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### Abstract

We study the consequences of age-dependent preferences for economic growth and structural change in a two-sector model with overlapping generations and non-dimishing returns to capital. Savings and accumulation rates depend on the relative price of services consumed by old agents and on the intergenerational distribution of income. The feedback effects originating in preferences and income distribution yield three possible long-run growth outcomes: sustained endogenous growth, decumulation traps, and bounded accumulation. In the endogenous growth scenario, the transition features rising savings and accumulation rates accompanied by distributional shifts in favor of young workers, growing employment and rising prices in the service sector. Traps are triggered by initially low capital in manufacturing and low employment in services. Bounded accumulation yielding zero long-run growth in per capita incomes is induced by preferences, not by diminishing returns to capital.

**Keywords:** Endogenous growth, structural change, overlapping generations.

**JEL:** O41, O14, D91

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# 1 Introduction

More than fifty years after Kuznets (1966) seminal work, understanding the interactions between economic growth and structural change remains a central topic in macroeconomics. Two major questions, in particular, still lack comprehensive answers. How are savings and accumulation – two crucial engines of growth – related to the determinants of structural change? And what triggers equilibrium paths where economic growth and structural change feed or counteract each other?

These questions are furthermore relevant in view of the recent growth experience of East Asian economies, in particular China, where the typical pattern of structural change – manufacturing sectors spark development but service sectors eventually dominate in later stages – was accompanied by the "savings puzzle" of very high and increasing household saving rates (Chamon and Prasad, 2010) and by substantial shifts in the income distribution in favor of young workers (Song and Yang, 2010). Most of the existing literature addressed these phenomena as separate topics without investigating possible causal links between sectoral shifts, saving behavior and distributional shifts.[1] Mehlum et al. (2016) provide a first comprehensive theory of saving behavior and structural change by assuming age-dependent preferences: young agents work and save in order to finance future purchases of old-age services (e.g., health care); capital accumulation increases wages and the relative price of services, causing structural change as well as saving effects that may be self-reinforcing or self-balancing over time. The analysis of Mehlum et al. (2016), however, abstracts from productivity growth: there is no technological change in either sector, and the economy exhibits a neoclassical steady state with constant per capita income. The economy converges to such steady state under both complementarity and substitutability between manufacturing goods and services. In this note, we endogeneize growth via technological change in the manufacturing sector, obtaining novel

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[1] On the one hand, theories of structural transformation suggest that the observed patterns of structural change reflect basic forces operating in the product markets – i.e., sectoral differences in productivity growth interacting with preferences for differentiated goods – but these explanations typically abstract from saving dynamics and income distribution (see Acemoglu, 2009: Ch.20). On the other hand, the saving puzzle has been rationalized by theories of relative consumption (Carroll et al. 2000, Alvarez-Cuadrado et al. 2004) that, abstracting from structural change, predict circular causality between growth and saving rates over time.

results about the long-term consequences of age-dependent preferences. Depending on the initial state and on the elasticity of substitution, there are three possible long-run growth outcomes: sustained endogenous growth, decumulation traps, and bounded accumulation. The scenario with sustained endogenous growth in the long run exhibits empirically-consistent transitional dynamics, i.e., rising accumulation rates accompanied by distributional shifts in favor of young workers, growing employment and rising prices in the service sector. Decumulation traps are triggered by initial capital scarcity in the manufacturing sector and are characterized by low employment in services. The scenario with bounded accumulation features zero long-run growth in per capita incomes caused by strict substitutability between manufacturing goods and services, despite the fact that capital exhibits non-diminishing returns in manufacturing production.

The general intuition for these results is that economic growth is both a cause and a consequence of structural change because savings depend on the relative price of services and on the intergenerational distribution of income. The two key mechanisms can be disentangled as follows. First, capital accumulation increases wages earned by the young and therefore the relative price of the labor-intensive services consumed by the old. The increase in service prices, in turn, induces structural change, the direction of which depends on the elasticity of substitution between manufacturing goods and services. Second, the changes in the relative prices of capital and labor inputs affect the intergenerational distribution of income and thereby savings. This mechanism originates in the overlapping-generations (OLG) demographic structure and is similar to, but conceptually different from, that found in one-sector OLG models of endogenous growth by Bertola (1996) and Uhlig and Yanagawa (1996). These contributions show that *exogenous* changes in the functional income distribution induced by capital income taxes affect young cohorts' savings (by modifying the shares of income captured by different generations) and thereby economic growth, which is endogenously determined by capital accumulation. In our two-sector OLG model, instead, labor reallocations between manufacturing and services acts as *endogenous* changes to the income distribution that affect savings of young agents and thereby growth in the capital-intensive manufacturing sector.

