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How much of the GPFG should NBIM actively invest in private real estate?

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Preface

The objective of this thesis is to assess the optimal allocation to private real estate in the Norwegian Government Pension Fund Global (GPFG) portfolio. We aim to do so by combining quantitative analysis with qualitative discussions. Inspired by the many academic papers encountered in the completion of our time as students, we have tried to structure the thesis as closely to a publishable academic paper as possible. Finally, we would like to thank our supervisor Bruno Gerard for giving guidance, constructive feedback and support.

"Real estate cannot be lost or stolen, nor can it be carried away. Purchased with common sense, paid for in full, and managed with reasonable care, it is about the safest investment in the world."

Franklin D. Roosevelt

U.S. President

Contents

Abbreviations

CAL: Capital Allocation Line

CAPM: Capital Asset Pricing Model

CML: Capital Market Line

EMH: Efficient Market Hypothesis

EPRA: European Public Real Estate Association

FTSE: Financial Times Stock Exchange

GPFG: Government Pension Fund Global

IPD: Investment Property Databank

MPT: Modern Portfolio Theory

MSCI: Morgan Stanley Capital International

NAREIT: North American Real Estate Investment Trust

NASDAQ: National Association of Securities Dealers Automated Quotations

NBIM: Norges Bank Investment Management

NBREM: Norges Bank Real Estate Management

NCREIF: National Council of Real Estate Fiduciaries

NPI: NCREIF Property Index

OCM: Opportunity Cost Model

REIT: Real Estate Investment Trust

U.S.: United States of America

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Abstract

In this thesis, we investigate private real estate investing in the Norwegian Pension Fund Global (GPFG). We test two hypotheses. Firstly, whether public and private real estate share similar characteristics in the long run. Secondly, we test whether the 7% private real estate allocation limit in the GPFG should stay as it is today or if it should be decreased or increased. We suggest an optimal range for NBIM to invest in private real estate. We retrieve global and/or U.S. data for equity, bonds and public and private real estate from broad indices. In addition, we incorporate qualitative discussions covering important aspects of real estate investing. We find that public and private real estate share similar characteristics in the long run, and that the real estate allocation limit in the GPFG should be decreased from the current 7%. Our analysis shows that the optimal allocation range to private real estate in the fund is 0-5%.

Keywords: real estate, optimal allocation, portfolio management

1 Introduction

Real estate is an important alternative asset class. The question of whether to invest in real estate is a decisive question for all long-term investors including pension funds, endowments and sovereign funds. Andonov, Kok, and Eichholtz (2013b) state that the three main reasons to add real estate to investment portfolios include: 1) diversification and reduction of overall risk of the portfolio; 2) hedging against inflation; and 3) delivering steady cash flows to the portfolio (i.e. rental income). The real estate market is not considered very efficient, which may create opportunities to earn greater returns. Real estate, and different aspects of it, is broadly discussed in past literature and the GPFG is an extensive part of the Norwegian economy. Hence, this has motivated us to assess the decision to invest in private real estate in the GPFG.

We aim to combine qualitative and quantitative analysis to assess the allocation of the GPFG to private real estate. Our thesis contributes to the literature quantitatively by updating the data period using methods from previous studies. We use data on total returns retrieved from broad indices and perform a mean-variance optimization to estimate the optimal allocation to private real estate. Qualitatively, we, first of all, give a more in-depth overview of the history of real estate investing in the GPFG compared to existing literature, forming a picture of suggestions and implementations dealt with in the fund. Moreover, we discuss other factors related to real estate investing affecting our final conclusion.

We find that public and private real estate share similar characteristics in the long run when adjusting for appraisal-based prices. This makes us reconsider whether private real estate should be in the GPFG portfolio as it is already exposed to real estate through the equity portfolio. Secondly, we recommend investing 0-5% of the GPFG portfolio in private real estate. This implies that the upper limit for real estate should be decreased from the current 7% to be within this range. Since NBIM is in an early stage of investing in real estate, the share invested as of 2017, is within this range. Diversification benefits, one of the three main arguments for adding real estate to a long-term portfolio, has weakened over time, making real estate appear less attractive. Additional qualitative

discussions of NBIM's history and management of the fund support our conclusion. Our analysis is consistent to Van Nieuwerburgh, Stanton, and de Bever (2015) results and advice on the topic and gives NBIM reason to go against own desires by decreasing instead of increasing the upper limit allocation to private real estate. It is up to the Ministry of Finance to recommend to the Storting to change the investment mandate.

The rest of the thesis is structured as follows. Section 2 gives an introduction to the GPFG, NBIM and its history regarding real estate from 2006 until today (2018). Section 3 explores relevant literature on factors related to investments by long-term funds, emphasizing real estate investments. Section 4 presents and explains main theories related to the research question. In section 5, we outline the methodology for our analysis. Section 6 reports the data used and presents and discusses the descriptive summary statistics of relevant variables. Section 7 goes through the results and analysis. Finally, section 8 summarizes and concludes the thesis.

2 The Norwegian GPFG and Real Estate Investing

This section starts with a short introduction to NBIM and the GPFG as well as a short summary of the fund's investment strategy and description of the governance framework of the fund. Furthermore, in section 2.4, we present the history of NBIM from 2006 to 2018, including events, actions and changes that have occurred since the inclusion of real estate to the fund was first considered.

2.1 NBIM and the GPFG

The Government Pension Fund Global (GPFG) has, since it was founded in 1990, been saving for current and future Norwegian generations and aims to ensure a return on the fund after the oil runs out. As of May 31st 2018, its market value is approximately NOK 8437 billion which is equivalent to approximately USD 103 trillion. The fund has three areas of investments; equity, fixed income and real estate. The allocations as of late May 2018 were 66.2% to equity investments, 31.2% to fixed income investments and 2.7% to private real estate investments (see Appendix A for historic shares and returns for the three asset classes). The fund is invested in more than 72 countries and in around 9000 companies.

Norges Bank Investment Management (NBIM) is a business unit within the central bank of Norway, Norges Bank, who manages the GPFG following the guidelines issued by the Ministry of Finance. Norges Bank Real Estate Management (NBREM) was established in 2014 and has taken shape as an established and self-sufficient entity within NBIM. NBREM is in charge of managing the private real estate investments within an upper limit of 7%.

2.2 The Investment Strategy of the Fund and its Strategic Benchmark

The GPFG's investment strategy has evolved over time and is based on expert reviews, practical experience and in-depth analysis. The fund invests long-term and has limited liquidity needs. Its investments are spread across markets, countries and currencies to achieve broad exposure to global growth and value creation. Other important aspects of the fund's management include ensuring that investments are responsible, transparent and cost-effective. The investment mandate of the fund specifies which markets the fund can invest in and sets limits

for allocations to the different asset classes. Hence, it reflects an understanding of how markets function, as well as the fund's purpose and characteristics.

The GPFG's strategic benchmark is defined in the investment mandate issued by the Ministry of Finance. The benchmark is decisive when managing the fund because it indicates the desired distribution of capital across asset classes, geography and currencies. The overall benchmark portfolio consists of 70% global equity and 30% global bonds and includes separate benchmark portfolios for the two asset classes[1]. For global equity, the index is computed by FTSE Russell and includes all countries, excluding Norway, that are classified as developed markets, advanced emerging markets or secondary emerging markets. The bond benchmark is provided by Bloomberg and comprises 70% government bonds and 30% corporate bonds.

2.3 The Governance Model

To properly assess NBIM's investment management decision it is crucial to understand the governance framework between the Storting (the Norwegian Parliament), the Ministry of Finance and Norges Bank (the central bank of Norway), which is illustrated in Figure 1. The Storting has the highest authority and is responsible for the Government Pension Fund Act, which describes the formal framework of the fund. It receives annual reports from the Ministry of Finance, who has the overall responsibility for the management of the fund. The Ministry issues the management mandate, which is a general investment framework, to Norges Bank and imposes requirements regarding risk management, reporting and responsible management. Norges Bank has an executive board who has delegated the operational management of the GPFG to Norges Bank Investment Management (NBIM). They provide the Ministry with quarterly and annual reports, as well as recommendations. Norges Bank Real Estate Management (NBREM) has in turn been given the specific responsibility to invest in private real estate.

[1] Real estate was removed from the benchmark in 2017.

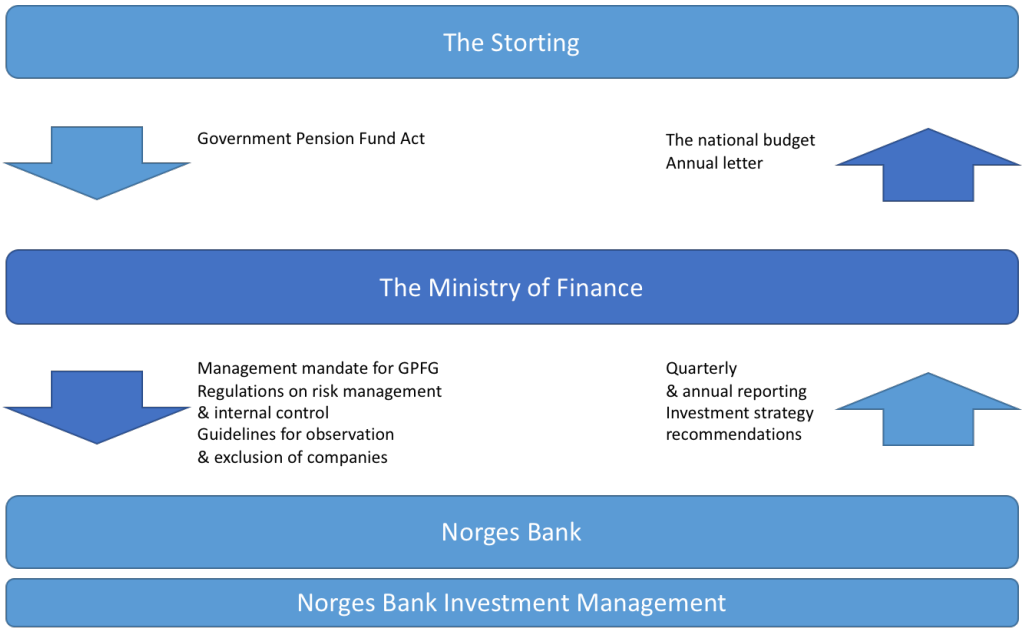

Figure 1: The Governance Model

2.4 The History of Real Estate Investing in the GPFG (2006-2018)

The following paragraphs describe the history of NBIM emphasizing real estate investing from 2006 until today (2018). We create a timeline by combining annual reports from NBIM, expert reports, letters from Norges Bank to the Ministry of Finance and reports from the Ministry of Finance to the Storting (Figure 2). Further, the main expert recommendations and implemented actions are presented briefly to provide an overall picture of the GPFG's real estate history.

