Modelling Occasionally Binding Constraints Using Regime-Switching

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Abstract

Occasionally binding constraints are part of the economic landscape: for instance recent experience with the global financial crisis has highlighted the gravity of the lower bound constraint on interest rates; mortgagors are subject to more stringent borrowing conditions when credit growth has been excessive or there is a downturn in the economy. In this paper we take four common examples of occasionally binding constraints in economics and demonstrate how to use regime-switching to incorporate them into DSGE models. In particular we investigate the zero lower bound constraint on interest rates, occasionally binding collateral constraints, downward nominal wage rigidities and irreversible investment. We compare our approach against some well-known methods for solving occasionally-binding constraints. We demonstrate the versatility of our regime-switching approach by combining multiple occasionally binding constraints to a model solved using higher-order perturbation methods, a feat that is difficult to achieve using alternative methodologies.

Keywords: Occasionally Binding Constraints; DSGE models; ZLB; Collateral Constraints

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1. Introduction

Occasionally binding constraints are part of the economic landscape: for instance recent experience with the global financial crisis has highlighted the gravity of the lower bound constraint on interest rates; emerging economies have been affected by sudden stops, or slowdowns in private capital inflows, that occur from time to time; mortgagors are subject to more stringent borrowing conditions when credit growth has been excessive or there is a downturn in the economy; workers have an aversion to taking pay cuts, a problem that is

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more acute during economic downturns. In this paper we demonstrate how to use regime-switching to incorporate them into DSGE models.

The problem of modelling occasionally binding constraints is not a new one. The RBC and DSGE literatures have addressed these problems using a range of different methodologies. These include: eternally binding constraints, smooth approximation of occasionally binding constraints, the extended path algorithm, piecewise linear methods, anticipated shocks, global solution methods, and regime-switching methods. We discuss each of these methods in turn, and how they are applied to occasionally binding constraints.

One of the simplest and most straightforward approaches to handling occasionally binding constraints is to assume that the constraint is binding at all times. Following Brzoza-Brzezina et al. (2015), we refer to this approach as eternally binding constraints (EBC). Eternally binding constraints are commonly used in the modelling of borrowing and collateral constraints as demonstrated by Iacoviello (2005) and Kiyotaki & Moore (1997). In order to pursue this strategy, the Blanchard Kahn conditions need to hold when the constraint is binding. While this is a reasonable strategy to pursue for some problems, like the modelling of borrowing constraints, it is not readily applicable to problems like the zero lower bound, which does not in isolation, satisfy the Blanchard Kahn conditions and is genuinely an occasionally binding constraint that would be poorly approximated as an eternally binding constraint.

Others have tried to solve occasionally binding constraints using smooth approximations. In particular Den Haan & De Wind (2012) propose using an exponential penalty function to prevent negative asset positions in a Deaton-type model. Brzoza-Brzezina et al. (2015) use the same type of penalty function to model occasionally binding collateral constraints. Kim & Ruge-Murcia (2011) use Linex adjustment costs to model downward nominal wage rigidities. While the use of smooth functions permits the use of derivatives, the effective application of these methods requires non-linear solution techniques in order to preserve the non-linearity and asymmetry introduced by the occasionally-binding constraint. When it comes to implementability, smooth approximations work well when the constraint is "soft", that is agents can violate the constraint but at some cost.

A range of non-linear solution techniques have been used to solve models with occasionally binding constraints. One of the oldest and most commonly used methods is the extended path algorithm due to Fair & Taylor (1983). The extended path algorithm is a certainty equivalent method that solves for the paths of all variables numerically, assuming perfect foresight at each point in time. It has been used by Coenen et al. (2007), Adjemian & Juillard (2010) and Braun & Köber (2011) to account for the zero lower bound on interest rates. A stochastic version by Adjemian & Juillard (2013) has been used to model irreversible investment in a DSGE model. While this method can be applied to a range of different occasionally binding constraints, certainty equivalence means agents' behavior does not change in the vicinity of the constraint binding. In fact the constraint only impacts agents' behavior when it binds.

Piecewise linear methods have been developed by Jung et al. (2005), Cagliarini & Kulish (2013) and Guerrieri & Iacoviello (2015b) to model occasionally binding constraints. Guerrieri & Iacoviello (2015b), Eggertsson & Woodford (2003) and Braun et al. (2015) have used

these methods to enforce the lower bound constraint on interest rates. Guerrieri & Iacoviello (2015a) and Akinci & Queralt (2014) have used piecewise linear methods to account for occasionally binding collateral constraints in DSGE models. Amano & Gnocchi (2017) model downward nominal wage rigidities as a non-negativity constraint on wage inflation and solve the model using piecewise linear methods. While this method is simple and applicable to a range of different occasionally binding constraints, it suffers from some of the same problems as the extended path algorithm, namely that agents' behavior does not change in the vicinity of the constraint binding.

Occasionally binding constraints can also be imposed through the addition of shocks. More specifically Holden & Paetz (2012) have proposed the use of news shocks to impose borrowing constraints and the lower bound on interest rates, while Lindé et al. (2016) have used anticipated shocks to enforce the lower bound on interest rates.[1] This method can suffer from sign reversals and the forward guidance puzzle.[2,3] Just like the extended path and piecewise linear methods, agents are unaware of the constraint until it is actually binding. Moreover the Kuhn-Tucker and complementary slackness conditions associated with many occasionally binding constraints are not easily incorporated into this methodology.[4]

Many practitioners have used global and projection methods to account for occasionally binding constraints. Fernández-Villaverde et al. (2015) and Judd et al. (2012), among others, have used global methods to impose the lower bound on interest rates. Christiano & Fisher

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[1]Anticipated shocks differ from the alternative approach commonly referred to as news shocks in many ways. Solving the model with anticipated shocks does not require a modification of the equations of the original system, in contrast to news shocks that are typically implemented by augmenting the law of motion of a shock process with additional shocks. An anticipated shock is genuinely a particular structural shock in the original system, while in the news shocks, it is a different iid shock with no other interpretation than a news shock and is unrelated to any structural shock in the system. Because it is unrelated, it will have its own distribution independently of other parts of the system. Under the anticipated shocks approach the policy functions are explicitly expressed in terms of leads of future shocks as opposed to lags in the news shocks approach (Maih, 2015).

[2]Binning & Maih (2016a) describe sign reversals as

> ... a phenomenon that occurs when anticipated monetary policy shocks are used to enforce the lower bound constraint. In the face of large negative demand shocks, the [effective lower bound (ELB)] is generally seen as a contractionary monetary policy since the interest rate is not able to decrease any further in order to give a boost to the economy. In that case, the anticipated shocks required to keep the interest rate from going below its lower bound are also expected to be positive. The positive monetary policy shocks then act as contractionary policy shocks. In some cases, however, if the ELB is expected to last for a very long time, some of the shocks in the sequence of shocks required to keep the interest rate at the ELB may be negative. Negative monetary policy shocks are expansionary, which leads to an improvement of economic conditions at the ELB.

[3]The forward guidance puzzle describes a situation where anticipated monetary policy shocks have a larger than reasonable impact on the current period due to the lack of discounting on the shocks themselves in the consumption Euler equation.

[4]Estimation using these methods requires specialized filters (see Juillard & Maih, 2010).

(2000) have used global methods to enforce the non-negativity constraint on investment. Mendoza & Smith (2004) use value function iteration to solve a model with an occasionally binding debt constraint. These methods are certainty non-equivalent which means agents' behavior will be affected in the neighborhood of the constraint binding even when the constraint is not binding. The computational cost of these methods can be heavy putting a low upper bound on the size of models that can be solved.[5]

Regime-switching can also be used to impose occasionally binding constraints. Bianchi & Melosi (2014), Chen (2014), Binning & Maih (2016b) and Binning & Maih (2016a) all use regime-switching to impose the lower bound constraint on interest rates. Benigno et al. (2015) use regime-switching to model an occasionally binding debt constraint in a small open economy model. Many occasionally binding constraints involve a policy aspect: the entry and exit of the zero lower bound are functions of the monetary policy regime, debt and collateral constraints can be affected by macroprudential policies like variable loan-to-value ratios. A desirable feature of any solution technique in such an environment is that it be robust to the Lucas critique: regime-switching methods exhibit such a feature.

In this paper we demonstrate how to impose occasionally binding constraints in DSGE models using regime-switching. A key feature of our approach relates to the use of endogenous transition probabilities from one regime to another, which helps capture the nature of the constraint and the probability of the constraint binding, and not binding. Interestingly our approach does not suffer from the problems outlined above. In particular, the framework we present can (1) be applied to large(r) models, (2) easily accommodate/handle complementary slackness problems, (3) be used to solve models at higher-orders of perturbations if more non-linearity is desirable, (4) accommodate multiple constraints simultaneously. These features make the strategy attractive also from an estimation standpoint: there are efficient non-linear filters for this type of problems (Binning & Maih, 2015).

We illustrate our approach by solving four common types of occasionally binding constraints: the lower bound constraint on interest rates, borrowing/collateral constraints, downward nominal wage rigidities and irreversible investment, and we walk through each example showing how to reformulate the constraints in a regime-switching framework. To further demonstrate the versatility of the methodology, we solve a model with multiple constraints using higher-order perturbation methods. We compare the solution results for all models against some common competing methods like piecewise linear methods and the extended path algorithm.

The remainder of the paper is laid out as follows. Section 2 outlines the different types of occasionally binding constraints and how they can be translated into a regime-switching problem. Section 3 describes the general setup for dynamic models with rational expectations and regime-switching, and its perturbation solution. In Section 4 we describe some common forms for endogenous transition probabilities and a general method for calibrating them. Our four examples of models with occasionally binding constraints are presented in Section 5. Section 6 presents simulation exercises for a model with multiple occasionally binding

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[5]Estimation requires non-linear filters which further restrict model size.

constraints. In Section 7 we demonstrate how occasionally binding constraints models with regime-switching can be solved to higher-orders of approximation. We compare alternative solution methods using the models from the applications section in Section 8. Section 9 concludes.

2. Types of Constraints & Conversion to a Regime-Switching Problem

In this section we investigate the general types of occasionally binding constraints and show how they can be translated into the regime-switching framework. We identify two main types of occasionally binding constraints: those that contain choice variables and those that do not contain choice variables.

When the occasionally binding constraint contains choice variables the constraint must be added to the agent's optimization problem. For example if there exists an inequality constraint of the form

$$A_t \leq B_t, \tag{2.1}$$

that only binds with equality intermittently, and A_t and/or B_t are choice variables, then (2.1) must be added as an additional constraint to the agent's optimization problem.[6] This results in the following Kuhn-Tucker and complementary slackness conditions

$$\lambda_t (A_t - B_t) = 0, \tag{2.2}$$

with

$$A_t < B_t, \quad \lambda_t = 0,$$

or

$$A_t = B_t, \quad \lambda_t > 0.$$

The constraint can be written as a simple minimum or maximum condition if it does not contain choice variables,.[7]

$$C_t = \max (D_t, F_t). \tag{2.3}$$

Some common examples of constraints that contain choice variables and can be added to agents's optimization problems include debt and collateral constraints. The zero lower bound is a common example of a constraint that does not normally involve choice variables and is therefore written as a maximum constraint.[8]

We can recast occasionally binding constraints of the form (2.2) and (2.3) into a regime-switching framework by introducing a Markov chain with two states, binding (B) and non-

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[6] In many cases A_t and/or B_t are functions of choice variables.

[7] The variables D_t and F_t can be functions of many variables.

[8] Typically interest rates are not considered choice variables, except during the calculation of optimal policy.

binding (N), and a regime-switching parameter $\phi(s_t)$ that takes the values

$$\phi\left(N\right)=0,\quad \phi\left(B\right)=1.$$

The complementary slackness condition (2.2) are then replaced by

$$\phi\left(s_t\right)\left(A_t - B_t\right) + \left(1 - \phi\left(s_t\right)\right)\lambda_t = 0. \tag{2.4}$$

While the min/max constraint in (2.3) is replaced with

$$C_t = \phi\left(s_t\right)D_t + \left(1 - \phi\left(s_t\right)\right)F_t. \tag{2.5}$$

This implies $C_t = D_t$ in the binding regime.

We assume the Markov chain evolves according to the transition matrix

$$\mathbb{Q}_{t,t+1} = \left[\begin{array}{cc} 1 - p_{NB,t} & p_{NB,t} \\ p_{BN,t} & 1 - p_{BN,t} \end{array}\right], \tag{2.6}$$

where $p_{NB,t}$ is the probability of transitioning from state N in period t to state B in period $t+1$ and $p_{BN,t}$ is the probability of transitioning from state B in period t to state N in period $t+1$. These transition probabilities are endogenous and their specification should ideally capture how the probability of the constraint binding depends on the state of the economy. More specifically the probability of the constraint binding when the constraint is not binding should be increasing as the distance to the constraint binding decreases. When the constraint is binding, a shadow variable should indicate when the constraint no longer binds. As the shadow variable approaches some threshold, the probability that the constraint no longer binds should increase.

As a general convention the RISE toolbox does not allow variables with switching steady states to enter into transition probabilities. To do so would require the solving a fixed point problem with unknown properties. Throughout this paper we assume the transition probabilities are logistic functions, but any other functional form assuming values in $[0, 1]$ would also work.

When the occasionally binding constraint contains choice variables, there is an associated Lagrange multiplier for the constraint and so we can write the transition probabilities as

$$p_{NB,t} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp\left(-\psi_{N,B}\left(A_t - B_t\right)\right)}, \quad p_{BN,t} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(\psi_{B,N}\lambda_t\right)}. \tag{2.7}$$

This specification ensures that when the constraint is not binding, i.e. $A_t < B_t$, the probability of switching to the binding regime increases as A_t approaches B_t from below. Likewise when the constraint is binding, i.e. $A_t = B_t$, the probability of exiting the binding regime increases as the Lagrange multiplier on the constraint (the shadow price) falls. When the occasionally binding constraint does not contain any choice variables, we can write the tran-

sition probabilities as

$$p_{NB,t} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp\left(\psi_{N,B}\left(F_t - D_t\right)\right)}, \quad p_{BN,t} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(-\psi_{B,N}\left(F_t - D_t\right)\right)}. \quad (2.8)$$

When the constraint is not binding, i.e. $C_t = F_t$ where $F_t > D_t$, the probability of the constraint binding increases as F_t approaches D_t from above. Likewise, when the constraint is binding, i.e. $C_t = D_t$ and $D_t > F_t$, the probability of exiting the binding constraint is increasing in F_t.

3. Generic Model & Perturbation Solution

In this section we outline the general form of dynamic regime-switching rational expectations models that we are concerned with and its perturbation solution.

We characterize history as made up of possibly different regimes, each with its distinctive properties. Following Maih (2015) the generic problem then takes the form

$$E_t \sum_{r_{t+1}=1}^{h} p_{r_t,r_{t+1}}\left(\mathcal{I}_t\right) f_{r_t}\left(x_{t+1}\left(r_{t+1}\right), x_t\left(r_t\right), x_{t-1}, \theta_{r_t}, \theta_{r_{t+1}}, \eta_t\right) = 0, \quad (3.1)$$

where r_t represents the switching process, with h different states, θ_{r_t} is the parameters in state r_t, $p_{r_t,r_{t+1}}\left(\mathcal{I}_t\right)$ is the transition probability for going from state r_t to state r_{t+1}, which depends on $\mathcal{I}_t$, the information at time t and $\eta_t \sim N\left(0, I\right)$ is a vector of shocks. An exact solution of the model, if it exists, takes the form

$$x_t\left(r_t\right) = \mathcal{T}^{r_t}\left(z_t\right), \quad (3.2)$$

where the state vector is defined as follows

$$z_t \equiv \left[\begin{array}{ccc} x'_{t-1} & \sigma & \eta'_t \end{array}\right]', \quad (3.3)$$

where σ is the perturbation parameter. In general an analytical/closed-form solution to this problem does not exist in which case we resort to finding an approximate solution. Following Maih (2015) and Foerster et al. (2014) we solve the model using perturbation methods. Perturbation methods are reasonably accurate, computationally more efficient than competing methods and less prone to the curse of dimensionality. The general p-th order perturbation solution takes the form

$$\mathcal{T}^{r_t}\left(z\right) \simeq \mathcal{T}^{r_t}\left(\bar{z}_{r_t}\right) + \mathcal{T}_z^{r_t}\left(z - \bar{z}_{r_t}\right) + \frac{1}{2!}\mathcal{T}_{zz}^{r_t}\left(z - \bar{z}_{r_t}\right)^{\otimes 2} + \ldots + \frac{1}{p!}\mathcal{T}_{z^{(p)}}^{r_t}\left(z - \bar{z}_{r_t}\right)^{\otimes p}. \quad (3.4)$$

We find this solution using the efficient solution method of Maih (2015) available as part of

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the RISE toolbox.[9]

4. Calibrating Endogenous Probabilities

Choosing and calibrating the endogenous transition probabilities is an important part of approximating occasionally binding constraints using regime-switching. We give a brief introduction and overview in this section.

Throughout this paper we use logistic functions to model the endogenous transition probabilities although any function bounded between $[0, 1]$ would also work. More specifically we concern ourselves with functions of the form

$$p\left(\alpha, \gamma, x\right) = \frac{\alpha}{\alpha + \exp\left(\pm\gamma\left(x - x_1\right)\right)}, \tag{4.1}$$

where x_1 is the x value of the sigmoid's midpoint, γ is the steepness of the logistic function and α is a scaling parameter that is related to x_1. We calibrate the parameters γ and α as follows:

$$\text{when } x \to x_1 \Rightarrow p\left(\alpha, \gamma, x\right) = \frac{\alpha}{\alpha + 1}. \tag{4.2}$$

If we want $p\left(\alpha, \gamma, x\right) \to 0$ as $x \to x_1$ then we need to choose α to be small. Likewise if we want $p\left(\alpha, \gamma, x\right) \to 1$ as $x \to x_1$ then we choose α to be large. Conversely

$$\text{when } x - x_1 \to N, \qquad p\left(\alpha, \gamma, x\right) = \frac{\alpha}{\alpha + \exp\left(\gamma \cdot N\right)}, \tag{4.3}$$

where N is a large number. If we want $p\left(\alpha, \gamma, x\right) \to 0$, then we need to assign a large value to γ. Likewise if we want $p\left(\alpha, \gamma, x\right) \to 1$, then we need to assign a negative and possibly large value to γ. This simple exercise guides our choices for α and γ, however we will still need to fine tune our choices to obtain the exact behavior we seek.

5. Applications

We demonstrate how to use regime-switching to model occasionally binding constraints in four common applications. In particular we examine the zero lower bound on interest rates, debt or constraints, downward nominal wage rigidities, and irreversible investment.

5.1. The Zero Lower Bound

The zero lower bound on interest rates is an occasionally binding constraint that has gained prominence in recent years. Its effects have been felt and continue to be felt in many developed economies. A number of approaches have been proposed in the literature to model this constraint in DSGE models. In particular Guerrieri & Iacoviello (2015b) have used piecewise linear methods, Coenen & Warne (2014) have used the extended path

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[9]In addition to implementing efficient algorithms, RISE is the only program available that can solve DSGE models with regime-switching using perturbations up to a fifth-order.

algorithm, Fernández-Villaverde et al. (2015) have used global approximations, and Lindé et al. (2016) have used anticipated shocks.

In this paper we demonstrate how to model the zero lower bound constraint using regime-switching. Regime-switching is a natural approach for modelling the lower bound for a range of reasons. It is natural to think of past and expected future implementations of policy in terms of regimes governed by separate policy parameters. Lucas (1976) has shown that ignoring (potential) changes in policy parameters can have severe consequences for both forecasting and policy analysis. Moreover there are many features of monetary policy and the transmission mechanism that can change when the economy is at the lower bound and can hence be captured in a regime-switching framework. For example there is evidence that central banks' policy objectives changed while at the ZLB (see Buiter, 2013). Monetary policy can also change at the lower bound with the implementation of unconventional monetary policies not normally used away from the lower bound. Changes in interest rate pass-through and agents behavior to risk-taking in the vicinity of the lower bound will also have consequences for the monetary policy transmission mechanism. Furthermore a regime-switching interpretation of our experience at the lower bound fits the data and is consistent with the stylized facts (see Binning & Maih, 2016a, for example).

Our model and our approach follow Binning & Maih (2016b) closely. We augment a simple New-Keynesian DSGE model with regime-switching to model the lower bound on interest rates. Apart from the modelling of the ZLB, the model is relatively standard. For this reason we focus on how the ZLB constraint is modelled using regime-switching in this section and leave a full derivation of the model to Appendix A.

The monetary authority sets policy according to

$$R_t = \max\left(R_{ZLB}, R_t^*\right), \tag{5.1}$$

where R_{ZLB} is the interest rate at the effective lower bound and R_t^* is set according to a Taylor-type rule of the form

$$R_t^* = R_{t-1}^{*\rho_r}\left(R^*\left(\frac{\pi_t}{\pi}\right)^{\kappa_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}\right)^{\kappa_y}\right)^{1-\rho_r}\exp(\varepsilon_{R,t}), \tag{5.2}$$

where R^* is the steady state Taylor-rule interest rate, π_t is the gross rate of inflation, $\tilde{Y}_t$ is detrended output and $\varepsilon_{R,t}$ is the monetary policy shock. Note that when the economy is at the lower bound, the Taylor-rule interest rate is the shadow interest rate.

We cast the zero lower bound on interest rates as an occasionally binding constraint and use regime-switching with a two state Markov chain to model the problem. When the constraint binds, the economy is in the ZLB state (Z) and when the constraint is not binding, the economy is in the normal state (N), so that

$$s_t = Z, N.$$

We introduce the regime-switching parameter $\mathbf{z}\left(s_t\right)$ which takes the values

$$\mathbf{z}\left(N\right) = 0, \quad \mathbf{z}\left(Z\right) = 1, \tag{5.3}$$

in each of the regimes. We replace equation (A.15) with

$$R_t = \mathbf{z}\left(s_t\right) R_{ZLB} + \left(1 - \mathbf{z}\left(s_t\right)\right) R_t^*. \tag{5.4}$$

The Markov chain is governed by the transition matrix

$$\mathbb{Q}_{t,t+1} = \left[\begin{array}{cc} 1 - p_{NZ,t} & p_{NZ,t} \\ p_{ZN,t} & 1 - p_{ZN,t} \end{array} \right], \tag{5.5}$$

where $p_{ZN,t}$ is the probability of transitioning from the ZLB state in period t to the normal state in period $t+1$ and $p_{NZ,t}$ is the probability of transitioning from the normal state in period t to the ZLB state in period $t+1$. We assume the following functions for the transition probabilities

$$p_{NZ,t} = \frac{\theta_{N,Z}}{\theta_{N,Z} + \exp\left(\psi_{N,Z}\left(R_t^* - R_{ZLB}\right)\right)}, \tag{5.6}$$

$$p_{ZN,t} = \frac{\theta_{Z,N}}{\theta_{Z,N} + \exp\left(-\psi_{Z,N}\left(R_t^* - R_{ZLB}\right)\right)}. \tag{5.7}$$

With these functional forms the probability of hitting the lower bound in normal times increases as the interest approaches the lower bound. When the economy is at the lower bound, the probability of exiting this regime increases with the shadow interest rate. We set $\theta_{N,Z} = \theta_{Z,N} = 1$ and $\psi_{N,Z} = \psi_{Z,N} = 2000$.

Finally we assume regime specific steady states, which implies separate steady states for interest rates in normal times and at the lower bound. More precisely we assume

$$R(Z) = R_{ZLB}, \quad R(N) = R^* = \frac{\pi_t \mu_t}{\beta}, \text{ where } R_{ZLB} < R^*. \tag{5.8}$$

where μ_t is the productivity growth rate. As highlighted by Binning & Maih (2016b) a regime specific steady state for interest rates at the lower bound requires a shift in either the discount factor, the steady state rate of inflation, the productivity growth rate, or some combination of these elements. Based on the estimation results of Binning & Maih (2016b) we assume that a shift in preferences d brings the economy to the lower bound. This implies that

$$d(Z) = \frac{R^*}{R_{ZLB}} > 1, \quad d(N) = 1. \tag{5.9}$$

Binning & Maih (2016b) interpret the shift in the reduced form parameter d as an increase in precautionary savings.

5.1.1. Simulated Transition Probabilities

To better understand how the endogenous transition probabilities behave and their relationship with the normal and ZLB regimes, we simulate artificial data from our simple NK DSGE. The calibrated parameters for the model are listed in Table A.1 and the simulation results are plotted in Figure 1.

Figure 1: Transition Probabilities
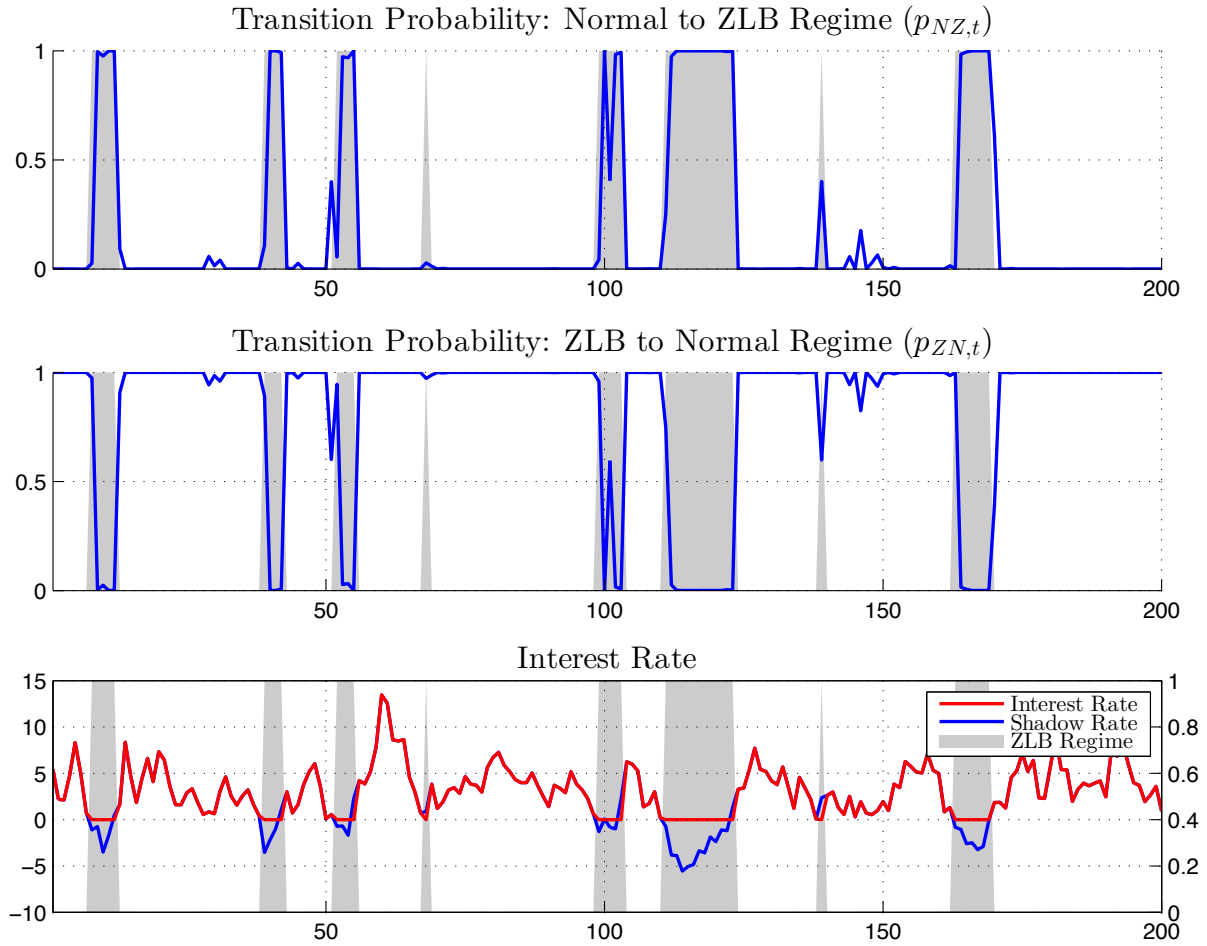
Note: The grey shaded areas indicate when the lower bound constraint on interest rates is binding. The top two panels plot the probability of switching to the ZLB regime from the normal regime, and the probability of switching to the normal regime from the ZLB regime. The bottom panel plots the shadow interest rate against the actual interest rate. Interest rates are reported in net annualized terms.

We make several observations about the simulation results. First, the interest rate never goes below zero, a key property of the constraint binding at the correct times. Second, we see that the transition probabilities track quite closely the actual regimes. Third, we see that the transition probabilities track closely the distance between shadow rate and the lower bound. There are periods where the shadow rate gets close to zero, but does not quite cross

the zero threshold and this is reflected in movements in the transition probabilities.

5.1.2. Asymmetric Impulse Response Functions

To illustrate the difference in dynamics of the economy when the lower bound constraint is binding and when the economy is away from the lower bound, we carry out both regime specific and generalized impulse responses. We conduct the analysis with the probabilities set at constant levels to make the exposition more clear and because the generalized impulse responses are better behaved under constant probabilities. We set $p_{NZ} = 0.04$ and $p_{ZN} = 0.2$, implying an expected duration at the lower bound of 5 quarters and expected duration in normal times of 25 quarters. When conducting the generalized impulse responses we assume the economy always remains in the normal regime unless interest rates fall below the lower bound, in which case we enforce the ZLB. The generalized impulse responses are conducted under a range of shock sizes, that is we keep the standard deviations of the shocks in the simulation constant but we alter the size of the shock used in the impulse responses, to illustrate the non-linearity and asymmetry the lower bound introduces into the model.

