



BI NORWEGIAN BUSINESS SCHOOL
DEPARTMENT OF FINANCE

PRELIMINARY MASTER THESIS REPORT

RISK PARITY VERSUS OTHER HEURISTIC ASSET ALLOCATION STRATEGIES

MASTER OF SCIENCE IN FINANCIAL
ECONOMICS

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BI Oslo

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Version: 16 January 2016

(Work in Progress)

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Chapter 1 – Introduction

The optimal distribution of wealth is a question that has been on the mind of academics and investors for a long time and will continue to be so in the future. It is a subject that is constantly being challenged and refined in order to gain a unique selling point in a fierce industry where performance is everything. The first mistake can easily become the last as reputation is vital for survival and the flow of capital from one competitor to another has become increasingly painless. In addition, the new millennia has brought with it two financial crisis which both caused incredible wealth destruction on a massive scale. These events have contributed to a fundamental change in investors' appetite for risk, consequently opening the door for more robust portfolios.

Traditional buy and hold strategies such as the 60/40 portfolio experienced large drawdowns during these extraordinary events. They might appear somewhat diversified at first glance due to having a solid portion of its assets in two different asset classes, which throughout most of the 20th century tended to be negatively correlated (Rankin et al. 2014). However, equities are much more volatile than bonds, thus representing 80-90% of the total portfolio risk, while bonds represent only 10-20%. Following this asset allocation strategy therefor produce a portfolio with a highly concentrated risk profile towards equities.

In response to recent developments, asset allocation strategies with risk as the only input has gained in popularity. These strategies do not require estimates of expected return in their models. This is in contrast to traditional strategies such as the mean-variance optimization framework developed by Markowitz (1952), which heavily relies on this input parameter in determining asset class weights. Estimating expected return with accuracy is rather difficult (Merton, 1980) and can therefore lead to significant variations in the composition of the portfolio.

In light of these discoveries came an asset allocation strategy called risk parity, which ignores expected return and instead allocate weights to each asset class such that they each contribute equally to the portfolios overall risk.

1.1 Research question

This thesis will address the growing concern facing investors in the search for a truly diversified portfolio in the light of recent market turmoil. In doing so we will construct one unlevered and one levered portfolio based on the principles of risk parity, and compare with the traditional buy and hold strategy, 60/40, which is a strategy that is widely used among practitioners and has a strong hold among institutional pension funds as well as private investors.

The results stemming from out-of-sample backtesting the portfolios against its peers will be analyzed by comparing Sharpe-ratios i.e. which strategy yields the best risk-return tradeoff. Moreover, the paper will also examine the drawdown of the portfolios in question. This is particularly interesting due to the fact that the risk parity strategy is expected to perform well relative to the benchmark strategy during periods that exhibits large negative returns or abnormal levels of volatility.

1.2 Motivation and contribution

After more than a decade of high macroeconomic uncertainties, increased political instability and repeated events of extreme wealth destruction, investors seem reluctant to continue with their traditional investment strategies. This has given rise to new approaches, among them Risk Parity. Traditional strategies have simply not provided satisfactory diversification benefits. The volatility and underperformance stemming from equities has dominated the returns in these portfolios. Furthermore, the correlation across markets and asset classes has increased due to globalization and quantitative easing programs, consequently magnified the absence of the sought after diversification effect. To top it all, historical low interest rate levels have made it extremely difficult for investors to get their desired returns, especially among pensions funds whose portfolios contain a significant portion of fixed income securities. The cocktail of all these factors has created a new economic regime, in which the question of the optimal distribution of wealth has become increasingly important. The Risk Parity approach to asset allocation has as a result been implemented by noteworthy asset managers who claim its supremacy. This thesis will therefore contribute with some much needed empirical research on the strategy in a horserace setting and hopefully provide some valuable insight on the field.