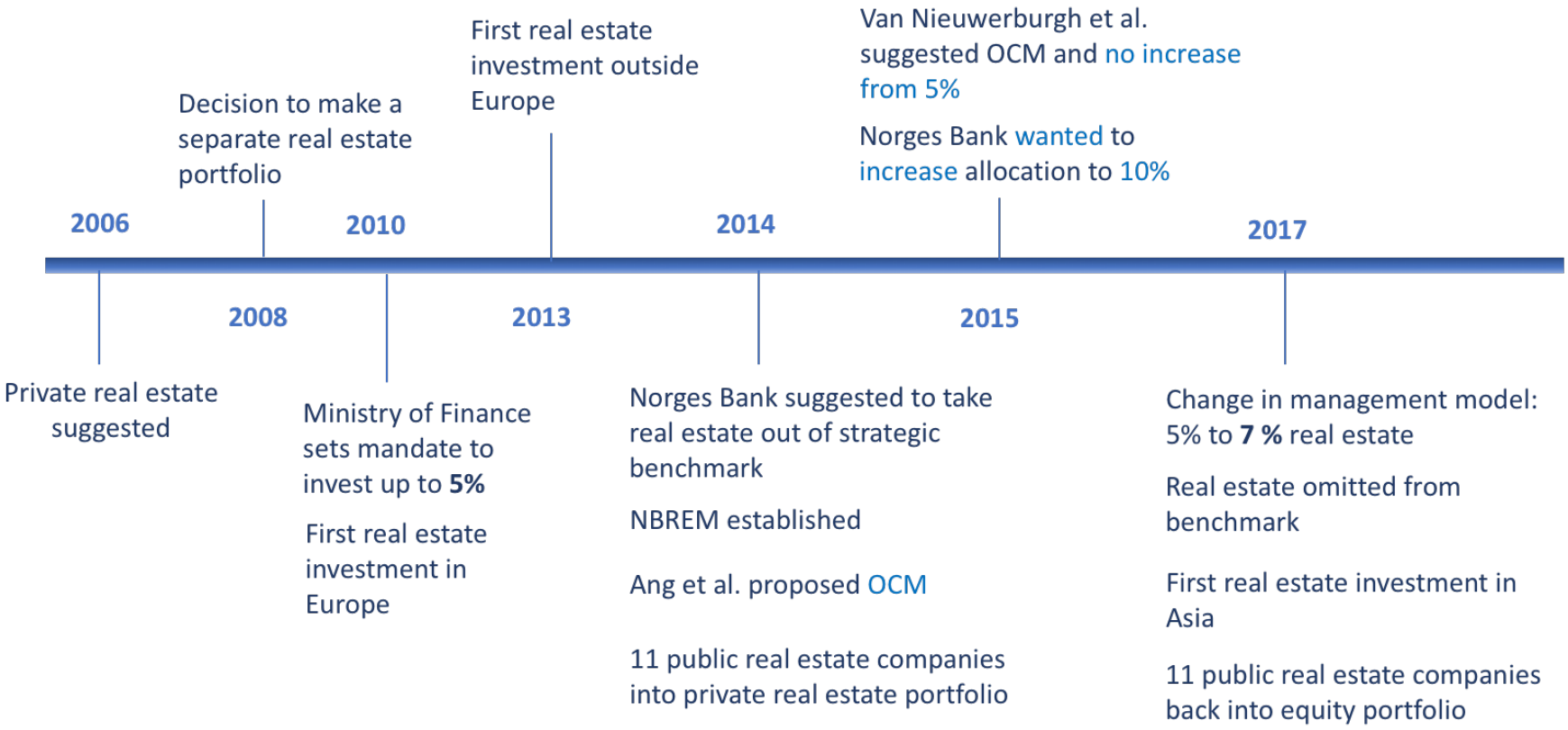

Figure 2: Timeline with the most important actions related to the GPFG´s real estate investing

Norges Bank proposed in the letter to the Ministry of Finance in **2006** to consider whether the GPFG's investment strategy should include more asset classes, such as real estate. The main reasons to include real estate in the portfolio were long-term liquidity premiums and diversification benefits. Norges Bank recommended a long-term strategic target of the GPFG to have an allocation of up to 10% invested in real estate and infrastructure.

In **2007**, Norges Bank provided the Ministry of Finance with an outline of how private real estate would be implemented by the fund. The fund already had, and still has, exposure to the real estate markets through public equity investments. However, the private real estate market is significantly larger than the public real estate market.

In **2008**, the Storting concluded that parts of the GPFG should be allocated to a separate portfolio for real estate by reducing the share of fixed income. It was decided that up to 5% of the fund could gradually be invested in real estate. Norges Bank built up expertise to start the implementation of the strategic decision during 2009.

In **2009**, the Ministry presented real estate investing guidelines, worked on developing management rules for the GPFG and continued to acquire more expertise. Private real estate is actively managed since it is illogical to use indices or passively manage it for two reasons. Firstly, it is not possible to purchase a share of properties included in the real estate index in the same manner as shares can be purchased in companies included in equity indices. Secondly, real estate indices are not equally representative of the overall real estate market. The same year, the Ministry asked an expert group, the consulting firm Mercer Ltd and Norges Bank to prepare reports on active management. The expert group, consisting of professors Ang, Goetzmann and Schaefer, stated that there is potential for active management because their studies proved violations of the Efficient Market Hypothesis (EMH). However, finding active managers who consistently deliver excess risk-adjusted returns is difficult. Mercer Ltd argued that other large funds of similar size and complexity like the GPFG benefit from active management. In addition, Norges Bank concluded that the active management of the GPFG has been largely positive.

On March 1st **2010**, the Ministry gave mandate to gradually invest up to 5% of the GPFG in real estate. In November, NBIM announced its first real estate investment. The IPD index was set as the return objective for private real estate investments.

In April **2011**, the fund carried out its first real estate investment in London and the second in Paris. Norges Bank sat up the subsidiary NBIM S.à r.l. in Luxembourg to channel through the fund's real estate investments in mainland Europe. In the fair value hierarchy, real estate investments were, and still are, characterized as particularly uncertain estimates (level 3 holdings). This generates ample uncertainty in the establishment of fair value.

In **2012**, the fund made its first investment in a shopping center and entered the logistics property market. Investments were made in the UK, Germany, Switzerland and France.

In **2013**, the first real estate investment was made in the U.S., as the mandate to invest outside Europe became effective from January 1st. Norges Bank's letter to the Ministry stated that the exposure of real estate in the stock market as a whole has varied over time and that all correlations of different sectors with the stock market have increased. In conclusion, real estate does not stand out in a systematic manner compared to other sectors.

In **2014**, an expert group (Ang, Brandt and Denison) and Norges Bank released their results after the Ministry requested advice on whether changes to the management mandate for Norges Bank could improve the ratio between expected return and risk compared to the benchmark index. The expert group proposed the use of the "Opportunity Cost Model" (OCM) already in use by other large funds including CPPIB and GIC Singapore, while Norges Bank suggested leaving real estate investments apart from the strategic benchmark noting that the IPD index suffered several shortcomings. NBIM established Norges Bank Real Estate Management (NBREM) as a separate entity to manage and invest in private real estate. They also transferred investments in 11 public real estate companies from the equity portfolio to the real estate portfolio to create a broader real estate portfolio and be able to exploit the fund's advantages within a flexible real estate mandate. Furthermore, they planned to continue the

expansion of the fund and invest 1% of the fund in real estate every year for the next few years. In December, the Ministry announced that it would assess whether the 5% cap on real estate investments in the GPFG should be increased, and asked an expert group (Van Nieuwerburgh, Stanton and de Bever) and Norges Bank for advice to be submitted in 2015.

2015 was the first year NBIM provided a separate report for real estate investments. Offices were opened in Tokyo and Singapore to get local presence, and the development and leader group of NBREM was expanded. The report to the Storting presented the recommendation of Norges Bank and the expert group to change the regulations of private real estate because of currently unsuitable target returns and advised to use a benchmark composed of public equity and fixed income to make it easier to manage risk in real estate. The Ministry suggested changing the upper limit of real estate from 5% to 7% while Norges Bank suggested an increase to 10%. The expert report by Van Nieuwerburgh, Stanton and de Bever (2015) did not warrant a recommendation to increase the real estate allocation from 5% but suggested that the Ministry of Finance should delegate the decision of the real estate allocation to Norges Bank. Further recommendations included the use of the OCM and to not use tracking error. They also suggested that the Ministry should demand a detailed report of costs related to real estate from Norges Bank and that investments should be broadened to developing countries.

In **2016**, the Ministry decided that they would not take the advice to use the OCM as suggested by both Ang et al. (2014) and Van Nieuwerburgh et al. (2015). They argued that the implementation of such a model would be too operationally demanding. However, the Ministry agreed with the recommendation by Ang et al., Van Nieuwerburgh et al. and Norges Bank to change the regulation of real estate from the IPD index to an index comprised of public equity and bonds. Norges Bank advised on the implementation of a new management model for the GPFG where Norges Bank would decide the allocation to real estate within the upper limit of 7%.

From **2017**, the 11 public real estate companies were no longer reported as part of the real estate portfolio but were transferred back into the equity portfolio. The GPFG changed the management model so that NBIM could invest up to 7%

of the investment portfolio in private real estate. The real estate portfolio was omitted from the benchmark index and added to Norges Bank's scope of deviations from the benchmark index, making the private real estate investments subject to tracking error limit. The benchmark index is comprised of public equity and bonds. Norges Bank decides what combination of equity and fixed income to sell in order to acquire real estate assets. NBIM made its first private investment in Asia (Tokyo). Gjedrem-utvalget was appointed by royal decree in 2015 to propose a new act for Norges Bank and consider the organization of Norges Bank and the management of the GPFG. In 2017, they recommended to separate the management of the GPFG from Norges Bank and establish a separate statutory entity to manage the fund.

Early in **2018,** an expert report by Dahlquist and Ødegaard, as requested by the Ministry of Finance, was published. It pointed out the difficulty of evaluating real estate because of the short return history and illiquid nature of real estate investments. Real estate returns are excessively smoothed since property values are appraisal-based, which leads to an underestimation of the volatility, correlations and factor exposures. The expert report also pointed out that the market portfolio is a natural starting point for real estate investing.

Considering the period from 2006 to 2018, one notices that the Ministry of Finance and Norges Bank have always desired to increase the allocation to private real estate in the GPFG portfolio. The Ministry has frequently asked expert groups, consulting firms and Norges Bank itself to evaluate different aspects of fund and its management. Some advice has been followed, but we note that the Ministry has not followed the advice by Van Nieuwerburgh et al. (2015) to not increase the allocation to real estate. Nor have they followed the advice of implementing an OCM as advised by both Ang, Brandt, and Denison (2014) and Van Nieuwerburgh et al. (2015).

3 Literature Review

In this section, we review the literature on real estate and factors related to the fashion of investing in real estate and real estate portfolio management. Section 3.1 to 3.6 present factors related to real estate which we consider important for further discussions. Section 3.7 briefly summarizes the most important findings in the literature review, and section 3.8 presents a critique of the literature in the context of our research topic.

3.1 Small versus Large Pension Funds

Andonov, Eichholtz, and Kok (2012) find that "the costs and performance of pension funds' real estate investments are driven by three main variables: size, the choice to invest internally or externally, and geography." Larger pension funds are often managed internally, and their size implies more favorable investment opportunities due to stronger negotiating power. This, in turn, leads to lower costs compared to smaller funds. Large funds are able to allocate more resources to monitoring external managers and can create internal units, which enhance performance.

