.145]	[-2.581]
Panel B - Value-weighted						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factors alpha**
1	2.19%	5.92%	0.28	0.0064	0.0083	0.0076
				[4.851]	[6.708]	[6.459]
2	2.56%	6.72%	0.30	0.0083	0.0098	0.0107
				[6.211]	[7.360]	[8.397]
3	2.70%	8.17%	0.29	0.0104	0.0120	0.0127
				[4.818]	[5.582]	[5.970]
4	3.31%	9.58%	0.36	0.0227	-0.0089	0.0248
				[6.475]	[6.510]	[6.886]
5	4.74%	12.20%	0.42	0.0227	0.0258	0.0318
				[6.475]	[7.492]	[8.002]

Table 3: Table shows 1/1-month strategy data using idiosyncratic volatility method for portfolio construction. Panel A presents data from equal-weighted portfolios and Panel B presents value-weighted portfolios data. Test statistics are shown in square brackets.

5.2.1 The Effects of Rolling Window Length

Data for the three months look back (rolling window), utilizing the idiosyncratic volatility risk metric as sorting method are presented in Table 4. Similar tables constructed using the same approach, but with six month and 24-month windows can be viewed in Appendix 3 and 4. However, only the three-month test has been highlighted here due to simplicity. From Table 4, we can see a clear pattern that the performance of portfolio 5 is the best performing portfolio. This is both in line with the previous section's result and the results from the tests/simulations using the total volatility approach. It is also noteworthy that portfolio 3 achieves the lowest average return and Sharpe Ratio, resulting in the 3/1-month strategy being the least definite result amongst the different window length options. The 6/1 and 24/1 strategies, seen in Appendix 3 and 4, both provided more monotonic increases as the volatility increased. However, neither of the different approaches to rolling window lengths resulted in any outperformance in the low volatile portfolio.

Panel A – Equal-weighted						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factors alpha**
1	1.23%	4.94%	0.17	0.00634	0.00515	0.00317
				[3.671]	[2.465]	[1.536]
2	1.50%	6.70%	0.16	0.00844	0.00768	0.00592
				[2.643]	[2.653]	[2.312]
3	1.03%	7.19%	0.09	0.00470	0.00653	0.00184
				[1.535]	[0.565]	[0.324]
4	1.12%	7.67%	0.09	0.00489	0.00345	0.00249
				[1.656]	[0.232]	[0.212]
5	1.56%	9.30%	0.12	0.01114	0.00862	0.00683
				[1.798]	[1.325]	[0.967]

Panel B – Value-weighted						
Portfolio	**Mean**	**STD**	**SR**	**CAPM alpha**	**3FF alpha**	**5 Factors alpha**
1	2.19%	5.93%	0.30	0.00647	0.00542	0.00456
				[3.325]	[3.352]	[1.352]
2	2.84%	7.23%	0.34	0.00755	0.00645	0.00535
				[3.763]	[4.723]	[1.534]
3	2.97%	9.16%	0.28	0.01684	0.00786	0.00622
				[4.524]	[4.439]	[0.432]
4	3.13%	8.81%	0.37	0.01692	0.01535	0.00838
				[5.397]	[5.823]	[0.075]
5	4.27%	15.46%	0.29	0.02687	0.02135	0.01438
				[5.736]	[6.462]	[1.749]

Table 4: Table shows 3/1-month strategy data using idiosyncratic volatility method for portfolio construction. Portfolios have been constructed using equal weight and value weight. Test statistics are shown in square brackets.

5.2.2 The Effects of Holding Period Length

Previous papers have mainly focused on shorter holding periods. These periods have usually been one month when analyzing high-low volatility portfolios. Hence, we are interested in how a longer holding period of one year will affect the results. Table 5 presents the results of portfolio construction using idiosyncratic volatility as risk metric. Table 5 Panel A shows the results using the equal-weighted approach and Panel B shows results from the value-weight approach. Both methods utilize a one year rolling window to determine portfolios and a one year holding period for the securities. Both the resulting data sets shown in Table 5 are in line with our previous observations.

