An equilibrium model estimated on pharmaceutical data¹.
by
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Abstract
The purpose of this paper is to estimate patients/doctors response to prices when making a choice between a brand name product and its generics. We account for how pharmacies respond to government regulation and to prices set by brand name producers. Data is unique in the sense that we observe prices set by pharmacies as well as by producers. Our results confirm that estimating only the demand side yields biased estimates of consumers’ price responses. We find much stronger price responses when demand and supply are jointly estimated.

Keywords: pharmaceuticals, discrete choice model, market equilibrium

¹ We gratefully acknowledge support from the Norwegian Research Council. We thank John K. Dagsvik and two anonymous reviewers for helpful suggestions and comments. Financial support from The Frisch Centre is gratefully acknowledged.

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Introduction

One important problem with estimating the demand for pharmaceuticals on microdata is that quality attributes of the product, observed and unobserved, may be correlated with the price. The reason why is that these quality attributes are most likely priced out in the markets by the producers. Since the papers by Berry (1994) and Berry et al (1995) the standard method in estimating demand on microdata has been to apply an algorithm proposed in these two papers. Basically, the endogeneity problem is controlled for using instrument variables. However, identifying suitable instrument is not an easy task. Recent examples in this tradition and related to demand for pharmaceuticals are Arcidiacono et al (2013) and Granlund (2010). More recent papers using demand data only to estimate how prices affect demand are Yeung et al (2016) who use natural experiment data and Skipper and Vejlin (2015) who focus on how prices affect the choice between generics and brand-name drugs.

Below we estimate patients/ doctors responses to prices when making a choice between a brand-name product and its generics. We take into account how pharmacies respond to government regulation and to prices set by brand-name producers. In contrast to the studies referred to above, we use aggregate data which means that demand is represented by market shares derived from discrete choices in micro, with prices of all relevant pharmaceutical products as explanatory variables. But also when using aggregate data we are facing endogeneity problems. If we do a regression of demand against prices, we ignore that prices may depend on demand. A straightforward demand analysis may thus bias our estimates of price responses.

Instead of applying an instrumental variable approach, we estimate jointly the demand side and the price equations following from the prices setting of producers of brand-name products and pharmacies, given the government regulation of brand-name retail prices. In the modelling of the price setting among brand-name producers, we assume that they take into account the responses by pharmacies with respect to their sales and price setting of generics versus brand-name products. We thus estimate an oligopoly model where market shares are derived from underlying discrete choice demand. Example of this theoretical construct is discussed in Anderson et al (1992) and in Schweitzer and Lu (2018) who are using the analytical framework of industrial organization in the analysis of the pharmaceutical markets.
The econometric approach to estimate equilibrium models was first presented in Haavelmo (1943, 1944), for which he was awarded the Nobel Prize in economics in 1989. A recent example of estimating an equilibrium model on pharmaceutical data is Daalen et al (2014). The new aspect in our paper is that we model the pricing behaviour of the producers of pharmaceuticals, not only the pricing behaviour of pharmacies. To include this pricing behaviour in the model requires that we observe the prices set by manufacturers.

The two drugs we are analysing are two best-sellers in the Norwegian market: An antidepressant drug, Paroxetine. This drug is used to treat major depression, obsessive-compulsive disorder, panic disorder, social anxiety, post-traumatic stress and generalized anxiety disorder in adult outpatients. The other drug is an anti-hypertensive used in the treatment of angina pectoris.

It should be noted that since 2001 the pharmacies in Norway are almost entirely owned by international firms that also are wholesalers. In the period analysed here, there were three pharmacy chains in Norway. The regulatory authority related to the pharmaceutical sector in Norway is the Norwegian Ministry of Health and Social Affairs. The Ministry, and its agency (Norwegian Medicines Agency), control the entry of new drugs, the wholesale prices, and the retail margins. The manufacturer price is not regulated, see Brekke et al (2012) for details of the markets structure and regulation in Nordic countries, and Vogler (2012) for an overview of pharmaceutical pricing and market regulation in 29 European countries, including Norway.

During the last decades there have been several policy initiatives by the Norwegian government to foster competition after patent expiration. From 1987 doctors were encouraged to prescribe the cheapest of the available versions of the drug. In 1991 this light-handed regulation was replaced by a law that instructed doctors to prescribe the cheapest available generic drug. Doctors could still prescribe a more expensive brand-name version, as long as a medical reason for this could be provided. In this period, generic competition was entirely based on the prescription-choice of the doctor. The pharmacy was required to dispense the exact product name written on the prescription. This changed in March 2001 when pharmacies were allowed to substitute a branded drug for a generic, independent of the product name prescribed by the doctor. Being permitted to intervene between the physician and the patient, the pharmacies now got an active role in the market for generics. The doctor can still guard against substitution, but this requires an explicit reservation to be added to the prescription note (“active substitution method”).
In Norway, the physicians objected substitution on 5.2 per cent of the prescriptions in 2005, and on 4.5 per cent in 2006, Brekke et al (2012). Even without such a reservation by the physician, the patient may insist on the branded drug, in which case the pharmacy is obligated to hand out the brand-name drug. In this case, the insurance scheme does not cover the price difference between the branded drug and the reference price. The difference has to be paid by the patient himself. In 2005, the patients refused to substitute on 4.0 per cent of the prescriptions (4.3 per cent in 2006). These come in addition to the reservations made by the physicians, bringing total number refusals to substitutes close to an average of 10 per cent of all prescriptions. The two drugs we analyse here are all approved for reimbursement for part of the expenses by the social insurance scheme. For both drugs patents have expired. Thus patients/doctor can freely choose between brand-name drugs and generics.