Andonov, Eichholtz, and Kok (2013a) add that U.S. pension funds underperform compared to global peers, partly because they are less likely to follow an internal investment approach. In another article (2013b) they study the contribution of real estate to the overall performance of pension funds using the CEM global database on pension fund investments, a large database covering almost 900 pension funds over a period of 20 years. They argue that larger funds have lower costs and that the performance is more benchmark-adjusted than for smaller funds. The study suggests that an external management approach is expensive, as well as arguing that such an approach does not add significant value to the fund's performance. Further, they suggest that an internal management approach is preferable, finding that the internal approach has a gross annual average return of 7.77% of which 7.51% is actually delivered to the pension plan, so annual costs are only 16 basis points. In addition, internal management approaches, on average, outperform their benchmarks. In comparison, the cost wedge between gross and net returns is higher for external management with an average of 84 basis points. This makes it difficult for

external managers to beat the net return of internal benchmarks. A combination of indirect and direct investments in real estate is often preferred by large pension funds, while smaller funds often focus solely on direct investments with less than one-fifth of them investing indirectly.

3.2 The Real Estate Allocation in Long-Term Funds

Numerous papers examine the role of real estate and the optimal real estate allocation in long-term funds. Bajtelsmit and Worzala (1995) investigate the actual role of real estate in pension plan investment portfolios. Using 1991 end-of-year asset allocations for 159 pension plans, they find that the allocation does not vary substantially in fund size or type and that the average corporate allocation to real estate is 4.48%. Among others, Steinert and Crowe (2001) argue that increased recognition of real estate's attractive risk-return profile and an increasing demand for annuity style income streams should drive increased allocations to the real estate sector. By forming efficient portfolios for various level of aggressiveness, the paper suggests an optimal weighting to international real estate of 10-20% based on comparisons of expected returns and standard deviations. Hoesli, Lekander, and Witkiewicz (2004) also document the benefits of including real estate in mixed-asset portfolios from the perspective of investors in seven countries on three continents. They desmooth returns from 1987 to 2001 using a variant devised by D. Geltner (1993) and estimate the weight allocated to real estate in the efficient portfolio by constructing efficient frontiers. Their results show that the optimal allocation to real estate is ranging from 15% to 20% considering both domestic and international real estate. They also find that real estate is an even more effective portfolio diversifier when both domestic and international real estate assets are considered.

Considering the GPFG, Van Nieuwerburgh et al. (2015) study historical return and risk measures of global real estate, stocks and bonds in the period 1994-2015 to estimate the optimal real estate allocation. They also use factor analysis to go beyond the univariate correlation analysis and find that the correlations between returns on real estate and returns on stocks and bonds have been rising over time, reducing the diversification benefits from real estate, which in turn leads to a higher required rate of return. They claim that the GPFG

real estate allocation of 5% is appropriate and should not be increased. Dahlquist and Ødegaard (2018) share this opinion, as well as arguing that the target allocation for real estate in the market portfolio is 6%.

3.3 Biases in Private Real Estate Returns

There has been a broad discussion in the literature about the treatment of appraisal-based private real estate returns. Edelstein and Quan (2006) examine "how individual parcel appraisal smoothing affects the accuracy of aggregate real estate performance indices". They find that it leads to a downward bias of appraisal-based rate of return indices, understating the true sample mean return of approximately 9%. Also, the variance of appraisal-based return indices underestimates the variance of the true underlying returns by more than 55%. These biases induce real estate returns to appear more attractive and less risky than what is the case.

Miles, Cole, and Guilkey (1990) support the fact that real estate returns are smoothed by the appraisal process, understating the volatility and creating biases in the correlations with other asset class returns. On the other hand, Gau and Wang (1990) argue that the return biases can be small and do not significantly impact the real estate return indices. However, they concede that the mean of a time-series of holding period returns may be a noticeably biased measure of real estate returns.

Many academics have attempted to control the appraisal smoothing effects by applying statistical filters to the appraisal-based returns to remove part of or all of the autocorrelation in the data. Among these are D. M. Geltner (1991), D. Geltner (1993), Ross and Zisler (1991) and Fisher, Geltner, and Webb (1994).

3.4 Public versus Private Real Estate

A majority of the literature reports differences in returns characteristics between public and private real estate[2]. Riddiough, Moriarty, and Yeatman (2005), Pagliari, Scherer, and Monopoli (2005) and Tsai (2007) all find evidence of favoring public market real estate investments. They document a difference in mean returns between 2.66% and 3.08%. Ling and Naranjo (2015) examine U.S.

[2] Also when adjusting for appraisal-based biases.

public and private real estate returns at the aggregate level and by the four major property types over the 1994-2012 time period. They find that expected returns on public real estate exceed those of private real estate by 49 basis points (annualized). Regarding volatility, several authors report that the total volatility of public and private real estate is very close[3]

Van Nieuwerburgh et al. (2015) demonstrate that the correlation between public and private real estate approaches one in the long run. This is supported by several other studies, including Kutlu (2010), Bond and Chang (2013), Boudry, Coulson, Kallberg, and Liu (2012), Hoesli and Oikarinen (2012), Stefek and Suryanarayanan (2012), and Yunus, Hansz, and Kennedy (2012). They conclude that public and private real estate are substitutes in the long run.

However, several papers report that public and private real estate can behave differently in short and medium horizons. Ang, Nabar, and Wald (2013) introduce a methodology for estimating common real estate cycles across public and private real estate and find that public and private real estate returns can diverge in the short run because of shocks and institutional features specific to different real estate markets. Hoesli and Oikarinen (2012) find short- and medium-term diversification benefits when holding both types of real estate in an institutional portfolio. Van Nieuwerburgh et al. (2015) argue that the private real estate market accounts for approximately 80 % of the real estate market and note that other studies identify several differences between public and private real estate at shorter horizon. They recommend keeping private real estate in the portfolio.

3.5 Management in Long-Term Funds

Clark and Urwin (2008) conduct case studies and use exemplars to illustrate best-governance developments inspired by a selected group of institutions who shared their governance strategies and practices. The paper claims that good governance[4] by institutional asset owners makes a significant difference to value

[3] For example, Shepard, Liu, and Dai (2014) in The Barra Private Real Estate Model (PRE2) and Hoffmann, Tiwari, Pedersen, and He (2012).

[4] Considering three aspects of asset owner best-practice: the ways in which the exemplars organized their governance practices with respect to institutional coherence, their people and their processes.

creation, as measured by their long-term risk-adjusted rate of return. By summarizing 12 important findings of global best practice, with implications for large and small institutions, the paper argues that aspects of the design and management of sovereign funds are increasingly important for national welfare in global financial markets. Many academics argue that the price of poor performance is high. K. P. Ambachtsheer (2007) is of the opinion that many pension institutions are not "fit-for-purpose". He suggests that the impact and practices of good governance may account for 100-300 basis points per year. The same conclusion is drawn by Watson Wyatt (2006). Lerner, Schoar, and Wongsunwai (2007) analyze whether there exist systematic differences in returns and investment strategies across institutional investors. They find that funds picked by public pension funds have a moderate IRR of 20% while corporate pension funds have very poor performance on average, with an IRR of 13%. To make a comparison, endowment funds have an average IRR of 44%.

Good governance of long-term funds is closely related to fund efficiency. It has been emphasized that ethical processes related to the GPFG have a highly valid political justification. Yet, the processes create a source of institutional contradiction since there is reason to question the functional efficiency of the fund (Clark & Monk, 2010). The GPFG has scored poorly on the Clark and Urwin (2008) best practice investments management framework as discussed above, indicating a low degree of efficiency.

Chambers, Dimson, and Ilmanen (2012) assess "the Norway Model" and discuss how the GPFG is managed. The paper compares the Norway Model to the Yale University Endowment Model to evaluate the investment model of the GPFG's effectiveness. The attention is drawn to seven aspects of the Norway model for endowment asset management and stresses that the Norway Model can be an appropriate alternative to the Yale Model. However, the authors further argue that the success of Norway's investments, in the long run, depends on the "fund's culture and competence, on building and retaining professionalism, and on clarity in line structures and delegated responsibilities". The fund needs to guarantee that its active management strategies are effective and needs to exploit the competitive edge of its long horizon in constructing more dynamic strategies.

3.6 Passive and Active Management

Cremers and Lizieri (2014) find that funds with a high segment active share on average outperform the real estate market by 1.9% per year, by employing proprietary IPD data for 256 UK real estate funds over 2002-2011. These funds do not seem to take on additional risk.

K. Ambachtsheer (2015) argues that Norway's and NBIM's historically mainly passive approach of managing the GPFG has been a defensible successful strategy. Furthermore, it is suggested that following an active approach that is built and implemented attentively may possibly lead to an increase in the long-term return of the GPFG portfolio without taking on additional long-term risk. This approach is known as "the Canada Model". To follow a more active management approach in the GPFG, both Van Nieuwerburgh et al. (2015) and Ang et al. (2014) recommend the Opportunity Cost Model (OCM) for evaluating fund performance[5]. The approach is also referred to as the Reference Portfolio Approach. Further, they comment on the usefulness of this approach in the management of the GPFG.

Dahlquist and Ødegaard (2018) examine Norges Bank's active management of the GPFG by looking at the return difference between the fund and the benchmark. Their findings imply that the performance of the fund's real estate investments is in line with the performance of the country benchmarks with outperformance in the UK and underperformance in the U.S. However, their results are limited due to short return history and the illiquid nature of real estate investments.

3.7 Summary of Literature Review

The literature has, in general, suggested an allocation to real estate between 4% and 20%, which is a broad range. The literature recognizes the problem of appraisal-based biased prices in private real estate returns and comes up with different methods to control for these effects so that returns can be compared and assessed more accurately. It is argued that the expected returns of public real

[5] OCM is already in use by Canada Pension Plan Investment Board (CPPIB) and the Singaporean GIC Private Limited.

estate exceed those of private real estate. In addition, studies show that the correlation between private and public real estate approaches one in the long run while varying in short- and medium horizons, affecting diversification benefits. Further, the literature asserts that active management, in general, pays off, but that good and tailored management is crucial for efficiency in long-term funds. In addition, it is argued that the GPFG should adopt the OCM to successfully actively manage the fund.

3.8 Critique of Literature

The finding that the correlation between public and private real estate approaches one in the long run suggests the possibility to omit private real estate from the GPFG portfolio while keeping the real estate exposure as before, i.e. through equity. Yet, the literature advice the inclusion of private real estate simply because of different short-term correlations. We think that this argument is inadequate to justify the inclusion of private real estate in the GPFG portfolio. Other determining factors should be the costs related to private versus public real estate, the existing real estate exposure through public real estate, manager ability to take on private real estate and further assessment of active management. Without assessing these aspects, the argument to include private real estate on the basis of short-term correlations is not valid or sufficiently justified. In addition, we do not think that examining the aspects will help to justify the inclusion of private real estate. It can, however, provide arguments to omit private real estate from the GPFG portfolio.

When assessing real estate based on literature findings it is important to recognize the existence of methodological differences. Biases are handled differently, and case-specific assumptions are made, making results less comparable. Therefore, it is essential to be cautious about drawing conclusions without identifying and accounting for these differences. The datasets, including indices and time horizons, also differ. It is desirable to get data for longer time horizons.