## 2  The Model

*Households.* The economy is populated by overlapping generations of selfish agents that live for two periods $(t, t+1)$. Total population $N_t$ consists of $N_t^y$ young and $N_t^o$ old agents, and grows at the exogenous rate $N_{t+1}/N_t = N_t^y/N_t^o = 1 + n$. Preferences are age-dependent since agents have different needs in different periods of life. Specifically, each agent purchases manufactured *goods* in both periods of life and old age-related *services* (e.g. health care, nursing) in the second period of life, in order to maximize lifetime utility

$$U_t \equiv u\left(c_t\right) + \beta v\left(d_{t+1}, h_{t+1}\right) = \ln c_t + \beta \ln \left[\gamma \cdot d_{t+1}^{\frac{\sigma-1}{\sigma}} + (1 - \gamma) \cdot h_{t+1}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad (1)$$

where $c_t$ and $d_{t+1}$ are consumed quantities of manufactured goods, $h_{t+1}$ is the purchased quantity of services, $\beta \in (0, 1)$ is the discount factor, $\gamma \in (0, 1)$ is a taste parameter, and $\sigma > 0$ is the elasticity of substitution: goods and services are complements if $\sigma < 1$, substitutes if $\sigma > 1$. The hypothesis of age-dependent needs in (1) assumes the absence of a comprehensive welfare system, which obliges retired agents to purchase $h_{t+1}$ on the market. These hypotheses capture important aspects of the actual structure of the intertemporal trade-offs faced by private agents in many fast developing countries – e.g., China, where the impact of rising private expenditures on old-age care and health is a well documented fact (Blanchard and Giavazzi, 2006). Concerning the elasticity of substitution, empirical evidence tends to support the case of strict complementarity, $\sigma > 1$ (Finkelstein et al. 2013). Taking the manufactured good as the numeraire, the budget constraints read

$$c_t = w_t - s_t, \qquad d_{t+1} + p_{t+1} h_{t+1} = s_t R_{t+1}, \qquad (2)$$

where $w_t$ is the wage rate earned by young agents, each of which supplies inelastically one unit of labor, $s_t$ is savings, and $p_{t+1}$ is the price of services. Savings consist of goods stored in period $t$ and used as capital in producing goods in period $t+1$ (with full capital depletion after use) so that the gross interest $R_{t+1}$ represents the capital rental rate. Labor is perfectly mobile between sectors: we will denote by $\ell_t$ the fraction of the work force $N_t^y$ employed in manufacturing, and by $(1 - \ell_t)$ the employment share of the service sector.

*Manufacturing sector.* Goods are produced by a continuum of firms, indexed by

$j \in [0, J]$, exploiting the technology

$$X_t^j \equiv (k_t^j)^\alpha (a_t \ell_t^j N_t^y)^{1-\alpha} \qquad \text{for each } j \in [0, J], \tag{3}$$

where $k_t^j$ and $\ell_t^j N_t^y$ are capital and labor units employed by the $j$-th firm to produce $X_t^j$ units of output, $\alpha \in (0, 1)$ is a constant, and $a_t$ is labor productivity in the manufacturing sector. Firms maximize profits taking $a_t$ and all prices as given. The standard neoclassical model can be obtained by setting $a_t$ equal to a constant, which would imply diminishing returns to aggregate capital. The present analysis, instead, considers non-diminishing returns generated by learning-by-doing externalities (Romer, 1986): labor productivity in manufacturing increases with the average amount of capital used by each worker according to the spillover function $a_t = A^{\frac{1}{1-\alpha}} \cdot K_t / (\ell_t N_t^y)$, where $K_t = J k_t^j$ is aggregate capital and $A > 0$ is a constant. Aggregating (3) across firms and substituting the spillover function, sectoral output $X_t = J X_t^j$ becomes linear in aggregate capital,

$$X_t = A K_t. \tag{4}$$

*Service sector.* Our analysis builds on the premise that the service sector is labor-intensive, and to model this in the simplest possible way we assume that services are produced by labor alone under constant returns to scale,

$$H_t \equiv \eta \cdot (1 - \ell_t) \cdot N_t^y, \tag{5}$$

where $H_t$ is total supply of services, and $\eta > 0$ is a productivity parameter. The zero-profit condition

$$w_t = p_t \eta \tag{6}$$

implies that the price of services will always be proportional to the market wage rate.

