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Breaking Down Anomalies: Comparative Analysis of the Q-factor and Fama-French Five-Factor Model Performance

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Content

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Abstract

The continuous development of the asset pricing supplies investors and researchers with the new empirical models and it might get challenging to pick one to use. This thesis provides an empirical comparison of the Fama-French 5-factor model and q-factor model. The analysis is performed on the value-weighted portfolios formed on the five anomalies. We perform intercept study and employ Fama-MacBeth methodology. Our findings suggest that q-factor model is superior in performance compared to the Fama-French revised model; however, none of the models is complete. Analyzed anomalies are likely to be proxies for some priced risk factors and might be used to improve the models.

KEYWORDS: asset pricing, stock returns, market anomalies, priced risk factors, Fama-French 5-factor model, q-factor model, priced risk.

1. Introduction

A precise measuring of portfolio performance and predicting future returns have always been challenging for scholars and practitioners. Jensen, Black, and Scholes (1972) were the first to notice a tight relationship between systematic risk and expected return for the assets, particularly stocks. Further developed capital asset pricing model found that around 70% of actual returns of the portfolio is explained by a portfolio's market risk factor. Thereafter, Fama and French noticed that small company and value stocks tend to outperform large company and growth stocks, so they came up with the Three Factor Model, which suggests that a model that combines market risk, company size and value factors provides a better tool for assessing portfolio performance (Fama & French, 1993). John Cochrane suggested another alternative for more correct asset pricing. According to his q-factor asset pricing model, real investment is maximized when the marginal benefit of investment – i.e., Tobin's q or the expected discounted cash-flows of investment – is equal to its marginal cost which is associated with the investment expense (Cochrane, 1991). According to Cochrane, expected return of the stock is driven mostly by the expected discounted profitability of the firm (Tobin's q) and the investment-to-assets ratio.

Many more explanations and theories arose afterward, yet, none of the existing asset pricing models have become truly fundamental, as investors keep finding certain strategies that offer superior performance to one predicted by the model.

Thus, one of the fundamental questions to be answered in the financial science is which risks should be priced. The definite answer will allow to determine the relation between risk and reward and price all financial assets, which in turn will help to make correct investment decisions. One should remember that parsimony is important and attempt to include all possible factors in the model is not scientifically correct.

Trying to tackle existing insufficiency, Fama and French recently presented a reviewed version of the factor model, which includes two more factors (profitability and investment) into the model (Fama & French, 2015). Researchers also applied the new

model in a new attempt to explain the anomalies and found that profitable firms that invest conservatively tend to have higher average returns (Fama & French, 2016).

In our research, we aim to evaluate new 5-factor Fama-French model and q-factor model in order to understand their similarities and differences. We also use two models to assess the list of five potential market anomalies and check whether they can potentially be priced risk factors omitted by the model. Hence, the research question can be formulated as follows:

"How do Fama-French five factor model and q-factor model perform empirically and do the market anomalies persist according to the models?"

In order to answer the questions, we will study in detail both models and conduct a statistical analysis on the data from the USA market from 1967 to 2015 with the focus on such market anomalies as accruals, market beta, net share issues, variance and residual variance. We are performing a wide intercept study and utilize Fama-MacBeth methodology, analyzing the set of portfolios jointly and separately by the portfolios formed on each anomaly.

The similar questions have already been raised by different scholars. However, the main assessment was made for three-factor model; hence, the novelty of the given paper would be in the expansion of the comparison to the latest models and in the methods used for the comparison.

We start the analysis by the overview of the literature related to the topic, which is followed by the review of the key-established asset pricing theories alongside with searching for theoretical intuition behind the analyzed empirical models. We further explain the construction of the factors and portfolios, followed by the details on the methodology of the study. This is followed by the description of the data set used with its summary statistics. Subsequent part presents the empirical findings and the analysis of the models. Finally, we conclude with a short summary of our study, its main findings, and some suggestions on further research.

2. Literature Review

From the beginning of the development of theoretical foundations of the Capital Asset Pricing Model, researchers started to look for the empirical proofs of the developed theory. However, often beta was not the only factor influencing realized returns of a security or a portfolio, so instead of proving they persistently found the cases that CAPM fails to explain.

Ball (1978) pointed out the relationship between stock prices and public announcements regarding company's earnings. Particularly, he presented the evidence that after the announcement, securities are priced in such a way that they yield systematic excess returns. Basu (1977) doubted the efficient market hypothesis and showed empirically that portfolios with low P/E ratio yielded more as compared to CAPM predicted returns even after the adjustment for risk.

Companies' specific betas also do not fully account for difference in return between small and large firms (Banz (1981), Reinganum (1981)). Yet, some other researchers, such as Roll (1981) attributed this not to CAPM failure, but rather to inability of correct measurement of beta for small firms' securities.

Statman (1980) and Rosenberg, Reid and Lanstein (1985) discovered another noteworthy exception from CAPM rules. They showed that stocks with high book-to-market ratios earn on average higher returns than stocks with approximately the same beta value, but lower B/M ratio.

Bhandari (1988) argued that debt-to-equity ratio is a natural proxy for the risk of firm's equity, and hence can be used in an asset pricing model. It follows that securities of the companies with high D/E ratios yield higher returns than expected with regards to the market betas.

Wide range of various CAPM anomalies triggered further research in the area. The three-factor model by Fama and French (1993) added to the market factor two additional factors – size and value. Even though their model performed much better in different empirical tests than CAPM, it lacked a solid theoretical foundation.

In their next article, Fama and French (1996) present the evidence that most of the average-return anomalies of the CAPM are captured by their three-factor model. However, since the publication of the abovementioned article, three-factor model has received a lot of criticism, too. Many academics claimed that model is still incomplete and further extensions may be needed to describe the cross-section of stock returns more accurately.

Various studies have presented evidence that Fama-French (1993) model cannot explain many capital market anomalies. For instance, Jegadeesh and Titman (1993), Asness (1994), and Chan, Jegadeesh, and Lakonishok (1996) prove that three-factor model is far from perfect, failing to capture the continuation of short-term returns. They found a noteworthy trend that is often referred to as a momentum, meaning that the stocks with high average returns for the last months continue to earn higher returns over so called 'losers' stocks (those with low returns for the same previous time period). Bernard and Thomas (1989) discovered similar phenomenon, showing that stocks with unexpected high earnings perform better than those with unexpected low earnings over the next six months; hence, there is a post-earnings-announcement drift. Later, Fama and French (1996) themselves admit that abovementioned issues remain unaccounted by their model. Carhart (1997) presented an updated version of the Fama-French three factor model adding momentum as the forth factor to control for the expected returns. However, such an extension was further criticized; for example, Zach (2003) provides evidence that adding a momentum factor increases the abnormal returns to the accrual anomaly. Lee and Swaminathan (2000) further developed the research and pointed out a positive relation between stock trading volume and momentum effects.

Various researchers tried to use share issuance to forecast future stock returns. Argument follows that share repurchases are often made by the companies that treat their stocks as undervalued, whereas new stocks are usually issued when the firm is overvalued on the market. Indeed, Ikenbery, Lakonishok, and Vermaelen (1995) show that average returns tend to be high after the repurchases of the shares, whereas, on the other hand, Loughran and Ritter (1995) demonstrate that low average returns are associated with share issues. Yet, some argue that there are many statistical issues to be taken into consideration, for example, Mitchell and Stafford (2000) showed that

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long-run results heavily depend on adequate control for cross-sectional correlation, as well as heteroskedasticity. Furthermore, Schultz (2003) proves that estimated regression results might be spurious if company's decision regarding stocks issuing policy correlates with previous period securities performance. However, even taking into consideration econometric challenges, Pontiff and Woodgate (2008) prove that a negative relation exists between net stock issues and average returns, which is proven not to be captured by the three-factor model.

There were also many studies (e.g. by Lakonishok, Shleifer and Vishny (1994), Kothari, Shanken, and Sloan (1995), Campbell, Hilscher and Szilagyi (2008)) proving that three-factor model overstates the average returns in many cases as it fails to account for the distress premium.

Idiosyncratic risk tends to be another factor not fully accounted for by traditional asset pricing models. However, researchers do not always agree on how it influences future stock returns. Traditionally, it was argued, for example by Merton (1987), that investors do not hold diversified enough portfolios and should be compensated for tolerating additional idiosyncratic volatility, hence a relationship is positive. When analyzing monthly returns, Malkiel and Xu (2002), Fu (2006) indeed prove that a positive relationship exists. However, Ang et al. (2006) using daily returns data demonstrate that stocks with high volatility earn low average return, which is not explained by exposures to size or book-to-market value.

