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Evaluating Modern Asset Pricing Models in the Norwegian Stock Market

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ABSTRACT

This master thesis evaluates three different asset pricing models in the Norwegian stock market. We chose to evaluate the CAPM, Fama-French three factor model and the q-factor model. This is the first time the q-factor model is tested in the Norwegian market.

We study the effectiveness of the three models accuracy in explaining cross-sectional returns in the Norwegian market in the last 20 years. We apply the models to real-world data and evaluate their performance based on cross-sectional regressions, intercept analysis and explanatory power.

We find the q-factor model to perform best in terms of explanatory power and intercept analysis. Hence, a new asset pricing model should be considered for practitioners when conducting analysis on the Norwegian stock market.

Contents

1 Introduction and motivation

One central issue of empirical finance has been the desire to explain the cross-sectional average stock returns. Asset pricing models are put to this test and have several other functions on a day-to-day basis. Other functions include calculating the cost of equity for a company and the measuring and evaluation of portfolio performance.

There are several asset pricing models that have been created to solve these tasks. Despite the many models and empirical studies testing them, no definite conclusions have been made regarding what model is best for explaining the cross-sectional average returns and the cost of equity. In this study, we aim to examine which of our three selected asset pricing models that best explains the risk factors that are priced in the Norwegian stock market. One of the three selected models have never been tested in the Norwegian market before. Hence, we aim to contribute to the understanding of the risk factors that drives the Norwegian market.

The Capital Asset Pricing Model (CAPM) is perhaps the most famous asset pricing model. It is based on the work of William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966). Although it is simple in construction, it is widely used to this day. The CAPM describes the relationship between systematic risk and expected return for financial assets. After the introduction of the CAPM, researchers observed several ”anomalies” in the US stock market. Anomalies included that small cap firms seemed to outperform large cap firms (Banz, 1981) and that value stocks (high book-to-market ratio) tended to outperform growth stocks (low book-to-market ratio) (Basu, 1983).

These observations led Eugene Fama and Kenneth French (1993) to extend the CAPM with two additional factors accounting for these anomalies. Small minus big (SMB), known as the size factor, and high minus low (HML), known as the value factor. After adding these two factors to the CAPM, the Fama French three-factor model was created. The search for more anomalies to better understand the cross-sectional average return continued and led to the inclusion of the momentum effect into the Carhart four-factor model in 1997 (1997). Fama and French (2015) added two additional factors to their three-factor model and subsequentially made a five-factor model. The two new factors were the profitability factor, robust minus weak, (RMW) and investment factor, conservative minus aggressive, (CMA).

A new generation of factor pricing models have been created in recent years to evaluate the cross-section of expected returns. Examples of new asset pricing models include the Hou-Xue-Zhang (2015) four factor q-model, the Hou et al. (2020) five factor q-model, the Fama and French (2018) six-factor model, the Stambaugh-Yuan (2017) four-factor model, the Barillas-Shanken (2018) six-factor model, and the Daniel-Hirshleifer-Sun (2018) three-factor model. These factors are compared in Hou et al. (2018). Findings suggest that the different asset pricing models are closely related. The q-factor and q^5-models largely subsume the Fama and French five- and six-factor models. Although the models are closely related, the main question in financial economics remains, which factor model is superior in including priced risks.

In this thesis we evaluate three asset pricing models. The models in our thesis are the CAPM, the Fama and French three-factor model, and the Hou-Xue-Zhang four-factor q-model. While the first two are established models, the latter is relatively new and has never been tested in the Norwegian market before. We include the CAPM as it is used daily despite its simple construction. As from findings in Hou et al. (2018) we include the Fama-French three-factor model instead of the Fama-French five-factor model as the q-factor model subsumes the five-factor model. We aim to find the model that is best suited to explain expected returns in the cross-section in the Norwegian market. Hence, our research question can be formulated as follows:

Are modern financial asset pricing models better suited in explaining the cross-section of expected returns in the Norwegian market compared to established asset pricing models?

To answer our research question we estimate our models using the methodology from Fama and MacBeth (1973). The methodology includes running time-series regressions before running cross-sectional regressions. The methodology allows us to analyze the coefficients and statistical significance of the risk premia estimates. When conducting two separate regressions in the Fama and MacBeth procedure, we can compare models with two different types of intercept tests. Further comparisons of the models consist of analyzing the explanatory power and stability in results when splitting the data sample in three time-periods.

Our thesis will supplement the understanding of which factors efficiently subsumes the vast majority of anomalies the Norwegian market experiences. To our knowledge, the q-factor model has not been tested in the Norwegian stock market. The original q-factor paper studies anomalies in the US market, which in recent years have been largely influenced by big tech firms. We find it interesting to study if this model could be equally suited to explain returns in a market which is largely driven by big value firms as the Norwegian market is. The theory predicts that investments and profitability are linked as firms that invest more, tend to be more profitable. As investments are scaled to assets, it is interesting to see if we get the same relationship in terms of priced in factors the Norwegian market.

With this fundament, we emphasize that our contribution to the empirical research on the Norwegian stock market are i) Creating the factor portfolios ME, I/A and ROE as described in (Hou et al., 2015) for the Norwegian stock market in the sample period and ii) Identifying whether a model built on the neoclassical q-theory of investment or a characteristic-based model is best in explaining the expected returns in the Norwegian market over the sample period. This contribution could potentially increase the understanding of which risk factors that drives the returns in the Norwegian stock market.

The findings in our thesis suggest the q-factor model is most preferred amongst the three tested models in explaining cross-section return on the OSEBX. As this is the first time the model is tested in the Norwegian market, we believe this should provide incentives to include the q-factor model when conducting analysis on the Norwegian market. Although the q-factor model is better suited in explaining cross-sectional returns in the entire sample period, our results show that it outperforms the other models when looking at a sample period after the oil price shock in 2014. However, all three of our models yield a significant intercept, implying that the models are missing priced risk factors in the Norwegian stock market. As a result of this, models with different risk factors should be considered when conducting analysis on the Norwegian stock market.

This paper is further structured as follows: Section two introduces the theory of the asset pricing models in this thesis, section three reviews previous literature, section four presents the methodology used for testing and comparing the models, section five describes the data used in the study, section six contains discussions on the empirical results, in the seventh and final chapter we present our conclusion.

2 Theory

2.1 CAPM

The Capital Asset Model was introduced by William Sharpe (1964) and John Lintner (1965) and marked the birth of asset pricing models. The model is a single factor model which aims to explain the relation between systematic risk and expected return for an asset over a given period.

The model assumes homogeneity of investors' expectations, resulting in mean-variance efficient portfolios. In a frictionless efficient market where all information is available, the mean-variance efficient portfolio implies the investors will hold the market portfolio. The market portfolio is the weighted portfolio of all assets available in the financial markets.

The CAPM is still taught to this day, due to its simplicity and economic reasoning. The model is widely used in investment literature and portfolio management. However, the validity of the CAPM is widely discussed as the empirical records of the model is poor. The poor empirical records are mainly due to the unobservable market portfolio. Thus, the market portfolio must be proxied for empirically testing of the Capital Asset Pricing Model.

The Sharpe and Linter CAPM assumes investors can borrow and lend at the pure rate of the market - the risk-free rate. The expected return for asset i, is thus given by:

$$E(R_i) = R_f + \beta_{im}[E(R_m) - R_f] \tag{1}$$

The expected return consists of the risk-free rate R_f and the risk premium $E(R_m) - R_f$. The risk premium expresses the expected excess return of the market, multiplied with the assets risk β_{im}. β_{im} is the regression coefficient between the asset and the market by:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{2}$$

The β_{im} is a risk measure that identify the amount of market risk the asset obtains. A beta higher than one implies higher market risk than the market portfolio, and a higher expected return than the market (given positive market returns). CAPM implies that investors require compensation for holding riskier assets in the form of a risk premium.

Only systematic risk - risk that cannot be eliminated through diversification, yields higher expected returns. β_{im} captures the systematic risk as it is correlated with the market, the un-systematic risk is not captured by β_{im} thus it is not rewarded.