## 3 Intratemporal Equilibrium

Denote capital per worker by $\kappa_t \equiv K_t / N_t^y$ and consider the intratemporal equilibrium arising in period $t$ for a given $\kappa_t$. Such an equilibrium is fully characterized by two intratemporal loci, denoted by $\Phi$ and $\Psi$, representing the labor-market equilibrium and the product-market equilibrium, respectively. Combining (6) with the labor demand of manufacturing firms, we obtain the equilibrium condition for the labor market

$$p_t = (A/\eta)(1 - \alpha)(\kappa_t / \ell_t) \equiv \Phi(\ell_t, \kappa_t), \tag{7}$$

where $\Phi\left(\ell_t, \kappa_t\right)$ is the price of services that, for each value of the employment share $\ell_t$, guarantees equal wages between the two sectors for a given $\kappa_t$. Next, combining households' demands for goods and services with the relative supplies of goods and services, we obtain the equilibrium condition for the product markets

$$p_t = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\sigma-1}} \left[\frac{(1-\alpha)\left(1-\ell_t\right)}{\ell_t - (1-\alpha)}\right]^{\frac{1}{1-\sigma}} \equiv \Psi\left(\ell_t\right), \tag{8}$$

where $\Psi\left(\ell_t\right)$ is the price of services that, for each value of the employment share $\ell_t$, clears both product markets. In (8), the restriction $\ell_t > 1-\alpha$ always holds in equilibrium since it is a necessary condition for positive second-period consumption (see Appendix). The properties of $\Phi$ and $\Psi$ are graphically described in Figures 1(a)-1(b). The labor-market locus $\Phi$ is strictly *decreasing* in $\ell_t$ because higher employment in manufacturing reduces the private marginal product of labor in that sector, putting downward pressure on the wage rate and on the price of services. The slope of the product-market locus depends on the elasticity of substitution between goods and services. When $\sigma < 1$, complementarity implies a strictly decreasing $\Psi\left(\ell_t\right)$ because a higher $p_t$ induce old agents to increase their expenditure share for services, which attracts labor in services and reduces $\ell_t$. When $\sigma > 1$, substitutability implies a strictly decreasing $\Psi\left(\ell_t\right)$ because agents respond to a higher $p_t$ by reducing their expenditure share for services, which attracts labor in the manufacturing sector and raises $\ell_t$.[2] In either case, the properties of $\Phi$ and $\Psi$ guarantee existence and uniqueness of the intratemporal equilibrium (see Appendix).

The equilibrium is characterized by an employment share of the manufacturing sector $\ell_t = \ell\left(\kappa_t\right)$ given by the fixed point

$$\ell\left(\kappa_t\right) \equiv \arg\text{solve}_{\{\ell_t \in (1-\alpha, 1)\}} \left[\Phi\left(\ell_t, \kappa_t\right) = \Psi\left(\ell_t\right)\right]. \tag{9}$$

As the sectoral division of labor determines the labor share of income, a key mechanism of the model is the response of employment shares to changes in capital per worker. From (7), the labor-market locus exhibits $\partial\Phi/\partial\kappa_t > 0$ because higher capital per worker increases the marginal product of labor, the wage rate, and the price of services. Graphically, an increase in $\kappa_t$ shifts $\Phi$ upwards in Figures 1(a)-1(b). This comparative statics

