The Impact of Monetary Policy on Leading Variables for Financial Stability in Norway

Navn: Harald Wieslander, Helene Olsen

Start: 15.01.2019 09.00
Finish: 01.07.2019 12.00
Acknowledgement

We would like to thank BI Norwegian Business School, including all academic and administrative staff, for five good years. We would like to express special thanks to our supervisor, Hilde C. Bjørnland, Provost for Research and Academic Resources and Professor of Economics at BI Norwegian Business School. Bjørnland has supported us throughout the process, giving insightful comments and suggestions for improvement. We are also grateful for good help provided by Jamie Cross, Postdoctoral Fellow at the Department of Economics, who has supported us with advice on both content and coding. Furthermore, we would like to thank Steffen Grønneberg, Associate Professor at the Department of Economics, for improving our understanding of econometrics, both prior to and during the thesis process. We would also like to thank Tommy Sveen for always being open to questions, answers, and discussions. Lastly, we would like to thank our friends and family for their great support throughout our years as students at BI Norwegian Business School.
Contents

1 Introduction 1

2 Methodology 3
  2.1 Research design .......................... 3
  2.2 Step 1 - The signaling approach ........ 4
    2.2.1 Candidate indicator variables .......... 5
    2.2.2 Crisis classification ................... 7
    2.2.3 Signal classification ................... 7
    2.2.4 Threshold values ...................... 8
  2.3 Step 2 - VAR and SVAR ................... 9
    2.3.1 Data .................................. 10
    2.3.2 SVAR ................................ 10
    2.3.3 VAR .................................. 11
  2.4 Identification of structural parameters ... 11
    2.4.1 Identification problem ................ 11
    2.4.2 Recursive restrictions ................ 12
    2.4.3 Sign restrictions ..................... 13
    2.4.4 Our identification approach .......... 15
  2.5 HP-filter ................................ 16

3 Step 1 - EWI analysis 18
  3.1 Results ................................ 18
    3.1.1 Credit-to-GDP ......................... 19
    3.1.2 Household credit ..................... 21
    3.1.3 House prices ......................... 22
    3.1.4 House prices-to-disposable income .... 24
    3.1.5 Banks' wholesale funding ratio ....... 25
  3.2 List of indicators ....................... 26

4 Step 2 - SVAR analysis 26
  4.1 Results ................................ 26
    4.1.1 Non-indicator variables .............. 27
    4.1.2 Indicator variables ................... 28

5 Limitations 32

6 Conclusion 33

A Data 41

B Indicators 42
  B.1 Summary statistics ...................... 42
B.2 Lagged correlations (1986-2018) ................................................. 42
B.4 Crisis predictability ................................................................. 43
  B.4.1 $\beta = 0.5$ ................................................................. 43
  B.4.2 $\beta = 0.6$ ................................................................. 44
  B.4.3 $\beta = 0.7$ ................................................................. 44
  B.4.4 $\beta = 0.8$ ................................................................. 45

C SVAR models .......................................................... 46
  C.1 Model 1 ................................................................. 46
  C.2 Model 2 ................................................................. 47
  C.3 Model 3 ................................................................. 48
  C.4 Model 4 ................................................................. 49
  C.5 Model 5 ................................................................. 51
  C.6 Model 6 ................................................................. 52
  C.7 Model 7 ................................................................. 53
  C.8 Model 8 ................................................................. 55
  C.9 Model 9 ................................................................. 56
  C.10 Model 10 ................................................................. 57
  C.11 Lag length ............................................................. 59

D VAR and SVAR theory ............................................. 63
  D.1 Lag length ............................................................. 63
    D.1.1 Akaike information criterion (AIC) ................................ 64
    D.1.2 Bayesian information criterion (BIC) ............................... 64
    D.1.3 Hannan-Quinn information criterion (HQC) ....................... 64
  D.2 Model diagnostics ....................................................... 64
    D.2.1 Stability ........................................................... 64
    D.2.2 Residuals .......................................................... 65
  D.3 Companion form ......................................................... 65
  D.4 Moving average representation ........................................... 66
  D.5 Impulse response ......................................................... 66
1 Introduction

On 2 March 2018, the Norwegian Government issued a new regulation for the conduct of monetary policy. The new regulation specifies that the inflation targeting regime shall contribute to the standard goals of monetary policy, high and stable output and employment, and in addition, counteract the build-up of financial imbalances (Forskrift for pengepolitikken, 2018). The Governor of Norges Bank, Øystein Olsen, in a hearing before the Norwegian parliament, highlights that counteracting build-ups of financial imbalances may contribute to the two other targets of monetary policy as well. However, he points out that the primary responsibility for financial stability lays with financial regulation and supervision, not monetary policy (Norges Bank, 2018). In this paper, we investigate the relationship between financial stability and monetary policy in light of the new regulation.

In the aftermath of the global financial crisis, a large literature has developed on how to combat financial imbalances. The macroeconomic research has focused mainly on macroprudential policy (Borio, 2003; Arnold et al., 2012; Detken et al., 2014; Shin, 2016; Akinci and Olmstead-Rumsey, 2018), and to some extent monetary policy (Assenmacher-Wesche and Gerlach, 2008; Bjørnland and Jacobsen, 2010; Svensson, 2013, 2017; Robstad, 2018). Both macroprudential policy and monetary policy influence the financial cycle through the financial intermediation process. They both affect the demand for credit by reallocating spending over time, and the supply of credit by influencing funding costs (Shin, 2016). While macroprudential policy is seen as the first line of defense against financial imbalances (Mester, 2017), monetary policy can act as a second line of defense by leaning against the wind. The policy paradigm after the global financial crisis is one in which both monetary policy and macroprudential policy are used to stabilize the financial cycle (Smets, 2014).

This paper attempts to build a bridge between studies on financial stability and monetary policy. If financial stability is important for welfare, and the central bank can affect financial stability, then a proxy for financial stability should be included in the loss function together with the other goals of monetary policy (Woodford, 2012). The result will be a higher interest rate than when conducting standard monetary policy. The practice of keeping interest rates higher, due to financial stability concerns, is called leaning against the wind (Svensson, 2017). Research on the interaction between financial imbalances and monetary policy has mainly focused on the normative question of whether or not to lean against the wind, see for example Gerdrup et al. (2016); Alpanda and Ueberfeldt (2016); Ajello et al. (2016); Svensson (2017); Krug (2018). In this thesis, we will answer the descriptive question raised by Smets (2014), how does monetary policy affect financial imbalances? Knowledge about the transmission from monetary policy to financial stability is important because it can aid policymakers in their goal of maintaining a stable financial cycle, and reduce the severe costs associated with financial crises (Jordà et al., 2013).

To answer this question, we proceed in two steps. In step 1, we explore measures of financial
stability. The primary difficulty is that financial stability is a latent state. Borio and Drehmann (2009) defines financial instability as a situation in which normal-sized shocks to the financial system are sufficient to produce financial distress, while financial stability is its converse. To pin down financial stability, we search for determinants of financial distress with the use of a signaling model. More specifically, we look for \( x_1, x_2, \ldots, x_n \) in the following relationship:

\[
\text{Financial Stability}_t = f(\Theta) + \sum_{i=1}^{n} \gamma_i x_{i,t-h}
\]

where \( \sum_{i=1}^{n} x_{i,t-h} \) are variables that are leading determinants of financial instability, \( h \) is the lag horizon, and \( f(\Theta) \) is an unknown function of all else relating to financial stability. In the search for indicator variables, we include three indicators Norges Bank use in their assessment of financial imbalances, and two additional variables from the early warning indicators (EWI) literature. The literature on EWI and early warning systems (EWS) search for variables that provide an indication of the future state of the financial system, see for example Davis and Karim (2008); Borio and Drehmann (2009); Gramlich et al. (2010); Alessi and Detken (2011); Oet et al. (2013); Drehmann and Juselius (2014); Azis and Shin (2015); Aldasoro et al. (2018).

To assess the candidate variables’ ability to provide good signals for financial distress, we use the signaling approach first suggested by Kaminsky and Reinhart (1999). The approach is to assess the candidate variables based on how they signal past crises, given a threshold value where the indication switches from normal to signaling. Borio and Drehmann (2009) build on the methodology of Kaminsky and Reinhart (1999), and find that the methodology is a step towards a better framework for financial stability. However, since all indicator variables will, at some point, provide a false signal, they highlight the role of judgment when interpreting the signals.

In step 2, when we have found our determinants of financial stability, we examine the transmission from the monetary policy instrument to these determinants, in order to establish how monetary policy affects financial stability. Previous studies on the effect of monetary policy on financial stability in Norway have mainly focused on asset prices and credit, for example Bjørnland and Jacobsen (2010), who investigates the role of house prices in the monetary policy transmission in Norway. They find that house prices react strongly to interest rate shocks, and therefore that the interest rate can be used to stabilize the housing market. Robstad (2018) builds on the research of Bjørnland and Jacobsen (2010) and includes household credit in his structural VAR model. In line with Bjørnland and Jacobsen (2010), he finds that house prices react to changes in the interest rate, while the response of credit is small. Assenmacher-Wesche and Gerlach (2008) finds that the interest rate has an impact on both property prices and equity prices in Norway. We notice that since we cannot directly observe financial stability, a broader set of variables might be useful. Therefore, we
contribute to the literature by using a broader set of financial stability indicator variables.

The remainder of the paper is structured as follows. Section 2 explains the research design, the signaling approach, and the VAR methodology. In section 3 we go through step 1 of our research, that is, assess indicators for financial stability, while in section 4 we assess the transmission from the monetary policy instrument to the indicators found in section 3. In section 5 we discuss some of the limitations of our approaches. Finally, our concluding remarks are presented in section 6.

2 Methodology

2.1 Research design

We conduct our research in two steps. In step 1, we assess potential financial stability indicators. In step 2, we assess the transmission from the interest rate to the financial stability indicators.

Step 1 of our research is to analyze indicators found in the EWI literature, and the indicators used by Norges Bank in their assessment of financial stability. We analyze whether these indicator variables are appropriate to use in the assessment of financial stability. We begin with a technical analysis, where we use the signaling approach proposed by Kaminsky and Reinhart (1999). This approach allows us to see whether the indicators presented in table 2 have provided good signals in the past. We combine the technical analysis with plots showing the development of these indicators, with particular focus on their behavior previous to crises.

The optimal result would be to find indicator variables capturing financial stability perfectly. However, this is not possible for several reasons. First, since financial stability is a latent state, it is difficult to measure. Second, this indicator is yet to be found in the EWI literature (Davis and Karim, 2008), and it is unlikely that one variable perfectly captures all aspects of financial stability. Since we find our candidates in the EWI literature, we will have to do with these non-perfect indicators. Furthermore, in the optimal research design, we would be able to see in what way, and by how much, different indicators, both alone and together, change the outlook for financial stability.

Step 2 of our research is to investigate the transmission from the short-term nominal interest rate to the indicators found and accepted in step 1. We want to get an indication of how much the interest rate can affect the indicator variables, since financial stability is a function of these variables, among others. This step is conducted with structural VAR models for the Norwegian economy. The structural VAR models consist of four core variables, output gap, interest rate, inflation and the foreign exchange rate, and different combinations of indicator variables we find and accept in step 1.
The optimal framework would allow us to see by exactly how much the policy instrument should change to mitigate financial risk, all else equal. Optimally, we would use an estimated time-varying regime-switching DSGE model. However, our analysis can be seen as a first step towards developing an empirical understanding of a complex topic.

2.2 Step 1 - The signaling approach

When assessing the candidate indicator variables’ ability to give good signals of financial distress, we will use the signaling approach. The signaling approach was first proposed by Kaminsky et al. (1998) and Kaminsky and Reinhart (1999), and has since become a workhorse model in the EWI literature. The idea is to consider each potential indicator variable, and see whether they provide signals in times before financial distress.

First, we look at each indicator variable, $V_i$, throughout the sample, and see whether the variable provides a signal or not. The signal $S_{i,t}$ is a binary variable taking on the value 1 if the variable signals a crisis, and 0 otherwise. Hence, if the threshold value is $\theta$, we have

$$S_{i,t} = \begin{cases} 
0, & \text{if } V_{i,t} > \theta \\
1, & \text{otherwise}
\end{cases} \quad (1)$$

Second, we gather the signals, $\sum_{t=1}^n S_t$ for each indicator $i$, in a vector and categorize the signals as either false or true. A signal is true if it appears within a specified time period before a crisis, and false otherwise. Hence, we can categorize the signal $S_t$ in one of two possible categories, and the variable at time $t$, $V_t$, in one of four possible categories:

<table>
<thead>
<tr>
<th>Signal</th>
<th>Crisis</th>
<th>No crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>True signal</td>
<td>False negative</td>
<td>True negative</td>
</tr>
<tr>
<td>False signal</td>
<td>Signal</td>
<td>No signal</td>
</tr>
</tbody>
</table>

Table 1: Classification of indicator variables at time $t$.

Now, we can assess the properties of each indicator by looking at two ratios:

$$\text{Ratio of true signals} = \frac{\text{True signal}}{\text{True signal} + \text{False negative}}$$

$$\text{Ratio of false signals} = \frac{\text{False signal}}{\text{False signal} + \text{True negative}}$$

The ratio of true signals tells us how many of the periods before the crisis the indicator actually signals, as a fraction of all the defined pre-crisis periods. We want this ratio to be as large as possible, as a higher ratio implies more signals given before a crisis. The ratio of
false signals tells us how many periods the indicator provides a false signal as a fraction of the total number of periods that are not followed by a crisis.

To use this approach, we need to make four sets of judgments (Kaminsky and Reinhart, 1999). First, we need a list of candidate indicator variables. Second, we need to define the periods of crises during our sample period. Third, we need to define a threshold value to indicate when the signal will go off. Fourth, we need to determine when the signal is true or provides a false alarm.

