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Theoretical and empirical analysis of volatility selling strategies

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Theoretical and empirical analysis of volatility selling strategies

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1 Introduction

Despite being the corner-stone of option-pricing, the Black-Scholes model is based on assumptions that do not hold in reality. For instance, one of them is that the volatility of the underlying asset's returns is constant until maturity. In addition, plenty of papers indicate that not only does the actual (realized) volatility of the underlying vary, but it is also persistently lower than the implied volatility, priced in the value of an option. As many researchers indicate, that explains why delta-hedged option strategies, which are modelled to be less sensitive to the movements in the price of the underlying, deliver returns significantly different from zero. There are some limitations to that observation mostly based on inconceivability of some strategies. For instance, delta-hedging, which is one of the focuses of our thesis, is generally accepted to be a good tool to reduce the market risk, however, that effect depends on how continuously an investor can rebalance the portfolio. And, of course, a perfectly continuous rebalancing is not attainable for regular practitioners. But, overall, this has inspired introducing a new term – volatility risk premium – and looking for its relationship to persistently non-zero returns on option selling. Quickly, researchers found that selling an option and holding the position until maturity, on average, produced positive excess returns, which can be considered a consequence of realized volatility being lower than implied volatility, as the latter positively affects the price of an option. Thus, appeared the term *volatility selling*.

That might sound like a completely new feature in financial economics, but, in fact, volatility selling can be allocated to a bigger family of strategies – the well-known carry. Those two share a lot in common, including carry's most famous trait – "going up by stairs, and going down by elevator".

The literature in this field has been experiencing a rapid growth since the early 1990s when researchers got access to abundant options data, however, there is little consensus on the determinants of the expected returns of option strategies. Among generally accepted reasons for the existence of the volatility risk premium are compensation for systematic risk, like volatility of volatility risk, correlation risk and risk of jumps in prices, lack of liquidity and peso problem.

Researchers in this field have come up with ways to assess these theories, thus, supporting the argument of the very existence of the volatility risk premium. In our thesis, we dig into details of the option strategies' returns to understand and clearly illustrate why certain patterns are natural to them and why they should not come as surprising. In addition, we use the existing approaches to check if the results of the previous researches hold up until now. Furthermore, we find ways to come up with other quantitative methods to explain the previously and newly observed (ir)regularities. But, in our thesis we systematically summarize features we observe in simulated strategies' returns, formulate concrete expectations, and, finally, test them empirically. The object of our analysis is the equity index S&P500 as well as options written on it.

We follow the common ways of winsorizing the obtained data to eliminate the effect of noise and unreliable prices on our results. Moreover, we propose some ways to check if those winsorizing methods, indeed, achieve the results they are used for and do not take them for granted.

One of the novel points of our research is connecting the theoretical construction of the strategies of interest to the identified empirical findings. Our aim is to not only quantify and compare statistics of interest, but also to try to justify it with the economic theory. As a part of that, we, first, formulate theoretical expectations on whether variance risk premium and market risk premium and their derivatives affect the returns, how that relationship changes during crises, etc. from the simulations that we run and, after that, we assess empirically if those expectations hold. We also check, as most researchers do, if volatility selling strategies are related to conventional risk factors.

The following sections of our thesis start with the literature review with the focus on the key findings of previous researches, concise explanations of their inferences and the points that were missed or not clearly emphasised. After that, we briefly introduce the notations and set up the research by describing the strategies of interest. Overall, we focus on six strategies – three for calls and three for puts – a *(1) simple short*, a *(2) statically* and a *(3) dynamically* delta-hedged short strategies. Furthermore, we divide each of them into eight brackets of moneyness levels to investigate different return patterns. In total, that gives us forty eight strategies. We spend a great deal in the theoretical part to closely elaborate on what type of return patterns are likely to be natural to the strategies of interest by construction. That allows us to formulate clear expectations. The subsequent section describes what data are available to us and what we are able do with it. In addition, we describe the ways we winsorize the data. Finally, we report the results of our empirical analysis and compare them with the expected ones that we describe in the theoretical part and with the results of the previous researches. As the last step, we propose how this field can be developed and what practitioners might get out of it.

2 Literature Review

2.1 Options as risk-hedges

Investing in the financial markets bears various types of risks. One of the common risks that most practitioners always seek to hedge against is the risk of abrupt shifts in the price of an asset. One generic tool serving that purpose is options. For instance, holding on to an asset exposes one's portfolio to the downside risk to hedge against which one can buy a put option. Similarly, a call option can protect against dramatic rises in the price of an asset. Returns delivered by options became a central point of a number of researches in the past decades. The commencement of this theory is closely tied to the central work of this topic by Black and Scholes (1973).

There are notable properties in the relationship between risks and return of options. They became interesting for many researches, including Scholes et al. (1982) and Merton et al. (1978) who propose investment strategies using options, Jackwerth (2000) who suggests mispricing of options in the market, and Coval and Shumway (2001) who show a thorough overview on characteristics of call, put and straddle returns.

Many previous studies show striking features of options. For instance, there is a persistent gap between realized volatility and implied volatility for most indexes, including equities and other asset classes like commodities. However, the gap is usually near zero for individual stocks. Another remarkable observation pointed out by many researchers is that mean return of holding put options on equity index until maturity leads to average negative returns (e.g. Coval and Shumway (2001), Jackwerth (2000), Broadie et al. (2007)). However, the average return of a long call option is, on average, positive (e.g. Coval and Shumway (2001), Wilkens (2007)). Those noteworthy observations indicate, as many researchers suggest, a connection between several types of risk premia and option-related trading strategies. For instance, Bakshi and Kapadia (2003) identify the volatility risk premium in delta-hedged options. They showed that delta-hedging decreases exposure to the market, while the volatility risk premium significantly affects that strategy, especially, in times of crises. Driessen et al. (2006) and Buraschi et al. (2013) point out correlation premium in the strategy of selling index options and buying options on its constituents that delivers positive returns, on average. Boyer and Vorkink (2014) show sign of skewness preferences in lottery-like options (out-of-the-money calls on single-stock options). Our thesis focuses on volatility risk premium and the returns of selling volatility strategies using an equity index (selling options, selling statically delta-hedged options and selling dynamically delta-hedged options). In this study, we summarize the theoretical background behind the nature of aforementioned strategies, that is missing in most papers related to this topic, propose expectations directly following from them and provide empirical assessment of the latter. Thus, our work contributes to the ever-growing field of studies dedicated to understanding the drivers behind the returns of the volatility selling strategies.

2.2 Volatility risk premium

Given an option and its underlying, the *volatility risk premium (VRP)* is defined as the difference between realized volatility and implied volatility:

$$VRP(t) = IV(t) - RV(t, t + \tau),$$

where $RV(t, t + \tau)$ is the (annualized) realized volatility of the underlying's returns over the holding period from t to $t + \tau$, $IV(t)$ is the (annualized) implied volatility at time moment t. Implied volatility computed from option prices via the Black-Scholes model (more details in section 3) is considered to be investors' expectation for the volatility of the underlying's return until maturity. On the other hand, realized volatility is the actual standard deviation of the asset's return over that period. There are different ways to calculate both figures, but for now we leave them at their most well-known forms.

Authors indicate that the strategy of selling volatility should be profitable when investors overestimate the risk of the underlying asset, which is represented by a positive spread between implied volatility at time t and realized volatility at time $t + \tau$. Indeed, overestimating the volatility leads to overpricing of an option, so *ceteris paribus* selling that option should provide a positive payoff. Some previous researchers provide evidences on effectiveness of implied volatility on predicting future volatility (e.g., Harvey and Whaley (1992), Day and Lewis (1988), Christensen and Prabhala (1998)). However, Eraker (2009) notes that up to year 2009, on average, annual implied volatility of at-the-money (ATM) options was about 19%, while the realized volatility of the index was only about 16%, suggesting the existence of a spread. This can also be seen in figure 1, where we visualize the gap between current VIX and over-next-30-days realized volatility of the S&P500 return over the period of 1996 – 2020. Thus, the blue line, that indicates the former, is shifted to the left by 30 days to clearly show the spread between the "forecast" and the "actual" volatilities.

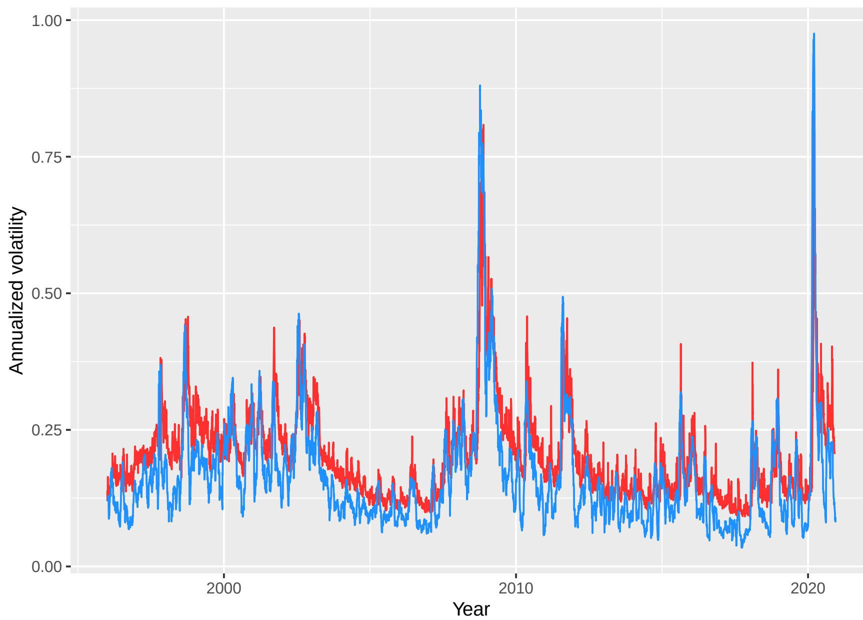


Figure 1: Current VIX (red) and over-next-30-days realized volatility (baby blue) over the period of 1996 – 2020.

Under Black-Scholes assumptions, implied volatility should be the same across different options on the same asset. However, in real-life, as Dumas et al. (2001) indicates, implied volatility of the S&P500 returns is recorded to have a ”smile” pattern before the crisis 1987. This means that *in-the-money* (ITM) and *out-of-the-money* (OTM) options have higher implied volatilities than *at-the-money* (ATM) options. Then, after the 1987 crash, there was detected an excess skew of the implied volatility, making its pattern look liek a *”smirk”* (OTM options have higher implied volatilities than that of ATM options, and ITM options have lower implied volatilities than that of ATM options). Both Fan et al. (2013) and Chen et al. (2016), in their studies about volatility risk premium, use the VIX volatility index as a proxy for implied volatility. VIX is a measure for expected volatility of S&P500 over the next 30 days, so using VIX will not be appropriate if our strategy of selling volatility has different-from-30-day holding period. Besides, only out-of-the-money call and put are selected in the calculation of VIX. As the implied volatility differs across strike prices, assuming the same value of volatility risk premium for different level of moneyness might lead to mismeasurment of its impact on returns of selling volatility strategies in our research. Hence, we decide to choose implied volatility instead of VIX, like Cao and Han (2013) and Goyal and Saretto (2009).

Table 1: Summary statistics of Volatility Risk Premium

Variable	S&P500	RV	VIX	VRP1	VRP2	VRP3
Mean	**0.0914**	**0.166**	**0.2033**	**0.0368**	**0.0096**	**0.0373**
Standard deviation	0.1951	0.1063	0.0855	0.0762	0.0616	0.0512
median	0.0007	0.1418	0.1862	0.0452	0.0123	0.0414
Skewness	-0.1827	3.0868	2.0875	-3.3415	-7.0949	-2.4911
Kurtosis	10.1685	14.665	7.4168	23.6496	76.9561	17.9602
Correlation with S&P500	**1**	**-0.3852**	**-0.4273**	**0.6484**	**0.5502**	**0.0855**

Summary statistics of the S&P500's return, 30-day-historical volatility, the CBOE Volatility Index, the gap between VIX and RV (*VRP1*), the gap between VIX^2 and RV^2 (*VRP2*), and the gap between VIX and lagged RV (*VRP3*). **Bold format** means that the respective statistics is significant at 5% significance level.

According to Ilmanen (2012), in the risk-neutral world, implied volatility reflects the market's volatility expectations, while in the real world when assumptions of Black-Scholes fail, there exists some risk premia in addition to volatility expectations in implied volatility. As we can see in table 1, VRP is statistically different from zero at 5% significance level. This premium, researchers suggest, can be partially explained by the willingness of investors to pay extra to protect their wealth against volatility. In other words, the favourite over the positive skewness may create a premium on assets that have a negative skewness. Theory also suggests that volatility selling should carry a positive risk premium if its losses tend to coincide with the equity market losses. Indeed, given the statistically positive spread between implied volatility and realized volatility, investors seem to overestimate the probability of a market crash.

2.3 VRP and selling volatility strategies

This research contributes to the literature that aims to study the behavior of selling volatility using an equity index and the roles of volatility risk premium in the profitability of the strategy. There have already been researchers working on that topic.

There is a relative consensus about the positive VRP in case of selling volatility using equity indexes – the difference between implied volatility and realized volatility is usually positive and drives the strategy's excess return. Coval and Shumway (2001) show that a strategy of buying zero-beta straddles on S&P500 has an average return of around -3% percent per week. This result is shown to be consistent under different robustness checks (like that of mismeasurement of the call option beta, altering sample period and frequency, sensitivity test to the inclusion of the October 1987 crash, and including the transaction costs). The fact that zero-beta straddles offer returns that are significantly different from the risk-free rate strongly suggests that there is another important factor besides the market risk. A long straddle thrives when the price of the underlying asset makes a big move in either direction. Even though the interim volatility is high, if the price at maturity is around the initial position, this strategy will not be profitable. Therefore, selling volatility by selling straddle is not a pure bet on interim volatility but on price change. However, Coval and Shumway (2001) still provide preliminary evidence that volatility helps in explaining their proposed strategy's returns. Then, Bakshi and Kapadia (2003) make a more thorough investigation on the volatility risk premium by examining the statistical properties of delta-hedged option portfolios. Before the appearance of a variance swap, which is a contract that pays the difference between the realized swap rate (actual volatility) and the contracted swap rate (market's expectation of volatility at the time the swap is entered into), delta-hedged option strategy is the purest bet on volatility. Using S&P500 index options, they find that the delta-hedged gains are non-zero, and consistent with a non-zero volatility risk premium. Benzoni et al. (2010) show that the implied volatility across strike prices exhibit a smile pattern – higher volatility for OTM puts and calls than for ATM options, but then after the 1987 crash, a highly asymmetric smirk or skew replaced asymmetric smile for index options. Therefore, a selling volatility strategy should exhibit the highest gains for OTM options but Bakshi and Kapadia (2003) indicate that gains are generally most pronounced for at-the-money options. Given those inconsistencies, in this study, we aim to understand more about the pattern of options' returns across different levels of moneyness. In an approach, similar to that of Bakshi and Kapadia (2003), Lin and Chen (2009) also provide evidence regarding non-zero volatility risk premium for FTSE 100 index options. In order to mitigate the mis-specification effect on delta calculated from Black-Scholes models, they use modified delta ratios that account for skewness and kurtosis. Even with the modification, returns of a long position in delta-hedged FTSE 100 index options are significantly negative, indicating the existence of volatility risk premium.

Even though selling volatility using the equity index is shown to be profitable in the past, there are inconsistent findings of selling volatility on individual securities. Duarte and Jones (2007) use Fama-MacBeth regressions to understand the effect of volatility risk on expected returns of delta-hedged options on individual equities. They cannot make a conclusion whether the price of the volatility risk is nonzero on average but provide strong evidence of a conditional risk premium that is increasing in the level of overall market volatility. Goyal and Saretto (2009), on the other hand, indicate that the difference between historical volatility and implied volatility is strongly statistically significant in explaining the pattern of returns of both straddles and delta-hedged options on individual stocks.

Cao and Han (2013) also provide evidence on existence of volatility risk premium for individual stocks by studying the long delta-hedged option strategies. Besides volatility risk premium, they also suggest that there is additional systematic risk factors that can explain the option returns. Specifically, securities with higher idiosyncratic volatility have lower returns than low idiosyncratic volatility stock, indicating a significant negative relation between long delta-hedged option return and idiosyncratic volatility. The findings are shown to be robust and to remain significant after controlling for jump risk, transaction cost, limit to arbitrage, volatility mis-pricing and stock characteristics. With these properties, Cao and Han (2013) propose a volatility-based trading strategy using options on individual stocks. Securities are sorted based on their idiosyncratic volatility. A strategy of longing the first group and shorting the last one shows significant performance that cannot be explained by common risk factors.

There are also some studies that use covariates other than volatility, such as skewness, kurtosis or correlation risk premium. For instance, Boyer and Vorkink (2014) find that there is a robust negative correlation between the total skewness of the underlying's returns and average option returns, even after controlling for option characteristics that can influence their expected returns. Differences in average returns for option portfolios sorted based on ex-ante skewness range from 10% to 50% per week, even after controlling for risk. Their findings suggest that these large premiums compensate intermediaries for bearing the risk that cannot be hedged when accommodating investor demand for lottery-like options.

A purer bet on volatility can be achieved via a variance swap whose profit is affected by the difference between implied volatility and realized volatility. One of the earliest study is conducted by Wu and Carr (2009). They use a set of European options and futures contracts to synthesize variance swap rates and investigate the historical behavior of variance risk premium, which is defined as the gap between realized volatility and implied volatility (an opposite to what we use), on five stock indexes and 35 individual stocks. Their results suggest that there is a negative variance risk premium on stock indexes, but neither the original capital asset pricing model nor the Fama-French factors can fully account for it. Nevertheless, this is not true for individual stocks where there is no consistency in the sign of premiums, and they are also not always statistically significant. The study also finds out that there is a positive correlation between volatility risk premium and riskiness return volatility. It suggests that a negative premium arises as compensation for the return uncertainty.

