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Testing, Disclosure and Approval[*]

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Abstract

Certifiers often base their decisions on a mixture of information, some of which is voluntarily disclosed by applicants, and some of which they acquire by way of tests or otherwise. We study the interplay between the information acquisition of certifiers and the information disclosure of applicants. We show that the inability of a certifier to commit to the amount of information to be acquired can result in a reduction of information disclosed. Among other consequences, given the choice between two information acquisition technologies, the certifier may prefer to commit to the inferior technology, in the sense of being either more expensive or less accurate.

JEL classification: C72, D82, D83, G24, G28
Keywords: Information Acquisition; Testing; Disclosure; Certification

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1 Introduction

In many markets, certifiers act as a screening device by approving or rejecting applicants based on information about their quality.[1] In order to make the right decision, certifiers often rely on a mixture of information. Part of this information is disclosed strategically by applicants trying to maximize their chances of being approved. Another part is acquired firsthand by the certifiers, through tests, interviews, or by contacting references. The aim of this paper is to examine the interplay between these information sources.

To fix ideas, consider a college student selecting a set of courses for his final year. The courses vary in their correlation between grades and talent, some courses generating more informative grades than others. The student's goal is to be accepted at a selective program after graduation. The admissions committee acts as a certifier. Based on transcripts, outstanding candidates are accepted outright, and unfit applicants immediately discarded. Candidates between these two extremes are subject to the most intense scrutiny; for them, the committee accesses additional (costly) information, perhaps by conducting an interview, requesting an aptitude test, or calling a reference. This process is anticipated by the student and influences his choice of courses.

In this paper, we ask the following questions: how is an applicant's information disclosure affected by the certifier's information acquisition possibilities? What implications does this have regarding social welfare and the quality of certification? To address these questions, we propose a simple model of the interaction between a certifier (she) and an applicant (he). The applicant is either good or bad; both he and the certifier are uncertain about his quality. To obtain a positive payoff the applicant requires the certifier's approval. The certifier would like to approve the applicant if he is good, but to reject him if he is bad. The applicant may voluntarily disclose information to the certifier by designing a signal of his quality.[2] The certifier, for her part, may choose to acquire additional information by testing the applicant at a cost.

We show that the inability of the certifier to commit to the amount of information to be collected (via the test) can result in a reduction of information disclosed by the applicant. The mechanism is as follows. The applicant is faced with two broad options. He can disclose sufficiently convincing evidence that he is good in order to avoid the test and secure approval

[1] Applicants may for instance be firms who need a certification in order to sell their products, workers who must be certified to carry out a certain task, or potential students seeking admission to a study program.

[2] As in Rayo and Segal (2010) and Kamenica and Gentzkow (2011).

outright (option 1), but then he risks revealing that he is bad. Or he can withhold information and let the certifier conduct the test (option 2). Commitment enables the certifier to steer the applicant's choice towards option 1. This, in turn, allows the certifier to improve certification quality all the while lowering testing expenditures.[3]

We first show that the certifier *always* benefits from the ability to commit to whether or not to conduct the test. We then show that the aforementioned mechanism leads to situations in which, given the choice between two tests, the certifier might wish to commit to the inferior test, in the sense of being either more expensive or less accurate. In a similar way, allowing the certifier to fine-tune the toughness of the test conducted can hurt the certifier. Intuitively, the better the test, the smaller the applicant's chances of securing approval outright (i.e. without being tested first). This, in turn, may lead the applicant to prefer withholding information.

The related literature is discussed in the next paragraphs. The baseline model is laid out in Section 2, analyzed in Section 3, and extended to allow for more flexible tests in Section 4. Section 5 concludes.

Related Literature. The bulk of the literature studying certification assumes that the certifier knows a priori the quality of the applicant (see, e.g., Lizzeri (1999), Strausz (2005), and Stahl and Strausz (2017)). Our paper belongs to a small but growing literature exploring the optimal ways of organizing certification processes when certifiers need to *learn* the quality of applicants. In a recent study, Henry and Ottaviani (2019) examine how control rights should be split between the applicant and the certifier. In contrast to the present paper, in their (dynamic) setting information collected by the applicant is the certifier's only source of information. Quigley and Walther (2019) examine the impact of public tests on the applicant's information disclosure strategy. Tests are exogenous in their setting, whereas in ours the certifier's decision to conduct the test is endogenous. Our finding that the certifier could be better off committing to relatively inaccurate tests is reminiscent of Perez-Richet and Skreta (2018), but the mechanisms are different: in their setting, the applicant can manipulate the test.

Our paper also contributes to the literature on Bayesian persuasion initiated by Kamenica and Gentzkow (2011). With a few exceptions, the literature assumes that the sender (the applicant in our model) is the only source of information available to the receiver. Notable exceptions include Kolotilin (2018) (where the receiver is privately informed) as well as Gentzkow

[3]Certification quality refers to the probability of making the (ex post) socially optimal certification decision, that is, approve a good applicant and reject a bad applicant.

and Kamenica (2017) and Li and Norman (2018, 2019) (where multiple senders disclose information to the receiver). We extend the standard Bayesian framework by adding information acquisition on the part of the receiver.[4]

There exists a well-established literature that analyzes the crowding-out effect of disclosure on information acquisition in financial markets. This literature argues that when firms disclose more precise information about their operations this motivates traders to cut back on their costly private information acquisition activities, either at the intensive margin, as in Verrecchia (1982), at the extensive margin, as in Diamond (1985), or both, as in Goldstein and Yang (2017).[5] On the surface, the findings of that literature are related to some of our results, however the underlying mechanism is very different. In the finance literature better public information limits the gains from informed trading (and through this channel decreases incentives to collect private information). By contrast, in our model, the applicant responds to the certifier's access to more efficient tests by disclosing less.

2 Baseline Model

The broad features of the model are as follows. An applicant (he) seeks approval from a certifier (she). The quality of the applicant is uncertain. The certifier would like to approve the applicant if he is good but to reject him if he is bad. The applicant may voluntarily disclose information to the certifier, who may for her part incur a cost in order to test the applicant.

Disclosure. Let $\omega \in \{G, B\}$ represent the unknown quality of the applicant. The common prior belief that $\omega = G$ is denoted ρ, where, for expositional simplicity, $\rho \in (0, \frac{1}{2})$.[6] The applicant can disclose information by choosing a signal π comprising a pair of conditional probability distributions $\pi(\cdot|\omega = G)$ and $\pi(\cdot|\omega = B)$ over the set of outcomes $\{s_1, s_2\}$.[7] We let s denote the random outcome of the signal π, and μ indicate the Bayes-updated belief that

[4]Our paper asks, among other things, whether the receiver benefits from having access to more information than that revealed by the sender. Others have considered whether the sender benefits from having access to information before choosing a signal (Hedlund, 2017; Alonso and Câmara, 2018; Degan and Li, 2019).

[5]Allen, Morris and Shin (2006) and Cespa and Vives (2015) examine traders who have short horizons and so care not only about fundamental asset values, but also about other traders' willingness-to-pay for the assets. A beauty contest thus emerges, in which traders put too much weight on public information, and this is exacerbated when the public information is more precise.

[6]The case $\rho \geq 1/2$ is analyzed in a previous version of this paper, see Bizzotto, Rüdiger and Vigier (2019).

[7]All our results are the same if there are more than two signal outcomes.

$\omega = G$ after observing both π and the realization of s. We refer to μ as the interim belief. We will say that the applicant splits ρ on μ_1 and μ_2 if $\mu = \mu_1$ following $s = s_1$ and $\mu = \mu_2$ following $s = s_2$. Throughout, we use the convention $\pi(s_1|B)\pi(s_2|G) \geq \pi(s_2|B)\pi(s_1|G)$, thus $\mu_1 \leq \rho \leq \mu_2$.

