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Buying First or Selling First in Housing Markets*

Espen R. Moen[†], Plamen T. Nenov[‡], and Florian Sniekers[§]

Abstract

Housing transactions by moving homeowners take two steps – buying a new house and selling the old one. This paper argues that the transaction sequence decisions of moving homeowners have important effects on the housing market. Moving homeowners prefer to buy first whenever there are *more* buyers than sellers in the market. However, this congests the buyer side of the market and increases the buyer-seller ratio, further strengthening the incentives of other moving owners to buy first. This endogenous strategic complementarity leads to multiple steady state equilibria and large fluctuations, which are broadly consistent with stylized facts about the housing cycle.

Keywords: search frictions, order of transactions, strategic complementarity, coordination, self-fulfilling fluctuations

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[†]Norwegian Business School (BI), e-mail: espen.r.moen@bi.no.

[‡]Norwegian Business School (BI), e-mail: plamen.nenov@bi.no.

[§]Tilburg University, e-mail: f.j.t.sniekers@uvt.nl.

1 Introduction

A large number of households move within the same local housing market. Many of these moves are by owner-occupiers who buy a new housing unit and sell their old unit. As it takes time to transact in the housing market, a moving homeowner may end up owning either two housing units or no housing units for some time, depending on whether she buys the new house before selling the old one or vice versa. Either of these two alternatives may be costly.

In this paper we argue that moving homeowners make a decision regarding the sequence of the two transactions they have to undertake. They either buy a new house before they sell the old one, or sell the old house before buying a new one – buy first or sell first. Moreover, we show that the transaction sequence decisions of moving owners have profound effects on housing market conditions and give rise to powerful equilibrium feedbacks with important consequences for housing market dynamics.

We start by documenting a number of novel facts about moving owners and their transaction sequences using a unique matched property-owner data set for the Copenhagen housing market. Among other things, we show that the share of buy-first owners is not constant but varies strongly with the state of the housing market, increasing during the Copenhagen housing boom of 2004-2006 and decreasing sharply during the subsequent bust.[1] Moreover, the share of buy-first owners is higher when selling is quick and buying is slow.

We use these facts to argue that moving owners actively choose their transaction sequence. We then analyze the aggregate implications of the individual transaction sequence decisions in a search model of the housing market. In the model, agents continuously enter and exit a local housing market. They have a preference for owning housing over renting. The housing market is characterized by a frictional trading process, and the rate at which buyers and sellers find a trading

[1] Anundsen and Røed Larsen (2014) document a similar relationship using survey evidence from Norway.

partner is affected by the tightness in the market – the ratio of buyers to sellers. An owner-occupier may be hit by an idiosyncratic preference shock and become "mismatched" with his current house, in which case he wants to move internally in the same housing market. To do that, the mismatched owner has to choose whether to buy first or to sell first. Given trading delays, the agent would then become a double owner (owning two housing units) or a forced renter (owning no housing) for some time, which is costly. The expected time in such a state depends on the time-on-market for sellers and buyers, respectively.

If the costs incurred by a double owner or a forced renter are high relative to the costs of mismatch (living in the old house), the mismatched owner prefers to do the most time-consuming transaction first. Hence he prefers buying first over selling first when the buyer-seller ratio is high, as the expected time-on-market then is low for a seller and high for the buyer. Conversely, if there are more sellers than buyers, the expected time-on-market is high for sellers and low for buyers, and the agent wants to sell first. We call this effect on a moving owner's transaction sequence, the *queue-length effect*.

The order of transactions by moving owner-occupiers affects the buyer-seller ratio. Specifically, when mismatched owners buy first, they congest the buyer side of the market, so the market ends with more buyers than sellers in steady state. Conversely, when all mismatched owners sell first, there are more sellers than buyers in steady state. Therefore, the queue-length effect interacts with the stock-flow conditions in the housing market to create a *strategic complementarity* in moving owners' transaction sequence decisions. That strategic complementarity may, in turn, lead to multiple steady state equilibria. In one steady state equilibrium (a "sell first" equilibrium), mismatched owners prefer to sell first, the market tightness is low and the expected time-on-market for sellers is high. In the other steady state equilibrium (a "buy first" equilibrium), mismatched owners prefer to buy first, the market tightness is high and the expected time-on-market for sellers is low. Therefore, this paper provides a formalization of the common idea of "buyer's markets" (a "sell first" equilibrium) and "seller's markets" (a "buy first" equilibrium) and shows how either of these can arise endogenously as the result of the choices of moving owners.

In our analysis, we initially assume that prices are fixed across steady states (or, equivalently, that the user price of housing is constant, in which case prices do not influence the transaction sequence decision). Then we endogenize house prices. First, we assume that the steady state price level is an increasing function of the buyer-seller ratio, while the rental price is constant. We show that multiplicity exists as long as the responsiveness of the house price to the buyer-seller ratio is not too high. Then we show that there can exist multiple equilibria when house prices are (endogenously) determined by Nash bargaining, in which case housing prices differ across trading pairs and respond to changes in the buyer-seller ratio. Specifically, the main channel that drives equilibrium multiplicity in our benchmark model – the queue-length effect and its interaction with the stock-flow conditions that determine the equilibrium market tightness – is still present in that environment. Moreover, we show that there can be equilibrium multiplicity even in an environment with competitive search where agents can trade off prices and time-on-market.

We also analyze dynamic equilibria of our economy. We first show that the dynamics of the stock variables that unfold when all mismatched owners either buy first at all time (the buy first trajectory) or sell first at all time (the sell first trajectory) are globally asymptotically stable and converge to the respective steady state equilibrium allocations of the stocks. We derive sufficient conditions under which the buy-first and the sell-first trajectories constitute dynamic equilibria, such that it is optimal for mismatched owners to buy first (sell first) along the buy first (sell first) trajectory.

We illustrate the quantitative relevance of our mechanism in a calibrated numerical example that matches the behavior of the buy-first share in Copenhagen during 2004-2008. Initially, the economy is on a buy-first trajectory and then it suddenly moves to a sell-first trajectory. We show that the behavior of our calibrated economy is broadly consistent with the housing cycle in Copenhagen. Quantitatively, the numerical model comes close to explaining the full change of time-on-market and explains almost half of the change in the for-sale inventory. The simulated change in transaction volume is also about half of that in the data. Interestingly, after the switch, the for-sale inventory and transaction volume move in opposite directions – a feature that search-based models of the housing market typically have trouble generating (Diaz and Jerez, 2013). Finally, we show that when prices are determined by Nash bargaining, there can be substantial house price fluctuations arising from such switches.

Related literature. The paper is related to the growing literature on search models of the housing market initiated by the seminal work of Wheaton (1990), and particularly, to the literature on search frictions and housing market dynamics (Krainer (2001), Novy-Marx (2009), Caplin and Leahy (2011), Diaz and Jerez (2013), Head et al. (2014), Ngai and Tenreyro (2014), Guren and McQuade (2019), Ngai and Sheedy (2015), Piazzesi et al. (forthcoming), Guren (2018), among others).[2] However, most of this literature abstracts away from the transaction sequence choices of moving owners by assuming that the actions of buying and selling are independent of each other.

In Wheaton (1990), mismatched owners must also both buy and sell a housing unit. However, the model implicitly assumes that the cost of becoming a forced renter with no housing is prohibitively large, so that mismatched owners always buy first.

Diaz and Jerez (2013) calibrate a model of the housing market in the spirit of Wheaton (1990) where mismatched owners must buy first, as well as a model where they must sell first. They show that each of the two models can explain some aspects of housing market cycles, which points to

[2]The paper is also broadly related to the Walrasian literature on house price dynamics and volatility (Stein (1995), Ortalo-Magné and Rady (2006), Glaeser et al. (2014), He et al. (2015)). Our extension to a model of the housing market with competitive search relates the paper to recent models of competitive search in housing and asset markets (Lester et al. (2015), Albrecht et al. (2016), Lester et al. (2017)).

the importance of a model that can accommodate an explicit transaction sequence choice. Ngai and Sheedy (2015) model an endogenous moving decision based on idiosyncratic match quality as an amplification mechanism of sales volume. The paper argues that the endogenous participation decisions of mismatched owners are important for explaining key patterns in the data during the housing boom of the late 90s and early 2000s. In our model we assume that mismatched owners always participate and instead focus on their transaction sequence decisions. The implications we draw from our analysis are, therefore, complementary to the insights in their paper.

In parallel and independent work, Anenberg and Bayer (first version 2013, revised 2015) also introduce a buy-sell decision of mismatched owners. In their paper, moving owners simultaneously search both on the buyer and seller side. In the event of meeting a buyer and a seller simultaneously, a moving owner faces a decision to buy first or sell first. In a rich calibrated model with many economic forces at play, they study the quantitative effects of shocks to the flow of new buyers on the timing of moving decisions of owners in Los Angeles. In contrast, our analysis focuses on the *ex ante* decision of a moving owner whether to first search for a seller or a buyer. Using data for Copenhagen, we argue that moving owners indeed tend to choose a particular sequence of transactions, rather than search on the buyer and seller side simultaneously and take whichever trading opportunity arrives first. Furthermore, we explore theoretically how that decision influences the stock-flow process in the economy and the buyer-seller ratio, which in turn feeds back into the buy-first/sell-first decisions of other moving owners, giving rise to the strategic complementarity and multiple equilibria. These findings are absent in Anenberg and Bayer's paper. We thus view the two papers as complementary.[3]

[3]Maury and Tripier (2014) study a modification of the Wheaton (1990) model, in which mismatched owners can buy and sell simultaneously, which they use to study price dispersion in the housing market. However, they do not consider the feedback from buying and selling decisions on the stock-flow process and on market tightness. This feedback is key for the mechanisms we explore in our paper. Novy-Marx (2009) describes the general feedback in a stock-flow process without free entry, in which an exogenous increase in e.g. buyers decreases time-to-sale and depletes the stock of sellers, further increasing the buyer-seller ratio. This feedback is purely mechanical and does not result in strategic complementarities or multiple equilibria. Moreover, in his model moving agents do not both

The paper is also related to the literature on multiple equilibria and self-fulfilling fluctuations as the result of search frictions. Multiple equilibria in that literature arise mainly from increasing returns to scale in matching (Diamond (1982)) or from the interactions between several frictional markets (Howitt and McAfee (1988)).[4] In contrast, multiplicity in our model arises in a single market with constant returns to scale in matching. Other sources of multiplicity in models with search frictions include an indeterminacy in the division of the match surplus (Howitt and McAfee (1987), Farmer (2012), Kashiwagi (2014)) or the interaction between the outside option of matched market participants and their endogenous separation decisions (Burdett and Coles (1998), Coles and Wright (1998), Burdett et al. (2004), Moen and Rosén (2013), Eeckhout and Lindenlaub (forthcoming)). In our paper, the division of the match surplus is determined by a fixed price, by Nash bargaining, or by competitive search, so that the indeterminacy of a bilateral monopoly is not exploited. Also, separation is exogenous in our framework.[5]

2 Motivating Facts

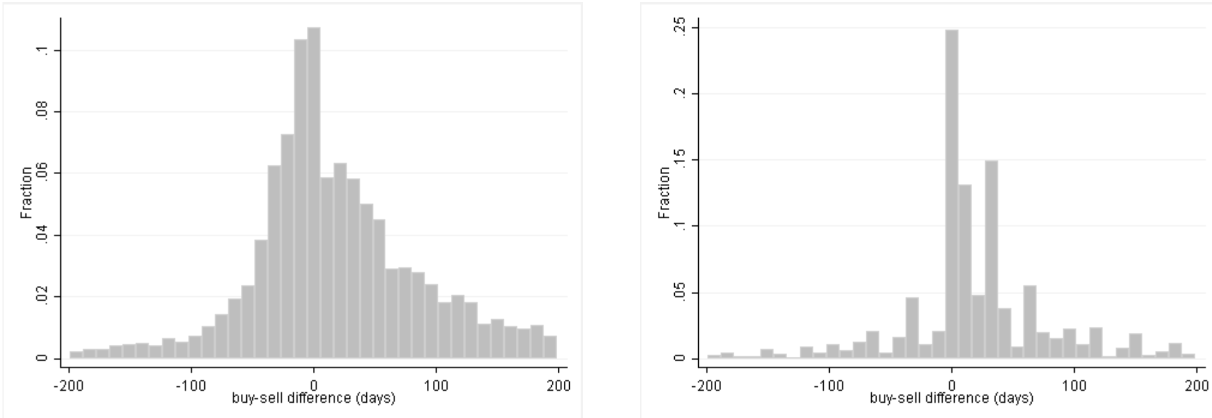
We combine information from the Danish ownership register with a record of property sales for each year. The unique owner and property identifiers give us a matched property-owner data set, which we use to keep track of the transactions of individuals over time. We use the ownership records of individual owners over time to identify owner-occupiers who buy and sell in Copenhagen.[6] We then use the property sales record to determine the agreement dates (the dates the sale agreement

buy and sell housing and, so, there is no transaction sequence decision.

[4]Similar papers include Drazen (1988), Diamond and Fudenberg (1989), Mortensen (1989), Howitt and McAfee (1992), Boldrin et al. (1993), Mortensen (1999), Kaplan and Menzio (2016), Chéron and Decreuse (2017), and Sniekers (2018), among others.

[5]The working paper version of our paper (Moen et al. (2015)) provides a discussion of the institutional details of transacting for several countries. There, we argue that our model captures essential elements of housing transactions for many countries, including Denmark, Norway, the Netherlands, and the United States. In these countries, the institutional set-up for the process of housing transactions is such that homeowners are concerned about the order of buying and selling, at least to some extent.

[6]The Online Appendix contains detailed information on the data used and on the procedure for identifying owner-occupiers that buy and sell. Given the way we identify these owner-occupiers, we have a consistent count for the number of owners who buy first or sell first for the period 1993-2008.



(a) (b)

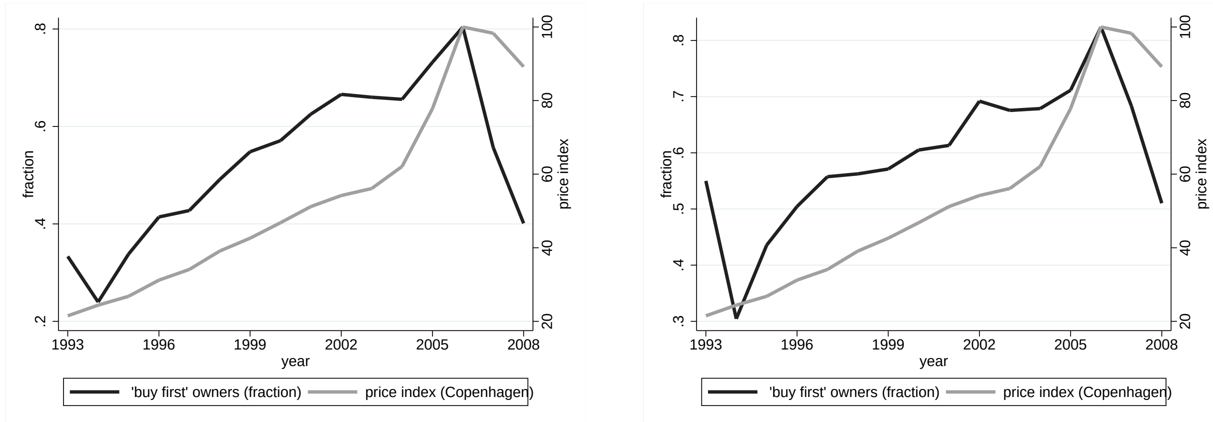
Figure 1: Distribution of the time difference between "sell" and "buy" agreement dates (a) and closing dates (b) for homeowners who both buy and sell in Copenhagen (1993-2008). The distributions are truncated at ±200 days. (Source: Statistics Denmark registry data.)

is signed) and closing dates (the dates the property formally changes ownership) for the two transactions. We use those to measure the time difference between the sale of the old property and purchase of the new property. If the difference is positive, the moving owner buys first, if it is negative – she sells first.

Figure 1 shows that there is substantial dispersion in the difference between both agreement (Panel 1a) and closing dates (Panel 1b). This suggests that a large fraction of moving owners cannot synchronize the two transactions on the same date and that the time difference between transactions can be substantial.

The two distributions are right-skewed, so moving owners tend to buy first on average during 1993-2008. However, as we show in Figure 2, the share of buy-first owners is not constant over time but exhibits large fluctuations. These fluctuations appear to be related to changes in housing market conditions proxied by changes in house prices.

A closer examination of the period 2004-2008 show this link more clearly. Figure 3 illustrates the fluctuations in housing market variables such as time-on-market, the for-sale inventory, sales volume, and prices for Copenhagen in that period. It also includes our constructed series for the



(a) (b)

Figure 2: Fraction of buy-first owners and house prices. Copenhagen, 1993-2008. Own calculations based on registry data from Statistics Denmark. Panel (a) is based on agreement dates, and panel (b) is based on closing dates. See the Online Appendix for a description on how we identify an owner that buys and sells in Copenhagen as a buy-first (sell-first) owner. The price index is a repeat sales price index for single family houses for Copenhagen (Region Hovedstaden) constructed by Statistics Denmark.

fraction of buy-first owners.

Finally, the negative relation between seller time-on-market and the fraction of buy-first owners holds beyond 2004-2008. Moreover, there is a *positive* relation between buyer time-on-market and the fraction of buy-first owners. To show this, we construct a proxy for time-to-buy by taking the average time between the sell and buy transactions for owners that sell first and who complete their second transaction in a given quarter. We also construct a similar proxy for time-to-sell.[7] Table 1 shows that there is a strong positive (negative) relation between time-to-buy (time-to-sell) and the fraction of buy-first owners. A 10 percentage point increase in the fraction of buy-first owners increases (decreases) time-to-buy (time-to-sell) by around 14% (10%).

To summarize, we show that there is a large time difference between the buying and selling transactions of moving owners, and that the fraction of buy first owners comoves with transaction rates and the state of the housing market.

[7]The proxy for time-to-sell constructed this way is strongly positively correlated with time-on-market during 2004-2008.

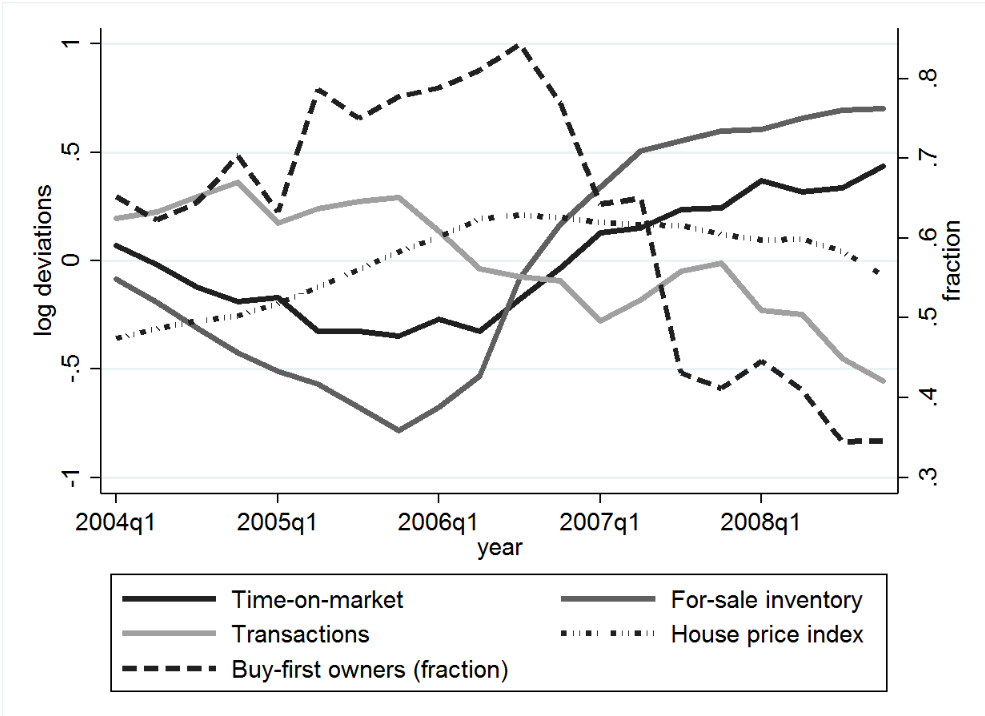


Figure 3: Housing market dynamics, Copenhagen Q12004-Q42008. (Sources: Seller time-on-market and for-sale inventory (in log-deviations from sample mean): from the Danish Mortgage Banks' Federation, available at http://statistik.realkreditforeningen.dk/BMSDefault.aspx); transactions and price index (in log-deviations from sample mean after controlling for seasonality effects) from Statistics Denmark. The fraction of buy first owners is based on registry data from Statistics Denmark.)

Table 1: Relation between time-to-sell (time-to-buy) and the fraction of buy-first owners. Copenhagen, Q3:1993-Q4:2008.

	time-to-sell (log)	time-to-buy (log)
buy-first owners	-0.969**	1.375**
(fraction)	(0.256)	(0.314)
Observations	62	62
Linear time trend	Yes	Yes

Notes. Newey-West standard errors with 5 lags in parenthesis. Quarterly data for Copenhagen for the period Q3:1993-Q4-2008. "Buy-first owners" denotes the fraction of transacting owners that are observed to buy a new property before selling their own house in a given quarter. Time-to-sell is the average time between buy and sell transactions for buy-first owners who complete the second transaction in the quarter. Time-to-buy is the average time between buy and sell transactions for sell-first owners who complete the second transaction in the quarter. ** denotes significance at 1%.

2.1 Why an Explicit Transaction Sequence Choice?

Before proceeding with our model, we make an important conceptual point. Suppose that rather than explicitly choosing how to conduct the sequence of transactions, moving owners always enter both sides of the market simultaneously, and simply take whichever trading opportunity comes first. Thus, they are *observed* to "buy first" whenever they happen to meet a seller before a buyer and vice versa. We call this a simultaneous search strategy.

Suppose that the market is frictional and there are trading delays. Suppose further that there is a (weakly) negative relation between time-to-buy and time-to-sell. With simultaneous search, a higher time-to-sell and a lower time-to-buy imply that buying happens more often and selling happens less often, so the fraction of owners observed to buy first *increases*. Put differently, with simultaneous search, the fraction of buy-first owners and time-to-sell (buy) should be positively (negatively) related – one should observe fewer buy-first owners whenever time-to-sell is low and time-to-buy is high and vice versa. However, this is counterfactual, in view of Figure 3 and Table 1.[8]

This simple example shows (without reference to the optimization decision of agents) that to be consistent with the data, moving owners must explicitly choose to (predominantly) search only on one side of the market, thus steering the sequence of their transactions, rather than to search simultaneously on both sides and having the sequence of their transactions be determined by the

[8] To show this formally, suppose we have a constant returns to scale matching function, and denote the buyer-seller ratio by θ, the rate at which sellers meet buyers by $\mu(\theta)$, where $\mu(\theta)$ is increasing in θ, and the rate at which a buyer meets a seller by $q(\theta) = \mu(\theta)/\theta$, where $q(\theta)$ is decreasing in θ. We assume that all meetings lead to a sale (or that the probability of a sale is independent of θ). If owners follow a simultaneous search strategy, in steady state, the ratio of buy-first owners relative to sell-first owners is

$$\frac{q(\theta)}{\mu(\theta)} = \frac{1}{\theta}. \tag{1}$$

Therefore, this ratio is decreasing in θ. Since θ and the average time-to-sell, $1/\mu(\theta)$, are negatively related, the fraction of buy-first owners and time-to-sell should move in tandem as θ changes. Similarly, since θ and the average time-to-buy, $1/q(\theta)$, are positively related, the fraction of buy-first owners and time-to-buy should be negatively related as θ changes.

exogenous arrival of trading counterparties. Moreover, as we show in our model, when agents' optimizing decisions are taken into account, under a naturally satisfied parametric assumption on preferences, moving owners rationally choose to bias their search towards one side of the market in a way that leads to aggregate behavior that is consistent with the data.

3 Model

In this section, we set up the basic model of a housing market characterized by trading frictions and re-trading shocks that will provide the main insights of our analysis.

3.1 Agents and Environment

Preferences. Time is continuous. The housing market consists of a unit measure of durable housing units that do not depreciate, and a unit measure of households, which we refer to as agents. The agents are risk neutral and can borrow and lend freely at interest rate $r > 0$. When an agent buys a house and becomes a homeowner, he receives a flow utility of $u > 0$. We say that the homeowner is *matched*. With a Poisson rate γ the matched homeowner is hit by a taste shock, and becomes *mismatched* with his current housing unit. In that case the homeowner obtains a flow utility of $u - \chi$, for $0 < \chi < u$. A mismatched owner has to move to another house to become matched again.

A mismatched owner can choose to *sell first* (and become a *mismatched seller*) – selling the housing unit he owns first and then buying a new one. Alternatively, he can choose to *buy first* (becoming a *mismatched buyer*) – buying a new housing unit first and then selling his old one.[9] He can also choose not to enter the housing market and to remain mismatched. A mismatched buyer

[9] In Section 5.3, we explicitly allow a mismatched owner to search as a buyer and seller simultaneously, subject to a fixed time endowment, and show that restriction to either only buying first or selling first is without loss of generality in this case.

ends up holding two housing units simultaneously for some period. In this case we say that he becomes a *double owner*. Similarly, a mismatched seller ends up owning no housing. In that case he becomes a *forced renter*.

The utility flows during the transaction period (when the agent is a double owner or a forced renter) are central for our results. We assume that a double owner receives a flow utility of $u_2 < u$, while a forced renter receives a flow utility of $u_0 < u$. These flows do not include the cost of renting a house for a forced renter, or rental income from renting out the second house for the double owner (as will be clear below, we assume that the double owner rents out the second housing unit).

The utility flows include effort costs of renting in/out a house caused by unmodelled frictions in the rental market and inconveniences from living in temporary dwellings. The utility flows also include financing costs in excess of the interest rate and uninsurable risk associated with housing price volatility, caused by unmodelled frictions in financial markets. We will particularly emphasize the latter, uninsurable risk associated with housing price volatility during the transaction period. If prices increase during the transaction period, a forced renter would experience a capital loss. A double owner, by contrast, would experience a capital gain. As long as the expected price stays constant, price risk would not influence the expected utility of risk neutral agents. However, in a richer framework, with risk-averse agents, this exposure to price changes in the transaction period would be costly. For tractability, we also assume that a double owner does not experience mismatch shocks. This ensures that an agent will not hold more than two housing units in equilibrium.

Agents are born (enter) and die (exit) at the same rate g. New entrants start out their life without owning housing, and receive a flow utility $u_n < u$. Also, we assume that $u_n \geq u_0$, so that forced renters do not obtain a higher utility flow than new entrants. After a death/exit shock, an agent exits the economy immediately and obtains a reservation utility normalized to 0. If he owns housing, his housing units are taken over by a real-estate firm, which immediately places them for sale on the market.[10] Real-estate firms are owned by all the agents in the economy, and the ownership shares of exiting agents are distributed to the rest of the agents in the economy. Given the exit shock, agents effectively discount future flow payoffs at rate $\rho \equiv r + g$.

Finally, agents without a house rent a unit. A landlord can simultaneously rent out a unit and have it up for sale. Hence, double owners rent out one of their units, as do real-estate firms. The rental price is denoted by R.