Cooper, Gulen, and Schill (2008) find that three-factor model doesn't capture the differences in expected returns across the growth-sorted portfolios, even though investment-related expected return is associated with a firm's size and BM-value, as proven by Berk, Green, and Naik (1999) and Anderson and Garcia-Feijoo (2006).

Possible relation between current and future profitability of the corresponding stocks forms another big group of possible anomalies of asset pricing models. Starting with Ball and Brown (1968), many studies have documented a relationship between stock returns and earnings, accruals, and cash flow. Sloan (1996) divided firm's earnings into cash flow and operating accruals and found that high accruals are often associated with low future profits and low returns; yet, the multifactor model does not capture this relationship. Hirshleifer et al. (2004) argue that investors are too limited in analyzing

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financials of the company and suggest looking at net operating assets (the difference between cumulative earnings and cumulative free cash flow over a time) rather than simple one period accruals. They show empirically that net operating assets is a good predictor of future returns. On the other hand, Fairfield, Whisenant & Yohn (2003) find the relationship between growth and accruals, showing that high-growth companies usually have higher accruals, hence there is no direct relationship between accruals and stock returns. However, recently Lewellen and Resutek (2016) proved that accruals have well-defined separate predictive power of stock returns.

Furthermore, many researchers (Frankel and Lee (1998), Dechow, Hutton and Sloan (1999), Piotroski (2000)) proved that one can earn higher average returns on stocks of the firms with higher future expected cash flows. They explained that finding by irrationality of stock pricing, as from their point of view traded security prices do not disclose all information regarding profitability. However, it is quite possible that found relation is the result of a poorly specified asset pricing model.

Based on the abovementioned critique, Fama and French (2015) have recently revised their primary model and improved it by adding two additional factors. Titman, Wei, and Xie (2004), Novy-Marx (2013) and others state that most of the variation left unexplained by three-factor model is related to profitability and investment. Therefore, Fama and French (2015) augment their model with the profitability factor (the difference between the returns on portfolios with robust and weak profitability, RMW) and the investment factor (the difference between the returns on portfolios of the stocks of "conservative" and "aggressive" investment firms, CMA). Authors conclude that this model explains between 71% and 94% of the cross-section variation of returns of the portfolios examined, capturing a number of anomalies unexplained by three-factor model.

Later, Fama and French (2016) consider anomalies not targeted by their five-factor model, which three-factor model failed to capture for sure, such as accruals, net share issues, and volatility. Authors prove that five-factor model performs much better than three-factor model, when applied for these anomalous portfolios, except for the one formed on accruals. Hence, the five-factor model is a big improvement and it indeed captures a great amount of variation unexplained by the former model.

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At the same time, together with Fama and French (2015), many other academics were trying to explain anomalies using various factors. For example, Chen, Novy-Marx, and Zhang (2011) build an alternative three-factor model to explain the cross section of returns. Their model consists of the market factor, an investment factor, and a return-on-equity factor. Authors state that highly profitable firms will invest a lot, so in their model they are basically controlling for both profitability and investment factors. However, authors find that their model does not outperform Fama-French three-factor model.

Among all the studies in this area, the paper of Hou, Xue, and Zhang (2015) stands out. Authors make use of investment-based asset pricing and the q-factor model in order to capture anomalies; and create a model that consists of four factors: market risk premium, size, investment and profitability. The paper provides solid evidence that the q-factor model outperforms the three-factor model in explaining anomalies.

Looking on the components of the model of Hou, Xue, and Zhang (2015) we can conclude that it is quite similar to Fama and French (2015) five-factor model, even though factor construction process and underlying theories are different. It is clear that both models perform significantly better in comparison to the three-factor model. Nevertheless, they differ noticeably and it is hard to define straight away, which of the two models does better work in capturing anomalies that were left unexplained before.

3. Theory

Asset pricing understanding in financial world developed with the advancement of asset pricing theory alongside with the empirical asset pricing. These two key elements mutually affected each other stimulating further research, as often either theoretical model was not sufficiently backed by empirical evidence or, vice versa, statistical correlation was used to explain causation without solid theoretical foundation. In this section, we focus not on the empirical models, but solely on the key established asset pricing theories.

3.1. CAPM

The capital asset pricing model of William Sharpe (1964) and John Lintner (1965) is said to mark the birth of asset pricing theory. Despite many advancements made and empirical problems detected during more than 50 years, CAPM is still most widely used model in different economic applications, such as estimating the cost of capital for firms or evaluating the performance of managed portfolios. Huge popularity of the empirically not proven enough model can be explained by its simplicity and intuitive predictions about how to measure risk and the relation between expected return and risk.

According to CAPM, agents carry only about the wealth and assess potential priced risk by beta only – β_{iM} – the market beta of asset i, which is the covariance of its return with the market return divided by the variance of the market return,

$$\beta_{iM} = \frac{cov\ (R_i, R_M)}{\sigma^2(R_M)} \tag{1}$$

The main CAPM equation states that the expected return on any asset i is a sum of the risk-free interest rate and risk premium, which is the asset's market beta times the premium per unit of beta risk:

$$E(R_i) = R_f + [E(R_M) - R_f] \cdot \beta_{iM},\ i = 1,\ \dots,\ N \tag{2}$$

One of the central ideas of CAPM is that return on the stock is in fact the reward for the investor for the risk he takes. Reorganizing the main equation, we can get the following:

$$E(Ri) - Rf = \frac{E(R_M) - Rf}{\sigma(R_M)} \cdot \beta_i \sigma(R_M) \tag{3}$$

Thus, risk premium from CAPM equals to the product of price of risk and quantity of risk. However, it means not the whole risk. Total risk is decomposed to systematic and non-systematic risks that are not correlated. Non-systematic risk is associated with specific stock and investor can get rid of it using benefits of diversification, or in other words, if an investor holds big enough portfolio of securities, their unsystematic risks are diversified away. On opposite, systematic risk is inherent to any stock and only this risk is awarded.

3.2. APT

Strict and partly implausible assumptions of CAPM motivated a development of the new alternative approach to the asset pricing by Stephen Ross (1976), which is now called Arbitrage Pricing Theory. On contrary to CAPM which is equilibrium analysis of a comparative static model, APT utilizes a linear return generating process. Arbitrage pricing theory takes the focus away from the efficient portfolios and starts with an assumption, unlike CAPM, that there is not one but many economic factors that influence the price, however, it neither highlights the most important, nor presents a comprehensive list of factors that have a significant effect. One of the main advantages of APT, especially when opposed to CAPM, is that it does not have limiting assumptions, but is rather based on the exact postulates.

APT is consistent with CAPM in its belief that any individual stock risk can be diversified away in the portfolio. Systematic risk that is left determines an additional expected return and volatility of the portfolio, meaning that the difference between realized return on the asset and expected return equals to the sum of products of factor loading (β) of the asset (sensitivities to the specific risk factors) and the realized values of those risk factors plus idiosyncratic risk (error term). Risk factors are associated with

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the shocks and at the beginning of the period are expected to equal to zero ($E[f_1(t)] = E[f_2(t)] = ... = E[f_K(t)] = E[\varepsilon_i(t)] = 0$).

APT further states that it is not plausible for an investor to earn a pure arbitrage profit. In other words, with no risk and with no additional net investment, one cannot get a positive expected rate of return. Hence, with an approximation error that can be neglected practically, given that P_i is a non-zero risk premium for ith risk factor, we got the main APT result:

$$R_i(t) - R_f = \beta_{i1}[P_1 + f_1(t)] + \beta_{i2} [P_2 + f_2(t)] + ... + \beta_{iK} [P_K + f_K(t)]) + \varepsilon_i(t) \qquad (4)$$

In practice, APT performance depends on how well researchers select the risk factors that they condition expected return upon. It is rather difficult to be sure that the chosen list is fully comprehensive, however, usually it is enough to study four or five factors to get rather stable results.

One might claim that Fama-French factor model is one of the alternative formulation of APT, however, it's not the case, since APT suggests pricing only external factors reflected in the covariance matrix and does not take into consideration firm-specific variables.