Testing. The certifier can perform a binary test of the applicant's quality, at a cost $c > 0$. The expertise $e \in (\frac{1}{2}, 1)$ determines the rate of false positives/negatives generated by the test: if $\omega = G$ the applicant passes the test with probability e and fails the test with probability $1 - e$; if $\omega = B$ he passes the test with probability $1 - e$ and fails the test with probability e. Regardless of π, the signal outcome and the test result are assumed uncorrelated conditional on ω. We refer to the vector (e, c) as the test technology, and say that (e_b, c_b) is more efficient than (e_a, c_a) if $e_b \geq e_a$ and $c_b \leq c_a$, at least one of which holds strictly. To make the analysis interesting we assume $e - c > \frac{1}{2}$.[8]

Timing. First, the applicant selects his signal π. Both π and the realized signal outcome are observed by the certifier, who then decides whether or not to perform the test. Finally, the certifier chooses between approval and rejection (after observing the test result, in case the test was performed).

Payoffs. The applicant's payoff is 1 in case of approval and 0 in case of rejection. The certifier's payoff is $\mathbf{1}_{\text{approval}}\big(\mathbf{1}_{\{\omega=G\}} - \mathbf{1}_{\{\omega=B\}}\big) - c\mathbf{1}_{\text{test}}$, where $\mathbf{1}_X$ denotes the indicator function of X; that is, the certifier pays c for conducting the test and, in case of approval, gets 1 when $\omega = G$ and -1 when $\omega = B$. The ex ante expectation of the certifier's payoff will be denoted W. For concreteness, we refer to W as social welfare; the idea being that the certifier acts in society's best interest.

Certification Quality. We define certification quality, denoted Q, as the ex ante probability that the certifier makes the (ex post) socially optimal certification decision, that is, approve if $\omega = G$ and reject if $\omega = B$. Straightforward algebra gives[9]

$$W + (1 - \rho) = Q - c\mathbb{P}(\text{test}),$$

[8] Otherwise the applicant is neither tested on the equilibrium path nor off it.

[9] Write $\mathbb{E}\big[\mathbf{1}_{\text{approval}}\big(\mathbf{1}_{\{\omega=G\}} - \mathbf{1}_{\{\omega=B\}}\big)\big] = \mathbb{E}\big[\mathbf{1}_{\text{approval}}\mathbf{1}_{\{\omega=G\}} + \big(1 - \mathbf{1}_{\text{approval}}\big)\mathbf{1}_{\{\omega=B\}} - \mathbf{1}_{\{\omega=B\}}\big] = Q - (1 - \rho)$.

that is, up to a constant term, social welfare is obtained by subtracting expected testing expenditures from certification quality. As social welfare and certification quality are of independent interest, our main results will be stated in terms of both.

Strategies and Equilibrium. A strategy of the applicant consists of a signal π. A strategy of the certifier specifies (i) whether or not to conduct the test as a function of π and the realized signal outcome, and (ii) whether to approve or reject the applicant as a function of the certifier's information at that stage. The equilibrium concept is perfect Bayesian equilibrium (henceforth referred to as equilibrium for short): that is, beliefs are updated using Bayes' rule, and each player chooses actions so as to maximize his or her expected payoff given the other player's strategy and his or her beliefs about ω at the corresponding stage of the game.

3 Analysis

In this section, we first characterize the equilibrium strategies. We then discuss the certifier's commitment problem and examine its implications for the certification process.

3.1 Equilibrium Characterization

We first examine the problem of the certifier. Conducting the test enables the certifier to make the (ex post) socially optimal certification decision with probability e at a cost c. Hence, in any equilibrium, the certifier (a) rejects without testing if $\mu < 1-e+c$, (b) conducts the test if $\mu \in (1-e+c, e-c)$, and (c) approves without testing if $\mu > e-c$. Intuitively, the certifier conducts the test if and only if she is sufficiently uncertain about the quality of the applicant.

Next, let $f(\mu)$ denote the applicant's equilibrium interim expected payoff. By virtue of the remarks in the previous paragraph,

$$f(\mu) = \begin{cases} 0 & \text{for } \mu \in [0, 1-e+c); \\ \mu e + (1-\mu)(1-e) & \text{for } \mu \in (1-e+c, e-c); \\ 1 & \text{for } \mu \in (e-c, 1]. \end{cases}$$

Concavifying $f(\cdot)$ (Aumann, Maschler and Stearns, 1995; Kamenica and Gentzkow, 2011) yields the following proposition.

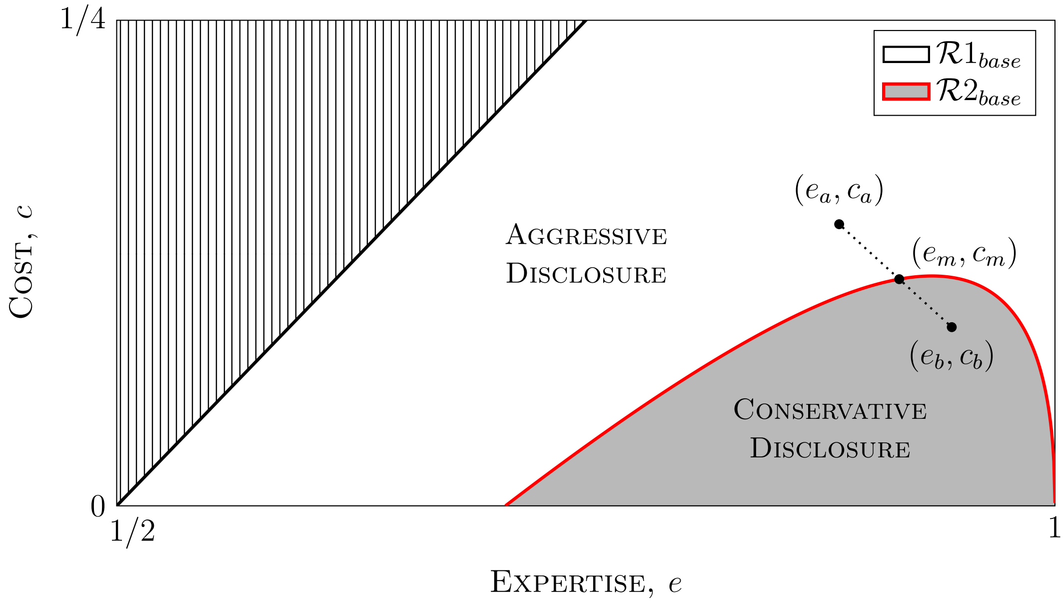

Figure 1: Test Technology and Equilibrium Disclosure

Lemma 1. *If*

$$\frac{1-e+c}{e-c} > (1-e+c)e + (e-c)(1-e), \tag{1}$$

then in equilibrium the applicant splits ρ *on* 0 *and* $e-c$*; in this case testing never occurs. If the inequality in* (1) *is reversed then in equilibrium the applicant splits* ρ *on*

$$\begin{cases} 0 \text{ and } 1-e+c & \text{for } \rho \in (0, 1-e+c); \\ 1-e+c \text{ and } e-c & \text{for } \rho \in (1-e+c, \frac{1}{2}); \end{cases}$$

while for $\rho = 1-e+c$ *the applicant discloses no information. Testing then occurs with positive probability.*

Lemma 1 identifies two parametric regions: the applicant's equilibrium disclosure strategy in one region differs qualitatively from his disclosure strategy in the other. Henceforth, we refer to the applicant's equilibrium disclosure as aggressive in case (1) holds, and as conservative whenever the reverse inequality holds.[10] Figure 1 depicts the parametric regions $\mathcal{R}1_{base}$, in which the applicant's equilibrium disclosure is aggressive, and $\mathcal{R}2_{base}$, in which the applicant's equilibrium disclosure is conservative, with e on the horizontal axis and c on the vertical axis.

