# Master thesis

curr\_age <- 25 # age when you start saving

retirement\_age <- 65 # age when you retire

death <- 90 # age when you die

ann\_income <- 36000 # income that you earn when you start saving

savings <- 0.1 # percentage of annual income intended for retirement saving

pay\_sum <- ann\_income \* savings # amount you annually put aside for retirement

income\_increase <- 0.008 # expected annual income increase

pens\_benefit <- 1 # amount of pension benefit as a fraction of last salary

min\_req\_per <- 0.5 # the lowest acceptable pension payout (as fraction of the last salary), before assigning disutility

min\_pen <- 6000 # the lowest pension payout, everything below counts as bankruptcy

benefit\_increase <- 0 # expected annual pension benefit increase

phi <- 0.25 # how do you value residual wealth compared to consumption when alive

rho <- 0.98 # discount factor

iterations <- 1000 # number of replications/scenarios

times\_salary <- 3.5 # max pension proceed as a fraction of the last salary

years\_active <- retirement\_age - curr\_age

library(readr)

input\_data\_v02 <- read\_delim("~/Desktop/input\_data\_v02.csv",

 ";", escape\_double = FALSE, trim\_ws = TRUE)

end\_values\_act <- c() # residual wealth, AM

end\_values\_pass <- c() # residual wealth, PM

TWR <- c() # Terminal Wealth Ratio, introduced by W. Sharpe

total\_wealth\_act <- c() # Retirement proceeds + residual wealth

total\_wealth\_pass <- c() # as above, PM

total\_wealth\_ratio <- c() # TW passive / TW active

utility\_c\_act <- c() # utilities for each simulation, AM

utility\_c\_pass <- c() # utilities for each simulation, PM

consumption\_act <- c() # (salary-savings) + retirement proceeds + residual wealth

consumption\_pass <- c() # as above, PM

consumption\_ratio <- c() # Consuption passive / Consumption active

years\_zero\_pass\_vec <- c() # no of years w/o retirement income for each observation, passive

years\_zero\_act\_vec <- c() # as above, active

utility\_ratio <- c() # utility passive / utility active

years\_good\_act <- c()

all\_ret\_proceeds\_act <- c()

all\_ret\_proceeds\_pass <- c()

years\_good\_pass <- c()

years\_bad\_act <- c()

years\_bad\_pass <- c()

utility\_ret\_act <- c()

utility\_ret\_pass <- c()

utility\_ret\_ratio <- c()

final\_values\_act <- rep(0, death - curr\_age + 2) # amount of funds

final\_values\_pass <- rep(0, death - curr\_age + 2)

for(j in 1:iterations){

 returns <- sample(1:40, death - curr\_age + 1, replace=TRUE)

 returns\_active <- input\_data\_v02[returns,2]

 returns\_pass <- input\_data\_v02[returns,3]

 value\_at\_end\_active <- pay\_sum\*returns\_active[1,1]$`Actively Managed`

 value\_at\_end\_pass <- pay\_sum\*returns\_pass[1,1]$`Passively Managed`

 values\_act <- c(value\_at\_end\_active)

 values\_pass <- c(value\_at\_end\_pass)

 for(i in 1:(years\_active)){

 value\_at\_end\_active <- (value\_at\_end\_active + pay\_sum\*(1+ income\_increase)^(i-1))\*(returns\_active[i+1,1]$`Actively Managed`)

 value\_at\_end\_pass <- (value\_at\_end\_pass + pay\_sum\*(1+ income\_increase)^(i-1))\*(returns\_pass[i+1,1]$`Passively Managed`)

 values\_act <- c(values\_act, value\_at\_end\_active)

 values\_pass <- c(values\_pass, value\_at\_end\_pass)

 }

 pension\_act <- value\_at\_end\_active + pay\_sum\*(1+ income\_increase)^(years\_active) # balance of plan when you retire, active

 pension\_pass <- value\_at\_end\_pass + pay\_sum\*(1+ income\_increase)^(years\_active) # as above, passive

