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Buyer Power and Price Formation in Intermediate Goods Markets: a Dynamic Perspective

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Buyer Power and Price Formation in Intermediate Goods Markets

a Dynamic Perspective

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Abstract

This thesis studies how buyer power affects input prices in intermediate goods markets. The aim is to investigate how a buyer can invest to increase buyer power, and we use Inderst and Wey (2007) as a baseline for our study. We further extend the model to allow for size-based investment downstream and use the framework to briefly touch on the topic of prohibiting price discrimination. Our paper discusses previous literature on buyer power and price formation and provides somewhat novel insight into buyer-side investment incentives, an area where extant research is relatively narrow. We use a downstream agent's total number of operated stores as a measure of buyer power and vary how the agents can acquire additional stores.

We find that large buyer discounts depend on whether or not investments introduce additional retail stores to the market. When agents only reallocate stores, large-buyer discounts are amplified through investment. On the contrary, introducing new stores raises input prices faced by all buyers, and smaller buyers may have a higher incentive to grow. Interestingly, when all buyers invest, it might reduce their individual profits. They may still choose to invest in order not to fall behind. Furthermore, we provide some evidence that forbidding price discrimination may be welfare-hindering and reduce investment incentives.

Keywords – Industrial Organisation, Price Formation, Buyer Power, Investment Incentives, Price Discrimination

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1 Introduction

It is a highly discussed topic whether large firms in vertical supply chains face lower wholesale prices than small firms do. Price differences may be explained by, for instance, economies of scale and quantity discounts. Some argue that differences are not necessarily evidence of price discrimination, while others say more systematic discrimination is in place. We say large retail firms have buyer power when they can negotiate for better prices than if they were smaller. In an increasingly competitive environment, coming out on the better end of negotiations is pivotal for a firm to succeed among its peers.

Although many industry-leading firms are vertically integrated, several downstream markets are dominated by large buyers. For instance, the three largest retailers in the Norwegian grocery market make up 96.2 percent of the total market shares (Wifstad et al., 2018). On the other hand, smaller grocery chain stores like Rimi and ICA were bought by their more powerful rivals in recent years, resulting in a more concentrated market with high barriers to entry.

The literature on buyer power stems from allegations of adverse effects that monopolist wholesalers can inflict on welfare by discriminating in prices toward competing retailers. The main accusations are typically that systematically differentiating input prices can lead to higher consumer prices and less competition, which can harm the economy. Input price discrimination in favor of large retailers is a well-known topic in modern industrial economics and the focus of antitrust authorities. In 2019, The Norwegian Competition Authority examined the purchasing conditions in the Norwegian grocery sector. Their investigation found significant differences in prices given to the retailers with variations up to more than 15 percent (Sørgard and Birkeland, 2019). In most cases, the suppliers gave the more prominent retailer the best conditions.

The US' Robinson-Patman Act of 1936 specifically targets big firms exploiting their market positions to gain "unfair" advantages on smaller firms through lower input prices. This motivated Katz (1987) to investigate whether the effects of a ban on price discrimination toward firms that differ in buyer power are beneficial in the grand scheme of things. His results were somewhat ambiguous and heavily dependent on a few key assumptions, but it sparked a debate that has not been fully settled yet. The literature on price discrimination has since branched out to study the factors that facilitate discrimination and determine which fundamental assumptions tug on the consensus in opposite directions. For instance, O'Brien (2014) extends Katz' framework to account for bargaining, DeGraba (1990), Inderst and Valletti (2009), and Akgün and Chioveanu (2019) study frameworks where agents can undertake investments to reduce costs, and Inderst and Wey (2007), Ellison and Snyder (2010), and Gaudin (2017) account for buyers' size as a means to obtain better negotiation terms. This thesis focuses on the latter assumption, in particular Inderst and Wey's model, to incorporate investments in size to examine the dynamic effects on input prices and profitability. We will also use the framework to study the implication of price discrimination on welfare.

1.1 Research question

We are interested in studying price formation in a vertical bilateral market in a dynamic setting. More specifically, we want to attempt to model buyer power, allowing the downstream firm to invest in its size. We want to examine how growth in buyer power influences key economic variables in intermediate goods markets.

The aim is to answer the following research question:

• How does buyer power affect input prices and size-based investment incentives in a bilateral negotiation structure in intermediate goods markets?

Our research question relates to different fields within the expanding literature on price discrimination. As expressed, our main focus is to discuss input prices and investment incentives. In addition, we will expand our discussion to see what happens under a ban on price discrimination.

As our thesis is solely based on literature and economic theory, the literature must be critically evaluated. The articles we present in the introduction and the literature review have been carefully selected as to truthfully reflect different

viewpoints on the topics. Even though there is an abundance of works that deserve their spotlights, in order to moderate our thesis, we have picked three well-known publications to review more thoroughly. For instance, virtually all modern literature on price discrimination is rooted in Katz (1987), and we have therefore chosen to emphasize particularly this article in the beginning of our thesis. This is important to gain valuable insight as to how theoretical research on wholesale price formation and discrimination is conducted. The reason we also chose to do an extensive review on O'Brien (2014), is that it provides a direct extension and critique to Katz. This result is valuable when moving onward with our own extension and contribution to the literature. The emphasis on DeGraba (1990) is to investigate a method of extending existing literature into the topic of investment incentives. We have selected subsequent literature to the review based on select criteria such as the number of citations, whether the works have been cited across relevant literature, and whether the articles have been published in critically acclaimed journals.

This thesis proceeds as follows. Chapter 2 introduces relevant literature regarding buyer power and price discrimination and discusses a few key articles in-depth. In chapter 3, we explain the methodology framework on how we will answer our thesis question. Chapter 4 presents a model based on Inderst and Wey (2007), which we further extend as our contribution to the literature and will be the **main part** of our thesis. In chapter 5, we discuss the plausibility of several key assumptions of the model in chapter 4. Chapter 6 provides concluding remarks to our principal findings, and finally, we show the most extensive derivations of our model in the appendix.

2 Literature Review

The proceeding sections will introduce the concept of buyer power and price discrimination and review relevant literature to provide a foundation to our theoretical model. In addition, we will focus on what determines input prices since a lot of the main results in the literature depend on the assumptions made on this topic. We will present different views and findings by various authors to give our model both credibility and critique. We begin by introducing several articles on buyer power and price discrimination before we review three front-runner papers to gain further intuition about how we will lay out our piece. Furthermore, we will append closely related works where we see fit to gain a more dynamic perception of the literature.

2.1 Buyer power and input prices

The debate on price discrimination has ushered in a literature growth on buyer power, and Galbraith (1952) was the first to define the term formally. In his book American Capitalism: The Concept of Countervailing Power, he states that what he calls "countervailing power" is the ability of large downstream firms to extract price concessions from upstream firms. As such, this buyerside effect will potentially offset or countervail suppliers' market power. In modern literature, the terms countervailing power and buyer power are often interchangeable. What determines buyer power, the welfare effects of price discrimination, and incentives to invest under different regimes, is not obvious in the literature. Several papers have discussed the origins and results of buyer power: In Katz (1987), buyer power emerges from a threat of integrating back into the supply chain. In O'Brien (2014), buyer power also depends on concession costs, bargaining weights, and disagreement profits. In Dobson and Waterson (1997) and Yoshida (2000), buyer power is studied using linear wholesale prices. On the other hand, Inderst and Shaffer (2009) examine price formation using non-linear pricing schemes such as two-part tariffs, while in O'Brien and Shaffer (1994), non-linear, non-observable contracts provoke largebuyer discounts. Ellison and Snyder (2010) further suggest that an important factor for downstream discounts to emerge is competition upstream. Gaudin

(2017) argues that buyer power does not generally translate into lower retail prices due to increased retail-level market concentration, while Inderst and Wey (2007) state that large buyers obtain discounts if the shape of the surplus function is concave. Furthermore, Inderst and Wey suggest that buyer power occurs from two different channels, which are either on the demand- or supply side.¹

In one dimension, the literature is generally divided into "take-it-or-leave-it", or mutual negotiation models. The former is often characterized by an established supplier who offers an "ultimatum" price that the downstream firms can either accept or find alternative business elsewhere. In such frameworks, finding business elsewhere is most often referred to as an "outside option", and is usually the core of these analyses. For instance, Katz (1987), Inderst and Shaffer (2009), and Chen (2017) employ such framework without room for mutual re-negotiations, but their models differ in various economic outcomes. Some analyze the relevance of outside options: Foros et al. (2018) argue that they matter a great deal for price outcomes, while Oslo Economics (2019) find the relevance to be negligible. Due to the different weighting of assumptions, they draw opposite conclusions about the welfare effects of forbidding price discrimination.

On the other hand, several other types of models employ negotiations to determine the economic outcomes. They often apply an array of principles from game theory in bargaining frameworks. Many of these bargaining structures originate from the axiomatic bargaining solution presented by Nash (1950), and others have branched out to more strategic approaches as in Binmore et al. (1986). In these bargaining models, there usually is one or several factors that drive the parties toward an agreement, which could be impatience, the fear of negotiations breaking down, or how great the operating capacity of the negotiators parallel to the dispute is, to name a few. For instance, Horn and Wolinsky (1988), Chipty and Snyder (1999), O'Brien (2014), and Inderst and Montez (2019) apply the Nash bargaining solution in solving their models.

In another dimension, the use of different pricing schemes seems to have important implications for economic findings. In one branch of the literature,

 $^{^{1}}$ We will present the two channels briefly when we introduce the model later.

the use of linear price contracts is common practice. These articles include, for instance, Katz (1987) and DeGraba (1990). When prices are linear, implications of a phenomenon known as double marginalization might play into vertical supply chains if both the up- and downstream firms have market power. In short, it is the failure to internalize vertical externalities, resulting in loss of efficiency in the supply chain. Another branch uses non-linear contracts, which are usually assumed to be sufficient to avoid double marginalization. Such articles include, for instance, Inderst and Wey (2007) and Inderst and Shaffer (2009). In the latter, the argument is that "with non-linear contracts such as two-part tariffs, the supplier can disentangle the objective of extracting surplus from that of providing downstream firms with the right incentives to choose a given retail price or quantity." (p. 659).

An interesting direction for our thesis is where agents, especially the buyers, can affect their market position through investments of different sorts. It is not immediately obvious what effects such investments will have on the price they face, or how the incentives can change under different pricing regulations. As we will provide a more extensive review for later, DeGraba (1990) examines buyers' incentives to invest in their profitability and the resulting price they obtain from the supplier. In Inderst and Wey (2007) and Inderst and Wey (2011) the supplier is allowed to invest in their production technology.

The next sections will provide a more in-depth insight into the pioneering article by Katz (1987). A lot of the advanced modeling that has been introduced in later years is built upon his framework. Further, we will review another front-runner article by O'Brien (2014) to introduce a broader and somewhat contrasting view to Katz. Before we present the model we extend, we will also review some articles that cover incentives to invest under discriminatory and non-discriminatory input prices, with primary focus on DeGraba (1990).

2.2 Katz (1987)

Katz's (1987) model is in a take-it-or-leave-it framework. This means that the supplier offers an input price, and the downstream firm can either accept the price or reject it entirely. The downstream firm's only option apart from accepting the price offer is to exert their outside option; to integrate backward into self-supply or finding other sources of supply. Downstream "bargaining power" solely depends on the threat of exerting the outside option. He considers a model with a monopolist supplier serving a representative downstream market of Cournot competitors. The downstream market has two sellers; one chain store also present in other markets, and one independent store only operating in one market. The chain store poses a credible threat of integrating into an alternative source of supply, while the independent store does not. Katz justifies this setup using the notion that a large firm is better suited to bear additional fixed costs associated with the outside option; or more easily obtain production technology that exhibits economies of scale than a smaller firm. The outside option will only be used if it raises profits in expectation.

The core of Katz's (1987) analysis is based on this: The supplier is incentivized to discriminate in prices due to the chain's threat of backward integration. He argues there are two types of welfare effects in the model that differ based on discriminatory and non-discriminatory pricing regimes. First, he argues that the quantity sold in the intermediate goods market may differ. Second, the cost of production may vary if one of the regimes induces backward integration. The arguably most interesting case Katz (1987) emphasizes, is where integration does not happen under either regime, but the outside option remains dormant. This allows for a more clear-cut comparison of the effects of a ban on price discrimination on wholesale prices and welfare.²

A crucial assumption in Katz's (1987) model is that each downstream firm's respective profit function depends positively on the other firm's input price level. Naturally, their profit functions also depend negatively on their own price of inputs, and they also face homogeneous demand in the final good markets. Katz further assumes the downstream firms have access to the same production technology as the incumbent supplier by their outside options but have to incur

²Katz also analyzes the cases where integration depends on the pricing regime and where integration happens regardless. Under the former, he argues that it will only happen under a ban. Since the production technology is characterized by increasing returns to scale, the incumbent supplier is likely to be at least as efficient, if not more so, than the integrated chain store. This means the chain may have to raise prices offered in the final goods market post-integration to make up for the potentially less efficient production. Since the vertical market served by the supplier ex-post only contains the independent store, the supplier may raise the price for final goods to the level that the integrated chain operates at. This implies a higher price level and lower total welfare as a result of socially inefficient integration.

a fixed cost. Since the chain store operates in several markets, it can divide the fixed costs of integration across all markets they operate in, as they will use the same means of production for all firms they operate. The marginal costs of the incumbent and well-established supplier are lower or equal to the marginal costs the chain would face in backward integration. Taking the fixed costs as well as, at best, equal marginal costs into consideration, it is more efficient to stay within the supply chain than to make use of the outside option. Katz stresses that doing so will increase the average production costs.

In Katz (1987), the equilibrium level of output in a single market is given by

$$X[m_1, m_2] = x[m_1, m_2] + x[m_2, m_1]$$

where X is the total quantity offered in the market, x is the quantity offered by one downstream firm, and m_1 , m_2 are the marginal costs of inputs faced by the chain store and the independent store, respectively. He defines the profit function for an individual downstream firm as:

$$\pi[m_i, m_j] = x[m_i, m_j] * \{P[X(m_i, m_j)] - m_i\}$$
(2.1)
for $i = 1, 2$ and $j = 1, 2$ and $i \neq j$

The intuition behind this specification is simple: The profit for a given downstream firm is equal to the difference between the price charged in the final goods market and the marginal cost of inputs the firm faces, times the quantity sold. The price in the final goods market is subsequently a function of the total output offered in equilibrium. Since m_i and m_j are positive numbers, and m_i enters negatively in the profit function, π is decreasing in m_i as a higher cost of inputs reduces profits and therefore also the quantity firm *i* purchases. On the other hand, a higher m_j implies that the other downstream firm *j* will purchase less inputs, leaving more space for firm *i* to serve the demand for final goods. Formally, Katz (1987) expresses this as for all input prices such that firm *i* sells a positive quantity: $x[m_i, m_j] > 0$, then

$$\frac{\partial \pi[m_i,m_j]}{\partial m_i} < 0, \ \frac{\partial \pi[m_i,m_j]}{\partial m_j} \ge 0$$

and the profit function is unchanged in m_j only if firm j purchases zero inputs: $x[m_j, m_i] = 0$. In Katz's model, one unit of input translates to one unit of output at no additional costs, such that m_i equals the input price w_i when the downstream firm purchases inputs from the supplier. If the chain store integrates backward, then the input price it faces, v, is greater or equal to the supplier's marginal cost of production, c. It follows from the increasing returns to scale of the production technology, generally favoring the incumbent producer. Additionally, the chain faces a fixed cost of integration F, such that the profit of an integrated firm is $\pi[v, m_j] - \frac{F}{k_i}$, where k_i is the number of final goods markets that firm i operates in. The chain store operates in all markets, denoted by K. In contrast, the independent firm only operates in a single market. This implies it would have lower profits from integrating than the chain store, given that everything else is constant.

Katz (1987) argues that the chain store will only integrate if expected postintegration profits (denoted π^e), less the fixed costs of integration, are greater than the profits it gets by being served by the supplier. This is expressed by the equation:³

$$\{\pi^{e}[w_{1}, w_{2}^{e}] - \frac{F}{K}\} - \pi[w_{1}, w_{2}] \ge 0$$
(2.2)

A fundamental concept in Katz's (1987) analysis of how input prices are formed, is what he refers to as the "integration frontier", that he denotes $I[w_2]$. Since the downstream firms' profits are functions of their competitor's price and their own, Katz describes the integration frontier as the chain's input price where the chain is indifferent between integration and not, for given values of the independent store's input price. In particular, Katz formally describes the integration frontier as the "price pair that satisfies the equation [above] with equality" (p. 158). Therefore, his intuition of the integration frontier is the line that the supplier freely can move the prices, w_1 and w_2 , without inducing the chain's outside option. He further shows that the price outcomes for the downstream firms depend crucially on whether the integration frontier is upor downward sloping, illustrating the chain's information about the market conditions upstream. Arguably the most interesting case to discuss for this thesis is where the chain store has full information about the upstream market

³For clarity, we denote the price the supplier charges the independent after the chain has integrated as w_2^e .

conditions. In this case, the integration frontier is upward sloping.⁴ The reason is as follows: In case the supplier raises w_1 input price, the chain's incentive to integrate increases because the profits without integration decline relative to the profits in integration. On the other hand, if the supplier raises w_2 , the chain's incentive to integrate decreases because the profits without integration increase relative to the profits in integration. Because the chain's profits are negatively dependent on w_1 and positively dependent on w_2 , it can justify not integrating if the supplier raises w_1 only if w_2 also increases. Hence, the integration frontier is upward sloping.