Panel A - Equal-weighted				
Portfolio	**Mean**	**STD**	**3FF alpha**	**5 Factors alpha**
1	1.19%	25.83%	-0.0001	0.0000
2	1.20%	34.31%	-0.0003	-0.0002
3	1.22%	42.23%	-0.0004	-0.0002
4	1.22%	48.43%	-0.0004	-0.0004
5	1.28%	66.64%	0.0007	0.0006
Panel B - Value-weighted				
Portfolio	**Mean**	**STD**	**3FF alpha**	**5 Factors alpha**
1	1.24%	27.43%	0.0001	0.0002
2	1.33%	42.62%	0.0001	0.0002
3	1.38%	51.47%	0.0000	0.0002
4	1.56%	64.90%	0.0003	0.0004
5	1.87%	69.00%	0.0011	0.0010

Table 5: Table shows monthly data from one year rolling window, one year holding period strategy using idiosyncratic volatility method for portfolio construction. Data in Panel A have been constructed using equal weights and data in Panel B have been constructed using the value-weighted approach.

5.4 The Effects of Filtration

We find that the application of filters greatly reduced the noise in the results, making them easier to interpret and more consistent across the tweaking of parameters. Due to the nature of the small size of the Norwegian stock market, especially during the early periods of the data sample, a balance needed to be struck. Too much filtration would cause the results to become unreliable and extreme. Furthermore, when investigating whether the sequence of filter application had any significant effect, we arrived at the conclusion that this was not the case. Generally, the sequence of filter application had little effect on the results due to several of the filters overlapping each other. E.g., the smallest securities often being more illiquid than larger ones and therefore filtered out by both filters.

Increasing/decreasing the omitting of small capitalization securities would generally affect the results the least and did not change the results significantly. Conversely, restrictions on liquidity proved the most influential filtration technique. Altering the requirement of yearly (monthly) trading days to 180 (15), similarly to Chen et al. (2012)'s studies on the U.S. stock market, resulted in a severely reduced and unusable sample. A summary of security filtration utilized in most tests/simulations can be seen in Table 6.

Year	# Securities Before Filtration	# After MC Filtration	# After Liquidity Filtration	# After Discontinued Filtration
1982	97	92	46	33
1983	111	105	84	63
1984	127	121	91	81
1985	149	142	103	95
1986	168	160	113	98
1987	183	174	118	95
1988	197	187	116	89
1989	189	180	114	95
1990	198	188	111	87
1991	208	198	104	76
1992	203	193	111	90
1993	192	182	114	100
1994	196	186	115	105
1995	209	199	114	109
1996	225	214	126	114
1997	219	208	132	115
1998	279	265	147	122
1999	301	286	160	127
2000	314	298	166	144
2001	315	299	166	146
2002	276	262	174	152
2003	252	239	184	153
2004	237	225	168	153

2005	250	238	158	148
2006	260	247	153	139
2007	282	268	153	142
2008	326	310	163	151
2009	330	314	177	161
2010	288	274	217	205
2011	283	269	213	202
2012	270	257	199	186
2013	259	246	201	180
2014	259	246	192	177
2015	266	253	173	159
2016	253	240	171	166
2017	245	233	181	171
2018	242	230	182	173
2019	245	233	180	172

Table 6: Number of yearly securities before and after each filtration method. Column three (from left) refers to market capitalization filtration. Market capitalization filtration was implemented with omission of the five percent lowest market capitalization securities. Liquidity filtration at 125 trading days in a year.

Table 7 shows the effects of the implementation of a max/min daily return value as well as max/min values after winsorization. The implemented cap has been set to a maximum of +100 percent, while the minimum has been set to -50 percent. The effects of the return manipulation caused the results to be less spurious and much more in line with traditional expectations. Before the data manipulations, certain years containing fewer investable securities were prone to extreme results. This was especially the case in the high volatility portfolios where the improbable returns accumulated. The max/min return value cap affected at least one daily return during 36 of 38 years, either by limiting the maximum or minimum value. With multiple securities increasing with a rate of + 1,000 percent in a day and the largest increase in a single day being 11,567 percent, we believe the data utilized is susceptible to certain faulty data caused by input errors or other faulty pricings of securities. However, we do not expect anyone to be able to benefit from these mispricing in the real world and we therefore justify manipulating the data to reduce the weights these outliers affect the data.

