

- Breaking Down Anomalies: Comparative Analysis of the Q-factor and Fama-French Five-Factor Model Performance -

Preliminary Master Thesis Report

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Introduction

A precise measuring of portfolio performance and predicting future returns have always been challenging for scholars and practitioners. A tight relationship between systematic risk and expected return for assets, in particular stocks was first noticed (Jensen, Black & Scholes, 1972). According to capital asset pricing model that was developed a portfolio's market risk factor explains around 70% of actual returns of the portfolio. Thereafter, Fama and French noticed that small company and value stocks tend to outperform large company and growth stocks, so they came up with the Three Factor Model, which suggests that a model that combines market risk, company size and value factors provides a better tool for assessing portfolio performance (Fama & French, 1993). Another suggestion for more correct asset pricing was made by John Cochrane. According to his q-factor asset pricing model, real investment is maximized when the marginal benefit of investment – i.e., Tobin's q or the expected discounted cash-flows of investment– is equal to its marginal cost which is associated with the investment expense (Cochrane, 1991). According to Cochrane, expected return of the stock is driven mostly by the expected discounted profitability of the firm (Tobin's q) and the investment-to-assets ratio.

Since neither of these models could be used for a perfect prediction of future stock returns, many more explanations and theories arose later on. Huge variety of anomalies have not given a chance to any model to become truly fundamental in the finance world. The anomalies found provided evidence that one or another assumption of model does not hold in practice. This, in turn stimulated scientific activity. Fama and French recently presented a reviewed version of the factor model, which includes two more factors (profitability and investment) into the model (Fama & French, 2015). Professors also applied the new model in a new try to explain the anomalies and found that profitable firms that invest conservatively tend to have higher average returns (Fama & French, 2016).

In our research we attempt to compare new five factor Fama French model with q-factor model to conclude which one works better for explaining anomalies. Hence, the research question can be formulated as follows:

“Does q-factor model outperform Fama-French five factor model in explaining anomalies?”

In order to answer this question we will study in details both models and conduct a statistical analysis on data from the USA market from 1975 to 2015.

The similar questions have already been raised by different scholars. However, the main assessment was made for three-factor model, hence, the novelty of the given paper would be an expansion of the comparison to the latest models.

Literature Review

The three-factor model by Fama and French (1993) was a great advancement to the CAPM, adding to the market factor two additional factors – size and value – to explain what CAPM could not. In their next article, Fama and French (1996) present the evidence that most of the average-return anomalies of the CAPM are captured by their three-factor model. However, since the publication of the abovementioned article, three-factor model has received a lot of criticism, too. Many academics claimed that model is still incomplete and further extensions may be needed to describe the cross-section of stock returns more accurately.

Various studies have presented evidence that Fama-French (1993) model cannot explain many capital market anomalies. For instance, Jegadeesh and Titman (1993), Asness (1994), and Chan, Jegadeesh, and Lakonishok (1996) prove that three-factor model is far from perfect, failing to capture the continuation of short-term returns. Later, Fama and French (1996) themselves admit that this issue remains unaccounted by their model.

Loughran and Ritter (1995) demonstrate that low average returns are associated with share issues; similarly, Ikenberry, Lakonishok, and Vermaelen (1995) show that, on the other hand, average returns tend to be high after the repurchases of the shares, and these relationships are proven not to be captured by the three-factor model.

There were also many studies (e.g. by Lakonishok, Shleifer and Vishny (1994), Kothari, Shanken, and Sloan (1995), Campbell, Hilscher and Szilagyi (2008)) proving that three-factor model overstates the average returns in many cases as it fails to account for the distress premium.

Ang et al. (2006) demonstrate that low average return of stocks with high volatility is not explained by exposures to size or book-to-market value. In addition, Cooper, Gulen, and Schill (2008) find that three-factor model doesn't capture the differences in expected returns across the growth-sorted portfolios, even though investment-related expected return is associated with a firm's size and BM-value, as proven by Berk, Green, and Naik (1999) and Anderson and Garcia-Feijoo (2006).