When the supplier sets the input prices as take-it-or-leave-it, Katz (1987) assumes that the profit-maximizing combination of prices that does not induce integration exceeds any combination that does. Therefore, the supplier faces the following maximization problem when price discrimination is allowed:

$$\max_{w_1,w_2} U^m[w_1,w_2] \equiv (w_1 - c)x[w_1,w_2] + (w_2 - c)x[w_2,w_1]$$
(2.3)

subject to the chain's integration constraint:

$$\pi[w_1, w_2] \ge \pi^e[w_1, w_2^e] - \frac{F}{K}$$

where $U^m[w_1, w_2]$ is the supplier's iso-profit function for a given set of input prices w_1, w_2 . The iso-profit function is simply the difference between the prices the supplier charges the chain and the independent store respectively, and its own marginal cost, multiplied by the respective quantities it sells to the downstream firms. The solution to the supplier's maximization problem is therefore the maximal value of U^m that does not induce integration from the chain.

⁴In Katz's model, when there is either incomplete information or prices are fixed, the integration frontier may become downward sloping. In this case, w_2^e depends on w_2 because the supplier has to commit to the price initially quoted. In this scenario, Katz finds that discrimination still reduces welfare, but the wholesale prices may move in opposite directions. In reality, prices are rarely fully fixed, so the assumption of a downward sloping integration frontier is less plausible. We later analyze a model by Inderst and Wey (2007) that we further extend. As the model we present is without uncertainty, Katz' full information analysis is most relevant for the purpose of comparison.

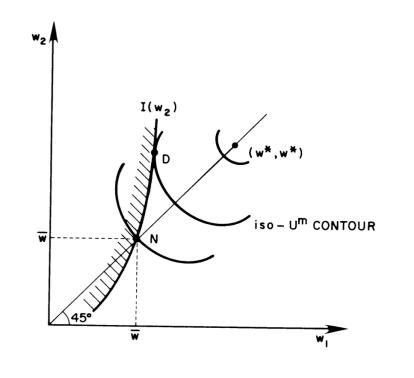


Figure 2.1: Upward Sloping Integration Frontier (Katz, 1987, p. 159)

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In figure 2.1, Katz (1987) shows the solution to the maximization problem when the chain has complete information about the upstream market conditions, with the chain's input price on the x-axis and the independent's input price on the y-axis. The supplier's profits increase with higher values of w_1 and w_2 , so it would like to set prices at (w^*, w^*) , but Katz assumes this point unattainable due to the chain's outside option. On the other hand, the chain's profits increase the lower w_1 and higher w_2 , and the opposite is true for the independent. The straight 45° line illustrates where the input prices for the chain and the independent store, w_1 and w_2 respectively, are set equal as under a ban on price discrimination. Katz represents the integration frontier as the $I(w_2)$ -line. The shaded area to the left is where the chain store finds integration undesirable because the input price it gets from the supplier here yields higher profits than if it were to integrate. In contrast, the chain store is better off integrating to the right of the line because the input price, w_1 , is too high. The integration frontier Katz (1987) draws is increasing and convex because if the supplier increases w_1 , the chain store requires increasingly more compensation in the form of an increased competitor's price, w_2 , to not integrate. Furthermore, the supplier may set the prices to the left of the frontier, but this is not optimal since it can increase its own profits by moving the prices back toward the

frontier. Also, setting the prices to the right of the frontier induces integration from the chain.

Point D in figure 4.2 illustrates the solution to the maximization problem in Katz' model when price discrimination is allowed. Because of the ordinal properties of the iso-profit curves, the supplier's profits increase the further out the curves are shifted. In order not to induce integration from the chain, the profit-maximizing set of prices (w_1, w_2) is where the integration frontier is tangent to the iso-profit curve. Any other combination would either reduce the supplier's profits or induce integration from the chain. At this point, the chain does not integrate, and the supplier serves both downstream firms. If the supplier is not allowed to discriminate in prices between the two downstream firms, then the prices have to be equal: $w_1 = w_2$. If the supplier so chooses not to induce integration under a ban on price discrimination, then the highest feasible price combination is given at point N by (\bar{w}, \bar{w}) . In contrast to when price discrimination is allowed, when the supplier raises the chain store's input price, it cannot compensate by also raising the independent store's price under a ban. Therefore, if the supplier does not want to induce integration, it has to offer both downstream firms a lower input price under a ban on price discrimination, otherwise, the chain will integrate backward. This result is interesting because it offers debate to the Robinson-Patman Act of 1936, despite Katz stating it is not meant to defend the Act. Up until the point of this article, the Act was almost unilaterally criticized for being anti-competitive and welfare-hindering; however, Katz sparked a healthy discussion from his findings.

2.3 O'Brien (2014)

O'Brien (2014) further extends Katz's (1987) model to incorporate four different sources of buyer power; outside options, concession costs, disagreement profits, and bargaining costs. He assumes a bilateral negotiation framework, as is common in the literature. Similar to Katz, O'Brien finds that if a chain store's outside option is binding, forbidding price discrimination reduces welfare if the chain would either way face prices that do not induce integration. However, O'Brien finds evidence of the opposite effect when other sources determine bargaining power.

O'Brien (2014) uses a similar setup to Katz (1987), in which there is one monopolist supplier selling to two Cournot-competing downstream firms. As in Katz, the chain store exhibits the possibility of integrating back into the supply of inputs, provided it sinks a substantial fixed cost. On the other hand, the smaller, independent store does not have this ability (it is at the very least not feasible to do so for the small firm). O'Brien's model relies on Nash's (1950) bargaining solution to determine the wholesale price outcomes for each downstream firm. Similar to Katz, the supplier in O'Brien (2014) has a utility function $U(w_1, w_2)$ that is expressed in terms of the input prices of both downstream firms; w_1 is the chain's input price, and w_2 is the independent's. Likewise, the chain and independent firms face profit functions expressed in terms of their own and their competitor's input prices. In particular, the chain's profit function is given by $\pi(w_1, w_2)$ if it has not integrated backward. It it has integrated, $\pi^{I}(v, w_{2}^{I})$ is the post-integration profit for the chain, with w_2^I as the independent's price, and v as the marginal cost of inputs for the integrated firm. The supplier goes on to negotiate with both the chain store and the independent individually for the respective input prices.⁵ As in Katz, O'Brien (2014) assumes an agreement without inducing outside options is the more efficient outcome. Therefore, the negotiation with the chain store is a constrained negotiation problem, subject to the chain's integration frontier; the prices have to be set at most such that the chain is indifferent between integration and non-integration. The negotiation problem with the independent store is unconstrained because it does not have a feasible threat of integration into self-supply.

O'Brien (2014) defines the asymmetric Nash bargaining problem between the supplier and chain as:

$$max_{w_1}\phi_1(w_1, w_2) = [U(w_1, w_2) - d_{u_1}(w_2)]^{1-\gamma_1} [\pi(w_1, w_2) - d_1(w_2)]^{\gamma_1}$$
(2.4)
subject to $\pi_1(w_1, w_2) \ge \pi^I(v, w_2^I)$

 $\pi_1 \geq \pi^I$ becomes the integration constraint for the chain store. Therefore, an intuition is that the supplier maximizes profits given that the chain store is

⁵When the supplier negotiates with the chain, both parties take the independent's input price as given, and vice versa for the negotiation with the independent.

either better off not integrating $(\pi_1 > \pi^I)$, or indifferent between integrating or not $(\pi_1 = \pi^I)$. If the supplier chooses prices such that $\pi_1 < \pi^I$, then the chain will integrate because profits from doing so exceed the non-integration profits. O'Brien (2014) employs the "outside option principle" as in Shaked and Sutton (1984) and Binmore et al. (1986) to model the chain store's bargaining problem. The principle states that in alternating-offer games, where discounting drives the parties toward agreement, the outside option is irrelevant unless it binds. This means that it can be modeled as a constraint on the maximization problem: If it binds, it defines the price outcome. If the constraint is slack, the parties are driven toward agreement from other bargaining sources than the threat of an outside option.

The bargaining problem between the supplier and the independent store is:

$$max_{w_2}\phi_2(w_1, w_2) = [U(w_1, w_2) - d_{u_2}(w_1)]^{1-\gamma_2} [\pi_2(w_1, w_2) - d_2(w_1)]^{\gamma_2}$$
(2.5)

The parameters, γ_1 and γ_2 , are the bargaining weights of the chain and the independent store, respectively. $d_{u1}(w_2)$ and $d_{u2}(w_1)$ are the supplier's respective payoffs from disagreement with the chain and the independent, as functions of the other downstream firm's agreed upon input prices. Lastly, $d_1(w_2)$ and $d_2(w_1)$ are the chain and independent's respective disagreement payoffs, also as functions of the competitor's price.

The chain's bargaining problem can be solved by specifying the Lagrangian and maximizing over the chain's input price, to which O'Brien (2014) shows the first-order conditions to be (with λ as the Lagrangian multiplier):

$$(1 - \gamma_1)\frac{\partial U}{\partial w_1}[\pi_1 - d_1] + \gamma_1 \frac{\partial \pi_1}{\partial w_1}[U - d_{u_1}] + \lambda \frac{\partial \pi_1}{\partial w_1} = 0$$
(2.6)
where $\lambda \ge 0$, and $\lambda[\pi_1 - \pi^I] = 0$

A key insight from these conditions is that they form the "reaction function" denoted $R_1(w_2)$, which O'Brien (2014) defines as "the wholesale price negotiated by the manufacturer and firm 1 as a function of the wholesale price negotiated with firm 2" (p. 97). This reaction function is crucial for O'Brien's analysis because it sheds some new light on the pioneering framework by Katz (1987).

The condition $\lambda[\pi_1 - \pi^I]$ states that either $\lambda = 0$, or $\pi_1 = \pi^I$ with $\lambda \ge 0$. This is important because it allows the model to differentiate between binding and non-binding outside options.⁶

He further shows that the first order condition to the independent store's bargaining problem is:

$$(1-\gamma_2)\frac{\partial U}{\partial w_2}[\pi_2 - d_2] + \gamma_2 \frac{\partial \pi_2}{\partial w_2}[U - d_{u2}] = 0$$

$$(2.7)$$

Analogously, this expression forms the reaction function $R_2(w_1)$ for the independent's problem as a function of the chain's input price.

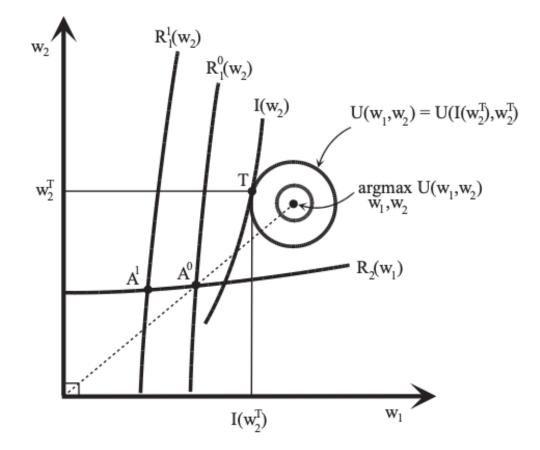


Figure 2.2: Bargaining Equilibrium when Price Discrimination is Allowed (O'Brien, 2014, p. 99)

Figure 2.2 illustrates the price outcomes in O'Brien (2014) when price discrimination is allowed. The negotiation between the supplier and the two

⁶Somewhat elegantly, when the chain's integration constraint binds ($\pi_1 = \pi^I$), then the analysis yields identical results as in Katz (1987). O'Brien (2014) demonstrates that when accounting for other sufficiently strong bargaining sources, the outside option may not bind, which interestingly might reverse Katz' results.

downstream firms is a result of profit maximization subject to different sources of bargaining power. The supplier's profits are increasing in both w_1 and w_2 . It would like to set the prices as far away from the origin as possible without inducing the retailers' outside options. On the other hand, the retailers' profits decrease in w_1 and w_2 respectively, presenting a clear conflict of interest in price setting. Analogous to Katz (1987), when the outside option binds, the equilibrium outcome T is achieved where the wholesale prices are set at the chain store's integration frontier. Suppose the downstream firms' bargaining power is from other sources and sufficiently strong (which means the outside option is slack). In that case, they require a lower input price relative to the integration constraint, captured by the reaction functions $R_2(w1)$, $R_1^0(w_2)$, and $R_1^1(w_2)$. Here, wholesale prices decrease for both downstream firms relative to where the outside option binds. O'Brien further shows that for equal negotiation power downstream and when the chain's outside option is slack, the equilibrium outcome is symmetric wholesale prices along the 45° line, in the intersect between $R_1^0(w_2), R_2(w_1)$ illustrated by A_0 . Consequently, he shows that when the bargaining power of the chain increases, shifting the reaction function from $R_1^0(w_2)$ to $R_1^1(w_2)$, the input price it faces is lower, as the equilibrium shifts from A_0 to A_1 .

To determine whether the chain obtains a discount or not, O'Brien (2014) considers under what conditions the shift from $R_1^0(w_2)$ to $R_1^1(w_2)$ happen. Here, the implication of a non-binding integration constraint comes into play. When the constraint is slack, the Lagrangian multiplier binds ($\lambda = 0$). Under this condition, the first order conditions' functional forms become symmetric. O'Brien lists three conditions under which the chain obtains a discount, where it is sufficient if at least one on the following is with strict inequality: $d_1 \geq d_2$, $d_{u_1} \leq d_{u_2}$, and $\gamma_1 \geq \gamma_2$. The latter reflects the bargaining weights of the chain store and the independent. The former two specify the relationship between the chain and independent's disagreement profits and the supplier's relative *inside options* (the interluding operating capacity of a firm parallel to a price negotiation) in negotiating with the downstream firms, respectively. If *all* of these conditions hold with equality, the chain store and independent have equal negotiation power and receive the same price.

To give some additional intuition to the expressions above, and since they are symmetric when λ binds, O'Brien (2014) rewrites either (2.6) or (2.7) in terms of buyer *i* to obtain:

$$\frac{\gamma_i \left[-\frac{\partial \pi_i(w_1^A, w_2^A)}{\partial w_i}\right]}{\pi_i(w_1^A, w_2^A) - d_i(w_i^A)} = \frac{(1 - \gamma_i) \left[\frac{\partial U(w_1^A, w_2^A)}{\partial w_i}\right]}{U(w_1^A, w_2^A) - d_{ui}(w_i^A)}$$
(2.8)

The left-hand side numerator of (2.8) is the weighted concession cost, or the cost of compromise to reach an agreement, for each of the downstream firms.⁷ The left-hand side denominator is the respective downstream firms' net profit, that is, the excess profits over the disagreement points. On the right-hand side, the numerator postulates the supplier's weighted concession cost, while the denominator is the supplier's net profit. Furthermore, the supplier's profit increases with the negotiated price, while the downstream negotiating firm's profit decreases with w_i . Because the concession costs are the gains or losses from a given change in price, O'Brien argues that the firm with the greater concession cost has to have greater profits in equilibrium relative to the firm with the lower concession cost for condition (2.8) to hold with equality. An intuition for this result is that a downstream firm with a low concession cost loses less from a slightly higher price. Therefore, they are more inclined to accept slightly worse terms than a firm that loses much more doing so.

O'Brien (2014) argues that inside options play a role in determining the patience of the negotiators, which influences their disagreement profits. As in Binmore et al., the disagreement profit is "identified with the agreement that gives parties the same income streams as they are receiving during the dispute" (1986, p. 185). For the chain store, O'Brien assumes the competing firm's input price influences the inside option because, during the chain's negotiations, the competitor may extract a higher surplus from the market. He further indicates that the supplier may have an inside option that "allows it to redeploy resources during negotiations" (2014, p. 96). O'Brien shows in equation (2.8) that if the supplier's inside options differ with respect to the downstream firms, it can translate to differences in prices. In the equation, it follows that a

⁷The concession costs are the derivatives of the profit functions with respect to the negotiated upon price. For downstream firm i, the profits decrease with a given increase in the price they pay, while the profit of the supplier increases with the price.

lower d_{ui} reflects a lower price because it enters negatively in the denominator. In particular, he argues that if the supplier has to devote more resources to the chain once an agreement is reached and that distributing large chunks of resources is more demanding, the chain will obtain a discount relative to its competitor. If this is true, the condition $d_{u_1} < d_{u_2}$ is met. Likewise, O'Brien argues that the chain may receive better terms if its inside option profitability exceeds its competitor's. If all else is equal, the differences in inside options reflect differences in disagreement profits, satisfying the condition $d_1 > d_2$. Lastly, O'Brien argues that if the chain has a greater bargaining weight, if $\gamma_1 > \gamma_2$, it will receive a discount relative to the independent. This condition is reflected by a "lower discount rate, which might be the case if it has lower capital costs than the independent" (2014, p. 100).

O'Brien (2014) further analyzes the effects of a ban on price discrimination. Under a ban, the price is set equal for both downstream firms, but the price outcome may depend on which firm negotiates with the supplier. O'Brien shows that if the firms are symmetric or that the weaker firm negotiates with the supplier, then Katz's (1987) result is reversed. When the stronger firm negotiates, O'Brien argues that the results are somewhat ambiguous. In this thesis, we will only review the case where the independent store negotiates.⁸

In the case the supplier negotiates with the independent store, O'Brien (2014) shows that the Nash bargaining problem becomes:

$$max_{w}\phi^{F}(w,\hat{w}) = [U(w,w) - d_{u_{2}}(\hat{w})]^{1-\gamma_{2}}[\pi_{2}(w,w) - d_{2}(\hat{w})]^{\gamma_{2}}$$
(2.9)
subject to $\pi_{1}(w,w) \ge \pi^{I}(v,w_{2}^{I})$

where \hat{w} is defined as the "wholesale price paid by the chain during negotiations with the independent" (2014, p. 100).