Similar to Wu and Carr (2009), Driessen et al. (2006) also use model-free implied variances and find a significantly negative variance risk premium for the S&P100 but no significant negative premium on variance risk in individual options. They argue that variance risk is not priced and instead emphasize the importance of priced correlation risk as a separate source of risk (a trading strategy that sells correlation risk by selling index options and buying individual options is shown to earn excess returns of 10% per month and has a large Sharpe ratio). Another important study also use variance swap is conducted by Schürhoff and Ziegler (2011). They decompose stocks' total variance into systematic and idiosyncratic return variances and find out that while systematic variance risk exhibits a negative price of risk, common shocks to the variances of idiosyncratic returns carry a large positive risk premium. Both of them are heavily priced and cannot be explained by other standard risk factors. Differently from the argument of Driessen et al. (2006), Schürhoff and Ziegler (2011) indicate that correlation risk premia is a combination of systematic and idiosyncratic variance risk premia in the sense that it increases when systematic variances rise or idiosyncratic variances drop, so they can offset each other with their opposite sign. A more recent study by Gourier (2016) also decomposes the risk premia of individual stocks into two components which are a systematic and an idiosyncratic risk. Both of them are assumed to contain a diffusive and a jump part, which indicates that investors may exhibit different levels of risk aversion towards small and large price movements. Different from the methodology of Schürhoff and Ziegler (2011) who use variance swaps with one-month time-to-maturity to summarize the information contents of options, Gourier (2016) use all available maturities to summarize the information contents of options. She finds a negative variance risk premium for all stocks that rises in absolute magnitude when the time to maturity increases. She also shows that idiosyncratic variance risk carries a negative risk premium whose contribution to the overall variance risk premium is substantial and amounts to 80% on average.

In general, there is a consistency in the results of the existing researches about the volatility risk premium of options on stock indexes, but that is not the case for single stocks. Even though variance swap is a better choice to study properties of volatility risk premium, it is an over-the-counter investment product and is not popular enough for a strategy that can be implemented in real life. Given those reasons, we decide to make a thorough study on the nature of selling volatility's returns through the aforementioned strategies: selling options, selling statically delta-hedged options and selling dynamically delta-hedged options. Options, in our research, are European calls and puts. Selling-volatility strategies are well-known for their concentrated loss (negative skew and fat tails) in market crash times. The crash in 1987 and financial crisis in 2008 have already eliminated a big portion of return of those strategies. Our data for empirical analysis include another severe crash which is 2020 stock market crash. Another big barrier for profitability of volatility selling is the presence of new participants. Consistent excess return of the strategy attracts more speculators. Moreover, even though investors often prefer positive skewness and always try to hedge high volatility, Taleb (2004) provides evidence on negative skewness preference of delegated fund managers. Such new features will have a big impact on the nature of returns of selling-volatility strategies which will be explained in our study. The research proceeds as follows. In section 3, we develop different simulations of volatility selling under Black-Scholes-Merton world to get a preliminary understanding about their patterns of returns. In section 4, we discuss our dataset and methods for structuring it. Then, expectations and hypothesis from section 3 will be tested in section 5.

3 Theoretical Analysis

3.1 Overview of theory and notations

Options are derivative products that give buyers the right to buy (call options) or the right to sell (put options) the underlying assets at predefined prices (strike prices). As calls allow buyers to capture upside and puts allow buyers to protect from downside, sellers of options are offering buyers a financial insurance against those respective occurrences.

We use most of the notations from the Black-Scholes-Merton world, like S as an underlying's price; C is the price of the call; P is the price of the put; K is the exercise price; T is the time to maturity. Prices for the European options can be deduced as:

$$C = \Phi(d_1)S - \Phi(d_2)Ke^{-rt} \tag{1}$$

$$P = \Phi(-d_2)Ke^{-rt} - \Phi(-d_1)S, \tag{2}$$

where:

$$d_1 = \frac{1}{\sigma\sqrt{t}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \tag{3}$$

$$d_2 = \frac{1}{\sigma\sqrt{t}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \tag{4}$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}z^2}dz \tag{5}$$

Having established the notation that we use in the subsequent research, we draw attention to the key corollaries of these calculations. Namely, two greeks of European options – delta (Δ) and vega (ν).

3.2 Set-up

In this thesis, we focus on three types of strategies for both calls and puts executed in the same fashion correspondingly. The first type is a simple short selling and closing the position in a predetermined number of days. The second strategy is a short position in an option *statically delta-hedged* with a long position in the underlying, i.e. long in *delta* parts of the underlying held until the end of the holding period without rebalancing. The third strategy is a short position in an option *dynamically delta-hedged* with a long position in the underlying, i.e. similar to the previous one, but rebalancing every working day.

Delta, Δ, is a measure of the rate of change of the option's calculated value, $C(.)$ or $P(.)$, with respect to the change of the underlying assets' price, S. Δ of a call and a put can be inferred through the Black-Scholes formula as follows:

$$\Delta_{call} = \Phi(d_1) \tag{6}$$

$$\Delta_{put} = \Phi(d_1) - 1 \tag{7}$$

As we formulate strategies of interest of this thesis, we need to keep in mind that delta of a call is always positive and grows with the strike price from 0 to 1. In contrast, delta of a put is always negative and comes closer to 0 from -1 with strike.

Vega, ν, is a measure of the rate of change of the option's value, $C(.)$ or $P(.)$, with respect to the change of the volatility of the underlying assets' return, σ. ν of a call and a put can also be calculated using the Black-Scholes formulae as follows:

$$\nu_{call} = \nu_{put} = S\phi(d_1)\sqrt{T} \tag{8}$$

One can make an important observation from the above ν-formula that $Vega$ is always positive. Indeed, higher volatility should increase the value of an option, ceteris paribus, because the latter allows risk hedging.

Most variables that are necessary for determining an option price are available in the market for any contract. Two exceptions are the expected return and volatility of the underlying asset's return. While expected return of an asset can be approximated from the market data, it is universally accepted to infer the volatility from the market prices of an option. Thus, we can obtain an *implied volatility (IV)*. Naturally, one would be interested in checking whether that IV matches the *realized volatility (RV)* for the period of the option's life.

In a perfectly fair market, they are supposed to be the same, on average. Of course, it is impossible (even in a perfectly fair market) that RV would be exactly the same as IV all the time for all options, because that would violate the very nature of financial markets – unpredictability. But that topic is not a focus of our research.

It is well-known that even the IV itself is not the same for different strike prices of an option written on a very same underlying. Nevertheless, even if IV and RV can not be equal all the time it is worth checking if there is a systematic (statistical) difference between the two. Many researchers found such statistical anomalies for options written on assets of various classes. For individual stocks, such anomaly differs from firm to firm, however, for equity indices the difference between IV and RV remains persistently different from zero.

The main observations in the literature dedicated to this topic can be summarized in two statements:

- IV of an equity index is statistically higher than its RV, but the difference between them $(IV - RV)$ has a negative skewness.
- Buying (selling) options on an equity index results in statistically significant negative (positive) returns.
- Positive (negative) returns of a short position in an option are associated with a positive (negative) difference between IV and RV.

Thus, it is natural to call long (short) position in an option *volatility buying (selling)*. Indeed, the data support the argument that IV usually exceeds RV and we showed that in graph 1 and provided a summary statistics in table 1.

A strategy of selling volatility, therefore, can be implemented by writing options. If traders sell only calls or puts, returns are mainly driven by underlying assets' returns rather than its volatility exposure. However, in the classic Black–Scholes–Merton world, option traders can continuously delta-hedge to remove the directional exposure. Delta-hedging for a short position of an option requires the trader buy offsetting amounts (delta) of the underlying securities. In reality, it is difficult and costly to continuously hedge. This together with inconstant volatility, makes delta-hedging for option selling imperfectly hedge the market risk and capture the volatility risk.

3.3 Payoff illustration

Using the above formulas, we will now present several simulations under different scenarios to get a preliminary understanding about the pattern of selling volatility in the classic Black–Scholes–Merton world. For that purpose we focus on two out of three strategy types as it is conceivable to depict only their dollar and relative payoffs on a graph:

- a simple short strategy,
- a statically delta-hedged short strategy.

We consider *two main dichotomies* of scenarios. The *first* dichotomy of scenarios of interest is inspired by all the previous researchers whose main unit of observation was the return of a short option strategy where an option was held until maturity. The only tuning parameter was the time to maturity – the time at which to enter the position. We propose a different way – set a holding time period fixed for all strategies and close the position at the end of each of them regardless of whether the options expire or not. Thus, we pursue our aim to make the strategies continuous. In the case of previous researches, the authors need to hold each option for n days until maturity and the day after, they then need to seek for options that expire in another n-day period. In times when options are not abundant, this might be troublesome. We consider two ways of holding on to a strategy for illustration: (1) holding until maturity, (2) holding for a specified number of working days. In section 5, we discuss which execution style is preferable and why we choose to count working days as opposed to calendar days.

The *second* dichotomy is set to showcase how those strategies perform in persistently calm periods and what patterns they show when the economy (the underlying) enters a crisis mode. We approximate those with different levels of volatility σ. Notably, two dichotomies should give four scenarios for each strategy type, however; holding an option until maturity makes the current volatility (at maturity) irrelevant, so we will have only three scenarios for each strategy. We describe the choice of particular values for the Black-Scholes variables in subsection 3.5.

In figure 2, we provide a detailed derivation of the final dollar payoff of a simple short call position. The grey solid line depicts the intrinsic value of the call. The dashed grey line shows the price of a call with strike $K_3 = 1962.5$ (as mentioned before, we describe why we choose such values in subsection 3.5) at time moment 0 for all possible spot prices of the underlying. The green dot indicates the call price at the assumed spot $S_0 = 2000$: $call_0 = 64.00881$. In case of a short call position, the green dot pins down the only cash inflow. And, the only cash outflow happens at the end of the holding period. Blue lines indicate those losses: if the short call position is closed at maturity (dashed line), after 10 working days closing at a low volatility (solid line) and a high volatility (dotted line). The total dollar payoff is the difference between $call_0$ and the loss (blue line). It is coloured in red and matches the line type of the blue counter-part.

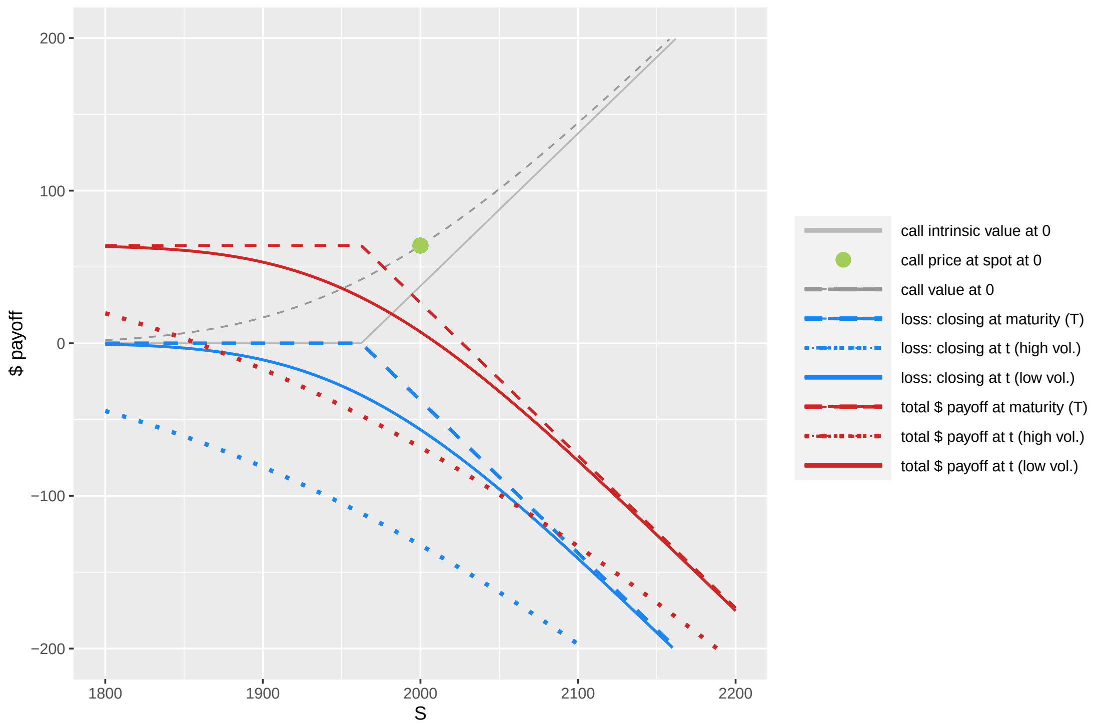


Figure 2: Dollar payoffs from a short position in a call option bought at spot $S_0 = 2000$, $K = 1962.5$, holding period$t = 10$ and time to maturity $T = 30$ working days, $\sigma_{low} = 0.15$, $\sigma_{high} = 0.5$, $r_f = 0.01$. Dashed grey line indicates the values of the call option at time t at all possible spots. The green dot is the call price at spot $S_0 = 2000$. Blue lines indicate the losses from the short call position at closing: at maturity (dashed), at t with low σ (solid), at t with high σ (dotted). Red lines are the corresponding total net payoffs.

Similarly, in figure 3, we illustrate a detailed derivation of the final dollar payoff of a statically delta-hedged short call position. The gray solid and dashed lines as well as the green dot are as before. Blue lines are the same as before. However, when entering the short call position at time moment t, we also buy Δ amount of the underlying (S&P500 index). Hence, the total cash flow at t is negative and equal to $(call_0 - \Delta S_0)$. In this case, $\Delta = 0.6603229$. At the end of the holding period, this position loses the value of the call, but receives Δ amount of the index. Blue lines indicate those losses as before. In addition, we have a green solid line that shows the value of the long index position less the initial cash outflow: $\Delta \cdot (S_t - S_0) + call_0$. Since, at $S_t = S_0$, that value is equal to $call_0$, this green line goes through the green dot (price of the call at 0). The total dollar payoff is the difference between the green line (long index position gain/loss plus the initial cash inflow from call selling) and the short call position loss at closing (blue lines). It is coloured in red and matches the line type of the blue counter-part as before.

In figure 2, it can be seen that a short call position on S&P500 is indeed a bet against the market, because it only pays off when the index price does not grow substantially by the date of closing the short position. Around S_0, the payoff from waiting until maturity is somewhat more sensitive to movements in the index price than that from holding for 10 working days, while far from S_0, their sensitivities converge. In addition, we can see that waiting until maturity can potentially deliver higher dollar payoff but at a risk of bigger index price moves. since its holding period is larger. Hence, this is in line with a regular rule in financial theory that higher expected return comes only with higher volatility.

One can also see that during "bad" times, when the volatility of the market increases (the volatility of the index returns increases as well), the room for a positive payoff shrinks, however crisis times are associated with downward market movements, so entering the crisis mode of the economy can potentially lead to substantial dollar gains for this strategy. Nevertheless, it is sensible to expect that the price of the underlying will not decrease dramatically enough to bring a positive payoff, thus, entering high volatility state should on average deliver negative returns.

In figure 3, one can clearly see that there is a tiny scope of S&P500 prices at time moment t that results in a positive payoff. Outside of that scope returns are negative and, most importantly, quite substantial compared to maximum possible gains. Notably, during a holding period of 10 working days the underlying's price is not expected to change much. That explains why this strategy is expected to give very frequent yet small positive returns and suffer rare but dramatic negative returns. In fact, most researchers indicate exactly that result in their papers. Ilmanen (2012) summarizes those observations by asserting that delta-hedged strategies are in the "carry" family, i.e. are expected to deliver small but steady positive returns with a substantial negative skewness. That is even more evident in times when the market enters a crisis. The dotted line is entirely beneath the zero-line, so if an option is shorted during benign times but bought in times of crisis (with high volatility and, hence, high option prices, ceteris paribus), that strategy will show negative returns. That what follows from the nature of the delta-hedged strategy by construction.

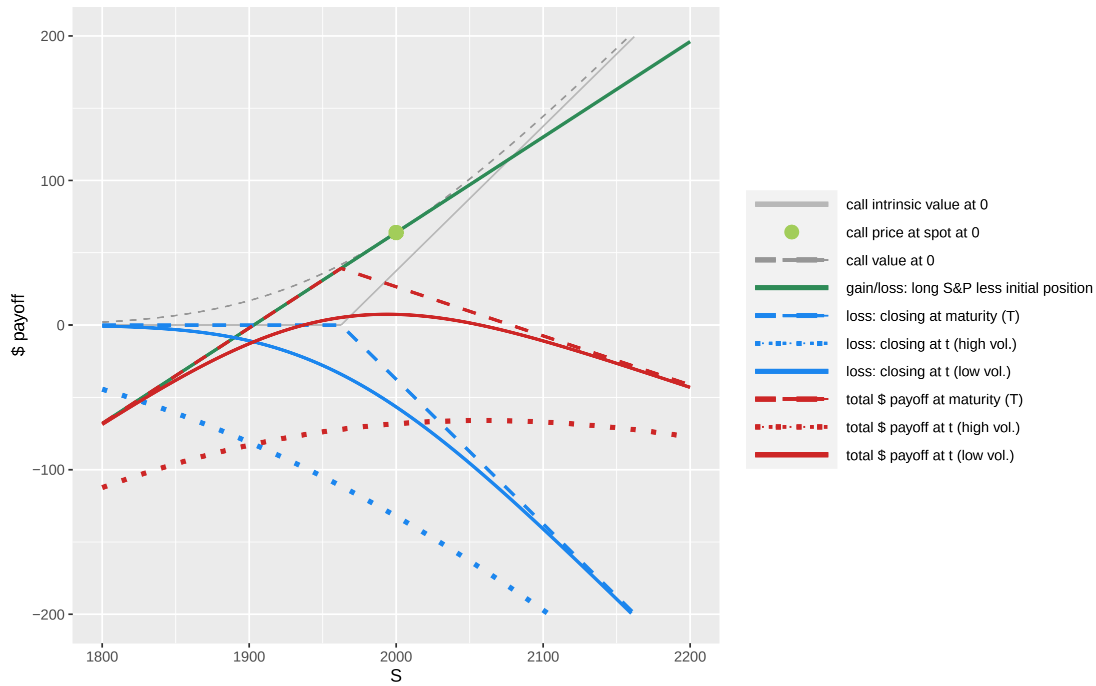


Figure 3: Derivation of a net dollar payoff from a statically delta-hedged short position in a call option with $S_0 = 2000$, $K = 1962.5$, $T - t = 10$ days, $\sigma_{low} = 0.15$, $\sigma_{high} = 0.5$, $r_f = 0.01$, days to maturity at t equal to 30 days. Dashed grey line indicates the values of the call option at time t at all possible spots. The green dot is the call price at spot $S_0 = 2000$. Blue lines indicate the losses from the short call position at closing: at maturity (dashed), at T with low IV (solid), at T with high IV (dotted). The solid green line indicates the gain/loss from the long position in $\Delta \cdot S$ bought for $S_0 = 2000$ at time t when $\Delta = 0.66$. Red lines are the corresponding total net payoffs.