The regime specific impulses for consumption preference and technology shocks are plotted in Figures 2

Figure 2: Impulse Responses (Regime Specific): Consumption Preference Shock
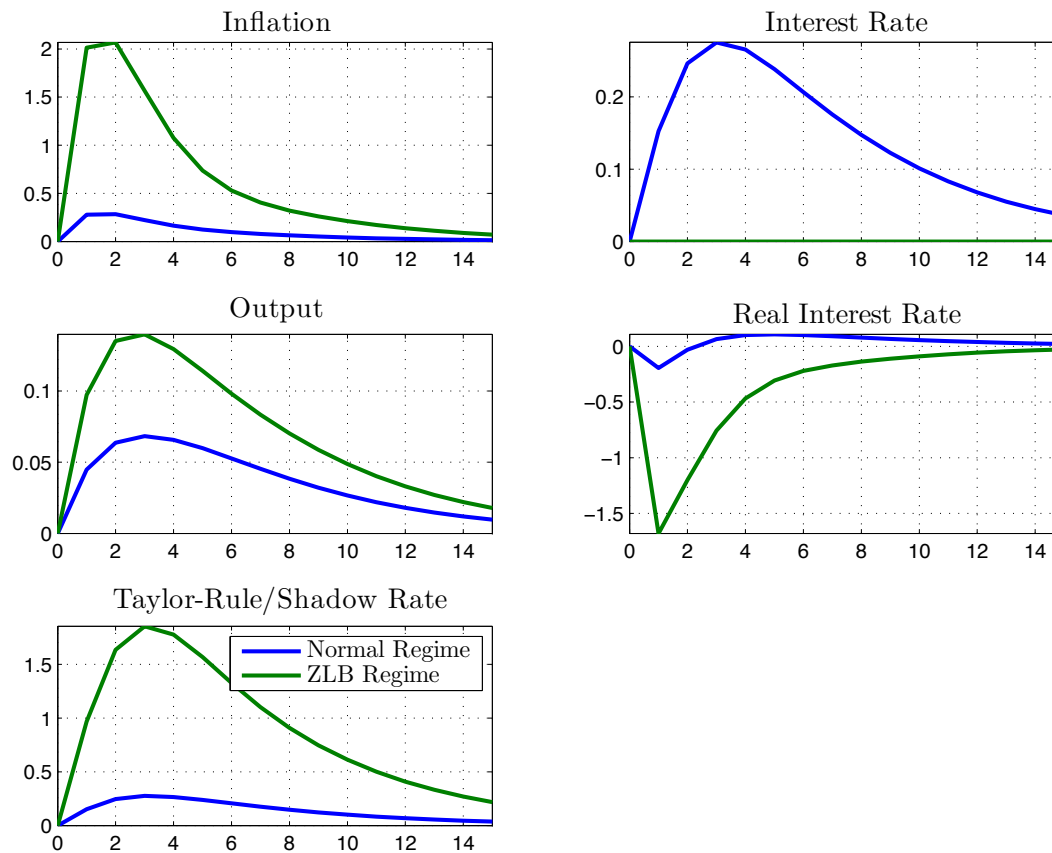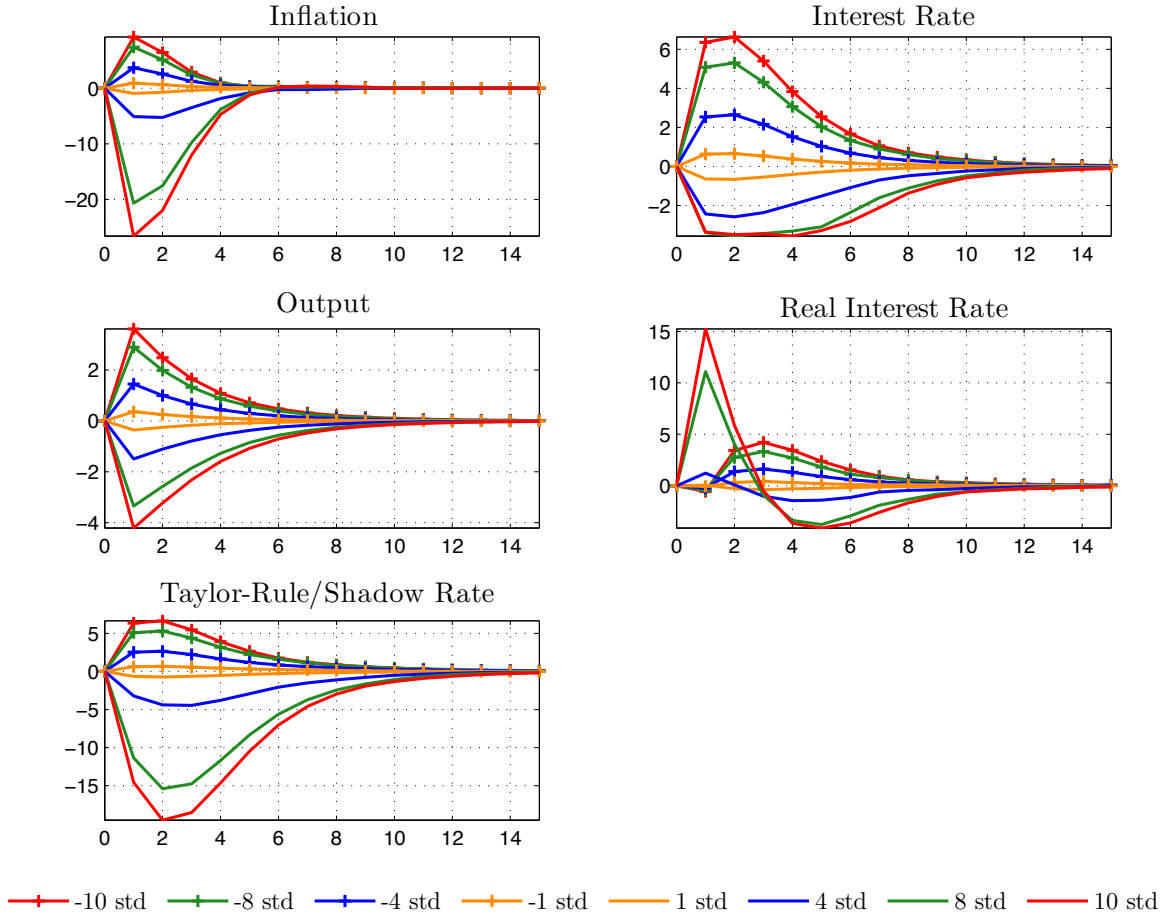

Note: All inflation and interest rate variables are reported in annualized terms. Output is expressed as a percentage deviation from steady state.

The responses are larger for inflation, output, the real interest and the shadow rate when the economy is at the lower bound. This is because the interest rate cannot adjust to dampen the impact of shocks. This implies that the economy is more volatile when at the lower bound.

The generalized impulse responses are constructed using a range of shock sizes, both positive and negative to illustrate the asymmetry and non-linearity the ZLB introduces into the model. More specifically we plot the generalized impulse responses for technology shocks that have standard deviations of -10, -8, -4, -1, 1, 4, 8 and 10 in Figure 3.

Figure 3: Impulse Responses (GIRF): Technology Shock

Inflation

Interest Rate

Output

Real Interest Rate

Taylor-Rule/Shadow Rate

-10 std -8 std -4 std -1 std 1 std 4 std 8 std 10 std

Note: All inflation and interest rate variables are reported in annualized terms. Output is expressed as a percentage deviation from steady state.

Note that detrended output falls after a technology shock because output is measured relative to technology and non-detrended output actually rises following the technology shock. Positive technology shocks have a larger impact on the economy than negative technology shocks do. Interest rates are able to fall by nearly 3% before the effects of the lower bound become particularly noticeable.

5.2. Borrowing Constraints

The global financial crisis and the years preceding it have highlighted important linkages between credit markets and the macroeconomy. In particular leverage plays a key role in amplifying the business cycle. As a consequence the financial accelerator is now a common feature of many DSGE models and is often modelled as a borrowing or collateral constraint à la Kiyotaki & Moore (1997) or Iacoviello (2005). Under this approach we typically assume there is an impatient agent that borrows from a more patient agent, and the borrowing

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constraint always binds, what we refer to as an eternally binding constraint. However this assumption is not always realistic: there have been many periods over history when the financial accelerator was less prominent and credit constraints were not binding (see Guerrieri & Iacoviello, 2015a, for example).

Occasionally binding borrowing constraints have also played an important role in the modelling of sudden stops in developing economies. The setup in these models mirrors the financial accelerator and collateral constraints literature in closed economies, namely the economy consists of an impatient developing country borrowing from a more patient developed country (see Mendoza, 2010). When the constraint is not binding, the developing economy increases its borrowing until the constraint binds, forcing a sudden stop and a rapid deleveraging. Moreover binding and occasionally binding borrowing constraints imply incomplete asset markets and provide a means for inducing stationary debt in both open and closed economy models.[10]

We build a simple DSGE model with housing, based on Iacoviello (2005), to demonstrate the modelling of debt and collateral constraints using regime-switching. The approach we take is applicable to a wide family of models or problems including Kiyotaki-Moore-Iacoviello type models with debt or collateral constraints and small open economy models with sudden stops like Mendoza (2010). Our model follows Iacoviello (2005) closely so we focus attention on the main difference in this section; i.e. the modelling of the occasionally binding constraint using regime-switching, and leave a full derivation of the model to Appendix B.

Following Iacoviello (2005), entrepreneurs are relatively more impatient than patient households. They borrow from patient households and are subject to a borrowing constraint which sets an upper limit on how much they can borrow

$$B_t \leq E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\}, \tag{5.10}$$

where B_t is debt, Q_t is the real house price, H_t is the housing stock owned by entrepreneurs, π_t is inflation and R_t is the policy interest rate. From the entrepreneur's optimization problem, we end up with the following Kuhn-Tucker and complementary slackness conditions

$$\Omega_t \left(B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} \right) = 0, \tag{5.11}$$

with

$$B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} = 0, \quad \Omega_t > 0, \tag{5.12}$$

or

$$B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} < 0, \quad \Omega_t = 0. \tag{5.13}$$

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[10] Schmitt-Grohe & Uribe (2003) cover a broad range of methods for inducing stationarity in small open economy DSGE models. However they do not consider eternally and occasionally binding debt constraints that can also be used to stationarize debt in both open and closed economy models.

In order to model the occasionally binding collateral constraint (5.10) using regime switching we need to introduce a Markov chain with two discrete states of nature: a binding state (B) and non-binding state (N), and a regime-switching parameter $\mathbf{o}(s_t)$ that takes the values

$$\mathbf{o}(N) = 0, \quad \mathbf{o}(B) = 1$$

Under the regime-switching representation, we can then replace (5.11) with

$$\mathbf{o}(s_t)\left(B_t - E_t\left\{m\frac{Q_{t+1}H_t\pi_{t+1}}{R_t}\right\}\right) + (1 - \mathbf{o}(s_t))\,\Omega_t = 0, \tag{5.14}$$

The Markov chain is governed by the transition matrix

$$\mathbb{Q}_{t,t+1} = \left[\begin{array}{cc} 1 - p_{NB,t} & p_{NB,t} \\ p_{BN,t} & 1 - p_{BN,t} \end{array}\right], \tag{5.15}$$

with the endogenous transition probabilities

$$p_{NB,t} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp\left(-\psi_{N,B}B_t^*\right)}, \quad p_{BN,t} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(\psi_{B,N}\hat{\Omega}_t\right)}, \tag{5.16}$$

where B_t^* is a measure of leverage defined as

$$B_t^* = B_t - E_t\left\{m\frac{Q_{t+1}H_t\pi_{t+1}}{R_t}\right\}, \tag{5.17}$$

and

$$\hat{\Omega}_t = \tilde{\Omega}_t - \tilde{\Omega}, \quad \tilde{\Omega}_t = \Omega_t, \tag{5.18}$$

is a gap measure created using the shadow of the shadow price on the collateral constraint. Our specification of the transition probabilities ensures that when the borrowing constraint is not binding, the probability of hitting the constraint increases as the economy nears the constraint. Likewise, when the collateral constraint is binding, the probability of the constraint no longer binding is inversely related to our measure of the shadow price on the collateral constraint.

The model can be solved around either the ergodic mean or the regime-specific steady state. We opt for the latter due to the complications of solving the model around the ergodic mean.[11] However solving the model around a regime-specific steady state introduces its own problems, namely that there can be large differences between the regime-specific steady states and there can be shifts in the steady state of variables that enter the transition probabilities. We get around this problem by introducing a tax on housing investment that is positive when

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[11] When we solve the model around the ergodic mean we lose control of the mean of the Lagrange multiplier which enters into the transition probability.

the constraint is binding and equal to 0 when the constraint is not binding. This tax offsets the over-borrowing and investment in housing by entrepreneurs in the presence of a borrowing constraint. In the absence of a borrowing constraint, entrepreneurs demand for loans would be limitless, just as patient household's supply of loans would be limitless. The entrepreneur's impatience ensures that the return from lending to them is always higher than what patient households would be able to achieve lending amongst themselves. This also causes a problem when defining the steady state behavior of entrepreneurs when the constraint is not binding. To get around this we replace the interest rate entrepreneurs face with an effective interest rate, where the steady state can change between the binding and non-binding regimes. This effectively sets entrepreneurs time preference to the same level as patient households when the constraint does not bind. When the constraint binds, entrepreneurs return to an effective time preference factor that is lower than patient households.

In the model with a binding collateral constraint, the regime-specific steady state ratio of housing to output for entrepreneurs is given by

$$\frac{H_t}{Y_t} = \left(\frac{\varepsilon - 1}{\varepsilon}\right)\left(\frac{\gamma\nu}{Q_t\left(1 - \gamma - (\beta - \gamma)m\right)}\right), \tag{5.19}$$

where H_t is entrepreneur's housing stock, Y_t is output, ε is the elasticity of substitution between differentiated intermediate goods, γ is the time preference for the entrepreneur, ν is housing's share of income, β is the patient household's time preference with $\beta > \gamma$, m is the loan to value ratio and Q_t is the real house price. The regime-specific steady state ratio of housing to output when the constraint does not bind

$$\frac{H_t}{Y_t} = \left(\frac{\varepsilon - 1}{\varepsilon}\right)\left(\frac{\gamma\nu}{(1 - \gamma)Q_t}\right), \tag{5.20}$$

we introduce a tax when the constraint binds to reduce the over-borrowing that occurs when the constraint binds and equate the housing steady states in both regimes. The entrepreneur's user cost of capital then becomes

$$(1 + \tau(s_t))\frac{Q_t}{C_t} = E_t\left\{\frac{\gamma}{C_{t+1}}\left(\nu MC_{t+1}\frac{Y_{t+1}}{H_t} + (1 + \tau(s_{t+1}))Q_{t+1}\right) + \Omega_t m Q_{t+1}\pi_{t+1}\right\}. \tag{5.21}$$

The tax on housing investment takes the value

$$\tau(s_t) = \mathbf{o}(s_t)m\left(\frac{\beta - \gamma}{1 - \gamma}\right), \tag{5.22}$$

We also replace the interest rate the entrepreneur faces with an effective interest rate R_t^* so that

$$\frac{1}{C_t} = E_t\left\{\gamma\frac{R_t^*}{\pi_{t+1}C_{t+1}}\right\} + \Omega_t R_t, \tag{5.23}$$

where

$$R_t^* = R_t\psi_1, \tag{5.24}$$

and

$$\psi_1 = \mathbf{o}\left(s_t\right) + \left(1 - \mathbf{o}\left(s_t\right)\right)\beta/\gamma, \tag{5.25}$$

This implies entrepreneurs have the same time preference as patient households when the constraint is not binding. When the constraint binds, entrepreneurs have a lower time preference than patient households.

5.2.1. Simulations From a Model With an Occasionally Binding Collateral Constraint

To better understand the role and the properties of an occasionally binding collateral constraint, we simulate the model under a range of different assumptions and compare the results. The parameter values for the baseline model calibration are based on Iacoviello (2005) and listed in Table B.2.

In our first exercise we compare a model with an eternally binding collateral constraint to a model with non-stationary debt. This exercise demonstrates the role debt/collateral constraints play in stationarizing debt and their consequences for the dynamics of other variables in the system. We impose eternally binding constraints on entrepreneurs by setting $\mathbf{o}\left(N\right) = \mathbf{o}\left(B\right) = 1$. Non-stationary debt is achieved in the second model by setting $\mathbf{o}\left(N\right) = \mathbf{o}\left(B\right) = 1$ and $\psi_2 = 0$. Both models are simulated for 1000 periods using the same sequence of shocks. The simulations are plotted in Figure 4.

Figure 4: Eternally Binding Constraint vs Non-stationary Debt
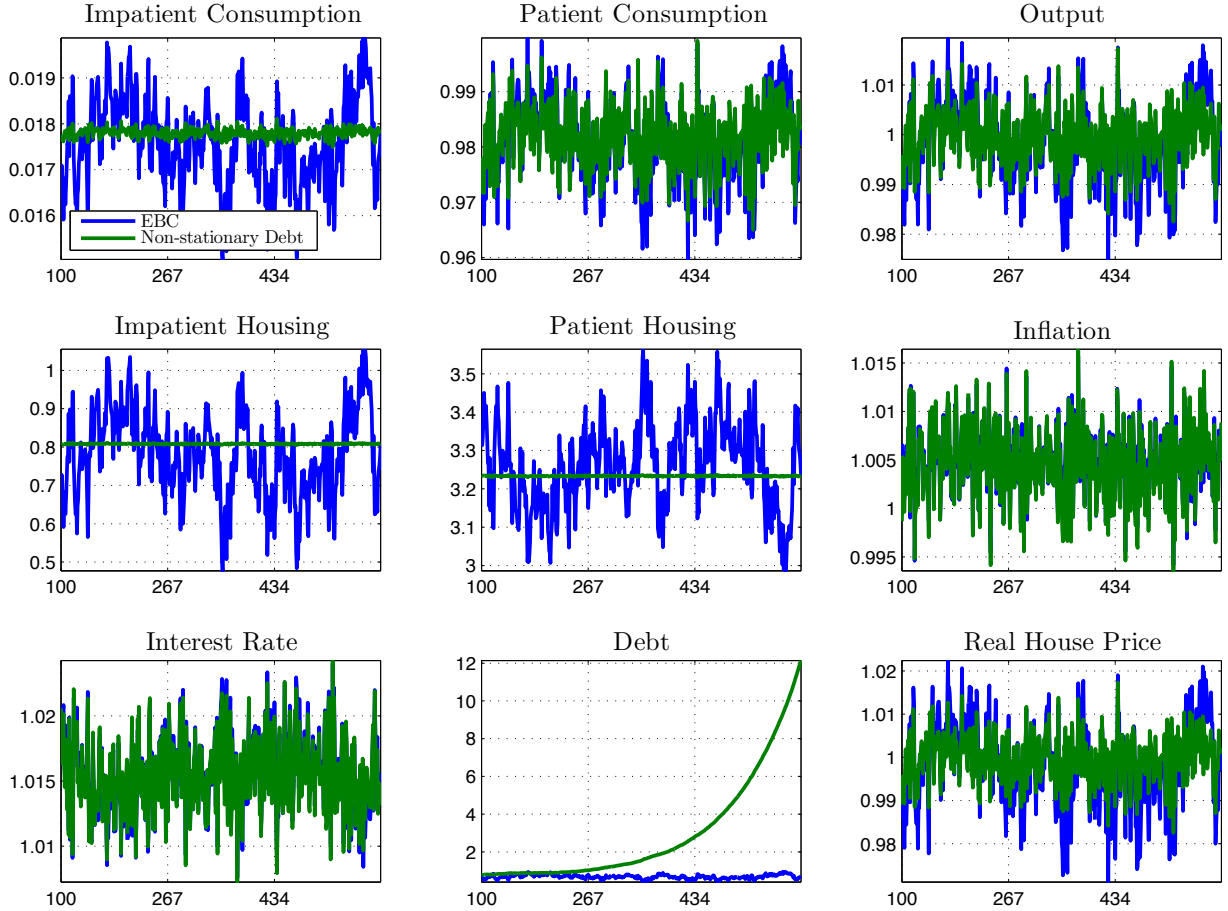
Note: Interest rates and inflation are reported in gross quarterly terms. All other variables are reported in levels.

We make several observations about the simulation results. First, in the model with non-stationary debt there is no mechanism to anchor or induce stationarity in debt. Furthermore there are no consequences for entrepreneurs perpetually increasing their borrowing. Accordingly, debt accumulates rapidly and explodes after a number of periods. Schmitt-Grohe & Uribe (2003) have outlined a number of strategies for stationarizing debt in small open economy models. Eternally binding borrowing or collateral constraints provide another method for stationarizing debt, as we observe from the simulations.

Second, in the model with non-stationary debt the variables are generally smoother and less volatile than in the model with eternally binding collateral constraints. While the collateral constraint provides an anchor for debt, it also imposes a straight jacket on housing and debt. This straight jacket makes it more difficult for entrepreneurs to use debt to smooth through shocks and results in increased volatility as we observe. Gelain et al. (2013) find that debt to income constraints result in higher welfare (lower loss) compared with collateral

constraints of the form used in our model, which is a direct result of the increased volatility a loan to value constraint introduces.

In our second exercise we use the model with an occasionally binding collateral constraint to simulate synthetic data. We then plot the endogenous transitions probabilities and their determinants against the actual regimes in Figure 5.

Figure 5: Transition Probabilities & Their Determinants
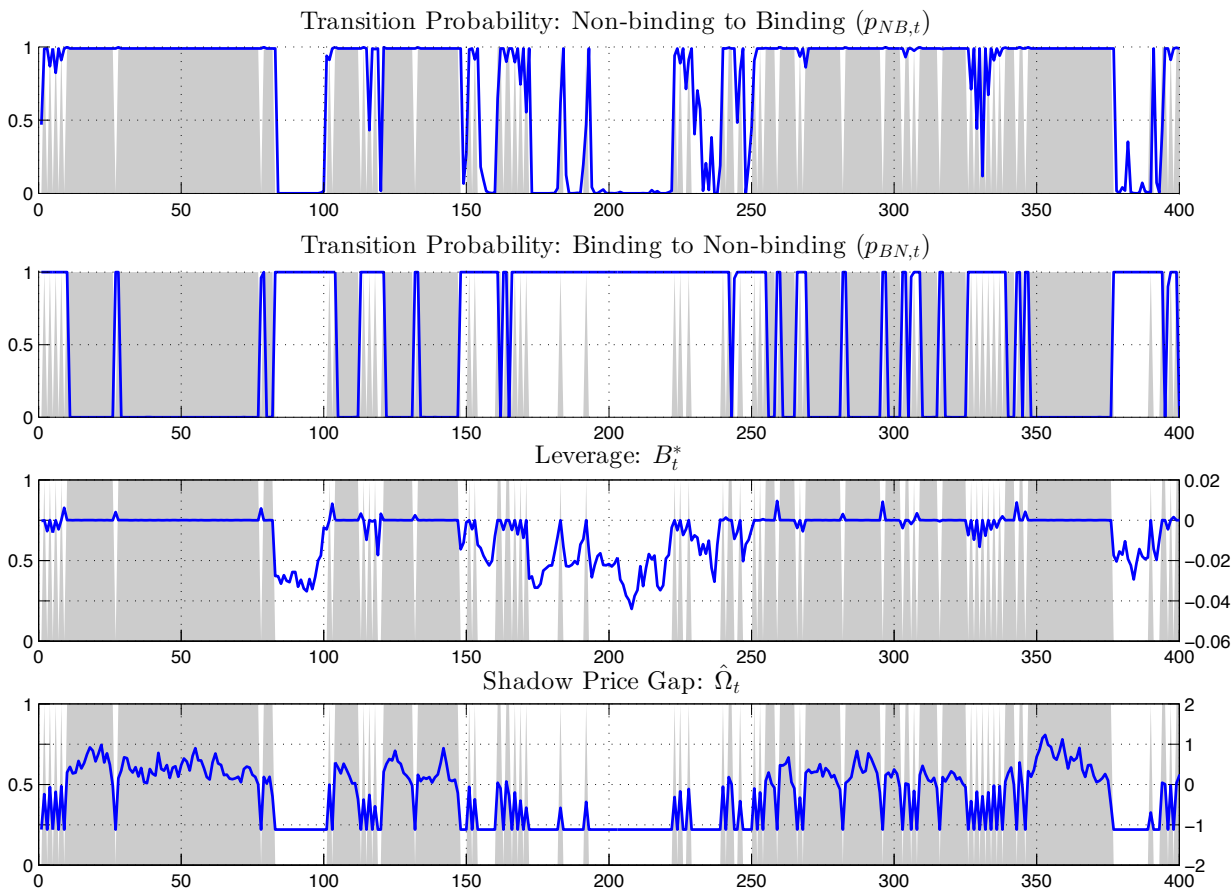
Note: The top two panels plot the transition probabilities for going from the non-binding to the binding regime, and the binding to the non-binding regime. The bottom two panels plot the determinants of the endogenous transition probabilities: leverage and the shadow price gap on the borrowing constraint. The grey shaded areas represent when the constraint is binding.

The model with occasionally binding constraints spends more time with the constraint binding than with it not binding. This should not be a surprise given that we solve the model around a steady state where the constraint is binding.

We also plot some key model variables against the binding regimes in Figure 6.

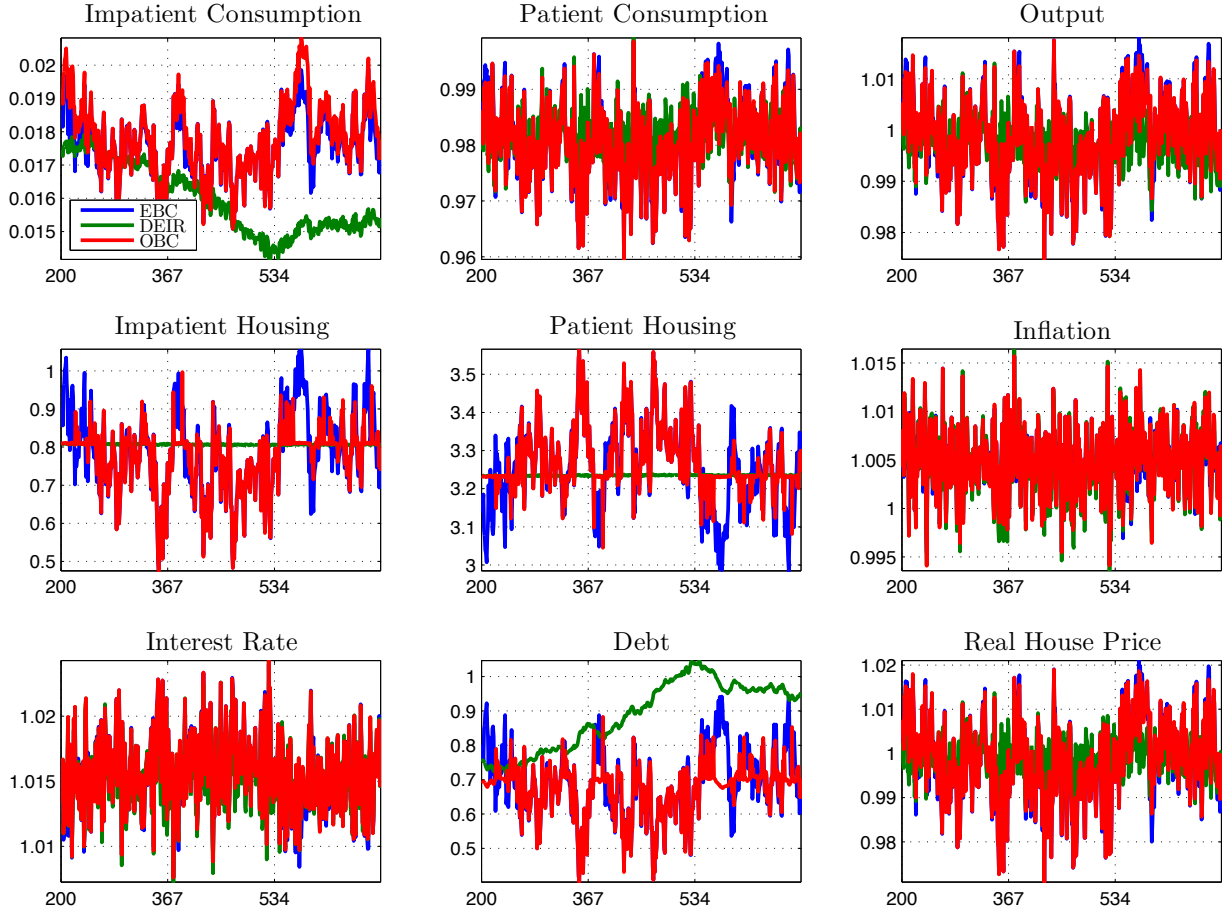Figure 6: Key Model Variables Against Regimes
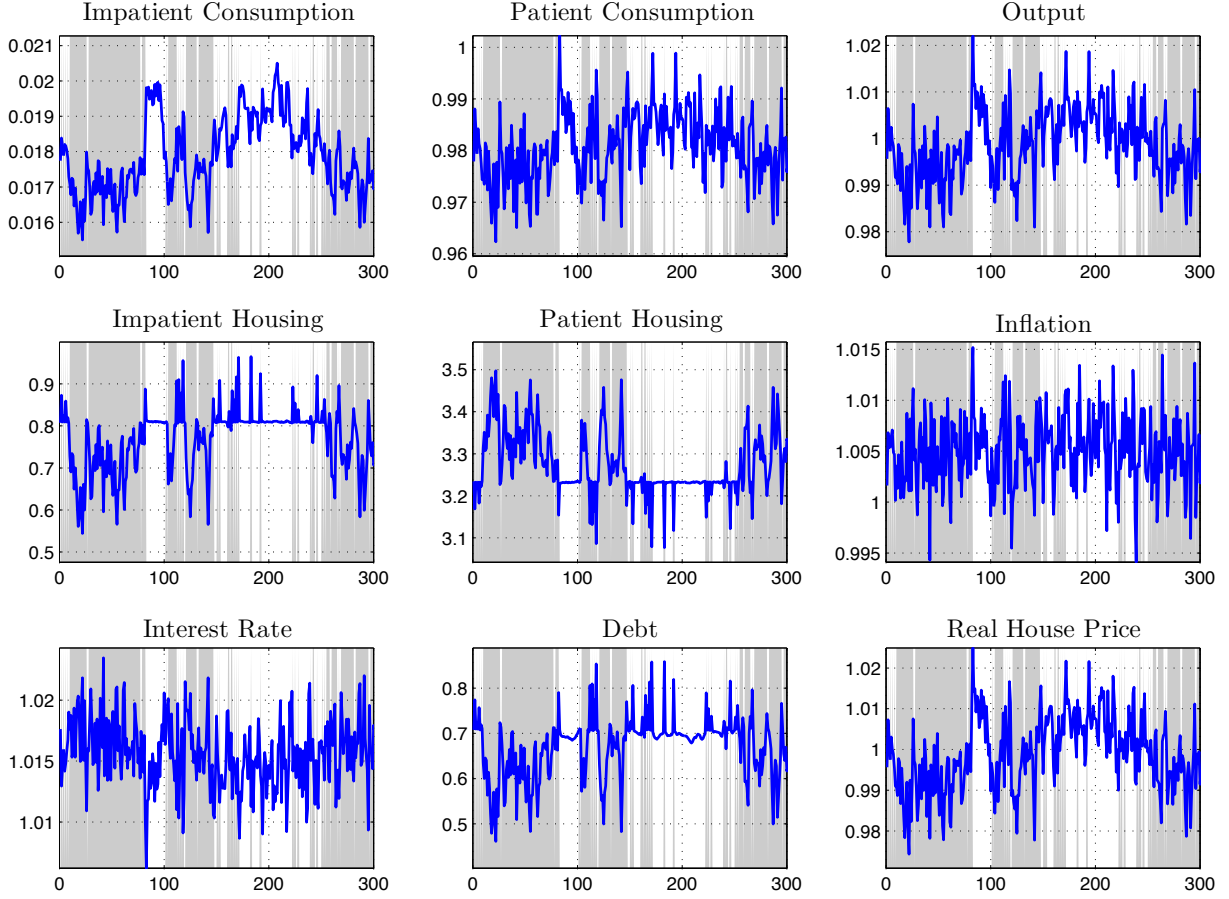
Note: Interest rates and inflation are reported in gross quarterly terms. All other variables are reported in levels. The grey areas represent when the constraint is binding.