O'Brien (2014) shows that the first order conditions for the Lagrangian

⁸To determine if forbidding price discrimination has advantageous or adverse welfare effects when the supplier negotiates with the strong firm (chain), O'Brien plots the chain's discount by a slack integration constraint on changes in welfare. He finds that the result is discontinuous at a threshold; when the chain would have otherwise gotten a discount from other sources of approximately 30 percent or more, a ban on price discrimination would raise welfare. When the discount would have been less than 30 percent, he shows that welfare would be reduced.

associated with the maximization problem are:

$$\frac{\partial \phi_2}{\partial w_2} + \left\{ (1 - \gamma_2) \frac{\partial U}{\partial w_1} [\pi_2 - d_2] + \gamma_2 \frac{\partial \pi_2}{\partial w_1} [U - d_{u2}] \right\} + \eta \sum_i \frac{\partial \pi_1}{\partial w_i} = 0 \qquad (2.10)$$

where

$$\eta \ge 0$$
 and $\eta[\pi_1 - \pi^I] = 0$

 η is the Lagrangian multiplier, and $\eta[\pi_1 - \pi^I] = 0$ is the integration constraint analogous to the discrimination case. Furthermore, O'Brien (2014) defined $\frac{\partial \phi_2}{\partial w_2}$ as the "derivative of the Nash product for negotiations between the supplier and [the independent firm] when discrimination is allowed" (p. 101). When the Nash product is maximized under discrimination, that means $\frac{\partial \phi_2}{\partial w_2} = 0$. This allows O'Brien in a similar manner as under price discrimination, assuming the integration constraint is slack (η binds), to rewrite the above condition as:

$$\frac{\gamma_2[\left(-\frac{\partial\pi_2}{\partial w_2}\right) + \left(-\frac{\partial\pi_2}{\partial w_1}\right)]}{\pi_2 - d_2} = \frac{(1 - \gamma_2)[\left(\frac{\partial U}{\partial w_2}\right) + \left(\frac{\partial U}{\partial w_1}\right)]}{U - d_{u_2}}$$
(2.11)

The intuition behind (2.11) is that now the independent store's concession cost also depends on the gain or loss that the chain would face from a change in the bargained price outcome because prices have to be set equal. Likewise, the supplier's concession cost depends on the profit change from a change in both prices, as opposed to (2.8). Furthermore, O'Brien (2014) argues that the supplier's concession cost in (2.11) is higher. This is because an agreement to lower the price charged to the independent firm implies the price charged to the chain also has to be lower. Therefore, it costs the supplier more to facilitate an agreement to lower prices under non-discrimination because it loses more profits doing so than under discrimination.

On the other hand, since the downstream firms' respective profit functions decrease with a higher input price faced and increase with a higher competitor's price, the derivatives in the numerator on the left-hand side of (2.11) pull in opposite directions. This lowers the concession cost of the independent firm, meaning it is more inclined to accept a higher price under a ban on price

discrimination. Therefore, O'Brien (2014) finds evidence that when the outside option of the chain store is slack, a sufficient amount of bargaining power stems from other sources, and the weaker firm negotiates for input prices on behalf of both downstream firms, a ban on price discrimination raises the average wholesale price.

Foros et al. (2018) extend the model derived by O'Brien (2014) to assume that retailers differ in size and that size is exogenously determined. They discuss why monopolistic suppliers discriminate in favor of bigger retailers and evaluate the effects that a ban on price discrimination on consumer prices. The authors find evidence that a ban would lead to lower final goods prices. Similar results are shown in a report and comment made by Foros and Kind (2018a,2018b) which also concern the Norwegian grocery sector. Between the supplier-retailer relationship, they argue that size is not the only reason why suppliers discriminate. In their model, and similarly to Katz (1987), they show that a reasonable explanation is that bigger retailers have a more significant threat of using an outside option to self-supply. Midttømme et al. (2019) conducts a similar analysis in light of the Norwegian grocery sector, but they are more careful as to draw conclusions about policy implications.

2.4 Investment incentives

Although most of the literature is in a static setting, a few papers analyze investment decisions. DeGraba (1990) discusses how price discrimination affects retailers' incentives to invest. He assumes that the retailers face some marginal costs by selling in the final good market, and it is possible to reduce these costs by investing in more efficient technology.

In the model, DeGraba (1990) assumes a monopolistic supplier serving two downstream firms producing a homogeneous final good. The downstream firms' production costs are determined from a combination of an input price, denoted w_i ,⁹ and a marginal cost of transforming an input into an output, denoted c_i , making the per-unit cost of output $c_i + w_i$. The key novelty of his article is that DeGraba assumes the downstream firms can affect their position in the

⁹DeGraba (1990) denotes prices as k_i in his model, but we use w_i to be more in line with previously presented literature.

market by investing in their production technologies, directly influencing their marginal production cost. As in Katz (1987), DeGraba assumes that the price contracts are linear, mutually observable, and in a similar take-it-or-leave-it environment.¹⁰

DeGraba (1990) analyzes the model in a three-stage sub-game perfect Nash equilibrium framework with complete information. In the first stage, he allows the downstream producers to choose a marginal cost of c_i by investing in a production technology at the fixed cost, F_i . A more efficient technology requires a higher fixed investment cost. In the second stage, the supplier quotes an input price of w_i to each of the downstream producers affected by their choice of investment. In the final stage of the game, the downstream producers observe the prices charged to them, and compete in Cournot. Since the game is over multiple stages, DeGraba solves it using backward induction. In the first section of his paper, which he refers to as "the short run", he solves for stages two and three, implying that the investment cost has already been sunk. In the latter part of the article, "the long run", he also solves for stage one in which the choice of investments is made.

As a beginning, DeGraba (1990) presents the latter two stages in the following framework: The supplier chooses a combination of prices $(w_1, w_2) \in \mathbb{R}^{2+}$, from which it obtains the payoff, $\pi_S = \sum_i *w_i * q_i$. After observing the respective input prices, the downstream producers choose a strategy Q_i that yields a Cournot quantity of final goods q_i , given the price and marginal cost they face.¹¹ He assumes the inverse demand for final goods is a linear function declining in total output, of the form $p = a - b(q_1 + q_2)$, where p is the price charged to final consumers, and a, b are some positive parameters.

When he solves the final stage, DeGraba (1990) assumes the final good output is chosen uniquely to maximize the producers' profits, such that the equilibrium quantities are given by (q_1^*, q_2^*) . For stage two, he maximizes the supplier's

¹⁰In contrast to both Katz (1987) and O'Brien (2014), the downstream firms do not have the option of integrating into self-supply. Therefore, the firms have to comply with the prices offered, and their respective sizes are determined by sales volume. This influences their production efficiencies.

¹¹Formally, DeGraba (1990) defines the producers' strategies as a function $Q_i : \mathbb{R}^{2+} \to \mathbb{R}^{1+}$. This means that for a given combination of two positive real number variables, given by the input prices, the strategy is to choose one positive real number quantity.

profit function over the price pair (w_1, w_2) given the equilibrium quantities. Intuitively, DeGraba shows that the production strategy of a downstream producer calls for a reduction in production when its own marginal cost and input price are high. Conversely, the equilibrium strategy is to produce more when the competitor faces a high input price and marginal cost.

If the supplier is allowed to discriminate in prices based on the effectiveness of the downstream producers' production technology, DeGraba (1990) shows that the prices offered by the supplier is:

$$w_i^{d*} = \frac{(a - c_i)}{2} \tag{2.12}$$

These are the results of a regular two-stage Cournot game in which $(w_1^*, w_2^*, Q_1^*, Q_2^*)$ represents the sub-game perfect equilibrium strategies for all players.

DeGraba (1990) shows that the best response for the input price quoted by the supplier, is decreasing in the marginal costs of the downstream producers. This means that a firm faced with a high production cost will face a lower input price, and vice versa. DeGraba argues that the reason for this is that a firm with a more effective production (a lower marginal cost) has a more inelastic demand given the input prices. Since there is no room for mutual negotiation and no outside options in his framework, the supplier can exploit the more effective firm and charge a higher price because its demand for inputs will not change too adversely following a price increase.

In the case where the supplier is not allowed to discriminate, DeGraba (1990) shows the price to be:

$$w^{u*} = \frac{(2a - c_1 - c_2)}{4} \tag{2.13}$$

Since the downstream producers are otherwise symmetric, their price is equally influenced by the respective marginal costs. Therefore, if the marginal costs differ, the uniform price as charged under a ban on price discrimination will be the average between the two discriminatory prices, as DeGraba shows by expression (2.13). Therefore, he argues the more efficient firm will produce more, and the less efficient firm will produce less under a ban on price discrimination, as reflected by the change in price under such a regime.

The more interesting aspect of DeGraba's (1990) for the purpose of our thesis is to review the downstream firms' incentives to invest. This happens in his model as the first stage of the game. He allows the downstream producers to each choose a level of marginal costs c_i by incurring a fixed cost that follows the function:

$$F_i = \alpha c_i^2 - \beta c_i + \gamma \tag{2.14}$$

on which he places a number of restrictions to feasibly reach a solution.¹²

DeGraba (1990) shows that when the downstream producers choose a technology to reflect their marginal costs in stage one, their profits given a strategy choice of (c_i, Q_i) is:

$$\pi_i = [a - b(q_1^* + q_2^*) - c_i - w_i]q_i^* - [\alpha c_i^2 - \beta c_i + \gamma]$$
(2.15)

such that the producers' respective profits are given by the markup (equilibrium price in the final good market less the input price and marginal cost of production) times the Cournot quantity sold in equilibrium, less the fixed cost of investment. The supplier's profit is in the same functional form as in the short-run section of the model.

To conclude his analysis, DeGraba (1990) compares the resulting discriminatory and non-discriminatory variables. To see the effects of a ban on price discrimination on marginal costs, input prices, and Cournot quanitities, he shows that the relevant comparisons become:

$$c_i^d = \frac{a - 9b\beta}{1 - 18b\alpha} \text{ and } c_i^u = \frac{(7/4)a - 9b\beta}{(7/4) - 18b\alpha}$$
$$w_i^d = \frac{-9b(2a\alpha - \beta)}{2(1 - 18b\alpha)} \text{ and } w_i^u = \frac{-9b(2a\alpha - \beta)}{2[(7/4) - 18b\alpha]}$$
$$q_i^d = \frac{-3(2a\alpha - \beta)}{2(1 - 18b\alpha)} \text{ and } q_i^u = \frac{-3(2a\alpha - \beta)}{2[(7/4) - 18b\alpha]}$$

Superscripts d and u denote the variables under discrimination and nondiscrimination, respectively. DeGraba's (1990) restrictions imposed on the first-

¹²In particular, the technical restrictions imposed by DeGraba guarantee that there is a negative relationship between the fixed cost incurred and the marginal cost of production, that profits are concave in c and strictly positive, and that there exist interior solutions.

order conditions result in all the denominators being negative. His restrictions result in all numerators also being negative. Then $q_i^u > q_i^d$, and $c_i^u < c_i^d$ if $a\alpha - \frac{\beta}{2} > 0$. These finding imply that under a ban on price discrimination, the equilibrium quantity produced is higher, and the marginal production costs are lower. The intuition behinds this is as follows: When the supplier is allowed to discriminate in prices, DeGraba showed that the price charged to the more efficient firm is higher due to a more inelastic demand of inputs. However, when price discrimination is banned, the supplier has to set prices equal, which DeGraba showed to be the average between the price outcomes under discrimination. Hence, when a firm invests in new production technology, the price does not increase as much under a ban on price discrimination.

On the other hand, the marginal costs of production are still reduced as much as they would have following an investment under discrimination. Therefore, the firms gain more from an investment under a ban on price discrimination, meaning the *incentives to invest in production technology increase when discrimination is not practiced*. Since the firms invest more and thus become more efficient under a ban, they also produce more output. This finding, somewhat counter-intuitively, suggests that welfare increases when discrimination is forbidden, as more consumers are served with the higher level of output sold. Consequently, DeGraba finds that the average price consumers pay decreases under a ban on discrimination, simply because the inverse demand for final goods is declining in total quantity. Hence, DeGraba's model defends a ban on price discrimination in intermediate goods markets.

Other articles that discuss incentives to invest in are, for instance, Akgün and Chioveanu (2019) and Inderst and Valletti (2009). Akgün and Chioveanu analyze a dynamic model where retailers sell two heterogeneous products offered by a strong and a weak supplier in a duopolist upstream market. The product offered by the weak supplier can be seen as a private label. Similar to Katz (1987), they assume linear prices and a take-it-or-leave-it environment. The retailers have the possibility to invest in reducing the marginal costs for the weak supplier's substitute. Similar to DeGraba (1990), the mechanism in Akgün and Chioveanu's (2019) paper shows that suppliers offer the retailer with the lowest cost of sales the highest price. By investing in the substitute product,

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they get reduced costs of sales for this product. The strong supplier reacts by reducing the price because the retailer will sell more of the competing product. In contrast to DeGraba (1990), the authors show that in the case of a ban on price discrimination, incentives to invest decrease.

Inderst and Valletti (2009) discuss how a ban on price discrimination will affect input prices when retailers have an investment opportunity and an outside option. They combine the ideas of both DeGraba (1990) and Katz (1987), and analyze a dynamic model, where both retailers have the possibility to integrate back into self-supply. The supplier offers take-it-or-leave-it prices to the retailers but they have to take the outside options into consideration. Inderst and Valletti (2009) show that when price discrimination is allowed, the supplier offers the lower price to the retailer that has invested the most. Contrary to DeGraba (1990), when the authors include a threat of an outside option, the most efficient retailer obtains the lowest price. Under a ban on price discrimination, the incentives to invest decrease. This is because the retailer that would otherwise face the best price, will no longer gain the same benefit of investing. At the same time, the authors argue that the non-discriminatory price will increase if the least efficient retailer were to invest.

Lastly, an interesting result from Inderst and Wey (2007), is that welfare and output increase if the supplier can invest in its production technology facing larger buyers. A potential loss against a disagreement with a large retailer could be minimized by increasing its capacity to sell to others. When the supplier can produce more efficiently and serve the remaining buyers, a decline in demand is not as costly as when the supplier is less effective. The supplier produces at lower incremental costs and can therefore offer lower prices, leading to higher output and welfare.

3 Methodology

We will make use of the model presented in Inderst and Wey (2007) as the foundation for our analysis. The choice of this specific article as a base is because we deem it particularly well-suited for our extensions. Among other aspects, the assumptions of non-linear price contracts, identical and independent downstream markets, and the abstract functional forms, facilitate an appropriate venture into our topics of interest. The aim is to capture the dynamic effects that stem from buyer power and extend to account for retailside incentives to invest in size, using a two-stage bargaining model. While Inderst and Wey define two sources of buyer power, in our extension, we will not explicitly distinguish between the channels. However, we will use their assumption that large buyers obtain discounts due to disagreement inflicting losses on the supplier, increasing more than proportionally with size. We assume in our model that the disagreement point is not to trade any inputs, which is independent of the buyer's size.

The method we will employ going forward is to take the generic functional forms of Inderst and Wey's (2007) model and insert our functions, which satisfy their underlying assumptions. Our contribution will therefore be twofold: First, we attempt to verify that Inderst and Wey's assumptions result in discounts for large buyers in our particular set of functions. Second, we make use of their framework to account for buyer-side investment in size. The expansion will seek to highlight the formation of large buyers and study the factors that make Inderst and Wey's assumption that large buyers obtain discounts hold. We use algebra to derive the key functions we need for our study. Since we specify the functions using parameters, we may lose some abstraction, but it makes the analysis more conceivable.

Finally, we will impede the supplier's ability to discriminate in prices toward the end to make another humble contribution to the literature on forbidding price discrimination. We will use the model and extensions to compare with existing literature on buyer power, input price formation, and price discrimination and present a few concluding remarks about our findings toward the end.

4 Analytical Framework

4.1 Background

Our model is based on the article Buyer power and supplier incentives by Inderst and Wey (2007). They analyze how buyers' sizes affect negotiations with a monopolist supplier. Furthermore, they discuss how buyer power arises from two different channels, assuming the supplier is either capacity constrained or has strictly convex costs.¹³ The two channels are supply-side and demand-side buyer power. They define supply-side power as: "if a supplier has strictly convex costs then a large buyer essentially negotiates over a range of production where average incremental costs are lower" (Inderst and Wey, 2007, p. 648). Inderst and Wey argue that with strictly convex costs in the supplier's production function, the average cost per unit is high when the supplier produces many units. The implication is that buyers with higher demand have more influence on suppliers' revenue than those who demand less. This is because the average costs of a high volume order is lower than the cost of small orders. Therefore, the supplier is more inclined to offer better prices to larger buyers. They present a principal finding that "under relatively standard conditions on demand, the supplier's loss from a disagreement increases more than proportionally with the size of the respective buyer" (2007, p. 648), which they define as the demandside channel. Their argument follows that when there is a large portion of excess quantity produced, the supplier must lower prices much further to dispose of it due to the buyers having downward-sloping demand curves. We want to examine what conditions make their assumptions of large-buyer discounts hold.

The following chapter is the **main contribution** to our master thesis and will present the findings to our research question. It proceeds as follows: In section 4.2, we present the assumptions of the economy before we derive the profit maximization conditions in section 4.3. We present the first case in section 4.4, where the total number of retailers is fixed. Further, we extend our model in

¹³Inderst and Wey (2007) further discuss in their paper how the increase of buyers with more power affects the supplier's investment incentives. The investment strategy in our model regards how buyers can influence buyer power by increasing their size. Therefore, the supplier's incentive to invest will not be included in our analysis.

section 4.5, where the numbers of retailers are endogenously determined but only allow for a single buyer to invest. In 4.6, we develop our model to assume that all buyers can invest. Lastly, we discuss the implication of forbidding price discrimination in section 4.7.