The detailed derivations of the dollar payoffs for a short put position and a short delta-hedged put position are presented in figures 10 and 11, respectively, in appendix A.1. The dynamically delta-hedged returns are impossible to depict on a graph, because they depend on the sequence of the index prices for each day in the holding period and require a multidimensional illustration.

3.4 Defining returns

Having described dollar payoffs of the short call strategies (and respective put strategies in the appendix) under different scenarios, we summarize returns of those strategies with different strike prices. The reason why we need to analyze percentage returns rather than dollar payoffs is because the latter differ quite substantially across strategies due to their construction, which can make the analysis unreliable.

But before doing that, we need to clarify how to calculate the return using the dollar payoff from a strategy. Let us, first, formalize the dollar payoffs of the two described strategies. Let $\pi_1(0,t)$ be the dollar payoff of the short call strategy held for t working days starting from time moment 0, assuming we can invest the initial proceeds into a deposit with a risk-free rate of return. Hence,

$$\pi_1(0,t) = C(0|X_0) \cdot e^{r_f \cdot (t-0)/250} - C(t|X_t), \tag{9}$$

where X_w is the universe of all relevant variables (state of the world) that determine the price of an option at time moment w.

In case of a simple short call, the calculation of the gross return is straightforward – just divide by the initial proceeds:

$$R_1(0,t) = \frac{\pi_1(0,t)}{C(0|X_0)} \tag{10}$$

It is important to point out that, in fact, $C(0|X_0) - \Delta_0 \cdot S_0$ can never be higher than zero. We can demonstrate that using the derived values of interest from equations 1 and 6:

$$C(0|X_0) - \Delta_0 \cdot S_0 = \Phi(d_1)S - \Phi(d_2)Ke^{-rt} - \Phi(d_1)S = -\Phi(d_2)Ke^{-rt} < 0 \tag{11}$$

In case of a statically delta-hedged short call strategy, assuming that an agent has an initial capital to invest in a long position, the dollar payoff is determined as follows:

$$\pi_2(0,t) = (C(0|X_0) - \Delta_0 \cdot S_0) - C(t|X_t) + \Delta_t \cdot S_t \tag{12}$$

For this strategy, most researchers use several types of *"scaling"* and show results with all possible approaches and sometimes arrive at similar results. The main ways to *"scale"* and arrive at a percentage return are:

- divide $\pi_2(0,t)$ by the initial call price $C(0|X_0)$ (Bakshi and Kapadia (2003), Fan et al. (2016), etc.)
- divide $\pi_2(0,t)$ by the initial price of the underlying S_0 (Bakshi and Kapadia (2003), etc.)
- divide $\pi_2(0,t)$ by the absolute value of the initial net cash flow $|C(0|X_0) - \Delta_0 \cdot S_0|$ (Cao and Han (2013), Ruan (2020), etc.)

That last approach is, coincidentally, the most sensible way to calculate the percentage return from the delta-hedged strategy. To prove that, assume we have a cash reserve of $Q(0)$ at time moment 0. Assume we want to invest all of it into the delta-hedged strategy, so after entering the position we will end up with 0 at hand. To do that we simultaneously sell z parts of a call option with premium $C(0|X_0)$ as well as buy z parts of $\Delta_0 \cdot S_0$ (of the underlying asset) such that we are left with 0 at hand:

$$Q(0) + z \cdot (C(0|X_0) - \Delta_0 \cdot S_0) = 0, \tag{13}$$

where z is such that $z \cdot (C(0|X_0) - \Delta_0 \cdot S_0) = Q(0)$.

By time moment t, when the position is closed, the value of the portfolio becomes:

$$Q(t) = z \cdot (-C(t|X_t) + \Delta_t \cdot S_t) \tag{14}$$

Therefore, we find the gross return of this strategy as the proportion between the ending and the beginning values of this portfolio. And, therefore, the net return is defined as follows:

$$\begin{aligned} R_2 &= \frac{Q(t)}{Q(0)} - 1 \\ &= \frac{z \cdot (-C(t|X_t) + \Delta_t \cdot S_t)}{z \cdot (-C(0|X_0) + \Delta_0 \cdot S_0)} - 1 \\ &= \frac{-C(t|X_t) + \Delta_t \cdot S_t + C(0|X_0) - \Delta_0 \cdot S_0}{-C(0|X_0) + \Delta_0 \cdot S_0} \\ &= \frac{\pi_2(0,t)}{\Delta_0 \cdot S_0 - C(0|X_0)} \end{aligned} \tag{15}$$

The other two approaches are admitted by the authors to be just some generic ways to scale the dollar returns to make them comparable across different strategies, because they can (and do) have very different dollar returns, which makes the analysis of pure dollar returns vulnerable to the sizes (premiums) of the options and deltas.

3.5 Comparing return patterns

As the next step we investigate possible return patterns not only under two main dichotomies but also for eight different levels of moneyness.

To simulate scenarios as close to the real-world cases as possible, we selected the following values for the Black-Scholes formula:

- Spot of S&P500 at time moment 0: $S_0 = 2000$,
- The risk-free rate: $r_f = 0.01$,

- Strike prices: $K = \{1912.5, 1937.5, 1962.5, 1987.5, 2012.5, 2037.5, 2062.5, 2087.5\}$,
- Time to maturity $T = 30$ working days. If held for a shorter period of time, the holding period is $t = 10w.d.$
- Volatility during "good" times is $\sigma_1 = 0.15$.
- Volatility during "bad" times is $\sigma_2 = 0.5$.

The choice of the particular strike prices comes from our decision to focus on the most liquid option with moneyness between 0.95 and 1.05. We define moneyness as:

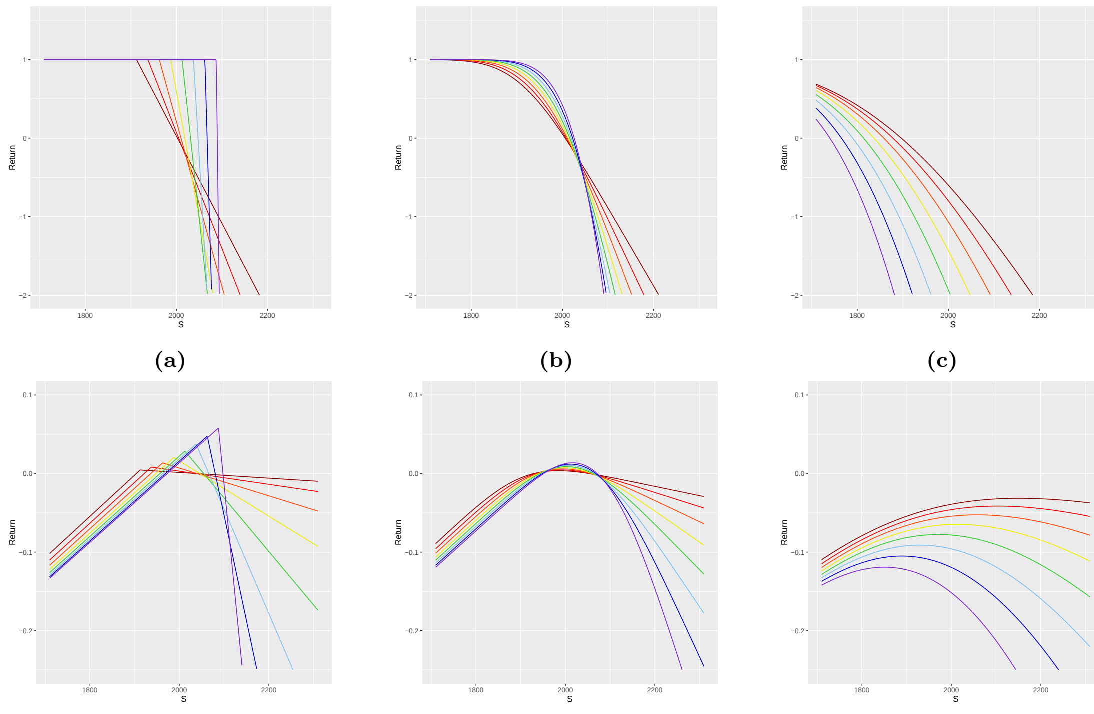
$$m = \frac{S_0 e^{r_f \cdot T/250}}{K}. \tag{16}$$

Omitting the $e^{r_f \cdot T/250}$ part for its negligible value and for simplicity, we divide $m \approx S_0/K$ into eight equally sized brackets: $[0.95; 0.9625], \cdots, [1.0375; 1.05]$. Subsequently, we choose such strikes that make an option have moneyness in the middle of those brackets. Hence, we get $K_1 = \frac{0.95+0.9625}{2} \cdot S_0 = 1912.5$ and so on. We assign colors from the warmest to the coldest (burgundy, red, orange, yellow, green, light blue, dark blue, purple) to the strike prices from the lowest to the highest, so it is easy to remember.

In figure 4, one can see returns on short call positions (4a, 4b, 4c) and delta-hedged call selling (4d, 4e, 4f), held until maturity (4a, 4d) and for 10 working days where volatility stays low (4b, 4e) and increases (4c, 4f).

In figure 4a, in which options are kept until maturity, we can see that OTM (purple, dark blue) options' returns are steeper than ATM (yellow, green) and ITM (red, burgundy) options, which demonstrates a higher level of sensitivity to the price increase of the underlying assets. This should translate into higher market beta for lower moneyness levels. However, with higher strike prices (lower moneyness), OTM options, even though with lower premium, also offer a bigger buffer against underlying asset's upside movement for sellers unless that shift in the price is large enough.

If a short position in call option is closed before maturity under the scenario that volatility remains unchanged (figure 4b), the buffer against loss when the index appreciates shrinks as the position is held only for 10 days. In a scenario similar to the one illustrated in figure 4b, but with an increase in volatility, 4c indicates that not only OTM options are still the most sensitive to the change in the price of the underlying asset, but also their high strike prices implying lower premiums aggravate the losses. So OTM options are the most vulnerable in periods of considerable increase in volatility which coincide with crisis times. One can see that from how far each line moved from where they were on figure 4b. Red lines (ITM) almost did not move, while blue ones (OTM) shifted the most.



(a) (b) (c)

(d) (e) (f)

Figure 4: Returns on short call positions (a, b, c) and delta-hedged short calls (d, e, f) held until maturity (a, d) and for 10 working days where *volatility* stays low (b, e) and increases (c, f). Each color corresponds to a call with a particular strike price: 1912.5 (burgundy), 1937.5 (red), 1962.5 (orange), 1987.5 (yellow), 2012.5 (green), 2037.5 (light blue), 2062.5 (dark blue), 2087.5 (purple). $S_0 = 2000$.

Statically delta-hedged call positions (hedged only at the beginning when options are sold) are shown in figures 4d, 4e, 4f. As anticipated, delta-hedge reduces the losses in cases of a large shift in the price of the underlying, but more so for ITM (as blue lines on the right side move only slightly). So, this strategy is profitable when stock price changes in either directions (increasing price still outweighs decreasing price because of the partial hedge), but only slightly. The expected return of OTM delta-hedged call strategies seem to be higher than ATM and ITM when price of the underlying asset do not change much. And, across all strikes we see that this strategy is profitable around the initial price of S&P500. This is the reason why in many papers, selling volatility is shown to have a positive mean return with a large negative skewness – even though its losses, that coincide with market down movements, can be severe, they happen rarely. Also worth noting that the exposures to the downside shifts is very close for all the moneyness levels, however, the exposure to the upside movements goes down with moneyness.

Figure 4f indicates such rare event when volatility increases dramatically (making VRP which is the difference between implied volatility and realized volatility go down). In this case, ITM options demonstrate the smallest losses thanks to their higher premiums. Also, in that scenario, we can see that ITM (OTM) calls show the smallest (highest) sensitivity both to the market movement as well as the volatility change. The latter can be seen from comparing figures 4b and 4c, where ITM call return levels (red lines) changed slightly compared to dramatic reduction of the OTM call returns (blue lines).

The reason for this is probably that when prices go up, there is a higher chance that buyers of calls will exercise their rights, making the OTM sellers lose the most, while when market goes down, strike price might not be attractive enough. Also, holding until maturity allows higher possible returns at the expense of the risk of larger shifts in the price of S&P500, which can be observed by comparing figures 4d and 4e. In normal times, it seems to be better to hold the position until expiration dates, while in periods with large shifts of in the price of the index, closing the short position before maturity is a wiser action. So holding on to a short position longer, obviously, bears more risks. However, it does not make almost any difference in case of a sharp fall of the index (or crisis period) as we described above. As other researchers, like Bakshi and Kapadia (2003) and Coval and Shumway (2001), often work with held-until-maturity returns, in this study, we would like to investigate whether closing the position before maturity diminish profitability of volatility selling strategy.

Similarly, we illustrate patterns of put returns under different scenarios in figure 5. In general, selling a put has identical characteristics to those of selling a call, however, while a call can give investors a lottery, put offers them hedge against crashes.

Figure 5a, which depicts returns of a put selling strategy held until expiration, indicates that OTM (burgundy, red) options' returns, again, are more sensitive to the fall of the price of the underlying asset than ATM (yellow, green) and ITM (purple, dark blue). However, also similar to call, an OTM put, even though with lower premium, also offers a bigger buffer against underlying asset's downside movement for sellers unless that shift in the price is large enough because of its low strike price. When volatility does not change and we buy back put option after 10 working days (figure 5b), OTM puts still have the highest sensitivity to the down movement of the underlying. To simulate a scenario which is identical to crisis times, volatility used in calculation is raised sharply. This can be seen in figure 5c. Characteristics of puts in different moneyness levels are still analogous to those of a short call strategy. OTM puts continue to suffer the most in periods of considerable increase in volatility.

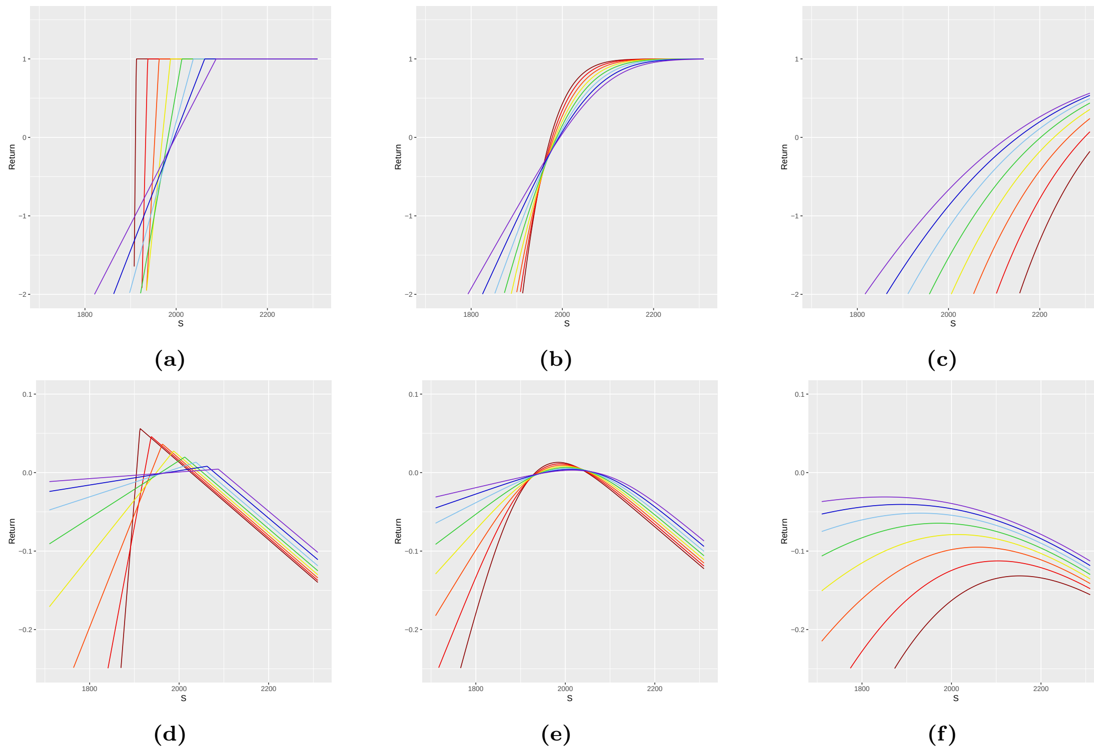


Figure 5: Returns on short put positions (a, b, c) and Δ-hedged put selling (d, e, f) held until maturity (a, d) and for 10 working days where *volatility* stays low (b, e) and increases (c, f). Each color corresponds to a put with a particular strike price: 1912.5 (burgundy), 1937.5 (red), 1962.5 (orange), 1987.5 (yellow), 2012.5 (green), 2037.5 (light blue), 2062.5 (dark blue), 2087.5 (violet). $S_0 = 2000$.