In the period consider here, there was a price cap on brand name products. Under this regulatory scheme, the regulator sets a maximum price level defined by the lowest observed prices in a selection of European countries. This price cap is first set when the brand-name drug enters the market. After patent expiration, generic drugs are given the exact same price cap, and this cap will only fall if generic competition triggers price reductions in the reference countries. However, competition from generics (made possible by generic substitution) was supposed to lower prices below the price cap.

In most studies of pharmaceuticals, information about prices at the different levels of the market has been lacking. Examples are Coscelli (2000) who was able to reveal the habit behaviour claimed by Hellerstein (1998). Lundin (2000) had access to retail prices (only) and found support for habit persistence among doctors and patients, but the results indicated that these are affected by the price differences, especially the share of price differences covered by the patient. If the price differences between generic and brand-name increases, the doctor becomes more inclined to prescribe a generic version.

The data we use here are quarterly register data for the period 2004 -2008. We have access to unique price data that give the prices set by the brand-name producers as well as by the retailers/wholesalers. Our results indeed clearly indicate that if only the demand side is estimated, the estimates of price responses are biased. It should be noted that what we analyse here are products which from a medical point of view are perfect substitutes. The chemical substance in
the brand-name products and the generics are identical. We should thus expect the demand elasticities to be numerically high.

**Demand and pricesetting**

The pharmaceuticals can be specified according to chemical substance. One specific substance, identified by atc code (Anatomical Therapeutic Chemical Classification System), is one market. In each market the patient/doctors can choose between the brand-name product and generics and between three pharmacy chains. The model deals therefore with the choice of brand-name products vs generic substitution.

**The demand side**

**Choice of drugs, given the chain**

Let $U_{ncd}$ be the utility for patient/doctor $n$ of using drug $d$ bought in retailer chain $c$, where $d=B,G$ and $c=1,2,3$. $B$ stands for brand-name product and $G$ for generics, of which there can be many different drugs but with the same chemical substance. Let $P_{cd}$ be the price of the drug $d$ in retailer chain $c$. We will assume that

$$U_{ncd} = a_{cd} + bP_{cd} + \varepsilon_{ncd}$$

Here $a_{cd}$ and $b$ ($<0$) are constants. $\varepsilon_{ncd}$ is assumed to be extreme value distributed with zero expectation and unit variance. The latter means that the coefficients $a_{cd}$ and $b$ are scaled with the standard deviation of the extreme value distributed taste shifter. For each chain we get the following choice probabilities, denoted $Y_{cd}$.

$$Y_{cd} = \frac{\exp(v_{cd})}{\sum_{s=B,G} \exp(v_{cs})}; c = 1,2,3; d = B,G$$

where

$$v_{cd} = a_{cd} + bP_{cd}$$

We note that the deterministic part of the random utility function in (1), $V_{cd}$, does not depend on individual characteristics.
The choice of chain

Let $Z_c$ denote the probability that the individuals choose chain $c$, $c=1,2,3$, and it is given by

$$Z_c = \frac{\exp(I_c)}{\sum_{j=1}^{3} \exp(I_j) \sum_{r=1}^{3} \sum_{j=1}^{3} \exp(v_{rj})}; \quad c = 1, 2, 3$$

$$I_c = \ln \left( \sum_{j=1}^{3} \exp(v_{cj}) \right)$$

$I_c$ is the expected value of the maximum of utility related to the choice of drugs in chain $c$, also called inclusive value, see Train (2009). $Z_c$ is thus the ratio of the expected value of the maximum utility of choosing drugs in chain $c$ to the sum of the expected value of maximum utility across the three chains.

The market shares for drugs

The unconditional probability of choosing a generic or a brand-name drug is then given by the product of $Y_{cd}$ and $Z_c$, which here will be denoted $X_{cd}$:

$$X_{cd} = Y_{cd}Z_c = \frac{\exp(v_{cd})}{\sum_{r=1}^{3} \sum_{j=1}^{3} \exp(v_{rj})}; \quad c = 1, 2, 3, d = B, G$$

Note that when the agents choose between generics/brand and chains, $X_{cG} + X_{cB}$ is not equal to one, but $\sum_{r=1}^{3} \sum_{j=1}^{3} X_{rj} = 1$.

