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Equal Risk Contribution Adopts Time Series Momentum

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Abstract

We study risk based portfolios with an emphasis on equal risk contribution, in a time series momentum setting. The benchmark strategies in which we compare the equal risk contribution includes inverse volatility, minimum variance, 1/N and 60/40. We perform an out of sample horserace of all strategies in a broad asset class environment. We then compare these portfolios to time series momentum long-only and long-short portfolios made up of the constituents of the broad asset classes. We find that risk based portfolios offers attractive traits mainly by controlling risk and avoiding large drawdowns. We also find that time series momentum portfolios add significant value with low market exposure and moderate momentum exposure, avoiding large drawdowns. We are skeptical of the suitability of the long-short portfolios, even though they clearly offer the best returns.

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Chapter 1 – Introduction

1.1 Background

The optimal allocation of wealth in a portfolio is a thoroughly researched subject in finance. The foundation of what is considered modern portfolio theory dates back to 1952 when Markowitz (1952, 1959) provided a framework to solve the problem of efficient asset allocation. The method has brought to light two central principles which has since inception been at the core of finance, both in academia and practice. The first is that diversification provides excellent risk management. He shows that the strong point of diversification not only comes from the number of assets in a portfolio, but also the correlations among them. The second principle refers to how investors should consider expected return desirable and variance of return undesirable. Hence, investors should maximize the expected return for a given volatility. A portfolio that satisfies this is called the mean-variance efficient. Although the method is sophisticated and powerful, it does present difficulties in its practical implementation. Firstly, it tends to create portfolios that are highly concentrated in a limited subset of the full set of assets or securities (Maillard, 2008). Secondly, the proposed solution is highly sensitive to the input parameters. According to Merton (1980), small changes to the expected return, which by default is very difficult to estimate with accuracy, can lead to significant variations of the portfolio composition.

Markowitz's portfolio theory would lay the foundation for the Capital Asset Pricing Model (CAPM) by William F. Sharpe (1964) has been at the core of asset allocation decisions for the past fifty years. Under certain assumptions, the model states that market-capitalization weighting is efficient for asset allocation, meaning that for a given level of risk, any two portfolios should have the same expected return. In practice, these assumptions do not hold because investors do not all have homogeneous expectations and they cannot sell short without penalty (Demey et al., 2010). Thus, the CAPM appears to be inefficient (see Haugen & Baker, 1991 and Hsu, 2006).

In light of these discoveries, a range of risk based asset allocation strategies have emerged. Instead of diversifying on the basis of capital invested in each asset,

these methods aim to diversify the risk of the portfolio. The simplest of them all is the equally weighted (EW) approach, which have only to be driven by the number of asset classes in the investment universe. It is highly improbable that this portfolio is efficient in a mean-variance framework as it requires that the ex-ante expected returns and volatility are equal and that correlations among asset classes are uniform (Braga, 2016). That said, Demiguel et al (2011) found that none of the theoretically more robust models performed consistently better out-of sample. Another approach is the minimum-variance (MV) portfolio, which is situated on the mean-variance efficient frontier with the lowest possible risk. It is recognized as being robust due to its unique solution and because it is the only mean-variance efficient portfolio that does not incorporate expected returns. However, the portfolio typically suffers from large drawdowns due to it being highly concentrated in a few asset classes (Demey et al., 2010). Inverse volatility (IV), also known as naïve risk parity is a third method which derives its portfolio weights by investing proportionally to the inverse of its volatility; it is then normalized to guarantee the weights sum to one. This strategy is computationally attractive because it obtains its weights with no regard to the correlation among assets. This is what separates it from the strategy that will be the main focus of this paper, namely equal risk contribution (ERC), also known as risk parity, originated by Maillard et al. (2008). Roughly speaking, the resulting portfolio is similar to a MV portfolio subject to a diversification constraint on the weights of its components. Although such a portfolio typically exhibits attractive risk-adjusted returns and diversification, it tends to provide somewhat low returns because of its concentration into low-risk assets. Investors address this issue by applying leverage to target a desired level of return and risk (Asness et. al., 2012). However, applying leverage introduces its own risks and practical concerns. As an alternative, we will combine the allocation scheme as proposed by the ERC framework with a trend following filter. More precisely, our methodology follows that of Moskowitz et al. (2012) who employ what they call time series momentum (TSM) to select what securities to buy or sell for each period. Moskowitz et al. (2012) relates TSM to the phenomenon known as “momentum” in the finance literature, which is primarily cross-sectional in nature. Cross-sectional momentum (CSM) focuses on the relative performance of securities, finding that securities that outperformed their peers over the past three to 12 months continue to do so

on average over the next month. TSM on the other hand selects securities based on their absolute performance, i.e. its own past return.

1.2 Research question

The aim of this paper is to extend previous work in the area of combining risk based allocation strategies with trend following in a multi-asset class context.

In theory, ERC appears to have very attractive qualities, however the amount of leverage needed to get equally attractive returns presents complications to most investors and institutions. We investigate whether combining the strategy with TSM can be a solution to the problem. Another area of interest is whether ERC is a superior method to IV which is the standard method to size time series momentum positions in the finance literature. To test this, we apply all four risk based allocation strategies to our TSM results to see which performs best both under a long only and long-short scenario. In addition, we create benchmark strategies without TSM to see if trend following itself is a solution to the return problem. We also compare all these results to the average investors' favorite portfolio, namely the traditional 60/40 portfolio.

1.2 Motivation and contribution

Over the last two decades, investors holding the market portfolio or the 60/40 portfolio have experienced large drawdowns during the financial crisis of 2007-2008 and the dot-com bubble. This has raised awareness around how traditional portfolios may be insufficiently diversified. The 60/40 portfolio tries to diversify by dividing its investments between equities and bonds, however looking at how much each asset contributes to the overall risk of the portfolio we see how equities dominates in terms of risk contribution. Equities are much more volatile than bonds and when viewed from a risk perspective, the 60/40 is mainly an equity portfolio since almost the entire variation in returns is explained by the variation in equity markets (Asness et al., 2011). ERC addresses this issue by diversifying by risk not by dollars. To achieve this, one typically need to invest more in low-risk assets than high-risk assets. Consequently, overall portfolio returns tend to be quite low. An alternative to using leverage to boost returns is to combine the strategy with another investment strategy that specifies what to invest in, but not how much in each asset. Rule based investment strategies such as trend

following and momentum does this and there exist substantial literature that find support for the concept that momentum in financial markets offer significant explanatory power with regard to future returns across markets and asset classes (Clare et al. 2015). Moskowitz et al. (2012), finds TSM to be superior to more traditional momentum rules, we will therefore combine this approach with ERC. TSM is a relatively new concept, this paper will therefore contribute with further research on the asset-pricing anomaly that it is in a multi asset class context. To our knowledge, little research has been done on combining the more sophisticated weighting scheme that is ERC with momentum trading. The vast majority of the finance literature employ IV to size momentum positions. IV differ from ERC as it ignores correlations between assets.

The rest of this paper is organized as follows: Chapter two digs deeper into the motivation behind the study and tries to bring forward why the issue at stake deserves more research and attention and what possible solutions it can solve. Discoveries from similar studies on the same subject will be highlighted and related to the thesis. The third chapter contains the theory and will define the relevant theories for the study and derive the mathematics behind them. Furthermore, chapter four explains the methodology for the thesis, i.e. how we go about solving the issue based on the above theoretical framework. Chapter five defines the investment universe in which the strategies will be tested and provide an overview of the data used. In chapter six we provide the results with a discussion of our findings and how they relate to relevant literature. Finally, in chapter seven we conclude the paper.

Chapter 2 – Literature Review

ERC is a relatively new approach to asset allocation that came out of the industry in a response to diversification challenges faced by traditional portfolios. The strategy takes a heuristic approach to asset allocation and has therefore not been subject to comprehensive examinations in academic literature. However, as of late the strategy has gained traction from fund managers. This has been reflected in an increased interest for the approach in the academic literature. Maillard et al.

(2008) was the first to derive the theoretical properties for the strategy and found some appealing characteristics with it. These include superior diversification benefits along with increased robustness due to its lack of dependence on expected return. Moreover, the paper shows that the ERC portfolio appears to be an attractive alternative to MV and EW portfolios and seems to offer good trade-off between the other two strategies regarding its absolute level of risk. The distinct risk budgeting characteristics that the approach offers leads to increasing diversification benefits. Chaves et al. (2011) find that the ERC strategy has a higher Sharpe ratio than both the MV and the mean-variance optimization; however, it does not consistently outperform an EW portfolio or a buy and hold approach. Furthermore, the ERC portfolio exhibits more balanced risk and thus lowers volatility over time. Anderson et al. (2012) did extensive out-of-sample backtesting on both an unlevered and levered ERC portfolio performance relative to other heuristic benchmark portfolios over an 85-year period (1926 – 2010). Over this horizon the levered ERC strategy substantially outperformed the 60/40 strategy, the unlevered ERC and value-weighted strategies. However, taking into account borrowing costs that exceed the risk-free rate, the risk parity barely outperformed the 60/40 portfolio. Furthermore, taking trading costs into account, it seemed that the 60/40 slightly outperformed ERC, but the results were not statistically significant. Overall, the unlevered risk parity strategy delivered superior risk-adjusted returns measured by the Sharpe-ratio. Poddig and Unger (2012) examined the resilience of ERC asset allocation and shows that the approach is more robust to changes in the input parameters. Furthermore, the ERC portfolio has a smaller estimation error than the mean-variance optimization model developed by Markowitz (1952).

TSM has its roots in the paper “Time series momentum” by Moskowitz et al (2012). They use forward and future contracts from four broad asset classes (equities, bonds, currencies and commodities) and find strong positive predictability from a security’s own past returns for almost all contracts. It appears as return patterns persist for one to 12 months and partially reverses over longer horizons. This is consistent with sentiment theories of initial under-reaction and delayed over-reaction. Further, they find that a diversified portfolio of 58 contracts provide abnormal returns with limited exposure to standard asset pricing factors. Moskowitz et al (2012) employ the IV method in determining the

portfolio weights. Baltas (2015) hypothesize that the IV weighting scheme leads to uneven risk allocation, especially under recent market conditions where we have seen a dramatic increase in correlations between assets. He finds that an ERC trend following portfolio outperforms an IV variant of the entire sample period. Moreover, during the post crisis period (2009-2013), the ERC variant excels and the Sharpe ratio more than doubles. Clare et al (2015) also finds that combining ERC with a trend following filter enhance the performance relative to a pure ERC strategy. There is controversy surrounding momentum and why the price anomaly persist. Jegadeesh and Titman (1993) point to underreaction and slow incorporation of new information by investors in explaining their findings. This explanation implies that investors are irrational and challenge the theory of efficient markets. Crombez (2001) proposes an alternative explanation where the momentum effect can persist even if investors are rational and markets efficient due to an information market imperfection in that the strength of the expert evidence is noisy. Hence, the costly public information they reflect through the forecasts is slowly diffused through the market and prices do not fully reflect all costly public information.

Chapter 3 – Theoretical Framework

In this chapter, the theoretical framework for conducting the analysis will be presented. The theory includes a general theoretical foundation of portfolio and asset allocation theory as well as strategy and performance specific theory. Throughout the thesis, most formulas will be computed using matrix formulas because of its enhanced convenience. Furthermore, matrix formulas are expressed in bold to not confuse them with the conventional formulas.

3.1 Asset allocation theoretical foundation

To understand the portfolio strategies as well as the asset allocation principles studied in the thesis, we need some background theory. For a portfolio consisting of n risky assets, the weight invested in each asset is denoted by $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$, forming a vector of weights in the portfolio. Furthermore, the portfolio satisfies the budget constraint of being fully invested, that is $\sum_{i=1}^n x_i = 1$ and short selling

is not allowed in the general case. Let $\mathbf{r} = (\mathbf{r}_1, \dots, \mathbf{r}_n)$ be the return of the assets forming a vector of returns where the return on asset i from period $[0, 1]$ is

$$r_i = \frac{P_{i,1} - P_{i,0}}{P_{i,0}}$$

Furthermore, we can write the return of the portfolio consisting of n assets as

$$r(\mathbf{x}) = \sum_{i=1}^n x_i r_i$$

In matrix form the return of the portfolio would be

$$\mathbf{r}(\mathbf{x}) = \mathbf{x}^T \mathbf{r}$$

Logarithmic returns are used throughout the thesis for the purpose of aggregating returns over time. Because logarithmic returns aggregate across time, the cumulative return of a particular return series at time t is

$$\sum_{t=1}^T r_t = \sum_{t=1}^T \ln(1 + r_t)$$

However, noting Meucci (2010), it is important to bear in mind that logarithmic returns do not aggregate across asset classes and that the two therefore should be used in a consistent manner.

We have decided to express average annual returns as an arithmetic average of returns. This is because of its superior statistical characteristics as well as to better reflect a constant dollar-exposure investor, i.e. one that invests and withdraws capital to keep constant exposure. The formula for arithmetic average is simply

$$R_{Arithmetic} = \frac{[r_i + r_{i+1} + \dots + r_T]}{T}$$

3.2 Equal risk contribution (ERC)

3.2.1 Theory behind MRC & TRC

To understand the intuition behind equal risk contribution we turn to the theoretical framework derived by Maillard et al. (2008). For each asset class to be correctly accounted for in the portfolio one must define its total risk contribution (TRC) as well as the marginal risk contribution (MRC). Take a portfolio $\mathbf{x} = (x_1, \dots, x_n)$ with n assets and variance σ_i^2 for asset i , let σ_{ij} be the covariance between asset i and j and $\mathbf{\Sigma}$ be the covariance matrix. Let $\sigma(\mathbf{x}) = \sqrt{\mathbf{x}^T \mathbf{\Sigma} \mathbf{x}}$ be the standard deviation of the portfolio.

The marginal risk contribution is the derivative of total portfolio standard deviation with respect to x_i and is the risk added to the portfolio by an infinitely small increase in the weight of asset i . Mathematically it is defined as:

$$\partial_{x_i} \sigma(\mathbf{x}) = \frac{\partial \sigma(\mathbf{x})}{\partial x_i} = \frac{x_i \sigma_1^2 + \sum_{j \neq i} x_j \sigma_{ij}}{\sigma(\mathbf{x})} = \mathbf{MRC} = \frac{\boldsymbol{\Sigma} \mathbf{x}}{\sqrt{\mathbf{x}^T \boldsymbol{\Sigma} \mathbf{x}}}$$

An asset i 's total risk contribution to the portfolio is simply its MRC multiplied by its weight.

$$\sigma_i(\mathbf{x}) = x_i \cdot \partial_{x_i} \sigma(\mathbf{x}) = \mathbf{TRC} = x_i \frac{\boldsymbol{\Sigma} \mathbf{x}}{\sqrt{\mathbf{x}^T \boldsymbol{\Sigma} \mathbf{x}}}$$

The risk of the portfolio is therefore the sum of the total risk contribution of each asset.

$$\sigma(\mathbf{x}) = \sum_{i=1}^n \sigma_i(\mathbf{x})$$

Furthermore, the optimal weight of assets \mathbf{x}^* is

$$\mathbf{x}^* = \{ \mathbf{x} \in [0,1]^n : \sum x_i = 1, x_i \cdot \partial_{x_i} \sigma(\mathbf{x}) = x_j \cdot \partial_{x_j} \sigma(\mathbf{x}) \text{ for all } i, j \}$$

Noting that $x_i \cdot \partial_{x_i} \sigma(\mathbf{x}) = \mathbf{TRC} = x_i \frac{\boldsymbol{\Sigma} \mathbf{x}}{\sqrt{\mathbf{x}^T \boldsymbol{\Sigma} \mathbf{x}}}$, and denoting $(\boldsymbol{\Sigma} \mathbf{x})_i$ as the covariance matrix of portfolio \mathbf{x} where i denotes the i^{th} row of the vector issued from the product of $\boldsymbol{\Sigma}$ with \mathbf{x} , we have that

$$\mathbf{x}^* = \{ \mathbf{x} \in [0,1]^n : \sum x_i = 1, x_i \cdot (\boldsymbol{\Sigma} \mathbf{x})_i = x_j \cdot (\boldsymbol{\Sigma} \mathbf{x})_j \text{ for all } i, j \}$$

The optimal portfolio construction denoted above takes into account the no short selling constraint, indicating weights between zero and one for each asset class. Furthermore, the sum of the weights of the individual asset classes satisfies the budget constraint, thus sums to one. The objective function is to equalize the total risk contribution of each asset class taking into account the covariance between the asset classes. That is, all asset classes should have the same behavioral impact on the portfolio risk.

Knowing that $\beta_i = \sigma_{ix}/\sigma^2(x)$ and $\sigma_i(x) = x_i\beta_i\sigma(x)$ the optimal weights $x_i = \frac{\beta_i^{-1}}{n}$. Therefore, the portfolio weight is inversely related to the beta of the individual asset classes. Note that the betas represent the covariance of each asset class with the constructed portfolio. However, in order to estimate the beta we need the portfolio weight, making the solution endogenous. We therefore need a numerical algorithm to deal with the endogeneity. Maillard et al. (2008) recommends using a sequential quadratic programming algorithm for solving the following optimization problem:

$$\begin{aligned} x^* &= \arg \min f(x) \\ \text{s.t.} \quad & \mathbf{1}^T \mathbf{x} = \mathbf{1} \text{ and } 0 \leq x \leq 1 \end{aligned}$$

where:

$$f(x) = \sum_{i=1}^n \sum_{j=1}^n (x_i(\Sigma x)_i - x_j(\Sigma x)_{jj})^2$$

For the thesis, the optimization problem will be solved using MATLAB's function "*fmincon*" with the option of sequential quadratic programming (*'sqp'*).

3.3 Benchmark investment strategies

3.3.1 60/40 Portfolio

The traditional 60/40 portfolio's objective is to maintain a 60% weight exposure to equities and 40% weight exposure to bonds. While this approach to asset allocation has long traditions, because equities have historically been much more volatile than bonds, the risk in the portfolio measured by the volatility has mainly been driven by equities. It is still widely used among both practitioners and private individuals.

3.3.2 Equally weighted portfolio (1/N)

The equally weighted portfolio seeks to equalize the weight of each asset class in the portfolio. The 1/N portfolio is viewed as a naïve portfolio strategy, however it is widely used in practice and some suggest that it even outperforms optimization strategies (DeMiguel et al. 2007).

The optimal portfolio weight in the 1/N strategy is simply:

$$x^* = \frac{1}{N}$$

3.3.3 Inverse volatility portfolio (IV)

The Inverse Volatility (sometimes referred to as Naïve Risk Parity) portfolio seeks to weight assets by the inverse of their volatility, hence the portfolio seeks to down weight more risky assets so that they have equal volatility impact to the portfolio. Mathematically, the weights of the portfolio are computed as follows

$$x^* = \frac{\frac{1}{\sigma_i}}{\sum_{n=1}^N \frac{1}{\sigma_i}}$$

The Inverse Volatility portfolio is easy to compute and is therefore computationally superior to the Equal Risk Contribution portfolio. However, it does not account for correlations between asset classes and thereby not correctly accounting for their risk behavior in a portfolio. Furthermore, because of this some asset classes might be unfavorably penalized in terms of their weighting in the portfolio. Moreover, looking at correlations between assets over time, we see that they vary quite severely leading to differences in their impact of a portfolio over time (see figure F & G).

3.3.4 Minimum variance portfolio (MV)

As stated earlier, the Minimum Variance portfolio is located at the leftmost of the efficient frontier. Chopra and Ziemba (1993) among others, suggests that portfolio weights are most sensitive to estimation errors related to the mean, while variance and covariance estimation affect less. The minimum variance portfolio is a good benchmark for our portfolio horserace because of the fact that it is estimated using only the covariance matrix, thus ignoring expected returns. Minimum variance aims to form optimal portfolio weights such that the overall portfolio variance is minimized. This portfolio is located on the efficient frontier in the sense that it offers the best possible return for a given level of risk. Theoretically, because it is the minimum variance portfolio on the efficient frontier, it should also have the lowest ex ante expected return. However, contrary to theory, the minimum variance outperforms in many cases other asset allocation strategies and delivers higher ex post returns. Following the theoretical framework of Clarke et. al (2012), the objective of the minimum variance function is to minimize ex-ante (i.e. estimated) portfolio risk. Portfolio risk is defined as

$$\sigma_p^2 = \mathbf{X}^T \Sigma \mathbf{X}$$

Where \mathbf{X} is an N -by- 1 vector of asset weights and $\mathbf{\Sigma}$ is an N -by- N asset covariance matrix. Braga (2012) proposes the following optimization program

$$\text{Min}_{\mathbf{w}^*} \left(\sum_{i=1}^N (w_i \sigma_i)^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N w_i w_j \sigma_i \sigma_j \rho_{ij} \right)$$

Using matrix notation, we get

$$\begin{aligned} & \text{Min}_{\mathbf{w}^*} \mathbf{w}' \mathbf{\Sigma} \mathbf{w} \\ & \text{s.t.} \\ & \mathbf{w}' \mathbf{e} = 1 \\ & [\mathbf{w}] \geq 0 \end{aligned}$$

The optimization programs are quadratic programming problems and can only be solved using a numerical iterative procedure. For that reason, the optimization problem will be solved using MATLAB's function "*fmincon*" with the option of *sequential quadratic programming* ('*sqp*').

3.4 Time series momentum (TSM)

The underlying assumption of time series momentum is that an assets' past returns predicts its future returns. This means that assets that previously have risen continue to rise and assets that have previously fallen continue to fall. We follow the time series momentum methodology proposed by Moskowitz et al (2012). The decision of whether to invest in a particular asset at time t is determined by the sign of the cumulative return over a lookback period minus the most recent return. The purpose of this thesis is not to optimize time series momentum and for those reasons we adopt fully the optimal lookback period and holding period developed by Moskowitz et al. (2012). They show that the optimal lookback period for most cases is 12 months and that the optimal holding period generally is 1 month. Thus, the TSM sign is calculated as follows:

$$TSM_sign = \sum_{k=1}^k (r_k) - r_k$$

And the return of the strategy is calculated as follows:

$$r_{t,t+1}^{TSMOM,s} = \text{sign}(r_{t-12,t}^s) \quad \text{for all } TSM_sign_i > 0$$

where $S_{i,t}$ is security i at time t .

3.4.1 Behavioral finance

The economic rationale behind time series momentum can be explained by the field of behavioral finance. Most of the theory behind this was developed by Kahneman and Tversky (1979) and relates to investors' behavioral biases. For momentum strategies to work, a security has to trade above or below its fundamental value. A common finding in academia is that major shifts in fundamental variables moving asset prices are met with under-reaction. Furthermore, the price trend often overextends fundamental value due to herding behavioral, which is the tendency for investors to buy and sell assets collectively. This type of behavior could for example be seen leading into the dot-com bubble of 2000 - 2002. Anchoring is a behavioral bias relating to an investors tendency to rely too heavily on initial information ("anchor") in decision making. Kahneman and Tversky (1979) finds that historical data provides a natural anchor for investors. Furthermore, Shefrin and Statman (1985) finds that investors tend to sell winners too early because they prefer to realize gains early and that they tend to delay selling losers in an attempt to avoid losses, generally referred to as the disposition effect. Kahneman and Tversky (1979) also points out that investors tend to seek out information which they already find to be true and that they regard recent price changes as representative of future return patterns.

3.5 Short selling

We allow for a long-short portfolio in the case of time series momentum. Further explanation is given in the methodology chapter.

3.6 Performance measurements

Portfolio evaluation involves the determination of how a portfolio has performed relative to some comparable benchmark. In order for the portfolio and the benchmark to be comparable, some risk adjustment has to be performed to capture return per unit of risk. Risk adjusted performance methods adjust the return to take into account the differences in risk levels between the portfolio and the benchmark. The performance evaluation methods regarded in this thesis will capture the essence of asset allocation from a long-term investor's perspective, namely risk adjusted return, portfolio risk as well as risk distribution.

3.6.1 Sharpe ratio

The Sharpe ratio is a portfolio evaluation measurement that computes the excess return of an asset or a portfolio per unit of total risk. The excess return is the return on the portfolio above the risk free rate (3 Month T-bill). Furthermore, the total risk is the standard deviation of returns on the portfolio or asset. The numerator captures the reward for investing in the portfolio or asset adjusted for the risk free rate while the denominator captures the variability of returns of the portfolio or asset (Encyclopedia of Finance). Furthermore, the Sharpe ratio is widely used for both practitioners and scholars. As the analysis in the thesis does not rely on expected returns because of its troubling features regarding its estimation, it is important to notice that the Sharpe ratio is solely used as a performance indicator using realized returns. Furthermore, it should be noted that the ratio is sensitive to both the sample period as well as the frequency of returns (i.e. daily, monthly, etc.) and should therefore be used primarily to decide upon dominance with regard to the risk-return relationship between two portfolios. The Sharpe ratio is a measure of the efficiency in terms of excess return per unit of risk and is given by

$$SR(x) = \frac{r(x) - r_f}{\sigma(x)}$$

Hence, for a portfolio with a higher Sharpe ratio than the benchmark we can conclude that the portfolio outperformed the benchmark in terms of excess return per unit of risk. However, we cannot say anything about the distribution of neither returns nor risk, which might reveal additional valuable information we might want to capture to evaluate the portfolio further. In order for us to be able to say something about total outperformance of two portfolios we need to turn to the maximum drawdown and rolling window returns.

3.6.2 Portfolio drawdown

While the drawdown of a portfolio does not say anything about the frequency of losses, it is an important measure of portfolio performance because it is a measure of capital preserved in the portfolio.

Using Chekhlov et al. (2005) for specifying maximum drawdowns we have that for a given time interval stretching from $[0, T]$, the maximum drawdown (MDD) of portfolio x will be equal to:

$$\text{MDD}(\mathbf{x}) = \max_{\tau \in [0, t]} (W_{\tau} - W_t)$$

3.6.3 Rolling window drawdowns

As it is an attractive trait of long-term asset allocation, we will measure the average of rolling window drawdowns as well as the max and min of rolling window drawdowns. Furthermore, we provide rolling window drawdowns over the entire sample period to fully grasp the drawdown of all portfolios and strategies. That way we can measure portfolios not only on their maximum drawdown but also in terms of their frequency of drawdowns.