4 Theory

In this section, we present and explain the main theories related to the research question *"How much of the GPFG should NBIM actively invest in private real estate?"*.

4.1 Definition of Real Estate

We define real estate investing as the purchase of property or land, which is non-moveable, with or without a building placed on it. The buyer receives ownership rights to build on the property. Investments in real estate can be either public or private. Public real estate is listed on exchange and hence often called listed real estate. Private or sometimes called commercial or unlisted real estate investments can take the form of direct investing, investing in unlisted funds or in fund-of-funds. Investment strategies can be mainly passive or active.

When referring to real estate in the GPFG, it means private real estate. It is important to note that "actual" exposure to real estate, i.e. public and private, is greater than the exposure to private real estate since public real estate is a part of the equity portfolio. The "actual" real estate exposure is not reported.

In the investigation of whether the allocation to real estate in the fund should remain the same, be decreased or increased, we refer to the upper limit allocation in the mandate from the Ministry of Finance. Today, what has been invested in (private) real estate has been and still is significantly lower than the upper limit (see Appendix A).

4.2 Modern Portfolio Theory

Modern Portfolio Theory (MPT) was pioneered by Harry Markowitz in the article "Portfolio Selection" published in the Journal of Finance (1952). In the article, Markowitz demonstrates how to reduce the risk of portfolios consisting of different assets by selecting assets whose values are not highly correlated. Diversification across uncorrelated assets is the cornerstone of Modern Portfolio Theory. Alternative investments, such as real estate, have exhibited low correlations with traditional assets, making real estate an attractive alternative to include in portfolios when evaluated on a MPT basis.

Further, Markowitz introduced the concept of the efficient frontier which is a graphical representation of all the optimal portfolios of risky assets for an investor, that offers the maximum possible expected return for a given level of risk, where risk is measured by standard deviation. Any other portfolio that lies outside the efficient frontier would be inefficient because it involves taking on additional risk without getting compensation.

To find the optimal portfolio among the candidates lying on the efficient frontier, one also need to consider an optimization plan involving a risk-free asset (Bodie, Marcus, & Kane, 2014). The Capital Allocation Line (CAL) is a straight line from the risk-free asset through a risky asset, with the standard deviation on the x-axis and the expected return on the y-axis. The optimal portfolio is chosen by choosing a point on the optimal and steepest CAL (the Capital Market Line: CML), i.e. the CAL with the highest Sharpe ratio, which is a combination of the risk-free asset and the tangency (market) portfolio to the efficient frontier. The chosen combination of the two assets depends on the risk preference of the investor.

4.3 The Capital Asset Pricing Model and the Market Portfolio

The Capital Asset Pricing Model (CAPM) was developed in the early 1960s by Treynor, Sharpe, Lintner and Mossin (Litterman, 2003) and became an important part of the MPT. The model focuses on the understanding of prices and individual assets. The investor wishes to maximize return while minimizing the volatility. The market risk (beta) is considered the only relevant risk since the investor is assumed to be fully diversified. The investor's compensation for taking on risk (having a positive beta) depends on the market risk premium.

The market portfolio is the portfolio consisting of all securities available to investors, where each security is held in proportion to its market value relative to the total market value of all assets. The market portfolio plays a central role in the CAPM since the efficient set consists of an investment in the market portfolio in addition to risk-free borrowing or lending. The CAPM tells us what share of the market portfolio that should be invested in each asset class, including real estate.

4.4 Market Efficiency

The Efficient Market Hypothesis (EMH) was developed by Eugene Fama in 1970 (Malkiel & Fama). It states that it is impossible to beat the market because all available information already is incorporated into the stock prices. This implies that no investor has an advantage of predicting a stock price return since all investors are subject to the same information. Alpha is equal to zero, meaning that active management does not improve returns. Investors who support this hypothesis tend to buy index funds and follow a more passive portfolio management approach.

In the real world, the EMH has been criticized for being subject to anomalies. Amongst others, Shiller (1980) challenged the EMH stating that with a rational stock market, investors base the prices on expected future dividends discounted to the present value. Later, Shiller extended his research to the real estate market and argued that it is inefficient and "it is far less rational than even the often irrational stock market". In his book "Market Volatility" (1992) he challenges the standard EMH related to real estate by assessing why real estate goes in and out of booms. Shiller has met support by several other researchers on this topic.

4.5 Liquidity Risk & the Liquidity Preference Theory

Liquidity risk is defined as "the risk arising from unpredictable changes in liquidity over time" (Acharya & Pedersen, 2005), and occurs when frequent transactions in the market are unavailable (Brueggeman & Fisher, 2016), which is the case for real estate transactions. Liquidity or marketability refers to how easily assets can be turned into cash, and it affects the size of the liquidity premium. Real estate can be difficult and time consuming to sell. As the illiquidity increases, a higher liquidity premium is required to bear the additional risk. This is called the Liquidity Preference Theory and was first introduced by economist John Maynard Keynes in his book "The General Theory of Employment, Interest and Money" (1936). To check whether the added risk of taking on illiquid investments are compensated, one can do a subjective assessment on how adequate the risk premiums earned on riskier assets are compared to the additional risk (Brueggeman & Fisher, 2016). Other risks that also affect real estate include business risk, financial risk, bankruptcy risk,

inflation risk, management risk, interest rate risk, legislative risk and environmental risk.

4.6 Hypotheses

We want to test whether public and private real estate share similar characteristics in the long run. In addition, we test whether the private real estate allocation in the GPFG should stay as it is today or if it should be decreased or increased. This provide us with information on whether the current upper limit target for private real estate on 7% is optimal.

5 Methodology

We do an empirical study based on the background, literature and theory discussed in previous sections. We combine quantitative analysis of summary statistics, the market portfolio and the mean-variance optimization, with qualitative discussions affecting the conclusion of our research question.

Hypothesis I Firstly, we test whether public and private real estate share similar characteristics in the long run. To test this hypothesis, we use data for public and private real estate on a global and/or U.S. basis. We aim to get data as far back in time as possible and split this period into equal subperiods. We compare quarterly returns and quarterly returns that are semi-annually and annually compounded to see how the results change. We compound the returns by taking the natural logarithm of the public and private adjusted real estate quarterly returns plus one separately, and then add up the returns to be semi-annual and annual. Further, we compare descriptive summary statistics to the academic paper findings discussed in the literature review (section 3). Especially, we examine the correlation between public and private real estate. If we find that the correlation between the two have been rising over time, and approaches one, the split between public and private real estate is less significant for the total return of the fund. Since the fund is already exposed to real estate through the equity portfolio, this can be an argument to reconsider the allocation to private real estate in the GPFG.

Investigating returns, we assess whether the returns of private real estate are equal to public real estate returns. To do this, we perform a univariate t-test over the full sample period. Because of the biases in private real estate returns, we have to be careful when interpreting the results. The null hypothesis states that the return means are not different, while the alternative hypothesis states that they are. If the null hypothesis is rejected, i.e. we accept the alternative hypothesis, the means are statistically significantly different. If the null is accepted, the means are statistically significantly the same. In both cases of private real estate returns not being different (accept null) or slightly higher (reject null) than those of public real estate, the costs related to active management of private real estate cannot be significantly higher than for the

passive management of public real estate, to justify investing in private real estate. Yet, we do not investigate the cost issue any further in this thesis and are therefore not able to draw a certain conclusion about this point.

Because of the biases that exist in private real estate returns, we adjust for autocorrelation to investigate how this affects the results. As mentioned in the literature section (section 3) several papers use different methods to diminish the autocorrelation. As a basis, we choose to use the method elaborated in Fisher et al. (1994) which suggests an unsmoothing model given by:

$$r_t^M = \frac{r_t^* - 0.6r_{t-1}^*}{0.4}$$

where r_t^M is the unsmoothed underlying market value appreciation return in calendar year "t", r_t^* is the smoothed return in calendar year "t" and r_{t-1}^* is the smoothed return in calendar year "t-1". To get more accurate estimations of the returns, we calculate the autocorrelation of our dataset using the CORREL function in Excel to be 0.782. We adjust the unsmoothing model to the following:

$$r_t^M = \frac{r_t^* - 0.782r_{t-1}^*}{0.218}$$

where 0.782 is the autocorrelation calculated over the total period and 0.218 is one minus the autocorrelation.

Hypothesis II The second hypothesis we test is whether the current share of private real estate in the GPFG on 7% should remain the same, be decreased or increased. For this, we use global data for equity, bonds and real estate retrieved from broad indices. We use public real estate to avoid the problem of appraisal-based returns and argue that this makes sense because there is evidence that public and private real estate share the same characteristics in the long run. We go as far back in time as possible, to increase the validity of the results. We assess summary statistics and the optimal allocation to real estate

according to the market portfolio, in addition to performing a mean-variance optimization.

When analyzing descriptive statistics, we compare returns and volatilities and focus on the correlation of real estate with equity and real estate with fixed income. If the correlations have been rising over time, it can indicate that the diversification benefits of adding both asset classes in a portfolio have been reduced. Since diversification benefits were one of the main reasons for including real estate in GPFG (and portfolios in general), this argument becomes weaker if this is the case.

The market portfolio can be a good indicator of what the optimal allocation to real estate should be according to theory, and work as a natural benchmark for investors. We do not spend time on estimating the market portfolio since it already exists good estimations performed in existing literature. In the paper "The Global Multi-Asset Market Portfolio 1959-2012", Doeswijk, Lam, and Swinkels (2014) estimate the invested global market portfolio from 1990 to 2012 by estimating the market capitalization of eight asset classes: equities, private equity, real estate, emerging-market debt, high-yield bonds, investment-grade credits, government bonds and inflation-linked bonds. The period is extended to 1959-2012 for the main asset categories: equities, real estate, non-government bonds and government bonds. We find additional updated data from 2012 to 2017 on the homepage of Laurens Swinkels[6]. We use this data to discuss the implications it has on our research question and also compare the weights of real estate in the market portfolio with the real weights in the GPFG over the period 2011-2017.

To find the optimal private real estate allocation in the GPFG, we use a mean-variance optimization. To get valid results, the optimization has to be done properly and the global data have to be in the right format. Using returns on global real estate, equity and bonds over the period 1999-2017, we form excess returns by subtracting the monthly global treasury rate from each of the raw returns, and the return covariance matrix to be used as input to the analysis. However, we recognize that a mean-variance optimization approach has limitations, but it gives us an indication of what is the optimal allocation to real

[6] https://personal.eur.nl/lswinkels/

estate in the GPFG. This is not in itself sufficient to state the final conclusion. In all mean-variance optimizations the weights have to equal to one. We do an unconstrained optimization, plus constrained optimizations including constraints for bonds and equity that are more in line with the mandate of the GPFG.

Another method previously used to find the optimal allocation to private real estate is index models (Van Nieuwerburgh et al., 2015), but we do not do this for the reason that we do not think it will affect our conclusion. Simulations have also been used to find the optimal allocation to equity in the GPFG (Norges offentlige utredninger (NOU), 2016), and this method could be extended to find the optimal private real estate allocation, but we do not have the resources to do so at this point.

In 2017, NBIM invested 2.6% of the fund in real estate, being significantly lower than the upper limit of 7%. The allocation has not been higher than 3.2% (2016). Hypothetically, if we find a mean-variance optimal allocation of 3% this is approximately the same as what is invested today. This implies that NBIM invests the optimal amount as of today and should continue to invest this amount. However, the upper limit of 7% is not optimal and should be changed to stay close to the allocation we find.