We observe that housing and debt are more volatile when the constraint is binding and much less volatile when it is not binding. This result squares up with our earlier observation in the first exercise comparing the model with the eternally binding constraint with the model with non-stationary debt, where the variables were more volatile under the eternally binding constraint.

In our final exercise we compare eternally binding constraints, as outlined in our first exercise, with occasionally binding constraints and a model with debt elastic interest rates. We introduce a debt elastic interest rate by setting $\mathbf{o}(N) = \mathbf{o}(B) = 0$ and $\psi_2 = 0.000001$, our calibration implies near random walk behavior for entrepreneur's debt. We plot the simulation results using the same sequence of shocks for all models in Figure 7.

Figure 7: A Comparison: EBC, DEIR & OBC

Note: Interest rates and inflation are reported in gross quarterly terms. All other variables are reported in levels.

From this exercise we learn that the model with occasionally binding constraints behaves very similarly to the model with eternally binding constraints when the collateral constraint is binding. When the constraint is not binding the eternally binding constraints model behaves more like the model with a debt elastic interest rate.

5.2.2. An Optimal Simple Implementable LTV Rule

We extend the model to investigate optimal simple implementable LTV rules. We assume the macro-prudential regulator can adjust the LTV ratio to smooth fluctuations over the business cycle. The constant LTV ratio is replaced with a time-varying LTV rule that responds to output, real house prices and debt

$$m_t = m \left(\frac{Y_t}{Y} \right)^{\phi_Y} \left(\frac{Q_t}{Q} \right)^{\phi_Q} \left(\frac{B_t}{B} \right)^{\phi_B}. \tag{5.26}$$

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With a time-varying LTV rule equations (5.10) and (B.25) become

$$B_t \leq m_t \frac{Q_{t+1} H_t \pi_{t+1}}{R_t},$$

(5.27)

and

$$(1 + \tau(s_t)) \frac{Q_t}{C_t} = E_t \left\{ \frac{\gamma}{C_{t+1}} \left(\nu MC_{t+1} \frac{Y_{t+1}}{H_t} + (1 + \tau(s_{t+1})) Q_{t+1} \right) + \ldots \\ \ldots + \Omega_t m_t Q_{t+1} \pi_{t+1} \right\}.$$

(5.28)

The parameters ϕ_Y, ϕ_Q and ϕ_B are chosen to minimize the macro-prudential regulator's ad-hoc loss function. We only consider implementable LTV rules, which we define as rules where the LTV ratio never exceeds 1 or falls below 0 and where the policy parameters are between -3 and 3. The macro-prudential regulator's loss function is equal to the present value of a weighted sum of the variance of inflation, output and debt. It is augmented with an indicator function that penalizes LTV rules that are not implementable. The macro-prudential regulator's loss function is calculated using a linear-quadratic approximation and simulated data which allows us to assess the implementability of candidate LTV rules. The macro-prudential regulator's loss function takes the form

$$\tilde{\mathbb{L}} = \sum_{t=0}^{T} [\mathbb{L}_t + \mathbf{I}(m_t) \mathbb{P}],$$

(5.29)

where $\mathbb{L}_t$ is similar to the loss function used in Gelain et al. (2013) and takes the form

$$\mathbb{L}_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \omega_Y \hat{Y}_t^2 + \omega_B \hat{B}_t^2 \right] \right\},$$

(5.30)

with $\hat{\pi}_t = \pi_t - \pi$, $\hat{Y}_t = Y_t - Y$, $\hat{B}_t - B$, T is the length of the simulation, $\mathbb{P}$ is a scalar penalty and $\mathbf{I}(m_t)$ is an indicator function with

$$\mathbf{I}(m_t) = \begin{cases} 1 & \text{if } m_t < 0, \\ 1 & \text{if } m_t > 1, \\ 0 & \text{if } 0 \leq m_t \leq 1. \end{cases}$$

(5.31)

We set T to 10,000, $\mathbb{P}$ to 1000 and following Gelain et al. (2013) we set ω_Y to 1 and ω_B to 0.25. We calculate the loss function using synthetic data generated from the model, holding the random number generator seed constant between simulations. The parameters that minimize 5.29 are $\phi_Y = 1.4279$, $\phi_Q = -2.9293$ and $\phi_B = -1.2839$. This implies that the LTV should be pro-cyclical with respect to output and counter-cyclical with respect to real house prices and debt. We plot artificial data generated by the model with a constant LTV ratio and the model with an optimized simple implementable LTV rule in Figure 8 to

compare the models' dynamic properties.

Figure 8: A Constant LTV Ratio vs. A Simple Implementable LTV Rule
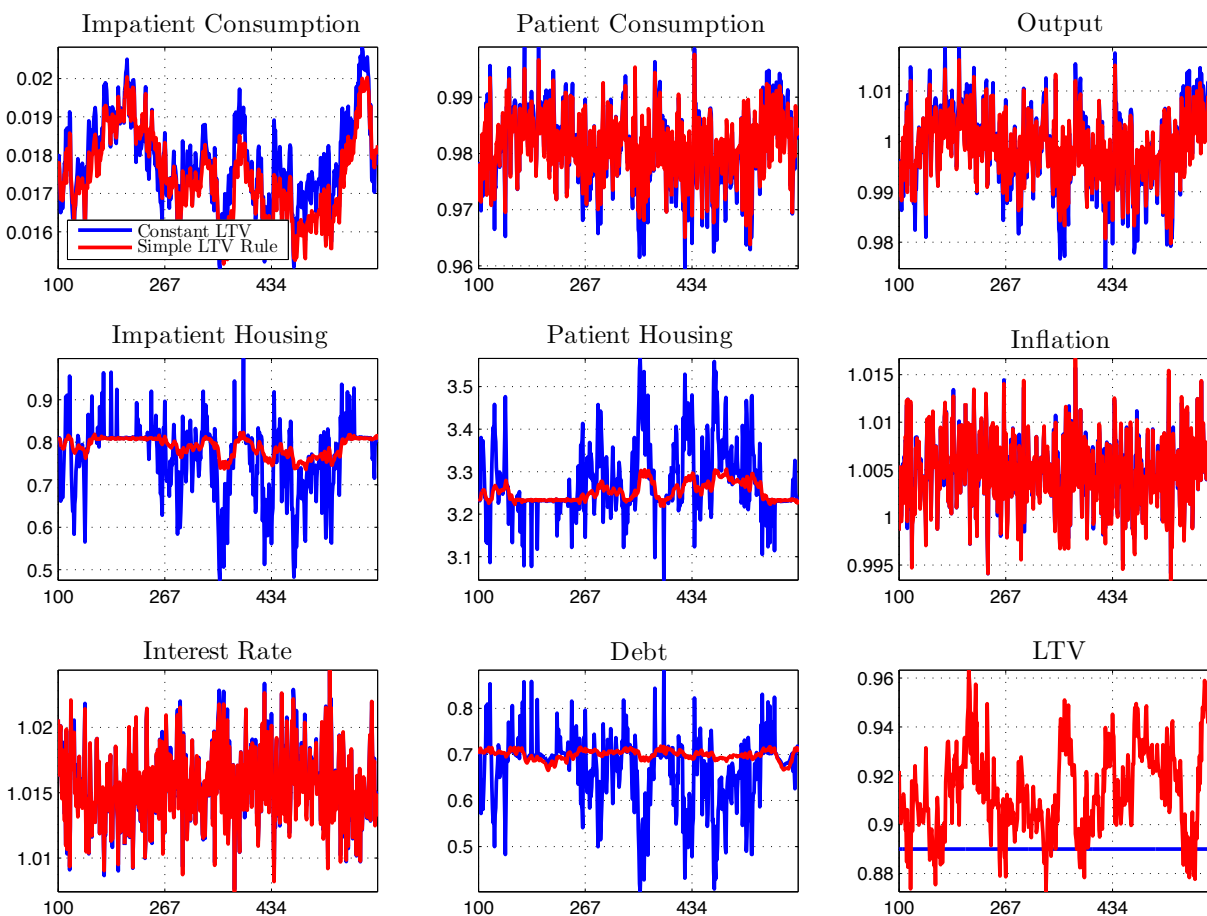
Note: Interest rates and inflation are reported in gross quarterly terms. All other variables are reported in levels. The grey areas represent when the constraint is binding.

Figure 8 illustrates how operating a simple implementable counter-cyclical LTV rule can reduce the volatility of housing and debt quite substantially. This results in a noticeable reduction in the volatility of GDP and consumption.

5.3. Downward Nominal Wage Rigidities

There is much evidence to suggest that nominal wages are more sticky downwards than upwards, a direct consequence of worker's aversion to taking pay cuts.[12] This will undoubtedly have consequences for firms hiring and firing decisions at different points of the business cycle, and ultimately have consequences for the symmetry of business cycles.

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[12]See Kahneman et al. (1986), Akerlof et al. (1996), Card & Hyslop (1997) for example.

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Downward nominal wage rigidities have been addressed by the DSGE literature in several different ways. Fagan & Messina (2009) treat downward nominal wage rigidities as a threshold problem with asymmetric adjustment costs. They solve their model using discrete-state dynamic programming. Kim & Ruge-Murcia (2011) and Fahr & Smets (2010) approximate downward nominal wage rigidities with a smooth approximation of the constraint. In particular they use Linex adjustment costs in place of Rotemberg adjustment costs and solve the model using higher order perturbation methods. Amano & Gnocchi (2017) set the problem up as a constraint on wage adjustment by households. They solve and simulate the problem using piecewise linear methods.

We introduce a non-negativity constraint on net changes in nominal wages in a simple and otherwise standard New-Keynesian DSGE model. We cover the modelling of the occasionally binding constraint using regime-switching in this section, and leave a full derivation of the model to Appendix C. The constraint is imposed on individual households and takes the form

$$\frac{W_t(i)}{W_{t-1}(i)} - 1 \geq 0, \tag{5.32}$$

where $W_t(i)$ is the wage level faced by the ith household. We can formulate this as part of the household's optimization problem. The household's optimization leads to the complementary slackness and Kuhn-Tucker conditions

$$\Omega_t \left(\frac{W_t(i)}{W_{t-1}(i)} - 1 \right) = 0, \tag{5.33}$$

with

$$\frac{W_t(i)}{W_{t-1}(i)} - 1 = 0 \text{ and } \Omega_t > 0, \tag{5.34}$$

or

$$\frac{W_t(i)}{W_{t-1}(i)} - 1 > 0 \text{ and } \Omega_t = 0. \tag{5.35}$$

We approximate the occasionally binding non-negativity constraint on wage setting by reformulating the complementary slackness conditions as a regime-switching problem. We do this by augmenting the model with a Markov chain that has two states: B when the constraint is binding and N when the constraint is not binding. We introduce a new state dependent parameter, $\mathbf{o}(s_t)$, which takes the values

$$\mathbf{o}(N) = 0, \quad \mathbf{o}(B) = 1, \tag{5.36}$$

in each of the states. We make use of (5.36) to replace the complementary slackness condition (5.33) with

$$\mathbf{o}(s_t)(\pi_{W,t} - 1) + (1 - \mathbf{o}(s_t))\hat{\Omega}_t = 0, \tag{5.37}$$

where

$$\hat{\Omega}_t = \tilde{\Omega}_t - \tilde{\Omega}, \qquad \tilde{\Omega}_t = \Omega_t, \tag{5.38}$$

The probability of switching between states is determined by the transition matrix

$$\mathbb{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{NB,t} & p_{NB,t} \\ p_{BN,t} & 1 - p_{BN,t} \end{bmatrix}. \tag{5.39}$$

The endogenous transition probabilities are specified as follows

$$p_{NB,t} = \frac{\theta_{NB}}{\theta_{NB} + \exp\left(\psi_{NB}\left(\pi_{W,t} - 1\right)\right)}, \tag{5.40}$$

$$p_{BN,t} = \frac{\theta_{BN}}{\theta_{BN} + \exp\left(-\psi_{BN}\hat{\Omega}_t\right)}. \tag{5.41}$$

When the constraint binds, a decrease in the Lagrange multiplier on the non-negativity constraint leads to an increase in the probability of wages increasing. Likewise, when the constraint does not bind, a fall in the rate of wage inflation increases the probability of the constraint binding.

5.3.1. Simulations and Impulse Responses

We simulate artificial data from the model to better understand how the endogenous transition probabilities track wage inflation and the shadow price on the non-negativity constraint. The model calibration is listed in Table C.3. We plot wage inflation and the shadow price simulations against the transition probabilities in Figure 9.

Figure 9: Transition Probabilities
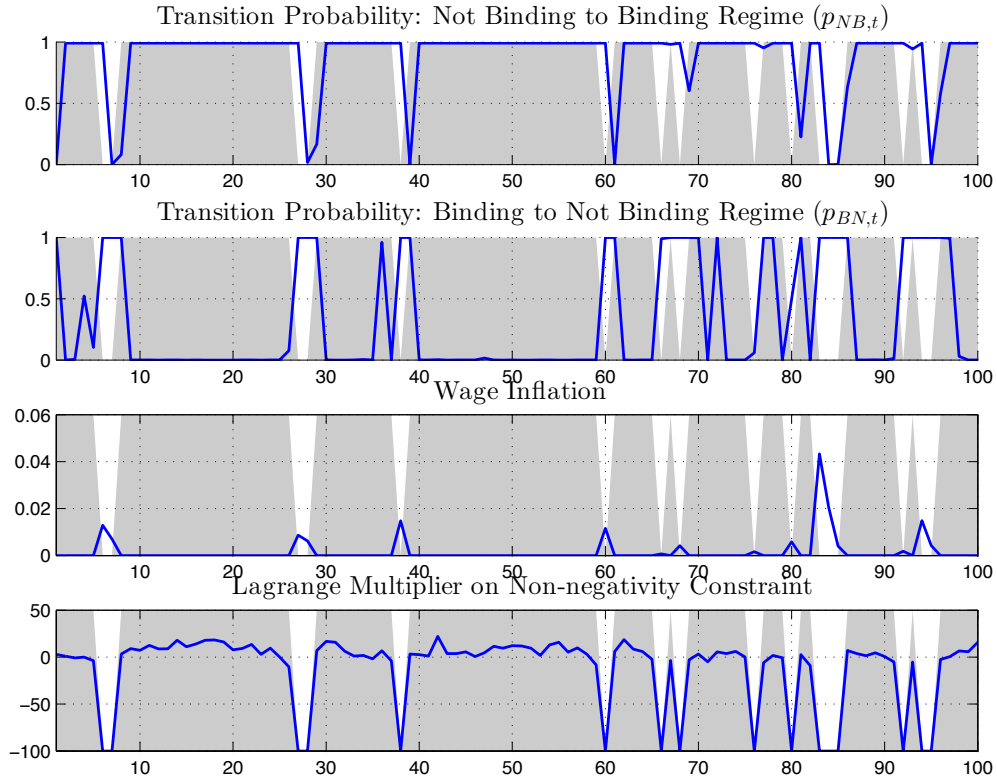
Note: The top two panels plot the transition probabilities for going from the not binding to the binding regime, and the binding to the not binding regime. The bottom panels plot wage inflation and the shadow price gap, both determinants of the transition probabilities. The grey areas represent when the constraint is binding.

We see that the transition probabilities track quite closely the binding and non-binding regimes. More specifically, as wage inflation falls, the probability of switching to the binding regime increases. When the constraint is binding, the probability of switching to the non-binding regime increases as the multiplier falls.

Gross steady state inflation is set to $\pi = 1$ in the baseline calibration. To understand how the downward nominal wage rigidity affects the ex post average level of inflation we reconstruct the price and wage level in Figure 10.

Figure 10: Evolution of the Wage and Price Level
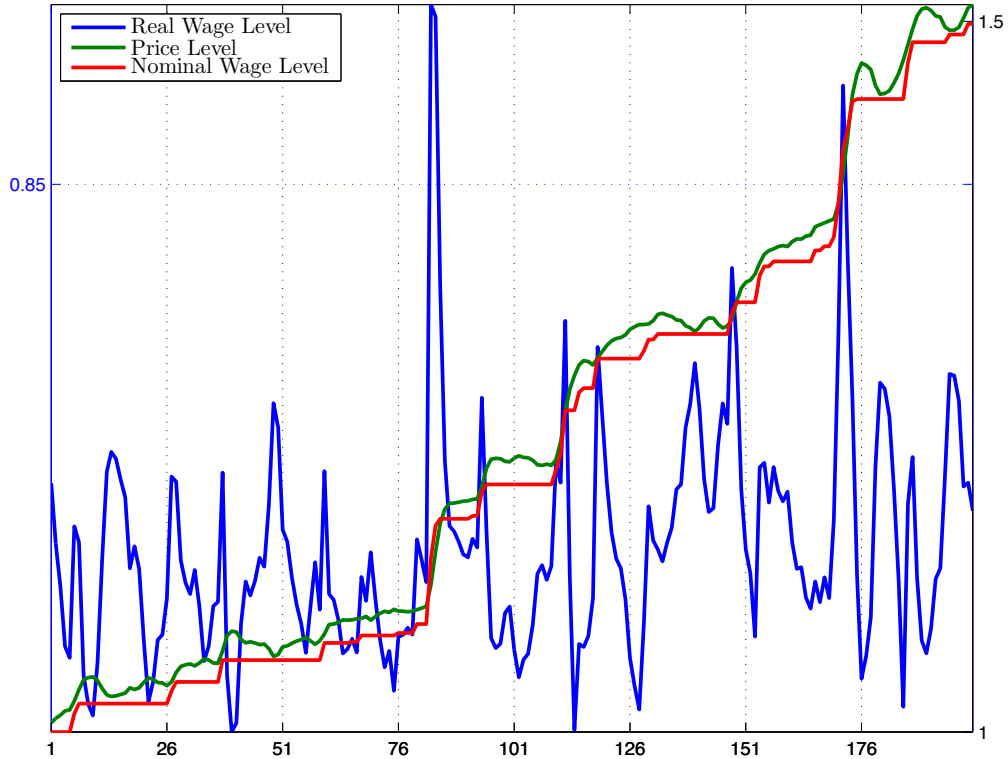
Note: The nominal wage and price levels are constructed by cumulating the wage and price inflation series.

Even though the steady state level of net inflation is 0, the model exhibits a positive average level of inflation. This should not be surprising given wages are prevented from falling. In fact the shift in the expected level of inflation is due to Jensen's inequality and is a direct consequence of using a non-linear model. Downward nominal wage rigidity is often cited as one of the reasons for having a non-negative inflation target (see Schmitt-Grohé & Uribe, 2010).

To better understand how the downward nominal wage rigidity affects the properties of the model economy, we simulate the model on a sequence of shocks with and without the downward nominal wage rigidity. We plot the results in Figures 11 and 12.

Figure 11: Model Simulations: No Constraint vs. Downward Nominal Wage Rigidity
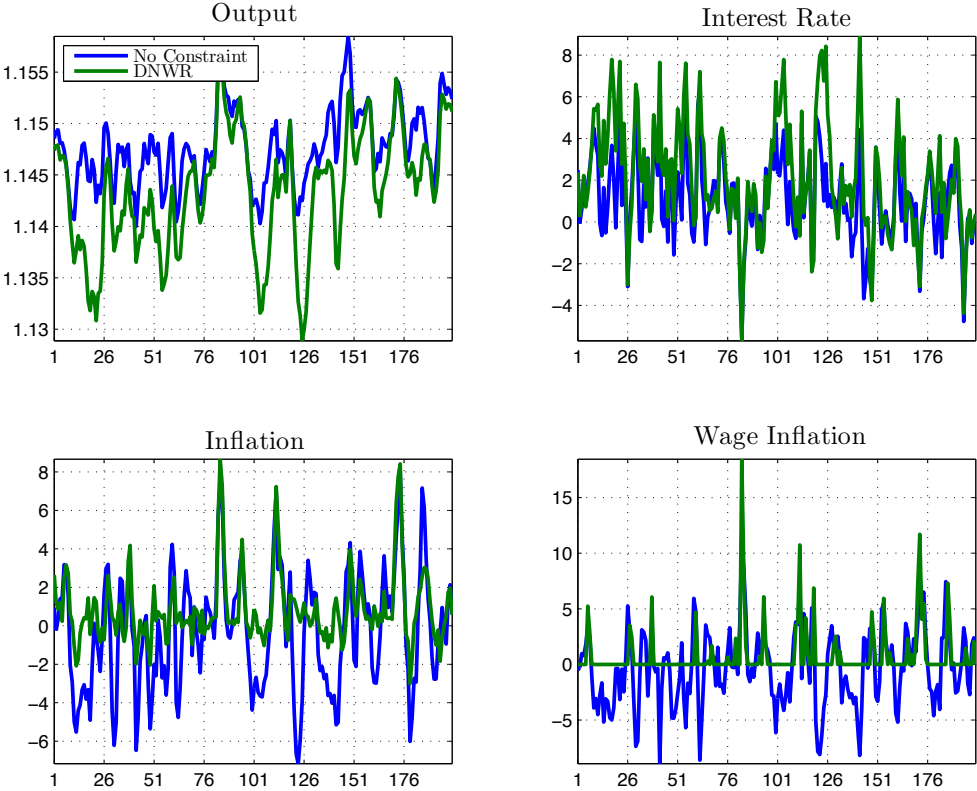
Note: Interest rates and inflation are reported in net annualized terms. Output is reported in levels.

Figure 12: Distributions: No Constraint vs. Downward Nominal Wage Rigidity
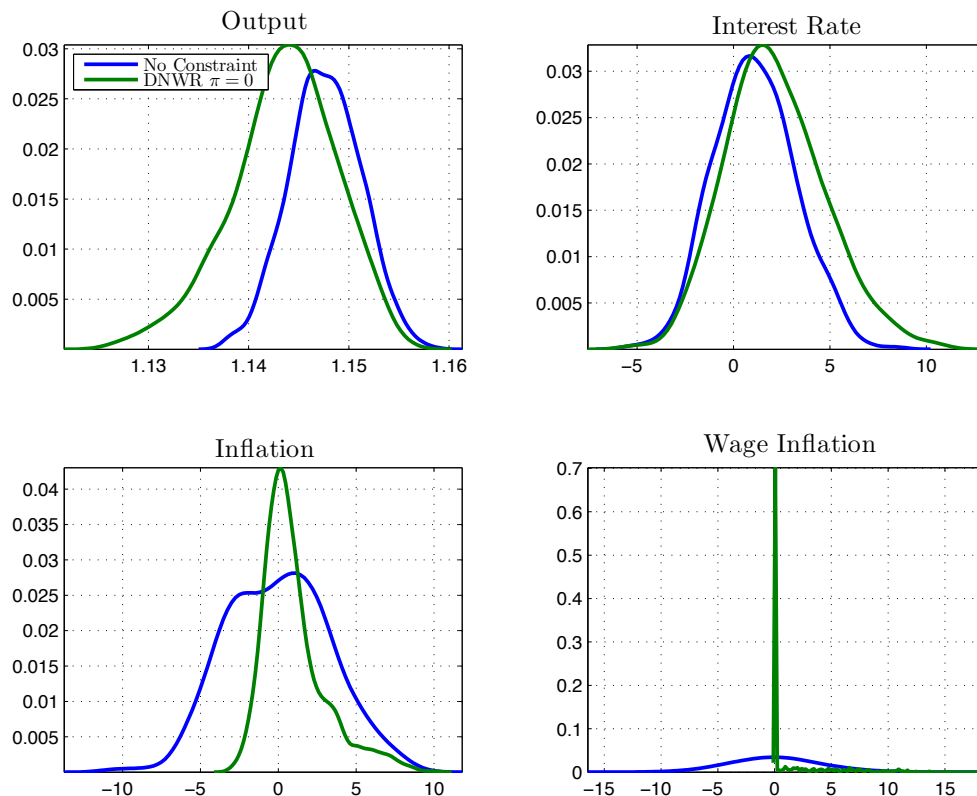
Note: The densities are constructed using the simulated model data and a kernel smoothing algorithm. Interest rates and inflation are reported in annualized terms. Output is reported in levels.

As expected the downward nominal wage rigidity prevents wage inflation from turning negative. This also has consequences for the other variables in the system. First, the model with the downward nominal wage rigidity leads to more upward movements in price inflation than downward movements. Second, downward nominal wage rigidities exacerbate downturns in output. This is because wages cannot fall in downturns, so firms demand less labor as a way of cutting costs.

We plot the regime-specific impulse responses for monetary policy shocks in Figure 13.

Figure 13: Regime Specific IRFs: Monetary Policy Shock
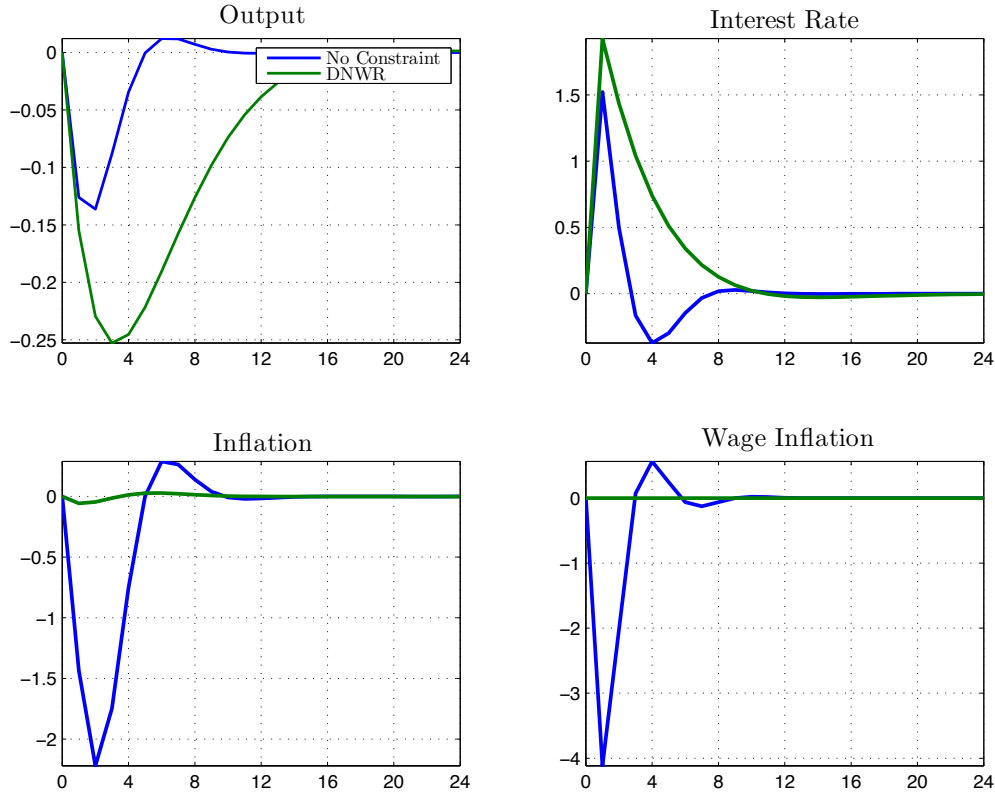
Note: Interest rates and inflation are reported in net annualized terms. Output is reported as percentage deviation from the steady state.

When wages are rigid downwards, we see that monetary policy shocks have a much larger impact on output and interest rates, but a much smaller impact on inflation. This is because downward nominal wage rigidity effectively flattens the Phillips curve.

5.4. Irreversible Investment

In our final example we investigate irreversible investment as an occasionally binding constraint in a simple real business cycle model. The model is simple and similar to the models used by Adjemian & Juillard (2013) and Christiano & Fisher (2000). For this reason we only focus on the modelling of the constraint in this section and leave a full description of the model to Appendix D. Irreversible investment can be expressed as a non-negativity constraint on investment

$$I_t \geq 0. \tag{5.42}$$

Incorporating this into the social planner's problem results in the following Kuhn-Tucker and complementary slackness conditions

$$\Omega_t \left(K_t - (1 - \delta) K_{t-1} \right) = 0, \tag{5.43}$$

with

$$I_t > 0 \text{ and } \Omega_t = 0, \tag{5.44}$$

or

$$I_t = 0 \text{ and } \Omega_t > 0. \tag{5.45}$$

In order to model the occasionally binding constraint using regime-switching, we introduce a Markov chain with a binding state (B) and a non-binding state (N). We also introduce a regime specific parameter $\mathbf{o}(s_t)$ which takes the following values in each of the states

$$\mathbf{o}\,(N) = 0, \quad \mathbf{o}\,(B) = 1. \tag{5.46}$$

We can then replace the complementary slackness condition (5.43) with

$$\mathbf{o}(s_t)I_t + (1 - \mathbf{o}(s_t))\,\Omega_t = 0. \tag{5.47}$$

To complete the system, we specify the transition probabilities

$$p_{N,B} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp\left(\psi_{N,B}\tilde{I}_t\right)}, \quad p_{B,N} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(\psi_{B,N}\hat{\Omega}_t\right)}, \tag{5.48}$$

so that the probability of the constraint binding increases as investment falls. Likewise the probability of exiting the binding regime increases as the shadow price on the constraint falls. Just as we did in the model with collateral constraints in Section 5.2 we introduce a tax on investment to ensure that capital has the same steady state in both states. This means the Euler equation becomes

$$\lambda_t\,(1 + \tau\,(s_t)) - \Omega_t = E_t\left\{\beta\left(\lambda_{t+1}\left(\alpha\frac{Y_{t+1}}{K_t} + (1 + \tau\,(s_{t+1}))\,(1 - \delta)\right) - \Omega_{t+1}\,(1 - \delta)\right)\right\}. \tag{5.49}$$

The tax rate $\tau(s_t)$ is set so that

$$\tau\,(s_t) = \frac{\Omega_t}{\lambda_t} = \frac{\mathbf{o}(s_t)\Omega}{\lambda_t}. \tag{5.50}$$

5.4.1. Simulations from a Model with Irreversible Investment

To illustrate how the occasionally binding irreversible investment constraint works, we simulate artificial data from the model and plot the investment and multiplier series against the endogenous transition probabilities in Figure 14.