Trading frictions and aggregate consistency. The housing market is subject to trading frictions. These frictions are captured by a standard constant returns to scale matching function $m(B(t), S(t))$, mapping a stock $B(t)$ of searching buyers and a stock $S(t)$ of searching sellers at time t to a flow m of new matches. We define the market tightness in the housing market as the buyer-seller ratio, $\theta(t) \equiv B(t)/S(t)$. Additionally, $\mu(\theta(t)) \equiv m(B(t)/S(t), 1) = m(B(t), S(t))/S(t)$ is the Poisson rate with which a seller meets a buyer. Similarly, $q(\theta(t)) \equiv m(B(t), S(t))/B(t) = \mu(\theta(t))/\theta(t)$ is the rate with which a buyer meets a seller.

Denote the stock of new entrants by $B_n(t)$, of matched owners by $O(t)$, of mismatched buyers by $B_1(t)$, of mismatched sellers by $S_1(t)$, of double owners by $S_2(t)$, of forced renters by $B_0(t)$, and of the housing units sold by real-estate firms by $A(t)$. The total measure of buyers is $B(t) = B_n(t) + B_0(t) + B_1(t)$ and the total measure of sellers is $S(t) = S_1(t) + S_2(t) + A(t)$. Since the total population is constant and equal to 1 in every instant, it follows that

$$B_n(t) + B_0(t) + B_1(t) + S_1(t) + S_2(t) + O(t) = 1. \tag{2}$$

Also, since the housing stock does not shrink or expand over time, the following housing ownership

[10] For simplicity, we assume that exiting agents are not compensated for their housing. In Section 5.3, we discuss the case where real-estate firms compensate them upon exit.

condition holds in every instant,

$$O(t) + B_1(t) + S_1(t) + A(t) + 2S_2(t) = 1. \tag{3}$$

House price and rental price determination. We begin our analysis by assuming that transactions take place at a common price p, and that there is a common rental price R. Furthermore, we assume that p and R are both independent of the market tightness or move in tandem so that $\rho p - R$ is constant, in which case price changes do not influence the transaction sequence decision. However, in the equilibria we consider, all actively trading pairs are willing to trade at price p. Endogenous price determination is analyzed in detail later on. The main insights of our analysis hold with endogenous prices as well, although at a significant reduction in tractability.[11][12]

3.2 Value Functions

We use $V^x(t)$ (and $\dot{V}^x$), to denote the value function (and its time derivative) for a new entrant ($x = Bn$), a forced renter ($x = B0$), a mismatched buyer or seller ($x = B1$ or $x = S1$), a double owner ($x = S2$) and real-estate firm holding one housing unit ($x = A$). Finally, we denote the value function of a matched owner by $V(t)$ (and $\dot{V}$ for its time derivative). Given this notation, we have a standard set of Hamilton-Jacobi-Bellman equations for the agents' value functions.

First of all, for a mismatched buyer we have

[11] Although the assumption that house prices are independent of θ is made for convenience, it may also be an equilibrium outcome in some environments. Given that the price is assumed to lie in the bargaining sets of all trading pairs, it can be derived as the market clearing price in a competitive market with frictional entry of traders. In particular, as in Duffie et al. (2005) or Rocheteau and Wright (2005), the total measure of participants in that competitive market is determined by the matching function $M(B, S)$. The transaction price in our case will be indeterminate, and this opens up for a price that is independent of θ. Also, under certain conditions, a unique fixed price that does not vary with tightness or across trading pairs can be microfounded as resulting from bargaining between heterogeneous buyers and sellers. See the working paper version of our paper (Moen et al., 2015) for this microfoundation.

[12] In this paper, we do not explicitly model the rental market. Since there are equally many houses as there are agents in the economy, and all houses are either occupied by the owner or rented out, the supply of houses for rent is equal to the demand for houses for rent independently of the price and independently of the transaction sequence of the agents. Thus, if the rental market is competitive, the rental price is indeterminate.

$$\rho V^{B1}(t) = u - \chi + q\left(\theta(t)\right) \max\left\{-p + V^{S2}(t) - V^{B1}(t), 0\right\} + \dot{V}^{B1}, \tag{4}$$

where $u - \chi$ is the flow utility from being mismatched. Upon matching with a seller, a mismatched buyer can purchase a housing unit at price p, in which case he becomes a double owner, incurring a utility change of $V^{S2}(t) - V^{B1}(t)$.

A double owner has a flow utility of $u_2 + R$ while searching for a counterparty. Upon finding a buyer, he can sell his second unit and become a matched owner. Therefore, his value function satisfies the equation

$$\rho V^{S2}(t) = u_2 + R + \mu\left(\theta(t)\right) \max\left\{p + V(t) - V^{S2}(t), 0\right\} + \dot{V}^{S2}. \tag{5}$$

The value function of a mismatched seller is analogous to that of a mismatched buyer apart from the fact that a mismatched seller enters on the seller side of the market first and upon transacting becomes a forced renter. Therefore,

$$\rho V^{S1}(t) = u - \chi + \mu\left(\theta(t)\right) \max\left\{p + V^{B0}(t) - V^{S1}(t), 0\right\} + \dot{V}^{S1}, \tag{6}$$

and

$$\rho V^{B0}(t) = u_0 - R + q\left(\theta(t)\right) \max\left\{-p + V(t) - V^{B0}(t), 0\right\} + \dot{V}^{B0}. \tag{7}$$

Analogously, the value function for a new entrant satisfies

$$\rho V^{Bn}(t) = u_n - R + q\left(\theta(t)\right) \max\left\{-p + V(t) - V^{Bn}(t), 0\right\} + \dot{V}^{Bn}. \tag{8}$$

Finally, the value functions for a matched owner and a real estate firm satisfy

$$\rho V(t) = u + \gamma \left(\max\left\{V^{B1}(t), V^{S1}(t)\right\} - V(t)\right) + \dot{V}, \tag{9}$$

and

$$\rho V^{A}(t) = R + \mu\left(\theta(t)\right) \max\left\{p - V^{A}(t), 0\right\} + \dot{V}^{A}. \tag{10}$$

3.3 Parametric Assumptions

We will characterize equilibria in Section 4 under three parametric assumptions.

Assumption A1: $\underline{p} \leq p \leq \bar{p}$, where $\rho\underline{p} = R$ and $\rho\bar{p} = R + \rho\tilde{V} - u_n$, with $\tilde{V} \equiv \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}$.

Here, $\tilde{V}$ denotes the value of a matched owner who never transacts upon becoming mismatched. Assumption A1 is a condition on the house price p that ensures that real estate firms prefer to sell their houses, while new entrants and forced renters prefer to buy a house.

We next define the effective utility flow for a forced renter as $\tilde{u}_0 \equiv u_0 + \Delta$, and for a double owner as $\tilde{u}_2 \equiv u_2 - \Delta$, where

$$\Delta \equiv \rho p - R. \tag{11}$$

Given Assumption A1, $\tilde{u}_0 \leq \bar{u}_0 \equiv u_0 + \rho\bar{p} - R$ and $\tilde{u}_2 \leq u_2$. We assume further that

Assumption A2: $u - \chi \geq \max\left\{\bar{u}_0, u_2\right\}$.

Assumption A2 is the most important parametric assumption that we make. It states that being mismatched gives a higher (effective) utility flow than being a double owner or a forced renter. Put differently, the utility flow to an agent during the transaction period is lower than when living mismatched in his own house. We view this assumption as empirically relevant and realistic, since anecdotal evidence points to the mismatch state as not particularly costly for the majority of homeowners. In contrast, delays between transactions can be particularly costly for moving homeowners and expose them to a range of costs, which we already discussed. In addition, together with Assumption A1 above, this assumption ensures that double owners prefer to sell one of their houses.

Note that a higher house price increases the effective utility flow of a forced renter and makes it more attractive to sell first (due to discounting), while a higher rental price makes it more attractive to buy first. When $R = \rho p$ and $\Delta = 0$, the two effects cancel out and the house price does not influence the flow value (including incomes/expenses from renting) of double owners or forced renters. In that case, Assumption A2 can be written as $u - \chi > \max\{u_0, u_2\}$. If, in addition, $u_0 = u_2 = c$, the assumption simplifies further to $u - \chi > c$.

Finally, we rule out uninteresting equilibria in which mismatched owners never transact, by making the following assumption

Assumption A3: $\frac{u-\chi}{\rho} < \frac{u_n+u_2}{\rho+\mu(\hat{\theta})} + \frac{\mu(\hat{\theta})-\rho}{\rho+\mu(\hat{\theta})}\tilde{V}$, with $\hat{\theta} \equiv \frac{u-\chi-u_2}{u-\chi-u_0}$.

Assumption A3 imposes a lower bound on the utility of mismatched owners during the transaction period when they have zero or two houses. That utility is lowest when the market tightness is $\hat{\theta}$.[13]

[13] All three assumptions are mutually consistent for a subset of parameters, since A1 does not depend on u_2 and neither A1 nor A2 depend on $\mu(.)$.

4 Equilibria

We start by characterizing steady state equilibria of this economy.[14] Section 4.1 contains our main theoretical result. We then proceed to discuss dynamic equilibria and equilibrium dynamics in Section 4.2. Throughout this section we maintain Assumptions A1 through A3.

4.1 Steady State Equilibria

We characterize steady state equilibria in two steps. First, we look at the optimal choice of mismatched owners. Then we examine the steady state stock-flow conditions.

4.1.1 Optimal Choice of Mismatched Owner

In a steady state equilibrium, the optimal decision of mismatched owners depends on the simple comparison

$$V^{B1} \gtreqless V^{S1}. \tag{12}$$

We define $D(\theta) \equiv V^{B1} - V^{S1}$ as the difference in value between buying first and selling first. Suppose it is optimal for a mismatched buyer and mismatched seller to transact (in both transactions that they have to undertake). Then we can write $D(\theta)$ as

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))}\left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2\right]. \tag{13}$$

In the case where $\tilde{u}_0 = \tilde{u}_2 = c$, equation (13) simplifies to

$$D(\theta) = \frac{(\mu(\theta) - q(\theta))(u - \chi - c)}{(\rho + q(\theta))(\rho + \mu(\theta))}. \tag{14}$$

[14] We define an equilibrium for this economy in Appendix A.

In this simple case, buying first is preferred whenever $\mu(\theta) > q(\theta)$. The (expected) time-on-market for a buyer and a seller are $1/q(\theta)$ and $1/\mu(\theta)$, respectively. Therefore, buying first is preferred, if and only if, time-on-market is higher for a buyer than for a seller. Intuitively, a mismatched owner has to complete two transactions on both sides of the market. Since it is more costly to be a double owner or a forced renter than to be mismatched, a mismatched owner wants to minimize the delay between the two transactions. A low seller time-on-market thus favors buying first.

We now formally characterize the optimal action of a mismatched owner given a steady state market tightness θ. We adopt the notation $\theta = \infty$ for the case where the buyer-seller ratio is unbounded. We define

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2}{u - \chi - \tilde{u}_0}. \tag{15}$$

Note that if $\tilde{u}_2 = \tilde{u}_0$, then $\tilde{\theta} = 1$, while if $\tilde{u}_2 > \tilde{u}_0$, then $\tilde{\theta} < 1$, and vice versa if $\tilde{u}_2 < \tilde{u}_0$.

The following lemma fully characterizes the incentives of mismatched owners to buy first or sell first given a steady state market tightness θ.

Lemma 1. *Let $\tilde{\theta}$ be as defined in (15). Then for $\theta \in (0, \infty)$, $\theta > \tilde{\theta} \Longleftrightarrow V^{B1} > V^{S1}$ and $\theta = \tilde{\theta} \Longleftrightarrow V^{B1} = V^{S1}$.*

Proof. See Appendix B. □

Lemma 1 shows that, in general, as θ increases, the incentives to buy first are strengthened. For sufficiently high values of θ, buying first dominates selling first, and vice versa for sufficiently low values of θ. We call this effect of tightness on the transaction sequence decision the queue-length effect.

4.1.2 Steady State Flows and Stocks

We turn next to a description of the steady state stocks and flows of this model. The full set of equations for these flows are included in Appendix A. Here we make some important observations on the stock-flow process. First, combining the population and housing ownership conditions (2) and (3) we get that at any moment

$$B_n(t) + B_0(t) = A(t) + S_2(t). \tag{16}$$

Since there are equally many agents and houses, the stocks of agents without a house (forced renters and new entrants) must be equal to the stock of double owners and real-estate firms, both in and out of steady state. This identity implies that if all mismatched owners buy first (so that there are no forced renters), the steady state market tightness, denoted by $\bar{\theta}$ satisfies

$$\bar{\theta} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n} > 1. \tag{17}$$

We call this steady state, the "Buy first" steady state and the market tightness associated with it the "Buy first" market tightness. Similarly, if $\underline{\theta}$ denotes the steady state market tightness when all mismatched owners sell first (so that there are no double owners), then

$$\underline{\theta} = \frac{B_n + B_0}{A + S_1} = \frac{A}{A + S_1} < 1. \tag{18}$$

We call this the "Sell first" steady state and the respective tightness, the "Sell first" market tightness. Therefore, $\underline{\theta} < 1 < \bar{\theta}$. This points to possibly wide variations in market tightness arising from changes in the behavior of mismatched owners. Lemma 2 characterizes the steady state market tightnesses, $\bar{\theta}$ and $\underline{\theta}$, and shows that the distance between them increases in the mismatch rate γ.

Lemma 2. *Let $\bar{\theta}$ and $\underline{\theta}$ denote the steady-state market tightness when all mismatched owners buy first and sell first, respectively. Then $\bar{\theta}$ and $\underline{\theta}$ are unique. Moreover, $\bar{\theta} > 1$, $\underline{\theta} < 1$, and $\bar{\theta}$ is increasing and $\underline{\theta}$ is decreasing in γ.*

Proof. See Appendix B. □

A small flows example. To illustrate the possibly large differences in market tightness across steady states implied by Lemma 2, it is illustrative to consider a limit economy with small flows, where $g \to 0$ and $\gamma \to 0$ but the ratio $\gamma/g = \kappa$ is kept constant in the limit. One can show that (see the proof of Lemma 2),

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \bar{\theta} = 1 + \kappa, \tag{19}$$

and

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \underline{\theta} = \frac{1}{1 + \kappa}. \tag{20}$$

To gain intuition, consider a buy-first steady state equilibrium. In this limit economy, the ratio of the flow of new entrants into the economy to the flow from B_n to O equals one, as the fraction of new entrants that exit before transacting is zero in the limit. Hence $B_n q(\theta) \approx g$. For the same reason, the ratio of the flow from O to B_1 relative to the flow from B_1 to S_2 equals one. Hence $B_1 q(\theta) \approx \gamma$ (as $O = 1$ in the limit economy). It follows that the ratio of B_1 to B_n is equal to the ratio of the respective inflows, γ/g. Since $\bar{\theta} = (B_n + B_1)/B_n$, the result follows. An analogous argument applies in the sell-first equilibrium.

This small flows example suggests that the more important mismatched owners are in housing transactions (the higher is $\kappa = \gamma/g$), the larger the variation in market tightnesses from changes in mismatched owners' actions. This example shows that the transaction order of existing homeowners can have potentially large effects on the stock-flow process of the housing market.

4.1.3 Equilibrium Characterization

We now combine the observations on the optimal choice of mismatched owners and the steady state market tightness from the previous two sections to characterize steady state equilibria of our model.

Proposition 1. *Consider the above economy. Let* $\tilde{\theta}$ *be defined by condition (15), and* $\bar{\theta}$ *and* $\underline{\theta}$ *be defined by (17) and (18), with* $\bar{\theta}$, $\underline{\theta} \in (0, \infty)$.

1. *If* $\tilde{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$, *the model exhibits multiple steady state equilibria: an equilibrium with* $\theta = \bar{\theta}$, *in which mismatched owners buy first (a "Buy first" equilibrium); an equilibrium with* $\theta = \underline{\theta}$, *in which mismatched owners sell first (a "Sell first" equilibrium); and an equilibrium with* $\theta = \tilde{\theta}$, *in which mismatched owners randomize between buying and selling.*

2. *If* $\tilde{\theta} < \underline{\theta}$, *there exists a unique steady state equilibrium in which all mismatched owners buy first.*

3. *If* $\tilde{\theta} > \bar{\theta}$, *there exists a unique steady state equilibrium in which all mismatched owners sell first.*

Proof. See Appendix B. □

In the special case in which $\tilde{u}_0 = \tilde{u}_2$, multiple steady state equilibria always exists:

Corollary 1. *Consider the above economy and suppose that* $\tilde{u}_0 = \tilde{u}_2 = c$. *Then there exist three steady state equilibria: one with* $\theta = \underline{\theta}$, *in which mismatched owners sell first; one with* $\theta = \bar{\theta}$, *in which mismatched owners buy first; and another with* $\theta = 1$, *in which mismatched owners are indifferent between buying first and selling first, and half of them buy first.*

Proof. See Appendix B. □

Therefore, depending on the flow payoffs $\tilde{u}_0$ and $\tilde{u}_2$, there can exist multiple steady state equilibria or a unique equilibrium. Intuitively, the equilibrium multiplicity arises because the feedback from the transaction sequence decisions of mismatched owners to the steady state equilibrium market tightness creates a form of *strategic complementarity* in their actions. When mismatched owners are buying first, the steady state buyer-seller ratio is high, so that it is individually rational for any mismatched owner to buy first. Conversely, when mismatched owners are selling first, the steady state buyer-seller ratio is low, and it is individually rational to sell first.

A unique steady state equilibrium obtains if flow payoffs $\tilde{u}_0$ and $\tilde{u}_2$ are sufficiently different, so that buying first or selling first is optimal irrespective of any obtainable market tightness.

4.2 Dynamic Equilibria

In a dynamic equilibrium, mismatched owners optimally choose to enter the market as a mismatched buyer or seller, and are free to switch at any time until their first transaction. The stocks of all different agent types evolve endogenously over time, depending on the agents' behaviour and the dynamics of the market tightness, and agents' expectations about market tightness are correct. The full equilibrium definition can be found in Appendix A.

We refer to a buy first trajectory as the trajectory for the stocks $(B_0(t), B_n(t), B_1(t), S_2(t), O(t), A(t))$ that unfolds from a given set of initial conditions (at $t = 0$), if all current and future mismatched owners buy first. Analogously, we define a sell first trajectory as the trajectory for the stocks $(B_0(t), B_n(t), S_1(t), S_2(t), O(t), A(t))$ that unfolds if all current and future mismatched owners sell first. Finally, we define $\theta_b(t)$ and $\theta_s(t)$ as the market tightnesses along the buy first and sell first trajectories, respectively.

A buy first (sell first) trajectory is globally asymptotically stable if for any set of initial conditions, the trajectory converges to the buy first (sell first) steady state equilibrium allocation of the stocks.

Proposition 2. *The buy first and the sell first trajectories are globally asymptotically stable. Moreover, $\theta_b(t)$ converges to $\bar{\theta}$ along the buy first trajectory, while $\theta_s(t)$ converges to $\underline{\theta}$ along the sell first trajectory.*

Proof. See Appendix B. □

A buy first (sell first) trajectory constitutes an equilibrium if agents perfectly foresee its path and if it is optimal for all mismatched owners to buy first (sell first) along it for all t. For the steady state equilibrium analysis we defined a threshold value $\tilde{\theta}$ (Equation 15) with the property that mismatched owners strictly prefer to buy first iff $\theta > \tilde{\theta}$. In a dynamic context, it follows readily that a sufficient condition for mismatched owners to buy first (sell first) along the buy first (sell first) trajectory is that $\theta(t) > \tilde{\theta}$ ($\theta(t) < \tilde{\theta}$), for all t. This is shown in the proof of Proposition 3.

From equations (17) and (18), it follows that $\theta_b(t) > 1$ and $\theta_s(t) < 1$ for all t, as long as there are mismatched owners in the economy (which there will always be with the possible exception at $t = 0$). Hence, in the special case, in which $\tilde{\theta} = 1$, both the buy first and sell first trajectories constitute an equilibrium for any set of initial conditions. Unfortunately, we are not able to derive strong properties for the dynamics of the tightnesses θ_b and θ_s. For instance, if θ_b after a switch to a buy first trajectory is lower than its steady state value, we cannot rule out that it will be decreasing for a period of time before it starts increasing again.

In order to obtain stronger results, we derive bounds on $\theta_b(t)$ and $\theta_s(t)$. In the Appendix (Lemma 1 in the proof of Proposition 3) we show that there exists an interval $\left(\theta^{lb}, \theta^{ub}\right)$ with the property that for any initial allocation of houses over agents, and independently of switches between trajectories, the tightness $\theta(t)$ will reach this interval after some finite amount of time and will always subsequently remain within it. We therefore say that generically, $\theta(t) \in \left(\theta^{lb}, \theta^{ub}\right)$. The values of θ^{ub} and θ^{lb} are equal to the buy first and sell first (steady state) equilibrium values $\bar{\theta}$ and $\underline{\theta}$ in the small flows economy, defined by (19) and (20), respectively.

Knowing that $\theta(t) \in \left(\theta^{lb}, \theta^{ub}\right)$ generically allows us to derive bounds on all the stocks in the economy, including the stock of mismatched owners (see Lemma 2 in the proof of Proposition 3). Again, for any initial allocation of houses over agents, and independently of switches between trajectories, the bounds on the stocks will be satisfied after a finite amount of time. From (17) and (18) we can then derive a lower bound $\theta_b^{\min} > 1$ for $\theta_b(t)$, and an upper bound $\theta_s^{\max} < 1$ for $\theta_s(t)$ (formally defined in Lemma 3 in the proof of Proposition 3), and these will be satisfied whenever the bounds on the stocks are satisfied. We, therefore, say that the bounds on $\theta_b(t)$ and $\theta_s(t)$ are generically satisfied.

Proposition 3. *Suppose we are in the generic situation in which $\theta_b(t) \geq \theta_b^{\min} > 1$ along the buy first trajectory and $\theta_s(t) \leq \theta_s^{\max} < 1$ along the sell first trajectory. Then if $\tilde{\theta} \leq \theta_b^{\min}$, the buy first trajectory constitutes a dynamic equilibrium. If $\tilde{\theta} \geq \theta_s^{\max}$, then the sell first trajectory constitutes a dynamic equilibrium. If $\tilde{\theta} \in \left[\theta_s^{\max}, \theta_b^{\min}\right]$, there exist multiple dynamic equilibria.*

Proof. See Appendix B. □

If there exist multiple dynamic equilibria, there will also exist a mixed equilibrium in which mismatched owners mix between buying first and selling first in such a way that $\theta(t) \equiv \tilde{\theta}$.

The bounds on the stocks, and hence on $\theta(t)$ are somewhat involved and clearly satisfied with a large slack in most situations. In the Online Appendix we, therefore, derive more restrictive bounds θ_b^{high} and θ_s^{low} that are satisfied if the economy is on a buy first or sell first trajectory for all future times.

4.3 Quantitative Relevance

In this section we provide a calibrated numerical example illustrating our mechanism by examining a permanent switch in mismatched owners' behavior from "buying first" to "selling first" and comparing the resulting dynamic path for our economy to the boom-bust episode in Copenhagen during 2004-2008.

The two most important parameters that determine the stock-flow process and the quantitative impact of the switch in the order of transactions are the rate of mismatch, γ, and the entry/exit rate, g. In terms of data counterparts, γ broadly corresponds to owner-owner transitions within a housing market, while g corresponds to moves outside the housing market as well as owner-renter transitions. Unfortunately, there is no available data of transitions within and across housing markets by housing tenure status for Denmark. Instead, there is data on the total transitions within and between municipalities. Similarly, there is no available data on owner-renter transitions. In the Online Appendix, we describe how we supplement the available mobility information from Denmark with information from the USA. Our approach results in a value of $g = 0.0371$ and $\gamma = 0.0322$. These parameter values imply an average duration of ownership of around 14 years and an annual turnover rate of around 7%.

To match the fact that the fraction of owners that "buy first" in Copenhagen never hits 1 or 0, we assume that a certain share of moving owners always enter one side of the market first. Specifically, we assume that 1/4 of newly mismatched owners always "buy first" and similarly that 1/4 always "sell first", while the remaining 1/2 can choose whether to "buy first" or "sell first". One can show (see the proof of Proposition 1) that a steady state market tightness $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ exists for any fraction $x_b \in [0, 1]$ of newly mismatched owners that "buy first".

Finally, we assume that the matching function is Cobb-Douglas, $m(B, S) = \mu_0 B^{\alpha} S^{1-\alpha}$, for $0 < \alpha < 1$, so that $\mu(\theta) = \mu_0 \theta^{\alpha}$. We choose $\alpha = 0.84$, following Genesove and Han (2012).

Table 2: Calibration parameters

Parameter	Value
entry/exit rate (g)	0.0371
mismatch rate (γ)	0.0322
matching function	$3.1\theta^{0.84}$
fraction of newly mismatched that always "buys first"	0.25
fraction of newly mismatched that always "sells first"	0.25

Additionally, we set $\mu_0 = 3.1$, which gives a seller time-on-market of 12 weeks for a steady state market tightness when all mismatched owners (that can switch) buy first, which matches seller time-on-market for Copenhagen during the boom period of 2005-2006. Table 2 summarizes the parameters we use in our numerical example. Here we do not make any assumptions on individual preference parameters (e.g. u, χ, u_0, u_n, and u_2). Consequently we only consider variables that result from the stock-flow process of our model. We examine the behavior of house prices in Section 5.2.

We use these numbers to quantify the effect of a permanent switch from "buying first" to "selling first" (for the owners that can switch). Figure 4 plots the simulated transition path of the economy. Initially, the economy is in a steady state, in which all mismatched owners (that can switch) "buy first". In year 2 these owners permanently switch to "sell first". Such a switch lowers the fraction of "buy first" owners from around 80% to around 20%. In addition, market tightness moves from 1.35 to 0.74. This is associated with an (almost) immediate 65% increase in seller time-on-market and a gradual increase in the for-sale stock of around 64%. In addition, the transaction rate falls by around 18% immediately after the switch. Comparing these changes against the fluctuations for Copenhagen in Figure 3, we see that all variables move together as in the data. A decrease in the fraction of mismatched owners that buy first is associated with an increase in time-on-market for sellers and an increase in the for-sale stock (inventory).[15] The volume of transactions tends to be

[15]Note that the Copenhagen data plots the for-sale inventory which may be quite different from the for-sale stock,

lower after the switch in the model just as in the data. It is particularly interesting to note that upon the switch, transaction volume falls while the for-sale stock increases. Search-based models of the housing market usually tend to make the opposite (and counterfactual) prediction (Diaz and Jerez, 2013).

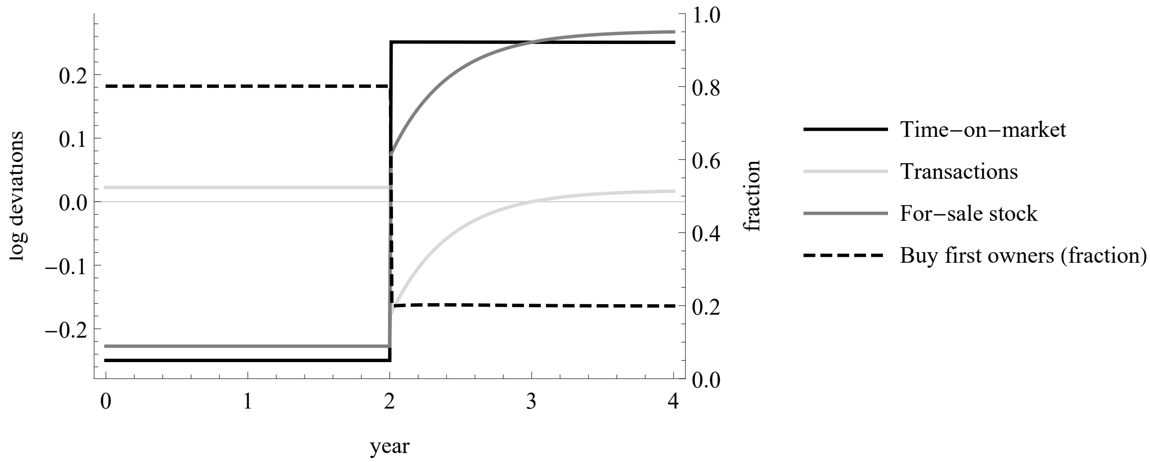


Figure 4: Simulated housing cycle with half of mismatched owners switching from buying first to selling first at $t = 2$.