2.2 ATP

Stephen Ross (1976) introduced the Arbitrage Pricing theory (APT). APT predicts an asset's expected return, by linking risk and expected return. The difference between the CAPM and APT is that APT allows for multiple risk factors. The APT uses observable portfolios as indexes, allowing the proxying of the unobservable market portfolio to be avoided.

The APT assumes that returns for an asset i could be described by the following K-Factor model:

$$E(R_i) = \lambda_0 + \lambda_1\beta_{i1} + \lambda_2\beta_{i2} + ... + \lambda_k\beta_{ik} + \epsilon_i \tag{3}$$

Where λ_0 is a constant, β_{ik} is the risk multiplier associated with risk factor K. λ_k Is the systematic risk associated with factor K, ϵ_i is the unsystematic risk associated with asset i. For an asset with no unsystematic risk, the APT can be rewritten to:

$$E(R_i) = R_f + \lambda_1\beta_{i1} + \lambda_2\beta_{i2} + ... + \lambda_k\beta_{ik} \tag{4}$$

In contrast to the CAPM, the APT has not defined which factors to include. A set of different factors could be included in the APT and the challenge is choosing the relevant factors. Factors has mainly been derived either from theory or from empirical exercises, Harvey, Liu, Zhu (2016) identified hundreds of papers attempting to explain cross sectional returns.

3 Literature review

3.1 CAPM

The first studies (Lintner, 1965) of the CAPM were based on individual securities and focused on the risk-return relationship. The empirical evidence was poor, whereas the intercept was significantly larger than risk-free rate and the beta coefficient significantly lower. Friend and Blume (1970) examined the relationship of the one-parameter (CAPM) on 200 random portfolios which were derived from a universe of 788 stocks in the period 1960-1968. The results were striking, and they identified an inverse relationship between risk and return, contradicting the results of the CAPM. Friend and Blume also criticized the assumptions behind the CAPM, more specifically that all investors have the same lending/borrowing rate, as this could be a possible source of the biasedness as it removes the possibility to move to a stock financed with borrowings portfolio (Friend and Blume, 1970).

Black (1972) address the issue of the borrowing rate in his paper ”Capital Market Equilibrium with Restricted Borrowing” where the risk-free rate is replaced with a proxied ”zero-beta portfolio” which replaces the risk-free rate. Doublas Breeden (1979) introduced the Consumption CAPM where the betas are measured relative to changes in the aggregate real consumption rate rather than market returns. Robert Merton (1973) introduced the Intertemporal Capital Asset Pricing model (ICAPM) and acts as an extension to the CAPM. The intertemporal nature of the model compared to the static CAPM allows the model to capture time-varying effects such as inflation.

Substantial empirical testing has been conducted on the CAPM. The empirical evidence are contradictive as some results support the CAPM such as Black, Jensen and Scholes (2006) and Fama and Macbeth (1973) which identifies a relation between the beta and return. On the other side, Rolf W Banz (1981) finds that smaller firms earns higher risk adjusted returns than large firms, suggesting that the CAPM is missing a factor. Fama and French (1992) supports Banz (1981) findings and identifies that other factors such as size and book-to-market equity combined captures the cross-sectional variation in average stock returns. Based on these findings, Fama and French introduced the three-factor model.

Roll (1977) challenge the opposition of the CAPM. Since the CAPM uses proxies for the market portfolios when conducting analyses, Roll argues that the that CAPM neither have nor will be correctly tested due to the unobservable market portfolio.

3.2 Fama-French three-factor model

Fama and French (1993) extended the CAPM by adding two risk factors related to firm size and book-to-market equity. They find that their model with two added factors captures more of the explained variability in stock returns compared to the CAPM in the US market (E. F. Fama and French, 1996). The factors are chosen by empirical evidence, which challenges the possibility to link the factors to economic theory. Fama and French found that two classes of stocks tended to outperform the market, namely the small-cap stocks and the high book-to-market ratio stocks. As a result, the Fama-French three-factor model included the size and book-to-market as risk factors to account for compensation to investors who hold less profitable - more volatile stocks. Fama and French (1998) found that the three-factor model outperformed the CAPM in explaining cross-sectional returns in 13 major markets.

3.3 Carhart four-factor model

Fama and French found in their ”Multifactor Explanations of Asset Pricing Anomalies” (E. F. Fama and French, 1996) that their three-factor model explains four of the five anomalies but is not capable of explaining the continuation of short-term returns identified by Jagadeesh and Titman (1993). Jagadeesh and Titman found that buying firms that has performed well in the past and shorting firms that has performed poorly in the past generate significant positive returns over 3-12 months (Jegadeesh and Titman, 1993). Based on these results, Carhart introduced a four-factor model which includes a momentum factor (Carhart, 1997).

3.4 Fama-French five-factor model

Fama and French (2015) introduced their five-factor model which adds two factors to the existing three-factor model (E. Fama and French, 1993). The two added factors were the profitability factor (RMW) and the investment factor (CMA). Fama and French argues that the five-factor model outperforms the three-factor model in capturing average stock returns for US stocks (E. F. Fama and French, 2015). On the other side the, the GRS-test rejects the five-factor model, implying imperfection of the model. Despite this, the model explains between 71 % and 94 % of the cross-sectional variance of returns. Thus, Fama and French concludes that the five-factor model outperforms the three-factor model.

In 2017, Fama and French studied their five-factor model in an international study and found that ”average stock returns for North America, Europe and Asia Pacific increase with book-to-market ratio and profitability, while being negatively related to investment. For Japan the book-to-market relation is strong but average returns show little relation to profitability and investment” (E. F. Fama and French, 2017). Fama and French found that the CMA factor is redundant for Europe and Japan from spanning tests. Thus for these two regions, dropping the CMA factor in the five-factor model will have little effect on the explanation of average returns in the time interval 1990-2015 (E. F. Fama and French, 2017).

3.5 Hou, Xue, Zhang's Q-factor model

Hou, Xue and Zhang introduced their four-factor model (2015) consisting of the market factor, a size factor, an investment factor and a profitability factor based on return on equity. The q-factor model was created as they identified many anomalies the Fama-French three-factor model had challenges capturing, and as an opposition to the empirical method of factor choosing, the q-factor is created based on investment-based asset pricing (Hou et al., 2015). Their investment-based asset pricing is built on the fundamentals of James Tobin's (1969) q-theory of investments. Tobin's Q is calculated by taking the market value of the company divided by its assets' replacement costs, resulting in equilibrium when market equals replacement costs. Implications of this is that if the market places a higher value on the firms installed capital than its replacement cost, it provides incentives for the firm to add to its installed capital, i.e. invest in the business.

Hou, Xue and Zhang argues that the negative investment-expected return is intuitive, due to the fact that ”firms will invest more when their marginal q (the net present value of future cash flows generated an additional unit of assets) is high” (Hou et al., 2015). With given expected profitability or cash flows, a low discount rate implies high marginal q and high investments with the inverse relation high discount rate implying low marginal q and low investments. ”Given expected cash flows, high cost of capital imply high net present value of new projects and high investment” (Hou et al., 2015). If the firm adds to its investments, it would in theory imply that the marginal product of capital is positive.

The model was tested on 80 anomalies, and the conclusion were that the q-factor model outperforms the Fama-French three-factor model and Carhart four-factor model. They identified that Carhart four-factor model captures many of the anomalies, where many of the anomalies might seem unrelated. Hou, Xue and Zhang highlights the importance of understanding the driving forces behind the q-model in order of explaining the capturing of the anomalies (Hou et al.,2015).

4 Methodology

The methodology used in this thesis is the two-step regression from Fama and MacBeth (1973). To determine which of the asset pricing models that best forecasts expected return, we use intercept analysis and evaluate the explanatory power of the models.