3.7 Turnover and transaction cost

Transaction costs are important to consider as there typically is large variation between the different strategies. Furthermore, a typical finding seems to be that trading costs generally eat up excess returns of more complex strategies, leaving them at par or even worse off than naïve or simple strategies. For the purpose of estimating transaction costs, the framework of Anderson et al. (2012) will be used. Moreover, the budget constraint of all portfolios throughout the thesis is that at all rebalancing dates, it needs to be fully invested, hence $\mathbf{x}^T \mathbf{1}$, that is the sum of the weights adds to one. At period $t+1$ the portfolio will be subject to rebalancing due to its affection to prices, hence for any strategy, the modification of weights needed to asset i at time t is:

$$\tilde{x}_{i,t}^* = \frac{(1 + r_{i,t})x_{i,t-1}}{\sum_j (1 + r_{j,t})x_{j,t-1}}$$

and the turnover required to balance is given by:

$$\text{Turnover}(\mathbf{x}) = \sum_j |\tilde{x}_{i,t}^* - x_{j,t}|$$

Trading costs for any given strategy at time t is therefore given by:

$$c_t = \text{Turnover}_t(\mathbf{x}) \cdot z_t$$

where z_t is the transaction cost measured in basis points (bp). The trading cost adjusted returns are given by:

$$r_{c,\lambda-adj} = r_t - c_t$$

Chapter 4 – Methodology

We will construct the relevant portfolios using the appropriate theoretical techniques developed, and using MATLAB for all computational purposes. The portfolio construction is divided into two cases, one using the broad asset classes (BAC), which will serve as a benchmark for both the ERC portfolio as well as the TSM strategies. Second, we will construct time series momentum (TSM) on the constituents of the broad asset classes, referred to as TSM strategies. The BAC case is a horserace going back approximately 45 years with progressive adding of asset classes. The TSM case is a horserace going back approximately 27 years.

4.1 Broad asset classes (BAC)

For the BAC case, we add asset classes progressively as they are available. Starting in 1973/01, Dev Eqty, Us Corp Debt, Commodities and US Real Estate are available. Global Sovereign Debt enters in 1985/12 and EM Eqty enters in 1987/12. For computational purposes, we therefore have three cases, one for four asset classes, one for five and one for six.

4.1.1 Covariance matrix estimation BAC

The covariance matrix is estimated using an estimation window of 24 months and rolled forward one month for each computation. All data is treated as an $n \times m$ matrix with n being the length of the data and m being the number of asset classes. For asset classes that have not yet entered, values are replaced with MATLAB's "nan", referring to "not a number". Furthermore, it is important to note that $\text{cov}(\text{nan}, r_{i,t}) = \text{nan}$ and that the covariance matrix is symmetric. Thereby we can easily remove all "nan" values after the covariance matrix is estimated to achieve the proper size for each of the three cases. We have programmed a rolling window covariance matrix in MATLAB for calculation of rolling window covariance matrices for the entire sample period (see Appendix Matlab Code).

4.1.2 Portfolio construction BAC

All portfolios are constructed using monthly rebalancing. The purpose of this thesis is not to develop a trading strategy, but rather to evaluate strategies. Therefore, because all strategies are treated similarly, they are comparable. It is

important to note that monthly rebalancing will incur higher portfolio turnover. Furthermore, to adjust for the frequent rebalancing, we have calculated average annual turnover for all portfolios. Performance will have to be seen in relation to this. All returns are logarithmic and computed in excess of the risk-free rate. The risk-free rate in our analysis is the 3-month T-Bill rate, which is de-annualized to monthly returns. Furthermore, all portfolios are always fully invested, that is all weights sum to one for all periods. To ensure out-of-sample testing, all estimates are used to invest in the next period. For example, the covariance matrix based on returns from period 1-24 is used to invest in period 25.

The 60/40 portfolio is a constant 60% weighting in developed equities and a 40% constant weight in US corporate debt. Furthermore, 1/N is simply 1/N multiplied by the return of the next period. For the IV portfolio, we calculate weights from the diagonal elements of the covariance matrices used and then multiply with the respective return in the next period. For ERC and MV, we use the rolling window covariance matrix to minimize the respective objective function for the two strategies using MATLAB's *fmincon*. We then make a looped program such that the process is carried forward to the next investable period. For a fair comparison between all strategies we have decided not to have any restrictions on the weights for any of the strategies. We find that ERC and MV are well diversified during most of the sample period. However, during extreme bull or bear markets, both tend to invest heavily in a few concentrated asset classes, thus not making them particularly diversified. Lastly, because 60/40 & 1/N do not rely on the covariance matrices, the returns of these strategies will be longer than that of the others. To adjust for this, we delete the returns exceeding the length of the other strategies.

4.2 Time series momentum

For the TSM case we use the constituents of the broad asset classes to investigate whether we can identify winners and losers within the broad asset classes. For this analysis, we follow closely the framework of Moskowitz et al. (2012). We have a total of 94 indices within the six broad asset classes that we will include in the TSM analysis. For a simpler computational process, we start our horserace where all the securities are available (in contrast to progressive adding). There is a somewhat equal number of securities within the broad asset classes to ensure a

fair comparison between asset classes. Furthermore, every period we calculate the cumulative returns for each security and subtract the last months return, consistent with the framework of Moskowitz et al. (2012). This variable is the TSM_sign variable in which investment decisions are based upon. We perform the TSM filter on all strategies and size our positions according to 1/N, ERC, IV and MV weighting scheme.

4.2.1 TSM long-only

For the TSM Long-Only case, we invest in only securities who's TSM_sign is positive. Furthermore, if the asset has a negative TSM_sign, we sell it and trade into cash. The only difference in the TSM case relative to the BAC is that the covariance matrix has to be calculated differently because the investable securities varies with the sign of TSM_sign. All computations and objective functions are equal.

4.2.2 Covariance matrix estimation TSM long-only

Covariance estimation for the TSM case is more complex than that of the BAC. For each period, we invest only in the assets who's TSM_sign is positive. However, because the covariance matrix is symmetric, we can first estimate it for the entire sample, and then remove all rows and columns relating to securities who's TSM_sign is negative. This method is unproblematic to implement in MATLAB. Furthermore, for each period we store the total number of securities in the period as well as the number of securities with positive TSM_sign, we call this variable $w_{\text{risky_assets}}$. This weight equals the weight in risky assets and one minus this weight equals the weight we invest in cash for each period. This way we can calculate returns by multiplying the return of the TSM strategy with the weight in risky assets and the return of the risk-free rate with one minus the weight in risky assets.

4.2.3 TSM long-short

We allow for one long-short scenario with the TSM filter. We have recognized that the variance of a short only portfolio is equal to the variance of the same portfolio being long only. Furthermore, using the fact that the return of a strategy is equal to the weight invested in the strategy multiplied by the return of the

strategy, we simply calculate a long and short weight for all periods and multiply by the returns. All optimization programs and objective functions are running on long only objectives, however weights for the short-only portfolio are reversed after optimization is complete. Finally, we multiply the long only and short only returns with the net long exposure fraction for every period. This enables the portfolio to always invest in all securities and be fully invested while allowing for long-short positions. The main drawback of this method is that the strategies are not always net long 100%. Performance characteristics has to be seen in relation to the net long exposure.

4.2.4 Covariance matrix estimation TSM long-short

We know that each period, we will either go long or short, depending on the sign of TSM_sign for each security. Knowing that, we create two portfolios within each strategy, one long only and one short only. That means that we estimate two covariance matrices, one based on the long only securities and another based on the short only securities. Again, this is unproblematic to implement in MATLAB (see Appendix Matlab Code). It is important to note that all securities are used every period and that none of the securities are used in both long only and short only portfolios.

4.3 Performance measurement

4.3.1 Mean

The average return over a period of time is calculated using the arithmetic mean. To annualize the mean monthly returns, we take the exponential of the monthly return multiplied by the number of months in the period and finally subtract one.

4.3.2 Standard deviation

The standard deviation of returns is calculated by taking the sample standard deviation of monthly returns and multiplying with the square-root of the number of months. That is for annual standard deviation, we multiply the sample standard deviation by the square-root of 12.

4.3.3 Rolling window returns

Rolling window returns are calculated on a 36-month (3yr) and 60-month (5yr) period. The rolling window returns reveals holding period returns over all 3- and 5 year holding periods for all strategies.

4.3.4 Maximum drawdown

The maximum drawdown of all portfolios are computed as the maximum loss from a peak to a bottom for a specific portfolio through time. MDD therefore shows the maximum drawdown of a portfolio through time and can be found in.

4.3.5 Cumulative return

Wealth plots are calculated as the cumulative sum of the logarithmic returns of a strategy through time. All strategies within a plot have the same investment period and length.

4.3.6 Net exposure long-short

For the purpose of estimating returns for the long-short strategy we needed to calculate net long exposure through time. Figure shows the net exposure of all TSM long-short strategies through time as well as the historical average.

4.3.7 Turnover and transaction costs

The turnover of each portfolio is calculated using the formula laid out in the theory part. Furthermore, transaction costs are calculated by multiplying the annual turnover with the average transaction cost for asset classes. We have used transaction costs proposed by Jones and Charles (2002), amounting to an average of 20bp. Transaction cost adjusted returns and Sharpe ratios can be found in appendix A.

4.3.8 Correlation

Figure F shows the average pairwise correlation of the broad asset classes over time as well as a historical average. Moreover, it is the average pairwise correlation based on a 36-month estimation window for Dev Eqty, Us Corp Debt, Commodities, US Real Estate, Global Sovereign Debt and EM Eqty. Figure G

shows the correlation matrix of broad asset classes over the entire investment period (1987/12 to 2017/06).

4.3.9 Total risk contribution

Figure K & L shows the total risk contribution of each asset class for a strategy in the BAC case. TRC for ERC is not constant through time because relative volatility of certain assets (say EM Eqty and Global Sovereign Debt) varies across time. In some periods, the volatility of a certain asset will be larger than what ERC can compensate in terms of weights. This is mainly a data issue and for further analysis we recommend that the variance of asset classes should not be significantly large.

4.3.10 Regression analysis

To get a better understanding of exposure of the strategies, we have performed several regression analyses. We follow the framework of Moskowitz et al (2012) and use Fama/French 5 Factor + UMD for US data and Fama/French 3 Factor + WML for global data. Fama/French 3 factor + WML, see Fama and French (1993) and Fama/French 5 factor + UMD, see Fama and French (2014), captures the following

- $R_{mkt} - r_f$ is the return on the market, value weighted return of all US incorporated firms listed on NYSE, NASDAQ or AMEX excess of the one month T-Bill rate.
- SMB (small minus big) is the average return on small stocks minus the average return on large stocks.
- HML (high minus low) is the average return on value portfolios minus the average return on growth portfolios.
- RMW (robust minus weak) is the average return on robust operating profitability portfolios minus the average return on weak operating profitability portfolios.
- CMA (conservative minus aggressive) is the average return on conservative investment portfolios minus the average return on aggressive investment portfolios.
- UMD (up minus down) is the average return of winners minus the average return of losers.

- WML (winners minus losers) is the average returns

The following regressions are performed

$$R_{i,t}^P - r_{f,t} = \alpha_i^P + \beta_{1,i}^P \text{MKT}_t + \beta_{2,i}^P \text{SMB}_t + \beta_{3,i}^P \text{HML}_t + \beta_{4,i}^P \text{UMD}_t + \varepsilon_{i,t}^P$$

$$R_{i,t}^P - r_{f,t} = \alpha_i^P + \beta_{1,i}^P \text{MKT}_t + \beta_{2,i}^P \text{SMB}_t + \beta_{3,i}^P \text{HML}_t + \beta_{4,i}^P \text{RMW}_t + \beta_{5,i}^P \text{CMA}_t + \beta_{6,i}^P \text{WML}_t + \varepsilon_{i,t}^P$$

$$R_{i,t}^P - r_{f,t} = \alpha_i^P + \beta_{1,i}^P \text{DEV}_t + \beta_{2,i}^P \text{CORP}_t + \beta_{3,i}^P \text{COM}_t + \beta_{4,i}^P \text{USRE}_t + \beta_{5,i}^P \text{GSOV}_t + \beta_{6,i}^P \text{EM}_t + \varepsilon_{i,t}^P$$

4.4 Limitations

We are aware of the fact that our covariance matrices weights all observations equally across the entire sample period. For further analysis, one should consider adjusting the estimation to weight recent observations more heavily to reflect proper volatility in markets, see Litterman (2003). Furthermore, for increased robustness, one should consider the use of shrinkage estimators to adjust the covariance matrix, see Ledoit (2003).

We are aware of the uncertainty regarding liquidity and thus tradability of the indices we have used. Some of the securities proposed might be illiquid and thus expensive to trade in, which might alter the performance of the strategies. Furthermore, we assume that short selling is as costly as buying securities, which might not be the case in markets.

Chapter 5 – Data

To evaluate the potential value of combining risk based weighting schemes and TSM to asset allocation we select six broad asset classes as represented by reputable financial market index providers to act as benchmark strategies. The asset classes are as follows with sources in brackets: developed economy equities represented by MSCI World (Datastream), emerging market equities by MSCI Emerging Markets (Datastream), global sovereign bonds by JP Morgan Global Government Bond Index (Datastream), commodities by S&P Goldman Sachs Commodity Index (Bloomberg) and U.S. real estate by FTSE NAREIT US Real Estate Index (Bloomberg). We use only monthly data; all indices are in total

return format denominated in US dollars and are unhedged. Descriptive statistics are presented in table A and B.

Moreover, to properly implement the combined TSM and risk based weighting strategies we select 94 individual country level indices within each broad asset class or in the case of commodities, individual commodities. For developed equities, we collected data from 23 markets, for emerging market equities we collected data from 21 markets, we used 22 commodities, 11 real estate markets and 17 sovereign debt markets. These are also in total return format and are as follows:

For developed economy equities, we use the following country level indices provided by MSCI (Datastream): Canada, United States, Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, Australia, Hong Kong, Japan, New Zealand and Singapore.

For emerging economy equities, we use the following country level indices provided by MSCI (Datastream): Brazil, Chile, Colombia, Mexico, Peru, Czech Republic, Egypt, Hungary, Poland, Qatar, Russia, South Africa, Turkey, United Arab Emirates, China, India, South Korea, Malaysia, Pakistan, Philippines and Thailand.

For sovereign debt, we use the following country level indices provided by Bank of America Merrill Lynch (Datastream): United States, Canada, Switzerland, France, Australia, Netherlands, United Kingdom, Japan, Germany, New Zealand, Sweden Italy, Ireland, Denmark, Belgium, Spain and Norway.

For commodities, we use the following commodity indices provided by S&P GSCI (Bloomberg): Crude Oil, Brent Crude, Unleaded Gasoline, Heating Oil, Gas Oil, Natural Gas, Aluminum, Copper, Lead, Nickel, Zinc, Gold, Silver, Wheat, Soybeans, Cotton, Sugar, Coffee, Cocoa, Live Cattle, Feeder Cattle and Lean Hogs.

For real estate, we use the following country level indices provided by FTSE EPRA/NAREIT (Bloomberg): United States, Japan, Hong Kong, Australia, Germany, Canada, Singapore, United Kingdom, Netherlands, France and Sweden.

More detailed overview of the country level data is presented in appendix B

Chapter 6 – Results and Analysis

6.1 Broad asset classes

We start our analysis by examining the six broad asset classes making up our benchmark portfolio for subsequent investment strategies. What follows are graphical display of the cumulative excess returns for the broad asset classes.

Figure A

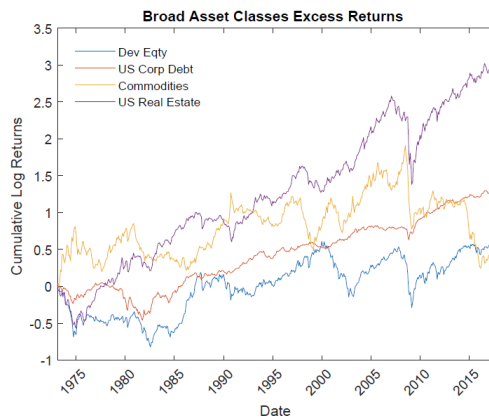


Figure B

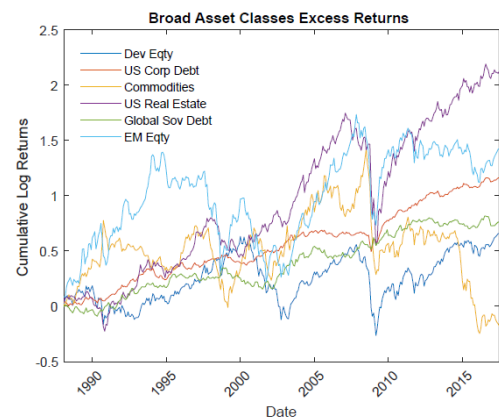
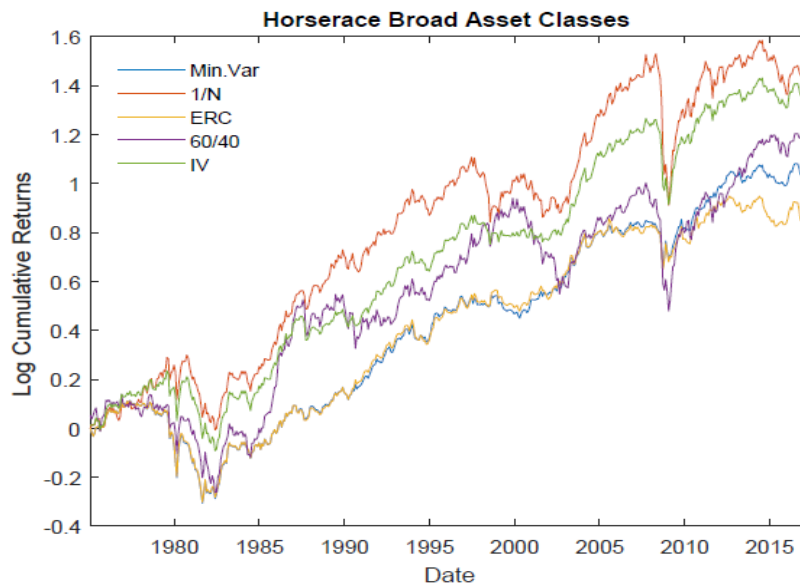


Figure C



Looking at table A, the overall mean excess return over the sample period range from -0.55% for commodities to 8.31% for US real estate. The rest of the asset classes provide relatively modest returns in the range of 2% and 4%. On a risk adjusted basis US corporate debt and US real estate perform best with Sharpe ratios of 0.49 and 0.48, respectively. Emerging market equities, US real estate and commodities suffers from large monthly negative returns in the -30% to -40% range.

Table A – Performance statistics for broad asset classes (1975.02-2017.06)

The table summarizes performance statistics for each benchmark constituent. Return are reported in excess of the risk-free rate (3-month T-bill from St. Louis FED). All numbers are annualized. Developed equities, US corporate debt, commodities and US real estate data starts in 1973.02, while global sovereign debt runs from 1986.01 and emerging market equities enter in 1988.01.

Performance statistics for broad asset classes (1973.02 - 2017.06)					
	Mean return	Stdev	Sharpe Ratio	Max	Min
Dev Eqty	2.77%	14.61%	0.19	10.49%	-21.14%
US Corp Debt	3.40%	6.90%	0.49	11.44%	-9.36%
Commodities	-0.55%	19.39%	-0.03	20.08%	-33.14%
US Real Estate	8.31%	17.15%	0.48	27.00%	-38.10%
Global Sov Debt	2.28%	5.68%	0.40	6.25%	-5.29%
EM Eqty	3.42%	19.42%	0.18	15.97%	-35.03%

Furthermore, table B provides comparable statistics as all asset classes are present over the whole period from 1990 to 2017. The statistics remain relatively similar for all asset classes except for US corporate debt. Over the period, returns increase, volatility decreases consequently increasing the sharpe ratio from 0.49 to 0.77, due to the recession in the early 1980s.

Table B – Performance statistics for broad asset classes (1990.01-2017.06)

The table summarizes performance statistics for each benchmark constituent. Return are reported in excess of the risk-free rate (3-month T-bill from St. Louis FED). All numbers are annualized. All asset classes run from 1990.01. This makes them comparable to both to each other and the strategies.

Performance statistics for broad asset classes (1990.01 - 2017.06)					
	Mean return	Stdev	Sharpe Ratio	Max	Min
Dev Eqty	2.26%	14.84%	0.15	10.34%	-21.14%
US Corp Debt	4.02%	5.23%	0.77	6.56%	-8.14%
Commodities	-0.61%	20.82%	-0.03	20.08%	-33.14%
US Real Estate	7.46%	18.50%	0.40	27.00%	-38.10%
Global Sov Debt	2.60%	6.41%	0.41	6.25%	-5.29%
EM Eqty	4.94%	23.28%	0.21	15.97%	-35.03%

6.2 TSM combined with risk based allocation strategies

Now, we turn to the strategies that will be applied in combination with TSM later. To see how robust the selected risk based allocation schemes are under different economic environments we chose to test them over the entire sample period and compare them to the more recent period of 1990 to 2017 as can be seen in table D. The two buy and hold strategies, 1/N and 60/40 perform worse in the more recent sample, while the risk based strategies performs better. The volatility of the buy and hold strategies are similar in both periods, however, returns drop resulting in lower Sharpe ratio. On the other hand, for the risk based strategies, volatility decreases while returns increases. This could be explained due to these strategies typically being more concentrated in debt instruments which aligns with our findings that US corporate debt suffers in the early 1980s. We note that the literature on risk based strategies typically have a shorter sample period, thus excluding this bond bear market. Moreover, the US interest rate peak in 1981 and has since been in a negative trend. We note that the interest rate environment has a significant impact on the strategies and consider this a potential pitfall going forward in an increasing rate environment.

Table C – Performance statistics for benchmark strategies (1975.02-2017.06)

The table summarizes performance statistics for each risk based allocation strategy applied to the six broad asset classes. Return are reported in excess of the risk-free rate (3-month T-bill from St. Louis FED). All numbers are annualized. Asset classes are added progressively as they become available.

Performance statistics for benchmark strategies on broad asset classes (1975.02 - 2017.06)							
	Mean return	Stdev	Sharpe Ratio	Max	Min	Skewnes	Kurtosis
1/N	3.59%	9.73%	0.37	9.08%	-22.12%	-1.49	11.87
MV	2.56%	6.27%	0.41	9.67%	-8.84%	-0.42	7.27
IV	3.38%	7.35%	0.46	6.04%	-14.25%	-1.16	8.94
ERC	2.09%	6.54%	0.32	10.05%	-8.73%	-0.37	6.85
60/40	3.02%	9.97%	0.30	7.77%	-15.36%	-0.70	5.44

Having provided some background information and the benchmark strategies, we now move to the case of combining TSM and our risk based strategies. The purpose of this paper is to test if adding TSM to ERC and other risk strategies can provide a solution to their typical low returns when not applying leverage. Table D summarizes our findings.

Table D

Performance statistics with TSM on country level data and benchmarks (1990.01 - 2017.06)								
	Mean return	Stdev	Sharpe Ratio	Turnover	Max	Min	Skewness	Kurtosis
1/N	2.83%	10.36%	0.27	25.71%	9.08%	-22.12%	-1.6349	12.7858
1/N TSM LO	4.23%	5.93%	0.71	26.31%	5.28%	-6.82%	-0.5309	5.1188
1/N TSM LS	5.01%	7.22%	0.69	26.31%	10.40%	-7.98%	-0.1666	5.8256
MV	3.34%	5.42%	0.62	12.11%	6.54%	-7.97%	-0.6903	6.8184
MV TSM LO	3.37%	3.10%	1.09	26.00%	2.59%	-3.10%	-0.2188	3.4981
MV TSM LS	3.87%	5.10%	0.76	26.64%	12.34%	-5.47%	1.6612	17.1559
IV	3.42%	6.98%	0.49	17.16%	5.90%	-14.25%	-1.3833	11.321
IV TSM LO	4.24%	5.93%	0.72	21.61%	5.43%	-4.42%	-0.3398	3.9813
IV TSM LS	2.54%	3.18%	0.80	57.97%	4.28%	-3.16%	-0.1924	5.2878
ERC	2.62%	5.86%	0.45	12.94%	6.67%	-8.18%	-0.6594	6.2384
ERC TSM LO	3.81%	3.84%	0.99	21.78%	4.71%	-3.65%	-0.1636	4.2107
ERC TSM LS	4.47%	5.38%	0.83	23.63%	12.98%	-5.63%	1.612	17.3637
60/40	2.65%	9.86%	0.27	25.73%	7.56%	-15.36%	-0.956	6.0956

Across, all strategies benefit significantly from the TSM filter. We start by looking at the long only scenarios and note that mean returns for all strategies increases, 1/N benefits the most when looking at returns, IV and ERC follows closely, however, MV lags behind. The volatility of the returns decreases significantly even though they by definition are low volatility strategies. The combination of higher returns and lower volatility boost the Sharpe ratios by a factor of approximately x2. In terms of risk-adjusted returns, MV perform the best, closely followed by ERC, however, the returns of these two are in the lower range.