To quantify how much of the boom-bust episode can be explained by our numerical model, Table 3 compares the log-change in time-to-sell, inventory, and sales in the data for the period 2006Q3-2008Q3 with our simulation. During that period the buy first share fell by 50 percentage points, which is comparable to the decrease in the buy first share in year 2 of our simulation. Quantitatively, the numerical model comes close to explaining the full change of time-on-market and explains almost half of the change in the for-sale inventory. The simulated change in transaction volume is also about half of that in the data. However, the time path for the volume of transactions is quite different. While in the model, transactions recover over time, in the data, transactions fall over time. The reason is that the switch from buying first to selling first, by immediately reducing

since the data is at a quarterly frequency. Still we believe that it is informative to compare the broad patterns of the for-sale inventory against the for-sale stock that the model generates.

Table 3: Data and model comparisons.

	Data	Numerical model
Time-on-market	0.51	0.50
For-sale inventory/stock	0.78	0.30
Volume of transactions	-0.38	-0.20

Notes: Log-differences. Data for Copenhagen, 2006Q3-2008Q3.

the total stock of buyers and increasing the total stock of sellers, only impacts transactions in the short-run. Intuitively, given the matching function we use, when some buyers switch to sellers, transactions fall even if there is the same total number of agents trading, since buyers are more important in matching than sellers ($\alpha > 1/2$). Over time, however, the stock of sellers increases given the low market tightness which helps transactions almost recover to their pre-switch level.[16]

Finally, it is interesting to point out that the tightness in the limit example introduced in Section 4.1.2 provides a very good approximation to the dynamic transitions of our economy (assuming again that everyone either buys first or sells first, just as in the limit economy). The calibrated values of γ and g give us values for market tightness in the limit economy of $\lim \bar{\theta} \approx 1.87$ and $\lim \underline{\theta} \approx 0.54$, respectively. This is compared against $\bar{\theta} \approx 1.84$ and $\underline{\theta} \approx 0.54$ in the true economy without mismatched agents exogenously buying or selling first. In addition, tightness converges quickly to its steady state level upon a switch in the behavior of mismatched owners, as can be inferred from the dynamics of seller time-on-market in Figure 3, which only depends on the dynamics of tightness (see also Figure C.3 in the Online Appendix). This should not be surprising in view of the small annual transition rates inherent in γ and g. In the next section we present some extensions that leverage on this property of the limit economy.

[16]The discrepancy between the dynamics of transactions in the model and in the data suggests that there is an additional mechanism that is important for driving transactions, which our model does not have. For example, heterogeneity in mismatch and an endogenous decision of mismatched owners whether to transact or not as in Ngai and Sheedy (2015) may be important for explaining the transaction volume.

5 Robustness and Extensions

In this section, we first clarify the existence of multiple steady state equilibria without the assumption that prices are fixed across steady states, and show that there can be equilibrium multiplicity in an environment where prices are determined by Nash bargaining. Afterwards, we discuss a number of additional extensions of the benchmark model.

5.1 Prices Depend on Market Tightness

In Section 4, we assumed that prices were exogenous and, hence, unaltered by the transaction sequence decision of the agents. Now, we allow steady state housing prices to depend on θ. We thus write $p = p(\theta)$, with $p'(\theta) > 0$. We may also write the rent R as a function of θ, $R = R(\theta)$, with $R'(\theta) \geq 0$. Recall that the effective utility flows of double owners and forced renters depend on $\Delta = \rho p - R$. Thus, we can write $\Delta = \Delta(\theta)$.

As already mentioned in Section 3.3, when $R = \rho p$, $\Delta'(\theta) = 0$ and, so, assuming that house prices are exogenous is without loss of generality, since different (steady state) prices do not influence the flow value of double owners or forced renters. However, a countervailing effect arises if $\Delta'(\theta) > 0$. Specifically, from Equation (13), the decision whether to buy or to sell first depends on the sign of the following expression:

$$\tilde{D}(\theta) = \frac{\theta - 1}{\theta}\left(u - \chi - u_2 + \Delta(\theta)\right) + u_2 - u_0 - 2\Delta(\theta).$$

We normalize $u_2 - u_0$ so that $\tilde{D}(1) = 0$, which requires that $u_2 - u_0 = 2\Delta(1)$.

In order for the "Buy first" and "Sell first" equilibria to exist, we must have that:

$$\frac{\bar{\theta} - 1}{\bar{\theta}}[u - \chi - u_2 + \Delta(\bar{\theta})] \geq 2(\Delta(\bar{\theta}) - \Delta(1)), \tag{21}$$

and

$$\frac{1-\underline{\theta}}{\underline{\theta}}[u-\chi-u_2+\Delta(\underline{\theta})] \geq 2(\Delta(1)-\Delta(\underline{\theta})). \tag{22}$$

In the two conditions above, the left-hand side broadly reflects the queue-length effect from Lemma 1. The right-hand sides of (21) and (22) reflect how a higher value of θ changes the difference in the flow values of a forced renter compared to a double owner. If $R \leq \rho p$, it is beneficial for the agent, everything else equal, to buy late and sell early. We refer to this as a *discounting effect*. A higher θ increases p, and, unless R increases at the same rate (so that $\Delta'(\theta) > 0$), this strengthens the discounting effect and makes it more attractive to sell first. Conversely, when rental prices are more responsive to θ than housing prices, so that $\Delta'(\theta) < 0$, a higher θ weakens the discounting effect and makes it more attractive to buy first, so that there is no countervailing effect in that case.

Therefore, our multiple equilibria result requires that housing prices should not be too sensitive to changes in θ compared to rental prices, so that the discounting effect is always weaker than the queue-length effect. Notice that conditions (21) and (22) always hold if u_2 (and u_0, given the normalization) is sufficiently low. In that case, the queue-length effect always dominates for values of θ that are consistent with a steady state equilibrium.

5.2 Prices Determined by Nash Bargaining

In this section we assume that prices are determined by symmetric Nash bargaining. Therefore, there is no longer a single transaction price, p, but prices depend on the types of trading counterparties. This leads to two complications. First, generally, match surpluses need not be positive for all counterparties so, depending on parameter values, there may be many possible equilibrium matching sets of counterparties that prefer trading over not trading. Second, the composition of buyers and sellers (and the expected payoff of agents) are determined by the stock-flow process of the economy.

To simplify the composition of buyers and sellers, we consider the small flows economy introduced in Section 4.1.2. In addition, we focus on parametric assumptions (Assumptions B2 and B3 in Proposition 4), which ensure that all trading surpluses are positive in both a "Buy first" and "Sell first" equilibrium, except for the surplus between mismatched buyers and sellers. Finally, for simplicity, we consider parameter values for which mismatched owners are indifferent between buying first and selling first at $\theta = 1$ (Assumption B1 in Proposition 4).[17]

In the Online Appendix we further discuss the characterization of the "Buy first" and "Sell first" equilibria and the relevant economic forces. There we argue that in the case of Nash bargaining two effects arise which strengthen the incentives to buy first as market tightness increases above one (and vice versa when market tightness decreases below one). The first effect is tightly linked to the queue-length effect from Section 4, while the second effect is a compositional effect that does not arise in that model, since it assumes a single transaction price.[18]

The following proposition establishes that the model with Nash bargaining may exhibit multiple equilibria.

Proposition 4. *Consider the limit economy with $g \to 0$, $\gamma \to 0$ and $\gamma/g = \kappa$, with prices determined by symmetric Nash bargaining, and suppose that the following conditions hold:*

Assumption B1: $u_2 - u_0 = u - u_n$.

Assumption B2: $r\left(u_2 - (u - \chi)\right) + \frac{1}{2}\mu_0\chi > 0$.

Assumption B3: $r\left(u_2 - (u - \chi)\right) + \frac{1}{2}\mu_0\chi \leq \frac{1}{2}r\left(u_2 - u_0\right)$.

Then there exists a $\kappa^* > 0$, such that for $\kappa \leq \kappa^*$, there exists a steady state equilibrium in which

[17] The reason that one needs an additional condition to "restore symmetry" under Nash bargaining even with $u_2 = u_0$ is that the real-estate firms are different from the other agents, as they receive no utility from owning a house, and their gain from transacting is the price, which is a transfer, and hence, does not affect match surplus. As a result, the equilibrium allocation becomes asymmetric, and tilts towards the "Buy first" equilibrium even with $u_2 = u_0$.

[18] A similar compositional effect arises in Albrecht et al. (2007). However, entry of buyers and sellers is exogenous in their model and there is no transaction sequence decision.

all mismatched owners buy first and the equilibrium market tightness is $\theta = 1 + \kappa$. *Also, for* $\kappa \leq \kappa^*$ *there exists a steady state equilibrium, in which all mismatched owners sell first and the equilibrium market tightness is* $\theta = 1/(1+\kappa)$.

Proof. See Online Appendix D. □

The restrictions in Proposition 4 are only sufficient conditions and the same matching sets may emerge for more general parameter values. Also, we conjecture that if these restrictions are not satisfied, and other matching sets emerge, these may also have multiple steady state equilibria with the structure described in Proposition 4.

To gain some additional insight into when multiple equilibria are possible in the case of Nash bargained prices, we consider a simple numerical example.[19] Figure 5 plots the difference $D(\theta) = V^{B1} - V^{S1}$ in a candidate "Sell first" equilibrium for $\theta \leq 1$ and a candidate "Buy first" equilibrium for $\theta > 1$.

The figure shows that multiple steady state equilibria can be sustained even if the market tightness is very different in the "Buy first" and "Sell first" equilibria. Using the stock-flow calibration from Section 4.3 produces substantial variation in market tightness across the two steady state equilibria as illustrated by the red and blue vertical lines in the figure. This also means that there can be large differences in house prices across the two steady state equilibria. Specifically, for this numerical example, average house prices decline by around 15% between the "Buy first" and the "Sell first" equilibrium.

The figure also illustrates the discounting effect discussed in Section 5.1. Specifically, for a very low market tightness, buying first starts to dominate selling first, and vice versa for a very high

[19] In this numerical example we do not consider the limit economy and, furthermore, condition B3 is not satisfied. This additionally shows that the conditions from Proposition 4 are only sufficient for multiplicity One can also relax the symmetry condition B1. Relaxing B1 changes the value of θ for which a mismatched owner is indifferent between buying first and selling first. Thus, it essentially shifts the curve in Figure 5 to the left or to the right.

market tightness. In those cases multiple equilibria cease to exist.[20]

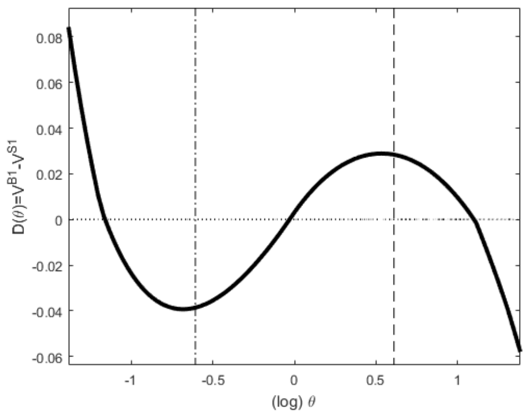


Figure 5: A plot of $D(\theta)$ against (log) θ and steady state market tightnesses in a "Buy first" (dash) and a "Sell first" (dash-dot) equilibrium. Preference parameters used for the example: $r = 0.05$, $u = 2$, $u_0 = 1.55$, $u_2 = 1.85$, $u_n = 1.7$, $\chi = 0.14$. Rental price $R = 0.1$. In addition, we use the calibration from Section 4.3 for the remaining parameters.

5.3 Additional Extensions

House price expectations. So far, we assumed that mismatched owners do not expect house prices to change. In the Online Appendix we examine the implications of expected changes in prices for the behavior of mismatched owners. There we show in a simple example that expected house price increases strengthen the incentives to buy first and expected house price decreases weaken those incentives. Intuitively, an owner that buys first expects to make a capital gain when prices increase in the future, while an owner that sells first expects to incur a capital loss.

This discussion points to a destabilizing effect of house price expectations on the housing market. When prices increase with tightness the destabilizing effect of expectations can give rise to dynamic equilibria with self-fulfilling fluctuations in prices and tightness, as we discuss next.

[20]As the numerical example illustrates, with Nash bargaining, the price response may be very large for the discounting effect to start dominating. Specifically, in the example in Figure 5, average house prices must decline by more than 50% between a candidate "Buy first" and "Sell first" equilibrium for the discounting effect to dominate.

Equilibrium switches. In the Online Appendix we use the small flows economy introduced in Section 4.1.2 to construct (approximate) dynamic equilibria with self-fulfilling fluctuations in prices and tightness. Specifically, we use the insight that when the flows of the economy are small (or when matching efficiency is high), changes in the buy first/sell first decisions of mismatched owners lead to a jump in θ from one steady state value to another with no dynamic adjustment in θ.[21] As in Section 5.1, we assume that prices are an exogenous increasing function of tightness.

We construct an (approximate) regime-switching dynamic equilibrium, in which the economy initially starts in a "Buy first" regime, in which mismatched owners prefer to buy first and the market tightness equals $\bar{\theta}$. With Poisson rate λ, the economy may permanently transition to a "Sell first" regime in which mismatched owners prefer to sell first and tightness equals (approximately) $\underline{\theta}$.

We show that such a dynamic equilibrium exists when agents expect the regime-switch to happen with sufficiently low probability (λ is sufficiently low). The reason that λ cannot be too high in such an equilibrium is that if agents expect the change in regimes to occur sufficiently soon, then by the preceding discussion, it cannot be optimal for mismatched owners to buy first in the "Buy first" regime despite the high market tightness. Instead mismatched owners speculate on house prices decreasing in between their two transactions and choose to sell first.

In this dynamic equilibrium prices and tightness move together upon the regime-switch. Specifically, upon the switch, the expectation that prices will imminently fall, induces mismatched owners to sell first, which leads to a fall in market tightness and thus prices. Finally, we show that for empirically-relevant values of the elasticity of matching with respect to buyers, transaction volume also falls upon the switch. Consistent with our numerical example in Section 4.3, we also show that transaction rates are (approximately) equal in the two steady state equilibria, so transaction

[21]The equilibrium is approximate since, away from the limit, when there is a dynamic adjustment in θ, over time it only gets (arbitrarily) close to the new steady state value.

volume eventually recovers (almost) fully.

Competitive search. Our multiplicity results are derived under the assumption of random search. In the Online Appendix we derive sufficient conditions on parameter values for multiple competitive search equilibria to exist. The economic mechanisms play out in a slightly different way with competitive search than with random search and bargaining. With bargaining, the strategic complementarity works through the market tightness – if more agents buy first, this increases the buyer-seller ratio in the market, and makes it more attractive to buy first. In competitive search equilibrium, the strategic complementarities are more involved. Single agents trade off waiting time and terms of trade. In that environment, more mismatched buyers influence the buyer-seller ratio in the submarket(s) for mismatched buyers. This does not directly affect the mismatched sellers, as they search in different submarkets. However, the asset values of all agents change, thereby influencing the value of selling first and the speed at which mismatched sellers transact.

Simultaneous search. Our benchmark model assumes that a mismatched owner has to choose to enter the housing market either as a buyer or as a seller. In the Online Appendix, we examine the optimal behavior of a mismatched owner that can choose to be both a buyer and a seller at the same time. We show that a mismatched owner strictly prefers to either only enter as a buyer or as a seller for any $\theta \neq \tilde{\theta}$, where $\tilde{\theta}$ is defined as in Equation (15). Intuitively, since the decision to enter as both a buyer and seller depends ultimately on the value from entering as a buyer only and the value from entering as a seller only, whenever entering as a buyer only is dominated by entering as a seller only, then entering as both a buyer and seller is also dominated by entering as a seller only, and vice versa.

Alternative assumptions on exiting the economy. Additionally, we show that our main results continue to hold even assuming that homeowners are compensated for the value of their housing units when they exit the economy when g is sufficiently small. In that case a modified version of Equation (15) determines the critical value of $\tilde{\theta}$ below which a mismatched owner is better off selling first. In the limit, as $g \to 0$, that modified version converges to the value of $\tilde{\theta}$ from Equation (15).

6 Concluding Comments

The transaction sequence decision of moving owners depends on housing market conditions, such as the expected time-on-market for buyers and sellers and expectations about future house price appreciation.[22] However, these decisions in turn influence the buyer-seller ratio of the housing market. This creates a coordination problem for moving owners, resulting in multiple equilibria. Equilibrium switches are associated with large fluctuations in the for-sale stock, time-on-market, transactions, and prices.

The tractable equilibrium model that we study in this paper is deliberately simplified to show these effects, and so lacks heterogeneity in many important dimensions. In particular, there is no heterogeneity in the costs of being a double owner versus a forced renter, which are likely to vary substantially across households and also to vary over time in response to aggregate shocks. In addition, we assumed constancy of the rate of mismatch and entry into and exit from the market. Nevertheless, endogenous fluctuations in these rates are likely to additionally amplify and propagate aggregate shocks. Enriching the model along these dimensions will allow for a detailed quantitative model of the housing market, which can be taken to the data. We view this as an important step for future research.

[22] Similar economic forces would influence the behavior of investors in the housing market provided that they have to rebalance their housing portfolio.

Appendix A: Equilibrium definitions

Before moving to our formal definition, it is necessary to describe the flow conditions that the aggregate stock variables defined in Section 3 must satisfy. In any equilibrium, given a market tightness $\theta(t)$, the values of $B_n(t)$, $B_0(t)$, $B_1(t)$, $S_1(t)$, $S_2(t)$, $O(t)$, and $A(t)$ must satisfy the following system of flow conditions:

$$\dot{B}_n = g - \left(q\left(\theta(t)\right) + g\right) B_n(t), \tag{A.1}$$

$$\dot{B}_0 = \mu\left(\theta(t)\right) S_1(t) - \left(q\left(\theta(t)\right) + g\right) B_0(t), \tag{A.2}$$

$$\dot{O} = \mu\left(\theta(t)\right) S_2(t) + q\left(\theta(t)\right)\left(B_n(t) + B_0(t)\right) - \left(\gamma + g\right) O(t), \tag{A.3}$$

$$\dot{B}_1 = \gamma x_b(t) O(t) - \left(q\left(\theta(t)\right) + g\right) B_1(t), \tag{A.4}$$

$$\dot{S}_1 = \gamma x_s(t) O(t) - \left(\mu\left(\theta(t)\right) + g\right) S_1(t), \tag{A.5}$$

$$\dot{S}_2 = q\left(\theta(t)\right) B_1(t) - \left(\mu\left(\theta(t)\right) + g\right) S_2(t), \tag{A.6}$$

$$\dot{A} = g\left(O(t) + B_1(t) + S_1(t) + 2S_2(t)\right) - \mu\left(\theta(t)\right) A(t). \tag{A.7}$$

$$x_b(t) + x_s(t) = 1, \tag{A.8}$$

where $x_b(t)$, and $x_s(t)$ are the equilibrium fractions of newly mismatched buyers and sellers, respectively. Finally, the equilibrium market tightness $\theta(t)$, satisfies

$$\theta(t) = \frac{B(t)}{S(t)} = \frac{B_n(t) + B_0(t) + B_1(t)}{S_1(t) + S_2(t) + A(t)}. \tag{A.9}$$

Definition 1. Given a house price p and a rental price R, a dynamic equilibrium consists of value functions $V^{Bn}(t)$, $V^{B0}(t)$, $V^{B1}(t)$, $V^{S2}(t)$, $V^{S1}(t)$, $V(t)$, $V^{A}(t)$, and functions for market tightness $\theta(t)$, expected market tightness $\theta(t)^e$, fractions of mismatched owners that choose to buy first and sell first, $x_b(t)$, and $x_s(t)$, and aggregate stock variables, $B_n(t)$, $B_0(t)$, $B_1(t)$, $S_1(t)$, $S_2(t)$, $O(t)$, and $A(t)$ such that for any time $t \geq 0$:

1. Agents have rational expectations, so that expected market tightness $\theta(t)^e = \theta(t)$ for all t;

2. The value functions satisfy equations (4)-(10) given $\theta(t)$;

3. Mismatched owners choose $x(t) \in \{b, s\}$, to maximize $\bar{V}(t) = \max\left\{V^{B1}(t), V^{S1}(t)\right\}$ and the fractions $x_b(t)$, and $x_s(t)$ reflect that choice, i.e.

$$x_b(t) = \int_i I\left\{x_i(t) = b\right\} di,$$

   where $i \in [0, 1]$ indexes the i-th mismatched owner, and similarly for $x_s(t)$;

4. The market tightness $\theta(t)$ solves (A.9) given $B_n(t)$, $B_0(t)$, $B_1(t)$, $S_1(t)$, $S_2(t)$, and $A(t)$;

5. The aggregate stock variables $B_n(t)$, $B_0(t)$, $B_1(t)$, $S_1(t)$, $S_2(t)$, $O(t)$, and $A(t)$ satisfy the population constancy and housing ownership conditions (2) and (3), and solve (A.1)-(A.7) given $\theta(t)$ and mismatched owners' optimal decisions reflected in $x_b(t)$ and $x_s(t)$.

We define a steady state equilibrium for this economy in the following way:

Definition 2. A steady state equilibrium is a dynamic equilibrium in which the value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^{A}, and functions for market tightness θ, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, and aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A are constant.

We therefore have that in a steady state equilibrium, given market tightness θ and equilibrium fractions of newly mismatched owners x_b and x_s, the steady state values of B_n, B_0, B_1, S_1, S_2, O, and A must satisfy the following system of flow conditions:

$$g = (q(\theta) + g) B_n, \tag{A.10}$$

$$\mu(\theta) S_1 = (q(\theta) + g) B_0, \tag{A.11}$$

$$\mu(\theta) S_2 + q(\theta)(B_n + B_0) = (\gamma + g) O, \tag{A.12}$$

$$\gamma x_b O = (q(\theta) + g) B_1, \tag{A.13}$$

$$\gamma x_s O = (\mu(\theta) + g) S_1, \tag{A.14}$$

$$q(\theta) B_1 = (\mu(\theta) + g) S_2, \tag{A.15}$$

$$g(O + B_1 + S_1 + 2S_2) = \mu(\theta) A. \tag{A.16}$$

Appendix B: Proofs

Proof of Lemma 1

We start by showing that given assumptions A1 and A2, new entrants, forced renters, double owners, and real estate firms prefer transacting at price p to not transacting. We then use this observation to establish the incentives of mismatched owners.

Note that sufficient conditions for new entrants, forced renters and double owners to prefer transacting and becoming matched owners are given by

$$\frac{u_n - R}{\rho} \leq \tilde{V} - p, \tag{B.1}$$

$$\frac{u_0 - R}{\rho} \leq \tilde{V} - p, \tag{B.2}$$

and

$$\frac{u_2 + R}{\rho} \leq \tilde{V} + p. \tag{B.3}$$

Since $u_n \geq u_0$, we can disregard (B.2), as it is implied by (B.1). Conditions (B.1) and (B.3) imply restrictions on the values of the house price, p, that are sufficient for these agents to be willing to transact at p, namely $p \in \left[\frac{u_2}{\rho} - \tilde{V} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]$. From (10), a real estate firm is willing to transact iff $p \geq \frac{R}{\rho}$. Therefore, all four agents types prefer to transact at price p, whenever

$$p \in \left[\max\left\{\frac{u_2}{\rho} - \tilde{V}, 0\right\} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right].$$

By Assumption A2, $\frac{u_2}{\rho} < \frac{u-\chi}{\rho} < \tilde{V}$ and, so, the set for prices p is as in Assumption A1.

Next, note that the function $D(\theta)$, defined in (13), crosses zero only at $\theta = \tilde{\theta}$. To see this, notice that

$$\lim_{\theta \to 0} D(\theta) = \frac{\tilde{u}_2 - (u - \chi)}{\rho} < 0,$$

and

$$\lim_{\theta \to \infty} D(\theta) = \frac{u - \chi - \tilde{u}_0}{\rho} > 0.$$

Away from these two limiting values, $D(\theta) > 0$, whenever

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 > 0,$$

which is equivalent to $\tilde{\theta} < \theta$. Therefore, $D(\theta) > 0$ iff $\theta \in \left(\tilde{\theta}, \infty\right)$ and $D(\theta) < 0$ iff $\theta \in \left(0, \tilde{\theta}\right)$.

Therefore, $D(\theta) = 0$, iff

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 = 0,$$

or $\theta = \tilde{\theta}$.

We next show that $D(\theta)$ fully summarizes the incentives of a mismatched owner to buy first/sell first apart from at $\theta = 0$ and $\theta = \infty$. To see this, let

$$\begin{aligned}
\tilde{V}^{B1} &= \frac{u-\chi}{\rho + q(\theta)} + \frac{q(\theta)\,\tilde{u}_2}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\,\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))}V - \frac{u-\chi}{\rho} \\
&= \frac{q(\theta)}{\rho + q(\theta)}\left(\frac{\tilde{u}_2}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)}V - \frac{u-\chi}{\rho}\right),
\end{aligned}$$

and

$$\begin{aligned}
\tilde{V}^{S1} &= \frac{u-\chi}{\rho + \mu(\theta)} + \frac{\mu(\theta)\,\tilde{u}_0}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\,\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))}V - \frac{u-\chi}{\rho} \\
&= \frac{\mu(\theta)}{\rho + \mu(\theta)}\left(\frac{\tilde{u}_0}{\rho + q(\theta)} + \frac{q(\theta)}{\rho + q(\theta)}V - \frac{u-\chi}{\rho}\right).
\end{aligned}$$

The functions $\tilde{V}^{B1}$ and $\tilde{V}^{S1}$ give the difference between the value of transacting and never transacting for a mismatched buyer and mismatched seller, respectively.

Consider the term

$$\frac{\tilde{u}_2}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)}V - \frac{u-\chi}{\rho}$$

in the expression for $\tilde{V}^{B1}$. Note that one can use the definition for $\tilde{u}_2$ to re-write it as

$$\begin{aligned}
\frac{u_2 - \rho p + R}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)} V - \frac{u - \chi}{\rho} &\geq \\
\geq \frac{u_2 - \rho \bar{p} + R}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)} V - \frac{u - \chi}{\rho} &\geq \\
\geq \frac{u_2 - R - \left(\rho \tilde{V} - u_n\right) + R}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)} V - \frac{u - \chi}{\rho} &\geq \\
\geq \frac{u_2 + u_n}{\rho + \mu(\theta)} + \frac{\mu(\theta) - \rho}{\rho + \mu(\theta)} \tilde{V} - \frac{u - \chi}{\rho} &> 0
\end{aligned}$$

by Assumption A3 for $\theta \geq \hat{\theta}$, since $\mu(.)$ is an increasing function of θ. Noting that $\tilde{\theta} \geq \hat{\theta}$, it follows that $\tilde{V}^{B1} > 0$ for $\theta \geq \tilde{\theta}$. Therefore, for any $\theta > \tilde{\theta}$, a mismatched buyer is better off transacting than not transacting. Similarly, at $\theta = \tilde{\theta}$, $V^{B1} = V^{S1}$, so $\tilde{V}^{B1} = \tilde{V}^{S1}$. Therefore, $\tilde{V}^{S1} > 0$ for $\theta \leq \tilde{\theta}$, since $\tilde{V}^{S1}$ is decreasing in θ. Therefore, for $\theta \leq \tilde{\theta}$, a mismatched seller is better off transacting than not transacting.