4.1 Fama and MacBeth - two-step regression

Fama and Macbeth (1973) provides us with one method of testing asset pricing models. The first step in the two-step regression involves running OLS time-series regressions for the monthly excess return (return of the financial asset less the risk-free rate) of each test portfolio i on all factors f_j.

$$r_{i,t} - r_{RF,t} = \alpha_i + \beta_{i,1} f_{1,t} + \beta_{i,2} f_{2,t} + ... + \beta_{i,k} f_{k,t} + \epsilon_{i,t}, t = 1, ...T \tag{5}$$

In 5 $\hat{\alpha}_i$ is the intercept and the estimated sensitivity of each portfolioÂ´s excess return to movements in the factors is captured in $\beta_{i,1}$,$\beta_{i,2}$,...,$\beta_{i,K}$. As these factor loadings are estimations of the population, we label them $\hat{\beta}_{i,1}$,$\hat{\beta}_{i,2}$,...,$\hat{\beta}_{i,K}$ going forward. Further on the above equation $\epsilon_{i,t}$ is the error term and T is the number of time steps observations.

The second step of the two-step regression is to regress the estimated factor loadings for each cross-sectional observation. The estimates from the second regression will yield the risk premiums for each of the K factors. The regression is as follows:

$$r_{i,t} - r_{RF,t} = \lambda_{0,t} + \lambda_{1,t}\hat{\beta}_{i,1} + \lambda_{2,t}\hat{\beta}_{i,2} + ... + \lambda_{k,t}\hat{\beta}_{i,k} + \epsilon_{1,t}, i = 1, ...N \tag{6}$$

In (6) $\lambda_{o,t}$ is the intercept and $\lambda_{1,t}$,$\lambda_{2,t}$,...,$\lambda_{K,t}$ is the estimate of the risk premium for the K factors. These factors are again estimations for the population hence we label them $\hat{\lambda}_{1,t}$, $\hat{\lambda}_{2,t}$, ..., $\hat{\lambda}_{K,t}$.

Further on in the Fama and MacBeth process we calculate the average risk premium $\overline{\lambda}_K$ k=1,...K

$$\overline{\lambda}_K = \frac{1}{T}\sum_{t=1}^{t}\hat{\lambda}_{k,t}, k = 1, ...K \tag{7}$$

When we have the average risk premium, we can calculate the t-ratio by the equation:

$$t(\overline{\hat{\lambda}}_K) = \frac{\sqrt{T} * \overline{\hat{\lambda}}_K}{\hat{\sigma}_{\lambda,k}}, k = 1, ...K \tag{8}$$

Where,

$$\hat{\sigma}_{\lambda,k} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(\hat{\lambda}_{k,t} - \overline{\hat{\lambda}}_k)^2}, k = 1...K \tag{9}$$

4.2 Fama-French three-factor model

If we transfer equation (5) to the models we are to test, we get the following equation for the fama-french model:

$$r_{i,t} - r_{RF,t} = \alpha_i + \beta_{MKT,i}(r_{Mt} - r_{Ft}) + \beta_{SMB,i}SMB_t + \beta_{HML,i}HML_t + \epsilon_{i,t} \quad (10)$$

In (10) $r_{i,t} - r_{RF,t}$ is the excess return on the value weighted market portfolio. SMB_t is the return on the portfolio of small market cap stocks minus the return of the portfolio of big market cap stocks.HML_t is the difference between the returns on the portfolio of high book-to-market ratios minus the portfolio with low book-to-market ratios, also referred to as the value minus growth factor.

4.3 q-factor model

Using equation (5) for the q-factor model we get:

$$r_{i,t} - r_{RF,t} = \alpha_i + \beta_{MKT,i}(r_{Mt} - r_{Ft}) + \beta_{ME,i}ME_t + \beta_{ROE,i}ROE_t + \beta_{I/A,i}I/A_t + \epsilon_{i,t} \quad (11)$$

In (11) $r_{i,t} - r_{RF,t}$ is the excess return on the value weighted market portfolio, same as in the Fama-French model. ME is the size factor in the q-model and measures the return of the portfolio with small market cap stocks minus the portfolio consisting of big market cap stocks, quite similar to the SMB in the Fama-French model, although it is calculated somewhat differently. The I/A is the investment factor and measures the return of the portfolio of shares that has low investments minus the portfolio of shares that has high investments scaled to the firms assets. The last factor ROE, the profitability factor, measures the difference in returns between a portfolio of shares with high return on equity minus the portfolio of shares with low return on equity.

4.4 Comparing models

4.4.1 Intercept analysis

The intercept analysis will be conducted using the GRS test statistic introduced in Gibbons, Ross and Shanken (1989). The GRS-test is commonly used when assessing more than one intercept at once. The test statistic can be seen as an F-test where the hypothesis of the test is that all alphas from the set of time-series regressions are jointly equal to zero. An unsignificant GRS statistic will indicate that the model tested is efficient in pricing risk factors. Hence, a model with alphas significantly different from zero is misspecified. The GRS statistic is defined as: that all alphas from the set of

$$GRS = (\frac{T-N-K}{N})\frac{\hat{\alpha}\hat{\sum}^{-1}\hat{\alpha}}{1+E_t(f)'\hat{\Omega}^{-}1E_T(f)} \sim F_{N,T-N-K} \tag{12}$$

In (12) N is the number of test portfolios, K is the number of factors in the model and T is the number of cross-sectional periods. $\hat{\alpha}$ is an $NX1$ vector with the estimated intercepts from the time-series regression, $\hat{\sum}$ is an NXN estimate of the residual covariance matrix, $E_T(f)$ is a $KX1$ vector of the factor portfolio means, and $\hat{\Omega}$ is an unbiased estimate of the factor portfolios covariance matrix.

When conducting the GRS test, one looks at the time-series intercepts. As we run cross-sectional regressions as well, we analyze the intercepts from these regressions for further comparisons of the models. An intercept estimate different from zero indicates that the model fails to include all priced risks. In that sense the test is somewhat equal to the GRS test. In the test, the values of the estimated intercepts and the corresponding statistical significance is analyzed. We expect that a well specified model yields a statistically insignificant intercept. (Adrian et al., 2014)

4.4.2 Explanatory power

As the second comparison method the asset pricing models in our thesis, we will report the explanatory power of the cross-sectional regressions. The explanatory power or goodness of fit is defined as R^2. R^2 describes how much of the variation in the dependent variable is explained by the independent variables. A high explanatory variable indicates that the estimated factor loadings account for the cross-sectional variation in average returns. From (Brooks, 2014) we know that adding more risk factors to a model, will always yield higher or at least the same value of R^2. That is because adding more explanatory variables will describe the variation in the dependent variable to some degree.

Since the three models in our test contains different numbers of independent variables, we compare the models with the adjusted R^2. The adjusted R^2 takes the number of independent variables into account and adjust for it, making the explanatory power valid for comparison. Although we use the adjusted R^2 to compare models in our thesis, we are careful to interpret this as the true explanatory power. We do this with the basis in the findings of (Kan et al., 2013), which tells us that the R^2 should be treated only as a descriptive statistic and not as a description of the actual explanatory power.

5 Data

The models compared in this thesis are the CAPM, Fama-French three-factor model (FF3) and the q-factor model. To compare these models, we have collected data to create each of the factors. The creation of the factors is described in detail later in this chapter. The data used to create the factors has been collected through the Refinitiv database. More specifically the variables collected are date, risk-free rate, adjusted closing price, number of shares, total assets (the I/A factor is calculated as the annual change in total assets T / total assets T-1), income before extraordinary items and book equity (the ROE factor is calculated as the income before extraordinary items / Book equity (t-1 quarter)) and market returns of a benchmark index (OSEBX). We have excluded firms with negative return on equity, and financial firms due to their high leverage. These types of firms were also excluded in the original q-factor paper (Hou et al., 2015).

Test assets are created based on three sets of test portfolios. The portfolios have different characteristics which is: Size, Investment over assets and Return on equity. The test portfolios are further split in quantiles resulting in a total of 30 test portfolios. Through Refinitiv we have collected data on every firm that has been in the OSEBX index from January 2001 until February 2021, resulting in 211 monthly observations after factor creations. We create portfolios in June each year from the financial data from last December. We do this to make sure all firms have reported their financial data in time for us to rebalance our portfolios. The portfolio creation and rebalancing are done in Microsoft Excel, while the regressions and rest of the analysis is conducted in MATLAB.

We have created two sets of portfolios; value- and equally-weighted. The analysis is conducted on both sets of portfolios. For our analysis and discussions in this thesis we have focused on the value-weighted portfolios. Reasons for this will be discussed in chapter six.