1.3 Outline

This thesis is structured into nine chapters, each containing subsections. The first chapter comprise of the introduction, the problem statement, a brief overview for the motivation of the study and its contribution, the structure and the limitations and the assumptions undertaken. The second chapter digs deeper into the motivation behind the study and tries to bring forward why the issue at stake deserves more research and attention and what possible solutions it can solve. Discoveries from similar studies on the same subject will be highlighted and related to the thesis. The third chapter contains the theory and will define the relevant theories for the study and derive the mathematics behind them. Furthermore, chapter four explains the methodology for the thesis, i.e. how we go about solving the issue based on the above theoretical framework. Chapter five defines the investment universe in which the strategies will be tested. It will describe the data used in the paper and the process in which it was selected. Furthermore, justifications for the specific data will be provided.

1.4 Limitations

Regarding limitations to the study, we assume a no short selling constraint, as it is not available to all markets participants. Although most pension funds, which the strategy are primarily targeted for are large enough to do so we cannot implement it as it would add an additional element of complexity which are beyond the scope of this study. Furthermore, the issue of currency risk that arises due to cross-currency trading will not be taken into account, as it also would be too complex of an issue.

The cost per transaction unit is assumed to be constant throughout, meaning that low volume and high volume transactions have the same per unit cost. Finally, we restrict the performance evaluation of the strategies to two measurement metrics, namely the Sharpe ratio and maximum drawdown even though other approaches could potentially reveal different results.

Chapter 2 – The Search for a Truly Diversified Portfolio

Two crises within the span of eight years gave rise to what is known as the “lost decade”. These events have made investors revise their strategies and ultimately affected their appetite for risk. Could this be the start of the end of a long period with overweight in equities in the portfolios of sophisticated investors?

2.1 The final chapter of Markowitz?

The optimal distribution of wealth in a portfolio construction framework is a subject that has been under the scope of academics for a long time. The foundation of what is considered modern portfolio theory dates back to 1952 when Markowitz (1952, 1959) provided a framework to solve the problem of efficient asset allocation, called mean-variance optimization. The method has brought to light two central principles which has since inception been at the core of finance, both in academia and practice. The first is that diversification provides excellent risk management. He shows that the strong point of diversification not only comes from the number of assets in a portfolio, but also the correlations/covariances among the assets in the portfolio. The second principle refers to how investors should consider expected return desirable and variance of return undesirable. Hence, investors should not simply choose the portfolio with the highest expected return. This principle is at the heart of Markowitz’s framework, which proclaims that investors should seek to maximize the expected return for a given volatility. A portfolio that satisfies this is called the mean-variance efficient portfolio as it provides the best possible return for a given level of risk. Although the method is sophisticated and powerful, it does present difficulties in its practical implementation. Firstly, it tends to create portfolios that are highly concentrated in a limited subset of the full set of assets or securities (Maillard, 2008). Secondly, the proposed solution is highly sensitive to the input parameters. According to Merton (1980), small changes to the expected return, which by default is very difficult to estimate with accuracy, can lead to significant variations of the portfolio composition.

2.1.1 Low yield environment

A key issue for long-term asset allocation strategies is the issue of the historically low yield environment currently seen throughout the world. Because of diversification benefits, most asset allocation strategies will want some exposure to the fixed income market. However, fear that yields cannot drop further and when yields rise, prices will fall is a concern that cannot be overlooked taking current market conditions into account. Furthermore, the risk parity strategy typically is more exposed in terms of weighting to the fixed income market, due to its risk balancing characteristics, than a traditional buy and hold portfolio such as the 60/40 strategy. Several adjustments to the strategy have been proposed by practitioners, including diversification across the yield curve among others, however for the practice of empirically testing the risk parity strategy, we need consistency across market conditions. We will therefore look comprehensively at the performance of the risk parity portfolio in previous low yield environments to examine whether diversification benefits of the risk parity portfolio exceeds its increased weight exposure to the fixed income market. More specifically, in the U.S. we have a low yield environment followed by rising interest rates both leading into the 1980's as well as in 1994. It should be noted that none of the historical environments are as extreme as the current market environment and that other conditions might influence the portfolio; however, the results should give an indication of the impact it has on the portfolio both in terms of return on the fixed income portion as well as the overall portfolio returns.