3.3. ICAPM

Intertemporal Capital Asset Pricing Model (ICAPM) is another alternative to classical CAPM offered by Robert Merton (1973). He has tried to weaken some of the assumptions of CAPM, thus making it more realistic. Both models discussed above are one-period model, meaning that investor takes a decision regarding the percentage of wealth he wants to allocate to a certain stock or portfolio and next period only observes the result, namely return earned. However, in real life investors keep rebalancing portfolios, buying certain securities or selling them probably for current consumption. Merton takes it into consideration and turns a static model into a dynamic one, by making specific investment amount in an asset, endogenous variable. If CAPM, assumed that there is a fixed fraction of total wealth that individual chooses to invest fixed, ICAPM set it as subject to change from period to period. In ICAPM setting value of wealth varies with the opportunity set (whether next period has relatively better or

worse investment opportunities), hence, one wants to manage and hedge against the risk that the value will be different.

Following an idea that investor is trying to maximize its lifetime utility (not one period as in CAPM), and applying the number of statistical properties and mathematical techniques, we can simplify Merton's conclusion to the following:

$$E(R_i) - R_f = \beta_i^1 \big(E[R_M] - R_f \big) + \sum_{j=1}^{m} \beta_i^j \big(E[R_j] - R_f \big) \qquad (5)$$

where m is the number of state variables, and $E[R_j]$ is the expected return on the hedging portfolio which is supposed to balance an anticipated risk from the main investment to stock i in the specific state realization; β_i^1 is CAPM beta and β_i^j shows the volatility with regards to the hedging security.

Hence, excess return is a compensation for not only systematic risk, but also a risk of negative change of the state variable. Thus, the model gives an explanation to the completely uncorrelated with the market portfolio stocks that yield a higher than a risk-free return.

Intertemporal investor (as opposed to the one period investor of CAPM) changes in the investment opportunity set should be priced, that is, all state variables that determine the investment opportunity set are priced risks.

Main challenge with empirical testing of ICAPM is defining state variables (similarly as defining risk factors in APT).

3.4 Theoretical intuition behind empirical models

Even though the models that are being tested in our research are mainly empirical and do not comprise solid theoretical foundation, there is some theoretical intuition behind them.

From the theoretical point of view, we assume that markets are rational and have no frictions. Even though there might be some non-rational investors, in general market participants try to benefit from mispriced opportunities; hence, market as a whole is

rational and the prices are set correctly. We further assume that assets with higher expected returns are riskier than ones with lower expected returns. After calculating the expected return with the help of any asset pricing model, any found pattern that is not explainable by the used model is called an anomaly, as noted by Fama and French (2008). Whenever a sensible explanation for it is found, an anomaly can be captured simply by including an extra risk factor to the previous model. Following this logic, Fama and French took the anomalies that were found when testing CAPM and came up with the three factor model.

Fama-French model is mostly motivated by dividend discount model. It relates the market value of a firm to the present value of all expected future dividends, as follows:

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(D_t + \tau)}{(1+r)^{\tau}} \qquad (6)$$

where M_t is the stock price at time t, $E(D_t + \tau)$ is the expected dividend for the period $t+\tau$, r is the required rate of return on expected dividends.

According to Miller and Modigliani (1961), expected dividend for the period can be expressed as the difference between earnings in period $t+\tau$ ($Y_{t+\tau}$), and the change in book equity for the period $t+\tau$ ($dB_{t+\tau}$), which transforms the equation (6) into the following:

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(Y_{t+\tau} - dB_{t+\tau})}{(1+r)^{\tau}} \qquad (7)$$

Equation (7) divided by book equity shows that one should expect a positive correlation between increase in book-to-market ratio and market value, which is ultimately stock returns. Furthermore, growing earnings ($Y_{t+\tau}$) should result into increased expected future earnings, while increase in investments should potentially lower them.

Q-factor model, in turn, has its conceptual roots in investment-based asset pricing. The neoclassical theory of corporate investment assumes that the management seeks to maximize the present net worth of the company, the market value of the outstanding common shares. Hence, the main rule for undertaking any new investment project is that it should increase the value of the shares. The securities markets evaluate the project, its expected contributions to the future earnings of the company and its risks.

When the project value anticipated by investors is greater than the costs, then current stockholders will benefit. It is the case when the market attributes more value to such project than the total value of the cash needed to finance it. In case that available cash is not enough and company raises extra by issuing equity securities or debt, it ultimately leads to an increase of share prices. Therefore, according to Brainard & Tobin (1968) the rate of investment (the speed at which investors want to grow the capital stock) should be related, if to anything – to q, the value of capital relative to its replacement cost.

The first order condition (Euler equation) of the q-factor model specifies that firms will continue to invest until the marginal cost of investment is equal to its marginal benefit – i.e., Tobin's q

$$1 + a \frac{I_{it}}{A_{it}} = E_t[M_{t+1} \, \pi_{it+1}] \tag{8}$$

where I_{it} is the investment level of firm i;

A_{it} is the level of firm's assets;

a is the marginal cost of adjusting the level of capital to its target value;

$E_t[.]$ is the expectation operator conditional on the information set available at time t;

M_{t+1} is the stochastic discount factor;

π_{it+1} is the time t+1 profitability of firm i.

The equation (8) can be further rearranged to the following:

$$E_0[r_{i1}^S] = \frac{E_0[\pi_{i1}]}{1 + a(\frac{I_{i0}}{A_{i0}})} \tag{9}$$

where r_{i1}^S is stock *i* return at date 1.

It follows that low investment stocks are expected to yield higher expected returns compared to high investment stocks, while higher profitability stocks should earn higher returns compared to lower ones.

Yet, although inspired and derived from the q-theory, q-factor model is significantly reduced and simplified empirical model.

Each theoretical model aims to yield the results that are consistent to the highest degree with reality; however, often it is not the case and realized stock returns frequently differ from those projected by one or another model. There are two possible reasons for the differences: either there is a market anomaly that might be also sometimes treated as behavioral bias, or there is a certain risk factor that is not included into the model. Some results might also be triggered by the data selection bias, and if that is the case, then the obtained results are not persistent throughout the different data samples.

In case the obtained difference is not connected to the data problems, and if the found anomaly can be put to the right-hand side of the model and this will improve its performance, it is not an anomaly, but rather the factor that is priced by the investors and omitted in the used model. Then, one can conclude that the model is misspecified and should be corrected. Another indication is that over the longer period of time omitted risk factors will be preserved, while market anomalies will be eventually arbitraged away.

Supporters of the behavioral finance explain found anomalies as the consequences of the market assumptions violations, therefore, treat anomalies as persistent and not avoidable. Firstly, investors can assess available information in the wrong way and make suboptimal investment decisions, secondly, market can be restrictive on short selling, can have transaction costs, hence, not allowing even rational arbitrageurs to fully benefit from the mispricing opportunities.

4. Research Methodology

If an asset pricing model explains the variation in returns perfectly, then in the time-series regression of any assets' excess returns on the model's factors, the intercept will be indistinguishable from zero. Hence, to define better asset pricing model, we test our two models on different sets of portfolios and pay special attention to the intercepts returned from all the regressions. Firstly, we compare different intercept measures for the two models from the estimated time-series regressions. Next, we employ Fama-MacBeth approach to estimate whether the models capture all the priced risk factors and then to check whether additional factors can improve the models. Here we also test if the anomalies studied are true anomalies or they are just missing priced risk factors.

4.1. Factor construction

To estimate both Fama-French five-factor model and q-factor model we need the factor returns. In the beginning, the market, size, and value factors are built as in Fama and French (1993). Here, the market factor ($R_M - R_F$) is the value-weight return on the market portfolio of all sample stocks net from the one-month T-bill rate.

In order to build size and value factors, stocks are allocated to two (*Big* and *Small*) size groups using the median NYSE market-cap breakpoints at the end of each June in the scope of our dataset. Independently, using NYSE percentile breakpoints, the stocks are also divided into three book-to-market equity groups: *Low* (bottom 30%), *Medium* (middle 40%), and *High* (top 30%). The intersection of the two sorts creates six Size-B/M portfolios.