2.2.1 Candidate indicator variables

As candidate indicator variables, we use three of the four key variables Norges Bank use in their assessment of financial imbalances. These are, private credit-to-GDP (ratio and gap), house price-to-disposable income (ratio and gap), and banks’ wholesale funding (ratio and gap). Data for the last key indicator the bank use, real commercial property prices, is not available. For these variables, we have data from 1983 to 2018. The gaps are provided by Norges Bank and are constructed using the one-sided HP-filter. In addition, we have found two variables from the EWI literature. These are household credit and house prices, and we assess these variables in both log-levels and growth rates. For these variables, we have data from 1985 to 2018, except for the growth rates, for which we lose one year at the beginning of the period, due to the transformation to growth rates. The candidate indicator variables are presented in table 2, and the justification for their inclusion is provided below. The data and its sources are listed in appendix A, while summary statistics and correlations can be found in appendix B.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Used by Norges Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private credit-to-GDP ratio</td>
<td>Yes</td>
</tr>
<tr>
<td>Private credit-to-GDP gap</td>
<td>Yes</td>
</tr>
<tr>
<td>Household credit growth</td>
<td>No</td>
</tr>
<tr>
<td>Household credit (log)</td>
<td>No</td>
</tr>
<tr>
<td>House price-to-income ratio</td>
<td>Yes</td>
</tr>
<tr>
<td>House price-to-income gap</td>
<td>Yes</td>
</tr>
<tr>
<td>House price growth</td>
<td>No</td>
</tr>
<tr>
<td>House prices (log)</td>
<td>No</td>
</tr>
<tr>
<td>Banks’ wholesale funding ratio</td>
<td>Yes</td>
</tr>
<tr>
<td>Banks’ wholesale funding gap</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 2: Candidate indicator variables, and whether they are among the main indicators Norges Bank use in their assessment of financial imbalances.

Credit-to-GDP is a widely used and accepted indicator in the EWI literature (Drehmann, 2013). The Bank of International Settlement (BIS) regularly publish and monitor credit-
to-GDP, which is in line with Norges Banks’ stand, namely that it is one of the most important determinants of financial instability (Norges Bank, 2019). Papers from BIS include Drehmann et al. (2010, 2011) who find that the credit-to-GDP gap is an appropriate indicator for the accumulation of capital, because it captures system-wide vulnerabilities typically leading to banking crises. Researchers have found that not only is credit-to-GDP among the best indicators for financial instability (Detken et al., 2014), but also that the signals come at an early stage, making it appropriate in a monetary policy framework (Giese et al., 2014). Furthermore, Alessi and Detken (2011) finds that using credit-to-GDP will reduce the crisis loss by 25 percentage points compared to when ignoring it.

For many countries, the credit-to-GDP gap is negatively correlated with GDP, this is also the case for Norway. In light of this fact, Repullo and Saurina (2011) propose that credit growth is a better indicator of banking crises. Sohn and Park (2016) build on this, by providing evidence that supports the findings of Repullo and Saurina (2011). For the Norwegian economy, Anundsen et al. (2016) have found that an increase in household credit contributes positively to the probability of a crisis. Additional studies support the effectiveness of household credit, due to the fact that household credit growth raises debt levels without much effect on future income (Büyükkarabacak and Valev, 2010). Furthermore, credit growth can be a good leading indicator variable, because a crisis can occur several years after the peak of the credit cycle (Davis and Karim, 2008).

House prices are important for the financial system, because a large fraction of households’ wealth is in housing, and a large fraction of banks’ assets are in mortgages. Anundsen et al. (2016) finds that house prices exercise a positive and significant impact on the probability of a crisis in Norway, and Barrell et al. (2010)’s study finds that a one percentage point increase in real house price growth in Norway increases the probability of a crisis with 0.31 percentage points. Furthermore, during the Norwegian banking crisis in the 1990s, the burst of the house price bubble was a significant contributor to the instability in the economy (Stamso, 2009). However, Ragnarsson et al. (2019) point out that housing cycles have almost twice the frequency of credit cycles, and that this high frequency can lead to a high level of noise, if used as an indicator for financial distress. Another problem with using house prices as an indicator, is that the prices can increase due to changes in fundamental values, and at the time of the increase it is hard to know whether the increase is due to fundamental causes, or due to the build-up of financial imbalances.

The ratio of house prices-to-disposable income may be a more suitable indicator for financial stability, because it captures changes in house prices, but takes an important potential fundamental cause for house price changes into account, namely disposable income. If disposable income increases, this provides an explanation for increased house prices, and an increase in house prices does not necessarily imply future distress. Hermansen and Röhn (2017) finds that the ratio of house price-to-income is the best performing indicator among the 23 indicators they test.
Banks’ wholesale funding ratio is the ratio of total liabilities net of customer deposits and equity, as a percentage of total liabilities. Deposits from households and firms finance a large share of banks’ lending, however these deposits grow in line with the size of the economy and the wealth of households and firms. When credit is growing faster than the pool of available deposits, the bank will turn to other sources of funding to support credit growth (Hahm et al., 2012; Shin, 2016). Therefore, an increase in the wholesale funding ratio may indicate an increase in household spending, and may reinforce an increase in debt and asset prices. In turbulent times, banks’ access to wholesale funding often dries up, or the costs increase substantially. This in turn may lead to a tightening in the banks’ lending policies (Norges Bank, 2013). Hahm et al. (2012) use wholesale funding as an indicator and finds that it has significant predictive power for credit crises. Gambacorta and Marques-Ibanez (2011) find that banks with greater dependence on market funding suffered greater losses during the recent crisis, and reacted more to changes in monetary policy.

2.2.2 Crisis classification

Our sample size goes from the first quarter of 1983 until the fourth quarter of 2018. This sample contains two crises for the Norwegian economy. First, there is the Norwegian recession from 1988Q2 to 1993Q2, and second the global recession from 2008Q3 to 2009Q3 (Anundsen et al., 2016; Moe et al., 2004). The Norwegian recession lasted from 1988 to 1993, but the banking crisis was in 1991-1992. The Global recession did not hit the Norwegian economy as hard as it hit other economies (Grytten and Hunnes, 2014), but it is still representing a decline in activity.

<table>
<thead>
<tr>
<th>Crisis Type</th>
<th>Start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norwegian recession</td>
<td>1988Q2</td>
<td>1993Q2</td>
</tr>
<tr>
<td>Global recession</td>
<td>2008Q3</td>
<td>2009Q3</td>
</tr>
</tbody>
</table>

Table 3: Classification of crises in our sample.

2.2.3 Signal classification

To conduct our analysis, we need to define time periods for which the signal should be accepted as true or false. The signals from house prices (log-levels and growth rate), house price-to-income (ratio and gap), wholesale funding (ratio and gap), and credit (log-levels) provides a true signal when it appears within four quarters prior to the crisis. The signals from credit-to-GDP (ratio and gap) and credit growth are accepted as true signals when they appear within eight quarters prior to the crisis. Credit variables have proven to provide timely signals before previous crises (Giese et al., 2014), and the crisis can occur several years after the peak of the credit cycle (Davis and Karim, 2008). Therefore, we allow the credit variables to provide signals at a longer horizon than the other variables. Our sample size
only allows for a four-quarter horizon for credit (log-levels), since this variable is compared to itself $h$ quarters earlier. The four-quarter period for the remaining indicators is following Kaminsky and Reinhart (1999)’s horizon for indicators for banking crises. However, they also allow signals given in the first year of the crisis. We acknowledge that it can be useful to know whether the economy is in a crisis or not, however, we look for leading variables that give the central bank time to respond with a contractionary monetary policy prior to crises, and not an expansionary monetary policy during a crisis.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Signal horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit-to-GDP gap</td>
<td>Within eight quarters prior to recession</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td></td>
</tr>
<tr>
<td>Credit growth</td>
<td></td>
</tr>
<tr>
<td>House price growth</td>
<td>Within four quarters prior to recession</td>
</tr>
<tr>
<td>House prices (log)</td>
<td></td>
</tr>
<tr>
<td>House price to income gap</td>
<td></td>
</tr>
<tr>
<td>House price to income ratio</td>
<td></td>
</tr>
<tr>
<td>Banks’ wholesale funding gap</td>
<td></td>
</tr>
<tr>
<td>Banks’ wholesale funding ratio</td>
<td></td>
</tr>
<tr>
<td>Credit (log)*</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: How many quarters prior to a crisis we accept the signal as true. *The data sample for credit (log-level) only allows us to use four periods.

2.2.4 Threshold values

The most important part of the set-up is to define threshold values for the indicators. Hence, the threshold value for when the indicator variable switches from normal to signaling. To evaluate the optimal threshold value, one can use the noise-to-signal ratio approach of Kaminsky and Reinhart (1999), where the threshold is found by minimizing a loss function consisting of the noise-to-signal ratio. The noise to signal ratio is the ratio of type 2 errors over one minus type 1 errors. Type 1 errors are the fraction of missed signals during the pre-crisis periods over all pre-crisis periods, while type 2 errors are the fraction of false signals given over all normal periods. However, as pointed out by Borio and Drehmann (2009), this approach often results in a too low percentage of crisis expected. We use the approach suggested by Demirgüç-Kunt and Detragiache (1998), where we minimize a loss function consisting of type 1 errors and type 2 errors in the following way:

$$\min L = \beta \text{ Type 1} + (1 - \beta) \text{ Type 2}$$

This approach allows us to determine the optimal threshold value based on how much weight the central bank puts on avoiding false signals versus missing a true signal. In our analysis,
we assume that $\beta \geq 0.5$, that is, the central banks care more about missing a crisis than getting a false signal when there is no crisis. Furthermore, we assume that the central bank put some value on avoiding false signals, namely $\beta \leq 0.8$, because if not, the signal would go off in almost every period, and thereby devalue the information provided by the indicator. We minimize the loss function for values of $\beta = [0.5, 0.6, 0.7, 0.8]$, and find the optimal threshold for each indicator given the values of $\beta$.

The candidate threshold values are the percentiles $X \in [1, 100]$. Hence, the signal, $S_{i,t}$, goes off when the following condition is fulfilled:

$$V_{i,t} > X(V_i)$$

and we determine the optimal threshold by minimizing the loss function corresponding to each candidate threshold, and choose the threshold value giving the lowest loss.

We will use the percentile method for all variables, except the level variables, as these are not stationary, but drifting upwards and it would not be sensible to find a threshold value based on percentiles. Instead, our candidate thresholds are that the variables are smaller than they were $h$ quarters ago, with $h$ ranging from one to four quarters back in time. That is, the signal goes off when the following condition is fulfilled:

$$V_{i,t} < V_{i,t-h}$$

and we determine the optimal threshold as we do under the percentile method. We realize that this is a weak method of determining the threshold values, but given the non-stationary properties of the level variables and the fact that we do not have sufficient data to compute trends, this method will do. If we were able to construct trends, we could have run the same test, but added an additional requirement, namely:

$$V_{i,t} < V_{i,t-h} \text{ AND } V_{i,t} > \text{trend}(V_i), \text{ for } h = 1, \ldots, 4$$

In section 3 we present the indicators together with the corresponding false signal ratio, true signal ratio, loss and threshold value, using $\beta = 0.6$, while the results for $\beta = [0.5, 0.7, 0.8]$ are in appendix B.4.

2.3 Step 2 - VAR and SVAR

The structural VAR methodology is mainly based on Bjørnland and Thorsrud (2015); Lütkepohl (2005) and Kilian and Lütkepohl (2017), while the Matlab code is based on code from Cesa-Bianchi (2015) and algorithms from Kilian and Lütkepohl (2017). For estimation of the reduced form VAR, we use the unmodified code of Cesa-Bianchi (2015), while for identification of the structural VAR, we have written our own code.
2.3.1 Data

In step 2, we use data from 1993 to 2018. This starting period is chosen, first because of the deregulation of the housing and credit markets, and second because of the completion of the disinflationary process, both spanning into the early 1990s. Thus after 1993, Norway has had a stable housing and credit regulation, and a relatively stable monetary policy regime, even though inflation targeting was not formally introduced before 2001 (Steigum, 2011). Following policy regime shifts, we might get structural breaks in the data, and these supposed breaks can cause misleading parameter estimation results, since the OLS estimates reflect the average over the sample (Bjørnland and Thorsrud, 2015). The limited sample size, with data from 1993 to 2018, containing only one recession, is not a problem in step 2, since we are assessing the effect of a monetary policy shock in normal times. It is in normal times, during the build-up of financial imbalances, the central bank should use contractionary monetary policy to address these imbalances.

In the structural VAR models, we include four non-indicator variables that feature in standard New Keynesian models for small open economies with an inflation-targeting central bank (Clarida et al., 2002; Galí and Monacelli, 2005). These are, the short-term nominal interest rate, inflation, the output gap, and the foreign exchange rate. In accordance with the literature, we use the 3-month Nibor for the short-term interest rate, since it should capture expectations about monetary policy within the quarter (Lund et al., 2016). For inflation, we use the annual growth rate of CPI, since this gives a more direct measure of the central banks’ inflation target, and in addition, we avoid the seasonal component of inflation (Bjørnland and Jacobsen, 2010). The use of the output gap is also motivated by being a better measure of the target of the central bank, and that it might be helpful in addressing the price puzzle (Giordani, 2004).

Woodford (2012) shows that such structural VAR models can be augmented taking financial stability into consideration, by adding indicators for financial imbalances. Therefore, in addition to the non-indicator variables, we add indicator variables from step 1 to the models. This specification is similar to Bjørnland and Jacobsen (2010) and Robstad (2018). The inclusion of multiple indicators in some of our models is motivated by the information content these variables can generate together. Especially of interest is the interaction between credit and asset prices, which in combination can give a better indication of financial stability (Borio and Lowe, 2002; Borio, 2014; Anundsen et al., 2016).