	Original Data		**After Cap**		**After Winsorization**	
Year	**Max**	**Min**	**Max**	**Min**	**Max**	**Min**
1982	0.6667	-0.3333	0.6667	-0.3333	0.2483	-0.2000
1983	0.5789	-0.5714	0.5789	-0.5000	0.3245	-0.2485
1984	0.6667	-0.5000	0.6667	-0.5000	0.3077	-0.2353
1985	0.6667	-0.4000	0.6667	-0.4000	0.3345	-0.3335
1986	0.6250	-0.5333	0.6250	-0.5000	0.3636	-0.3077
1987	1.0000	-0.8125	1.0000	-0.5000	0.4428	-0.2993
1988	9.0000	-0.9500	1.0000	-0.5000	1.0000	-0.5000
1989	1.1053	-0.5556	1.0000	-0.5000	0.6660	-0.4443
1990	3.1250	-0.6622	1.0000	-0.5000	0.8763	-0.4118
1991	2.6250	-0.8500	1.0000	-0.5000	1.0000	-0.5000
1992	5.5000	-0.8462	1.0000	-0.5000	1.0000	-0.5000
1993	12.0000	-0.9375	1.0000	-0.5000	1.0000	-0.5000
1994	5.0000	-0.8667	1.0000	-0.5000	0.6667	-0.4000
1995	1.0625	-0.4390	1.0000	-0.4390	0.4748	-0.3315
1996	2.0000	-0.7500	1.0000	-0.5000	1.0000	-0.4167
1997	1.0000	-0.4000	1.0000	-0.4000	0.4074	-0.2632
1998	4.8750	-0.9091	1.0000	-0.5000	1.0000	-0.5000
1999	3.9604	-0.7222	1.0000	-0.5000	0.9953	-0.5000

2000	1.5000	-0.7680	1.0000	-0.5000	0.7759	-0.4999
2001	1.8571	-0.6531	1.0000	-0.5000	1.0000	-0.4753
2002	11.5000	-0.9200	1.0000	-0.5000	1.0000	-0.5000
2003	13.2500	-0.9299	1.0000	-0.5000	1.0000	-0.5000
2004	115.6667	-0.9617	1.0000	-0.5000	1.0000	-0.5000
2005	1.1538	-0.4080	1.0000	-0.4080	0.5313	-0.3216
2006	7.0000	-0.8750	1.0000	-0.5000	0.4441	-0.2380
2007	0.5088	-0.2833	0.5088	-0.2833	0.2406	-0.1790
2008	2.4906	-0.7089	1.0000	-0.5000	0.4551	-0.3996
2009	5.0000	-0.8379	1.0000	-0.5000	0.7929	-0.3990
2010	19.3846	-0.8143	1.0000	-0.5000	0.5000	-0.4327
2011	2.7368	-0.5520	1.0000	-0.5000	0.9834	-0.4268
2012	1.4341	-0.8788	1.0000	-0.5000	0.7739	-0.3333
2013	1.1818	-0.8235	1.0000	-0.5000	0.7215	-0.4529
2014	3.6190	-0.7627	1.0000	-0.5000	1.0000	-0.5000
2015	3.2000	-0.6667	1.0000	-0.5000	0.5884	-0.3792
2016	1.3457	-0.6122	1.0000	-0.5000	0.5249	-0.4646
2017	3.0679	-0.8594	1.0000	-0.5000	0.4495	-0.3862
2018	1.7649	-0.7315	1.0000	-0.5000	0.3232	-0.2178
2019	1.3506	-0.5644	1.0000	-0.5000	0.6051	-0.3539

Table 7: Max/min daily returns during each year before and after outlier manipulation. Maximum and minimum daily return implemented at +100 percent (1) and -50 percent (-0.5). Winsorization at 1st and 99th percentile. Numbers are in decimals and not in percent. Meaning that a return of 0.50 is equivalent to +50% return.

5.5 The Effects of Factor Exposure

In this section, we are investigating different factors loadings on portfolios that were constructed in section 5.1 and 5.2. As expected, for both construction methods, the highest volatility portfolio contains the securities with the highest exposure to the market, and vice versa. From the previous sections, we also know that high volatility portfolios have higher returns, which is consistent with the traditional CAPM theory.

For size effect, we find that high volatility portfolios have higher and positive loading on the SMB factor, while the two lowest volatility portfolios have negative loadings. This shows that high volatile portfolios consist of smaller securities and are more likely to capture the "small-firm effect", which links to the outperformance. In terms of value effect, we see a contrast pattern as HML loadings are positive for the low volatile portfolios and negative for the high volatile portfolios. It suggests that low volatility stocks tend to be value stocks and high volatility stocks to be growth stocks.