Figure 14: Transition Probabilities & Their Determinants
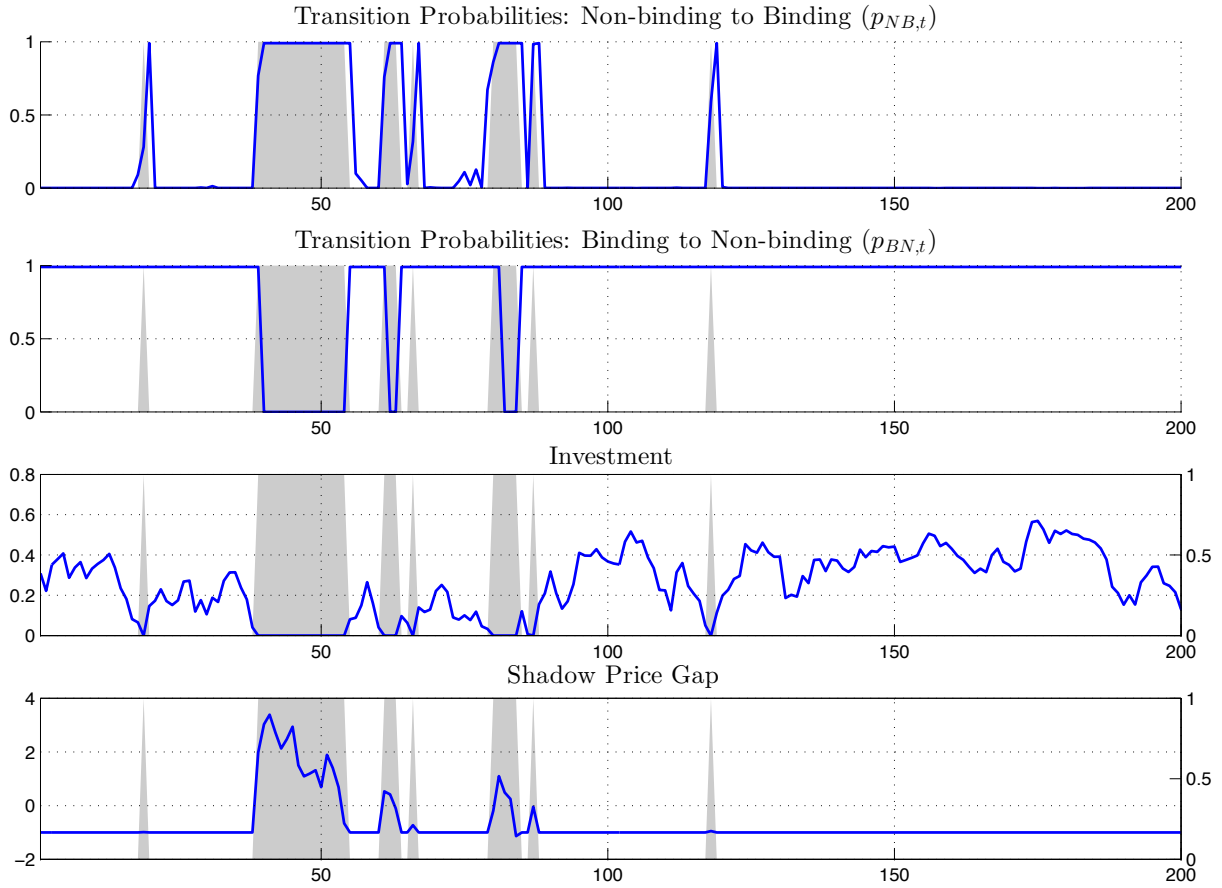
Note: The top two panels plot the transition probabilities for the non-binding to binding regimes, and binding to non-binding regimes. The bottom panels plot investment and shadow price gap on the investment constraint, both determinants of the transition probabilities. The grey shaded areas represent when the constraint is binding.

We observe that when investment falls the probability of switching from the non-binding to the binding states increases. Likewise when the Lagrange multiplier on the constraint falls the probability of exiting the binding state increases.

In Figure 15 we plot the output, consumption and investment series to better understand when the irreversible investment constraint binds and how it affects the other variables in the system.

Figure 15: A Simulation With Irreversible Investment
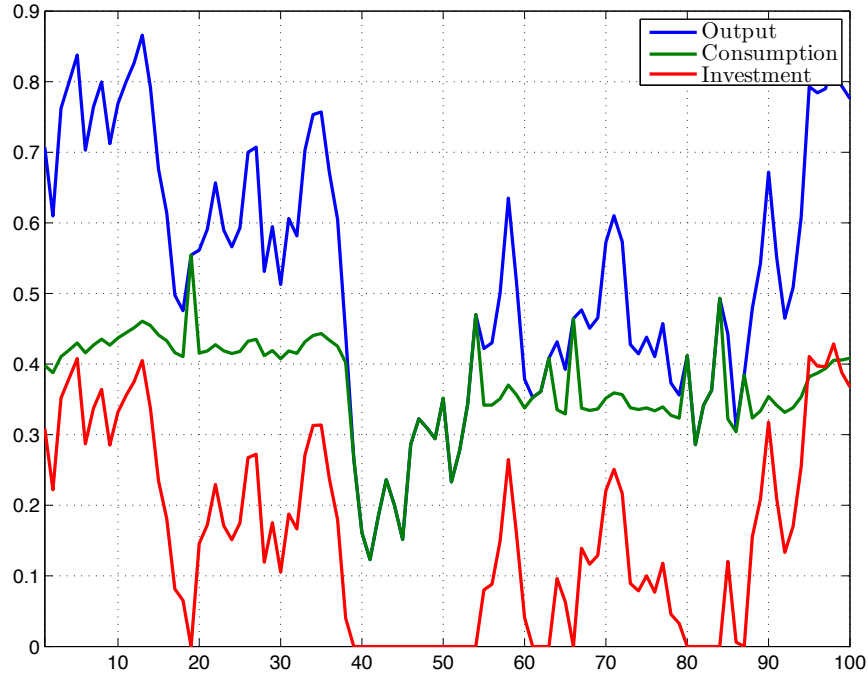
We note that output and investment co-move reasonably tightly and that when the constraint is not binding, consumption is relatively smooth. The irreversible investment constraint tends to bind when both investment and output fall. In period 40 there is quite a sharp fall in investment and output, which also causes consumption to fall. While there are some other episodes where the constraint binds and consumption actually rises.

6. All in One & Scalability of the Regime-Switching Approach

To demonstrate the versatility and scalability of the regime-switching framework we take the model from Section 5.2 and add a lower bound constraint on interest rates, and a non-negativity constraint on net wage inflation. The full derivation of the model with three occasionally binding constraints can be found in Appendix E.

We simulate the model for a number of periods to understand the implications of the model with three occasionally binding constraints and plot the model simulations in Figure 16.

Figure 16: All in One Simulations
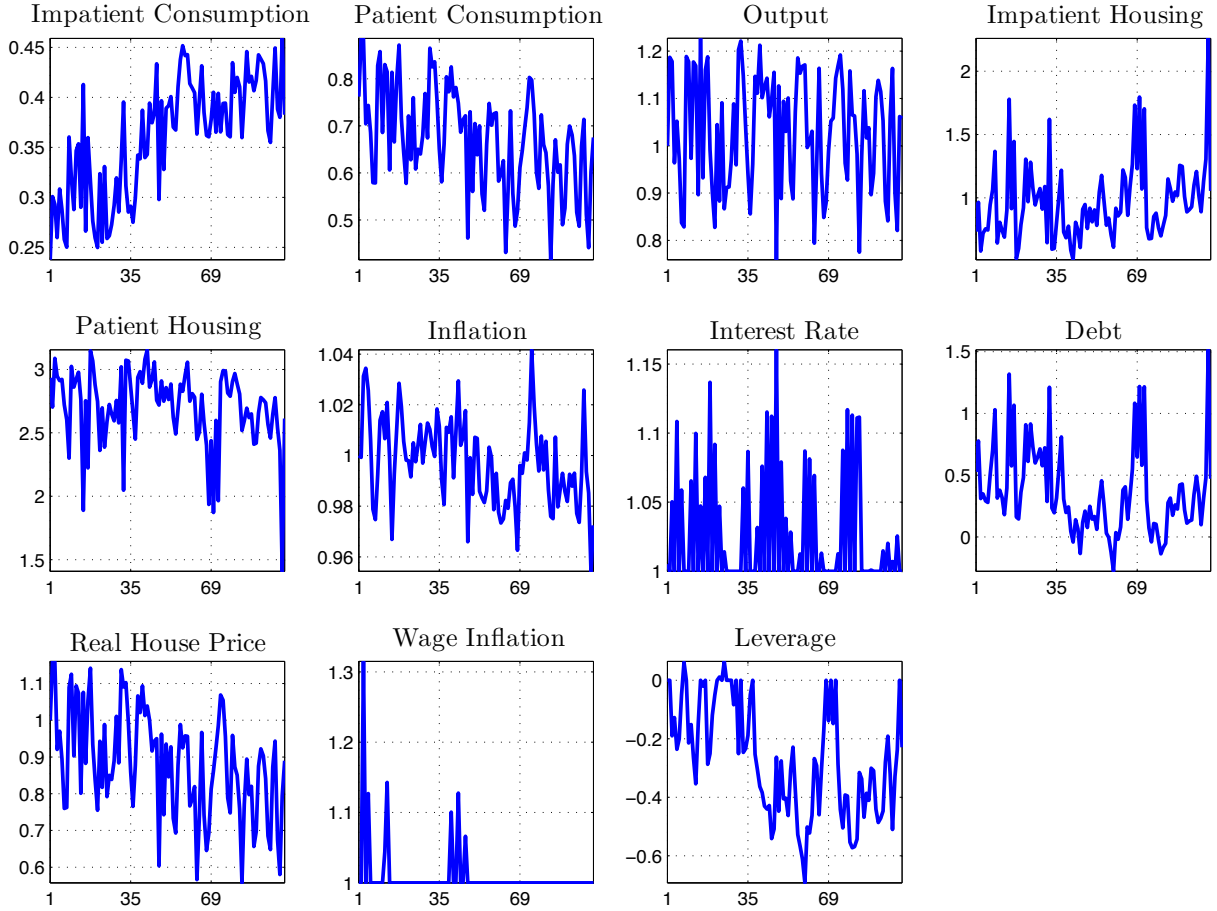
Note: Interest rates and inflation are measured in gross quarterly terms. All other variables are reported in levels.

We make several observations. The non-negativity constraint on wage inflation binds with a high frequency. There are quite prolonged periods where the collateral constraint does not bind. The ZLB constraint binds intermittently.

7. Higher-Order Approximations

We take the model with the occasionally binding collateral constraint in Section 5.2.1 and solve it using both a second and third-order perturbation solution method to illustrate how easily these models can be further extended into the non-linear domain. We simulate the models solved at first, second and third-order, using the same sequence of shocks, and plot the simulation results in Figures 17 and 18.
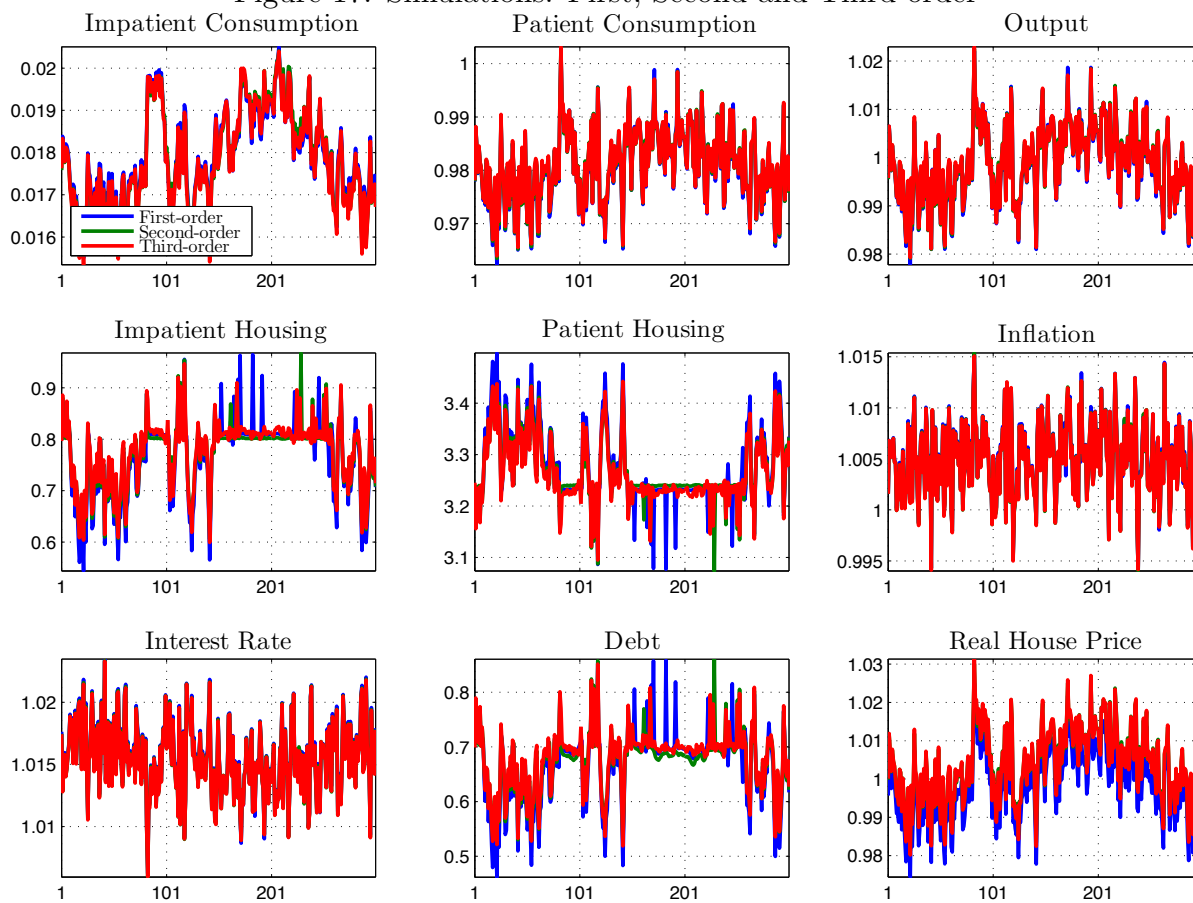
Figure 17: Simulations: First, Second and Third-order

Note: Interest rates and inflation are measured in gross quarterly terms. All other variables are reported in levels.

Figure 18: Simulations: First, Second and Third-order
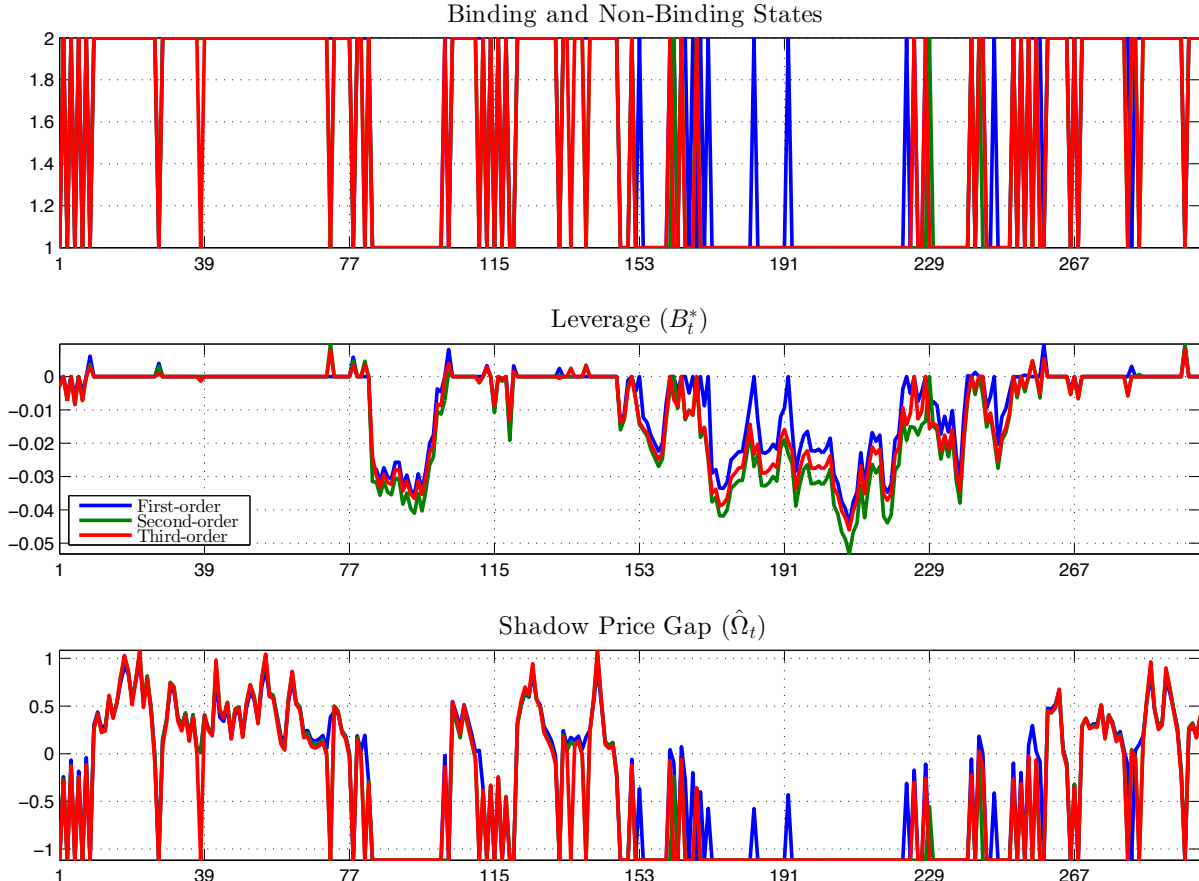
Note: The top panel plots the binding and non-binding regimes for each of the models. The bottom two panels plot leverage and the shadow price gap for the borrowing constraint. These variables are the determinants of the transition probabilities.

We focus attention on several features of the simulated data. First, there is a noticeable difference in the means of the simulated series for some variables like impatient consumption, impatient housing, patient housing and debt. This is because the higher-order approximations take into account the agents' perception of risk and shift the stochastic steady states accordingly. Second, we note that there are periods when patient housing, impatient housing and debt are more volatile at first-order than they are at second and third-order. This is because there are a couple of periods where the collateral constraint binds with the first-order approximation, where the constraint is not binding for the higher orders of approximation, and when the collateral constraint binds, the system is generally more volatile than when it does not bind.

8. Comparison With Alternatives

A number of methods have been suggested for imposing occasionally binding constraints in DSGE models. We compare the implementation of occasionally binding constraints using

regime-switching with two commonly used methods, namely, piecewise linear methods and the extended path method. We conduct various simulations using the four models from the applications section.

8.1. A New Keynesian DSGE Model with an Occasionally Binding ZLB Constraint

We carry out several exercises to compare the properties of the different solution methods when an occasionally binding ZLB constraint is imposed.

In our first exercise we compare the impulse responses from the model solved using regime-switching with constant probabilities against the model solved using piecewise linear methods. In particular we look at the regime-specific impulses when the economy is in normal times. We compare the regime-switching model under two different calibrations against the model solved using piecewise linear methods. In the first calibration we set $p_{NZ} = 0$ and $p_{ZN} = 1$ so that agents do not believe they will ever hit the lower bound, and if they do, they believe they will return to the normal state with certainty in the following period. In the second calibration we set $p_{NZ} = 0.05$ and $p_{ZN} = 0.2$ so that agents believe there is a small chance the lower bound constraint could bind and when it does bind, they expect they will be there for a duration of 8 quarters. The results are plotted in Figure 19.

Figure 19: Impulse Responses (Normal Times): Consumption Preference Shock
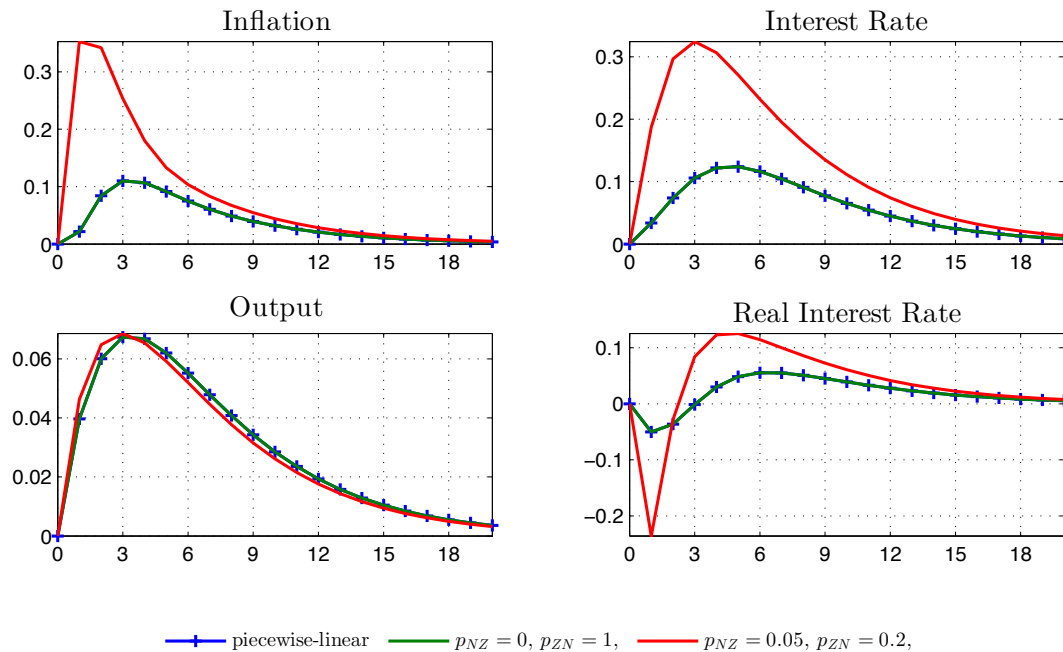
Note: Interest rates and inflation are reported in net annualized terms. Output is reported as a percentage deviation from steady state.

The impulse responses for the regime-switching model where agents believe there is no chance of hitting the lower bound are identical to the model solved using piecewise linear methods. The response of inflation and nominal and real interest rates is more pronounced when agents believe there is a non-negligible chance they could hit the lower bound, when they are away from the lower bound. This illustrates how agents' behavior depends on the likelihood of hitting the lower bound when they are away from the lower bound under a regime-switching solution. The same cannot be said for piecewise linear methods.

In our second exercise we simulate the New Keynesian DSGE model solved under a number of different assumptions using the same sequence of shocks. In particular we solve the model using regime-switching with endogenous transition probabilities, piecewise linear methods, the extended path method using the non-linear model and the extended path method using the linearized model. We repeat the simulations for a number of different shock standard deviations to illustrate some of the non-linear properties of the solution methods.[13] When the shocks are small, it is only the regime-switching model that hits the lower bound during the simulation. All the other models have very similar dynamics away from the lower bound. The interest rate and inflation series generated by the model with regime-switching are more volatile than the same series generated by other models. This is likely due to agents' knowledge of the lower bound constraint and the destabilizing nature

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[13]Larger shocks tend to exacerbate non-linearities.

Figure 20: Comparing Solution Methods: $\sigma = 0.00375$
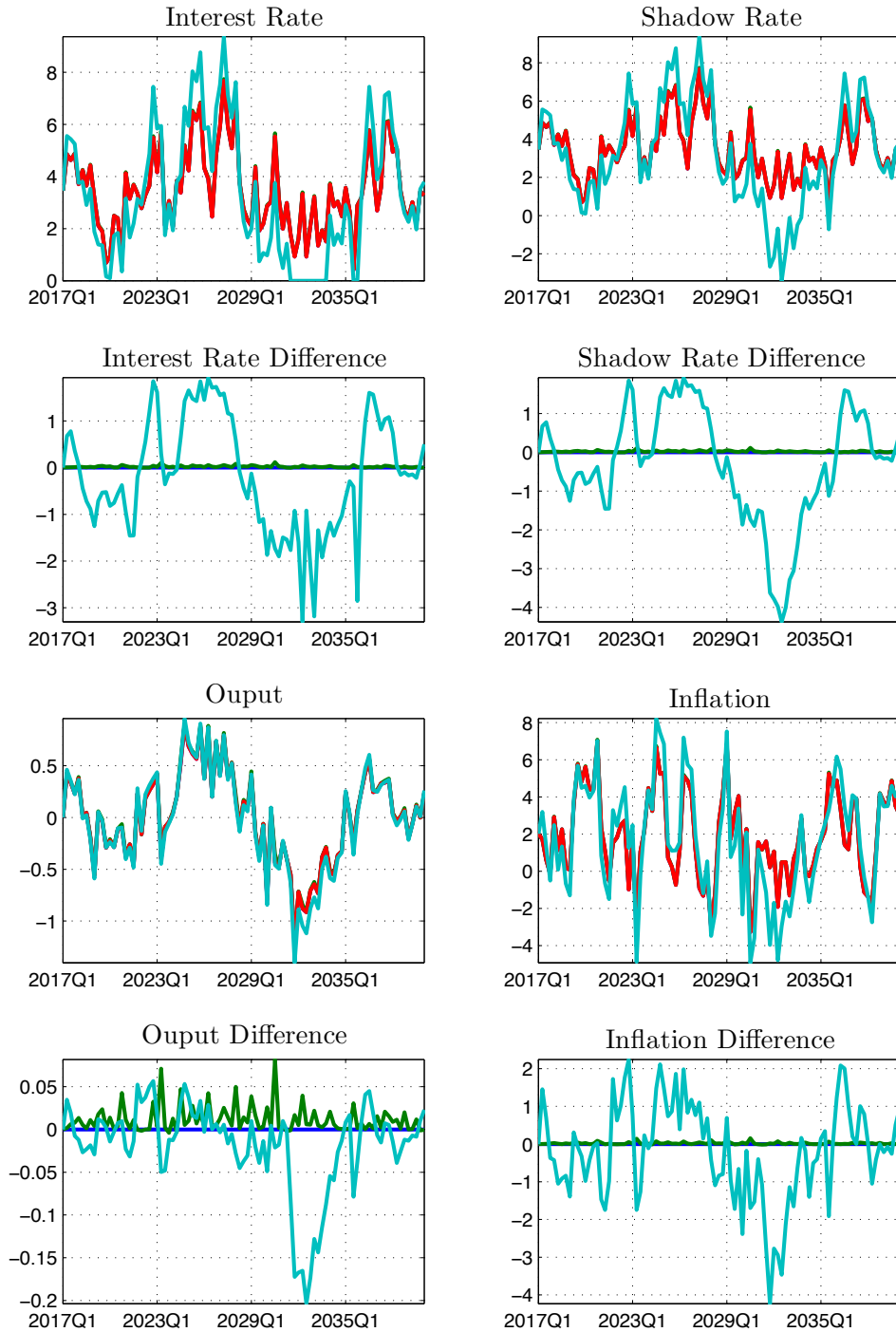
Key:
Blue = linearized model solved using the extended path algorithm,
Green = non-linear model solved using the extended path algorithm,
Red = piecewise linear solution,
Turquoise = Regime-switching with endogenous transition probabilities.

of this constraint binding. With all solution methods, the economy spends some time at the lower bound when the size of the shocks is increased. We observe that there are similarities between the extended path solutions and the piecewise linear solution. However there is a period when only the non-linear extended path solution is away from the lower bound while all other methods are at the lower bound. Again the volatility of interest rates and inflation is larger under the regime-switching model.

Figure 21: Comparing Solution Methods: $\sigma = 0.0075$

Interest Rate	Shadow Rate

Interest Rate Difference	Shadow Rate Difference

Ouput	Inflation

Ouput Difference	Inflation Difference

Key:

Blue = linearized model solved using the extended path algorithm,

Green = non-linear model solved using the extended path algorithm,

Red = piecewise linear solution,

Turquoise = Regime-switching with endogenous transition probabilities.

