Anmol Juneja, Stig Bordvik

# BI Norwegian Business School Preliminary Thesis Report

# Working Title:

# The Beta Anomaly: The Conditional and Unconditional Value Premium in the Norwegian Stock Market –

Supervisor:

Bruno Gerard

Hand-In Date:

16.01.17

Examination Code and Name: GRA 19002 – Preliminary Thesis Report

> Programme: MSc in Financial Economics

"This thesis is part of the MSc programme at BI Norwegian Business School. The school takes no responsibility for the methods used, results found and conclusions drawn.

# ABSTRACT

In this Master thesis we investigate the relation between systematic risk and returns in the Norwegian Stock Market between 1985-2016. In an efficient market, market participants realize above average returns only by taking on above average risks. However, prior studies find that strategies that buy low-beta stocks and sell high-beta stocks have significantly positive unconditional capital asset pricing model (CAPM) alpha. We will first examine whether this relationship is present in Norway. Second, if present, by utilizing the methodology in Cederburg & O'Doherty (2016) we will try to resolve the anomaly by using the conditional CAPM.

# **Table of Contents**

1	I	NTRODUCTION	1	
2	L	ITERATURE REVIEW	3	
	2.1	The Low Beta Anomaly	3	
	2.	.1.1 Evidence against the unconditional CAPM	3	
	2.	.1.2 Findings of Frazzini and Pedersen	4	
	2.2	The Idiosyncratic Volatility Puzzle	5	
	2.3	The Minimum Volatility Anomaly	5	
	2.4	Conditional Beta	6	
	2.5	Possible explanations of the low risk anomaly	7	
	2.	.5.1 Explanations on the basis of behavior elements	7	
	2.	.5.2 Explanations on the basis of rational elements	8	
3	N	IODELS AND THEORY	. 10	
	3.1	The Capital Asset Pricing Model	. 10	
	3.2	The CAPM with funding constraints and the Low-Beta Anomaly	. 10	
	3.3	The conditional CAPM	. 12	
	3.4	Mismeasurement of the conditional CAPM and possible consequences	. 12	
4	N	IETHODOLOGY	. 13	
	4.1	Empirical method applied by Frazzini and Pedersen (2014)	. 13	
	4.2	Empirical method applied by Cederburg and O'Doherty (2016)	. 14	
	4.	.2.1 One-step IV method (IV1)	. 14	
	4.	.2.2 Two-step IV method (IV2)	. 15	
5	D	ОАТА	. 16	
R	<b>REFERENCE</b>			

#### **1 INTRODUCTION**

One of the main results of the Capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) is that expected return on any asset is proportional to its non-diversifiable risk. The model assumes that everyone has a portfolio on the same efficient frontier, where the portfolio has the lowest risk for a given level of expected return. The most efficient portfolio is the market portfolio, because the model assumes that everyone has the same optimization problem. The CAPM beta of an asset serves as the only measure of risk one should be compensated for.

However, early studies by Friend and Blume (1970) and Black, Jensen & Scholes (1972) shows that high-beta portfolios earn lower returns than predicted by the CAPM. Moreover, high-beta portfolios earn negative alphas, whereas portfolio of low-beta portfolios earn positive alphas. Furthermore, Fama & Macbeth (1973) and Black (1972) found that the security market line (SML) also known as beta-return relation is too flat relative to the predictions of the CAPM. They find a positive CAPM alpha, indicating that low-beta stocks produce higher risk-adjusted returns than high-beta stocks. This is also characterized as the "low volatility puzzle" in academic literature as low volatility stocks generate significantly higher Sharpe ratio than stocks with higher volatility. Fama and French (2006) extended this argument by showing that security market line becomes even flatter when controlling for size and book-to-market factors.

Frazzini & Pedersen (2014) did the groundbreaking work that has drawn interest of many academics and practitioners in this field. They developed a "bettingagainst-beta" (BAB) strategy that primarily focused on the US market and confirmed the underperformance of high beta stocks. Their focused on the unconditional CAPM and found a significantly negative unconditional alpha for their high-minus-low strategy. In particular, they discovered that high-beta portfolios earn negative alphas, while low-beta portfolios earn positive alphas over 1926-2012 period; confirming the presence of beta anomaly in the US stock market. However, Grant (1977), Jagannathan and Wang (1996), Lewellen and Nagel (2006) and Boguth et al. (2011) showed that the unconditional alpha is a biased estimate of the true portfolio alpha, if portfolios beta varies systematically with the market risk premium or market volatility. Thus, large swings in portfolio beta over the sample period can lead to a bias in its unconditional alpha.

Cederburg and O'Doherty (2016) contrasted the performance of conditional and unconditional beta. They show that there is a negative bias in unconditional alpha if beta is negatively correlated with expected excess return (such as dividend yield) or positive correlated with the market volatility. In contrast to previous study of unconditional alpha by Frazzini and Pedersen (2014), they found conditional alphas for BAB strategy to be statistically insignificant and substantially smaller in magnitude. This shows that market risk is rewarded in consistency with the assumption of the CAPM if properly accounted for predictable time-series variation in portfolio betas, namely due to market and volatility timing.