2.3.2 SVAR

The premise of the structural VAR approach is that the data generating process can be approximated by the $K$-dimensional structural VAR($p$) process:
\[ B_0 y_t = \mu + B_1 y_{t-1} + \cdots + B_p y_{t-p} + w_t \]  \hspace{1cm} (3)

Where \( K \) is the number of variables within the system, \( \mu \) is a constant, \( p \) is the number of lags, \( y_t \) is a \((K \times 1)\) vector of variables and \( B_i \) for \( i = 1, \ldots, p \) are \((K \times K)\) parameter matrices, \( w_t \) is a \((K \times 1)\) vector of structural shocks, with \( E[w_t] = 0 \) and \( E[w_t w'_t] = \Sigma_w = I_K \). This means that the number of structural shocks equals the number of variables, that the structural shocks by definition are uncorrelated, that \( \Sigma_w \) is diagonal and that the variance of all structural shocks are normalized to one. For equation (3) to be considered a structural VAR, the shocks must be economically interpretable.

### 2.3.3 VAR

In the structural VAR in equation (3), all variables are endogenous. Contemporaneous values of all variables enter as explanatory variables for all other variables in the system. This is the problem of simultaneous causality. If we use the OLS parameter estimates from equation (3), these will be inconsistent, meaning they will not converge to their true value. Hence, we will follow the general modelling strategy of first estimating the reduced form VAR from equation (4) and then recover the structural VAR from equation (3) (Lütkepohl, 2005). By premultiplying both sides of equation (3) by \( B_0^{-1} \), we obtain the corresponding reduced form VAR:

\[ y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t \]  \hspace{1cm} (4)

Where \( A_i = B_0^{-1} B_i \) for \( i = 1, \ldots, p \), \( u_t = B_0^{-1} w_t \) and \( \nu = B_0^{-1} \mu \). In the reduced form VAR, \( u_t \) is a vector of error terms, with \( E[u_t] = 0 \) and \( E[u_t u'_s] = \Sigma_u \) if \( t = s \) and 0 if \( t \neq s \). For derivation of the companion form and moving average representation, see appendix D. For each of the \( K \) variables we have a sample size of \( T \), in other words, \( \{y_t\}_{t=1}^T \). We can estimate the reduced form VAR equation-by-equation for all \( K \) equations in (4) by OLS.

### 2.4 Identification of structural parameters

#### 2.4.1 Identification problem

After estimating the reduced form VAR in equation (4), we want to recover the structural parameters of the structural VAR in equation (3), to use them for impulse response analysis. We can clearly see that knowledge of \( B_0 \) or \( B_0^{-1} \) enable us to identify \( w_t \) and \( B_i \) for \( i = 1, \ldots, p \) through:

\[ w_t = B_0 u_t \]  \hspace{1cm} (5)
We call $B_0^{-1}$ the structural impact multiplier matrix. We know that $u_t = B_0^{-1} w_t$ and hence, the variance of $u_t$ is:

$$\Sigma_u = \mathbb{E}[u_t u_t'] = B_0^{-1} \mathbb{E}[w_t w_t'] B_0^{-1} = B_0^{-1} \Sigma_w B_0^{-1} = B_0^{-1} B_0^{-1}'$$

(6)

Where we have used the fact that $\Sigma_w = I_K$, and consider $\Sigma_u$ as known from the estimation. $B_0^{-1} B_0^{-1'}$ have $K^2$ elements and $\Sigma_u$ only have $K(K+1)/2$ free elements due to the symmetry of the covariance matrix. A necessary condition for identification is thus that $B_0^{-1}$ only have $K(K+1)/2$ free elements, since this is the maximum number of elements in $B_0^{-1}$ that can be uniquely identified from $\Sigma_u$. To fully ensure identification of the structural shocks, the system also has to satisfy another necessary condition, the rank condition. We will now discuss ways to disentangle the structural shocks, $w_t$, from the reduced form errors, $u_t$, by putting restrictions on $B_0^{-1}$.

In doing so, we solve the structural identification problem, which is useful to distinguish from the model identification problem (Fry and Pagan, 2011). The first refers to identifying the structural relationship untangling the simultaneity, for example using recursive restrictions or sign restrictions. The second refers to identifying the model among all the possible structural models consistent with the estimated reduced form VAR parameters, for example a particular recursive structure.

2.4.2 Recursive restrictions

A way to disentangle $w_t$ from $u_t$, first proposed by Sims (1980), is to orthogonalize $u_t$, in other words, make the shocks uncorrelated. To do this, we define the lower-triangular ($K \times K$) matrix $P$ with positive main diagonal such that $PP' = \Sigma_u$, in other words, $P$ is the lower-triangular Cholesky decomposition of $\Sigma_u$. Taking a Cholesky decomposition of the variance-covariance matrix is the matrix analogue of taking the square root of the scalar variance. We can see from $\Sigma_u = B_0^{-1} B_0^{-1'}$ that $P = B_0^{-1}$ is the unique solution recovering $w_t$. Since $P$ is lower triangular, it has $K(K-1)/2$ zero parameters, and as a result the order condition is satisfied. And if $B_0^{-1}$ is lower triangular, so is $B_0$.

When applying the Cholesky decomposition, we impose a particular causal chain (recursive structure), rather than learning about the causal relationship from the data. By doing so, we solve the structural identification problem, namely which structural shock, $w_t$, causes the variation in the error term, $u_t$, by imposing a particular solution. Meaning that $P$ is not unique, since there is a different solution for $P$ for each ordering of the $K$ variables, hence we are still left with the model identification problem, and the question of which recursive ordering to choose.

Notice that the orthogonalization is appropriate only if the recursive structure in $P$ can be justified by economic rationale (Cooley and Leroy, 1985). There are several ways of
rationalizing the recursive structure. One method is to impose the structure provided by a specific economic model. In that case, the results will only be as credible as the underlying model. Another method is to use extraneous information or selective insight from economic theory. This could, for example, be information delays, physical constraints and institutional knowledge (Kilian and Lütkepohl, 2017).

The recursive structure poses a problem when there are multiple asset prices in the model, since asset price and the interest rate may respond simultaneously to news (Bjørnland and Jacobsen, 2010). The normal procedure is either to assume that asset prices are restricted from responding contemporaneously to monetary policy shocks, or the opposite, that the central bank does not respond contemporaneously to asset price shocks. The first approach is problematic, since asset prices are forward looking and therefore will respond immediately to monetary policy news. The second approach is problematic, because it prohibits the policymaker from using all current available information when designing monetary policy. Given the previous discussion, we should allow for both the interest rate and the exchange rate to simultaneously respond to each other. This highlights the cost of including multiple asset prices in a recursive model. Previous studies have tackled this problem by ignoring additional asset prices (Iacoviello, 2005), assumed the exchange rate rate to be exogenous (Giuliodori, 2005) or assumed a recursive order so that all asset prices respond with a lag to monetary policy shocks (Assenmacher-Wesche and Gerlach, 2008). The idea is that to identify the monetary policy shock, the interest rate must be ordered last (or close to last), so that all variables that are part of the central banks reaction function is ordered above the interest rate. The last equation of the system can then be interpreted as a linear monetary policy reaction function, and the interest rate as the monetary policy instrument. If all the endogenous variation in the interest rate is captured by the equation, then the residual can be interpreted as an exogenous monetary policy shock (Kilian and Lütkepohl, 2017). Solutions to the asset price problem is to use long-run restrictions (Bjørnland and Jacobsen, 2010) or sign restrictions (Uhlig, 2005; Peersman, 2005; Vargas-Silva, 2008; Scholl and Uhlig, 2008; Robstad, 2018), and thereby let asset prices and monetary policy respond simultaneously to each other. However, these identification schemes have their own problems.

### 2.4.3 Sign restrictions

Following Faust (1998), Canova and Nicoló (2002) and Uhlig (2005) one way to disentangle \( w_t \) from \( u_t \) is to impose a sign structure on \( B_0^{-1} \). We define the lower-triangular \((K \times K)\) matrix \( P \) with positive main diagonal, such that \( PP' = \Sigma_u \), in other words, \( P \) is the lower-triangular Cholesky decomposition of \( \Sigma_u \). We let \( u_t = P \eta_t \), where the elements of \( \eta_t \) are uncorrelated and have unit variance. There is no reason why \( \eta_t \) should correspond to economically interpretable structural shocks, however, we can search for candidate solutions for the unknown economically interpretable structural shocks, \( w_t^* \), by constructing a large number of combinations of \( \eta_t \) on the form \( w_t^* = Q' \eta_t \). We define \( Q' \) to be a square orthogonal
matrix, such that \( Q'Q = QQ' = I_K \), and hence:

\[
  u_t = P\eta_t = PQQ'\eta_t = PQw^*_t
\]  

Each candidate solution, \( w^*_t \), consists of uncorrelated shocks with unit variance. Whether any of these candidate solutions, \( w^*_t \), is an admissible solution for \( w_t \), depends on whether \( PQ \) satisfies the sign restrictions on \( B^{-1}_0 \). Retaining the admissible solutions allows us to characterize the set of all structural models that are consistent with the sign restrictions on \( B^{-1}_0 \), and the reduced form parameters from \( P \).

We generate a large number of candidate matrices, \( Q \), also called rotation matrices, from the set of all orthogonal \( K \times K \) matrices \( O(K) \equiv \{ Q \mid QQ' = I_K \} \), by using the Householder transformation approach. This approach is taken from Kilian and Lütkepohl (2017), but first proposed by Rubio-Ramírez et al. (2010). Any square matrix can be decomposed as \( W = QR \), where \( Q \) is an orthogonal matrix, and \( R \) is an upper-triangular matrix. We cover the space \( O(K) \) of \( K \times K \) orthogonal matrices, \( Q \), by drawing \( W \) at random from a \( N(0, I_K) \) distribution. If \( W \) is invertible, then the factorization will be unique, and with this method of drawing \( W \), this will always result in a nonsingular matrix (Rubio-Ramírez et al., 2010).

We then apply the \( QR \) decomposition to each draw \( W \). This factorization can easily be done in Matlab using the \( qr() \) function, however, Kilian and Lütkepohl (2017) point out that this function does not ensure positive diagonal elements of \( R \). Hence we follow their advice, and perform a normalization on the output of the \( qr() \) function by reversing all signs of each row of \( R \) corresponding to a negative diagonal element, and adjust \( Q \) accordingly to ensure that \( W = QR \).

For a fully identified structural VAR model, the requirement for identification is that each shock has its own sign pattern. When the number of identified shocks are less than \( K \), the model is called a partially identified VAR model. In the beginning we assumed that given \( K \) variables, there are \( K \) structural shocks to be identified, but in our sign restriction model this is not the case, hence we have a partially identified VAR model. For a partially identified VAR model, the requirement for identification is that the sign pattern of each of the identified shocks are different from the sign pattern of each unidentified shock. However, Kilian and Lütkepohl (2017) points out that there is no consensus in the literature on whether this requirement should be imposed. For example, Uhlig (2005) consider one column of \( PQ \) at the time (he actually only draws one column \( q \)), looking for solutions that satisfy the sign restrictions, while ignoring the other columns. All columns satisfying the sign pattern are considered admissible solutions, and the possibility that other shocks in the same structural model may have the same sign pattern is ignored. Fry and Pagan (2011) points out that we need to be sure that no two shocks have the same sign pattern, if not, this might lead to the multiple shocks problem, where we have failed to specify enough information to discriminate between the shocks. In our sign restriction model, we are interested in the
effect of a monetary policy shock, we therefore look to identify one shock column \( b \), given our sign restrictions, within each draw of \( PQ \), and retain all models containing our identified shock column \( b \). Hence, we follow Uhlig (2005)’s approach regarding the unidentified shocks, but follow Fry and Pagan (2011)’s suggestion of drawing the entire \( PQ \) matrix to ensure orthogonality.

Having identified a set of admissible models, and thus solved the structural identification problem, the model identification problem is still present. One way to deal with this, is to summarize the information from the impulse response functions from the admissible models, to get a view of the possible range of responses as the model varies. This can be done by reporting a central tendency and the magnitude of the spread of the responses, an approach similar to extreme bound analysis (Uhlig, 2005). However, this does not solve the model identification problem, since it is simply a summary of the admissible models. Fry and Pagan (2011) point out that it is important to recognize that the distribution is across models and has nothing to do with sampling uncertainty, and thus, referring to this range as if it is a confidence interval, is false. Any solution to the model identification problem, in a sense that we restrict the number of admissible models further, has to be one that incorporates more information that enables us to discriminate between the models. However, there is no single answer to what extra information to be used (Fry and Pagan, 2011), hence we only provide some examples: First, one can add on dynamic sign restrictions for the impulse responses beyond the impact period. Second, one can add knowledge of the likely magnitude of responses (elasticity constraints) as in Kilian and Murphy (2012). Third, one can use the penalty function approach as in Uhlig (2005) and Faust (1998). Fourth, one can use the median or mean target approach. This last approach finds the best model closest to a central tendency measure (Fry and Pagan, 2011). We follow Fry and Pagan (2011), and identify a single model using the mean target approach.

### 2.4.4 Our identification approach

We use two identification approaches, the agnostic sign restriction approach of Uhlig (2005), and the recursive restriction approach of Sims (1980). Considering the problem of having multiple asset prices under the recursive approach, we want to complement it with an approach where the responses of the indicators are unrestricted, and where all the variables can respond contemporaneously to each other.