6 Data

In our empirical investigation, we use total returns throughout the analysis. All returns are expressed in USD. This makes the analysis more straightforward but can lead to additional correlation between the various asset classes. We use quarterly data for U.S. public and private real estate and monthly data for global real estate, equity and bonds. Further, in this section, we first present our main sources of data within public and private real estate, equity and bonds. Then we include two subsections containing discussions of the descriptive summary statistics.

6.1 Public and Private Real Estate Data

We use both global and U.S. data for public real estate. For public U.S. real estate data, we use monthly and quarterly data of total returns retrieved from FTSE NAREIT All Equity REITS Total Return Index (FNERTR) on Bloomberg. We retrieve returns back to 1978. The index is a free-float adjusted, market capitalization-weighted index and contains all tax-qualified REITs with more than 50% of total assets in qualifying real estate assets other than mortgages secured by real property that also meet minimum size and liquidity criteria.

To get a complete global public real estate index we combine FTSE EPRA/NAREIT Developed Ex. North America Index and FTSE EPRA/NAREIT North America Index. Both contain data from 1999 and share the same characteristics as the one used for public U.S. real estate data. We assume that North America covers 45% of the market capitalization and hence Ex North America covers 55%. We compute a weighted average of the two indices to get a proxy for a global public real estate index in the following manner:

$$R_{\substack{Global \\ Constructed}} = 0.55 \times R_{ExNA} + 0.45 \times R_{NA}$$

where $R_{\substack{Global \\ Constructed}}$ is a proxy for global public real estate return, R_{ExNA} is the global public real estate return excluding North America and R_{NA} is the public real estate return for North America.

For private real estate, the available data is limited, especially on a global basis. Like the majority of academic literature, we have therefore excluded global private real estate from our analysis and use only U.S. private real estate quarterly data from NCREIF Property Index (NPI) received by email from the index provider. The index goes back to 1977 and is an unleveraged composite total return for private commercial real estate properties held for investment purposes only. It is comprised exclusively of operating properties acquired on behalf of tax-exempt institutions and held in a fiduciary environment. The property returns are weighted by its market value.

When comparing public and private real estate data, REIT versus NCREIF, over 40 years and with four subperiods, there are some data issues that have to be taken into consideration. Firstly, as discussed in section 3.3 (biases in private real estate returns), privately held real estate trades infrequently compared to transaction-based REITS, which can be observed daily. Thus, the NPI index is based on appraised prices which have a tendency to exhibit significant smoothing, serial correlation and lags relative to REIT returns. Secondly, NCREIF returns are unlevered while REIT returns are calculated for levered equity. Thirdly, the mix of different property types can differ between the indices. Because of these issues, especially concerning the return biases, we perform the analysis with adjusted returns applying a desmoothing method, as discussed in the methodology section (section 5).

6.2 Stocks and Bonds Data

We have chosen to use MSCI All-Cap World Index (MSCI ACWI), retrieving monthly returns back to 1987 from Bloomberg, as the global equity index. This index is MSCI's flagship global equity index and is designed to represent the performance of the full opportunity set of large and mid-cap stocks across 23 developed and 24 emerging markets. As of March 2018, it covers more than 2,400 components across eleven sectors and roughly 85% of the free-float-adjusted market capitalization in the market.

We use Bloomberg Barclays Global-Aggregate Total Return Index Value Unhedged USD (earlier Barclays Capital Aggregate Bond Index and Lehman Aggregate Bond Index) retrieved from Datastream as a global bond index in our

analysis. The data goes back to 1990 and the market-capitalization weighted index includes treasury, corporate, government-related and securitized fixed-rate bonds from both emerging and developed markets issuers. It measures global investment grade debt from 24 local currency markets and consists of approximately 17,000 bonds.

6.3 Descriptive Statistics: Public versus Private U.S. Real Estate

Recall that for this assessment we use only U.S. data. Figure 3 shows total returns on REITs (NAREIT) and NCREIF Property Index (NPI) retrieved quarterly from 1978Q1-2017Q4, while Figure 4 displays cumulative returns for the same data. By observing the two figures, note that the total return on REITs has been considerably higher over the period, both for biased and adjusted private real estate returns. We observe that the adjusted private real estate returns follow public real estate returns more closely.

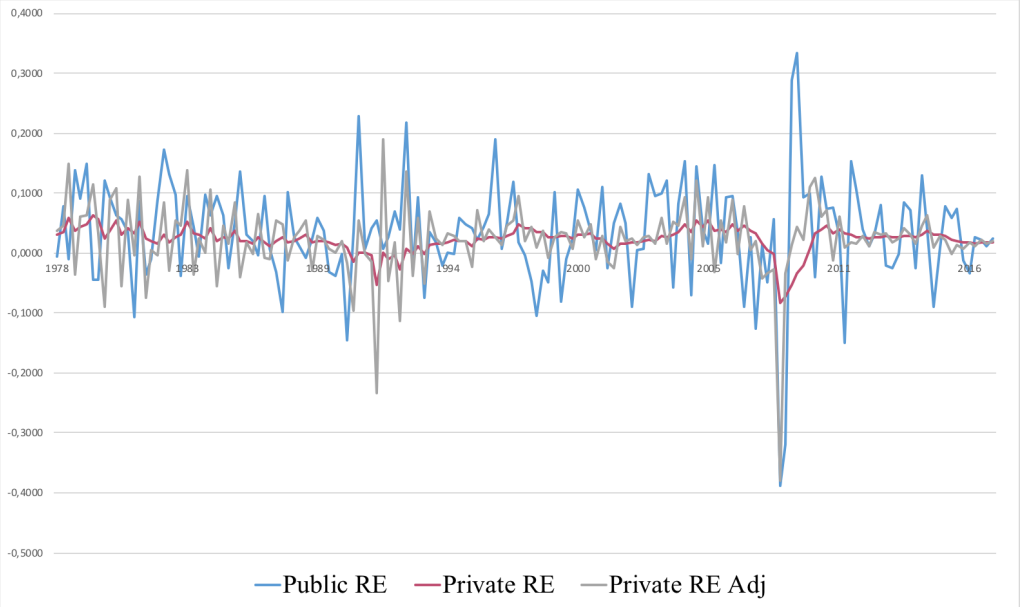

Figure 3: Full sample historical quarterly returns on public, private non-adjusted and private adjusted real estate

The figure shows quarterly returns on REITs (public real estate) and NPI (private real estate) non-adjusted and adjusted for autocorrelation. The full sample period is from 1978Q1 to 2017Q4, summing up to 160 observations. The Y-axis displays the quarterly returns and the X-axis displays the time.

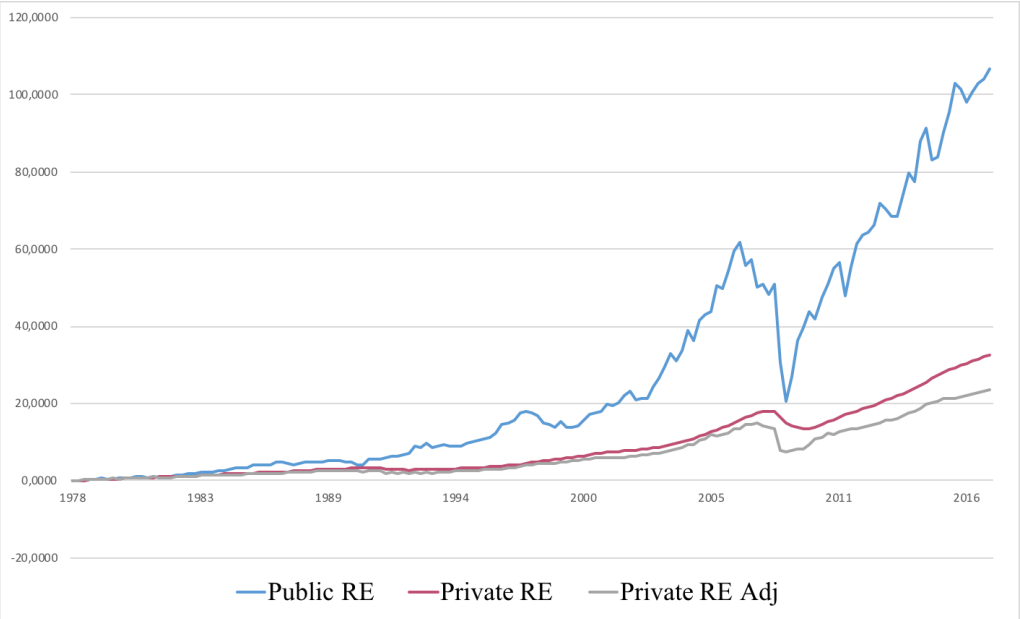

Figure 4: Full sample cumulative quarterly returns on public, private non-adjusted and private adjusted real estate

The figure displays cumulative quarterly returns on REITs (public real estate) and NPI (private real estate) non-adjusted and adjusted for autocorrelation. The full sample period is from 1978Q1 to 2017Q4, i.e. 160 observations. The Y-axis displays the cumulative returns and the X-axis displays the time.

Table 1 displays important descriptive summary statistics for public and private real estate quarterly returns, for the full sample. Full descriptive summary statistics are presented in Appendix B. Comparing the volatility of the returns, the annualized volatility of REITs on 17.41% is substantially higher than the annualized volatility of NCREIF on 4.20%. Notice that these results are affected by smoothing of prices in the NCREIF data and the leverage of REITs. However, adjusting for biases in returns, the result is significantly different, and the volatilities are much closer. In this case, the annualized volatility of NCREIF is 12.00%.

	Public RE	Private RE	Private RE Adj
Annualized Mean	14,21 %	9,34 %	9,23 %
Annualized Stdev.	17,41 %	4,20 %	12,00 %

Table 1: Full sample (1978-2017) annualized means and standard deviations of public, private and private adjusted real estate quarterly returns

By performing a t-test, we get a t-statistic of 1.59 and with a critical value of 1.97 we accept the null hypothesis of equal return means (Appendix D). This implies that public and private real estate offer the same return on average over the sample period from 1978 to 2017. If the investor only cares about the return, the investor is indifferent of the choice of real estate type. We also perform the t-test with adjusted private real estate returns and get a t-statistic of 1.53, and hence get the same conclusion as with the biased returns.

Table 2 shows the annual means and standard deviations of public and private adjusted real estate for the full sample period using different return frequencies: quarterly (159 observations), semi-annual (80 observations) and annual (40 observations). Both means and standard deviations become more similar at lower frequencies, as the difference between the means and between the standard deviations become smaller. We observe that the annual mean difference changes from 0.050 using quarterly returns to 0.038 using annual returns, while the standard deviation difference changes from 0.054 to 0.020. Appendix C shows complete descriptive summary statistics at the three different frequencies.