As previous studies conducted up to date has mostly focused on the US data- and large international markets, our main contribution is to test whether we can obtain significant positive unconditional alpha by implementing the BAB strategy by Frazzini and Pedersen (2014) on the Norwegian stock market. Furthermore, we want to contrast the unconditional and conditional performance of high-minus-low beta equity portfolios by implementing the framework by Cederburg and O'Doherty (2016).

As we have not done any analysis yet, we do not present any results in this preliminary report. However, our hypothesis is that we would be able to obtain positive significant unconditional alpha by using betting-against-beta strategy in Norwegian market, as it is highly significant in other markets. Further, we expect that this effect will disappear, or become negative, when the CAPM beta are conditioned upon its lagged state variables.

The remainder of this preliminary report is organized as follows: Section 2. review existing literature on the topic. Section 3. introduces the models and theory behind models. Section 4. outlines our methodological approach and finally section 5. provides a description of data to be used.

#### 2 LITERATURE REVIEW

The cornerstone in finance theory is the relationship between risk and return. It has been studied broadly, both by academics and practitioners. Our thesis focuses on the low beta anomaly, and we will dedicate most space to research addressing this version of the puzzle, which is found in section 2.1. In section 2.2 and section 2.3 we examine several studies that uses idiosyncratic and total volatility as risk measures. Section 2.4 discusses literature of the puzzle with emphasize on the conditional CAPM. Section 2.5 reviews possible explanations of the anomaly, using both behavior finance theory and rational explanations.

# 2.1 The Low Beta Anomaly

The findings that high beta stocks have long outperformed low beta stocks conflicts with the unconditional Capital Asset Pricing Model, and are therefore referred to as an anomaly. The predictions of the CAPM are that asset returns are proportional to its systematic risk, which is the only risk measure in the model.

## 2.1.1 Evidence against the unconditional CAPM

The early empirical investigations of the unconditional CAPM by Black, Jensen and Scholes (1972), Fama and MacBeth (1973), and Haugen and Heins (1975), reveals that the security market line is much flatter than predicted by theory. Their findings show that low-beta assets have higher risk-adjusted returns than highbeta assets, thus violates the CAPM and Fama's (1970) Efficient Capital Markets theory. Two decades later, Fama and French (1992) expands the model by adding size and value factors to the market risk factor in the CAPM, in an attempt to better measure market returns. Investigating the period 1963-1991 in the US, they find that the market beta is unpriced, after controlling for size. This implies that firms with higher average beta, are not compensated with higher average returns. While the Fama-French 3 factor model (FF-3) explains assets returns better than the CAPM, it is considered to be an empirical factor pricing model which lacks convincing theoretical explanations of the introduced additional risk factors. The extension of the FF-3 factor model is the Carhart (1997) four factor model that includes a momentum component. Momentum is described as the tendency for a stock to continue rise (fall) if the price direction is positive (negative). However, after controlling for the FF-3 and Carhart risk factors, the superior performance between low and high beta stocks is still present in international markets (see e.g.

Baker et al. (2011,2014) and Frazzini and Pedersen (2014)). In the five factor model, Fama & French (2015,2016), adds profitability (RMW<sup>1</sup>) and investment (CMA<sup>2</sup>) factors to the three-factor model. The study from July 1963 to December 2014 for US stocks claims that the five factor model is able to explain the returns of portfolios with different betas. The low beta stocks have positive exposure to profitability and investment factors while high beta stocks have the opposite exposure. Thus, low (high) beta stock returns behave like profitable (less profitable) firms that invest conservatively (aggressively). However, Blitz & Vidojevic (2016) disagrees and claims that the rejection of the low-beta anomaly are solely supported by time-series spanning tests. Using Fama-MacBeth regressions the study finds that all five factors, except market beta are rewarded with a significant risk premia, thus bringing further evidence of the anomaly.

# 2.1.2 Findings of Frazzini and Pedersen

Frazzini and Pedersen (2014) constructs market-neutral betting-against-beta (BAB) portfolios, which buys low-beta stocks and sells high-beta stocks. The paper finds that high-beta stocks have both lower FF-3 factor alphas and Sharpe ratios than low beta stocks. When the beta increase in the portfolios, the alpha declines, which is documented for the US stock market and also in 18 of 19 international markets. The flatness of the SML is not only found in stock markets, but also in Treasury, corporate bond and in futures markets, thus supporting the presence of the low-beta anomaly among different asset classes.

Frazzini and Pedersen (2014) constructs BAB factors attempting to capture the anomaly. A BAB factor is a portfolio holding low-beta stocks leveraged to beta of one, and sells high-beta stocks deleveraged to beta of one. Combined with a position in the risk-free asset, the portfolio is self-financing.

Further, Frazzini and Pedersen (2014) show that when funding constraints tighten e.g. as the banks reduce credit availability, the BAB strategy should lead to losses. This could be explained by the increase in the future required return, because when it is credit shortage in the economy, investors might need to rebalance their

<sup>1</sup> RMW (Robust Minus Weak) is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios,

<sup>&</sup>lt;sup>2</sup> CMA (Conservative Minus Aggressive) is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios

BAB positions. Moreover, the authors argue that a funding shock makes all stock prices drop together, thereby compressing all securities betas towards one.