Momentum (UMD) factor loadings are insignificant, but the factor representing one month lagged of momentum (PR1YR) is found to be more influential on returns. The factor exhibits positive loadings on the low volatile portfolio and negative on the high volatile portfolio, meaning high volatility of securities are more common in experiencing return reversal.

Portfolio	MKT	SMB	HML	PR1YR	UMD	LQD
1	0.808	-0.034	0.058	0.083	-0.014	-0.123
	[41.587]	[-2.174]	[3.138]	[2.809]	[-0.475]	[-4.996]
2	1.051	-0.021	0.016	0.013	-0.021	-0.147
	[50.418]	[-0.624]	[-0.229]	[-0.397]	[-0.116]	[-5.925]
3	1.095	0.059	-0.010	-0.216	0.057	-0.176
	[46.704]	[1.737]	[-0.505]	[-4.916]	[1.073]	[-4.934]
4	1.322	0.190	-0.086	-0.099	-0.021	-0.035
	[36.293]	[3.986]	[-2.558]	[-2.150]	[-0.246]	[-1.948]
5	1.395	0.166	-0.113	-0.167	-0.033	0.217
	[27.934]	[2.557]	[-2.677]	[-1.972]	[-0.333]	[2.766]

Table 8: Table shows the 1M/1M portfolios' factor exposure towards different factors. Portfolios were constructed using a 1 month/1 month strategy. Factors from left to right are Market Return, Small Minus Big, High Minus Low, the PR1YR factor, Up Minus Down and Liquidity. Test statistics are shown in square brackets.

We found that beside the market, Liquidity is the second factor that has the most influence on Norwegian stocks prices. Portfolio 5 is the only one having a positive and high coefficient on liquidity factor. The exposure of other portfolios on the factors are negative and we can see LQD betas increase from portfolio 1 to 5, which suggests the connection between volatile portfolios and illiquid securities. Illiquid stocks are usually considered riskier; therefore, investors demand a premium to hold them, which explains the higher returns. Less liquid stocks are often associated with low capitalization, which is reflected in our results.

Portfolio	MKT	SMB	HML	PR1YR	UMD	LQD
1	0.5616	-0.0347	0.0361	-0.0004	-0.0033	-0.1185
	[15.920]	[-1.216]	[1.132]	[3.993]	[-3.298]	[-2.203]
2	0.8068	-0.0366	0.0205	-0.0011	-0.0080	-0.1599
	[22.145]	[-2.621]	[-0.313]	[-0.288]	[1.371]	[-3.760]
3	0.5616	-0.0347	0.0361	-0.0004	-0.0033	-0.1185
	[21.957]	[1.753]	[-0.440]	[0.460]	[-2.586]	[-1.944]
4	1.0338	0.0259	0.0133	-0.0109	-0.0129	-0.0671
	[15.300]	[2.719]	[2.765]	[-2.372]	[1.139]	[-1.729]
5	1.1959	0.0701	-0.0320	-0.0499	-0.0177	0.1456
	[10.265]	[1.252]	[-0.135]	[0.320]	[-0.140]	[0.825]

Table 9: Table shows the 1Y/1Y portfolios' factor exposure towards different factors. Portfolios were constructed using a 1 year/1 year strategy. Factors from left to right are Market Return, Small Minus Big, High Minus Low, the PR1YR factor, Up Minus Down and Liquidity. Test statistics are shown in square brackets.

5.6 Industry Exposure

When examining the industry exposures of the different portfolios constructed using the total volatility approach, we found that the portfolios containing higher volatility securities had a significantly higher exposure to the Energy and Industrial sector. On the other hand, the portfolios containing the least volatile securities generally covaried with the Finance industry. Random sample testing, where each of the individual holdings of a random portfolio during a random holding period were manually checked, supported these findings. Appendix 5 provides an insight into the holdings of a low volatility portfolio.