[10]To streamline the exposition, we ignore the knife-edge case in which the two sides of (1) are equal.

3.2 The Certifier's Commitment Problem

In this subsection we first analyze a simple commitment benchmark. We then compare the equilibrium of the baseline model to the commitment benchmark.

Consider as a benchmark the hypothetical case in which the certifier is able to commit at the onset to a testing rule as a function of the information disclosed by the applicant via the signal π; formally, a testing rule is a mapping $T : [0, 1] \to \{\text{test, no test}\}$. For example, the certifier might commit to conduct the test if and only if the interim belief $\mu \in [\frac{1}{2}, e)$. Let T^* denote this particular testing rule, and $f_{T^*}(\mu)$ denote the applicant's interim expected payoff resulting from this testing rule. Given T^*, the certifier rejects at $\mu < \frac{1}{2}$ and approves at $\mu \geq e$. For $\mu \in [\frac{1}{2}, e)$ the certifier first conducts the test. Notice that the posterior belief is then strictly greater than $\frac{1}{2}$ after passing the test but strictly less than $\frac{1}{2}$ after failing the test. So the certifier approves if and only if the applicant passes the test (this occurs with probability $\mu e + (1 - \mu)(1 - e)$). We thus obtain

$$f_{T^*}(\mu) = \begin{cases} 0 & \text{for } \mu \in [0, \frac{1}{2}); \\ \mu e + (1 - \mu)(1 - e) & \text{for } \mu \in [\frac{1}{2}, e); \\ 1 & \text{for } \mu \in [e, 1]. \end{cases}$$

The applicant's optimal disclosure (given the testing rule T^*) is now readily obtained by concavifying $f_{T^*}(\cdot)$ (see Figure 2). We conclude that, given T^*, (a) the applicant splits ρ on 0 and e and (b) testing never occurs. As it turns out, the certifier can do no better.[11]

Lemma 2. *Any certifier-optimal testing rule is such that (a) the applicant splits ρ on* 0 *and* e *and (b) testing never occurs.*

Comparing the baseline model (Lemma 1) to the commitment benchmark (Lemma 2) illustrates the certifier's commitment problem, as summarized in the next proposition. Henceforth, say that the applicant discloses more (respectively less) information if the signal chosen is more (resp. less) informative in the sense of Blackwell.

Proposition 1. *Relative to the commitment benchmark, the baseline model (a) reduces the amount of information disclosed by the applicant in equilibrium, and (b) lowers social welfare and certification quality.*

[11] We say that a testing rule is certifier optimal if it maximizes social welfare over all testing rules.

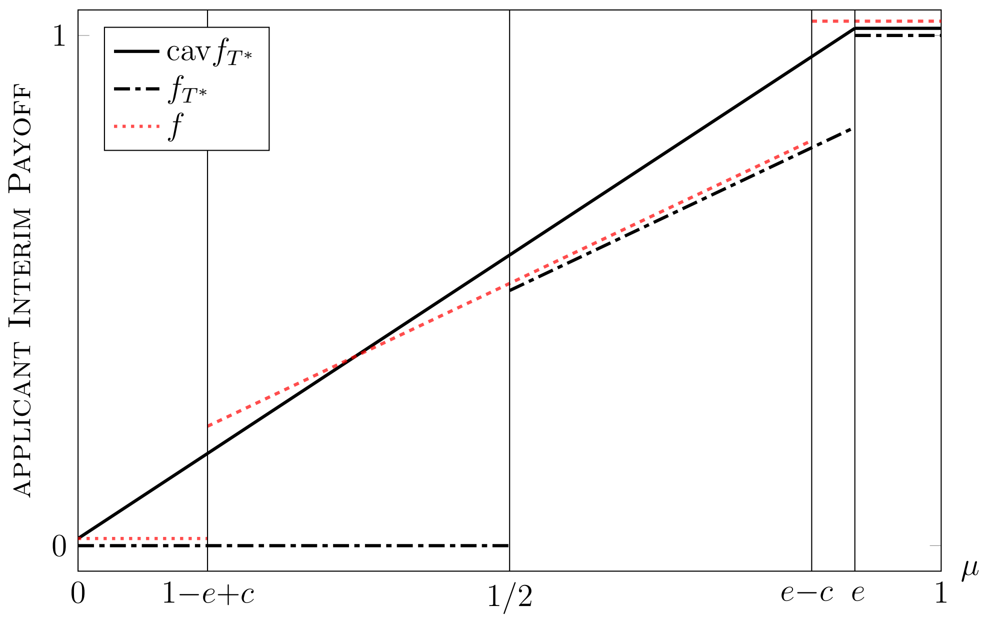

Figure 2: The Commitment Problem
($e = 0.9$ and $c = 0.05$)

The applicant's interim expected payoffs given the certifier-optimal testing rule T^* and those obtained in the baseline model are depicted in Figure 2, for $e = 0.9$ and $c = 0.05$. The testing rule T^* minimizes the applicant's interim expected payoffs at all $\mu \in (0, e)$, which serves the certifier by providing incentives for the applicant to split ρ on 0 and e.[12] In the baseline model, the certifier's inability to commit to a testing rule leads her to conduct (i) "too many" tests at pessimistic interim beliefs, and (ii) "too few" tests at optimistic interim beliefs.

3.3 Comparative Statics

In this subsection we pursue the analysis of the baseline model and examine the consequences of varying the efficiency of the test. Consider Figure 1: starting from the technology (e_a, c_a) and gradually improving the efficiency of the test in the direction of (e_b, c_b), the applicant's disclosure switches from aggressive to conservative at (e_m, c_m), causing a drop in the amount of information disclosed (Lemma 1). This remark yields the next result.

[12]Note that if the certifier could commit to an approval rule $A : [0, 1] \to \{\text{approve}, \text{reject}\}$, she could do even better and induce the applicant to split ρ on 0 and 1 by committing to reject the applicant unless $\mu = 1$.

Proposition 2. *Increasing e, decreasing c, or both at once, can reduce the amount of information disclosed by the applicant in equilibrium.*

The basic intuition behind the proposition is as follows. The applicant is faced with two broad options. One option is to disclose a large amount of information (i.e. disclose aggressively); the applicant then avoids the tests, and might secure approval outright. Another option is to withhold information (i.e. disclose conservatively); the certifier then conducts the test. The certifier's access to more efficient tests lowers the applicants chances of securing approval outright. This, in turn, may lead the applicant to prefer withholding information.

We next explore the implications of this finding for social welfare and certification quality. The social welfare resulting from the two disclosure regimes, aggressive and conservative, is depicted in Figure 3, for $e = 0.9$ and $c = 0.05$.[13] The solid curve indicates social welfare given aggressive disclosure; the dash-dotted curve shows social welfare given conservative disclosure; the dotted curve shows social welfare in the absence of disclosure. Notice that whereas with aggressive disclosure the certifier benefits from information which the applicant discloses, with conservative disclosure social welfare is as if the applicant did not disclose any information.[14] So social welfare drops whenever the applicant's disclosure goes from aggressive to conservative. These observations underlie the following result.

Proposition 3. *Equilibrium social welfare and certification quality are non-monotonic functions of the test technology. In particular, increasing e, reducing c, or both at once, can induce lower social welfare and certification quality.*

Proposition 3 in particular shows that, given the choice between two test technologies, the certifier might prefer to commit to use only the least efficient technology.[15]

4 Extension: Fine-Tuning the Toughness of the Test

In this section, we extend the baseline model by considering the possibility for the certifier to choose what kind of information to acquire through the test. We follow the approach of Gill and Sgroi (2012) and consider general binary tests such that the applicant passes the

[13] Notice that Figure 3 depicts social welfare for $\rho \in [0, 1]$ even though we focus on $\rho < 1/2$ in the paper; the case $\rho \geq 1/2$ is analyzed in a previous version of this paper, see Bizzotto *et al.* (2019).