Afterwards, the size factor (SMB) is created as the difference between the returns of the three portfolios of the small stocks (*Small/Low, Small/Medium,* and *Small/High)* and the average returns on the three portfolios of the big stocks (*Big/Low, Big/Medium,* and *Big/High)*. Similarly, the value (HML) factor is constructed as the difference between the average of the returns on the two high Size-B/M portfolios (*Big/High* and *Small/High*) and the average of the returns on the two low Size-B/M portfolios (*Big/Low* and *Small/Low*).

Following Fama and French (2015), the profitability and investment factors, RMW (robust minus weak) and CMA (conservative minus aggressive), are constructed in the similar way as HML factor, except the second sort in not book-to-market equity value, but, respectively, operating profitability and investment (defined as the annual change in total assets).

Thereafter, using all abovementioned factors, for each month *t* we can estimate the following five-factor model for the excess returns on our portfolios ($\bar{R}_{it}$):

$$\bar{R}_{it} = \alpha_i^{FF} + \beta_{i,RM}(R_{Mt} - R_{Ft}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,RMW}RMW_t + \\ \beta_{i,CMA}CMA_t + e_{it}^{FF} \tag{10}$$

Second, we are moving to the estimation of the q-factor model. Its factors are quite similar to the corresponding factors in Fama-French model: the market factor ($R_M - R_F$), the size factor (noted as ME), the investment factor (I/A), and the profitability factor (ROE). However, these factors for the q-factor model are built not from the double (as in Fama and French (1993, 2015)), but from the triple sorts on size, investment-to-assets, and ROE. Another important difference is that while Fama and French (2015) use the change in the total assets as the measurement of investment, Hou, Xue, and Zhang (2015) argue that past investment does not forecast future investment and use investment-to-assets measure instead (the annual change in total assets divided by 1-year-lagged total assets).

As we can see, Hou, Xue, and Zhang (2015) construct the profitability factor using not operating profitability as Fama and French (2015), but return on equity (ROE). To create the portfolios, the NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked values of ROE are used. Moreover, the ROE portfolios are meant to be constructed monthly rather than annually. As the portfolios for anomalies we expect ROE factor to capture (e.g. earnings surprise, financial distress) are typically constructed monthly, it seems natural to use the same frequency for the factor as well.

Other significant difference between the two models is that Hou, Xue, and Zhang (2015) do not include HML factor to their model. Authors claim that, due to its high correlation with the investment factor, including HML may just add the noise to the model.

Finally, we estimate the q-factor model in the following way:

$$\bar{R}_{it} = \alpha_i^Q + \beta_{i,M}(R_{Mt} - R_{Ft}) + \beta_{i,ME}ME_t + \beta_{i,I/A}I/A_t + \beta_{i,ROE}ROE_t + e_{it}^Q \quad (11)$$

For all our models and tests, we make an assumption of no market frictions (taxes, transaction costs, etc.). We also estimate all the regression slopes as constants, which may be a potential problem and leaves room for further research.

4.2. Left-hand-side (LHS) portfolios

Researchers often focus their empirical tests of asset pricing models on the portfolios constructed on size and value variables, as those two are seen as the most important (Lewellen, Nagel, and Shanken, 2010). While such regressions return high R-squared and small pricing errors, they usually provide little economic meaning due to the high correlation between LHS portfolios and factors. The solution is then to consider portfolios of anomalies not targeted by the two models. Based on our research of the literature on anomalies, the most interesting anomalies for us to investigate are accruals, market beta, net share issues, volatility and residual volatility. Returns on portfolios formed on anomalies from January 1967 to December 2015 were extracted from Kenneth French Data Library. We are testing this anomalous portfolios both separately and altogether to get a better insight on the explanatory power of the two models and to account for the possible sample selection bias.

LHS portfolios are constructed similarly to the factor portfolios. At the end of each June all stocks are sorted into deciles, using NYSE breakpoints for the market capitalization and the second sort, which is the respective anomalous variable. For each June of year t, accruals (AC) are the change (from the year t-2 to t-1) in operating working capital per split-adjusted share divided by book equity per share in year t-1. Similarly, net share issues (NI) for each t are the change in the natural log of outstanding split-adjusted shares from the year-end of t-2 to t-1. Market beta (β) for each year is estimated on the monthly returns for the preceding five years. While portfolios for AC, NI and β are formed annually, the portfolios for the variance (VAR) and residual variance (ResVar) are constructed monthly. Here, variance is the variance

of daily returns, and residual variance is the variance of the residuals from the FF 3-factor model, both estimated using 60 days of lagged returns.

We can perform our analysis on either value-weighted (VW) or equally weighted (EW) portfolios. The main problem with VW portfolios is that they may be dominated by few big stocks, while small stocks are left underweighted. However, Fama and French (2008) argue that the most serious issue with EW portfolios is that they can be dominated by very small stocks (microcaps). Even though they are on average only about 3% of the market capitalization, they usually account for up to 60% of the total number of stocks. Moreover, microcaps tend to have largest cross-section dispersion of anomaly variables, which leads to them being determinant in extreme sort portfolios. Hence, due to the specifics of our test assets, using EW portfolios can bias the results of our analysis significantly, so we are using VW portfolios in our research.

4.3. Intercepts Study

If an asset-pricing model explains expected return completely, then the regression of any assets' excess returns on the model's factor returns produces the intercept, which is indistinguishable from zero. Thus, after the estimation of the time-series regressions for the two models for each set of portfolios, we are going to compare the performance of the two models at this point of our analysis based on the intercepts estimated.

To begin with, we are comparing the average value of the intercepts (A(α)) and the average absolute value of the intercepts (A$|\alpha|$) for all the models as following:

$$A(\alpha) = \frac{\sum_{i=1}^{10} \alpha_i}{10},$$ (12)

$$A(|\alpha|) = \frac{\sum_{i=1}^{10} |\alpha_i|}{10},$$ (13)

where i=1,…10 represents each of the 10 decimal portfolios for each anomaly studied.

Moreover, we compute the average standard errors of the intercepts (A(SE)) for all the regressions to compare them between the models. Here, we are looking for the lowest

(absolute) values of the intercept and lowest SE, which will show that the model explains the most variation in excess returns.

Furthermore, we are going to test the hypothesis that the intercepts are indistinguishable from zero for our models and combinations of LHS portfolios with GRS statistics. Introduced by Gibbons, Ross, and Shanken (1989), GRS statistic tests whether the estimated intercepts from multiple linear regressions are jointly zero (null hypothesis) and it is used to judge the efficiency of a given portfolio. We can expect that GRS statistics will demonstrate that our models are incomplete. However, we are interested in a relative performance of the two and want to identify the model that is the best description of the returns, despite being imperfect.

In addition, Barillas and Shanken (2016) suggest that we should judge competing models by their maximum squared Sharpe ratio for the intercepts from time series regressions, which is computed as following:

$$\text{Sh}^2(\alpha) = \alpha' \Sigma^{-1} \alpha, \tag{14}$$

where α is the vector of intercepts and Σ is the residual covariance matrix.

Then, the best model among competing will be the one with the smallest $\text{Sh}^2(\alpha)$.

4.4. Fama-MacBeth Approach

In order to further examine the explanatory power of the factors of the two models, we use two-pass regression approach introduced by Fama and MacBeth (1973).

The first step of this method is to estimate factor loadings by performing a set of time series regressions for the excess returns of each investigated portfolio ($\bar{R}_{it}$) on the five factors from Fama-French model and, similarly, on the factors of q-factor model. Thus, using ordinary least squares (OLS) approach, for each portfolio i=1,…n, we successively estimate two following time-series regressions:

$$\bar{R}_{it} = \alpha_i^{FF} + \beta_{i,RM}(R_{Mt} - R_{Ft}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,RMW}RMW_t +$$
$$\beta_{i,CMA}CMA_t + e_{it}^{FF} \tag{15}$$

$$\bar{R}_{it} = \alpha_i^Q + \beta_{i,M}(R_{Mt} - R_{Ft}) + \beta_{i,ME}ME_t + \beta_{i,I/A}I/A_t + \beta_{i,ROE}ROE_t + e_{it}^Q \tag{16}$$

where t=1,…,T; $\beta_{i,RM}$, $\beta_{i,SMB}$, $\beta_{i,HML}$, $\beta_{i,RMW}$, $\beta_{i,CMA}$ are the regression coefficients for the Fama-French factors; $\beta_{i,M}$, $\beta_{i,ME}$, $\beta_{i,I/A}$, $\beta_{i,ROE}$ are the coefficients of the q-factor model; α_{it}^{FF} and α_{it}^{Q} are the intercepts and e_{it}^{FF} and e_{it}^{Q} are the error terms for the two regressions respectively.