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[2] The special case $\sigma = 1$ implies that $\Psi$ reduces to a vertical locus whereby $\ell_t$ is independent of $p_t$; the resulting intertemporal equilibrium is characterized by constant employment shares that do not depend on service price and capital per worker.
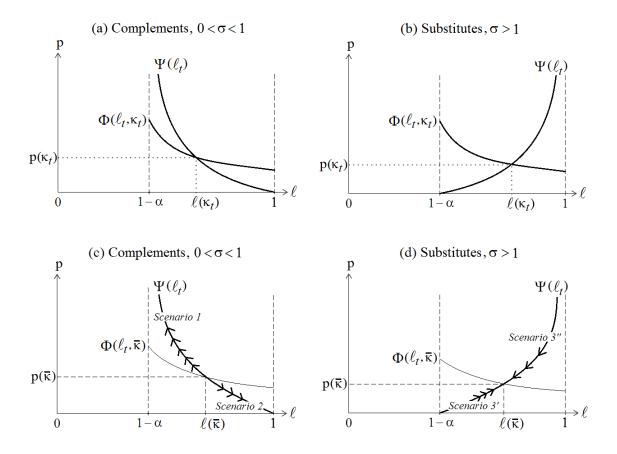
Figure 1: Upper panel: determination of the intratemporal equilibrium under (a) complementarity and (b) substitutability. Lower panel: equilibrium dynamics under (c) complementarity and (d) substitutability.

exercise shows that changes in capital per worker push employment in different directions depending on the elasticity of substitution between goods and services:

$$\ell'_{\kappa_t} \equiv \frac{\mathrm{d}\ell\left(\kappa_t\right)}{\mathrm{d}\kappa_t} \begin{cases} < 0 & \text{if } \sigma < 1 \\ > 0 & \text{if } \sigma > 1 \end{cases}.$$ (10)

Therefore, preferences determine the direction of the structural change generated by capital accumulation: higher capital always implies higher price of services but employment will move towards services, increasing the labor share of income, under complementarity, and towards manufacturing, lowering the labor share, under substitutability.

## 4  Equilibrium Dynamics

The aggregate constraint $K_{t+1} = N_t^y s_t$ and the utility-maximizing conditions for savings yield the dynamic law

$$\kappa_{t+1}\left(1+n\right) = \left(1-\alpha\right)A\kappa_t \cdot \frac{\beta}{1+\beta} \cdot \frac{1}{\ell_t}$$ (11)

where the left hand side is next-period capital per worker adjusted for population growth. The right hand side of (11) shows immediately the role of the income distribution: $\left(1-\alpha\right)A\kappa_t$ is the share of manufacturing production accruing to each young worker, and $\beta/(1+\beta)$ is the fraction of income that is saved. The last term in (11) captures the fact that the economy's aggregate wage earnings are larger, by a factor $1/\ell_t$, than the total wage earnings of workers employed in manufacturing. The fact that $1/\ell_t > 1$ introduces a multiplier effect which evolves endogenously over time and makes structural change, income distribution and capital accumulation intimately linked to each other.

In the literature on income distribution and endogenous growth, Bertola (1996) and Uhlig and Yanagawa (1996) show that taxing the old and distributing the proceeds to wage earners may stimulate growth in one-sector OLG models. This result is based on accumulation laws that are similar to (11) but where the 'multiplier' factor is replaced by a 'policy' factor determined by exogenous fiscal instruments, which is larger than one when the redistributive policy raises the income share of young workers. In our two-sector model, a similar redistribution effect is *endogenously* determined by the demand of old agents for labor-intensive services, which modifies the income share captured by young workers; the strength of this effect, measured by $1/\ell_t$, is generally time-varying

because the allocation of labor is subject to structural change as the economy develops. By substituting the equilibrium level $\ell_t = \ell(\kappa_t)$ defined by (9) into equation (11), we obtain the autonomous equation

$$\frac{\kappa_{t+1}}{\kappa_t} = \frac{1}{1+n}(1-\alpha)A\frac{\beta}{1+\beta}\frac{1}{\ell(\kappa_t)}, \tag{12}$$

which fully describes the accumulation path and includes the feedback effects that $\kappa_t$ exerts on the labor share. The sign of such feedback effects is determined by the value of $\sigma$ that determines the sign of $\ell'_{\kappa_t}$: from (12) and (10), complementarity (substitutability) between goods and services accelerates (curbs) accumulation over time;

$$\frac{d(\kappa_{t+1}/\kappa_t)}{d\kappa_t} = -\frac{1}{1+n}(1-\alpha)A\frac{\beta}{1+\beta}\cdot\frac{\ell'_{\kappa_t}}{\ell(\kappa_t)^2}\begin{cases} >0 & \text{if } \sigma<1 \\ <0 & \text{if } \sigma>1 \end{cases}. \tag{13}$$