The simplification that the deterministic parts of the utility function in (1) do not depend on individual characteristics implies that empirical parallels to the aggregate demand probabilities in (6) are market shares. Thus, $X_{cd}$ is the share chain $c$ has in market for drug $d$. In the empirical part we will come back to how we deal with the heterogeneity in the market share equations.
The supply side: A non-cooperative game

There are three stages in this game. In the first stage the brand-name producer sets the price. In doing so, he takes into account the demand structure and the price setting of the generic producers and the retailers. In the second stage the generic producers set their prices and in third stage the retailer set his prices, given the prices set by the generic producers and brand-name producer (see Figure 1).

Figure 1. Stage representation of the game

As common in these games, we start backwards. The model we employ combines a demand model derived from logit probabilities and monopolistic price setting, see Anderson et al (1992, chapter 7, for a theoretical outline of similar models.

Pricing decisions of three retailer/wholesalers chains

For expository reason we specify one supplier of generics. The expected profit of the chain \( c \) is given by

\[
\pi_c = (P_{cb} - q_{cb})X_{cb} + (P_{cG} - q_{cG})X_{cG} \quad c = 1,2,3
\]

As mentioned above, in the Norwegian market the retailer and the wholesaler are vertically integrated. Here \( q_{cb} \) is what retailer \( c \) thus has to pay the producer for the brand-name product, while \( q_{cG} \) is what he has to pay the producer for the generic product.
In the expression above we have set the number of potential users equal to 1, which is a normalisation without any implications for the results. Thus, the two market shares given in eq. (6), \( X_cB \) and \( X_cG \), equal the expected volume of brand-name and generics sold by chain \( c \). In Norway brand-name prices are regulated with a price-cap:

(8) \[ P_{cB} = \bar{P}_{cB} \]

In accordance with the situation in the Norwegian market, we assume that the price cap is binding.

We assume that the retailer/wholesaler set a price of the generics that maximize profits. Maximizing expected profit with respect to the price of generics yields the following first order condition:

(9) \[ \frac{\partial \pi_c}{\partial P_{cG}} = (\bar{P}_{cB} - q_{cB}) \frac{\partial X_{cB}}{\partial P_{cG}} + X_{cG} + (P_{cG} - q_{cG}) \frac{\partial X_{cG}}{\partial P_{cG}} = 0 \quad \text{for } c = 1, 2, 3 \]

Given the structure of the market shares that we assume that the retailer knows, we then get the following price setting of generics:

(10) \[ P_{cG} = q_{cG} + \frac{1}{(b)(1-X_{cG})} + \frac{X_{cB}}{1-X_{cG}} (P_{cB} - q_{cB}); \quad c = 1, 2, 3 \]

The first element on the right hand side, \( q_{cG} \), is the direct cost to the retailer chain \( c \) of buying generics. The second element is the standard mark-up in these types of models (see Anderson et al (1992)), while the third element captures the opportunity cost related to the fact that the retailer can sell brand-name products instead of generics.

**Pricing decisions of the generic producers**

Without introducing too strong assumptions, we assume that the prices of generics are set equal to marginal cost, \( k_G \). The marginal cost is not observed but will be estimated together with the other unknown parameters of the model.

(11) \[ q_{cG} = k_G \]

**Pricing decision of the brand producer**

The expected profit of the brand producer is given by

(12) \[ \pi_B = \sum_{c=1,2,3} (q_{cB} - k_B) X_{cB} \]
In maximizing expected profit with respect to the price, $q_{cB}$, the brand-name producer takes into account how the retailers set their price of generics in response to an increase in the producers’ price of brand-name product. According to (6), $X_{cB}$ depends on $P_{cG}$. From (6) and (10), and after some time-consuming but straightforward calculations, we find that

$$\frac{\partial P_{cG}}{\partial q_{cB}} = -X_{cB} < 0$$

Thus, if the brand-name producer increases his price, the retailer lowers the price of generics in order to shift the sale in pharmacies towards more generics. The marginal cost is $k_B$. The first order condition becomes:

$$q_{cB} = k_B + \frac{1}{(-b)X_{cG}X_{cB}}$$

Most likely, the marginal cost of producing the drug is low and equal across generics and brand-name producers. Hence, we set $k_G=k_B=k$.