Starting with Ball and Brown (1968), many studies have documented a relationship between stock returns and earnings, accruals, and cash flow. For instance, Sloan (1996) finds that high accruals are usually followed by low returns; yet, the multifactor model does not capture this relationship.

Based on the abovementioned critique, Fama and French (2015) have recently revisited their primary model and improved it by adding two additional factors. Titman, Wei, and Xie (2004), Novy-Marx (2013) and others state that most of the variation left unexplained by three-factor model is related to profitability and investment. Therefore, Fama and French (2015) augment their model with the profitability factor (the difference between the returns on portfolios with robust and weak profitability, RMW) and the investment factor (the difference between the returns on portfolios of the stocks of "conservative" and "aggressive" investment firms, CMA). Authors conclude that this model explains between 71% and 94% of the cross-section variation of returns of the portfolios examined, capturing a number of anomalies unexplained by three-factor model.

Later on, Fama and French (2016) consider anomalies not targeted by their five-factor model, which three-factor model failed to capture for sure, such as accruals, net share issues, and volatility. Authors prove that five-factor model performs much better than three-factor model, when applied for these anomalous portfolios, except for the one formed on accruals. Hence, the five-factor model is a big improvement and it indeed captures a great amount of variation unexplained by the former model.

At the same time, together with Fama and French (2015), many other academics were trying to explain anomalies using various factors. For example, Chen, Novy-Marx, and Zhang (2011) build an alternative three-factor model to explain the cross section of returns. Their model consists of the market factor, an investment factor, and a return-on-equity factor. Authors state that highly profitable firms will invest a lot, so in their model they are basically controlling for both profitability and investment factors. However, authors find that their model does not outperform Fama-French three-factor model.

Among all the studies in this area, the paper of Hou, Xue, and Zhang (2015) stands out. Authors make use of investment-based asset pricing and the q-factor model in order to capture anomalies; and create a model that consists of four factors: market, size, investment and profitability. The paper provides solid evidence that the q-factor model outperforms the three-factor model in explaining anomalies.

Just from the components of the model of Hou, Xue, and Zhang (2015) we can conclude that it is quite similar to Fama and French (2015) five-factor model, even though factor construction process and underlying theories are different. It is clear that both models are a great improvement in comparison to the three-factor model. Nevertheless, they differ noticeably and it is hard to define straight away, which of the two models does better work in capturing anomalies that were left unexplained before. Hence, in our study we aim to compare the performance of the two models in order to define superior one and possibly combine the best from the two in order to construct even better model.

Theory

The Fama-French Three Factor Model says that the expected return on a portfolio in excess of the risk-free rate $[E(R_i) - R_f]$ is explained by the sensitivity of its return to three factors:

- (i) the excess return on a broad market portfolio ($R_m - R_f$);
- (ii) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB, small minus big);

- (iii) the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks (HML, high minus low).

Specifically, the expected excess return on portfolio i is,

$$E(R_i) - R_f = \beta_i [E(R_m) - R_f] + s_i E(SMB) + h_i E(HML)$$

where $E(R_m) - R_f$, $E(SMB)$ and $E(HML)$ are expected premiums, and the factor sensitivities or loadings;

β_i , s_i and h_i are the slopes in the time-series regression (Fama & French, 1993).

Size (SMB) factor is designed in a way to mimic the risk factor in returns related to size and it is based on the company's market capitalization. It is the difference between the returns on small- and big-stock portfolios with about the same weighted-average book-to-market equity. The factor is designed in a way so that it is largely free of the influence of book-to-market equity differences, focusing instead on the different return behaviors of small and big stocks.

HML factor is meant to mimic the risk factor in returns related to book-to-market equity, is defined similarly. HML is the difference, each month, between the simple average of the returns on the two high-BE/ME portfolios and the average of the returns on the two low- BE/ME portfolios. The two components of HML are returns on high- and low-BE/ME portfolios with about the same weighted-average size. Thus the difference between the two returns should be largely free of the size factor in returns, focusing instead on the different return behaviors of high- and low-BE/ME firms (Fama & French, 1993).