It follows that for $\theta \in (0, \infty)$, if $D(\theta) > 0$, a mismatched owner is better off buying first (and transacting) compared to selling first (and transacting or not transacting) and similarly, if $D(\theta) < 0$, a mismatched owner is better off selling first (and transacting) compared to buying first (and transacting or not transacting). At $D(\theta) = 0$, he is indifferent between buying first (and transacting) and selling first (and transacting).

Finally, clearly if $\theta \to \infty$, $\tilde{V}^{B1} \to 0$, so $V^{B1} \to \frac{u-\chi}{\rho} = V^{S1}$. Similarly, if $\theta \to 0$, $\tilde{V}^{S1} \to 0$, and so $V^{S1} \to \frac{u-\chi}{\rho} = V^{B1}$. □

Proof of Lemma 2

We show that $\bar{\theta}$ solves

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma}\right)\theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g}\right) = \frac{1}{g} + \frac{1}{\gamma}, \tag{B.4}$$

and $\underline{\theta}$ solves

$$\left(\frac{1}{\mu(\theta)+g}+\frac{1}{\gamma}\right)\frac{1}{\theta}=\frac{1}{g}+\frac{1}{\gamma}. \tag{B.5}$$

These two equations arise from the flow conditions and population and housing conditions if all mismatched owners buy first and sell first, respectively. For the case where mismatched owners buy first ($x_s = 0$), the steady state conditions are

$$g=(q(\theta)+q)\,B_n,$$

$$\gamma O=(q(\theta)+g)\,B_1,$$

$$q(\theta)\,B_1=(\mu(\theta)+g)\,S_2,$$

$$g=(\mu(\theta)+g)\,A,$$

$$B_n+B_1+S_2+O=1,$$

and

$$B_n=A+S_2.$$

It follows that $B_n = \frac{g}{q(\theta)+g}$ and $A = \frac{g}{\mu(\theta)+g}$, or $A = \frac{q(\theta)+g}{\mu(\theta)+g}B_n$, so $S_2 = \frac{g}{q(\theta)+g} - \frac{g}{\mu(\theta)+g}$. Therefore, from the equation for θ, we have that $B_1 = (\theta-1)\,B_n$ and so $O = \frac{1}{\gamma}(q(\theta)+g)(\theta-1)\,B_n$. Substituting into the population constancy condition, we have that

$$\theta B_n+B_n-\frac{q(\theta)+g}{\mu(\theta)+g}B_n+\frac{1}{\gamma}(q(\theta)+g)(\theta-1)\,B_n=1,$$

which, after substituting for B_n and re-arranging we can write as

$$\left(\frac{1}{q(\theta)+g}+\frac{1}{\gamma}\right)\theta+\left(\frac{1}{q(\theta)+g}-\frac{1}{\mu(\theta)+g}\right)=\frac{1}{g}+\frac{1}{\gamma}.$$

This is exactly equation (B.4). At $\theta = 1$, the left-hand side equals

$$\frac{1}{q(1)+g}+\frac{1}{\gamma}<\frac{1}{g}+\frac{1}{\gamma}.$$

Furthermore, note that $\left(\frac{1}{q(\theta)+g}+\frac{1}{\gamma}\right)\theta$ is strictly increasing in θ and also unbounded. Similarly, $\left(\frac{1}{q(\theta)+g}-\frac{1}{\mu(\theta)+g}\right)$ is strictly increasing in θ as well. Therefore, the left-hand side of (B.4) is strictly increasing in θ, unbounded, and lower than the right-hand side for $\theta = 1$. Therefore, it has a unique solution for $\theta > 1$. We call this solution $\bar{\theta}$. Furthermore, by the Implicit Function Theorem, it immediately follows that $\bar{\theta}$ is increasing in γ.

For the case where mismatched owners sell first ($x_s = 1$) the steady state conditions become

$$g=(q(\theta)+g)\,B_n,$$

$$\mu(\theta)\,S_1=(q(\theta)+g)\,B_0,$$

$$\gamma O=(\mu(\theta)+g)\,S_1,$$

$$g=(\mu(\theta)+g)\,A,$$

$$B_n+B_0+S_1+O=1,$$

and

$$B_n+B_0=A.$$

It follows that $A = \frac{g}{\mu(\theta)+g} = B_0 + B_n$, $S_1 = \frac{1-\theta}{\theta}A$ and $O = \frac{1}{\gamma}\left(\mu\left(\theta\right)+g\right)\frac{1-\theta}{\theta}A$. Therefore, substituting for these in the population constancy condition, we have that

$$\frac{1}{\theta}A + \frac{1}{\gamma}\left(\mu\left(\theta\right)+g\right)\frac{1-\theta}{\theta}A = 1.$$

Substituting for A, we obtain an equation for θ of the form

$$\left(\frac{1}{\mu\left(\theta\right)+g} + \frac{1}{\gamma}\right)\frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma},$$

which is equation (B.5). At $\theta = 1$, the left-hand side equals

$$\frac{1}{\mu\left(1\right)+g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}.$$

Note also that $\left(\frac{1}{\mu(\theta)+g} + \frac{1}{\gamma}\right)\frac{1}{\theta}$ is strictly decreasing in θ and goes to 0 as $\theta \to \infty$. Also it asymptotes to ∞ as $\theta \to 0$. Therefore, the equation has a unique solution for $\theta < 1$. We call this solution $\underline{\theta}$. By the Implicit Function Theorem, it immediately follows that $\underline{\theta}$ is decreasing in γ.

Moreover, the limiting values for $\bar{\theta}$ and $\underline{\theta}$ in (19) and (20) follow directly from multiplication by γ and taking these limits in equations (B.4) and (B.5). □

Proof of Proposition 1

Observe that Lemma 2 that determines the values of $\bar{\theta}$ and $\underline{\theta}$ is independent of the agents' payoffs. With regard to Item 1, a direct application of Lemma 1 shows that if $\tilde{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$, then at $\theta = \underline{\theta}$ a mismatched owner is (weakly) better off selling first, at $\theta = \bar{\theta}$ he is (weakly) better off buying first, and at $\theta = \tilde{\theta}$ he is indifferent. Also, by Lemma 1, they are strictly better off from transacting than not transacting.

If mismatched owners are indifferent, they can randomize, such that $\theta = \tilde{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$. To show this, we show that there exists a steady state tightness $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ for every share $x_b \in [0,1]$ of newly mismatched agents that choose to buy first. Remember from (16) that

$$\theta = \frac{B_n + B_0 + B_1}{S_1 + S_2 + A} = \frac{B_n + B_0 + B_1}{B_n + B_0 + S_1}. \tag{B.6}$$

The steady state stock-flow conditions (A.10)-(A.16) yield

$$\theta S_2 + B_n + B_0 = \frac{(\gamma + g)}{q(\theta)} O,$$

so combining this with $S_2 = \frac{q(\theta)}{\mu(\theta)+g} B_1$ yields

$$B_n + B_0 = \frac{(\gamma + g)}{q(\theta)} O - \frac{\mu(\theta)}{\mu(\theta) + g} B_1.$$

Furthermore,

$$B_1 = \frac{\gamma x_b}{q(\theta) + g} O,$$

and

$$S_1 = \frac{\gamma(1 - x_b)}{\mu(\theta) + g} O.$$

Substituting for these into (B.6), we get

$$\theta = \frac{\frac{(\gamma+g)}{q(\theta)} O + \left(1 - \frac{\mu(\theta)}{\mu(\theta)+g}\right) \frac{\gamma x_b}{q(\theta)+g} O}{\frac{(\gamma+g)}{q(\theta)} O + \frac{\gamma(1-x_b)}{\mu(\theta)+g} O - \frac{\mu(\theta)}{\mu(\theta)+g} \frac{\gamma x_b}{q(\theta)+g} O}.$$

Canceling O and rearranging, we arrive at a continuous relation between the fraction of agents buying first, x_b, and θ given by

$$x_b = \frac{\left(1+\frac{g}{\gamma}\right)\frac{(\theta-1)}{\mu(\theta)} + \frac{1}{\mu(\theta)+g}}{\frac{\frac{1}{\theta}+1}{q(\theta)+g}}.$$

Since $\theta = \underline{\theta}$ if and only if $x_b = 0$, and $\theta = \bar{\theta}$ if and only if $x_b = 1$, by the intermediate value theorem, it must be the case that for $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, $x_b \in [0,1]$.

Therefore, agents' actions are optimal given θ and the steady state value of θ is consistent with agents' actions.

Considering Item 2, by the same logic a steady state equilibrium in which mismatched owners buy first and $\theta = \bar{\theta}$ exists. To see that it is the only symmetric steady state equilibrium, remember from Lemma 1 that mismatched owners only sell first for $\theta < \tilde{\theta}$, which contradicts $\tilde{\theta} < \underline{\theta}$. The same logic applies to Item 3. □

Proof of Corollary 1

For $\tilde{u}_0 = \tilde{u}_2 = c$, $\tilde{\theta} = 1$. Because Lemma 2 shows that $\bar{\theta} > 1$ when all mismatched owners buy first, and $\underline{\theta} < 1$ if they sell first, $\tilde{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$. Then an application of Proposition 1 gives the buy first equilibrium and the sell first equilibrium. To see that $\theta = 1$ if half of the mismatched owners buy first, take $x_s = x_b = \frac{1}{2}$. We have

$$\gamma \frac{1}{2} O = \left(q\left(\theta\right) + g\right) B_1, \tag{B.7}$$

and

$$\gamma \frac{1}{2} O = \left(\mu\left(\theta\right) + g\right) S_1. \tag{B.8}$$

At $\theta = 1$, $\mu(\theta) = q(\theta) = \mu(1)$, so $B_1 = S_1$. Also, $B_n = A = \frac{g}{\mu(1)+g}$ and $B_0 = S_2 = S_1 \frac{\mu(1)}{\mu(1)+g}$. Finally, population constancy implies that

$$2S_1 \frac{\mu(1)}{\mu(1)+g} + 2S_1 + 2S_1 \frac{\mu(1)+g}{\gamma} = \frac{\mu(1)}{\mu(1)+g},$$

which is satisfied for some $S_1 \in \left(0, \frac{1}{2}\right)$. Because mismatched owners are indifferent at $\theta = 1$ for $\tilde{u}_0 = \tilde{u}_2 = c$ by Lemma 1, an equilibrium in which half of the mismatched owners buy first exists at $\theta = 1$. □

Proof of Proposition 2

Consider the dynamics of an economy in which all mismatched owners sell first from time zero and onward, so that $x_s(t) = 1$ and $B_1(t) = 0$ for all $t \geq 0$. Defining $B_{n0}(t) = B_n(t) + B_0(t)$, the system of flow conditions in (A.1)-(A.7) can then be reduced to a system of three equations

$$\dot{S}_1 = \gamma \left(1 - S_1(t) - B_{n0}(t) - S_2(t)\right) - \left(\mu\left(\theta(t)\right) + g\right) S_1(t), \tag{B.9}$$

$$\dot{B}_{n0} = g + \mu\left(\theta(t)\right) S_1(t) - \left(q\left(\theta(t)\right) + g\right) B_{n0}(t), \tag{B.10}$$

$$\dot{S}_2 = -\left(\mu\left(\theta(t)\right) + g\right) S_2(t), \tag{B.11}$$

with

$$\theta(t) = \frac{B(t)}{S(t)} = \frac{B_{n0}(t)}{S_1(t) + A(t) + S_2(t)} = \frac{B_{n0}(t)}{S_1(t) + B_{n0}(t)} \leq 1,$$

using that population and housing constancy conditions (2) and (3) imply $B_{n0}(t) = A(t) + S_2(t)$, $\forall t$. On top of that, the state space is restricted to the 3-simplex with vertices $S_1(t) = 1$, $B_{n0}(t) = 1$, $S_2(t) = 1$ and $O(t) = 1 - B_{n0}(t) - S_1(t) - S_2(t)$.

Clearly, market tightness is strictly smaller than one for any positive measure of mismatched sellers. Suppose that at $t = 0$, there are none. Then (B.9) shows there is no outflow and either $O(t) > 0$ and $\dot{S}_1 > 0$, or $B_{n0}(t) + S_2(t) = 1$, with $\theta(t) = 1$. Then (A.3) shows that $\dot{O} > 0$, resulting in $O(t) > 0$ and $\dot{S}_1 > 0$. Therefore, the instant after $t = 0$, $S_1(t) > 0$. The same argument implies that $S_1(t) \neq 0$ for any $t > 0$. Moreover, (B.10) shows that $\dot{B}_{n0} > 0$ for $B_{n0}(t) = 0$, since the outflows stops, so that $B_{n0}(t) > 0$ for all $t > 0$. Consequently, $\theta(t) \in (0, 1)$ for all $t > 0$.

To show global asymptotic stability, we use the sufficient condition in Hartman (1961) that requires the Hermitian part of the Jacobian matrix to be negative definite. Since our Jacobian matrix is real, this amounts to the condition that its first principal minor is negative and subsequent principal minors alternate in sign. Denoting the elasticity of the matching function by $\alpha(\theta(t)) = \frac{d\mu(\theta(t))}{d\theta(t)} \frac{\theta(t)}{\mu(\theta(t))}$ with $\alpha(\theta(t)) \in [0, 1]$, the first principal minor is

$$\mu(\theta(t)) \left(\frac{\alpha(\theta(t))S_1(t)}{B_{n0}(t) + S_1(t)} - 1 \right) - \gamma - g < 0,$$

the second is given by

$$g(\gamma + g) + \mu(\theta(t))(2g + (1 + \alpha(\theta(t)))\gamma) + (\mu(\theta(t)))^2 > 0,$$

and the determinant is

$$\begin{array}{c} -g^2(\gamma + g) - \mu(\theta(t))(gy + (\mu(\theta(t)) + g)(1 + \alpha(\theta(t)))\gamma + 3g)) \\ -\mu(\theta(t))(\mu(\theta(t)) + g)\alpha(\theta(t))\gamma \frac{S_2(t)}{B_{n0}(t)}\theta(t) - (\mu(\theta(t)))^3 \end{array} < 0.$$

It follows that the economy must converge to the "sell first" steady state equilibrium stocks from any initial condition. Finally, since $\theta(t)$ is a continuous function of these stocks it must also converge and, furthermore, by Lemma 2, $\theta(t) \to \underline{\theta}$.

The proof of Item 2 is analogous. With $x_b(t) = 1$ and $S_1(t) = 0$ for all $t \geq 0$, the system can be reduced to

$$\dot{B}_1 = \gamma\left(1 - B_1(t) - B_{n0}(t) - S_2(t)\right) - \left(q\left(\theta(t)\right) + g\right) B_1(t), \tag{B.12}$$

$$\dot{B}_{n0} = g - \left(q\left(\theta(t)\right) + g\right) B_{n0}(t), \tag{B.13}$$

$$\dot{S}_2 = q\left(\theta(t)\right) B_1(t) - \left(\mu\left(\theta(t)\right) + g\right) S_2(t), \tag{B.14}$$

defined on the analogous 3-simplex with $B_{n0}\left(t\right) = A\left(t\right) + S_2\left(t\right)$ and

$$\theta(t) = \frac{B(t)}{S(t)} = \frac{B_{n0}(t) + B_1(t)}{A\left(t\right) + S_2\left(t\right)} = \frac{B_{n0}(t) + B_1(t)}{B_{n0}(t)} \geq 1.$$

The same argument as above implies that $B_{n0}\left(t\right) > 0$ and $B_1\left(t\right) > 0$ for all $t > 0$, and therefore $\theta(t) \in (1, \infty)$ for all $t > 0$. To see global asymptotic stability, note that the first principal minor of the Jacobian matrix is

$$q\left(\theta(t)\right)\left(\frac{(1 - \alpha(\theta(t)))B_1(t)}{B_{n0}(t) + B_1(t)} - 1\right) - \gamma - g < 0$$

the second is given by

$$g(\gamma + g) + q\left(\theta(t)\right)\left(2g + \left(2 - \alpha(\theta(t))\right)\gamma\right) + \left(q\left(\theta(t)\right)\right)^2 > 0,$$

and the determinant is

$$-g^2(\gamma+g) - g\frac{q(\theta(t))}{B_{n0}(t)+B_1(t)}\left[\begin{array}{c} 4(\gamma+g)B_1(t) - \alpha(\theta(t))\gamma S_2(t) \\ +B_{n0}(t)\left(3g+(4-\alpha(\theta(t)))\gamma\right) \\ +\frac{B_1(t)}{B_{n0}(t)}\left((\gamma+g)B_1(t)-\alpha(\theta(t))\gamma S_2(t)\right) \end{array}\right]$$
$$-\frac{(q(\theta(t)))^2}{B_{n0}(t)}\left[\begin{array}{c} \left(2g+(2-\alpha(\theta(t)))\gamma\right)\left(B_{n0}(t)+B_1(t)\right) \\ +\left(\mu(\theta(t))+g+\gamma\right)B_{n0}(t)-\alpha(\theta(t))\gamma S_2(t) \end{array}\right].$$

Remember that population and housing constancy guarantee that $S_2(t) \leq B_{n0}(t)$, $\forall t$. Given that $\alpha(\theta(t)) \leq 1$, we, therefore, simply note that

$$\gamma B_{n0}(t) > \alpha(\theta(t))\gamma S_2(t)$$

and

$$\gamma B_1(t) > \alpha(\theta(t))\gamma B_1(t)S_2(t)/B_{n0}(t),$$

so that the determinant is negative. Consequently, the stocks of agents will converge to the buy first steady state equilibrium stocks for any initial condition, and by Lemma 2, $\theta(t) \to \bar{\theta}$. □

Proof of Proposition 3

The proof consists of two parts. The first part regards optimal buyer behaviour. We show that if $\theta(t) \leq \tilde{\theta}$ $\forall t$, it is optimal to sell first. If $\theta(t) \geq \tilde{\theta}$ $\forall t$, it is optimal to buy first. The second part regards bounds on the trajectory of $\theta(t)$. We show that along any trajectory (independently of the agents' sequence choice), there exist two values $\theta_s^{\max} < 1$ and $\theta_b^{\min} > 1$, such that after a finite amount of time, $\theta(t) < \theta_s^{\max}$ at a point in time at which all mismatched owners sell first, and that $\theta(t) > \theta_b^{\min}$ at a point in time where they buy first. With these two results at hand, it follows that if $\tilde{\theta} \in [\theta_b^{\min}, \theta_s^{\max}]$, then after a finite amount of time the buy-first trajectory along which all the mismatched agents buy first constitutes an equilibrium, as does the sell-first trajectory along which all the mismatched agents sell first.

Proof of the first part: We first show that if $\theta_s(t) < \tilde{\theta}\ \forall t$, mismatched owners prefer to sell first. To show this, suppose that $V^{B1} \geq V^{S1}$ for some t along the transition path for $\theta_s(t)$. We will show that this leads to a contradiction. First, since $\theta_s(t) \to \underline{\theta}$, where $\underline{\theta} < \tilde{\theta}$ and $D(\underline{\theta}) = V^{B1} - V^{S1} < 0$ by Lemma 1, it follows that $D(\theta_s(t)) < 0$ for t sufficiently large. In addition, since both V^{B1} and V^{S1} are continuous in $\theta(t)$, there is at least one time instant,$\hat{t}$, such that at $\hat{t}$, $D\left(\theta_s\left(\hat{t}\right)\right) = 0$. If the equation has more than one solution, let $\hat{t}$ denote the biggest one. We can write $D\left(\theta_s\left(\hat{t}\right)\right) = 0$ as

$$0 = D\left(\theta_s\left(\hat{t}\right)\right) = \frac{\mu\left(\theta_s\left(\hat{t}\right)\right)\left[\left(1 - \frac{1}{\theta_s\left(\hat{t}\right)}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2\right]}{\left(\rho + q\left(\theta_s\left(\hat{t}\right)\right)\right)\left(\rho + \mu\left(\theta_s\left(\hat{t}\right)\right)\right)} + \dot{D}\left(\hat{t}\right),$$

where $\dot{D}\left(\hat{t}\right) \equiv \dot{V}^{B1}\left(\hat{t}\right) - \dot{V}^{S1}\left(\hat{t}\right)$. However, note that the right-hand side of the above expression is less than zero, since the first term is easily shown to be negative for $\theta_s\left(\hat{t}\right) < \tilde{\theta}$. Furthermore, since $D(\theta_s(t))$ crosses the zero line from above at $\hat{t}$, $\dot{D}\left(\hat{t}\right) < 0$. The right-hand side is therefore strictly negative, a contradiction. An analogous argument shows that for $\theta_b(t) > \tilde{\theta}$, $\forall t$, mismatched owners prefer to buy first $\forall t \geq 0$.

Proof of the second part: We derive the bounds in three steps. First we derive Lemma 1, stating that after a finite amount of time, the tightness in the housing market is within a well defined interval. We do this by comparing the inflow of agents to the mismatched state and to the stock of new agents B_0 / houses to A. Given that the tightness is within this interval, we derive Lemma 2, which gives bounds on the measure of the stocks of agents/houses in the different states. Finally, given the bounds on the stock of agents in the different states, we derive bounds on the tightness when the mismatched agents buy first and when they sell first, stated in Lemma 3.

Let $\theta^{ub} = 1+\gamma/g$ and $\theta^{lb} = \frac{g}{g+\gamma} = (\theta^{ub})^{-1}$. Note that θ^{ub} (θ^{lb}) is equal to $\bar{\theta}$ ($\underline{\theta}$) in the small-flow limit case in which γ and g go to zero while $\kappa \equiv \gamma/g$ is constant.

Lemma 1. *For any arbitrary initial allocation of houses over agents, and along any dynamic path, there exists a $t' < \infty$ such that $\theta(t) \in [\theta^{lb}, \theta^{ub}]$ for all $t \geq t'$.*

Proof. Consider a time interval in which the system is on a buy first trajectory. Let $B_{n0}(t) \equiv B_0(t) + B_n(t)$ and $A_s(t) \equiv A(t) + S_2(t)$. Recall from equation (16) that $B_{n0}(t) \equiv A_s(t)$ for all t. Hence

$$\theta_b\,(t) = \frac{B_{n0}(t) + B_1(t)}{S_2(t) + A_s(t)} = 1 + \frac{B_1\,(t)}{B_{n0}\,(t)}, \forall t \tag{B.15}$$

We have that

$$\begin{aligned} \left(\frac{\dot{B_1}}{B_{n0}}\right) &= \frac{\dot{B}_1 B_{n0}(t) - \dot{B}_{n0} B_1(t)}{\left(B_{n0}(t)\right)^2} \\ &= \frac{\left(\gamma O(t) - (g+q)\,B_1(t)\right) B_{n0}(t) - \left(g - (g+q)\,B_{n0}(t)\right) B_1(t)}{\left(B_{n0}(t)\right)^2}. \end{aligned} \tag{B.16}$$

It follows that

$$\left(\frac{\dot{B_1}}{B_{n0}}\right) = 0 \Leftrightarrow B_1(t)/B_{n0}(t) = \gamma O(t)/g. \tag{B.17}$$

Furthermore, the derivative (and hence also $\dot{\theta}$) is positive iff $B_1(t)/B_{n0}(t) < \gamma O(t)/g$. Since $O(t) < 1$ (possibly with the exception of $t = 0$), it follows that $(B_1 \dot{/} B_{n0}) < 0$ at $B_1(t)/B_{n0}(t) = \gamma/g$, i.e. that $\dot{\theta} < 0$ at $\theta(t) = 1 + \gamma/g$. Hence along a buy first trajectory, if $\theta(t) < 1 + \gamma/g$, it will stay below, and if $\theta(t) > 1 + \gamma/g$, then $\dot{\theta} < 0$ and bounded away from zero.

Along the sell first trajectory, we have that (with $A_s(t) = A(t) + S_2(t)$)

$$\theta_s\,(t) = \frac{B_0(t) + B_n(t)}{S_1(t) + A_s(t)} \equiv 1 - \frac{S_1(t)}{S_1(t) + A_s(t)}. \tag{B.18}$$

Note that $\dot{A} = g - (g + \mu(\theta(t)))\, A(t)$. Furthermore, note that

$$\frac{S_1(t)}{S_1(t) + A_s(t)} = \frac{S_1(t)/A_s(t)}{1 + S_1(t)/A_s(t)}$$

which increases iff $S_1(t)/A_s(t)$ increases. It follows readily that

$$\left(\frac{\dot{S_1}}{A_s}\right) \geq 0 \Leftrightarrow S_1(t)/A_s(t) \leq \gamma O(t)/g. \tag{B.19}$$

Since $O(t) < 1$, it follows that at $S_1(t)/A_s(t) = \gamma/g$, $S_1(t)$ is increasing. Equivalently, at $\theta(t) = \frac{g}{g+\gamma}$, and for all higher values of $\theta(t)$, $\dot{\theta} > 0$. Hence, along a sell first trajectory, if $\theta(t) > \frac{g}{g+\gamma}$, it will stay above, and if $\theta(t) < \frac{g}{g+\gamma}$, then $\dot{\theta} > 0$ and bounded away from zero.

In addition, consider a switch from the buy first to the sell first trajectory at time t'. Define $\theta_b(t') = \lim_{t \to t'-} \theta_b(t)$, and $\theta_s(t') = \lim_{t \to t'+} \theta_s(t)$. It follows that

$$\begin{aligned} \theta_s\left(t'\right) &= \frac{B_{n0}\left(t'\right)}{S_1\left(t'\right) + A_s\left(t'\right)} \\ &= \frac{B_{n0}\left(t'\right)}{B_{n0}\left(t'\right) + B_1\left(t'\right)} \\ &= \frac{1}{\theta_b\left(t'\right)}. \end{aligned}$$

Therefore, if $\theta(t) > \theta^{ub}$ along a buy first trajectory and moves towards θ^{ub}, it will be below θ^{lb} and move towards θ^{lb} after a switch to a sell first equilibrium (and vice versa).