Figures 5d, 5e, 5f illustrate returns of a statically delta-hedged put position for different moneyness levels. As anticipated, delta-hedge reduces the effect of the exposure to the changes in the price of the underlying asset. The return of OTM strategies continues to be higher than ATM and ITM in non-crisis periods when the price of the underlying asset is around its initial position. More precisely, the return grows with moneyness. These can be seen in figures 5d and 5e. OTM puts, on the other hand, suffer the most from the impact of volatility surge (figure 5f). In this case, similar to call, ITM options deliver the smallest losses, due to their higher premiums. Also, in that scenario, we can see that ITM (OTM) puts show the smallest (highest) sensitivity both to the market movement as well as the volatility change. The latter can be seen from comparing figures 5b and 5c, where the profit level of ITM strategy changed slightly compared to the significant fall of the OTM strategy's profit. It is worth noting, that all of delta-hedged puts, from ITM to OTM, have quite similar pattern in case of market up movement, but are considerably different when the market goes down. That is a reverted pattern from that of the calls. Also, holding until maturity allows higher possible returns at the period of small shifts in the price of the underlying asset.

3.6 Main expectations and hypotheses

After reviewing previous studies and observing patterns of simulated options returns in Black-Scholes-Merton world, we outline what type of results we expect in our empirical analysis.

To begin with, selling volatility (in case of all six strategies) is expected to deliver returns higher than the risk-free rate, on average; especially the delta-hedged ones which are partially eliminated the exposure to the market movement. Overall, there can be several explanations for the existence of this premium. It may reflect the compensation for the systematic risk since the strategies' losses coincide with market's crashes. Therefore, all those strategies are expected to show negative skewness, with the most negative one for the OTM strategies. We also expect OTM strategies to have higher standard deviations and kurtosis. Therefore, it is not clear what to expect from their Sharpe ratios.

Secondly, with respect to different levels of moneyness, we expect the returns of the OTM option strategies, on average, exceed those of the ITM and ATM, as their payoffs are higher during calm periods which are the most frequent. In other words, the mean return for call (put) strategies should go down (up) with moneyness. In addition, demand from investors for preventing the downturn risk may be a partial explanation for the OTM options to be expensive and, thus, deliver higher returns than ITM and ATM. Benzoni et al. (2010) show evidence on the expensiveness of OTM put options after the crash in 1987 due to a higher risk aversion. They indicate that buyers are willing to pay a high price for those options as a tool for the tail risk protection. Hence, the larger profitability of OTM options may imply the demand for the insurance in the market turmoil. However, we expect that in cases of market crashes OTM option strategies deliver the most negative returns. The trade off between the two is ambiguous. Therefore, at this point we are not sure if average returns are statistically significant and which moneyness levels or hedging decision delivers higher Sharpe ratio. However, usually, when a strategy is characterized with small frequent gains and rare large losses, the mean tends to be significantly positive with a negative skewness. So, our projection is that OTM strategies will show more desirable mean and Sharpe ratio that the others.

Thirdly, in comparing the graphs of returns for call and put strategies, we can observe that calls suffer in period of increasing price of the underlying asset more than puts; conversely, put strategies struggle in a bearish market more than calls. Hence, their reactions to downside market are also different. We expect that selling volatility strategies using put have higher exposure to downside market factor than call strategies. Also, there should be bigger discrepancy between downside market exposures between different moneyness levels for puts than that for calls. Nevertheless, in general, the exposures to those factors should not be strikingly different for delta-hedged call and put strategies, because, overall, the graphs of their returns are quite similar.

Next, delta-hedged options strategies are theoretically better at capturing volatility risk than simple option selling. So, we conjecture that in a regression of the strategies' returns against volatility risk premium, the coefficient of VRP in the case of delta-hedged strategies, should be higher than in the case of simple option selling strategies. We also anticipate that VRP explains higher share of the target variable's volatility which should be reflected in the coefficient of determination. However, as we can see in figures 4 and 5, returns and standard deviation for a simple short option and a delta-hedged option strategies are not comparable, which many researchers ignore. Hence, it is difficult to compare the impact of VRP on each strategy. To solve this problem, we implement a risk-parity approach, such that each strategy has the same standard deviation with the market. We explain it in detail in section 5.3. Also, we expect that a dynamic delta-hedge will increase the exposure of the strategy to VRP, as the theory suggests, even though we cannot provide an illustration for that.

Last but not least, we also expect that OTM call strategies have higher exposure to VRP than ATM, and ITM should have the lowest, since OTM calls payoff illustrations show higher sensitivity to it. The same pattern is expected for the puts. In other words, the exposure to VRP should go down (up) with moneyness for call (put) strategies.

4 Data

4.1 Sources of data

In this paper, we use data from the equity market and the equity option market. Our focus is on the call and put options written on the equity index S&P500. The most popular provider of this data is OptionMetrics via Wharton research data services (WRDS). We use all available data, i.e. from 1^{st} of January, 1996 to 31^{st} of December, 2020. Through the Center for Research in Security Prices (CRSP) via WRDS we also obtained data on the index itself. Finally, Keneth-French data library provides daily Fama-French factor returns and risk-free rates. In particular, we use the following information from each of those data sources:

- OptionMetrics via WRDS:
  - daily closing option prices (the best closing bids and the best closing asks),
  - expiration date,
  - exercise style (we use only European),
  - security IDs (unique for each option with a specific expiration date and a specific strike price),
  - implied volatility,
  - annualized realized volatility of the daily S&P500 returns for different number of days,
  - the greeks (Δ, Γ, Θ)
  - daily trading volumes,
  - open interest,
  - date of observation etc.
- CRSP via WRDS:
  - daily closing prices of S&P500,
  - daily returns on S&P500,
- Keneth-French data library:
  - market excess return,
  - SMB (return on long nine small-capitalization stock portfolios, short nine big stock portfolios)

  - HML (return on long two value portfolios, short two growth portfolios)
  - RMW (return on long two robust-income portfolios, short two weak-income portfolios,
  - CMA (return on long two conservative investment portfolios, short two aggressive investment portfolios)

In this thesis we use the following notation:

- S_x is the price of the underlying (S&P500) at time moment x.
  - $x = 0$ is the moment of entering a position,
  - $x = t$ is the end of a specified holding period, in our case, 10 working days,
  - $x = T$ is the moment of maturity,
- K is the strike price
- C_x and P_x are the prices of a call and a put options at time moment x,
- r_f is the risk-free rate

4.2 Data Cleaning

When we investigated the obtained data set on option prices, we tried to collect observations belonging to each option separately. Quickly, we noticed that *security ID* is the variable that serves that purpose. There are observations that have missing parameters, like greeks and implied volatility. When asked about the reasons for that, the WRDS consultant replied that it indicates no trading occurring that day. In fact, if an observation does not have at least one of those variables reported it does not have the rest of that group. However, it was not a suitable step to get rid of them all in the beginning because that would have caused breaking the consistency of the data flow.

That problem is a part of a bigger one. Cleaning data from illiquid assets. One of the necessary steps in our research is to narrow the available data set to mostly liquid options, so that their prices are not distorted by the lack of trading and our results do not suffer from unreliable prices. Most papers, like Bakshi and Kapadia (2003) and Coval and Shumway (2001), use a forward looking approach, i.e. they dropped entire options from analysis if they had at least one day suspicious of being illiquid. For instance, getting rid of stale options fits into that category. Stale options are the ones whose prices have not changed from the day before.

If such options are eradicated that makes the further analysis biased. We avoid that step and perform our analysis as if at each point in time in the data set we do not know what will happen next.

Similarly, we do not exclude options that have low or zero trading volumes at least one day during a specific holding period. Instead, we winsorize the data set at each given point in time based on the available information on that day. We build our analysis around three types of strategies, each of which require slightly different data cleaning.

One of our objectives is to demonstrate a time-varying profitability of those strategies and their cumulative return through time. For that reason, we decided to divide the available time frame into windows of 10 working days and enter a position (short sell a certain value of all available and relevant options) on the first day of each window and close those positions on its 10^{th} day. The next window start on the last day of the previous window. Thus, we can generate a seamlessly continuous flow of returns. This goes in contrast with other papers, since they held option positions until maturity. Their approach causes breaks in the sequence of returns, because not always after the day of maturity there are options in the market that will expire in an exact number of days. Some researchers approached this problem differently. They were looking for options that expired *close* to the end of the month. The problem occurs when there are several non-working days.

Importantly, we avoid forward-looking and, hence, winsorize the data set at the first days of each window.

First, we exclude all options that violate no-arbitrage rules:

$$S_0 \geq C_0 \geq max(0, S_0 - K), \tag{17}$$

for calls and

$$K \cdot e^{r_f T} \geq P_0 \geq max(0, K - S_0), \tag{18}$$

for puts.

Secondly, following the example of Coval and Shumway (2001), Cao and Han (2013), and most other researchers, we remove all options that:

- expire in more than 60 calendar days,
- have the best bid lower than 10 cents,
- have a price (an average between the best bid and ask prices) lower than 12.5 cents,
- have a bid-ask spread higher than 40% of the price,

- have implied volatility lower than 1% or higher than 100%.

Overall, this process helps identify options which are liquid in a given moment (the first day of a window) and use those indicators as a proxy for fair market prices. Even for the dynamically delta-hedged strategies we do not get rid of options that do not have a delta available at least on one of the days within a window. We checked that sticking to that method allows for maximum of 4 days with missing deltas. We decided that in those case the rebalancing will be executed only on days with available deltas.

Our approach is based on the current available data and does not look into the future. This is either ignored or not emphasized in most of the previous papers. For instance, they get rid of stale options and perform other similar forward-looking winsorizing actions. That makes the results of their analysis impossible to replicate in the real world. We keep our focus on strategies that are conceivable for a common practitioner and continue our analysis accordingly.

5 Empirical Analysis

5.1 Strategy execution and methodology

To begin with, we focus on six strategies to compare their performance and analyse whether the expectations we indicated in the theoretical part hold in empirical data. In this section, we describe how exactly each strategy can be executed to best fit our interest.

As we mentioned, most researches in the field of analysing return structure of options attempt to create a time series of returns (or a panel data) to make it easier to apply the i.i.d.r.v. (independent identically distributed random variables) assumption and all the research methods that follow from that.

As we emphasised above, a crucial part of that approach lies in how to divide the available time frame so that the continuity of the time series is not compromised. The most popular way that most researchers use to execute those or similar strategies is to start on the 1^{st} of each month (or every 4^{th} Monday) and look for all options that are *close* to being at the money and expire *close to* the end of that month. In addition to this, options for any individual stock or even the most liquid indexes were relatively scarce before 2008 and only became abundant after 2015. In figure 6, one can see how many options written on the S&P500 index expired on each day. One can see that holding on to such an approach to choose options to invest in might make the results of the analysis vulnerable in moments with very few options. Indeed, in case of all the options ever written on S&P500 after 1996, fixing time periods in which to execute a strategy results in many empty observations.

To sum up, that operation is usually impossible to execute precisely, because some moments in time options are very scarce and they used to have set dates of issue and maturity which moved due to weekends and holidays. This leads to breaks in the time series of the returns themselves. That might not be a big problem for the research results due to the law of large numbers, but we resolve that by fixing *time windows* of 10 working days in which we execute the strategies. With that approach, on the first day of the first *window*, we create an equally-weighted portfolio of all the options (calls and puts separately) that expire not earlier than at the end of that *window*. At the end of the first window we close all the positions. After that we open a new set of short positions on the last day of a previous window and hold it until the last day of the current window (which consists of only working days, hence, always has observations).

We fix the size of a *window* on 10 working days. This leads to window size

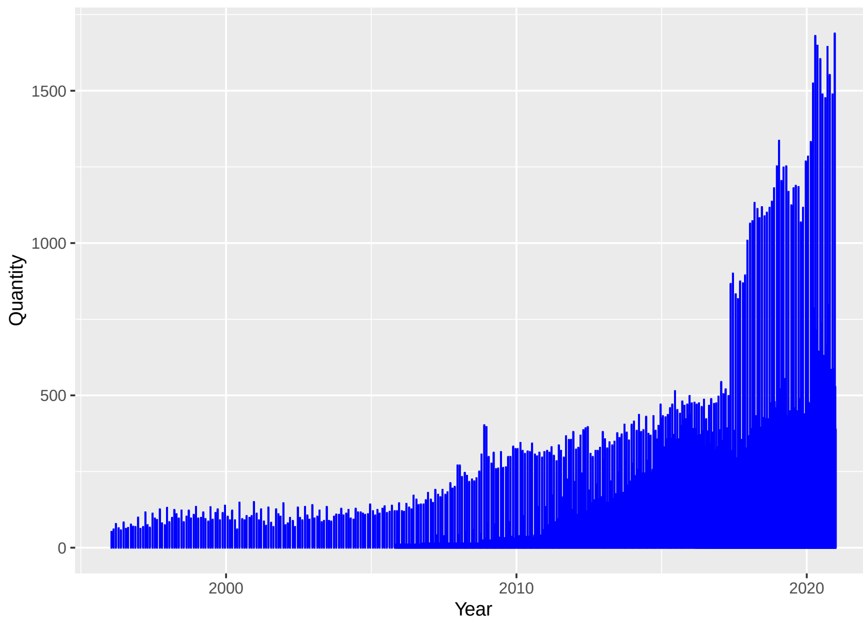


Figure 6: Number of options expiring on each day from 1996 to 2020.

of 14 (and sometimes of 15 or 16, when there are holidays on top of weekends) calendar days, but we consider that negligible.

For a simple short strategy, the selection process does not go beyond of that described above in subsection 4.2. In case of the statically delta-hedged strategy (short an option and long the index and do not change that position until the end of the window), we only select the options that have IV and Δ available on the first day of that window and winsorize further as described in subsection 4.2.

In case of a dynamically delta-hedged option (short option and long the index while changing the latter every day). Given the data set that is available to us, we cannot execute a perfectly continuous delta-hedging nor are we interested in doing that for the reason that we focus on strategies that are doable by a regular investor. However, we pursue this path to see the effect of a dynamic delta-hedge as it is asserted to reduce the exposure to the market and reveal the exposure to the volatility risk.

Overall, this selection happens from the set of available options. In figure 7, one can see the number of calls and puts circulating on each day. We focus on options that have moneyness levels between 0.95 and 1.05 and show in the perspective what share of all the circulating options they account for. The number of options of interest on a single working day was at least 30, and 60 on average before 2012. And that number exploded afterwards.

In subsection 5.2.1, we report how many of those available options actually satisfy our winsorizing methodology and other descriptive statistics.

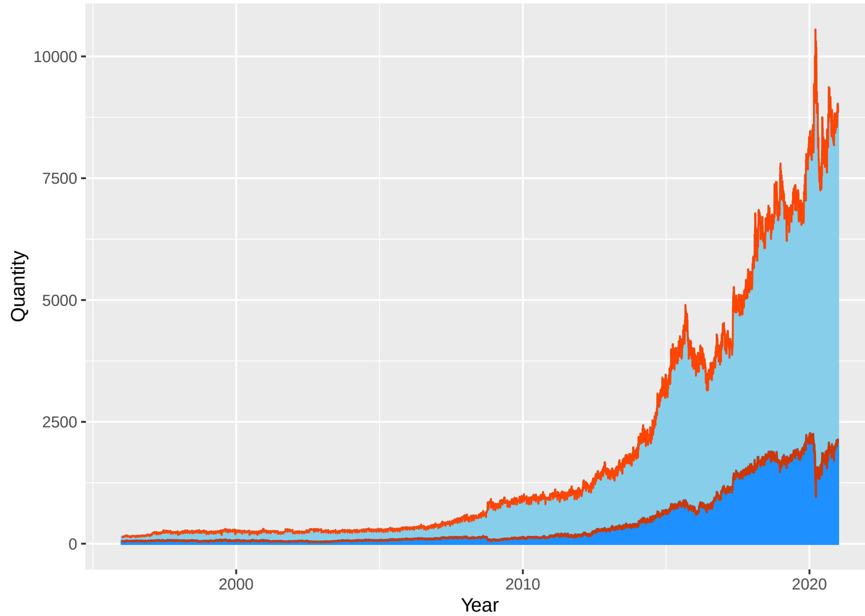


Figure 7: Number of calls (all – orange line, with moneyness between 0.95 and 1.05 – red) and puts (all – light blue area, with moneyness between 0.95 and 1.05 – bark blue ares) circulating on each day from 1996 to 2020.

Next, we showcase what parameters are needed to be calculated for our research and present different ways for that. Those ways are inspired by the previous researches in this topic. We already touched this issue in section 3 where we speak about different ways to calculate the percentage return from a delta-hedged strategy.

5.1.1 Moneyness

Most researchers (with a notable exception of Broadie et al. (2007)) focus on ATM call and put option, whereas it might be worth investigating two extremes, deeply in the money and out of the money, and also evaluate the dependence of the characteristics of interest with the strike price. Those papers usually indicate that ATM options are the most liquid and abundant. But to do that we, first, need to define moneyness. There are several ways used in this field. For instance, $S - K$ and $\frac{S}{K}$. The former one has a problem of unreliable grouping when index price and, hence, strikes, change substantially. Probably, for that reason that approach was dropped after early 2000s. The latter is reliable when considering options with similar days to maturity.

Following the notation of Chen et al. (2016), we define the moneyness of an option for each specific strike price K of a specific option (call/put) written for a specific stock index as in equation 16.

5.1.2 Volatility risk premium

The volatility risk premium is an important measure for our research. In some papers, it is defined as the difference between the realized volatility of the underlying asset (realized over the period of holding on to a particular strategy) and the implied volatility inferred from the option price at the maturity. Many researchers (including us) think that it is more appropriate to use the implied volatility at the beginning of the holding period. The latter approach makes sure that we correctly attribute the error of the market's consensus assessment of the underlying's volatility to the dates when that happened.

There are other ways to calculate the realized volatility and, hence, VRP. The most conventional way is to calculate the (annualized) standard deviation of daily returns over the holding period. Data source CRSP provides that measure for different numbers of days over which it is calculated. Another way can be to calculate *an idiosyncratic volatility* which is the standard deviation of the residuals from the Fama-French regression. Another way to define RV is the square root of the sum of squared daily returns. All these different definitions are useful for the robustness check.