Figure 22: Comparing Solution Methods: $\sigma = 0.0125$
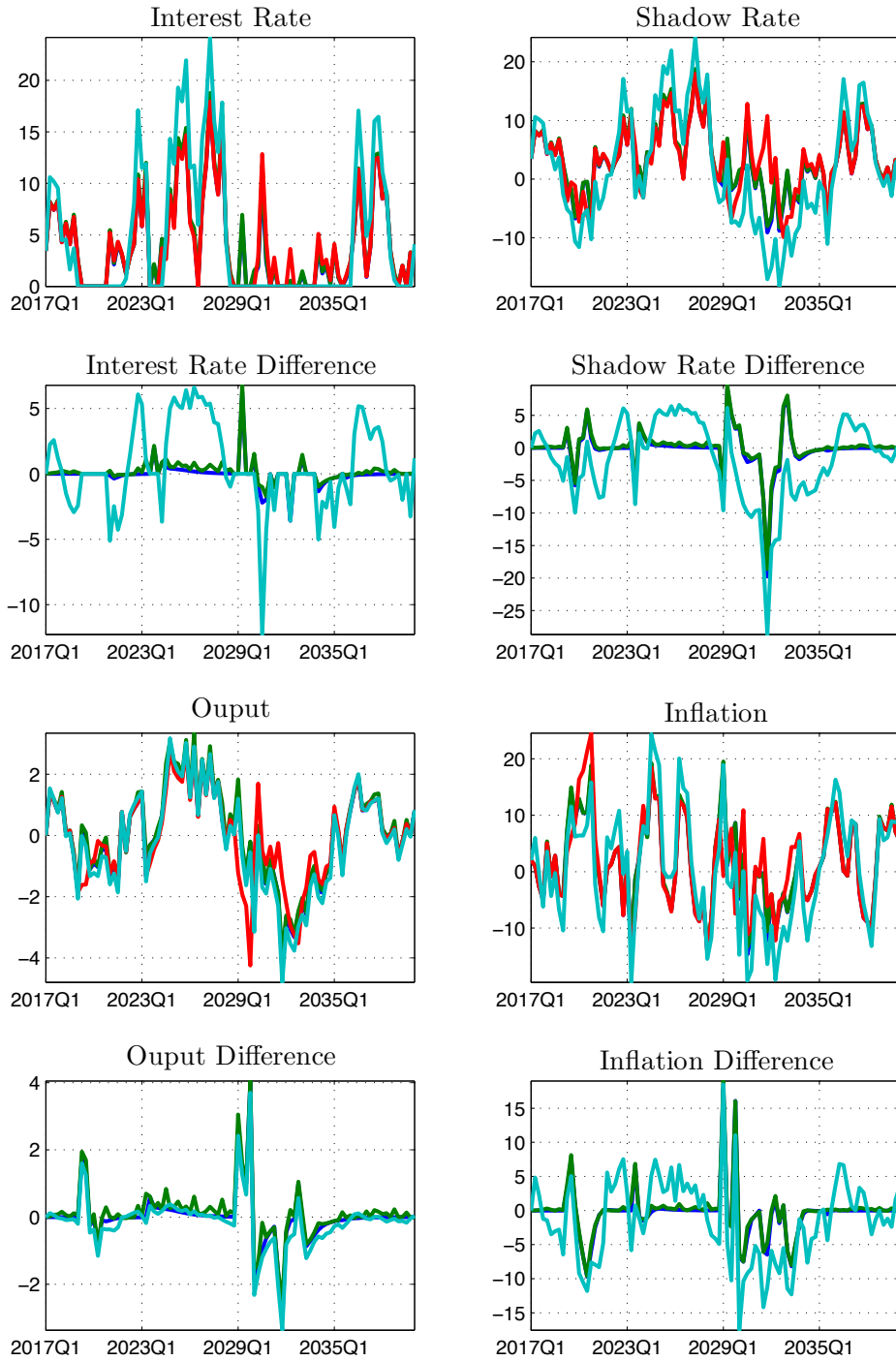
Key:
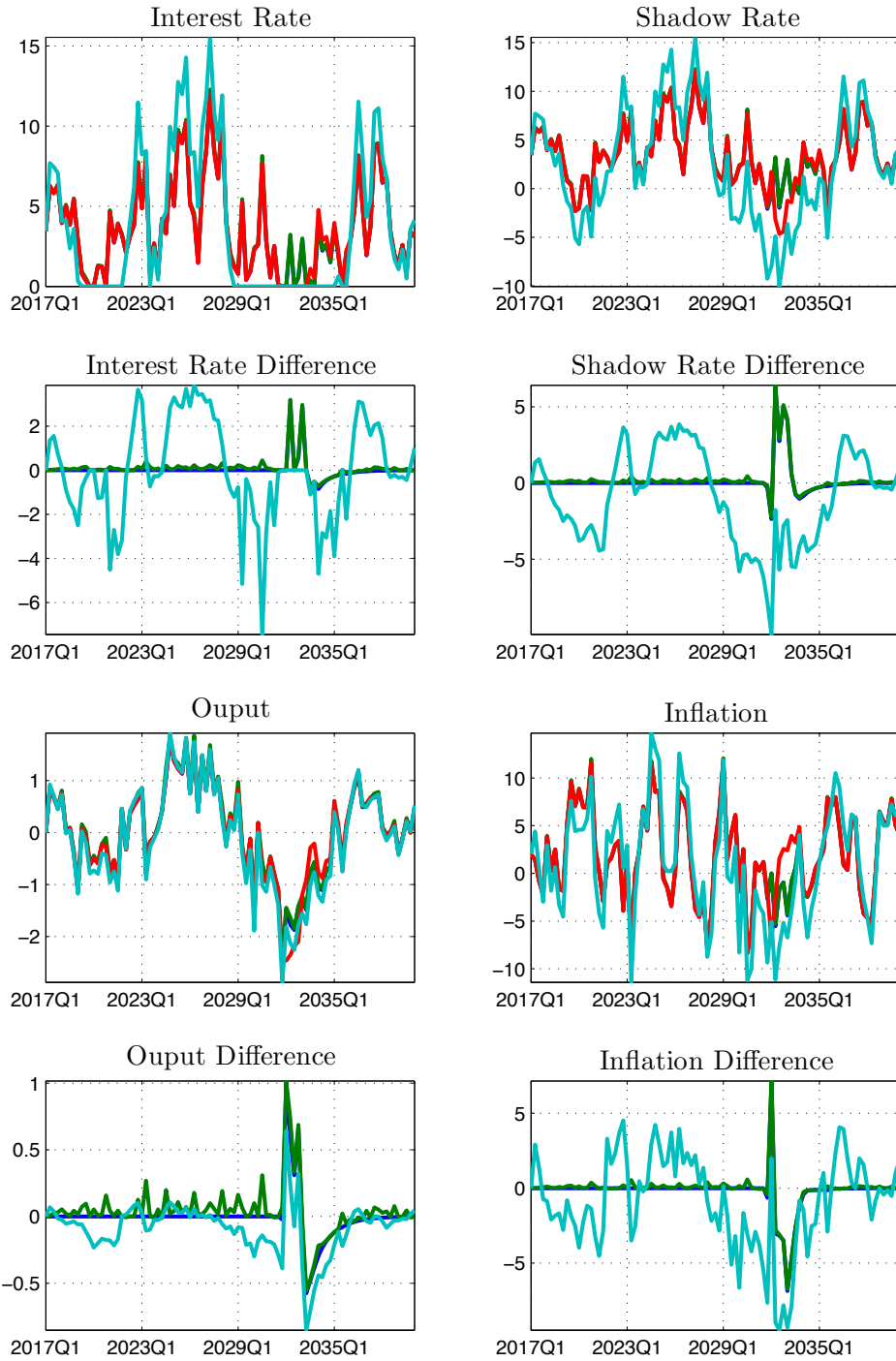Blue = linearized model solved using the extended path algorithm,
Green = non-linear model solved using the extended path algorithm,
Red = piecewise linear solution,
Turquoise = Regime-switching with endogenous transition probabilities.

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8.2. Collateral Constraints

We solve the model with an occasionally binding collateral constraint using both regime-switching and piecewise linear methods, then we carry out some simulation exercises and compare the results. We simulate both models on the same sequence of shocks and plot the series for leverage and the shadow price on the constraint from the model using piecewise linear methods in Figure 23.

Figure 23: Piecewise Linear Regimes
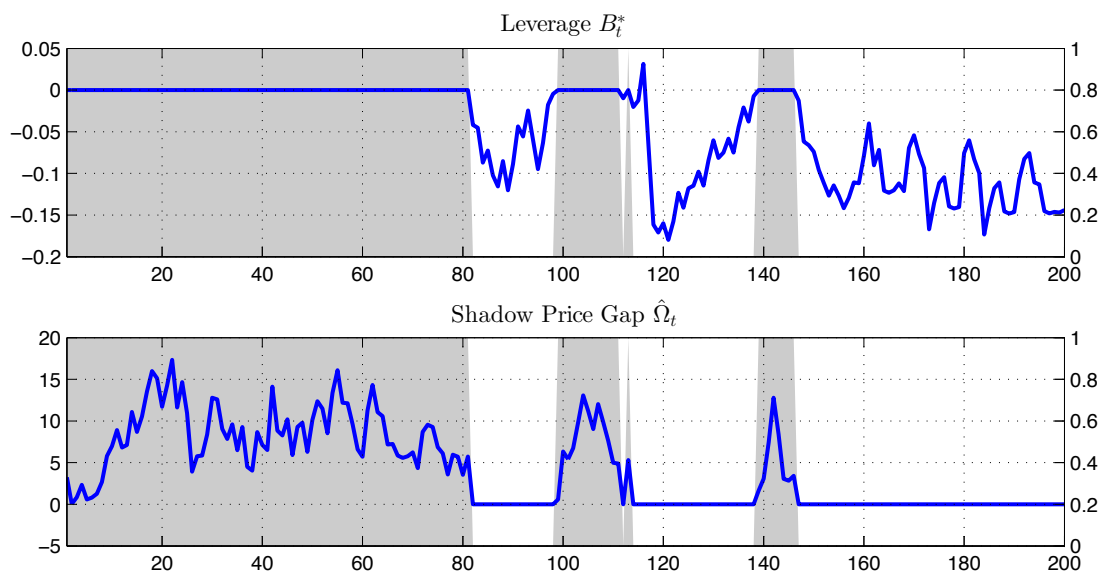
Note: The grey areas represent when the constraint is binding.

We plot the simulation results for the model solved under both methods in Figure 24.

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Figure 24: Comparing Solution Methods
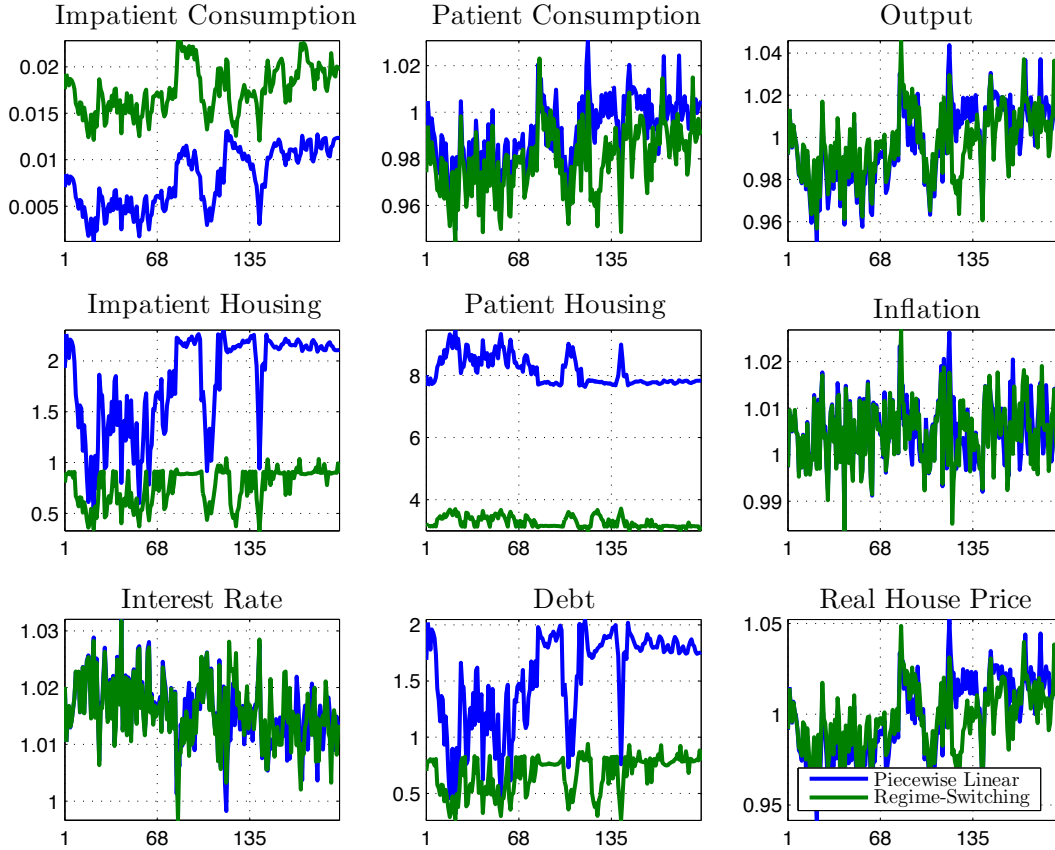
Note: Interest rates and inflation are reported in gross quarterly terms. All other variables are reported in levels.

We make several observations about the simulations. First, the means of impatient housing, patient housing, debt and impatient consumption differ between the models because the tax in the regime-switching model is chosen to center the housing stock on the steady state when the constraint is not binding.[14] The model solved using piecewise linear methods is solved around a steady state (the reference regime) where the constraint is binding. Second, the volatility of housing and debt is noticeably larger in the piecewise linear model.

We also compare the models to see when the binding and non-binding regimes occur. The results are plotted in Figure 25.

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[14] This was an arbitrary choice, the tax could have been chosen to center the steady states around the case where the constraint is binding.

Figure 25: Comparing Solution Methods

Binding Collateral Constraint Regime
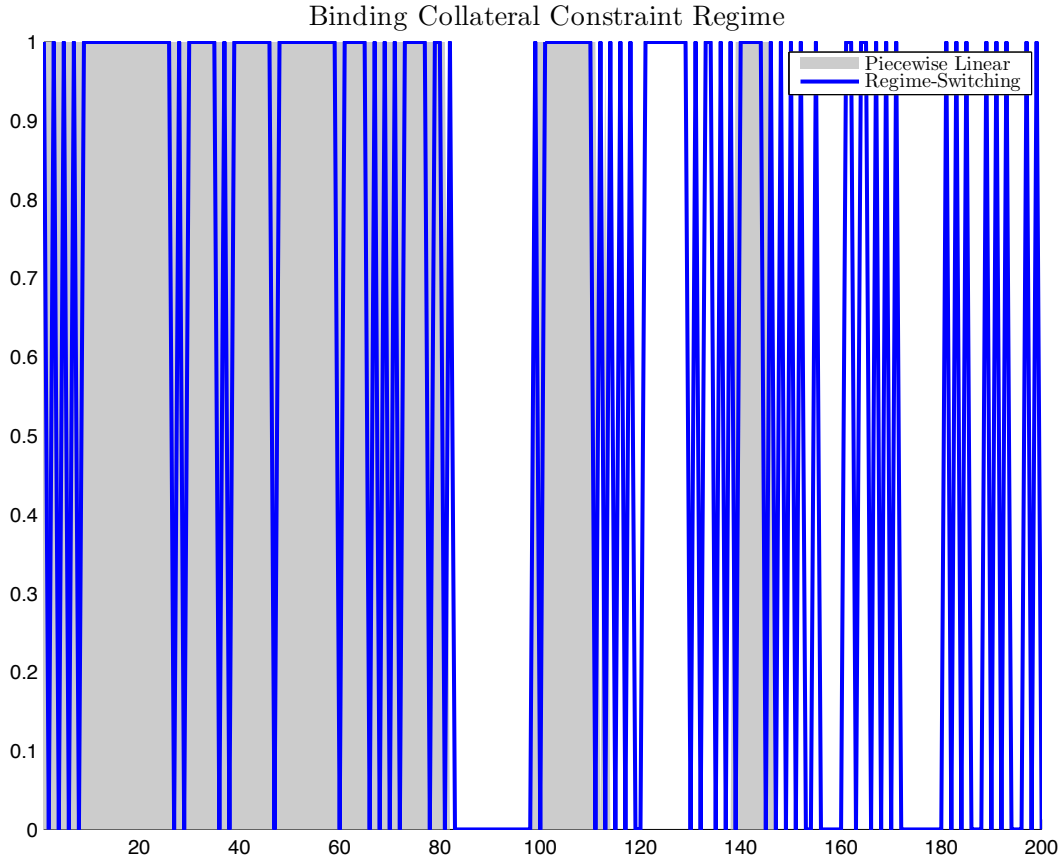
Note: The grey areas represent when the constraint is binding for the piecewise linear model and the blue line represents when the constraint is binding for the regime-switching model, with a 1 indicating when the constraint is binding and 0 when it is not binding.

There are some common periods when the constraint binds for the model solved under both methods, but there are also periods where the constraint is binding in one model and not in the other. In general there is much more switching between the binding and non-binding regimes in the regime-switching model than there is with the model solved using piecewise linear methods. This is because there is slightly more flexibility in the regime-switching method to determine when a constraint should and should not bind.

8.3. Downward Nominal Wage Rigidity

We compare simulation results for the model with downward nominal wage rigidities solved using piecewise linear methods against the same model solved using regime-switching methods. The simulation results and unconditional distributions from the simulations are plotted in Figures 26 and 27.

Figure 26: Comparing Solution Methods
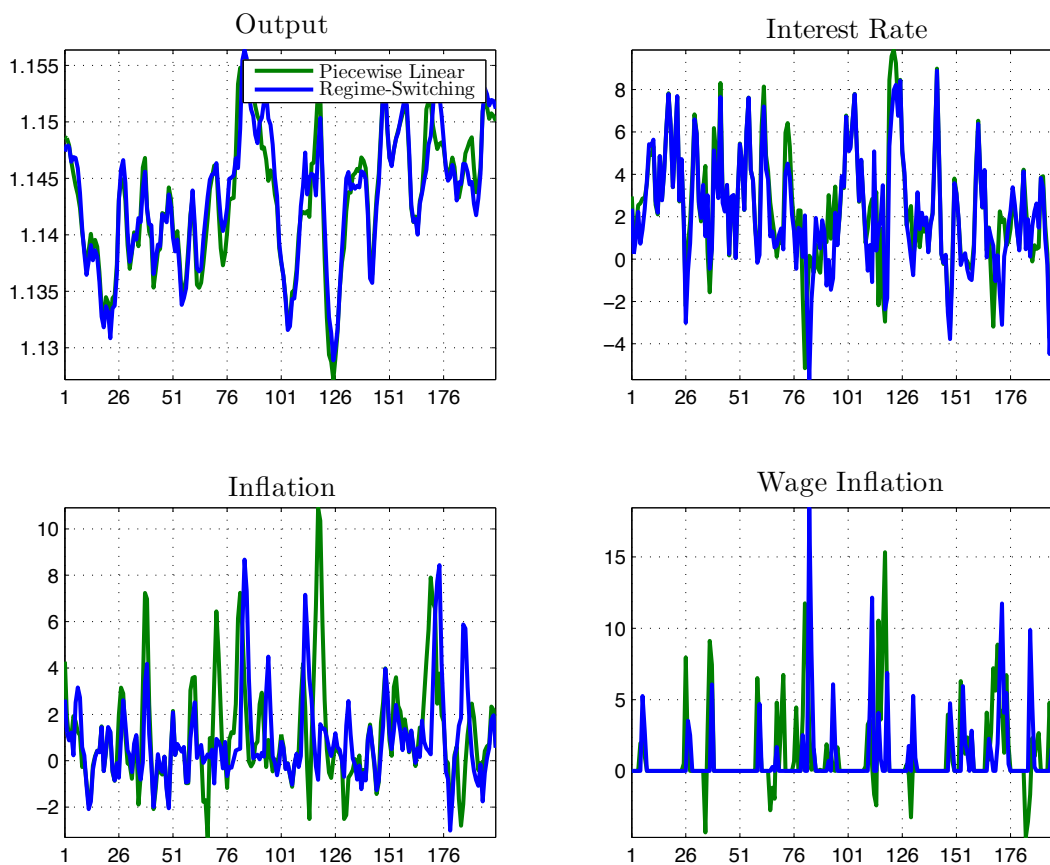
Note: Interest rates and inflation are reported in net annualized terms. Output is reported in levels.

Figure 27: Comparing Solution Methods
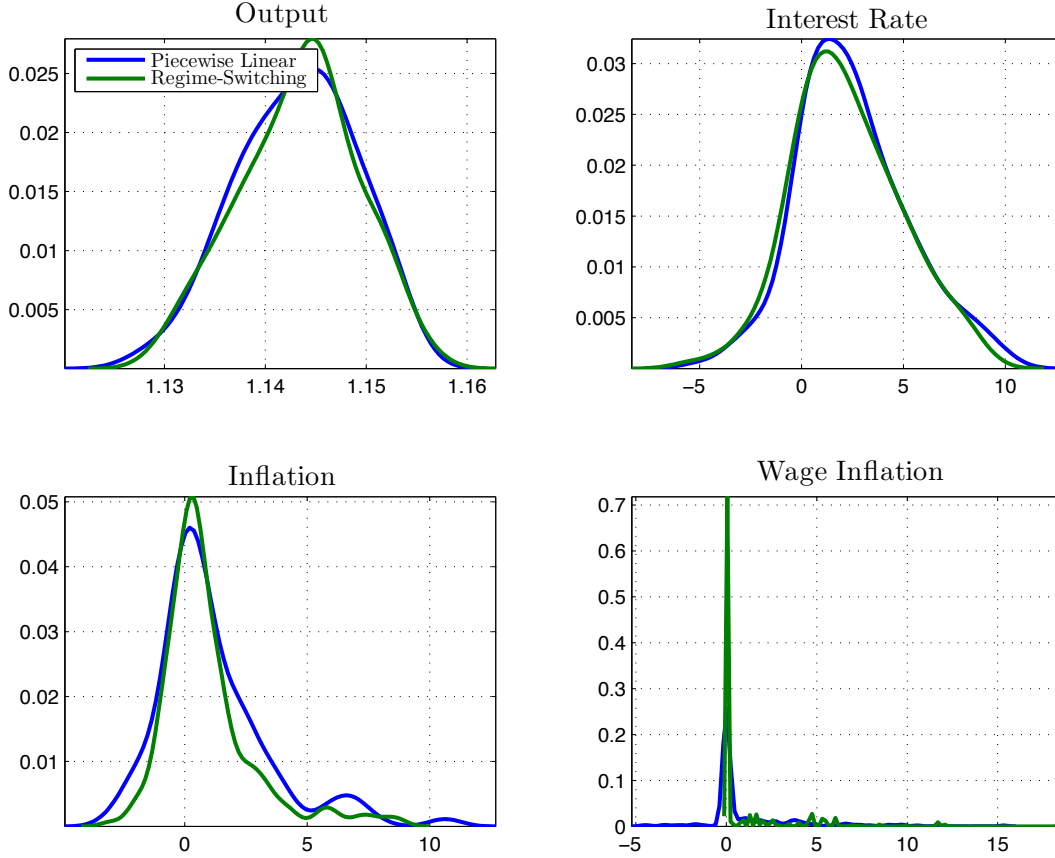
Note: Interest rates and inflation are reported in net annualized terms. Output is reported in levels.

The overall results are quite similar between the models. However we do note that there is more "leakage" with the piecewise linear model, that is there are periods where the downward nominal wage rigidity constraint does not bind. This problem arises because there are periods where either both regimes are possible, or neither of the regimes are possible. When neither of the regimes are possible the procedure is forced to violate the constraint. Leakage is ultimately due to the approximation of the non-linear model and the fact that the Kuhn-Tucker problems are designed for static as opposed to dynamic problems. Many of the overall patterns we observed with the model solved using regime-switching are also present with the model solved using piecewise linear methods. For example there is still a pronounced long left tail to the distribution for output. Likewise the long right tail on price inflation is also present in the model solved using piecewise linear methods.

8.4. Irreversible Investment

Finally, we compare extended path and piecewise linear solution methods against regime-switching, using the real business cycle model from Section Appendix D. Using the same

sequence of shocks we simulate artificial data from each of the models and plot the results in Figure 28.

Figure 28: Comparing Solution Methods
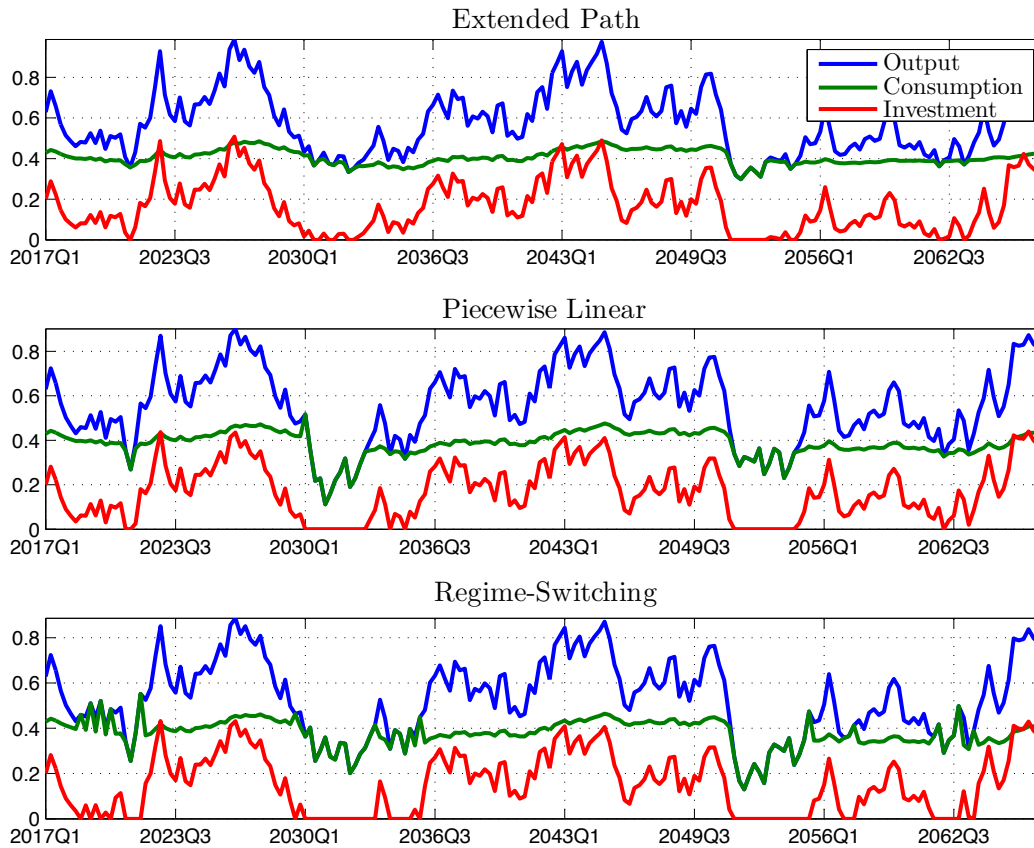
Overall the results from these simulations are qualitatively similar. The constraint binds for similar periods and durations for the simulations from the different models.

9. Conclusion

In this paper we show how to model occasionally binding constraints in DSGE models using regime-switching. Our approach can be applied to large models, can easily accommodate/handle complementary slackness problems, can be used to solve models at higher-orders of perturbations and can accommodate multiple constraints simultaneously. We have illustrated our method by solving four well known problems in economics that involve occasionally binding constraints: the zero lower bound on interest rates, occasionally binding collateral constraints, downward nominal wage rigidities and irreversible investment, and reformulated them as regime-switching problems. To demonstrate the versatility of the approach, we have combined multiple occasionally binding constraints into a single model, a feat that is difficult

to achieve using alternative methodologies. We have also solved the model with collateral constraints using higher-order perturbation methods. All codes have been implemented in Matlab using the RISE toolbox.

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Appendix A. A Simple NK DSGE Model With an Occasionally Binding ZLB

In this section we develop a simple New Keynesian DSGE model with an occasionally binding ZLB constraint.

Appendix A.1. Households

The representative infinitely lived household seeks to maximize lifetime utility

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} d_j \right) \left[A_t \frac{(\mathscr{C}_t/Z_t)^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t^{1+\eta}}{1+\eta} \right] \right\},$$

where $\mathscr{C}_t = C_t - \chi \bar{C}_{t-1}$ is consumption adjusted for the external habit stock, C_t is consumption, Z_t is the level of technology in the economy and N_t is labor. Following Braun et al. (2015), βd_{t+1} is the time discount factor, where d_{t+1} is a preference shock whose value is revealed at the beginning of period t and $d_0 = 1$. A_t is a consumption preference shock process such that

$$\log (A_t) = \rho_A \log (A_{t-1}) + \varepsilon_{A,t}. \tag{A.1}$$

The representative household maximizes current and expected utility by choosing allocations of consumption, labor and bond holdings (B_t) subject to the resource constraint

$$B_t + C_t = \frac{B_{t-1} R_{t-1}}{\pi_t} + W_t N_t + \Phi_t, \tag{A.2}$$

where R_t is the gross nominal interest rate, π_t is the gross rate of inflation, W_t the real wage and Φ_t is profits from firms, who are owned by the household.

Setting up the Lagrangian

$$\mathscr{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} d_j \right) \left[\begin{array}{c} A_t \frac{(\mathscr{C}_t/Z_t)^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t^{1+\eta}}{1+\eta} - \ldots \\ \ldots - \lambda_t \left[B_t + C_t - \frac{B_{t-1} R_{t-1}}{\pi_t} - W_t N_t - \Phi_t \right] \end{array} \right] \right\}.$$

This results in the following set of first order conditions

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = A_t \left(C_t - \chi C_{t-1} \right)^{1-\sigma} Z_t^{\sigma-1} - \lambda_t = 0, \tag{A.3}$$

$$\frac{\partial \mathcal{L}_t}{\partial N_t} = \kappa N_t^{\eta} - \lambda_t W_t = 0, \tag{A.4}$$

$$\frac{\partial \mathcal{L}_t}{\partial B_t} = \lambda_t - E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1} R_t}{\pi_t} \right\} = 0. \tag{A.5}$$

Equation (A.3) equates the marginal utility of consumption with the shadow value of wealth

$$\lambda_t = A_t \left(C_t - \chi C_{t-1} \right)^{-\sigma} Z_t^{\sigma-1}. \tag{A.6}$$

From equation (A.4) we obtain the real marginal rate of substitution

$$W_t = \kappa N_t^{\eta} / \lambda_t. \tag{A.7}$$

We define the nominal stochastic discount factor as

$$\mathcal{M}_{t,t+1} \equiv \beta d_{t+1} \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}. \tag{A.8}$$

Combining (A.8) and (A.5) we obtain the consumption Euler equation

$$E_t \left\{ \mathcal{M}_{t,t+1} R_t \right\} = 1. \tag{A.9}$$

Appendix A.2. Firms

The economy consists of a continuum of firms indexed by i and normalized to unit mass. The ith firm produces output according to the production technology

$$Y_t(i) = Z_t N_t(i), \tag{A.10}$$

where $Y_t(i)$ is output produced by the ith firm, $N_t(i)$ is labor demanded by the ith firm and neutral technology evolves according to

$$Z_t = Z_{t-1} \exp(g_Z + \varepsilon_{Z,t}), \tag{A.11}$$

where g_Z is the productivity growth rate.