5.1 Market risk premium

The market risk premium is the return of the market in excess of risk-free return:

$$ERM_t = R_{market,t} - R_{f,t} \tag{13}$$

The monthly return of the market is the monthly return of the OSEBX index, which is collected from Refinitiv. The risk-free rate is collected from Norges Bank Investment Managements homepage.

5.2 HML and SMB factors

The factors in the FF3 model are created as follows: First we collected monthly data of price, number of shares and book to market (B/M) from Refinitiv. The price and number of shares is used to calculate the market capitalization for the SMB factor while B/M is used for the HML factor. When the financial data is calculated we sort the companies into three B/M portfolios: High, Medium and Low. The High portfolio consists of the companies that are in the 70% highest quantiles when it comes to B/M metrics. The Low portfolio consists of the companies within the lowest 30% quantile. Thereafter the companies in each of the three B/M portfolios is sorted into two size portfolios: Small and Big. The Small and Big portfolios separate the dataset in two equal parts. Doing this we get six portfolios of cross-sorted companies (Small-High, Small-Medium, Small-Low, Big-High, Big-Medium, Big-Low). At the end we create each factor with the formulas:

$$SMB = \frac{SH + SM + SL}{3} + \frac{BH + BM + BL}{3} \tag{14}$$

$$HML = \frac{SH + BH}{2} + \frac{SL + BL}{2} \tag{15}$$

5.3 ME, ROE and I/A factors

The factors in the Q-model are created as follows: The data for the ME factor is the same as for the SMB factor in the FF3 model, while the construction of the factor is done slightly different. Further we collected financial data on income before extraordinary items, book equity and total assets. To calculate ROE and I/A we use the respective formulas:

$$ROE = \frac{Income\ before\ extraordinary\ items}{Book\ equity_{t-1quarter}} \tag{16}$$

$$I/A = \frac{Total\ assets_t - Total\ assets_{t-1}}{Total\ assets_{t-1}} \tag{17}$$

After we calculated the financials for each company, we created the factors in the same way as for the FF3 model, with separating the companies into three portfolios using the 70% and 30% quantiles for ROE and I/A. The ME factor still splits the sample in half. This gives us 2x2x3 portfolios.

$$ME = \frac{SH_{ROE} + SM_{ROE} + SL_{ROE} + SH_{I/A} + SM_{I/A} + SL_{I/A}}{6} \tag{18}$$

$$- \frac{BH_{ROE} + BM_{ROE} + BL_{ROE} + BH_{I/A} + BM_{I/A} + BL_{I/A}}{6}$$

$$ROE = \frac{SH_{ROE} + BH_{ROE}}{2} - \frac{SL_{ROE} + BL_{ROE}}{2} \tag{19}$$

$$I/A = \frac{SH_{I/A} + BH_{I/A}}{2} - \frac{SL_{I/A} + BL_{I/A}}{2} \tag{20}$$

5.4 Descriptive statistics

Descriptive statistic of the factors is presented in table 1. We identify that the market risk factor ERM has the highest mean with a value of 0.91% while ROE has the lowest with 0.01%. It can also be noted that ROE has the highest standard deviation. The size factor of the Q-model (ME) has the lowest standard deviation, while the size factor in the FF3 model also has a relatively low standard deviation.

Table 1: Summary statistics of the factors

Table 1 shows statistics for monthly excess returns for the factors

Variable	**Mean**	**Max**	**Min**	**Std.Dev**	**Skewness**	**Kurtosis**	**N-Observations**
				Q-Factor			
ERM	0.91%	15.49%	-25.58%	5.62%	-1.1	4.03	
ME	0.46%	17.27%	-13.34%	4.43%	0.18	0.60	**211**
ROE	-0.01%	21.71%	-30.72%	6.41%	-0.35	2.43	
I/A	0.68%	20.54%	-20.48%	5.29%	-0.08	2.61	
				Fama			
SMB	0.38%	18.77%	-16.92%	4.82%	0.09	1.48	
HML	0.25%	20.95%	-19.55%	6.36%	0.11	0.62	**211**

6 Results and analysis

In this section we describe the results of the tests we have conducted on the different asset pricing models. In the first subsection we discuss the reasoning behind the use of the value-weighted portfolios. In the second subsection we discuss the correlation between the created factors, while the rest of the chapter describes and discusses the results from our cross-sectional regressions.

6.1 The use of value-weighted portfolios

In our analysis we have used value-weighted portfolios to better compare the models against the benchmark index. The OSEBX index which we use to test our portfolios against is value-weighted, making it logical for us to create value-weighted factors and test portfolios. As described in chapter five, we have created equally-weighted portfolios. Our findings suggest that the value-weighted portfolios are better suited than the equally-weighted portfolios in describing the cross-sectional returns in the Norwegian market. We believe reasons for this could include the fact that there are a lot of listings and delisting's from the OSEBX in the time-period we are examining. Delistings often occurs when companies are taken private, hence we are more likely to experience small firms being delisted. If these small firms are given more weight in the portfolios, as is the case for the equally-weighted portfolios, we experienced less significant values and less explanatory power than in the case of value-weighted. Further in this chapter we focus on the results from the value-weighted portfolios, while the results from the analysis conducted on the equally-weighted portfolios are presented in the appendix.

6.2 Correlation between the factors

When using regressions to estimate the factors, one takes the assumption that the explanatory variables are not correlated with each other. If this assumption does not hold, the problem of multicollinearity may occur. If the dataset contains multicollinear variables, the standard errors of the coefficients will be affected, and the results are biased. Additionally, multicollinearity within the explanatory variables will make the regression highly sensitive to small changes in the variables. This sensitivity could lead to extensive changes in the coefficients values or the significance of the other variables when adding or removing variables from the regression. (Brooks. 2014)

The correlation matrix for our explanatory variables is presented in table 2. We identify that the SMB and ME factor have a correlation of 0.8877. Although both are size factors, the correlation indicates that there are some differences in the two in the way they are calculated. Although their respective correlation is high, the p-value describes that the correlation is not significant. We identify that the HML factor has low p-values regarding the two size factors, indicating significant correlation.

We believe this could be caused by the historically overweight of value companies within the oil and telecom industry in the Norwegian market. The value factor seems to indicate through its correlation with size that the Norwegian market are dominated by the big value firm companies and that the growth companies seems to remain small in comparison. As a result of this, we can link this together with the discussion about value- and equally-weighted portfolios as well. When the correlation is significant, giving these big companies a higher weight in the portfolios, as in the case of value-weighted portfolios, we believe we get a better representation of the Norwegian market in comparison of us using equally-weighted portfolios.

The original q-factor model paper (Hou et al., 2015) finds significant correlation between the HML and the I/A factor. In our test of the Norwegian market, we find no such significant correlation. This result strengthens our theory that the US and Norwegian markets could have different characteristics, implying we may not find the same results as the original paper did later in our tests.

Table 2: Correlation between the factors

Panel A shows the correlations between the factors in the Q-factor model and the Fama-French three-factor model in the time period 2003-2020

Panel B shows the corresponding p-values to the correlation matrix.

Panel A	ERM	ME	SMB	I/A	ROE	HML
ERM	1.00					
ME	-0.028	1.00				
SMB	0.001	0.888	1.00			
I/A	-0.169	-0.0200	-0.06	1.00		
ROE	-0.293	-0.142	-0.135	-0.168	1.00	
HML	0.322	0.145	0.147	-0.039	-0.281	1.00

Panel B	ERM	ME	SMB	I/A	ROE	HML
ERM	1.00					
ME	0.229	1.00				
SMB	0.170	0.986	1.00			
I/A	0.366	0.214	0.182	1.00		
ROE	0.569	0.216	0.305	0.257	1.00	
HML	0.537	0.005	0.049	-0.227	-0.646	1.00

6.3 Results from the Fama-MAcBeth two step regression

6.3.1 CAPM

Table 3 shows the results for the CAPM. Looking at the t-stat of the market risk factor we identify that it is significant at the 1% level. The significance of the coefficients implies that the CAPM predictions hold. When the CAPM predictions hold, we have a reasonably well-specified model and the market is a priced in risk factor in the Norwegian market. Looking at the sign of the estimated risk premium for the CAPM model, it is negative. A negative market risk premium implies that higher risk in the market would lead to lower returns. The results conflict with modern portfolio theory and intuition that investors want compensation for higher risk. Although the results are conflicting with theory, (Asness et al., 2012) found that "Leverage aversion changes the predictions of modern portfolio theory. It implies that safer assets must offer higher risk-adjusted returns than riskier assets because leverage-averse investors tilt their portfolio toward riskier assets to achieve high unleveraged retuns, thus pushing up the prices of risky assets and reducing the expected return on those assets."