	Annual Mean	**Annual Standard Deviation**
Quarterly returns (159 obs)		
Public RE	0,142	0,174
Private RE Adj	0,092	0,120
Difference	0,050	0,054
Semi-annual returns (80 obs)		
Public RE	0,121	0,161
Private RE Adj	0,082	0,189
Difference	0,040	-0,029
Annual returns (40 obs)		
Public RE	0,118	0,162
Private RE Adj	0,080	0,142
Difference	0,038	0,021

Table 2: Full sample (1978-2017) annual means and volatilities of public and adjusted private real estate quarterly, semi-annually and annually returns

The table displays annual means and annual standard deviations of public and adjusted private real estate at different frequencies: quarterly, semi-annual and annual returns over the full sample period from 1978Q2 to 2017Q4. The table provides the differences between the means and between the standard deviations.

Table 3 displays correlations of public and private real estate returns at different frequencies, with and without adjusted private real estate returns. The subsample correlations are also illustrated in Figure 5.

Date	Quartely RE Corr	Quaterly RE Adj Corr	Semi-annual RE Adj Corr	Annual RE Adj corr
Full sample	0,150	0,313	0,429	0,543
1978-1987	-0,043	-0,070	-0,044	0,181
1988-1997	0,022	0,225	0,175	0,092
1998-2007	0,029	0,201	0,353	0,288
2008-2017	0,285	0,593	0,755	0,904

Table 3: Full sample and subsample correlations between public and private real estate returns at different frequencies

The full sample from 1978 to 2017 is split into four subsamples of ten years: 1978-1987, 1988-1997, 1998-2007 and 2008-2017. All columns show correlations of public and private real estate returns. The dark pink column displays the correlations between quarterly public real estate returns and quarterly unadjusted private real estate returns in the different samples. The bright blue column shows the correlations between quarterly public real estate and quarterly adjusted private real estate returns. The same applies to the two last columns, only that the returns are semi-annually (blue) and annually compounded (dark blue/grey). The same description and color coding apply to Figure 5.

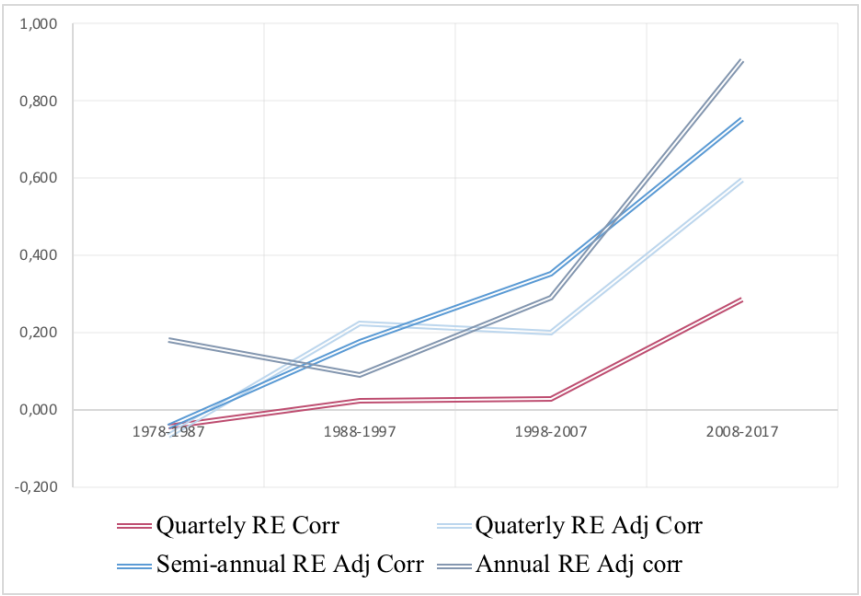

Figure 5 – Subsamples correlations between public and private real estate returns at different frequencies

See description of table 3. There are two differences: Columns in Table 3 are lines in this figure and the full sample correlations are not included.

Using unadjusted private real estate, the full sample correlation between quarterly NAREIT and NCREIF returns from 1978 to 2017 is 0.150. This indicates potential diversification benefits by investing in both public and private real estate. A comparison of the four subperiods of ten years shows that the correlations between quarterly public and unadjusted private real estate returns have been increasing over time with the highest correlation of 0.285 in the last subperiod (2008-2017).

Nonetheless, as discussed with the means and volatilities, the results are affected by data issues. Hence, we adjust the private real estate returns to get a different and more accurate picture of the relationship between public and private real estate over time. The full sample correlation when using adjusted quarterly returns is more than doubled to 0.313 and the subsample correlations are higher in each period, with the highest quarterly correlation of 0.593 in the last subperiod. We see that when the returns are adjusted, public and private real estate returns become more similar.

Examining the correlations between public and adjusted real estate returns at a quarterly, semi-annual and annual basis we see that the full sample correlations increase from 0.313 (quarterly) to 0.429 (semi-annually) to 0.543 (annually) as we lower the frequency, i.e. decrease the number of observations in each sample. The highest subsample correlation is 0.904, observed in the last subsample when the quarterly returns are annually compounded. This indicates that public and private real estate returns characteristics become even more similar at lower frequencies.

6.3 Descriptive Statistics: Global Public Real Estate, Bonds and Equity

In this assessment, we use 19 years of monthly global data on equity, fixed income and public real estate. We split the total data period from February 1999 to December 2017 (227 months) into two subperiods of nine and ten years: February 1999 - December 2007 (107 months), January 2008 - December 2017 (120 months). Splitting the total dataset into two subperiods is useful to gauge the stability of the statistical measures. In addition, the first year of the second subperiod is when real estate was first included in the GPFG.

Full sample Appendix E shows the full sample (1999-2017) descriptive summary statistics of monthly global equity, bonds and public real estate returns and Table 4 below displays the most important summary statistics and correlations.

1999-2017	Equity	Fixed Income	Public US RE	Global RE Constructed
Annualized Mean	7,1 %	4,3 %	13,3 %	10,2 %
Annualized Stdev	15,4 %	5,7 %	20,6 %	17,8 %
Annualized Sharpe Ratio	0,438	0,706	0,632	0,555
Skewness	-0,689	-0,036	-0,798	-0,926

1999-2017	Equity	Fixed Income	Public US RE	Global RE Constructed
Equity	100 %			
Fixed Income	26,9 %	100 %		
Public US RE	60,6 %	31,6 %	100 %	
Global RE Constructed	79,2 %	43,6 %	89,4 %	100 %

Table 4: Full sample (1999-2017) important descriptive summary statistics and correlations

The table displays annualized mean, annualized standard deviation, annualized Sharpe ratio and skewness, and correlations of monthly global equity, global fixed income, public U.S. real estate and global constructed real estate returns from 1999 to 2017.

Comparing the returns for the three asset classes, we notice that the constructed global public real estate index has an annualized average return of 10.2% which is higher than that of global equity whose return average 7.1%. The higher return comes at a cost of higher volatility: 17.8% for real estate compared to 15.4% for equity. To compare the return per unit of risk, we utilize the Sharpe ratio using an annual T-bill average of 0.33% over the period from 1999 to 2017. This gives a Sharpe ratio on global public real estate of 0.555 compared to 0.438 for global equity, 0.706 for global bonds and 0.632 for U.S. public real estate. This implies that public global real estate outperform equity. Public U.S. real estate perform better than global public real estate.

The volatility is a symmetric measure of risk which ignores the possibility that there may be more downside than upside risk. Therefore, we also report the skewness of the returns. We see that the skewness of global real estate is -0.926

which compared to -0.689 for global equity and -0.798 for U.S. public real estate is somewhat higher. However, notice that real estate returns have suffered from large downside risk, especially in the financial crisis, making the skewness more negative.

The volatility of an asset is not the only thing that matters for the risk of a well-diversified portfolio. One also needs to consider the asset's covariance with the other assets in the portfolio. Recall that Table 4 reports full sample correlations between global and U.S. real estate, global equity and global bonds. First, we notice the correlation of 89.4% between U.S. public real estate with global public real estate returns. This implies that there are potential gains from international diversification in real estate. Further, we see that global real estate has 79.2% correlation with global equity and a 43.6% correlation with global bonds. This suggests that combining stocks and bonds with real estate can result in substantial gains from diversification.

Subsamples Appendix F shows descriptive summary statistics of monthly global equity, bonds and public real estate returns for the two subperiods. Table 5 and Table 6 display the most important summary statistics and the correlations over the two subperiods.

1999-2007	Equity	Fixed Income	Public US RE	Global RE Constructed
Annualized Mean	7,4 %	5,5 %	15,8 %	14,1 %
Annualized Stdev	13,6 %	5,5 %	14,9 %	13,4 %
Annualized Sharpe Ratio	0,517	0,946	1,040	1,024
Skewness	-0,446	0,096	-0,724	-0,740

1999-2007	Equity	Fixed Income	Public US RE	Global RE Constructed
Equity	100 %			
Fixed Income	2,7 %	100 %		
Public US RE	31,9 %	13,4 %	100 %	
Global RE Constructed	60,9 %	26,5 %	80,9 %	100 %

Table 5: Subsample 1 (1999-2007) important descriptive summary statistics and correlations of monthly global equity, bonds and real estate returns

Over the earliest period from February 1999 to December 2007 global real estate returns 14.1% with a volatility of 13.4%. This gives a Sharpe ratio of 1.024 which is higher than the full sample Sharpe ratio (0.555). In comparison, global equity returns only 7.4% with a volatility of 13.6%. Its Sharpe ratio is 0.517 compared to the full sample Sharpe of 0.438. Throughout the period, global real estate has a correlation of 60.9% with global equity and 26.5% with global bonds. This emphasizes the greater diversification benefits of adding real estate to equity and bonds compared to the full sample. The correlation between global and U.S. public real estate is 80.9%.

2008-2017	Equity	Fixed Income	Public US RE	Global RE Constructed
Annualized Mean	6,8 %	3,3 %	11,2 %	6,9 %
Annualized Stdev	16,8 %	5,8 %	24,6 %	21,0 %
Annualized Sharpe Ratio	0,384	0,505	0,441	0,312
Skewness	-0,790	-0,119	-0,705	-0,835

2008-2017	Equity	Fixed Income	Public US RE	Global RE Constructed
Equity	100 %			
Fixed Income	43,7 %	100 %		
Public US RE	73,9 %	41,8 %	100 %	
Global RE Constructed	88,6 %	53,7 %	92,4 %	100 %

Table 6: Subsample 2 (2008-2017) important descriptive summary statistics and correlations for monthly global equity, bonds and real estate returns

Looking at subsample 2, from January 2008 to December 2017, real estate has on average earned 6.9% annually the past ten years with an average annual volatility of 21.0%. It gives a Sharpe ratio of 0.312 which is lower than both the full sample (0.555) and the first subsample (1.204). Global equity provides an annual return of 6.8% which is lower than in subperiod 1 (7.4%) and has a volatility of 16.8%. This gives a Sharpe ratio of 0.384 which is lower than both the full sample (0.438) and the first subsample (0.517). Global real estate has a correlation of 88.6% with global equity and 53.7% with global bonds, which is substantially higher than those in the first subperiod (60.9% and 26.5% respectively). The correlation between global and U.S. public real estate

increases to 92.4% compared to subperiod 1 (80.9%). We see higher volatility, correlations and skewness for the second subsample, and this can be attributable to the financial crisis. However, subperiod 1 include the unstable years ahead of the crisis, and therefore the impact should not be too noticeable when comparing the two periods.