Finally, suppose that agents are on a mixing trajectory (where they mix between buying first and selling first in every instant). Let $M(t)$ denote the measure of mismatched owners $B_1(t)+S_1(t)$. The outflow of mismatched owners is higher along a mixing trajectory than on a buy first or sell first trajectory. Furthermore, $B_{n0}(t)$ grows more quickly than along the buy first trajectory (since the outflow is lower and the inflow is higher). Hence, after a switch from the mixing to a buy first trajectory, it follows from (B.15) that $\theta_b(t)$ is lower than if the economy were to follow a buy first trajectory. Similarly, on a mixing trajectory $A_s(t)$ grows more quickly than on the sell first trajectory. Hence, after a switch from the mixing to the sell first trajectory, it follows from (B.18) that $\theta_s(t)$ is higher than if the economy were following the sell first trajectory. □

Define

$$\nu = \min\left\{q\left(\tilde{\theta}\right), \mu\left(\tilde{\theta}\right)\right\} > \max\left\{\mu\left(\theta_s\left(t\right)\right), q\left(\theta_b\left(t\right)\right)\right\}, \forall t,$$

and

$$\nu^{\min} = \min\left\{q\left\{\theta^{ub}\right\}, \mu\left(\theta^{lb}\right)\right\}.$$

Also, let $M = S_1 + B_1$. Bounds on the stocks are then as follows:

Lemma 2. *For any arbitrary initial allocation of houses over agents, and along any equilibrium path, there exists a* $\hat{t} < \infty$ *such that* $B_n^{\min} \leq B_n(t) \leq B_n^{\max}$, $A^{\min} \leq A(t) \leq A^{\max}$, $O^{\min} \leq O\left(t\right)$, $M^{\min} \leq M\left(t\right) \leq M^{\max}$, $B_0\left(t\right) \leq B_0^{\max}$, *and* $S_2\left(t\right) \leq S_2^{\max}$ *for all* $t \geq \hat{t}$, *where*

$$B_n^{\min} = \frac{g}{g + q\left(\theta^{lb}\right)} \tag{B.20}$$

$$A^{\min} = \frac{g}{g + \mu\left(\theta^{ub}\right)}$$

$$B_n^{\max} = \frac{g}{g + q\left(\theta^{ub}\right)}$$

$$A^{\max} = \frac{g}{g + \mu\left(\theta^{lb}\right)}$$

$$O^{\min} = \frac{B^{\min} q\left(\theta^{ub}\right)}{g + \gamma} \tag{B.21}$$

$$M^{\min} = \frac{\gamma}{g + \nu} O^{\min} \tag{B.22}$$

$$M^{\max} = \frac{\gamma}{g + \nu^{\min}}$$

$$B_0^{\max} = \frac{\mu\left(\tilde{\theta}\right) M^{\max}}{g + q\left(\theta^{ub}\right)} \tag{B.23}$$

$$S_2^{\max} = \frac{q\left(\tilde{\theta}\right) M^{\max}}{g + \mu\left(\theta^{lb}\right)} \tag{B.24}$$

Proof. Consider $B_n^{\min}$. We have that

$$\dot{B}_n = g - \left(g + q\left(\theta(t)\right)\right) B_n\left(t\right).$$

From Lemma 1 we know that within an amount of time t', $\theta(t) > \theta^{lb}$. Furthermore, since $\dot{\theta}$ is bounded away from zero at $\theta = \theta^{lb}$, the result follows for $B_n^{\min}$. An analogous argument follows for $A^{\min}$, $A^{\max}$, and $B_n^{\max}$. Next, consider $O^{\min}$. All agents without a house leave to be matched agents. Hence a lower bound on the inflow is $q(\theta^{ub})B^{\min}$. The outflow rate is $(\gamma + g)$. This gives the expression for $O^{\min}$. The inflow to $M(t)$ is bounded below by $\gamma O^{\min}$, while the outflow is bounded above by $g + \nu M(t)$. The expression for $M^{\min}$ thus follows. Next, $\mu\left(\tilde{\theta}\right) M^{\max}$ is an upper bound on the inflow into $B_0(t)$, obtained on a mixing trajectory in which all the agents choose to buy first. The lowest possible outflow is $g + q\left(\theta^{ub}\right)$ and the expression for $B_0^{\max}$ follows. $S_2^{\max}$ is derived in an analogous way. Since the dynamics of all the variables are bounded away from zero when outside the bounds, it follows that for any initial allocation of houses over agents there exists a $\hat{t}$ such that all the variables are within the bounds by time $\hat{t}$. □

The next lemma follows immediately.

Lemma 3. *For any arbitrary initial allocation of houses over agents, and along any dynamic path, there exists a $\hat{t} < \infty$ such that $\theta_b(t) \geq \theta_b^{\min}$ along the buy first trajectory, and $\theta_s(t) \leq \theta_s^{\max}$ along* *the sell first trajectory, where*

$$\theta_b^{\min} = 1 + \frac{M^{\min}}{B_n^{\max} + B_0^{\max}} \tag{B.25}$$

$$\theta_s^{\max} = \frac{A^{\max} + S_2^{\max}}{M^{\min} + A^{\max} + S_2^{\max}}. \tag{B.26}$$

Proof. Recall that $B_1(t) = M(t)$ and $S_1(t) = 0$ on the buy first trajectory, while $S_1(t) = M(t)$ and $B_1(t) = 0$ on the sell first trajectory. Furthermore, θ_b is given by (B.15) and θ_s by (B.18). Hence, it follows that (B.25) and (B.26) constitute lower and upper bounds for θ_b and θ_s, respectively. □

Finally, note that if we only focus on buy first and sell first trajectories, and not allow for the mixing trajectory, we get substantially tighter bounds. In this case ρ in (B.22) is replaced by $\tilde{\nu} = \min[q(\theta_b^{\min}), \mu(\theta_s^{\min})] < \rho$, and $\tilde{\theta}$ in (B.23) and (B.24) replaced with $\theta_b^{\min}$ and $\theta_s^{\max}$, respectively. Furthermore, it follows that we can write the lower bounds recursively, as

$$\left(\theta_b^{\min}, \theta_s^{\max}\right)' = \Gamma\left(\left(\theta_b^{\min}, \theta_s^{\max}\right)\right),$$

where Γ is an increasing and continuous mapping defined as above. A tighter mapping $\left(\theta_b^{\min}, \theta_s^{\max}\right)^*$ is thus given by the fixed-point

$$\left(\theta_b^{\min}, \theta_s^{\max}\right)^* = \Gamma\left(\left(\theta_b^{\min}, \theta_s^{\max}\right)^*\right).$$

□

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Online Appendix for "Buying First or Selling First in Housing Markets"

Appendix C: Data and Calibration

C.1 Data description

We use two data sets. The first (EJER) is an ownership register which contains the owners (private individuals and legal entities) of properties in Denmark as of the end of a given calendar year. The data set contains unique identifiers for owners (which, unfortunately, cannot be matched with other data-sets beyond EJER for different years). It also contains unique identifiers for each individual property. The second data set (EJSA) contains a record of all property sales in a given calendar year. The majority of transactions include information on the sale price, sale (agreement), and takeover (closing) dates. Furthermore, they contain the property identifiers used in the EJER data-set, which allows for linking of the two data-sets. The first data set is available from 1986 (recording ownership in 1985) until 2010 (recording ownership at the end of 2009), while the second is available from 1992 to 2010. Therefore, we effectively use data from 1991 (for ownership as of January 1, 1992) to 2009 (for ownership as of January 1, 2010).

We focus on the Copenhagen urban area (Hovedstadsområdet). We take the definition of the Copenhagen urban area as containing the following municipalities (by number): 101, 147, 151, 153, 157, 159, 161, 163, 165, 167, 173, 175, 183, 185, 187, 253, 269.[23]

We restrict attention to private owners and also to the primary owner of a property in a given year (whenever a property has more than one owners). Furthermore, we examine transactions where the new owner is a private individual and which have a non-missing agreement date. We drop

[23] Due to a reform in 2007, which merged some municipalities and created a new one, we omit municipality 190 for consistency.

properties that are recorded to transact more than once in a given year. We also remove property-year observations for which no owner is recorded. This leaves us with a total of 3,312,520 property-year observations. These comprise 199,812 unique properties and 345,943 unique individual owners over our sample period.

To identify an individual owner as a buyer-and-seller we rely on the information from the ownership register across consecutive years. First of all, we use the information on ownership over consecutive years to determine the counterparties for each recorded transaction in our sample. We then identify an individual owner as a buyer-and-seller if he is recorded to buy a new property and sell an old property within the same year or over two consecutive years. An old property is defined as a property which an individual is registered as owning over at least 2 consecutive years.[24] Also, we do not count individuals that are recorded as holding two properties for two or more consecutive years, which we treat as purchases for investment purposes.

We conduct this for individuals that are recorded as owning at most 2 properties at the end of any calendar year in our sample. This comprises the large majority of individual owners in our sample. In particular, in a given year in our sample from 1991-2009 there are on average only around 0.4% of individual owners who own more than two properties in the Copenhagen. Therefore, the majority of individuals hold at most 1 or 2 properties over that period. In particular, on average, around 1.6% of individual owners hold two properties at the end of a calendar year in our sample. Interestingly, around 5% of the recorded owners of two properties at the end of a calendar year are also identified as a buyer-and-seller according to our identification procedure described above with that number going up to almost 14% at the peak of the housing boom in 2006.

For each individual owner that has been identified as buyer-and-seller, we compute the time period (in days) between the agreement data for sale of the old property and the agreement date

[24]We make this restriction in order not to misclassify as a buyer-and-seller an individual who acquires a house, for example as a bequest (which is not recorded as a transaction), which he ends up selling quickly and then buys a new house with the proceeds from the sale. Adding back those agents has a very small effect on the pattern we uncover.

for the purchase of the new property. Similarly, we compute the time period (in days) between the closing date that of the buyer-and-seller's old property by the new owner and the closing date for his new property. We then denote a buyer-and-seller for which the time period between agreement dates is negative (sale date is before purchase date) as "selling first" and a buyer-and-seller for which the time period is positive (sale date is after purchase date) as "buying first". We also do the same classification but based on closing dates rather than agreement dates. Given the way we identify a buyer-and-seller, we have a consistent count for the number of owners who "buy first" vs. "sell first" in a given year for the years 1993 to 2008.

In principle, and as Figures 1 and 2 show, working with either of the two identifications produces similar results. This is not surprising given that the time difference between the agreement dates and closing dates are highly correlated with a correlation coefficient of 0.9313.

C.2 Calibration of γ and g

To calibrate γ and g, we need to remedy the lack of information on owner-renter transitions and on transitions within and across housing markets by housing tenure status in Denmark. We therefore supplement the available mobility information from Denmark with information from the USA in the following way. We first take the one-year mobility rates of owners and renters within and across counties from the 2012 American Community Survey (ACS). We then assume that the ratio of the mobility rates of renters relative to owners within a US county and within a Danish municipality are the same. We assume the same for the renter-owner mobility ratio across US counties and Danish municipalities. These assumptions are reasonable since US counties and Danish municipalities tend to be of similar size and roughly correspond to local housing markets. Also, there is no *a priori* reason to think that Danish owners are more or less mobile within (or across) housing markets relative to renters compares to US owners and renters.

Table C.1: Internal migration by housing tenure for Denmark (a) and the USA (b).

(a)

	Within municipality	Outside municipaity	Total
Owners	4.1*	2.6*	6.7*
Renters	21.4*	10.2*	31.6*
All	9.9	5.1	

(b)

	Within county	Outside county	Total
Owners	3.8	2.7	6.5
Renters	19.7	10.7	30.4
All	9.2	6.1	

Notes. Data source: Statistics Denmark, 2012 ACS and own calculations. An asterisk denotes that the mobility rate is imputed. See text for details.

We then combine these assumptions with the home ownership rate in Denmark and the total mobility of individuals within and across municipalities in 2012 to impute the mobility of owners and renters within and across municipalities. Put differently, we solve for the mobility rate of owners m_{own}^{within} implicitly given in the equation

$$m_{own}^{within} own + \rho^{within,US} m_{own}^{within} (1 - own) = m_{tot}^{within}, \tag{C.1}$$

where own is the home-ownership rate, $\rho^{within,US}$ is the ratio of the mobility of renters to owners in the USA, and m_{tot}^{within} is the total mobility rate within municipalities, and similarly for m_{own}^{across}. Table C.1 shows the resulting mobility rates and compares them against the same mobility rates in the USA. As the table shows, the overall mobility rates in the US and Denmark are quite similar, which given our assumption implies that the imputed mobility rates of owners and renters are also similar.

Second, to get owner-renter transitions, we use the 2003 American Housing Survey (AHS) to compute the fraction of movers that owned a housing unit 2 years before and are renters as of the 2003 survey.[25] We use the 2003 AHS to minimize the effects of the 2002-2010 housing boom-bust cycle. We find that approximately 0.248 of moving owners switch to renters. We assume that this

[25]We do not count movers who lived in an owned unit but were not the owners themselves. This omits some groups of moves that would artificially inflate the number of owner-renter transitions, for example, young people that move out of their parents' house and establish a new household as renters.

is the fraction of owner-renter switches in Denmark as well.

Finally, to get owner-owner transitions within municipalities we reduced the owner mobility rate of 4.1 by the owner-renter transition rate, which is approximately $0.248 \times 4.1 = 1$ percentage point. Similarly, we inflate the owner transition rate across municipalities of 2.6 percentage points with that number. Therefore, we arrive at a 1 year owner-owner transition rate within a housing market of 3.1% and a 1 year transition out of owning housing or across housing markets of 3.6%. We use these transition rates to calibrate γ and g, respectively. Therefore, we set $g = 0.0371$ and $\gamma = 0.0322$.

C.3 Additional results

Figure 1a plots the distribution of the time difference between "sell" and "buy" agreement dates over the whole period 1993-2008. As Figure 2a shows, the share of moving owners that buy first has fluctuated substantially over this period, so it is very likely that the distribution of the difference in sell and buy dates is not stable over time but varies as moving owners switch from selling first to buying first and vice versa. To illustrate this, in Figure C.1 we plot kernel densities for the distribution of the time between "sell" and "buy" transactions in two periods: first, between 2004-2006 when the share of buy first owners was the highest in the sample and, second, between 2007-2008 when the share of buy first owners fell substantially. As the Figure shows, the distribution is shifted to the left during 2007-2008 relative to 2004-2006, which is a direct implication of more homeowners buying first in the first period relative to the second period.

Additionally, in Table 1 we used proxies for average time-to-buy and average time-to-sell based on the average time between the sell (buy) and buy (sell) transactions for owners that sell first (buy first) and who complete their second transaction in a given quarter. We can examine whether the time-to-sell proxy constructed this way comoves with seller time-on-market for the period 2004-

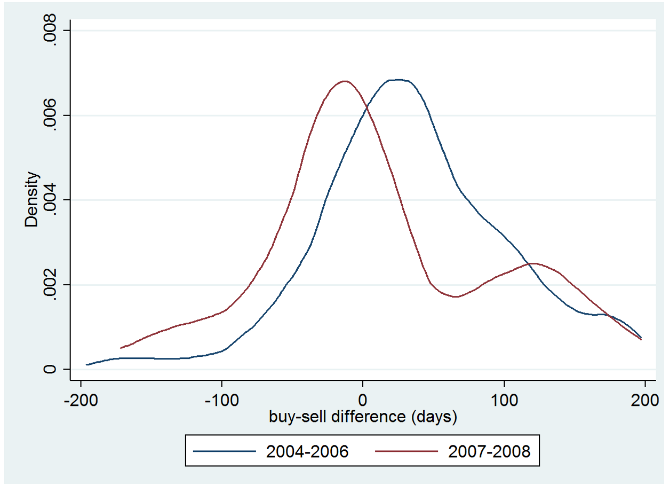

Figure C.1: Distribution of the time difference between "sell" and "buy" agreement dates for 2004-2006 (blue) and 2007-2008 (red). Own calculations based on registry data from Statistics Denmark. The distributions are truncated at ±200 days.

2008. Figure C.2 plots the two series for the period 2004-2008. The two series are nearly identical in the first half of this period. Subsequently, the time-to-sell proxy is consistently above seller time-on-market, though the two series continue to fluctuate closely. One explanation for this difference is that our measure of seller time-on-market does not account for property withdrawals and relistings, which are known to be particularly prevalent during a housing downturn. In that case, the time-to-sell proxy would be a more accurate measure of the true underlying seller time-on-market.

Finally, Figure C.3 plots the dynamics of market tightness θ for a numerical example in which *all* mismatched owners switch from buying first to selling first, with γ, g, and the matching function from Table 2.

Appendix D: Omitted Proofs

Proof of Proposition 4

Below, we use the notation Σ_{ij} to denote the surplus from trade between agents of type i and type j. Also, for brevity, we use the notation $\underline{\theta}$ for $\lim_{g,\gamma\to 0,\frac{g}{\gamma}=\kappa}\underline{\theta}$ and $\bar{\theta}$ for $\lim_{g,\gamma\to 0,\frac{g}{\gamma}=\kappa}\bar{\theta}$.

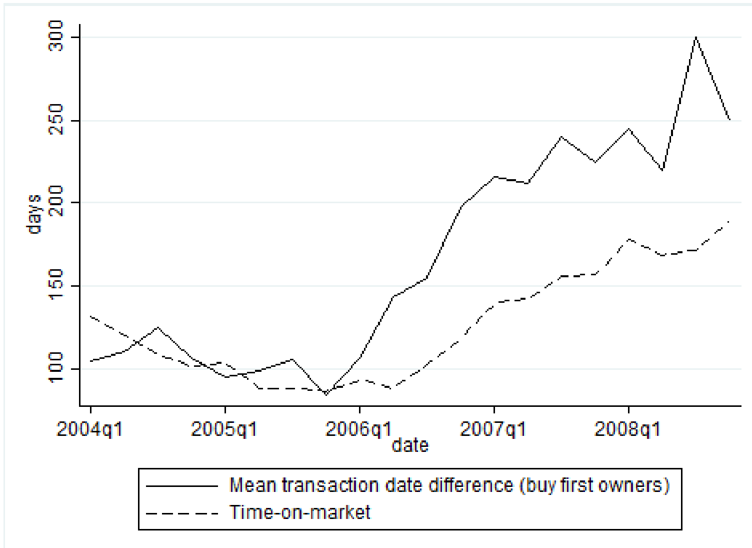

Figure C.2: Comparison of seller time-on-market (dashed line) against time-to-sell proxy (solid line) based on the mean transaction data difference of buy-first owners. Copenhagen, Q1:2004-Q4:2008. Seller time-on-market for Copenhagen is from the Danish Mortgage Banks' Federation (available at http://statistik.realkreditforeningen.dk/BMSDefault.aspx). The time-to-sell proxy is based on own calculations based on registry data from Statistics Denmark. Specifically it is given by the average time between buy and sell transactions for buy-first owners who complete the second transaction in the quarter.

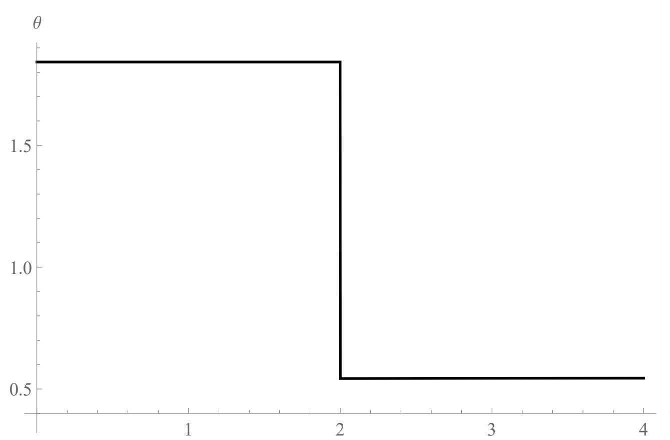

Figure C.3: Dynamics of market tightness θ.

"Sell first" equilibrium existence.

We first show that a "Sell first" equilibrium exists with $\theta = \underline{\theta} < 1$. We proceed in two steps. First, we show that no mismatched owner has an incentive to deviate and buy first when $\theta = \underline{\theta} < 1$. This is verified under the conjecture that $\Sigma_{ij} \geq 0$ for all buyer-seller pairs except for Σ_{S1B1}. Second, we verify the conjecture on the different surpluses.

Step 1. In the limit economy with small flows of a "Sell first" equilibrium candidate, the faction of buyers who are forced renters is given by

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{B_0}{B} = \lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{q\left(\underline{\theta}\right) - \mu\left(\underline{\theta}\right)}{g + q\left(\underline{\theta}\right)} = 1 - \underline{\theta},$$

where $\underline{\theta} = \frac{1}{1+\kappa}$. Similarly,

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{A}{S} = \underline{\theta}.$$

Thus,

$$\begin{aligned} rV^{B0} &= u_0 - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\frac{A}{S}\Sigma_{AB0} + \frac{S_1}{S}\Sigma_{S1B0}\right) \\ &= u_0 - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{AB0} + \left(1-\underline{\theta}\right)\Sigma_{S1B0}\right), \end{aligned}$$

and similarly,

$$rV^{Bn} = u_n - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{ABn} + \left(1-\underline{\theta}\right)\Sigma_{S1Bn}\right),$$

so

$$V^{Bn} - V^{B0} = \frac{u_n - u_0}{r + \frac{1}{2} q\left(\underline{\theta}\right)}. \tag{D.1}$$

Also,

$$
\begin{aligned}
rV^{A} &= R+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\frac{B_{n}}{B}\left(V-V^{Bn}-V^{A}\right)+\frac{B_{0}}{B}\left(V-V^{B0}-V^{A}\right)\right) \\
&= R+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta}\left(V-V^{Bn}-V^{A}\right)+\left(1-\underline{\theta}\right)\left(V-V^{B0}-V^{A}\right)\right),
\end{aligned}
$$

or

$$
V^{A}=\frac{R}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V-V^{B0}-\underline{\theta}\frac{u_{n}-u_{0}}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right).
$$

Analogous to (D.1)

$$
V^{S2}-V^{A}=\frac{u_{2}+\frac{1}{2}\mu\left(\underline{\theta}\right)V}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}.
$$

This in turn implies that

$$
V-V^{S2}=\frac{rV-u_{2}}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^{A}=\frac{u-u_{2}}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^{A}.
$$

Turning to the value functions of mismatched owners, a mismatched seller has a value function given by

$$
rV^{S1}=u-\chi+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V-V^{S1}-\underline{\theta}\frac{u_{n}-u_{0}}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right),
$$

which can be re-written as

$$
V^{S1}=\frac{u-\chi}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}V-\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\underline{\theta}\frac{u_{n}-u_{0}}{r+\frac{1}{2}q\left(\underline{\theta}\right)}.
$$

For the value function of a deviating mismatched buyer, assuming that trade takes place when he meets a real-estate firm but not when he meets a mismatched seller, writes

$$rV^{B1} = u - \chi + \frac{1}{2} q\left(\underline{\theta}\right) \underline{\theta} \Sigma_{AB1}.$$

Or

$$\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right) V^{B1} = u - \chi + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2} - V^{A}\right).$$

Consider the difference between the utilities from buying first compared to selling first. In the limit we consider, we have that

$$\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)\left(V^{B1} - V^{S1}\right) = \frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2} - V^{A} - V + \underline{\theta}\frac{u_n - u_0}{\rho + \frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Substituting for $V^{S2} - V^{A} - V$, we get that

$$V^{B1} - V^{S1} = \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\frac{u_2 - u}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Note that at $\underline{\theta} = 1$ (i.e. for $\kappa = 0$),

$$\frac{u_2 - u}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)} = 0,$$

given Assumption B1. As $\underline{\theta}$ moves away from 1 toward 0 (κ increases), we have that $\frac{u_2 - u}{r + \frac{1}{2}\mu(\underline{\theta})} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})}$ decreases, so $V^{B1} < V^{S1}$ for $\underline{\theta} < 1$. Therefore, it is not optimal for a mismatched owner to deviate and buy first in an equilibrium in which mismatched owners sell first and $\theta < 1$.

Step 2. We verify that our conjectures for the surpluses are correct. It is clear given our assumptions that $\Sigma_{S2B1} = V - V^{B1} > 0$ and $\Sigma_{S1B0} = V - V^{S1} > 0$. Also, $\Sigma_{ABn} \geq 0$. Next, we show that $\Sigma_{S1Bn} > 0$. In the limit we consider,

$$
\begin{aligned}
\Sigma_{S1Bn} &= V - V^{Bn} + V^{B0} - V^{S1} = V - V^{S1} - \frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} \\
&= \frac{\chi}{r + \frac{1}{2}\mu(\underline{\theta})} + \frac{\frac{1}{2}\mu(\underline{\theta})(\underline{\theta} - 1) - r}{r + \frac{1}{2}\mu(\underline{\theta})} \frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} \\
&= \frac{r(\chi + u_0 - u_n) + \frac{1}{2}q(\underline{\theta})\chi + \frac{1}{2}\mu(\underline{\theta})(\underline{\theta} - 1)(u_n - u_0)}{\left(r + \frac{1}{2}q(\underline{\theta})\right)\left(r + \frac{1}{2}\mu(\underline{\theta})\right)}.
\end{aligned}
$$

Therefore, at $\underline{\theta} = 1$, $\Sigma_{S1Bn} > 1$ if

$$
r(\chi + u_0 - u_n) + \frac{1}{2}\mu_0\chi > 0.
$$

Note that given Assumption B1, this is equivalent to

$$
r(u_2 - (u - \chi)) + \frac{1}{2}\mu_0\chi > 0,
$$

which holds by Assumption B2. Therefore, by continuity of the value functions with respect to θ, it follows that there is a $\kappa_1 > 0$, such that for $\kappa < \kappa_1$, $\Sigma_{S1Bn} > 0$. Next, we show that $\Sigma_{ABn} > 0$. To show, this suppose, toward a contradiction, that $\Sigma_{ABn} < 0$. Then

$$
rV^{Bn} + rV^{A} \leq u_n + \frac{1}{2}q(\underline{\theta})\Sigma_{ABn} + \frac{1}{2}\mu(\underline{\theta})\Sigma_{ABn},
$$

where the inequality comes from $\Sigma_{ABn} < 0 < \Sigma_{S1Bn}$ and $\Sigma_{ABn} < \Sigma_{AB0}$, since $V^{Bn} > V^{B0}$. Therefore,

$$
\Sigma_{ABn} \geq \frac{rV - u_n}{r + \frac{1}{2}q(\underline{\theta}) + \frac{1}{2}\mu(\underline{\theta})} > 0,
$$

so we arrive at a contradiction. $\Sigma_{ABn} > 0$ also implies that $\Sigma_{AB0} > 0$, since $V^{Bn} > V^{B0}$. Next notice that

$$\Sigma_{S2Bn} = \Sigma_{ABn} + V - V^{S2} + V^{A} = \Sigma_{ABn} + \frac{rV - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} > 0.$$

Again, this also implies that $\Sigma_{S2B0} > 0$. Next, we show that $\Sigma_{AB1} > 0$. In the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2} - V^{B1} - V^{A} = V^{S2} - V^{A} - \frac{u-\chi}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V^{S2} - V^{A}\right) \\
&= \frac{r\left(V^{S2} - V^{A}\right) - (u-\chi)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} = \frac{\frac{r}{r+\frac{1}{2}\mu(\underline{\theta})}u_2 + \frac{\frac{1}{2}\mu(\underline{\theta})}{r+\frac{1}{2}\mu(\underline{\theta})}u - (u-\chi)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} \\
&= \frac{r\left(u_2 - (u-\chi)\right) + \frac{1}{2}\mu\left(\underline{\theta}\right)\chi}{\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)^2}.
\end{aligned}$$

At $\underline{\theta} = 1$, $\Sigma_{AB1} > 0$ if $r\left(u_2 - (u-\chi)\right) + \frac{1}{2}\mu_0\chi > 0$, which is our parametric Assumption B2. Therefore, by continuity of the value functions with respect to $\underline{\theta}$, it follows that there is a $\kappa_2 > 0$, such that for $\kappa < \kappa_2$, $\Sigma_{AB1} > 0$. Finally, in the limit we consider

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A} \\
&= \Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.
\end{aligned}$$

At $\underline{\theta} = 1$,

$$\begin{aligned}
\frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} &= \frac{\frac{r}{r+\frac{1}{2}\mu_0}\left(u_0 - R\right) + \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}u - \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}rV^{A} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} \\
&= \frac{ru_0 + \frac{1}{2}\mu_0 u - \frac{1}{2}\mu_0\left(rV^{A} - R\right) - \left(r + \frac{1}{2}\mu_0\right)(u-\chi)}{\left(r + \frac{1}{2}\mu_0\right)^2}.
\end{aligned}$$

Substituting for Σ_{AB1}, we get

$$\Sigma_{S1B1} = \frac{r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu\chi - \frac{1}{2}\mu_0\left(rV^A - R\right)}{\left(r + \frac{1}{2}\mu_0\right)^2}.$$

Therefore, a sufficient condition for $\Sigma_{S1B1} < 0$ at $\underline{\theta} = 1$ is

$$r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu_0\chi \leq 0,$$

or

$$r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi\right],$$

which is our parametric Assumption B3. Again by continuity of the value functions with respect to $\underline{\theta}$, we have that there is a $\kappa_3 > 0$, s.t. for $\kappa < \kappa_3$, $\Sigma_{S1B1} < 0$. Taking $\underline{\kappa} = \min\{\kappa_1, \kappa_2, \kappa_3\}$, we have that for $\kappa < \underline{\kappa}$, there is a "Sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$.