#### 2.2 The Idiosyncratic Volatility Puzzle

Since the beginning of classical asset pricing theory there has been conducted numerous research to validate if expected returns depend on idiosyncratic volatility (IVOL) factors.

Earlier studies find no documentation of a negative relation between IVOL and stock returns. The classic study by Fama and MacBeth (1973), who acknowledge the methodological issues raised by Miller and Scholes (1972), concludes that the coefficients and residuals of the risk-return regressions are consistent with the Efficient Market Hypothesis. In a more recent paper, Bali and Cakici (2008) also dismisses existence of the IVOL puzzle. They argue that portfolio construction and different IVOL measures play a critical role in determining the relationship between risk and returns. From the sample period 1958-2004 on NYSE they conclude that there exists no robust evidence between IVOL and returns.

However, Ang et al. (2006), finds that stocks with high IVOL relative to FF-3 model have significantly lower average returns. They uncover a robust result and argues that the findings cannot be explained by exposure to size, book-to-market, leverage or liquidity characteristics. Moreover, the effect perseveres both in bull and bear markets. Ang et al. (2009) studies stock returns in 23 developed markets, including Norway. The average return between the difference of the extreme quintiles portfolios sorted by IVOL was -1.307 % per month for all countries and -0.723 % for European countries, after controlling for the FF-3 factors. However, they do not specifically comment on the Norwegian market.

# 2.3 The Minimum Volatility Anomaly

Several studies look at the risk-return relation that aggregates both the systematic and nonsystematic risk factors. Minimum volatility portfolios tend to hold low beta and low residual risk stocks. Therefore, these investigations are particularly relevant in relation to the low beta anomaly. Scherer (2010) constructs a minimum variance portfolio using a standard multifactor regression with HAC<sup>3</sup> adjusted errors. His findings show that minimum variance investing implicitly picks up risk-based anomalies. Near 83 % of the variation of the minimum variance portfolio excess returns can be attributed to the FF-3 model. This result favors a view that minimum variance strategies provides significant improvement over the market-cap weighted benchmark, simply because the portfolios are a more efficient way to exploit the anomalies.

The study by Baker and Haugen (2012) covers stocks from 1990-2011 in 21 developed countries, including Norway. The volatility is computed of the total return for each company over the previous 24 months. Stocks in each country are then ranked by their volatility and formed into deciles. The difference in total return, low risk minus high risk decile is positive across all equity markets, and even more dramatic is the positive difference in Sharpe ratios. According to the paper, this provides significant evidence of the minimum volatility anomaly in Norway, and in the 20 other developed countries. The findings are consistent with the results of Ang et al. (2006). We will revisit the Norwegian result with our own research.

Sullivan and Feijóo (2016) challenges the conclusions of Scherer (2010). Using stock returns from 1963-2011 in the US, they find that high returns on low volatility portfolios are not solely compensation for bearing systematic risk factors. The results from their cross-sectional analyses suggest that the low volatility anomaly is not related to some systematic risk factor and there is no value premium associated with it. The empirical findings indicate that the abnormal returns most likely arise from market mispricing. This stems from investors preference for high volatility stocks and thus provides a behavior explanation of the anomaly.

# 2.4 Conditional Beta

Most empirical studies of the static CAPM assume that betas remain constant over time. However, Cederburg & O'Doherty (2016) revaluated the performance of beta-sorted portfolios while carefully conditioning for predictable time-series variation in portfolio betas with respect to market risk premium and market

<sup>&</sup>lt;sup>3</sup> Heteroscedasticity and Autocorrelation Consistent

volatility. This is also characterized as conditional beta as portfolio betas are modeled (conditioned upon) as a function of lagged state variables, namely either "market timing" and "volatility timing".

They found that although high-minus-low hedge portfolio earns a statistically significant negative monthly unconditional alpha of -0.59%, their most comprehensive conditional model earns a conditional alpha of merely -0.18%. This represents a circa 70% reduction in the economic magnitude of alpha and most importantly alpha become statistically insignificant. Similarly, while analyzing the performance of unconditional and conditional versions of the Fama-French (1993) three-factor model, they found that unconditional alpha estimate was -0.75% per month while conditional alpha was -0.26%.

# 2.5 **Possible explanations of the low risk anomaly**

#### 2.5.1 Explanations on the basis of behavior elements

Baker, Bradley and Wurgler (2011) looks at behavioral factors that affects the financial decisions of individual investors. In their paper, three biases that attracts investors towards high-volatility stocks are examined.