A revised model adds RMW and CMA as two additional factors. The choice is suggested by the dividend discount model, where the market value of a share of stock is the discounted value of expected dividends per share. Miller and Modigliani (1961) show that time t total market value of the firm's stock is:

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau}) / (1 + r)^\tau$$

Where $Y_{t+\tau}$ is total equity earnings for period $t + \tau$ and $dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$ is the change in total book equity.

If we divide by time t book equity, we get:

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^\tau}{B_t}$$

As one can see from the equation, higher expected earnings imply a higher expected return, while higher expected growth in book equity – investment – implies a lower expected return (provided other factors are fixed).

Therefore, the model can be described with the following equation:

$$E(R_i) - R_f = \beta_i [E(R_m) - R_f] + s_i E(SMB) + h_i E(HML) + r_i E(RMW) + c_i E(CMA)$$

where RMW is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and CMA is the difference between the returns on diversified portfolios of the stocks of low and high investment firms (named conservative and aggressive) (Fama & French, 2015).

In contrast, the neoclassical theory of corporate investment is based on the assumption that the management seeks to maximize the present net worth of the company, the market value of the outstanding common shares. An investment project should be undertaken if and only if it increased the value of the shares. The securities markets appraise the project, its expected contributions to the future earnings of the company and its risks. If the value of the project as appraised by investors exceeds the cost, then the company's shares will appreciate to the benefit of existing stockholders. That is, the market will value the project more than the cash used to pay for it. If new debt or equity securities are issued to raise the cash, the prospectus leads to an increase of share prices. Thus, the rate of investment – the speed at which investors wish to increase the capital stock – should be related, if to anything – to q , the value of capital relative to its replacement cost (Brainard & Tobin, 1968).

The first order condition (Euler equation) of the q -factor model stipulates that firms will continue to invest until the marginal cost of investment is equal to its marginal benefit – i.e., Tobin's q

$$1 + a \frac{I_{it}}{A_{it}} = E_t[M_{t+1} \pi_{it+1}]$$

where I_{it} is the investment level of firm i ;

A_{it} is the level of firm's assets;

a is the marginal cost of adjusting the level of capital to its target value;

$E_t[.]$ is the expectation operator conditional on the information set available at time t ;

M_{t+1} is the stochastic discount factor (Gregoriou, Racicot & Théoret, 2016).

Data and Methodology

In this paper, we are going to study the returns of the U.S. companies from January 1975 to December 2015. All the returns and companies' accounting data will be taken from the Center for Research in Security Prices (CRSP) and the Compustat. We are also planning to exploit the data on factors from Kenneth French Data Library when it is convenient. We exclude financial companies and firms with negative book equity.

In order to perform our analysis we first to estimate the two models we are about to compare: five-factor model and q-factor model. Our empirical methodology for this part of the study is from the two articles representing the models: Fama and French (2015) and Hou, Xue, and Zhang (2015) respectively.

First, we build the factors needed and estimate Fama-French five-factor model. In the beginning, we build the market, size, and value factors following Fama and French (1993). Here, the market factor ($R_M - R_F$) is the value-weight return on the market portfolio of all sample stocks net from the one-month T-bill rate.

To build size and value factors we need to construct specific stock portfolios. Hence, at the end of each June, stocks are allocated to two (*Big* and *Small*) size groups using the median NYSE market-cap breakpoints. Independently, using NYSE percentile breakpoints, the stocks are also divided into three book-to-market equity groups: *Low*

(bottom 30%), *Medium* (middle 40%), and *High* (top 30%). The intersection of the two sorts creates six Size-B/M portfolios.