 TWR <- c(TWR, pension\_pass / pension\_act)

 salary\_last <- ann\_income\*(1+ income\_increase)^(years\_active)

 values\_pass <- c(values\_pass, pension\_pass )

 values\_act <- c(values\_act, pension\_act )

 ret\_proceeds\_act <- c()

 ret\_proceeds\_pass <- c()

 for(i in 1:(death - retirement\_age - 1)){

 pension\_act <- max(0, (pension\_act-min(pension\_act/(death-retirement\_age-i), times\_salary\*salary\_last\*pens\_benefit))\*(returns\_active[i+years\_active,1]$`Actively Managed`))

 pension\_pass <- max(0, (pension\_pass-min(pension\_pass/(death-retirement\_age-i), times\_salary\*salary\_last\*pens\_benefit))\*(returns\_pass[i+years\_active,1]$`Passively Managed`))

 ret\_proceeds\_act <- c(ret\_proceeds\_act, min(pension\_act/(death-retirement\_age-i), times\_salary\*salary\_last\*pens\_benefit))

 ret\_proceeds\_pass <- c(ret\_proceeds\_pass, min(pension\_pass/(death-retirement\_age-i), times\_salary\*salary\_last\*pens\_benefit))

 values\_pass <- c(values\_pass, pension\_pass)

 values\_act <- c(values\_act, pension\_act)

 }

 pension\_act <- max(0, (pension\_act- salary\_last\*pens\_benefit)\*(returns\_active[death-retirement\_age+years\_active,1]$`Actively Managed`))

 pension\_pass <- max(0, (pension\_pass - salary\_last\*pens\_benefit)\*(returns\_pass[death-retirement\_age+years\_active,1]$`Passively Managed`))

 values\_pass <- c(values\_pass, pension\_pass)

 values\_act <- c(values\_act, pension\_act)

 ret\_proceeds\_act <- c(ret\_proceeds\_act, min(pension\_act, salary\_last\*pens\_benefit))

 ret\_proceeds\_pass <- c(ret\_proceeds\_pass, min(pension\_pass, salary\_last\*pens\_benefit))

 end\_values\_act <- c(end\_values\_act, pension\_act)

 end\_values\_pass <- c(end\_values\_pass, pension\_pass)

 years\_good\_act <- c(years\_good\_act, sum(ret\_proceeds\_act > salary\_last\*pens\_benefit))

 years\_good\_pass <- c(years\_good\_pass, sum(ret\_proceeds\_pass > salary\_last\*pens\_benefit))

 years\_bad\_act <- c(years\_bad\_act, sum(ret\_proceeds\_act < salary\_last\*min\_req\_per))

 years\_bad\_pass <- c(years\_bad\_pass, sum(ret\_proceeds\_pass < salary\_last\*min\_req\_per))

 ret\_proceeds\_act\_vec <- ret\_proceeds\_act

 ret\_proceeds\_pass\_vec <- ret\_proceeds\_pass

 ret\_proceeds\_act <- sum(ret\_proceeds\_act)

 ret\_proceeds\_pass <- sum(ret\_proceeds\_pass)

 all\_ret\_proceeds\_act <- c(all\_ret\_proceeds\_act, ret\_proceeds\_act)

 all\_ret\_proceeds\_pass <- c(all\_ret\_proceeds\_pass, ret\_proceeds\_pass)

 total\_wealth\_act <- c(total\_wealth\_act, ret\_proceeds\_act + pension\_act)

 total\_wealth\_pass <- c(total\_wealth\_pass, ret\_proceeds\_pass + pension\_pass)

 total\_wealth\_ratio <- c(total\_wealth\_ratio, (ret\_proceeds\_pass + pension\_pass)/(ret\_proceeds\_act + pension\_act))

 consumption\_working <- ann\_income \* (1- savings) \* ((1 - (1+ income\_increase)^(years\_active)) / (-income\_increase))