The view of stocks as lottery tickets: In a gamble with 50/50 percent chance of winning \$110 versus losing \$100, according to extensive studies by Kahneman and Tversky (1979), the possibility of losing \$100 is enough to make people shy away from the gamble. This behavior is called "loss aversion", where a dollar lost is more valuable than a dollar gained. However, in a gamble with a near-certain loss of \$1 and 0.12 % chance of winning \$5,000, people are much more likely to participate, even if the two gambles have the same positive expected payoff of \$5. This impose a behavior inconsistency. Mitton and Vorkink (2007) connects this irrationality to the behavior of investors in the stock market. Since low-priced volatile stocks have the same characteristics as in the second example, it is similar as buying lottery tickets. Blitz and van Vliet (2007) discusses the preference for lottery tickets related to behavioral portfolio theory mentioned in Shefrin and Statman (2000), where private investors think in terms of a two-layer portfolio. The low aspiration layer (first layer) is designed to avoid poverty, while the high aspiration layer (second layer) is designed to obtain riches. A private investor can make rational risk averse asset allocations (first layer), but he can increase the risk willingness in a specific stock or asset class (second layer). Buying few volatile

stocks keeps a potential upside intact compared to a well-diversified portfolio, which limits it. This behavior increases the demand for risky stocks, causing them to be overpriced, and hence, offers investors with lower expected returns.

*Representativeness:* Described first by Kahneman and Tversky (1972), the representativeness heuristic is a decision making shortcut when making judgments about the probability of uncertain events. The fact that people may overestimate their ability to accurately predict the likelihood of an event can be extended to the financial markets. Discussed in Baker, Bradley and Wurgler (2011), an investor might have the belief that the road to riches is by making speculative investments in new technologies, for example Microsoft Corporation in the 1980s. However, the fallacy of this logic is to not recognize that a large sample of speculative investments fail, and that investors might be inclined to overpay for volatile stocks.

*Overconfidence:* There exist extensive literature that both common individuals and market participants tend to exhibit irrationally high level of overconfidence (see e.g. Fischhoff, Slovic, and Lichtenstein (1977), Alpert and Raiffa (1982) and Barber and Odean (2001)). According to Cornell (2009), overconfidence plays an important part of demand for volatile stocks. Investors who consider themselves to have superior stock selection skills are more likely to invest heavily in volatile stocks, to capitalize on their perceived skills. Baker, Bradley and Wurgler (2011) points out that one needs to connect overconfidence with one extra assumption about the market participants. That is, either the pessimists in the stock market must act less aggressively that the optimists, or pessimists have reluctance or inability to short stocks instead of buying them. In many cases this is a reasonable assumption, and it has been investigated empirically by Diether, Malloy, and Scherbina (2002). This indicates that overconfident investors will tend to overvalue risky stocks, thus leading future expected return to be lower.

### 2.5.2 Explanations on the basis of rational elements

*Leverage constraints:* Black (1972) discovered that the security market line is flatter than predicted by CAPM, and notes the relevance of borrowing constraints for the beta-return relationship. Frazzini and Pedersen (2014) argues that in absence of leverage, investors that seeks higher expected returns will need to tilt their portfolios towards risky high beta assets to achieve their goals. The

increasing demand for high beta assets will cause the prices to rise, and hence they will exhibit lower risk adjusted expected returns than low beta assets.

*Benchmarking:* Ang (2014) and Baker, Bradley and Wurgler (2011) blames the agency problems for the risk anomaly. Many contracts for institutional equity management specifies that the portfolio manager cannot have a large tracking error relative to the benchmark index, for instance S&P 500. Shorting small capitalized volatile stocks are costly, and volumes of shares to borrow might be limited. Therefore, institutional investors cannot take bets on the anomaly without increasing their tracking error to the benchmark.

#### **3 MODELS AND THEORY**

We first present the Capital Asset Pricing Model in section 3.1. In section 3.2 we then expand the CAPM model used in Frazzini and Pedersen (2014). In section 3.3 we present the model of the conditional CAPM used by Cederburg O'Doherty (2016).

# 3.1 The Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM), first proposed by Sharpe (1964) and Lintner (1965), follows the mean variance optimization problem from Markowitz (1952). When investors can borrow and lend at the risk-free rate, the model predicts that the expected return of an asset above the risk-free rate is proportionate to the nondiversifiable risk. Thus, the required return of any individual asset's expected return can be formulated as:

$$E[r_i] = r_f + \beta_i [E[r_m] - r_f], \qquad (1)$$

where  $E[r_i]$  is the expected return for the individual asset,  $r_f$  is the risk-free rate,  $E[r_m]$  is the expected return on the market portfolio and  $\beta_i = \frac{Cov[r_i,r_m]}{Var[r_m]}$  is the nondiversifiable risk, measured by the covariance between the asset and the market divided by the variance of the market.