4.2 Setup

As in Inderst and Wey (2007), we consider an intermediate economy where a monopolistic supplier is producing and selling a total quantity of inputs, x, that is used by downstream retailers. The supplier serves $n \in [0, N]$ identical downstream firms that each operate in independent final goods markets, and where $N \ge 2$ is the total number of downstream retail stores. In equilibrium, the supplier serves all N stores. Further, we formally define the inverse demand function in the final goods markets as $P(x) = a - \frac{bx}{2}$, which satisfies P(0) > 0such that a > 0, and demand is declining in x such that also b > 0. The revenue function is therefore given by $R(x) = ax - \frac{bx^2}{2}$, where R'(x) = a - bx. A common assumption in the literature is convex costs of production.¹⁴ We assume the supplier faces a quadratic cost function in production of inputs, $C(x) = sx^2$, where C(0) = 0, C'(x) > 0, C''(x) > 0, and s > 0 is a cost parameter. Moreover, as in Katz (1987), we assume that one unit of input produces one unit of output at no additional costs, but given symmetric production functions for all retail stores, this specification will not impact our results. We further assume that the downstream stores are controlled by a set of buyers i = 1, ..., I, and k_i is the number of firms buyer i controls. Hence, the difference in buyer size is due to the difference in the number of stores k_i that each buyer possesses.¹⁵ As such, the definitions used onward in this thesis will be that a *retail store*. retailer, or store, is the singular downstream store operating in an independent final goods market, while the term *buyer* will be used for the equivalent of a "chain store"; an agent that owns and operates a number of retailers. Since all retail stores operate in individual and independent markets, the number of retailers and the number of markets are interchangeable in our thesis.

 $^{^{14}}$ As described, Inderst and Wey (2007) list this condition as important for discounts to emerge.

¹⁵Inderst and Wey (2007) denote the number of stores as r_i .

4.3 Profit maximization

A well-known problem in vertical relations is a loss of efficiency due to double marginalization. To avoid this, we assume that the contract space is sufficiently large; for instance, a two-part tariff may suffice in some settings. Hence, bargaining is about splitting the gain from trade and does not in itself influence that gain. Therefore, the buyer and supplier maximize their profits as if they were one entity, and they divide the surplus ex-post. The retailers' revenue function and the supplier's cost function enter into the total industry profits, which are maximized by choosing x_n^* . As in Inderst and Wey (2007), we denote n as the number of retailers the supplier agrees with out of the total N and that x_n^* is the uniquely determined total quantity that maximizes industry profits. Therefore, they define the revenue at each retail store as, R_n^* , and the total aggregate industry profits, Π_n^* are (given that retail stores are identical):

$$R_n^* = \left(\frac{x_n^*}{n}\right) * P\left(\frac{x_n^*}{n}\right)$$
$$\Pi_n^* = nR_n^* - C(x_n^*)$$

If we insert our demand function, we obtain:

$$R_n^* = \left(\frac{x_n^*}{n}\right) * \left(a - b * \left(\frac{x_n^*}{2n}\right)\right)$$

Total industry profits are then given by aggregating retail revenues less the supplier's cost of production. Using $C(x_n^*) = sx_n^{*2}$ we get:

$$\Pi_n^* = n(\frac{x_n^*}{n})(a - b * \frac{x_n^*}{2n}) - sx_n^{*2}$$

The industry profits are maximized by choosing x_n^* , such that $\frac{\partial \Pi_n^*}{\partial x_n^*} = 0$, which yields the equilibrium quantity:

$$x_n^* = \frac{na}{b+2ns} \tag{4.1}$$

Proof: See A.1 in the Appendix

Total industry quantity expressed in equation (4.1) is decreasing in the supplier's cost parameter s because a high s makes production costly. It is also decreasing in b, because a high b means the price elasticity of demand in the final goods market is low. On the other hand, x_n^* is increasing in a because a high a implies

a high reservation price of the consumers, making it more lucrative to sell more units. x_n^* is increasing and concave in n, because a higher n means the supplier can serve more markets at the same price, but since it has a convex cost function, it will only be profitable to serve the markets up to a certain point.

We will not consider the case where agents exert outside options, as we are only interested in equilibrium. As the equilibrium holds for any value n, it is also valid for N in which case the supplier serves all buyers, implying that $x_N^* = \frac{Na}{b+2Ns}$.¹⁶

Solving for the industry profits in equilibrium:

$$\Pi_{N}^{*} = a(\frac{Na}{b+2Ns}) - b(\frac{(\frac{Na}{b+2Ns})^{2}}{2N}) - s(\frac{Na}{b+2Ns})^{2}$$
$$\Pi_{N}^{*} = \frac{Na^{2}}{2(2Ns+b)}$$
(4.2)

Proof: See A.2 in the Appendix

Industry profits in (4.2) increase in N, as a larger N means more retailers contribute to profits. A higher a increases profits as the consumers' willingness to pay increases. We can also see that an increase in the cost parameter sreduces Π_N^* because of higher costs of production. A higher b also decreases profits, as the demand curve for final goods will be steeper.

4.4 Case 1: Total number of retailers is fixed

First, we want to look into a setting where N is held constant to see the effects of a reallocation of retailers between buyers. To see that, we need to extract buyer *i*'s contribution to industry profits to investigate the differences in a buyer's profits due to differences in size (the quantity of identical retail stores that a given buyer possesses). The industry profits less the contribution of buyer *i* is:

$$\Pi_{N-k_i}^* = ax_{N-k_i}^* - b\left(\frac{x_{N-k_i}^{*2}}{2(N-k_i)}\right) - sx_{N-k_i}^{*2}$$

¹⁶Inderst and Wey (2007) specify differences in n and N because they examine what happens out of equilibrium. Our thesis is only interested in equilibrium analysis, and we, therefore, specify n = N for all intents and purposes.

The first order condition for profit maximization is:

$$\frac{\partial \Pi_{N-k_i}^*}{\partial x_{N-k_i}^*} = 0$$
$$x_{N-k_i}^* = \frac{a(N-k_i)}{b+2Ns-2k_is}$$

As the maximization problem is the same as the previous, the method is the same with $N - k_i$ instead of N.

In equilibrium, the industry profits less the contribution of buyer i is therefore:

$$\Pi_{N-k_i}^* = \frac{(N-k_i)a^2}{2(2(N-k_i)s+b)}$$
(4.3)

Proof: See A.3 in the Appendix

As in Inderst and Wey (2007), buyer *i*'s contribution to the industry profits in equilibrium is the total industry profits less the total contribution of all other buyers apart from buyer *i*. Combining equation (4.2) and (4.3) yields:

$$\Pi_N^* - \Pi_{N-k}^* = \frac{Na^2}{2(2Ns+b)} - \frac{(N-k_i)a^2}{2(2(N-k_i)s+b)}$$
$$\Pi_N^* - \Pi_{N-k}^* = \frac{a^2bk_i}{2(2Ns+b)(2s(N-k_i)+b)}$$
(4.4)

Proof: See A.4 in the Appendix

An increase in k_i simultaneously increases the numerator and decreases the denominator. Therefore, an increase in k_i leads to a more than proportional increase in the buyer's contribution to industry profits. We can also show this by taking the second derivative:¹⁷

$$\frac{\partial^{2}(\Pi_{N}^{*} - \Pi_{N-k_{i}}^{*})}{\partial^{2}k_{i}} = \frac{2a^{2}bs}{(2Ns - 2k_{i}s + b)^{3}}$$

Proof: See A.5 in the Appendix

This expression is positive if $N \ge k_i$, which it must be by construction (as an agent cannot own more stores than there exists). Therefore, a buyer's

¹⁷In reality, the profit function is not continuous in the amount of stores k_i ; an agent cannot own fractional stores. For this thesis, we assume it is a continuous function such that the derivative is well-defined. One can justify this simplification by assuming buyers can obtain fractional shares of a store, which does not violate retail stores being a discrete variable when the shares are continuously defined.

contribution to industry profits is convex in k_i , meaning it will contribute increasingly more by additional k_i stores the bigger the initial value of k_i is.

4.4.1 Buyer *i*'s share of surplus and input price

We further use Inderst and Wey's (2007) notation of buyers' and the supplier's profit. Buyer *i* obtains a share $\rho \in [0, 1]$ of their contribution to industry profits. A $\rho = 0$ states that the supplier has all power, and $\rho = 1$ indicates the opposite where "each buyer extract its full net contribution" (Inderst and Wey, 2007) (p. 651). Therefore the buyer's profits are given by:

$$\rho[\Pi_N^* - \Pi_{N-k_i}^*]$$

The supplier obtains the rest of this contribution for all buyers, so the supplier's total profits are:

$$\Pi_N^* - \rho \sum_{i=1}^{I} [\Pi_N^* - \Pi_{N-k_i}^*]$$
, where $i = 1, 2, ..., I$

Buyer *i*'s margin per unit is the difference between the revenue one unit generates $P(\frac{x_N^*}{N})$ and the unit price τ_i the buyer pays the supplier. This has to be equal to the buyer's profits per unit. We assume the supplier is allowed to differentiate input prices based on the respective buyers' size, which is this model's definition of price discrimination. In equilibrium, buyer *i* purchases $\tilde{x}_i = k_i \frac{x_N^*}{N}$ units as in Inderst and Wey (2007). Therefore, the buyer's profit per unit is equal to:

$$O\frac{\Pi_N^* - \Pi_{N-k_i}^*}{\tilde{x}_i}$$

With $\tilde{x}_i = k_i \frac{x_N^*}{N}$, we obtain that:

$$P(\frac{x_N^*}{N}) - \tau_i = \rho \frac{\prod_{N=1}^{*} - \prod_{N=k_i}^{*}}{k_i \frac{x_N^*}{N}}$$

$$P(\frac{x_N^*}{N}) - \tau_i = \rho \frac{\Pi_N^* - \Pi_{N-k_i}^*}{k_i} \frac{N}{x_N^*}$$
(4.5)

In their paper, Inderst and Wey (2007) show that equation (4.5) is a buyer's margin per unit, where $P(\frac{x_N^*}{N})$ is the price the buyer obtains by selling a quantity of $\frac{x_N^*}{N}$ in its respective market, and τ_i is the average unit price it pays to the supplier.

Rewriting the equation in terms of the input price yields:

$$\tau_i = P(\frac{x_N^*}{N}) - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* N}{k_i x_N^*}$$

Inderst and Wey (2007) show that a large buyer obtains a discount (a decrease in τ_i) if an increase in k_i leads to a bigger increase in $\Pi_N^* - \Pi_{N-k_i}^*$. In other words, if a buyer controlling more firms leads to an increased contribution to total industry profits, the buyer requires a lower average input price from the supplier.

As buyer *i*'s incremental contribution to industry profits is the expression (4.4), this contribution is equal to the revenue generated by each buyer downstream. As we previously have established, equation (4.4) is over-proportionate in k_i , meaning that a buyer controlling more retail stores pays a lower average unit price than buyers controlling fewer stores, due to receiving a discount from the supplier. If we insert the equilibrium quantity, x_N^* , and the demand curve for final goods, we obtain:

$$\tau_{i} = \frac{a(b+4sN)}{2b+4sN} - \rho \frac{\Pi_{N}^{*} - \Pi_{N-k_{i}}^{*}}{k_{i}} \frac{b+2sN}{a}$$

The wholesale price τ_i is decreasing in k_i if and only if equation (4.4) is increasing more than proportionally in k_i . In other words, a larger buyer obtains a discount if their contribution to industry profits increases convexly in the number of stores owned.

If we insert the buyer's contribution to industry profits from equation (4.4), we get:

$$\tau_i = \frac{a(b+4sN)}{2b+4sN} - \rho \frac{ab}{2(2Ns-2k_is+b)}$$
(4.6)

Proof: See A.6 in the Appendix

Here, the input price, τ_i , is expressed in terms of only exogenous variables and k_i . If buyer *i* were to expand their portfolio of stores by one unit, they would face a lower input price. The change in input price for buyer *i* following an increase by one additional retail store is $\frac{\partial \tau_i}{\partial k_i} < 0$. This means that the margin

per unit increases in k_i , because the input price decreases.¹⁸ It also means that a buyer reducing its stock of retailers would face a higher input price. Another noteworthy observation is that the price depends negatively on the bargaining power parameter, ρ . A buyer with a large ρ , or much buyer power, faces a lower input price. As mentioned, the parameter s enters in the supplier's cost function, which employs a positive relationship with the input price. A higher s means the supplier faces higher costs, requiring a higher markup from the downstream firms. Further, the price is decreasing in the slope parameter of the demand curve, b. A high b implies a steep demand curve, suggesting that consumers have a low price elasticity of demand. Then, the industry can charge a high price in the final goods markets without losing too many customers to cover the increased costs of producing more units. It is not immediately clear what the effects of a change in a has on τ_i . This is because a is the point of intersect of the demand function for final goods. It illustrates the highest willingness to pay out of all the consumers but has no implication on how the surplus is shared between the supplier and buyers.

The first term in equation (4.6), given by $\frac{a(b+4sN)}{2b+4sN}$, is the price in the final goods market, which also encompasses the effect on the supplier's marginal costs. Since demand remains unchanged due to the number of markets being exogenously given, there is no change in the supplier's marginal costs for which no additional markup is required. The second term, $\rho_{\frac{ab}{2(2Ns-2k_is+b)}}$, captures the effect of owning more stores. Since the contribution to industry profits is convex in k_i , buyer *i* gains more leverage toward the supplier the bigger k_i is. When buyer *i* grows large, it costs the supplier increasingly more to disagree in the negotiations for a given increase in k_i .

Furthermore, if all else is equal but the total number of retail stores N increases by one, the change in input price for a given buyer is $\frac{\partial \tau_i}{\partial N} > 0$. This is because an increase in N implies an increase in the number of buyers, which dilutes the incumbent buyers' bargaining power. We will make use of this result later in

¹⁸Equation (4.6) is expressed in terms of the input price but is easily rearranged to the margin per unit by moving the demand curve for final goods to the left hand side of the equation and multiplying through by (-1). Inderst and Wey (2007) consistently express it as margin per unit, which in our model is given by $\rho \frac{ab}{2(2Ns-2k_is+b)}$, and it will always have opposite signs to the input price τ_i .

the paper.¹⁹

Proposition 1: When the total number of retail stores is fixed, a buyer controlling a larger set of stores obtains a lower input price. If all else is constant, changing the total number of retail stores affects the input price all buyers pay in the same direction as the change in N.

Proof: See A.7 in the Appendix

4.4.2 Increasing capacity by increasing in k_i

Up until this point, we have followed Inderst and Wey's (2007) results closely, but applied our functions. From here on, we add some new assumptions to examine how buyers can invest. By proposition 1, buyers controlling more retail stores receive lower input prices when the number of markets is exogenous. Suppose now the buyer can affect its bargaining power by investing in k_i . We assume there is a linear cost function related to increasing the buyer's capacity:

$$C(k_i^{new}) = F * k_i$$
, where $C'_k(k_i^{new}) = F$

F is the fixed price of one store, and by investing, the buyer will obtain $k_i = k_i^{new} + k_i^{old}$ retail stores in the next stage. The buyer will gain excess profits from investing if the profits exceed the costs of doing so, hence:

$$\rho(\Pi_N - \Pi_{N-k_i}) \ge C(k_i^{new})$$

To determine whether or not to invest in one additional store, the buyer considers the following (assuming, as mentioned, that the profit function is continuous in k_i):

$$\frac{\partial}{\partial k_i} (\rho(\Pi_N - \Pi_{N-k_i})) \ge C'_k(k_i^{new})$$

The left-hand side is the buyer's additional profits by investing in one additional store. As long as the marginal increase in profits is larger than the marginal cost of investing in k_i , the buyer will invest. The buyer's contribution to industry profits when owning one additional store is:

$$\frac{\partial \rho(\Pi_N^* - \Pi_{N-k_i}^*)}{\partial k_i} = \frac{\rho a^2 b}{2(2Ns - 2k_i s + b)^2}$$

¹⁹A change in N is not particularly interesting when it is exogenously given. Still, we show later that there is an application of this result when the total number of retailers is endogenously determined, and all buyers are allowed to invest.

The first order condition is:

$$\frac{\partial(\rho(\Pi_N^* - \Pi_{N-k}^*) - Fk_i)}{\partial k_i} = 0$$

Which yields:

$$\tilde{k}_i = N - \frac{\sqrt{\frac{\rho a^2 b}{2F}}}{2s} + \frac{b}{2s} \tag{4.7}$$

with
$$F \leq \frac{\rho a^2}{2b}$$

Proposition 2: Since the buyer's profits, $\rho(\Pi_N - \Pi_{N-k_i})$, are convex in k_i , $\tilde{k_i}$ is in fact a minimum. This means that the amount of retail stores $\tilde{k_i}$ minimizes profits for buyer *i*. If $F = \frac{\rho a^2}{2b}$, then $\tilde{k_i} = N$ because a retail store price at this level will imply higher profits from selling off all stores rather than operating them.

Proof: See A.8 in the Appendix

4.4.3 Analysis

Buyer *i* is strictly better off for any value k_i to diverge from the minimum at \tilde{k}_i , as the amount of stores \tilde{k}_i minimizes profits. This means small buyers that own an initial amount of stores $k_i < \tilde{k}_i$ are better off selling all their stores, while big buyers owning an initial amount $k_i > \tilde{k}_i$ will want to acquire as many stores as possible. A small buyer's profit from selling all retail stores exceeds the profit they would get from operating those stores due to facing a high input price. On the other hand, a large buyer can operate additional stores at greater profits than the cost of investment due to facing a low input price. We obtain corner solutions to the maximization problem, where the optimal amount of stores to invest in is discontinuous at the threshold, \tilde{k}_i .