2.1.2 Pension funds promises

The 2008 crisis adversely affected pension funds everywhere. They reported unrealized losses on an unprecedented scale, which has challenged their governance mechanisms and their capability to provide the promised benefits. Social security funds in high-income countries suffered the most from the crisis and posted negative returns in the range of -30.6 per cent to -16.4 per cent. The magnitude of the losses was highly correlated to the exposure to equities in their portfolios, while countries with higher fixed income exposure were less adversely affected. The losses from the crisis are not only of great concern due to their magnitude, but also because they came so quickly after the previous stock market

crash at the beginning of the millennia. Together, these two events has led to an underperformance among much of the pension funds in the OECD area, consequently caused them to have fallen below their long-term, target investment returns. Private pension funds were even worse of because they often only rely on accumulated returns to meet their obligations. Hence, short-term negative events directly affect the amount of available to meet their liabilities (Pino and Yermo, 2010).

In a reaction to the recession following the crisis, central banks immediately lowered interest rates to stimulate the economy. This has made it difficult for investors to reach their required returns and pension funds has not been a pleasant place to be since. Public pension funds usually set their discount rate for their future obligations based on the expected returns of its assets. This rate is usually somewhere between seven and eight per cent and is heavily based strategies that largely depend on equity risk premium (Qian, 2012). This poses issues due to the underperformance of equity markets over the last decade while at the same time, pension liabilities remain for most the most part fixed. According to Novy-Marx and Rauh (2009), this has led to the majority of public pensions to be underfunded.

In 2006, the US president signed the Pension Protection Act, which forced changes in how corporate pensions estimate their future commitments by going away from evaluating them based on the expected return on its asset and instead use high quality bond yields. The decreased discount rate paints a more accurate picture of a higher present value of its liabilities. This means that corporations either must increase their pension contributions or implement liability-matching investment strategies or a combination of both, especially if equities continue to underperform.

Qian (2012), suggest that a risk parity approach to asset allocation can be a solution to match pension liabilities either through an asset-only or an asset-liability management framework. This claim, if correct could potentially be invaluable to future pension funds and therefore provides huge motivation for this thesis. In brief, what Qian (2012) claims is that a risk parity strategy through proper risk budgeting among different asset classes will provide superior diversification benefits and a more stable long-term Sharpe ratio compared to traditional buy and hold approaches. In such, it will be less sensitive to different

macroeconomic environments and yield more robust returns in different market cycles.

2.2 Empirical studies on the subject

Risk parity is a relatively new approach to asset allocation that came out of the industry in a response to diversification challenges faced by traditional portfolios. The strategy takes a heuristic approach to asset allocation and has therefore not been subject to comprehensive examinations in academic literature. However, as of late the strategy has gained traction from fund managers. This has been reflected in an increased interest for the approach in the academic literature. Maillard et al. (2008) was the first to derive the theoretical properties for the strategy and found some appealing characteristics with it. These include superior diversification benefits along with increased robustness due to its lack of dependence on expected return. Moreover, the paper shows that the risk parity portfolio appears to be an attractive alternative to minimum variance and equally weighted portfolios and seems to offer good trade-off between the other two strategies with regard to its absolute level of risk. The distinct risk budgeting characteristics that the approach offers leads to increasing diversification benefits. Chaves et al. (2011) find that the risk parity strategy has a higher Sharpe ratio than both the minimum variance and the mean-variance optimization; however, it does not consistently outperform an equally weighted portfolio or a buy and hold approach. Furthermore, the risk parity portfolio exhibits more balanced risk and thus lowers volatility over time. Anderson et al. (2012) did extensive out-of-sample backtesting on both an unlevered and levered risk parity portfolio performance relative to other heuristic benchmark portfolios over an 85 year period (1926 – 2010). Over this horizon the levered parity strategy substantially outperformed the 60/40 strategy, the unlevered risk parity and value-weighted strategies. However, taking into account borrowing costs that exceed the risk-free rate, the risk parity barely outperformed the 60/40 portfolio. Furthermore, taking trading costs into account, it seemed that the 60/40 slightly outperformed risk parity, but the results were not statistically significant. Overall, the unlevered risk parity strategy delivered superior risk-adjusted return measured by the Sharpe-ratio. Poddig and Unger (2012) examined the resilience of risk parity asset allocation

and shows that the approach is more robust to changes in the input parameters. Furthermore, the risk parity portfolio has a smaller estimation error than the mean-variance optimization model developed by Markowitz (1952).