3. The econometric model

To take the market shares and the pricing equations to data, requires that we specify how unobserved variables are coming into the model. First, we let the brand-name product sold by chain no 1 to be the reference case. Dividing through the markets shares in (6) by $X_{1B}$, we get

$$\begin{align*}
\frac{X_{cB}}{X_{1B}} &= \frac{\exp(v_{cB})}{\exp(v_{1B})} = \exp(v_{cB} - v_{1B}); c = 2, 3 \\
\frac{X_{cG}}{X_{1B}} &= \frac{\exp(v_{cG})}{\exp(v_{1B})} = \exp(v_{cG} - v_{1B}); c = 1, 2, 3,
\end{align*}$$

Taking logs and applying (3) we get

$$\begin{align*}
\ln \frac{X_{cB}}{X_{1B}} &= (a_{cB} - a_{1B}) + b(P_{cB} - \bar{P}_{1B}); c = 2, 3 \\
\ln \frac{X_{cG}}{X_{1B}} &= (a_{cG} - a_{1B}) + b(P_{cG} - \bar{P}_{1B}); c = 1, 2, 3
\end{align*}$$

The term to the left, the log of the relative market shares, is the log-odds ratio, with brand-name drug sold by chain 1 as the reference case. We observe from $\sum_{r=1}^{3} \sum_{j=B,G} X_{rj} = 1$ that

$$X_{1B} = 1 - \sum_{r=2,3} X_{cB} - \sum_{r=1,2,3} X_{rG}$$
We add a stochastic term to the log-odds ratios, denoted $\varepsilon_{cd}$. The justification for this is that there may be unobserved variables affecting the choices. With a reference to the central limit theorem we assume that these stochastic terms are normally distributed with zero expectation and standard deviation $\sigma_{cd}$.

The demand model with random parts added is

\[
\ln \frac{X_{cb}}{X_{1b}} = (a_{cb} - a_{1b}) + b(P_{cb} - \bar{P}_{1b}) + \varepsilon_{cb}; c = 2, 3
\]

\[
\ln \frac{X_{cg}}{X_{1b}} = (a_{cg} - a_{1b}) + b(P_{cg} - \bar{P}_{1b}) + \varepsilon_{cg}; c = 1, 2, 3
\]

(18)

Altogether there are two equations covering the log-odds ratio for brand-name product, i.e. sold by chain 2 and 3 and three equations for the log-odds ratios of generics, i.e. sold by all three chains.

Now let

\[
\begin{align*}
D_{cbt} &= \ln \frac{X_{cbt}}{X_{1bt}} - \alpha_{cb} - b(P_{cbt} - \bar{P}_{1bt}) = \varepsilon_{cbt}; c = 2, 3, t = 1, 2, \ldots, T \\
D_{cg} &= \ln \frac{X_{cgt}}{X_{1bt}} - \alpha_{cg} - b(P_{cgt} - \bar{P}_{1bt}) = \varepsilon_{cgt}; c = 1, 2, 3, t = 1, 2, \ldots, T
\end{align*}
\]

(19)

Here we have added the subscript $t$. The model will be estimated on quarterly data and with a total of $T$ quarters. Moreover, $\alpha_{cd} = a_{cd} - a_{1B}$. Note that $D_{cbt}$ depends on observed markets shares and observed prices, as well as on the unknown coefficients $\alpha_{cd}$ and $b$. The latter coefficient captures the effect of price on demand for drugs. Now let

\[
\begin{align*}
D_{cbt}(X_{cbt}, X_{1bt}, (P_{cbt} - \bar{P}_{1bt}); \alpha_{cb}, b) &= \ln \frac{X_{cbt}}{X_{1bt}} - \alpha_{cb} - b(P_{cbt} - \bar{P}_{1bt}), c = 2, 3, t = 1, 2, \ldots, T \\
D_{cgt}(X_{cgt}, X_{1bt}, (P_{cgt} - \bar{P}_{1bt}); \alpha_{cG}, b) &= \ln \frac{X_{cgt}}{X_{1bt}} - \alpha_{cG} - b(P_{cgt} - \bar{P}_{1bt}), c = 1, 2, 3, t = 1, 2, \ldots, T
\end{align*}
\]

(20)

Dividing through (19) with the standard deviation $\sigma_{cd}$, using (20), we get

\[
\begin{align*}
\frac{D_{cbt}(X_{cbt}, X_{1bt}, (P_{cbt} - \bar{P}_{1bt}); \alpha_{cb}, b)}{\sigma_{cd}} &= \frac{\varepsilon_{cbt}}{\sigma_{cd}}; c = 2, 3, t = 1, 2, \ldots, T \\
\frac{D_{cgt}(X_{cgt}, X_{1bt}, (P_{cgt} - \bar{P}_{1bt}); \alpha_{cG}, b)}{\sigma_{cd}} &= \frac{\varepsilon_{cgt}}{\sigma_{cd}}; c = 1, 2, 3, t = 1, 2, \ldots, T
\end{align*}
\]

(21)
The random term to the right is then a standard normal distributed random term with zero expectation and unit variance. Let \( f(.) \) be the standard normal density function. For a given drug, we then have the following joint probability function for the sample we observe, based on demand data only:

\[
(22) L_D = \prod_{c=1}^{C} \prod_{t=1}^{T} \frac{1}{\sigma_{cb}} f \left( \frac{D_{cb}(X_{cb}, X_{1b}, (P_{cb} - \bar{P}_{cb}); \alpha_{cb}, b)}{\sigma_{cb}} \right) \prod_{c=1}^{C} \frac{1}{\sigma_{cg}} f \left( \frac{D_{cg}(X_{cg}, X_{1b}, (P_{cg} - \bar{P}_{1b}); \alpha_{cg}, b)}{\sigma_{cg}} \right)
\]

\( L_D \) is the likelihood for the sample (of market shares) we observe and we estimate the unknown coefficients \((\alpha_{cd}, b, \sigma_{cd})\) by maximizing this likelihood, or rather the log of this likelihood, with respect to the unknown coefficients. In doing so, we let our theory has the highest possible chance to explain data. This approach we call demand side estimates, DSE.