Result (13) establishes that capital accumulation does not proceed at a constant rate over time (unless in the special case $\sigma = 1$). Specifically, the accumulation process is self-reinforcing when $\sigma < 1$ and self-balancing when $\sigma > 1$. This dichotomy implies several possible scenarios of growth and structural change. Since $\sigma < 1$ can be argued to be the empirically most plausible case (Finkelstein et al. 2013), our discussion begins with the case of complementarity.

## 4.1 Complementarity, Sustained Growth and Traps

The accumulation law (12) admits the existence of an interior steady state $(\bar{\kappa}, \ell(\bar{\kappa}))$ in which capital per worker equals $\kappa_{t+1} = \kappa_t = \bar{\kappa}$ and the employment share of manufacturing $\ell_t = \ell(\kappa_t)$ equals

$$\ell(\bar{\kappa}) = \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}. \tag{14}$$

Since $1-\alpha < \ell_t < 1$ must hold in equilibrium, the interior steady state $(\bar{\kappa}, \ell(\bar{\kappa}))$ exists only if

$$1-\alpha < \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)} < 1. \tag{15}$$

Suppose that (15) holds and assume $\sigma < 1$. From (13), complementarity implies that the steady state $\bar{\kappa}$ is globally unstable. Consequently, depending on initial endowments, two scenarios may arise: the economy may undertake a permanent accumulation path, or remain trapped in a permanent decumulation path, as established in the next Proposition.

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**Proposition 1** *(Complementarity) Assume $\sigma < 1$. If (15) holds, the interior steady state $(\bar{\kappa}, \ell(\bar{\kappa}))$ acts as a separating threshold and determines two possible scenarios. First, if $\kappa_0 > \bar{\kappa}$, net accumulation per worker is positive and self-reinforcing: employment in services and the saving rate increase during the transition, capital per worker and the price of health care grow forever yielding sustained endogenous growth in the long run with*

$$\lim_{t \to \infty} \frac{\kappa_{t+1}}{\kappa_t} = \frac{A\beta}{(1+\beta)(1+n)} > 1 \ and \ \lim_{t \to \infty} \ell(\kappa_t) = 1 - \alpha. \tag{16}$$

*Second, if $\kappa_0 < \bar{\kappa}$, net accumulation per worker is persistently negative, with opposite transitional dynamics: in the long run, employment in services and capital per worker vanish asymptotically,*

$$\lim_{t \to \infty} \kappa_t = 0 \ and \ \lim_{t \to \infty} \ell(\kappa_t) = 1. \tag{17}$$

**Proof.** See Appendix. ∎

The reason for these results is that, under complementarity, capital accumulation induces positive feedback effects on savings via structural change and distributional shifts. The case with sustained endogenous growth in the long run – henceforth called 'Scenario 1' – is due to the fact that growing capital per worker drives up the health-care price and reduces the employment share of the manufacturing sector, increasing the income share of young workers and thereby savings, boosting subsequent capital accumulation. This equilibrium path is graphically described in Figure 1, diagram (c). Given $\kappa_0 > \bar{\kappa}$, the employment share of manufacturing is below the critical level, $\ell(\kappa_0) < \ell(\bar{\kappa})$, and positive net accumulation of capital per worker in the manufacturing sector drives manufacturing employment further down, feeding sustained growth in the long run.

Symmetrically, the case with permanent decumulation – henceforth called 'Scenario 2' – results from an initial decline in capital per worker that causes self-reinforcing feedback effects. When $\kappa_0 < \bar{\kappa}$, the employment share of manufacturing is above the critical level, $\ell(\kappa_0) > \ell(\bar{\kappa})$, and net accumulation per worker is strictly negative. The manufacturing sector experiences falling capital per worker while the service sector progressively disappears: wages and the relative price of services decline over time, and labor fully migrates to manufacturing in the long run.[3]

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[3] When (15) is violated, no interior steady state exists and two subcases may arise. When $1 - \alpha < 1 < \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}$, the economy exhibits endogenous growth as in Scenario 1 for any $\kappa_0 > 0$. When $\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)} <$