Figure D

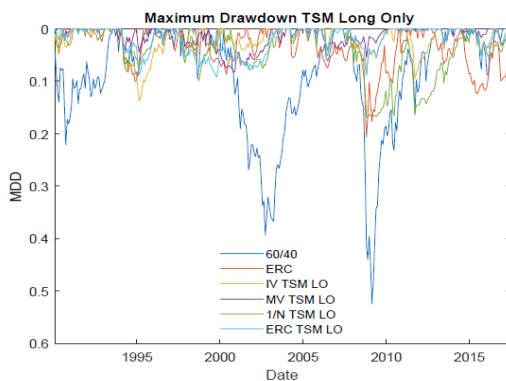
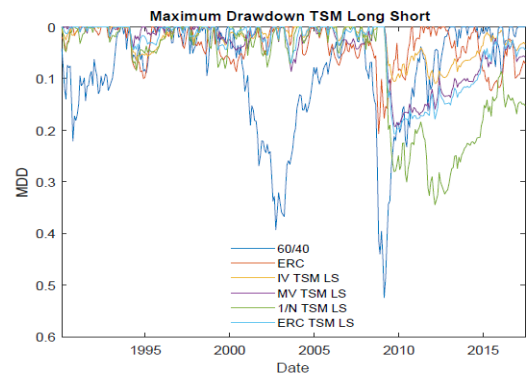


Figure E



It is worth mentioning that the MV strategy barely improve its returns, while the ERC strategy improves its returns by 1.19%. Since TSM indirectly implement a market timing component, we expected to see less extreme observations for both the maximum and minimum monthly return. It turns out that this is true, however to a much lesser extent for the maximum observations. We suspect that the

increased mean returns are mainly driven by avoiding large drawdown rather than providing large positive returns. We base this argument on the observation related to significantly decreased minimum monthly returns. We can also relate this to figure D & E, showing that not only minimum monthly returns are lower, but portfolio drawdowns are significantly reduced throughout.

For all long-short strategies, but IV, mean returns increase thus being the strategy that provides the highest returns. However, the volatility increases as well, but proportionally to the increase in returns its does not add value on a risk-adjusted basis. Hence, Sharpe ratios decrease across. We note that the IV portfolio behaves differently than the other portfolios. The IV portfolio effectively ignores pairwise correlations, an explanation to this could be the dramatic increase in average correlations following the financial crisis (see figure F). Baltas (2015) argue that the IV portfolio leads to uneven and therefore suboptimal risk allocation under such conditions as can be seen in figure E.

Figure F

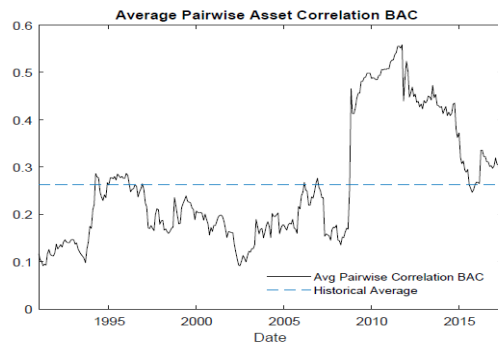
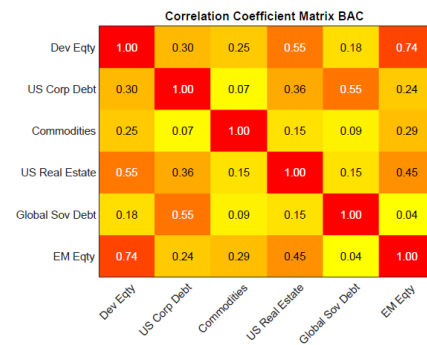


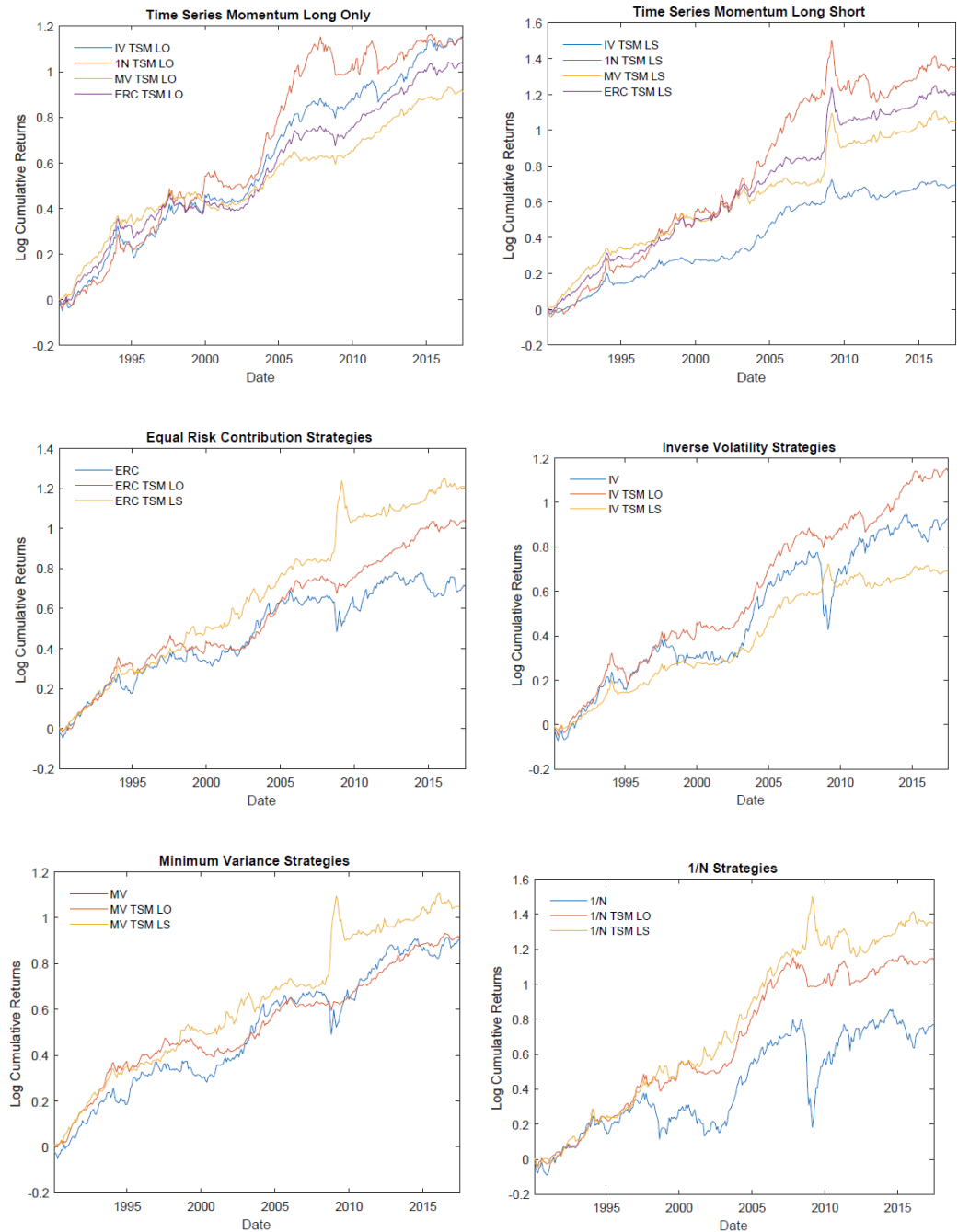
Figure G



It seems that more sophisticated methods such as ERC and MV add value under such environments. The main benefit with the long-short strategy is that it significantly increases returns in distressed markets that exhibit large variation in returns. This is evident from Figure H where all long-short strategies are able position themselves advantageously when markets turn for the worse. Allowing short selling appears to increase returns at the cost of drawdowns. Comparing figure D and E, drawdowns increase primarily from the financial crisis and onwards. Adding TSM to risk strategies provides very attractive benefits to the portfolios, however, turnover and thus transaction cost almost doubles for the MV and ERC case to approximately 25% annual portfolio turnover. 1/N is less affected with only moderate increase in turnover, while IV suffer the greatest (see

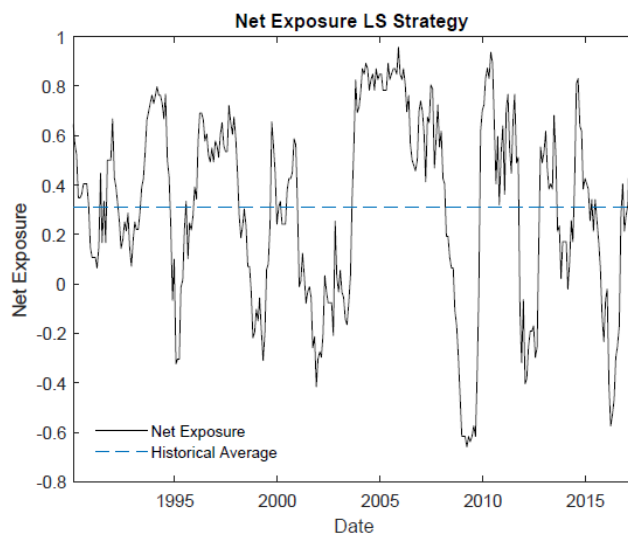
appendix A). Compared to the 60/40, the main benefits of these strategies appear to be the ability to control and lower risk.

Figure H



Regarding the feasibility of the long-short strategies we examine both the net exposure over time as well as the average. Because the strategy attempts to time the market both when it is going up and down, one must expect the net exposure to vary over time depending on the current state of the market. As can be seen in figure I, the net exposure for the long-short strategy vary from almost being entirely net long to 60% net short. Firstly, the variation in the exposure can provide challenges for investors seeking a stable exposure. Furthermore, the periods with large negative exposure may not be doable for the majority of market participants. Even though there is a great variation in the net exposure, the strategy has over the last 27 years been on average approximately 33% net long. Investors that is invested in the strategy for a short period of time cannot expect the average net exposure since the current market conditions determines the positions. However, since markets tend to trend upwards over the long run, investors sitting long can expect a net long exposure.

Figure I



6.3 On the robustness of the strategies

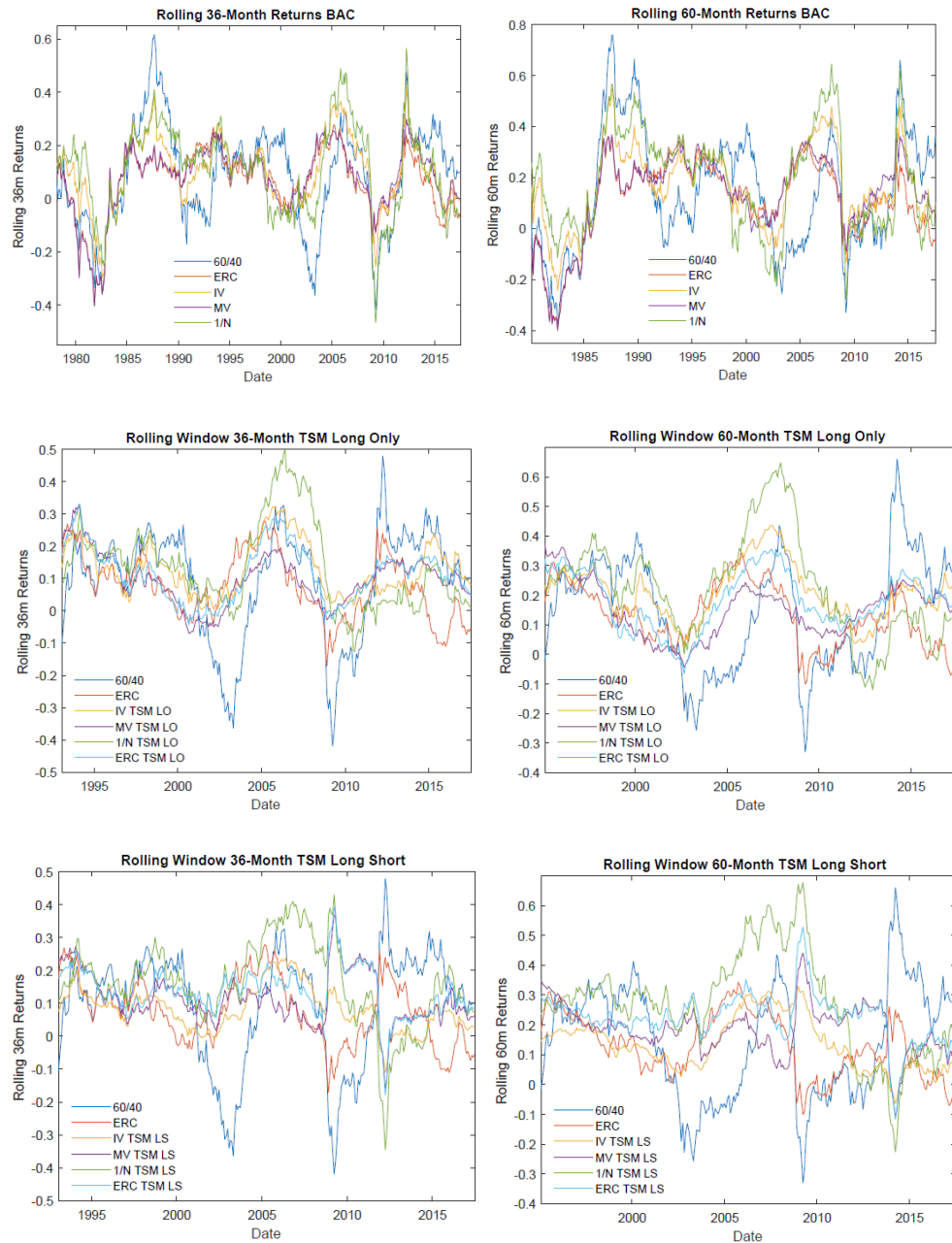
To get a better grasp of the portfolio performances we calculate the 36 and 60-month rolling window returns. As opposed to the above performance analysis, rolling window returns allows us to evaluate the performance over a wide range of holding periods, thus putting the robustness of the strategies through a more comprehensive test. We have chosen to compute the 36-month rolling window as well as the 60-month return with rolling intervals of 1 month (see table E and figure J).

Table E

	Performance statistics rolling window returns							
	Mean return		Stdev		Max		Min	
	36m	60m	36m	60m	36m	60m	36m	60m
1/N	10.25%	18.00%	16.36%	19.27%	56.55%	64.45%	-46.64%	-27.96%
1/N TSM LO	13.57%	22.64%	13.16%	17.22%	49.77%	64.77%	-12.58%	-11.99%
1/N TSM LS	16.16%	26.88%	12.76%	17.14%	42.95%	67.67%	-34.44%	-22.67%
ERC	6.23%	11.12%	12.67%	15.99%	27.78%	36.32%	-40.13%	-39.32%
ERC TSM LO	11.60%	18.32%	8.53%	9.85%	32.92%	35.57%	-4.45%	-6.56%
ERC TSM LS	13.69%	22.52%	7.23%	8.28%	39.22%	52.75%	-17.85%	-11.73%
IV	9.85%	16.75%	13.01%	15.45%	42.32%	54.66%	-29.66%	-24.35%
IV TSM LO	13.26%	21.09%	8.41%	9.35%	32.79%	43.64%	0.37%	2.22%
IV TSM LS	8.27%	13.56%	6.71%	8.72%	23.72%	32.86%	-10.96%	-6.40%
MV	7.48%	13.07%	12.81%	16.35%	30.46%	37.30%	-40.41%	-40.03%
MV TSM LO	9.77%	15.14%	7.96%	9.12%	32.61%	37.33%	-6.96%	-4.45%
MV TSM LS	11.51%	18.60%	7.74%	8.21%	36.75%	44.06%	-15.94%	-10.57%
60/40	7.66%	12.89%	19.11%	23.57%	61.61%	75.92%	-41.84%	-37.08%

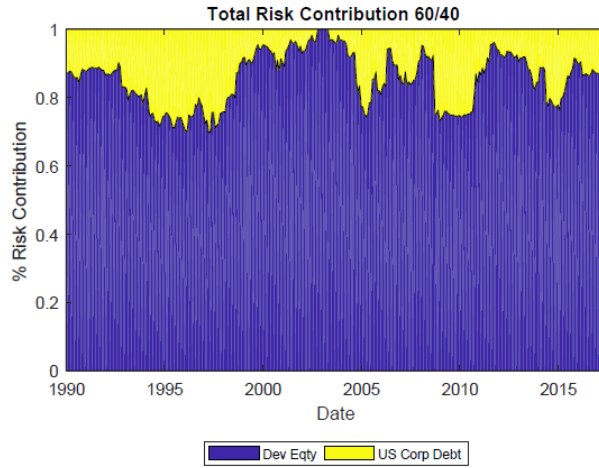
For all portfolios, but the IV, average rolling window returns increase consistently from the benchmark case through the long-only to the long-short portfolio. This is consistent for the portfolio volatilities as well, only these are decreasing. Worth noticing is that maximum rolling window returns for both holding periods are quite different for the two cases. It seems that the longer you hold a strategy, the higher maximum return you get. However, this not true for the minimum returns. In fact, rolling window minimum returns are quite similar between the two holding periods for ERC, IV, MV and the 60/40 portfolio suggesting that strategies have the ability to avoid drawdowns even in the fairly short time periods. A strategy that stands out is the IV TSM LO which both in the 36 and 60-month case never experience negative rolling returns. During both holding periods, all strategies except for the simple ERC have higher mean returns than the 60/40 portfolio at a lower volatility.

Figure J



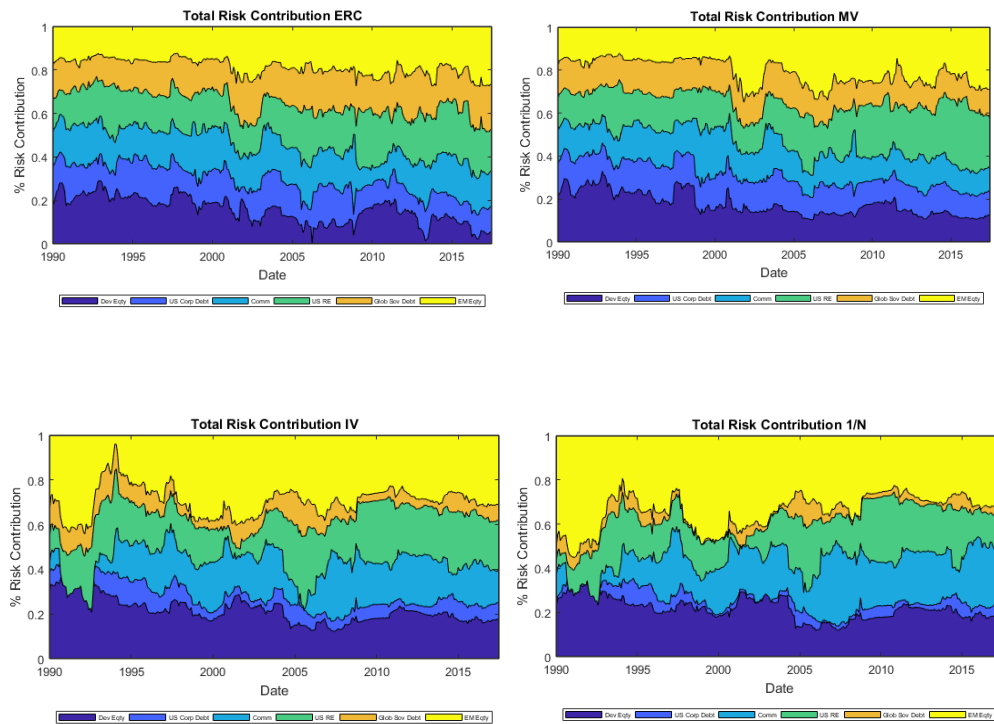
In order to assess how efficiently each of the portfolio strategies balance risk between the asset classes we have computed total risk contribution (TRC) for all portfolios in the benchmark strategies. For the 60/40 portfolio, the portfolio risk is mainly driven by equities. As we can see in figure K, during certain periods the portfolio risk tend to mimic that of a pure equity portfolio. This is of great concern regarding the portfolios ability to diversify between stocks and bonds.

Figure K



From figure L, we see that the ERC and the MV portfolio are superior in allocating risk evenly among the assets. For the IV and 1/N portfolios, the more risky assets (developed market equities, commodities, US real estate and emerging market equities) contribute much more to the portfolios total risk.

Figure L



6.4 Risk factor exposures

In table F we regress the benchmark strategies on the Fama French US 5-factor + UMD, as well as Fama French global 3-factor + WML. In panel C we regress the respective portfolios on the benchmark constituents. From panel A, all strategies are significantly exposed to the market factor for both the US and the global data. 60/40 and 1/N are the most exposed strategies, while ERC and MV are the least exposed. Both for US and global data, the 60/40 portfolio is moderately negatively exposed to the SMB factor. From panel B, MV, IV and 1/N is moderately exposed to the SMB and HML factors. Not surprisingly, in panel C, the ERC and MV portfolio are heavily exposed to the debt instruments. ERC are also moderately exposed to emerging market equities. IV is heavily exposed to the debt instruments, while also being moderately exposed to all benchmark constituents.

Table F

Panel A: Broad Asset Classes with Fama French US 5-Factor + UMD									
		Intercept	MKT	SMB	HML	RMW	CMA	UMD	R ²
ERC	Coefficient	0.0011	0.1360	-0.0325	-0.0044	0.0121	0.0590	0.0143	9.19%
	<i>(t-statistic)</i>	(1.16)	(5.31***)	(-0.98)	(-0.10)	(0.28)	(0.94)	(0.71)	
MV	Coefficient	0.0016	0.1430	-0.0114	-0.0019	0.0244	0.0575	0.0078	12.00%
	<i>(t-statistic)</i>	(1.84**)	(6.14***)	(-0.38)	(0.05)	(0.61)	(1.00)	(0.42)	
IV	Coefficient	0.0002	0.3380	0.0318	0.0617	0.0368	0.0382	-0.0005	47.50%
	<i>(t-statistic)</i>	(0.29)	(14.52***)	(1.06)	(1.53*)	(0.93)	(0.67)	(-0.03)	
1/N	Coefficient	0.0015	0.5340	0.0958	0.1410	0.0046	0.0100	-0.0090	60.30%
	<i>(t-statistic)</i>	(-1.39*)	(17.76***)	(2.46***)	(2.7***)	(0.07)	(0.14)	(-0.38)	
60/40	Coefficient	-0.0017	0.6160	-0.0876	0.0129	-0.0028	0.0739	-0.0082	79.40%
	<i>(t-statistic)</i>	(-2.27**)	(29.87***)	(-3.28***)	(0.36)	(-0.08)	(1.47*)	(-0.51)	

Panel B: Broad Asset Classes with Fama French Global 3-Factor + WML							
		Intercept	MKT	SMB	HML	WML	R ²
ERC	Coefficient	0.0009	0.1660	0.0526	0.0647	0.0377	15.90%
	<i>(t-statistic)</i>	(0.95)	(7.59***)	(1.18)	(1.59*)	(1.56*)	
MV	Coefficient	0.0016	0.1680	0.0595	0.0606	0.0276	19.20%
	<i>(t-statistic)</i>	(1.93**)	(8.46***)	(1.48*)	(1.65**)	(1.26)	
IV	Coefficient	0.0005	0.3860	0.1008	0.1330	0.0236	62.00%
	<i>(t-statistic)</i>	(0.69)	(21.92***)	(2.83***)	(4.10***)	(1.22)	
1/N	Coefficient	-0.0014	0.6270	0.2090	0.1990	0.0160	75.50%
	<i>(t-statistic)</i>	(-1.60*)	(29.77***)	(4.92***)	(5.11***)	(0.69)	
60/40	Coefficient	0.0004	0.6390	-0.1070	0.0105	0.0036	94.40%
	<i>(t-statistic)</i>	(-1.13)	(69.05***)	(-5.74***)	(0.61)	(0.35)	

Panel C: Broad Asset Classes with Benchmark Constituents									
		Intercept	Dev Eqty	US Corp Debt	Comm	US RE	Sov Debt	EM Eqty	R ²
ERC	Coefficient	-0.0004	-0.0227	0.5450	0.0563	0.0139	0.3750	0.3690	79.60%
	<i>(t-statistic)</i>	(-0.99)	(-1.50*)	(15.87***)	(8.00***)	(1.49*)	(14.05***)	(4.01***)	
MV	Coefficient	0.0000	-0.0050	0.6170	0.0414	0.0176	0.2580	0.0309	83.40%
	<i>(t-statistic)</i>	(0.29)	(-0.39)	(21.46***)	(7.04***)	(2.26**)	(11.57***)	(4.01***)	
IV	Coefficient	0.0001	0.1090	0.3530	0.0849	0.1050	0.2740	0.0670	98.10%
	<i>(t-statistic)</i>	(0.77)	(20.23***)	(28.82***)	(33.79***)	(31.70***)	(28.79***)	(20.39***)	
1/N	Coefficient	0.0000	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	100.00%
	<i>(t-statistic)</i>	(0.00)	(146.00***)	(648.00***)	(316.00***)	(238.00***)	(834.00***)	(241.00***)	
60/40	Coefficient	0.0000	0.6000	0.4000	0.0000	0.0000	0.0000	0.0000	100.00%
	<i>(t-statistic)</i>	(Inf)	(Inf)	(Inf)	(Inf)	(Inf)	(Inf)	(Inf)	

In table G we regress the TSM long portfolios on the Fama French US 5-factor + UMD, the Fama French global 3-factor + WML as well as the benchmark constituents. From panel A, all strategies are moderately exposed to the market factor with MV having the lowest exposure and 1/N having largest. Furthermore, all strategies exhibit small, but positive momentum exposure captured by the UMD variable. These results are consistent for the global data and the momentum factor WML seen in panel B. In comparison to the benchmark strategy, the TSM LO filter reduces market exposure, while increasing momentum exposure. From panel C, ERC, MV and IV is still moderately exposed to US corporate debt.