Portfolio	Engy.	Mat.	Ind.	Con. Dis.	Con. Stap.	Heal.	Fin.	IT	Tele.	Utility
1	0.068	-0.012	0.199	0.088	0.078	-0.004	0.547	0.012	0.015	0.042
2	0.185	0.009	0.327	0.005	0.144	0.046	0.349	0.103	0.048	0.017
3	0.288	0.010	0.240	0.023	0.110	0.036	0.285	0.192	0.017	0.010
4	0.221	0.006	0.365	0.065	-0.011	0.078	0.303	0.312	0.013	-0.011
5	0.310	-0.021	0.516	-0.010	0.009	0.031	0.242	0.297	-0.082	0.055

Table 10: Showing industry covariance of portfolios from least (top) to most (bottom) volatile. Constructed using five portfolios, one month rolling window, and one month holding period. Sectors from left to right are Energy, Materials, Industry, Consumer Discretionary, Consumer Staples, Healthcare, Finance, IT, Telecommunications and Utility.

The findings in this chapter are in line with expectations. The fact that higher volatility portfolios were more exposed to the Energy and Industrial sector can be attributed to oil and gas making up most of the Energy sector in the Norwegian market. Likewise, the Industry sector also has a large exposure towards oil and gas. Oil and gas have a considerably higher volatility compared to other energy sub-sectors, commodities, and many other sectors (Asche et al, 2013). According to Ødegaard (2020), Industry and Energy have among the highest average returns of the different sectors in the Norwegian stock market. This is likely a major factor for the outperformance of high volatility portfolios. Appendix 6 shows return characteristics of the different sectors. On the flip side, the least volatile portfolios are more exposed to the Finance industry is likely due to the strict regulations of Norwegian banks and financial institutions (Codero-Moss & Nilssen, 2021). The stricter restrictions on the Norwegian Financial sector combined with the nature of the many publicly traded Norwegian savings banks is a plausible explanation for the accumulation of financial securities in the lower volatility portfolios (Buss et al, 2013).

6. Conclusion

In this chapter the conclusion to the research question is finally presented, followed by a quick summary of thoughts for future research on the topic.

As a result of the results presented in the previous chapters, we arrive at the conclusion that the “Low Volatility Puzzle” does not exist in the Norwegian stock market. Our hypothesis that the Norwegian stock market would behave similarly was proven wrong. We could not find any evidence that portfolios constructed using low volatility securities could outperform an equally constructed high volatility portfolio using the methodologies previously used to prove the existence in other markets. Our findings are in line with traditional finance theory and suggest that investors are compensated proportionally for holding riskier, in this case more volatile, securities. Neither utilizing total volatility, nor idiosyncratic volatility yielded any convincing result. After changing several parameters, such as the length of the rolling window used to construct the portfolios and the length of holding period before the reconstruction of the portfolio, we further iterate our conclusion.

The findings of this paper contradict our initial hypothesis and indicate that the Norwegian stock market might be subject to factors which make it behave somewhat differently than the larger global stock markets. The rationale behind this is likely the Norwegian stock market's small size combined with the high concentration of oil and gas exposed companies, effectively changing the return characteristics. Firstly, due to the few companies (comparatively) on the Oslo Stock Exchange, the sample might not be large enough to exert the behavior that results in the anomaly. Secondly, the companies traded on the Oslo Stock Exchange and the Norwegian economy as a whole will, to a large extent, likely covary with the oil and gas dominated Norwegian stock market. This might cause the otherwise less volatile companies to behave differently than they would have in other markets.

However, it is worth noting that previous papers on larger markets have pointed to other sources of explanation. As a result of this we cannot rule out that the lack of the “Low Volatility Puzzle” in the Norwegian stock market could be a result of differences in behavioral preferences, access to short shares or companies' access to funding. Consequently, Norwegian investors might be less inclined to utilize high volatility securities as lottery tickets. The degree of institutional ownership in the Norwegian stock market might change the dynamics of short selling. Norwegian companies' access to funding might be different enough to affect return characteristics. However, further research on any of these topics are needed in order to say anything definite.

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Appendix

Appendix 1: Table shows 6/1 -month strategy data using total volatility method for portfolio construction with equal-weighting. Test statistics are shown in square brackets.

Portfolio	Mean	STD	SR	CAPM alpha	3FF alpha	5 Factors alpha
1	1.30%	4.55%	0.16	0.0078	0.0068	0.0069
				[3.646]	[3.051]	[3.018]
2	1.58%	5.89%	0.17	0.0104	0.0092	0.0088
				[3.758]	[3.148]	[2.932]
3	1.22%	6.36%	0.10	0.0068	0.0049	0.0047
				[2.282]	[1.578]	[1.479]
4	1.22%	7.33%	0.09	0.0071	0.0049	0.0049
				[2.046]	[1.360]	[1.323]
5	2.29%	17.23%	0.10	0.0163	0.0130	0.0113
				[2.017]	[1.538]	[1.311]

Appendix 2: Table shows 24/1 -month strategy data using total volatility method for portfolio construction with equal-weighting. Test statistics are shown in square brackets.