Firms choose prices subject to a quadratic cost à la Rotemberg (1982). Firm i's real profit is defined as follows

$$\Phi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) \exp(\varepsilon_{\pi,t}) - W_t N_t(i) - \frac{\phi}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right]^2, \tag{A.12}$$

where P_t is the aggregate price level and $P_t(i)$ is the price level faced by the ith firm. Following

[Lombardo & Vestin](2008), $\exp(\varepsilon_{\pi,t})$ is a stochastic subsidy to firms. The quadratic price adjustment cost is relative to the inflation reference index

$$\tilde{\pi}_t = \pi_t^\xi \pi^{1-\xi}.$$

Firm i's expected discounted sum of future profits is given by

$$\Xi_0(i) = E_0 \left\{ \sum_{t=0}^\infty \mathcal{M}_{0,t} \left(\frac{P_t}{P_0} \right) \left[\frac{P_t(i)}{P_t} Y_t(i) \exp(\varepsilon_{\pi,t}) - W_t N_t(i) - \frac{\phi}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right]^2 \right] \right\}.$$
$$(A.13)$$

Firms maximize profits by choosing prices subject to the demand constraint

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t,$$

and the production function (A.10). Substituting these into equation (A.13) gives

$$\Xi_0(i) = E_0 \left\{ \sum_{t=0}^\infty \mathcal{M}_{0,t} \left(\frac{P_t}{P_0} \right) \left[\begin{array}{c} \left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t \exp(\varepsilon_{\pi,t}) - \frac{W_t}{Z_t} \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t - \dots \\ \dots - \frac{\phi}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right]^2 \end{array} \right] \right\}.$$

We obtain the firm's first-order condition with respect to prices

$$(1 - \varepsilon) Y_t(i) \exp(\varepsilon_{\pi,t}) + \varepsilon \frac{W_t}{Z_t} Y_t(i) \frac{P_t}{P_t(i)} - \phi \frac{P_t Y_t}{P_{t-1}(i)} \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right] + \dots$$
$$\dots + E_t \left\{ \phi \mathcal{M}_{t,t+1} P_{t+1} Y_{t+1} \frac{P_{t+1}(i)}{P_t(i)^2} \left[\frac{P_{t+1}(i)}{P_t(i)} - \tilde{\pi}_{t+1} \right] \right\}.$$

Assuming symmetry of the firms (a symmetric equilibrium) allows us to write the non-linear Philips curve as follows

$$\left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{W_t}{Z_t} - \exp(\varepsilon_{\pi,t}) - \left(\frac{\phi}{\varepsilon - 1} \right) \pi_t \left[\pi_t - \tilde{\pi}_t \right] + \dots$$
$$\dots + E_t \left\{ \left(\frac{\phi}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1} \pi_{t+1}^2 \frac{Y_{t+1}}{Y_t} \left[\pi_{t+1} - \tilde{\pi}_{t+1} \right] \right\}. \quad (A.14)$$

Appendix A.3. Monetary Policy

The monetary authority sets policy according to

$$R_t = \max \left(R_{ZLB}, R_t^* \right), \quad (A.15)$$

where R_{ZLB} is the interest rate at the effective lower bound and R_t^* is set according to a Taylor-type rule of the form

$$R_t^* = R_{t-1}^{*\rho_r} \left(R^* \left(\frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left(\frac{Y_t Z_{t-1}}{Y_{t-1} Z_t} \right)^{\kappa_y} \right)^{1-\rho_r} \exp(\varepsilon_{R,t}), \tag{A.16}$$

where R^* is the steady state Taylor-rule interest rate.

Appendix A.4. Market Clearing

In a symmetric equilibrium; $Y_t = \int_0^1 Y_t(i)\, di$, $N_t = \int_0^1 N_t(i)\, di$, $\Phi_t = \int_0^1 \Phi_t(i)\, di$, and $B_t = 0$. Substituting (A.12) into the budget constraint (A.2) gives the market clearing condition

$$Y_t = C_t + \frac{\phi}{2} Y_t \left[\pi_t - \tilde{\pi}_t \right]^2. \tag{A.17}$$

Appendix A.5. The ZLB as an Occasionally Binding Constraint Under Regime-Switching

We cast the zero lower bound on interest rates as an occasionally binding constraint and use regime-switching with a two state Markov chain to model the problem. When the constraint binds, the economy is in the ZLB state (Z) and when the constraint is not binding, the economy is in the normal state (N), so that

$$s_t = Z, N.$$

We introduce the regime-switching parameter $\mathbf{z}(s_t)$ which takes the values

$$\mathbf{z}(N) = 0, \quad \mathbf{z}(Z) = 1, \tag{A.18}$$

in each of the regimes. We replace equation (A.15) with

$$R_t = \mathbf{z}(s_t) R_{ZLB} + (1 - \mathbf{z}(s_t)) R_t^*. \tag{A.19}$$

We assume regime specific steady states for the policy interest rate

$$R(Z) = R_{ZLB}, \quad R(N) = R^*, \text{ where } R_{ZLB} < R^*. \tag{A.20}$$

This implies restrictions on the time preference shock or shift term

$$d(Z) = \frac{R^*}{R_{ZLB}} > 1, \quad d(N) = 1. \tag{A.21}$$

The Markov chain is governed by the transition matrix

$$\mathbb{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{ZN_t} & p_{ZN_t} \\ p_{NZ_t} & 1 - p_{NZ_t} \end{bmatrix}, \tag{A.22}$$

where p_{ZN_t} is the probability of transitioning from the ZLB state in period t to the normal state in period $t+1$ and p_{NZ_t} is the probability of transitioning from the normal state in period t to the ZLB state in period $t+1$. We assume the following functions for the transition probabilities

$$p_{NZ,t} = \frac{\theta_{N,Z}}{\theta_{N,Z} + \exp\left(\psi_{N,Z}\left(R_t^* - R_{ZLB}\right)\right)}, \tag{A.23}$$

$$p_{ZN,t} = \frac{\theta_{Z,N}}{\theta_{Z,N} + \exp\left(-\psi_{Z,N}\left(R_t^* - R_{ZLB}\right)\right)}. \tag{A.24}$$

Our choice of functional forms for the transition probabilities means that when the economy is in the normal state, the probability of hitting the lower bound increases as the Taylor-rule rate gets closer to the lower bound. Likewise when the economy is at the lower bound, the probability of returning to the normal interest rate regime is increasing in the Taylor-rule interest rate (the shadow interest rate).

Appendix A.6. Stationarizing the Model

The model exhibits trend growth, so the non-stationary variables need to be stationarized in order to solve the model. There is only one trend process in the model so we stationarize everything relative to the productivity stock as follows; $\tilde{\lambda}_t = \lambda_t Z_t$, $\tilde{C}_t = C_t/Z_t$, $\tilde{Y}_t = Y_t/Z_t$, $\tilde{W}_t = W_t/Z_t$, $\mu_t = \exp(g_Z + \varepsilon_{Z,t})$. Equations (A.6), (A.7), (A.8), (A.10), (A.14), (A.16) and (A.17) become

$$\tilde{\lambda}_t = A_t \left(\tilde{C}_t - \chi \tilde{C}_{t-1}/\mu_t\right)^{-\sigma}, \tag{A.25}$$

$$\tilde{W}_t = \kappa N_t^\eta / \tilde{\lambda}_t, \tag{A.26}$$

$$\mathcal{M}_{t,t+1} = E_t \left\{ \beta d_{t+1} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \pi_{t+1} \mu_{t+1}} \right\}, \tag{A.27}$$

$$\tilde{Y}_t = N_t, \tag{A.28}$$

$$\left(\frac{\varepsilon}{\varepsilon - 1}\right) \tilde{W}_t - \exp\left(\varepsilon_{\pi,t}\right) - \left(\frac{\phi}{\varepsilon - 1}\right) \pi_t \left[\pi_t - \tilde{\pi}_t\right] + \dots$$
$$\dots + E_t \left\{ \left(\frac{\phi}{\varepsilon - 1}\right) \mathcal{M}_{t,t+1} \pi_{t+1}^2 \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \mu_{t+1} \left[\pi_{t+1} - \tilde{\pi}_{t+1}\right] \right\}, \tag{A.29}$$

$$R_t^* = R_{t-1}^{*\rho_r} \left(R^* \left(\frac{\pi_t}{\pi}\right)^{\kappa_r} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}\right)^{\kappa_y} \right)^{1-\rho_r} \exp\left(\varepsilon_{R,t}\right), \tag{A.30}$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\phi}{2} \tilde{Y}_t \left[\pi_t - \tilde{\pi}_t\right]^2. \tag{A.31}$$

Appendix A.7. Model Equations

The model economy is described by the following set of equations

$$\tilde{\lambda}_t = A_t \left(\tilde{C}_t - \chi \tilde{C}_{t-1}/\mu_t \right)^{-\sigma}, \tag{A.32}$$

$$\tilde{W}_t = \kappa N_t^{\eta}/\tilde{\lambda}_t, \tag{A.33}$$

$$\mathscr{M}_{t,t+1} = E_t \left\{ \beta d_{t+1} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \pi_{t+1} \mu_{t+1}} \right\}, \tag{A.34}$$

$$E_t \left\{ \mathscr{M}_{t,t+1} R_t \right\} = 1, \tag{A.35}$$

$$\tilde{Y}_t = N_t, \tag{A.36}$$

$$\left(\frac{\varepsilon}{\varepsilon - 1} \right) \tilde{W}_t - \exp\left(\varepsilon_{\pi,t} \right) - \left(\frac{\phi}{\varepsilon - 1} \right) \pi_t \left[\pi_t - \tilde{\pi}_t \right] + \dots$$

$$\dots + E_t \left\{ \left(\frac{\phi}{\varepsilon - 1} \right) \mathscr{M}_{t,t+1} \pi_{t+1}^2 \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \mu_{t+1} \left[\pi_{t+1} - \tilde{\pi}_{t+1} \right] \right\}, \tag{A.37}$$

$$R_t^* = R_{t-1}^{*\rho_r} \left(R^* \left(\frac{\pi_t}{\pi} \right)^{\kappa_r} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} \right)^{\kappa_y} \right)^{1-\rho_r} \exp\left(\varepsilon_{R,t} \right), \tag{A.38}$$

$$R_t = \mathbf{z}\left(s_t \right) R_{ZLB} + \left(1 - \mathbf{z}\left(s_t \right) \right) R_t^*, \tag{A.39}$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\phi}{2} \tilde{Y}_t \left[\pi_t - \tilde{\pi}_t \right]^2, \tag{A.40}$$

$$\log\left(A_t \right) = \rho_A \log\left(A_{t-1} \right) + \varepsilon_{A,t}, \tag{A.41}$$

$$\mu_t = \exp(g_Z + \varepsilon_{Z,t}). \tag{A.42}$$

Appendix A.8. Steady State

We set all time subscripts equal to t and solve for the deterministic steady state.

$$A_t = 1, \tag{A.43}$$

$$\pi_t = \pi, \tag{A.44}$$

$$\tilde{Y}_t = \tilde{Y}, \tag{A.45}$$

$$\tilde{C}_t = \tilde{Y}_t, \tag{A.46}$$

$$N_t = \tilde{Y}_t, \tag{A.47}$$

$$R_t^* = \frac{\pi_t \exp(g_Z)}{\beta}, \tag{A.48}$$

$$R_t = \mathbf{z}\left(s_t \right) R_{ZLB} + \left(1 - \mathbf{z}\left(s_t \right) \right) R_t^*, \tag{A.49}$$

$$d_{t+1} = \frac{\pi_t \exp(g_Z)}{R_t \beta}, \tag{A.50}$$

$$\mu_t = \exp(g_Z), \tag{A.51}$$

$$\tilde{\lambda}_t = \left(\tilde{C}_t \left(1 - \chi/\mu_t \right) \right)^{-\sigma}, \tag{A.52}$$

$$\mathscr{M}_{t,t+1} = 1/R_t, \tag{A.53}$$

$$\tilde{W}_t = \left(\frac{\varepsilon - 1}{\varepsilon} \right), \tag{A.54}$$

$$\kappa = \tilde{\lambda} \tilde{W}_t / N_t^\eta, \tag{A.55}$$

Appendix A.9. Calibration

Table A.1: Calibrated Parameters

Parameter	Value	Parameter	Value
χ	0.8	ρ_R	0.7
g_Z	0.001	ρ_A	0.8
π	1.005	R_{ZLB}	1
σ	2	σ_R	0.01
β	0.9975	σ_A	0.01
κ_π	1.5	σ_π	0.01
κ_Y	0.5	σ_Z	0.01
ε	6	$\psi_{N,Z}$	2000
ϕ	10	$\psi_{Z,N}$	2000
η	2	$\theta_{N,Z}$	1
ξ	0.5	$\theta_{Z,N}$	1

Appendix B. A Simple Housing DSGE Model with a Collateral Constraint

In this appendix we take the "toy" model from Iacoviello (2005) and make the minimum set of modifications to allow for occasionally binding constraints with regime-switching.

Appendix B.1. Patient Households

The model economy is inhabited by a representative patient household. The representative patient household's utility is given by

$$U_0' = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log C_t' + j \log H_t' - \frac{L_t'^{1+\eta}}{1+\eta} \right] \right\} \tag{B.1}$$

The patient household derives positive utility from the consumption of final goods (C_t') and housing services (H_t'), and disutility from working (L_t'), The patient household maximizes

the expected present value of their current and future utilities by choosing allocations of consumption, housing, hours worked and loans issued $(-B'_t)$, subject to the following period by period budget constraint

$$C'_t + Q_t \left(H'_t - H'_{t-1} \right) + \frac{R_{t-1}B'_{t-1}}{\pi_t} = B'_t + W'_t L'_t + F'_t + T'_t \tag{B.2}$$

where Q_t is the real house price, R_t is the gross nominal interest rate, π_t is the gross rate of inflation, W'_t is the real wage, F'_t is dividends paid to patient households by firms (patient household's own firms) and T'_t is adjustment costs rebated to patient households. We can write the patient household's problem more formally as the Lagrangian

$$\mathcal{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \log C'_t + j \log H'_t - \frac{L'^{1+\eta}_t}{1+\eta} - \dots \\ \dots - \lambda'_t \left[C'_t + Q_t \left(H'_t - H'_{t-1} \right) + \frac{R_{t-1}B'_{t-1}}{\pi_t} - B'_t - W'_t L'_t - F'_t - T'_t \right] \end{array} \right] \right\} \tag{B.3}$$

Optimization results in the following set of first order conditions

$$\frac{\partial \mathcal{L}_t}{\partial C'_t} = \frac{1}{C'_t} - \lambda'_t = 0, \tag{B.4}$$

$$\frac{\partial \mathcal{L}_t}{\partial H'_t} = \frac{j}{H'_t} - \lambda'_t Q_t + E_t \left\{ \beta \lambda'_{t+1} Q_{t+1} \right\} = 0, \tag{B.5}$$

$$\frac{\partial \mathcal{L}_t}{\partial B'_t} = \lambda'_t - E_t \left\{ \beta \lambda'_{t+1} \frac{R_t}{\pi_{t+1}} \right\} = 0, \tag{B.6}$$

$$\frac{\partial \mathcal{L}_t}{\partial L'_t} = -L'^{\eta}_t + \lambda'_t W'_t = 0. \tag{B.7}$$

Combining (B.4) and (B.6) gives the consumption Euler equation

$$\frac{1}{C'_t} = E_t \left\{ \beta \frac{R_t}{\pi_{t+1} C'_{t+1}} \right\}. \tag{B.8}$$

Combining (B.4) and (B.5) gives the user cost of housing capital for patient households

$$\frac{Q_t}{C'_t} = \frac{j}{H'_t} + E_t \left\{ \beta \frac{Q_{t+1}}{C'_{t+1}} \right\}. \tag{B.9}$$

Combining (B.4) and (B.7) gives the real marginal rate of substitution

$$W'_t = \frac{L'^{\eta}_t}{C'_t}. \tag{B.10}$$

Appendix B.2. Entrepreneurs

The model economy is also inhabited by a representative entrepreneur that produces an intermediate good using neutral technology, housing services and labor supplied by patient households. Entrepreneurs produce intermediate goods according to the following Cobb Douglas production technology

$$Y_t = A_t H_{t-1}^{\nu} L_t^{1-\nu}, \tag{B.11}$$

with

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A) \log A + \varepsilon_{A,t}, \tag{B.12}$$

where Y_t is output, A_t is neutral technology, H_t is the entrepreneurs demand for housing services and L_t is entrepreneurs demand for labor. The representative entrepreneur gains utility from consuming the final good. The expected present value of their current and future utility is given by

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \gamma^t \log C_t \right\}, \tag{B.13}$$

where C_t is the entrepreneur's consumption of the final good and the entrepreneurs's rate of time preference satisfies $\gamma < \beta$, that is entrepreneurs are more impatient that patient households. The representative entrepreneur maximizes their lifetime utility by choosing quantities of consumption, housing, debt holdings (B_t) and labor demanded subject to the following period by period budget constraint

$$MC_t Y_t + B_t = C_t + (1 + \tau(s_t)) Q_t (H_t - H_{t-1}) + \frac{R_{t-1}^* B_{t-1}}{\pi_t} + W_t' L_t + T_t, \tag{B.14}$$

where MC_t is the real marginal cost of producing output, T_t is a lump sum tax or transfer and R_t^* is the effective interest rate faced by entrepreneurs.

$$R_t^* = R_t \psi_1 \exp \left(\psi_2 \left(B_t R_t - m Q_{t+1} H_t \pi_{t+1} \right) \right) \tag{B.15}$$

We include a time varying tax, $\tau(s_t)$, to offset the externalities of the collateral constraint in the binding equilibrium. As a consequence our specification of the entrepreneur's problem differs from Iacoviello (2005). Entrepreneurs are also subject to the following borrowing/collateral constraint

$$B_t \leq E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} \tag{B.16}$$

We formalize the entrepreneur's optimization problem as the following Lagrangian:

$$
\mathscr{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \gamma^t \left[\ldots + \lambda_t \left[\begin{array}{c} \log C_t + \ldots \\ MC_t A_t H_{t-1}^{\nu} L_t^{1-\nu} + B_t - C_t - \ldots \\ \ldots - (1 + \tau(s_t)) Q_t (H_t - H_{t-1}) - \ldots \\ \ldots - \dfrac{R_{t-1}^* B_{t-1}}{\pi_t} - W_t' L_t - T_t \\ \ldots - \Omega_t [R_t B_t - E_t \{ m Q_{t+1} H_t \pi_{t+1} \}] \end{array} \right] + \ldots \right] \right\} \quad \text{(B.17)}
$$

We obtain the following set of first-order conditions for the entrepreneur

$$
\frac{\partial \mathscr{L}_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0, \quad \text{(B.18)}
$$

$$
\frac{\partial \mathscr{L}_t}{\partial H_t} = -\lambda_t (1 + \tau(s_t)) Q_t + \ldots \quad \text{(B.19)}
$$

$$
\ldots + E_t \left\{ \gamma \lambda_{t+1} \left(\nu MC_{t+1} \frac{Y_{t+1}}{H_t} + (1 + \tau(s_{t+1})) Q_{t+1} \right) + \Omega_t m Q_{t+1} \pi_{t+1} \right\} = 0, \quad \text{(B.20)}
$$

$$
\frac{\partial \mathscr{L}_t}{\partial B_t} = \lambda_t - \Omega_t R_t - E_t \left\{ \gamma \lambda_{t+1} \frac{R_t^*}{\pi_{t+1}} \right\} = 0, \quad \text{(B.21)}
$$

$$
\frac{\partial \mathscr{L}_t}{\partial L_t} = (1 - \nu) MC_t \frac{Y_t}{L_t} - W_t' = 0. \quad \text{(B.22)}
$$

$$
\text{(B.23)}
$$

Combining (B.18) and (B.21) results in the entrepreneur's consumption Euler equation

$$
\frac{1}{C_t} = E_t \left\{ \gamma \frac{R_t^*}{\pi_{t+1} C_{t+1}} \right\} + \Omega_t R_t \quad \text{(B.24)}
$$

Combining (B.18) and (B.20) gives us an expression for the entrepreneur's user cost of housing capital

$$
(1 + \tau(s_t)) \frac{Q_t}{C_t} = E_t \left\{ \frac{\gamma}{C_{t+1}} \left(\nu MC_{t+1} \frac{Y_{t+1}}{H_t} + (1 + \tau(s_{t+1})) Q_{t+1} \right) + \Omega_t m Q_{t+1} \pi_{t+1} \right\} \quad \text{(B.25)}
$$

From (B.18) and (B.22) we obtain the usual result that workers are paid their marginal product of labor

$$
W_t' = (1 - \nu) MC_t \frac{Y_t}{L_t} \quad \text{(B.26)}
$$

Finally we have the complementary slackness conditions

$$
\Omega_t \left(B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} \right) = 0, \quad \text{(B.27)}
$$

with

$$B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} = 0, \quad \Omega_t > 0, \tag{B.28}$$

or

$$B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} < 0, \quad \Omega_t = 0. \tag{B.29}$$

Appendix B.3. Retailers

There is a continuum of retailers, with unit mass, that buy the intermediate good from entrepreneurs, differentiate the good at no additional cost and sell the good to a perfectly competitive final goods producer. Differentiation gives the retailers a degree of market power and the ability to set their prices. Retailers choose prices to maximize profits subject to a quadratic cost on adjusting prices. The profit function for the ith retailer is given by

$$\Lambda_0(i) = E_0 \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \left[\frac{P_t(i)}{P_t} Y_t(i) - MC_t Y_t(i) - \frac{\phi_P}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right]^2 \right] \right\}$$

Retailers also take into account how their pricing decisions impact the demand for their own variety of intermediate good. We capture this behavior by making the following substitution $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$, for the firm's own output.

$$\Lambda_0(i) = E_0 \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \left[P_t(i)^{1-\varepsilon} P_t^{\varepsilon-1} Y_t - MC_t P_t(i)^{-\varepsilon} P_t^{\varepsilon} Y_t - \frac{\phi_P}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right]^2 \right] \right\}$$

The ith retailer's first-order condition with respect to prices

$$\frac{\partial \Lambda_t(i)}{\partial P_t(i)} = (1 - \varepsilon) \frac{Y_t(i)}{P_t} + \varepsilon MC_t \frac{Y_t(i)}{P_t(i)} - \phi_P \frac{Y_t}{P_{t-1}(i)} \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right] + \dots$$
$$\dots + E_t \left\{ \phi_P \mathcal{M}_{t,t+1} Y_{t+1} \frac{P_{t+1}(i)}{P_t(i)^2} \left[\frac{P_{t+1}(i)}{P_t(i)} - \tilde{\pi}_{t+1} \right] \right\} = 0$$

In a symmetric equilibrium we obtain the price Phillips curve

$$\left(\frac{\phi_P}{\varepsilon - 1} \right) \pi_t [\pi_t - \tilde{\pi}_t] = \left(\frac{\varepsilon}{\varepsilon - 1} \right) MC_t - 1 + E_t \left\{ \left(\frac{\phi_P}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1} \frac{Y_{t+1}}{Y_t} \pi_{t+1} [\pi_{t+1} - \tilde{\pi}_{t+1}] \right\} \tag{B.30}$$

Appendix B.4. Monetary Policy

The monetary authority sets interest rates according to the following Taylor-type rule

$$R_t = R_{t-1}^{\rho_R} \left(R \left(\frac{\pi_t}{\pi} \right)^{1+r_\pi} \left(\frac{Y_t}{Y} \right)^{r_Y} \right)^{1-\rho_R} \exp(\varepsilon_{R,t}) \tag{B.31}$$

Appendix B.5. Equilibrium

In equilibrium we obtain

$$Y_t = C_t + C_t'. \tag{B.32}$$

We also assume that the entire stock of housing is fixed so that

$$h = H_t + H_t'. \tag{B.33}$$

Finally the lump sum tax transfer is set so that

$$T_t = -\tau(s_t) Q_t (H_t - H_{t-1}) \tag{B.34}$$

Appendix B.6. An Occasionally Binding Collateral Constraint Under Regime-Switching

In order to model the occasionally binding constraint we augment the model with a Markov chain with two states, a binding state (B) and a non-binding state (N). We include an additional regime-switching parameter $\mathbf{o}(s_t)$ which takes the values

$$\mathbf{o}(N) = 0, \quad \mathbf{o}(B) = 1 \tag{B.35}$$

We then replace the complementary slackness condition (B.27) with

$$\mathbf{o}(s_t) \left(B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} \right) + (1 - \mathbf{o}(s_t)) \Omega_t = 0, \tag{B.36}$$

and we define the transition probabilities as follows

$$p_{NB,t} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp(-\psi_{N,B} B_t^*)}, \quad p_{BN,t} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(\psi_{B,N} \hat{\Omega}_t\right)} \tag{B.37}$$

where

$$B_t^* = B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\}, \tag{B.38}$$

is a measure of leverage and

$$\hat{\Omega}_t = \tilde{\Omega}_t - \tilde{\Omega}, \text{ and } \tilde{\Omega}_t = \Omega_t, \tag{B.39}$$

is a measure of the shadow price on the collateral constraint. The tax rate "τ" is chosen so that

$$\tau(s_t) = \mathbf{o}(s_t) m \left(\frac{\beta - \gamma}{1 - \gamma} \right). \tag{B.40}$$

The parameter ψ_1 is determined according to

$$\psi_1 = \mathbf{o}(s_t) + (1 - \mathbf{o}(s_t)) \beta/\gamma. \tag{B.41}$$

Appendix B.7. Model Equations

The 18 model variables: Y_t, C_t, C'_t, R_t, π_t, B_t, W'_t, L_t, H'_t, H_t, Q_t, MC_t, A_t, Ω_t, R^*_t, B^*_t, $\hat{\Omega}_t$ and $\tilde{\Omega}_t$, can be explained by the 18 model equations:

$$Y_t = C_t + C'_t, \tag{B.42}$$

$$\frac{1}{C'_t} = E_t\left\{\beta\frac{R_t}{\pi_{t+1}C'_{t+1}}\right\}, \tag{B.43}$$

$$MC_tY_t + B_t = C_t + Q_t(H_t - H_{t-1}) + \frac{R^*_{t-1}B_{t-1}}{\pi_t} + W'_tL_t, \tag{B.44}$$

$$\frac{Q_t}{C'_t} = \frac{j}{H'_t} + E_t\left\{\beta\frac{Q_{t+1}}{C'_{t+1}}\right\}, \tag{B.45}$$

$$(1 + \tau(s_t))\frac{Q_t}{C_t} = E_t\left\{\frac{\gamma}{C_{t+1}}\left(\nu MC_{t+1}\frac{Y_{t+1}}{H_t} + (1 + \tau(s_{t+1}))Q_{t+1}\right) + \Omega_t m Q_{t+1}\pi_{t+1}\right\}, \tag{B.46}$$

$$B^*_t = B_t - E_t\left\{m\frac{Q_{t+1}H_t\pi_{t+1}}{R_t}\right\}, \tag{B.47}$$

$$\mathbf{o}(s_t)\left(B_t - E_t\left\{m\frac{Q_{t+1}H_t\pi_{t+1}}{R_t}\right\}\right) + (1 - \mathbf{o}(s_t))\Omega_t = 0, \tag{B.48}$$

$$\hat{\Omega}_t = \tilde{\Omega}_t - \tilde{\Omega}, \tag{B.49}$$

$$\tilde{\Omega}_t = \Omega_t, \tag{B.50}$$

$$Y_t = A_tH^\nu_{t-1}L^{1-\nu}_t, \tag{B.51}$$

$$W'_t = \frac{L'^\eta_t}{C'_t}, \tag{B.52}$$

$$W'_t = (1 - \nu)MC_t\frac{Y_t}{L_t}, \tag{B.53}$$

$$h = H_t + H'_t, \tag{B.54}$$

$$\left(\frac{\phi_P}{\varepsilon - 1}\right)\pi_t[\pi_t - \tilde{\pi}_t] = \left(\frac{\varepsilon}{\varepsilon - 1}\right)MC_t - 1 + E_t\left\{\left(\frac{\phi_P}{\varepsilon - 1}\right)\mathcal{M}_{t,t+1}\frac{Y_{t+1}}{Y_t}\pi_{t+1}[\pi_{t+1} - \tilde{\pi}_{t+1}]\right\}, \tag{B.55}$$

$$\frac{1}{C_t} = E_t\left\{\gamma\frac{R^*_t}{\pi_{t+1}C_{t+1}}\right\} + \Omega_tR_t, \tag{B.56}$$

$$R_t = R^{\rho_R}_{t-1}\left(R\left(\frac{\pi_t}{\pi}\right)^{\kappa_\pi}\left(\frac{Y_t}{Y}\right)^{\kappa_Y}\right)^{1-\rho_R}\exp(\varepsilon_{R,t}) \tag{B.57}$$

$$R^*_t = R_t\psi_1\exp(\psi_2(B_tR_t - mQ_{t+1}H_t\pi_{t+1})), \tag{B.58}$$

$$\log A_t = \rho_A\log A_{t-1} + (1 - \rho_A)\log A + \varepsilon_{A,t}. \tag{B.59}$$

Appendix B.8. Steady State

Following the strategy taken by Guerrieri & Iacoviello (2015a) we will solve the steady state model for the following ratios: $h\_y \equiv H_t/Y_t$, $h\_p\_y \equiv H'_t/Y_t$, $c\_y \equiv C_t/Y_t$, $c\_p\_y \equiv C'_t/Y_t$ and $b\_y \equiv B_t/Y_t$, before solving for the level variables.

$$Q_t = 1, \tag{B.60}$$

$$MC_t = \left(\frac{\varepsilon - 1}{\varepsilon} \right), \tag{B.61}$$

$$\pi_t = \pi, \tag{B.62}$$

$$R_t = \frac{\pi_t}{\beta}, \tag{B.63}$$

$$h\_y = \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\gamma \nu}{(1 - \gamma) Q_t} \right), \tag{B.64}$$

$$\tau(s_t) = \mathbf{o}(s_t) \left(\frac{\beta - \gamma}{1 - \gamma} \right), \tag{B.65}$$

$$b\_y = \beta m Q_t \times h\_y, \tag{B.66}$$

$$c\_y = b\_y \left(1 - \frac{1}{\beta} \right) + \mu MC_t, \tag{B.67}$$

$$c\_p\_y = 1 - c\_y, \tag{B.68}$$

$$j = 4 \times Q_t (1 - \beta) h\_y / c\_p\_y, \tag{B.69}$$

$$h\_p\_y = c\_p_y \times \frac{j}{Q_t (1 - \beta)}, \tag{B.70}$$

$$L_t = \left(\frac{(1 - \nu) MC_t}{c\_p\_y} \right)^{\frac{1}{\varepsilon}}, \tag{B.71}$$

$$Y_t = 1, \tag{B.72}$$

$$H_t = h\_y \times Y_t, \tag{B.73}$$

$$H'_t = h\_p\_y \times Y_t, \tag{B.74}$$

$$B_t = b\_y \times Y_t, \tag{B.75}$$

$$C_t = c\_y \times Y_t, \tag{B.76}$$

$$C'_t = c\_p\_y \times Y_t, \tag{B.77}$$

$$h = H_t + H'_t, \tag{B.78}$$

$$A_t = \frac{Y_t}{H_t^\nu L_t^{1-\nu}}, \tag{B.79}$$

$$W'_t = L_t^{\eta-1}C'_t, \tag{B.80}$$

$$\psi_1 = \mathbf{o}\,(s_t) + (1 - \mathbf{o}\,(s_t))\,\beta/\gamma, \tag{B.81}$$

$$\psi_2 = (1 - \mathbf{o}\,(s_t))\,\tilde{\psi}_2, \tag{B.82}$$

$$R_t^* = R_t\psi_1, \tag{B.83}$$

$$\Omega_t = \mathbf{o}\,(s_t)\left(\frac{\beta-\gamma}{\pi_t C_t}\right), \tag{B.84}$$

$$B_t^* = 0, \tag{B.85}$$

$$\tilde{\Omega}_t = \left(\frac{\beta-\gamma}{\pi_t C_t}\right), \tag{B.86}$$

$$\hat{\Omega}_t = 0. \tag{B.87}$$

Note that in the original model by Iacoviello (2005) the impatient housing to output ratio is given by

$$h\_y = \left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\gamma\nu}{Q_t\left(1 - \gamma - (\beta-\gamma)\,m\right)}\right) \tag{B.88}$$

In our setup we have

$$h\_y = \left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\gamma\nu}{(1-\gamma)Q_t}\right), \tag{B.89}$$

which is the level of housing that would occur in a world without a debt constraint. This is level consistent with the regime where the constraint is not binding. When the constraint binds the

$$\tau = 1 + \mathbf{o}\,(s_t)\,m\left(\frac{\beta-\gamma}{1-\gamma}\right), \tag{B.90}$$

Appendix B.9. Calibration

Table B.2: Calibrated Parameters

Parameter	Value	Parameter	Value	Parameter	Value
β	0.99	σ_A	0.005	$\psi_{N,B}$	2000
γ	0.99	σ_π	0.005	$\psi_{B,N}$	2000
ν	0.03	$\bar{\psi}_2$	0	$\theta_{N,B}$	1
m	0.89	ϕ_Y	0	$\theta_{B,N}$	1
η	1.01	ϕ_Q	0		
ρ_R	0.73	ϕ_B	0		
r_π	0.27	ω_R	0.01		
r_Y	0	ω_Y	0.5		
ρ_A	0.5	ϕ	10		
σ_R	0.005	ε	6		

Appendix C. A Simple Model With Downward Nominal Wage Rigidities

In this section we develop a simple New Keynesian DSGE model where households choose wages subject to both a quadratic adjustment cost and a non-negativity constraint on changing wages. Such non-negativity constraints on adjusting wages have also been explored by Amano & Gnocchi (2017) using piecewise linear methods.