Chapter 3 – Theoretical Framework

Throughout the thesis, all formulas will be computed using matrix formulas because of its enhanced convenience. Furthermore, matrix formulas are expressed in bold to not confuse them with the conventional formulas.

3.1 Asset allocation theoretical foundation

To understand the portfolio strategies as well as the asset allocation principles studied in the thesis, we need some background. For a portfolio consisting of n risky assets, the weight invested in each asset is denoted by $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$, forming a vector of weights in the portfolio. Furthermore, the portfolio satisfies the budget constraint of being fully invested, that is $\sum_{i=1}^n x_i = 1$ and short selling is not allowed. Let $\mathbf{r} = (\mathbf{r}_1, \dots, \mathbf{r}_n)$ be the return of the assets forming a vector of returns where the return on asset i from period $[0, 1]$ is

$$r_i = \frac{P_{i,1} - P_{i,0}}{P_{i,0}}$$

Furthermore, we can write the return of the portfolio consisting of n assets as

$$r(\mathbf{x}) = \sum_{i=1}^n x_i r_i$$

In matrix form the return of the portfolio would be

$$\mathbf{r}(\mathbf{x}) = \mathbf{x}^T \mathbf{r}$$

3.2 Risk Parity

3.2.1 Naïve Risk Parity (Inverse Volatility)

The naïve Risk Parity portfolio seeks to weight assets by the inverse of their volatility, hence the portfolio seeks to down weight more risky assets so that they have equal volatility impact to the portfolio. Mathematically, the weights of the portfolio are computed as follows

$$x^* = \frac{\frac{1}{\sigma_i}}{\sum_{n=1}^N \frac{1}{\sigma_i}}$$

The naïve risk parity portfolio is easy to compute and is therefore computationally superior to the risk parity portfolio. However, it does not account for correlations between asset classes and thereby not correctly accounting for their risk behavior in a portfolio. Furthermore, because of this some asset classes might be unfavorably penalized in terms of their weighting in the portfolio. Moreover, looking at correlations between assets over time, we see that they vary quite severely leading to differences in their impact of a portfolio over time.

3.2.2 Theory behind MRC & TRC

To understand the intuition behind risk parity we turn to the theoretical framework derived by Maillard et al. (2008). For each asset class to be correctly accounted for in the portfolio one must define its total risk contribution (TRC) as well as the marginal risk contribution (MRC). Take a portfolio $\mathbf{x} = (x_1, \dots, x_n)$ with n assets and variance σ_i^2 for asset i , let σ_{ij} be the covariance between asset i and j and Σ be the covariance matrix. Let $\sigma(\mathbf{x}) = \sqrt{\mathbf{x}^T \Sigma \mathbf{x}}$ be the standard deviation of the portfolio.

The marginal risk contribution is the derivative of total portfolio standard deviation with respect to x_i and is the risk added to the portfolio by an infinitely small increase in the weight of asset i . Mathematically it is defined as:

$$\partial_{x_i} \sigma(\mathbf{x}) = \frac{\partial \sigma(\mathbf{x})}{\partial x_i} = \frac{x_i \sigma_i^2 + \sum_{j \neq i} x_j \sigma_{ij}}{\sigma(\mathbf{x})} = \mathbf{MRC} = \frac{\Sigma \mathbf{x}}{\sqrt{\mathbf{x}^T \Sigma \mathbf{x}}}$$

An asset i 's total risk contribution to the portfolio is simply its MRC multiplied by its weight.

$$\sigma_i(\mathbf{x}) = x_i \cdot \partial_{x_i} \sigma(\mathbf{x}) = \mathbf{TRC} = \sqrt{\mathbf{x}^T \Sigma \mathbf{x}}$$

The risk of the portfolio is therefore the sum of the total risk contribution of each asset.

$$\sigma(\mathbf{x}) = \sum_{i=1}^n \sigma_i(\mathbf{x})$$

Furthermore, the optimal weight of assets \mathbf{x}^* is

$$\mathbf{x}^* = \{ \mathbf{x} \in [0,1]^n : \sum x_i = 1, x_i \cdot \partial_{x_i} \sigma(\mathbf{x}) = x_j \cdot \partial_{x_j} \sigma(\mathbf{x}) \text{ for all } i, j \}$$