Table 3,4 and 5 reports the results from the cross-sectional regressions using the Fama and MacBeth procedure. The models are estimated using monlthly excess returns on the three different sets of test portfolios (Size, ROE and I/A). Column 2 reports the intercept, column 3-6 reports the estimated risk premia and corresponding t-statistic, column 7-8 reports the R^2 and adjusted R^2 for the estimated model. *,**,*** indicates significance level at the 10, 5 and 1 percent, respectively

Table 3: CAPM cross-sectional regression

2003-2020	λ_0	λ_{ERM}	R^2	R^2_{adj}
CAPM	0.028	-0.0285	0.5082	0.5082
T-ratio	(3.0989)**	(-3.4762)***		

6.3.2 FF3

Looking at the results from the Fama-MacBeth regression on the FF3 model we see that the market factor is priced in at the 1% level and the SMB factor is priced in at a 5% significance level. The HML factor is not significant. We identify that by adding SMB and HML to the CAPM, it decreases the importance of the market factor although it remains significant and negative. These results can be argued towards the implication that the HML factor which can be described as the value factor, is redundant for describing the average return in the OSEBX index.

Table 4: FF3 cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{SMB}	λ_{HML}	R^2	R^2_{adj}
FF3	0.0295	-0.0292	-0.0074	-0.0011	0.6486	0.6225
T-ratio	(3.0307)**	(-3.8005)***	(-1.6419)	(-0.0268)		

6.3.3 Q-factor

The results from the q-factor model is presented in table 5. We identify that only one factor is significant at the 5% level, the market factor. The factors for size (ME), ROE and I/A are not significant. We have similar results in this model compared to CAPM and FF3, that the market factor is negative. We can also see that adding the other factors in the model to the market factor makes it less significant than in the CAPM model and the FF3 model.

Table 5: Q-factor cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{ME}	λ_{ROE}	$\lambda_{I/A}$	R^2	R^2_{adj}
Q-factor	0.0252	-0.0235	-0.0058	0.0135	0.0163	0.7044	0.6703
T-ratio	(2.194)**	(-1.8106)*	(-1.2354)	(0.6862)	(1.1592)		

6.4 Stability in results

Table 6: CAPM Stability results

Table 6,reports the results from the cross-sectional regressions using the Fama and MacBeth procedure on different time samples for the CAPM model. Column 3 reports the intercept, column 4 reports the estimated risk premia and corresponding t-statistic, column 5-6 reports the R^2 and adjusted R^2 for the estimated model, column 7 reports the number of observations. *,**,*** indicates significance level at the 10, 5 and 1 percent, respectively,

FF3		λ_0	λ_{ERM}	R^2	R^2_{adj}	N
2003-2008	Coefficient	0.0216	-0.0218	0.163	0.163	67
	T-ratio	(2.0333)**	(-1.6623)**			
2008-2014	Coefficient	0.0158	-0.0146	0.1397	0.1397	66
	T-ratio	(2.0328)**	(-1.4774)*			
2014-2020	Coefficient	0.0103	-0.013	0.1584	0.1584	78
	T-ratio	(1.9882)**	(-2.2891)**			

Table 7: FF3 Stability results

Table 7,reports the results from the cross-sectional regressions using the Fama and MacBeth procedure on different time samples for the FF3 model. Column 3 reports the intercept, column 4-6 reports the estimated risk premia and corresponding t-statistic, column 7-8 reports the R^2 and adjusted R^2 for the estimated model, column 9 reports the number of observations. *,**,*** indicates significance level at the 10, 5 and 1 percent, respectively,

FF3		λ_0	λ_{ERM}	λ_{SMB}	λ_{HML}	R^2	R^2_{adj}	N
2003-2008	Coefficient	0.0195	-0.0205	0.0042	-0.0053	0.1755	0.1144	67
	T-ratio	(1.3641)*	(-1.0575)	(0.5135)	(-0.2692)			
2008-2014	Coefficient	0.0193	-0.0161	-0.0089	-0.0045	0.278	0.2189	66
	T-ratio	(2.5167)***	(-1.5332)*	(-1.4235)*	(-0.3551)			
2014-2020	Coefficient	0.0095	-0.0117	-0.0121	0.0175	0.4939	0.4564	78
	T-ratio	(1.2852)*	(-1.1084)	(-2.4046)**	(0.3168)			

Table 8: Q-factor Stability results

Table 7,reports the results from the cross-sectional regressions using the Fama and MacBeth procedure on different time samples for the Q-factor model. Column 3 reports the intercept, column 4-7 reports the estimated risk premia and corresponding t-statistic, column 8-9 reports the R^2 and adjusted R^2 for the estimated model, column 10 reports the number of observations. *,**,*** indicates significance level at the 10, 5 and 1 percent, respectively,

Q-factor		λ_0	λ_{ERM}	λ_{ME}	λ_{ROE}	$\lambda_{I/A}$	R^2	R^2_{adj}	N
2003-2008	Coefficient	0.0098	-0.0104	-0.0021	-0.0106	0.023	0.3879	0.3173	67
	T-ratio	(0.8094)	(-0.6135)	(-0.3549)	(-0.7381)	(2.1812)***			
2008-2014	Coefficient	0.0224	-0.0109	-0.0081	0.0037	-0.002	0.2954	0.2124	66
	T-ratio	(2.6284)***	(-1.7389)**	(-1.2158)	(0.2909)	(-0.3084)			
2014-2020	Coefficient	-0.0026	0.0045	-0.0024	0.0194	0.0252	0.7253	0.6937	78
	T-ratio	(-0.1361)	(0.1712)	(-0.0978)	(0.4394)	(1.1201)			

In table 6, 7 and 8 the results from the split sample test are depicted. The results indicate that the q-factor model is the better suited among the three models in explanatory power in the period stretching from 2003-2008 and 2014-2020, while the Fama-French three factor model performs is slightly better in 2008-2014. The period between 2008-2014 can be described as highly volatile. The financial crises in 2008 saw the OSEBX index drop more than 50% over the course of six months, and there was another drop of more than 25% three years later.

Our three models all yield a significant intercept in the time-period between 2008-2014, implying that the models are missing some priced in risk factors. Although all three of our models yield a significant intercept in this period, we find that all three models have a significant market risk factor. Implying that the market was a significant factor in the period between 2008-2014. Equal to the results discussed when regressing the entire sample period, we find the market risk factor to be negative in the 2008-2014 time-period. Again, implying a negative relationship between risk and reward. Interestingly we find that the q-factor model has statistically insignificant intercepts in the first and last time-period. A statistically insignificant intercept implies that the risk factors explain the cross-sectional returns. In other words, the model can be seen as well specified.

In the time-period spanning from 2003-2008 we find the I/A factor in the q-factor model to be significant at a 1% level. This statistically significant coefficient implies that the investment channel was priced in prior to the financial crises in the Norwegian market. After the financial crises, we see no significance. We interpret the change in significance levels as a change in investors preferences after the financial crises. High investments may imply high leverage, especially in the Norwegian market. As discussed, the Norwegian market historically consist of big value firms. These value firms often need high leverage to finance their investments as they are of a substantial scale. After the financial crises high leverage could be seen as risky, and there could be trouble getting financing. We believe these are the reasons for why we do not see any significance within the investment factor in the last two time-periods.

6.5 Intercept analysis

6.5.1 GRS

In the intercept analysis we rely on the GRS test statistic obtained in our time-series regression. The GRS-test is designed with the null hypothesis being that the intercepts of the explanatory variables are jointly equal to zero. If the null hypothesis is not rejected, the intercepts is not significantly different from zero and all risk factors are priced in. If the null is rejected, the model indicates that there are some factors that are not priced in, and we have unexplained abnormal returns.