Table G

Panel A: TSM LO with Fama French US 5-factor + UMD									
		Intercept	MKT	SMB	HML	RMW	CMA	UMD	R ²
ERC	Coefficient	0.00212	0.119	-0.00192	-0.00965	0.00323	0.0197	0.0408	19.30%
	<i>(t-statistic)</i>	(3.60***)	(7.54***)	(-0.09)	(-0.35)	(0.12)	(0.51)	(3.27***)	
MV	Coefficient	0.0022	0.0541	-0.00838	-0.00722	0.0198	0.0383	0.0172	4.73%
	<i>(t-statistic)</i>	(4.27***)	(3.88***)	(-0.46)	(-0.30)	(0.84)	(1.12)	(1.57*)	
IV	Coefficient	0.0017	0.188	0.00781	-0.0177	0.0279	0.0448	0.0764	33.80%
	<i>(t-statistic)</i>	(2.71***)	(11.16***)	(0.36)	(-0.60)	(0.97)	(1.08)	(5.76***)	
1/N	Coefficient	0.00101	0.278	0.0108	0.0108	-0.0051	0.0503	0.103	44.30%
	<i>(t-statistic)</i>	(1.35*)	(13.67***)	(0.41)	(0.31)	(-0.15)	(1.01)	(6.47***)	

Panel B: TSM LO with Fama French Global 3-Factor + WML							
		Intercept	MKT	SMB	HML	WML	R ²
ERC	Coefficient	0.0020	0.1400	0.0016	0.0301	0.0752	27.40%
	<i>(t-statistic)</i>	(3.56***)	(10.52***)	(0.06)	(1.22)	(5.11***)	
MV	Coefficient	0.0022	0.0557	-0.0007	0.0385	0.0339	6.68%
	<i>(t-statistic)</i>	(4.27***)	(4.55***)	(-0.03)	(1.70**)	(2.51***)	
IV	Coefficient	0.0018	0.1960	-0.0279	0.0434	0.1280	41.90%
	<i>(t-statistic)</i>	(3.11***)	(14.14***)	(-0.99)	(1.69**)	(8.39***)	
1/N	Coefficient	0.0006	0.3200	0.0294	0.0976	0.1790	59.20%
	<i>(t-statistic)</i>	(0.96)	(20.59***)	(0.93)	(3.40***)	(10.47***)	

Panel C: TSM LO with Benchmark Constituents									
		Intercept	Dev Eqty	Corp Debt	Comm	US RE	Sov Debt	EM Eqty	R ²
ERC	Coefficient	0.0021	0.0032	0.2330	0.0082	-0.0109	0.0541	0.0762	43.40%
	<i>(t-statistic)</i>	(4.45***)	(0.18)	(5.88***)	(1.04)	(-1.04)	(1.77**)	(6.86***)	
MV	Coefficient	0.0018	-0.0283	0.2170	0.0004	0.0021	0.0792	0.0349	28.80%
	<i>(t-statistic)</i>	(4.18***)	(-1.73**)	(6.04***)	(0.05)	(0.22)	(2.85***)	(3.47***)	
IV	Coefficient	0.0024	0.0591	0.2250	-0.0083	-0.0105	0.0473	0.0774	46.10%
	<i>(t-statistic)</i>	(4.36***)	(2.85***)	(4.95***)	(-0.92)	(-0.88)	(1.34*)	(6.07***)	
1/N	Coefficient	0.0031	0.1006	0.0787	0.0424	-0.0173	-0.0587	0.1270	58.20%
	<i>(t-statistic)</i>	(4.85***)	(4.19***)	(1.49*)	(4.03***)	(-1.24)	(-1.44*)	(8.65***)	

In table H we regress the TSM long-short portfolios on the Fama French US 5-factor + UMD, the Fama French global 3-factor + WML as well as the benchmark constituents. From panel A, ERC and MV have negative exposure to the market factor while having moderate momentum exposure. Panel B reveal similar results, however, with negative exposure to the SMB factor as well. From panel C, ERC and MV have negative exposure to developed equities, commodities and US real estate while still being moderately exposed to global sovereign debt.

Table H

Panel A: TSM LS with Fama French US 5-factor + UMD									
		Intercept	MKT	SMB	HML	RMW	CMA	UMD	R ²
ERC	Coefficient	0.00367	-0.109	-0.018	-0.0807	0.0488	0.0271	0.129	38.10%
	<i>(t-statistic)</i>	(5.09***)	(-5.62***)	(-0.72)	(-2.39***)	(1.48*)	(0.57)	(8.44***)	
MV	Coefficient	0.00343	-0.129	-0.0262	-0.066	0.0514	0.0267	0.101	38.90%
	<i>(t-statistic)</i>	(5.04***)	(-7.02***)	(-1.10)	(-2.07**)	(1.65**)	(0.59)	(7.07***)	
IV	Coefficient	0.00134	0.0383	-0.0068	-0.0279	0.0275	0.0288	0.083	20.00%
	<i>(t-statistic)</i>	(2.76***)	(2.92***)	(-0.40)	(-1.23)	(1.24)	(0.90)	(8.08***)	
1/N	Coefficient	0.00252	0.0304	-0.0154	-0.0576	0.0467	0.0668	0.24	32.50%
	<i>(t-statistic)</i>	(2.49***)	(1.11)	(-0.44)	(-1.22)	(1.01)	(1.00)	(11.21***)	

Panel B: TSM LS with Fama French Global 3-Factor + WML							
		Intercept	MKT	SMB	HML	WML	R ²
ERC	Coefficient	0.0034	-0.1400	-0.1220	-0.0265	0.1880	44.90%
	<i>(t-statistic)</i>	(4.99***)	(-8.49***)	(-3.66***)	(-0.87)	(10.39***)	
MV	Coefficient	0.0033	-0.1680	-0.1100	-0.0174	0.1410	45.10%
	<i>(t-statistic)</i>	(5.00***)	(-10.77***)	(-3.49***)	(-0.60)	(8.23***)	
IV	Coefficient	0.0013	0.0279	-0.0516	0.0234	0.1260	25.30%
	<i>(t-statistic)</i>	(2.73***)	(2.47***)	(-2.26**)	(1.12)	(10.16***)	
1/N	Coefficient	0.0019	0.0177	-0.1190	0.0697	0.3630	41.80%
	<i>(t-statistic)</i>	(1.96**)	(0.78)	(-2.60***)	(1.66**)	(14.52***)	

Panel C: TSM LS with Benchmark Constituents									
		Intercept	Dev Eqy	Corp Debt	Comm	US RE	Sov Debt	EM Eqty	R ²
ERC	Coefficient	0.0038	-0.0933	-0.0037	-0.0604	-0.0683	0.1530	-0.0070	32.50%
	<i>(t-statistic)</i>	(5.17***)	(-3.37***)	(-0.06)	(-4.98***)	(-4.26***)	(3.26***)	(-0.41)	
MV	Coefficient	0.0032	-0.1040	-0.0022	-0.0655	-0.0567	0.1690	-0.0208	43.30%
	<i>(t-statistic)</i>	(5.07***)	(-4.34***)	(-0.04)	(-6.21***)	(-4.07***)	(4.15***)	(-1.41*)	
IV	Coefficient	0.0019	-0.0058	0.0367	-0.0217	-0.0307	0.0616	0.0289	6.88%
	<i>(t-statistic)</i>	(3.79***)	(0.30)	(0.87)	(-2.58***)	(-2.77***)	(1.89**)	(2.45***)	
1/N	Coefficient	0.0045	-0.0182	-0.0486	-0.0278	-0.0793	0.0813	0.0254	5.27%
	<i>(t-statistic)</i>	(3.86***)	(-0.41)	(-0.50)	(-1.44*)	(-3.11***)	(1.09)	(0.94)	

Chapter 7 – Conclusion

With increased awareness around weaknesses in traditional portfolio theory, a group of risk based asset allocation strategies has gained in popularity. However, these tend to be highly concentrated in low risk asset and for that reason typically provide somewhat low returns, but at an attractive risk-adjusted basis. The most common solution to the problem is to apply leverage to these portfolios in order to tailor the investors preferred return target and desired risk level. However, applying leverage introduces its own risks and practical concerns. We investigate an alternative solution by combining TSM which filters out what positions to take in each asset in the specific investment universe. We then use the framework of the risk based allocation strategies so size the positions. We find that combining the strategies adds significant value to all strategies with low market exposure and moderate momentum exposure. The implied market timing traits of the strategy contribute to control risk in already low risk portfolios by lowering volatility and reducing drawdowns. The finance literature provides some research on the combination of TSM and inverse volatility, however, we found that equal risk contribution in combination with TSM can add further value, especially when correlation between assets drift away from normal levels.

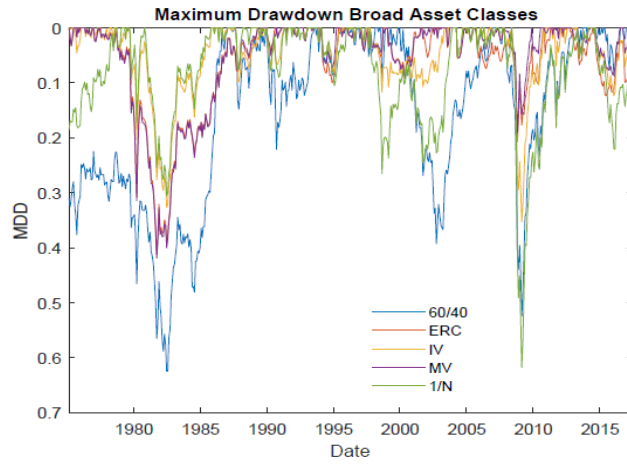
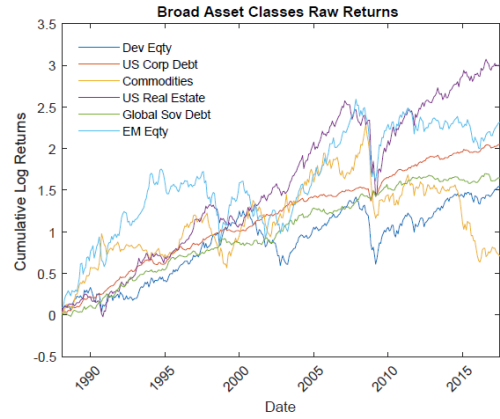
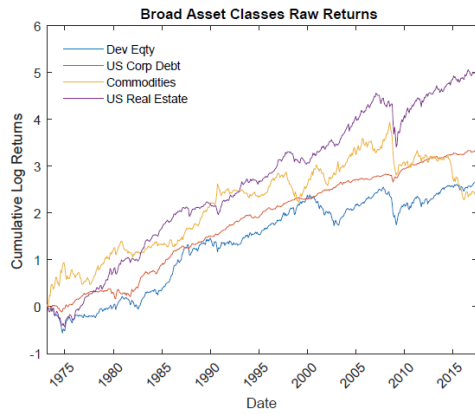
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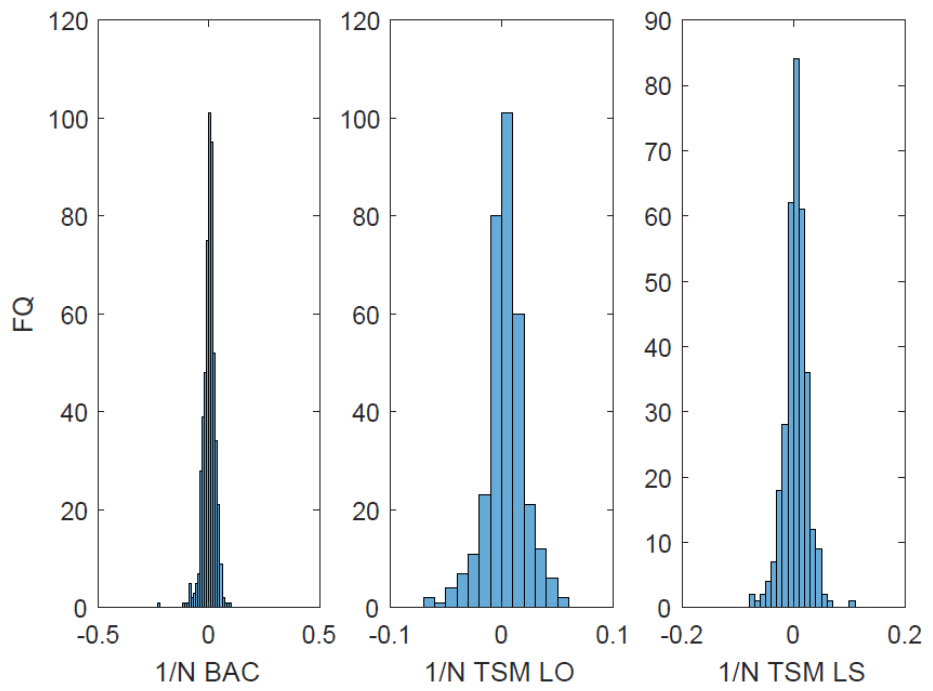
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<http://us.spindices.com/indices/equity/sp-500>
- [36] Stuart, A., & Markowitz, H. M. (1959). Portfolio Selection: Efficient Diversification of Investments. *Or*, 10(4), 253.
- [37] US Aggregate Index. (n.d.). Retrieved January 14, 2017, from <https://data.bloomberglp.com/indices/sites/2/2016/08/Factsheet-US-Aggregate.pdf>

Appendix A – Results graphs

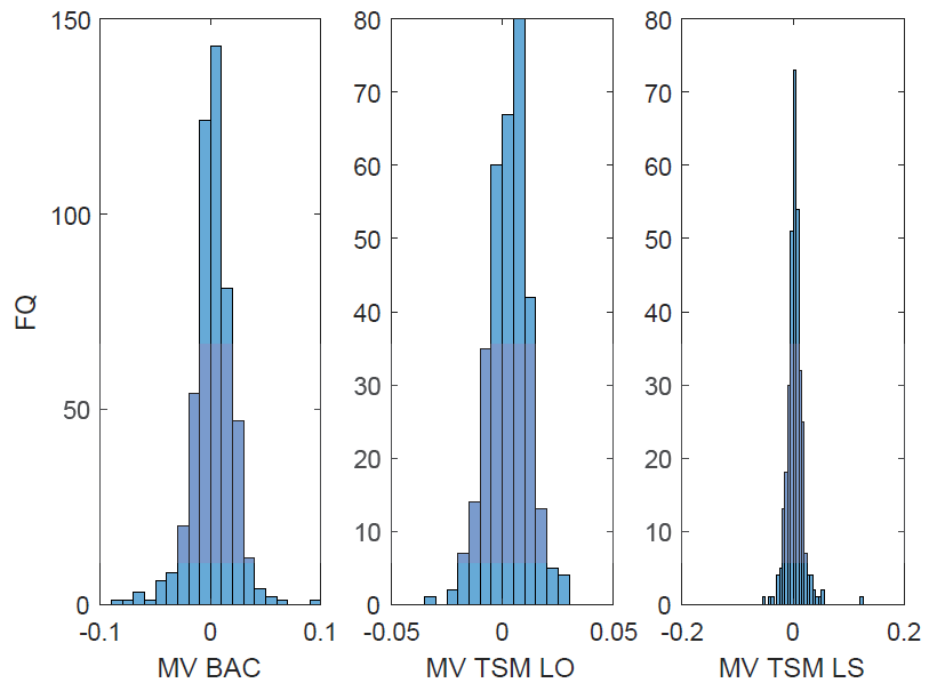


Performance statistics adjusted for trading costs (TC)					
	Mean Return	TC	Turnover	TC adj. returns	Sharpe Ratio
1/N	2.83%	20bp	25.71%	2.78%	0.27
1/N TSM LO	4.23%	20bp	26.31%	4.18%	0.70
1/N TSM LS	5.01%	20bp	26.31%	4.96%	0.69
MV	3.34%	20bp	12.11%	3.32%	0.61
MV TSM LO	3.37%	20bp	26.00%	3.32%	1.07
MV TSM LS	3.87%	20bp	26.64%	3.82%	0.75
IV	3.42%	20bp	17.16%	3.39%	0.49
IV TSM LO	4.24%	20bp	21.61%	4.20%	0.71
IV TSM LS	2.54%	20bp	57.97%	2.42%	0.76
ERC	2.62%	20bp	12.94%	2.59%	0.44
ERC TSM LO	3.81%	20bp	21.78%	3.77%	0.98
ERC TSM LS	4.47%	20bp	23.63%	4.42%	0.82
60/40	2.65%	20bp	25.73%	2.60%	0.26

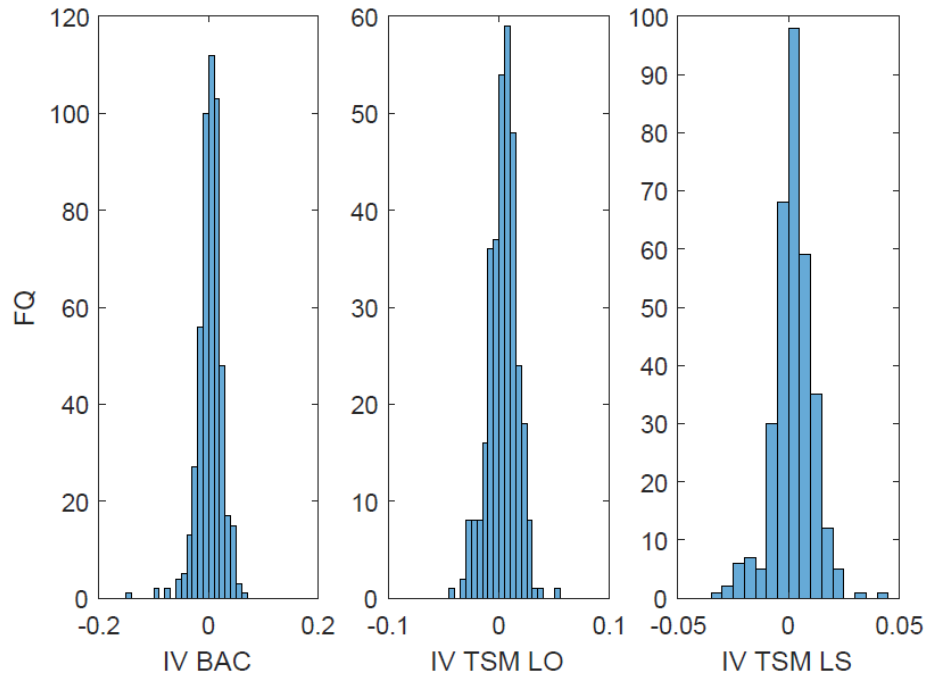
Log Returns



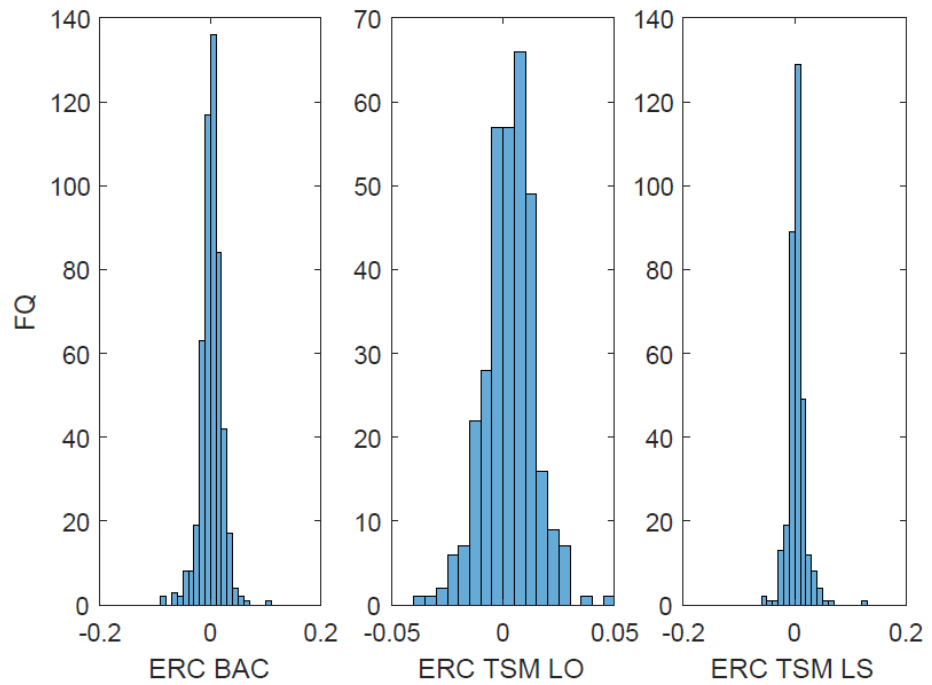
Log Returns



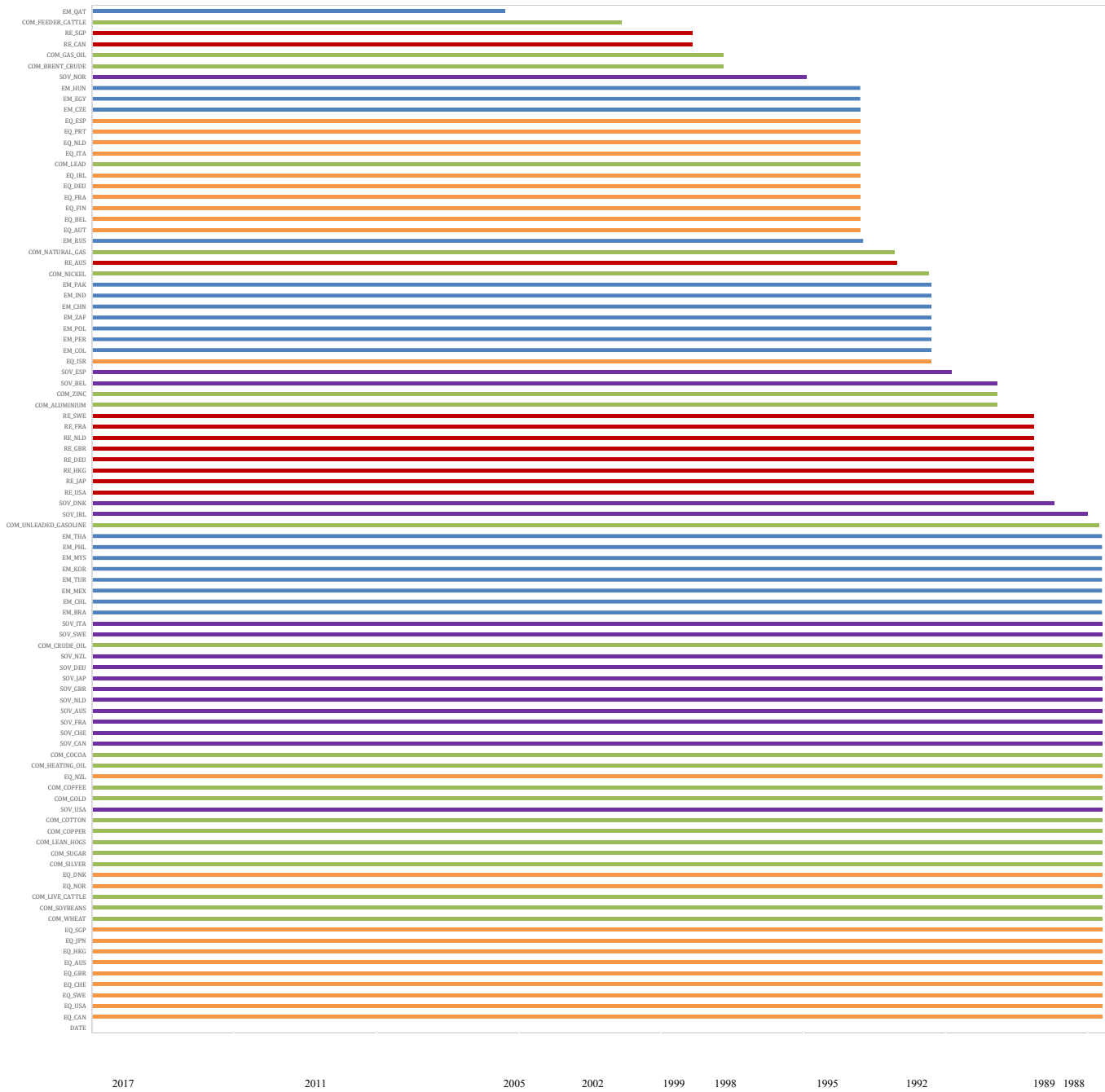
Log Returns



Log Returns



Appendix B – Markets and inception dates



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----- BROAD ASSET CLASSES (BAC) -----
Load data/extract date & prices

```
data_load
date1 = table2array(data(:,1));
prices1 = table2array(data(:,2:end));
```

Calculate returns (continously compounded // default method @ price2ret)

```
ret_raw = price2ret(prices1);

% load risk free (rf)
rf;
r_f = log((1+r_f/100).^(1/12));

ret_excess = bsxfun(@minus, ret_raw, r_f);
```

Descriptives raw return & plot

```
%raw returns
mean_monthly_raw = nanmean(ret_raw);
stdev_monthly_raw = nanstd(ret_raw);
mean_annual_raw = ((1+mean_monthly_raw).^12)-1;
stdev_annual_raw = sqrt(12)*stdev_monthly_raw;
maxdd_ret_raw = min(ret_raw);

%excess returns
mean_monthly = nanmean(ret_excess);
stdev_monthly = nanstd(ret_excess);
mean_annual = ((1+mean_monthly).^12)-1;
stdev_annual = sqrt(12)*stdev_monthly;
maxdd_ret = min(ret_excess);
%export to excel
descriptives_raw_return = [mean_monthly_raw; mean_annual_raw; stdev_monthly_raw; stdev_annual_raw; maxdd_ret_raw; mean_monthly; stdev_monthly; mean_annual];

%log cumulative returns for asset classes excluding EM Eqty & Global Debt
```

```

plot(datel(1:end,:), cumsum(ret_excess));
title('Broad Asset Classes Excess Returns')
legend('Dev Eqty', 'US Corp Debt', 'Commodities', 'US Real Estate', 'Location', 'northwest')
ylabel('Cumulative Log Returns')
xlabel('Date')
xtickangle(45)
legend boxoff

% raw returns
figure
plot(datel(2:end,:), cumsum(ret_raw))
title('Broad Asset Classes Raw Returns')
legend('Dev Eqty', 'US Corp Debt', 'Commodities', 'US Real Estate', 'Location', 'northwest')
ylabel('Cumulative Log Returns')
xlabel('Date')
xtickangle(45)
legend boxoff

%log cumulative returns for all asset classes

%fix: cum(1:1,:) = 0 : excess returns
figure
plot(datel(181:end,:), cumsum(ret_excess(180:end,:)))
title('Broad Asset Classes Excess Returns')
legend('Dev Eqty', 'US Corp Debt', 'Commodities', 'US Real Estate', 'Global Sov Debt', 'EM Eqty', 'Location', 'northwest')
ylabel('Cumulative Log Returns')
xlabel('Date')
xtickangle(45)
legend boxoff

%fix: cum(1:1,:) = 0 : raw returns
figure
plot(datel(181:end,:), cumsum(ret_raw(180:end,:)))
title('Broad Asset Classes Raw Returns')
legend('Dev Eqty', 'US Corp Debt', 'Commodities', 'US Real Estate', 'Global Sov Debt', 'EM Eqty', 'Location', 'northwest')
ylabel('Cumulative Log Returns')
xlabel('Date')
xtickangle(45)
legend boxoff