Under the agnostic sign restriction approach, we identify the monetary policy shock by restricting the signs of all non-indicator variables, and leave the signs of the indicators unrestricted. The impulse response functions are constructed using ten thousand accepted draws of the impact multiplier matrix. Following a contractionary monetary policy shock, captured by a one percentage point increase in the interest rate, we assume a decline in output, a decline in inflation, and an appreciation of the foreign exchange rate. The restrictions derives from economic theory and empirical evidence, which follows below:
According to Walsh (2017), empirical evidence from VAR models suggest that following a contractionary monetary policy shock, output will fall and follow a hump-shaped pattern, he refers to both Sims (1992) and Christiano et al. (1999). This response is also suggested in theoretical models, for example Christiano et al. (2005). The restriction on inflation is not as clear cut, even though this assumption is often made in sign restriction models, see for example (Uhlig, 2005; Vargas-Silva, 2008; Rafiq and Mallick, 2008; Carstensen et al., 2009). Many empirical studies find that inflation initially increases following a contractionary monetary policy shock. An explanation for this price puzzle is that monetary policy acts in anticipation of inflation (Walsh, 2017). From this, it follows that the solution is to add forward-looking variables to the VAR model, which are supposed to proxy for expected inflation and capture more of the central banks information set (Sims, 1992). An alternative explanation for the price puzzle, is that the increased interest rate increase firms’ costs, and when the costs increase, prices increase as well (Barth and Ramey, 2001). Furthermore, another alternative explanation is that that using output, as opposed to the output gap, spuriously produce a price puzzle (Giordani, 2004). The overshooting model of Dornbusch (1976) is consistent with the fact that the foreign exchange rate should appreciate following a contractionary monetary policy shock. Empirical evidence is provided by Bjørnland (2009) who finds that a contractionary monetary policy shock has a strong effect on the foreign exchange rate which appreciates on impact. Other puzzles related to the foreign exchange rate, like the forward discount puzzle, and the delayed overshooting puzzle, has more to do with the shape of the response, than the sign of the response (Scholl and Uhlig, 2008).

For the sign restriction approach we report two impulse responses. First, the median impulse response for each variable, notice that this response might come from multiple models. Second, the mean target of Fry and Pagan (2011), that uniquely identifies the model with the impulse response function closest to the mean impulse responses over all variables. The bands represent impulse responses within the 16-84 percentiles of responses.

Under the recursive restriction approach we identify the monetary policy shock by ordering the short-term nominal interest rate last in a recursive structure, so that all variables above it is part of the monetary policy function, and the residual variation is treated as a monetary policy shock. As we are only interested in identifying the monetary policy shock, the ordering of the variables above the interest rate does not matter. For the recursive approach, we report the point estimate of the uniquely identified model. The bands are 84 percentage confidence intervals constructed around the point estimate using bootstrapping.

### 2.5 HP-filter

Following Borio and Lowe (2002), much of the EWI literature focuses on macroeconomic imbalances, see for example (Alessi and Detken, 2011; Csortos and Szalai, 2014; Borio, 2014; Hermansen and Röhn, 2017). To capture imbalances, a gap measure is often constructed, and the imbalances are defined as the gap between the original series and its trend. A
time series $y_t$ can be decomposed into four components, a trend component $g_t$, a cycle component $c_t$, a seasonal component $s_t$ and noise $e_t$. To filter out desired components we use Matlab code on one-sided HP-filters provided by Meyer-Gohde (2010). While the normal double-sided HP-filter is both backward and forward looking, the one-sided HP-filter is only backward looking and runs recursively while expanding the sample each period (Drehmann and Yetman, 2018). This is the recommended approach since at each point in time, the one-sided HP-filter only use information known at the time, to construct $g_t$ (Stock and Watson, 1999), which mimics the information available to the policymaker at any point in time, given that the data itself is not revised. Norges Bank use the one-sided HP-filter in their assessment of financial stability, and their main argument for using it is that it puts more weight on recent observations, which can effectively capture structural breaks (Norges Bank, 2013). We are aware of the problems associated with the HP-filter. There is the end-of sample problem, which comes about because the solution $g_t$ will be more responsive to transitory shocks at the end of the sample (Bjørnland and Thorsrud, 2015). Also, as noted by Hamilton (2018), the HP-filter can produce spurious cycles, which have no basis in the true data-generating process. Regardless, for consistency, we follow Norges Bank. We are interested in decomposing $y_t$ into a trend component, $g_t$, and a cycle component, $c_t$, such that $y_t = g_t + c_t$. The HP-filter filters out $g_t$ as the solution to:

$$
\arg\min_{g_t} \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda [(g_{t+1} - 2g_t + g_{t-1})]^2
$$

(8)

The solution, $g_t$, varies for different values of the smoothing parameter $\lambda$. Choosing a small value allows for large variations in the trend, for example $\lambda = 0$ results in $g_t = y_t$, while choosing a large value allows for small variations in the trend, for example letting $\lambda \rightarrow \infty$ gives a linear deterministic trend (Bjørnland and Thorsrud, 2015). When choosing $\lambda$, one has to consider the frequency of the cycles. The Basel Committee recommends using $\lambda = 400,000$ for financial cycles, where the underlying assumption is that the length of a financial cycle is approximately four times the length of a business cycle (Norges Bank, 2013).

We use three gaps in step 1, all provided by Norges Bank, and constructed using one-sided HP-filters with $\lambda = 400,000$ (Norges Bank, 2019). It can be argued that we should use more gap variables, however due to our limited data period, we will lose too many observations by applying the filter on data not provided by Norges Bank. In step 2, we use the gaps from step 1, and construct the output gap using the one-sided HP-filter. We set $\lambda = 40,000$ following Hagelund et al. (2018), and discard the first 40 observations, so that we get filtered data from 1993 and onward, as with the rest of the data in step 2.
3 Step 1 - EWI analysis

In step 1, we analyze indicators from the EWI and EWS literature, along with the indicators used by Norges Bank in their assessment of financial stability. A good indicator needs to be a leading indicator of financial distress because policymakers need time to react to imbalances. In addition, if the economy has already entered financial distress, it will be evident in several economic variables and the indicator will not add new information. For each indicator, we assess the findings from the technical assessment using the signaling approach described in section 2.2. To aid our assessment, we plot each indicator variable against the two recessions that have hit Norway during our sample period. For additional stylized facts of the candidate indicators, see appendix B.

3.1 Results

The results from the signaling approach using $\beta = 0.6$, meaning that the central bank puts 60 percent weight on not missing a crisis, and 40 percent weight on avoiding false signals, is presented in table 5. The results are similar when using $\beta = [0.5, 0.7, 0.8]$, as can be seen from appendix B.4. In what follows, we will go through the results of our assessment.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>True signals</th>
<th>False signals</th>
<th>Loss</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale funding gap</td>
<td>1.00</td>
<td>0.23</td>
<td>0.09</td>
<td>71</td>
</tr>
<tr>
<td>Credit growth</td>
<td>1.00</td>
<td>0.32</td>
<td>0.13</td>
<td>64</td>
</tr>
<tr>
<td>House price to income ratio</td>
<td>1.00</td>
<td>0.33</td>
<td>0.13</td>
<td>66</td>
</tr>
<tr>
<td>House price to income gap</td>
<td>1.00</td>
<td>0.35</td>
<td>0.14</td>
<td>65</td>
</tr>
<tr>
<td>Credit-to-GDP gap</td>
<td>1.00</td>
<td>0.42</td>
<td>0.17</td>
<td>49</td>
</tr>
<tr>
<td>Wholesale funding ratio</td>
<td>1.00</td>
<td>0.54</td>
<td>0.21</td>
<td>46</td>
</tr>
<tr>
<td>House prices (log)</td>
<td>0.70</td>
<td>0.13</td>
<td>0.23</td>
<td>2**</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>1.00</td>
<td>0.67</td>
<td>0.27</td>
<td>23</td>
</tr>
<tr>
<td>House price growth *</td>
<td>0.70</td>
<td>0.24</td>
<td>0.28</td>
<td>41</td>
</tr>
<tr>
<td>House price growth</td>
<td>0.56</td>
<td>0.92</td>
<td>0.40</td>
<td>1-17</td>
</tr>
<tr>
<td>Credit (log)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.59</td>
<td>1**</td>
</tr>
</tbody>
</table>

Table 5: True signals is the periods a true signal is provided, divided by the number of periods we want a signal. False signals is the periods of false signals made divided by the number of periods we do not want a signal. Loss is the loss stemming from the loss function in section 2.2.4. Threshold shows the percentile value for which the signal goes off. * The result for house price growth when using an alternative threshold method, the signal goes off when the growth rate is below the threshold percentile. **The threshold of the logarithms is by how many quarters back we compare the logarithms with when determining the signal value. The loss function is minimized using $\beta = 0.6$. 


3.1.1 Credit-to-GDP

The first key indicator Norges Bank use in their assessment of financial stability is the ratio of private credit-to-GDP. Norges Bank takes both the ratio and gap into consideration when assessing the financial situation in Norway. The gap can be a better indicator than the ratio because it removes long-term changes in the ratio, which can be due to financial developments (Aldasoro et al., 2018).

![Credit-to-GDP ratio for mainland Norway](image1)

![Credit-to-GDP gap for mainland Norway](image2)

Figure 1: Credit-to-GDP mainland Norway. The ratio is plotted against its average value. The yellow shaded areas are the two recessions that have hit Norway in our sample period. The Norwegian recession lasted from 1988Q2 to 1993Q2. The Global recession lasted from 2008Q3 to 2009Q3. One-sided Hodrick-Prescott filter. Lambda=400,000.

As seen in figure 1, the credit-to-GDP ratio has increased before both recessions in Norway. Before the Norwegian recession, the ratio had grown at the same pace of approximately five percent per year for at least five years preceding the peak. From 1996 up until the peak of the Global recession in 2009, the ratio again increased steadily. Following the Global recession, the growth of the ratio stagnated for 1.5 years before it picked up yet again. The ratio provides good lagging indications, that is, a crisis implies a stagnation or decline in the growth of the ratio. However, an increase in the ratio does not necessarily imply a crisis, and in real-time, it is hard to assess whether the indicator is indicating financial distress, in other words, if the change is due to fundamentals or imbalances. The assessment of the plot is in line with the technical assessment of the ratio. We find that the ratio could have provided true signals in all sixteen quarters preceding the two recessions in Norway. However, it would also have provided a false signal in two-thirds of the quarters not followed.
by a crisis. The high level of type 2 errors is probably due to the fact that the ratio has yet to decline after the Global recession. This is also reflected in a high standard deviation and the highest coefficient of variation in our sample, as can be seen in appendix B.1. Due to the high variation of the ratio, the signaling approach finds a low threshold value, namely that the signal goes off whenever the ratio is in its 23rd percentile or above. As the situation is today, with the ratio still being at alleviated levels compared to historical levels, the ratio does not provide clear enough signals to act as an early warning indicator.

From figure 1, we see that the gap provides a clearer signal of past recessions compared to the ratio. Both recessions follow years where the gap is above zero. Before the Norwegian recession, the gap rose from 5.59 percent in 1983 to 11.02 percent in 1987, and before the Global recession, the gap rose from 8.91 percent in 2004 to 20.20 percent in 2008. This is in line with the results of the technical assessment. We find that the credit-to-GDP gap provides a true signal in all sixteen quarters preceding a crisis, and a false signal in 42 percent of the normal quarters. The gap thereby provides less false signals than the ratio does. Using the gap measure, rather than the ratio, is also supported by the lower coefficient of variation of the credit-to-GDP gap, and the higher lagged correlation between the credit-to-GDP gap and GDP growth, see appendix B.

Despite serving as a good indicator before the Norwegian banking crisis, the credit-to-GDP ratio has increased steadily since 1996 and did not provide a good warning signal before the Global recession. In line with the literature (Drehmann et al., 2011), we find the credit-to-GDP gap to be a good indicator. We proceed with the credit-to-GDP gap in step 2.
### 3.1.2 Household credit

The financial liberalization, preceding the Norwegian recession, started in 1984-1985 when credit regulations were abolished. This combined with increased economic activity, low real interest rates, and favorable tax deductions, resulted in a credit-fueled boom (Stamso, 2009). Despite our limited data sample, where we have the household credit growth rate from 1986 and onward, we still see this credit boom in the build-up to the Norwegian recession. From figure 2, we see that the growth rate was at its highest, at 19.13 percent, in April 1986, which is one year prior to the Norwegian recession. The growth rate was also at alleviated levels one year before the Global recession, with a high of 11.89 percent in the third quarter of 2007. Prior to the crises, the correlation between credit growth and GDP one year ahead was negative 9.5 percent, reflecting that a decrease in the growth rate decreases GDP, see appendix B.3. From the technical assessment, we find that credit growth provides a true signal in all sixteen quarters prior to both recessions, while it provides a false signal in one-third of the quarters not preceded by a crisis. The indicator proves to be the second-best of all the potential indicators we test, in terms of loss. Our assessment of credit growth is in line with Repullo and Saurina (2011) and Sohn and Park (2016), who also find credit growth to provide more reliable signals than the credit-to-GDP gap.

From figure 2, we see that following a period of decrease after the Norwegian recession, household credit increases steadily from 1996 and up until the Global recession. Household

---

**Figure 2**: Household credit measures Norway. The growth rate is plotted against its average value. The yellow shaded areas are the two recessions that have hit Norway in our sample period. The Norwegian recession lasted from 1988Q2 to 1993Q2. The Global recession lasted from 2008Q3 to 2009Q3.
credit slowly flattens at the breach of the Global recession. We find household credit to be among the poorest of the indicators we test. It provides a true signal in only six percent of the quarters before the recessions and a false signal in seven percent of the non-crisis quarters. However, as discussed in section 2.2.4, the threshold for this indicator is not optimal and would have been improved if we had the trend of the data. Yet, given the data we have at hand, credit in log-levels does not act as a good indicator for financial imbalances.

Our assessment is that the household credit growth rate is a good indicator of financial imbalances. We proceed with household credit growth in step 2.

### 3.1.3 House prices

![House price growth rate for Norway](image1)

![House prices in log-levels for Norway](image2)

Figure 3: House prices Norway. The growth rate is plotted against its average value. The yellow shaded areas are the two recessions that have hit Norway in our sample period. The Norwegian recession lasted from 1988Q2 to 1993Q2. The Global recession lasted from 2008Q3 to 2009Q3.