Most researchers who focused on options on equity indexes always calculated VRP as the difference between VIX and RV. We introduce another approach where we take an average of all IVs in a given moneyness bracket and calculate different VRPs for each of them. This approach will take into account the volatility smile/smirk.

5.1.3 Delta

As we have presented in section 3 about theoretical analysis, implied volatility can be computed from option prices via the Black-Scholes model. Then, this obtained implied volatility is used in formulas 6 and 7 to calculate Δ for the delta-hedged strategies.

For robustness, beside implied volatility, we also adopt another measure of volatility, GARCH(1,1), to calculate a model-free volatility that takes into account its time-varying nature.

The GARCH model was introduced by Bollerslev (1986) and Taylor (1986). With GARCH(1,1), we can estimate the volatility (σ_t) using historical stock returns (r_t) and its previous lag (σ_{t-1}). To do that, we need to use the maximum likelihood approach to estimate the following specifications:

$$r_t = c + \sigma_t \cdot z_t,$$

$$z_t \sim N(0, 1),$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 * r_{t-1}^2 + \beta_1 * \sigma_{t-1}^2, \tag{19}$$

where $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$, and $\alpha_1 + \beta_1 < 1$.

Then, following Bakshi and Kapadia (2003), the τ-period GARCH volatility estimate is:

$$\sigma_\tau^G = \sqrt{\frac{252}{\tau} \sum_{i=t-\tau}^{t} \hat{\sigma_t}^2} \tag{20}$$

We then can use this σ_τ^G, beside IV, to calculate delta for the delta-hedged strategies.

5.2 Preliminary analysis

To begin with, we decided to check how many options are available in each *window* to invest in to see if the strategy proposition is even conceivable. The time frame from 1996 to 2020 comprises 629 windows of 10-working-day size. In table 2, we report the number of available options to invest in in each moneyness bracket that satisfy all the winsorizing conditions. The columns of those tables report:

- the number of windows that have *more* than (or equal to) 20 options (because 20 is isually enough to run a t-test).
  - For each such window we run a t-test for the significance of the mean of the strategies returns based on those options, so the next three columns report: how many of those windows the respective strategy for all the available options delivered returns that were significantly *positive*, *negative* or *insignificant*
- the number of windows that have *fewer* than 20 options,
- the number of windows that have *none* options available.

From those tables we can see that most windows have fewer than 20 options that we could invest in based on our winsorizing procedure. That is because before 2012 the options were not as abundant as they became after that year. But those that have more than 20 usually have a significantly negative or positive means and very rarely insignificant ones. That means that usually options in one window deliver returns of the same sign. Also, out of 629 windows only few do not have any observations.

5.2.1 Descriptive statistics

As a usual step in such research papers, we provide a summary statistics for the strategies of interest the way we defined them. Notably, returns from different options in one window cannot be considered i.i.d., because all of them have slightly different strike prices and number of days until maturity. However, that issue is smoothed out if we take an average of all the returns in one window, because, thus, return of each window can be regarded as i.i.d.

Table 4 and table 5 report the summary statistics for call and put strategies respectively.

Most of the reported returns are statistically positive, except for the call strategy (which can be seen in Panel A of table 4 and Panel C of table 4, in which p-value for some of call strategies are not below the 5% significance level and their values are also not positive). The average return of a short position in an ATM call is about -87%, while it is more than 300% for ATM put. Mean returns of other strategies (statically delta-hedged and dynamically delta-hedged) for call options are also lower than put options for each type of moneyness.

Also, as expected, in general, OTM options deliver higher returns than ATM options, and returns of ITM options are the lowest. This applies for results of both call and put strategies. All of the strategies also exhibit negative skewness and quite high positive excess kurtosis, which are the prominent feature of a typical carry strategy.

From these tables we can already see that static delta-hedge reduces the standard deviation of a short option strategy dramatically. And standard deviation of a dynamically delta-hedged strategy is at least 50% lower than that of a statically delta-hedged for different brackets of moneyness.

Another observation is that means and standard deviations usually rise when moving from ITM to OTM options. Exception is again a short call strategy. In addition, put-strategies demonstrate higher Sharpe ratios than the respective call-strategies. And for both, statically delta-hedged ones have the highest Sharpe ratios while also demonstrating an increasing trend while moving from ITM to OTM.

A clear way to depict the returns patterns of those strategies is to plot their cumulative returns along with that of the S&P500 to see how they perform compared to the benchmark, which, coincidentally, is also their underlying. But, one important step before that is to *scale* two of them. In tables 4 and 5, one can clearly see that all the strategies have different standard deviations. This makes the the graphs of cumulative returns for the short call and put strategies too volatile for S&P500. To resolve that issue, we *scale* all window returns for those two strategies by respective values so that their annualized standard deviations become equal to the annual standard deviation of the S&P500 return, i.e. 0.1951. The summary statistics of those *scaled* strategies can be seen in table 6. We can clearly observe that for the *original* strategies, OTM usually outperforms ITM and ATM. However, after calibrating volatility, there gaps in mean returns shrink.

Performance of each *scaled* strategy, compared to the market, is illustrated in figure 8. For visualisation purpose, we do not scale returns of 8b, 8c, 8e and 8f because our aim with this is to show how a portfolio being invested in those strategies would grow and it turns out that those four strategies perform comparable to the index while having lower standard deviations. We will need those scaled returns once again when we analyse their exposures to different types of risk.

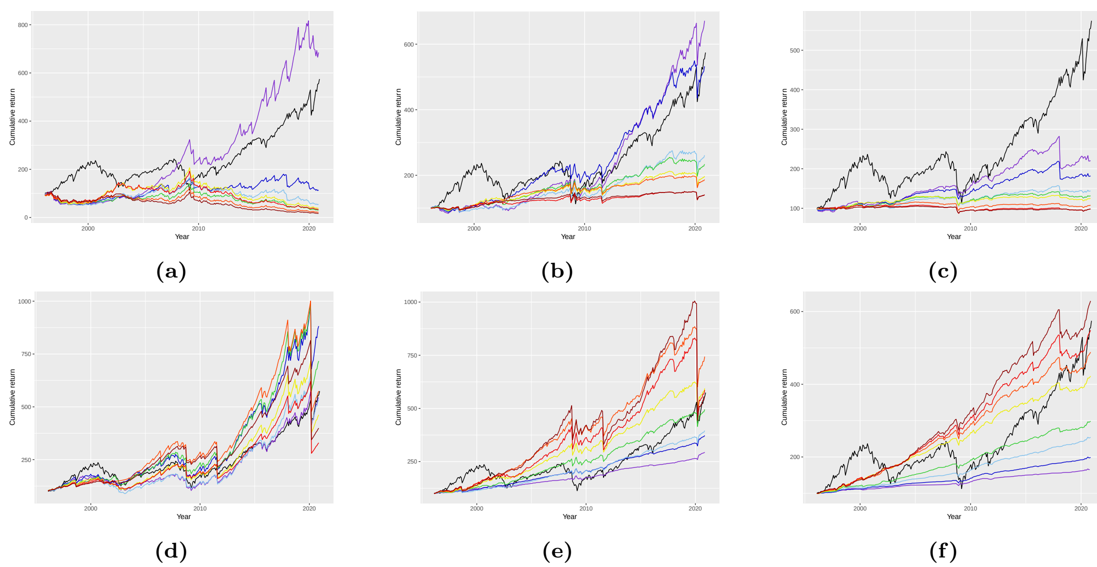


Figure 8: Cumulative returns of the S&P500 and the six strategies of interest for 8 brackets of moneyness: (a) short call (scaled), (b) statically, and (c) dynamically delta-hedged call, (d,e,f) are the same for puts. Each color (from warm to cold, as explained before) corresponds to a level of moneyness from the lowest to highest.

Cumulative returns in graphs 8 provide several interesting observations. In the usual order, the short call strategy performs better than the market only for the deeply OTM calls, whereas all the other moneyness-level-call strategies virtually stay on the same level after 24 years. Both statically and dynamically delta-hedged call strategies perform better for lower levels of moneyness (OTM, blue lines).

As expected, the short put strategies resembles the market movement the

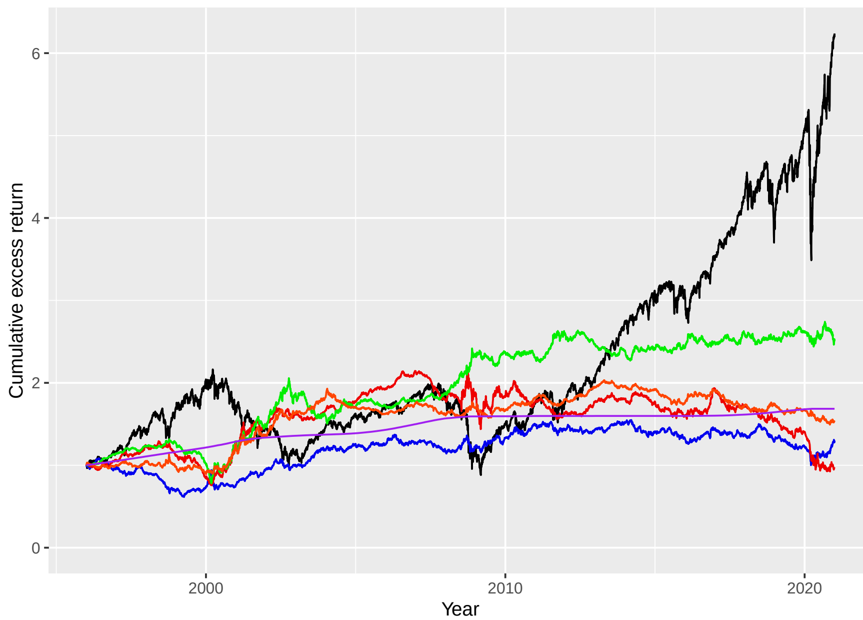


Figure 9: Cumulative market excess (black), SMB (blue), HML (red), RMW (green), CMA (orange), and risk-free (purple) returns.

closest among all six strategies. Before the latest major 2020 crisis, it performed better than the market for all moneyness brackets, however, is clearly overly sensitive to the market crashes. Hence, some portfolios were pushed lower than the market eventually. Both dynamically and statically delta-hedged put strategies perform steadily with rare but dramatic crashes, especially, those sold OTM, as they are the most sensitive to the market crashes. That is also in line with our expectations in the theoretical part. From those graphs it can also be seen that ITM put strategies are almost not sensitive to the market crashes, or at least less sensitive to them than OTM and ATM. But we can formally check that in the following subsections.

Overall, OTM strategies tend to perform better, while also being the most volatile. Put strategies have clearer patterns in their cumulative returns from moneyness to moneyness. In addition, as previous researchers claim and Ilmanen (2012) summarizes, short option strategies do behave like a typical carry-strategy. They "go up by stairs and go down by elevators".

5.2.2 Fama-French 5-factor models

The next step in our analysis will be the attempt to describe the volatility risk premium of the delta-hedged strategy returns through conventional market factors and to assess how market- and other factor-betas can explain the volatility of the strategies of interest.

5.3 Empirical evidence

5.3.1 Market exposure and downside beta

For each strategy, the vector (the time series) of its returns consists of r_t for each time moment (window) t. After preliminary descriptive analysis, we want to understand the relationship between the strategies of interest and the conventional risk factors. Therefore, we start with evaluating a base-line Fama-French-5-factor model:

$$r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t, \tag{21}$$

where $r^{ex}_{M,t}$ is the excess return of the market over the risk-free rate, ε_t is the error term, and $X'_t \cdot \beta$ is a vector of control variables, in this case, the other 4 Fama-French factors:

$$X'_t \cdot \beta = \beta_1 \cdot SMB_t + \beta_2 \cdot HML_t + \beta_3 \cdot RMW_t + \beta_4 \cdot CMA_t.$$

The estimation of that regression equation (and all the following regressions) is run for each moneyness bracket (8) for each strategy (6) and is presented in tables 7 and 8 (in total 48 strategies). We do not report the estimated coefficients for the 4 Fama-French factors to save space and because they are not the focus of our research, but we should mention that they are usually insignificant, and there is no particular pattern in their values or significance levels.

In those tables, one can clearly see the general result of a positive and significant market beta for all strategies except the short call. As described in section 3, the selling call option strategy should move in the the direction opposite to that of the market; while for put it is to the contrary. All strategies with OTM options are estimated to have higher exposure to market than ITM and ATM.

Also interesting to compare exposures across different levels of moneyness and see if the results are in line with the theory. But to do so we need to take into account that all 6 strategies have different volatilities for each moneyness bracket. Hence, to check if differences in estimated coefficients are just caused by different volatilities or some intrinsic nature of those strategies, we run the same regression but for *scaled* strategies. We already mentioned scaling before, when we illustrated the cumulative returns of the short option strategies. Here, we scale all 48 strategies, so that their resulting volatilities are all equal to the annualized standard deviation of the index (0.1951), which can be found in summary table 1. The scaling procedure is straightforward: we divide the vector of returns by its standard deviation and multiply it by that of the underlying.

As the next step, we seek to assess the exposure of those strategies to the downside movements of the market, i.e. the crisis times. To do so, we follow the logic used by Dobrynskaya (2014) and run the following regression:

$$r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \theta \cdot D^M_t \cdot r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t, \tag{22}$$

where we define the second factor's dummy-component as follows:

$$D^M_t = \begin{cases} 1, & when \quad r^{ex}_{M,t} \geq mean(r^{ex}_M) - st.dev.(r^{ex}_M) \\ 0, & when \quad r^{ex}_{M,t} < mean(r^{ex}_M) - st.dev.(r^{ex}_M) \end{cases} .$$

Thus, in *"bad"* times, i.e. when $r^{ex}_{M,t} < mean(r^{ex}_M) - st.dev.(r^{ex}_M)$, that dummy is equal to 0 and, hence, the exposure to the market is measured only by β_M, which is why in this specification it is called the *downside beta*. In contrast, in *"good"* times, i.e. when $r^{ex}_{M,t} \geq mean(r^{ex}_M) - st.dev.(r^{ex}_M)$, the exposure to the market is measured by $\beta_M + \theta$, which is why that is called *upside beta*. From the estimated value of β_M in regressions 21 and 22, we can judge if a strategy return is more exposed to the downside market risk. From the sign (and significance criterion) of θ we can judge to what risk, upside or downside, a strategy is more exposed to.

In a part of tables 7, 8, 9 and 10 under the Reg. 22, we can see the results of exposure to downside market by looking at β_M. Both *original* and *scaled* put strategies have higher exposure to down movement of the market for OTM options than ATM and ITM ones. In other words, downside market beta increases with moneyness. For call strategies, this only applies for the *original* strategies (downside market beta decreases with moneyness). The reason might be that when the market experiences a dramatic downturn the returns of call strategies with different moneyness do not differ as much as those for put strategies, however the call strategies with lower moneyness have substantially higher volatility. Overall, scaling helps keep in check our inference on the exposures but the regressions with scaled returns cannot be claimed to be superior and more important to the original ones because they neglect some features that are specific to each strategy.

Also interesting and in line with theoretical expectations that delta-hedged strategies have dramatically lower estimated market betas (in absolute terms, since short call's beta is always negative). Further more, that holds even when we scale them. Same applies for the downside market beta. However, the scaled statically delta-hedged and the scaled short put have comparable downside betas. That might be explained by the fact that the former still has a significant downside risk, as a static delta hedge implies a long position in the index.

5.3.2 VRP exposure. Robustness

After that, we evaluate the exposure of the six strategies to VRP. To do that, we run the following regression:

$$r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot VRP_t + X'_t \cdot \beta + \varepsilon_t, \tag{23}$$

where $VRP_t = IV_{t-1} - RV_t$. In tables 7 and 8, one can see that $\hat{\phi}$ is positive and statistically significant for all 48 strategies. Notably, adding the VRP as a factor increases R^2 very slightly for a short and a statically delta-hedged strategies, while more than doubling it for the dynamically delta-hedged ones. This supports the argument that the latter is the most related to the VRP. We can also look at the actual values of those estimated coefficients for the *scaled* regressions in tables 9 and 10. As the theory suggests, delta-hedging decreases the exposure to the market $\hat{\beta}_M$, and increases that to the VRP $\hat{\phi}$. And that is the whole point of delta-hedging.

As for the discrepancy within one strategy for different moneyness levels, we can see that VRP exposure decreases (increases) with moneyness for call (puts). In other words, it is higher for OTM option-strategies. That supports our expectation that OTM stratgeies are the most exposed to the VRP. As before, if we check the *scaled version* of those strategies in tables 9 and 10, we can see that this observation holds for all puts but only for the short call. The exposure of delta-hedged calls are more or less the same across moneyness. When, as a robustness check, we decreased the number of moneyness brackets to 5 and 3 we saw the theoretically backed pattern once again.

As previously mentioned, the downside market factor explains a lot of the target variable's volatility in case of a short and a statically delta-hedged short strategies. However, that effect is not that pronounced in case of the dynamically delta-hedged one. Hence, to check the robustness of the results in regression 23, we include the downside market factor:

$$r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot VRP_t + \theta \cdot D^M_t r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t. \tag{24}$$

For regression 24, where we add the downside market factor, we can see that the exposure stays significant but reduces slightly for the dynamically delta-hedged strategies, and completely loses significance for the other strategies. The same applies to the regressions with scaled returns. We do not report the values for the downside market beta not to overwhelm the readers, but we should say that it always stays significant but goes down in value when VRP is added. The fact that VRP exposure stays significant and positive for the dynamically delta-hedged strategies is not surprising and only supports the argument that VRP plays one of the central roles in their return structure. To check in more detail the exposure to VRP, we introduce the upside factor as we did for the market factor and run the following regression:

$$r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot VRP_t + \lambda \cdot D^V_t VRP_t + \theta \cdot D^M_t r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t, \quad (25)$$

where analogously to the market upside factor

$$D^V_t = \begin{cases} 1, & when \quad VRP_t \geq mean(VRP) - st.dev.(VRP) \\ 0, & when \quad VRP_t < mean(VRP) - st.dev.(VRP) \end{cases} .$$

As reported in tables 7 and 8, the downside VRP exposure is significant and positive for most strategies. They are also higher than those in regressions 23 for puts with some mixed directions for calls, probably because even though the strategies might be more sensitive to VRP during bad times, some of that effect is taken on by the downside market exposure, as the two are highly correlated.