```

□□□□

1/N BAC

```

w_1N = (1./sum(~isnan(ret_excess),2)).*(~isnan(ret_excess));
ret_1N = nansum(w_1N.*ret_excess,2);
ln_cum_ret_1N = cumsum(ret_1N(25:end,1));

```

1/N Performance Characteristics BAC

```

mean_1N = ((1+mean(ret_1N))^12-1);
stdev_1N = std(ret_1N)*sqrt(12);
maxdd_1N = min(ret_1N);
kurt_1N = kurtosis(ret_1N);
skew_1N = skewness(ret_1N);
summary_1N = table(mean_1N, stdev_1N, maxdd_1N, kurt_1N, skew_1N);

```

Var-Cov Estimation BAC

```

%Inputs(estimation window)

% Estimation window
ew = 24;

% calculate var-cov matrices @ rolling window (t+1) w/estimation window
% (ew)
sz = size(ret_excess);
covC = cell(sz(1)-(ew-1),1);
covM = zeros(sz(1)-(ew-1),sz(2));
for n = 1:sz(1)-(ew-1)
    covC{n} = cov(ret_excess(0+n:(ew-1)+n,:));
    covM(n,:) = covC{n}(1,:);
end

% load var-cov matrix from excel
load('matlab_thesis.mat')
clear ans i j lb naive_cumulative_naive_returns port_var1 ub w w1 weights returns;

```

Minimum Variance BAC

```

%setting up min-var portfolio for [4 5 6] asset class
x0_1 = zeros(4,1);
x0_2 = zeros(5,1);
x0_3 = zeros(6,1);

% upper bound portfolio weights
ub = 1;
ub1(1:4,:) = ub;
ub2(1:5,:) = ub;
ub3(1:6,:) = ub;

% lower bound portfolio weights (for short selling allowed: 'lb = -1;')

```

```

% weight constraints. For sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for N = # of asset classes
Aeq1 = ones(1,4);
Aeq2 = ones(1,5);
Aeq3 = ones(1,6);
beq = 1;

%estimation length
n_rows1 = length(varcovmatrixS1);
n_rows2 = length(varcovmatrixS2);
n_rows3 = length(varcovmatrixS3);
n_cols1 = size(varcovmatrixS1,1)/size(varcovmatrixS1,2);
n_cols2 = size(varcovmatrixS2,1)/size(varcovmatrixS2,2);
n_cols3 = size(varcovmatrixS3,1)/size(varcovmatrixS3,2);

% run fmincon (optimization program) for all covariance matrices: 4 asset case

options = optimset('Display', 'off', 'Algorithm', 'sqp');
for i = 1:4:n_rows1
    cov1 = varcovmatrixS1(i:i+3,1:4);
    fun_mv = @(x1)x1'*cov1*x1;
    [w1_mv(i:i+3,1), port_var1(i,1)] = fmincon(fun_mv, x0_1,[],[],Aeq1,beq, lb1, ub1, [], options);
end
port_var1(port_var1==1) = [];

% run fmincon (optimization program) for all covariance matrices: 5 asset case
for i = 1:5:n_rows2
    cov2 = varcovmatrixS2(i:i+4,1:5);
    fun_mv = @(x2)x2'*cov2*x2;
    [w2_mv(i:i+4,1), port_var2(i,1)] = fmincon(fun_mv, x0_2,[],[],Aeq2,beq, lb2, ub2, [], options);
end
port_var2(port_var2==0) = [];

% run fmincon (optimization program) for all covariance matrices: 6 asset case
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    fun_mv = @(x3)x3'*cov3*x3;
    [w3_mv(i:i+5,1), port_var3(i,1)] = fmincon(fun_mv, x0_3,[],[],Aeq3,beq, lb3, ub3, [], options);
end
port_var3(port_var3==0) = [];

port_var = [port_var1;port_var2;port_var3];

%reshape matrix
w1_mv = reshape(w1_mv,[4,155]);
w2_mv = reshape(w2_mv,[5,24]);
w3_mv = reshape(w3_mv,[6,331]);
n_cols1 = size(w1_mv,2);
n_cols2 = size(w2_mv,2);
n_cols3 = size(w3_mv,2);

```

Inverse Volatility BAC

```

options = optimset('Display', 'off');
for i = 1:4:n_rows1
    cov1 = varcovmatrixS1(i:i+3,1:4);
    stdev1_iv = diag(sqrt(cov1));
    w1_iv(i:i+3,1) = 1./stdev1_iv./sum(1./stdev1_iv);
end

% run fmincon (optimization program) for all covariance matrices: 5 asset case
for i = 1:5:n_rows2
    cov2 = varcovmatrixS2(i:i+4,1:5);
    stdev2_iv = diag(sqrt(cov2));
    w2_iv(i:i+4,1) = 1./stdev2_iv./sum(1./stdev2_iv);
end

% run fmincon (optimization program) for all covariance matrices: 6 asset case
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    stdev3_iv = diag(sqrt(cov3));
    w3_iv(i:i+5,1) = 1./stdev3_iv./sum(1./stdev3_iv);
end

w1_iv = reshape(w1_iv,[4,155]);
w2_iv = reshape(w2_iv,[5,24]);
w3_iv = reshape(w3_iv,[6,331]);

ret_iv = [sum(w1_iv'.*ret_excess(25:179,1:4),2); sum(w2_iv'.*ret_excess(180:203,1:5),2); sum(w3_iv(:,1:330)'.*ret_excess(204:end,:),2)];
ln_cum_ret_iv = cumsum(ret_iv);

```

Minimum Variance Performance Characteristics BAC

```

ret_mv = [sum(w1_mv'.*ret_excess(25:179,1:4),2); sum(w2_mv'.*ret_excess(180:203,1:5),2); sum(w3_mv(:,1:330)'.*ret_excess(204:end,:),2)];
ln_cum_ret_mv = cumsum(ret_mv);
mean_mv = ((1+mean(ret_mv))^12-1);
stdev_mv = std(ret_mv)*sqrt(12);
maxdd_mv = min(ret_mv);
kurt_mv = kurtosis(ret_mv);
skew_mv = skewness(ret_mv);
summary_mv = table(mean_mv, stdev_mv, maxdd_mv, kurt_mv, skew_mv);

```

```

bond_ret = ret_excess(:,2).*bond_w;

ret_60_40 = eqty_ret + bond_ret;
ln_cum_ret_60_40 = cumsum(ret_60_40(25:end,1));

```

60/40 Performance Characteristics BAC

```

mean_60_40 = ((1+mean(ret_60_40))^12-1);
stdev_60_40 = std(ret_60_40)*sqrt(12);
maxdd_60_40 = min(ret_60_40);
kurt_60_40 = kurtosis(ret_60_40);
skew_60_40 = skewness(ret_60_40);
summary_60_40 = table(mean_60_40, stdev_60_40, maxdd_60_40, kurt_60_40, skew_60_40);

```

Equal Risk Contribution BAC

```

options = optimset('Display', 'off', 'Algorithm', 'sqp');

%setting up risk parity portfolio for [4 5 6] asset class
x0_1 = zeros(4,1);
x0_2 = zeros(5,1);
x0_3 = zeros(6,1);

% set lb/ub // lower bound 0 // upper bound = 1
LB1 = ones(1,4)*0.0001;
UB1 = ones(1,4);
LB2 = ones(1,5)*0.0001;
UB2 = ones(1,5);
LB3 = ones(1,6)*0.0001;
UB3 = ones(1,6);

% for sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for N = # of asset
% classes
Aeq1 = ones(1,4);
Aeq2 = ones(1,5);
Aeq3 = ones(1,6);
beq = 1;

%estimation length
n_rows1 = length(varcovmatrixS1);
n_rows2 = length(varcovmatrixS2);
n_rows3 = length(varcovmatrixS3);

% run fmincon for all covariance matrices 4 asset case & run risk parity
% objective function (fun_erc) ref. Roncalli(2012)
for i = 1:4:n_rows1
    cov1 = varcovmatrixS1(i:i+3,1:4);
    fun_erc = @(w1_erc) MRC2(w1_erc, cov1);
    [w1_erc(i:i+3,1), port_var_erc_1(i,1)] = ...
        fmincon(fun_erc, x0_1, [], [], Aeq1, beq, LB1, UB1, [], options);
end
for i = 1:5:n_rows2
    cov2 = varcovmatrixS2(i:i+4,1:5);
    fun_erc = @(w2_erc) MRC2(w2_erc, cov2);
    [w2_erc(i:i+4,1), port_var_erc_2(i,1)] = ...
        fmincon(fun_erc, x0_2, [], [], Aeq2, beq, LB2, UB2, [], options);
end
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    fun_erc = @(w3_erc) MRC2(w3_erc, cov3);
    [w3_erc(i:i+5,1), port_var_erc_3(i,1)] = ...
        fmincon(fun_erc, x0_3, [], [], Aeq3, beq, LB3, UB3, [], options);
end

port_var_erc_1(port_var_erc_1 == 0) = [];
port_var_erc_2(port_var_erc_2 == 0) = [];
port_var_erc_3(port_var_erc_3 == 0) = [];
port_var_erc = [port_var_erc_1;port_var_erc_2;port_var_erc_3];

```

Equal Risk Contribution Performance Characteristics BAC

```

% reshape weight matrices
w1_erc = reshape(w1_erc, [4,155]);
w2_erc = reshape(w2_erc, [5,24]);
w3_erc = reshape(w3_erc, [6,331]);

%calculate returns for risk parity
ret_erc = [sum(w1_erc'.*ret_excess(25:179,1:4),2); sum(w2_erc'.*ret_excess(180:203,1:5),2); sum(w3_erc(:,1:330)'.*ret_excess(204:end,:),2)];
ln_cum_ret_erc = cumsum(ret_erc);

%descriptives
mean_erc = ((1+mean(ret_erc))^12-1);
stdev_erc = std(ret_erc)*sqrt(12);
maxdd_erc = min(ret_erc);
kurt_erc = kurtosis(ret_erc);
skew_erc = skewness(ret_erc);
summary_erc = table(mean_erc, stdev_erc, maxdd_erc, kurt_erc, skew_erc);

```

Analytics Broad Asset Classes

```

plot(date1(25:end-1,:),exp(ln_cum_ret_mv)-1);
hold on
plot(date1(25:end-1,:),exp(ln_cum_ret_1N)-1)
plot(date1(25:end-1,:),exp(ln_cum_ret_erc)-1)
plot(date1(25:end-1,:),exp(ln_cum_ret_60_40)-1)
plot(date1(25:end-1,:),exp(ln_cum_ret_iv)-1)
title('Horseshoe Broad Asset Classes')
legend('Min.Var', '1/N', 'ERC', '60/40', 'IV', 'Location', 'northwest')
ylabel('Log Cumulative Returns')
xlabel('Date')
legend boxoff

```

----- MOMENTUM STRATEGIES (TSM) -----

Load data/extract dates & prices TSM

```

data1
date2 = table2array(data2(:,1));
prices = table2array(data2(:,2:end));

%load risk free (rf)
rf1

% calculate log returns (ret_raw = returns // ret = excess returns)
ret_raw = price2ret(prices);
rf = log((1+rf/100).^(1/12));

ret_exc = bsxfun(@minus, ret_raw, rf);

% inputs (holding period (j) // lookback period (p))
j = 1;
p = 12;
[m,n] = size(ret_exc);
tsm_sign = ones(m-12,n);

for i = 1:1:m-p
tsm_sign(i,1:n) = sum(ret_raw(i:i+(p-1),:))-ret_raw(i+(p-1),:);
end

% TSM return/rf matrix
% ret_tsm is ret minus 12 first rows % minus last row
ret_tsm = ret_exc;
% rf_tsm is same as ret_tsm for risk free asset
rf_tsm = rf;
ret_raw_tsm = ret_exc;

```

Var-Cov Matrix Estimation overlapping rolling covariance TSM

```

[n,m] = size(ret_tsm); % n = no. of dates // m = no. of assets
rolling_window = 24;
cov_tsm = nan(m*(n - rolling_window + 1),m);
for i = rolling_window:n
start_index = m*(i - rolling_window) + 1; % aggregate covariance matrix start index
end_index = m*(i - rolling_window + 1); % aggregate covariance matrix end index
covariance_mtx = cov(ret_tsm(i-rolling_window+1:i,:));
cov_tsm(start_index:end_index,:) = covariance_mtx;
end

```

1/N TSM LO

```

w_1N_tsm_lo = (1./sum(~isnan(ret_tsm),2)).*(~isnan(ret_tsm));
w_1N_tsm_lo = w_1N_tsm_lo(25:end,:);

idx_long = tsm_sign>0;
idx_long = idx_long(rolling_window-11:end,:);

ret_long = idx_long.*ret_tsm(rolling_window+1:end,:).*w_1N_tsm_lo;

idx_short = tsm_sign<0;
idx_short = idx_short(rolling_window-11:end,:);
ret_short = idx_short.*rf_tsm(rolling_window+1:end,:).*w_1N_tsm_lo;

ret_1N_tsm_lo = ret_long + ret_short;

ret_1N_tsm_lo = nansum(ret_1N_tsm_lo,2);
mean_1N_lo = (1+mean(ret_1N_tsm_lo))^12-1;
stdev_1N_lo = sqrt(12)*std(ret_1N_tsm_lo);

```

MV TSM LO

```

options = optimset('Display', 'off', 'Algorithm', 'sqp');

w_mv_tsm_lo = zeros(m,n);
returns_mv_tsm_lo = zeros(m,n);
for i=rolling_window:n-1

start_index = m*(i - rolling_window) + 1;
end_index = m*(i - rolling_window + 1);
covariance_mtx = cov_tsm(start_index:end_index,:);

```

```

covariance_mtx(condition,:) = [];
w_risky_assets(i-(rolling_window-1),:) = sum(~isnan(covariance_mtx(1,:)),2)/no_riskyassets;
k = sum(~isnan(covariance_mtx(1,:)),2);

columns = isnan(covariance_mtx(1,:));
rows = isnan(covariance_mtx(:,1));
covariance_mtx(rows,:) = [];
covariance_mtx(:,columns) = [];

% initial weight guess = 0
x0 = zeros(k,1);
% upper bound = 1
ub = ones(k,1);
% lower bound = 0
lb = ones(k,1)*0.0001;
% weight constraints. for sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for
% N = # of asset classes
Aeq = ones(1,k);
beq = 1;
% function objective // choose weights s.t. portfolio variance is minimized
fun_mv = @(x)x'*covariance_mtx*x;
[w_mv_tsm_lo(1:k,i-(rolling_window-1)), port_var_mv_lo(i-(rolling_window-1),1)] = fmincon(fun_mv, x0, [], [], Aeq, beq, lb, ub, [], options);
stdev_mv_short(i-(rolling_window-1),1) = sqrt(w_mv_tsm_lo(1:k,i-(rolling_window-1))*covariance_mtx*w_mv_tsm_lo(1:k,i-(rolling_window-1)))*sqrt(12);

%returns
returns_mv_tsm_lo = ret_raw_tsm(i+1,1:m);
returns_mv_tsm_lo(:,condition) = [];
returns_mv_tsm_lo(:,columns) = [];
a_mv_lo = returns_mv_tsm_lo*w_mv_tsm_lo(1:k,i-(rolling_window-1));
ret_mv_tsm_lo(i-(rolling_window-1),1) = a_mv_lo;

end

% For trading between cash and risky asset according to TSM sign, enable
% ret_mv_tsm_lo = w_risky_assets.*ret_mv_tsm_lo + (1-w_risky_assets).*rf_tsm(rolling_window+1:end,:);

```

ERC TSM LO

```

options = optimset('Display', 'off', 'Algorithm', 'sqp');

w_erc_tsm_lo = zeros(m,n);
returns_erc_tsm_lo = zeros(m,n);

for i=rolling_window:n-1
    start_index = m*(i - rolling_window) + 1;
    end_index = m*(i - rolling_window + 1);
    covariance_mtx = cov_tsm(start_index:end_index,:);
    no_riskyassets = sum(~isnan(covariance_mtx(1,:)),2);
    condition = tsm_sign(i-1,1,:) < 0;
    covariance_mtx(condition,:) = [];
    covariance_mtx(:,condition) = [];
    w_risky_assets(i-(rolling_window-1),:) = sum(~isnan(covariance_mtx(1,:)),2)/no_riskyassets;
    k = sum(~isnan(covariance_mtx(1,:)),2);

    columns = isnan(covariance_mtx(1,:));
    rows = isnan(covariance_mtx(:,1));
    covariance_mtx(rows,:) = [];
    covariance_mtx(:,columns) = [];
    % initial weight guess = 0
    x0 = zeros(k,1);
    % upper bound = 1
    ub = ones(k,1);%*ub
    % lower bound = 0
    lb = ones(k,1)*0;%*lb;
    % weight constraints. for sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for
    % N = # of asset classes
    Aeq = ones(1,k);
    beq = 1;
    % function objective // choose weights s.t. portfolio variance is minimized
    fun_erc = @(w_erc) fm_fitnessERC(covariance_mtx, w_erc);
    [w_erc_tsm_lo(1:k,i-23), port_var_erc_lo(i-23,1)] = fmincon(fun_erc, x0, [], [], Aeq, beq, lb, ub, [], options);
    stdev_erc_tsm_lo(i-23,1) = sqrt(w_erc_tsm_lo(1:k,i-23)*covariance_mtx*w_erc_tsm_lo(1:k,i-23))*sqrt(12);

    %returns
    returns_erc_tsm_lo = ret_raw_tsm(i+1,1:m);
    returns_erc_tsm_lo(:,condition) = [];
    returns_erc_tsm_lo(:,columns) = [];
    a_erc_lo = returns_erc_tsm_lo*w_erc_tsm_lo(1:k,i-23);
    ret_erc_tsm_lo(i-23,1) = a_erc_lo;

end

% For trading between cash and risky asset according to TSM sign, enable
% returns_minvar_tsm
%ret_erc_tsm_lo = w_risky_assets.*ret_erc_tsm_lo + (1-w_risky_assets).*rf_tsm(rolling_window+1:end,:);

```

IV TSM LO

```

w_iv_tsm_lo = zeros(m,n);
returns_iv_tsm_lo = zeros(m,n);

for i=rolling_window:n-1

```

```

condition = tsm_sign(i-11,:) < 0;
covariance_mtx(condition,:) = [];
covariance_mtx(:,condition) = [];
k = sum(~isnan(covariance_mtx(1,:)),2);

columns = isnan(covariance_mtx(1,:));
rows = isnan(covariance_mtx(:,1));
covariance_mtx(rows,:) = [];
covariance_mtx(:,columns) = [];
stdev_iv_lo = diag(sqrt(covariance_mtx));
w_iv_tsm_lo(1:k,i-23) = 1./stdev_iv_lo./sum(1./stdev_iv_lo);

%returns
returns_iv_tsm_lo = ret_raw_tsm(i+1,1:m);
returns_iv_tsm_lo(:,condition) = [];
returns_iv_tsm_lo(:,columns) = [];

a_iv_lo = returns_iv_tsm_lo*w_iv_tsm_lo(1:k,i-23);
ret_iv_tsm_lo(i-23,1) = a_iv_lo;

end

```

1/N TSM LS

```

w_1N_ls = (1./sum(~isnan(ret_tsm),2)).*(~isnan(ret_tsm));
w_1N_ls = w_1N_ls(25:end,:);
idx_long_ls = tsm_sign>0;
idx_long_ls = idx_long_ls(rolling_window-11:end,:);
ret_long_ls = idx_long_ls.*ret_tsm(rolling_window+1:end,:).*w_1N_ls;

idx_short_ls = tsm_sign<0;
idx_short_ls = idx_short_ls(rolling_window-11:end,:);
ret_short_ls = idx_short_ls.*ret_tsm(rolling_window+1:end,:).*-w_1N_ls;

ret_1N_tsm_ls = ret_long_ls + ret_short_ls;

ret_1N_tsm_ls = nansum(ret_1N_tsm_ls,2);
mean_1N_ls = (1+mean(ret_1N_tsm_ls))^12-1;
stdev_1N_ls = sqrt(12)*std(ret_1N_tsm_ls);

```

MV TSM LS

```

options = optimset('Display', 'off', 'Algorithm', 'sqp');

% Long Positions

w_mv_long = zeros(m,n);
returns_mv_tsm_long = zeros(m,n);
for i=rolling_window:n-1
    start_index = m*(i - rolling_window) + 1;
    end_index = m*(i - rolling_window + 1);
    covariance_mtx = cov_tsm(start_index:end_index,:);
    no_riskyassets = sum(~isnan(covariance_mtx(1,:)),2);
    condition = tsm_sign(i-11,:) < 0;
    covariance_mtx(condition,:) = [];
    covariance_mtx(:,condition) = [];
    w_risky_assets(i-(rolling_window-1),:) = sum(~isnan(covariance_mtx(1,:)),2)/no_riskyassets;
    k = sum(~isnan(covariance_mtx(1,:)),2);

    columns = isnan(covariance_mtx(1,:));
    rows = isnan(covariance_mtx(:,1));
    covariance_mtx(rows,:) = [];
    covariance_mtx(:,columns) = [];

    % initial weight guess = 0
    x0 = zeros(k,1);
    % upper bound = 1
    ub = ones(k,1);
    % lower bound = 0
    lb = ones(k,1)*0.0001;
    % weight constraints. for sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for
    % N = # of asset classes
    Aeq = ones(1,k);
    beq = 1;
    % function objective // choose weights s.t. portfolio variance is minimized
    fun_mv = @(x)x'*covariance_mtx*x;
    [w_mv_long(1:k,i-(rolling_window-1)), port_variance_minvar(i-(rolling_window-1),1)] = fmincon(fun_mv, x0, [], [], Aeq, beq, lb, ub, [], options)
    stdev_mv_short(i-(rolling_window-1),1) = sqrt(w_mv_long(1:k,i-(rolling_window-1))*covariance_mtx*w_mv_long(1:k,i-(rolling_window-1)))*sqrt(12);

%returns
returns_mv_tsm_long = ret_raw_tsm(i+1,1:m);
returns_mv_tsm_long(:,condition) = [];
returns_mv_tsm_long(:,columns) = [];
a = returns_mv_tsm_long*w_mv_long(1:k,i-(rolling_window-1));
ret_mv_tsm_long(i-(rolling_window-1),1) = a;

end

% Short Positions

w_mv_short = zeros(m,n);
returns_mv_tsm_short = zeros(m,n);
for i=rolling_window:n-1
    start_index = m*(i - rolling_window) + 1;
    end_index = m*(i - rolling_window + 1);
    covariance_mtx = cov_tsm(start_index:end_index,:);

```



```

covariance_mtx(1,condition) = [];
w_mv_risky_short(i-(rolling_window-1),:) = sum(~isnan(covariance_mtx(1,:)),2)/no_riskyassets;
k = sum(~isnan(covariance_mtx(1,:)),2);

columns = isnan(covariance_mtx(1,:));
rows = isnan(covariance_mtx(:,1));
covariance_mtx(rows,:) = [];
covariance_mtx(:,columns) = [];

% initial weight guess = 0
x0 = ones(k,1);
% upper bound = 1
ub = ones(k,1)*4/k;
% lower bound = 0
lb = ones(k,1)*0.3/k;
% weight constraints. for sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for
% N = # of asset classes
Aeq = ones(1,k);
beq = 1;
% function objective // choose weights s.t. portfolio variance is minimized
fun_mv = @(x)x'*covariance_mtx*x;
[w_mv_short(1:k,i-(rolling_window-1))] = fmincon(fun_mv, x0, [], [], Aeq, beq, lb, ub, [], options);
stdev_mv_short(i-(rolling_window-1),1) = sqrt(w_mv_short(1:k,i-(rolling_window-1))*covariance_mtx*w_mv_short(1:k,i-(rolling_window-1))*sqrt(12));

%returns
returns_mv_tsm_short = ret_raw_tsm(i+1,1:m);
returns_mv_tsm_short(:,condition) = [];
returns_mv_tsm_short(:,columns) = [];
a = returns_mv_tsm_short*w_mv_short(1:k,i-(rolling_window-1));
ret_mv_tsm_short(i-(rolling_window-1),1) = a;

end

ret_mv_tsm_short = ret_mv_tsm_short.*-1;
ret_mv_tsm_ls = w_risky_assets.*ret_mv_tsm_long + (1-w_risky_assets).*ret_mv_tsm_short;