From figure 3, it is evident that the house price growth reached a peak one year prior to both recessions, and then stayed below its average growth rate throughout them. Prior to both recessions, the growth rate was above 15 percent, which is far above the average rate of 3.46 percent over the period. However, in non-crisis periods such as 1994 and 2000, growth rates were above 14 percent. It is hard to know whether this high growth rate was due to changes in fundamentals or imbalances, and with large and frequent deviations from average, this indicator has the potential to produce false warning signals. In fact, the house price growth proves to be the indicator with the highest false signal ratio of 92 percent. This reflects the fact that the optimal threshold value is the first percentile. In the search for
the optimal threshold, we found that all percentiles between the first and the seventeenth provided the same loss. This reflects the fact that house price growth is a volatile variable, as pointed out by Ragnarsson et al. (2019). Another potential problem is that the variable only signals distress when it is above the optimal percentile, since the plot shows that house price growth has decreased substantially one year prior to both recessions. A more reliable signal might be that house price growth is below some percentile, not above. Therefore, we test the performance of the indicator when changing the signs of the condition, namely that the signal goes off when house price growth is below the threshold value. The results of this alternative approach are also in table 5. It yields a higher ratio of true signals and a lower ratio of false signals, and thereby a lower loss. However, the variable still provides poor indications compared to the other variables in our sample.

House prices in log-levels peak approximately one year before both recessions. However, after the Norwegian recession, the house price index was above its trend up until the Global recession. Moreover, it is difficult to find an appropriate threshold value. The plot of house prices shows that house prices decrease prior to both recessions. Therefore, the signal goes off when the house prices are lower than they were 2 quarters ago. We find that the house prices provide a true signal in 70 percent of the quarters before crises, while it gives a false signal in 12 percent of the quarters which is not followed by a crisis. The false signal ratio is among the lowest in our sample, but the true signal ratio is not high compared to the other variables.

We find house price growth to give the second-largest loss among our indicator variables, and the house prices in log-levels, despite the low ratio of false signals, provide a too low ratio of true signals. Based on our discussion, we will not proceed with either of the two indicators, although we will recommend including the house price gap in future research.
3.1.4 House prices-to-disposable income

Figure 4: House prices to disposable income Norway. The ratio is plotted against its average value. The yellow shaded areas are the two recessions that have hit Norway in our sample period. The Norwegian recession lasted from 1988Q2 to 1993Q2. The Global recession lasted from 2008Q3 to 2009Q3.  One-sided Hodrick-Prescott filter. Lambda=400,000.

Figure 4 indicates that the ratio of house prices-to-disposable income is a leading indicator of crises. In the time preceding both recessions, the ratio increases extensively above its average level. This is also evident in the technical assessment, where we see that the house price-to-income ratio provides true signals in all the pre-crisis quarters. Furthermore, the ratio only provides a false signal in one-third of the quarters not succeeded by a crisis.

The house price-to-income gap also peaked prior to the two recessions. However, this gap was high during the period of 1997-2004, and could, therefore, provide false warnings. In our analysis, we see that the house-price-to-income gap provides false signals in 35 percent of the normal quarters, while it gives a true signal in all pre-crisis quarters.

Both the house price-to-income gap and ratio proves to be good indicators for the previous recessions in Norway. These indicators have also had a high correlation with GDP one year ahead, prior to both recessions in Norway, see appendix B.3. For our further assessment, we will only include the house price-to-income ratio. This is because the ratio has a lower rate of false signals compared to the gap, and monetary policy will work through the same channels for both indicators.
3.1.5 Banks’ wholesale funding ratio

From figure 5, we see that the wholesale funding ratio increased from 20 percent in 1983 to over 40 percent when the Norwegian recession started in 1987. The ratio increased before the Global recession as well, although it did not reach a peak as it did during the Norwegian recession. The high rate of false signals in the technical analysis reflects the fact that the ratio has yet not decreased after the Global recession. To find a threshold that gives a high rate of true signals, we also need to accept a high rate of false signals. The wholesale funding ratio provides false signals in 54 percent of the cases.

The wholesale funding gap was well above zero in the time preceding the Norwegian recession, before it declined during the 1990s. Prior to the Global recession, the gap was at its highest level since before the Norwegian banking crisis. The alleviated level of the gap prior to the Norwegian recession is reflected in the high correlation coefficient between GDP growth one year ahead, and the wholesale funding gap, as can be seen in appendix B.3. As opposed to the ratio, the gap has a downward sloping curve after the global recession. The distinction of the gap in periods followed by distress and other periods makes it an excellent indicator of financial distress. This is evident in the technical analysis, where the funding gap is the highest performing indicator variable of all variables tested. With a true signal ratio of 100 percent, and giving false signals in less than one-fourth of the normal quarters,
the loss associated with this indicator is the smallest among all indicator variables we test. The high threshold value of the 71st percentile also reflects the distinction of the properties of the gap in normal periods and distress periods.

Based on our assessment, we will proceed with the wholesale funding gap in step 2.

### 3.2 List of indicators

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Proceed to step 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private credit-to-GDP ratio</td>
<td>No</td>
</tr>
<tr>
<td>Private credit-to-GDP gap</td>
<td>Yes</td>
</tr>
<tr>
<td>Household credit growth</td>
<td>Yes</td>
</tr>
<tr>
<td>Household credit (log)</td>
<td>No</td>
</tr>
<tr>
<td>House price-to-income ratio</td>
<td>Yes</td>
</tr>
<tr>
<td>House price-to-income gap</td>
<td>No</td>
</tr>
<tr>
<td>House price growth</td>
<td>No</td>
</tr>
<tr>
<td>House prices (log)</td>
<td>No</td>
</tr>
<tr>
<td>Banks’ wholesale funding ratio</td>
<td>No</td>
</tr>
<tr>
<td>Banks’ wholesale funding gap</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 6: Assessment of indicator variables.

### 4 Step 2 - SVAR analysis

In step 2 we assess whether monetary policy can affect the indicators from step 1. The rationale for this assessment is that the central bank wants to minimize a loss function consisting of the output gap, inflation and financial stability (Woodford, 2012). To do so, the central bank needs to consider future financial distress to minimize the loss function. In step 1, we found four financial stability indicators and following our reasoning above the central bank needs to know how it influences these variables. Therefore, we will look into whether, and to which extent, the central bank can influence these variables using the monetary policy instrument.

#### 4.1 Results

In the following, we will see how the accepted variables from step 1 react to a monetary policy shock. An overview of the structural VAR models is presented in table 7.
Table 7: Overview of structural VAR models. All ten models include the non-indicator variables, output gap, inflation, foreign exchange rate, and interest rate.

<table>
<thead>
<tr>
<th>Model</th>
<th>Indicator 1</th>
<th>Indicator 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>Credit-to-GDP gap</td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>Credit-to-GDP gap</td>
<td>Credit growth</td>
</tr>
<tr>
<td>Model 3</td>
<td>Credit-to-GDP gap</td>
<td>House price-to-income</td>
</tr>
<tr>
<td>Model 4</td>
<td>Credit-to-GDP gap</td>
<td>Wholesale funding gap</td>
</tr>
<tr>
<td>Model 5</td>
<td>Credit growth</td>
<td></td>
</tr>
<tr>
<td>Model 6</td>
<td>Credit growth</td>
<td>House price-to-income</td>
</tr>
<tr>
<td>Model 7</td>
<td>Credit growth</td>
<td>Wholesale funding gap</td>
</tr>
<tr>
<td>Model 8</td>
<td>House price-to-income</td>
<td></td>
</tr>
<tr>
<td>Model 9</td>
<td>House price-to-income</td>
<td>Wholesale funding gap</td>
</tr>
<tr>
<td>Model 10</td>
<td>Wholesale funding gap</td>
<td></td>
</tr>
</tbody>
</table>

Below we focus on the main results for the indicator variables. The impulse response functions show the mean target response from the sign restriction models and the point estimate for the recursive models. Appendix C shows all models with impulse responses and bands.

### 4.1.1 Non-indicator variables

Starting with the non-indicators variables, we go through the responses of the output gap, foreign exchange rate, and inflation to a contractionary monetary policy shock, throughout the 10 models.

In all models, there seems to be signs of money neutrality with a negative output gap turning positive before the effect dies out. In the sign models, the *output gap* decreases instantly between -1.5 and -2 percentage points and in the recursive models after one quarter with approximately -0.5 percentage points. In all sign restriction models, the *foreign exchange rate* appreciates on impact, where the initial appreciation of the exchange rate is followed by a gradual depreciation back to baseline. These results are in line with Bjørnland (2009) who find evidence in support of Dornbusch’s overshooting hypothesis. In the recursive model, there is a delayed response, followed by either a small appreciation or a non-significant response. *Inflation* decrease around 2 percentage points on impact. This effect dies out rather quickly without a reversal in the price level, which is consistent with the effect in standard New Keynesian models with inflation-targeting central banks. In the recursive models that usually produce the price puzzle, we find no evidence of it. The initial increase in inflation is non-significant in all recursive models. Overall, the effect on the non-indicators is consistent over all model specifications, and in line with previous literature.
4.1.2 Indicator variables

Continuing with the indicator variables, we look at the responses of household credit growth, house price-to-income ratio, private credit-to-GDP gap and the wholesale funding gap, to a contractionary monetary policy shock, in the models where they are included.

The main results for credit growth are presented in figure 6. Both restriction schemes show that the change in credit growth is small. The small effect is in line with the previous VAR study of credit and monetary policy in Norway (Robstad, 2018). All recursive models show small but significant declines following a hump-shaped pattern. The response of the sign models show an initial increase before decreasing below baseline, however, the spread of responses differs substantially, see appendix C.

Figure 6: Response of credit growth to a one percentage point contractionary monetary policy shock. Solid lines show the impulse response function using sign restrictions. Dashed lines show the impulse response function using recursive restrictions. All models include credit growth and the core variables, output gap, inflation, foreign exchange rate, and nominal interest rate. Model 2 includes credit-to-GDP gap. Model 6 includes house price-to-income. Model 7 includes wholesale funding gap.
The main results for the house price-to-income ratio are presented in figure 7. In line with Robstad (2018) and Bjørnland and Jacobsen (2010), we find that house prices react quite strongly to a monetary policy shock in Norway. In all recursive specifications, the ratio declines significantly during the first 3-4 years. For the sign models, the response decreases on impact and stays below baseline in all models, except model 9, where the ratio instead returns to baseline after 10 years. Remarkably, all responses of the sign models show declines in the ratio, see appendix C.
The main results for the credit-to-GDP gap are presented in figure 8. Our results are contrary to the findings of Robstad (2018), who find that the credit-to-GDP ratio increase slightly but insignificant following a monetary tightening. We also debunk the argument of Svensson (2013), who claims that following a contractionary monetary policy shock, this ratio increase. Our results give support to the argument of Borio and Lowe (2004), that one can lean by targeting the credit-to-GDP ratio. In all models, the indicator first decreases before returning to baseline at the end of the horizon. The results are significant under recursive restrictions in model 1 and 3, while under the sign restriction, all responses falls below baseline in model 1, 3 and 4, see appendix C.
The main results for the wholesale funding gap are presented in figure 9. Previous results from Halvorsen and Jacobsen (2016) find that following an expansionary monetary policy shock, the wholesale funding ratio increases. Given symmetry, their result implies that a contractionary monetary policy shock should lead to a decrease in the ratio. We find non-significant effects in all recursive models while we get mixed results from the sign models. In model 7 all sign responses decrease below baseline briefly after 3 quarters. In model 9 all sign responses increase above baseline after 6 years, similar for model 10 where all sign responses increase above baseline after 8 years, see appendix C.

Overall, the effects on credit growth are small. We find that both the credit-to-GDP gap and the house price-to-income ratio decreases. And that a monetary policy shock has no effect on the wholesale funding gap in the short run, but seems to increase it in the long run. Additionally, the identification method seems to be important for the magnitude of the responses, where the effect seems to be stronger with sign restrictions in all models. Furthermore, the combination of indicators seems to matter in some cases. The house price-to-income ratio responds differently when combined with the wholesale funding gap compared to when alone or combined with the other indicators. The credit-to-GDP gap is significant when alone, and when combined with the house price-to-income ratio, but not significant when combined with credit growth or the wholesale funding gap.

To summarize, we find that monetary policy influences the credit-to-GDP gap, which falls. In addition, the house price-to-income ratio falls in all our models. Nevertheless, the effects of household credit growth and the wholesale funding gap are either small or insignificant.
5 Limitations

In our view, the main limitation of this, and similar, papers is the use of historical crises when addressing future crises. In many cases, new crises occurs because of new developments that are hard to pin down before the crisis. This problem is most evident in the choice of indicator variables in step 1. First, the candidate variables we include in the assessment are chosen based on existing EWI literature, and we do not explore new indicator variables. Second, we base our assessment of the indicator variables on their performance prior to past crises, and our results, thereby, does not consider future financial development.

As all studies of financial crises, this paper suffers from the infrequency of crises in the data. With only one financial crisis originating in Norway the technical assessment in step 1 is somewhat limited. The problem of few crises does not apply to step 2 where structural changes in the data does not permit us to use data going back further than the early 1990s. In addition, using the two crises in our data set, we do not differentiate between the Norwegian recession and the Global recession. The Norwegian recession was endogenous to Norway following the build-up of domestic financial imbalances, while the Global recession was exogenous following the build-up of financial imbalances abroad. Although it is plausible that the indicator variables behave differently prior to endogenous and exogenous crises, the build-up of financial imbalances require somewhat similar policies in both cases. This is due to the global financial system being intertwined, and domestic financial imbalances can trigger endogenous financial distress, but also leave the domestic financial system vulnerable to exogenous financial distress. Nevertheless, we cannot be sure that the same indicator variables and thresholds apply when assessing both endogenous and exogenous crises.

A longer data sample would be beneficial in step 1 because we would have been able to construct more gap variables, such as the house price gap and credit gap. In addition, with a longer data sample, we could have used different filtering methods than the HP filter.

There are several limitations to the signaling approach. First, the periods when the signals are accepted as true, are chosen somewhat arbitrary. Second, the threshold values depends on $\beta$ when minimizing the loss of type 1 and type 2 errors. We address this problem by checking the robustness of the results with different $\beta$ values.