Appendix C.1. Households

The model economy is populated by a continuum of households, indexed by i and normalized to unity. The net present value of the ith household's utility is given by

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t - \chi \bar{C}_{t-1}\right)^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t(i)^{1+\eta}}{1+\eta} \right] \right\}. \tag{C.1}$$

The ith household's utility is increasing in consumption (C_t), relative to last period's aggregate consumption ($\bar{C}_t$), and decreasing in hours worked ($N_t(i)$). Labor is differentiated with each household supplying their own variety of labor to a labor union that aggregates the labor inputs into a homogenous labor output. Labor is aggregated according to the following Dixit-Stiglitz aggregator function

$$N_t = \left[\int_0^1 N_t(i)^{1-\frac{1}{\upsilon}} di \right]^{\frac{\upsilon}{\upsilon-1}},$$

where N_t is aggregate labor and $\upsilon > 1$ the elasticity of substitution between differentiated labor inputs. Cost minimization by the labor union implies the following demand function for the ith household's labor variety

$$N_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\upsilon} N_t,$$

where $W_t(i)$ is the nominal wage paid for labor from the ith household and W_t is the aggregate nominal wage. The ith household maximizes their utility subject to a period by period budget constraint, which includes a quadratic cost on adjusting wages

$$C_t + B_t = \frac{B_{t-1} R_{t-1}}{\pi_t} + \frac{W_t(i)}{P_t} N_t(i) - \frac{\phi_W}{2} \frac{W_t}{P_t} N_t \left[\frac{W_t(i)}{W_{t-1}(i)} - \tilde{\pi}_{W,t} \right]^2 + \Phi_t + \Psi_t, \tag{C.2}$$

where B_t is debt, R_t the gross interest rate, π_t the gross rate of inflation, P_t the price level, $\pi_{W,t}$ wage inflation, $\tilde{\pi}_{W,t}$ is a wage inflation reference index, such that

$$\tilde{\pi}_{W,t} = \pi_{W,t-1}^{\xi_W} \pi_W^{1-\xi_W},$$

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and Ψ_t is price and wage adjustment costs rebated to households,

$$\Psi_t = \frac{\phi_W}{2}\frac{W_t}{P_t}N_t\left[\pi_{W,t} - \tilde{\pi}_{W,t}\right]^2 + \frac{\phi_P}{2}Y_t\left[\pi_t - \tilde{\pi}_t\right]^2,$$

where $\tilde{\pi}_t$ is an inflation reference index, such that

$$\tilde{\pi}_t = \pi_{t-1}^{\xi_P}\pi^{1-\xi_P}.$$

Following Amano & Gnocchi (2017) we formulate the downward nominal wage rigidity as an inequality constraint on households wage setting problem. This implies households maximize utility subject to the following non-negativity constraint

$$\frac{W_t(i)}{W_{t-1}(i)} - 1 \geq 0. \tag{C.3}$$

Households maximize their utility subject to the budget constraint and the non-negativity constraint on adjusting wages, by choosing consumption, debt holdings and the wage level for their variety of labor. This can be expressed more concretely as the Lagrangian

$$\mathscr{L}_0 = E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}\left[\begin{array}{c}\frac{\left(C_t - \chi\bar{C}_{t-1}\right)^{1-\sigma}}{1-\sigma} - \kappa\frac{N_t(i)^{1+\eta}}{1+\eta} - \dots \\ -\lambda_t\left[\begin{array}{c}C_t + B_t - \frac{B_{t-1}R_{t-1}}{\pi_t} - \frac{W_t(i)}{P_t}N_t(i) + \dots \\ \dots + \frac{\phi_W}{2}W_tN_t\left[\frac{W_t(i)}{W_{t-1}(i)} - \tilde{\pi}_{W,t}\right]^2 - \Phi_t - \Xi_t\end{array}\right] \\ \dots + \Omega_t\left(\frac{W_t(i)}{W_{t-1}(i)} - 1\right)\end{array}\right] + \dots\end{array}\right]\right\} \tag{C.4}$$

The inequality constraint results in the Kuhn-Tucker conditions, where

$$\Omega_t\left(\frac{W_t(i)}{W_{t-1}(i)} - 1\right) = 0, \tag{C.5}$$

is the complementary slackness condition,

$$\frac{W_t(i)}{W_{t-1}(i)} - 1 \geq 0, \tag{C.6}$$

is the primal feasibility condition and

$$\Omega_t \geq 0, \tag{C.7}$$

is the dual feasibility condition. Labor is differentiated giving each household a degree of market power when setting wages. As a consequence households take into account how their

choice of wage affects the demand for their variety of labor. Substituting in the labor union's demand for the ith variety of labor gives

$$
\mathscr{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \dfrac{(C_t - \chi \bar{C}_{t-1})^{1-\sigma}}{1-\sigma} - \kappa \dfrac{\left(\left(\frac{W_t(i)}{W_t} \right)^{-\upsilon} N_t \right)^{1+\eta}}{1+\eta} - \dots \\ \dots - \lambda_t \left[\begin{array}{c} C_t + B_t - \dfrac{B_{t-1} R_{t-1}}{\pi_t} - \dfrac{W_t(i)^{1-\upsilon} W_t^\upsilon}{P_t} N_t + \dots \\ \dots + \dfrac{\phi_W}{2} \dfrac{W_t}{P_t} N_t \left[\dfrac{W_t(i)}{W_{t-1}(i)} - \tilde{\pi}_{W,t} \right]^2 - \Phi_t - \Xi_t \\ \dots + \Omega_t \left(\dfrac{W_t(i)}{W_{t-1}(i)} - 1 \right) \end{array} \right] + \dots \end{array} \right] \right\} \quad \text{(C.8)}
$$

Optimization by the ith household results in the following set of first-order conditions:

$$
\frac{\partial \mathscr{L}_t}{\partial C_t} = (C_t - \chi C_{t-1})^{-\sigma} - \lambda_t = 0, \quad \text{(C.9)}
$$

$$
\frac{\partial \mathscr{L}_t}{\partial B_t} = -\lambda_t + E_t \left\{ \beta \frac{\lambda_{t+1}}{\pi_{t+1}} \right\} = 0, \quad \text{(C.10)}
$$

$$
\frac{\partial \mathscr{L}_t}{\partial W_t(i)} = \upsilon \kappa \frac{N_t(i)^{1+\eta}}{W_t(i)} + \frac{\lambda_t}{P_t} (1 - \upsilon) N_t(i) - \frac{\lambda_t}{P_t} \phi_W \frac{W_t}{W_{t-1}(i)} N_t \left[\frac{W_t(i)}{W_{t-1}(i)} - \tilde{\pi}_{W,t} \right] + \frac{\Omega_t}{W_{t-1}(i)} + \dots
$$
$$
\dots + E_t \left\{ \beta \phi_W \frac{\lambda_{t+1}}{P_{t+1}} \frac{W_{t+1} W_{t+1}(i)}{W_t(i)^2} N_{t+1} \left[\frac{W_{t+1}(i)}{W_t(i)} - \tilde{\pi}_{W,t+1} \right] - \beta \Omega_{t+1} \frac{W_{t+1}(i)}{W_t(i)^2} \right\} = 0 \quad \text{(C.11)}
$$

From equation (C.9) we get

$$
\lambda_t = (C_t - \chi C_{t-1})^{-\sigma}. \quad \text{(C.12)}
$$

We define the real stochastic discount factor as follows

$$
\mathscr{M}_{t,t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}, \quad \text{(C.13)}
$$

which when combined with (C.10) leads to

$$
E_t \left\{ \mathscr{M}_{t,t+1} \frac{R_t}{\pi_{t+1}} \right\} = 1. \quad \text{(C.14)}
$$

Rearranging (C.11) gives

$$\left(\frac{\upsilon}{\upsilon-1}\right)\kappa\frac{N_t^{\eta}}{\lambda_t\tilde{W}_t}-1-\left(\frac{\phi_W}{\upsilon-1}\right)\pi_{W,t}\left[\pi_{W,t}-\tilde{\pi}_{W,t}\right]+\left(\frac{1}{\upsilon-1}\right)\frac{P_t\Omega_t}{\lambda_t W_{t-1}N_t}+\dots$$

$$\dots+E_t\left\{\beta\left(\frac{\phi_W}{\upsilon-1}\right)\frac{\lambda_{t+1}}{\lambda_t}\frac{\pi_{W,t+1}^2}{\pi_{t+1}}\frac{N_{t+1}}{N_t}\left[\pi_{W,t+1}-\tilde{\pi}_{W,t+1}\right]-\left(\frac{\beta}{\upsilon-1}\right)P_t\Omega_{t+1}\frac{W_{t+1}}{\lambda_t N_t W_t^2}\right\}=0,$$
(C.15)

assuming a symmetric equilibrium and substituting in (C.15) gives

$$\left(\frac{\upsilon}{\upsilon-1}\right)\kappa\frac{N_t^{\eta}}{\lambda_t\tilde{W}_t}-1-\left(\frac{\phi_W}{\upsilon-1}\right)\pi_{W,t}\left[\pi_{W,t}-\tilde{\pi}_{W,t}\right]+\left(\frac{1}{\upsilon-1}\right)\frac{\pi_t\Omega_t}{\lambda_t\tilde{W}_{t-1}N_t}+\dots$$

$$\dots+E_t\left\{\left(\frac{\phi_W}{\upsilon-1}\right)\mathcal{M}_{t,t+1}\frac{\pi_{W,t+1}^2}{\pi_{t+1}}\frac{N_{t+1}}{N_t}\left[\pi_{W,t+1}-\tilde{\pi}_{W,t+1}\right]-\left(\frac{\beta}{\upsilon-1}\right)\frac{\pi_{W,t+1}\Omega_{t+1}}{\lambda_t\tilde{W}_t N_t}\right\}=0.$$
(C.16)

Appendix C.2. Intermediate Goods Producers

The economy is inhabited by a continuum of firms, indexed by j and normalized to unit mass. Each firm produces a differentiated intermediate input ($Y_t(j)$) using labor ($N_t(j)$) and a common neutral technology (A_t), according to the Cobb Douglas production technology

$$Y_t(j)=A_t N_t(j),$$
(C.17)

where the neutral technology evolves according to the following process

$$\log A_t=\rho_A\log A_{t-1}+(1-\rho_A)\log A+\varepsilon_{A,t}.$$
(C.18)

Firms supply their differentiated intermediate input to a perfectly competitive final goods producing firm. Cost minimization by the final goods producer gives the demand function for the jth firm's output

$$Y_t(j)=\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}Y_t.$$
(C.19)

The net present value of the jth firm's profits can be expressed as follows

$$\Upsilon_0(j)=E_0\left\{\sum_{t=0}^{\infty}\mathcal{M}_{0,t}\left[\frac{P_t(j)}{P_t}Y_t(j)-\frac{W_t}{P_t}N_t(j)-\frac{\phi_P}{2}Y_t\left[\frac{P_t(j)}{P_{t-1}(j)}-\tilde{\pi}_t\right]^2\right]\right\},$$
(C.20)

where $P_t(j)$ is the price for goods produced by the jth firm, P_t is the aggregate price level and firms face a quadratic cost when changing prices. Firms maximize profits by choosing prices subject to their production technology (C.17) and the demand for their variety of output. Substituting the production function (C.17) and demand for the jth firm's intermediate

(C.19) good into the profit function (C.20) gives

$$\Upsilon_0(j) = E_0 \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \left[P_t(j)^{1-\varepsilon} P_t^{\varepsilon-1} Y_t - \left(\frac{W_t}{P_t A_t} \right) P_t(j)^{-\varepsilon} P_t^{\varepsilon} Y_t - \frac{\phi_P}{2} Y_t \left[\frac{P_t(j)}{P_{t-1}(j)} - \tilde{\pi}_t \right]^2 \right] \right\}. \tag{C.21}$$

Optimization by the jth firm leads to

$$\frac{\partial \Upsilon_t(j)}{\partial P_t(j)} = (1 - \varepsilon) \frac{Y_t(j)}{P_t} + \varepsilon \left(\frac{W_t}{P_t A_t} \right) \frac{Y_t(j)}{P_t(j)} - \phi_P \frac{Y_t}{P_{t-1}(j)} \left[\frac{P_t(j)}{P_{t-1}(j)} - \tilde{\pi}_t \right] + \dots$$

$$\dots + E_t \left\{ \phi_P \mathcal{M}_{t,t+1} Y_{t+1} \frac{P_{t+1}(j)}{P_t(j)^2} \left[\frac{P_{t+1}(j)}{P_t(j)} - \tilde{\pi}_{t+1} \right] \right\} = 0. \tag{C.22}$$

Assuming a symmetric equilibrium results in the Phillips curve

$$\left(\frac{\phi_P}{\varepsilon - 1} \right) \pi_t \left[\pi_t - \tilde{\pi}_t \right] = \left(\frac{\varepsilon}{\varepsilon - 1} \right) MC_t - 1 + E_t \left\{ \left(\frac{\phi_P}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1} \frac{Y_{t+1}}{Y_t} \pi_{t+1} \left[\pi_{t+1} - \tilde{\pi}_{t+1} \right] \right\}, \tag{C.23}$$

where $MC_t \equiv \left(\frac{\tilde{W}_t}{A_t} \right)$.

Appendix C.3. Monetary Policy

The monetary authority sets interest rates according a Taylor-type rule of the form

$$R_t = R_{t-1}^{\rho_R} \left(R \left(\frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left(\frac{Y_t}{Y} \right)^{\kappa_Y} \right)^{1-\rho_R} \exp(\varepsilon_{R,t}). \tag{C.24}$$

Appendix C.4. Equilibrium

In a symmetric equilibrium we have $N_t = \int_0^1 N_t(i)di$, $Y_t = \int_0^1 Y_t(j)dj$, $N_t = \int_0^1 N_t(j)dj$, $B_t = 0$ and from (C.2) we obtain

$$Y_t = C_t. \tag{C.25}$$

Appendix C.5. Occasionally Binding Constraints Under Regime-Switching

We approximate the occasionally binding non-negativity constraint on wage setting by reframing the problem as a regime-switching problem. To do so we augment the model with a Markov chain with two states: B when the constraint is binding and N when the constraint is not binding. We introduce a new state dependent parameter, $\mathbf{o}(s_t)$, which takes the values

$$\mathbf{o}(N) = 0, \quad \mathbf{o}(B) = 1, \tag{C.26}$$

in each of the states. Making use of (C.26) allows us to replace the complementary slackness condition (C.5) with

$$\mathbf{o}(s_t)(\pi_{W,t} - 1) + (1 - \mathbf{o}(s_t))\hat{\Omega}_t = 0, \tag{C.27}$$

where

$$\hat{\Omega}_t = \tilde{\Omega}_t - \tilde{\Omega}, \quad \text{and} \quad \tilde{\Omega}_t = \Omega_t, \tag{C.28}$$

The probability of switching between states is determined by the transition matrix

$$\mathbb{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{NB,t} & p_{NB,t} \\ p_{BN,t} & 1 - p_{BN,t} \end{bmatrix}, \tag{C.29}$$

with the endogenous transition probabilities

$$p_{NB,t} = \frac{\theta_{NB}}{\theta_{NB} + \exp\left(\psi_{NB}\left(\pi_{W,t} - 1\right)\right)}, \quad p_{BN,t} = \frac{\theta_{BN}}{\theta_{BN} + \exp\left(-\psi_{BN}\hat{\Omega}_t\right)}. \tag{C.30}$$

We assume these forms for the transition probabilities because they capture the nature of the primal feasibility (C.6) and dual feasibility (C.7) conditions. Namely, when the constraint is binding, we would expect that any decrease in the Lagrange multiplier on the constraint would imply that the probability of the constraint not binding would be increasing. Likewise, when the constraint is not binding, we would assume that if the rate of wage inflation were falling, the probability of the constraint binding should increase.

Appendix C.6. Model Equations

The model consists of 13 endogenous variables: λ_t, Y_t, $\mathcal{M}_{t,t+1}$, π_t, R_t, W_t, MC_t, N_t, A_t $\pi_{W,t}$, Ω_t $\tilde{\Omega}_t$, $\hat{\Omega}_t$, and 13 equations:

$$\lambda_t = (Y_t - \chi Y_{t-1})^{-\sigma}, \tag{C.31}$$

$$\mathcal{M}_{t,t+1} \equiv E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\right\}, \tag{C.32}$$

$$E_t\left\{\mathcal{M}_{t,t+1}\frac{R_t}{\pi_{t+1}}\right\} = 1, \tag{C.33}$$

$$\left(\frac{\upsilon}{\upsilon-1}\right)\kappa\frac{N_t^\eta}{\lambda_t\tilde{W}_t} - 1 - \left(\frac{\phi_W}{\upsilon-1}\right)\pi_{W,t}\left[\pi_{W,t} - \tilde{\pi}_{W,t}\right] + \left(\frac{1}{\upsilon-1}\right)\frac{\pi_t\Omega_t}{\lambda_t\tilde{W}_{t-1}N_t} + \dots$$

$$\dots + E_t\left\{\left(\frac{\phi_W}{\upsilon-1}\right)\mathcal{M}_{t,t+1}\frac{\pi_{W,t+1}^2}{\pi_{t+1}}\frac{N_{t+1}}{N_t}\left[\pi_{W,t+1} - \tilde{\pi}_{W,t+1}\right] - \left(\frac{\beta}{\upsilon-1}\right)\frac{\pi_{W,t+1}\Omega_{t+1}}{\lambda_t\tilde{W}_tN_t}\right\} = 0, \tag{C.34}$$

$$\left(\frac{\phi_P}{\varepsilon-1}\right)\pi_t\left[\pi_t - \tilde{\pi}_t\right] = \left(\frac{\varepsilon}{\varepsilon-1}\right)MC_t - 1 + E_t\left\{\left(\frac{\phi_P}{\varepsilon-1}\right)\mathcal{M}_{t,t+1}\frac{Y_{t+1}}{Y_t}\pi_{t+1}\left[\pi_{t+1} - \tilde{\pi}_{t+1}\right]\right\}, \tag{C.35}$$

$$R_t = R_{t-1}^{\rho_R}\left(R\left(\frac{\pi_t}{\pi}\right)^{\kappa_\pi}\left(\frac{Y_t}{Y}\right)^{\kappa_Y}\right)^{1-\rho_R}\exp\left(\varepsilon_{R,t}\right), \tag{C.36}$$

$$\tilde{W}_t = \frac{\pi_{W,t}}{\pi_t}\tilde{W}_{t-1}, \tag{C.37}$$

$$Y_t = A_t N_t, \tag{C.38}$$

$$MC_t = \frac{\tilde{W}_t}{A_t}, \tag{C.39}$$

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A) \log A + \varepsilon_{A,t}, \tag{C.40}$$

$$\mathbf{o}\left(s_t\right)\left(\pi_{W,t} - 1\right) + \left(1 - \mathbf{o}\left(s_t\right)\right)\Omega_t = 0, \tag{C.41}$$

$$\left(\frac{\upsilon}{\upsilon - 1}\right)\kappa\frac{N_t^\eta}{\lambda_t \tilde{W}_t} - 1 + \left(\frac{1}{\upsilon - 1}\right)\pi_t\left(\frac{\tilde{\Omega}_t - \left(1 - \mathbf{o}\left(s_t\right)\right)\tilde{\Omega}}{\lambda_t \tilde{W}_{t-1} N_t}\right) - \ldots$$

$$\ldots - E_t\left\{\left(\frac{\beta}{\upsilon - 1}\right)\pi_{W,t+1}\left(\frac{\Omega_{t+1} - \left(1 - \mathbf{o}\left(s_t\right)\right)\tilde{\Omega}}{\lambda_t \tilde{W}_t N_t}\right)\right\} = 0, \tag{C.42}$$

$$\hat{\Omega}_t = \tilde{\Omega}_t - \tilde{\Omega}. \tag{C.43}$$

Appendix C.7. Steady State

We set all time subscripts to the current period and after some rearranging we obtain the steady state model:

$$\tilde{\Omega}_t = \Omega, \tag{C.44}$$

$$\hat{\Omega}_t = 0, \tag{C.45}$$

$$\pi_t = \pi, \tag{C.46}$$

$$R_t = \frac{\pi_t}{\beta}, \tag{C.47}$$

$$\mathscr{M}_{t,t+1} = \beta, \tag{C.48}$$

$$\pi_{W,t} = \pi, \tag{C.49}$$

$$MC_t = \left(\frac{\varepsilon - 1}{\varepsilon}\right), \tag{C.50}$$

$$A = 1, \tag{C.51}$$

$$\tilde{W}_t = MC_t, \tag{C.52}$$

$$\Omega_t = \mathbf{o}\left(s_t\right)\Omega, \tag{C.53}$$

Find Y_t such that

$$N_t = Y_t, \tag{C.54}$$

$$\lambda_t = \left(Y_t - \chi Y_t\right)^{-\sigma}, \tag{C.55}$$

$$\left(\frac{\upsilon}{\upsilon - 1}\right)\kappa\frac{N_t^\eta}{\lambda_t \tilde{W}_t} - 1 + \left(\frac{1 - \beta}{\upsilon - 1}\right)\frac{\pi_t \Omega_t}{\lambda_t \tilde{W}_t N_t} = 0. \tag{C.56}$$

Appendix C.8. Calibration

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Table C.3: Calibrated Parameters

Parameter	Value	Parameter	Value	Parameter	Value
π	1.005	σ_R	0.001	α_R	1000
χ	0.8	σ_A	0.001	ψ_{π_W}	20
σ	2	σ_P	0.001	$\psi_{\hat{\Omega}}$	0.5
β	0.9975	ρ_A	0.5	γ_{π_W}	1
ϕ_P	10	κ	10	$\gamma_{\hat{\Omega}}$	1
ϕ_W	10	Ω	100	ψ_π	1
ε	6	ξ_W	1	ψ_Y	0.5
υ	6	ξ_P	1	ψ_R	0.01
η	2	$\omega_d r$	0.01	$\psi_{N,B}$	85
ρ_R	0.8	ω_y	0.5	$\psi_{B,N}$	85
κ_π	1.5	α_Y	1000	$\theta_{N,B}$	100
κ_Y	0.5	α_π	1000	$\theta_{B,N}$	100

</div>

Appendix D. A Simple RBC Model With Irreversible Investment

We set up a simple RBC model with irreversible investment. We can write the social planner's problem as

$$\max_{C_t,K_t,L_t} \quad \mathscr{W}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \kappa \frac{L_t^{1+\eta}}{1+\eta} \right] \right\},$$

$$Y_t = C_t + I_t, \tag{D.1}$$

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha}, \tag{D.2}$$

$$I_t = \left(K_t - (1-\delta) K_{t-1} \right) \left(1 + \tau\left(s_t\right) \right), \tag{D.3}$$

$$I_t \geq 0, \tag{D.4}$$

$$\log A_t = \rho_A \log A_{t-1} + (1-\rho_A) \log A + \varepsilon_{A,t}, \tag{D.5}$$

where C_t is consumption, L_t is hours worked, Y_t is output, I_t is investment, K_t is capital, A_t is technology and we assume that investment cannot go negative. We set this up as the Lagrangian

$$\mathscr{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{C_t^{1-\sigma}}{1-\sigma} - \kappa \frac{L_t^{1+\eta}}{1+\eta} - \dots \\ \dots - \lambda_t \left[\begin{array}{c} C_t + \left(1+\tau\left(s_t\right)\right) K_t - A_t K_{t-1}^\alpha L_t^{1-\alpha} - \dots \\ \dots - \left(1+\tau\left(s_t\right)\right) \left(1-\delta\right) K_{t-1} \end{array} \right] - \dots \\ \dots - \Omega_t \left[K_t - (1-\delta) K_{t-1} \right] \end{array} \right] \right\}, \tag{D.6}$$

Optimization by the social planner leads to the first order conditions

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = C_t^{-\sigma} - \lambda_t = 0, \tag{D.7}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = -\lambda_t \kappa L_t^{\eta} + (1 - \alpha) \frac{Y_t}{L_t} = 0, \tag{D.8}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = -\lambda_t (1 + \tau(s_t)) + \Omega_t + \ldots \tag{D.9}$$

$$\ldots + E_t \left\{ \beta \left(\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + (1 + \tau(s_{t+1})) (1 - \delta) \right) - \Omega_{t+1} (1 - \delta) \right) \right\} = 0, \tag{D.10}$$

From (D.7) we equate the shadow price of wealth to the marginal utility of consumption

$$\lambda_t = C_t^{-\sigma}. \tag{D.11}$$

Equation D.8 equates the marginal rate of substitution with the marginal product of labor

$$\kappa \lambda_t L_t^{\eta} = (1 - \alpha) \frac{Y_t}{L_t}, \tag{D.12}$$

from equation (D.10) we get an expression for the user cost of capital

$$\lambda_t (1 + \tau(s_t)) - \Omega_t = E_t \left\{ \beta \left(\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + (1 + \tau(s_{t+1})) (1 - \delta) \right) - \Omega_{t+1} (1 - \delta) \right) \right\}, \tag{D.13}$$

and we have the complementary slackness condition

$$\Omega_t (K_t - (1 - \delta) K_{t-1}) = 0, \tag{D.14}$$

with

$$\Omega_t > 0, \quad I_t = 0, \text{ or } \Omega_t = 0, \quad I_t > 0. \tag{D.15}$$

Finally we assume the lump sum tax/transfer takes the values

$$T_t = -\tau(s_t) (K_t - (1 - \delta) K_{t-1}) \tag{D.16}$$

Appendix D.1. Approximating the Occasionally Binding Constraint in a Regime-Switching Model

In order to model the occasionally binding constraint using regime-switching, we introduce a Markov chain with a binding state (B) and a non-binding state (N). We also introduce a regime specific parameter $\mathbf{o}(s_t)$ which takes the value

$$\mathbf{o}(N) = 0, \quad \mathbf{o}(B) = 1 \tag{D.17}$$

We can then replace the complementary slackness condition (D.14) with

$$\mathbf{o}(s_t)I_t + (1 - \mathbf{o}(s_t))\,\Omega_t = 0. \tag{D.18}$$

We specify the transition probabilities as

$$p_{N,B} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp\left(\psi_{N,B}\tilde{I}_t\right)}, \quad p_{B,N} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(\psi_{B,N}\hat{\Omega}_t\right)}, \tag{D.19}$$

so that the probability of the constraint binding increases as investment falls. Likewise the probability of exiting the binding regime increases as the shadow price on the constraint falls.

Appendix D.2. Model Equations

The 11 endogenous variables in the model: Y_t, C_t, I_t, K_t, L_t, A_t, Ω_t, λ_t, $\tilde{\Omega}_t$, $\hat{\Omega}_t$ and I_t^* are then explained by the 11 model equations

$$\kappa L_t^\eta = (1 - \alpha)\frac{Y_t}{L_t}, \tag{D.20}$$

$$\lambda_t = C_t^{-\sigma}, \tag{D.21}$$

$$\lambda_t\left(1 + \tau\left(s_t\right)\right) - \Omega_t = E_t\left\{\beta\left(\lambda_{t+1}\left(\alpha\frac{Y_{t+1}}{K_t} + \left(1 + \tau\left(s_{t+1}\right)\right)\left(1 - \delta\right)\right) - \Omega_{t+1}\left(1 - \delta\right)\right)\right\}, \tag{D.22}$$

$$\mathbf{o}(s_t)I_t + (1 - \mathbf{o}(s_t))\,\Omega_t = 0, \tag{D.23}$$

$$\tilde{\Omega}_t = \Omega_t, \tag{D.24}$$

$$\hat{\Omega}_t = \tilde{\Omega}_t - \Omega, \tag{D.25}$$

$$K_t = I_t + (1 - \delta)\,K_{t-1}, \tag{D.26}$$

$$I_t^* = I_t, \tag{D.27}$$

$$Y_t = C_t + I_t, \tag{D.28}$$

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha}, \tag{D.29}$$

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A)\log A + \varepsilon_{A,t}. \tag{D.30}$$

Appendix D.3. Steady State

The steady state of the model can be characterized by the following set of equations

$$k\_y = \frac{\alpha\beta}{1 - \beta\left(1 - \delta\right)}, \tag{D.31}$$

$$i\_y = (1 - \mathbf{o}(s_t))\,\delta k\_y, \tag{D.32}$$

$$i\_y^* = \delta k\_y, \tag{D.33}$$

$$c\_y = 1 - i\_y, \tag{D.34}$$

$$l\_y = k\_y^{\frac{\alpha}{1-\alpha}}, \tag{D.35}$$

$$L_t = \left(\frac{1-\alpha}{\kappa l\_y} \right)^{\frac{1}{\eta}}, \tag{D.36}$$

$$Y_t = L_t / l\_y, \tag{D.37}$$

$$I_t = i\_y \times Y_t, \tag{D.38}$$

$$I_t^* = i\_y^* \times Y_t, \tag{D.39}$$

$$K_t = k\_y \times Y_t, \tag{D.40}$$

$$A_t = \frac{Y_t}{K_t^\alpha L_t^{1-\alpha}}, \tag{D.41}$$

$$\lambda_t = C_t^{-\sigma}, \tag{D.42}$$

$$\Omega_t = \mathbf{o}(s_t)\Omega, \tag{D.43}$$

$$\tilde{\Omega}_t = \Omega, \tag{D.44}$$

$$\hat{\Omega}_t = 0, \tag{D.45}$$

$$\tau(s_t) = \frac{\Omega_t}{\lambda_t}. \tag{D.46}$$

The tax rate $\tau(s_t)$ is chosen so that the capital stock has the same steady state in both regimes

$$\lambda_t \left(1 + \tau(s_t)\right) - \Omega_t = \beta \left(\lambda_t \left(\alpha \frac{Y_t}{K_t} + (1 + \tau(s_t)) (1 - \delta) \right) - \Omega_t (1 - \delta) \right), \tag{D.47}$$

$$\frac{\alpha \beta \lambda_t}{k\_y} = \lambda_t \left(1 + \tau(s_t)\right) - \Omega_t - \beta \lambda_t \left(1 + \tau(s_t)\right) (1 - \delta) + \beta \Omega_t (1 - \delta), \tag{D.48}$$

$$k\_y = \frac{\alpha \beta \lambda_t}{\left(\lambda_t \left(1 + \tau_t(s_t)\right) - \Omega_t\right) \left(1 - \beta (1 - \delta)\right)}, \tag{D.49}$$

$$\lambda_t = \lambda_t \left(1 + \tau_t(s_t)\right) - \Omega_t, \tag{D.50}$$

$$\tau(s_t) = \frac{\Omega_t}{\lambda_t} = \frac{\mathbf{o}(s_t)\Omega}{\lambda_t}. \tag{D.51}$$

Table D.4: Calibrated Parameters

Parameter	Value
η	2
α	0.35
β	0.9975
σ	2
δ	0.025
ρ_A	0.9
σ_A	0.1
κ	0.01
$\psi_{N,B}$	85
$\psi_{B,N}$	85
$\theta_{N,B}$	100
$\theta_{B,N}$	100

Appendix E. All in One

In this section we take the model based on Iacoviello (2005) in Appendix B with an occasionally binding collateral constraint, and we add an occasionally binding lower bound constraint on interest rates, and an occasionally binding non-negativity constraint on net wage inflation.