```

ERC TSM LS

```

options = optimset('Display', 'off', 'Algorithm', 'sqp');

% Long Positions
w_erc_long = zeros(m,n);
returns_erc_tsm_long = zeros(m,n);

for i=rolling_window:n-1
start_index = m*(i - rolling_window) + 1;
end_index = m*(i - rolling_window + 1);
covariance_mtx = cov_tsm(start_index:end_index,:);
condition = tsm_sign(i-11,:) < 0;
covariance_mtx(condition,:) = [];
covariance_mtx(:,condition) = [];
k = sum(~isnan(covariance_mtx(1,:)),2);

columns = isnan(covariance_mtx(1,:));
rows = isnan(covariance_mtx(:,1));
covariance_mtx(rows,:) = [];
covariance_mtx(:,columns) = [];
% initial weight guess = 0
x0 = zeros(k,1);
% upper bound = 1
ub = ones(k,1)*ub;
% lower bound = 0
lb = ones(k,1)*0;*lb;
% weight constraints. for sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for
% N = # of asset classes
Aeq = ones(1,k);
beq = 1;
% function objective // choose weights s.t. portfolio variance is minimized
fun_erc = @(w_rp) fm_fitnessERC(covariance_mtx, w_rp);
[w_erc_long(1:k,i-23), port_variance_erc(i-23,1)] = fmincon(fun_erc, x0, [], [], Aeq, beq, lb, ub, [], options);
stdev_erc_tsm_lo(i-23,1) = sqrt(w_erc_long(1:k,i-23)*covariance_mtx*w_erc_long(1:k,i-23))*sqrt(12);

%returns
returns_erc_tsm_long = ret_raw_tsm(i+1,1:m);
returns_erc_tsm_long(:,condition) = [];
returns_erc_tsm_long(:,columns) = [];
a = returns_erc_tsm_long*w_erc_long(1:k,i-23);
ret_erc_long(i-23,1) = a;

end

% Short Positions
w_erc_short = zeros(m,n);
returns_erc_tsm_short = zeros(m,n);

for i=rolling_window:n-1
start_index = m*(i - rolling_window) + 1;
end_index = m*(i - rolling_window + 1);
covariance_mtx = cov_tsm(start_index:end_index,:);
no_riskyassets = sum(~isnan(covariance_mtx(1,:)),2);
condition = tsm_sign(i-11,:) > 0;
covariance_mtx(condition,:) = [];
covariance_mtx(:,condition) = [];
k = sum(~isnan(covariance_mtx(1,:)),2);

columns = isnan(covariance_mtx(1,:));
rows = isnan(covariance_mtx(:,1));

```

```

x0 = zeros(k,1);
% upper bound = 1
ub = ones(k,1);%*ub
% lower bound = 0
lb = ones(k,1)*0;%*lb;
% weight constraints. for sum(weights) = 1 // Aeq = ones(1,N), beq = 1 for
% N = # of asset classes
Aeq = ones(1,k);
beq = 1;
% function objective // choose weights s.t. portfolio variance is minimized
fun_erc = @(w_rp) fm_fitnessERC(covariance_mtx, w_rp);
[w_erc_short(1:k,i-23), port_variance_rp(i-23,1)] = fmincon(fun_erc, x0, [], [], Aeq, beq, lb, ub, [], options);
stdev_erc_tsm_lo(i-23,1) = sqrt(w_erc_short(1:k,i-23)*covariance_mtx*w_erc_short(1:k,i-23))*sqrt(12);

%returns
returns_erc_tsm_short = ret_raw_tsm(i+1,1:m);
returns_erc_tsm_short(:,condition) = [];
returns_erc_tsm_short(:,columns) = [];
a = returns_erc_tsm_short*w_erc_short(1:k,i-23);
ret_erc_short(i-23,1) = a;

end
ret_erc_short = ret_erc_short.*-1;

ret_erc_tsm_ls = w_risky_assets.*ret_erc_long + (1-w_risky_assets).*ret_erc_short;

```

IV TSM LS

Long Positions

```

w_iv_long = zeros(m,n);
returns_iv_tsm_long = zeros(m,n);
w_iv_short = zeros(m,n);
returns_iv_tsm_short = zeros(m,n);

for i=rolling_window:n-1
    start_index = m*(i - rolling_window) + 1;
    end_index = m*(i - rolling_window +1);
    covariance_mtx_long = cov_tsm(start_index:end_index,:);
    condition_long = tsm_sign(i-11,:) < 0;
    covariance_mtx_long(condition_long,:) = [];
    covariance_mtx_long(:,condition_long) = [];
    k_long = sum(~isnan(covariance_mtx_long(1,:)),2);

    columns_long = isnan(covariance_mtx_long(1,:));
    rows_long = isnan(covariance_mtx_long(:,1));
    covariance_mtx_long(rows_long,:) = [];
    covariance_mtx_long(:,columns_long) = [];

    start_index = m*(i - rolling_window) + 1;
    end_index = m*(i - rolling_window +1);
    covariance_mtx_short = cov_tsm(start_index:end_index,:);
    condition_short = tsm_sign(i-11,:) > 0;
    covariance_mtx_short(condition_short,:) = [];
    covariance_mtx_short(:,condition_short) = [];
    k_short = sum(~isnan(covariance_mtx_short(1,:)),2);

    columns_short = isnan(covariance_mtx_short(1,:));
    rows_short = isnan(covariance_mtx_short(:,1));
    covariance_mtx_short(rows_short,:) = [];
    covariance_mtx_short(:,columns_short) = [];

    % stdev
    stdev_iv_long = diag(sqrt(covariance_mtx_long));
    stdev_iv_short = diag(sqrt(covariance_mtx_short));

    %returns
    returns_iv_tsm_long = ret_raw_tsm(i+1,1:m);
    returns_iv_tsm_long(:,condition_long) = [];
    returns_iv_tsm_long(:,columns_long) = [];
    w_iv_long(1:k_long,i-23) = 1./stdev_iv_long./(sum(1./stdev_iv_long)+(sum(1./stdev_iv_short)));
    a_iv_tsm_long = returns_iv_tsm_long*w_iv_long(1:k_long,i-23);
    ret_iv_tsm_long(i-23,1) = a_iv_tsm_long;

    %returns
    returns_iv_tsm_short = ret_raw_tsm(i+1,1:m);
    returns_iv_tsm_short(:,condition_short) = [];
    returns_iv_tsm_short(:,columns_short) = [];
    w_iv_short(1:k_short,i-23) = 1./stdev_iv_short./(sum(1./stdev_iv_long)+(sum(1./stdev_iv_short)));
    a_iv_tsm_short = returns_iv_tsm_short*w_iv_short(1:k_short,i-23);
    ret_iv_tsm_short(i-23,1) = a_iv_tsm_short;
end
% Short Positions

ret_iv_tsm_short = ret_iv_tsm_short.*-1;
ret_iv_tsm_ls = w_risky_assets.*ret_iv_tsm_long + (1-w_risky_assets).*ret_iv_tsm_short;

```

Plot TSM Strategies

```

% TSM LS
figure
hold on
plot(date1(205:end,:), cumsum(ret_iv_tsm_ls))

```

```

title ('Time Series Momentum Long Only')
legend('IV TSM LS', '1N TSM LS', 'MV TSM LS', 'ERC TSM LS', 'Location', 'northwest')
ylabel('Log Cumulative Returns')
xlabel('Date')
legend boxoff
box on

% TSM LO
figure
hold on
plot(datel(205:end,:), cumsum(ret_iv_tsm_lo))
plot(datel(205:end,:), cumsum(ret_1n_tsm_lo))
plot(datel(205:end,:), cumsum(ret_mv_tsm_lo))
plot(datel(205:end,:), cumsum(ret_erc_tsm_lo))
title ('Time Series Momentum Long Only')
legend('IV TSM LO', '1N TSM LO', 'MV TSM LO', 'ERC TSM LO', 'Location', 'northwest')
ylabel('Log Cumulative Returns')
xlabel('Date')
legend boxoff
box on

% Plot ERC
figure
hold on
plot(datel(205:end,:), cumsum(ret_erc(180:end,:)))
plot(datel(205:end,:), cumsum(ret_erc_tsm_lo))
plot(datel(205:end,:), cumsum(ret_erc_tsm_ls))
title ('Equal Risk Contribution Strategies')
legend('ERC', 'ERC TSM LO', 'ERC TSM LS', 'Location', 'northwest')
ylabel('Log Cumulative Returns')
xlabel('Date')
legend boxoff
box on

% Plot IV
figure
hold on
plot(datel(205:end,:), cumsum(ret_iv(180:end,:)))
plot(datel(205:end,:), cumsum(ret_iv_tsm_lo))
plot(datel(205:end,:), cumsum(ret_iv_tsm_ls))
title ('Inverse Volatility Strategies')
legend('IV', 'IV TSM LO', 'IV TSM LS', 'Location', 'northwest')
ylabel('Log Cumulative Returns')
xlabel('Date')
legend boxoff
box on

% Plot MV
figure
hold on
plot(datel(205:end,:), cumsum(ret_mv(180:end,:)))
plot(datel(205:end,:), cumsum(ret_mv_tsm_lo))
plot(datel(205:end,:), cumsum(ret_mv_tsm_ls))
title ('Minimum Variance Strategies')
legend('MV', 'MV TSM LO', 'MV TSM LS', 'Location', 'northwest')
ylabel('Log Cumulative Returns')
xlabel('Date')
legend boxoff
box on

% Plot 1N
figure
hold on
plot(datel(205:end,:), cumsum(ret_1n(204:end,:)))
plot(datel(205:end,:), cumsum(ret_1n_tsm_lo))
plot(datel(205:end,:), cumsum(ret_1n_tsm_ls))
title ('1/N Strategies')
legend('1/N', '1/N TSM LO', '1/N TSM LS', 'Location', 'northwest')
ylabel('Log Cumulative Returns')
xlabel('Date')
legend boxoff
box on

```

□□□□□□

Maximum Drawdown

```

% BAC

%ret_erc
n = max(size(ret_erc));
cr = cumsum(ret_erc);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_erc(i) = mx - cr(i);
end

%ret_mv
n = max(size(ret_mv));
cr = cumsum(ret_mv);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end

```

```

ret_iv
n = max(size(ret_iv));
cr = cumsum(ret_iv);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_iv(i) = mx - cr(i);
end

%ret_1N
n = max(size(ret_1N));
cr = cumsum(ret_1N);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_1N(i) = mx - cr(i);
end

%ret_mv
n = max(size(ret_60_40));
cr = cumsum(ret_60_40);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_60_40(i) = mx - cr(i);
end

% TSM lo

%ret_1N_tsm_lo
n = max(size(ret_1N_tsm_lo));
cr = cumsum(ret_1N_tsm_lo);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_1N_tsm_lo(i) = mx - cr(i);
end

%ret_erc_tsm_lo
n = max(size(ret_erc_tsm_lo));
cr = cumsum(ret_erc_tsm_lo);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_erc_tsm_lo(i) = mx - cr(i);
end

%ret_iv_tsm_lo
n = max(size(ret_iv_tsm_lo));
cr = cumsum(ret_iv_tsm_lo);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_iv_tsm_lo(i) = mx - cr(i);
end

%ret_mv_tsm_lo
n = max(size(ret_mv_tsm_lo));
cr = cumsum(ret_mv_tsm_lo);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_mv_tsm_lo(i) = mx - cr(i);
end

% TSM ls

%ret_1N_tsm_ls
n = max(size(ret_1N_tsm_ls));
cr = cumsum(ret_1N_tsm_ls);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_1N_tsm_ls(i) = mx - cr(i);
end

%ret_erc_tsm_ls
n = max(size(ret_erc_tsm_ls));
cr = cumsum(ret_erc_tsm_ls);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_erc_tsm_ls(i) = mx - cr(i);
end

%ret_iv_tsm_ls
n = max(size(ret_iv_tsm_ls));
cr = cumsum(ret_iv_tsm_ls);
mx = 0;
for i = 1:n
if cr(i) > mx mx = cr(i); end
dd_iv_tsm_ls(i) = mx - cr(i);
end

%ret_mv_tsm_ls
n = max(size(ret_mv_tsm_ls));
cr = cumsum(ret_mv_tsm_ls);
mx = 0;
for i = 1:n

```

Plot Maximum Drawdown

```

% BAC

figure
hold on
plot(datel(26:end,:),dd_60_40(:,25:end))
plot(datel(26:end,:),dd_erc)
plot(datel(26:end,:),dd_iv)
plot(datel(26:end,:),dd_mv)
plot(datel(26:end,:),dd_1N(:,25:end))
ylabel('MDD')
xlabel('Date')
legend('60/40', 'ERC', 'IV', 'MV', '1/N', 'Location', 'best')
set(gca, 'Ydir', 'reverse')
title ('Maximum Drawdown Broad Asset Classes')
legend boxoff
box on

% Plot TSM lo

figure
hold on
plot(datel(205:end,:),dd_60_40(:,204:end))
plot(datel(205:end,:),dd_erc(:,180:end))
plot(datel(205:end,:),dd_iv_tsm_lo)
plot(datel(205:end,:),dd_mv_tsm_lo)
plot(datel(205:end,:),dd_1N_tsm_lo)
plot(datel(205:end,:),dd_erc_tsm_lo)
ylabel('MDD')
xlabel('Date')
legend('60/40', 'ERC', 'IV TSM LO', 'MV TSM LO', '1/N TSM LO', 'ERC TSM LO', 'Location', 'best')
set(gca, 'Ydir', 'reverse')
title ('Maximum Drawdown TSM Long Only')
legend boxoff
box on

% Plot TSM ls

figure
hold on
plot(datel(205:end,:),dd_60_40(:,204:end))
plot(datel(205:end,:),dd_erc(:,180:end))
plot(datel(205:end,:),dd_iv_tsm_ls)
plot(datel(205:end,:),dd_mv_tsm_ls)
plot(datel(205:end,:),dd_1N_tsm_ls)
plot(datel(205:end,:),dd_erc_tsm_ls)
ylabel('MDD')
xlabel('Date')
legend('60/40', 'ERC', 'IV TSM LS', 'MV TSM LS', '1/N TSM LS', 'ERC TSM LS', 'Location', 'best')
set(gca, 'Ydir', 'reverse')
title ('Maximum Drawdown TSM Long Short')
legend boxoff
box on

```

□□□

Correlation

```

% Correlation Function

% function Cor = MovCorr1(Data1,Data2,k)
% y = zscore(Data2);
% n = size(y,1);
%
% if (n<k)
%     Cor = NaN(n,1);
% else
%     x = zscore(Data1);
%     x2 = x.^2;
%     y2 = y.^2;
%     xy = x .* y;
%     A=1;
%     B = ones(1,k);
%     Stdx = sqrt((filter(B,A,x2) - (filter(B,A,x).^2)*(1/k))/(k-1));
%     Stdy = sqrt((filter(B,A,y2) - (filter(B,A,y).^2)*(1/k))/(k-1));
%     Cor = (filter(B,A,xy) - filter(B,A,x).*filter(B,A,y)/k)/((k-1)*Stdx.*Stdy);
%     Cor(1:(k-1)) = NaN;
% end
% end

ret_excess(isnan(ret_excess)) = 0;

for i = 1:5
    A(:,i) = moving_correlation(ret_excess(180:end,1), ret_excess(180:end,i+1), 36);
end

for i = 2:5
    B(:,i-1) = moving_correlation(ret_excess(180:end,2), ret_excess(180:end,i+1), 36);
end

for i = 3:5
    C(:,i-2) = moving_correlation(ret_excess(180:end,3), ret_excess(180:end,i+1), 36);
end

```

```

end

for i = 5
    E(:,i-4) = moving_correlation(ret_excess(180:end,i), ret_excess(180:end,i+1), 36);
end

avg_correlation = nanmean([A B C D E],2);
avg_correlation(isnan(avg_correlation)) = [];
hst_avg = mean(avg_correlation);

figure
hold on
plot(date1(216:end,:), avg_correlation, 'k')
plot(get(gca, 'xlim'), [hst_avg hst_avg], '--')
xlabel('Date')
legend('Avg Pairwise Correlation BAC', 'Historical Average', 'Location', 'southeast')
title ('Average Pairwise Asset Correlation BAC')
legend boxoff
box on
% Plot Correlation Coefficient Matrix

correl_mtx = corrcoef(ret_excess(180:end,:));
figure
imagesc(correl_mtx);
colormap((flipud(autumn)))
textStrings = num2str(correl_mtx(:), '%0.2f'); %# Create strings from the matrix values
textStrings = strtrim(cellstr(textStrings)); %# Remove any space padding
[x,y] = meshgrid(1:6); %# Create x and y coordinates for the strings
hStrings = text(x(:),y(:),textStrings(:), 'HorizontalAlignment', 'center');
midValue = mean(get(gca, 'CLim')); %# Get the middle value of the color range
textColors = repmat(correl_mtx(:) > midValue, 1, 3); %# Choose white or black for the
%# text color of the strings so
%# they can be easily seen over
%# the background color
set(hStrings, {'Color'}, num2cell(textColors, 2)); %# Change the text colors

set(gca, 'XTick', 1:6, ... %# Change the axes tick marks
'XTickLabel', {'Dev Eqty', 'US Corp Debt', 'Commodities', 'US Real Estate', 'Global Sov Debt', 'EM Eqty'}, ... %# and tick labels
'YTick', 1:6, ...
'YTickLabel', {'Dev Eqty', 'US Corp Debt', 'Commodities', 'US Real Estate', 'Global Sov Debt', 'EM Eqty'}, ...
'TickLength', [0 0]);
title('Correlation Coefficient Matrix BAC')
xtickangle(45)
box on

```

□□

Sharpe Ratio

```

% Sharpe Ratio BAC
sr_ret_erc = ((1+(mean(ret_erc(180:end,:)))^12-1)/(std(ret_erc(180:end,:))*sqrt(12)));
sr_ret_mv = ((1+(mean(ret_mv(180:end,:)))^12-1)/(std(ret_mv(180:end,:))*sqrt(12)));
sr_ret_iv = ((1+(mean(ret_iv(180:end,:)))^12-1)/(std(ret_iv(180:end,:))*sqrt(12)));
sr_ret_lN = ((1+(mean(ret_lN(204:end,:)))^12-1)/(std(ret_lN(204:end,:))*sqrt(12)));
sr_ret_60_40 = ((1+(mean(ret_60_40(204:end,:)))^12-1)/(std(ret_60_40(204:end,:))*sqrt(12)));

% Sharpe Ratio TSM LO
sr_ret_lN_tsm_lo = ((1+(mean(ret_lN_tsm_lo))^12-1)/(std(ret_lN_tsm_lo)*sqrt(12)));
sr_ret_erc_tsm_lo = ((1+(mean(ret_erc_tsm_lo))^12-1)/(std(ret_erc_tsm_lo)*sqrt(12)));
sr_ret_iv_tsm_lo = ((1+(mean(ret_iv_tsm_lo))^12-1)/(std(ret_iv_tsm_lo)*sqrt(12)));
sr_ret_mv_tsm_lo = ((1+(mean(ret_mv_tsm_lo))^12-1)/(std(ret_mv_tsm_lo)*sqrt(12)));

% Sharpe Ratio TSM LS
sr_ret_lN_tsm_ls = ((1+(mean(ret_lN_tsm_ls))^12-1)/(std(ret_lN_tsm_ls)*sqrt(12)));
sr_ret_erc_tsm_ls = ((1+(mean(ret_erc_tsm_ls))^12-1)/(std(ret_erc_tsm_ls)*sqrt(12)));
sr_ret_iv_tsm_ls = ((1+(mean(ret_iv_tsm_ls))^12-1)/(std(ret_iv_tsm_ls)*sqrt(12)));
sr_ret_mv_tsm_ls = ((1+(mean(ret_mv_tsm_ls))^12-1)/(std(ret_mv_tsm_ls)*sqrt(12)));

```

Net Exposure Long Short

```

net_exposure_long_short = w_risky_assets-(1-w_risky_assets);
hst_exp = mean(net_exposure_long_short);
figure
hold on
plot(date1(205:end,:), net_exposure_long_short, 'k')
plot(get(gca, 'xlim'), [hst_exp hst_exp], '--')
legend('Net Exposure', 'Historical Average', 'Location', 'southwest')
title('Net Exposure LS Strategy')
ylabel('Net Exposure')
xlabel('Date')
legend boxoff
box on

```

□

Rolling Window Cumulative Returns

36month

```

j = 1;
p = 36;

```

```

roll_ret_erc = ones(m-p,n);

for i = 1:1:m-p
roll_ret_erc(i,1:n) = sum(ret_erc(i:i+(p-1),:));
end

% ret_mv
[m,n] = size(ret_mv);
roll_ret_mv = ones(m-p,n);

for i = 1:1:m-p
roll_ret_mv(i,1:n) = sum(ret_mv(i:i+(p-1),:));
end

% ret_iv
[m,n] = size(ret_iv);
roll_ret_iv = ones(m-p,n);

for i = 1:1:m-p
roll_ret_iv(i,1:n) = sum(ret_iv(i:i+(p-1),:));
end

% ret_1N
[m,n] = size(ret_1N);
roll_ret_1N = ones(m-p,n);

for i = 1:1:m-p
roll_ret_1N(i,1:n) = sum(ret_1N(i:i+(p-1),:));
end

% ret_60_40
[m,n] = size(ret_60_40);
roll_ret_60_40 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_60_40(i,1:n) = sum(ret_60_40(i:i+(p-1),:));
end

% ret_1N_tsm_lo
[m,n] = size(ret_1N_tsm_lo);
roll_ret_1N_tsm_lo = ones(m-p,n);

for i = 1:1:m-p
roll_ret_1N_tsm_lo(i,1:n) = sum(ret_1N_tsm_lo(i:i+(p-1),:));
end

% ret_erc_tsm_lo
[m,n] = size(ret_erc_tsm_lo);
roll_ret_erc_tsm_lo = ones(m-p,n);

for i = 1:1:m-p
roll_ret_erc_tsm_lo(i,1:n) = sum(ret_erc_tsm_lo(i:i+(p-1),:));
end

% ret_iv_tsm_lo
[m,n] = size(ret_iv_tsm_lo);
roll_ret_iv_tsm_lo = ones(m-p,n);

for i = 1:1:m-p
roll_ret_iv_tsm_lo(i,1:n) = sum(ret_iv_tsm_lo(i:i+(p-1),:));
end

% ret_mv_tsm_lo
[m,n] = size(ret_mv_tsm_lo);
roll_ret_mv_tsm_lo = ones(m-p,n);

for i = 1:1:m-p
roll_ret_mv_tsm_lo(i,1:n) = sum(ret_mv_tsm_lo(i:i+(p-1),:));
end

% ret_1N_tsm_ls
[m,n] = size(ret_1N_tsm_ls);
roll_ret_1N_tsm_ls = ones(m-p,n);

for i = 1:1:m-p
roll_ret_1N_tsm_ls(i,1:n) = sum(ret_1N_tsm_ls(i:i+(p-1),:));
end

% ret_erc_tsm_ls
[m,n] = size(ret_erc_tsm_ls);
roll_ret_erc_tsm_ls = ones(m-p,n);

for i = 1:1:m-p
roll_ret_erc_tsm_ls(i,1:n) = sum(ret_erc_tsm_ls(i:i+(p-1),:));
end

% ret_iv_tsm_ls
[m,n] = size(ret_iv_tsm_ls);
roll_ret_iv_tsm_ls = ones(m-p,n);

for i = 1:1:m-p
roll_ret_iv_tsm_ls(i,1:n) = sum(ret_iv_tsm_ls(i:i+(p-1),:));
end

% ret_mv_tsm_ls
[m,n] = size(ret_mv_tsm_ls);
roll_ret_mv_tsm_ls = ones(m-p,n);

for i = 1:1:m-p

```

```

j = 1;
p = 60;

% BAC
% ret_erc
[m,n] = size(ret_erc);
roll_ret_erc_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_erc_5(i,1:n) = sum(ret_erc(i:i+(p-1),:));
end

% ret_mv
[m,n] = size(ret_mv);
roll_ret_mv_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_mv_5(i,1:n) = sum(ret_mv(i:i+(p-1),:));
end

% ret_iv
[m,n] = size(ret_iv);
roll_ret_iv_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_iv_5(i,1:n) = sum(ret_iv(i:i+(p-1),:));
end

% ret_1N
[m,n] = size(ret_1N);
roll_ret_1N_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_1N_5(i,1:n) = sum(ret_1N(i:i+(p-1),:));
end

% ret_60_40
[m,n] = size(ret_60_40);
roll_ret_60_40_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_60_40_5(i,1:n) = sum(ret_60_40(i:i+(p-1),:));
end

% ret_1N_tsm_lo
[m,n] = size(ret_1N_tsm_lo);
roll_ret_1N_tsm_lo_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_1N_tsm_lo_5(i,1:n) = sum(ret_1N_tsm_lo(i:i+(p-1),:));
end

% ret_erc_tsm_lo
[m,n] = size(ret_erc_tsm_lo);
roll_ret_erc_tsm_lo_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_erc_tsm_lo_5(i,1:n) = sum(ret_erc_tsm_lo(i:i+(p-1),:));
end

% ret_iv_tsm_lo
[m,n] = size(ret_iv_tsm_lo);
roll_ret_iv_tsm_lo_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_iv_tsm_lo_5(i,1:n) = sum(ret_iv_tsm_lo(i:i+(p-1),:));
end

% ret_mv_tsm_lo
[m,n] = size(ret_mv_tsm_lo);
roll_ret_mv_tsm_lo_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_mv_tsm_lo_5(i,1:n) = sum(ret_mv_tsm_lo(i:i+(p-1),:));
end

% ret_1N_tsm_ls
[m,n] = size(ret_1N_tsm_ls);
roll_ret_1N_tsm_ls_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_1N_tsm_ls_5(i,1:n) = sum(ret_1N_tsm_ls(i:i+(p-1),:));
end

% ret_erc_tsm_ls
[m,n] = size(ret_erc_tsm_ls);
roll_ret_erc_tsm_ls_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_erc_tsm_ls_5(i,1:n) = sum(ret_erc_tsm_ls(i:i+(p-1),:));
end

% ret_iv_tsm_ls
[m,n] = size(ret_iv_tsm_ls);
roll_ret_iv_tsm_ls_5 = ones(m-p,n);

for i = 1:1:m-p
roll_ret_iv_tsm_ls_5(i,1:n) = sum(ret_iv_tsm_ls(i:i+(p-1),:));
end