Noting that $x_i \cdot \partial_{x_i} \sigma(\mathbf{x}) = \mathbf{TRC} = \sqrt{\mathbf{x}^T \boldsymbol{\Sigma} \mathbf{x}}$, and denoting $(\boldsymbol{\Sigma} \mathbf{x})_i$ as the covariance matrix of portfolio \mathbf{x} where i denotes the i^{th} row of the vector issued from the product of $\boldsymbol{\Sigma}$ with \mathbf{x} , we have that

$$\mathbf{x}^* = \{ \mathbf{x} \in [0,1]^n : \sum x_i = 1, x_i \cdot (\boldsymbol{\Sigma} \mathbf{x})_i = x_j \cdot (\boldsymbol{\Sigma} \mathbf{x})_j \text{ for all } i, j \}$$

The optimal portfolio construction denoted above takes into account the no short selling constraint, indicating weights between zero and one for each asset class. Furthermore, the sum of the weights of the individual asset classes satisfies the budget constraint, thus sums to one. The objective function is to equalize the total risk contribution of each asset class taking into account the covariance between the asset classes. That is, all asset classes should have the same behavioral impact on the portfolio risk.

Knowing that $\beta_i = \sigma_{ix} / \sigma^2(\mathbf{x})$ and $\sigma_i(\mathbf{x}) = x_i \beta_i \sigma(\mathbf{x})$ the optimal weights $x_i = \frac{\beta_i^{-1}}{n}$. Therefore, the portfolio weight is inversely related to the beta of the individual asset classes. Note that the betas represent the covariance of each asset class with the constructed portfolio. However, in order to estimate the beta we need the portfolio weight, making the solution endogenous. We therefore need a numerical algorithm to deal with the endogeneity. Maillard et al. (2008) recommends using a sequential quadratic programming algorithm for solving the following optimization problem:

$$\begin{aligned} \mathbf{x}^* &= \arg \min f(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{1}^T \mathbf{x} = 1 \text{ and } 0 \leq x \leq 1 \end{aligned}$$

where:

$$f(\mathbf{x}) = \sum_{i=1}^n \sum_{j=1}^n (x_i (\boldsymbol{\Sigma} \mathbf{x})_i - x_j (\boldsymbol{\Sigma} \mathbf{x})_j)^2$$

For the thesis, the optimization problem will be solved using MATLAB's function "*fmincon*".

3.2.3 Unlevered vs. Levered Risk Parity

In order for the risk parity to equalize the TRC of each asset in a portfolio it typically down weights risky assets and over weights less risky assets. This causes the portfolio to exhibit less risk in terms of portfolio standard deviation and will subsequently typically suffer from lower returns. Even though risk parity portfolios are not measured in terms of expected returns we know that risk and return is closely correlated in finance. In order for us to test portfolio superiority, we need both portfolios to exhibit somewhat equal risk. The typical approach to dealing with this problem is to leverage up the risk parity portfolio. The use of leverage will naturally add other unwanted risks to the portfolio which will be thoroughly discussed in the limitations chapter. Nevertheless, a risk parity's leverage ratio is computed as follows

$$RP_{\text{Leverage Ratio}} = \frac{\sigma_{\text{Benchmark}}}{\sigma_{\text{Unlevered RP}}}$$

That is, if the $\sigma_{\text{Benchmark}} = 8\%$ and the $\sigma_{\text{Unlevered RP}} = 5\%$, the leverage ratio in the Levered Risk Parity portfolio would be $RP_{\text{Leverage Ratio}} = \frac{8\%}{5\%} = 1.6$, hence 60% leverage would be applied to the portfolio.

3.3 Benchmark investment strategy

3.3.1 60/40 Portfolio

The traditional 60/40 portfolio's objective is to maintain a 60% weight exposure to equities and 40% weight exposure to bonds. While this approach to asset allocation has long traditions, because equities have historically been much more volatile than bonds, the risk in the portfolio measure by the volatility has mainly been driven by equities. It is still widely used among both practitioners and private individuals.