The results obtained under complementarity deserve two remarks. First, Scenario 1 predicts increasing accumulation rates in conjunction with the typical path of structural change observed in developing economies, e.g., the East Asian miracles: rising wages and service prices, growing employment in services, distributional shifts in favor of young workers, and increasing savings accumulation rates during the transition. Second, as shown in Figure 1, diagram (c), escaping the decumulation trap of Scenario 2 requires having a sufficiently large stock of aggregate capital at time zero so that manufacturing firms have sufficiently high capital per worker, the resulting equilibrium wage is sufficiently high, and the service sector displays a sufficiently *high* price and a sufficiently *high* employment share – that is, achieving sustained growth in *manufacturing* is associated with having a *large service sector* at time zero.

## 4.2 Substitutability and Bounded Accumulation

When $\sigma > 1$, the interior steady state $(\bar{\kappa}, \ell(\bar{\kappa}))$ is globally stable due to self-balancing accumulation: from (13), labor reallocations reduce $\kappa_{t+1}/\kappa_t$ over time until capital per worker and the manufacturing employment share reach the stationary levels $\bar{\kappa}$ and $\ell(\bar{\kappa})$.

**Proposition 2** *(Substitutability) Assume $\sigma > 1$. If (15) holds, the interior steady state $(\bar{\kappa}, \ell(\bar{\kappa}))$ is globally stable: if $\kappa_0 < \hat{\kappa}$ ($\kappa_0 > \hat{\kappa}$), the economy follows a self-balancing accumulation (decumulation) path during the transition, and converges from below (above) to the stationary long-run equilibrium featuring*

$$\lim_{t \to \infty} \kappa_t = \bar{\kappa} \qquad and \qquad \lim_{t \to \infty} \ell(\kappa_t) = \ell(\bar{\kappa}). \tag{18}$$

**Proof.** See Appendix. ∎

The main result established in Proposition 2 may be restated as follows: when manufacturing and services are substitutes, the existence of a steady-state level of capital per worker compatible with positive production in both sectors implies that the AK model behaves similarly to a neoclassical model. Starting from relatively low capital, capital per worker grows over time but at decreasing rates, until the economy reaches a stable steady state representing the long-run equilibrium. However, this result is not due to decreasing returns to capital in production: differently from the neoclassical model,

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$1 - \alpha < 1$, the economy is trapped in permanent decumulation as in Scenario 2 for any $\kappa_0 > 0$.

the convergence towards $(\bar{\kappa}, \ell(\bar{\kappa}))$ is determined by the reaction of sectoral employment shares to capital accumulation. Since capital growth increases employment in the generic sector, accumulation under substitutability is self-balancing. This conclusion is opposite to that obtained under complementarity, where accumulation is self-reinforcing. Figure 1, diagram (d), shows that the economy may approach the steady state from below or from above. If $\kappa_0 < \bar{\kappa}$, both capital per worker and manufacturing employment grow over time along the trajectory termed Scenario 3′. If $\kappa_0 > \bar{\kappa}$, instead, capital per worker and manufacturing employment decline along the trajectory termed Scenario 3″.[4]

The long-run predictions for the case of substitutability, summarized in expression (18), may resemble but do not match those of neoclassical models: accumulation is eventually bounded by preference-induced structural change, not by diminishing returns to capital. Scenario 3 thus signals that demand-side forces may impede sustained endogenous growth in the long run despite constant returns to capital in the manufacturing sector.[5] This conclusion has links with recent findings of the literature on endogenous growth and resource scarcity, which shows that different degrees of substitutability between natural and man-made inputs alters the stability properties of steady states (Peretto, 2012) and that poor substitutability may promote sectoral change and enhance investment activities (Bretschger and Smulders, 2012).

## 5  Conclusion

Age-dependent preferences can play a critical role in shaping the interactions between economic growth and structural change because they create explicit causal links between changes in sectoral employment levels and accumulation rates. On the one hand, age-dependent needs create feedbacks from capital accumulation – which increases wages and

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[4]When (15) is violated, no interior steady state exists and two subcases may arise. When $1 - \alpha < 1 < \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}$, the equilibrium path is similar to Scenario 3′ but the service sector disappears because the manufacturing employment share approaches $\lim_{t\to\infty} \ell(\kappa_t) = 1$. When $\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)} < 1 - \alpha < 1$, the equilibrium path is similar to Scenario 3″ but the manufacturing employment share eventually reaches the lower bound $\lim_{t\to\infty} \ell(\kappa_t) = 1 - \alpha$.