Table 9: Intercept statistics

The first column shows which model the data is reported for. The second column reports the average of absolute estimated time-series alphas. The third column presents the standard deviation of the estimated time-series alphas. The fourth column reports the GRS-statistics, while the fifth column reports the corresponding p-value.

	Intercept	Std.Dev	GRS	p-GRS
CAPM	0.011	0.007	5.378	0.000
FF3	0.012	0.008	3.986	0.000
Q-Factor	0.010	0.007	2.972	0.000

The results from the analysis indicates that all models yield GRS statistics that rejects the null hypothesis on all commonly used significant levels. This indicates that all our models are incomplete in describing the expected time-series returns. Although we reject the null hypothesis in all our models, this thesis aims to compare the models. We find that CAPM has the highest GRS statistic, with the corresponding highest absolute alpha. We find that the q-factor model yields the lowest absolute alpha and the lowest GRS statistic. While we find the q-factor model to be the better suited amongst the three, although it is still not sufficient in describing the returns in the Norwegian market.

Table 10: Cross-sectional intercepts

The first column shows which model the data is reported for. The second column reports the intercept of the Fama-macbeth regression. The third column presents the corresponding T-ratio.

	λ_0	T-ratio
CAPM	0.0280	(3.0989)***
FF3	0.0295	(3.0307)***
Q	0.0252	(2.194)**

6.5.2 Cross-sectional intercept

Table 10 provides statistics for all intercepts from the cross-sectional regressions. We observe intercepts which are statistically significantly different from 0 for all models, implying that the included risk factors do not explain all the cross-sectional returns. As all our intercepts are statistically different from 0, we favor a small intercept, to explain most of the cross-sectional return. We find that the largest intercepts is for CAPM and FF3, with an intercept of 0.0280 and 0.0295 respectively. The q-factor has the smallest intercept of 0.0252. These intercepts are high, and the smallest intercept of the q-factor model has an average excess return of 2.52 %.

Based on the analysis of cross-sectional intercept, we conclude that q-factor is the most favored model. Firstly, the q-factor produces the lowest intercept of all the models. Secondly, the t-stat of the intercept is lowest amongst the models as well. However, we emphasize that differences are marginal between the models, and that the q-factor does not produce superior cross-sectional intercepts compared to the other models.

6.6 Goodness of fit

In this section we focus on the adjusted R^2. We focus on the adjusted measure as we have tested asset pricing models with different numbers of explanatory variables. If we were to look at the actual R^2 and not the adjusted, we should experience higher R^2 for the model with the most explanatory variables as adding more variables should explain more of the variation on average returns. As we are focusing on the explanatory power we are using the measurement as comparison purposes only, as the R should not be viewed as the true explanatory power (Kan et al., 2013).

In our Fama-MacBeth regressions we find that the q-factor has the highest adjusted R^2 of 0.6777. The FF3 model has an adjusted R^2 of 0.6564 and the CAPM has a value of 0.5082. We see that the q-factor and FF3 are similar, while the CAPM model has low R^2 relative to the other models.

When we tested the three models we wanted to find out if the models could be improved with the factors we already had created. We decided to run tests on the q-factor model including factors from FF3 and vice versa. Our initial hypothesis was that we would see a higher R^2 as we included more explanatory variables and we expected the adjusted R^2 to remain the same or lower. Our hypothesis was correct when adding the HML factor to the q-factor model. The results showed a higher R^2, but slightly lower adjusted R^2. When adding the SMB factor as well we interestingly saw a higher adjusted R^2. We thought that adding a second size factor which is correlated with the size factor already in the regression would provide no further explanatory power, but the results proved false. In fact, this was the model which had the highest adjusted R^2 when comparing it to models where we inserted q-factor factors into the FF3 as well. The results from the Fama-MacBeth regressions when adding more coefficients is presented in appendix A.4

7 Conclusion

This study compares the asset pricing models CAPM, Fama-French three factor model and the q-factor model. The models are tested to evaluate which model is superior in describing the cross-section of expected returns in the Norwegian stock market. This is done by collecting financial data from all corporations that have been in the OSEBX index from January 2001 until February 2021. With the data we have created the characteristic-based factors RME, SMB, HML, ME, I/A and ROE. The factors are tested to see which risk factors are priced in for the Norwegian market.

Firstly, we created value and equal weighted test portfolios based on three characteristics (Size, ROE and I/A). The test portfolios are split in ten, based on quantiles resulting in a total of 30 test portfolios. Secondly, we regress the performance of the test portfolios based on the Fama-MacBeth two step regression to determine factor coefficients for the Norwegian stock market. Thirdly, we evaluate the models based on intercept analysis, the goodness of fit statistic and stability in results..

Our findings indicates that the market portfolio is a priced risk factor in all three of our models. The market factor is significant in all models, but the sign of the factor indicates a negative risk-return relationship. This is not consistent with Modern Portfolio Theory, which could indicate that safer assets must offer higher risk-adjusted returns compared to riskier assets. This could be because leverage-averse investors tilt their portfolios towards riskier assets for higher unlevered returns according to (Asness et al., 2012). We believe this finding could be a result of the high weight a few of the big oil-related firms have in the Norwegian market. As big value firms often have smaller returns than smaller cap firms, investors could seek smaller cap firms. As the market is oil-heavy it will move with the oil price, and the smaller firms may not benefit from oil-price rise as they are consumers and not producers of the oil. If we believe the findings in (Asness et al., 2012), this would lead to lower returns for the unlevered investors in small cap firms when the market rises and vice versa, hence a negative market risk premium.

We further find that the SMB factor is priced in in the Norwegian market. The other factors in the q-factor and the HML factor in FF3 seem to be not significant in the Norwegian stock market.

We find that the q-factor is marginally superior to the other models in terms of intercept, both in the time-series GRS statistic and in the cross-sectional intercept analysis. Although the q-factor is marginally superior in both cases, all models yield a statistically significant intercept. Hence, the models do not include all priced risk factor to fully explain the cross-sectional excess returns. For further research it could be interesting to evaluate additional factors to the model, such as the fifth factor introduced to the q-factor in 2020, namely the expected growth factor (Hou et al., 2020), in order to try to explain some of the risk which are not fully subsumed by the models.

The Goodness of fit analysis shows that the q-factor model is superior compared to the CAPM and FF3 models. It has the highest R^2, which is natural due to its numbers of explanatory variables, and it also has the highest adjusted R^2. Interestingly, we find a model with higher adjusted R^2 when we add SMB to the q-factor model. When testing the explanatory power of the equally-weighted portfolios we see lower levels of adjusted R^2. This result tells us that when smaller stocks are given larger weight in the portfolios, the models explain less of the expected returns in the Norwegian market. This could be a result of the characteristics the big and small firms in the Norwegian market entails, as exposure to the oil-price and the possibility of being delisted from the market.

Our findings indicate that amongst the tested models, the q-factor model is best suited to explain the expected returns in the Norwegian stock market. The q-factor model performs best when it comes to explanatory power, and it has the marginally smallest intercept. As this is the first time the q-factor model is tested in the Norwegian market, we believe there is ground to say that asset pricing models containing risk factors derived from investment and profitability channels could be beneficial for understanding the Norwegian stock market. Hence, although the q-factor model seems to miss some priced in risk factors, we believe practitioners should consider including the q-factor model when conducting analysis on the Norwegian stock market.

For future research it would be interesting to see how the q-factor model is able to explain the expected returns using the financial data stemming from the covid-19 pandemic year. As our last portfolio was created in June 2020 from the financial data from December 2019, we have not been able to regress the consequences of the Covid-19 pandemic. Looking at the implications of the pandemic could be interesting as we are dealing with financial data that in theory should provide information about further growth and profitability. As the q-factor model is based on the profitability- and investment channel, one could think that business models that are more profitable would perform better than less profitable businesses in the pandemic.