[14] To see that this must be, consider e.g. $\rho \in (0, 1-e+c)$: in this case, without disclosure the certifier chooses outright rejection; with conservative disclosure, outright rejection remains a best response with probability 1.

[15] In contrast, notice that, by Lemma 2, in the commitment benchmark social welfare and certification quality are increasing in e and constant in c.

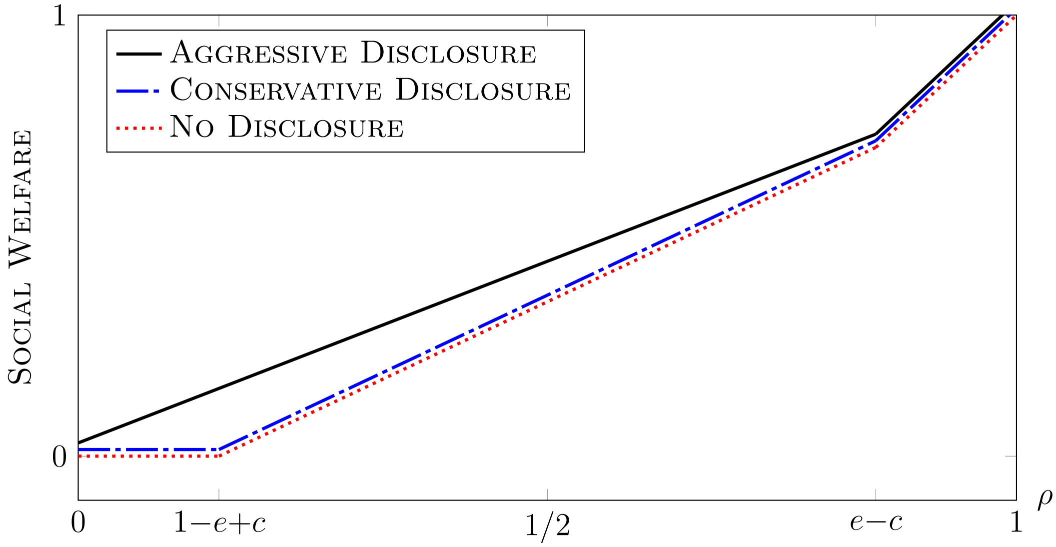

Figure 3: Disclosure Regimes and Social Welfare
($e = 0.9$ and $c = 0.05$)

test with probability $q_G \geq \frac{1}{2}$ conditional on $\omega = G$ and fails the test with probability $q_B \geq \frac{1}{2}$ conditional on $\omega = B$. Define $e(q_G, q_B) := \frac{q_G+q_B}{2}$. We fix the expertise e of the certifier but allow her to choose any test satisfying $e(q_G, q_B) = e$. For expository convenience we suppose that the certifier chooses the toughness $t := q_B$ of the test conducted; hence $q_G = 2e - t$. All other aspects of the model are as in Section 2.

The timing is as follows. First, the applicant selects his signal π. Both π and the realized signal outcome are observed by the certifier, who then decides whether or not to perform the test; if she decides to perform the test, she also chooses the toughness t of said test. Lastly, the certifier chooses between approval and rejection (after observing the test result, in case the test was conducted). We refer to the setting described above as the GS model.

We show in the Online Appendix that Propositions 2 and 3 hold unchanged in the GS model. In what follows, our focus is on comparing equilibrium outcomes in the baseline and GS models.

Proposition 4. *Relative to the baseline model, the GS model can reduce the amount of information disclosed by the applicant in equilibrium.*

The logic behind Proposition 4 is the following. When the certifier can choose the toughness of the test, to minimize the chances of making mistakes the certifier chooses (a) the toughest possible test whenever she believes B to be the most likely state, and (b) the softest possible

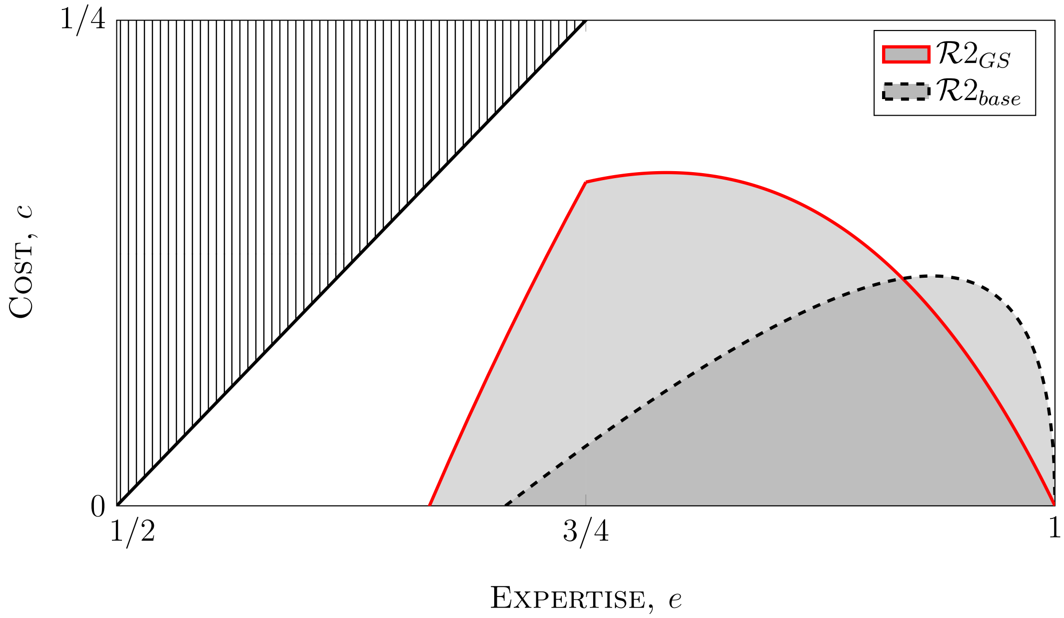

Figure 4: Test Technology and Equilibrium Disclosure: Baseline vs GS

test whenever she believes G to be most likely. The type of test performed by the certifier thus becomes (discontinuously) more favorable to the applicant at $\mu = \frac{1}{2}$. This, in turn, generates incentives for the applicant to split ρ on 0 and $\frac{1}{2}$. In Figure 4, the parametric region $\mathcal{R}2_{GS}$ (below the solid curve) is such that in equilibrium the applicant splits ρ on 0 and $\frac{1}{2}$; the parametric region $\mathcal{R}1_{GS}$ (above the solid curve) on the other hand is such that in equilibrium the applicant splits ρ on 0 and some belief $\overline{\mu} > \frac{1}{2}$. We also indicate the parametric regions $\mathcal{R}1_{base}$ (above the dashed curve) and $\mathcal{R}2_{base}$ (below the dashed curve) obtained in the baseline model. For test technologies in the set $\mathcal{R}2_{GS} \cap \mathcal{R}1_{base}$, the applicant splits ρ on 0 and $\frac{1}{2}$ in the GS model *albeit* in the baseline model the applicant splits ρ on 0 and $e - c$. As $e - c > \frac{1}{2}$, we conclude that allowing the certifier to choose the toughness of the test reduces the amount of information disclosed by the applicant whenever $(e, c) \in \mathcal{R}2_{GS} \cap \mathcal{R}1_{base}$. This effect in turn lowers social welfare and certification quality.[16]

Proposition 5. *Relative to the baseline model, the GS model can induce lower equilibrium social welfare and certification quality.*

[16] We show in the Online Appendix that relative to the baseline model, flexible tests decrease equilibrium social welfare at any technology in the non-empty set $\mathcal{R}2_{GS} \cap \mathcal{R}1_{base}$, and increase it at any interior technology in the complement of this set.