 consumption\_act <- c(consumption\_act, consumption\_working + ret\_proceeds\_act + pension\_act)

 consumption\_pass <- c(consumption\_pass, consumption\_working + ret\_proceeds\_pass + pension\_pass)

 consumption\_ratio <- c(consumption\_ratio, (consumption\_working + ret\_proceeds\_pass + pension\_pass)/(consumption\_working + ret\_proceeds\_act + pension\_act))

 years\_zero\_act <- sum(ret\_proceeds\_act\_vec < min\_pen)

 years\_zero\_act\_vec <- c(years\_zero\_act\_vec, years\_zero\_act )

 years\_zero\_pass <- sum(ret\_proceeds\_pass\_vec < min\_pen)

 years\_zero\_pass\_vec <- c(years\_zero\_pass\_vec, years\_zero\_pass )

 dis\_act <- sum(sqrt(-1\*(ret\_proceeds\_act\_vec\*(ret\_proceeds\_act\_vec/salary\_last\*min\_req\_per - 1))\*((ret\_proceeds\_act\_vec-salary\_last\*min\_req\_per) < 0))\*rho^(years\_active:(death-curr\_age-1)))

 dis\_pass <- sum(sqrt(-1\*(ret\_proceeds\_pass\_vec\*(ret\_proceeds\_pass\_vec/salary\_last\*min\_req\_per - 1))\*((ret\_proceeds\_pass\_vec-salary\_last\*min\_req\_per) < 0))\*rho^(years\_active:(death-curr\_age-1)))

 dis\_ret\_act <- sum(sqrt(-1\*(ret\_proceeds\_act\_vec\*(ret\_proceeds\_act\_vec/salary\_last\*min\_req\_per - 1))\*((ret\_proceeds\_act\_vec-salary\_last\*min\_req\_per) < 0))\*rho^(0:(death-retirement\_age-1)))

 dis\_ret\_pass <- sum(sqrt(-1\*(ret\_proceeds\_pass\_vec\*(ret\_proceeds\_pass\_vec/salary\_last\*min\_req\_per - 1))\*((ret\_proceeds\_pass\_vec-salary\_last\*min\_req\_per) < 0))\*rho^(0:(death-retirement\_age-1)))

 util\_con <- sqrt(ann\_income \* (1- savings)) \* (1 - (rho\*sqrt(1+ income\_increase))^(years\_active)) / (1-rho\*sqrt(1+income\_increase))

 ut\_act <- util\_con + sum(sqrt(ret\_proceeds\_act\_vec)\*rho^(years\_active:(death-curr\_age-1))) - dis\_act + rho^(death-curr\_age) \* sqrt(phi \* pension\_act)

 ut\_pass <- util\_con + sum(sqrt(ret\_proceeds\_pass\_vec)\*rho^(years\_active:(death-curr\_age-1))) - dis\_pass + rho^(death-curr\_age) \* sqrt(phi \* pension\_pass)

 ut\_ret\_act <- sum(sqrt(ret\_proceeds\_act\_vec)\*rho^(0:(death-retirement\_age-1))) - dis\_ret\_act + rho^(death-retirement\_age) \* sqrt(phi \* pension\_act)

 ut\_ret\_pass <- sum(sqrt(ret\_proceeds\_pass\_vec)\*rho^(0:(death-retirement\_age-1))) - dis\_ret\_pass + rho^(death-retirement\_age) \* sqrt(phi \* pension\_pass)

 utility\_c\_act <- c(utility\_c\_act, ut\_act)

 utility\_c\_pass <- c(utility\_c\_pass, ut\_pass)

 utility\_ratio <- c(utility\_ratio, ut\_pass/ut\_act)

 utility\_ret\_act <- c(utility\_ret\_act, ut\_ret\_act)

 utility\_ret\_pass <- c(utility\_ret\_pass, ut\_ret\_pass)

 utility\_ret\_ratio <- c(utility\_ret\_ratio, ut\_ret\_pass/ut\_ret\_act)