We now turn to the specification of the econometric model when the demand side and pricing behaviour of retailer/wholesaler and producers of drugs are jointly estimated. Adding error terms to the pricing equations, and applying (11), we get

\[
(23) p_{cGt} = k + \frac{1}{(1-X_{cGt})} + \frac{X_{cb}}{1-X_{cGt}}(P_{cb} - q_{cb}) + \eta_{cGt}; c = 1,2,3, t = 1,2,..,T
\]

Similar to above, we assume \( \eta_{cGt} \) to be normally distributed with zero expectation and standard deviation \( \tau_{cG} \). Now let

\[
(24) s_{cG}(X_{cb}, X_{cg}, P_{cg}, q_{cb}; k, b) = P_{cGt} - k - \frac{1}{(1-X_{cGt})} - \frac{X_{cb}}{1-X_{cGt}}(P_{cb} - q_{cb}); c = 1,2,3, t = 1,2,..,T
\]

From (23) and (24), and dividing through with \( \tau_{cG} \), we get,

\[
(25) \frac{s_{cG}(X_{cb}, X_{cg}, P_{cg}, q_{cb}; k, b)}{\tau_{cG}} = \frac{\eta_{cGt}}{\tau_{cG}}; c = 1,2,3; t = 1,2,..,T
\]

For the prices set by the producers which chain \( c \) has to pay, we have

\[
(26) q_{cbt} = k + \frac{1}{(P_{cGt}X_{cb})} + \eta_{cbt}
\]

Again, we let

\[
(27) s_{cb}(X_{cGt}, X_{cbt}, q_{cbt}; k, b) = q_{cbt} - k - \frac{1}{(P_{cGt}X_{cb})}; c = 1,2,3; t = 1,2,..,T
\]

Then from (26) and (27) we get
Let $LS$ denote the joint probability covering the pricing decisions:

$$L_s = \prod_{c=1}^{3} \prod_{t=1}^{T} \frac{1}{\tau_{ct}} f \left( \frac{S_{ct}(X_{ct}, X_{ct}, P_{ct}, q_{ct}, k, b)}{\tau_{ct}} \right)$$

Altogether we have 5 demand equations (in terms of market shares), 3 equations for the prices of generics set by the pharmacies and 3 equation for the prices on brand-name products set by the producers. We thus have a total of 11 equations. Note that demand depend on all prices in pharmacies, on brand-name products as well as on generics. Prices of generics set by pharmacies depend on the price-caps on brand and the producer prices of brand-name products.

The joint probability covering the demand and pricing decisions observed in our sample is given by the product $LDL_s$. Before we proceed to the empirical part of the paper we have to do one more thing.

At any point in time the observed dependent variables are

$$(X_{2Bt}, X_{3Bt}, X_{1Gt}, X_{2Gt}, X_{3Gt}, P_{1Gt}, P_{2Gt}, P_{3Gt}, q_{1Bt}, q_{2Bt}, q_{3Bt})$$.

The market share for $X_{1Bt}$ follows from eq. (17). Because there are the following 11 unobserved normally distributed random variables in the model,

$$(\varepsilon_{2Bt}, \varepsilon_{3Bt}, \varepsilon_{1Gt}, \varepsilon_{2Gt}, \varepsilon_{3Gt}, \eta_{1Gt}, \eta_{2Gt}, \eta_{3Gt}, \eta_{1Bt}, \eta_{2Bt}, \eta_{3Bt})$$

the observed dependent variables will also be random and normally distributed variables. What is not so common in econometrics, but common in statistics and also discussed at length in Haavelmo (1944), is to apply a Jacobian transformation. This transformation means that we make a bridge from the distribution of the unobserved random variables present in the model to the distribution of the observed dependent variables.

To clarify what this transformation is, we give an example. Let $F(\varepsilon^*)$ be the cumulative distribution for the random variable $\varepsilon$, evaluated in the point $\varepsilon^*$ and let $\varepsilon = g(\alpha)$. Then taking the differentials we get $dF(\varepsilon^*) = f(g(\alpha^*))g'(\alpha^*)d\alpha$. Here $f(.)$ is the derivative of $F(.)$, i.e. the probability density function, and $g'(.)$ is the derivative of $g(.)$.