#### **3.2** The CAPM with funding constraints and the Low-Beta Anomaly

As described earlier, the low-beta anomaly is the empirical finding that investors earn higher risk-adjusted returns investing in low-beta assets than high-beta assets. This flatness in the security market line is explained by Black (1972), who argues that leverage and margin constraints results in a lower price of risk than predicted by CAPM. However, investors are able to borrow, just not unlimited amount, in contrast to the CAPM assumptions. This idea is developed further by Frazzini and Pedersen (2014), where they present an equilibrium model with leverage and margin constraints:

$$E_t[r_{t+1}^i] = r^f + \psi_t + \beta_t^i \lambda_t, \tag{2}$$

where in equation (2) the risk premium is  $\lambda_t = E_t[r_{t+1}^M] - r^f - \psi_t$ , and  $\psi_t \ge 0$ measures the tightness of leverage constraints. Equation (2) shows that when borrowing is more difficult (i.e. higher  $\psi_t$ ), the intercept will increase and the slope of the security market line will flatten. When  $\psi_t = 0$ , equation (2) is reduced to the standard CAPM form. The asset's alpha with respect to the market portfolio is:

$$\alpha_t^i = \psi_t [1 - \beta_t^i] \tag{3}$$

Equation (3) shows that when funding constraint tighten the intercept increases, suggesting that larger mispricing occur when borrowing becomes more difficult. Further, with increasing systemic risk of the asset, the lower the alpha should be. This implies that assets with higher betas will have lower excess returns than predicted by the model.

Frazzini and Pedersen (2014) uses this model to motivate a "betting-against-beta" (BAB) strategy. They construct a self-financing portfolio (BAB factor), with  $w_L$  and  $w_H$  as the relative portfolio weights that goes long low-beta assets, and similarly a portfolio that sells high-beta assets:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L} [r_{t+1}^L - r^f] - \frac{1}{\beta_t^H} [r_{t+1}^H - r_f],$$
(4)

where in equation (4) the expected return for the low-beta portfolio is  $r_{t+1}^{L} = w_{L}r_{t+1}$  and likewise for the high-beta portfolio  $r_{t+1}^{H} = w_{H}r_{t+1}$ . The betas for these portfolios are  $\beta_{t}^{L}$  and  $\beta_{t}^{H}$ , where  $\beta_{t}^{L} < \beta_{t}^{H}$ .

The long and the short legs of equation (4) are weighted by their betas. The lowbeta portfolio is leveraged to one and the high-beta portfolio is deleveraged to one, making the strategy beta neutral. By substituting equation (2) into equation (4), we can rewrite the expected return of the BAB factor:

$$E_t[r_{t+1}^{BAB}] = \frac{\beta_t^H - \beta_t^L}{\beta_t^L \beta_t^H} \psi_t \ge 0$$
(5)

Equation (5) indicates that a self-financing portfolio that goes long low-beta assets and sells high-beta assets earns a positive expected return on average. The size of the expected return depends on the beta spread between the portfolios, in addition to the magnitude of the leverage constraint. In absence of leverage constraints as in the assumptions of the CAPM, the expected return of the BAB strategy is zero because the leverage constraint is zero.

#### **3.3** The conditional CAPM

Following Cederburg & O'Doherty (2016), the conditional CAPM is defined:

$$a_{i,t} = E(R_{i,t}|I_{t-1}) - \beta_{i,t}(E(R_{m,t}|I_{t-1})) = 0,$$
(6)

where in equation (6)  $R_{i,t}$  is the portfolio's excess return during period t,  $R_{m,t}$  is the excess market return,  $I_{t-1}$  is the investor's information set at end of period t-1, and  $\beta_{i,t} = \frac{Cov(R_{i,t},R_{m,t}|I_{t-1})}{Var(R_{m,t}|I_{t-1})}$  is the conditional beta of the asset.

The traditional implementation of the conditional CAPM follows classical instrumental variable (IV) approach suggested by Shanken (1990), Ferson and Schadt (1996), and Ferson and Harvey (1999). Under this method, portfolio betas are modelled as a linear function of instruments such as aggregate dividend yields and default spreads.

This approach was improved by Boguth et al. (2011) who incorporates lags of realized portfolio betas as additional state variables. These lagged realized betas are known to investors ex ante. Thus, incorporating them as instruments avoids the over-conditioning bias in the estimation of alphas.

3.4 Mismeasurement of the conditional CAPM and possible consequences In Cederburg & O'Doherty (2016) they compute the adjusted alphas from implementing in-sample BAB portfolios. The tests are conducted using ex-post returns on the test portfolios, using perfect foresight (PF) conditional betas. As the PF conditional betas is not part of an investors information set yet, this is an inappropriate way to evaluate a low volatility portfolio strategy. Some authors argue that the CAPM is overstated because of mismeasurement of the market portfolio, improper neglect of conditional information or sample-selection bias. In later version of our thesis, we will try to circumvent these issues.

#### 4 METHODOLOGY

The objective of our thesis is to contrast the performance of unconditional and conditional beta portfolios. We use the methodologies from both Frazzini and Pedersen (2014) and Cederburg and O'Doherty (2016) to conduct our empirical investigations in the Norwegian stock market.