[5]The manufacturing sector in our model satisfies the standard conditions that, in the absence of the service sector, would guarantee persistent endogenous growth in one-sector OLG economies: see Tvede (2010: Ch.8).

thereby the relative price of labor-intensive services consumed by the old – to structural change. On the other hand, the same changes in the relative prices of capital and labor inputs modify savings via changes in the intergenerational distribution of income, and savings in turn affect economic growth by changing the speed of accumulation. Our model shows that the feedback effects originating in age-dependent preferences determine growth prospects, with three possible outcomes in the long run, as well as the qualitative dynamics of sectoral employment and relative prices. In particular, the scenario with sustained endogenous growth in the long run exhibits the typical path of structural change observed in developing countries coupled with the phenomenon of rising accumulation rates during the transition. More generally, our results suggest that preferences and the demographic structure are key ingredients to build a theory of circular causality in which economic growth is both a cause and a consequence of structural change. Investigating the role played by age-dependent needs and intergenerational income shares in driving the process of economic development is our main suggestion for future research.

## Appendix

**Derivation of (7)**. Aggregating the profit-maximizing conditions $\partial X_t^j / \partial \ell_t^j = w_t N_t^y$ and $\partial X_t^j / \partial k_t^j = R_t$ across firms and substituting $a_t = A^{\frac{1}{1-\alpha}} (\kappa_t / \ell_t)$ gives

$$w_t = a_t^{1-\alpha} (1-\alpha) (\kappa_t / \ell_t)^\alpha = A (1-\alpha) (\kappa_t / \ell_t), \qquad (19)$$

$$R_t = a_t^{1-\alpha} \alpha (\ell_t / \kappa_t)^{1-\alpha} = \alpha A. \qquad (20)$$

Combining (6) with (19) gives (7).

**Derivation of (8)**. Maximizing (1) subject to (2) yields

$$s_t = \frac{\beta}{1+\beta} w_t, \qquad (21)$$

$$p_{t+1} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\sigma-1}} \cdot \left(\frac{p_{t+1} h_{t+1}}{d_{t+1}}\right)^{\frac{1}{1-\sigma}}. \qquad (22)$$

From $H_t = N_t^o h_t$ and (5)-(6),

$$p_{t+1} h_{t+1} = w_{t+1} (1 - \ell_{t+1}) (1+n). \qquad (23)$$

From (2), (23) and $K_{t+1} = N_t^y s_t$,

$$d_{t+1} = s_t R_{t+1} - p_{t+1} h_{t+1} = (1+n) \cdot [\kappa_{t+1} R_{t+1} - w_{t+1} (1 - \ell_{t+1})]. \qquad (24)$$

Substituting (19)-(20) in (24) gives

$$d_{t+1} = (1+n)\, A \cdot (\kappa_{t+1}/\ell_{t+1}) \cdot [\ell_{t+1} - (1-\alpha)]\,, \tag{25}$$

which implies that $d_{t+1} > 0$ requires $\ell_{t+1} > 1-\alpha$. Inserting (25) and (23) into (22), and using (19)-(20), yields (8) at time $t+1$.

**Existence and uniqueness of the fixed point (9).** From (7) and (8), the loci $\Phi$ and $\Psi$ exhibit

$$\lim_{\ell_t \to 1-\alpha} \Phi = (A/\eta)\,\kappa_t \quad \text{and} \quad \lim_{\ell_t \to 1} \Phi = (A/\eta)\,(1-\alpha)\,\kappa_t,$$

$$\lim_{\ell_t \to 1-\alpha} \Psi = \infty \qquad \text{and} \quad \lim_{\ell_t \to 1} \Psi = 0 \qquad \text{if } \sigma < 1,$$

$$\lim_{\ell_t \to 1-\alpha} \Psi = 0 \qquad \text{and} \quad \lim_{\ell_t \to 1} \Psi = \infty \qquad \text{if } \sigma > 1.$$