Propositions 4 and 5 in particular show that the inability of the certifier to commit to the kind of information acquired through the test can result in less information being disclosed by the applicant, lower social welfare, and lower certification quality.

5 Conclusion

This paper has examined the interplay between information acquisition on the part of a receiver and information disclosure from a sender. Important applications of our analysis include products requiring approval before they can be sold: complex financial products, organic food products, cars and drugs among others. As sellers are typically better able than certifiers to retrieve information about their own products, certifiers often rely on information that sellers disclose voluntarily. However, to the extent that it can raise chances of approval, sellers have a strategic incentive to conceal information. Certifiers therefore make their decisions based on a mixture of information, some of which they acquire firsthand by way of tests.

We show that the inability of the certifier to commit to the amount of information to be collected can result in a reduction of information disclosed, in turn lowering social welfare and certification quality. Among other consequences, given the choice between two test technologies, the certifier might prefer to commit to use only the least efficient technology. These insights are robust to several modifications of the setup. For example, when deciding whether or not to conduct the test, the certifier may be able to choose the expertise e given an increasing and convex cost function $c(e)$; or the applicant may be able to observe his quality with some small probability; finally, the applicant may be constrained to disclose information via some signal of bounded accuracy.[17]

[17] These and several other extensions are explored in a previous working paper version. See Bizzotto *et al.* (2019).

Appendix

Proof of Lemma 1: We provide the main arguments in the text. ■

Proof of Lemma 2: Note first that, regardless of the testing rule used by the certifier, any optimal signal of the applicant is (weakly) less informative (in the sense of Blackwell) than if he were to split ρ on 0 and e. As T^* induces the applicant to split ρ on 0 and e, we conclude that T^* induces the applicant to disclose the maximum amount of information over all testing rules. Moreover, notice that any testing rule inducing the agent to disclose less information lowers social welfare relative to T^*. So any certifier-optimal testing rule must be such that the applicant splits ρ on 0 and e. Finally, whether $\mu = 0$ or $\mu = e$, testing the applicant never improves certification quality. Hence, any certifier-optimal testing rule is such that testing never occurs. ■

Proof of Proposition 1: The result follows from Lemmata 1 and 2. ■

Proof of Proposition 2: Straightforward algebra establishes that (1) holds with equality if and only if $e \in (\frac{1}{\sqrt{2}}, 1)$ and $c = c^*(e)$, where

$$c^*(e) := \frac{4e^2 - 3e - 1 + \sqrt{e^2 - 6e + 5}}{2(2e - 1)}.$$

In particular, $c^*(\frac{1}{\sqrt{2}}) = 0$ and $c^*(\cdot)$ is strictly increasing over an interval $[\frac{1}{\sqrt{2}}, e^*]$, where $\frac{1}{\sqrt{2}} < e^* < 1$. Now, let $\mathcal{F}_{base}$ denote the frontier between the parametric regions $\mathcal{R}1_{base}$ and $\mathcal{R}2_{base}$, where (1) holds with equality. Crossing $\mathcal{F}_{base}$ from $\mathcal{R}1_{base}$ into $\mathcal{R}2_{base}$ (as illustrated in Figure 1), the applicant's disclosure switches from aggressive to conservative, reducing the amount of information disclosed by the applicant (in Blackwell's sense).

■

Proof of Proposition 3: We first derive a useful identity. Let A_G (resp. A_B) represent the approval probability conditional on $\omega = G$ (resp. $\omega = B$), and $\overline{A}$ the ex ante probability of approval. Then $Q = \rho A_G + (1-\rho)(1 - A_B)$ and, since $\overline{A} = \rho A_G + (1-\rho)A_B$, we obtain

$$Q = 2\rho A_G - \overline{A} + (1 - \rho). \tag{2}$$

Next, Lemma 1 implies $A_G = 1$ within the parametric region $\mathcal{R}1_{base}$ and $A_G \leq k(e, c)$

within $\mathcal{R}2_{base}$, where $k(e,c) < 1$ as long as $e < 1$. Hence, using (2), in equilibrium,

$$Q = \begin{cases} 2\rho - \overline{A} + (1-\rho) & \text{within } \mathcal{R}1_{base}; \\ 2\rho k - \overline{A} + (1-\rho) & \text{within } \mathcal{R}2_{base}. \end{cases} \tag{3}$$

By Lemma 1, in equilibrium the ex ante probability of approval $\overline{A}$ is continuous in e and c at any test technology in $\mathcal{R}1_{base} \cup \mathcal{R}2_{base}$. On the frontier $\mathcal{F}_{base}$ the applicant is indifferent between aggressive and conservative disclosure. So in equilibrium $\overline{A}$ is continuous in e and c at any point of $\mathcal{R}1_{base} \cup \mathcal{F}_{base} \cup \mathcal{R}2_{base}$. Hence, by (3), Q jumps downward when we cross $\mathcal{F}_{base}$ from $\mathcal{R}1_{base}$ into $\mathcal{R}2_{base}$. The remarks made earlier in the proof of Proposition 2 therefore establish the part of the proposition concerning Q. The part of the proposition concerning W is now immediate since $W = Q - c\mathbb{P}(\text{test}) - (1-\rho)$ and, in equilibrium, $\mathbb{P}(\text{test}) = 0$ at any test technology in $\mathcal{R}1_{base}$. ■

Proofs of Propositions 4 and 5: These proofs and the rest of the analysis of the GS model are relegated to the Online Appendix.

■

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Online Appendix to "Testing, Disclosure and Approval" by Jacopo Bizzotto, Jesper Rüdiger and Adrien Vigier

Online Appendix: Analysis of the GS Model

This appendix contains the analysis of the GS model presented in Section 4. Let $t_{max}(e)$ denote the maximum toughness given expertise e, thus

$$t_{max}(e) = \begin{cases} 2e - \frac{1}{2} & \text{for } e \in (\frac{1}{2}, \frac{3}{4}]; \\ 1 & \text{for } e \in (\frac{3}{4}, 1). \end{cases}$$

Similarly let $t_{min}(e) := 2e - t_{max}(e)$ denote the minimum toughness among tests of expertise e. We refer to $t = t_{max}(e)$ as the toughest possible test, and to $t = t_{min}(e)$ as the softest possible test (given expertise e). The following lemma characterizes the certifier's optimal strategy.