#### 4.1 Empirical method applied by Frazzini and Pedersen (2014)

Following Frazzini and Pedersen (2014), the ex-ante betas is estimated using rolling regressions of assets excess returns on market excess returns. To increase the accuracy of the covariance estimates, they use daily data instead of monthly, when possible. We will follow the same approach in our study of the Norwegian stock market if the data allows it. In Frazzini and Pedersen (2014), the estimated beta for asset *i* is defined as:

$$\widehat{\beta_{l}^{ts}} = \widehat{\rho} \frac{\widehat{\sigma_{l}}}{\widehat{\sigma_{m}}},\tag{7}$$

where in equation (7)  $\hat{\sigma}_{l}$  and  $\hat{\sigma}_{m}$  are the estimated volatilities of the asset and the market, and  $\hat{\rho}$  is the estimated correlation between them. We will collect the volatilities using one year rolling estimates, which is calculated from the one-day log returns. For the volatility we require at least six months (120 trading days) of non-missing data. For the correlation we use a five-year horizon because correlations appear to move more slowly than the volatilities for individual assets. As in Frazzini and Pedersen (2014), we will estimate the correlation by an overlapping three-day log returns, and require at least 3-years (750 trading days) of non-missing return data. If we only have access to monthly data, we use one and five-year rolling windows, which means that we require at least 12 and 36 observations. If this method should provide us with poor results, we will try to employ different time horizon windows, either suggested by existing literature or by consultation by our supervisor.

Reducing the influence due to outliers, our plan is to follow Vasicek (1973) and Elton et.al (2003) and shrink the beta estimates,  $\widehat{\beta_{\iota}^{ts}}$ , toward the cross sectional mean ( $\widehat{\beta^{Xs}}$ ):

$$\widehat{\beta}_{i} = w_{i} \widehat{\beta}_{i}^{TS} + [1 - w_{i}] \widehat{\beta}^{\overline{XS}}, \tag{8}$$

where in equation (8)  $w_i$  is the Bayesian shrinkage factor given by:

 $w_i = 1 - \frac{\sigma_{i,TS}^2}{\sigma_{i,TS}^2 + \sigma_{XS}^2}$ . Frazzini and Pedersen set  $w_i = 0.6$  and  $\widehat{\beta}^{XS} = 1$  for all periods and assets. We will use the same weights for our research for the Norwegian stock market. If the reduction of outliers is not satisfying, we will consult with our supervisor.

After this, we will construct BAB factors. First, we estimate the median beta from our dataset in every point in time. Then, we rank all assets on the basis of their estimated betas and allocate them to one of two portfolios: low-beta and highbeta. The low (high) beta portfolio consist of all stocks that have a beta lower (higher) than the beta median. If it turns out that constructing additional beta portfolios is more suitable, for instance beta quintile portfolios, we will investigate that to see if different strategies exploit the anomaly even better. We will rebalance the portfolios every month, if the data on the Norwegian stock market allows it. The excess return of the BAB factor is given by equation (4).

From our BAB factors, we will regress them against the FF-3 factors and Carhart model to investigate the significance of the portfolio alpha. Frazzini and Pedersen (2014) have explicitly tested the beta-anomaly in Norway using data from 1984-2012. We expect to find similar results.

#### 4.2 Empirical method applied by Cederburg and O'Doherty (2016)

In order to assess the performance of beta-sorted portfolios, IV instrument variable approach explained in the previous section namely 'conditional CAPM' will be followed. This will be used with both the one-step (IV1) and two-step (IV2) methods.

#### 4.2.1 One-step IV method (IV1)

This method is based on the conditional return regression and is given as follows:

$$R_{i,\tau} = a_i^{IV1} + \beta_{i,\tau}^{IV1} R_{m,\tau} + u_{i,\tau},$$
(9)

where in equation (9)  $\tau$  is the quarter,  $R_{i,\tau}$  is the quarterly buy-and-hold excess return of the portfolio,  $R_{m,\tau}$  is the quarterly buy-and-hold excess return for market portfolio,  $\beta_{i,\tau}^{IV1} = (\gamma_{i,o} + \gamma'_{i,1}Z_{i,\tau-1})$  is the conditional beta, and  $Z_{i,\tau-1}$  is a k x 1 vector of instruments for the investor's information set at start of  $\tau - 1$ . From above, the conditional beta is a linear function of portfolio specific state variable vector and the conditional alpha is constant. In other words, if  $Z_{i,\tau-1}$  is zero, the portfolio beta will be constant and equation (9) will be an application of the unconditional CAPM.

Equation (9) is estimated using generalized method of moments (GMM) where GMM parameter estimates relate to ordinary least square estimates. The moment conditions will be used following Boguth et al. (2011) that is

$$E[(R_{i,\tau} - a_i^{IV1} - (\gamma_{i,o} + \gamma_{i,1}' Z_{i,\tau-1}) R_{m,\tau}) X_{i,\tau}] = 0$$
(10)

where in equation (10),  $X_{i,\tau} = \begin{bmatrix} 1 & R_{m,\tau} & Z'_{i,\tau-1}R_{m,\tau} \end{bmatrix}'$ .

Finally, to statistically assess the portfolio performance, Newey-West (1987) standard errors will be used. In order to assess the difference in the performance of high-beta and low-beta firm's portfolios,  $a_{HL}^{IV1} = a_{H}^{IV1} - a_{L}^{IV1}$  will be set to zero. Moreover, the null hypothesis of  $a_{HL}^{IV1} \leq a_{HL}^{U}$  will be tested to assess whether the difference in conditional alpha is significantly larger than the corresponding difference in unconditional alphas.