When $\sigma > 1$, existence and uniqueness of (9) follow from combining the above limits with $\partial\Phi/\partial\ell_t < 0$ and $\partial\Psi/\partial\ell_t > 0$. When $\sigma < 1$, we have $\partial\Phi/\partial\ell_t < 0$ and $\partial\Psi/\partial\ell_t < 0$, and the above limits combined with the elasticities

$$\frac{\partial\Phi/\partial\ell_t}{\Phi/\ell_t} = -1 \qquad \text{and} \qquad \frac{\partial\Psi/\partial\ell_t}{\Psi/\ell_t} = -\frac{1}{1-\sigma} \cdot \frac{\alpha}{1-\ell_t} \cdot \frac{\ell_t}{\ell_t - (1-\alpha)} > 1$$

guarantee existence of a unique intersection $\Phi = \Psi$ where $\Psi$ cuts $\Phi$ from above (cf. Figure 1(a)).

**Derivation of (11).** Substituting (21) in $K_{t+1} = N_t^y s_t$, and using (19), yields (11).

**Proof of Proposition 1.** Assuming that (15) holds, equation (12) implies that there exists an interior steady state $(\bar{\kappa}, \ell(\bar{\kappa}))$ characterized by $\kappa_t = \bar{\kappa}$ and $\ell(\kappa_t) = \ell(\bar{\kappa})$ satisfying (14) and the feasibility condition $1-\alpha < \ell(\bar{\kappa}) < 1$. From (13), complementarity implies that $\frac{\mathrm{d}(\kappa_{t+1}/\kappa_t)}{\mathrm{d}\kappa_t} > 0$, so that the steady state $\kappa_t = \bar{\kappa}$ is dynamically unstable. First, consider the case $\kappa_0 > \bar{\kappa}$. From (10), having $\kappa_0 > \bar{\kappa}$ under complementarity implies an employment share of manufacturing strictly below the critical level associated with the steady state, $\ell(\kappa_0) < \ell(\bar{\kappa})$. This in turn implies $\kappa_1/\kappa_0 > 1$ from (12). In the subsequent period, $\kappa_1 > \kappa_0$ drives the employment share of manufacturing further down, $\ell(\kappa_1) < \ell(\kappa_0)$, implying again positive growth in $\kappa_t$ in all subsequent periods $t \geqslant 2$. In the limit as $t \to \infty$, equation (12) and the definition of equilibrium employment share in (9) imply, respectively, the two asymptotic results reported in expression (16). Considering the opposite case $\kappa_0 < \bar{\kappa}$, all the above mechanisms operate in reverse, implying the asymptotic results reported in expression (17). ∎

**Proof of Proposition 2**. Assuming that (15) holds, equation (12) implies that there exists an interior steady state $(\bar{\kappa}, \ell(\bar{\kappa}))$ characterized by $\kappa_t = \bar{\kappa}$ and $\ell(\kappa_t) = \ell(\bar{\kappa})$ satisfying (14) and the feasibility condition $1 - \alpha < \ell(\bar{\kappa}) < 1$. From (13), substitutability implies that $\frac{\mathrm{d}(\kappa_{t+1}/\kappa_t)}{\mathrm{d}\kappa_t} < 0$, so that the steady state $\kappa_t = \bar{\kappa}$ is dynamically stable. Therefore, the asymptotic results reported in expression (18) hold for any $\kappa_0 \gtreqless \bar{\kappa}$. If $\kappa_0 < \bar{\kappa}$, capital per worker grows over time but at vanishing rates in view of (12) and (13), and the same qualitative path is follows by the employment share of manufacturing in view of (10) under substitutability. For the same reasons, if $\kappa_0 > \bar{\kappa}$, both capital per worker and the employment share of manufacturing decline over time but at vanishing rates, until $\kappa_t = \bar{\kappa}$ eventually holds asymptotically. ∎

**Note on non-interior steady states**. The scenarios arising when (15) is violated can be verified by Figures 1(c)-1(d). If $1 - \alpha < 1 < \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}$, the critical share $\ell(\bar{\kappa})$ is pushed above the unity upper-bound; if $\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)} < 1 - \alpha < 1$, the critical share $\ell(\bar{\kappa})$ is pushed below the lower-bound $(1 - \alpha)$.

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