3.3.2 Equally Weighted Portfolio (1/N)

The equally weighted portfolio seeks to equalize the weight of each asset class in the portfolio. The 1/N portfolio is viewed as a naïve portfolio strategy, however it is widely used in practice and some suggest that it even outperforms optimization strategies (DeMiguel et al. 2007).

The optimal portfolio weight in the 1/N strategy is simply:

$$x^* = \frac{1}{N}$$

3.4 Performance measurements

Portfolio evaluation involves the determination of how a portfolio has performed relative to some comparable benchmark. In order for the portfolio and the benchmark to be comparable, some risk adjustment has to be performed to capture return per unit of risk. Risk adjusted performance methods adjust the return to take into account the differences in risk levels between the portfolio and the benchmark. The performance evaluation methods regarded in this thesis will capture the essence of asset allocation from a long-term investor's perspective, namely risk adjusted return, portfolio risk as well as risk distribution.

3.4.1 Sharpe Ratio

The Sharpe ratio is a portfolio evaluation measurement that computes the excess return of an asset or a portfolio per unit of total risk. The excess return is the return on the portfolio above the risk free rate (reference to risk free rate). Furthermore, the total risk is the standard deviation of returns on the portfolio or asset. The numerator captures the reward for investing in the portfolio or asset adjusted for the risk free rate while the denominator captures the variability of returns of the portfolio or asset (Encyclopedia of Finance). Furthermore, the Sharpe ratio is widely used for both practitioners and scholars. As the analysis in the thesis does not rely on expected returns because of its troubling features regarding its estimation, it is important to notice that the Sharpe ratio is solely used as a performance indicator using realized returns. Furthermore, it should be noted that the ratio is sensitive to both the sample period as well as the frequency of returns (i.e. daily, monthly, etc.) and should therefore be used primarily to

decide upon dominance with regard to the risk-return relationship between two portfolios. The Sharpe ratio is a measure of the efficiency in terms of excess return per unit of risk and is given by

$$SR(x) = \frac{r(x) - r_f}{\sigma(x)}$$

Hence, for a portfolio with a higher Sharpe ratio than the benchmark we can conclude that the portfolio outperformed the benchmark in terms of excess return per unit of risk. However, we cannot say anything about the distribution of neither returns nor risk, which might reveal additional valuable information we might want to capture to evaluate the portfolio further. In order for us to be able to say something about total outperformance of two portfolios we need to turn to the maximum drawdown and frequency of drawdowns.

3.4.2 Portfolio drawdown

While the drawdown of a portfolio does not say anything about the frequency of losses, it is an important measure of portfolio performance because it is a measure of capital preserved in the portfolio.

Using Chekhlov et al. (2005) for specifying maximum drawdowns we have that for a given time interval stretching from $[0, T]$, the maximum drawdown (MaxDD) of portfolio x will be equal to:

$$\text{MaxDD}(x) = \max_{\tau \in [0, t]} (W_\tau - W_t)$$

While the maximum drawdown is an absolute measure of how much loss the portfolio experienced in a specified time interval k , the portfolio relative drawdown (RelDD) is a measure of how much the portfolio loss was on a relative basis during a specified time interval k . The relative maximum drawdown is a better measure because it captures the relative preservation of capital in the portfolio. The relative maximum drawdown is defined as

$$\text{RelDD}(x) = \frac{\max_{\tau \in [0, t]} (W_\tau - W_t)}{W_\tau}$$

3.4.3 Frequency of portfolio drawdown

As it is an attractive trait of long-term asset allocation, we will measure the frequency of portfolio drawdowns above 5% in any given trading time interval.

That way we can measure portfolios not only on their maximum drawdown but also in terms of their frequency of drawdowns. The frequency of drawdowns will be expressed as a percentage of trading intervals where the portfolio drawdown was greater than 5% relative to all trading intervals.