#### 4.2.2 Two-step IV method (IV2)

A more direct evidence on the relation between conditioning variables and portfolio betas can be obtained by two-step IV method, introduced by Boguth et al. (2011). Under this approach separate CAPM regression for each quarter is estimated to obtain a time series of non-overlapping conditional CAPM regression parameters. The regression model is:

$$r_{i,j} = a_i + \beta_{i,1} r_{m,j-1} + \beta_{i,2} \left[ \frac{r_{m,j-2} + r_{m,j-3} + r_{m,j-4}}{3} \right] + \varepsilon_{i,j}$$
(11)

where in equation (11)  $r_{i,j}$  is the excess return of portfolio,  $r_{m,j}$  is the excess market return, and  $\hat{\beta}_{i,\tau} = \hat{\beta}_{i,0} + \hat{\beta}_{i,1} + \hat{\beta}_{i,2}$  is the portfolio beta estimate for quarter  $\tau$ .

As suggested by the name, IV2 method is broken into two steps. In the first step, estimated quarterly portfolios are regressed on a set of lagged instruments, i.e:

$$\hat{\beta}_{i,\tau} = \delta_{i,0} + \delta'_{i,1} Z_{i,\tau-1} + \epsilon_{i,\tau}$$
(12)

The estimates and  $R^2$  from equation (12) reflects the ability of instruments to describe the beta dynamics. Hence, the IV2 approach tends to produce more precise estimates of the beta coefficients. Therefore, IV2 may be preferred over IV1 approach that is described in section 4.2.1.

In the second step, fitted betas  $\tilde{\beta}_{i,\tau}$  from the previous regression, is used in the following regression:

$$R_{i,\tau} = a_i^{IV2} + (\phi_{i,o} + \phi_{i,1}\tilde{\beta}_{i,\tau})R_{m,\tau} + v_{i,\tau}$$
(13)

Hence, IV2 is the restricted version of the IV approach where  $\beta_{i,\tau}^{IV2} = \phi_{i,o} + \phi_{i,1}\tilde{\beta}_{i,\tau}$  is constrained to be linear in the fitted first stage beta.

# 5 DATA

We will obtain prices for the Norwegian stock market from OBI (Oslo Børs Informasjon AS), where the returns are adjusted for corporate events such as dividends, stock splits, etc. We have not gained access to the database yet. A detailed analysis of the market data will therefore be given at a later version of the thesis. The Fama French pricing factors; HML, SMB and UMD, in addition to Carhart momentum factor; PR1YR, and liquidity factor; LIQ, will be attained from professor Bernt Arne Ødegaard's (UiS) website. In addition to pricing factors, Ødegaard provides risk-free rates using both monthly and yearly NIBOR<sup>4</sup> as an estimate. Before 1986 we lack monthly NIBOR rates. From 1982-1986 Ødegaard use overnight NIBOR as a proxy. Before 1982 for the monthly data, and before 1986 for the annual data, we use the shortest possible bond yield for treasuries in Eitrheim et al. (2006) as estimates for interest rates (see Ødegaard (2015) Chapter 14).

<sup>&</sup>lt;sup>4</sup> Nibor - Norwegian Interbank Offered Rate is a collective term for Norwegian money market rates at different maturities.

#### REFERENCE

Akram, Q. F., Ben-Haim, Y., & Eitrheim, Ø. (2006). Managing uncertainty through robust-satisficing monetary policy (No. 2006/10). Norges Bank.

Alpert, M., & Raiffa, H. (1982). A progress report on the training of probability assessors.

Ang, A. (2014). Asset Management: A Systematic Approach to Factor Investing. Oxford University Press.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The cross-section of volatility and expected returns. The Journal of Finance, 61(1), 259-299.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2009). High idiosyncratic volatility and low returns: International and further US evidence. Journal of Financial Economics, 91(1), 1-23.

Baker, M., Bradley, B., & Taliaferro, R. (2014). The low-risk anomaly: A decomposition into micro and macro effects. Financial Analysts Journal, 70(2), 43-58.

Baker, M., Bradley, B., & Wurgler, J. (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. Financial Analysts Journal, 67(1), 40-54.

Baker, N. L., & Haugen, R. A. (2012). Low risk stocks outperform within all observable markets of the world. Available at SSRN 2055431.

Bali, T. G., & Cakici, N. (2008). Idiosyncratic volatility and the cross section of expected returns. Journal of Financial and Quantitative Analysis, *43*(01), 29-58.

Barber, B. M., & Odean, T. (2001). Boys will be boys: Gender, overconfidence, and common stock investment. Quarterly journal of Economics, 261-29

Black, F, Jensen, M., and Scholes, M. (1972). The Capital Asset Pricing Model: Some Empirical Tests. In Studies in the Theory of Capital Markets, edited by Michael C. Jensen, 79-121. New York: Praeger Publishers Inc.

Black, F., Jensen, M. C., & Scholes, M. (1972). Studies in the theory of capital markets. Studies in the theory of capital markets.