Afterwards, we create the size factor (SMB) as the difference between the returns of the three portfolios of the small stocks (*Small/Low*, *Small/Medium*, and *Small/High*) and the average returns on the three portfolios of the big stocks (*Big/Low*, *Big/Medium*, and *Big/High*). Similarly, the value (HML) factor is constructed as the difference between the average of the returns on the two high Size-B/M portfolios (*Big/High* and *Small/High*) and the average of the returns on the two low Size-B/M portfolios (*Big/Low* and *Small/Low*).

Following Fama and French (2015), the profitability and investment factors, RMW (robust minus weak) and CMA (conservative minus aggressive), are constructed in the similar way as HML factor, except the second sort in not book-to-market equity value, but, respectively, operating profitability and investment.

Thereafter, using all abovementioned factors, for each month t we are estimating the following five-factor model:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$$

Second, we are moving to the construction and estimation of the q-factor model.

Three of its factors are quite similar to the corresponding factors in Fama-French model: the market factor ($R_M - R_F$), the size factor (noted as ME), and the investment factor (I/A). However, in order to construct the profitability factor, Hou, Xue, and Zhang (2015) use slightly different approach. The way authors construct portfolios for the factor is similar, but as the second sort they use not operating profitability as Fama and French (2015), but return on equity (ROE). To create the portfolios, the NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked values of ROE are used. Moreover, the ROE portfolios are meant to be constructed monthly rather than annually. As the portfolios for anomalies we expect ROE factor to capture (e.g. earnings surprise, financial distress) will be constructed monthly, it seems natural to use the same frequency for the factor as well.

Other significant difference between the two models is that Hou, Xue, and Zhang (2015) do not include HML factor to their model. Authors claim that, due to its high correlation with the investment factor, including HML may just add the noise in the

model. Following the advice of the two articles, we first estimate the average returns on each factor in both models and test if they are significantly different from zero.

Finally, we estimate the q-factor model in a following way:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + m_iME_t + h_iI/A_t + r_iROE_t + e_{it}$$

The next step of our research is to estimate both models as Fama and French (2015), where the left-hand-side (LHS) assets are portfolios constructed using size as the first sort and book-to-market value/profitability/investment as the second sort. Frankly speaking, these portfolios are just better sorts on the variables used to build the factors. Here, to construct size-profitability sorted portfolios we use operating profitability measure as Fama and French (2015) and not ROE.

Afterwards, following Fama and French (2016) and Hou, Xue, and Zhang (2015), we are going to consider anomalies not targeted by the two models and form LHS portfolios on size and each of the anomalous variables studied. The most interesting anomalies for us to investigate are, for instance, net share issues, volatility, market beta, accruals, and momentum. In the scope of our study we exploit both time-series and cross-sectional regressions.

For the next and, probably, the most important part of our analysis we need to compare the performance of the two models among themselves and with Fama-French three-factor model in order to find out, which one is the best explanation of the stock returns.

To begin with, we are going compare the usual adjusted R^2 and residual standard errors of all the models estimated, and then proceed to the testing of our models with more sophisticated tools.

If an asset-pricing model explains expected return completely, then the regression of an asset's excess returns on the model's factor returns produces the intercept, which is indistinguishable from zero. We are going to test this hypothesis for our models and combinations of LHS portfolios with GRS statistics. Introduced by Gibbons, Ross, and Shanken (1989), GRS statistic tests whether the estimated intercepts from a multiple linear regression are jointly zero and it is used to judge the efficiency of a given portfolio. We can expect that GRS statistics will demonstrate that our models

are incomplete. However, we are interested in a relative performance of the two and want to identify the model that is the best description of the returns, despite being imperfect. In addition to the GRS statistics, we follow Fama and French (2015) and compute the average absolute value of the intercepts ($A|\alpha_i|$) for all the regressions to compare them between the models.

Together with the intercept-based metrics, we are planning to use Hansen-Jagannathan distance for model comparison. Introduced by Hansen and Jagannathan (1997), this measure became quite popular in the empirical asset pricing studies. We also consider following Chen and Ludvigson (2009), who propose a method for comparison of HJ distance between the two models based on the White (2000) test.

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