"Buy first" equilibrium existence.

We follow the same two steps to show the existence of a "Buy first" equilibrium with $\theta = \bar{\theta} > 1$. Again, we make the same conjectures on the different surpluses as in the case of the "Sell first" equilibrium. In the limit economy, the fraction of buyers who are new entrants is

$$\lim_{g\to 0, \gamma\to 0, \frac{g}{\gamma}=\kappa} \frac{B_n}{B} = \frac{1}{\bar{\theta}},$$

where $\bar{\theta} = 1 + \kappa$. Also,

$$\lim_{g\to 0, \gamma\to 0, \frac{g}{\gamma}=\kappa} \frac{A}{S} = \frac{1}{\bar{\theta}}$$

as well. Therefore, similarly to the value functions in the "Sell first" equilibrium, we have that

$$\left(r + \frac{1}{2}\mu\left(\bar{\theta}\right)\right)V^A = R + \frac{1}{2}\mu\left(\bar{\theta}\right)\left(\frac{1}{\bar{\theta}}\left(V - V^{Bn}\right) + \frac{\bar{\theta} - 1}{\bar{\theta}}\left(V^{S2} - V^{B1}\right)\right),$$

and

$$\left(r+\frac{1}{2}\mu\left(\bar{\theta}\right)\right)V^{S2}=u_2+\left(r+\frac{1}{2}\mu\left(\bar{\theta}\right)\right)V^{A}+\frac{1}{2}\mu\left(\bar{\theta}\right)V.$$

Therefore, as in the "Sell first" equilibrium,

$$V-V^{S2}=\frac{rV-u_2}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}-V^{A}.$$

Also, as in the "Sell first" equilibrium,

$$V^{Bn}-V^{B0}=\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)}.$$

Turning to the value functions of a mismatched buyer, we have that

$$rV^{B1}=u-\chi+\frac{1}{2}q\left(\bar{\theta}\right)\left(\frac{1}{\bar{\theta}}\left(V^{S2}-V^{B1}-V^{A}\right)+(1-\frac{1}{\bar{\theta}})\left(V-V^{B1}\right)\right),$$

For the value function of a deviating agent who chooses to sell first, we have that

$$rV^{S1}=u-\chi+\frac{1}{2}\mu\left(\bar{\theta}\right)\left(\frac{1}{\bar{\theta}}\Sigma_{S1Bn}\right),$$

since $\Sigma_{S1B1}<0$. Then,

$$\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)V^{S1}=u-\chi+\frac{1}{2}q\left(\bar{\theta}\right)V+\frac{1}{2}q\left(\bar{\theta}\right)\frac{u_0-u_n}{r+\frac{1}{2}q\left(\bar{\theta}\right)}.$$

Therefore, the difference between $V^{B1}-V^{S1}$ satisfies

$$\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)\left(V^{B1}-V^{S1}\right)=\frac{1}{2}q\left(\bar{\theta}\right)\left(\frac{1}{\bar{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\right).$$

At $\bar{\theta}=1$, we have that

$$\frac{1}{\bar{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)}=0,$$

by Assumption B1. As $\bar{\theta}$ increases, we have that $\frac{1}{\bar{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu(\bar{\theta})}+\frac{u_n-u_0}{r+\frac{1}{2}q(\bar{\theta})}$ increases, so $V^{B1}>V^{S1}$ for $\bar{\theta}>1$. Therefore, it is not optimal for a mismatched owner to deviate and sell first in an equilibrium in which mismatched owners buy first and $\theta>1$.

Finally, we verify that our conjectures for the surpluses are correct. As in the "Sell first" case, $\Sigma_{S1B0}>0$ and $\Sigma_{S2B1}>0$. Also, as in the "Sell first" case, in the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2}-V^{B1}-V^{A}=V^{S2}-V^{A}-V+V \\
&- \frac{u-\chi}{r+\frac{1}{2}q\left(\bar{\theta}\right)}-\frac{\frac{1}{2}q\left(\bar{\theta}\right)}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\left[\frac{1}{\bar{\theta}}\left(V^{S2}-V^{A}-V\right)+V\right] \\
&= \frac{\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\frac{\bar{\theta}-1}{\bar{\theta}}\right)}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}+\frac{\chi}{r+\frac{1}{2}q\left(\bar{\theta}\right)} \\
&= \frac{r\left(u_2-\left(u-\chi\right)\right)+\frac{1}{2}\mu\left(\bar{\theta}\right)\chi+\frac{1}{2}q\left(\bar{\theta}\right)\frac{\bar{\theta}-1}{\bar{\theta}}\left(u_2-u\right)}{\left(r+\frac{1}{2}\mu\left(\bar{\theta}\right)\right)\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)}.
\end{aligned}$$

Note that at $\bar{\theta}=1$, Σ_{AB1} in the "Buy first" case is the same as the "Sell first" case. Therefore, there is a $\kappa_4>0$, such that for $\kappa<\kappa_4$ and $\bar{\theta}=1+\kappa$, $\Sigma_{AB1}>0$. Similarly,

$$\begin{aligned}
\Sigma_{S1Bn} &= V-V^{Bn}+V^{B0}-V^{S1}=V-V^{S1}-\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)} \\
&= \frac{\chi}{r+\frac{1}{2}q\left(\bar{\theta}\right)}-\frac{r}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)} \\
&= \frac{r\left(\chi+u_0-u_n\right)+\frac{1}{2}q\left(\bar{\theta}\right)\chi}{\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)^2},
\end{aligned}$$

which at $\bar{\theta}=1$ is again the same as for the "Sell first" case. Therefore, there is a $\kappa_5>0$, such that for $\kappa < \kappa_5$, $\Sigma_{S1Bn} > 0$. A similar argument to the one for the "Sell first" case also confirms that $\Sigma_{ABn} > 0$, $\Sigma_{AB0} > 0$, $\Sigma_{S2Bn} > 0$ and $\Sigma_{S2B0} > 0$. Finally,

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R + \frac{1}{2}q\left(\bar{\theta}\right)\left(\bar{\theta}-1\right)\left(V^{S2} - V^{B1} - V^{A}\right)}{r + \frac{1}{2}q\left(\bar{\theta}\right)} - V^{A} \\
&= \left(1 + \frac{\frac{1}{2}q\left(\bar{\theta}\right)\left(\bar{\theta}-1\right)}{r + \frac{1}{2}q\left(\bar{\theta}\right)}\right)\Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}q\left(\bar{\theta}\right)}.
\end{aligned}$$

At $\bar{\theta} = 1$, showing that $\Sigma_{S1B1} < 0$ in the "Buy first" case therefore follows the "Sell first" case, so that $\Sigma_{S1B1} < 0$ for $\kappa < \kappa_6$, for some $\kappa_6 > 0$. Taking $\bar{\kappa} = \min\{\kappa_4, \kappa_5, \kappa_6\}$, we have that for $\kappa < \bar{\kappa}$, there is a "Buy first" equilibrium with a market tightness given by $\bar{\theta} = 1 + \kappa$. Finally, taking $\kappa^* = \min\{\bar{\kappa}, \underline{\kappa}\}$, we arrive at the desired result. □

Appendix E: Additional Extensions

E.1 Tighter bounds on θ_b and θ_s

We want to derive a set of conditions on the stocks such that if the conditions are satisfied initially, they will be satisfied at all later points in time along the buy-first trajectory. We will then do the same for the sell first trajectory.

Our ultimate goal is to derive a lower bound $\theta_b^{low} > \theta_b^{\min}$. First, we can redefine the lower bound on $B_n(t)$ as

$$B_n^{low} = \frac{g}{g + q\left(\theta_b^{low}\right)} \tag{E.1}$$

To derive a lower bound O_b^{low}, we focus on the the inflow into $O(t)$ from the pool of new entrants only. Define O_b^{low} by the equation

$$B_n^{low} q\left(\theta^{ub}\right) = O_b^{low} (\gamma + g). \tag{E.2}$$

If $B_{n0}(0) \geq B_n^{low}$ and $O(0) \geq O_b^{low}$, then $O(t) > O_b^{low}$, for all t.

Define $\theta_b^{low} = 1 + \frac{\gamma}{g} O_b^{low} > 1$. From (B.17) it follows that along any buy first trajectory, $\theta(t) > \theta^{low}$ within a finite amount of time. Substituted into (E.2) this gives

$$1 + \frac{q\left(\theta^{ub}\right)}{g + q\left(\theta_b^{low}\right)} \frac{\gamma}{\gamma + g} = \theta_b^{low}. \tag{E.3}$$

This is one equation in one unknown θ_b^{low}. At $\theta_b^{low} = 1$, the left-hand side is strictly greater than the right-hand side. At $\theta_b^{low} = \theta^{ub}$, the opposite is true.[26] Hence $\theta_b^{low} \in (1, \theta^{ub})$. Substituting θ_b^{low} into (E.2) gives O_b^{low}. Note also that for any $\theta < \theta_b^{low}$, the left-hand side of (E.3) is less than the right-hand side.

In Proposition 3 we showed that if $\theta \geq \tilde{\theta}$ along a buy first trajectory, then the buy first trajectory constitutes a dynamic equilibrium. The next lemma follows:

Lemma E.1. *Consider a switch to a buy first trajectory at $t = t'$. Suppose $B_{n0}(t') \geq B_n^{low}$ and that $O(t') \geq O^{low}$. Then the following is true:*

1. *If $\theta_b(t') \geq \theta_b^{low} \geq \tilde{\theta}$, the buy first trajectory constitutes a dynamic equilibrium from t' onward.*
2. *If $\tilde{\theta} \leq \theta_b(t') \leq \theta_b^{low}$, the buy first trajectory constitutes a dynamic equilibrium from t' onward.*

Proof. The first part follows directly from the discussion above. We therefore turn to Item 2. Define $\hat{\theta}^{low} = \theta(t')$, and redefine θ_b^{low} to $\hat{\theta}^{low}$, and redefine O^{low} to $\hat{O}^{low} < O^{low}$ as the lower bound on $O(t)$ when $\theta \geq \hat{\theta}^{low}$. From (E.3) and its properties it follows that the inflow to $O(t)$ is greater than

[26] Since $1 + \frac{q(\theta^{ub})}{g+q(\theta_b^{low})} \frac{\gamma}{\gamma+g} < 1 + \gamma/(g+\gamma) < 1 + \gamma/g = \theta^{ub}$.

$\gamma\hat{\theta}^{low}$, so that $O(t)$ will not decrease below $\hat{O}^{low}$. Hence $\theta(t)$ cannot fall below $\theta(t')$, and the result follows. $\square$

As should be clear from the derivation, θ_b^{low} is a lower bound, and certainly not a greatest lower bound. To get a simpler expression, suppose $q(\theta_b^{low})$ is large relative to g. Then (E.3) simplifies to

$$\mu\left(\theta_b^{low}\right) - q\left(\theta_b^{low}\right) = \frac{\gamma}{g+\gamma} q\left(\theta^{ub}\right) \tag{E.4}$$

where $\theta^{ub} = 1 + \gamma/g$. For example, with $\gamma/g = 1$ and $q(\theta) = \theta^{-1/2}$, we get that $\theta_b^{low} = 1.42$.

We now derive tighter bounds along the sell first trajectory. Hence our goal is to derive an upper bound $\theta_s^{high} < \theta_s^{\max}$ along the sell first trajectory. We use exactly the same approach as in the buy first case. First we define a lower bound on $A_s(t)$ as a function of θ_s^{high}.

$$A_s^{low} = \frac{g}{g + \mu\left(\theta_s^{high}\right)}. \tag{E.5}$$

We proceed to derive a lower bound on $O(t)$, denoted O_s^{low}, given by

$$A_s^{low}\mu\left(\theta^{lb}\right) = O_s^{low}\left(\gamma + g\right). \tag{E.6}$$

If $O(0) > O_s^{low}$, $O(t) > O_s^{low}$, for all t. It follows that $O_s^{low}\gamma/g \leq S_1(t)/A_s(t)$, provided that the inequality holds at $t = 0$. Hence an upper bound on $\theta(t)$ is $\frac{g}{g+\gamma O_s^{low}}$, or substituting for O_s^{low} from (E.6),

$$1 + \frac{\mu\left(\theta^{lb}\right)}{g + \mu\left(\theta_s^{high}\right)}\frac{\gamma}{\gamma + g} = \frac{1}{\theta_s^{\max}}. \tag{E.7}$$

At $\theta_s^{\max} = 1$, the left-hand side is strictly larger than the right-hand side. At $\theta_s^{\max} = \theta^{lb}$, the opposite is true.[27] Furthermore, for any $\theta > \theta_s^{high}$, the left-hand side is greater than the right-hand

[27] At this point, the right-hand side reads $1 + \gamma/g$, which is strictly greater than the left-hand side.

side.

Lemma E.2. *Consider a switch to a sell first trajectory at $t = t'$. Suppose $A_s(t') \geq A_s^{low}$ and that $O(t') \geq O_s^{low}$. Then the following is true:*

1. *If $\theta_s(t') \leq \theta_s^{high} \leq \tilde{\theta}$, the sell first trajectory constitutes a dynamic equilibrium from t' onward.*

2. *If $\tilde{\theta} \geq \theta_s(t') \geq \theta_s^{high}$, the sell first trajectory constitutes a dynamic equilibrium from t' onward.*

Proof. The proof is analogous to the proof of Lemma E.1. □

Also in this case we can get a simpler expression by assuming that $\mu(\theta_s^{high})$ is large relative to g. Then (E.7) simplifies to

$$q\left(\theta_s^{high}\right) - \mu\left(\theta_s^{high}\right) = \frac{\gamma}{g+\gamma}\mu\left(\theta^{lb}\right), \tag{E.8}$$

where $\theta^{lb} = \frac{g}{g+\gamma}$. Continuing the example, with $\gamma/g = 1$ and $q(\theta) = \theta^{-1/2}$, we get that $\theta^{lb} = 1/2$ and $\theta_s^{high} = 1/1.42 = 0.70$.

Finally, if the matching function is symmetric, it follows that $\theta_s^{high} = 1/\theta_b^{low}$. To see this, recall that with a symmetric matching function, it follows that $\mu(\theta) = q(1/\theta)$. Inserting θ_b^{low} into the left-hand side of (E.7) reads (using that $\theta^{lb} = (\theta^{ub})^{-1}$)

$$1 + \frac{\mu\left(\theta^{lb}\right)}{g + \mu\left(\left(\theta^{ub}\right)^{-1}\right)}\frac{\gamma}{\gamma+g} = 1 + \frac{q\left(\theta^{ub}\right)}{g + q\left(\theta_b^{low}\right)}\frac{\gamma}{\gamma+g} = \theta_b^{low}, \tag{E.9}$$

from (E.3). Hence $1/\theta_b^{low}$ satifies (E.7).

E.2 Prices determined by Nash bargaining – additional discussion

In this section we provide an informal discussion of the characterization of the "Buy first" and "Sell first" steady state equilibria when prices are determined by Nash bargaining and of the underlying economic forces.

Consider a "Buy first" steady state equilibrium candidate with a market tightness of $\theta = \bar{\theta} > 1$. In that candidate equilibrium, the sellers with positive measure are the double owners and real-estate firms, while the buyers with positive measure are the mismatched owners and new entrants. In the small flows economy from Section 4.1.2, the outflow rate of mismatched owners is equal to the outflow rate of new entrants, so $B_1/B_n = \gamma/g = \kappa$. Hence, the shares of new entrants and mismatched buyers in the pool of buyers are $1/\bar{\theta}$ and $1 - 1/\bar{\theta}$, respectively. Furthermore, in the limit, as there is no death, the shares of real-estate firms and double owners in the pool of sellers are also $1/\bar{\theta}$ and $1 - 1/\bar{\theta}$, respectively.

Given these shares and since buyers and sellers split the match surplus evenly, the value function of a mismatch buyer is (given $\rho \to r$ in the limit)

$$rV^{B1} = u - \chi + \frac{1}{2}q(\bar{\theta})\left[\frac{1}{\bar{\theta}}\Sigma_{AB1} + (1 - \frac{1}{\bar{\theta}})\Sigma_{S2B1}\right],$$

where $\Sigma_{AB1} = V^{S2} - V^{B1} - V^{A}$ is the match surplus when a mismatched buyer meets a real-estate firm, and $\Sigma_{S2B1} = V - V^{B1}$ is the match surplus when a mismatched buyer meets a double owner.

Consider a mismatched owners who deviates (permanently) and sells first.[28] Since a meeting between a mismatched buyer and a mismatched seller is assumed to lead to negative surplus, the value function of a deviator is simply

$$rV^{S1} = u - \chi + \frac{1}{2}\mu\left(\bar{\theta}\right)\frac{1}{\bar{\theta}}\Sigma_{S1Bn},$$

and so, the difference between the value of buying first and selling first, $D(\bar{\theta}) = V^{B1} - V^{S1}$, can be

[28]Studying permanent deviations is without loss of generality, since a temporary deviation can dominate a permanent deviation if and only if no deviation dominates the permanent deviation.

written as

$$D(\bar{\theta}) = \frac{\frac{1}{2}q\left(\bar{\theta}\right)}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\left(\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)} - \frac{1}{\bar{\theta}}\frac{u-u_2}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}\right). \tag{E.10}$$

Given our assumptions on utility flows, $D\left(\bar{\theta}=1\right)=0$ for $\kappa=0$. An increase in $\bar{\theta}$ (equivalently, an increase in κ) leads to an increase in $D\left(\bar{\theta}\right)$, since the expression in parenthesis increases. This increase comes from two effects. First, $\mu\left(\bar{\theta}\right)$ increases and $q\left(\bar{\theta}\right)$ decreases, so the second term in the parenthesis decreases (given that $u_2 < u-\chi < u$) and the first term increases (since then $u_n > u_0$). This effect is tightly linked to the queue-length effect from Section 4. Specifically, as before, an increase in $\bar{\theta}$ increases the value of buying first given a lower expected time-on-market for double owners, while it decreases the value of selling first, given a higher expected time-on-market for forced renters.

Second, the fraction of new entrants and real-estate firms, $1/\bar{\theta}$, decreases. Therefore, buyers are more likely to meet double owners and sellers are more likely to meet mismatched buyers. However, the trading surplus for a buyer is higher when matched with a double owner compared to a match with a real-estate firm. Similarly, the trading surplus is lower for a seller when matched with a mismatched buyer compared to a new entrant. This compositional effect on both sides of the market strengthens the incentives to buy first.

Finally, when trading between a mismatched buyer and seller is not profitable for $\bar{\theta}$ close to 1, the discounting effect arising from higher prices is dominated by both the queue-length and compositional effects. Thus, $D\left(\bar{\theta}\right)$ in (E.10) unambiguously increases in $\bar{\theta}$.

Consider a "Sell first" equilibrium candidate with a market tightness of $\theta = \underline{\theta} < 1$. In that candidate equilibrium, the sellers with positive measure are the mismatched owners and real-estate firms, while the buyers are the forced renters and new entrants. In the limit economy, the shares of forced renters and new entrants in the pool of buyers are $\underline{\theta}$ and $1-\underline{\theta}$, respectively. These are also the respective shares of real-estate firms and mismatched owners.

In this equilibrium candidate, the gain from deviating to (permanently) buying first for a mismatched owner is $D(\underline{\theta}) = V^{B1} - V^{S1}$, which is given by

$$D(\underline{\theta}) = \frac{\frac{1}{2}\mu(\underline{\theta})}{r + \frac{1}{2}\mu(\underline{\theta})}\left(\underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} - \frac{u - u_2}{r + \frac{1}{2}\mu(\underline{\theta})}\right). \tag{E.11}$$

Given our assumptions on the utility flows, $D(\underline{\theta} = 1) = 0$ for $\kappa = 0$. Decreasing $\underline{\theta}$ (increasing κ) decreases D, and hence, makes it more attractive to sell first.

E.3 House Price Expectations

So far, we assumed that mismatched owners do not expect house prices to change. In this section we examine the implications of expected changes in prices for the behavior of mismatched owners. To focus on the effect of expected capital gains or losses rather than the discounting effect explained above, we study the benchmark case in which $R = \rho p$. To simplify the exposition, we also assume that $u_0 = u_2 = c$.

Consider a simple, exogenous process for the price p. With rate λ, the house price p changes to a permanent new level p_N.[29] We compare the utility from buying first relative to selling first for a mismatched owner before the price change. If the price change occurs between the two transactions, the mismatched owner will make a capital gain of $p_N - p$ if he buys first and a capital loss of the same amount if he sells first. If the shock happens before the first or after the second transaction, it will not influence the decision to buy first or sell first.

The price risk associated with the transaction sequence decision creates asymmetry in the payoff from buying first or selling first. Specifically, at $\theta = 1$, the difference between the two value functions

[29] Since we assume that $p = \frac{R}{\rho}$, one can think of a permanent change in the equilibrium rental rate to R_N, which leads to a house price change to $p_N = \frac{R_N}{\rho}$. Also, for this exercise, we implicitly assume that $\gamma \to 0$, so that V is independent of p.

$D(\theta) = V^{B1} - V^{S1}$ is

$$D(1) = \frac{\mu(1)}{(\rho + q(1) + \lambda)(\rho + \mu(1) + \lambda)} 2\lambda (p_N - p). \tag{E.12}$$

An expected price decrease leads to a higher value of V^{S1} relative to V^{B1}, even if matching rates for a buyer and a seller are the same. Consequently, $V^{S1} > V^{B1}$ even for some values of $\theta > 1$. If the expected price decrease (increase) is sufficiently large, so that $D(\bar{\theta}) < 0$ ($D(\underline{\theta}) > 0$), then selling (buying) first will dominate buying (selling) first for any value of θ that is consistent with equilibrium.

Proposition E.1. *Consider the economy with $u_0 = u_2$, $R = \rho p$, and an exogenous and permanent house price change to p_N at rate λ. Then for every $\lambda > 0$ and every steady state market tightness, a mismatched owner prefers to "sell first" for sufficiently low values of p_N. Analogously, a mismatched owner prefers to "buy first" for sufficiently high values of p^N.*

Proof. Consider the difference between the two value functions, $D(\theta) = V^{B1} - V^{S1}$ assuming that the mismatched owner transacts in both cases, and denote the value of a mismatched owner after the price change by $\bar{V}_N$:

$$\begin{aligned} D(\theta) = & \frac{\mu(\theta)\left(1 - \frac{1}{\theta}\right)\left(u - \chi - c + \lambda\left(\bar{V}_N - v^{B0}\right)\right)}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)} \\ & + \frac{\frac{\lambda\mu(\theta)\left(1 - \frac{1}{\theta}\right)q(\theta)}{(r + \mu(\theta))(r + q(\theta))}\left[\rho V - c\right] + \mu(\theta)\left(1 + \frac{1}{\theta}\right)\lambda(p_N - p)}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)}. \end{aligned} \tag{E.13}$$

The proof of Proposition 1 implies that $\bar{\theta}$ is the highest possible steady state market tightness, so consider the case of $1 < \theta \leq \bar{\theta}$. In this case, $\bar{V}_N = V_N^{B1}$, where V_N^{B1} denotes the value of buying first after the price change, this difference simplifies further to

$$D(\theta) = \frac{\mu(\theta)\left[\left(1 - \frac{1}{\theta}\right)\left(1 + \frac{\lambda}{\rho + q(\theta)}\right)(u - \chi - c) + \left(1 + \frac{1}{\theta}\right)\lambda(p_N - p)\right]}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)}. \tag{E.14}$$

Suppose that $p_N < p$ and define θ_{B1}^{PR} as the solution to

$$\frac{\theta_{B1}^{PR}-1}{\theta_{B1}^{PR}+1}\left(1+\frac{\lambda}{\rho+q\left(\theta_{B1}^{PR}\right)}\right)=\frac{\lambda\left(p-p_N\right)}{\left(u-\chi-c\right)}. \tag{E.15}$$

Therefore, θ_{B1}^{PR} is the value of θ that leaves a mismatched owner indifferent between buying first and selling first he anticipates a price change of $p_N - p$ and a market tightness of $\theta > 1$ after the price change. Note that θ_{B1}^{PR} is increasing in $p - p_N$ if $\theta_{B1}^{PR} \geq 1$. Therefore, a sufficient condition for mismatched owners to prefer to sell first, given $1 < \theta \leq \bar{\theta}$, is that $\theta_{B1}^{PR} > \bar{\theta}$.

Similarly, the proof of Proposition 1 implies that $\underline{\theta}$ is the lowest possible steady state market tightness, so consider the case of $\underline{\theta} \leq \theta < 1$. In this case, $\bar{V}_N = V_N^{S1}$, where V_N^{S1} denotes the value of selling first after the price change. In that case the difference in value functions can be written as

$$D\left(\theta\right)=\frac{\mu\left(\theta\right)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+\mu(\theta)}\right)\left(u-\chi-c\right)+\left(1+\frac{1}{\theta}\right)\lambda\left(p_N-p\right)\right]}{\left(\rho+q\left(\theta\right)+\lambda\right)\left(\rho+\mu\left(\theta\right)+\lambda\right)}. \tag{E.16}$$

Suppose that $p_N > p$ and define θ_{S1}^{PR} as the solution to

$$\frac{\theta_{S1}^{PR}-1}{\theta_{S1}^{PR}+1}\left(1+\frac{\lambda}{\rho+\mu\left(\theta_{S1}^{PR}\right)}\right)=\frac{\lambda\left(p-p_N\right)}{\left(u-\chi-c\right)}. \tag{E.17}$$

Similarly, to the case of θ_{B1}^{PR}, θ_{S1}^{PR} is increasing in $p - p_N$ if $\theta_{S1}^{PR} \leq 1$. Then, a sufficient condition for mismatched owner to prefer to buy first, given $\underline{\theta} \leq \theta < 1$ is that $\theta_{S1}^{PR} < \underline{\theta}$. □

In the next section we show that such house price expectations can exert a destabilizing force on the housing market when prices move with market tightness, and study dynamic equilibria that feature switches in the transaction sequence decision.