```



```

roll_ret_mv_tsm_ls_5 = ones(m-p,n);

for i = 1:m-p
roll_ret_mv_tsm_ls_5(i,1:n) = sum(ret_mv_tsm_ls(i:i+(p-1),:));
end

```

Statistics Rolling Window 36/60m Returns

```

%36m
mean_rolling_36m = [mean(roll_ret_1N);mean(roll_ret_erc); mean(roll_ret_iv); mean(roll_ret_mv);mean(roll_ret_60_40);
mean(roll_ret_1N_tsm_lo);mean(roll_ret_erc_tsm_lo);mean(roll_ret_iv_tsm_lo);mean(roll_ret_mv_tsm_lo);0;mean(roll_ret_1N_tsm_ls)...
;mean(roll_ret_erc_tsm_ls);mean(roll_ret_iv_tsm_ls);mean(roll_ret_mv_tsm_ls);0];

std_rolling_36m = [std(roll_ret_1N);std(roll_ret_erc); std(roll_ret_iv); std(roll_ret_mv);std(roll_ret_60_40);
std(roll_ret_1N_tsm_lo);std(roll_ret_erc_tsm_lo);std(roll_ret_iv_tsm_lo);std(roll_ret_mv_tsm_lo);0;std(roll_ret_1N_tsm_ls)...
;std(roll_ret_erc_tsm_ls);std(roll_ret_iv_tsm_ls);std(roll_ret_mv_tsm_ls);0];

max_rolling_36m = [max(roll_ret_1N);max(roll_ret_erc); max(roll_ret_iv); max(roll_ret_mv);max(roll_ret_60_40);
max(roll_ret_1N_tsm_lo);max(roll_ret_erc_tsm_lo);max(roll_ret_iv_tsm_lo);max(roll_ret_mv_tsm_lo);0;max(roll_ret_1N_tsm_ls)...
;max(roll_ret_erc_tsm_ls);max(roll_ret_iv_tsm_ls);max(roll_ret_mv_tsm_ls);0];

min_rolling_36m = [min(roll_ret_1N);min(roll_ret_erc); min(roll_ret_iv); min(roll_ret_mv);min(roll_ret_60_40);
min(roll_ret_1N_tsm_lo);min(roll_ret_erc_tsm_lo);min(roll_ret_iv_tsm_lo);min(roll_ret_mv_tsm_lo);0;min(roll_ret_1N_tsm_ls)...
;min(roll_ret_erc_tsm_ls);min(roll_ret_iv_tsm_ls);min(roll_ret_mv_tsm_ls);0];

mean_rolling_36m = reshape(mean_rolling_36m,5,3);
std_rolling_36m = reshape(std_rolling_36m,5,3);
max_rolling_36m = reshape(max_rolling_36m,5,3);
min_rolling_36m = reshape(min_rolling_36m,5,3);

%60m
mean_rolling_60m = [mean(roll_ret_1N_5);mean(roll_ret_erc_5); mean(roll_ret_iv_5); mean(roll_ret_mv_5);mean(roll_ret_60_40_5);
mean(roll_ret_1N_tsm_lo_5);mean(roll_ret_erc_tsm_lo_5);mean(roll_ret_iv_tsm_lo_5);mean(roll_ret_mv_tsm_lo_5);0;mean(roll_ret_1N_tsm_ls_5)...
;mean(roll_ret_erc_tsm_ls_5);mean(roll_ret_iv_tsm_ls_5);mean(roll_ret_mv_tsm_ls_5);0];

std_rolling_60m = [std(roll_ret_1N_5);std(roll_ret_erc_5); std(roll_ret_iv_5); std(roll_ret_mv_5);std(roll_ret_60_40_5);
std(roll_ret_1N_tsm_lo_5);std(roll_ret_erc_tsm_lo_5);std(roll_ret_iv_tsm_lo_5);std(roll_ret_mv_tsm_lo_5);0;std(roll_ret_1N_tsm_ls_5)...
;std(roll_ret_erc_tsm_ls_5);std(roll_ret_iv_tsm_ls_5);std(roll_ret_mv_tsm_ls_5);0];

max_rolling_60m = [max(roll_ret_1N_5);max(roll_ret_erc_5); max(roll_ret_iv_5); max(roll_ret_mv_5);max(roll_ret_60_40_5);
max(roll_ret_1N_tsm_lo_5);max(roll_ret_erc_tsm_lo_5);max(roll_ret_iv_tsm_lo_5);max(roll_ret_mv_tsm_lo_5);0;max(roll_ret_1N_tsm_ls_5)...
;max(roll_ret_erc_tsm_ls_5);max(roll_ret_iv_tsm_ls_5);max(roll_ret_mv_tsm_ls_5);0];

min_rolling_60m = [min(roll_ret_1N_5);min(roll_ret_erc_5); min(roll_ret_iv_5); min(roll_ret_mv_5);min(roll_ret_60_40_5);
min(roll_ret_1N_tsm_lo_5);min(roll_ret_erc_tsm_lo_5);min(roll_ret_iv_tsm_lo_5);min(roll_ret_mv_tsm_lo_5);0;min(roll_ret_1N_tsm_ls_5)...
;min(roll_ret_erc_tsm_ls_5);min(roll_ret_iv_tsm_ls_5);min(roll_ret_mv_tsm_ls_5);0];

mean_rolling_60m = reshape(mean_rolling_60m,5,3);
std_rolling_60m = reshape(std_rolling_60m,5,3);
max_rolling_60m = reshape(max_rolling_60m,5,3);
min_rolling_60m = reshape(min_rolling_60m,5,3);

```

Plot Rolling Returns

```

% Plot BAC 36month

figure
hold on
plot(date1(62:end,:),roll_ret_60_40(25:end,:))
plot(date1(62:end,:),roll_ret_erc)
plot(date1(62:end,:),roll_ret_iv)
plot(date1(62:end,:),roll_ret_mv)
plot(date1(62:end,:),roll_ret_1N(25:end,:))
ylabel('Rolling 36m Returns')
xlabel('Date')
legend('60/40', 'ERC', 'IV', 'MV', '1/N', 'Location', 'best')
title('Rolling 36-Month Returns BAC')
legend boxoff
ylim([-0.55 0.65])
box on

% Plot TSM lo 36month

figure
hold on
plot(date1(241:end,:),roll_ret_60_40(204:end,:))
plot(date1(241:end,:),roll_ret_erc(180:end,:))
plot(date1(241:end,:),roll_ret_iv_tsm_lo)
plot(date1(241:end,:),roll_ret_mv_tsm_lo)
plot(date1(241:end,:),roll_ret_1N_tsm_lo)
plot(date1(241:end,:),roll_ret_erc_tsm_lo)
ylabel('Rolling 36m Returns')
xlabel('Date')
legend('60/40', 'ERC', 'IV TSM LO','MV TSM LO', '1/N TSM LO', 'ERC TSM LO', 'Location', 'best')
title('Rolling Window 36-Month TSM Long Only')
legend boxoff
box on

% Plot TSM LS 36month

figure
hold on
plot(date1(241:end,:),roll_ret_60_40(204:end,:))
plot(date1(241:end,:),roll_ret_erc(180:end,:))
plot(date1(241:end,:),roll_ret_iv_tsm_ls)
plot(date1(241:end,:),roll_ret_mv_tsm_ls)

```

```

date1 = date(1986,1,1);
legend('60/40', 'ERC', 'IV TSM LS', 'MV TSM LS', '1/N TSM LS', 'ERC TSM LS', 'Location', 'best')
title ('Rolling Window 36-Month TSM Long Short')
legend boxoff
box on
    % Plot BAC 60month

figure
hold on
plot(date1(86:end,:),roll_ret_60_40_5(25:end,:))
plot(date1(86:end,:),roll_ret_erc_5)
plot(date1(86:end,:),roll_ret_iv_5)
plot(date1(86:end,:),roll_ret_mv_5)
plot(date1(86:end,:),roll_ret_1N_5(25:end,:))
ylabel('Rolling 60m Returns')
xlabel('Date')
legend('60/40', 'ERC', 'IV', 'MV', '1/N', 'Location', 'best')
title ('Rolling 60-Month Returns BAC')
legend boxoff
ylim([-0.45 0.8])
box on
    % Plot TSM lo 60month

figure
hold on
plot(date1(265:end,:),roll_ret_60_40_5(204:end,:))
plot(date1(265:end,:),roll_ret_erc_5(180:end,:))
plot(date1(265:end,:),roll_ret_iv_tsm_lo_5)
plot(date1(265:end,:),roll_ret_mv_tsm_lo_5)
plot(date1(265:end,:),roll_ret_1N_tsm_lo_5)
plot(date1(265:end,:),roll_ret_erc_tsm_lo_5)
ylabel('Rolling 60m Returns')
xlabel('Date')
legend('60/40', 'ERC', 'IV TSM LO', 'MV TSM LO', '1/N TSM LO', 'ERC TSM LO', 'Location', 'best')
title ('Rolling Window 60-Month TSM Long Only')
legend boxoff
ylim([-0.4 0.7])
box on
    % Plot TSM LS 60month

figure
hold on
plot(date1(265:end,:),roll_ret_60_40_5(204:end,:))
plot(date1(265:end,:),roll_ret_erc_5(180:end,:))
plot(date1(265:end,:),roll_ret_iv_tsm_ls_5)
plot(date1(265:end,:),roll_ret_mv_tsm_ls_5)
plot(date1(265:end,:),roll_ret_1N_tsm_ls_5)
plot(date1(265:end,:),roll_ret_erc_tsm_ls_5)
ylabel('Rolling 60m Returns')
xlabel('Date')
legend('60/40', 'ERC', 'IV TSM LS', 'MV TSM LS', '1/N TSM LS', 'ERC TSM LS', 'Location', 'best')
title ('Rolling Window 60-Month TSM Long Short')
legend boxoff
box on
ylim([-0.4 0.7])

```

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Risk Contribution BAC

```

% 1/N
w3_1N = ones(6,331)*1/6;

k=0;
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    k=k+1;
    mrc_1N(1:6,k) = (cov3*w3_1N(1:6,k))/sqrt(w3_1N(1:6,k)'*cov3*w3_1N(1:6,k));
    trc_1N(1:6,k) = mrc_1N(1:6,k)./sum(mrc_1N(1:6,k));
end

% 60/40
w_60_40_1 = ones(1,331)*0.6;
w_60_40_2 = ones(1,331)*0.4;
w_60_40 = [w_60_40_1; w_60_40_2];

k=0;
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    cov3 = cov3(1:2,1:2);
    k=k+1;
    mrc_60_40(1:2,k) = (cov3*w_60_40(1:2,k))/sqrt(w_60_40(1:2,k)'*cov3*w_60_40(1:2,k));
    trc_60_40(1:2,k) = mrc_60_40(1:2,k)./sum(mrc_60_40(1:2,k));
end

% Minimum Variance
k=0;
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    k=k+1;
    mrc_mv(1:6,k) = (cov3*w3_mv(1:6,k))/sqrt(w3_mv(1:6,k)'*cov3*w3_mv(1:6,k));
    trc_mv(1:6,k) = mrc_mv(1:6,k)./sum(mrc_mv(1:6,k));
end

```

```

% Equal Risk Contribution
k=0;
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    k=k+1;
    mrc_erc(1:6,k) = (cov3*w3_erc(1:6,k))/sqrt(w3_erc(1:6,k)*cov3*w3_erc(1:6,k));
    trc_erc(1:6,k) = mrc_erc(1:6,k)./sum(mrc_erc(1:6,k));
end

% Inverse Volatility
k=0;
for i = 1:6:n_rows3
    cov3 = varcovmatrixS3(i:i+5,1:6);
    k=k+1;
    mrc_iv(1:6,k) = (cov3*w3_iv(1:6,k))/sqrt(w3_iv(1:6,k)*cov3*w3_iv(1:6,k));
    trc_iv(1:6,k) = mrc_iv(1:6,k)./sum(mrc_iv(1:6,k));
end

% 60/40
figure
area(datel(204:end,:), trc_60_40')
ylim ([0 1]);
legend({'Dev Eqty', 'US Corp Debt'}, 'FontSize', 5, 'Location', 'southoutside','Orientation', 'horizontal')
title('Total Risk Contribution 60/40')
ylabel('% Risk Contribution')
xlabel('Date')

% IV
figure
area(datel(204:end,:), trc_iv')
ylim ([0 1]);
legend({'Dev Eqty', 'US Corp Debt', 'Comm', 'US RE', 'Glob Sov Debt', 'EM Eqty'}, 'FontSize', 5, 'Location', 'southoutside','Orientation', 'horizontal')
title('Total Risk Contribution IV')
ylabel('% Risk Contribution')
xlabel('Date')

% 1/N
figure
area(datel(204:end,:), trc_1N')
ylim ([0 1]);
legend({'Dev Eqty', 'US Corp Debt', 'Comm', 'US RE', 'Glob Sov Debt', 'EM Eqty'}, 'FontSize', 5, 'Location', 'southoutside','Orientation', 'horizontal')
title('Total Risk Contribution 1/N')
ylabel('% Risk Contribution')
xlabel('Date')

% ERC
figure
area(datel(204:end,:), trc_erc')
ylim ([0 1]);
legend({'Dev Eqty', 'US Corp Debt', 'Comm', 'US RE', 'Glob Sov Debt', 'EM Eqty'}, 'FontSize', 5, 'Location', 'southoutside','Orientation', 'horizontal')
title('Total Risk Contribution ERC')
ylabel('% Risk Contribution')
xlabel('Date')

% min variance
figure
area(datel(204:end,:), trc_mv')
ylim ([0 1]);
legend({'Dev Eqty', 'US Corp Debt', 'Comm', 'US RE', 'Glob Sov Debt', 'EM Eqty'}, 'FontSize', 5, 'Location', 'southoutside','Orientation', 'horizontal')
title('Total Risk Contribution MV')
ylabel('% Risk Contribution')
xlabel('Date')
%

```

□□□□

Turnover & Transaction Costs

```

bp = 15*0.01/100;

% BAC

% 60/40
x_60_40 = w_60_40.*(1+ret_excess(203:end,1:2)');
trading_60_40 = sum(abs((x_60_40./sum(x_60_40))-x_60_40));
avg_trading_60_40 = mean(trading_60_40)*12;

% 1/N
x_1N = w_1N(203:end,:)' .* (1+ret_excess(203:end,:));
trading_1N = sum(abs((x_1N./sum(x_1N))-x_1N));
avg_trading_1N = mean(trading_1N)*12;

% ERC
x_erc = w3_erc.*(1+ret_excess(203:end,:));
trading_erc = sum(abs((x_erc./sum(x_erc))-x_erc));
avg_trading_erc = mean(trading_erc)*12;

% MV
x_mv = w3_mv.*(1+ret_excess(203:end,:));
trading_mv = sum(abs((x_mv./sum(x_mv))-x_mv));
avg_trading_mv = mean(trading_mv)*12;

```

```

trading_iv = sum(abs(x_iv./ (sum(x_iv); x_iv)));
avg_trading_iv = mean(trading_iv)*12;

% TSM LO

% 1/N
x_1N_tsm_lo = w_1N_tsm_lo.*(1+ret_exc(25:end,:));
trading_1N_tsm_lo = nansum(abs((x_1N_tsm_lo./ (nansum(x_1N_tsm_lo)))-x_1N_tsm_lo));
avg_trading_1N_tsm_lo = mean(trading_1N_tsm_lo)*12;

% ERC
x_erc_tsm_lo = w_erc_tsm_lo(:,25:end).*(1+ret_exc(25:end,:));
trading_erc_tsm_lo = nansum(abs((x_erc_tsm_lo./ (nansum(x_erc_tsm_lo)))-x_erc_tsm_lo));
avg_trading_erc_tsm_lo = mean(trading_erc_tsm_lo)*12;

% MV
x_mv_tsm_lo = w_mv_tsm_lo(:,25:end).*(1+ret_exc(25:end,:));
trading_mv_tsm_lo = nansum(abs((x_mv_tsm_lo./ (nansum(x_mv_tsm_lo)))-x_mv_tsm_lo));
avg_trading_mv_tsm_lo = mean(trading_mv_tsm_lo)*12;

% IV
x_iv_tsm_lo = w_iv_tsm_lo(:,25:end).*(1+ret_exc(25:end,:));
trading_iv_tsm_lo = nansum(abs((x_iv_tsm_lo./ (nansum(x_iv_tsm_lo)))-x_iv_tsm_lo));
avg_trading_iv_tsm_lo = mean(trading_iv_tsm_lo)*12;

% TSM LS

% 1/N
x_1N_tsm_ls = w_1N_ls'.*(1+ret_exc(25:end,:));
trading_1N_tsm_ls = nansum(abs((x_1N_tsm_ls./ (nansum(x_1N_tsm_ls)))-x_1N_tsm_ls));
avg_trading_1N_tsm_ls = mean(trading_1N_tsm_ls)*12;

% ERC Long
x_erc_long = w_erc_long(:,25:end).*(1+ret_exc(25:end,:));
trading_erc_long = nansum(abs((x_erc_long./ (nansum(x_erc_long)))-x_erc_long));
avg_trading_erc_long = mean(trading_erc_long)*12;

% ERC Short
x_erc_short = w_erc_short(:,25:end).*(1+ret_exc(25:end,:));
trading_erc_short = nansum(abs((x_erc_short./ (nansum(x_erc_short)))-x_erc_short));
avg_trading_erc_short = mean(trading_erc_short)*12;

avg_trading_erc_tsm_ls = (avg_trading_erc_long*(mean(w_risky_assets))+avg_trading_erc_short*(1-mean(w_risky_assets)));

% MV Long
x_mv_long = w_mv_long(:,25:end).*(1+ret_exc(25:end,:));
trading_mv_long = nansum(abs((x_mv_long./ (nansum(x_mv_long)))-x_mv_long));
avg_trading_mv_long = mean(trading_mv_long)*12;

% MV Short
x_mv_short = w_mv_short(:,25:end).*(1+ret_exc(25:end,:));
trading_mv_short = nansum(abs((x_mv_short./ (nansum(x_mv_short)))-x_mv_short));
avg_trading_mv_short = mean(trading_mv_short)*12;

avg_trading_mv_tsm_ls = (avg_trading_mv_long*(mean(w_risky_assets))+avg_trading_mv_short*(1-mean(w_risky_assets)));

% IV Long
x_iv_long = w_iv_long(:,25:end).*(1+ret_exc(25:end,:)).*w_risky_assets;
trading_iv_long = nansum(abs(((x_iv_long./ (nansum(x_iv_long)))).*w_risky_assets)-x_iv_long));

% IV Short
x_iv_short = w_iv_short(:,25:end).*(1+ret_exc(25:end,:)).*(1-w_risky_assets);
trading_iv_short = nansum(abs(((x_iv_short./ (nansum(x_iv_short)))).*(1-w_risky_assets))-x_iv_short));
avg_trading_iv_tsm_ls = mean(trading_iv_short)*mean(w_risky_assets) + mean(trading_iv_long)*(1-mean(w_risky_assets));

```

Regression

```

% import Fama French 5-Factor data US Data(1990.01)
FF5F_USDATA

% import Fama French Global 3-Factor + WML Data(1990.11)
FF3FWML

FF5F1 = FF5F1(:,2:end)/100;
FF3FWMLglobal = FF3FWMLglobal(:,2:end)/100;

% BAC

fitlm(FF5F1, ret_erc(180:end,:))
fitlm(FF5F1, ret_60_40(204:end,:))
fitlm(FF5F1, ret_mv(180:end,:))
fitlm(FF5F1, ret_iv(180:end,:))
fitlm(FF5F1, ret_1N(204:end,:))

% TSM LO
fitlm(FF5F1, ret_1N_tsm_lo)
fitlm(FF5F1, ret_erc_tsm_lo)
fitlm(FF5F1, ret_iv_tsm_lo)
fitlm(FF5F1, ret_mv_tsm_lo)

% TSM LS

```

```

fitlm(FF3F1, ret_mv_tsm_ls)

% BAC Global
fitlm(FF3FWMLglobal, ret_erc(190:end,:))
fitlm(FF3FWMLglobal, ret_60_40(214:end,:))
fitlm(FF3FWMLglobal, ret_mv(190:end,:))
fitlm(FF3FWMLglobal, ret_iv(190:end,:))
fitlm(FF3FWMLglobal, ret_1N(214:end,:))

% TSM LO Global
fitlm(FF3FWMLglobal, ret_1N_tsm_lo(11:end,:))
fitlm(FF3FWMLglobal, ret_erc_tsm_lo(11:end,:))
fitlm(FF3FWMLglobal, ret_iv_tsm_lo(11:end,:))
fitlm(FF3FWMLglobal, ret_mv_tsm_lo(11:end,:))

% TSM LS Global
fitlm(FF3FWMLglobal, ret_1N_tsm_ls(11:end,:))
fitlm(FF3FWMLglobal, ret_erc_tsm_ls(11:end,:))
fitlm(FF3FWMLglobal, ret_iv_tsm_ls(11:end,:))
fitlm(FF3FWMLglobal, ret_mv_tsm_ls(11:end,:))

% BAC
fitlm(ret_excess(180:end,:), ret_erc(156:end,:))
fitlm(ret_excess(180:end,:), ret_mv(156:end,:))
fitlm(ret_excess(180:end,:), ret_iv(156:end,:))
fitlm(ret_excess(180:end,:), ret_1N(180:end,:))
fitlm(ret_excess(180:end,:), ret_60_40(180:end,:))

% TSM LO
fitlm(ret_excess(204:end,:), ret_1N_tsm_lo)
fitlm(ret_excess(204:end,:), ret_erc_tsm_lo)
fitlm(ret_excess(204:end,:), ret_iv_tsm_lo)
fitlm(ret_excess(204:end,:), ret_mv_tsm_lo)

% TSM LS
fitlm(ret_excess(204:end,:), ret_1N_tsm_ls)
fitlm(ret_excess(204:end,:), ret_erc_tsm_ls)
fitlm(ret_excess(204:end,:), ret_iv_tsm_ls)
fitlm(ret_excess(204:end,:), ret_mv_tsm_ls)

```

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00110371579994349	0.000954167255937249	1.15673200172788	0.248236642883502
x1	0.136813939970383	0.0257547702922462	5.31217861459912	2.02156061094402e-07
x2	-0.032520600771226	0.0333294060070407	-0.975732983790835	0.32992698869718
x3	-0.00437143352543096	0.0446065448295216	-0.097999823616418	0.921993218728343
x4	0.0120894997859588	0.0436652645972347	0.276867663518622	0.782058816449495
x5	0.0590641849324248	0.0629667206973135	0.93802248551762	0.348933844047028
x6	0.0143728487910289	0.0201885691418459	0.711930037737915	0.477021984185569

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.0163

R-squared: 0.0919, Adjusted R-Squared 0.0751

F-statistic vs. constant model: 5.45, p-value = 2.18e-05

GRA 19502

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00173116652567314	0.0007642324443634099	-2.26523558387687	0.024160194137381
x1	0.616251699001763	0.0206280721888162	29.8744203220247	5.79392335424399e-95
x2	-0.0876182616130696	0.0266949145856132	-3.28220797755578	0.00114268951092217
x3	0.0129395741699188	0.0357272465021386	0.362176642108266	0.717456873618988
x4	-0.00278128383567579	0.0349733358144798	-0.0795258379249104	0.936663628137836
x5	0.0739693469488758	0.0504326788901027	1.46669478157331	0.14343202148596
x6	-0.00822871243774654	0.0161698689959693	-0.508891719518427	0.611175559382608

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.013

R-squared: 0.794, Adjusted R-Squared 0.79

F-statistic vs. constant model: 208, p-value = 1.07e-107

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00159396874882637	0.000868417517658846	1.83548663680061	0.0673528683799583
x1	0.143969684213293	0.0234402234470904	6.14199282435439	2.3929293647799e-09
x2	-0.0114300656095414	0.0303341367559794	-0.376805369524429	0.706565669512898
x3	-0.00192599571392567	0.0405978141580002	-0.0474408722210983	0.962191173331613
x4	0.0244058972523129	0.0397411255243714	0.614121943711581	0.539567046080546
x5	0.0575603438098107	0.0573079855159849	1.00440354501303	0.315935984546413
x6	0.00778951028044618	0.0183742493678685	0.423936244931338	0.671894496391113

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.0148

R-squared: 0.12, Adjusted R-Squared 0.104

F-statistic vs. constant model: 7.36, p-value = 2.19e-07

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000248578443533358	0.000864016782550397	0.287700943492795	0.773760170318012
x1	0.338711142540022	0.0233214393228922	14.5235951285196	3.92198907786802e-37
x2	0.0318689445502197	0.0301804174931916	1.05594777002038	0.291781234183491
x3	0.0617678019163057	0.0403920833632421	1.52920564559233	0.12719217806504
x4	0.0368105800013493	0.0395397360282038	0.930976877920789	0.352560821472216
x5	0.0382098300154822	0.0570175753633493	0.670141263846925	0.503246563381927
x6	-0.000517553781343183	0.0182811372384603	-0.0283108088185203	0.977431743627766

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.0147

R-squared: 0.475, Adjusted R-Squared 0.465

F-statistic vs. constant model: 48.6, p-value = 2.18e-42

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00154460652825513	0.00111505623355038	-1.38522747264239	0.16693881075782
x1	0.534622707441501	0.0300974666436426	17.7630467630879	9.96784776705024e-50
x2	0.0958971904906482	0.038949315958424	2.46210204546371	0.0143343781007649
x3	0.140886204143262	0.0521279739582405	2.70269863655405	0.00724154658884948
x4	0.00457947179200792	0.0510279777217333	0.0897443323539251	0.928546008341725
x5	0.0100094311528773	0.0735839906293985	0.136027294351148	0.891884441542961
x6	-0.00901832969240399	0.023592708435552	-0.382250716022677	0.702526789827107