7 Results and Analysis

Hypothesis I Our first hypothesis tests whether public and private real estate share the same characteristics in the long run. Our simple analysis (without adjusting for return biases) displays that returns on public real estate have been higher than returns on private real estate over the period 1978Q1-2017Q4 and that the volatility of public real estate has been much higher than for private real estate. We find a full sample correlation of 0.150 which indicates potential diversification benefits of investing in both public and private real estate.

However, recall the data issues that arise in private real estate returns causing the necessity of adjusting the biased returns to obtain comparable results between public and private real estate. After adjusting for the return biases, the volatilities are closer than what the unadjusted results show, and the sample correlations are significantly higher. Hence, both the mean and volatility for private real estate are underestimated. In addition, a univariate t-test shows that the means of public and private real estate over the period 1978Q1-2017Q4 are statistically significantly equal since the t-statistic is less than the critical value. We get approximately the same result both for biased and unbiased returns.

By compounding quarterly returns to semi-annual and annual, our findings imply that public and adjusted private real estate share more similar characteristics in the long run compared to shorter horizons. Their means and volatilities become more similar when we go from quarterly to semi-annual and lastly to annual returns, and the correlations between them increase in the same manner. The correlation approaches 1 (0.904) in the most recent time period (2008-2017) when compounding the returns annually. These results confirm our hypothesis. It supports the argument of using global public real estate and not private real estate in hypothesis II.

The results above support the findings in a majority of previous literature. The findings can indicate a possibility of omitting private real estate from the total portfolio because the correlation between public and private real estate has been increasing over time and approaches one in the long run. The costs related to private real estate investments are significantly higher than for passive management of public real estate, a fact that can be an additional argument to

exclude private real estate. Therefore, it can be sufficient for the GPFG to keep the real estate exposure that already exists through the equity portfolio.

Hypothesis II Our second hypothesis is related to the allocation choice of private real estate in the GPFG portfolio. The full sample correlation of global public real estate with equity is 79.2% and global public real estate with bonds is 43.6%, indicating potential gains of combining stocks and bonds with real estate. By splitting the full data sample into two subperiods, we find that the correlations of global public real estate with equity and bonds have declined over time, which is also supported in existing literature. This implies declining diversification benefits when real estate is added to a portfolio of stocks and bonds, like the GPFG. In addition, the correlation between U.S. and global public real estate is higher in subperiod 2 than in subperiod 1, meaning lowered diversification benefits of investing abroad.

Moreover, looking at the Sharpe ratios, global public real estate outperforms global equity over the full sample. The Sharpe ratio for global real estate in sample 1 (1999-2007) is substantially higher than in the full sample, while it is substantially lower than the full sample and subsample 1 in subsample 2 (2008-2017). In addition, global equity outperforms real estate with a slightly higher Sharpe ratio in subsample 2.

The market portfolio The market portfolio can be an indicator of what the optimal allocation to private real estate should be in long-term portfolios. In 2017, the market portfolio[7] consisted of 5.8% private real estate compared to the GPFG with 2.6% (See Table 7). However, the GPFG's real estate portfolio is still in an early and developing phase, hence comparing the shares directly is not reasonable. Since the market portfolio can serve as a useful benchmark for investors and private real estate has been close to 5-6% of the world portfolio over time, it can give an indication of what the optimal upper limit for private real estate should be in the GPFG.

[7] As estimated by Doeswijk et al. (2014)

	2011	2012	2013	2014	2015	2016	2017
Market portfolio	4.7%	5.1%	5.3%	5.9%	6.1%	5.9%	5.8%
GPFG	0.3%	0.7%	1.0%	2.2%	3.1%	3.2%	2.6%

Table 7: End-of-year allocations to private real estate in the market portfolio and the GPFG portfolio (2011-2017)

Mean-variance optimization Further, we estimate the optimal allocation to real estate by using the mean-variance optimization approach. The main input of the optimization is shown in Appendix G. We once again emphasize that the mean-variance optimization is not in itself be sufficient to state the final conclusion. The approach has its limitations, but it can serve as an indicator to answer hypothesis II. The optimization, maximizing the Sharpe ratio, gives an unconstrained mean-variance efficient portfolio consisting of 4.8% real estate, 7.1% equity and 88.1% bonds (Table 8). It gives an annualized average return of 4.44% with a volatility of 5.52% and a Sharpe ratio of 0.23.

	Unconstrained	Constrained 30 % FI	Constrained 60 % Equity
Weight Equity	7,1 %	11,4 %	60,0 %
Weight Fixed Income	88,1 %	30,0 %	40,0 %
Weight Real Estate	4,8 %	58,6 %	0,0 %
Sum Weights	100 %	100 %	100 %
Mean	0,36 %	0,62 %	0,46 %
Standard Deviation	1,59 %	3,64 %	2,88 %
Sharpe Ratio	22,74 %	17,05 %	15,83 %

Table 8: Mean-variance optimization outputs

This table displays the outputs from using Solver in Excel to retrieve mean-variance optimal allocations in equity, fixed income and public real estate. The Sharpe ratio is maximized in all three cases, and weights are constrained to equal 1 (even in the unconstrained case). The first column shows optimal weights, mean, standard deviation and the Sharpe ratio for the unconstrainted mean-variance optimization. The second column is constrained with a fixed allocation to fixed income of 30%, and the third column with a fixed equity allocation of 60%.

An undesirable feature of the optimization above is that the weight is not in line with the mandate of the GPFG. Recall that the strategic benchmark index consists of 70% equity and 30% bonds and that it is further up to Norges Bank to determine the scope of real estate investments within the limit of 7%. Therefore, we perform the optimization with additional constraints on equity and bonds. Constraining equity to equal any number in the interval 30-70% of the portfolio gives a zero weight to real estate. For example, a constrained optimization with 60% fixed to equity allocates 60% to equity and 40% to bonds. However, all of the combinations give an allocation equal to or above 30% towards bonds, which is not in line with the GPFG's mandate. Furthermore, constraining fixed income to equate 30% gives an optimal allocation of 58.6% to real estate, 11.4% to equity and 30% to bonds.

As expected, equity is given a small allocation in the mean-variance optimization due to poor equity performance in the time period in which the optimization is performed (1999-2017). Hence, we can expect to get a higher allocation to fixed income in the same period. It is not reasonable that this poor performance applies for future equity performance, nor is it reasonable with respect to current mandate weights. Looking into the future, expectations give reason to hold a portfolio of more equity, like in the mandate.

Additional insights The real estate market has been characterized as inefficient, creating a potential for earning abnormal returns from active management. It has also been argued that active management pays off, but this is not necessarily always the case and depends on several factors.

For NBIM, it has always been an objective to increase the allocation to real estate in the fund. NBIM established a separate entity, NBREM, to be in charge of the real estate investments, and have focused on building, expanding and improving the real estate portfolio. Two important export reports have recommended the implementation of the OCM to succeed with the active management of the GPFG. This method has not been adopted by NBIM because they argue it is too operational demanding to implement. This stated reason is somewhat surprising due to the amount of time and resources that have been allocated to the real estate portfolio. Further, since NBIM is not following expert

advice, it can be argued that NBIM is less likely to succeed in its active management of private real estate.

We note that good management is crucial for the success of long-term funds and that the GPFG needs to guarantee that its active management strategies are effective[8]. NBIM, as an active actor in a global market, do not necessarily get the best deals when investing privately because the best deals are taken by local and professional market participants. We recognize this as a tendency of the Market of Lemons phenomenon. Hence, the GPFG ends up with less cost-effective transactions compared to these participants. In comparison to other large funds, the GPFG invests a significantly smaller share in real estate (Norges offentlige utredninger (NOU), 2016). However, we find no justifications to recommend the GPFG to increase the allocation up to a level equal to that of other large long-term funds.

Lastly, one can also question the true transparency of the management of the fund, although NBIM states that their management is transparent. This is hard to control and justify. NBIM has the power to release information which endorses certain aspects of their management and hold back information that they do not necessarily want to share.

According to the aforementioned arguments, there is no reason to stress that the upper limit of private real estate investments should be increased from its current 7% in the GPFG.

[8] Recall that real estate is managed actively.

8 Conclusion

In this thesis we have addressed: *How much of the GPFG should NBIM actively invest in private real estate?*

We have been testing two hypotheses. Firstly, whether public and private real estate share similar characteristics in the long run. Secondly, whether the private real estate allocation limit on 7% in the GPFG should stay as it is today or if it should be decreased or increased. In addition, we have incorporated qualitative discussions. We have retrieved global and/or U.S. data for equity, bonds and public and private real estate from broad indices.

In hypothesis I, we find that U.S. public and private real estate share similar characteristics in the long run when we adjust for appraisal-based biased prices. When we go from quarterly to semi-annual and lastly to annual returns, i.e. lowering the frequency, correlations increase, and the means and volatilities become more similar. With annually compounded returns, the correlation converges to one. These observations make us reconsider whether private real estate should be in the GPFG portfolio since it is already exposed to real estate through the equity portfolio. It also implies that one can use public real estate as a substitute for private real estate in the long run, supporting what we do further in hypothesis II.

In hypothesis II, using global data on U.S. and global public real estate, global equity and global bonds, we find that the current upper limit of 7% to be invested in real estate in the GPFG is too high and should be decreased. We recommend a range to be invested in private real estate between 0 and 5%. The amount invested in private real estate today is within this range, but not the upper limit. The theory states that it is beneficial to include real estate in large fund portfolios because of its nature as an important alternative asset class. However, diversification benefits, one of the three main arguments of adding real estate to a long-term portfolio, has been weakened over time. The market portfolio has allocated 5-6% to real estate the last years, and a mean-variance optimization gives an unconstrained allocation of 4.8% to real estate. We question whether the fund's active management strategies are effective, whether the right people are used to build up proper expertise to complete the transactions and whether the management of the fund actually is as transparent as what is stated by NBIM.

Moreover, important expert advice to achieve successful active management have not been followed and have neither been argued against well enough by NBIM. Our analysis is consistent with Van Nieuwerburgh et al. (2015) advice on the topic and gives NBIM reason to go against own desires and decrease the upper limit allocation to real estate.

Even though this thesis highlights interesting aspects related to private real estate investments in large long-term funds, looking specifically at the GPFG, there are still several issues we suggest being addressed in future studies.

As mentioned earlier in the thesis, private real estate returns have to be substantially higher compared to public real estate returns to justify the higher costs associated with private real estate investing. To investigate if this is the case, a possibility is to do a cost-benefit analysis retrieving real costs on NBIM's real estate investments and compare them with real returns. If the case is that the additional costs of investing in private real estate cannot be justified by additional expected returns compared to public real estate, this can be an argument for excluding private real estate from the portfolio because passive investing is better in terms of returns and costs. Within this same topic, a possibility is to do case studies looking at specific transactions NBIM has carried out and the costs related to them. One can also compare transactions executed by NBIM to transactions performed by other large long-term (pension) funds.

Further and more in-depth analysis could also be done related to the OCM. Both Ang et al. (2014) and Van Nieuwerburgh et al. (2015) discuss how the model can be implemented and discuss advantages and disadvantages of it. It can be performed additional case studies looking at benefits of the model compared to the cost of operationally incorporating it[9] and also compare this to other large funds as the CPPIB and GIC Singapore that use the model.