E.4 Equilibrium switches

Consider the limit economy introduced in Section 4.1.2, where $g \to 0$ and $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, $\bar{\theta} = 1 + \kappa$, and $\underline{\theta} = \frac{1}{1+\kappa} = \frac{1}{\bar{\theta}}$. Suppose that the economy starts in a "Buy first" equilibrium. In that case

$$\theta = \bar{\theta} = \frac{\bar{B}}{\bar{S}} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n}, \tag{E.18}$$

where $\bar{B}$ and $\bar{S}$ denote the stocks of buyers and sellers in the "Buy first" equilibrium. Suppose that the whole stock of mismatched owners, B_1, decide to sell first rather than buy first, and so, moves to the seller side of the market. In that case, the new market tightness becomes

$$\theta' = \frac{B'}{S'} = \frac{B_n}{B_n + B_1} = \underline{\theta},$$

where B' and S' denote the stocks of buyers and sellers immediately after the switch. Hence, the tightness jumps directly to its new steady state value with no dynamic adjustment in θ.

We can use this property of the limit economy to construct (approximate) dynamic equilibria, in which prices and rents move with tightness and in tandem according to $R = \rho p$. Suppose that $X(t) \in \{0, 1\}$ follows a two-state Markov chain. $X(t)$ starts in $X(t) = 0$ and with Poisson rate λ transitions permanently to $X(t) = 1$. The realization of $X(t)$ plays the role of a sunspot variable. The price in state $X(t) = 1$ is given by a smooth and increasing function $p_1 = f(\theta_1)$. The price in state 0 is implicitly given by a smooth function $p_0 = f(\theta_0, \lambda(p_1 - p_0))$, increasing in both arguments, and with $f(\theta, 0) \equiv f(\theta)$. As in Section 5.1, we take these relationships as exogenous and reduced-form to illustrate the equilibrium consequences of the interaction of housing prices and market liquidity conditions with the transaction decisions of mismatched owners.

We consider a regime-switching equilibrium in which the economy starts out in a "Buy first" regime ($X(t) = 0$), in which 1) mismatched owners prefer to buy first and the market tightness is $\theta_0 = \bar{\theta}$, and 2) agents expect that with rate λ, the economy permanently switches to a "Sell first" regime with market tightness $\theta_1 = \underline{\theta}$. In that second regime, 1) mismatched owners strictly prefer to sell first, and 2) agents expect that the economy will remain in the "Sell first" regime forever. As $\lambda \to 0$, the payoffs from buying first and selling first converge to the payoffs without regime switching. Hence, in the limit, buying first in state 0 is an equilibrium strategy if $\bar{\theta} > \tilde{\theta}$, while selling first is an equilibrium strategy in state 1 if $\underline{\theta} < \tilde{\theta}$, where $\tilde{\theta}$ is defined by Proposition 1. The following proposition therefore shows that self-fulfilling fluctuations in prices and tightness can exist if $\underline{\theta} < \tilde{\theta} < \bar{\theta}$ and agents don't expect them to happen too often.

Proposition E.2. *Consider the limit economy with $g \to 0$, $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, and the sunspot process described above. Suppose further that $R = \rho p$ and that $\underline{\theta} < \tilde{\theta} < \bar{\theta}$. Then there is a $\bar{\lambda}$, such that for $\lambda < \bar{\lambda}$, there exists a regime-switching equilibrium characterized by two regimes $x \in \{0, 1\}$. In the first regime, $\theta_0 = \bar{\theta}$ and mismatched owners buy first. In the second regime, tightness is $\theta_1 = \underline{\theta}$, mismatched owners sell first, and $p_1 < p_0$. The economy starts in regime 0 and transitions to regime 1 with rate λ.*

Proof. Consider the first regime in which tightness $\theta_0 = \bar{\theta}$. The value function of a mismatched buyer (who transacts) in the first regime is given by

$$V_0^{B1} = \frac{u - \chi}{\rho + q\left(\bar{\theta}\right) + \lambda} + \frac{q\left(\bar{\theta}\right)}{\rho + q\left(\bar{\theta}\right) + \lambda}\left(V_0^{S2} - p_0\right) + \frac{\lambda}{\rho + q\left(\bar{\theta}\right) + \lambda} V^{S1},$$

where

$$V_0^{S2} = v^{S2}\left(\bar{\theta}\right) + \frac{\lambda}{\rho + \mu\left(\bar{\theta}\right) + \lambda}\left(v^{S2}\left(\underline{\theta}\right) - v^{S2}\left(\bar{\theta}\right) + p_1 - p_0\right) + p_0,$$

with

$$v^{S2}(\theta) = \frac{c}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)} V,$$

where V^{S1} is given in (6) with (7) substituted in, which arises since in the second regime a mismatched owner sells first. For the value of selling first we have

$$V_0^{S1}(\bar{\theta}) = \frac{u - \chi}{\rho + \mu(\bar{\theta}) + \lambda} + \frac{\mu(\bar{\theta})}{\rho + \mu(\bar{\theta}) + \lambda}\left(V_0^{B0} + p_0\right) + \frac{\lambda}{\rho + \mu(\bar{\theta}) + \lambda} V^{S1},$$

where

$$V_0^{B0} = v^{B0}(\bar{\theta}) + \frac{\lambda}{\rho + q(\bar{\theta}) + \lambda}\left(v^{B0}(\underline{\theta}) - v^{B0}(\bar{\theta}) + p_0 - p_1\right) - p_0,$$

with

$$v^{B0}(\theta) = \frac{c}{\rho + q(\theta)} + \frac{q(\theta)}{\rho + q(\theta)} V.$$

Consider the difference $D_0(\bar{\theta}) = V_0^{B1}(\bar{\theta}) - V_0^{S1}(\bar{\theta})$, and note that

$$\lim_{\lambda \to 0} D_0(\bar{\theta}) = \frac{\mu(\theta) - q(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))}(u - \chi - c) > 0.$$

Since $V_0^{B1}(\bar{\theta})$ and $V_0^{S1}(\bar{\theta})$ are continuous in λ, it follows that $D_0(\bar{\theta})$ is continuous in λ, as well, so that $D_0(\bar{\theta}) > 0$ will also be the case for λ sufficiently close to 0. Therefore, there exists a $\bar{\lambda}$ such that for $\lambda < \bar{\lambda}$, $V_0^{B1}(\bar{\theta}) > V_0^{S1}(\bar{\theta})$ and mismatched owners prefer to buy first. Also, by Lemma 2, $\bar{\theta}$ is consistent with the behavior of mismatched owners and given by $\bar{\theta} = (B_n + B_1)/B_n$.

Upon $X(t) = 1$, the whole stock of mismatched owners, B_1, sells first and, so, moves to the seller side of the market. In that case, the new market tightness becomes $B_n/(B_n + B_1) = 1/\bar{\theta} = 1/(1 + \kappa) = \underline{\theta}$. Since Lemma 2 shows that $\underline{\theta}$ obtains in steady state when all mismatched owners sell first, tightness jumps directly to its value θ_1 without any dynamic adjustment in θ. In that regime agents' payoffs are as in Section 3.2, and therefore, by Lemma 1, mismatched owners prefer to sell first.

Finally, since $\bar{\theta} > \underline{\theta}$, it follows that $p_0 > p_1$. To see this, suppose $p_0 \leq p_1$. Then $p_0 = f(\bar{\theta}, \lambda(p_1 - p_0)) \geq f(\bar{\theta})$. But then $p_0 \geq f(\bar{\theta}) > f(\underline{\theta}) = p_1$, which is a contradiction. □

As a result, there exist dynamic equilibria in which prices and tightness move together. The expectation that prices will fall, induces mismatched owners to sell first, which leads to a fall in market tightness and thus prices. The reason that λ cannot be too high is that if agents expect the change in regimes to occur sufficiently soon, then from Proposition E.1, it can be optimal for mismatched owners to sell first in the first regime despite the high market tightness, speculating on regimes changing in between their two transactions. This, however, is inconsistent with equilibrium. Therefore, a regime-switching equilibrium exists only for (sufficiently) low values of λ.

Upon the switch, average seller time-on-market for sellers, $\frac{1}{\mu(\theta)}$, increases. Second, consider the ratio of the stock of sellers before and after the switch. That ratio is exactly $\underline{\theta}$, which is less than 1. Therefore, there is an increase in the for-sale stock, since some of the previous buyers become sellers. Finally, transaction volume may also fall depending on the shape of the matching function. Specifically, consider a Cobb-Douglas matching function, $m(B, S) = \mu_0 B^{\alpha} S^{1-\alpha}$, for $0 < \alpha < 1$. The ratio of transaction volumes before and after the switch is

$$\frac{\mu(\bar{\theta})}{q(\underline{\theta})} = \frac{\mu_0 \bar{\theta}^{\alpha}}{\mu_0 \underline{\theta}^{\alpha-1}} = (1+\kappa)^{2\alpha-1}.$$

Hence, transaction volume falls after the switch if $\alpha > \frac{1}{2}$ and increases if $\alpha < \frac{1}{2}$. The reason is that for $\alpha > \frac{1}{2}$ buyers are more important than sellers in generating transactions. When mismatched owners switch from buying first to selling first, this leads to a reduction in the number of buyers and an increase in the number of sellers, and hence, to a fall in the transaction rate. As discussed in Section 4.3, Genesove and Han (2012) estimate a value of $\alpha = 0.84$. At that value, transaction volume would drop after the switch.

Although transaction volume falls immediately after the switch, it fully recovers over time. To see this, consider the ratio of transaction volumes in the buy first and sell first steady state equilibria in the limit economy. Denoting the total mass of buyers and sellers in the buy first and sell first steady state equilibria by $\bar{B}$ and $\underline{S}$, respectively, we can write that ratio as

$$\frac{q\left(\bar{\theta}\right)\bar{B}}{\mu\left(\underline{\theta}\right)\underline{S}} = \frac{q\left(\bar{\theta}\right)\left(g+\gamma\bar{O}\right)/\left(g+q\left(\bar{\theta}\right)\right)}{\mu\left(\underline{\theta}\right)\left(g+\gamma\underline{O}\right)/\left(g+\mu\left(\underline{\theta}\right)\right)}$$
$$= \frac{q\left(\bar{\theta}\right)}{\left(g+q\left(\bar{\theta}\right)\right)}\frac{\left(g+\mu\left(\underline{\theta}\right)\right)}{\mu\left(\underline{\theta}\right)}\frac{\left(1+\kappa\bar{O}\right)}{\left(1+\kappa\underline{O}\right)},$$

where $\bar{O}$ and $\underline{O}$ denote the stock of matched owners in the buy first and sell first steady state equilbiria, respectively. Next, note that

$$\lim_{\gamma,g\to0,\gamma/g=\kappa}\frac{q\left(\bar{\theta}\right)\bar{B}}{\mu\left(\underline{\theta}\right)\underline{S}} = \lim_{\gamma,g\to0,\gamma/g=\kappa}\frac{q\left(\bar{\theta}\right)}{\left(g+q\left(\bar{\theta}\right)\right)}\frac{\left(g+\mu\left(\underline{\theta}\right)\right)}{\mu\left(\underline{\theta}\right)}\frac{\left(1+\kappa\bar{O}\right)}{\left(1+\kappa\underline{O}\right)} = 1.$$

Therefore, in an economy with small flows, transaction volumes in the two steady state equilibria are (approximately) the same. Consequently, even if transaction volume falls upon a switch in mismatched owners' behavior, it eventually recovers (almost) fully. This property of the small flows economy is consistent with the transitional dynamics in our numerical example in Section 4.3.

E.5 A model with competitive search

In competitive search equilibrium, sellers post prices, and buyers direct their search towards the sellers they find most attractive, taking into account that better terms of trade mean a longer expected waiting time before trade occurs. The market splits up in submarkets, and the different agents choose which submarket to enter. As shown in Garibaldi et al. (2016), the most patient buyers (who are most willing to trade off a short waiting time for a low price) will search for the most impatient sellers (who are most willing to trade off a low price for a short waiting time). Analogously, the least patient buyers search for the most patient sellers.

We first define a competitive search equilibrium for our economy. Let $(\mathcal{P}, \Theta)$ denote the active market segments in the economy, i.e. segments that attract a positive measure of buyers and sellers. The following equations describe the steady state value functions of agents. For new entrants we have:

$$\rho V^{Bn} = u_n - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta) \left(-p + V - V^{Bn} \right) \right\}. \tag{E.19}$$

Similarly, for a real estate firm, we have

$$\rho V^{A} = R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu(\theta) \left(p - V^{A} \right) \right\}. \tag{E.20}$$

For mismatched owners that buy first, we have

$$\rho V^{B1} = u - \chi + \max \left\{ 0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta) \left(-p + V^{S2} - V^{B1} \right) \right\} \right\}, \tag{E.21}$$

where the value function takes into account the possibility that a mismatched buyer may be better off not searching. Similarly, if the mismatched owner sells first, we have

$$\rho V^{S1} = u - \chi + \max \left\{ 0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu(\theta) \left(p + V^{B0} - V^{S1} \right) \right\} \right\}. \tag{E.22}$$

A double owner solves

$$\rho V^{S2} = u_2 + R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu(\theta) \left(p + V - V^{S2} \right) \right\}, \tag{E.23}$$

while a forced renter solves

$$\rho V^{B0} = u_0 - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta) \left(-p + V - V^{B0} \right) \right\}. \tag{E.24}$$

Finally, for a matched owner we have

$$\rho V = u + \gamma \left(\max \left\{ V^{B1}, V^{S1} \right\} - V \right). \tag{E.25}$$

Next, we describe the steady state stock-flow conditions. Let

$$\left(p^{Bn}, \theta^{Bn} \right) \in \left(\mathcal{P}^{Bn}, \Theta^{Bn} \right) \equiv \arg\max_{(p,\theta)} \left\{ q(\theta) \left(-p + V - V^{Bn} \right) \right\} \subset (\mathcal{P}, \Theta) \tag{E.26}$$

denote a market segment that maximizes the value of searching for a new entrant. We define $\left(p^j, \theta^j \right)$ and $\left(\mathcal{P}^j, \Theta^j \right)$ analogously for an agent type $j \in \{A, B1, S1, B0, S2\}$. For agents $j \in \{B1, S1\}$, we adopt the convention that $\Theta^j = \emptyset$ if they choose not to search.

We have the following stock-flow conditions

$$g = \left(\sum_{\theta\in\Theta} x^{Bn}(\theta) q(\theta) + g \right) B_n, \tag{E.27}$$

$$\sum_{\theta\in\Theta} x^{S1}(\theta) \mu(\theta) S_1 = \left(\sum_{\theta\in\Theta} x^{B0}(\theta) q(\theta) + g \right) B_0, \tag{E.28}$$

$$\gamma x_b O = \left(\sum_{\theta\in\Theta} x^{B1}(\theta) q(\theta) + g \right) B_1, \tag{E.29}$$

$$\gamma x_s O = \left(\sum_{\theta\in\Theta} x^{S1}(\theta) \mu(\theta) + g \right) S_1, \tag{E.30}$$

$$\sum_{\theta\in\Theta} x^{B1}(\theta) q(\theta) B_1 = \left(\sum_{\theta\in\Theta} x^{S2}(\theta) \mu(\theta) + g \right) S_2, \tag{E.31}$$

$$g\left(O+B_1+S_1+2S_2\right)=\sum_{\theta\in\Theta}x^{A}\left(\theta\right)\mu\left(\theta\right)A, \tag{E.32}$$

$$x_b+x_s=1, \tag{E.33}$$

with

$$\sum_{\theta\in\Theta}x^{j}\left(\theta\right)=1\ \forall j\in\left\{Bn,A,B0,S2\right\}, \tag{E.34}$$

where $x^{j}\left(\theta\right)=0$ if $\theta\notin\Theta^{j}$ and, if a mismatched buyer/seller chooses to search,

$$\sum_{\theta\in\Theta}x^{j}\left(\theta\right)=1 \text{ for } j\in\left\{B1,S1\right\}, \tag{E.35}$$

with $x^{j}\left(\theta\right)=0$ if $\theta\notin\Theta^{j}$. In the above expressions $\mathbf{x}^{j}\left(\theta\right)\geq 0$ is the vector of mixing probabilities over segments in Θ for an agent $j\in\left\{Bn,A,B1,S1,B0,S2\right\}$. Market tightnesses in each segment are given by

$$\theta=\frac{x^{Bn}\left(\theta\right)B_n+x^{B1}\left(\theta\right)B_1+x^{B0}\left(\theta\right)B_0}{x^{A}\left(\theta\right)A+x^{S2}\left(\theta\right)S_2+x^{S1}\left(\theta\right)S_1}, \tag{E.36}$$

where $x^{j}\left(\theta\right)=0$ if $\theta\notin\Theta^{j}$.

Finally, we have the population constancy and housing ownership conditions

$$B_n+B_0+B_1+S_1+S_2+O=1, \tag{E.37}$$

and

$$O+B_1+S_1+A+2S_2=1. \tag{E.38}$$

Following Moen (1997), we additionally require that the active market segments $(\mathcal{P},\Theta)$ are such that the equilibrium allocation is a "no-surplus allocation". Formally, we make the following requirement.

No-surplus allocation Let $\mathcal{B} \subset \{Bn, B1, B0\}$ and $\mathcal{S} \subset \{A, S1, S2\}$ denote the sets of *active* buyers and sellers in a steady state equilibrium, that is agents that have a strictly positive measure in steady state. Given the set of active segments $(\mathcal{P}, \Theta)$ and agents' steady state value functions $\{V^{Bn}, V^{B1}, V^{B0}, V^{A}, V^{S1}, V^{S2}\}$, there exists no pair $(p, \theta) \notin (\mathcal{P}, \Theta)$, such that $V^i(p, \theta) > V^i$, for some $i \in \{Bn, B1, B0\}$, and $V^j(p, \theta) \geq V^j$ for some $j \in \mathcal{S}$, or $V^i(p, \theta) > V^i$, for some $i \in \{A, S1, S2\}$, and $V^j(p, \theta) \geq V^j$ for some $j \in \mathcal{B}$, where $V^i(p, \theta)$ denotes the steady state value function of an agent that trades in segment (p, θ), for $i \in \{Bn, B1, B0, A, S1, S2\}$.

Informally, the no-surplus allocation condition requires that in equilibrium there are no agents that would be strictly better off from deviating and opening a new market segment that would be at least as attractive for some active agents (buyers or sellers) compared to their equilibrium values.

We can now define a symmetric steady state competitive search equilibrium of this economy as follows

Definition E.1. A symmetric steady state competitive search equilibrium of this economy consists of a set of active market segments $(\mathcal{P}, \Theta)$, steady state value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^{A}, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A, distributions of agent types over active market segments $\{\mathbf{x}^j\}_{j \in \{Bn, A, B1, S1, B0, S2\}}$, and set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, such that

1. The value functions satisfy equations (E.19) - (E.25) and the mixing distributions $\{\mathbf{x}^j\}_j$ are consistent with the agents' optimization problems.

2. Mismatched owners choose to buy first or sell first, to maximize $\bar{V} = \max\{V^{B1}, V^{S1}\}$ and

the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\{x_i = b\}\, di,$$

where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;

3. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (E.27)-(E.32) and (E.37)-(E.38) given Θ, $\left\{\mathbf{x}^j\right\}_j$ and mismatched owners' optimal decisions, reflected in x_b and x_s.

4. Every $\theta \in \Theta$ satisfies equation (E.36) given B_n, B_0, B_1, S_1, S_2, O, A, and $\left\{\mathbf{x}^j\right\}_j$;

5. The set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, is consistent with mismatched owners' optimal decisions;

6. $(\mathcal{P}, \Theta)$ and agents' steady state value functions satisfy the no-surplus allocation condition.

Next, we characterize competitive search equilibria when the cost of being mismatched, χ, is low, and so is the flow utility of being a double owner, u_2.[30] Also, as in Section 5.2 we assume that new entrants enjoy a strictly higher flow utility than forced renters: $u_n > u_0$.

In the "Buy first" equilibrium, the buyers are mismatched owners and new entrants, while the sellers are real estate firms and double owners. Figure E.1a shows the market constellations in this equilibrium, where blue indicates sellers and red buyers. The lightly-shaded rectangles and dashed lines indicate a deviating agent. The most patient buyer is the mismatched owner, while the most impatient sellers are the double owners. Hence, these agents always transact. The least patient buyers are the new entrants, while the most patient sellers are the real estate firms. Hence, submarkets for real estate firms and new entrants will always exist. In addition, a market for new entrants and double owners will also open.

[30] We have no reason to believe that the condition on χ is necessary to obtain multiple equilibria. However, without it the model becomes less tractable, as it is not clear from the outset what market constellations will then be realized.

Note that the match surplus Σ_{B1S2} between a mismatched buyer and a double owner is given by

$$\Sigma_{B1S2} = V + V^{S2} - V^{B1} - V^{S2} = V - V^{B1} > 0,$$

so that there is trade in this market. Now, consider a mismatched owner that deviates and sells first. For a small χ, this seller will be more patient than both the real estate firms and the double owners, and will, therefore, transact with the most impatient buyers among the non-deviating buyers, namely, the new entrants. He will then become a new buyer type – a forced renter – that is even more impatient than the new entrant because $u_n > u_0$. A forced renter will , therefore, transact with the most patient sellers, which are the real estate firms. Given that a forced renter is more impatient than a new entrant, real estate firms are willing to open a new submarket for the deviating agent. In particular, the value from being a forced renter, V^{B0}, maximizes his gain from search given the value of the real estate firm.

The match surplus between the deviating mismatched owner and the new entrant, Σ_{BnS1}, can be written as

$$\Sigma_{BnS1} = V + V^{B0} - V^{Bn} - V^{S1}.$$

Note that also $\lim_{\chi \to 0} V^{S1} = \lim_{\chi \to 0} V$. Moreover, for $u_0 < u_n$, V^{B0} is strictly lower than V^{Bn}, also in the limit as $\chi \to 0$. As a result, the match surplus Σ_{BnS1} is negative for small values of χ, so that the mismatched owner cannot gain by deviating and the "Buy first" equilibrium exists.

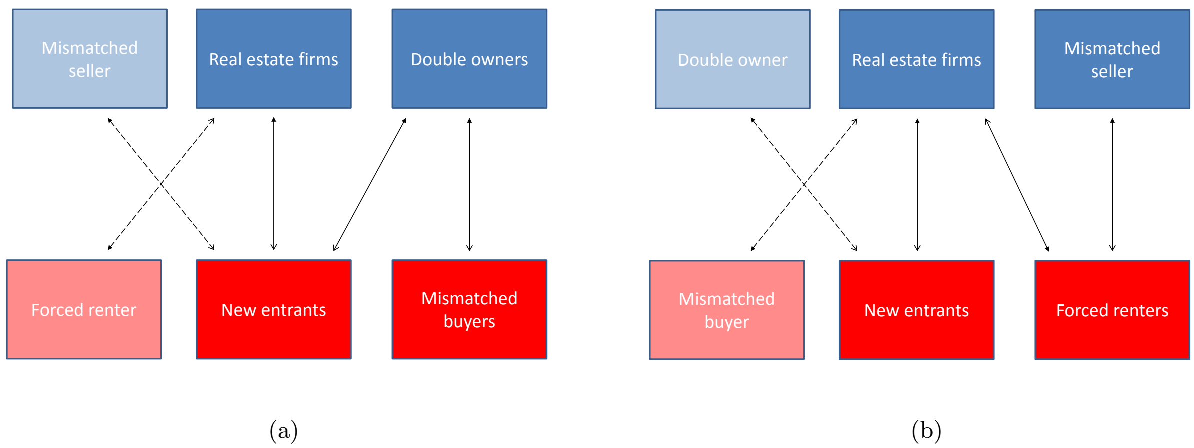

Figure E.1: Equilibrium market segments (solid colors) and deviators (weaker colors) for "Buy first" (a) and "Sell first" (b) competitive search equilibria.

In the "Sell first" equilibrium, the sellers are mismatched owners and real estate firms, while the buyers are new entrants and forced renters. The most patient buyers are the new entrants, and the most patient sellers are the mismatched owners. The active submarkets will be between new entrants and real estate firms, mismatched buyers and forced renters, and forced renters and real estate firms. The markets (together with a deviating agent) are illustrated in Figure E.1b.

The asset values in the market segment for real estate firms and new entrants are as in the "Buy first" equilibrium, so this market is active. For a real estate firm, the surplus of a transaction with a (more impatient) forced renter is even larger than with a new entrant, so that there are benefits to trading in that market as well. Hence, a forced renter obtains the same value V^{B0} as the (deviating) forced renter in the "Buy first" equilibrium. The match surplus between a forced renter and a mismatched seller, Σ_{B0S1}, is given by

$$\Sigma_{B0S1} = V + V^{B0} - V^{S1} - V^{B0} = V - V^{S1} > 0.$$

Now, consider a mismatched agent that deviates and buys first. This buyer will be more patient than both new entrants and forced renters, and will thus transact with the real estate firm – the most impatient seller. He will then become a double owner, thus, becoming more impatient than the real estate firm, and will therefore transact with the new entrants. A new submarket will open up, where the asset value V^{S2} is the same as for a double owner in the "Buy first" equilibrium. However, for the same reasons as in the "Buy first" equilibrium, the match surplus between the mismatched buyer and the real estate firm, Σ_{B1A}, is negative for low values of u_2. It follows that the deviation is unprofitable. We conclude that the model still exhibits multiple equilibria, as stated in the following proposition.

Proposition E.3. *Consider the economy with competitive search, and suppose that χ and u_2 are small and that $u_0 < u_n$. Then the economy exhibits multiple equilibria. In one equilibrium, all mismatched owners buy first. In another equilibrium, all mismatched owners sell first.*

Proof. Consider first the "Buy first" equilibrium as described above. In this "Buy first" equilibrium there are three active market segments characterized by prices $p_1^{B1} > p_2^{B1} > p_3^{B1}$ and market tightnesses $\theta_1^{B1} < \theta_2^{B1} < \theta_3^{B1}$. New entrants trade with real estate agents in market 1 and with double owners in market 2, while the latter also trade with mismatched buyers in market 3. Let x^{Bn} denote the probability with which a new entrant visits segment $\left(p_1^{B1}, \theta_1^{B1}\right)$, and x^{S2} the probability with which a double owner visits segment $\left(p_2^{B1}, \theta_2^{B1}\right)$. The stock-flow conditions for this equilibrium are

$$B_n = \frac{g}{x^{Bn} q\left(\theta_1^{B1}\right) + \left(1 - x^{Bn}\right) q\left(\theta_2^{B1}\right) + g}, \tag{E.39}$$

$$A = \frac{g}{\mu\left(\theta_1^{B1}\right) + g}, \tag{E.40}$$

$$B_1 = \frac{\gamma O}{q\left(\theta_3^{B1}\right) + g}, \tag{E.41}$$

$$S_2 = \frac{q\left(\theta_3^{B1}\right) B_1}{x^{S2}\mu\left(\theta_2^{B1}\right) + \left(1 - x^{S2}\right)\mu\left(\theta_3^{B1}\right) + g}, \tag{E.42}$$

$$B_n + B_1 + S_2 + O = 1, \tag{E.43}$$

and

$$B_n = A + S_2. \tag{E.44}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{B1} = \frac{x^{Bn} B_n}{A}, \tag{E.45}$$

$$\theta_2^{B1} = \frac{\left(1 - x^{Bn}\right) B_n}{x^{S2} S_2}, \tag{E.46}$$

and

$$\theta_3^{B1} = \frac{B_1}{\left(1 - x^{S2}\right) S_2}. \tag{E.47}$$

Observe that (E.39), (E.40), (E.44), and (E.45) imply that $x^{Bn} < 1$, as otherwise, (E.39), (E.40) and (E.45) give

$$\theta_1^{B1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{B1}\right) + g}{q\left(\theta_1^{B1}\right) + g}, \tag{E.48}$$

which has a unique solution at $\theta_1^{B1} = 1$. However, this is inconsistent with (E.44).