In our case we have 11 equations. In each equation we have one unobserved random variables and a varying number of observed dependent variables. For example, in equation number 1 we have
\[ \varepsilon_{2Br} = \ln \frac{X_{2Br}}{X_{1Br}} - \alpha_{cb} - b(P_{2Br} - \overline{P}_{2Br}) = \ln X_{2Br} - \ln(1 - \sum_{r=2,3} X_{rBr} - \sum_{r=1,2,3} X_{rG}) - \alpha_{cb} - b(P_{2Br} - \overline{P}_{2Br}) \]

Taking the derivatives wrt to all 11 observed variables, starting with \( X_{2Br} \) and ending with \( q_{3Br} \), we get the following line of derivatives:

\[
X_{1o}^{-1} + X_{2o}^{-1}, X_{1o}^{-1}, X_{1o}^{-1}, X_{1o}^{-1}, X_{1o}^{-1}, 0, 0, 0, 0, 0.
\]

Now let, \( J_t \) denote the matrix of the derivatives of all 11 equation, and let \( |J_t| \) denote the numerical value of the determinant, called the Jacobian determinant.

\[
\left( \begin{array}{c}
0, 0, -b^{-1}(1 - X_{1o})^2(\overline{P}_{1o} - q_{1o}), 0, 0, 1, 0, 0, -b^{-1}(1 - X_{1o})^2, 0, 0 \\
0, 0, 0, -b^{-1}(1 - X_{2o})^2(\overline{P}_{2o} - q_{2o}), 0, 0, 1, 0, 0, -b^{-1}(1 - X_{2o})^2, 0, 0 \\
0, 0, 0, 0, -b^{-1}(1 - X_{3o})^2(\overline{P}_{3o} - q_{3o}), 0, 0, 1, 0, 0, -b^{-1}(1 - X_{3o})^2, 0, 0 \\
b^{-1}X_{1o}^{-1}X_{1o}^{2}X_{2o}^{2}X_{3o}^{-1}X_{h1o}(X_{h1o}^{2} - X_{h1o}^{2}), b^{-1}X_{1o}^{-1}X_{1o}^{2}X_{2o}^{2}X_{3o}^{-1}X_{h1o}(X_{h1o}^{2} - X_{h1o}^{2}), b^{-1}X_{1o}^{-1}X_{1o}^{2}X_{2o}^{2}X_{3o}^{-1}X_{h1o}(X_{h1o}^{2} - X_{h1o}^{2}), 0, 0, 1, 0, 0 \\
b^{-1}X_{1o}^{-1}X_{1o}^{2}X_{2o}^{2}X_{3o}^{-1}X_{h1o}(X_{h1o}^{2} - X_{h1o}^{2}), b^{-1}X_{1o}^{-1}X_{1o}^{2}X_{2o}^{2}X_{3o}^{-1}X_{h1o}(X_{h1o}^{2} - X_{h1o}^{2}), b^{-1}X_{1o}^{-1}X_{1o}^{2}X_{2o}^{2}X_{3o}^{-1}X_{h1o}(X_{h1o}^{2} - X_{h1o}^{2}), 0, 0, 0, 1, 0 \\
-0, -b^{-1}X_{1o}^{-1}X_{1o}^{2}X_{2o}^{2}X_{3o}^{-1}X_{h1o}(X_{h1o}^{2} - X_{h1o}^{2}), 0, 0, 0, 0, 0, 1
\end{array} \right)
\]

From an econometric point of view, the importance of accounting for the Jacobian transformation is that \( |J_t| \) may depend on the coefficients that we like to estimate. We observe that \( |J_t| \) depends on the coefficient \( b \). Without taking into account the Jacobian transformation when estimating the model, we may get a biased estimated of \( b \). The expanded likelihood we will use in estimating the unknown coefficients of our model is therefore

\[
L(\theta; X_t, P_t, q_t) = L_0 L_1 |J_t|
\]

where

\[
\theta = \{ b, \alpha_{2B}, \alpha_{3B}, \alpha_{1G}, \alpha_{2G}, \alpha_{3G}, \sigma_{2B}, \sigma_{3B}, \sigma_{1G}, \sigma_{2G}, \sigma_{3G}, \tau_{2B}, \tau_{3B}, \tau_{1G}, \tau_{2G}, \tau_{3G} \}
\]

is the vector of 13 unknown coefficients that we like to estimate and \( X_t, P_t, q_t \) are vector of our observed variables at
each point in time. The functions $L_D, L_S$ and their dependency on coefficients and observed variables are given in Eqs. (22), (29) and (30). This approach to estimate price response we call market equilibrium estimates, MEE.