Lemma 3. *There exists $\overline{\mu} \in (e - c, 1)$ such that in any equilibrium the certifier (a) rejects without testing if $\mu < \underline{\mu} := 1 - \overline{\mu}$, (b) conducts the test if $\mu \in (1 - \overline{\mu}, \overline{\mu})$, and (c) approves without testing if $\mu > \overline{\mu}$. Whenever testing the applicant, the certifier chooses the toughest possible test if $\mu < \frac{1}{2}$ and the softest possible test if $\mu > \frac{1}{2}$.*

Proof: The certifier's interim expected payoff from conducting test t and conditioning approval on passing the test is

$$\mu q_G - (1-\mu)(1-q_B) - c = \mu(2e - t) - (1-\mu)(1-t) - c = \mu(1+2e) - (1+c) + t(1-2\mu).$$

The right-hand side of the last equation is increasing in t if $\mu < \frac{1}{2}$ and decreasing in t if $\mu > \frac{1}{2}$. Therefore in equilibrium, whenever testing the applicant, the certifier chooses $t = t_{max}$ if $\mu < \frac{1}{2}$ and $t = t_{min}$ if $\mu > \frac{1}{2}$. For $e < \frac{3}{4}$, $t_{max} = 2e - \frac{1}{2}$. Substituting $t = t_{max}$ into the penultimate highlighted equation, and equating with the certifier's interim expected payoff from outright rejection (i.e. 0), yields

$$\frac{\mu}{2} - (1-\mu)\left(\frac{3}{2} - 2e\right) - c = 0.$$

Solving for μ gives

$$\underline{\mu} = \frac{2(c - 2e) + 3}{4(1-e)}. \tag{4}$$

For $e \geq \frac{3}{4}$, $t_{max} = 1$, and so substituting $t = t_{max}$ yields

$$\mu(2e - 1) - c = 0,$$

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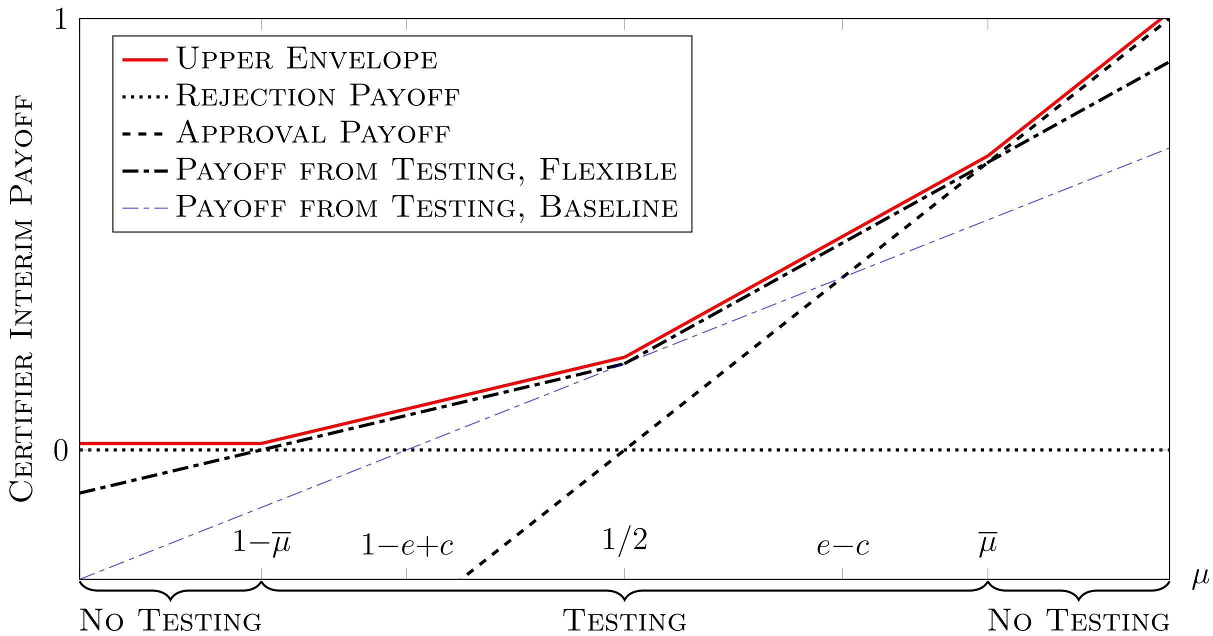

Figure 5: Optimal Certifier Strategy (GS Model)
($e = 0.8$ and $c = 0.1$)

followed by

$$\underline{\mu} = \frac{c}{2e-1}. \tag{5}$$

■

The thick dash-dotted curve in Figure 5 shows the certifier's interim expected payoffs from conducting the optimal test and conditioning approval on passing the test; it crosses the dotted horizontal axis (representing the certifier's interim expected payoffs from outright rejection) at $\mu = 1 - \overline{\mu}$ and the dashed line (representing the certifier's interim expected payoffs from outright approval) at $\mu = \overline{\mu}$. Notice that $[1 - e + c, e - c] \subset [1 - \overline{\mu}, \overline{\mu}]$, that is, relative to the baseline model flexible tests enlarge the interval of interim beliefs at which the certifier chooses to conduct the test. In the figure, the light dash-dotted curve shows the certifier's interim expected payoffs from conducting the baseline test and conditioning approval on passing this test.

We turn next to the applicant. By Lemma 3, the applicant's equilibrium interim expected

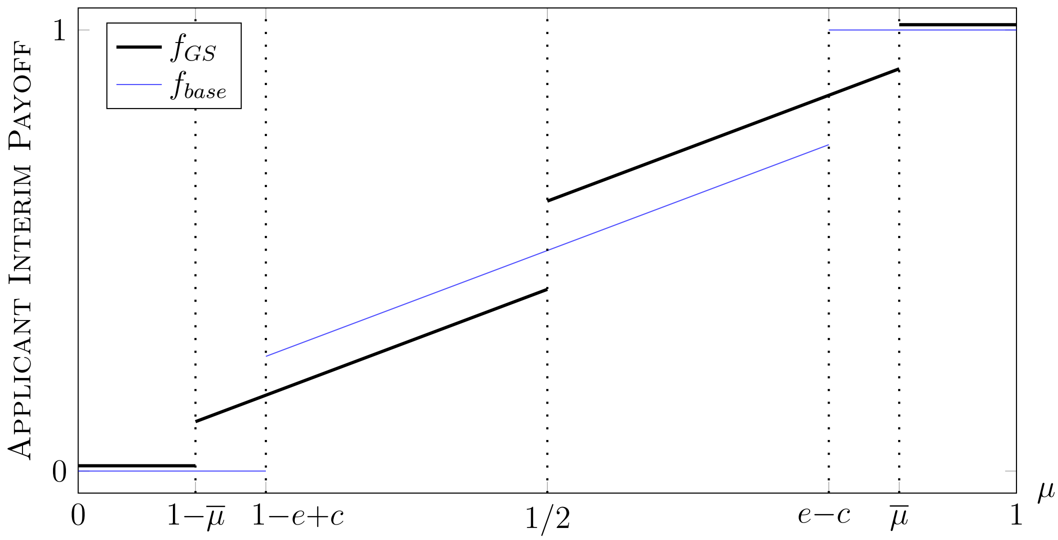

Figure 6: Equilibrium Disclosure (GS Model)
($e = 0.9$ and $c = 0.1$)

payoff, $f_{GS}(\mu)$, can be written as

$$f_{GS}(\mu) = \begin{cases} 0 & \text{for } \mu \in [0, 1-\overline{\mu}); \\ \mu(2e - t_{max}) + (1-\mu)(1-t_{max}) & \text{for } \mu \in (1-\overline{\mu}, \frac{1}{2}); \\ \mu(2e - t_{min}) + (1-\mu)(1-t_{min}) & \text{for } \mu \in (\frac{1}{2}, \overline{\mu}); \\ 1 & \text{for } \mu \in (\overline{\mu}, 1]. \end{cases}$$

In addition to $f_{GS}(\mu)$, we depict in Figure 6 the equilibrium interim expected payoffs $f_{base}(\mu)$ obtained in the baseline model. The following lemma sums up the implications of the certifier's optimal use of flexible tests for the applicant's choice of disclosure.

Lemma 4. *Define*

$$c^*_{GS}(e) := \begin{cases} \dfrac{3e-2}{2e} & \textit{for } e \in (\frac{1}{2}, \frac{3}{4}); \\ \dfrac{6e-4e^2-2}{3-2e} & \textit{for } e \in [\frac{3}{4}, 1). \end{cases}$$

If $c > c^*_{GS}(e)$ then in any equilibrium the applicant splits ρ on 0 and $\overline{\mu}$. If instead $c < c^*_{GS}(e)$ then in any equilibrium the applicant splits ρ on 0 and $\frac{1}{2}$.