3.5 Turnover and transaction cost

Transaction costs are important to consider as there typically is large variation between the different strategies. Furthermore, a typical finding seems to be that trading costs generally eat up excess returns of more complex strategies, leaving them at par or even worse off than naïve or simple strategies. For the purpose of estimating transaction costs, the framework of Anderson et al. (2012) will be used. Moreover, the budget constraint of all portfolios throughout the thesis is that at all rebalancing dates, it needs to be fully invested, hence $\mathbf{x}^T \mathbf{1}$, that is the sum of the weights adds to one. At period $t+1$ the portfolio will be subject to rebalancing due to its affection to prices, hence for any strategy, the modification of weights needed to asset i at time t is:

$$\tilde{x}_{i,t}^* = \frac{(1 + r_{i,t})x_{i,t-1}}{\sum_j (1 + r_{j,t})x_{j,t-1}}$$

and the turnover required to balance is given by:

$$\text{Turnover}(\mathbf{x}) = \sum_j |\tilde{x}_{i,t}^* - x_{j,t}|$$

Moreover, since the risk parity strategy relies heavily on the use of leverage, the leverage adjusted turnover would be :

$$\text{Turnover}_{leverage\ adj.}(\mathbf{x}) = \sum_j |\tilde{x}_{i,t}^* \cdot \lambda_{t-1} - x_{j,t} \cdot \lambda_t|$$

Trading costs for any given strategy at time t is therefore given by:

$$c_t = \text{Turnover}_{leverage\ adj.}(\mathbf{x}) \cdot z_t$$

where z_t is the transaction cost measured in basis points (bp). The trading cost adjusted returns are given by:

$$r_{c,\lambda-adj} = r_t - c_t$$

Chapter 4 – Methodology

We will construct the relevant portfolios using the appropriate theoretical techniques, presumably using excel with additional programming where needed. Further, the portfolios will be applied in a horserace over a period of approximately 40 years. At this stage, we presume rebalancing of the portfolios every quarter. The results will be a time series of returns for all subsequent portfolios. We will analysis their performance consistent with the performance evaluation techniques laid out in the theoretical part.

Chapter 5 – Data

5.1 The investment universe

We define the investment universe for the purpose of achieving relatively good diversification benefits. We therefore seek global exposure with more focus on the U.S. market and European market. For practical purposes we will restrict the asset classes to equities and bonds and in order to provide a fair comparison to the 60/40 equity/bond strategy. As proxies for the relevant markets we choose to construct the portfolios based on indices that has certain appealing characteristics. For the simulation to be relevant for investors on a practical level they must offer a certain level of liquidity. The ability to buy and sell large positions quickly and without significantly affecting prices is very important. Furthermore, the length of the historical data must be adequate so that we are able to test the strategies through different market environments. This comes at a cost. There is a trade off between the level of diversification and the historical length of the data. In terms of periods, we expect positions to be held for quite some time and therefore needs a sufficient horizon (aim to provide backtesting for the last 40 years, 1976-2016) so that the strategies can be tested through different economic environments.

5.2 The data (subject to change)

We use the MSCI Europe index to represent the development in the European equity market over the period. It represents the performance of large and mid-cap equities across 15 developed countries in Europe.

Austria	Finland	Ireland	Norway	Sweden
Belgium	France	Italy	Portugal	Switzerland
Denmark	Germany	Netherlands	Spain	United Kingdom

The weights are as follows: UK – 31%, France – 15%, Switzerland – 15%, Germany – 14%, Netherlands – 5%, Spain – 5%, Sweden – 4%, Others – 12%.

Source: (MSCI Europe - MSCI)

For the US equity market we use the S&P 500 which includes the 500 largest companies in terms of market capitalization that are listed on either NYSE or NASDAQ. It has over USD 7.8 trillion benchmarked to the index and is one of the most followed indices and a very good representation of the US stock market.

Source: (S&P Dow Jones Indices)

As a proxy for the bonds we use the Bloomberg Barclays US Aggregate Bond Index, which is a broad-based flagship benchmark that measure investment grade, US dollar-denominated, fixed-rate taxable bond market. The index includes Treasuries, government-related and corporate securities, MBS (agency fixed-rate and hybrid ARM pass-throughs), ABS and CMBS (agency and non-agency).

Note: if possible, will include the Bloomberg Barclays Global Aggregate Index

Source: (US Aggregate Index), (Global Aggregate Index)

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