After estimating the two regressions, we obtain estimates of their coefficients for each portfolio i: $\hat{\beta}_{i,RM}, \hat{\beta}_{i,SMB}, \hat{\beta}_{i,HML}, \hat{\beta}_{i,RMW}, \hat{\beta}_{i,CMA}$ and $\hat{\beta}_{i,M}, \hat{\beta}_{i,ME}, \hat{\beta}_{i,I/A}, \hat{\beta}_{i,ROE}$ respectively. The second stage of our analysis is to run cross-sectional regressions of the excess returns of each portfolio on the factor loadings we have obtained from the first-pass regressions. This way we determine the estimated risk prices for each factor. Therefore, for each period t=1…T, we estimate the following cross-sectional regressions using OLS for each set of the factor loadings from the two models:

$$\bar{R}_{it} = \gamma_i^{FF} + \lambda_{RM}\hat{\beta}_{i,RM} + \lambda_{SMB}\hat{\beta}_{i,SMB} + \lambda_{HML}\hat{\beta}_{i,HML} + \lambda_{RMW}\hat{\beta}_{i,RMW} +$$

$$\lambda_{CMA}\hat{\beta}_{i,CMA} + u_{it}^{FF} \tag{17}$$

$$\bar{R}_{it} = \gamma_i^{Q} + \lambda_{M}\hat{\beta}_{i,M} + \lambda_{ME}\hat{\beta}_{i,ME} + \lambda_{I/A}\hat{\beta}_{i,I/A} + \lambda_{ROE}\hat{\beta}_{i,ROE} + u_{it}^{Q} \tag{18}$$

where λ_n are the risk prices for each respective factor n; γ_i^{FF} and γ_i^{Q} are the intercepts, and u_{it}^{FF} and u_{it}^{Q} are the error terms for the two regressions.

Estimating the two latter regressions (17) and (18) returns T estimates of the risk prices for each factor $\widehat{\lambda_n}$. Then, we calculate time-series average risk prices for each factor n:

$$\overline{\widehat{\lambda_n}} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\lambda_{n,t}} \tag{19}$$

Similarly, we calculate time-series average intercepts for the two models $\overline{\widehat{\gamma^{FF}}}$ and $\overline{\widehat{\gamma^{Q}}}$.

Then, as in Fama and MacBeth (1973), we calculate the test-staticstics for the test that each time-series average estimated coefficient is different from zero in the following way:

$$t\left(\overline{\widehat{\lambda_n}}\right) = \frac{\overline{\widehat{\lambda_n}}}{\widehat{\sigma_{\lambda_n}}/\sqrt{T}} \tag{20}$$

where:

$$\widehat{\sigma_{\lambda_n}} = \sqrt{\frac{1}{(T-1)}\sum_{t=1}^{T}(\widehat{\lambda_{n,t}} - \overline{\widehat{\lambda_n}})^2} \tag{21}$$

The main problem with Fama-MacBeth approach is that the true factor loadings are not directly observable and we estimate them from the data in the first-pass regressions.

This may lead to errors-in-variables problem meaning that in the second-pass regressions the explanatory variables are measured with error (Kim 1995). Luckily for us, Fama and MacBeth (1973) suggest that one of the ways to address the problem is to group the stocks in portfolios, which increases the precision of the coefficients' estimates and helps to circumvent the problem.

Another possible problem with this method is that its standard errors tend to be biased downward due to the firm effect – the fact, that observations on the same firm in different years may be correlated (see, among others, Graham, Lemmon, and Schallheim, 1998; Fama and French, 2002; Cochrane, 2009). Hence, to avoid autocorrelation problem in our analysis, we follow the advice of Petersen (2009) and calculate adjusted standard errors for all our Fama-MacBeth regressions. In order to do so, we first estimate the correlation between the yearly estimated coefficients in the following way:

$$\text{Corr}[\beta_t \ \beta_{t-1}] = \theta \qquad\qquad (22)$$

Then, we adjust the estimated variance by $(1+\theta)/(1-\theta)$ to mitigate the serial correlation of the coefficients (see Fama and French, 2002; Chakravarty, Gulen, Mayhew, 2004).

In this section of the analysis, we focus on the comparison of the intercepts from the second-pass regressions. If they are indistinguishable from zero, it means that all the priced risk factors are included in the model. However, we do not expect to find the perfect model straight away, but rather want to compare which of the two is closer to perfection.

In addition, we attempt to improve each of the two models with factors build on the anomalies studied in our thesis. Thus, while testing each of the anomalous portfolios, we augment each model with one factor that is supposed to eliminate the anomaly. For instance, while running regressions on portfolios formed on accruals, we add to both Fama-French and q-factor models a new accruals factor; and repeat the procedure for each anomaly.

To do so, we build new factors – Accruals, Market Beta, Net Share Issues, Variance, and Residual Variance – with respect to each anomaly. Here, returns on each factor are simply the difference of the returns on high (10th) decimal portfolio and low (1st) decimal portfolio in the set of 10 decimal portfolios for each anomaly.

These new extended models will also help us to check if the anomalies we study are anomalies or just missing priced risk factors. Thus, for any particular regression, if the new factor makes the intercept indistinguishable from zero, then the original (Fama-French or q-factor) model is incomplete, and we have not an anomaly, but a missing factor.

5. Data Description

In order to effectively evaluate the performance of Fama-French 5-factor model and q-factor model, we examine monthly data from the U.S. market from January 1967 to December 2015 with total of 588 observations. SMB, HML, RMW and CMA factor values were extracted from Kenneth French Data Library, while data from q-factor model was kindly provided by Kewei Hou and Lu Zhang. Both factor data sets include the excess market return factor (as a difference between return on a market portfolio and risk-free rate), however they differ slightly.

Fama-French excess market return factor is a value weighted return that consists of incorporated American firms from Center for Research in Security Prices, which are listed on NYSE, AMEX or NASDAQ and have good shares, price, and return data for period t. Included firms are NYSE, AMEX or NASDAQ companies with available market equity data for December of t-1 and June of t, non-negative book equity data for t-1, non-missing revenues and minimum one of the following: COGS, selling, general and administrative expenses, or interest expense for t-1 and total assets for 2 previous periods.

All 5 factors (2x3) are constructed using the 6 value-weight portfolios formed on size and book-to-market, the 6 value-weight portfolios formed on size and operating profitability, and the 6 value-weight portfolios formed on size and investment (Kenneth R. French Data Library).

Q-factor model factors data is also extracted from the Center for Research in Security Prices for companies only with positive book equity, while accounting data is take from the Compustat. Financial companies are excluded.

Following the advice of the analyzed articles, we first estimate the average returns on each factor in both models and test if they are significantly different from zero.

As we can see from the Table 5.1, all the factors have quite high average monthly returns from 0.24% to 0.56%. For both models, excess market return factor is the most volatile factor with the highest average excess return. Almost all factors have high enough t-statistics value to reject the hypothesis that their mean is zero, however, for

SMB factor from FF 5-factor model, we can reject it only on 90% confidence level with t-statistic of 1.9051.

Table 5.1. Summary statistics for the models' factors, January 1967 – December 2015

Model	Explanatory Variables	Mean	Standard deviation	t-statistic
Fama-French	R_M- R_f	0.5091	4.5499	2.7134
	SMB	0.2428	3.0899	1.9051
	HML	0.3449	2.8771	2.9066
	RMW	0.2577	2.2922	2.7264
	CMA	0.3225	2.0308	3.8511
q-Factor	R_M- R_f	0.4941	4.5486	2.6340
	ME	0.3054	3.0998	2.3892
	IA	0.4114	1.8815	5.3017
	ROE	0.5613	2.5299	5.3794

From the Table 5.2, we can see that despite difference in ideas behind two models, their factors are indeed very similar. Since companies that are used to form market portfolios differ slightly, so do excess market return factors from two models, thus they are not perfectly correlated.