As we mentioned in subsection 5.1.2, we can use other ways to calculate VRP. When using the same VIX instead of different IVs for each moneyness bracket at each point in time, we receive quite similar results, however, in some regressions exposures to the VRP lose significance and, overall, the coefficient of determination decreases slightly. For instance, when we add downside market risk into the regression with VRP, we can see that the latter, similarly, becomes insignificant, but in the regression with the upside VRP it does not have a clear pattern of significance. When using the other ways to calculate RV, there are no major changes to the results. One piece of the results is reported as the last part in the tables 7, 8, 9, 10 and they refer to the following regression equation:

$$r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot (IV^2_t - RV^2_{t-1}) + X'_t \cdot \beta + \varepsilon_t, \quad (26)$$

One of the reasons for winsorizing is to make sure that we work with liquid options such that unfair prices do not distort our analysis. For each regression, we also ran a different version with additional factors like bid-ask spread, volume, and open interest. They were always insignificant. This suggests that the winsorizing that we execute is enough to leave mostly liquid options that have prices close to the fair ones.

As a robustness check we also try different window sizes, like 3,5,(10 – presented here), 20, 30, and different number of moneyness brackets, like 3, 4, 5, 6, (8 – presented here). The main inferences from the results do not change with that choice.

Another way to check robustness is to use IVs calculated with the GARCH model. From those we inferred deltas and calculated every step again. In addition, we tried to move a little some variable thresholds for winsorizing. We saw that the minimum best bid price for an option that we allow in each window might be a quite significant feature, as there are quite a lot of option with very small bid prices, lower than 25 cents. Including too many of them distorts some of the results.

All in all, our empirical analysis supports the expectations formulated after the theoretical part in subsection 3.6. This suggests that the (ir)regularities and anomalies observed not only by us, but by most researchers in this field are natural to the volatility selling strategies by construction. Starting from mere positive and significant mean returns and negative skewness (varying across moneyness) to the exposures to the market risk and VRP, especially, the downside exposures to them. Most of our robustness checks only support the main conclusions. Overall, our study is a contribution to the theoretical and empirical analysis that helps understand the drivers behind the volatility selling strategies.

6 Conclusion

In this thesis, we analyze the theoretical implications of the six strategies of interest: a short call (put), a statically delta-hedged call (put), and a dynamically delta-hedged call (put) – written on S&P500. All of them (except for the short call) were shown to deliver positive returns, on average, by previous researchers. Our preliminary analysis supports that assertion. We also show that another statistics of wide interest, VRP, is persistently higher than 0 on the new longer available time frame, which is consistent with prior papers in this topic, and is also positively correlated with the volatility selling returns.

One important step that we take in this work is to summarize a theoretical foundation to perform an empirical analysis. This is one of the features of our paper that sets us apart, as most researchers, if not all, in this field usually omit such elaborations and dive into discussing empirical irregularities and try to explain them by some arbitrary phenomena, like excess demand for *certain types* of put options as they are regarded by most investors as *some type* of special hedge. We start from the opposite side and abstain from such industry-exclusive judgement. First, we begin with analyzing the basis of option pricing and simulate several scenarios to illustrate return patterns in the strategies of interest. This helps us formulate expectations and assumptions based on the theoretical composition that is relevant for each strategy by construction, and only then test them empirically, rather than bringing the theory after we see the empirical results.

Thus, we show that volatility selling belongs to the family of carry strategies as it can be clearly seen in its payoffs simulations. We also discuss what moneyness levels drive higher or lower returns for each strategy. In addition, we elaborate on what role can be played by the market exposure in such strategies and how volatility risk premium affects those strategies to a different extent depending on moneyness and the hedging type.

All in all, our empirical analysis suggests that the variance risk premium is definitely linked to the returns of volatility selling strategies. Moreover, VRP tends to positively correlate with the returns of the strategies of interest even when the market exposure is not positive. That effect is robust under most changes of measure and only slightly fades away when a downside market beta is introduced into the relationship structure. The latter, in fact, can describe most of the volatility of the simple short and statically delta-hedged strategies, whereas only VRP is able to significantly increase the coefficient of determination of the dynamically delta-hedged strategies.

Furthermore, our approach discovered that in times of crises the exposure of the volatility selling strategies to the market and VRP (the downside beta) are usually higher than in normal time across moneyness levels and strategies. In the meantime, the classical Fama-French factors are not distinguished with any time of consistent significant relationship with volatility selling.

Overall, the main inference from our empirical analysis is robust to different specifications of the factors and returns, the winsorizing methods work well and make sure that illiquidity and unfair prices are not of concern for this paper, the empirical results are usually in line with the theoretical propositions.

Furthermore, the persistent negative returns of the delta-hedged options might be attributed to the lags in the volatility. And those lags can also be calculated in different ways, like an absolute value difference or log-differences between implied (or realized) volatilities in consecutive months or a deviation from a historical mean.

Further research might find fruitful results in applying a similar logic to other indexes, but most importantly, to individual stocks. The IV of certain indexes is well-known to have strong correlation with its lags, so many researchers tried to find strategies to capture the volatility risk premium. This might be a task for machine learning, as it can sometimes find connections not visible for a human perception. One of the things most researchers did to create a strategy was to long-short different levels (or quantiles) of moneyness or IV. Machine learning might be much better in that than a human. The duty of a researcher in this case is to come up with the covariates that can potentially improve a model, so analyzing the topic of volatility selling is not only interesting for research, but can potentially deliver returns for investors.

References

G. Bakshi and N. Kapadia. Delta-hedged gains and the negative market volatility risk premium. *Review of Financial Studies*, 16:527–566, 02 2003. doi: 10.2139/ssrn.267106.

L. Benzoni, P. Collin-Dufresne, and R. Goldstein. Explaining asset pricing puzzles associated with the 1987 market crash. *Journal of Financial Economics*, 101: 552–573, 06 2010. doi: 10.2139/ssrn.1543467.

F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, 1973. ISSN 00223808, 1537534X. URL http://www.jstor.org/stable/1831029.

B. H. Boyer and K. Vorkink. Stock options as lotteries. *The Journal of Finance*, 69(4):1485–1527, 2014. doi: https://doi.org/10.1111/jofi.12152. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12152.

M. Broadie, M. Chernov, and M. Johannes. Understanding index option returns. *Review of Financial Studies*, 22, 06 2007. doi: 10.2139/ssrn.965739.

A. Buraschi, R. Kosowski, and F. Trojani. When there is no place to hide: Correlation risk and the cross-section of hedge fund returns. *Review of Financial Studies*, 27, 03 2013. doi: 10.2139/ssrn.1639984.

J. Cao and B. Han. Cross section of option returns and idiosyncratic stock volatility. *Journal of Financial Economics*, 108(1):231 – 249, 2013. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2012.11.010. URL http://www.sciencedirect.com/science/article/pii/S0304405X12002450.

Y. Chen, J. Shu, and J. E. Zhang. Investor sentiment, variance risk premium and delta-hedged gains. *Applied Economics*, 48(31):2952–2964, 2016. doi: 10.1080/00036846.2015.1133894. URL https://doi.org/10.1080/00036846.2015.1133894.

B. Christensen and N. Prabhala. The relation between implied and realized volatility. *Journal of Financial Economics*, 50(2):125–150, 1998. ISSN 0304-405X. doi: https://doi.org/10.1016/S0304-405X(98)00034-8. URL https://www.sciencedirect.com/science/article/pii/S0304405X98000348.

J. Coval and T. Shumway. Expected option returns. *The Journal of Finance*, 56: 983 – 1009, 06 2001. doi: 10.1111/0022-1082.00352.

T. Day and C. Lewis. The behavior of volatility implicit in option prices. *Journal of Financial Economics*, 22:103–122, 10 1988. doi: 10.1016/0304-405X(88)90024-4.

V. Dobrynskaya. Downside Market Risk of Carry Trades. *Review of Finance*, 18(5):1885–1913, 2014. URL `https://ideas.repec.org/a/oup/revfin/v18y2014i5p1885-1913..html`.

J. Driessen, P. Maenhout, and G. Vilkov. The price of correlation risk: Evidence from equity options. *Journal of Finance*, 64:1377–1406, 12 2006. doi: 10.2139/ssrn.673425.

J. Duarte and C. Jones. The price of market volatility risk. *SSRN Electronic Journal*, 10 2007. doi: 10.2139/ssrn.1106960.

B. Dumas, J. Fleming, and R. Whaley. Implied volatility functions: Empirical tests. *The Journal of Finance*, 53, 07 2001. doi: 10.1109/CIFER.1996.501845.

B. Eraker. The volatility premium. *Quarterly Journal of Finance*, 01 2009. doi: 10.1142/S2010139221500142.

J. Fan, M. Imerman, and W. Dai. What does the volatility risk premium say about liquidity provision and demand for hedging tail risk? *SSRN Electronic Journal*, 34, 03 2013. doi: 10.2139/ssrn.2234438.

J. Fan, M. B. Imerman, and W. Dai. What does the volatility risk premium say about liquidity provision and demand for hedging tail risk? *Journal of Business & Economic Statistics*, 34(4):519–535, 2016. doi: 10.1080/07350015.2016.1152968. URL `https://doi.org/10.1080/07350015.2016.1152968`.

E. Gourier. Pricing of idiosyncratic equity and variance risks. 2016.

A. Goyal and A. Saretto. Cross-section of option returns and volatility. *Journal of Financial Economics*, 94:310–326, 11 2009. doi: 10.1016/j.jfineco.2009.01.001.

C. R. Harvey and R. E. Whaley. Market volatility prediction and the efficiency of the s p 100 index option market. *Journal of Financial Economics*, 31(1):43–73, 1992. ISSN 0304-405X. doi: https://doi.org/10.1016/0304-405X(92)90011-L. URL `https://www.sciencedirect.com/science/article/pii/0304405X9290011L`.

A. Ilmanen. Expected returns: An investor's guide to harvesting market rewards. *Expected Returns: An Investor's Guide to Harvesting Market Rewards*, 05 2012. doi: 10.1002/9781118467190.

J. Jackwerth. Recovering risk aversion from option prices and realized returns. *Review of Financial Studies*, 13:433–51, 02 2000. doi: 10.2139/ssrn.7745.

B.-H. Lin and Y.-J. Chen. Negative market volatility risk premium: Evidence from the liffe equity index options*. *Asia-Pacific Journal of Financial Studies*, 38:773 – 800, 10 2009. doi: 10.1111/j.2041-6156.2009.tb00030.x.

R. C. Merton, M. S. Scholes, and M. L. Gladstein. The returns and risk of alternative call option portfolio investment strategies. *The Journal of Business*, 51(2):183–242, 1978. ISSN 00219398, 15375374. URL `http://www.jstor.org/stable/2352527`.

X. Ruan. Volatility-of-volatility and the cross-section of option returns. *Journal of Financial Markets*, 48:100492, 2020. ISSN 1386-4181. doi: https://doi.org/10.1016/j.finmar.2019.03.002. URL `http://www.sciencedirect.com/science/article/pii/S1386418118300818`.

M. Scholes, R. Merton, and M. Gladstein. The returns and risks of alternative put-option portfolio investment strategies. *The Journal of Business*, 55:1–55, 02 1982. doi: 10.1086/296153.

N. Schürhoff and A. Ziegler. Variance risk, financial intermediation, and the cross-section of expected option returns. 02 2011. doi: 10.5167/uzh-48748.

N. Taleb. Bleed or blowup? why do we prefer asymmetric payoffs? *The Journal of Behavioral Finance*, 5:2–7, 03 2004. doi: 10.1207/s15427579jpfm0501_1.

S. Wilkens. Option returns versus asset-pricing theory: Evidence from the european option market. *Journal of Derivatives Hedge Funds*, 13, 08 2007. doi: 10.1057/palgrave.jdhf.1850065.

L. Wu and P. Carr. Variance risk premiums. *Review of Financial Studies*, 22:1311–1341, 03 2009. doi: 10.1093/rfs/hhn038.

A Appendix

A.1 Short put position dollar payoff

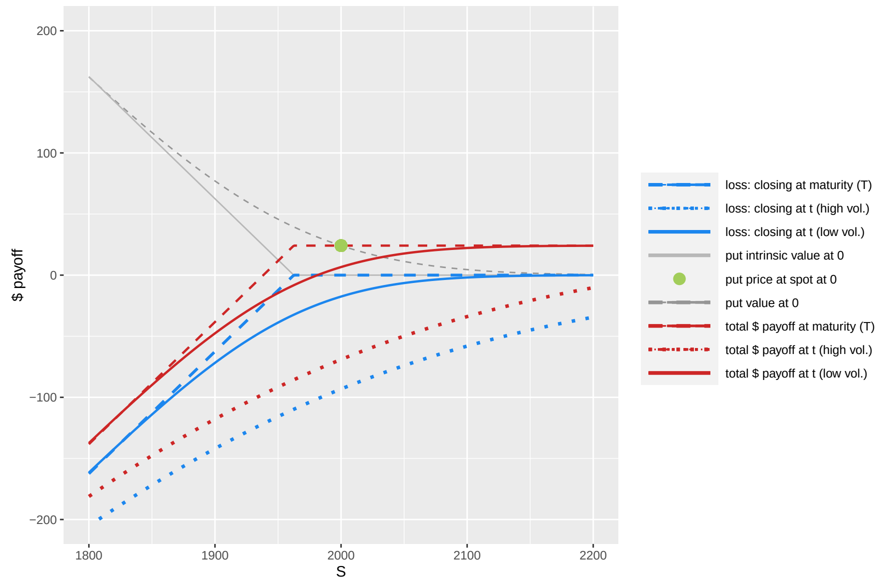


Figure 10: Derivation of a net dollar payoff from a short position in a put option with $S_0 = 2000$, $K = 1962.5$, $T - t = 10$ days, $\sigma_{low} = 0.15$, $\sigma_{high} = 0.5$, $r_f = 0.01$, days to maturity at t equal to 30 days. Dashed grey line indicates the values of the put option at time t at all possible spots. The green dot is the put price at spot $S_0 = 2000$. Blue lines indicate the losses from the short put position at closing: at maturity (dashed), at T with low IV (solid), at T with high IV (dotted). Red lines are the corresponding total net payoffs.

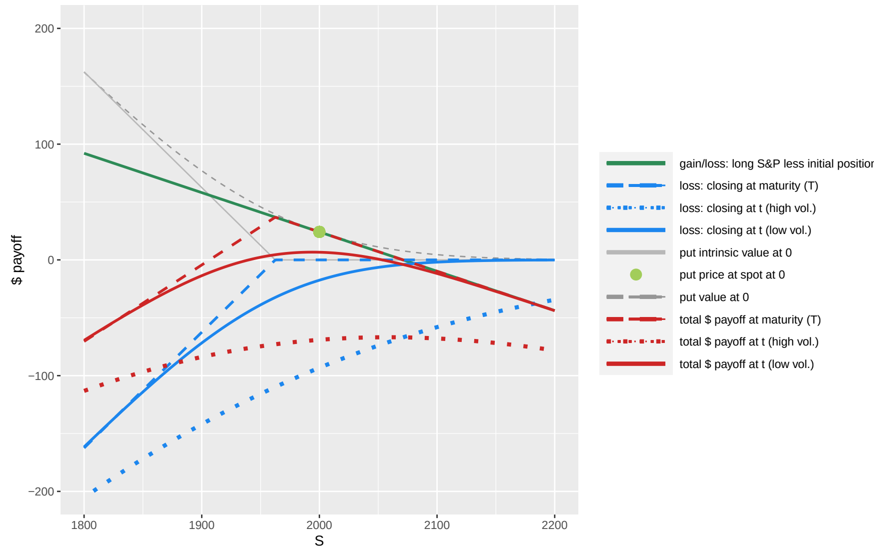


Figure 11: Derivation of a net dollar payoff from a statically delta-hedged short position in a put option with $S_0 = 2000$, $K = 1962.5$, $T - t = 10$ days, $\sigma_{low} = 0.15$, $\sigma_{high} = 0.5$, $r_f = 0.01$, days to maturity at t equal to 30 days. Dashed grey line indicates the values of the put option at time t at all possible spots. The green dot is the put price at spot $S_0 = 2000$. Blue lines indicate the losses from the short put position at closing: at maturity (dashed), at T with low IV (solid), at T with high IV (dotted). The solid green line indicates the gain/loss from the long position in $\Delta \cdot S$ bought for $S_0 = 2000$ at time t when $\Delta = 0.66$. Red lines are the corresponding total net payoffs.

A.2 Descriptive statistics

Table 2: Summary for windows of call option strategies

Panel A: Short call strategy						
Moneyness	more	signif. posit.	signif. negat.	insignif.	fewer	none
[0.95; 0.9625]	170	112	41	17	426	33
[0.9625; 0.975]	184	111	51	22	430	15
[0.975; 0.9875]	185	102	68	15	428	16
[0.9875; 1]	185	91	80	14	426	18
[1; 1.0125]	185	86	92	7	429	15
[1.0125; 1.025]	184	75	101	8	432	13
[1.025; 1.0375]	180	71	103	6	434	15
[1.0375; 1.05]	180	67	107	6	414	35
Panel B: Statically delta-hedged short call strategy						
Moneyness	more	signif. posit.	signif. negat.	insignif.	fewer	none
[0.95; 0.9625]	170	113	51	6	426	33
[0.9625; 0.975]	184	116	57	11	430	15
[0.975; 0.9875]	185	120	55	10	428	16
[0.9875; 1]	185	112	58	15	426	18
[1; 1.0125]	185	104	63	18	429	15
[1.0125; 1.025]	184	103	63	18	432	13
[1.025; 1.0375]	180	102	65	13	434	15
[1.0375; 1.05]	180	95	69	16	414	35
Panel C: Dynamically delta-hedged short call strategy						
Moneyness	more	signif. posit.	signif. negat.	insignif.	fewer	none
[0.95; 0.9625]	170	98	57	15	426	33
[0.9625; 0.975]	184	103	67	14	430	15
[0.975; 0.9875]	185	96	76	13	428	16
[0.9875; 1]	185	97	71	17	426	18
[1; 1.0125]	185	100	70	15	429	15
[1.0125; 1.025]	184	96	73	15	432	13
[1.025; 1.0375]	180	91	72	17	434	15
[1.0375; 1.05]	180	81	76	23	414	35

The number of 10-working-day windows out of 629 that have *more* than (or equal to) 20 calls that we can invest in (that satisfy the winsorizing procedure), *fewer* than 20, and *none*. Also reported how many of those *"more"* windows have returns whose mean is significantly *positive*, *negative* or *insignificant*. Reported for each strategy and each moneyness bracket separately.