Proof: We consider the cases $e < \frac{3}{4}$ and $e \geq \frac{3}{4}$ separately.

Case 1: $e < \frac{3}{4}$. We established in the proof of Lemma 3 that in equilibrium, whenever testing the applicant, the certifier chooses the toughest possible test if $\mu < \frac{1}{2}$ and the softest possible test if $\mu > \frac{1}{2}$. For $e < \frac{3}{4}$, the applicant's equilibrium interim expected payoffs are thus given by

$$f_{GS}(\mu) = \begin{cases} 0 & \text{for } \mu \in [0, \underline{\mu}); \\ \dfrac{\mu}{2} + (1-\mu)\left(\dfrac{3}{2} - 2e\right) & \text{for } \mu \in (\underline{\mu}, \frac{1}{2}); \\ \mu\left(2e - \dfrac{1}{2}\right) + \dfrac{1-\mu}{2} & \text{for } \mu \in (\frac{1}{2}, \overline{\mu}); \\ 1 & \text{for } \mu \in (\overline{\mu}, 1]. \end{cases}$$

Thus $f_{GS}(\cdot)$ is piecewise linear, with upward jumps at $\underline{\mu}$, $\frac{1}{2}$ and $\overline{\mu}$. It ensues that $\text{cav} f_{GS}(\mu) > f_{GS}(\mu)$ for all $\mu \in (0, \underline{\mu}) \cup (\underline{\mu}, \frac{1}{2}) \cup (\frac{1}{2}, \overline{\mu})$.

We next show that in any equilibrium $\text{cav} f_{GS}(\underline{\mu}) > f_{GS}(\underline{\mu})$ as well. Note first that in any equilibrium, $f_{GS}(\underline{\mu}) \leq \frac{\mu}{2} + (1-\underline{\mu})\left(\frac{3}{2} - 2e\right)$. So it is enough to show

$$\text{cav} f_{GS}(\underline{\mu}) > \frac{\underline{\mu}}{2} + (1-\underline{\mu})\left(\frac{3}{2} - 2e\right).$$

In turn, since $\text{cav} f_{GS}(\underline{\mu}) \geq \lim_{\varepsilon \downarrow 0}\left\{\frac{\underline{\mu}}{(\frac{1}{2}+\varepsilon)} f_{GS}(\frac{1}{2}+\varepsilon)\right\} = 2\underline{\mu}\left[\frac{1}{2}\left(2e - \frac{1}{2}\right) + \frac{1}{4}\right]$, a sufficient condition for this to hold is

$$2\underline{\mu}\left[\frac{1}{2}\left(2e - \frac{1}{2}\right) + \frac{1}{4}\right] > \frac{\underline{\mu}}{2} + (1-\underline{\mu})\left(\frac{3}{2} - 2e\right).$$

Using (4), tedious but straightforward algebra shows that the latter inequality is equivalent to

$$\frac{5}{8} - \frac{\sqrt{16c+1}}{8} < e < \frac{5}{8} + \frac{\sqrt{16c+1}}{8}.$$

The left-hand side of this sequence of inequalities is bounded from above by $\frac{1}{2}$ (when evaluated at $c = 0$); the right-hand side is bounded from below by $\frac{3}{4}$ (again, when evaluated at $c = 0$). As $\frac{1}{2} < e < \frac{3}{4}$, these inequalities are therefore satisfied. This finishes to show that, in any equilibrium, $\text{cav} f_{GS}(\underline{\mu}) > f_{GS}(\underline{\mu})$, and so $\text{cav} f_{GS}(\mu) > f_{GS}(\mu)$ for all $\mu \in (0, \frac{1}{2}) \cup (\frac{1}{2}, \overline{\mu})$. We conclude that either

$$\frac{1/2}{\overline{\mu}} > \frac{1}{2}\left(2e - \frac{1}{2}\right) + \frac{1}{4}, \tag{6}$$

in which case cavf_{GS} comprises two linear pieces, or (6) is violated and then cavf_{GS} has three linear pieces. Substituting $\overline{\mu}$ using (4) combined with $\overline{\mu} = 1 - \underline{\mu}$ establishes that (6) is equivalent to $c > \frac{3e-2}{2e}$.

Case 2: $e \geq \frac{3}{4}$. The applicant's equilibrium interim expected payoffs are in this case given by

$$f_{GS}(\mu) = \begin{cases} 0 & \text{for } \mu \in [0, \underline{\mu}); \\ \mu(2e-1) & \text{for } \mu \in (\underline{\mu}, \frac{1}{2}); \\ \mu + (1-\mu)(2-2e) & \text{for } \mu \in (\frac{1}{2}, \overline{\mu}); \\ 1 & \text{for } \mu \in (\overline{\mu}, 1]. \end{cases}$$

Thus $f_{GS}(\cdot)$ is piecewise linear, with upward jumps at $\underline{\mu}$, $\frac{1}{2}$ and $\overline{\mu}$. It ensues that cav$f_{GS}(\mu) > f_{GS}(\mu)$ for all $\mu \in (0, \underline{\mu}) \cup (\underline{\mu}, \frac{1}{2}) \cup (\frac{1}{2}, \overline{\mu})$.

We next show that in any equilibrium cav$f_{GS}(\underline{\mu}) > f_{GS}(\underline{\mu})$ as well. Note first that in any equilibrium, $f_{GS}(\underline{\mu}) \leq \underline{\mu}(2e-1)$. So it is enough to show

$$\text{cav} f_{GS}(\underline{\mu}) > \underline{\mu}(2e-1).$$

In turn, since cav$f_{GS}(\underline{\mu}) \geq \lim_{\varepsilon \downarrow 0} \left\{ \frac{\mu}{(\frac{1}{2}+\varepsilon)} f_{flex}(\frac{1}{2}+\varepsilon) \right\} = 2\underline{\mu}\left[\frac{1}{2} + \frac{1}{2}(2-2e)\right]$, a sufficient condition for this to hold is

$$2\underline{\mu}\left[\frac{1}{2} + \frac{1}{2}(2-2e)\right] > \underline{\mu}(2e-1).$$

Using tedious but straightforward algebra the latter is equivalent to

$$4e < 4,$$

which evidently holds. This finishes to show that, in any equilibrium, cav$f_{GS}(\underline{\mu}) > f_{GS}(\underline{\mu})$, and so cav$f_{GS}(\mu) > f_{GS}(\mu)$ for all $\mu \in (0, \frac{1}{2}) \cup (\frac{1}{2}, \overline{\mu})$. We conclude that either

$$\frac{1/2}{\overline{\mu}} > \frac{1}{2} + \frac{1}{2}(2-2e), \tag{7}$$

in which case cavf_{GS} comprises two linear pieces, or (7) is violated and then cavf_{GS} has three linear pieces. Substituting $\overline{\mu}$ using (5) combined with $\overline{\mu} = 1 - \underline{\mu}$ establishes that (7) is equivalent to $c > \frac{6e-4e^2-2}{3-2e}$. ■