Table 5.2. Correlation matrix of the models' factors

	R_M- R_f (FF)	SMB	HML	RMW	CMA	R_M- R_f (Q)	ME	IA	ROE
R_M- R_f (FF)	1.0000								
SMB	0.2764	1.0000							
HML	-0.2734	-0.0919	1.0000						
RMW	-0.2443	-0.3689	0.0965	1.0000					
CMA	-0.3986	-0.0944	0.6990	-0.0181	1.0000				
R_M- R_f (Q)	0.9988	0.2701	-0.2760	-0.2375	-0.3996	1.0000			
ME	0.2666	0.9736	-0.0479	-0.3755	-0.0549	0.2612	1.0000		
IA	-0.3859	-0.1880	0.6760	0.0945	0.9107	-0.3844	-0.1473	1.0000	
ROE	-0.2076	-0.3672	-0.1368	0.6682	-0.0902	-0.1970	-0.3112	0.0360	1.0000

Among three other q-model factors we observe extremely high correlation for two of them with FF factors, namely excess return on size is 97.36% correlated with SMB excess return, while excess return on investment is 91.07% correlated with CMA. Fourth factor ROE can be named the most unique in the q-factor model compared to FF 5-factor model, though it is also highly correlated with RMW factor ($\rho = 0.6882$).

Within the q-model, degree of correlation is relatively small (not higher than 0.3844 in absolute terms), hence multicollinearity should not be the issue. For the FF 5-factor model, we observe quite high degree of correlation between CMA and HML factor (0.6990), which might potentially cause multicollinearity, but not severe one.

6. Empirical Results

In our study, we aim to investigate how well Fama-French 5-factor model and q-factor model explain excess returns on portfolios with large differences in accruals, market beta, share issuance, variance and residual variance.

Before proceeding to our findings, we want to take a glimpse on the average return patterns and on how much variation there is in our tested portfolios (see Appendix A). Here we can see that the presence and strength of each of our anomalies have strong effect on the returns of the companies of all sizes: thus, the companies tend to have higher (but also more variable) average returns when the values of accruals, market beta, share issuance, variance and residual variance are lower. This comes in accordance with the findings of the previous studies on this topic, which we have shown in the Literature Review.

There is also a clear size effect in every "anomalous" quantile: given the value of the anomaly measure, average returns are higher for the smallest companies and lower for the biggest. In addition, big companies tend to have more variable returns.

6.1. Intercepts Study

We start our empirical analysis by comparing the intercepts of the two models. As we can see from the Table 6.1, both models performed rather poorly for all the portfolios studied and it is quite hard to tell which one returns better results even in the relative sense.

Both models have quite high average values of the intercepts and average absolute values of the intercepts. Noteworthy, for four out of five portfolios on anomalies, as well as for the regressions on all portfolios together, average values of the intercepts are negative, suggesting that both models overestimate average excess returns based on the factor loadings.

Table 6.1. Summary statistics for the intercept tests of Fama-French 5-factor model and q-factor model, January 1967 – December 2015

	A(α)	A|α|	A(SE)	GRS	p(GRS)	Sh²(α)
			FF 5-factor model			
Accruals	0.0262	0.1267	0.0721	4.2967	0.0000	0.0828
Market Beta	-0.0766	0.0997	0.0759	2.3371	0.0105	0.0450
Net Share Issues	-0.0135	0.1242	0.0771	3.5334	0.0002	0.0681
Variance	-0.0653	0.1357	0.0754	6.2502	0.0000	0.1204
Residual Variance	-0.0993	0.1123	0.0805	3.2352	0.0005	0.0623
ALL	-0.0457	0.1197	0.0762	3.2956	0.0000	0.5841
			Q-factor model			
Accruals	0.0590	0.1356	0.0762	4.7124	0.0000	0.0976
Market Beta	-0.0497	0.0792	0.0818	1.9881	0.0324	0.0412
Net Share Issues	-0.0056	0.1279	0.0822	3.6500	0.0001	0.0756
Variance	-0.0218	0.1353	0.0808	5.3443	0.0000	0.1107
Residual Variance	-0.0720	0.1018	0.0874	2.6181	0.0040	0.0542
ALL	-0.0180	0.1159	0.0817	3.0045	0.0000	0.5782

This table shows how well Fama-French 5 factor model and q-factor model explain monthly excess returns on the 10 Size-Accruals decimal portfolios, the 10 Size-Market Beta decimal portfolios, the 10 Size-Net Share Issues decimal portfolios, the 10 Size-Variance decimal portfolios, the 10 Size-Residual Variance decimal portfolios, and the 100 total (ALL) portfolios. The table (by column) shows: (1) the test assets for each regression model; (2) the average value of the intercepts, A(α); (3) the average absolute value of the intercepts, A\|α\|; (4) the average standard errors of the intercepts, A(SE); (5) the GRS statistics, testing the hypothesis that the expected values of all 10 (100 in case of testing ALL portfolios regressions) intercept estimates are jointly zero; (6) the p-value of the GRS statistics, p(GRS); (7) maximum from the 10 (100) squared Sharpe ratios of the intercept, Sh²(α).

The only significant difference between the two models is that the average standard errors of the intercepts are persistently lower for all the portfolios for Fama-French model comparing to q-factor model, making the former one slightly better. The value of the maximum squared Sharpe ratio is relatively the same for the two models.

Both models have quite high and statistically significant (almost all at 1% level) values of GRS statistics, which shows that estimated intercepts from multiple linear regressions are not jointly zero, regardless of the test assets used.

From this preliminary analysis, we can conclude with no doubts that both analyzed models are incomplete and fail to account for the previously found anomalies.

6.2. Fama-MacBeth Approach

The results of the Fama-MacBeth 2-pass regression analysis are presented in Table 6.2. Here, intercepts are of the most importance for us. Second-pass regression intercept indistinguishable from zero will signal that all the priced risk factors are included in the model; hence, this model is the best explanation of expected returns.

As we can see from the Table 6.2, almost all the regressions returned small insignificant intercepts, with the exception of the Fama-French 5-factor model for Beta and ResVar portfolios, and the q-factor model for ResVar portfolios. What is even more important, when testing all portfolios together, Fama-French 5-factor model returns an intercept significant at 5% level, while q-factor model gives small intercept that is insignificant at all acceptable levels. Overall, we see that q-factor model on average performs better than Fama-French 5-factor model, producing intercepts that are closer to 0 and have lower t-statistics.

Proceeding to analyzing slopes, we can see that only market factor is statistically significant for both models for all test assets. Looking at the FF5 model, it is worth noting that the coefficients for the value factor loadings are significant for all the portfolios tested except NI, despite this factor being seen as redundant by many researchers. The estimated coefficients on the size and investment factor loadings appear to be statistically insignificant for all the portfolios, while the coefficient on the profitability beta is significant only for portfolios formed on Beta and only at 10% level. Here, we can make a conclusion that new factors of the model are either not priced risk factors at all or there are flows in their construction process and there are still adjustments to be made. The insignificance of the coefficient for the size factor here might be a sign of a selection bias, as while tested on all portfolios it turns out to be significant at 1% level. To sum up, from testing Fama-French 5-factor model on all our portfolios, we see that the three "original" factors are significant priced risk factors,

while the intercept signals that there are still factors missing; however, two new factors offered by Fama and French appeared to be not significant priced risk factors.

Table 6.2. Fama-MacBeth 2-pass regressions output for Fama-French 5-factor model and q-factor model, January 1967 – December 2015

	$\widehat{\overline{\lambda_{RM}}}$	$\widehat{\overline{\lambda_{SMB}}}$	$\widehat{\overline{\lambda_{HML}}}$	$\widehat{\overline{\lambda_{RMW}}}$	$\widehat{\overline{\lambda_{CMA}}}$	$\widehat{\overline{\gamma^{FF}}}$
	Panel A: FF 5-factor model					
Accruals	1.0013	0.0746	-0.0923	0.0397	0.0130	0.0262
	(44.2)***	(1.18)	(-6.08)***	(1.02)	(0.29)	(0.51)
Market Beta	1.0787	0.1162	0.0802	0.1426	0.0052	-0.0766
	(17.57)***	(1.44)	(3.81)***	(2.06)*	(0.07)	(-2.78)**
Net Share Issues	1.0270	0.0376	0.0198	-0.0306	-0.1073	-0.0135
	(82.35)***	(1.29)	(0.51)	(-0.38)	(-1.36)	(-0.24)
Variance	1.1148	0.1947	0.0648	-0.0002	-0.0827	-0.0653
	(24.65)***	(1.77)	(2.58)**	(0.00)	(-1.03)	(-1.06)
Residual Variance	1.1129	0.1645	0.1025	0.0560	-0.0663	-0.0993
	(17.42)***	(1.65)	(6.34)***	(0.43)	(-0.71)	(-2.09)*
ALL	1.0669	0.1175	0.0350	0.0415	-0.0476	-0.0457
	(51.41)***	(3.27)***	(2.42)**	(0.99)	(-1.42)	(-2.04)**