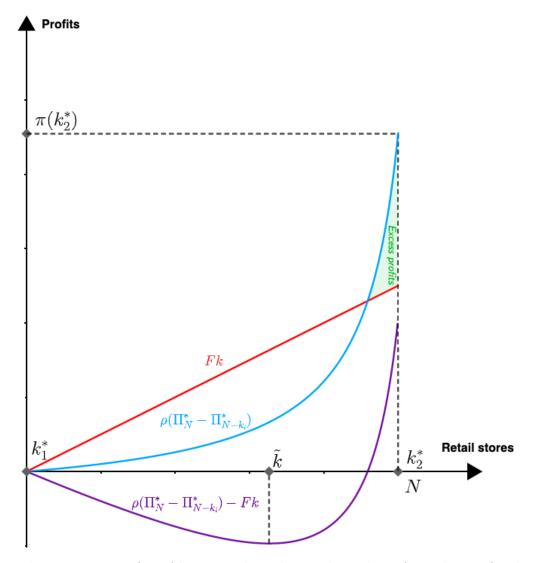


Figure 4.1: Profits of buyer *i* when the total number of retailers is fixed

Figure 4.1 illustrates the profitability of investing in additional retail stores, with the number of retail stores, k_i , controlled by buyer *i* on the x-axis and the profit level for a given value of k_i on the y-axis. The red line illustrates the cost of investing that increases linearly with the number of retail stores. The blue line is the buyer's profit curve, which increases convexly with the number of stores owned. The purple line shows the difference between the profit and cost curves, which is positive when the profit curve exceeds the cost of investing. \tilde{k} is the amount of retail stores the minimizes profitability, given that any buyer is strictly better off moving away from this point. k_1^* and k_2^* show the equilibrium levels of k_i . Any buyer below the threshold \tilde{k} would be better off selling all their stores since the price of a store, F, is high enough, such that the reduction in input price from owning more stores does not make up for the costs of investing in this interval. Conversely, a buyer owning stores

above the threshold k before investing is better off buying stores until they own $k_2^* = N$ stores. In this interval, the gain of better input price surpasses the cost F, yielding excess profits from investing when the profit curve transcends the cost curve. The price, F, is calibrated to satisfy $F < \frac{\rho a^2}{2b}$, as equality would imply no excess profits moving towards k_2^* .

As we have established, the contribution in equation (4.4) is convex in k_i , suggesting big buyers want to grow bigger due to receiving a discount from the supplier. A buyer controlling more retailers can better distribute the costs of acquiring an additional retailer across several markets than does a small buyer. This implies that when the number of total retailers is fixed, and the purchase of k_i reallocates the distribution of retailers, we end up with fewer buyers controlling more retailers each. These buyers then get better terms from the supplier (as in lower average input prices τ_i), meaning the supplier is worse off and the buyers are better off. Additionally, when retail stores are scarce, big buyers may squeeze small buyers out of the market due to the reallocation of retail stores, resulting in big chain stores controlling many downstream markets. One could also look at this as a divergence from symmetrical buyers; differences in size amplify when buyers are allowed to invest in stores, leading to greater asymmetries across markets. If one additional buyer enters the market without owning any retail stores initially, but purchases for instance, one store from all other buyers, then it is equivalent to all buyers reducing their k without changing N. In this scenario, the supplier will be better off because all buyers have diluted their buyer power.

Another noteworthy observation is that when N is constant, investing in additional retailers does not have any welfare effects. This is because the total equilibrium quantity remains unchanged $(x_N^*$ is not a function of k_i); it is only the distribution of profits between the supplier and buyers that changes.

4.4.4 Changes in supplier profits following an investment in k_i

As previously shown, the supplier's profit function is given by:

$$\Pi_N^* - \rho \sum_{i=1}^{I} [\Pi_N^* - \Pi_{N-k_i}^*]$$

Total industry profits, given by equation (4.2), are unchanged when buyer *i* invests in k_i , as Π_N^* is not a function of k_i since *N* is held constant. As we have established, the corner solutions impose a tendency toward monopsony downstream, where one buyer controls all retail stores in the limit. Since (4.4) is convex in k_i , a reallocation of stores suggests that the contribution to industry profits is greater with a single buyer controlling all retailers than the sum of contributions from many buyers together controlling the same amount of retailers. Therefore, we can infer that the supplier's profits are lower the fewer buyers it serves, even though the total amount of retailers is unchanged. Since total industry profits are unchanged, the single buyer's profits increase exactly as much as the supplier's profits decrease. The only cases where this is not true are with $\rho = 0$ or $\rho = 1$, which does not reallocate any profits between buyers and the supplier. With $\rho = 0$, the supplier's surplus is exactly Π_N^* , and the buyers get zero, and with $\rho = 1$ the allocation is reversed.²⁰

4.5 Case 2: Total number of retailers is endogenously determined and a single buyer invests

Suppose now that only buyer *i* can invest in additional retail stores k_i , and doing so does not reallocate the distribution of stores but rather creates additional ones. Total markets increase in the amount invested in k_i , which means $N = Z + k_i$, where N is the total number of downstream markets, $Z \ge 0$ is the number of markets controlled by the buyers except those controlled by buyer *i*, and $k_i \ge 0$ is the number of markets controlled by buyer *i*.

The equilibrium quantity, total industry profits, and total industry profits less buyer i's contribution are given by respectively:

$$x_{Z+k_i}^* = \frac{(Z+k_i)a}{b+2(Z+k_i)s}$$
$$\Pi_{Z+k_i}^* = \frac{(Z+k_i)a^2}{2(2(Z+k_i)s+b)}$$
$$\Pi_Z^* = \frac{Za^2}{2(2Zs+b)}$$

 $^{^{20}}$ In models where the supplier can invest, for instance, in production technology, their profits affect their incentives to do so. However, this analysis is beyond the scope of our thesis.

We obtain buyer *i*'s contribution to industry profits, $\Pi_{Z+k_i}^* - \Pi_Z^*$, by using equation (4.4) and inserting for the profit functions above, which becomes:

$$\Pi_{Z+k_i}^* - \Pi_Z^* = \frac{a^2 b k_i}{2(2(Z+k_i)s+b)(2Zs+b)}$$
(4.8)

Proof: By inserting $Z + k_i$ instead of N we get the equivalent results as A.4 in the Appendix

To determine the properties of this function, we can take the first and second derivatives:

$$\frac{\partial(\Pi_{Z+k_i}^* - \Pi_Z^*)}{\partial k_i} = \frac{a^2b}{2(2s(Z+k_i)+b)^2} \text{ and } \frac{\partial^2(\Pi_{Z+k_i}^* - \Pi_Z^*)}{\partial^2 k_i} = -\frac{2a^2bs}{(2s(Z+k_i)+b)^3}$$

Proofs: See B.1 in the Appendix:

The first derivative is positive, and therefore (4.8) is increasing in k_i . The second derivative is negative, and therefore the contribution to industry profits is concave in k_i . An increase in k_i increases the contribution *less than proportionally*, as opposed to when the total number of retailers is exogenous.

4.5.1 Input prices for buyer i

In equilibrium, buyer *i* purchases $\tilde{x}_i = k_i \frac{x_{Z+k_i}^*}{Z+k_i}$. When inserting $N = Z + k_i$, the input price is given by:

$$\tau_{i} = P(\frac{x_{Z+k_{i}}}{Z+k_{i}}) - \rho \frac{\Pi_{Z+k_{i}}^{*} - \Pi_{Z}^{*}}{k_{i}} \frac{Z+k_{i}}{x_{Z+k_{i}}^{*}}$$
$$\tau_{i} = \frac{a(b+4s(Z+k_{i}))}{2b+4s(Z+k_{i})} - \rho \frac{ab}{2(2Zs+b)}$$
(4.9)

Proof: See B.2 in the Appendix

When buyer *i* invests in one additional unit of k_i , the change in input price the buyer faces is given by $\frac{\partial \tau_i}{\partial k_i} > 0$. The change in input price for all other buyers is captured by $\frac{\partial \tau_i}{\partial Z} > 0$. We therefore obtain the following:

Proposition 3: When the total number of retail stores is endogenously determined through k_i , the input price all buyers face changes in the same direction as a change in k_i .

Proof: See B.3 in the Appendix

When the investment in retail stores endogenously determines the total number of markets, the demand for final goods is affected by a change in k_i . This further implies that the supplier must produce more to comply with increased demand following an increase in the number of markets. The nature of the convex costs leads the supplier to raise the input price if either Z or k_i increase, reflected by the first term in equation (4.9). As in Inderst and Wey (2007), a discount from the supplier stipulates a convex contribution to industry profits. When the number of markets is endogenously given, the contribution to industry profits is concave in the number of retailers controlled by buyer i. The second term in equation (4.9) shows that buyer *i* no longer gets any discount by growing larger, as it is no longer a function of k_i . It is, however, a function of Z, such that all other buyers are affected by buyer i's investment.²¹ Increasing k_i leads to an overall higher price charged to buyer i, and even higher to all other buyers due to the denominator of the second term in equation (4.9) is increasing in Z. Furthermore, the margin per unit for buyer *i*, given by $P(\frac{x_{Z+k_i}}{Z+k_i}) - \tau_i = \rho \frac{ab}{2(2Zs+b)}$ is not a function of k_i and therefore remains unchanged. The price increase buyer i faces is only due to an increase in the final goods price following more production of inputs, leaving the buyer's margin per unit unaltered. On the contrary, all other buyers except the one investing see their margin per unit decrease. As opposed to buyer i, they do not benefit from an increased contribution to industry profits; they only face a higher input price due to the supplier's increased production costs.

4.5.2 Increasing capacity by investing in k_i

As we studied in the first case, it is interesting to examine the incentives for buyer *i* to invest here as well. The excess profits from investment is given by $\rho(\Pi_{Z+k_i} - \Pi_Z) \ge C(k_i^{new})$, where $C(k_i^{new}) = F * k_i$. Buyer *i*'s marginal profits when owning an additional store are given by the derivative:

$$\frac{\partial \rho(\Pi_{Z+k_{i}}^{*} - \Pi_{Z}^{*})}{\partial k_{i}} = \frac{\rho a^{2} b}{2(2s(Z+k_{i})+b)^{2}}$$

Proof: See B.4 in the Appendix

²¹When buyer *i* invests such that their amount of stores k_i increases, in the perspective of all other buyers, it is Z that increases.

The first order condition for profit maximization therefore becomes:

$$\frac{\partial}{\partial k_i}\rho(\Pi_{Z+k_i} - \Pi_Z) - C'_k(k_i^{new}) = 0$$
$$\frac{\rho a^2 b}{2(2s(Z+k_i)+b)^2} - F = 0$$

Solving for k_i yields:

$$k_{i}^{*} = \frac{\sqrt{\frac{\rho a^{2}b}{2F}} - b}{2s} - Z$$
(4.10)
with $F \leq \frac{\rho a^{2}b}{2(2Zs+b)^{2}}$

Proposition 4: Since equation (4.8) is concave in k_i , k_i^* is a maximum, which means buyer i's profits are maximized when owning k_i^* stores, and the maximum is reached if and only if the the cost of investing in an additional store satisfies $F \leq \frac{\rho a^2 b}{2(2Zs+b)^2}$. Additionally, if $F = \frac{\rho a^2 b}{2(2Zs+b)^2}$, then $k_i^* = 0$.

Proof: See B.5 in the Appendix.

4.5.3 Analysis

Buyer *i*'s profit function, given by $\rho(\Pi_{Z+k_i} - \Pi_Z)$, is increasing and concave in k_i as shown. This means the solution k_i^* is a maximum, as opposed to the case where N is fixed. It further implies that k_i^* is the optimal amount of stores a given buyer would like to own. For this to be an economically valid solution, the maximum has to be reached with $k_i \in [0, N)$,²² which further implies that $F \leq \frac{\rho a^2 b}{2(2Zs+b)^2}$. As in the first case, a buyer is more capable of handling higher costs of investing, F, the more buyer power, ρ , it exhibits. A large ρ means the buyer receives a larger portion of the surplus shared between the buyer and the supplier which, in turn, can be used to cover the costs of investment F.

The concavity of $\rho(\Pi_{Z+k_i} - \Pi_Z)$ ensures that k_i^* is the unique optimal solution for positive values of k_i . However, as we have previously established, a requirement to obtain a discount from the supplier through the average input price, the marginal contribution to industry profits has to be over-proportionate, or

²²We assume in this chapter that $k_i = N$ is unattainable when there are more than a single buyer, as there is no reallocation of stores.

convex, in k_i . As all agents are profit-maximizing entities, they require better terms in order to invest in additional stores. This requirement is not fulfilled on the price side, as input price, τ_i , is *increasing* in k_i when $\rho(\Pi_{Z+k_i} - \Pi_Z)$ is concave in k_i . On the contrary, the incentive to invest stems from a larger relative revenue stream from owning additional stores. A buyer's profits are still increasing in k_i , and the increase in profits is greater than the cost of investing in additional stores for any value $k_i < k_i^*$, and is equal at $k_i = k_i^*$. In other words, a buyer's marginal gain from owning more stores is greater for lower initial values of k_i . Because of not receiving a discounted input price, the relative increase in profits does not make up for the costs of investing for buyers owning $k_i > k_i^*$ stores initially.

Initial values of k_i for any given buyer i where $k_i < k_i^*$, will maximize buyer i's profits by investing in additional stores until $k_i = k_i^*$, provided the marginal cost of investment satisfies $F \leq \frac{\rho a^2 b}{2(2Zs+b)^2}$. A large buyer would like to divest in stores for all $k_i > k_i^*$. However, the very assumption that $N = Z + k_i$ is restrictive on the reallocation of stores. For a large buyer to converge to k_i^* , it would imply selling stores to small buyers or re-purposing the stores otherwise. If large buyers can re-purpose the additional stores, for instance, by selling them to other industries or making profitable use of them, otherwise, they would do so. We assume they are not allowed to because it implies fixing the total amount of stores. When buyers are not allowed to re-purpose excess stores, welfare increases when small buyers increase the number of owned stores. A small buyer creates additional stores until they own k_i^* , while a large buyer does not alter their position due to the restrictiveness mentioned. In sum, the market tends to symmetric buyers when N is endogenous because the marginal willingness to pay for additional markets is declining.²³

It is important to state that an initially large buyer is always more profitable than a small buyer because $\rho(\Pi_{Z+k_i} - \Pi_Z)$ is strictly increasing in k_i and has no maximum when we disregard the costs of investing F. The model intends not to compare the profitability of large and small buyers; only to compare the relative optimal investment decision. A large buyer optimally divests (if

 $^{^{23}\}mathrm{If}$ buyers were allowed to sell stores to each other, the scenario would become more similar to the case of a fixed N.

allowed to) only because the profits from selling exceed the profits of operating the excess amount of stores. A small buyer does not exceed k_i^* when investing because the costs exceed the profits of operating stores above the optimum.

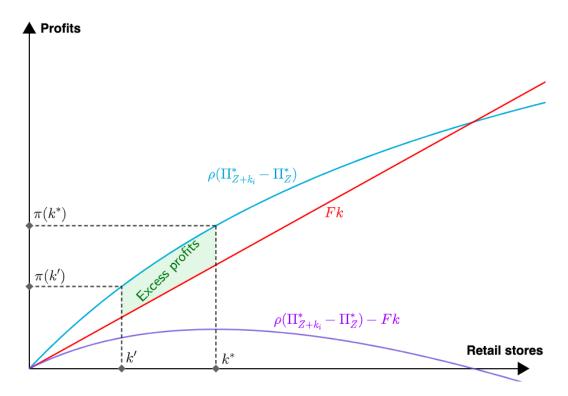


Figure 4.2: Profits of buyer *i* when the total amount of retailers increases with k_i

Figure 4.2 illustrates the profits and costs of investing in retail stores. The number of controlled retail stores is on the x-axis, and the profit for a given level of stores is on the y-axis. The blue line represents the profit function for buyer *i*, which is increasing and concave in the number of retailers controlled by buyer *i*, k_i . The red line is the cost function associated with investing in additional retailers. The cost, *F*, is calibrated in the same manner as with exogenous *N*, with $F < \frac{\rho a^2 b}{2(2Zs+b)^2}$, since $F = \frac{\rho a^2 b}{2(2Zs+b)^2}$ yields $k_i^* = 0$. The purple line is the difference between the profit function and the cost function. The profits from operating retailers is maximized in k^* given by $\pi(k^*)$. The green shaded area shows the excess profit going from an arbitrarily chosen number of retailers $k' < k^*$ to the equilibrium amount k^* .

4.5.4 Effects on all other agents

The equilibrium quantity, $x_{Z+k_i}^*$, and the total industry profits, $\Pi_{Z+k_i}^*$, are increasing in the number of retailers. On the other hand, equilibrium quantity

per retail store and the total industry profit per retail store decrease in both Z and k_i .²⁴ The change in contribution to industry profits is the same across all retail stores, meaning that also buyer i is affected by a decrease in the profits per store. Unlike buyer i, the other buyers do not benefit from increased revenue from operating more stores. As previously shown, the input price paid by all other buyers than buyer i increases when buyer i invests in additional stores, so they are worse off following buyer i's investment. The contribution to industry profits by all buyers except the one who invests, given by equation (4.8), decreases as Z only appears in the denominator.

In the case where a buyer creates additional retailers by investing, the supplier's profits are given by:

$$\Pi_{Z+k_i}^* - \rho \sum_{i=1}^{I} [\Pi_{Z+k_i}^* - \Pi_{Z}^*]$$

The change in the supplier's profit depends on the relative changes in total industry profits and buyer *i*'s contribution relative to all other buyers. Total industry profits, $\Pi_{Z+k_i}^*$, increase in k_i , as does buyer *i*'s contribution to industry profits, $\Pi_{Z+k_i}^* - \Pi_Z^*$. The contribution to industry profits by all other buyers, Π_Z^* , is not a function of k_i and remains unchanged. The increase in total industry profits solely stems from the increase in buyer *i*'s contribution. Because the supplier extracts some of the contributions from buyer *i* through ρ , the supplier's profits increase when $\rho < 1$. When $\rho = 1$, buyer *i* can extract its full contribution such that the supplier's profits are unchanged.