Blitz, D., & Van Vliet, P. (2007). The volatility effect: Lower risk without lower return. Journal of Portfolio Management, 102-113.

Blitz, D., & Vidojevic, M. (2016). The Profitability of Low Volatility. Available at SSRN: https://ssrn.com/abstract=2811144

Blitz, D., Hanauer, M. X., Vidojevic, M., & van Vliet, P. (2016). Five Concerns with the Five-Factor Model.

Boguth, O., Carlson, M., Fisher, A., & Simutin, M. (2011). Conditional risk and performance evaluation: Volatility timing, overconditioning, and new estimates of momentum alphas. *Journal of Financial Economics*, *102*(2), 363-389.

Carhart, M. M. (1997), On Persistence in Mutual Fund Performance. The Journal of Finance, 52: 57–82

Cederburg, S., & O'Doherty, M. S. (2015). Does it pay to bet against beta? On the conditional performance of the beta anomaly. *Journal of Finance, forthcoming*.

Cornell, B. (2009). The Pricing of Volatility and Skewness: A New Interpretation. The Journal of Investing, 18(3), 27-30.

Diether, K. B., Malloy, C. J., & Scherbina, A. (2002). Differences of opinion and the cross section of stock returns. The Journal of Finance, 57(5), 2113-2141.

Elton, E. J., Gruber, M. J., Brown, S. J., & Goetzmann, W. N. (2003). Evaluation of portfolio performance. Modern PortfoUo Theory and Investment Analysis, 618-659.

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. The Journal of Finance, Vol. 25, No. 2.

Fama, E., and French, K. R. (1992). The Cross-Section of Expected Stock Returns. Journal of Finance, 47, pp. 427-465.

Fama, E. F., & French, K. R. (2006). The value premium and the CAPM. *The Journal of Finance*, *61*(5), 2163-2185.

Fama, E. F., & French, K. R. (2016). Dissecting anomalies with a five-factor model. Review of Financial Studies, 29(1), 69-103.

Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. The journal of political economy, 607-636.

Ferson, Wayne E., and Campbell R. Harvey, 1999, Conditioning variables and the cross section of stock returns, *Journal of Finance* 54, 1325–1360.

Ferson, Wayne E., and Rudi W. Schadt, 1996, Measuring fund strategy and performance in changing economic conditions, *Journal of Finance* 51, 425–462.

Fischhoff, B., Slovic, P., & Lichtenstein, S. (1977). Knowing with certainty: The appropriateness of extreme confidence. Journal of Experimental Psychology: Human perception and performance, 3(4), 552.

Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, *111*(1), 1-25.

Friend, I., & Blume, M. (1970). Measurement of portfolio performance under uncertainty. *The American Economic Review*, 60(4), 561-575

Grant, D. (1977). Portfolio performance and the "cost" of timing decisions. *The Journal of Finance*, *32*(3), 837-846.

Haugen, R. A., and Heins, J.A. (1975). Risk and the Rate of Return on Financial Assets: Some Old Wine in New Bottles. The Journal of Financial and Quantitative Analysis, 10 (5): 775-784.

Jagannathan, R., & Wang, Z. (1996). The conditional CAPM and the crosssection of expected returns. *The Journal of finance*, *51*(1), 3-53.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. Econometrica: Journal of the econometric society, 263-291.

Kahneman, D., & Tversky, A. (1982). Variants of uncertainty. Cognition, 11(2), 143-157.

Lewellen, J., & Nagel, S. (2006). The conditional CAPM does not explain assetpricing anomalies. *Journal of financial economics*, 82(2), 289-314.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The review of economics and statistics*, 13-37.

Li, X., Sullivan, R. N., & Garcia-Feijóo, L. (2016). The Low-Volatility Anomaly: Market Evidence on Systematic Risk vs. Mispricing. Financial analysts journal, 72(1), 36-47.

Newey, W. K., & West, K. D. (1987). Hypothesis testing with efficient method of moments estimation. *International Economic Review*, 777-787.

Markowitz, H. (1952). Portfolio selection. The journal of finance, 7(1), 77-91.

Miller, M. H., & Scholes, M. (1972). Rates of return in relation to risk: A reexamination of some recent findings. Studies in the theory of capital markets, 23.

Mitton, T., & Vorkink, K. (2007). Equilibrium underdiversification and the preference for skewness. Review of Financial studies, 20(4), 1255-1288.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the econometric society*, 768-783.

Scherer, B. (2010). A new look at minimum variance investing. Available at SSRN 1681306.

Shanken, Jay A., 1990, Intertemporal asset pricing: An empirical investigation, *Journal of Econometrics* 45, 99–120.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442.

Shefrin, H., & Statman, M. (2000). Behavioral portfolio theory. Journal of financial and quantitative analysis, 35(02), 127-151.

Vasicek, O. A. (1973). A Note on Using Cross-sectional Information in Bayesian Estimation of Security Betas. The Journal of Finance, 28(5), 1233-1239

Ødegaard, B. A. (2015). Empirics of the Oslo Stock Exchange: Basic, descriptive, results 1980-2015. University of Stavanger.