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.019

R-squared: 0.603, Adjusted R-Squared 0.595

F-statistic vs. constant model: 81.7, p-value = 8.66e-62

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

GRA 19502

(Intercept)	0.00101846181205761	0.000755854822817276	1.3474304606029	0.17878604818738
x1	0.278811002793903	0.0204019444335508	13.6659034486635	7.24147013489338e-34
x2	0.0108959189096202	0.0264022812722821	0.412688539950489	0.680108616759928
x3	0.0108859803555994	0.0353355995280834	0.308074024524407	0.758224578949569
x4	-0.00510408860510493	0.0345899533127374	-0.147559858174928	0.882782203140716
x5	0.050342561510098	0.0498798289502221	1.00927694760817	0.313597295909297
x6	0.103405227330707	0.0159926126752853	6.46581202397951	3.72129837991786e-10

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0129
 R-squared: 0.443, Adjusted R-Squared 0.433
 F-statistic vs. constant model: 42.9, p-value = 2.11e-38

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00212289485810769	0.000589257248102671	3.6026622751661	0.000364568759683019
x1	0.119994843617262	0.015905162300941	7.54439604870698	4.6585746962826e-13
x2	-0.00192684461369796	0.0205829679675123	-0.0936135457597394	0.925474189642089
x3	-0.0096577920465548	0.0275472981178689	-0.350589448200372	0.72612490543562
x4	0.00323155266798609	0.0269659994032887	0.119838045668427	0.904685946397287
x5	0.0197414043252977	0.0388858413755802	0.50767589505457	0.612027134708198
x6	0.0408247404634431	0.0124676890991919	3.27444325396992	0.00117351047852314

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0101
 R-squared: 0.193, Adjusted R-Squared 0.178
 F-statistic vs. constant model: 12.9, p-value = 4.36e-13

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00170132666218406	0.000627579285788108	2.71093501763295	0.0070680675496362
x1	0.188984423266682	0.0169395462326656	11.156404113249	1.14820274872359e-24
x2	0.00781757872577149	0.0219215705501176	0.356615813994657	0.72161228215801
x3	-0.017701386507751	0.029338822278164	-0.603343458708827	0.546703662648528
x4	0.0279785072620471	0.0287197191049736	0.974191535780091	0.330690452075083
x5	0.0448629699261611	0.0414147617807565	1.08326036411024	0.279500844794543
x6	0.0764411355624925	0.013278518754742	5.75675171111942	1.99471590053982e-08

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0107
 R-squared: 0.338, Adjusted R-Squared 0.326
 F-statistic vs. constant model: 27.5, p-value = 1.79e-26

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00220738595229397	0.000516663827825836	4.27238338240715	2.54740149099542e-05
x1	0.0541215914774818	0.0139457292431361	3.88087209595867	0.000126187667675867
x2	-0.00838026278882426	0.018047253644063	-0.464351139187382	0.642709013627333
x3	-0.00722130219063985	0.0241536146354838	-0.298973975515495	0.765152168213763
x4	0.019815792466082	0.0236439288913504	0.838092203590209	0.40259866556803
x5	0.0383449698573546	0.0340953084888231	1.12464064872516	0.261576396521578
x6	0.0172066704617549	0.0109317348150948	1.57401096466368	0.116463869828689

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.00881
 R-squared: 0.0473, Adjusted R-Squared 0.0296
 F-statistic vs. constant model: 2.67, p-value = 0.0152

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00252399757051422	0.00101264756554463	2.49247384420141	0.0131863578085399

GRA 19502

```

x1      0.0668208917887305    0.0668259113145007    0.999924886534706    0.318095358792563
x5      0.240188159642512     0.0214259138176339     11.2101710894045    7.41818527398223e-25

```

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0173
 R-squared: 0.325, Adjusted R-Squared 0.312
 F-statistic vs. constant model: 25.9, p-value = 4.03e-25

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0036790424664919	0.000722610233761925	5.0913235027668	6.05197495129869e-07
x1	-0.109648364267083	0.0195046103977693	-5.6216639056591	4.09327206966231e-08
x2	-0.0182301065784883	0.0252410357995744	-0.722240827327524	0.470668986895856
x3	-0.0807598355893602	0.0337814419704759	-2.39065684821691	0.0173906425945343
x4	0.0488861307121208	0.0330685913413477	1.47832516382382	0.140295241767645
x5	0.0271310629065054	0.047685975890688	0.568952661652533	0.569783660426021
x6	0.12901262642231	0.015289213265423	8.43814682826587	1.09726198010277e-15

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0123
 R-squared: 0.381, Adjusted R-Squared 0.37
 F-statistic vs. constant model: 33.2, p-value = 4.04e-31

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00134215165531671	0.000485518959700249	2.76436507473433	0.00603085672959248
x1	0.0383215689377437	0.0131050706276098	2.92417874170077	0.00369777079113288
x2	-0.00680073540060743	0.0169593521798969	-0.40100207416347	0.688683735057555
x3	-0.0279441367913342	0.0226976173272458	-1.23114846763191	0.219163023716932
x4	0.0275987227192246	0.0222186558073208	1.24214187206281	0.215085336612895
x5	0.0288142387949147	0.0320400187058047	0.899320286279808	0.369152034429061
x6	0.0830348808499386	0.0102727619571869	8.08301420747385	1.27935073586429e-14

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.00828
 R-squared: 0.2, Adjusted R-Squared 0.185
 F-statistic vs. constant model: 13.4, p-value = 1.27e-13

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00343573004548229	0.000681455004663878	5.04175627439545	7.70176882686424e-07
x1	-0.129076553907117	0.0183937532968266	-7.01741247825622	1.33473108071742e-11
x2	-0.026263317618021	0.0238034688202148	-1.10333992983902	0.270700909828339
x3	-0.0660590260692693	0.0318574684110101	-2.07358052488687	0.0389105331410238
x4	0.0514829615964273	0.0311852171667007	1.6508771230043	0.0997355916784891
x5	0.0267250509307049	0.0449700895513426	0.5942850280561	0.552737503617401
x6	0.101874273857045	0.0144184380601072	7.06555546671244	9.89226716108401e-12

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0116
 R-squared: 0.389, Adjusted R-Squared 0.378
 F-statistic vs. constant model: 34.3, p-value = 5.67e-32

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000869139064204338	0.000916180531566229	0.948654805749395	0.34352342685139
x1	0.166933391050321	0.0219943168304421	7.58984206407676	3.65387385061143e-13
x2	0.0526517178897173	0.0445006891508607	1.18316634853056	0.237635409295219
x3	0.0647237939897354	0.0406804246174118	1.59103044273615	0.112605760333609
x4	0.0377758631782721	0.0242418286573228	1.55829264005878	0.12016798630347

GRA 19502

F-statistic vs. constant model: 14.9, p-value = 0.00011

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000437289287514182	0.000385608475244694	-1.13402405700936	0.257646737481661
x1	0.639158951483042	0.00925712202434241	69.0451038457004	2.79672781308014e-192
x2	-0.107538302418129	0.0187297615476138	-5.74157349225997	2.20737481741965e-08
x3	0.010519746160611	0.0171218618695271	0.61440433527464	0.539391680605558
x4	0.00356128692062502	0.010203070534268	0.349040703841466	0.727291950814313

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.00658
 R-squared: 0.944, Adjusted R-Squared 0.943
 F-statistic vs. constant model: 1.33e+03, p-value = 8.19e-196

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00160172921083404	0.000828788690736945	1.93261470473229	0.0541799362302389
x1	0.168348793454682	0.019896341846943	8.4612937700397	1.0095314299561e-15
x2	0.059561131973597	0.0402558956750409	1.47956295530956	0.139989283944743
x3	0.0606842726575569	0.036800035250313	1.64902756871791	0.100138871803928
x4	0.0276516716136695	0.021929469948052	1.26093661539347	0.208264793220056

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.0141
 R-squared: 0.192, Adjusted R-Squared 0.182
 F-statistic vs. constant model: 18.8, p-value = 7.5e-14

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000507105667281371	0.000734129824726455	0.690757479401309	0.490226645137147
x1	0.386367032838697	0.017623910794205	21.9228885887087	2.42305743915977e-65
x2	0.10087578709217	0.0356581285029917	2.82897031692806	0.00496926761319038
x3	0.133594935007858	0.0325969740299152	4.09838455818794	5.29795589618529e-05
x4	0.0236040786973397	0.0194248281971517	1.21514993377397	0.225218919199317

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.0125
 R-squared: 0.62, Adjusted R-Squared 0.615
 F-statistic vs. constant model: 128, p-value = 7.08e-65

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00140778525111363	0.000877741596078797	-1.60387209333902	0.109744390639469
x1	0.627359996931702	0.0210715313131705	29.7728716346106	1.41166936619048e-93
x2	0.209831821465371	0.0426336345033537	4.92174368687391	1.38561638150022e-06
x3	0.199146486395457	0.0389736515922343	5.1097723271878	5.6023306869064e-07
x4	0.0160556597821139	0.0232247473553842	0.691316875763205	0.489875594505374

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.015
 R-squared: 0.755, Adjusted R-Squared 0.752
 F-statistic vs. constant model: 242, p-value = 8.62e-95

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.00140778525111363	0.000877741596078797	-1.60387209333902	0.109744390639469
x1	0.627359996931702	0.0210715313131705	29.7728716346106	1.41166936619048e-93
x2	0.209831821465371	0.0426336345033537	4.92174368687391	1.38561638150022e-06
x3	0.199146486395457	0.0389736515922343	5.1097723271878	5.6023306869064e-07
x4	0.0160556597821139	0.0232247473553842	0.691316875763205	0.489875594505374

GRA 19502

x2	0.027412070010754	0.00174012000000001	0.00000000000000000	0.00000000000000000
x3	0.0976687574296804	0.0287602995144512	3.39595759010106	0.000771521200902448
x4	0.179454574678931	0.0171385195587216	10.4708329131969	3.26215455811459e-22

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.011
 R-squared: 0.592, Adjusted R-Squared 0.587
 F-statistic vs. constant model: 114, p-value = 4.09e-60

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00198021463458747	0.000556371884845049	3.55915654353915	0.000429361360236657
x1	0.140475357465604	0.0133565592033623	10.5173312472753	2.26511641059938e-22
x2	0.00160246864959191	0.0270240759836297	0.0592978146806069	0.952752502357907
x3	0.0301335886109129	0.0247041317086208	1.21977930519199	0.223461023532801
x4	0.0752098202866518	0.0147214129065898	5.10887241352935	5.62702013155039e-07

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.00949
 R-squared: 0.274, Adjusted R-Squared 0.265
 F-statistic vs. constant model: 29.7, p-value = 5.42e-21

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00180596200922021	0.000579899093363751	3.11426941322599	0.002013432835465
x1	0.196788158962484	0.0139213658767933	14.1356933438929	1.79593237165149e-35
x2	-0.0279105425651553	0.028166838743594	-0.990900782982008	0.322494569257854
x3	0.0434493649980707	0.0257487913577044	1.68743318451218	0.0925099668801769
x4	0.128702909942883	0.0153439349293188	8.38786859666364	1.68404846847315e-15

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.00989
 R-squared: 0.419, Adjusted R-Squared 0.412
 F-statistic vs. constant model: 56.9, p-value = 4.41e-36

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00218144954970085	0.000510748728959573	4.27108170028069	2.57883117663213e-05
x1	0.0557851801785433	0.0122613054724907	4.54969336696669	7.67825083611815e-06
x2	-0.000691681448606591	0.0248080696309624	-0.0278813087392869	0.977774471539409
x3	0.0385960814773034	0.0226783635440924	1.70189005931857	0.089762633789132
x4	0.0339513908445463	0.0135142395497282	2.51226794668065	0.0124953087689282

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.00871
 R-squared: 0.0668, Adjusted R-Squared 0.0549
 F-statistic vs. constant model: 5.63, p-value = 0.000216

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00185005775249642	0.000945576280730244	1.95653993252418	0.0512848487006765
x1	0.0177973866542917	0.0227000068099882	0.784025608594118	0.433614233076446
x2	-0.119360888230761	0.0459284984644556	-2.59884150846201	0.00979387267968945
x3	0.0697484795549495	0.0419856603397806	1.66124526780073	0.0976591028647807
x4	0.3633117540036	0.025019630291319	14.521068048302	6.24504043154122e-37

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.0161
 R-squared: 0.418, Adjusted R-Squared 0.411
 F-statistic vs. constant model: 56.7, p-value = 5.68e-36

ans =

GRA 19502

y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00343155020640408	0.000687571276108533	4.99082833393757	9.96538872091008e-07
x1	-0.140057743615702	0.0165062015281966	-8.48515894928639	8.54349498103667e-16
x2	-0.122194395660262	0.0333966882868157	-3.6588776291481	0.000296877023760795
x3	-0.026539347456156	0.0305296723769215	-0.869296831243365	0.385346674372572
x4	0.188959953753303	0.0181929046632606	10.3864642425624	6.31101605745002e-22

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.0117
 R-squared: 0.449, Adjusted R-Squared 0.442
 F-statistic vs. constant model: 64.2, p-value = 1.13e-39

ans =

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00128326672390199	0.000469854352530996	2.7312010987859	0.00666597845191414
x1	0.0279023402011983	0.0112795733348127	2.47370528768868	0.0138988683215173
x2	-0.0516209424469116	0.0228217493908288	-2.26191873212209	0.024383686313159
x3	0.0234286956963971	0.0208625635567964	1.12300176498514	0.262291925465768
x4	0.126297618068334	0.0124321881065101	10.1589210995125	3.69425226162075e-21

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.00801
 R-squared: 0.253, Adjusted R-Squared 0.244
 F-statistic vs. constant model: 26.7, p-value = 4.4e-19

ans =

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00325730076553813	0.000651667659443402	4.99840788220216	9.60933494057228e-07
x1	-0.168489441093602	0.015644280222205	-10.7700347155923	3.08009569389818e-23
x2	-0.110545838807031	0.0316527790576212	-3.49245286190484	0.000547035883212536
x3	-0.017414528104153	0.0289354730669434	-0.601840103455845	0.547713495576695
x4	0.141960764343187	0.0172429070444656	8.23299481793311	4.91158156675329e-15

Number of observations: 320, Error degrees of freedom: 315
 Root Mean Squared Error: 0.0111
 R-squared: 0.451, Adjusted R-Squared 0.444
 F-statistic vs. constant model: 64.8, p-value = 6.3e-40

ans =

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-0.000407623655631414	0.000412404193366005	-0.988408125301607	0.323641509450231
x1	-0.0227359069978335	0.0151855208404436	-1.49720956144494	0.135247773661341
x2	0.54572882479429	0.0343854427738227	15.8709262051367	4.96305344605032e-43
x3	0.0563430903251719	0.00704148806382154	8.00158855834139	1.86065655287025e-14
x4	0.0139105558765697	0.00934405143917284	1.48870711672806	0.137472749289055
x5	0.375498022378794	0.0267186430353998	14.0537834156208	7.94001565786914e-36
x6	0.0369275370165985	0.00921311514060491	4.00814886746047	7.49365688321305e-05

Number of observations: 354, Error degrees of freedom: 347
 Root Mean Squared Error: 0.00756
 R-squared: 0.796, Adjusted R-Squared 0.792
 F-statistic vs. constant model: 225, p-value = 2.36e-116

ans =

Linear regression model:

y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	9.92959764022123e-05	0.000344958681511748	0.287848898213134	0.773634246325663
x1	-0.00498149053661893	0.0127020465151759	-0.392180152282329	0.695165872568816
x2	0.617153396753262	0.0287619699151035	21.4572714794887	1.31345983808952e-65

GRA 19502

ans =

Number of observations: 354, Error degrees of freedom: 347
 Root Mean Squared Error: 0.00633
 R-squared: 0.834, Adjusted R-Squared 0.831
 F-statistic vs. constant model: 290, p-value = 8.05e-132

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.000113320377866632	0.000147295108645674	0.769342437156077	0.442213410613723
x1	0.109743999806957	0.00542369107301776	20.2341907622507	1.10980483649531e-60
x2	0.353931168132147	0.0122811736899699	28.8190019184571	4.36481196290114e-94
x3	0.0849846763085608	0.00251495199629876	33.7917687628362	8.80090087585983e-112
x4	0.105792065325249	0.0033373401484845	31.6995153680364	1.68266681711384e-104
x5	0.274740095709713	0.00954288412211041	28.7900483956578	5.58345138626801e-94
x6	0.0670860001924686	0.00329057467753763	20.3873203821862	2.67095761304557e-61

Number of observations: 354, Error degrees of freedom: 347
 Root Mean Squared Error: 0.0027
 R-squared: 0.981, Adjusted R-Squared 0.981
 F-statistic vs. constant model: 3.05e+03, p-value = 7.38e-297

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	-4.93724008905093e-19	3.08426094659625e-11	-1.60078546353207e-08	0.999999987236779
x1	0.166666666666667	1.13568459378727e-09	146754360.830826	0
x2	0.166666666666667	2.57159553624131e-09	64810606.6128382	0
x3	0.166666666666667	5.26614107967959e-10	316487280.049868	0
x4	0.166666666666667	6.98816680344718e-10	238498409.317379	0
x5	0.166666666666667	1.99821603625145e-09	83407731.5180218	0
x6	0.166666666666667	6.89024303868893e-10	241887935.92741	0

Number of observations: 354, Error degrees of freedom: 347
 Root Mean Squared Error: 5.66e-10
 R-squared: 1, Adjusted R-Squared 1
 F-statistic vs. constant model: 1.57e+17, p-value = 0

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	1.3317808072087e-18	0	Inf	0
x1	0.6	0	Inf	0
x2	0.4	0	Inf	0
x3	1.52333121226092e-17	0	Inf	0
x4	1.43281740865119e-16	0	Inf	0
x5	4.74364415261965e-18	0	Inf	0
x6	9.43732870506571e-17	0	Inf	0

Number of observations: 354, Error degrees of freedom: 347
 R-squared: 1, Adjusted R-Squared 1
 F-statistic vs. constant model: Inf, p-value = 0

ans =

Linear regression model:
 $y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00306564752157346	0.00063174217780249	4.85268774080795	1.8988312075994e-06
x1	0.100690614285863	0.0240197396178393	4.19199441325668	3.57415886165575e-05
x2	0.0787457939171731	0.0527619557163522	1.49247299210268	0.136551267503556
x3	0.0424382401552106	0.0105360514754798	4.02790744274321	7.02039828730666e-05
x4	-0.017310617551626	0.0139148562765483	-1.24403854467407	0.214387412455723
x5	-0.0587204870932361	0.0407088424332664	-1.44245042559233	0.150144312813881
x6	0.127613257528096	0.0147550682898497	8.6487744428729	2.47974148311431e-16

Number of observations: 330, Error degrees of freedom: 323

GRA 19502

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00211778255296412	0.000475849316389646	4.45053188062161	1.18079649030538e-05
x1	0.00322431519792295	0.0180924704390714	0.178213097475064	0.858667342982815
x2	0.233712609382928	0.0397420679530066	5.88073599137528	1.01952375797064e-08
x3	0.00822706086824349	0.00793610600687269	1.03666216921987	0.300668786242164
x4	-0.0109217480741096	0.0104811346772635	-1.0420387114958	0.298173041121705
x5	0.0541295626289001	0.0306632603038576	1.76529051681077	0.0784598068309741
x6	0.076205481895194	0.0111140104392438	6.85670418538664	3.59049983010463e-11

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.00842

R-squared: 0.434, Adjusted R-Squared 0.424

F-statistic vs. constant model: 41.3, p-value = 2.76e-37

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0023829861153781	0.000546258884562779	4.36237502532417	1.73295678121641e-05
x1	0.0591067200479334	0.0207695427536128	2.84583636477273	0.00471255986761401
x2	0.22595220481625	0.0456225468073404	4.9526433885952	1.1824325895496e-06
x3	-0.00834964561654467	0.00911038067255815	-0.916497994611254	0.360089523680244
x4	-0.0105587860521245	0.0120319873131141	-0.877559606517874	0.380835019995646
x5	0.0473088916843974	0.0352003833854995	1.34398796644601	0.179895535517611
x6	0.0774058695095941	0.0127585072342294	6.06700047964265	3.64511087947544e-09

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.00966

R-squared: 0.461, Adjusted R-Squared 0.451

F-statistic vs. constant model: 46, p-value = 1.42e-40

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00180167908196286	0.00043074950654537	4.18266081466327	3.71624345929096e-05
x1	-0.0283859980376187	0.0163777112741194	-1.73320908901815	0.0840128674143999
x2	0.217176338058487	0.0359754140023446	6.03679885502731	4.31367960228964e-09
x3	0.000361479789229381	0.00718394169878971	0.0503177509486582	0.959900279746171
x4	0.00205681393606238	0.00948775890763278	0.216786066771543	0.828511930176991
x5	0.0791998859805052	0.0277570730692055	2.85332267501835	0.00460564740032753
x6	0.0349524176079888	0.010060652285406	3.47417012500182	0.000582359046652082

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.00762

R-squared: 0.288, Adjusted R-Squared 0.275

F-statistic vs. constant model: 21.8, p-value = 1.76e-21

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00446028705775563	0.00115667159995069	3.85613951094309	0.000139026771455234
x1	-0.0182301533500056	0.043978305755693	-0.414526049531722	0.678764055035118
x2	-0.0486337223098574	0.096603104683064	-0.503438501996548	0.614999159108901
x3	-0.0278030079778138	0.0192907042548553	-1.44126453915315	0.150478709212876
x4	-0.0792998235021183	0.0254770373706329	-3.11259988155163	0.00202019845671601
x5	0.0813287797090329	0.0745347763120992	1.09115212700827	0.276019222589996
x6	0.0254415511460176	0.0270154012916611	0.941742485012456	0.34702829593404

Number of observations: 330, Error degrees of freedom: 323

Root Mean Squared Error: 0.0205

R-squared: 0.0527, Adjusted R-Squared 0.0351

F-statistic vs. constant model: 3, p-value = 0.00732

ans =

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0037662069296195	0.000727826471476987	5.17459460079363	4.01982212498094e-07
x1	-0.0933292670669107	0.0276730016549784	-3.37257476548883	0.000835199655081747
x2	-0.00371825928794457	0.0607867408676702	-0.0611689199794251	0.951262512701087
x3	-0.0604302305369707	0.0121385233377529	-4.97838401389586	1.04535810292427e-06
x4	-0.0683674575031495	0.0160312246051044	-4.26464348090919	2.63242453661721e-05
x5	0.153060976918503	0.0469004194862864	3.26353108554302	0.00121813064617108
x6	-0.00702410567671213	0.0169992279558715	-0.413201452145125	0.679733200239563

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0129
 R-squared: 0.325, Adjusted R-Squared 0.313
 F-statistic vs. constant model: 25.9, p-value = 3.8e-25

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00191314683619244	0.000505073505985884	3.78785822958196	0.000181194488724368
x1	-0.00583243181107616	0.0192036158545726	-0.303715292747193	0.761540353517941
x2	0.0367458595720308	0.0421828190244114	0.871109622871952	0.384341283611024
x3	-0.0217120477524952	0.0084235003538632	-2.5775564600964	0.0103931483015949
x4	-0.0307710789234436	0.0111248314452153	-2.76598158587634	0.00600174270577679
x5	0.0616460292925283	0.03254643823833	1.89409448865368	0.0591056896067121
x6	0.028942514971318	0.0117965751442125	2.45346760542762	0.0146765256979433

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.00893
 R-squared: 0.0688, Adjusted R-Squared 0.0515
 F-statistic vs. constant model: 3.98, p-value = 0.000739

ans =

Linear regression model:

$$y \sim 1 + x1 + x2 + x3 + x4 + x5 + x6$$

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.0032110465441726	0.000632765719227355	5.07462153305884	6.56544548299612e-07
x1	-0.104339415179861	0.0240586561242515	-4.33687628440247	1.93410080199191e-05
x2	-0.00218927192813147	0.0528474400313625	-0.0414262625934623	0.966981671315006
x3	-0.0655187887420018	0.0105531218651396	-6.20847457077431	1.64246823399872e-09
x4	-0.0567462614269165	0.0139374009669624	-4.07150957064589	5.87999619532213e-05
x5	0.169262986544712	0.0407747984324902	4.15116672679466	4.23649552713765e-05
x6	-0.0208467886995556	0.0147789742821231	-1.41057074067529	0.159333549221613

Number of observations: 330, Error degrees of freedom: 323
 Root Mean Squared Error: 0.0112
 R-squared: 0.433, Adjusted R-Squared 0.423
 F-statistic vs. constant model: 41.2, p-value = 3.53e-37

Histogram

```
%Histogram Ln Returns
%BAC
figure
subplot(1,3,1); histogram(ret_ln); xlabel('1/N BAC');ylabel('FQ');
subplot(1,3,2); histogram(ret_ln_tsm_lo); xlabel('1/N TSM LO');
subplot(1,3,3); histogram(ret_ln_tsm_ls); xlabel('1/N TSM LS');
suptitle('Log Returns')

figure
subplot(1,3,1); histogram(ret_mv); xlabel('MV BAC');ylabel('FQ');
subplot(1,3,2); histogram(ret_mv_tsm_lo); xlabel('MV TSM LO');
subplot(1,3,3); histogram(ret_mv_tsm_ls); xlabel('MV TSM LS');
suptitle('Log Returns')

figure
subplot(1,3,1); histogram(ret_iv); xlabel('IV BAC');ylabel('FQ');
subplot(1,3,2); histogram(ret_iv_tsm_lo); xlabel('IV TSM LO');
subplot(1,3,3); histogram(ret_iv_tsm_ls); xlabel('IV TSM LS');
suptitle('Log Returns')

figure
subplot(1,3,1); histogram(ret_erc); xlabel('ERC BAC');ylabel('FQ');
subplot(1,3,2); histogram(ret_erc_tsm_lo); xlabel('ERC TSM LO');
subplot(1,3,3); histogram(ret_erc_tsm_ls); xlabel('ERC TSM LS');
suptitle('Log Returns')
```