There are also several limitations to the structural VAR approach and one should always be careful when interpreting the results from these models. First, the results can be questioned from a specification perspective where misspecification and omitted variables make the interpretation of the residuals less useful since we do not know what kind of variation we are looking at. We try to solve this problem by using the same four non-indicator variables in all models that should capture the monetary policy reaction function, and then adding on indicator variables. In addition, we employ three different tests to find the optimal lag length. Second, one could question how well we have solved the structural identification and model identification problems. There are problems with both of our identification ap-
approaches. First, a problem with the recursive approach is that the structural identification makes little economic sense when including multiple asset prices in the VAR model. Second, a problem with the sign restriction approach is that choosing one model using the mean target approach does not ensure that we pick the most probable model. These problems could have been improved upon by using different identification approaches or a combination of different identification approaches. Other improvements to the structural VAR methodology could have been the use of Bayesian estimation or the use of nonlinear structural VAR models.

6 Conclusion

In 2018, the Norwegian Government issued a new regulation for the conduct of monetary policy. It explicitly states that the central bank shall counteract the build-up of financial imbalances. For monetary policy to do so, it is important to know how monetary policy affects financial imbalances. This paper attempts to build a bridge between financial stability and monetary policy by answering the descriptive question of how monetary policy affects financial stability. In order to do so, we first find good indicators for financial stability, and second, assess the impact of a monetary policy shock on these indicators.

We have investigated the predicative ability of five indicators of financial stability for the past two Norwegian recessions, using a signaling approach. Our results show that the private credit-to-GDP gap, household credit growth, house price-to-income ratio, and banks' wholesale funding gap works well as indicators of financial distress in Norway. Particularly, the wholesale funding gap performs best of all indicator variables in our sample. Furthermore of interest, Norges Bank uses the credit-to-GDP gap, however, we found credit growth to be a more reliable indicator variable. House prices-to-income, both in gap and ratio, also showed good predicative abilities over the past 35 years in Norway. Since these indicators are determinants of financial stability, we further ask the descriptive question of whether the central bank can affect these variables using the monetary policy instrument.

We have examined the effect of monetary policy on financial stability using ten structural VAR models, with a fixed set of non-indicator variables, and different combinations of indicator variables. Compared to previous studies on financial stability and monetary policy in Norway, we use a broader set of indicators. Our findings suggest that monetary policy is not neutral when it comes to financial stability. We show that monetary policy can affect some of these important determinants of financial stability, by using the short-term nominal interest rate. In particular, we find that monetary policy influences the credit-to-GDP gap, which falls. In addition, the house price-to-income ratio falls in all our models. Nevertheless, the effects of household credit growth and the wholesale funding gap are either small or insignificant.

We found variables that provide good signals for periods of financial instability and showed
that the central bank can use monetary policy to dampen the probability and cost of future financial distress. The central bank can rely on the four variables mentioned when assessing the probability of financial distress. However, our results show that a contractionary monetary policy shock will affect neither credit growth nor the wholesale funding gap. In a framework where the determinants of financial stability are included in the loss function, we advise the central bank to put particular importance on the credit-to-GDP gap and the house price-to-income ratio.
References


## A Data

We use quarterly data for Norway. The data and its sources are reported in table 8.

<table>
<thead>
<tr>
<th>Data</th>
<th>Source</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>FRED</td>
<td>Output gap constructed using one-sided HP-filter.</td>
</tr>
<tr>
<td>3-month Nibor</td>
<td>OSE</td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>SSIB</td>
<td>Inflation as annual growth rate of CPI.</td>
</tr>
<tr>
<td>Nominal foreign exchange rate I-44</td>
<td>Norges Bank</td>
<td></td>
</tr>
<tr>
<td>Nominal house price index</td>
<td>OECD</td>
<td>Deflated by CPI. In log and growth.</td>
</tr>
<tr>
<td>House price index over disposable income ratio</td>
<td>Norges Bank</td>
<td></td>
</tr>
<tr>
<td>House price index over disposable income gap</td>
<td>Norges Bank</td>
<td></td>
</tr>
<tr>
<td>Household credit (C2)</td>
<td>BIS</td>
<td>Deflated by CPI and population growth. In log and growth.</td>
</tr>
<tr>
<td>Private credit (C2 and C3) to real GDP</td>
<td>Norges Bank</td>
<td></td>
</tr>
<tr>
<td>Banks’ wholesale funding ratio</td>
<td>Norges Bank</td>
<td></td>
</tr>
<tr>
<td>Banks’ wholesale funding gap</td>
<td>Norges Bank</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Quarterly data. Mainland Norway when applicable.
B  Indicators

B.1  Summary statistics

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Mean</th>
<th>Median</th>
<th>Std</th>
<th>CoV</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit-to-GDP</td>
<td>151.84</td>
<td>143.53</td>
<td>28.88</td>
<td>5.26</td>
<td>108.76</td>
<td>200.23</td>
</tr>
<tr>
<td>Credit-to-GDP gap</td>
<td>2.43</td>
<td>4.51</td>
<td>11.34</td>
<td>0.21</td>
<td>-23.18</td>
<td>20.20</td>
</tr>
<tr>
<td>Household credit</td>
<td>1547.89</td>
<td>1178.43</td>
<td>817.79</td>
<td>1.89</td>
<td>684.27</td>
<td>3193.84</td>
</tr>
<tr>
<td>Household credit growth</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>1.15</td>
<td>-0.04</td>
<td>0.19</td>
</tr>
<tr>
<td>House prices</td>
<td>62.56</td>
<td>56.78</td>
<td>25.27</td>
<td>0.45</td>
<td>28.06</td>
<td>107.81</td>
</tr>
<tr>
<td>House price growth</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
<td>0.13</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>House price-to-income ratio</td>
<td>122.66</td>
<td>128.53</td>
<td>23.41</td>
<td>5.24</td>
<td>71.67</td>
<td>154.75</td>
</tr>
<tr>
<td>House price-to-income gap</td>
<td>0.73</td>
<td>0.40</td>
<td>14.40</td>
<td>0.05</td>
<td>-33.85</td>
<td>25.89</td>
</tr>
<tr>
<td>Wholesale funding ratio</td>
<td>40.66</td>
<td>40.09</td>
<td>9.39</td>
<td>0.33</td>
<td>20</td>
<td>53.64</td>
</tr>
<tr>
<td>Wholesale funding gap</td>
<td>-0.88</td>
<td>0.57</td>
<td>6.31</td>
<td>-0.14</td>
<td>-15.43</td>
<td>12.91</td>
</tr>
</tbody>
</table>

Table 9: Summary statistics of all indicator variables. Mean is the mean of each series. Median is the median of each series. Std is standard deviation of each series. CoV is the standard deviation divided by the mean of each series, commonly referred to as the coefficient of variation. Min is the smallest number in each series. Max is the largest number in each series.

B.2  Lagged correlations (1986-2018)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP growth *</td>
<td>1</td>
<td>-0.1984</td>
<td>-0.1821</td>
<td>-0.1538</td>
<td>-0.0991</td>
<td>0.0927</td>
<td>-0.0772</td>
<td>-0.0772</td>
<td>-0.0518</td>
<td>-0.0495</td>
<td></td>
</tr>
<tr>
<td>House price-to-income ratio (2)</td>
<td>-0.1984</td>
<td>1</td>
<td>0.6684</td>
<td>0.6636</td>
<td>0.7358</td>
<td>0.7613</td>
<td>0.1586</td>
<td>0.2014</td>
<td>0.3385</td>
<td>0.6415</td>
<td></td>
</tr>
<tr>
<td>Wholesale funding gap (3)</td>
<td>-0.1821</td>
<td>0.6684</td>
<td>1</td>
<td>0.5678</td>
<td>0.9114</td>
<td>0.2109</td>
<td>0.2757</td>
<td>0.4506</td>
<td>0.1019</td>
<td>0.7176</td>
<td></td>
</tr>
<tr>
<td>Wholesale funding ratio (4)</td>
<td>-0.1628</td>
<td>0.8836</td>
<td>0.5678</td>
<td>1</td>
<td>0.6799</td>
<td>0.9124</td>
<td>-0.0228</td>
<td>0.2745</td>
<td>0.7239</td>
<td>0.3928</td>
<td></td>
</tr>
<tr>
<td>Credit-to-GDP gap (5)</td>
<td>-0.1538</td>
<td>0.7358</td>
<td>0.6804</td>
<td>0.6799</td>
<td>1</td>
<td>0.4754</td>
<td>-0.0294</td>
<td>0.4195</td>
<td>0.6038</td>
<td>0.2812</td>
<td></td>
</tr>
<tr>
<td>Credit-to-GDP (6)</td>
<td>-0.0991</td>
<td>0.7613</td>
<td>0.2109</td>
<td>0.9124</td>
<td>0.4754</td>
<td>1</td>
<td>-0.0922</td>
<td>0.941</td>
<td>0.0791</td>
<td>0.9557</td>
<td></td>
</tr>
<tr>
<td>House price growth (7)</td>
<td>0.0927</td>
<td>0.1586</td>
<td>0.2275</td>
<td>-0.0228</td>
<td>-0.0294</td>
<td>-0.0922</td>
<td>1</td>
<td>0.1813</td>
<td>0.4086</td>
<td>0.6642</td>
<td></td>
</tr>
<tr>
<td>House prices (8)</td>
<td>-0.0772</td>
<td>0.8055</td>
<td>0.2745</td>
<td>0.8843</td>
<td>0.4195</td>
<td>0.941</td>
<td>0.1813</td>
<td>1</td>
<td>0.2864</td>
<td>0.9576</td>
<td></td>
</tr>
<tr>
<td>House price-to-income gap (9)</td>
<td>-0.0772</td>
<td>0.7372</td>
<td>0.7239</td>
<td>0.3294</td>
<td>0.6998</td>
<td>0.0791</td>
<td>0.4086</td>
<td>0.2864</td>
<td>1</td>
<td>0.0943</td>
<td></td>
</tr>
<tr>
<td>Household credit (10)</td>
<td>-0.0518</td>
<td>0.6928</td>
<td>0.1019</td>
<td>0.8357</td>
<td>0.2812</td>
<td>0.9537</td>
<td>0.0642</td>
<td>0.9764</td>
<td>0.0943</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Household credit growth (11)</td>
<td>-0.0495</td>
<td>0.6415</td>
<td>0.776</td>
<td>0.3928</td>
<td>0.6677</td>
<td>0.1649</td>
<td>0.5388</td>
<td>0.3283</td>
<td>0.804</td>
<td>0.1392</td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Correlation coefficients. * GDP growth is one year ahead

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP growth *</td>
<td>1</td>
<td>-0.2138</td>
<td>0.2054</td>
<td>0.2019</td>
<td>0.0982</td>
<td>-0.0952</td>
<td>0.0936</td>
<td>0.0838</td>
<td>-0.0743</td>
<td>0.0199</td>
<td>0.0169</td>
</tr>
<tr>
<td>Wholesale funding gap (2)</td>
<td>-0.2138</td>
<td>1</td>
<td>-0.6755</td>
<td>-0.549</td>
<td>-0.9393</td>
<td>0.3058</td>
<td>-0.9522</td>
<td>-0.9402</td>
<td>0.9604</td>
<td>-0.8842</td>
<td>-0.7443</td>
</tr>
<tr>
<td>House price-to-income gap (3)</td>
<td>0.2054</td>
<td>-0.6755</td>
<td>1</td>
<td>0.9578</td>
<td>0.6934</td>
<td>-0.4788</td>
<td>0.7025</td>
<td>0.7218</td>
<td>-0.3507</td>
<td>0.7009</td>
<td>0.8413</td>
</tr>
<tr>
<td>House price-to-income ratio (4)</td>
<td>0.2019</td>
<td>-0.509</td>
<td>0.9578</td>
<td>1</td>
<td>0.5162</td>
<td>-0.53</td>
<td>0.5383</td>
<td>0.5784</td>
<td>-0.5174</td>
<td>0.5769</td>
<td>0.7836</td>
</tr>
<tr>
<td>House prices (5)</td>
<td>0.0982</td>
<td>-0.9393</td>
<td>0.6934</td>
<td>0.5162</td>
<td>1</td>
<td>-0.3408</td>
<td>0.9948</td>
<td>0.9819</td>
<td>0.013</td>
<td>0.9392</td>
<td>0.8458</td>
</tr>
<tr>
<td>Household credit growth (6)</td>
<td>-0.0952</td>
<td>-0.9393</td>
<td>0.6934</td>
<td>0.5162</td>
<td>1</td>
<td>-0.3408</td>
<td>0.9948</td>
<td>0.9819</td>
<td>0.013</td>
<td>0.9392</td>
<td>0.8458</td>
</tr>
<tr>
<td>Household credit (7)</td>
<td>0.0936</td>
<td>-0.9522</td>
<td>0.7025</td>
<td>0.5383</td>
<td>0.9948</td>
<td>-0.3985</td>
<td>1</td>
<td>0.9946</td>
<td>-0.0762</td>
<td>0.9546</td>
<td>0.8686</td>
</tr>
<tr>
<td>Credit-to-GDP gap (8)</td>
<td>0.0838</td>
<td>-0.9402</td>
<td>0.7218</td>
<td>0.5784</td>
<td>0.9819</td>
<td>-0.4565</td>
<td>0.9946</td>
<td>1</td>
<td>-0.4199</td>
<td>0.9742</td>
<td>0.9024</td>
</tr>
<tr>
<td>House price growth (9)</td>
<td>-0.0743</td>
<td>0.0904</td>
<td>-0.3507</td>
<td>-0.5174</td>
<td>0.013</td>
<td>0.8177</td>
<td>-0.0702</td>
<td>-0.1499</td>
<td>1</td>
<td>-0.1448</td>
<td>-0.417</td>
</tr>
<tr>
<td>Credit-to-GDP gap (10)</td>
<td>0.0199</td>
<td>-0.8842</td>
<td>0.7009</td>
<td>0.5769</td>
<td>0.9392</td>
<td>-0.4284</td>
<td>0.9546</td>
<td>0.9742</td>
<td>-0.1448</td>
<td>1</td>
<td>0.8992</td>
</tr>
<tr>
<td>Wholesale funding ratio (11)</td>
<td>0.0169</td>
<td>-0.7443</td>
<td>0.8413</td>
<td>0.7836</td>
<td>0.8458</td>
<td>-0.6703</td>
<td>0.8686</td>
<td>0.9024</td>
<td>-0.417</td>
<td>0.8992</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 11: Correlation coefficients. * GDP growth is one year ahead.