	$\widehat{\overline{\lambda_M}}$	$\widehat{\overline{\lambda_{ME}}}$	$\widehat{\overline{\lambda_{IA}}}$	$\widehat{\overline{\lambda_{ROE}}}$	$\widehat{\overline{\gamma^Q}}$
	Panel B: Q-factor model				
Accruals	1.0024	0.0443	-0.1020	0.0194	0.0590
	(40.76)***	(0.77)	(-1.97)*	(0.91)	(1.17)
Market Beta	1.0707	0.0854	0.0792	0.0515	-0.0497
	(16.62)***	(1.17)	(0.9)	(0.86)	(-1.63)
Net Share Issues	1.0350	0.0324	-0.0857	0.0044	-0.0056
	(63.47)***	(1.03)	(-1.2)	(0.1)	(-0.1)
Variance	1.1182	0.1607	-0.0198	-0.0471	-0.0218
	(22.79)***	(1.59)	(-0.17)	(-0.45)	(-0.4)
Residual Variance	1.1147	0.1324	0.0484	-0.0061	-0.0720
	(16.15)***	(1.4)	(0.42)	(-0.06)	(-1.96)*
ALL	1.0682	0.0910	-0.0160	0.0044	-0.0180
	(48.30)***	(2.73)***	(-0.39)	(0.14)	(-0.85)

The table displays the results of Fama-MacBeth 2-pass regression analysis of Fama-French 5-factor model and q-factor model on the portfolios formed on Accruals, Market Beta, Net Share Issues, Variance, Residual Variance, and on all portfolios together. First column of the table shows the test assets for each regression model. Panel A shows regressions output for Fama-French 5-factor model, with columns (2) to (6) being slope coefficients from the second-pass regression for each of the factors of this model, and column (7) being the intercepts from these regressions. Panel B shows regressions output for q-factor model, with columns (2) to (5) being slope coefficients from the second-pass regression for each of the factors of this model, and column (6) being the intercepts from these regressions. For both Panel A and B, value of t-statistics for each coefficient is provided in the parenthesis below it; here, * stands for significance at 10% level, ** - at 5% level, *** - at 1% level.

As for the q-factor model, the estimated coefficient on the market beta is the only statistically significant coefficient for Beta, NI, Var and ResVar VW portfolios analyzed, while for the AC portfolios there is only one other coefficient (on the investment factor loading) that is significant at least at 10% level. From the test of all portfolios together, we also see that the size factor is significant priced risk factor. Moreover, risk prices for the investment and profitability factors are very low for all the portfolios tested, reaching maximum 0.1%. Here, we can conclude that all new factors suggested by Hou, Xue, and Zhang (2015) are not proven to be priced risk factors in the scope of our analysis. However, the intercept for the model shows that the model is good, so we are getting some mixed signals here. Possible explanation might be that even though new investment and profitability factors of this model are not priced risk factors, the good-old size and value factors are built in a way that helps the model to better explain the variation in excess returns of the assets.

Summing up, after using Fama-MacBeth methodology and analyzing both intercepts and coefficients on factor loadings, we can conclude that both models are still incomplete; however, q-factor model performs relatively better than Fama-French 5-factor model.

6.3. Extending the models with the new factors

In this section of the analysis, we attempt to improve each of the two models with factors build on the anomalies studied in our thesis. This should also help us to check if they are anomalies or just missing priced risk factors.

Firstly, we provide the summary statistics for the new factors. As we can see from the Table 6.3, 4 out of 5 new factors – Accruals, Net Share Issues, Variance, and Residual Variance – have large factor premiums for 1967-2015, as big as -0.71% per month for the Variance factor, but their monthly standard deviation is also substantial: from 2.75% to 7.89%, resulting in large (absolute) values of t-statistics. Unlike these factors, Market Beta has very small average return, -0.01% per month, which, together with high standard deviation of 6.62% per month, results in very low t-statistics of -0.0382.

This means that Market Beta factor has mean that is indistinguishable from zero, so there might be no premium for the risk associated with higher market beta of an asset.

Table 6.3. Summary statistics of the new factors, January 1967 – December 2015

Factor	Mean	Standard deviation	t-statistic
Accruals	-0.3986	2.7506	-3.5137
Market Beta	-0.0104	6.6248	-0.0382
Net Share Issues	-0.4653	3.2604	-3.4608
Variance	-0.7122	7.2287	-2.3891
Residual Variance	-0.5795	7.8933	-1.7801

It is worth noting that all our factors have negative average returns, which is logical due to the nature of our factors and their construction process. We have built our factors as the difference of the returns on the portfolio with high value of the "anomaly" measure (e.g. high accruals, substantial net share issues, etc.) and with its low value. As the stocks with high accruals, share issues, volatility etc. tend to have lower average returns, it is no surprise that these factors have negative premiums.

Variance and Residual Variance factors strongly correlate with Market Beta factor as well as among themselves, but it is expected judging from the nature of these factors (see Appendix B). Similarly, Market Beta factor has relatively high correlation with the market excess return factors of the two studied models.

The results of the Fama-MacBeth analysis for the extended models are shown in the Table 6.4. As we can see from the table, new models produce slightly better results, as in most of the cases the intercepts have become smaller as well as their t-statistics. As for the most of the portfolios we have got quite good results with our original models, now we are mostly interested in the regressions on Residual Variance portfolios for the two augmented models and in the regressions on Market Beta portfolios for the extended Fama-French model. As for the Residual Variance, here we can see little, but significant improvement, as the intercept changes from being significant at 10% level to being statistically insignificant. It can be a signal that Residual Variance is a priced

risk factor. Looking at Market Beta regressions, we see that Fama-French model still can't account for the variation of returns in these portfolios, even when extended with an additional factor.

Table 6.4. Fama-MacBeth 2-pass regressions output for Fama-French 5-factor and q-factor models augmented with new factors, January 1967 – December 2015

	Original model's intercept	Augmented model's intercept	New factor's risk price
FF 5-factor model			
Accruals	0.0262	0.0321	0.0125
	(0.51)	(0.84)	(0.16)
Market Beta	-0.0766	-0.0580	0.0959
	(-2.78)**	(-2.26)**	(1.07)
Net Share Issues	-0.0135	-0.0094	0.0128
	(-0.24)	(-0.20)	(0.16)
Variance	-0.0653	0.0212	0.1490
	(-1.06)	(0.41)	(1.65)
Residual Variance	-0.0993	-0.0472	0.1183
	(-2.09)*	(-1.31)	(1.34)
Q-factor model			
Accruals	0.0590	0.0634	0.0112
	(1.17)	(1.55)	(0.14)
Market Beta	-0.0497	-0.0514	0.0740
	(-1.63)	(-1.73)	(0.83)
Net Share Issues	-0.0056	0.0026	0.0261
	(-0.1)	(0.05)	(0.33)
Variance	-0.0218	0.0046	0.1010
	(-0.4)	(0.08)	(1.11)
Residual Variance	-0.0720	-0.0568	0.0744
	(-1.96)*	(-1.51)	(0.84)

The table displays the results of Fama-MacBeth 2-pass regression analysis of Fama-French 5-factor and q-factor models augmented with the new anomalous factors on the portfolios formed on Accruals, Market Beta, Net Share Issues, Variance, and Residual Variance. The table (by column) shows: (1) both the test assets for each regression model and the new factor added (e.g. portfolio on Accruals is tested with an Accruals factor added to each model, and so on); (2) the intercepts from the second-pass regressions of original FF 5-factor or q-factor model on each portfolio; (3) the intercepts from the second-pass regressions of the augmented models on each portfolio; (4) new factors' slope coefficients from each second-pass regression. For all intercepts and slope coefficients the value of t-statistics is provided in the parenthesis below; here, * stands for significance at 10% level, ** - at 5% level, *** - at 1% level.

It is also important to note that none of the new factors appears to carry a substantial risk premium and be statistically significant. It may be a sign that our anomalies are in

fact priced risk factors not yet included in the asset pricing models, but not in the form in which we included them in our models.