As mentioned above the models outlined above will be estimated on aggregated quarterly data. First we estimate the demand side only, DSE estimates. Second we estimate jointly the demand side and the pricing equations (i.e. the equilibrium model, MEE), and accounting for Jacobian transformation which gives the transformation from the distribution of the unobserved random variables to the distribution of the observed endogenous random variables. In both cases the unknown coefficients are estimated by maximizing the implied log-likelihood for the observed sample. We also report demand elasticities, denoted $E_{cdt}$, given by

\begin{equation}
E_{cdt} = b(1-X_{cdt})P_{cdt}; \quad c=1,2,3, d=B,G, \quad t=1,2,,T.
\end{equation}

Data

Using quarterly data from “National Prescription Data Base” for the period 2004-2008, we estimate the model for two important drugs in the Norwegian market: Seroxat/Paroxetine and Amlodipin/Norvasc. Seroxat/Paroxetine is an antidepressant drug. It is used to treat major depression, obsessive-compulsive disorder, panic disorder, social anxiety, post-traumatic stress and generalized anxiety disorder in adult outpatients. Amlodipin/Norvasc is an anti-hypertensive drug used in the treatment of angina pectoris.

In the Norwegian market there were three pharmacy chains. They were all retailers and wholesalers; Apokjeden, Holtung and NMD. Below they are named chain 1-3, not necessarily in the order mentioned here5.

Estimates

Tables 1 and 2 give the estimates. They are based on the maximum likelihood approach outlined above. In addition to the estimates we report asymptotically $z$-test.

---

5 We give a description of the variables and report summary statistics including market shares in http://folk.uio.no/steinast/supplements/An%20Equilibrium%20Model-Sum.stat.pdf
We first observe that the coefficient attached to price is negative and highly significant both in the demand model and in the equilibrium model. The numerical value of the price coefficient is significantly higher when the equilibrium process is accounted for (MEE), relative to the result of using the demand side only, i.e. market shares, in estimating price responses (DSE). It clearly seems that it is important to account for price formation when demand responses are estimated.

Second, the unobserved heterogeneity, as measured by the estimated standard deviations in demand and price equations, is a significant factor in explaining the observed choices. In particular, this is the case for the producer price formation for brand-name drugs. The constants, $\alpha_{cd}$, are fixed effects across chains and brand-name and generics. Most of them are significantly different from zero and reflects also the heterogeneity in the demand equations.

The estimates of the marginal cost of producing generics as well as brand ($k_G=k_B=k$), here assumed to equal the producer price of generics, are considerably lower than the retail prices of generics. This indicates a substantial margin for the generics sold by the retailers.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Demand model (DES)</th>
<th>Equilibrium model (MEE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates</td>
<td>$z$</td>
<td>Estimates</td>
</tr>
</tbody>
</table>

Table 1. Estimates. ATC code NO6AB05: Seroxat, Paroxetine
<table>
<thead>
<tr>
<th></th>
<th>Demand model (DES)</th>
<th>Equilibrium model (MEE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{2B} )</td>
<td>-0.1441</td>
<td>-0.3341</td>
</tr>
<tr>
<td>( \alpha_{3B} )</td>
<td>0.3148</td>
<td>0.3469</td>
</tr>
<tr>
<td>( \alpha_{1G} )</td>
<td>-0.4669</td>
<td>-0.9227</td>
</tr>
<tr>
<td>( \alpha_{2G} )</td>
<td>0.1061</td>
<td>-0.3638</td>
</tr>
<tr>
<td>( \alpha_{3G} )</td>
<td>0.1498</td>
<td>-0.3261</td>
</tr>
<tr>
<td>( b )</td>
<td>-0.5503</td>
<td>-0.7495</td>
</tr>
<tr>
<td>( k )</td>
<td>2.0257</td>
<td>5.9830</td>
</tr>
<tr>
<td>( \sigma_{B2} )</td>
<td>0.5309</td>
<td>0.8227</td>
</tr>
<tr>
<td>( \sigma_{B3} )</td>
<td>0.2482</td>
<td>0.2284</td>
</tr>
<tr>
<td>( \sigma_{G1} )</td>
<td>1.0311</td>
<td>1.0062</td>
</tr>
<tr>
<td>( \sigma_{G2} )</td>
<td>0.4191</td>
<td>0.4251</td>
</tr>
<tr>
<td>( \sigma_{G3} )</td>
<td>0.7883</td>
<td>0.8645</td>
</tr>
<tr>
<td>( \tau_{G1} )</td>
<td>2.0861</td>
<td>5.2481</td>
</tr>
<tr>
<td>( \tau_{G2} )</td>
<td>2.0485</td>
<td>5.2837</td>
</tr>
<tr>
<td>( \tau_{G3} )</td>
<td>1.9399</td>
<td>5.2715</td>
</tr>
<tr>
<td>( \tau_{B1} )</td>
<td>164.0017</td>
<td>4.9224</td>
</tr>
<tr>
<td>( \tau_{B2} )</td>
<td>59.5189</td>
<td>4.9068</td>
</tr>
<tr>
<td>( \tau_{B3} )</td>
<td>78.0807</td>
<td>4.9118</td>
</tr>
</tbody>
</table>