Looking further into economic downturns as the Covid-19 pandemic, it could be interesting to see how the models have performed isolated in previous crises. As we did not isolate the crises when splitting our data sample in three time-periods, we see the potential for further understanding of the Norwegian market. Methods like rolling window approaches for the factor loadings could be used to allow for time-varying betas in times of crises and out of crises.

A APPENDIX

A.1 Results and Analysis for Equal Weighted

Table 11: Summary statistics of the factors

Table 10 shows statistics for monthly excess returns for the factors

Variable	Mean	Max	Min	Std.Dev	Skewness	Kurtosis	N-Observations
				Q-Factor			
ERM	0.91%	15.49%	-25.58%	5.62%	-1.10	4.03	
ME	0.91%	15.81%	-8.05%	3.92%	0.66	1.28	**211**
ROE	0.05%	17.49%	-18.29%	6.22%	-0.40	0.89	
I/A	0.40%	15.64%	-20.26%	5.61%	-0.21	1.23	
				Fama			
SMB	0.86%	16.78%	-9.01%	3.75%	0.57	1.18	
HML	0.26%	22.96%	-17.73%	5.73%	0.37	2.13	**211**

Table 12: Correlation between the factors

Panel A shows the correlations between the factors in the Q-factor model and the Fama-French three-factor model in the time period 2003-2020

Panel B shows the corresponding p-values to the correlation matrix.

Panel A	**ERM**	**ME**	**SMB**	**I/A**	**ROE**	**HML**
ERM	1.00					
ME	-0.06	1.00				
SMB	0.01	0.91	1.00			
I/A	-0.16	-0.06	0.03	1.00		
ROE	-0.31	0.08	-0.16	-0.20	1.00	
HML	0.30	-0.07	-0.12	0.05	-0.24	1.00

Panel B	**ERM**	**ME**	**SMB**	**I/A**	**ROE**	**HML**
ERM	1.00					
ME	0.32	1.00				
SMB	0.20	0.97	1.00			
I/A	0.31	0.26	0.12	1.00		
ROE	0.58	0.00	0.21	0.36	1.00	
HML	0.51	0.49	0.43	0.00	0.52	1.00

Table 12,13 and 14 reports the results from the cross-sectional regressions using the Fama and MacBeth procedure. The models are estimated using monlthly excess returns on the three different sets of test portfolios (Size, ROE and I/A). Column 2 reports the intercept, column 3-6 reports the estimated risk premia and corresponding t-statistic, column 7-8 reports the R^2 and adjusted R^2 for the estimated model. *,**,*** indicates significance level at the 10, 5 and 1 percent, respectively

Table 13: CAPM cross-sectional regression

2003-2020	λ_0	λ_{ERM}	R^2	R^2_{adj}
CAPM	0.0202	-0.0208	0.1961	0.1961
T-ratio	(2.954)**	(-2.7066)***		

Table 14: FF3 cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{SMB}	λ_{HML}	R^2	R^2_{adj}
FF3	0.0217	-0.0219	-0.0038	-0.0045	0.2965	0.2443
T-ratio	(2.383)***	(-2.6704)***	(-1.1599)	(-0.549)		

Table 15: Q-factor cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{ME}	λ_{ROE}	$\lambda_{I/A}$	R^2	R^2_{adj}
Q-factor	0.0205	-0.0191	-0.0022	0.0112	0.0054	0.2702	0.186
T-ratio	(2.4035)***	(-2.42o1)***	(-0.6330)	(1.3342)*	(1.2238)		

A.2 Descriptive statistics for the three sets of constructed test portfolios

A.2.1 Value Weighted

Table 15-18 reports the statistics for the Size, I/A and ROE value weighted portfolios respectively, where the portfolios are split into quintiles. The first column display calculated properties of the portfolios. Where min (max) is the minimum (maximum) excess return, mean is the average excess return and Std.Dev is the standard deviation of the excess return. The last column reports the average across all the quintiles.

Table 16: CAPM cross-sectional regression

Table 17: Size sorted portfolios

2003-2020	Big	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	Small	**Average**
Min	-0.230	-0.281	-0.344	-0.236	-0.278	-0.263	-0.310	-0.184	-0.238	-0.279	**-0.263**
Max	0.295	0.222	0.184	0.367	0.249	0.274	0.250	0.635	0.851	0.546	**0.387**
Mean	0.005	0.007	0.006	0.008	0.004	0.006	0.006	0.017	0.025	0.025	**0.011**
Std.Dev	0.055	0.069	0.066	0.074	0.073	0.078	0.084	0.097	0.103	0.135	**0.08**

Table 18: Investment/Assets Portfolios

2003-2020	High	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	Low	**Average**
Min	-0.443	-0.434	-0.418	-0.259	-0.313	-0.427	-0.209	-0.223	-0.389	-0.343	**-0.346**
Max	0.371	0.408	0.218	0.257	0.194	0.281	0.264	0.271	0.301	0.750	**0.332**
Mean	0.003	-0.004	-0.002	0.01	0.007	0.002	0.011	0.015	0.013	0.007	**0.005**
Std.Dev	0.096	0.103	0.077	0.071	0.069	0.079	0.068	0.068	0.089	0.133	**0.09**

Table 19: ROE Portfolios

2003-2020	High	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	Low	**Average**
Min	-0.249	-0.426	-0.308	-0.270	-0.257	-0.357	-0.417	-0.301	-0.475	-0.416	**-0.348**
Max	0.231	0.267	0.263	0.211	0.289	0.288	0.391	0.672	0.389	0.847	**0.384**
Mean	0.008	0.006	0.001	0.008	0.003	0.009	0.009	0.008	0.011	0.004	**0.007**
Std.Dev	0.069	0.078	0.075	0.063	0.068	0.082	0.094	0.110	0.125	0.136	**0.09**

A.2.2 Equal Weighted

Table 15-18 reports the statistics for the Size, I/A and ROE equal weighted portfolios respectively, where the portfolios are split into quintiles. The first column display calculated properties of the portfolios. Where min (max) is the minimum (maximum) excess return, mean is the average excess return and Std.Dev is the standard deviation of the excess return. The last column reports the average across all the quintiles.

Table 20: Size Constructed Portfolios

2003-2020	High	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	Low	**Average**
Min	-0.299	-0.285	-0.343	-0.240	-0.270	-0.279	-0.294	-0.196	-0.255	-0.275	**-0.274**
Max	0.191	0.232	0.184	0.357	0.250	0.262	0.275	0.497	0.796	0.482	**0.353**
Mean	0.007	0.007	0.007	0.008	0.004	0.006	0.006	0.016	0.025	0.029	**0.012**
Std.Dev	0.061	0.069	0.067	0.074	0.073	0.081	0.084	0.093	0.102	0.119	**0.08**

Table 21: Investment/Assets Constructed Portfolios

2003-2020	High	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	Low	**Average**
Min	-0.297	-0.336	-0.409	-0.294	-0.255	-0.273	-0.326	-0.226	-0.289	-0.270	**-0.274**
Max	0.376	0.565	0.567	0.288	0.209	0.284	0.284	0.388	0.414	0.197	**0.357**
Mean	0.014	0.013	0.004	0.010	0.011	0.012	0.010	0.018	0.006	0.011	**0.011**
Std.Dev	0.091	0.099	0.088	0.073	0.066	0.071	0.071	0.077	0.085	0.063	**0.08**

Table 22: ROE Constructed Portfolios

2003-2020	High	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	Low	**Average**
Min	-0.297	-0.225	-0.217	-0.230	-0.257	-0.340	-0.268	-0.291	-0.339	-0.362	**-0.281**
Max	0.291	0.184	0.198	0.201	0.289	0.752	0.306	0.505	0.434	0.606	**0.377**
Mean	0.013	0.009	0.008	0.009	0.003	0.018	0.014	0.010	0.015	0.010	**0.011**
Std.Dev	0.079	0.061	0.059	0.057	0.068	0.090	0.082	0.101	0.113	0.123	**0.08**

A.3 Cross-sectional regressions

A.3.1 Value Weighted

Table 22-24 reports descriptive statistics for the different value weighted sets of test assets describe in section ”Data”. The second column display the minimum, maximum, average and standard deviation for the factor loadings for CAPM. Third to fifth column displays values for the factors loading of FF3, sixth to ninth column displays for the q-factor. The last column displays the average for all the 8 factor loadings.