Because of the weaknesses related to mean-variance optimizations in estimating the optimal share to real estate, another possibility is to perform simulations with costs and values of the portfolio. This has been done earlier with equity (NOU, 2016) and it should be possible to translate this method to private real estate. However, this requires expertise and is more advanced compared to other methods used.

[9] NBIM's argument to not implement the OCM.

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10 Appendices

Appendix A: End-of-year allocations and returns of real estate, equity and fixed income in the GPFG

	2010	2011	2012	2013	2014	2015	2016	2017
RE upper limit	5 %	5 %	5 %	5 %	5 %	5 %	5 %	7 %
RE allocation	0 %	0.3%	0.7%	1.0%	2.2%	3.1%	3.2%	2.6%
RE return	0 %	-4.4%	5.8%	11.8%	10.4%	10.0%	0.8%	7.5%
Private RE return					9.6%	10.8%	1.7%	
FI allocation	38.5%	41.0%	38.1%	37.3%	36.5%	35.7%	34.3%	30.8%
FI return	4.1 %	7 %	7 %	0.1 %	6.9 %	0.3 %	4.3 %	3.3 %
E allocation	61.5%	58.7%	61.2%	61.7%	61.3%	61.2%	62.5%	66.6%
E return	13.3%	-8.8%	18 %	26.3%	7.9%	3.8%	8.7%	19.4%
Annual total fund return	9.6%	-2.5%	13.4%	15.9%	7.6%	2.7%	6.9%	13.7%

RE: Real estate, FI: Fixed Income, E: Equity

- RE upper limit is the maximum amount NBIM can invest in private real estate on behalf of the GPFG.
- RE allocation is the private real estate allocation in the fund.
- RE return is the total return of the (private) real estate portfolio.
- From 2014 to 2016, 11 listed real estate companies were included in the real estate portfolio. Therefore, we included a separate row to show only the private real estate returns for the three years ("Private RE return").
- FI/E allocation is the allocation invested in fixed income/equity in the fund for every year.
- FI/E return is the fixed income/equity return in the respective year.

Appendix B: Full sample (1978-2017) descriptive summary statistics of public, private and private adjusted quarterly real estate returns

	Public RE	Private RE	Private RE Adj
Mean	0,0338	0,0226	0,0223
Annualized Mean	0,1421	0,0934	0,0923
Standard Error	0,0069	0,0017	0,0048
Median	0,0346	0,0254	0,0223
Mode	#N/A	0,0175	#N/A
Standard Deviation	0,0871	0,0210	0,0600
Annualized Stdev.	0,1741	0,0420	0,1200
Sample Variance	0,0076	0,0004	0,0036
Kurtosis	5,0027	7,7787	13,9051
Skewness	-0,7409	-2,1436	-2,1626
Range	0,7208	0,1459	0,5691
Minimum	-0,3880	-0,0840	-0,3793
Maximum	0,3328	0,0619	0,1899
Sum	5,3715	3,5882	3,5477
Count	159	159	159
Confidence Level (95,0%)	0,0136	0,0033	0,0094

The annualized means and standard deviations are calculated using the following formula:

$$(1 + r)^{12} - 1$$

The white rows are displayed in in-text tables and are the most important descriptive summary statistics. This applies to the rest of the appendices' white rows.

Appendix C: Full sample (1978-2017) descriptive summary statistics of public, private and private adjusted quarterly, semi-annually and annually private adjusted real estate returns

	Public RE Quartely	Private RE Adj Quaterly
Mean	0,0338	0,0223
Annualized mean	0,1421	0,0923
Standard Error	0,0069	0,0048
Median	0,0346	0,0223
Mode	#N/A	#N/A
Standard Deviation	0,0871	0,0600
Annualized stdev.	0,1741	0,1200
Sample Variance	0,0076	0,0036
Kurtosis	5,0027	13,9051
Skewness	-0,7409	-2,1626
Range	0,7208	0,5691
Minimum	-0,3880	-0,3793
Maximum	0,3328	0,1899
Sum	5,3715	3,5477
Count	159	159
Confidence Level(95,0%)	0,0136	0,0094

	Public RE Semi-annual	Private RE Adj Semi-annual
Mean	0,0590	0,0400
Annualized mean	0,1214	0,0817
Standard Error	0,0127	0,0106
Median	0,0591	0,0493
Mode	#N/A	#N/A
Standard Deviation	0,1135	0,0947
Annualized stdev.	0,1605	0,1895
Sample Variance	0,0129	0,0090
Kurtosis	3,9141	14,4248
Skewness	-0,8862	-2,8282
Range	0,8141	0,7270
Minimum	-0,4370	-0,5052
Maximum	0,3770	0,2218
Sum	4,7174	3,2031
Count	80	80
Confidence Level(95,0%)	0,0253	0,0211

	Public RE Annual	Private RE Adj Annual
Mean	0,1179	0,0801
Standard Error	0,0257	0,0224
Median	0,1391	0,0977
Mode	#N/A	#N/A
Standard Deviation	0,1624	0,1417
Sample Variance	0,0264	0,0201
Kurtosis	3,4324	12,1584
Skewness	-1,5351	-2,7853
Range	0,7894	0,9310
Minimum	-0,4737	-0,5808
Maximum	0,3158	0,3502
Sum	4,7174	3,2031
Count	40	40
Confidence Level(95,0%)	0,0520	0,0453

The semi-annual and annual returns are quarterly returns that are semi-annually and annually compounded. All private real estate returns are adjusted for autocorrelation.

Appendix D: Full sample (1978-2017) t-Test output of public and private real estate quarterly returns before and after adjusting for autocorrelation

Before adjusting private real estate for autocorrelation:

t-Test: Two-Sample Assuming Unequal Variances		
	Public RE	Private RE
Mean	0,0338	0,0226
Variance	0,0075	0,0004
Observations	160	160
Hypothesized Mean Difference	0	
df	177	
t Stat	1,5875	
P(T<=t) one-tail	0,0571	
t Critical one-tail	1,6535	
P(T<=t) two-tail	0,1142	
t Critical two-tail	1,9735	

After adjusting private real estate for autocorrelation: Quarterly real estate returns *after* adjusting private real estate for autocorrelation.

t-Test: Two-Sample Assuming Unequal Variances		
	Public RE	Private RE
Mean	0,0338	0,0223
Variance	0,0076	0,0036
Observations	159	159
Hypothesized Mean Difference	0	
df	205	
t Stat	1,5273	
P(T<=t) one-tail	0,0641	
t Critical one-tail	1,6523	
P(T<=t) two-tail	0,1282	
t Critical two-tail	1,9716	

Appendix E: Full sample (1999-2017) descriptive summary statistics of monthly global equity, bonds and public U.S. and global constructed real estate returns

1999-2017	Equity	Fixed Income	Public US RE	Global RE Constructed
Mean	0,0057	0,0035	0,0105	0,0081
Annualized Mean	0,0706	0,0432	0,1335	0,1021
Standard Error	0,0029	0,0011	0,0039	0,0034
Median	0,0095	0,0037	0,0128	0,0109
Mode	#N/A	-0,0090	0,0033	#N/A
Standard Deviation	0,0443	0,0163	0,0594	0,0514
Annualized Stdev	0,1536	0,0565	0,2059	0,1780
Annualized Sharpe Ratio	0,4382	0,7055	0,6321	0,5551
Sample Variance	0,0020	0,0003	0,0035	0,0026
Kurtosis	1,8592	0,5964	7,8468	5,2597
Skewness	-0,6888	-0,0364	-0,7978	-0,9256
Range	0,3168	0,1019	0,6269	0,4928
Minimum	-0,1979	-0,0397	-0,3167	-0,2741
Maximum	0,1189	0,0621	0,3102	0,2188
Sum	1,2947	0,8018	2,3821	1,8472
Count	227	227	227	227
Confidence Level(95,0%)	0,0058	0,0021	0,0078	0,0067

The Sharpe ratio is calculated in the following manner:

$$Sharpe\ ratio = \frac{Annualized\ mean - Average\ annual\ Rf}{Annualized\ standard\ deviation}$$

where Rf is the average annual risk-free rate calculated over the full sample period (1999-2017).

Appendix F: Subsample 1 and 2 descriptive summary statistics of monthly global equity, bonds and public U.S. and global constructed real estate returns

1999-2007	Equity	Fixed Income	Public US RE	Global RE Constructed
Mean	0,0060	0,0045	0,0123	0,0110
Annualized Mean	0,0739	0,0552	0,1580	0,1407
Standard Error	0,0038	0,0015	0,0042	0,0037
Median	0,0095	0,0029	0,0179	0,0183
Mode	#N/A	0,0123	0,0033	#N/A
Standard Deviation	0,0394	0,0158	0,0430	0,0387
Annualized Stdev	0,1364	0,0549	0,1488	0,1342
Annualized Sharpe Ratio	0,5170	0,9459	1,0397	1,0241
Sample Variance	0,0016	0,0003	0,0018	0,0015
Kurtosis	0,1106	-0,0309	0,9393	0,5104
Skewness	-0,4462	0,0959	-0,7236	-0,7402
Range	0,1989	0,0846	0,2407	0,1981
Minimum	-0,1096	-0,0366	-0,1458	-0,1067
Maximum	0,0893	0,0481	0,0949	0,0915
Sum	0,6372	0,4804	1,3163	1,1805
Count	107	107	107	107
Confidence Level(95,0%)	0,0075	0,0030	0,0082	0,0074

2008-2017	Equity	Fixed Income	Public US RE	Global RE Constructed
Mean	0,0055	0,0027	0,0089	0,0056
Annualized Mean	0,0678	0,0326	0,1119	0,0687
Standard Error	0,0044	0,0015	0,0065	0,0055
Median	0,0095	0,0041	0,0106	0,0072
Mode	#N/A	-0,0090	#N/A	#N/A
Standard Deviation	0,0485	0,0168	0,0711	0,0605
Annualized Stdev	0,1680	0,0580	0,2463	0,2097
Annualized Sharpe Ratio	0,3835	0,5045	0,4409	0,3120
Sample Variance	0,0024	0,0003	0,0051	0,0037
Kurtosis	2,3386	1,0517	6,7170	4,6425
Skewness	-0,7904	-0,1195	-0,7054	-0,8347
Range	0,3168	0,1019	0,6269	0,4928
Minimum	-0,1979	-0,0397	-0,3167	-0,2741
Maximum	0,1189	0,0621	0,3102	0,2188
Sum	0,6575	0,3214	1,0658	0,6667
Count	120	120	120	120
Confidence Level(95,0%)	0,0088	0,0030	0,0129	0,0109

Appendix G: Mean-variance optimization inputs

Variance-Covariance Matrix			
	Equity	Fixed Income	Global RE
Equity	0,0020	0,0002	0,0018
Bonds	0,0002	0,0002	0,0003
Global	0,0018	0,0003	0,0026
	Mean	Stdev.	Sharpe
Equity	0,0054	0,0441	0,1230
Bonds	0,0033	0,0147	0,2209
Global RE	0,0079	0,0508	0,1545

The appendix displays the variance-covariance matrix, and the mean, standard deviation and Sharpe ratio of the excess returns of equity, fixed income and global real estate. Excess returns are calculated by subtracting the monthly global treasury rate from each of the raw returns.