Table 2. Estimates. ATC code C08CA01: Amlodipin, Norvasc

Log-likelihood 8.4541 53.8313
We have tested whether there is autocorrelation in the demand part of the equilibrium model, the MEE approach. We have applied a modified Portmanteau statistic suggested by Ljung and Box (1978), which has a larger power when the sample is finite. Given the null hypothesis is true (there are no autocorrelation), the statistic $Q$ in eq. (33) converges to the Chi-square distribution with $m$ degree of freedom. In (33) $m$ is the number of lags specified and $\rightarrow$ indicates convergence in distribution to a $\chi^2$ distribution with $m$ degrees of freedom. $\hat{\rho}_j$ is the estimated autocorrelation for lag $j$.

$$Q = n(n+2)\sum_{j=1}^{m} \frac{1}{n-j} \hat{\rho}^2(j) \rightarrow \chi^2_m,$$
If the p-value of $Q$ is less than 0.05 in Table 3 below, then there is significant autocorrelation. As seen from the Table, there is no significant autocorrelation in the demand part of the equilibrium model. The same is also the case in the DSE approach.

### Table 3. Test of autocorrelation in the estimated log-odds ratio equations in the MEE model.

<table>
<thead>
<tr>
<th></th>
<th>Portmanteau (Q) statistic</th>
<th>p-value for Q</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Atc code N06AB05 : Seroxat,Paroxetine</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chain 1</td>
<td>5.9570</td>
<td>0.1137</td>
</tr>
<tr>
<td>Chain 2</td>
<td>1.3006</td>
<td>0.7290</td>
</tr>
<tr>
<td>Chain 3</td>
<td>1.7386</td>
<td>0.6284</td>
</tr>
<tr>
<td><strong>Atc code C08CA01 : Amlodipin,Norvasc</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chain 1</td>
<td>4.1967</td>
<td>0.5215</td>
</tr>
<tr>
<td>Chain 2</td>
<td>4.0058</td>
<td>0.54860</td>
</tr>
<tr>
<td>Chain 3</td>
<td>8.4436</td>
<td>0.1334</td>
</tr>
</tbody>
</table>

### Own price elasticities

Table 4 give the elasticities for the two drugs. In both cases the brand-name elasticities tend to be higher than the generic elasticities, which follow from the fact that the price of brand-named drugs exceeds the price of generics. Moreover, the brand-named market shares are lower than the market shares for generics in almost all periods and for all three chains. According to the formula (32) higher prices and lower market shares will contribute to higher elasticities. Most important here, however, is the result that the elasticities are numerically higher when the equilibrium process is accounted for compared to when only a demand model is used in estimating the elasticities.
Table 4. Own price elasticities.

N06AB05: Seroxat and Paroxetin.

<table>
<thead>
<tr>
<th>Demand model</th>
<th>Equilibrium model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brand</td>
</tr>
<tr>
<td>Chain 1</td>
<td>1</td>
</tr>
<tr>
<td>Mean, all periods</td>
<td>-3.29</td>
</tr>
<tr>
<td>Last period</td>
<td>-1.63</td>
</tr>
</tbody>
</table>

C08CA01: Amlodpin, Norvasc.

<table>
<thead>
<tr>
<th>Demand model</th>
<th>Equilibrium model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brand</td>
</tr>
<tr>
<td>Chain 1</td>
<td>1</td>
</tr>
<tr>
<td>Mean, all periods</td>
<td>-2.29</td>
</tr>
<tr>
<td>Last period</td>
<td>-1.95</td>
</tr>
</tbody>
</table>

Conclusions

When estimating price response based on the demand model only, the risk is that the estimates of price elasticities can be biased. Price responses could be underestimated, as demonstrated above. This will be the case if there are unobserved elements in the demand model that correlates with price. There are two ways of dealing with this problem: One could either employ an instrument variable approach or as done here, modelling the assumed whole data generating approach of demand and price setting. The advantage of an instrument variable approach is that it is rather straightforward to estimate the model. The disadvantage is that it is hard to find good instruments. The advantage of estimating jointly the demand and the price formation is that one avoids the search for proper instruments. The disadvantage is that it could be hard to estimate the model. But as shown here, with the software and computers available today this joint estimation of demand and price setting is manageable.
Although the estimation gives significant and reliable results, longer time series, covering periods with some regulatory shocks, may add some interesting results compared to what is found here. Another interesting next step could be to estimate the model on data from countries with a different market structure from the Norwegian oligopoly market, for instance a more competitive market. Finally, use of individual register-data will make it possible to model microeconomic behavior combined with price setting among pharmacies and producers.

Funding: This study was funded by The Ragnar Frisch Centre of Economic Research, Oslo, Norway and by Small Research Grants from Department of Economics, University of Oslo.

Conflict of Interest: The authors have received research grants from The Ragnar Frisch Centre of Economic Research, Oslo, Norway. The authors declare that they have no conflict of interest.

References