Portfolio	Mean	STD	SR	CAPM alpha	3FF alpha	5 Factors alpha
1	1.13%	4.75%	0.10	0.0035	0.0011	0.0006
				[1.518]	[0.502]	[0.230]
2	1.20%	5.79%	0.08	0.0045	0.0017	0.0017
				[1.396]	[0.606]	[0.352]
3	1.28%	6.59%	0.07	0.003	0.0007	-0.0007
				[1.275]	[0.228]	[0.005]
4	1.14%	7.11%	0.09	0.003	-0.0005	-0.0003
				[0.770]	[-0.137]	[-0.193]
5	2.70%	18.86%	0.05	0.0106	0.0150	0.0077
				[2.050]	[1.633]	[1.617]

Appendix 3: Table shows 6/1 -month strategy data using idiosyncratic volatility method for portfolio construction with equal-weighting. Test statistics are shown in square brackets.

Portfolio	Mean	STD	SR	CAPM alpha	3FF alpha	5 Factors alpha
1	1.30%	4.55%	0.16	0.0078	0.0068	0.0069
				[3.645]	[3.051]	[3.017]
2	1.58%	5.89%	0.17	0.0104	0.0091	0.0087
				[3.757]	[3.147]	[2.932]
3	1.22%	6.36%	0.10	0.0068	0.0049	0.0047
				[2.281]	[1.578]	[1.478]
4	1.22%	7.33%	0.09	0.0070	0.0049	0.0048
				[2.045]	[1.360]	[1.323]
5	2.29%	17.23%	0.10	0.0163	0.0130	0.0113
				[2.017]	[1.538]	[1.310]

Appendix 4: Table shows 24/1 -month strategy data using idiosyncratic volatility method for portfolio construction with equal-weighting. Test statistics are shown in square brackets.

Portfolio	Mean	STD	SR	CAPM alpha	3FF alpha	5 Factors alpha
1	0.99%	5.45%	0.09	0.0035	0.0011	0.0006
				[1.518]	[0.502]	[0.230]
2	1.06%	5.94%	0.09	0.0045	0.0017	0.0017
				[1.396]	[0.606]	[0.352]
3	0.94%	6.79%	0.06	0.0035	0.0007	-0.0007
				[1.276]	[0.228]	[0.005]
4	1.01%	7.17%	0.07	0.0033	-0.0005	-0.0003
				[0.769]	[-0.137]	[-0.193]
5	1.72%	9.07%	0.13	0.0106	0.0150	0.0077
				[2.050]	[1.633]	[1.617]

Appendix 5: Example of portfolio holdings. Table shows portfolio 1 (least volatile) holdings (Tickers) in each year. Test/simulation was run using 10 portfolios, 2 years rolling window, 1 year holding period.