 final\_values\_act <- final\_values\_act + values\_act

 final\_values\_pass <- final\_values\_pass + values\_pass

}

# Plots (Comparison Passive vs Active, TWR, Consumption ratio, Utility ratio)

plot((curr\_age - 1):death, final\_values\_act/iterations, main = "The Mean Value of Retirement Portfolio", xlab = "Years", ylab = "Value", type = "l", col="gray70", ylim=c(0,max(final\_values\_pass/iterations)))

lines((curr\_age - 1):death, final\_values\_pass/iterations, col="black")

abline(v = retirement\_age, col = "black", lwd = 1, lty=3)

abline(v = death)

plot(quantile(utility\_ret\_ratio, seq(0, 1, 0.01)), 1 - seq(0,1, 0.01), type = "l", main = "Utility ratio at retirement", xlab = "X", ylab = "Probability")

plot(quantile(utility\_ratio, seq(0, 1, 0.01)), 1 - seq(0,1, 0.01), type = "l", main = "Utility ratio", xlab = "X", ylab = "Probability")

plot(quantile(consumption\_ratio, seq(0, 1, 0.01)), 1 - seq(0,1, 0.01), type = "l", main = "Consumption ratio", xlab = "X", ylab = "Probability")

plot(quantile(TWR, seq(0, 1, 0.01)), 1 - seq(0,1, 0.01), type = "l", main = "Probability That Terminal Wealth Ratio Exceeds X", xlab = "X", ylab = "Probability")

# Output from data

cat("Mean residual wealth, AM:", mean(end\_values\_act), "\n")

cat("Mean residual wealth, PM:", mean(end\_values\_pass), "\n")

cat("Mean residual wealth PM/AM:", mean(end\_values\_pass)/mean(end\_values\_act), "\n")

cat("Percentage of scenarios with at least 1 payout < min, AM: ", sum(years\_zero\_act\_vec > 0 )/iterations, "\n")

print(summary(years\_zero\_act\_vec))

cat("Percentage of scenarios with at least 1 payout < min, PM: ", sum(years\_zero\_pass\_vec >0 )/iterations, "\n")

print(summary(years\_zero\_pass\_vec))

cat("Sum of pension payouts, AM: ", "\n")

print(summary(all\_ret\_proceeds\_act))

cat("Sum of pension payouts, PM:", "\n")

print(summary(all\_ret\_proceeds\_pass))

cat("Pension payouts, PM/AM:", (mean(all\_ret\_proceeds\_pass)/mean(all\_ret\_proceeds\_act)), "\n")

cat("Probability Consumption PM > Consumption AM:", sum(consumption\_ratio > 1)/iterations, "\n")

cat("TWR > 1 & measure statistics:", sum(TWR > 1)/iterations, "\n")

print(summary(TWR))

cat("Percentage of good years, AM:", mean(years\_good\_act)/(death-retirement\_age), "\n")

cat("Percentage of good years, PM:", mean(years\_good\_pass)/(death-retirement\_age), "\n")

cat("Percentage of bad years, AM:", mean(years\_bad\_act)/(death-retirement\_age), "\n")

cat("Percentage of bad years, PM:", mean(years\_bad\_pass)/(death-retirement\_age), "\n")

cat("Utility ratio, PM/AM:", "\n")

print(summary(utility\_ratio))

cat("Utility ratio > 1, PM/AM:", sum(utility\_ratio > 1)/iterations, "\n")

cat("Utility retirement ratio, PM/AM:", "\n")

print(summary(utility\_ret\_ratio))

cat("Mean pension payouts as percentage of last salary, AM: ", (mean(all\_ret\_proceeds\_act)/(death - retirement\_age)/(salary\_last)), "\n")

cat("Mean pension payouts as percentage of last salary, PM: ", (mean(all\_ret\_proceeds\_pass)/(death - retirement\_age)/(salary\_last)), "\n")