B.4 Crisis predictability

B.4.1 $\beta = 0.5$

<table>
<thead>
<tr>
<th></th>
<th>True signals</th>
<th>False signals</th>
<th>Loss</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale funding gap</td>
<td>1.00</td>
<td>0.23</td>
<td>0.12</td>
<td>71</td>
</tr>
<tr>
<td>Credit growth</td>
<td>0.83</td>
<td>0.15</td>
<td>0.16</td>
<td>79</td>
</tr>
<tr>
<td>House price to income ratio</td>
<td>1.00</td>
<td>0.33</td>
<td>0.17</td>
<td>66</td>
</tr>
<tr>
<td>House price to income gap</td>
<td>1.00</td>
<td>0.35</td>
<td>0.18</td>
<td>65</td>
</tr>
<tr>
<td>Credit-to-GDP gap</td>
<td>0.94</td>
<td>0.36</td>
<td>0.21</td>
<td>55</td>
</tr>
<tr>
<td>House prices (log)</td>
<td>0.70</td>
<td>0.13</td>
<td>0.21</td>
<td>2*</td>
</tr>
<tr>
<td>Wholesale funding ratio</td>
<td>0.90</td>
<td>0.41</td>
<td>0.25</td>
<td>59</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>0.94</td>
<td>0.61</td>
<td>0.33</td>
<td>29</td>
</tr>
<tr>
<td>House price growth</td>
<td>0.20</td>
<td>0.11</td>
<td>0.45</td>
<td>88</td>
</tr>
<tr>
<td>Credit (log)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.51</td>
<td>1*</td>
</tr>
</tbody>
</table>

Table 12: True signals is the periods a true signal is provided, divided by the number of periods we want a signal. False signals is the periods of false signals made divided by the number of periods we do not want a signal. Loss is the loss stemming from the loss function in section 2.2.4. Threshold shows the percentile value for which the signal goes off. *The threshold of the logarithms is by how many quarters back we compare the logarithms with when determining the signal value. The loss function is minimized using $\beta = 0.5$. 

43
### B.4.2 $\beta = 0.6$

<table>
<thead>
<tr>
<th></th>
<th>True signals</th>
<th>False signals</th>
<th>Loss</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale funding gap</td>
<td>1.00</td>
<td>0.23</td>
<td>0.09</td>
<td>71</td>
</tr>
<tr>
<td>Credit growth</td>
<td>1.00</td>
<td>0.32</td>
<td>0.13</td>
<td>64</td>
</tr>
<tr>
<td>House price to income ratio</td>
<td>1.00</td>
<td>0.33</td>
<td>0.13</td>
<td>66</td>
</tr>
<tr>
<td>House price to income gap</td>
<td>1.00</td>
<td>0.35</td>
<td>0.14</td>
<td>65</td>
</tr>
<tr>
<td>Credit-to-GDP gap</td>
<td>1.00</td>
<td>0.42</td>
<td>0.17</td>
<td>49</td>
</tr>
<tr>
<td>Wholesale funding ratio</td>
<td>1.00</td>
<td>0.54</td>
<td>0.21</td>
<td>46</td>
</tr>
<tr>
<td>House prices (log)</td>
<td>0.70</td>
<td>0.13</td>
<td>0.23</td>
<td>2**</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>1.00</td>
<td>0.67</td>
<td>0.27</td>
<td>23</td>
</tr>
<tr>
<td>House price growth</td>
<td>0.70</td>
<td>0.24</td>
<td>0.28</td>
<td>41</td>
</tr>
<tr>
<td>House price growth</td>
<td>0.56</td>
<td>0.92</td>
<td>0.40</td>
<td>1-17</td>
</tr>
<tr>
<td>Credit (log)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.59</td>
<td>1**</td>
</tr>
</tbody>
</table>

Table 13: True signals is the periods a true signal is provided, divided by the number of periods we want a signal. False signals is the periods of false signals made divided by the number of periods we do not want a signal. Loss is the loss stemming from the loss function in section 2.2.4. Threshold shows the percentile value for which the signal goes off. *The threshold of the logarithms is by how many quarters back we compare the logarithms with when determining the signal value. The loss function is minimized using $\beta = 0.6$.

### B.4.3 $\beta = 0.7$

<table>
<thead>
<tr>
<th></th>
<th>True signals</th>
<th>False signals</th>
<th>Loss</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale funding gap</td>
<td>1.00</td>
<td>0.23</td>
<td>0.07</td>
<td>71</td>
</tr>
<tr>
<td>Credit growth</td>
<td>1.00</td>
<td>0.32</td>
<td>0.10</td>
<td>64</td>
</tr>
<tr>
<td>House price to income ratio</td>
<td>1.00</td>
<td>0.33</td>
<td>0.10</td>
<td>66</td>
</tr>
<tr>
<td>House price to income gap</td>
<td>1.00</td>
<td>0.35</td>
<td>0.11</td>
<td>65</td>
</tr>
<tr>
<td>Credit-to-GDP gap</td>
<td>1.00</td>
<td>0.42</td>
<td>0.13</td>
<td>49</td>
</tr>
<tr>
<td>Wholesale funding ratio</td>
<td>1.00</td>
<td>0.54</td>
<td>0.16</td>
<td>46</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>1.00</td>
<td>0.67</td>
<td>0.20</td>
<td>23</td>
</tr>
<tr>
<td>House prices (log)</td>
<td>0.70</td>
<td>0.13</td>
<td>0.25</td>
<td>2*</td>
</tr>
<tr>
<td>House price growth</td>
<td>0.56</td>
<td>0.92</td>
<td>0.30</td>
<td>1-17</td>
</tr>
<tr>
<td>Credit (log)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.68</td>
<td>1*</td>
</tr>
</tbody>
</table>

Table 14: True signals is the periods a true signal is provided, divided by the number of periods we want a signal. False signals is the periods of false signals made divided by the number of periods we do not want a signal. Loss is the loss stemming from the loss function in section 2.2.4. Threshold shows the percentile value for which the signal goes off. *The threshold of the logarithms is by how many quarters back we compare the logarithms with when determining the signal value. The loss function is minimized using $\beta = 0.7$.  


### B.4.4 $\beta = 0.8$

<table>
<thead>
<tr>
<th></th>
<th>True signals</th>
<th>False signals</th>
<th>Loss</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale funding gap</td>
<td>1.00</td>
<td>0.23</td>
<td>0.05</td>
<td>71</td>
</tr>
<tr>
<td>Credit growth</td>
<td>1.00</td>
<td>0.32</td>
<td>0.06</td>
<td>64</td>
</tr>
<tr>
<td>House price to income ratio</td>
<td>1.00</td>
<td>0.33</td>
<td>0.07</td>
<td>66</td>
</tr>
<tr>
<td>House price to income gap</td>
<td>1.00</td>
<td>0.35</td>
<td>0.07</td>
<td>65</td>
</tr>
<tr>
<td>Credit-to-GDP gap</td>
<td>1.00</td>
<td>0.42</td>
<td>0.08</td>
<td>49</td>
</tr>
<tr>
<td>Wholesale funding ratio</td>
<td>1.00</td>
<td>0.54</td>
<td>0.11</td>
<td>46</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>1.00</td>
<td>0.67</td>
<td>0.13</td>
<td>23</td>
</tr>
<tr>
<td>House price growth</td>
<td>0.56</td>
<td>0.92</td>
<td>0.20</td>
<td>1-17</td>
</tr>
<tr>
<td>House prices (log)</td>
<td>0.70</td>
<td>0.13</td>
<td>0.27</td>
<td>2*</td>
</tr>
<tr>
<td>Credit (log)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.77</td>
<td>1*</td>
</tr>
</tbody>
</table>

Table 15: True signals is the periods a true signal is provided, divided by the number of periods we want a signal. False signals is the periods of false signals made divided by the number of periods we do not want a signal. Loss is the loss stemming from the loss function in section 2.2.4. Threshold shows the percentile value for which the signal goes off. *The threshold of the logarithms is by how many quarters back we compare the logarithms with when determining the signal value. The loss function is minimized using $\beta = 0.8$. 

---

45
C  SVAR models

In the sign restriction models, the solid line represents the median over impulse responses for each variable, while the thick dashed line represents the mean target model. The thin dashed lines and the shaded area between them, represents all impulse responses within the 16-84 percentiles. The sign restrictions are applied to the last column of the $B_0^{-1}$ matrix, and follow from the signs of the right hand side vector. In the recursive models, the solid line represents the point estimate, and the thin dashed lines and the shaded area between them, are 84 percentage confidence intervals. The recursive restrictions on the $B_0^{-1}$ matrix follow from the ordering of the left hand side vector.

C.1  Model 1

A five variable model, including the indicator credit-to-GDP gap. We use two lags since that is the overall best performer in the information criterion tests, see table 16 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9382.

\[
\begin{bmatrix}
\text{Output gap} \\
\text{Credit-to-GDP gap} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{Interest rate}
\end{bmatrix}_t =
\begin{bmatrix}
- \\
* \\
- \\
- \\
+
\end{bmatrix}_t \text{ Monetary policy shock}_t
\]

![Figure 10: Model 1](image)

Sign restrictions
Figure 11: Model 1
Recursive restrictions - Cholesky

C.2 Model 2

A six variable model, which includes the indicators credit-to-GDP gap and credit growth rate. We use two lags since that is the overall best performer in the information criterion tests, see table 17 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9229.

\[
\begin{bmatrix}
\text{Output gap} \\
\text{Credit-to-GDP gap} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{Credit growth rate} \\
\text{Interest rate}
\end{bmatrix}_t = \begin{bmatrix}
- \\
* \\
- \\
- \\
* \\
+
\end{bmatrix}_t \text{ Monetary policy shock}_t
C.3 Model 3

A six variable model, which includes the indicators credit-to-GDP gap and house price-to-income. We use two lags since that is the overall best performer in the information criterion tests, see table 18 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9808.
\[
\begin{bmatrix}
\text{Output gap} \\
\text{Credit-to-GDP gap} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{House price-to-income} \\
\text{Interest rate}
\end{bmatrix}_t = 
\begin{bmatrix}
- \\
* \\
\text{Monetary policy shock}_t \\
* \\
+ \\
\end{bmatrix}_t
\]

Figure 14: Model 3
Sign restrictions

Figure 15: Model 3
Recursive restrictions - Cholesky

C.4 Model 4

A six variable model, which includes the indicators private sector credit-to-GDP gap and house price-to-income. We use two lags since that is the overall best performer in the
information criterion tests, see table 19 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9523.

\[
\begin{bmatrix}
\text{Output gap} \\
\text{Credit-to-GDP gap} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{Banks’ wholesale funding gap} \\
\text{Interest rate}
\end{bmatrix}_t = \begin{bmatrix}
\vdots \\
\star \\
\vdots \\
\star \\
\vdots \\
\end{bmatrix} + \begin{bmatrix}
\text{Monetary policy shock}_t
\end{bmatrix}_t
\]

Figure 16: Model 4
Sign restrictions

Figure 17: Model 4
Recursive restrictions - Cholesky
C.5 Model 5

A five variable model, which includes the indicator household credit growth rate. We use two lags since that is the overall best performer in the information criterion tests, see table 20 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9331.

\[
\begin{bmatrix}
\text{Output gap} \\
\text{Credit growth rate} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{Interest rate}
\end{bmatrix}
_t = \begin{bmatrix}
\text{−} \\
\ast \\
\text{−} \\
\text{−} \\
\text{+}
\end{bmatrix}
_t \text{ Monetary policy shock}_t
\]

Figure 18: Model 5
Sign restrictions
C.6 Model 6

A six variable model, which includes the indicators household credit growth rate and house price-to-income. We use two lags since that is the overall best performer in the information criterion tests, see table 21 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9749.

$$
\begin{bmatrix}
\text{Output gap} \\
\text{Credit growth rate} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{House price-to-income} \\
\text{Interest rate}
\end{bmatrix}_{t} = 
\begin{bmatrix}
- \\
* \\
- \\
- \\
* \\
+
\end{bmatrix}_{t} \text{ Monetary policy shock}_{t}
$$

Figure 19: Model 5
Recursive restrictions - Cholesky
C.7 Model 7

A six variable model, which includes the indicators credit growth rate and banks’ wholesale funding gap. We use two lags since that is the overall best performer in the information criterion tests, see table 22 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9482.
\[
\begin{bmatrix}
\text{Output gap} \\
\text{Credit growth rate} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{Wholesale funding gap} \\
\text{Interest rate}
\end{bmatrix}
\begin{bmatrix}
* \\
* \\
* \\
- \\
- \\
+
\end{bmatrix}
t
= 
\begin{bmatrix}
\text{Monetary policy shock}_t
\end{bmatrix}
\]

Figure 22: Model 7
Sign restrictions

Figure 23: Model 7
Recursive restrictions - Cholesky
C.8 Model 8

A five variable model, which includes the indicator house price to disposable income. We use two lags since that is the overall best performer in the information criterion tests, see table 23 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9797.

\[
\begin{bmatrix}
\text{Output gap} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{House price-to-income} \\
\text{Interest rate}
\end{bmatrix}_t = \begin{bmatrix}
- \\
- \\
* \\
+
\end{bmatrix}_t \text{ Monetary policy shock}_t
\]

Figure 24: Model 8

Sign restrictions
C.9 Model 9

A six variable model, which includes the indicators house price-to-income gap and banks' wholesale funding gap. We use two lags since that is the overall best performer in the information criterion tests, see table 24 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9752.

\[
\begin{bmatrix}
\text{Output gap} \\
\text{House price-to-income gap} \\
\text{Wholesale funding gap} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{Interest rate}
\end{bmatrix}
t = \begin{bmatrix}
- \\
* \\
* \\
- \\
- \\
+
\end{bmatrix} t + \begin{bmatrix}
\text{Monetary policy shock}_t
\end{bmatrix}
\]
C.10 Model 10

A five variable model, which includes the indicator banks’ wholesale funding gap. We use two lags since that is the overall best performer in the information criterion tests, see table 25 in appendix C.11. The model is stable with a maximum eigenvalue of 0.9470.
\[
\begin{bmatrix}
\text{Output gap} \\
\text{Wholesale funding gap} \\
\text{Inflation} \\
\text{Foreign exchange rate} \\
\text{Interest rate}
\end{bmatrix}
\begin{pmatrix}
t \\
* \\
\text{Monetary policy shock}_t \\
+ 
\end{pmatrix}
\]

Figure 28: Model 10
Sign restrictions

Figure 29: Model 10
Recursive restrictions - Cholesky
C.11 Lag length

We have found the optimal lag lengths according to AIC, BIC and HQC. The figures below are the ranking of the lag lengths in descending order from best fit to worst fit. For methodology, see appendix D.1.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 16: Model 1. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 17: Model 2. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.
<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 18: Model 3. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 19: Model 4. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 20: Model 5. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.
### Table 21: Model 6. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

### Table 22: Model 7. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

### Table 23: Model 8. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Rank</td>
<td>AIC</td>
<td>BIC</td>
<td>HQC</td>
</tr>
<tr>
<td>------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 24: Model 9. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.