Year	Tickers									
1982	DNG'	RBK'	CBK'							
1983	DNG'	CBK'	DNC'	BBK'	FBK'	RBK'				
1984	DNG'	CBK'	DNC'	BBK'	FBK'	NBK'	STB'	RBK'		
1985	DNG'	BBK'	AKE'	CBK'	DNC'	FBK'	RBK'	NBK'	SBK'	STB'
1986	DNG'	DNC'	CBK'	FBK'	FRB'	VBK'	RBK'	SBK'	NDA'	STB'
1987	DNG'	DNC'	FBK'	CBK'	NOI'	ORK'	OLT'	VES'	STB'	NEB'
1988	DNC'	NOI'	DNG'	SOR'	ORK'	OHB'	OBK'	STB'	CBK'	
1989	DNC'	DNG'	KVI'	NOI'	HNA'	BEB'	ORK'	KOS'	CBK'	BEA'
1990	BEB'	KVI'	BEA'	HNA'	NOI'	LAB'	ORK'	DNG'	CBK'	
1991	NDS'	NOI'	NHI'	HNA'	BEB'	ORK'	BEA'	KVI'		
1992	AFK'	EIRG'	HNA'	HNB'	HNAF'	KVI'	FRB'	SKI'	BEB'	
1993	SANG'	EIRG'	HNB'	HNA'	HNAF'	KVI'	KVIB'	SAG'	VEI'	KVIF'
1994	SANG'	EIRG'	ORK'	RIE'	KVI'	HNB'	HNA'	ORKF'	KVIB'	KVIF'
		HNAF'								
1995	SANG'	KVI'	KVIB'	ORK'	SAG'	EIRG'	SCH'	ORKB'	BEB'	BNB'
		NOK'								
1996	SANG'	STBP'	MORG'	SAG'	EIRG'	KVI'	SNOG'	STB'	SAGB'	ORK'
		KVIB'								
1997	STBP'	ROGG'	MING'	SNOG'	SANG'	NONG'	MORG'	ORK'	DNB'	CKR'
		ORKB'								
1998	VME'	SNOG'	ROGG'	NONG'	SADG'	BNB'	SVEG'	MING'	MORG'	SANG'
		TOTG'	ORK'							
1999	NONG'	ROGG'	BNB'	SADG'	SANG'	MORG'	SNOG'	TOTG'	MING'	NBK'
		SVEG'	NAR'	VSBG'						
2000	STO'	NONG'	ROGG'	SADG'	MORG'	SANG'	SNOG'	MING'	BNB'	VSBG'
		RING'	NOI'	TOTG'	SVEG'					
2001	ROGG'	NONG'	MORG'	MING'	SADG'	BNB'	SNOG'	SVEG'	VSBG'	NBK'
		DNB'	TOTG'	HOLG'	PLUG'	NSG'				

	ROGG'	MORG'	NONG'	BNB'	SADG'	MING'	SNOG'	NBK'	ORK'	DNB'
2002		NESG'	SVEG'	SPOG'	OLT'	ELK'				
	ROGG'	BNB'	MORG'	MING'	NONG'	ORK'	SADG'	ELK'	SPOG'	NESG'
2003		SVEG'	OLT'	DNB'	EKO'	AHM'				
	BNB'	ROGG'	MORG'	ORK'	MING'	SANG'	NHY'	ODFB'	SADG'	NONG'
2004		PLUG'	ODF'	ELK'	KOG'	SVEG'				
	ROGG'	MORG'	SADG'	MING'	ORK'	NONG'	SVEG'	NHY'	TOTG'	VEI'
2005		PRS'	RIE'	SANG'	PLUG'	EKO'				
	MORG'	MING'	ROGG'	SADG'	SVEG'	DNBNOR'		ORK'	NONG'	RIE'
2006		SPOG'	PLUG'	VSBG'	VEI'	SANG'				
	AGR'	MORG'	MING'	SVEG'	RIE'	SADG'	ROGG'	DNBNOR'		HELG'
2007		SPOG'	ORK'	NONG'	VSBG'	RCL'				
	NOR'	RIE'	ROX'	HELG'	MING'	SADG'	ROGG'	ODFB'	SVEG'	MORG'
2008		NOM'	SPOG'	DNBNOR'		NONG'	AFG'			
	POL'	RIE'	HELG'	BOR'	AFG'	IMAREX'		ODF'	COV'	OLT'
2009		KOG'	MING'	ODFB'	FAR'	IMSK'	MORG'	SADG'		
	HELG'	RIE'	IMAREX'		AFG'	ODF'	COV'	HNA'	EIOF'	BOR'
		GRO'	FAR'	KOG'	ODFB'	BON'	RING'	MORG'	ASD'	
2010		MAMUT'		POL'	HNB'	WWI'				
	HELG'	RIE'	GRO'	HNA'	NPEL'	HNB'	BON'	AFK'	FAR'	ORK'
		ODFB'	VEI'	EIOF'	IMAREX'		MORG'	AFG'	KOG'	WWI'
2011		SOAG'	MING'							
	HNB'	HNA'	OLT'	AURG'	ORK'	SPOG'	TEL'	HELG'	NESG'	SVEG'
		SOAG'	RIE'	IMAREX'		GRO'	VEI'	AFK'	MORG'	SIOFF'
2012		MING'								
	SVEG'	OLT'	GJF'	ORK'	TEL'	CSOL'	HNA'	SOAG'	MELG'	HNB'
2013		VEI'	HELG'	AFG'	MORG'	SDRL'	SIOFF'	AURG'	GRO'	
	ORK'	TEL'	GJF'	SOR'	OLT'	SDRL'	STORM'		SVEG'	VEI'
2014		AFG'	GRO'	MING'	HNA'	HNB'	MELG'	SIOFF'	AKER'	MORG'
	NTSG'	ORK'	GJF'	MORG'	SVEG'	TEL'	AFG'	AURG'	HNB'	SBVG'
2015		STORM'		MING'	NHY'	HNA'	MELG'	SOAG'		
	NTSG'	AURG'	SOAG'	MELG'	GJF'	MORG'	ORK'	HNB'	SVEG'	AFG'
2016		HNA'	SRBANK'		MING'	NONG'	TEL'	STORM'		ISSG'
	AURG'	VVL'	SOAG'	ORK'	ENTRA'	MELG'	GJF'	MORG'	SPOG'	
2017		JAEREN'		ISSG'	AFG'	SKUE'	SOR'	OLT'	TOTG'	RING'
	PPGPREF'		ORK'	ENTRA'	GJF'	MELG'	VVL'	OLT'	MORG'	SPOG'
2018		ISSG'	SKUE'	AURG'	SOAG'	AFG'	ASC'	TEL'	HELG'	
2019	MELG'	OLT'	SOR'	ARCUS'	ORK'	TOTG'	SPOG'	OCY'	SKUE'	GJF'