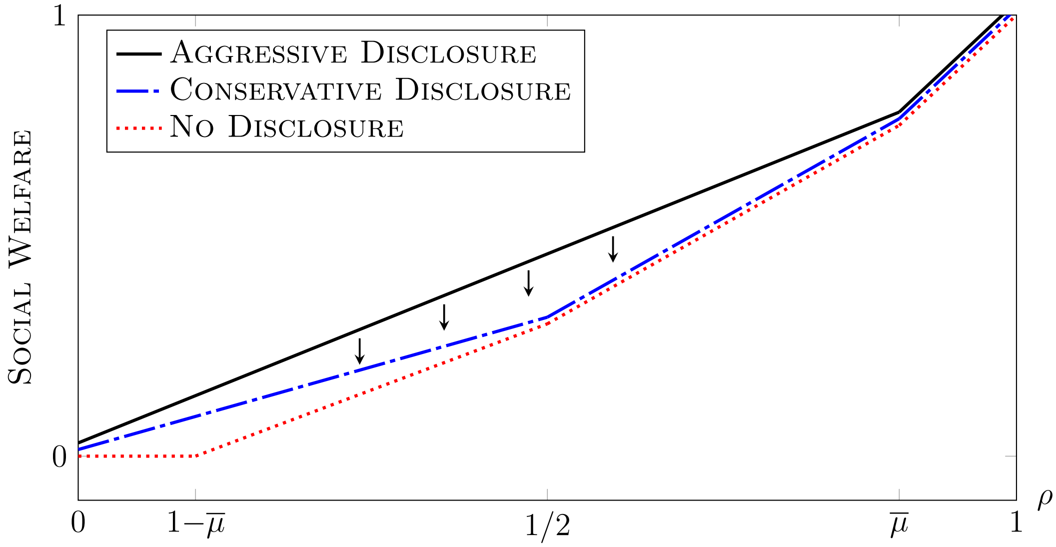

Figure 7: Disclosure Regimes and Social Welfare (GS Model)
($e = 0.9$ and $c = 0.1$)

Since at $\mu = \frac{1}{2}$ the certifier switches from using the toughest possible test to using the softest possible test, and the applicant prefers softer tests, the applicant's equilibrium strategy always entails disclosing information generating $\mu \geq \frac{1}{2}$ with positive probability: either the applicant splits ρ on 0 and $\frac{1}{2}$, or the applicant splits ρ on 0 and $\overline{\mu}$.

Using Lemmata 3 and 4 we depict, in Figure 7, the social welfare as a function of reputation ρ resulting from the two possible disclosure regimes.[18] The solid curve represents social welfare given that the applicant splits ρ on 0 and $\overline{\mu}$; the dash-dotted curve represents social welfare given that the applicant splits ρ on 0 and $\frac{1}{2}$; the dotted curve indicates social welfare in the absence of disclosure. In contrast to the baseline model, here with flexible tests the certifier gains from disclosure irrespective of the disclosure regime: at all $\rho \in (0, \frac{1}{2})$, the solid curve lies above the dash-dotted curve which itself lies above the dotted curve. The downward arrows indicate the drop in social welfare resulting from crossing the frontier $c^*_{GS}(e)$ "from above" in Figure 4: the parametric region $\mathcal{R}1_{GS}$, above the solid curve representing $c^*_{GS}(e)$, is such that in equilibrium the applicant splits ρ on 0 and $\overline{\mu}$; the parametric region $\mathcal{R}2_{GS}$, below $c^*_{GS}(e)$, is such that in equilibrium the applicant splits ρ on 0 and $\frac{1}{2}$. We can now extend Propositions 2 and 3.

[18]By extension of the terminology used in the baseline model we say that the applicant's equilibrium disclosure is aggressive whenever the applicant splits ρ on 0 and $\overline{\mu}$ and that it is conservative otherwise.

Proposition 6. *Increasing e, decreasing c, or both at once, can (i) reduce the amount of information disclosed by the applicant in equilibrium, (ii) lower social welfare, and (iii) lower certification quality.*

Proof: The proof is almost identical to the counterparts in the baseline model, and therefore omitted. However, note that now, for $e > \frac{3}{4}$, the equilibrium certification quality is continuous at the frontier between $\mathcal{R}1_{GS}$ and $\mathcal{R}2_{GS}$, since on both sides of the frontier the equilibrium probability of approval conditional on $\omega = G$ is equal to 1. ■

Proofs of Propositions 4 and 5: We first show that equilibrium social welfare is greater in the baseline model than in the GS model for all technologies in $\mathcal{R}2_{GS} \cap \mathcal{R}1_{base}$. Consider an arbitrary test technology in $\mathcal{R}2_{GS} \cap \mathcal{R}1_{base}$.

Case 1: $e < \frac{3}{4}$. Then, in any equilibrium of the GS model,

$$W = \frac{\rho}{1/2}\left[\frac{1}{2}\left(2e - \frac{1}{2}\right) - \frac{1}{4} - c\right].$$

On the other hand, in any equilibrium of the baseline model,

$$W = \frac{\rho}{e-c}[2(e-c) - 1].$$

Hence, equilibrium social welfare is greater in the baseline model than in the GS model whenever either $0 < c < e - \frac{1}{2}$ or $c > e$. The former condition is always satisfied.

Case 2: $e \geq \frac{3}{4}$. Then, in any equilibrium of the GS model,

$$W = \frac{\rho}{1/2}\left[\frac{1}{2} - \frac{1}{2}(2 - 2e) - c\right].$$

On the other hand, in any equilibrium of the baseline model,

$$W = \frac{\rho}{e-c}[2(e-c) - 1].$$

Hence, as in the previous case, equilibrium social welfare is greater in the baseline model than in the GS model whenever either $0 < c < e - \frac{1}{2}$ or $c > e$. The former condition is always satisfied.

Next, we show that equilibrium certification quality can be larger in the baseline model than in the GS model. Consider a test technology in $\mathcal{R}2_{GS} \cap \mathcal{R}1_{base}$ with $e < \frac{3}{4}$. We have in this case, in any equilibrium of the GS model,

$$Q = \frac{\rho}{1/2}\left[\frac{1}{2}\left(2e - \frac{1}{2}\right) - \frac{1}{4}\right] + (1-\rho).$$

On the other hand, in any equilibrium of the baseline model,

$$Q = \frac{\rho}{e-c}[2(e-c)-1] + (1-\rho).$$

Hence, equilibrium certification quality is greater in the baseline model than in the GS model whenever

$$c < \frac{2e^2 - 3e + 1}{2e - 3} =: \hat{c}(e).$$

Now, one verifies that there exist $\varepsilon > 0$ and $\eta > 0$ such that[19]

$$\left\{(e,c) : \frac{2}{3} + \varepsilon < e < \frac{1}{\sqrt{2}} - \varepsilon,\ c < \eta\right\} \subset \ \mathcal{R}2_{GS} \cap \mathcal{R}1_{base}.$$

Since $\hat{c}(\frac{1}{\sqrt{2}}) > 0$, we therefore established the existence of a non-empty set of test technologies such that certification quality is greater in the baseline model than in the GS model.

We now show for completeness that flexible tests increase equilibrium social welfare at any interior technology in the complement of $\mathcal{R}2_{GS} \cap \mathcal{R}1_{base}$. Pick one such technology. Either $(e,c) \in \mathcal{R}1_{GS}$ or $(e,c) \in \mathcal{R}2_{GS} \cap \mathcal{R}2_{base}$. If $(e,c) \in \mathcal{R}1_{GS}$ then any equilibrium of the GS model is such that the applicant splits ρ on 0 and $\overline{\mu}$. Since $\overline{\mu} > e - c$, we conclude that in this case flexible tests increase equilibrium social welfare relative to the baseline model. If $(e,c) \in \mathcal{R}2_{GS} \cap \mathcal{R}2_{base}$, any equilibrium of the GS test model is such that the applicant splits ρ on 0 and $\frac{1}{2}$. On the other hand we saw that in this case, in the baseline model, equilibrium social welfare is as if the applicant did not disclose any information. So once again flexible tests increase equilibrium social welfare relative to the baseline model. ■

[19] To obtain the lower bound, solve $c^*_{GS}(e) = 0$ for $e < \frac{3}{4}$.