Table 23: Size cross-sectional regression

Size	**CAPM**	**FF3**			**Q**				
2003-2020	β_{ERM}	β_{ERM}	β_{SMB}	β_{HML}	β_{ERM}	β_{ME}	β_{ROE}	$\beta_{I/A}$	**Average**
Min	0.8958	0.868	-0.1742	-0.2794	0.7874	-0.1695	-0.4459	-0.2751	**0.1509**
Max	1.1153	1.1118	0.8906	0.2696	1.0387	0.9194	-0.0289	0.1038	**0.6775**
Mean	0.9937	0.9776	0.3714	0.0433	0.9231	0.3644	-0.2074	-0.0586	**0.4259**
Std.Dev	0.0714	0.0823	0.3693	0.1482	0.0791	0.3711	0.1218	0.1103	**0.1692**

Table 24: ROE cross-sectional regression

ROE	**CAPM**	**FF3**			**Q**				
2003-2020	β_{ERM}	β_{ERM}	β_{SMB}	β_{HML}	β_{ERM}	β_{ME}	β_{ROE}	$\beta_{I/A}$	**Average**
Min	0.8054	0.7558	-0.1967	-0.0355	0.8170	-0.1622	-0.3784	-0.4684	**0.1421**
Max	1.5494	1.5439	0.3498	0.2100	1.4636	0.3932	0.0887	0.3160	**0.7393**
Mean	1.1099	1.0863	0.0779	0.0649	1.0726	0.0834	-0.1057	-0.0245	**0.4206**
Std.Dev	0.2205	0.2239	0.1720	0.0914	0.2015	0.1613	0.1305	0.2833	**0.1856**

Table 25: I/A cross-sectional regression

I/A	**CAPM**	**FF3**			**Q**				
2003-2020	β_{ERM}	β_{ERM}	β_{SMB}	β_{HML}	β_{ERM}	β_{ME}	β_{ROE}	$\beta_{I/A}$	**Average**
Min	0.8637	0.8326	-0.098	-0.0189	0.8084	-0.1122	-0.4043	-0.1542	**0.2146**
Max	1.5494	1.2504	0.4901	0.1765	1.1910	0.3013	0.1728	0.0652	**0.6496**
Mean	1.1099	1.0202	0.1078	0.0631	0.9966	0.1036	-0.1333	-0.0281	**0.4050**
Std.Dev	0.2205	0.1351	0.1689	0.0622	0.1479	0.1341	0.1736	0.0598	**0.1379**

A.3.2 Equal weight

Table 25-27 reports descriptive statistics for the different value weighted sets of test assets describe in section "Data". The second column display the minimum, maximum, average and standard deviation for the factor loadings for CAPM. Third to fifth column displays values for the factors loading of FF3, sixth to ninth column displays for the q-factor. The last column displays the average for all the 8 factor loadings.

Table 26: Size cross-sectional regression

Size	**CAPM**	**FF3**			**Q**				
2003-2020	β_{ERM}	β_{ERM}	β_{SMB}	β_{HML}	β_{ERM}	β_{ME}	β_{ROE}	$\beta_{I/A}$	**Average**
Min	0.8916	0.8517	-0.1061	-0.0628	0.8602	-0.1690	-0.4246	-0.2260	**0.2019**
Max	1.0972	1.0905	1.1693	0.3860	1.0802	1.0614	0.0406	0.3480	**0.7842**
Mean	1.0125	0.9776	0.4613	0.1149	0.9653	0.3982	-0.1923	0.0057	**0.4679**
Std.Dev	0.0633	0.0766	0.4861	0.1434	0.0839	0.4577	0.1390	0.1611	**0.2014**

Table 27: ROE cross-sectional regression

ROE	**CAPM**	**FF3**			**Q**				
2003-2020	β_{ERM}	β_{ERM}	β_{SMB}	β_{HML}	β_{ERM}	β_{ME}	β_{ROE}	$\beta_{I/A}$	**Average**
Min	0.8630	0.8066	0.1521	0.0009	0.8000	0.1370	-0.4120	-0.5037	**0.2305**
Max	1.2790	1.2337	1.3827	0.2175	1.1661	1.1907	0.0597	0.7710	**0.9126**
Mean	1.0208	0.9821	0.4583	0.1274	0.9736	0.4032	-0.1978	0.0155	**0.4729**
Std.Dev	0.1294	0.1330	0.3617	0.0678	0.1129	0.3086	0.1353	0.3947	**0.2054**

Table 28: I/A cross-sectional regression

I/A	**CAPM**	**FF3**			**Q**				
2003-2020	β_{ERM}	β_{ERM}	β_{SMB}	β_{HML}	β_{ERM}	β_{ME}	β_{ROE}	$\beta_{I/A}$	**Average**
Min	0.8420	0.8354	0.1771	0.0219	0.7566	0.1611	-0.5406	-0.1444	**0.2636**
Max	1.1958	1.1228	0.8792	0.2404	1.2142	0.8417	-0.0551	0.1521	**0.6989**
Mean	1.0377	0.9987	0.4678	0.1283	0.9875	0.3976	-0.2068	0.0178	**0.4786**
Std.Dev	0.1111	0.0942	0.2582	0.0723	0.1457	0.2263	0.1590	0.1008	**0.1459**

A.4 Added factors to the models

Table 28-33 displays the q-factor and ff3 with added factors from each other. The last two columns displays the R^2 and adjusted R^2 while the second to the third last columns represent estimated risk premia and it's corresponding t-statistics. *,**,*** indicates significance level at the 10, 5 and 1 percent, respectively.

A.4.1 Q-factor with added HML factor

Table 29: Q + HML cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{ME}	λ_{ROE}	$\lambda_{I/A}$	λ_{HML}	R^2	R^2_{adj}
$Q + HML$	0.0255	-0.0242	-0.0057	0.130	0.152	-0.0047	0.7167	0.6714
T-ratio	(2.7243)***	(-2.5631)**	(-1.164)	(0.8081)	(1.9345)*	(-0.1058)		

A.4.2 Q-factor with added HML and SMB factor

Table 30: Q + HML + SMB cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{ME}	λ_{ROE}	$\lambda_{I/A}$	λ_{HML}	λ_{SMB}	R^2	R^2_{adj}
$Q + Fama$	0.0266	-0.0257	-0.0035	0.0120	0.0139	-0.0075	-0.0103	0.7681	0.7198
T-ratio	(2.8178)***	(-2.6512)***	(-0.545)	(0.7839)	(1.6787)*	(-0.2052)	(-1.7111)*		

A.4.3 Fama with added ROE factor

Table 31: FF3 + ROE cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{SMB}	λ_{HML}	λ_{ROE}	R^2	R^2_{adj}
$FF3 + ROE$	0.0305	-0.0298	-0.0068	-0.0013	0.0129	0.6554	0.6156
T-ratio	(4.2250)***	(-4.2040)***	(-1.7124)**	(-0.0318)	(0.9083)		

A.4.4 Fama with added ROE and I/A Factor

Table 32: FF3 + ROE + I/A cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{SMB}	λ_{HML}	λ_{ROE}	$\lambda_{I/A}$	R^2	R^2_{adj}
$FF3 + ROE + I/A$	0.0258	-0.0245	-0.0075	-0.0048	0.0122	0.015	0.7369	0.6948
T-ratio	(2.7794)***	(-2.6056)***	(-1.9344)*	(-0.1094)	(0.7903)	(1.8839)*		

A.4.5 Comparison of size factors

CAPM + ME

Table 33: CAPM + ME cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{ME}	R^2	R^2_{adj}
$CAPM + ME$	0.0293	-0.0287	-0.0054	0.5786	0.5635
T-ratio	(3.1065)***	(-3.4576)***	(-1.3429)		

CAPM + SMB

Table 34: CAPM + SMB cross-sectional regression

2003-2020	λ_0	λ_{ERM}	λ_{SMB}	R^2	R^2_{adj}
$CAPM + SMB$	0.0300	-0.0291	-0.0073	0.6145	0.6007
T-ratio	(3.3557)***	(-3.5377)***	(-1.6844)*		

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