4.6 Case 3: Total number of retailers is endogenously determined and all buyers invest

In equilibrium where the total amount of retailers is endogenously determined by the investment in k_i , buyer *i* chooses a level of stores:

$$k_i^* = \frac{\sqrt{\frac{\rho a^2 b}{2F}} - b}{2s} - Z$$

²⁴Equilibrium quantity per retail store is given by $\frac{x_{Z+k_i}^*}{Z+k_i} = \frac{\frac{(Z+k_i)a}{b+2(Z+k_i)s}}{Z+k_i} = \frac{a}{b+2s(Z+k_i)}$, and the total industry profits per retail store in equilibrium is given by $\frac{\Pi_{Z+k_i}^*}{Z+k_i} = \frac{\frac{(Z+k_i)a^2}{2(2(Z+k_i)s+b)}}{Z+k_i} = \frac{a^2}{2(2s(Z+k_i)+b)}$. Both decrease in Z and k_i due to both entering positively in the denominators.

The remainder of the retail stores, Z, is constant if only buyer i is allowed to invest. However, if all other buyers in addition to buyer i invest, Z is increasing in the amount of retail stores k_i^* . In fact, if the buyers are symmetric and face the same maximization problem, the amount of stores controlled by buyers $I - \{i\}$ is given by $k_{I-\{i\}}(z-1)$, where z is the total number of buyers in the market. Assuming that the buyers have symmetric profit functions, $k_i^* = k^* \forall i$, such that the total amount of retail stores is given by zk^* in equilibrium.

Each buyer then chooses a level of k^* in equilibrium:

$$k^* = \frac{\sqrt{\frac{\rho a^2 b}{2F}} - b}{2sz}$$
(4.11)

The equilibrium amount, k^* , is decreasing in z. A higher z means that more total buyers in the market choose the same level of k^* , implying a less relative revenue increase of owning an additional retail store due to not receiving a lower input price. Since k^* is a non-negative number, the following has to be satisfied:

$$F \leq \frac{\rho a^2}{2b}$$

Where $F = \frac{\rho a^2}{2b}$ yields $k^* = 0$.

Proposition 5: Assuming that the initial level of stores controlled by all buyers is less than the equilibrium amount, and if $F < \frac{\rho a^2}{2b}$ (such that $k^* > 0$), there will be created additional markets. This means the supplier will serve more retailers, producing a larger quantity, as x^* increases in zk^* . However, due to the convexity of the supplier's costs, the marginal increase in production is lower at higher levels of k; it will not be profitable to increase production indefinitely.

Proof: See C.1 in the Appendix.

4.6.1 Analysis

The price faced by all buyers in equilibrium is derived by inserting zk^* for N in equation (4.6):

$$\tau_i = \frac{a(b+4s(zk^*))}{2b+4s(zk^*)} - \rho \frac{ab}{2(2(k^*z-k^*)s+b)}$$
(4.12)

If all (or a portion of) buyers initially own more than k^* , those who do will not invest for the same reason as discussed in case two. The interesting analysis is for those who own less than k^* before investment. A change in the product of z and k^* is the equivalent to changing N in equation (4.6). We have shown by proposition 1 that this has a positive relationship with the input price. An increase in k^* implies an increase in the profit-maximizing amount of stores controlled by each buyer, while an increase in z suggests more buyers in total. An increase in the product of these parameters increases the demand for inputs, and the supplier has to raise the price to cover the growth in marginal costs. Consequently, the buyers' margin per unit given by

$$P(\frac{x_{zk}^*}{zk}) - \tau_i = \rho \frac{ab}{2(2sk(z-1)+b)}$$

and is decreasing in the total amount of retail stores. Retail stores become less profitable to operate following an increase in the total amount of retail stores. Equilibrium quantity and total industry profits, given by

$$x_{zk}^* = \frac{kza}{b+2skz}$$
 and $\Pi_{zk}^* = \frac{kza^2}{2(2kzs+b)}$

respectively, are increasing in z and k. The implication is that the supplier serves more markets, leading to a higher required amount of total units produced. This implies that there are more consumers with sufficient willingness to pay for the goods, resulting in an overall increase in revenue streams. However, due to the convex costs of the supplier, the increase in industry profits is concave in both k and z.

Interestingly, an individual buyer's contribution to profits is decreasing in zk^* , or equivalently in N when we specify the function, which we showed in the first case in equation (4.4), as:

$$\Pi_N^* - \Pi_{N-k}^* = \frac{a^2 b k_i}{2(2Ns+b)(2(N-k)s+b)}$$

This means that when all buyers initially own $k < k^*$ and choose k^* , it reduces their profits individually because all buyers expand their portfolio of stores. This is somewhat similar to a prisoner's dilemma, where the buyers would have been better off by colluding on not investing but strictly prefer to invest if all others agree not to do so. Also, if all other buyers invest, then the last buyer would strictly prefer also to invest. Otherwise, he is even worse off. This means that when all buyers invest the same amount, they are worse off than if not allowed to invest at all. The supplier's profits are given by:

$$\Pi_{zk}^* - \rho \sum_{i=1}^{I} [\Pi_{zk}^* - \Pi_{k(z-1)}^*]$$

Since total industry profits increase in zk^* , and the buyers' contributions to profit are decreasing in zk^* , the supplier is much better off when the buyers invest "too much". Also, the consumers are better off since there will be a larger quantity sold at a lower price in the final goods markets to meet the falling demand curve.

4.7 Case 4: Ban on price discrimination

This section compares the incentives to invest for buyer i with price discrimination to the results with a ban on price discrimination. We previously defined price discrimination as the ability of the supplier to charge different prices to buyers based on size differences. To compare discrimination with non-discrimination, we find it useful to define non-discrimination as the price the supplier charges all buyers as if all buyers were the same size.

The input price that follows from the case under price discrimination and the total amount of retailers is endogenous, expressed by equation (4.9), is:

$$\tau_i = \frac{a(b+4s(Z+k_i))}{2b+4s(Z+k_i)} - \rho \frac{ab}{2(2Zs+b)}$$

The increase in price for buyer i when price discrimination is allowed, and only buyer i invests in one additional retail store was shown to be $\frac{\partial \tau_i}{\partial k_i} > 0$. In equilibrium, buyers choose a level of retail stores at $k_i = k^*$. Therefore, under a ban on price discrimination, using our definition specified above, the price in equilibrium is equivalent to as if all agents have chosen k^* .

It is interesting to examine what happens at the margin when approaching

 k^* , when all agents except buyer *i* are in equilibrium and *i* invests toward k^* . To compare the incentives to invest under price discrimination and non-discrimination, we can compare the change in price when approaching equilibrium in both cases. Under non-discrimination, all buyers receive equal prices in equilibrium, which is equivalent to each buyer owning k^* retail stores. The amount of stores owned in total by all other buyers apart from buyer *i* is then $Z = k^*(z-1)$.

When all buyers own the equilibrium amount of retail stores, the input price per unit charged to all buyers is:²⁵

$$\tau^* = \frac{a(b+4sk^*z)}{2b+4sk^*z} - \rho \frac{ab}{2(2k^*(z-1)s+b)}$$
(4.13)

The non-discriminatory input price out of equilibrium is the price $\hat{\tau} \neq \tau^*$, where the total amount of retail stores is $k^*z - 1$. Out of equilibrium, buyer *i* owns $k^* - 1$ stores. To implement non-discrimination, the price is determined as if all buyers own $\hat{k} < k^*$ stores, where $\hat{k} = k^* - \frac{1}{z}$. In other words, the price, $\hat{\tau}$, is determined as if all buyers own \hat{k} stores each, which is $\frac{1}{z}$ fewer than the equilibrium amount k^* . This implies that $\sum_{j=1}^{z} (k_j^* - \hat{k}_j) = z \cdot \frac{1}{z} = 1$, or that the sum of what each buyer hypothetically "lacks" toward the equilibrium amount of stores is equal to the one store that buyer *i* is short of equilibrium. To determine the incentives to invest under a ban on price discrimination, we need to examine how much the price changes when all buyers go from owning \hat{k} to k^* stores each.

Since buyer *i* is the one who invests, it is still k_i that increases by one unit. The total number of stores out of equilibrium is one less than the equilibrium amount, and therefore buyer *i* owns $k_i = k^* - 1$ stores before investing. When this deviation from equilibrium is split equally among all buyers to obtain equal prices, we get $\frac{1}{z} = \frac{k^* - k_i}{z}$, such that $\hat{k} = k^* - \frac{k^* - k_i}{z}$, which can be written as $\hat{k} = k^* + \frac{k_i - k^*}{z}$.

The non-discrimination input price that all buyers face when buyer i is one store short of the equilibrium amount, k^* , is therefore given by:

 $^{^{25}}$ When all buyers own the equilibrium amount, they are the same size which yields the same result as when all buyers invest toward the equilibrium amount.

$$\hat{\tau} = \frac{a(b+4s(k^*+\frac{k_i-k^*}{z})z)}{2b+4s(k^*+\frac{k_i-k^*}{z})z} - \rho \frac{ab}{2(2(k^*+\frac{k_i-k^*}{z})(z-1)s+b)}$$
(4.14)

If buyer *i* invests in one additional store, k_i increases by 1. When this happens, $k^* - k_i = 0$, and the price is as if all buyers own $k^* - \frac{0}{z} = k^*$ stores, and we return to equilibrium. To examine the incentives to invest for buyer *i*, we need to determine how much the price changes from $\hat{\tau}$ to τ^* , which is captured by $\frac{\partial \hat{\tau}}{\partial k_i}$:

$$\begin{aligned} &\frac{\partial}{\partial k_i} \left(\frac{a(b+4s(k^* + \frac{k_i - k^*}{z})z)}{2b+4s(k^* + \frac{k_i - k^*}{z})z} - \rho \frac{ab}{2(2(k^* + \frac{k_i - k^*}{z})(z-1)s+b)} \right) \\ &= \frac{4abs}{(2b+4s(k^*z+k_i - k^*))^2} + \rho \frac{absz(z-1)}{(-4k^*sz-2s(k_i - k^*)+2sk_iz+2k^*sz^2+bz)^2} \end{aligned}$$

Proof: See D.1 in the Appendix

When buyer *i* increases k_i by one unit, we return to equilibrium where $k_i = k^*$. We can therefore the expression above as:

$$\frac{4abs}{(2b+4s(k^*z))^2} + \rho \frac{absz(z-1)}{(-4k^*sz+2k^*sz+2k^*sz^2+bz)^2}$$
$$= \frac{4abs}{(2b+4sk^*z)^2} + \rho \frac{absz(z-1)}{(2k^*sz(z-1)+bz)^2} > 0$$
(4.15)

When price discrimination is allowed, we showed in B.3 in the Appendix that:

$$\frac{\partial \tau_i}{\partial k_i} = \frac{4abs}{(2b+4s(Z+k_i))^2}$$

In equilibrium where all buyers are the same size, we have that $Z + k_i = k^* z$. We can therefore write the derivative around equilibrium when price discrimination is allowed as

$$\frac{4abs}{(2b+4sk^*z)^2}$$

This expression is the same as the first term of the derivative when price discrimination is banned. Since the second term in (4.13) also is a positive number given that z > 1 (which holds by construction), prices increase more for buyer i under a ban on price discrimination.

Proposition 6: Given that all else is held constant, a change in input prices captures changes in profits one-to-one. Therefore, buyer i's incentive to invest in additional retail stores is lower under a ban on price discrimination than when price discrimination is practiced.

Buyer i does not get a relatively better price than its competitors since the supplier has to set prices equally for all agents. As under price discrimination, the increase in demand for inputs following an increase in stores means the supplier has to raise prices to accommodate the growth in marginal costs to produce more. In sum, buyer i is worse off when price discrimination is not allowed. Furthermore, when the total amount of stores is endogenously determined, investment in additional stores leads to more units produced. This serves a broader part of the final goods market, increasing consumer welfare. When price discrimination is banned, the incentive to invest in k is not as big, and there will not be created as many additional markets as under price discrimination. Therefore, by proposition 6, a ban on price discrimination leads to lower consumer welfare in this model.

5 Discussion

5.1 Pricing scheme

A key assumption in Inderst and Wey (2007) and our extension is that negotiations between the supplier and buyers are over a menu of prices. Therefore, the profits are maximized over the industry as a whole. The main idea behind this specification is to rule out double marginalization, a known problem when both the upstream and downstream firms are monopolists, which results in a loss of efficiency. Since the model assumes a bilateral relationship with a monopolist supplier and buyers operating in individual and independent markets, the assumption of such contract prices is imperative for our setup. For instance, DeGraba (1990) uses a linear pricing scheme, albeit under an array of different but crucial assumptions. He finds that less efficient buyers get the better end of the deal when the supplier discriminates in prices. However, it is not clear what implications different pricing rules would have for our results, but it would be interesting to examine.

5.2 Functional forms

As mentioned, our choice of model setup is to explicitly define a set of functions that satisfy Inderst and Wey's (2007) lemmas and propositions. While we lose some abstraction, we can more clearly outline the mechanisms of the key drivers in the model. We see this trade-off as well justified because Inderst and Wey's model has been properly peer-reviewed, and the specifications hold for particular sets of functions such as our own. Additionally, we have chosen quite simple representations of the functions to more confidently determine the properties of an arguably otherwise complicated setup. Our fall-back from some of the abstraction could, however, be a critique but is not unheard of in the literature; for instance, DeGraba (1990) specifies a demand curve of a similar form in his model.

5.3 Alternative sources of buyer power

We simplify Inderst and Wey's (2007) model in that we do not explicitly separate between the channels of buyer power. This is an important aspect of their study as they also conduct analyses where they shut down either of the channels. Furthermore, the consideration that negotiations may break down and outside options bind are widely studied in the literature. It goes back as the main driver of prices in the avant-garde article by Katz (1987). In Inderst and Wey, the role of outside options is primarily identified on the supplier's side, but they show that it has no implication for the equilibrium quantities. Our model does not venture out of equilibrium, and therefore we view outside options (at least on the supply side) to be not particularly relevant for our analysis. Inderst and Wey do briefly discuss outside options on the buyer-side as an alternative source of buyer power, in conjunction with endogenization of the sharing rule ρ , but assume the value of the option is inferior to an agreement with the supplier. They find somewhat ambiguous results in whether the presence of such outside options warrant large buyer discounts or not. Outside option analysis is an intrinsic part the model in O'Brien (2014), but he finds that if a chain store's outside option does not bind, "then forbidding price discrimination raises the average wholesale price for a wide range of parameters that determine relative bargaining powers" (p. 93), similar to our results. An interesting exercise and extension to our thesis would be to include binding outside options on either the upstream or downstream side of the market.

5.4 Competition

In Inderst and Wey (2007), and by the extension of our model, there is no direct competition, as downstream firms operate in independent markets. There is, however, indirect competition in that the buyers negotiate over prices from a supplier that produces a scarce good due to facing convex costs. While a monopolist supplier is the most common assumption in the literature on buyer power, some articles allow for competition upstream. For instance, in Inderst and Montez (2019), several buyers and sellers negotiate bilaterally over prices of substitute goods, and in Katz (1987), O'Brien (2014) and others, the exertion of a buyer's outside option leads to direct competition upstream. It seems to be much more common to have direct competition downstream Cournot-style, as in Katz (1987), O'Brien (2014), DeGraba (1990), and Foros and Kind (2018a), to name a few. In Inderst and Wey (2007), the choice of "restriction to a single supplier and to independent downstream markets allow [the authors] to focus exclusively on the interaction on the upstream market" (p. 650). Similarly, it is likely to be a resourceful specification in our extension to focus on the choice of investment, but it would be interesting with an expansion to direct competition.

5.5 Size as a form of buyer power

A significant assumption in our thesis is that a buyer's size is the sole factor that determines buyer power. In the literature, there is some ambiguity as to whether size equates to better negotiation terms or not. For instance, in Inderst and Montez (2019), "size is a source of mutual dependency and not an unequivocal source of power". Furthermore, Foros and Kind (2018a) argue that size alone cannot explain differences in input prices paid by competing retailers in the Norwegian grocery market. Foros et al. (2018) extend O'Brien (2014) to account for exogenous differences in size, yet they find no evidence that buyer size should induce discounts. In particular, they argue that even though an increase in a buyer's size suggests it becomes more costly for the supplier not to reach an accord on the price, the equivalent can be said from the buyer's perspective, unless there exist economies of scale in finding alternative supply. In Katz (1987), the relevance of size is that a large buyer can more credibly threaten to integrate backward.

On the other hand, size creates buyer power due to risk aversion in DeGraba (2005). Chipty and Snyder (1999) proclaim that buyer size induces an advantage in bargaining by referring to several empirical studies on the matter. They further allow for mergers which in their model endogenously determines buyer power, and obtain similar results as Inderst and Wey (2007); that a positive bargaining effect is dependent on the curvature of the supplier's surplus function. Furthermore, a large buyer typically takes more of the market demand, resulting in less demand to allocate across the remaining buyers. Oslo Economics and

Oeconomica (2017) highlight that large-volume discounts erect barriers to entry, leaving the large incumbents with better negotiation terms. Midttømme et al. (2019) claim that a volume discount in itself is not sufficient to explain price advantages. Instead, they point to additional factors such as increasing marginal costs to better explain different terms.

As we have discussed, in Inderst and Wey (2007), discounts originate from a threat of disagreement with a large buyer, which under standard assumptions increases more than proportionally with size. Despite disagreement happening out of equilibrium, and our analysis is restricted to what happens in equilibrium, the threat of disagreement is latent in our model. In our analysis, we take for granted that the channels of buyer power are present, as in Inderst and Wey, to infer under what assumptions large-buyer discounts emerge.

5.6 Investment incentives

There are several ways of modeling investments in the literature. In DeGraba (1990) and Inderst and Valletti (2009), downstream firms can invest in reducing their own marginal costs to produce more efficiently. Akgün and Chioveanu (2019) allow retail firms to invest in reducing the marginal costs of a substitute product, while the supplier can invest in production technology in Inderst and Wey (2007). In our thesis, the role of investment is to examine conditions under which size-induced buyer power arises, but the analysis is restricted to exclusively measure size as the number of retail stores controlled. If the investments would imply a change in other variables, for instance, the production efficiency, it is beyond the scope of our model to further give answers to what a change in size then would imply.