<table>
<thead>
<tr>
<th>Rank</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 25: Model 10. Optimal lag length using the Akaike information criterion, Bayesian information criterion and Hannan-Quinn information criterion.
D VAR and SVAR theory

D.1 Lag length

We do not know the true data generating process, hence we do not know the correct lag length. For example, if a standard New Keynesian model is the true data-generating process we should use infinitely many lags (Clarida et al., 2002; Galí and Monacelli, 2005). There are pitfalls when including both too few and too many lags in the analysis. First, a too short lag length will imply that the model is misspecified and the OLS estimates will be biased. Including too few lags omit valuable information. Since everything not included as an independent regressor ends up in the residual, this might lead to autocorrelated residuals and biased estimators. Second, a large lag length relative to the number of observations will typically lead to poor and inefficient estimates of the parameters. Including too many lags and hence estimate more coefficients than needed might lead to estimation errors and less precise estimates (Bjørnland and Thorsrud, 2015).

Information criteria used for VAR lag-order selection have the general form:

\[ C(m) = \text{log} \left( \text{det} \left( \sum_{i=u}^{m} (m) \right) \right) + c_T \phi(m) \tag{9} \]

where \( \sum_{i=u}^{m} = T^{-1} \sum_{i=u}^{m} u_t u_t' \) is the residual covariance matrix estimator for a reduced form VAR model of order \( m \) based on least square residuals, \( u_t \), and \( m \) is the candidate lag order at which the criterion function is evaluated. \( \phi(m) \) is a function of order \( m \) that penalizes larger lag orders, and \( c_T \) is a sequence of weights that may depend on the sample size. The function \( \phi(m) \) corresponds to the total number of regressors in the system of VAR equations. Since there are \( mK \) lagged regressors in each equation, and there are \( K \) equations in the VAR model in absence of any deterministic regressors, \( \phi(m) = mK^2 \), and when including an intercept \( \phi(m) = mK^2 + K \). Information criteria are based on the premise that there is a trade-off between the improved fit of the VAR model as \( m \) increases, and the parsimony of the model. Given \( T \), the effective sample size, the fit of the model by construction improves with larger \( m \), which is indicated by a reduction in the first term. The second term however, increases unambiguously with \( m \). Hence, we are looking for the value \( m \), which is the lag order that balances the objectives of model fit and parsimony (Kilian and Lütkepohl (2017)).

The VAR order is chosen such that the respective criterion is minimized over the possible orders \( m = p_{min}, ..., p_{max} \). A key issue in implementing information criteria is the choice of the upper and lower bound, \( p_{min} \) and \( p_{max} \). In the context of a model of unknown finite lag order, the default is to set \( p_{min} = 0 \) or sometimes \( p_{min} = 1 \), reducing the problem to one of choosing a suitable upper bound. The value of \( p_{max} \) must be chosen large enough to allow for delays in the response of the variables to the shocks. In practice, common choices would be 4-8 lags for quarterly data (Kilian and Lütkepohl, 2017).
We have employed three methods in the search for the optimal lag length for the VAR models, the Akaike information criterion (AIC), Bayesian information criterion (BIC) and the Hannan-Quinn information criterion (HQC). We use equations (10), (11) and (12) for implementation in Matlab.

\[ \text{D.1.1 Akaike information criterion (AIC)} \]

The Akaike information criterion was proposed by Akaike (1973, 1974).

\[ AIC(m) = \log \left( \det \left( \sum_{i=0}^{m} \right) \right) + \frac{2}{T} (mK^2 + K) \]  \hspace{1cm} (10)

where \( c_T = 2/T \).

\[ \text{D.1.2 Bayesian information criterion (BIC)} \]

\[ BIC(m) = \log \left( \det \left( \sum_{i=0}^{m} \right) \right) + \frac{\log(T)}{T} (mK^2 + K) \]  \hspace{1cm} (11)

where \( c_T = \log(T)/T \).

\[ \text{D.1.3 Hannan-Quinn information criterion (HQC)} \]

\[ HQC(m) = \log \left( \det \left( \sum_{i=0}^{m} \right) \right) + \frac{2\log(\log(T))}{T} (mK^2 + K) \]  \hspace{1cm} (12)

where \( c_T = 2\log(\log(T))/T \).

\[ \text{D.2 Model diagnostics} \]

\[ \text{D.2.1 Stability} \]

A stochastic process, \( y_t \), is stationary (covariance-stationary) if its first and second moments are time invariant. In other words, \( y_t \) is stationary if \( y_t \) have a finite mean vector and the autocovariances of the process do not depend on time \( t \) but only on the time period \( h \) for which the two vectors \( y_t \) and \( y_{t-h} \) are apart. The stationarity condition from Lütkepohl (2005) states that: A stable VAR process \( y_t \) is for all \( t \) stationary. In other words, if the VAR is stable it is also stationary. To assess the stability of a VAR system we check that the eigenvalue of the \( A \) matrix is less than one in the complex plane. From equation (14) we see that this is equivalent to checking that the elements of \( A^i \rightarrow 0 \) as \( i \rightarrow \infty \).

Stationarity requires time-invariant first and second unconditional moments. That assumption may be violated if the parameters change over time. This is a question of whether or
not there are structural breaks in the data, for discussion, see section ???. Going forward we assume stationarity.

In accordance with the literature we use logarithms as opposed to the levels when applicable. Log-transformation reduces the impact of outliers, as large but few outliers will be of less significance when the numbers are in logarithms. In addition, it reduces the increasing variance of trending time series (Ariño and Franses, 2000). There are some conflicting views on whether one should take the first difference of the logarithms or not. The main argument for taking first differences in a VAR model is to make the variables stationary. The eigenvalues of the companion form matrix in our case is less than one in absolute value for all models, see appendix C. This means that the VAR models are stable, and we do not need to use first differences of the variables.

D.2.2 Residuals

Given our assumption that the reduced form residuals $u_t$ are white noise, we need to make sure that they are normally distributed and that they are neither autocorrelated nor heteroskedastic (Kilian and Lütkepohl, 2017).

Normally distributed residuals are not required for the validity of most asymptotic procedures related to VAR modelling. However, if residuals do not have normal distribution this may signal model defects. To make sure that the condition for no autocorrelation is at least approximately satisfied, we use information criterion to choose the lag order of the model, see appendix D.1. Unmodeled conditional heteroskedasticity in $u_t$ does not invalidate the consistency of standard estimators of the parameters, they will still converge to their true value given that the unconditional error variance remains finite. However, unmodeled conditional heteroskedasticity undermines the efficiency of the estimator and affects how to conduct inference about the parameters (Kilian and Lütkepohl, 2017).

D.3 Companion form

Any $K$-dimensional VAR($p$) process can be written in companion form as a $pK$-dimensional VAR(1) model. By stacking $p$ consecutive $y_t$ variables in a $pK$-dimensional vector, $Y_t = (y_t', \ldots, y_{t-p+1}')$ we get:

$$Y_t = N + AY_{t-1} + U_t$$

(13)

Where $A$ is the companion matrix.
D.4 Moving average representation

If $y_t$ is covariance stationary, then starting from a VAR($p$) process we can derive the process infinite moving average representation MA($\infty$):

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t$$
$$y_t = A(L)^{-1}\nu + A(L)^{-1}u_t$$
$$y_t = \mu + \sum_{i=0}^{\infty} JA^i J' J U_{t-i}$$

Where $J \equiv [I_K, 0_{K \times (p-1)}]$ is a $(K \times Kp)$ matrix, $\mu = A(L)^{-1}\nu$ and $A(L)^{-1} = \sum_{i=0}^{\infty} \Phi_i L^i$ where $\Phi_i = JA^i J'$ for $i = 1, \ldots$, where $A$, as before, is the companion matrix. And the reduced form MA($\infty$) is:

$$y_t = \mu + \sum_{i=0}^{\infty} \Phi_i u_{t-i}$$

($14$)

$y_t$ is here expressed as a weighted average of current and past shocks, with weights $\Phi_i$. The structural MA($\infty$) representation can be found by using $w_t = B_0 u_t$ and define $\Theta$ as $\Theta_i \equiv \Phi_i B_0^{-1}$.

$$y_t = \mu + \sum_{i=0}^{\infty} \Phi_i B_0^{-1} B_0 u_{t-i}$$
$$y_t = \mu + \sum_{i=0}^{\infty} \Theta_i w_{t-i}$$

($15$)

D.5 Impulse response

Given $B_0$ and $u_t$ we obtain the structural shocks $w_t = B_0 u_t$, see section 2.4. After identifying the structural shocks we can perform impulse response analysis on each element of $y_t$ to a one time impulse in $w_t$. Starting from the structural MA representation in equation (15) we see that the impulse response of $y_{t+i}$, for propagation horizon $i = 0, 1, \ldots, H$, to a $w_t$ impulse in time period $t$, are given by $\Theta_i$:

$$\frac{\delta y_{t+i}}{\delta w_t^i} = \Theta_i, \quad i = 0, 1, \ldots, H$$

The elements of the $\Theta_i$ for any given time period $i$ are:
\[
\theta_{jk,i} = \frac{\delta y_{j,t+i}}{\delta u'_{k,t}}
\]

Such that \( \Theta_i = [\theta_{jk,i}] \).

Kilian and Lütkepohl (2017) mentions two implications of the linearity of our VAR model for the impulse response functions. First, responses to positive and negative shocks are the same but with opposite sign. Second, the magnitude of the structural shock does not matter for constructing impulse response functions, rescaling of the shock only rescale the entire response function. Since the magnitude does not matter, one can choose \( B_0^{-1} \) such that the structural shocks represent one standard deviation of the time series of the structural shocks. Or, as we do, choose \( B_0^{-1} \) such that the structural shock represents a one percentage point change in the interest rate.

Since there are \( K \) variables and \( K \) structural shocks, there are \( K^2 \) impulse response functions each of length \( H + 1 \). To find the impulse responses we need to find \( \Theta_i \). We start by finding the responses of \( y_{t+i} \) to the reduced form errors \( u_t \) captured in \( \Phi_i \) and then use the relationship \( w_t = B_0 u_t \) to get the responses of \( y_{t+i} \) to the structural shocks \( w_t \) captured in \( \Theta_i \).

We start by re-writing the companion form (13) using recursive substitution for \( Y_{t-i} \):

\[
Y_{t+i} = A^{i+1}Y_{t-1} + \sum_{s=0}^{i} A^s U_{t+i-s}
\]

(16)

Then left-multiply with \( J \equiv [I_K, 0_{K \times K(p-1)}] \) to unstack the variables \( JY_{t-i} = y_{t-i} \) and \( JU_{t+i-j} = u_{t+i-j} \), and multiply in \( J'J \):

\[
y_{t+i} = JA^{i+1}Y_{t-1} + \sum_{j=0}^{i} JA^j U_{t+i-j}
\]

\[
y_{t+i} = JA^{i+1}Y_{t-1} + \sum_{j=0}^{i} JA^j J'JU_{t+i-j}
\]

\[
y_{t+i} = JA^{i+1}Y_{t-1} + \sum_{j=0}^{i} JA^j J' u_{t+i-j}
\]

\[
y_{t+i} = JA^{i+1}Y_{t-1} + \sum_{j=0}^{i} \Phi_i u_{t+i-j}
\]

We see that the response of \( y_{k,t+i} \) for \( k = 1, \ldots, K \) to a unit shock in \( u_{k,t} \) for \( k = 1, \ldots, K, \) \( i \) periods ago is given by \( \Phi_i = [\phi_{jk,i}] \equiv JA^i J' \). Using the the reduced form MA representation (14) and re-writing it as the structural MA representation (15) we get:

\[
y_t = \sum_{j=0}^{\infty} \Phi_i u_{t-i} = \sum_{j=0}^{\infty} \Phi_i B_0^{-1} B_0 u_{t-i} = \sum_{j=0}^{\infty} \Theta_i w_{2-i}
\]
And we see that $\Theta_i$ is:

$$\Theta_i = \Phi_i B_0^{-1} = J A' B_0^{-1}$$

(17)

Where the $jk$th element of $\Theta_i$, $\theta_{jk,i}$ represents the response of variable $j$ to a structural shock $k$ at horizon $i = 0, 1, \ldots, H$. We use (17) to calculate the impulse response functions in Matlab.