To sum up, both Fama-French 5-factor model and q-factor model produce quite good results while tested on the portfolios formed on our five anomalies studied. In most cases, q-factor model shows better results than its opponent does.

As for the anomalies, we can see that all of them can be explained with the addition of the specific factor to the models. Hence, we can conclude that those are not anomalies, but missing priced risk factors, and adding the right proxy factors can both improve the models and eliminate the anomalies.

7. Conclusions

This thesis compares the performance of the Fama-French 5-factor and q-factor empirical asset pricing models and questions whether previously found anomalies persist while being tested on the above-mentioned models. We analyze value-weighted portfolios built on the five chosen anomalies (accruals, market beta, net share issues, variance and residual variance).

Despite the differences in the theoretical foundations, both models' performance turned out to be relatively similar. The models' factors are strongly correlated with each other across two comparable models, yet are constructed in a different fashion.

While testing the intercepts from the time-series multiple linear regressions, we find that they are not jointly zero, regardless of the test assets used. Hence, none of the model is complete. We also find that Fama-French and q-factor models tend to overestimate excess average returns for the analyzed data. It is worth noting that time-variation of the loadings might be a potential problem. Hence, future research should be connected with testing the models while accounting for this issue.

Fama-MacBeth analysis of the two models again proves that they are incomplete; however, in relative terms, q-factor model outperforms Fama-French 5-factor model, generating less statistically significant intercepts with values closer to 0. Two new factors suggested by these models – profitability and investment – are not proven to be significant priced risk factors in the scope of our analysis.

The results of our analysis also showed that none of the analyzed empirical models can fully account for all the previously found anomalies. However, it is quite likely that the analyzed anomalies can be included into the model as they capture certain priced risk. Thus, another suggestion for the further research might be to look for a new factor or factors to add to the models to improve them. Ideally, we do not want to add a separate factor for each anomaly, but want to look for one or two new proxy factors that will account for all the anomalies studied without adding much noise to the models. It might be done both by studying the theoretical background and by empirical testing of the models with new factors.

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9. Appendices

Appendix A: Descriptive statistics of the LHS portfolios

Table A1. Average excess return and standard deviation of returns of stocks in the 25 Size-Accruals portfolios, January 1967 – December 2015

	Mean				
	Low AC	**2**	**3**	**4**	**High AC**
Small	1.2224	1.1979	1.2172	1.1096	1.0219
2	1.3165	1.2305	1.1954	1.0679	0.9235
3	1.1951	1.2309	1.2060	1.0183	0.9267
4	1.3450	1.1341	1.1512	1.1519	0.8945
Big	0.9411	1.0149	0.9455	1.0867	0.6418
	Standard Deviation				
	Low AC	**2**	**3**	**4**	**High AC**
Small	7.2160	6.6369	6.2851	5.7445	5.2249
2	6.5629	5.8951	5.5138	5.2086	4.2856
3	6.3186	5.8987	5.2934	5.0286	4.1277
4	6.5744	6.2729	5.7446	5.2200	4.6131
Big	7.3176	6.8922	6.7085	6.3214	5.2474

Table A2. Average excess return and standard deviation of returns of stocks in the 25 Size-Market Beta portfolios, January 1967 – December 2015

	Mean				
	Low β	**2**	**3**	**4**	**High β**
Small	1.1351	1.1128	1.1006	1.0724	0.9175
2	1.2900	1.2640	1.2715	1.1929	0.9460
3	1.2846	1.3468	1.2294	1.1412	0.8948
4	1.3602	1.2514	1.1913	0.9501	0.8741
Big	1.1378	1.0513	1.0710	1.0830	0.7227
	Standard Deviation				
	Low β	**2**	**3**	**4**	**High β**
Small	4.4337	4.3375	3.9017	3.9460	3.6804
2	5.1579	4.8469	4.7155	4.6988	4.3310
3	6.0017	5.5263	5.2959	5.2547	4.9758
4	6.5878	6.2842	6.0671	5.9232	5.8118
Big	8.3372	8.0394	7.7985	7.6907	7.2159

Table A3. Average excess return and standard deviation of returns of stocks in the 25 Size-Net Share Issues portfolios, January 1967 – December 2015

	Low NI	2	3	4	High NI
			Mean		
Small	1.2748	1.2655	1.1903	1.0343	0.8896
2	1.3474	1.2323	1.2729	1.0849	0.8916
3	1.4028	1.2376	1.2062	1.2041	1.0327
4	1.0561	1.2074	1.1569	1.0069	0.8147
Big	0.6176	0.7043	0.6585	0.7056	0.6240
			Standard Deviation		
	Low NI	2	3	4	High NI
Small	6.3572	5.8835	5.2938	5.1591	4.3292
2	6.5234	6.1513	5.6961	5.4276	4.6162
3	6.9350	6.3862	5.9097	5.5394	4.9742
4	7.1735	6.6110	6.0760	5.8931	5.5087
Big	7.8530	7.2251	6.6522	6.2708	5.2380

Table A4. Average excess return and standard deviation of returns of stocks in the 25 Size-Variance portfolios, January 1967 – December 2015

	Low VAR	2	3	4	High VAR
			Mean		
Small	1.4248	1.3152	1.1557	1.0998	0.8565
2	1.5532	1.4382	1.2437	1.1482	0.9508
3	1.4621	1.4576	1.3562	1.1780	0.9239
4	1.1588	1.2920	1.2665	1.1493	0.8770
Big	0.1215	0.6356	0.7744	0.8240	0.8181
			Standard Deviation		
	Low VAR	2	3	4	High VAR
Small	4.1205	4.0833	3.7262	3.7663	3.4646
2	5.7591	5.3405	4.8634	4.5609	4.0674
3	6.6167	5.9960	5.4658	5.1955	4.5440
4	7.6486	6.9111	6.3130	5.8924	5.1966
Big	9.3692	8.9027	8.1628	7.8034	6.8275

Table A5. Average excess return and standard deviation of returns of stocks in the 25 Size-Residual Variance portfolios, January 1967 – December 2015

	Mean				
	Low RESVAR	**2**	**3**	**4**	**High RESVAR**
Small	1.4329	1.3269	1.1588	1.1456	0.8891
2	1.5564	1.4376	1.3264	1.1598	0.9442
3	1.4346	1.4604	1.2841	1.1537	0.9141
4	1.1767	1.3392	1.3146	1.1603	0.8847
Big	0.1146	0.5616	0.7037	0.7716	0.8182
	Standard Deviation				
	Low RESVAR	**2**	**3**	**4**	**High RESVAR**
Small	4.2523	4.1901	3.8737	3.9000	3.7205
2	5.7215	5.3930	4.9378	4.6030	4.2291
3	6.5947	6.0584	5.4962	5.2700	4.6336
4	7.6456	6.8994	6.2740	5.8058	5.1286
Big	9.1249	8.5659	7.8784	7.5311	6.5225

Appendix B: Correlation tables for the new factors

Table B1. Correlation matrix of the new "anomalous" factors

	Accruals	Market Beta	Net Share Issues	Variance	Residual Variance
Accruals	1				
Market Beta	0.1836	1			
Net Share Issues	0.1254	0.5894	1		
Variance	0.1300	0.8251	0.6186	1	
Residual Variance	0.1415	0.8524	0.6052	0.9542	1

Table B2. Correlation of the new factors with the factors of Fama-French 5-factor model

	R_M- R_f (FF)	*SMB*	*HML*	*RMW*	*CMA*
Accruals	0.1473	0.2040	-0.0930	0.0485	-0.1576
Market Beta	0.6841	0.5830	-0.3708	-0.4086	-0.4362
Net Share Issues	0.3933	0.4942	-0.2730	-0.4114	-0.3984
Variance	0.5591	0.6882	-0.3522	-0.6044	-0.3833
Residual Variance	0.6219	0.6109	-0.3806	-0.5605	-0.4374

Table B3. Correlation of the new factors with the factors of q-factor model

	R_M- R_f (Q)	*ME*	*I/A*	*ROE*
Accruals	0.1493	0.1944	-0.2071	-0.0154
Market Beta	0.6888	0.5457	-0.4620	-0.3920
Net Share Issues	0.3941	0.4773	-0.4121	-0.2982
Variance	0.5689	0.6462	-0.4481	-0.5465
Residual Variance	0.6298	0.5739	-0.4770	-0.4788