Table 3: Summary for windows of put option strategies

Moneyness	more	signif. posit.	signif. negat.	insignif.	fewer	none
Panel A: Short put strategy						
[0.95; 0.9625]	164	105	56	3	425	40
[0.9625; 0.975]	184	115	62	7	429	16
[0.975; 0.9875]	185	120	55	10	427	17
[0.9875; 1]	185	124	50	11	426	18
[1; 1.0125]	185	135	46	4	429	15
[1.0125; 1.025]	184	132	38	14	432	13
[1.025; 1.0375]	180	135	36	9	435	14
[1.0375; 1.05]	181	139	31	11	427	21
Panel B: Statically delta-hedged short put strategy						
Moneyness	more	signif. posit.	signif. negat.	insignif.	fewer	none
[0.95; 0.9625]	164	127	31	6	425	40
[0.9625; 0.975]	184	133	40	11	429	16
[0.975; 0.9875]	185	123	49	13	427	17
[0.9875; 1]	185	120	52	13	426	18
[1; 1.0125]	185	125	50	10	429	15
[1.0125; 1.025]	184	124	47	13	432	13
[1.025; 1.0375]	180	123	45	12	435	14
[1.0375; 1.05]	181	123	37	21	427	21
Panel C: Dynamically delta-hedged short put strategy						
Moneyness	more	signif. posit.	signif. negat.	insignif.	fewer	none
[0.95; 0.9625]	164	143	16	5	425	40
[0.9625; 0.975]	184	144	31	9	429	16
[0.975; 0.9875]	185	110	42	33	427	17
[0.9875; 1]	185	114	46	25	426	18
[1; 1.0125]	185	119	49	17	429	15
[1.0125; 1.025]	184	122	44	18	432	13
[1.025; 1.0375]	180	127	39	14	435	14
[1.0375; 1.05]	181	131	33	17	427	21

The number of 10-working-day windows out of 629 that have *more* than (or equal to) 20 puts that we can invest in (that satisfy the winsorizing procedure), *fewer* than 20, and *none*. Also reported how many of those *"more"* windows have returns whose mean is significantly *positive*, *negative* or *insignificant*. Reported for each strategy and each moneyness bracket separately.

Table 4: Summary statistics of call option strategies

Panel A: Short call strategy					
Moneyness	Mean	Standard deviation	Skewness	Kurtosis	Sharpe ratio
[0.95; 0.9625]	1.24	6.84	-19.02	20.43	0.18
[0.9625; 0.975]	0.17	6.45	-16.71	18.19	0.02
[0.975; 0.9875]	-1.24	5.97	-17.57	23.76	-0.21
[0.9875; 1]	-0.87	4.4	-7.33	3.78	-0.2
[1; 1.0125]	-0.87	3.49	-3.42	-0.06	-0.26
[1.0125; 1.025]	-0.63	2.95	-1.87	-0.54	-0.22
[1.025; 1.0375]	-0.45	2.57	-1.14	-0.54	-0.18
[1.0375; 1.05]	-0.61	2.25	-0.4	-0.5	-0.28
Panel B: Statically delta-hedged short call strategy					
Moneyness	Mean	Standard deviation	Skewness	Kurtosis	Sharpe ratio
[0.95; 0.9625]	**0.08**	0.12	-10.44	7.44	0.5
[0.9625; 0.975]	**0.07**	0.1	-8.96	5.77	0.46
[0.975; 0.9875]	**0.05**	0.09	-8.48	6.08	0.39
[0.9875; 1]	**0.05**	0.07	-10.27	10.58	0.34
[1; 1.0125]	**0.04**	0.06	-9.18	11.48	0.39
[1.0125; 1.025]	**0.04**	0.06	-12.39	18.29	0.31
[1.025; 1.0375]	**0.03**	0.05	-12.25	19.88	0.22
[1.0375; 1.05]	**0.03**	0.05	-11.59	22.8	0.24
Panel C: Dynamically delta-hedged short call strategy					
Moneyness	Mean	Standard deviation	Skewness	Kurtosis	Sharpe ratio
[0.95; 0.9625]	**0.04**	0.07	-7.62	10.26	0.25
[0.9625; 0.975]	**0.02**	0.06	-9.16	12.27	0.06
[0.975; 0.9875]	0.01	0.05	-11.59	18.1	-0.13
[0.9875; 1]	0.0075	0.05	-13.92	22.98	-0.28
[1; 1.0125]	0.0053	0.04	-15.82	28.67	-0.37
[1.0125; 1.025]	0.003	0.04	-17.1	32.47	-0.46
[1.025; 1.0375]	0	0.03	-18.55	35.04	-0.57
[1.0375; 1.05]	-0.0006	0.03	-18.97	36.63	-0.6

Annualized means, standard deviations, skewness, kurtosis and Sharpe ratios calculated for each strategy type and each moneyness bracket separately. A number being in **bold** denotes that it is statistically significant at 5%.

Table 5: Summary statistics of put option strategies

Panel A: Short put strategy					
Moneyness	Mean	Standard deviation	Skewness	Kurtosis	Sharpe ratio
[0.95; 0.9625]	**1.55**	2.52	-5.39	2.1	0.61
[0.9625; 0.975]	**1.9**	2.97	-6.91	3.38	0.63
[0.975; 0.9875]	**2.45**	3.44	-8.99	6.29	0.71
[0.9875; 1]	**2.76**	4.13	-12.94	12.84	0.66
[1; 1.0125]	**3.6**	4.72	-18.84	26.72	0.76
[1.0125; 1.025]	**4.19**	5.43	-26.67	49.12	0.77
[1.025; 1.0375]	**4.58**	6.2	-34.33	74.08	0.74
[1.0375; 1.05]	**5.52**	6.95	-43	105.86	0.79
Panel B: Statically delta-hedged short put strategy					
Moneyness	Mean	Standard deviation	Skewness	Kurtosis	Sharpe ratio
[0.95; 0.9625]	**0.05**	0.03	-2.93	8.1	0.9
[0.9625; 0.975]	**0.05**	0.04	-4.14	8	0.88
[0.975; 0.9875]	**0.06**	0.05	-4.1	5.25	0.98
[0.9875; 1]	**0.08**	0.06	-8.03	8.51	0.97
[1; 1.0125]	**0.09**	0.07	-12.07	16.5	1.04
[1.0125; 1.025]	**0.11**	0.09	-19.79	34.47	1.03
[1.025; 1.0375]	**0.13**	0.11	-27.32	57.14	0.99
[1.0375; 1.05]	**0.14**	0.12	-35.42	86.84	0.99
Panel C: Dynamically delta-hedged short put strategy					
Moneyness	Mean	Standard deviation	Skewness	Kurtosis	Sharpe ratio
[0.95; 0.9625]	**0.03**	0.02	-21.2	49.53	0.33
[0.9625; 0.975]	**0.03**	0.03	-19.24	46.99	0.43
[0.975; 0.9875]	**0.03**	0.03	-18.63	43.18	0.43
[0.9875; 1]	**0.04**	0.04	-18.47	36.41	0.49
[1; 1.0125]	**0.04**	0.04	-18.06	32.52	0.54
[1.0125; 1.025]	**0.05**	0.05	-19.09	28.02	0.54
[1.025; 1.0375]	**0.06**	0.06	-19.09	25.5	0.66
[1.0375; 1.05]	**0.07**	0.07	-20.42	26.09	0.75

Annualized means, standard deviations, skewness, kurtosis and Sharpe ratios calculated for each strategy type and each moneyness bracket separately. A number being in **bold** denotes that it is statistically significant at 5%.

Table 6: Summary statistics of scaled option strategies

Panel A			**Call**			**Put**
Moneyness	Mean	Standard deviation	Sharpe Ratio	Mean	Standard deviation	Sharpe Ratio
[0.95; 0.9625]	0.03	0.19	0.06	**0.1**	0.19	0.49
[0.9625; 0.975]	0.0042	0.19	-0.1	**0.1**	0.19	0.51
[0.975; 0.9875]	-0.03	0.19	-0.33	**0.11**	0.19	0.59
[0.9875; 1]	-0.03	0.19	-0.33	**0.11**	0.19	0.54
[1; 1.0125]	-0.04	0.19	-0.38	**0.12**	0.19	0.63
[1.0125; 1.025]	-0.03	0.19	-0.34	**0.12**	0.19	0.64
[1.025; 1.0375]	-0.03	0.19	-0.3	**0.12**	0.19	0.61
[1.0375; 1.05]	-0.04	0.19	-0.4	**0.13**	0.19	0.67
Panel B			**Call**			**Put**
Moneyness	Mean	Standard deviation	Sharpe Ratio	Mean	Standard deviation	Sharpe Ratio
[0.95; 0.9625]	**0.11**	0.19	0.55	**0.25**	0.19	1.41
[0.9625; 0.975]	**0.11**	0.19	0.53	**0.23**	0.19	1.3
[0.975; 0.9875]	**0.1**	0.19	0.5	**0.23**	0.19	1.31
[0.9875; 1]	**0.1**	0.19	0.49	**0.21**	0.19	1.2
[1; 1.0125]	**0.12**	0.19	0.59	**0.22**	0.19	1.21
[1.0125; 1.025]	**0.11**	0.19	0.55	**0.2**	0.19	1.14
[1.025; 1.0375]	**0.1**	0.19	0.5	**0.19**	0.19	1.06
[1.0375; 1.05]	**0.11**	0.19	0.56	**0.19**	0.19	1.03
Panel C			**Call**			**Put**
Moneyness	Mean	Standard deviation	Sharpe Ratio	Mean	Standard deviation	Sharpe Ratio
[0.95; 0.9625]	**0.09**	0.19	0.41	**0.17**	0.19	0.97
[0.9625; 0.975]	**0.06**	0.19	0.27	**0.19**	0.19	1.03
[0.975; 0.9875]	0.04	0.19	0.14	**0.18**	0.19	0.98
[0.9875; 1]	0.03	0.19	0.04	**0.17**	0.19	0.92
[1; 1.0125]	0.02	0.19	0.0026	**0.17**	0.19	0.9
[1.0125; 1.025]	0.01	0.19	-0.05	**0.15**	0.19	0.8
[1.025; 1.0375]	-0.0002	0.19	-0.13	**0.19**	0.19	0.87
[1.0375; 1.05]	-0.003	0.19	-0.14	**0.17**	0.19	0.91

Annualized means, standard deviations, skewness, kurtosis and Sharpe ratios calculated for each strategy type and each moneyness bracket separately for strategies that were scaled to have the same standard deviation as the market. A number being in **bold** denotes that it is statistically significant at 5%. *Panel A* is for the simple short strategies; *Panel B* is for the statically delta-hedged short option strategies; and *Panel C* is for the dynamically short option strategies.

A.3 Regression results

Tables 7 and 8 exhibit the regression results for our 6 original selling volatility strategies using call and put options with different levels of moneyness. There are six parts, each of which corresponds to a different regression that is denoted by the order of the regression equation in the text of this thesis. In the tables, all values in **bold** statistically significant at 5%.

The list of regressions:

$$(21) \quad r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t,$$

$$(22) \quad r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \theta \cdot D^M_t \cdot r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t,$$

$$(23) \quad r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot VRP_t + X'_t \cdot \beta + \varepsilon_t,$$

$$(24) \quad r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot VRP_t + \theta \cdot D^M_t r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t.$$

$$(25) \quad r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot VRP_t + \lambda \cdot D^V_t VRP_t + \theta \cdot D^M_t r^{ex}_{M,t} + X'_t \cdot \beta + \varepsilon_t,$$

$$(26) \quad r_t = \alpha + \beta_M \cdot r^{ex}_{M,t} + \phi \cdot (IV^2_t - RV^2_{t-1}) + X'_t \cdot \beta + \varepsilon_t,$$

Tables 9 and 10 report the results of the same regressions but with the dependent variable scaled so that it has a standard deviation equal to that of the index.

Table 7: Regression results for call strategies

	Reg. 21			Reg. 22				Reg. 23		Reg. 24		Reg. 25			Reg. 26	
Panel A: Short call returns																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	**0.15**	**-27.94**	0.41	**0.34**	**7.16**	**-46.1**	0.5	**2.34**	0.42	0.04	0.5	**2.82**	-0.98	0.51	0.71	0.41
[0.9625; 0.975]	**0.11**	**-27.84**	0.44	**0.29**	5.17	**-43.45**	0.53	**2.19**	0.45	0.13	0.53	**2.95**	-1.52	0.55	0.59	0.44
[0.975; 0.9875]	0.06	**-26.25**	0.46	**0.22**	3.34	**-38.14**	0.53	**1.71**	0.46	-0.26	0.53	**2.96**	-2.39	0.54	0.72	0.46
[0.9875; 1]	**0.05**	**-22.57**	0.62	**0.17**	0.07	**-29.89**	0.72	**1.11**	0.63	-0.35	0.72	**2.25**	-2.12	0.73	0.46	0.63
[1; 1.0125]	**0.04**	**-19.96**	0.77	**0.14**	**-2.51**	**-22.64**	0.84	**0.88**	0.77	-0.31	0.84	**2.03**	**-2.05**	0.87	0.25	0.77
[1.0125; 1.025]	**0.04**	**-17.13**	0.83	**0.11**	**-4.6**	**-16.44**	0.89	**0.55**	0.83	-0.24	0.89	**1.44**	**-1.52**	0.91	0.06	0.83
[1.025; 1.0375]	**0.03**	**-15.11**	0.86	**0.08**	**-5.17**	**-13.06**	0.92	**0.46**	0.87	-0.13	0.92	**1.1**	**-1.05**	0.94	0.04	0.87
[1.0375; 1.05]	**0.03**	**-13.46**	0.9	**0.07**	**-6.33**	**-9.33**	0.93	**0.34**	0.9	-0.09	0.93	**0.92**	**-0.93**	0.95	0.17	0.9
Panel B: Statically delta-hedged short call strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	0.0013	**0.26**	0.19	**0.0061**	**1.12**	**-1.14**	0.36	**0.05**	0.17	-0.0091	0.36	**0.02**	0.07	0.37	**0.05**	0.19
[0.9625; 0.975]	0.0011	**0.2**	0.13	**0.0058**	**1.06**	**-1.13**	0.38	**0.05**	0.14	-0.005	0.38	**0.02**	0.06	0.4	**0.05**	0.13
[0.975; 0.9875]	0.0007	**0.13**	0.07	**0.0053**	**1.01**	**-1.13**	0.38	**0.05**	0.09	-0.01	0.39	**0.03**	0.03	0.4	**0.06**	0.07
[0.9875; 1]	0.0004	**0.13**	0.08	**0.0048**	**0.92**	**-1.05**	0.48	**0.04**	0.1	-0.02	0.49	**0.01**	0.04	0.5	**0.05**	0.09
[1; 1.0125]	0.0005	**0.08**	0.05	**0.0045**	**0.81**	**-0.96**	0.47	**0.03**	0.06	**-0.02**	0.48	0.02	0.03	0.51	**0.05**	0.05
[1.0125; 1.025]	0.0003	**0.07**	0.05	**0.004**	**0.73**	**-0.87**	0.53	**0.03**	0.06	**-0.02**	0.53	**0.006**	0.04	0.55	**0.04**	0.06
[1.025; 1.0375]	0.0001	**0.06**	0.04	**0.0033**	**0.65**	**-0.78**	0.52	**0.03**	0.06	-0.0077	0.52	0.0015	**0.04**	0.54	**0.02**	0.06
[1.0375; 1.05]	0.0002	**0.04**	0.03	**0.003**	**0.55**	**-0.67**	0.47	**0.02**	0.04	-0.01	0.47	0.0022	0.03	0.49	**0.02**	0.06
Panel C: Dynamically delta-hedged short call strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	0	**0.17**	0.21	0.0007	**0.3**	**-0.19**	0.22	**0.09**	0.33	**0.09**	0.33	**0.05**	**0.08**	0.34	**0.06**	0.26
[0.9625; 0.975]	-0.0004	**0.14**	0.19	0.0003	**0.28**	**-0.18**	0.21	**0.09**	0.34	**0.08**	0.34	**0.06**	**0.06**	0.36	**0.06**	0.25
[0.975; 0.9875]	**-0.0008**	**0.11**	0.17	0.0002	**0.29**	**-0.23**	0.21	**0.08**	0.34	**0.08**	0.35	**0.06**	**0.04**	0.36	**0.05**	0.25
[0.9875; 1]	**-0.0009**	**0.1**	0.19	0.0001	**0.27**	**-0.23**	0.24	**0.08**	0.36	**0.07**	0.38	**0.06**	0.03	0.39	**0.05**	0.27
[1; 1.0125]	**-0.001**	**0.08**	0.19	0.0001	**0.28**	**-0.26**	0.24	**0.07**	0.34	**0.06**	0.36	**0.07**	0.004	0.39	**0.04**	0.26
[1.0125; 1.025]	**-0.0011**	**0.08**	0.17	0	**0.27**	**-0.25**	0.26	**0.06**	0.34	**0.05**	0.38	**0.07**	-0.0073	0.4	**0.04**	0.27
[1.025; 1.0375]	**-0.0011**	**0.07**	0.19	-0.0001	**0.26**	**-0.26**	0.27	**0.06**	0.34	**0.05**	0.39	**0.07**	-0.02	0.41	**0.05**	0.28
[1.0375; 1.05]	**-0.0011**	**0.06**	0.15	-0.0001	**0.25**	**-0.25**	0.26	**0.05**	0.31	**0.04**	0.36	**0.07**	**-0.03**	0.38	**0.04**	0.27