5.7 Additional remarks

This thesis is not meant to criticize any market policy, nor is it meant as a defense. Since we use several simplifying assumptions, we request that it is viewed in light of other models to gain a more nuanced perception of what the literature has to offer.

6 Conclusion

Our analysis extends Inderst and Wey's (2007) analytical framework to examine dynamic effects from buyer power and buyer-side incentives to invest in size. A key finding in our thesis is that how large buyers are formed can determine whether the respective buyers obtain discounted input prices or not. An important assumption is that the supplier and buyers negotiate over price contracts so that they maximize profits with respect to the quantity sold for the industry as a whole. When the total number of retail stores controlled by a given number of buyers is fixed, we obtain the same results as outlined in Inderst and Wey; they do indeed obtain a discount when growing in size. We show that a reallocation of retail stores among buyers will not violate the assumption that a buyer's contribution to industry profits is convex in the number of stores they control, which is Inderst and Wey's criterion for large buyers to obtain discounts. We find that if allowed to, large buyers whose size is initially above a given threshold will buy as many retail stores from smaller peers as they can get their hands on, such that the market tends toward greater asymmetries in buyer power. If unrestricted, we may even end up with a single monopsonist left downstream. We do not find support for any welfare effects under these assumptions because the total quantity sold to the final good markets is constant with the number of retail stores, and therefore unchanged. We find that unless the supplier has all the negotiation power, it is worse off the fewer buyers it serves. This follows as the supplier has to offer discounted input prices to larger buyers.

The second extension we present is when the total amount of retail stores increases as buyers invest. We can imagine this scenario as buyers opening new stores that were idle before their investment. First, we impose a restriction such that only one of the buyers can introduce new stores. This changes the premise of the buyer's contribution to industry profit function, and it turns out it becomes concave in the amount of retail stores operated. As the function no longer follows Inderst and Wey's criterion, buyers do not obtain discounts from growing larger. However, we show there are still incentives to invest in size up to a certain equilibrium point. This stems from the finding that a buyer's profitability increases from the increased revenue streams it obtains from operating more stores, in addition to discounted prices.

Interestingly, due to the convex production costs of the supplier, we find that a buyer introducing new stores increases the input price it faces. When initially controlling few stores, the gain from more revenue streams outweighs the price increase, and the buyer will profitably invest. In contrast, we find that an initially large buyer has lower incentives to invest because the marginal gain in revenue does not make up for increased prices. We find that for a given buyer that initially controls fewer than the equilibrium amount of stores invests, welfare increases. This follows as a larger quantity of salable goods will be produced by the supplier, which will serve a broader final market segment. Additionally, the supplier's profits increase with a higher demand for inputs, provided the buyers cannot extract their full contribution to industry profits.

We continue by lifting the restriction that only one buyer can invest to study a case where all buyers have the same opportunities. An interesting finding is that buyers may still have incentives to invest, even though it reduces their profits. Increased demand for inputs requires the supplier to raise prices, but if all buyers increase their size, their relative size to each other does not change. Therefore, they would be better off to collude on not investing, but then one buyer can profitably deviate, as seen in the second case of the model. Over-investing has positive welfare implications for the same reasons as the restricted case, and the supplier is also better off because it can extract a larger portion of the industry profits.

Our last contribution to the literature is to impose a ban on price discrimination in the restricted case. We compare incentives to introduce new stores for a single buyer when price discrimination is allowed and banned. A key finding of this exercise is that in our model, input prices for a given buyer increase more under a ban on price discrimination following an investment. This reduces the incentives to invest, implying that banning price discrimination reduces welfare in this framework.

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Appendix

A Case 1 - Total amount of retail stores is exogenous

A.1 Total industry quantity in equilibrium

Inserting the demand and cost functions into the industry profit yields:

$$\Pi_n^* = n(\frac{x_n^*}{n})(a - b * \frac{x_n^*}{2n}) - sx_n^{*2}$$
$$\Pi_n^* = x_n^*(a - b(\frac{x_n^*}{2n})) - sx_n^{*2}$$
$$\Pi_n^* = ax_n^* - b(\frac{x_n^{*2}}{2n}) - sx_n^{*2}$$

The industry profits are maximized by choosing x_n^* :

$$\frac{\partial \Pi_n^*}{\partial x_n^*} = 0$$
$$a - b\frac{x_n^*}{n} - 2sx_n^* = 0$$
$$x_n^*(\frac{1}{n}b + 2s) = a$$
$$x_n^* = \frac{a}{\frac{1}{n}b + 2s}$$
$$x_n^* = \frac{na}{b + 2ns}$$

A.2 Industry profits in equilibrium

We insert the equilibrium quantity into the industry profit function to obtain:

$$\Pi_N^* = a\left(\frac{Na}{b+2Ns}\right) - b\left(\frac{\left(\frac{Na}{b+2Ns}\right)^2}{2N}\right) - s\left(\frac{Na}{b+2Ns}\right)^2$$
$$\Pi_N^* = \frac{Na^2}{b+2Ns} - \frac{Na^2b}{2(b+2Ns)^2} - \frac{N^2a^2s}{(b+2Ns)^2}$$

Least common multiple is $2(b+2sN)^2$, which yields:

$$\Pi_N^* = \frac{Na^2(2(b+2sN)) - Na^2b - 2N^2a^2s}{2(b+2Ns)^2}$$
$$\Pi_N^* = \frac{2Na^2b + 4N^2a^2s - Na^2b - 2N^2a^2s}{2(b+2Ns)^2}$$
$$\Pi_N^* = \frac{Na^2b + 2N^2a^2s}{2(b+2Ns)^2}$$

$$\Pi_N^* = \frac{Na^2(b+2Ns)}{2(b+2Ns)^2}$$
$$\Pi_N^* = \frac{Na^2}{2(2Ns+b)}$$

A.3 Industry profits and quantity less buyer *i*'s contribution in equilibrium

This section is equivalent to simply inserting $N - k_i$ instead of N and employ the same method as above. For clarity, we also show the proof with $N - k_i$ instead below:

We subtract k_i from N in the industry profit function and solve for $x_{N-k_i}^*$ to obtain:

$$\Pi_{N-k_{i}}^{*} = ax_{N-k_{i}}^{*} - b\left(\frac{x_{N-k_{i}}^{*2}}{2(N-k_{i})}\right) - sx_{N-k_{i}}^{*2}$$

$$\frac{\partial \Pi_{N-k_{i}}^{*}}{\partial x_{N-k_{i}}^{*}} = a - \frac{bx_{N-k_{i}}^{*}}{N-k_{i}} - 2sx_{N-k_{i}}^{*}$$

$$\frac{\partial \Pi_{N-k_{i}}^{*}}{\partial x_{N-k_{i}}^{*}} = 0$$

$$a - \frac{bx_{N-k_{i}}^{*}}{N-k_{i}} - 2sx_{N-k_{i}}^{*} = 0$$

$$a = \frac{bx_{N-k_{i}}^{*}}{N-k_{i}} + \frac{2sx_{N-k_{i}}^{*}(N-k_{i})}{N-k_{i}}$$

$$a = \frac{bx+2sx_{N-k_{i}}^{*}(N-k_{i})}{N-k_{i}}$$

$$a(N-k_{i}) = bx_{N-k_{i}}^{*} + 2sx_{N-k_{i}}^{*}(N-k_{i})$$

$$a(N-k_{i}) = x_{N-k_{i}}^{*}(b+2s(N-k_{i}))$$

$$x_{N-k_{i}}^{*} = \frac{a(N-k_{i})}{b+2(N-k_{i})s}$$

$$x_{N-k_{i}}^{*} = \frac{aN-ak_{i}}{b+2Ns-2k_{i}s}$$

Inserting $x_{N-k_i}^*$ back into the industry profits yields:

$$\Pi_{N-k_i}^* = a \frac{aN - ak_i}{b + 2Ns - 2k_i s} - b \left(\frac{\left(\frac{aN - ak_i}{b + 2Ns - 2k_i s}\right)^2}{2(N - k_i)}\right) - s \left(\frac{aN - ak_i}{b + 2Ns - 2k_i s}\right)^2$$
$$\Pi_{N-k_i}^* = \frac{a(aN - ak_i)}{2Ns - 2k_i s + b} - \frac{b(aN - ak_i)^2}{2(2Ns - 2k_i s + b)^2(N - k_i)} - \frac{s(aN - ak_i)^2}{(2Ns - 2k_i s + b)^2}$$

Least common multiple is $2(2Ns - 2k_is + b)^2(N - k_i)$, which yields:

$$\Pi_{N-k_i}^* = \frac{a(aN-ak_i)2(N-k_i)(2Ns-2k_is+b)}{2(N-k_i)(2Ns-2k_is+b)^2} - \frac{b(aN-ak_i)^2}{2(N-k_i)(2Ns-2k_is+b)^2} - \frac{s(aN-ak_i)^22(N-k_i)}{2(N-k_i)(2Ns-2k_is+b)^2}$$
$$\Pi_{N-k_i}^* = \frac{a(aN-ak_i)2(N-k_i)(2Ns-2k_is+b)-b(aN-ak_i)^2-s(aN-ak_i)^22(N-k_i)}{2(N-k_i)(2Ns-2k_is+b)^2}$$

$$\begin{split} \Pi_{N-k_{i}}^{*} &= \frac{a(aN-ak_{i})2(N-k_{i})(2Ns-2k_{i}s+b)-b(aN-ak_{i})(aN-ak_{i})-s(aN-ak_{i})(aN-ak_{i})2(N-k_{i})}{2(N-k_{i})(2Ns-2k_{i}s+b)^{2}} \\ \Pi_{N-k_{i}}^{*} &= \frac{(aN-ak_{i})(2(aN-ak_{i})(2Ns-2k_{i}s+b)-b(aN-ak_{i})-2s(aN-ak_{i})(N-k_{i}))}{2(N-k_{i})(2Ns-2k_{i}s+b)^{2}} \\ \Pi_{N-k_{i}}^{*} &= \frac{(aN-ak_{i})(aN-ak_{i})(2(2Ns-2k_{i}s+b)-b-2s(N-k_{i}))}{2(N-k_{i})(2Ns-2k_{i}s+b)^{2}} \\ \Pi_{N-k_{i}}^{*} &= \frac{(aN-ak_{i})(aN-ak_{i})(4Ns-4k_{i}s+2b-b-2Ns+2k_{i}s)}{2(N-k_{i})(2Ns-2k_{i}s+b)^{2}} \\ \Pi_{N-k_{i}}^{*} &= \frac{a(aN-ak_{i})(N-k_{i})(2Ns-2k_{i}s+b)}{2(N-k_{i})(2Ns-2k_{i}s+b)^{2}} \\ \Pi_{N-k_{i}}^{*} &= \frac{a(aN-ak_{i})(N-k_{i})(2Ns-2k_{i}s+b)}{2(N-k_{i})(2Ns-2k_{i}s+b)^{2}} \\ \Pi_{N-k_{i}}^{*} &= \frac{a(aN-ak_{i})}{2(2Ns-2k_{i}s+b)} \\ \Pi_{N-k_{i}}^{*} &= \frac{a(aN-ak_{i})}{2(2Ns-2k_{i}s+b)} \\ \Pi_{N-k_{i}}^{*} &= \frac{a(aN-ak_{i})}{2(2Ns-2k_{i}s+b)} \\ \Pi_{N-k_{i}}^{*} &= \frac{a(aN-ak_{i})}{2(2Ns-2k_{i}s+b)} \\ \end{split}$$

A.4 Buyer *i*'s contribution to industry profits

We subtract industry profits less buyer i's contribution from the industry profits to obtain:

$$\begin{aligned} \Pi_N^* - \Pi_{N-k}^* &= \frac{Na^2}{2(2Ns+b)} - \frac{(N-k_i)a^2}{2(2(N-k_i)s+b)} \\ \Pi_N^* - \Pi_{N-k}^* &= \frac{Na^2 2(2(N-k_i)s+b) - (N-k_i)a^2 2(2Ns+b)}{2(2Ns+b) 2(2(N-k_i)s+b)} \\ \Pi_N^* - \Pi_{N-k}^* &= \frac{Na^2 (2(N-k_i)s+b) - (N-k_i)a^2 (2Ns+b)}{2(2Ns+b) (2(N-k_i)s+b)} \\ \Pi_N^* - \Pi_{N-k}^* &= \frac{2Na^2 s(N-k_i) + Na^2 b - Na^2 (2Ns+b) + a^2 k_i (2Ns+b)}{2(2Ns+b) (2(N-k_i)s+b)} \\ \Pi_N^* - \Pi_{N-k}^* &= \frac{2N^2 a^2 s - 2Na^2 s k_i + Na^2 b - 2N^2 a^2 s - Na^2 b + 2Na^2 s k_i + a^2 b k_i}{2(2Ns+b) (2Ns-2k_is+b)} \\ \Pi_N^* - \Pi_{N-k}^* &= \frac{a^2 b k_i}{2(2Ns+b) (2s(N-k_i)+b)} \end{aligned}$$

A.5 Curvature of buyer *i*'s contribution to industry profits

The curvature of the function can be shown by taking the second derivative:

$$\frac{\partial^2 (\Pi_N^* - \Pi_{N-k_i}^*)}{\partial^2 k_i} = \frac{\partial^2}{\partial^2 k_i} \left(\frac{a^2 b k_i}{2(2Ns+b)(2Ns-2k_is+b)} \right)$$
$$= \frac{\partial}{\partial k_i} \frac{a^2 b}{2(2Ns-2k_is+b)^2}$$
$$= \frac{a^2 b}{2} \frac{\partial}{\partial k_i} \frac{1}{(2Ns-2k_is+b)^2}$$
$$= \frac{a^2 b}{2} \frac{\partial}{\partial k_i} \left((2Ns-2k_is+b)^{-2} \right)$$
$$= \frac{a^2 b}{2} \frac{\partial}{\partial u} (u^{-2}) \frac{\partial}{\partial k_i} \left((2Ns-2k_is+b) \right), \text{ with } u = 2Ns - 2k_is + b$$

$$= \frac{a^2b}{2} (-2u^{-3}(-2s))$$
$$= \frac{a^2b}{2} \frac{-2(-2s)}{u^3}$$

Substituting back for u:

$$= \frac{a^2b}{2} \frac{4s}{(2Ns - 2k_i s + b)^3}$$
$$= \frac{2a^2bs}{(2Ns - 2k_i s + b)^3}$$

A.6 Input price paid by buyer i

We have that: $x_N^* = \frac{Na}{b+2sN}$ and $\tau_i = P(\frac{x_N^*}{N}) - \rho \frac{\prod_{k=1}^{N} - \prod_{k=i}^{N} \frac{N}{x_N^*}}{k_i x_N^*}$ Inserting x_N^* into τ_i yields: $\tau_i = P(\frac{x_N^*}{N}) - \rho \frac{\prod_{k=1}^{N} - \prod_{k=i}^{N} \frac{N}{b+2sN}}{k_i x_N^*}$ $\tau_i = P(\frac{x_N^*}{N}) - \rho \frac{\prod_{k=1}^{N} - \prod_{k=k=i}^{N} \frac{N(b+2sN)}{N}}{k_i x_N^*}$

$$\tau_i = P(\frac{x_N^*}{N}) - \rho \frac{\prod_{k=1}^N - \prod_{k=k_i}^N b + 2sN}{k_i}$$

Inserting $P(\frac{x_N^*}{N}) = a - b(\frac{x_N^*}{2N})$ yields:

$$\begin{split} \tau_i &= a - b(\frac{x_N^*}{2N}) - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* N}{k_i} \frac{N}{x_N^*} \\ \tau_i &= a - b(\frac{x_N^*}{2N}) - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* \frac{b+2sN}{a}}{k_i} \\ \tau_i &= a - b(\frac{Na}{2N}) - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* \frac{b+2sN}{a}}{k_i} \\ \tau_i &= a - b(\frac{Na}{2N(b+2sN)}) - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* \frac{b+2sN}{a}}{k_i} \\ \tau_i &= a - \frac{ab}{2b+4sN} - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* \frac{b+2sN}{a}}{k_i} \\ \tau_i &= \frac{a(2b+4sN)-ab}{2b+4sN} - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* \frac{b+2sN}{a}}{k_i} \\ \tau_i &= \frac{2ab+4asN-ab}{2b+4sN} - \rho \frac{\Pi_N^* - \Pi_{N-k_i}^* \frac{b+2sN}{a}}{k_i} \end{split}$$

$$\tau_{i} = \frac{ab + 4asN}{2b + 4sN} - \rho \frac{\prod_{N}^{*} - \prod_{N-k_{i}}^{*} \frac{b + 2sN}{a}}{k_{i}}$$
$$\tau_{i} = \frac{a(b + 4sN)}{2b + 4sN} - \rho \frac{\prod_{N}^{*} - \prod_{N-k_{i}}^{*} \frac{b + 2sN}{a}}{k_{i}}$$

Inserting $\Pi_N^* - \Pi_{N-k_i}^*$ yields:

$$\tau_{i} = \frac{a(b+4sN)}{2b+4sN} - \rho \frac{\frac{a^{2}bk_{i}}{2(2Ns+b)(2Ns-2k_{i}s+b)}}{k_{i}} \frac{b+2sN}{a}}{a}$$
$$\tau_{i} = \frac{a(b+4sN)}{2b+4sN} - \rho \frac{\frac{abk_{i}}{2(2Ns-2k_{i}s+b)}}{k_{i}}$$

$$\tau_i = \frac{a(b+4sN)}{2b+4sN} - \rho \frac{ab}{2(2Ns-2k_is+b)}$$

A.7 Properties of τ_i when the number of retail stores is exogenous

The change in input price for a one unit change in k_i is shown by the derivative:

$$\frac{\partial \tau_i}{\partial k_i} = \frac{\partial}{\partial k_i} \left(\frac{a(b+4sN)}{2b+4sN} - \rho \frac{ab}{2(2Ns-2k_is+b)} \right)$$
$$= -\rho \frac{ab}{2} \frac{\partial}{\partial k_i} \left(\frac{1}{2Ns-2k_is+b} \right)$$
$$= -\rho \frac{ab}{2} \frac{\partial}{\partial k_i} \left((2Ns - 2k_is + b)^{-1} \right)$$
$$= -\rho \frac{ab}{2} \left(-\frac{1}{(2Ns-2k_is+b)^2} (-2s) \right)$$
$$= -\rho \frac{abs}{(2Ns-2k_is+b)^2} < 0$$