Appendix E.1. Patient Households

There are now a continuum of patient households, indexed by i, and normalized to unit mass. Each household derives positive utility from consumption, C'_t and housing, H_t and disutility from working $L'_t(i)$. The ith household's utility function is given by

$$U'_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} d_j \right) \left[\log C'_t + j \log H'_t - \frac{L'_t(i)^{1+\eta}}{1+\eta} \right] \right\}. \tag{E.1}$$

Households choose consumption, housing, wages $\mathcal{W}'_t$ and loans $-B'_t$ to maximize their lifetime utility, subject to the budget constraint

$$C'_t + Q_t \left(H'_t - \frac{H'_{t-1}}{d_t} \right) + \frac{R_{t-1} B'_{t-1}}{\pi_t} = \dots$$

$$\dots B'_t + (1 - \tau_{\mathcal{W}} (s_{\mathcal{W},t})) \frac{\mathcal{W}'_t}{P_t} L'_t(i) - \frac{\phi_{\mathcal{W}}}{2} \frac{\mathcal{W}'_t}{P_t} L'_t \left[\frac{\mathcal{W}'_t(i)}{\mathcal{W}'_{t-1}(i)} - \pi \right]^2 + F'_t + T'_t, \tag{E.2}$$

and a non-negativity constraint on wage inflation

$$\frac{\mathcal{W}'_t(i)}{\mathcal{W}'_{t-1}(i)} \geq 1. \tag{E.3}$$

Patient household's also take into account how their choice of wage also affects the demand for their variety of labor. Demand for the ith household's labor is given by

$$L'_t(i) = \left(\frac{\mathcal{W}'_t(i)}{\mathcal{W}'_t}\right)^{-\upsilon} L'_t. \tag{E.4}$$

The Lagrangian for the ith household is given by

$$\mathcal{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} d_j\right) \left[\begin{array}{c} \log C'_t + j \log H'_t - \dfrac{L'_t(i)^{1+\eta}}{1+\eta} - \cdots \\ -\lambda'_t \left[\begin{array}{c} C'_t + Q_t \left(H'_t - \dfrac{H'_{t-1}}{d_t} \right) + \dfrac{R_{t-1} B'_{t-1}}{\pi_t} - B'_t - \cdots \\ \cdots - (1 - \tau_{\mathcal{W}} (s_{\mathcal{W},t})) \dfrac{\mathcal{W}'_t}{P_t} L'_t(i) + \cdots \\ \cdots + \dfrac{\phi_{\mathcal{W}}}{2} \dfrac{\mathcal{W}'_t}{P_t} L'_t \left[\dfrac{\mathcal{W}'_t(i)}{\mathcal{W}'_{t-1}(i)} - \pi \right]^2 - F'_t - T'_t \\ \cdots + \Omega'_t \left(\dfrac{\mathcal{W}'_t(i)}{\mathcal{W}'_{t-1}(i)} - 1 \right) \end{array} \right] + \cdots \end{array} \right] \right\}. \tag{E.5}$$

Substituting in E.4 gives

$$\mathcal{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} d_j\right) \left[\begin{array}{c} \log C'_t + j \log H'_t - \dfrac{\left(\left(\dfrac{\mathcal{W}'_t(i)}{\mathcal{W}'_t}\right)^{-\upsilon} L'_t \right)^{1+\eta}}{1+\eta} - \cdots \\ -\lambda'_t \left[\begin{array}{c} C'_t + Q_t \left(H'_t - \dfrac{H'_{t-1}}{d_t} \right) + \dfrac{R_{t-1} B'_{t-1}}{\pi_t} - B'_t - \cdots \\ \cdots - (1 - \tau_{\mathcal{W}} (s_{\mathcal{W},t})) \mathcal{W}'_t(i)^{1-\upsilon} \mathcal{W}'^{\upsilon}_t L'_t / P_t + \cdots \\ \cdots + \dfrac{\phi_{\mathcal{W}}}{2} \dfrac{\mathcal{W}'_t}{P_t} L'_t \left[\dfrac{\mathcal{W}'_t(i)}{\mathcal{W}'_{t-1}(i)} - \pi \right]^2 - \cdots \\ \cdots - F'_t - T'_t \\ \cdots + \Omega'_t \left(\dfrac{\mathcal{W}'_t(i)}{\mathcal{W}'_{t-1}(i)} - 1 \right) \end{array} \right] + \cdots \end{array} \right] \right\}. \tag{E.6}$$

Optimization results in the following set of first-order conditions

$$\frac{\partial \mathscr{L}_t}{\partial C_t'} = \frac{1}{C_t'} - \lambda_t' = 0, \tag{E.7}$$

$$\frac{\partial \mathscr{L}_t}{\partial H_t'} = \frac{j}{H_t'} - \lambda_t' Q_t + E_t\left\{\beta \lambda_{t+1}' Q_{t+1}\right\} = 0, \tag{E.8}$$

$$\frac{\partial \mathscr{L}_t}{\partial B_t'} = \lambda_t' - E_t\left\{\beta d_{t+1} \lambda_{t+1}' \frac{R_t}{\pi_{t+1}}\right\} = 0, \tag{E.9}$$

$$\frac{\partial \mathscr{L}_t}{\partial \mathcal{W}_t'(i)} = v\frac{L_t'(i)^{1+\eta}}{\mathcal{W}_t(i)} + (1 - \tau_{\mathcal{W}}(s_{\mathcal{W},t}))(1-v)\lambda_t'\frac{L_t'(i)}{P_t} - \dots$$
$$\dots - \phi_{\mathcal{W}}\frac{\mathcal{W}_t'}{\mathcal{W}_{t-1}'(i)}\frac{L_t'}{P_t}\left[\frac{\mathcal{W}_t'(i)}{\mathcal{W}_{t-1}'(i)} - \pi\right] + \frac{\Omega_t'}{\mathcal{W}_{t-1}'(i)} + \dots$$
$$\dots + E_t\left\{\beta d_{t+1}\phi_{\mathcal{W}}\lambda_{t+1}'\frac{\mathcal{W}_{t+1}'\mathcal{W}_{t+1}'(i)}{\mathcal{W}_t'(i)^2}\frac{L_{t+1}'}{P_{t+1}}\left[\frac{\mathcal{W}_{t+1}'(i)}{\mathcal{W}_t'(i)} - \pi\right] - \beta d_{t+1}\Omega_{t+1}'\frac{\mathcal{W}_{t+1}'(i)}{\mathcal{W}_t'(i)^2}\right\} = 0. \tag{E.10}$$

From the patient household's first-order conditions we obtain

$$\frac{1}{C_t'} = E_t\left\{\beta d_{t+1}\frac{R_t}{\pi_{t+1}C_{t+1}'}\right\}, \tag{E.11}$$

$$\frac{Q_t}{C_t'} = \frac{j}{H_t'} + E_t\left\{\beta\frac{Q_{t+1}}{C_{t+1}'}\right\}, \tag{E.12}$$

$$\left(\frac{v}{v-1}\right)\frac{L_t'^{\eta}}{\lambda_t'W_t} - (1 - \tau_{\mathcal{W}}(s_{\mathcal{W},t})) - \left(\frac{\phi_{\mathcal{W}}}{v-1}\right)\pi_{\mathcal{W},t}[\pi_{\mathcal{W},t} - \pi] + \left(\frac{1}{v-1}\right)\frac{\pi_t\Omega_t'}{\lambda_t W_{t-1}L_t'} + \dots$$
$$\dots + E_t\left\{\beta d_{t+1}\left(\frac{\phi_{\mathcal{W}}}{v-1}\right)\frac{\lambda_{t+1}'}{\lambda_t'}\frac{\pi_{\mathcal{W},t+1}^2}{\pi_{t+1}}\frac{L_{t+1}'}{L_t'}[\pi_{\mathcal{W},t+1} - \pi] - \left(\frac{\beta d_{t+1}}{v-1}\right)\frac{\pi_{\mathcal{W},t+1}\Omega_{t+1}'}{\lambda_t'W_tL_t'}\right\} = 0, \tag{E.13}$$

where $W_t \equiv \frac{\mathcal{W}_t'}{P_t}$. We also obtain the complementary slackness condition from the patient household's non-negativity constraint

$$\Omega_t'(\pi_{\mathcal{W},t} - 1) = 0. \tag{E.14}$$

Appendix E.2. Entrepreneurs

The representative entrepreneur produces output Y_t, using housing H_{t-1}, labor L_t, and a common neutral technology A_t

$$Y_t = A_t H_{t-1}^\nu L_t^{1-\nu}, \tag{E.15}$$

where neutral technology evolves according to

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A) \log A + \varepsilon_{A,t}, \tag{E.16}$$

The entrepreneur's utility takes the form

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \gamma^t \left(\prod_{j=0}^{t} d_j \right) \log C_t \right\}, \tag{E.17}$$

where C_t is the entrepreneur's consumption. The entrepreneur maximizes utility subject to the budget constraint

$$MC_t Y_t + B_t = C_t + (1 + \tau(s_{B,t})) Q_t \left(H_t - \frac{H_{t-1}}{d_t} \right) + \frac{R^*_{t-1} B_{t-1}}{\pi_t} + W_t L_t + T_t, \tag{E.18}$$

where MC_t is real marginal cost and

$$R^*_t = R_t \psi_1. \tag{E.19}$$

The entrepreneur is also subject to the borrowing constraint

$$B_t \leq m Q_t H_t. \tag{E.20}$$

Setting up the Lagrangian for the entrepreneur

$$\mathscr{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} \gamma^t \left(\prod_{j=0}^{t} d_j \right) \left[\ldots + \lambda_t \left[\begin{array}{c} \log C_t + \ldots \\ MC_t A_t H_{t-1}^{\nu} L_t^{1-\nu} + B_t - C_t - \ldots \\ \ldots - (1 + \tau(s_{B,t})) Q_t \left(H_t - \frac{H_{t-1}}{d_t} \right) - \ldots \\ \ldots - \frac{R^*_{t-1} B_{t-1}}{\pi_t} - W_t L_t - T_t \\ \ldots - \Omega_t \left[B_t - m Q_t H_t \right] \end{array} \right] + \ldots \right] \right\}. \tag{E.21}$$

The entrepreneur's first-order conditions

$$\frac{\partial \mathscr{L}_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0, \tag{E.22}$$

$$\frac{\partial \mathscr{L}_t}{\partial H_t} = -\lambda_t \left(1 + \tau(s_{B,t})\right) Q_t + \dots \tag{E.23}$$

$$\dots + E_t \left\{ \gamma \lambda_{t+1} \left(\nu M C_{t+1} \frac{Y_{t+1}}{H_t} + \left(1 + \tau(s_{B,t+1})\right) Q_{t+1} \right) + \Omega_t m Q_t \right\} = 0, \tag{E.24}$$

$$\frac{\partial \mathscr{L}_t}{\partial B_t} = \lambda_t - \Omega_t - E_t \left\{ \gamma d_{t+1} \lambda_{t+1} \frac{R_t^*}{\pi_{t+1}} \right\} = 0, \tag{E.25}$$

$$\frac{\partial \mathscr{L}_t}{\partial L_t} = (1 - \nu) M C_t \frac{Y_t}{L_t} - W_t = 0. \tag{E.26}$$

From the entrepreneur's first-order conditions we obtain

$$\frac{1}{C_t} = E_t \left\{ \gamma d_{t+1} \frac{R_t^*}{\pi_{t+1} C_{t+1}} \right\} + \Omega_t, \tag{E.27}$$

$$(1 + \tau(s_{B,t})) \frac{Q_t}{C_t} = E_t \left\{ \frac{\gamma}{C_{t+1}} \left(\nu M C_{t+1} \frac{Y_{t+1}}{H_t} + \left(1 + \tau(s_{B,t+1})\right) Q_{t+1} \right) + \Omega_t m Q_t \right\}, \tag{E.28}$$

$$W_t = (1 - \nu) M C_t \frac{Y_t}{L_t}. \tag{E.29}$$

We also obtain the entrepreneur's complementary slackness condition

$$\Omega_t \left(B_t - m Q_t H_t \right) = 0. \tag{E.30}$$

Appendix E.3. Retailers

There is a continuum of retailers normalized to unit mass, that buys intermediate goods from the entrepreneur, differentiates them, and sells them to a perfectly competitive final goods producer. Profits for the ith retailer are given by

$$\Lambda_0(i) = E_0 \left\{ \sum_{t=0}^{\infty} \mathscr{M}_{0,t} \left[\frac{P_t(i)}{P_t} Y_t(i) - M C_t Y_t(i) - \frac{\phi_P}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right]^2 \right] \right\}, \tag{E.31}$$

where $\mathscr{M}_{t,t+1} \equiv E_t \left\{ \beta d_{t+1} \frac{\lambda'_{t+1}}{\lambda'_t} \right\}$. Substituting in the demand function for the ith firm's demand, $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$, gives

$$\Lambda_0(i) = E_0 \left\{ \sum_{t=0}^{\infty} \mathscr{M}_{0,t} \left[P_t(i)^{1-\varepsilon} P_t^{\varepsilon-1} Y_t - M C_t P_t(i)^{-\varepsilon} P_t^{\varepsilon} Y_t - \frac{\phi_P}{2} Y_t \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right]^2 \right] \right\}. \tag{E.32}$$

The ith firm's first-order condition

$$
\frac{\partial \Lambda_t(i)}{\partial P_t(i)} = (1 - \varepsilon) \frac{Y_t(i)}{P_t} + \varepsilon MC_t \frac{Y_t(i)}{P_t(i)} - \phi_P \frac{Y_t}{P_{t-1}(i)} \left[\frac{P_t(i)}{P_{t-1}(i)} - \tilde{\pi}_t \right] + \dots
$$

$$
\dots + E_t \left\{ \phi_P \mathcal{M}_{t,t+1} Y_{t+1} \frac{P_{t+1}(i)}{P_t(i)^2} \left[\frac{P_{t+1}(i)}{P_t(i)} - \tilde{\pi}_{t+1} \right] \right\} = 0. \quad \text{(E.33)}
$$

Rearranging gives the Rotemberg Phillips curve

$$
\left(\frac{\phi_P}{\varepsilon - 1} \right) \pi_t \left[\pi_t - \tilde{\pi}_t \right] = \left(\frac{\varepsilon}{\varepsilon - 1} \right) MC_t - 1 + E_t \left\{ \left(\frac{\phi_P}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1} \frac{Y_{t+1}}{Y_t} \pi_{t+1} \left[\pi_{t+1} - \tilde{\pi}_{t+1} \right] \right\}.
$$
$$
\text{(E.34)}
$$

Appendix E.4. Monetary Policy

The monetary authority sets policy according to

$$
R_t = \max \left(R_{ZLB}, \tilde{R}_t \right), \quad \text{(E.35)}
$$

where R_{ZLB} is interest rate set in the lower bound state and $\tilde{R}_t$ is the interest rate set in normal times, set according to the following Taylor-type rule

$$
\tilde{R}_t = \tilde{R}_{t-1}^{\rho_{\tilde{R}}} \left(\tilde{R} \left(\frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left(\frac{Y_t}{Y} \right)^{\kappa_Y} \right)^{1 - \rho_{\tilde{R}}} \exp \left(\varepsilon_{\tilde{R},t} \right). \quad \text{(E.36)}
$$

Appendix E.5. Equilibrium

In equilibrium we obtain

$$
Y_t = C_t + C_t'. \quad \text{(E.37)}
$$

The housing stock of the economy is fixed so that

$$
h = H_t + H_t'. \quad \text{(E.38)}
$$

Appendix E.6. Approximating the Occasionally Binding Constraints in a Regime-Switching Model

We approximate the three occasionally binding constraints using regime-switching. In particular we introduce three two-state Markov chains, one for each of the occasionally binding constraints. Each Markov chain has a binding state, B_k, and a non-binding state, N_k, where $k = R, \mathcal{W}, B$, with R representing the ZLB chain, B on the collateral constraint chain and $\mathcal{W}$ is the downward nominal wage rigidity chain, so that

$$
s_{k,t} = N_k, B_k. \quad \text{(E.39)}
$$

We introduce three state dependent parameters, one for each of the Markov chains. These parameters take following values depending on the state

$$\mathbf{o}_k\left(N_1\right) = 0, \quad \mathbf{o}_k\left(B_k\right) = 1. \tag{E.40}$$

We replace equations E.14, E.30 and E.35 with equations C.26, B.36 and A.19 so that

$$\mathbf{o}_{\mathcal{W}}\left(s_{\mathcal{W},t}\right)\left(\pi_{\mathcal{W},t} - 1\right) + \left(1 - \mathbf{o}\left(s_{\mathcal{W},t}\right)\right)\hat{\Omega}_t' = 0, \tag{E.41}$$

$$\mathbf{o}_B(s_{B,t})\left(B_t - E_t\left\{m\frac{Q_{t+1}H_t\pi_{t+1}}{R_t}\right\}\right) + \left(1 - \mathbf{o}_B(s_{B,t})\right)\Omega_t = 0, \tag{E.42}$$

$$R_t = \mathbf{o}_R\left(s_{R,t}\right)R_{ZLB} + \left(1 - \mathbf{o}_R\left(s_{R,t}\right)\right)\tilde{R}_t. \tag{E.43}$$

The transition probabilities that govern these Markov chains are determined by A.23, A.24, C.30 and B.37 so that

$$p_{NB,t}^{\mathcal{W}} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp\left(\psi_{N,B}\left(\pi_{W,t} - 1\right)\right)}, \quad p_{BN,t}^{\mathcal{W}} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(-\psi_{B,N}\hat{\Omega}_t\right)}, \tag{E.44}$$

$$p_{NB,t}^{B} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp\left(-\psi_{N,B}B_t^*\right)}, \quad p_{BN,t}^{B} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp\left(\psi_{B,N}\hat{\Omega}_t\right)}, \tag{E.45}$$

$$p_{NZ,t}^{R} = \frac{\theta_{N,Z}}{\theta_{N,Z} + \exp\left(\psi_{N,Z}\left(R_t^* - R_{ZLB}\right)\right)}, \quad p_{ZN,t}^{R} = \frac{\theta_{Z,N}}{\theta_{Z,N} + \exp\left(-\psi_{Z,N}\left(R_t^* - R_{ZLB}\right)\right)}. \tag{E.46}$$

Appendix E.7. Model Equations

The 23 model variables: Y_t, C_t, C_t', R_t, π_t, $\pi_{\mathcal{W},t}$, B_t, W_t, L_t, H_t', H_t, Q_t, MC_t, A_t, Ω_t, Ω_t', $\tilde{R}_t$, R_t^*, B_t^*, $\hat{\Omega}_t$, $\tilde{\Omega}_t$, $\hat{\Omega}_t'$ and $\tilde{\Omega}_t'$ can be explained by the 23 model equations:

$$Y_t = C_t + C_t', \tag{E.47}$$

$$\frac{1}{C_t'} = E_t\left\{\beta d_{t+1}\frac{R_t}{\pi_{t+1}C_{t+1}'}\right\}, \tag{E.48}$$

$$MC_tY_t + B_t = C_t + Q_t\left(H_t - H_{t-1}\right) + \frac{d_t R_{t-1}^* B_{t-1}}{\pi_t} + W_tL_t, \tag{E.49}$$

$$\frac{Q_t}{C_t'} = \frac{j}{H_t'} + E_t\left\{\beta\frac{Q_{t+1}}{C_{t+1}'}\right\}, \tag{E.50}$$

$$\left(1 + \tau(s_{B,t})\right)\frac{Q_t}{C_t} = E_t\left\{\frac{\gamma}{C_{t+1}}\left(\nu MC_{t+1}\frac{Y_{t+1}}{H_t} + \left(1 + \tau(s_{B,t+1})\right)Q_{t+1}\right) + \Omega_t m Q_{t+1}\pi_{t+1}\right\}, \tag{E.51}$$

$$B_t^* = B_t - E_t\left\{m\frac{Q_{t+1}H_t\pi_{t+1}}{R_t}\right\}, \tag{E.52}$$

$$\mathbf{o}_B(s_{B,t})\left(B_t - E_t\left\{m\frac{Q_{t+1}H_t\pi_{t+1}}{R_t}\right\}\right) + (1 - \mathbf{o}_B(s_{B,t}))\,\Omega_t = 0, \qquad \text{(E.53)}$$

$$\mathbf{o}_{\mathcal{W}}(s_{\mathcal{W},t})\,(\pi_{\mathcal{W},t} - 1) + (1 - \mathbf{o}_{\mathcal{W}}(s_{\mathcal{W},t}))\,\Omega'_t = 0, \qquad \text{(E.54)}$$

$$R_t = \mathbf{o}_R\,(s_{R,t})\,R_{ZLB} + (1 - \mathbf{o}_R\,(s_{R,t}))\,\tilde{R}_t, \qquad \text{(E.55)}$$

$$\hat{\Omega}_t = \tilde{\Omega}_t - \tilde{\Omega}, \qquad \text{(E.56)}$$

$$\hat{\Omega}'_t = \tilde{\Omega}'_t - \tilde{\Omega}', \qquad \text{(E.57)}$$

$$\tilde{\Omega}_t = \Omega_t, \qquad \text{(E.58)}$$

$$\tilde{\Omega}'_t = \Omega'_t, \qquad \text{(E.59)}$$

$$Y_t = A_t H_{t-1}^\nu L_t^{1-\nu}, \qquad \text{(E.60)}$$

$$\left(\frac{\upsilon}{\upsilon - 1}\right)\frac{C'_t L'^\eta_t}{W_t} - (1 - \tau_{\mathcal{W}}\,(s_{\mathcal{W},t})) - \left(\frac{\phi_{\mathcal{W}}}{\upsilon - 1}\right)\pi_{\mathcal{W},t}\,[\pi_{\mathcal{W},t} - \pi] + \left(\frac{1}{\upsilon - 1}\right)\frac{\pi_t \Omega'_t C'_t}{W_{t-1} L'_t} + \dots$$

$$\dots + E_t\left\{\beta d_{t+1}\left(\frac{\phi_{\mathcal{W}}}{\upsilon - 1}\right)\frac{C'_t}{C'_{t+1}}\frac{\pi^2_{\mathcal{W},t+1}}{\pi_{t+1}}\frac{L'_{t+1}}{L'_t}\,[\pi_{\mathcal{W},t+1} - \pi] - \left(\frac{\beta d_{t+1}}{\upsilon - 1}\right)\frac{\pi_{\mathcal{W},t+1}\Omega'_{t+1}C'_t}{W_t L'_t}\right\} = 0,$$
$$\text{(E.61)}$$

$$W_t = \frac{\pi_{\mathcal{W},t}}{\pi_t}W_t, \qquad \text{(E.62)}$$

$$W_t = (1 - \nu)\,MC_t\frac{Y_t}{L_t}, \qquad \text{(E.63)}$$

$$h = H_t + H'_t, \qquad \text{(E.64)}$$

$$\left(\frac{\phi_P}{\varepsilon - 1}\right)\pi_t\,[\pi_t - \tilde{\pi}_t] = \dots$$

$$\dots \left(\frac{\varepsilon}{\varepsilon - 1}\right)MC_t - 1 + E_t\left\{\left(\frac{\phi_P}{\varepsilon - 1}\right)\mathcal{M}_{t,t+1}\frac{Y_{t+1}}{Y_t}\pi_{t+1}\,[\pi_{t+1} - \tilde{\pi}_{t+1}]\right\}, \quad \text{(E.65)}$$

$$\frac{1}{C_t} = E_t\left\{\gamma d_{t+1}\frac{R^*_t}{\pi_{t+1}C_{t+1}}\right\} + \Omega_t R_t, \qquad \text{(E.66)}$$

$$R^*_t = R_t\psi_1, \qquad \text{(E.67)}$$

$$\tilde{R}_t = \tilde{R}^{\rho_{\tilde{R}}}_{t-1}\left(\tilde{R}\left(\frac{\pi_t}{\pi}\right)^{\kappa_\pi}\left(\frac{Y_t}{Y}\right)^{\kappa_Y}\right)^{1-\rho_{\tilde{R}}}\exp\left(\varepsilon_{\tilde{R},t}\right), \qquad \text{(E.68)}$$

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A)\log A + \varepsilon_{A,t}. \qquad \text{(E.69)}$$

Following the strategy taken by Guerrieri & Iacoviello (2015a) we will solve the steady state model for the following ratios: $h\_y \equiv H_t/Y_t$, $h\_p\_y \equiv H'_t/Y_t$, $c\_y \equiv C_t/Y_t$, $c\_p\_y \equiv C'_t/Y_t$ and $b\_y \equiv B_t/Y_t$, before solving for the level variables.

$$Q_t = 1, \tag{E.70}$$

$$MC_t = \left(\frac{\varepsilon - 1}{\varepsilon}\right), \tag{E.71}$$

$$\pi_t = \pi, \tag{E.72}$$

$$\tilde{R}_t = \frac{\pi_t}{\beta}, \tag{E.73}$$

$$R_t = \mathbf{o}_R\left(s_{R,t}\right) R_{ZLB} + \left(1 - \mathbf{o}_R\left(s_{R,t}\right)\right) \tilde{R}_t, \tag{E.74}$$

$$d_t = \frac{\pi_t}{\beta R_t}, \tag{E.75}$$

$$h\_y = \left(\frac{\varepsilon - 1}{\varepsilon}\right)\left(\frac{\gamma\nu}{(1 - \gamma)Q_t}\right), \tag{E.76}$$

$$\tau\left(s_{B,t}\right) = 1 + \mathbf{o}_B\left(s_{B,t}\right)\left(\frac{\beta - \gamma}{1 - \gamma}\right), \tag{E.77}$$

$$b\_y = mQ_t \times h\_y, \tag{E.78}$$

$$c\_y = b\_y\left(1 - \frac{1}{\beta}\right) + \mu MC_t, \tag{E.79}$$

$$c\_p\_y = 1 - c\_y, \tag{E.80}$$

$$j = 4 \times Q_t\left(1 - \beta\right) h\_y/c\_p\_y, \tag{E.81}$$

$$h\_p\_y = c\_p_y \times \frac{j}{Q_t\left(1 - \beta\right)}, \tag{E.82}$$

$$L_t = \left(\frac{(1 - \nu) MC_t}{c\_p\_y}\right)^{\frac{1}{\varepsilon}}, \tag{E.83}$$

$$Y_t = 1, \tag{E.84}$$

$$H_t = h\_y \times Y_t, \tag{E.85}$$

$$H'_t = h\_p\_y \times Y_t, \tag{E.86}$$

$$B_t = b\_y \times Y_t, \tag{E.87}$$

$$C_t = c\_y \times Y_t, \tag{E.88}$$

$$C'_t = c\_p\_y \times Y_t, \tag{E.89}$$

$$h = H_t + H'_t, \tag{E.90}$$

$$A_t = \frac{Y_t}{H_t^\nu L_t^{1-\nu}}, \tag{E.91}$$

$$W_t = L_t^{\eta-1} C_t', \tag{E.92}$$

$$\psi_1 = \mathbf{o}_B\left(s_{B,t}\right) + \left(1 - \mathbf{o}_B\left(s_{B,t}\right)\right)\beta/\gamma, \tag{E.93}$$

$$\psi_2 = \left(1 - \mathbf{o}_B\left(s_{B,t}\right)\right)\tilde{\psi}_2, \tag{E.94}$$

$$R_t^* = R_t\psi_1, \tag{E.95}$$

$$\Omega_t = \mathbf{o}_B\left(s_{B,t}\right)\left(\frac{\beta - \gamma}{\pi_t C_t}\right), \tag{E.96}$$

$$B_t^* = 0, \tag{E.97}$$

$$\tilde{\Omega}_t = \left(\frac{\beta - \gamma}{\pi_t C_t}\right), \tag{E.98}$$

$$\hat{\Omega}_t = 0, \tag{E.99}$$

$$\tilde{\Omega}_t' = \Omega', \tag{E.100}$$

$$\Omega_t' = \mathbf{o}_\mathcal{W}(s_{\mathcal{W},t})\Omega', \tag{E.101}$$

$$\hat{\Omega}_t' = 0, \tag{E.102}$$

$$\tau_\mathcal{W}(s_{\mathcal{W},t}) = \left(\frac{\beta d_t}{\upsilon - 1}\right)\pi_{\mathcal{W},t}\frac{\Omega_t' C_t'}{W_t L_t} - \left(\frac{1}{\upsilon - 1}\right)\frac{\pi_t\Omega_t' C_t'}{W_t L_t}. \tag{E.103}$$

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