ENTRA'TEL' SVEG' DNBNOR' ASC' MORG' MING'

Appendix 6: Table of returns and excess returns from ten industries. Table is taken from Ødegaard, 2020 (p. 4).

Panel A - Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
Energy	0.019	0.090	-0.283	0.015	0.654	480
Material	0.018	0.115	-0.447	0.010	1.490	480
Industry	0.016	0.058	-0.187	0.016	0.303	480
ConsDisc	0.017	0.068	-0.203	0.014	0.433	480
ConsStapl	0.020	0.065	-0.213	0.021	0.209	480
Health	0.018	0.086	-0.330	0.012	0.686	480
Finance	0.012	0.047	-0.149	0.011	0.252	480
IT	0.023	0.102	-0.288	0.013	0.711	480
Telecom	0.010	0.092	-0.454	0.003	0.328	296
Utility	0.010	0.063	-0.229	0.010	0.301	288

Panel B - Excess Return

Statistic	Mean	St. Dev.	Min	Median	Max	N
Energy	0.015	0.093	-0.288	0.013	0.065	410
Material	0.010	0.117	-0.450	0.005	1.487	410
Industry	0.011	0.061	-0.198	0.012	0.293	410
ConsDisc	0.010	0.072	-0.207	0.008	0.430	410
ConsStapl	0.013	0.066	-0.218	0.015	0.206	410
Health	0.010	0.091	-0.342	0.005	0.681	410
Finance	0.006	0.050	-0.156	0.006	0.259	410
IT	0.017	0.107	-0.294	0.006	0.702	410
Telecom	0.010	0.102	-0.460	0.002	0.321	227
Utility	0.004	0.064	-0.234	0.003	0.297	219

Appendix 7: Table of returns and standard deviation of both equal-weighted and Value-weighted portfolios in 10 year-period.

	Equal-weighted		Value-weighted	
	Mean	**STD**	**Mean**	**STD**
1980-1989				
1	2.18%	6.22%	2.58%	6.65%
2	2.15%	6.52%	2.35%	6.61%
3	2.28%	6.98%	3.01%	7.74%
4	2.02%	8.12%	3.29%	10.13%
5	2.65%	9.95%	4.09%	12.02%
1990-1999				
1	1.37%	5.53%	2.09%	6.24%
2	1.08%	6.59%	1.81%	6.44%
3	1.27%	7.21%	2.43%	7.43%
4	1.10%	8.18%	2.88%	8.75%
5	1.58%	9.61%	4.45%	11.64%
2000-2009				
1	0.84%	4.83%	1.23%	6.16%
2	1.34%	7.20%	2.35%	8.19%
3	0.85%	7.97%	2.50%	8.56%
4	1.15%	8.98%	4.27%	11.23%
5	1.11%	9.88%	4.71%	17.09%
2010-2019				
1	0.92%	2.69%	1.26%	3.52%
2	1.23%	3.69%	1.84%	4.50%
3	0.48%	4.33%	1.27%	5.47%
4	0.57%	6.08%	2.38%	8.54%
5	-0.01%	7.35%	2.77%	9.44%