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Table 8: Regression results for put strategies

	Reg. 21			Reg. 22				Reg. 23		Reg. 24		Reg. 25			Reg. 26	
Panel A: Short put strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	0.0066	**15.24**	0.93	**0.02**	**17.35**	**-2.75**	0.93	**0.3**	0.93	0.17	0.93	-0.4	**1.25**	0.94	**0.35**	0.93
[0.9625; 0.975]	0.01	**17.88**	0.89	**0.03**	**21.86**	**-5.24**	0.9	**0.44**	0.89	0.19	0.9	-0.31	**1.38**	0.92	**0.59**	0.89
[0.975; 0.9875]	0.01	**20.15**	0.82	**0.06**	**28.57**	**-10.85**	0.84	**0.78**	0.82	0.18	0.84	-0.22	**1.77**	0.85	**1.07**	0.82
[0.9875; 1]	0.02	**23.27**	0.75	**0.08**	**34.62**	**-14.97**	0.78	**1.21**	0.76	0.42	0.78	-0.33	**2.69**	0.79	**1.3**	0.75
[1; 1.0125]	0.04	**25.4**	0.66	**0.15**	**46.53**	**-27.42**	0.72	**1.68**	0.67	0.06	0.72	-0.24	**3.28**	0.75	**2.27**	0.66
[1.0125; 1.025]	0.06	**26.79**	0.57	**0.2**	**53.52**	**-35.09**	0.66	**2.26**	0.58	0.33	0.66	0.43	2.99	0.68	**2.36**	0.58
[1.025; 1.0375]	**0.08**	**28.33**	0.5	**0.27**	**63.49**	**-46.24**	0.61	**2.99**	0.51	0.43	0.61	0.71	3.63	0.63	**2.72**	0.5
[1.0375; 1.05]	**0.09**	**29.4**	0.42	**0.33**	**73.36**	**-57.65**	0.56	**3.38**	0.43	0.15	0.56	0.8	4.02	0.58	**3.46**	0.42
Panel B: Statically delta-hedged short put strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	**0.0009**	**0.05**	0.1	**0.0022**	**0.28**	**-0.3**	0.28	**0.01**	0.12	-0.0006	0.28	-0.0019	**0.03**	0.29	**0.02**	0.1
[0.9625; 0.975]	**0.0011**	**0.06**	0.1	**0.0029**	**0.38**	**-0.42**	0.35	**0.02**	0.12	-0.0045	0.35	0.0017	0.03	0.37	**0.03**	0.1
[0.975; 0.9875]	**0.0015**	**0.06**	0.07	**0.0039**	**0.52**	**-0.59**	0.38	**0.03**	0.09	-0.0069	0.38	0.02	0.02	0.39	**0.04**	0.07
[0.9875; 1]	**0.0018**	**0.1**	0.08	**0.0052**	**0.7**	**-0.8**	0.47	**0.04**	0.1	-0.0088	0.48	0.02	0.03	0.49	**0.04**	0.09
[1; 1.0125]	**0.0024**	**0.09**	0.05	**0.0068**	**0.92**	**-1.08**	0.49	**0.05**	0.08	-0.02	0.49	**0.05**	0.0011	0.52	**0.06**	0.06
[1.0125; 1.025]	**0.003**	**0.11**	0.05	**0.0086**	**1.13**	**-1.34**	0.54	**0.06**	0.07	-0.02	0.54	**0.06**	-0.002	0.56	**0.06**	0.05
[1.025; 1.0375]	**0.0035**	**0.13**	0.05	**0.01**	**1.39**	**-1.66**	0.55	**0.08**	0.07	-0.02	0.55	**0.07**	0.0048	0.57	**0.07**	0.05
[1.0375; 1.05]	**0.0042**	**0.13**	0.03	**0.01**	**1.57**	**-1.89**	0.51	**0.08**	0.06	-0.02	0.52	**0.1**	-0.02	0.54	**0.08**	0.04
Panel C: Dynamically delta-hedged short put strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	-0.0001	**0.04**	0.13	**0.0007**	**0.17**	**-0.19**	0.18	**0.03**	0.22	**0.02**	0.25	**0.04**	-0.02	0.26	**0.02**	0.19
[0.9625; 0.975]	0.0001	**0.05**	0.12	**0.0009**	**0.21**	**-0.2**	0.19	**0.04**	0.24	**0.03**	0.28	**0.04**	-0.01	0.3	**0.03**	0.19
[0.975; 0.9875]	0.0002	**0.06**	0.15	**0.001**	**0.25**	**-0.23**	0.22	**0.04**	0.27	**0.04**	0.3	**0.04**	-0.0024	0.31	**0.03**	0.21
[0.9875; 1]	0.0004	**0.08**	0.17	**0.0014**	**0.29**	**-0.27**	0.25	**0.06**	0.34	**0.05**	0.37	**0.07**	-0.01	0.38	**0.04**	0.27
[1; 1.0125]	0.0005	**0.09**	0.19	**0.0018**	**0.35**	**-0.33**	0.26	**0.08**	0.34	**0.07**	0.37	**0.09**	-0.01	0.4	**0.05**	0.27
[1.0125; 1.025]	0.0005	**0.13**	0.19	**0.0025**	**0.5**	**-0.48**	0.33	**0.1**	0.38	**0.08**	0.44	**0.12**	**-0.05**	0.46	**0.06**	0.3
[1.025; 1.0375]	**0.001**	**0.14**	0.17	**0.0034**	**0.59**	**-0.59**	0.34	**0.12**	0.37	**0.09**	0.44	**0.15**	**-0.05**	0.46	**0.07**	0.29
[1.0375; 1.05]	**0.0013**	**0.15**	0.15	**0.004**	**0.64**	**-0.64**	0.32	**0.13**	0.34	**0.1**	0.42	**0.18**	**-0.08**	0.44	**0.09**	0.27

Table 9: Regression results for scaled call strategies

	Reg. 21			Reg. 22				Reg. 23		Reg. 24		Reg. 25			Reg. 26	
Panel A: Short call returns																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	**0.0027**	**-0.66**	0.41	**0.0072**	**0.17**	**-1.08**	0.5	**0.05**	0.42	0	0.5	**0.06**	-0.02	0.51	0.02	0.41
[0.9625; 0.975]	0.002	**-0.7**	0.44	**0.0065**	0.13	**-1.08**	0.53	**0.05**	0.44	0.0024	0.53	**0.07**	-0.04	0.55	0.02	0.44
[0.975; 0.9875]	0.0008	**-0.71**	0.45	**0.005**	0.09	**-1.03**	0.53	**0.04**	0.46	-0.0084	0.53	**0.08**	-0.06	0.54	0.02	0.46
[0.9875; 1]	0.0009	**-0.83**	0.62	**0.0055**	0.002	**-1.09**	0.72	**0.04**	0.63	-0.01	0.72	**0.08**	-0.08	0.73	0.02	0.63
[1; 1.0125]	0.0012	**-0.92**	0.77	**0.0055**	**-0.12**	**-1.04**	0.84	**0.04**	0.77	-0.01	0.84	**0.09**	**-0.09**	0.87	0.01	0.77
[1.0125; 1.025]	**0.0013**	**-0.93**	0.82	**0.005**	**-0.25**	**-0.9**	0.89	**0.03**	0.83	-0.01	0.89	**0.08**	**-0.08**	0.91	0.0045	0.83
[1.025; 1.0375]	**0.0011**	**-0.95**	0.86	**0.0044**	**-0.33**	**-0.82**	0.92	**0.03**	0.87	-0.0085	0.92	**0.07**	**-0.06**	0.94	0.0016	0.87
[1.0375; 1.05]	**0.0012**	**-0.96**	0.9	**0.0039**	**-0.45**	**-0.67**	0.93	**0.02**	0.9	-0.006	0.93	**0.06**	**-0.06**	0.95	0.01	0.9
Panel B: Statically delta-hedged short call strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	0.0022	**0.36**	0.19	**0.0088**	**1.57**	**-1.6**	0.36	**0.07**	0.17	-0.01	0.36	**0.02**	0.09	0.37	**0.08**	0.19
[0.9625; 0.975]	0.0022	**0.32**	0.13	**0.0096**	**1.68**	**-1.79**	0.38	**0.08**	0.14	-0.0073	0.38	**0.03**	0.09	0.4	**0.08**	0.13
[0.975; 0.9875]	0.002	**0.25**	0.07	**0.01**	**1.9**	**-2.13**	0.39	**0.09**	0.09	-0.02	0.39	**0.06**	0.06	0.4	**0.11**	0.07
[0.9875; 1]	0.0019	**0.28**	0.08	**0.01**	**2**	**-2.28**	0.49	**0.08**	0.1	-0.03	0.49	0.03	0.09	0.5	**0.11**	0.09
[1; 1.0125]	**0.0027**	**0.2**	0.04	**0.01**	**2.11**	**-2.49**	0.47	**0.09**	0.06	**-0.05**	0.48	**0.05**	0.07	0.51	**0.12**	0.05
[1.0125; 1.025]	0.0024	**0.19**	0.05	**0.01**	**2.08**	**-2.48**	0.53	**0.08**	0.06	**-0.04**	0.54	0.02	0.09	0.56	**0.11**	0.06
[1.025; 1.0375]	0.0022	**0.18**	0.04	**0.01**	**2.07**	**-2.49**	0.53	**0.09**	0.06	-0.02	0.53	0.0094	0.13	0.55	**0.08**	0.06
[1.0375; 1.05]	**0.0027**	**0.13**	0.03	**0.01**	**1.93**	**-2.36**	0.47	**0.07**	0.04	**-0.04**	0.48	0.01	0.09	0.5	**0.06**	0.06
Panel C: Dynamically delta-hedged short call strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	0.0011	**0.39**	0.21	**0.0026**	**0.68**	**-0.38**	0.22	**0.21**	0.33	**0.21**	0.33	**0.12**	**0.18**	0.34	**0.14**	0.26
[0.9625; 0.975]	0.0002	**0.36**	0.19	0.0023	**0.74**	**-0.5**	0.21	**0.24**	0.34	**0.23**	0.34	**0.15**	**0.17**	0.36	**0.15**	0.25
[0.975; 0.9875]	-0.0007	**0.34**	0.17	0.0023	**0.91**	**-0.74**	0.21	**0.27**	0.35	**0.25**	0.35	**0.2**	**0.14**	0.36	**0.17**	0.25
[0.9875; 1]	-0.0011	**0.34**	0.18	0.0024	**0.98**	**-0.85**	0.24	**0.27**	0.37	**0.25**	0.38	**0.22**	0.1	0.39	**0.18**	0.27
[1; 1.0125]	-0.0016	**0.33**	0.19	**0.0027**	**1.14**	**-1.06**	0.24	**0.27**	0.35	**0.24**	0.37	**0.27**	0.0098	0.4	**0.18**	0.26
[1.0125; 1.025]	-0.0019	**0.32**	0.17	**0.0026**	**1.15**	**-1.09**	0.26	**0.27**	0.35	**0.23**	0.39	**0.29**	-0.04	0.41	**0.19**	0.28
[1.025; 1.0375]	-0.0021	**0.31**	0.19	**0.0027**	**1.23**	**-1.21**	0.27	**0.27**	0.34	**0.23**	0.39	**0.33**	-0.1	0.41	**0.21**	0.28
[1.0375; 1.05]	-0.0024	**0.29**	0.14	0.0024	**1.2**	**-1.2**	0.26	**0.25**	0.31	**0.21**	0.37	**0.36**	**-0.17**	0.39	**0.21**	0.28

Table 10: Regression results for scaled put strategies

	Reg. 21			Reg. 22				Reg. 23		Reg. 24		Reg. 25			Reg. 26	
Panel A: Short put returns																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	-0.0003	**0.98**	0.93	0.0004	**1.11**	**-0.17**	0.93	**0.02**	0.93	0.01	0.93	**-0.03**	**0.08**	0.94	**0.02**	0.93
[0.9625; 0.975]	-0.0002	**0.97**	0.89	**0.001**	**1.19**	**-0.28**	0.9	**0.02**	0.89	0.0092	0.9	-0.02	**0.08**	0.92	**0.03**	0.89
[0.975; 0.9875]	-0.0001	**0.95**	0.82	**0.002**	**1.34**	**-0.51**	0.84	**0.03**	0.82	0.0069	0.84	-0.01	**0.08**	0.85	**0.05**	0.82
[0.9875; 1]	0.0001	**0.91**	0.75	**0.0025**	**1.35**	**-0.58**	0.78	**0.05**	0.76	0.02	0.78	-0.01	**0.11**	0.79	**0.05**	0.75
[1; 1.0125]	0.0004	**0.87**	0.66	**0.0043**	**1.59**	**-0.93**	0.72	**0.06**	0.67	0.0014	0.72	-0.01	**0.11**	0.75	**0.08**	0.66
[1.0125; 1.025]	0.0009	**0.8**	0.57	**0.0052**	**1.59**	**-1.04**	0.66	**0.07**	0.58	0.0093	0.66	0.01	0.09	0.68	**0.07**	0.58
[1.025; 1.0375]	0.0012	**0.74**	0.5	**0.0061**	**1.65**	**-1.2**	0.61	**0.08**	0.51	0.01	0.61	0.02	0.1	0.63	**0.07**	0.5
[1.0375; 1.05]	0.0013	**0.68**	0.42	**0.007**	**1.7**	**-1.34**	0.56	**0.08**	0.43	0.0037	0.56	0.02	0.1	0.58	**0.08**	0.42
Panel B: Statically delta-hedged short put strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
[0.95; 0.9625]	**0.008**	**0.27**	0.1	**0.01**	**1.45**	**-1.54**	0.28	**0.08**	0.11	0.0022	0.28	-0.0023	**0.15**	0.29	**0.09**	0.1
[0.9625; 0.975]	**0.0074**	**0.27**	0.1	**0.01**	**1.61**	**-1.77**	0.35	**0.08**	0.12	-0.02	0.35	0.01	0.12	0.37	**0.11**	0.1
[0.975; 0.9875]	**0.0074**	**0.22**	0.07	**0.02**	**1.86**	**-2.11**	0.38	**0.1**	0.09	-0.02	0.38	0.07	0.05	0.39	**0.13**	0.07
[0.9875; 1]	**0.0066**	**0.27**	0.08	**0.02**	**1.97**	**-2.25**	0.48	**0.1**	0.1	-0.02	0.48	0.06	0.08	0.49	**0.12**	0.09
[1; 1.0125]	**0.0067**	**0.21**	0.05	**0.02**	**2.15**	**-2.51**	0.49	**0.12**	0.08	-0.04	0.49	**0.12**	-0.0002	0.52	**0.14**	0.06
[1.0125; 1.025]	**0.0063**	**0.2**	0.05	**0.02**	**2.1**	**-2.49**	0.54	**0.11**	0.07	-0.03	0.54	**0.12**	-0.0064	0.56	**0.11**	0.05
[1.025; 1.0375]	**0.0058**	**0.2**	0.04	**0.02**	**2.12**	**-2.53**	0.55	**0.12**	0.07	-0.03	0.55	**0.11**	0.0053	0.57	**0.11**	0.05
[1.0375; 1.05]	**0.0057**	**0.17**	0.03	**0.02**	**2.05**	**-2.46**	0.51	**0.11**	0.06	-0.03	0.52	**0.13**	-0.03	0.54	**0.1**	0.04
Panel C: Dynamically delta-hedged short put strategy																
Moneyness	α	β_M	R^2	α	β_M	θ	R^2	ϕ	R^2	ϕ	R^2	ϕ	λ	R^2	ϕ	R^2
?? [0.95; 0.9625]	**0.0038**	**0.25**	0.13	**0.0087**	**1.1**	**-1.03**	0.18	**0.18**	0.22	**0.19**	0.25	**0.25**	-0.13	0.26	**0.14**	0.19
[0.9625; 0.975]	**0.0043**	**0.27**	0.12	**0.0095**	**1.25**	**-1.2**	0.19	**0.23**	0.25	**0.21**	0.29	**0.27**	-0.06	0.31	**0.17**	0.19
[0.975; 0.9875]	**0.0044**	**0.31**	0.15	**0.0091**	**1.31**	**-1.21**	0.22	**0.24**	0.27	**0.21**	0.31	**0.25**	-0.02	0.32	**0.17**	0.22
[0.9875; 1]	**0.0043**	**0.33**	0.17	**0.0091**	**1.29**	**-1.21**	0.26	**0.28**	0.35	**0.25**	0.38	**0.31**	-0.05	0.39	**0.2**	0.28
[1; 1.0125]	**0.0041**	**0.34**	0.19	**0.009**	**1.32**	**-1.24**	0.26	**0.29**	0.35	**0.25**	0.38	**0.33**	-0.06	0.41	**0.2**	0.27
[1.0125; 1.025]	**0.0033**	**0.39**	0.19	**0.0092**	**1.5**	**-1.44**	0.33	**0.3**	0.38	**0.24**	0.44	**0.38**	**-0.14**	0.46	**0.19**	0.31
[1.025; 1.0375]	**0.0039**	**0.36**	0.17	**0.01**	**1.56**	**-1.55**	0.34	**0.3**	0.37	**0.24**	0.44	**0.39**	**-0.14**	0.46	**0.19**	0.29
[1.0375; 1.05]	**0.0041**	**0.35**	0.15	**0.01**	**1.45**	**-1.44**	0.32	**0.29**	0.34	**0.23**	0.42	**0.41**	**-0.18**	0.44	**0.2**	0.27