The change in input price for a one unit change in N is shown by the derivative:

$$\begin{aligned} &\frac{\partial \tau_i}{\partial N} = \frac{\partial}{\partial N} \left(\frac{a(b+4sN)}{2b+4sN} - \rho \frac{ab}{2(2Ns-2k_is+b)} \right) \\ &= a \frac{\partial}{\partial N} \frac{(b+4Ns)(2b+4Ns) - \frac{\partial}{\partial N}(2b+4Ns)(b+4Ns)}{(2b+4Ns)^2} - \rho \frac{ab}{2} \frac{1}{(2Ns-2k_is+b)^2} \frac{\partial}{\partial N} (2Ns-2k_is+b) \\ &= a \frac{4s(2b+4Ns) - 4s(b+4Ns)}{(2b+4Ns)^2} - \rho \frac{ab}{2} \left(-\frac{1}{(2Ns-2k_is+b)^2} 2s \right) \\ &= \frac{4abs}{(2b+4Ns)^2} + \rho \frac{abs}{(2Ns-2k_is+b)^2} > 0 \end{aligned}$$

A.8 Amount of retail stores that minimizes profits

The buyer's contribution to industry profits when owning one additional store is:

$$\frac{\partial \rho(\Pi_N^* - \Pi_{N-k_i}^*)}{\partial k_i} = \frac{\partial}{\partial k_i} \left(\frac{\rho a^2 b k_i}{2(2Ns+b)(2Ns-2k_is+b)} \right)$$
$$\frac{\partial \rho(\Pi_N^* - \Pi_{N-k_i}^*)}{\partial k_i} = \frac{\rho a^2 b}{2(2Ns+b)} \frac{\partial}{\partial k_i} \frac{k_i}{2Ns-2k_is+b}$$
$$\frac{\partial \rho(\Pi_N^* - \Pi_{N-k_i}^*)}{\partial k_i} = \frac{\rho a^2 b}{2(2Ns+b)} \frac{2Ns-2k_is+b-k_i(-2s)}{(2Ns-2k_is+b)^2}$$
$$\frac{\partial \rho(\Pi_N^* - \Pi_{N-k_i}^*)}{\partial k_i} = \frac{\rho a^2 b}{2(2Ns+b)} \frac{2Ns+b}{(2Ns-2k_is+b)^2}$$
$$\frac{\partial \rho(\Pi_N^* - \Pi_{N-k_i}^*)}{\partial k_i} = \frac{\rho a^2 b}{2(2Ns-2k_is+b)^2}$$

The optimality condition is:

$$\frac{\partial (\rho(\Pi_N^* - \Pi_{N-k}^*) - Fk_i)}{\partial k_i} = 0$$

$$\frac{\rho a^2 b}{2(2Ns - 2k_i s + b)^2} - F = 0$$

$$\rho a^2 b = 2F(2Ns - 2k_i s + b)^2$$

$$\frac{\rho a^2 b}{2F} = (2Ns - 2k_i s + b)^2$$

$$\pm \sqrt{\frac{\rho a^2 b}{2F}} = 2Ns - 2k_i s + b$$

$$-2k_i s = \pm \sqrt{\frac{\rho a^2 b}{2F}} - 2Ns - b$$

$$k_i = -\frac{\pm \sqrt{\frac{\rho a^2 b}{2F}} - 2Ns - b}{2s}$$

$$\tilde{k}_i = N - \frac{\pm \sqrt{\frac{\rho a^2 b}{2F}} - 2Ns - b}{2s}$$

It is only economically meaningful to consider the case where $k_i \leq N$, as you cannot own more stores than there are in total. For this to happen, we need to place a few constraints on the equation: 1) The first term has to be with a negative sign, which means only $-\frac{+\sqrt{\frac{\rho a^2 b}{2F}}}{2s}$ is a valid solution. 2) Further, we have to impose that $|-\frac{\sqrt{\frac{\rho a^2 b}{2F}}}{2s}|\geq \frac{b}{2s}$ This implies:

$$\frac{\sqrt{\frac{\rho a^2 b}{2F}}}{2s} \ge \frac{b}{2s}$$

$$\sqrt{\frac{\rho a^2 b}{2F}} \ge b$$

$$\frac{\rho a^2 b}{2F} \ge b^2$$

$$2Fb^2 \le \rho a^2 b$$

$$F \le \frac{\rho a^2 b}{2b^2}$$

$$F \le \frac{\rho a^2}{2b}$$

B Case 2 - Total amount of retail stores is endogenous

B.1 Properties of $\Pi_{Z+k_i}^* - \Pi_Z^*$

The first derivative is:

 $\frac{\partial (\Pi^*_{Z+k_i} - \Pi^*_Z)}{\partial k_i} = \frac{\partial}{\partial k_i} \frac{a^2 b k_i}{2(2(Z+k_i)s+b)(2Zs+b)}$

$$= \frac{a^2b}{2(2Z+b)} \frac{\partial}{\partial k_i} \frac{k_i}{2(Z+k_i)s+b}$$
$$= \frac{a^2b}{2(2Z+b)} \frac{2(Z+k_i)s+b-2sk_i}{(2(Z+k_i)s+b)^2}$$

 $= \frac{a^2b}{2(2s(Z+k_i)+b)^2} > 0$

The second derivative is:

$$\frac{\partial^{2}(\Pi_{Z+k_{i}}^{*}-\Pi_{Z}^{*})}{\partial^{2}k_{i}} = \frac{\partial}{\partial k_{i}} \frac{a^{2}b}{2(2s(Z+k_{i})+b)^{2}}$$

$$= \frac{a^{2}b}{2} \frac{\partial}{\partial k_{i}} \frac{1}{(2s(Z+k_{i})+b)^{2}}$$

$$= \frac{a^{2}b}{2} \frac{\partial}{\partial u} \left(\frac{1}{u^{2}}\right) \frac{\partial}{\partial k_{i}} \left(\left(2s(Z+k_{i})+b\right)\right) \text{ with } u = 2s(Z+k_{i})+b$$

$$= \frac{a^{2}b}{2} \left(\frac{-2}{u^{3}}\right) 2s$$

$$= -\frac{2a^{2}bs}{u^{3}}$$

$$= -\frac{2a^{2}bs}{(2s(Z+k_{i})+b)^{3}} < 0$$

B.2 Input price paid by buyer i

$$\begin{aligned} \tau_i &= P(\frac{x_{Z+k_i}^*}{Z+k_i}) - \rho \frac{\Pi_{Z+k_i}^* - \Pi_Z^*}{\tilde{x}_i} \\ \tau_i &= P(\frac{x_{Z+k_i}^*}{Z+k_i}) - \rho \frac{\Pi_{Z+k_i}^* - \Pi_Z^*}{k_i} \frac{Z+k_i}{x_{Z+k_i}^*} \\ \tau_i &= \frac{a(b+4s(Z+k_i))}{2b+4s(Z+k_i)} - \rho \frac{\Pi_{Z+k_i}^* - \Pi_Z^*}{k_i} \frac{b+2s(Z+k_i)}{a}, \text{ Proof: See A.6 in the Appendix} \end{aligned}$$

Inserting buyer i's contribution to industry profits yields:

$$\begin{aligned} \tau_i &= \frac{a(b+4s(Z+k_i))}{2b+4s(Z+k_i)} - \rho \frac{\frac{a^2bk_i}{2(2(Z+k_i)s+b)(2Zs+b)}}{k_i} \frac{b+2s(Z+k_i)}{a} \\ \tau_i &= \frac{a(b+4s(Z+k_i))}{2b+4s(Z+k_i)} - \rho \frac{ab}{2(2(Z+k_i)s-2k_is+b)} \\ \tau_i &= \frac{a(b+4s(Z+k_i))}{2b+4s(Z+k_i)} - \rho \frac{ab}{2(2Zs+b)} \end{aligned}$$

B.3 Properties of τ_i when the number of retail stores is endogenous

The change in price for a one unit change in k_i is given by:

$$\frac{\partial \tau_i}{\partial k_i} = \frac{\partial}{\partial k_i} \left(\frac{a(b+4s(Z+k_i))}{2b+4s(Z+k_i)} - \rho \frac{ab}{2(2Zs+b)} \right)$$
$$= a \frac{\frac{\partial}{\partial k_i} (b+4s(Z+k_i))(2b+4s(Z+k_i)) - \frac{\partial}{\partial k_i} (2b+4s(Z+k_i))(b+4s(Z+k_i))}{(2b+4s(Z+k_i))^2}$$

$$= a \frac{4s(2b+4s(Z+k_i))-4s(b+4s(Z+k_i))}{(2b+4s(Z+k_i))^2}$$

 $= \frac{4abs}{(2b+4s(Z+k_i))^2} > 0$

The change in price for a one unit change in Z is given by:

$$\begin{aligned} &\frac{\partial \tau_i}{\partial Z} = \frac{\partial}{\partial Z} \Big(\frac{a(b+4s(Z+k_i))}{2b+4s(Z+k_i)} - \rho \frac{ab}{2(2Zs+b)} \Big) \\ &= a \frac{\partial}{\partial Z} (b+4s(Z+k_i))(2b+4s(Z+k_i)) - \frac{\partial}{\partial Z}(2b+4s(Z+k_i))(b+4s(Z+k_i))}{(2b+4s(Z+k_i))^2} - \rho \frac{ab}{2} \frac{\partial}{\partial Z} \Big(\frac{1}{2Zs+b} \Big) \\ &= a \frac{4s(2b+4s(Z+k_i)) - 4s(b+4s(Z+k_i))}{(2b+4s(Z+k_i))^2} - \rho \frac{ab}{2} \Big(-\frac{1}{(2Zs+b)^2} 2s \Big) \\ &= \frac{4abs}{(2b+4s(Z+k_i))^2} + \rho \frac{abs}{(2Zs+b)^2} > 0 \end{aligned}$$

B.4 Marginal contribution to industry profits

The change in buyer *i*'s profits for a one unit change in k_i is given by:

$$\begin{aligned} \frac{\partial \rho(\Pi_{Z+k_{i}}^{*}-\Pi_{Z}^{*})}{\partial k_{i}} &= \frac{\partial}{\partial k_{i}} \left(\frac{\rho a^{2} b k_{i}}{2(2(Z+k_{i})s+b)(2Zs+b)}\right) \\ &= \rho \frac{a^{2} b}{2(2Zs+b)} \frac{\partial}{\partial k_{i}} \left(\frac{k_{i}}{2(Z+k_{i})s+b}\right) \\ &= \rho \frac{a^{2} b}{2(2Zs+b)} \frac{\frac{\partial}{\partial k_{i}} (k_{i})(2(Z+k_{i})s+b) - \frac{\partial}{\partial k_{i}} (2(Z+k_{i})s+b)k_{i}}{(2(Z+k_{i})s+b)^{2}} \\ &= \rho \frac{a^{2} b}{2(2Zs+b)} \frac{2(Z+k_{i})s+b - 2sk_{i}}{2(Z+k_{i})s+b)^{2}} \\ &= \rho \frac{a^{2} b}{2(b+2Zs)} \frac{b+2Zs}{(b+2s(Z+k_{i}))^{2}} \\ &= \rho \frac{a^{2} b(2Zs+b)}{2(2Zs+b)(2(Z+k_{i})s+b)^{2}} \\ &= \rho \frac{a^{2} b(2Zs+b)}{2(2Zs+b)(2(Z+k_{i})s+b)^{2}} \\ &= \rho \frac{a^{2} b}{2(2(Z+k_{i})s+b)^{2}} > 0 \end{aligned}$$

B.5 Profit maximizing amount of retail stores k_i^*

The investment condition is given by:

$$\frac{\partial}{\partial k_i} \rho(\Pi_{Z+k_i} - \Pi_Z) - C'_k(k_i^{new}) = 0$$

$$\frac{\rho a^2 b}{2(2s(Z+k_i)+b)^2} - F = 0$$

$$\rho a^2 b = 2F(2s(Z+k_i)+b)^2$$

$$\frac{\rho a^2 b}{2F} = (2s(Z+k_i)+b)^2$$

$$\pm \sqrt{\frac{\rho a^2 b}{2F}} = 2Zs + 2sk_i + b$$

$$k_i^* = \frac{\pm \sqrt{\frac{\rho a^2 b}{2F}} - b}{2s} - Z$$

For $k_i^* \ge 0$, the only valid solution implies:

1)
$$|\pm\sqrt{\frac{\rho a^2 b}{2F}}| -b \ge 0 \rightarrow \sqrt{\frac{\rho a^2 b}{2F}} \ge b$$

2) $Z \le \frac{\sqrt{\frac{\rho a^2 b}{2F}} -b}{2s}$

These conditions imply that:

$$2Zs \le \sqrt{\frac{\rho a^2 b}{2F}} - b$$
$$(2Zs + b)^2 \le \frac{\rho a^2 b}{2F}$$
$$2F(2Zs + b)^2 \le \rho a^2 b$$
$$F \le \frac{\rho a^2 b}{2(2Zs + b)^2}$$

C Case 3 - Endogenous amount of retail stores where all buyers can invest

C.1 Criteria for k^* to be a non-negative number:

When all buyers can invest, assuming symmetric production functions, they choose k^* in equilibrium:

$$k^* = \frac{\sqrt{\frac{\rho a^2 b}{2F}} - b}{2s} - k^* (z - 1)$$
$$k^* + k^* z - k^* = \frac{\sqrt{\frac{\rho a^2 b}{2F}} - b}{2s}$$
$$k^* = \frac{\sqrt{\frac{\rho a^2 b}{2F}} - b}{2sz}$$

In order to have $k^* > 0$, the numerator has to be positive. For this to happen, we need the following to hold:

$$\sqrt{\frac{\rho a^2 b}{2F}} \ge b$$
$$\frac{\rho a^2 b}{2F} \ge b^2$$
$$\frac{\rho a^2}{2F} \ge b$$
$$2Fb \le \rho a^2$$
$$F \le \frac{\rho a^2}{2b}$$

D Case 4 - Price discrimination

D.1 Price increase under a ban on price discrimination

The price increase for a one unit change in k_i captures the change in price going from $\hat{\tau}$ to τ^* , which is given by:

$$\begin{split} &\frac{\partial}{\partial k_i} \left(\frac{a(b+4s(k^*+\frac{k_1-k^*}{2})z)}{2b+4s(k^*+\frac{k_1-k^*}{2})z)} - \rho \frac{ab}{2(2(k^*+\frac{k_1-k^*}{2})(z-1)s+b)}\right) \\ &= \frac{\partial}{\partial k_i} \left(\frac{a(b+4k^*sz+4k_is-4k^*s)}{2b+4k^*sz+4k_is-4k^*s} - \rho \frac{ab}{2(2s(-2k^*-\frac{k_i-k^*}{2}+k_i+k^*z)+b)}\right) \\ &= \frac{\partial}{\partial k_i} \left(\frac{a(b+4k^*sz+4k_is-4k^*s)}{2b+4k^*sz+4k_is-4k^*s} - \rho \frac{ab}{2(-4k^*sz-2s(k_i-k^*)+2sk_iz+2k^*sz^2+bz)}\right) \\ &= \frac{\partial}{\partial k_i} \left(\frac{a(b+4k^*sz+4k_is-4k^*s)}{2b+4k^*sz+4k_is-4k^*s} - \rho \frac{ab}{2(-4k^*sz-2s(k_i-k^*)+2sk_iz+2k^*sz^2+bz)}\right) \\ &= \frac{\partial}{\partial k_i} \left(\frac{a(b+4k^*sz+4k_is-4k^*s)}{2b+4k^*sz+4k_is-4k^*s} - \rho \frac{ab}{2(-4k^*sz-2s(k_i-k^*)+2sk_iz+2k^*sz^2+bz)}\right) \\ &= a \frac{\frac{\partial}{\partial k_i}(b+4k^*sz+4k_is-4k^*s)(2b+4k^*sz+4k_i-4k^*s) - \frac{\partial}{\partial k_i}(2b+4k^*sz+4k_is-4k^*s)}{(2b+4k^*sz+4k_is-4k^*s)^2} - \rho \frac{abz}{2k_i} \left(\frac{2b+4k^*sz}{2} - \frac{ab}{2k_i}((-4k^*sz-2s(k_i-k^*)+2sk_iz+2k^*sz^2+bz))\right) \\ &= a \frac{4s(2b+4k^*sz+4k_is-4k^*s)-4s(b+4k^*sz+4k_is-4k^*s)}{(2b+4k^*sz+4k_is-4k^*s)^2} - \rho \frac{abz}{2k_i} \frac{\partial}{\partial k_i} \left((-4k^*sz-2s(k_i-k^*)+2sk_iz+2k^*sz^2+bz)\right) \\ &= \frac{4sbs}{(2b+4k^*sz+4sk_i-4k^*s)^2} \\ &= \frac{4sbs}{(2b+4k^*sz+4sk_i-4k^*s)^2} + \rho \frac{abz}{(-4k^*sz-2s(k_i-k^*)+2sk_isz+2k^*sz^2+bz)^2} \\ &= \frac{4abs}{(2b+4k^*sz+4sk_i-4k^*s)^2} + \rho \frac{abz}{(-4k^*sz-2s(k_i-k^*)+2sk_iz+2k^*sz^2+bz)^2} \\ &= \frac{abs}{(2b+4k^*sz+4sk_i-4k^*s)^2} + \rho \frac{abz}{(-4k^*sz$$