Let Σ_{ij}, for $i \in \{B_n, B_0, B_1\}$ and $j \in \{A, S_1, S_2\}$ denote the match surplus from trading between a buyer i and seller j. The no-surplus allocation condition determines the equilibrium prices in each segment as a function of the steady state values of agents. Define

$$\begin{aligned}\bar{V}^{Bn} &= q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right)\\ &= q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),\end{aligned}$$

$$\bar{V}^A = \mu\left(\theta_1^{B1}\right)\left(p_1^{B1} - V^A\right),$$

$$\bar{V}^{B1} = q\left(\theta_3^{B1}\right)\left(-p_3^{B1} + V^{S2} - V^{B1}\right),$$

and

$$\begin{aligned}\bar{V}^{S2} &= \mu\left(\theta_2^{B1}\right)\left(p_2^{B1} + V - V^{S2}\right) \\ &= \mu\left(\theta_3^{B1}\right)\left(p_3^{B1} + V - V^{S2}\right),\end{aligned}$$

as the maximized value of searching for each trader. The no-surplus allocation condition implies that

$$\left(p_1^{B1}, \theta_1^{B1}\right) = \arg\max_{p,\theta} \mu\left(\theta\right)\left(p - V^A\right),$$

$$\text{s.t. } q\left(\theta\right)\left(-p + V - V^{Bn}\right) \geq \bar{V}^{Bn}.$$

Denote the elasticity of the matching function with respect to buyers by α (which may depend on θ). Solving for p_1^{B1} and θ_1^{B1} gives the well-known Hosios rule (Hosios (1990)),

$$p_1^{B1} - V^A = (1-\alpha)\,\Sigma_{BnA},$$

or equivalently,

$$p_1^{B1} = (1-\alpha)\left(V - V^{Bn}\right) + \alpha V^A.$$

Therefore,

$$\bar{V}^{Bn} = \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2},$$

or

$$q\left(\theta_1^{B1}\right)\Sigma_{BnA} = q\left(\theta_2^{B1}\right)\Sigma_{BnS2}. \tag{E.49}$$

We have similar surplus sharing rules between the other trading pairs, which determine p_2^{B1} and p_3^{B1}. There is one more indifference condition for a double owner that relates θ_2^{B1} and θ_3^{B1}. Specifically,

$$\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \mu\left(\theta_3^{B1}\right)\Sigma_{B1S2}. \tag{E.50}$$

These surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{E.51}$$

$$\rho V^{A} = R + (1-\alpha)\,\mu\left(\theta_1^{B1}\right)\Sigma_{BnA}, \tag{E.52}$$

$$\rho V^{B1} = u - \chi + \alpha q\left(\theta_3^{B1}\right)\Sigma_{B1S2}, \tag{E.53}$$

$$\rho V^{S2} = u_2 + R + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{E.54}$$

and

$$\rho V = u + \gamma\left(V^{B1} - V\right). \tag{E.55}$$

Finally, use V^{Bn} and V^{S2} from (E.51) and (E.54) to solve for

$$\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\rho + \alpha q\left(\theta_2^{B1}\right) + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)}. \tag{E.56}$$

Similarly, using V^{Bn} and V^{A} from (E.51) and (E.20), combined with indifference condition (E.49), to solve for

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{B1}\right) + (1-\alpha)\,\mu\left(\theta_1^{B1}\right)}. \tag{E.57}$$

Solving for V^{B1} from equation (E.53), we get

$$V^{B1} = \frac{u-\chi}{\rho+\alpha q\left(\theta_3^{B1}\right)} + \frac{\alpha q\left(\theta_3^{B1}\right)}{\rho+\alpha q\left(\theta_3^{B1}\right)} V,$$

so

$$\Sigma_{B1S2} = V - V^{B1} = \frac{\rho V - (u-\chi)}{\rho+\alpha q\left(\theta_3^{B1}\right)}. \tag{E.58}$$

Therefore, equations (E.39)-(E.47), combined with the two indifference conditions (E.49) and (E.50), and the value function equations (E.51)-(E.55) with surpluses (E.56)-(E.58) jointly determine the equilibrium stocks of agents, market tightnesses, mixing probabilities x^{Bn} and x^{S2}, and active agent value functions in a "Buy first" equilibrium.

We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n. Note that $\Sigma^{S2B1} = V - V^{B1} > 0$ for any u_2, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{B1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0}\Sigma_{B1S2} = 0$. This in turn implies that $\lim_{\chi\to 0}\theta_3^{B1} = \infty$ and $\lim_{\chi\to 0} x^{S2} = 1$. To see this, suppose to the contrary that as $\chi \to 0$, θ_3^{B1} remains bounded and thus x^{S2} is strictly below one. Therefore, $\mu\left(\theta_3^{B1}\right)\Sigma_{B1S2} \to 0$, so indifference condition (E.50) implies that $\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} \to 0$. Given (E.56), this in turn means that $\theta_2^{B1} \to 0$ and thus $x^{Bn} \to 1$. However,

$$\lim_{\theta_2^{B1}\to 0} q\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\alpha},$$

which is inconsistent with $x^{Bn} \to 1$. To see this, remember from (E.48) that $\theta_1^{B1} \to 1$ as $x^{Bn} \to 1$. Because

$$\lim_{\theta_1^{B1}\to 1} q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \frac{\rho V - u_n}{\frac{\rho}{q(1)}+1} < \rho V - u_n < \frac{2\rho V - u_n - u_2}{\alpha},$$

in this case new entrants would be strictly better off participating in the second market segment. Thus, we arrive at a contradiction.

As $\theta_3^{B1} \to \infty$, $q\left(\theta_3^{B1}\right) \to 0$, and mismatched owners do not buy to become double owners: $S_2 \to 0$. Without trading partners in market 2, all new entrants visit market 1: $x^{Bn} \to 1$ and thus $\theta_1^{B1} \to \frac{B_n}{A} \to 1$. In this case, V^{Bn} from (E.51) is given by

$$\lim_{\chi \to 0} \rho V^{Bn} = u_n + \frac{\alpha q(1)}{\rho + q(1)}(u - u_n) - R, \tag{E.59}$$

which is strictly between 0 and ρV, as long as R is not too large. Similarly, using that $\mu(1) = q(1)$, V^A is given by

$$\lim_{\chi \to 0} \rho V^{A} = R + \frac{(1-\alpha) q(1)}{\rho + q(1)}(u - u_n), \tag{E.60}$$

which is also strictly between 0 and ρV if R is not too large. As a result,

$$\lim_{\chi \to 0} \left(V^A + V^{Bn}\right) = \frac{u_n}{\rho} + \frac{q(1)}{\rho + q(1)} \frac{u - u_n}{\rho},$$

which is strictly between 0 and V. By continuity, there exists a $\bar{\chi}_1 > 0$ such that for $\chi \in (0, \bar{\chi}_1)$, it is the case that $\Sigma_{BnA} > 0$, $x^{Bn} \in (0,1)$ and $\theta_1^{B1} \in (0,1)$, but also $\Sigma_{B1S2} > 0$.

With V^{Bn} as defined in (E.59) above, it follows that V^{S2} is uniquely determined as

$$\rho V^{S2} = \max_{p,\theta} \left\{u_2 + R + \mu(\theta)(V + p - V^{S2})\right\},$$

subject to $u_n - R + q(\theta)(V - p - V^{Bn}) = \rho V^{Bn}$. Note that V^{S2} goes to negative infinity for any $\chi > 0$ when u_2 does. To see this, suppose to the contrary that V^{S2} remains bounded when u_2 goes to negative infinity. Then $\mu(\theta)$ must go to infinity, and hence θ must go to infinity. But then $q(\theta)$ goes to zero, and for the new entrants to get their outside option, p must go to $-\infty$. In this case V^{S2} still goes to negative infinity, so that we arrive at a contradiction. Consequently, for any $\chi > 0$ there exists a $\bar{u}_2^{B1}$ such that for $u_2 < \bar{u}_2^{B1}$, V^{S2} is sufficiently low such that both

$\Sigma_{B1A} = V^{S2} - V^{A} - V^{B1} < 0$ and $\Sigma_{BnS2} = 2V - V^{Bn} - V^{S2} > \Sigma_{BnA} > 0$. We can then conclude that there is trade in markets 1, 2, and 3, but that real estate agents and mismatched buyers do not open a fourth market.[31]

Furthermore, it follows that for any $u_2 < \bar{u}_2^{B1}$ there exists a $\bar{\chi}_2 > 0$ such that for $\chi \in (0, \bar{\chi}_2)$, it is the case that $\Sigma_{BnS2} > \Sigma_{B1S2}$. Given this ranking and the fact that $\Sigma_{BnA} < \Sigma_{BnS2}$, the ranking of tightnesses across segments then follows from the indifference conditions (E.49) and (E.50). Having established the ranking of tightnesses, the ranking of prices across segments immediately follows from the indifference conditions as well. Specifically, (E.49) implies that

$$q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) = q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),$$

or

$$\frac{q\left(\theta_1^{B1}\right)}{q\left(\theta_2^{B1}\right)} = \frac{-p_2^{B1} + V - V^{Bn}}{-p_1^{B1} + V - V^{Bn}}.$$

$\theta_1^{B1} < \theta_2^{B1}$ and $q(.)$ decreasing imply that $p_1^{B1} > p_2^{B1}$. Similarly, (E.50) implies that $p_2^{B1} > p_3^{B1}$. Finally, note that $\Sigma_{BnS2} > \Sigma_{B1S2}$ implies that $V - V^{Bn} > V^{S2} - V^{B1}$, so a new entrant is more impatient than a mismatched buyer in the sense that the direct utility gain from transacting is higher for a new entrant compared to a mismatched buyer.[32]

Consider now a mismatched owner that deviates and sells first, and upon trade becomes a forced renter. We allow both a mismatched seller and a forced renter to open new market segments with active agents as counterparties. First, observe that $V^{Bn} > V^{B0}$ for any $\chi > 0$, that is, a new entrant is always better off than a forced renter. This ranking comes from the assumption that $u_0 < u_n$ and from a revealed preference argument. Specifically, suppose to the contrary that

[31] For market 2 to be active, it is sufficient for u_2 to be low enough to ensure $\Sigma_{BnS2} > 0$, even if $\Sigma_{BnS2} < \Sigma_{BnA}$. However, $u_2 < \bar{u}_2^{B1}$ ensures the ranking of tightnesses and prices proven next.

[32] This also implies that a new entrant has steeper sloped indifference curves in the $\theta - p$ space, so he is willing to trade-off a higher price for the same decrease in market tightness compared to a mismatched buyer.

$V^{B0} > V^{Bn}$. Suppose also that it is optimal for a forced renter to trade with a real estate firm (the argument for the case where the forced renter trades with a double owner is analogous). The no-surplus allocation condition again implies that the Hosios condition holds, so

$$\rho V^{B0} = u_0 - R + \alpha q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right),$$

where $\tilde{\theta}$ is such that a real estate firm is indifferent between trading in this new segment and trading in the segment with a tightness of θ_1^{B1} and a price of p_1^{B1}. In contrast, we have that

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right).$$

Since $u_0 < u_n$ but $V^{B0} > V^{Bn}$, it follows that $q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$ and so $\tilde{\theta} < \theta_1^{B1}$. But then a new entrant is better off deviating and trading in the segment with tightness $\tilde{\theta}$, since $q\left(\tilde{\theta}\right)\left(V - V^{Bn} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$. Furthermore, given that $V^{B0} > V^{Bn}$, $\Sigma_{BnA} > \Sigma_{B0A}$, so a real estate firm is in fact also strictly better off trading with a new entrant in the segment with tightness $\tilde{\theta}$. However, this is not consistent with $\left(p_1^{B1}, \theta_1^{B1}\right)$ not violating the no-surplus allocation condition. Therefore, in an equilibrium where $\left(p_1^{B1}, \theta_1^{B1}\right)$ are consistent with the no-surplus allocation, we must have $q\left(\tilde{\theta}\right) < q\left(\theta_1^{B1}\right)$. However, this means that $V^{B0} < V^{Bn}$, and we arrive at a contradiction.

We conclude that $V^{B0} > V^{Bn}$ and $\Sigma_{BnA} < \Sigma_{B0A}$, so that a forced renter is the most impatient of the buyers. The forced renter will therefore trade with a real estate agent, the most patient of the sellers. A new submarket opens up, and real estate firms flow into this submarket up to the point where they are indifferent between selling to the deviator and to a new agent. Now suppose the deviating mismatched owner sells to a new entrant. Then the match surplus reads

$$\Sigma_{BnS1} = V - V^{Bn} + V^{B0} - V^{S1} \leq V - V^{Bn} + V^{B0} - \frac{u-\chi}{\rho}.$$

Given that $V^{Bn} - V^{B0}$ is bounded away from zero for any $\chi > 0$, there exists a $\bar{\chi}_3 > 0$ such that for $\chi \in (0, \bar{\chi}_3)$, it is the case that $\Sigma_{BnS1} < 0$. Note, however, that $\Sigma_{BnS1} > \Sigma_{B1S1}$ for $\chi < \bar{\chi}_2$, since, as shown above, in that case $V - V^{Bn} > V^{S2} - V^{B1}$, meaning that a new entrant is more impatient than a mismatched buyer. Therefore, for $\chi < \min\{\bar{\chi}_1, \bar{\chi}_2, \bar{\chi}_3\}$, $\Sigma_{B1S1} < \Sigma_{BnS1} < 0$ and $\Sigma_{B1S2} > 0$. In that case a mismatched owner that deviates and sells first is better off not trading. However, not trading is dominated by buying first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from buying first in a "Buy first" equilibrium.

Constructing a "Sell first" equilibrium follows similar steps. In this equilibrium there are three active market segments characterized by prices $p_1^{S1} < p_2^{S1} < p_3^{S1}$ and market tightnesses $\theta_1^{S1} > \theta_2^{S1} > \theta_3^{S1}$. Real estate agents trade with new entrants in market 1 and with forced renters in market 2, while the latter also trade with mismatched sellers in market 3. Let x^A denote the probability with which a real estate firm visits segment $\left(p_1^{S1}, \theta_1^{S1}\right)$, and x^{B0} the probability with which a forced renter visits segment $\left(p_2^{S1}, \theta_2^{S1}\right)$. The stock-flow conditions in this case become

$$B_n = \frac{g}{q\left(\theta_1^{S1}\right) + g}, \tag{E.61}$$

$$A = \frac{g}{x^A \mu\left(\theta_1^{S1}\right) + \left(1 - x^A\right)\mu\left(\theta_2^{S1}\right) + g}, \tag{E.62}$$

$$S_1 = \frac{\gamma O}{\mu\left(\theta_3^{S1}\right) + g}, \tag{E.63}$$

$$B_0 = \frac{\mu\left(\theta_3^{S1}\right) S_1}{x^{B0} q\left(\theta_2^{S1}\right) + \left(1 - x^{B0}\right) q\left(\theta_3^{S1}\right) + g}, \tag{E.64}$$

$$B_n + B_0 + S_1 + O = 1, \tag{E.65}$$

and

$$B_n + B_0 = A. \tag{E.66}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{S1} = \frac{B_n}{x^A A}, \tag{E.67}$$

$$\theta_2^{S1} = \frac{x^{B0} B_0}{\left(1 - x^A\right) A}. \tag{E.68}$$

and

$$\theta_3^{S1} = \frac{\left(1 - x^{B0}\right) B_0}{S_1}, \tag{E.69}$$

Similarly to before, observe that (E.61), (E.62), (E.66), and (E.69) imply that $x^A < 1$, as otherwise, (E.61), (E.62) and (E.69) give

$$\theta_1^{S1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{S1}\right) + g}{q\left(\theta_1^{S1}\right) + g},$$

which has a unique solution at $\theta_1^{S1} = 1$. However, this is inconsistent with (E.66). As before, the no-surplus allocation implies that the match surpluses between trading pairs are split according to the Hosios rule. Consequently, there are two indifference conditions for real estate firms and forced renters given by

$$\mu\left(\theta_1^{S1}\right) \Sigma_{BnA} = \mu\left(\theta_2^{S1}\right) \Sigma_{B0A}, \tag{E.70}$$

and

$$q\left(\theta_2^{S1}\right) \Sigma_{B0A} = q\left(\theta_3^{S1}\right) \Sigma_{B0S1}, \tag{E.71}$$

respectively. In addition, the surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{S1}\right)\Sigma_{BnA}, \tag{E.72}$$

$$\rho V^{A} = R + (1-\alpha)\,\mu\left(\theta_1^{S1}\right)\Sigma_{BnA}, \tag{E.73}$$

$$\rho V^{S1} = u - \chi + (1-\alpha)\,q\left(\theta_3^{S1}\right)\Sigma_{B0S1}, \tag{E.74}$$

$$\rho V^{B0} = u_0 - R + \alpha\mu\left(\theta_3^{S1}\right)\Sigma_{B0S1}, \tag{E.75}$$

and

$$\rho V = u + \gamma\left(V^{S1} - V\right). \tag{E.76}$$

Finally, the above value functions allow us to solve for the surpluses as follows:

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{S1}\right) + (1-\alpha)\,\mu\left(\theta_1^{S1}\right)}. \tag{E.77}$$

$$\Sigma_{B0S1} = V - V^{S1} = \frac{\rho V - (u-\chi)}{\rho + (1-\alpha)\,\mu\left(\theta_3^{S1}\right)}, \tag{E.78}$$

and

$$\Sigma_{B0A} = \frac{\rho V - u_0}{\rho + \alpha q\left(\theta_2^{S1}\right) + (1-\alpha)\,\mu\left(\theta_2^{S1}\right)}. \tag{E.79}$$

The stock-flow and market tightness equations (E.61)-(E.69), combined with the two indifference conditions (E.70) and (E.71), and value functions and surpluses (E.72)-(E.79) fully characterize the equilibrium stocks of agents, market tightnesses, mixing probabilities x^A and x^{B0}, and active agent value functions in a "Sell first" equilibrium. We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n.

Note that $\Sigma_{B0S1} = V - V^{S1} > 0$ for any u_0, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0} \Sigma_{B0S1} = 0$. Then, a set of arguments similar to the case of the "Buy first" equilibrium shows that $\lim_{\chi\to 0} \theta_3^{B1} = 0$ and $\lim_{\chi\to 0} x^{B0} = 1$, so that $\lim_{\chi\to 0} \mu\left(\theta_3^{B1}\right) = 0$ and $\lim_{\chi\to 0} B_0 = 0$, implying that $\lim_{\chi\to 0} x^A = 1$ and $\lim_{\chi\to 0} \theta_1^{B1} = 1$. As a result, $\lim_{\chi\to 0} V^{Bn}$ and $\lim_{\chi\to 0} V^A$ are the same as in the "Buy first" equilibrium, and there exists a $\bar{\chi}_4 > 0$ such that for $\chi \in (0, \bar{\chi}_4)$, it is the case that $\Sigma_{BnA} > 0$, $x^A \in (0,1)$ and $\theta_1^{S1} > 1$ but remains bounded, while $\Sigma_{B0S1} > 0$. As a result, markets 1 and 3 are active.

Following the same arguments as in the "Buy first" equilibrium, it is then the case that $V^{Bn} > V^{B0}$, also as $\chi \to 0$, so that $0 < \Sigma_{BnA} < \Sigma_{B0A}$ and market 2 is active. Furthermore, there must exist a $\bar{\chi}_5 > 0$ such that for $\chi \in (0, \bar{\chi}_5)$, it is the case that $\Sigma_{B0A} > \Sigma_{B0S1}$, since $\lim_{\chi\to 0} \Sigma_{B0S1} = 0$. This ranking implies that $-V^A > V^{B0} - V^{S1}$, so that a real estate firm is more impatient than a mismatched seller in the sense that the direct utility gain from transacting is higher for a real estate firm compared to a mismatched seller. The ranking of tightnesses and prices across segments then follows from the indifference conditions (E.70) and (E.71), similar to the case of a "Buy first" equilibrium. The fact that $V^{Bn} - V^{B0}$ is bounded away from zero also implies that there exists a exist a $\bar{\chi}_6 > 0$ such that for $\chi \in (0, \bar{\chi}_6)$, a mismatched owner and a new entrant will not open a fourth market, because $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$ and thus $\Sigma_{BnS1} = V - V^{S1} + V^{B0} - V^{Bn} < 0$ for a sufficiently small χ.

Now consider a mismatched owner that deviates and buys first. Potential sellers are real estate firms and mismatched homeowners, and upon trade the deviator becomes a double owner, who can open up new market segments with new entrants and forced renters. Note that V^{S2} falls without bounds as u_2 does, because a deviating double owner has to offer new entrants or forced renters their market value, following a similar argument as in the "Buy first" equilibrium. Then there exists a $\bar{u}_2^{S1}$ such that for all $u_2 < \bar{u}_2^{S1}$ it is the case that $\Sigma_{B1A} = V^{S2} - V^A - V^m < 0$, so that a deviating mismatched owner does not buy from a real estate agent. Note, however, that $\Sigma_{B1S1} < \Sigma_{B1A} < 0$ for $\chi < \bar{\chi}_5$ since, as shown above, in that case $-V^A > V^{B0} - V^{S1}$, meaning that a new entrant is more impatient than a mismatched buyer. As a result, a mismatched owner that deviates and buys first is better off not trading. However, not trading is dominated by selling first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from selling first in a "Sell first" equilibrium.

Finally, setting $\bar{\chi} = \min\{\bar{\chi}_1, \bar{\chi}_2, \bar{\chi}_3, \bar{\chi}_4, \bar{\chi}_5, \bar{\chi}_6\}$ and $\bar{u}_2 = \min\{\bar{u}_2^{B1}, \bar{u}_2^{S1}\}$, we arrive at our result. □

E.6 Simultaneous Entry as Buyer and Seller

We assume that a mismatched owner can allocate a fixed amount of time (normalized to 1 unit) to search in the housing market as a buyer or a seller. A mismatched owner that chooses to enter as a buyer or seller only allocates all of his time to one activity. Otherwise, a mismatched owner that enters as both a buyer and a seller can allocate a fraction $\phi \in (0,1)$ of his time to searching as buyer, and searches the remaining $1-\phi$ of his time as seller. For a given market tightness θ, the value function V^{SB} for a mismatched owner that enters as both buyer and seller satisfies the following equation in a steady state equilibrium:

$$\rho V^{SB} = u - \chi + (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} + \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}.$$

We then show the following

Proposition E.4. *For $\theta \in \left(0, \tilde{\theta}\right)$, $V^{S1} > V^{SB}$, for any $\phi \in (0,1)$. Also, for $\theta \in \left(\tilde{\theta}, \infty\right)$, $V^{B1} > V^{SB}$, for any $\phi \in (0,1)$.*

Proof. To show the first part, suppose the opposite, so $V^{S1} \leq V^{SB}$. Then

$$\begin{aligned}\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}.\end{aligned}$$

Under the assumption that $V^{S1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{B1} < V^{S1}$ for $\theta \in \left(0, \tilde{\theta}\right)$, it must then be the case that

$$\begin{aligned}\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{B1}\right\},\end{aligned}$$

which does not hold because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > 0$ for $\theta \in \left(0, \tilde{\theta}\right)$ by Assumption A3, and because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > q(\theta)\left(-p + V^{S2} - V^{B1}\right)$ for $\theta \in \left(0, \tilde{\theta}\right)$ by Lemma 1.

To show the second part, suppose the opposite, so $V^{B1} \leq V^{SB}$. Then

$$\begin{aligned}q(\theta)\max\left\{0, p + V^{S2} - V^{B1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}.\end{aligned}$$

Under the assumption that $V^{B1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{S1} < V^{B1}$ for $\theta \in \left(\tilde{\theta}, \infty\right)$, it must then be the case that

$$\begin{aligned}q(\theta)\max\left\{0, p + V^{S2} - V^{B1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{B1}\right\},\end{aligned}$$

which does not hold because $q(\theta)\left(-p + V^{S2} - V^{B1}\right) > 0$ for $\theta \in \left(\tilde{\theta}, \infty\right)$ by Assumption A3, and because $\mu(\theta)\left(p+V^{B0}-V^{S1}\right)<q\left(\theta\right)\left(-p+V^{S2}-V^{B1}\right)$ for $\theta\in\left(\tilde{\theta},\infty\right)$ by Lemma 1.[33] □

Finally, note that under payoff symmetry (i.e. $\tilde{u}_0=\tilde{u}_2=c$) the possibility to enter as both buyer and seller while allocating each an equal amount of time can result in an equilibrium with a market tightness of $\theta=1$. Specifically, at $\theta=1$, $\mu\left(\theta\right)=q\left(\theta\right)=\mu\left(1\right)$. At these flow rates it can easily be seen that if $\tilde{u}_0=\tilde{u}_2=c$, then $V^{B1}=V^{S1}=V^{SB}$ for any ϕ. Finally, a tightness of $\theta=1$ can result from mismatched owners entering as buyers and sellers simultaneously and allocating each an equal amount of time (so $\phi=0.5$).

This is analogous to the equilibrium described in Proposition 1, with the only difference that now agents follow symmetric strategies compared to asymmetric strategies with one half of mismatched owners buying first and the other half selling first.

E.7 Homeowners compensated for their housing unit upon exit

Suppose that upon exit homeowners receive bids for their housing unit(s) from a set of competitive real estate firms. Therefore, given that the value of a housing unit to a real estate firm is $V^A\left(\theta\right)$, homeowners receive $V^A\left(\theta\right)$ for each housing unit that they own. Again, we consider a steady state equilibrium with a fixed market tightness θ. We define $\tilde{u}_0\left(\theta,g\right)\equiv u_0+\triangle-gV^A\left(\theta\right)$ and $\tilde{u}_2\left(\theta,g\right)=u_2-\triangle+gV^A\left(\theta\right)$. Note that $V^A\left(\theta\right)$ is (weakly) increasing in θ, so $\tilde{u}_2$ is increasing in θ and $\tilde{u}_0$ is decreasing in θ;

Given this definition, the difference between the values from buying first and selling first (assuming a mismatched owner transacts in both cases), $D\left(\theta\right)\equiv V^{B1}-V^{S1}$, is equal to

[33]Note also that for $\theta=0$ and $\theta\to\infty$, mismatched owners are indifferent between remaining mismatched and any search strategy, because $V^{B1}=V^{S1}=V^{SB}=\frac{u-\chi}{\rho}$, but that such tightnesses cannot occur in steady state by Lemma 2.

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))} \left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2(\theta, g)) - \tilde{u}_0(\theta, g) + \tilde{u}_2(\theta, g) \right].$$

Let $\tilde{\theta}$ be defined implicitly by

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2\left(\tilde{\theta}, g\right)}{u - \chi - \tilde{u}_0\left(\tilde{\theta}, g\right)},$$

whenever that equation has a solution.[34] Note that in the limit as $g \to 0$, assumption A3 will hold. Therefore, for g sufficiently close to zero, we will have that $u - \chi > \max\{\tilde{u}_0(\theta, g), \tilde{u}_2(\theta, g)\}$, for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, and so a version of Lemma 1 will hold in this case as well. Given this result one can then easily construct multiple steady state equilibria as in Proposition 1.

[34]Note that the above equation for $\tilde{\theta}$, whenever it has a solution, has a unique solution for any $g \geq 0$, since given the properties of $\tilde{u}_0$ and $\tilde{u}_2$, it follows that the right hand side of this expression is (weakly) decreasing in θ. Furthermore, the right hand side is strictly decreasing in g for any $\theta > 0$, so by the implicit function theorem, $\tilde{\theta}$ is decreasing in g.