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A Merton Approach to Predicting Defaults Amongst Public Firms in Norway

01.09.2016

BI Norwegian Business School – Thesis

GRA 19003 Master Thesis

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Master of Science in Business with Major in Finance

Acknowledgements

This thesis concludes five years of study at BI Norwegian Business School, where the authors have completed the programme of MSc. in business. The thesis is written in cooperation with Janis Berzins, whom the authors would like to thank for his rewarding supervision and support. The authors would also like to show their gratitude to the finance faculty at BI Norwegian Business school, having received guidance by multiple academicians.

Special thanks is made to Martin Engebretsen, who has contributed with constructive feedback.

The research conducted in the thesis could not have been completed without the contribution from the Brønnøysund Register Centre and Stamdata, of which the authors would like to express their gratefulness.

The authors would also like to express their thankfulness to their families, friends and colleges, for having showed support and understanding throughout the times of study.

The authors hope that future students of finance at BI Norwegian Business school will be equally inspired by the school's faculty.

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Abstract

The primary objective of this thesis is to investigate how well Merton's corporate debt model performs in predicting defaults amongst public firms in Norway. The study concludes that the model performs adequately in predicting defaults amongst public firms in in this market. However, the application of the model is dependent on historical market values of equity, making it unable to estimate default probabilities for young public firms.

In this thesis, Merton's corporate debt model is used by applying the generated distance to default as the covariate in the logit model to estimate the probability of default. Three analyses are conducted to validate the model's performance: regression analysis, discriminatory power analysis and a calibration analysis of the probability of default quantification. Two different sets of historical defaults are used, each with its own definition of default. The first dataset contains data from The Brønnøysund Register Centre on historical bankruptcies from 1996-2015. The second dataset is based on Stamdata's registry of bond issuers' financial failures from 2007-2015.

The model is tested with different types of equity volatility. The logistic regression analysis concludes that while no single version of the model exhibits exceptionally high explanatory power, models based on equity volatility with an estimation window of 90 days seem to be adequate candidates in the application of the model. Moreover, the study shows that winsorizing equity returns does not add much explanatory power, while a logarithmic functional form in the logit model yields higher fit. The discriminatory analysis finds that the model's ability to discriminate between defaulting and non-defaulting firms exceeds that of pure statistical models, achieving accuracy levels on par with previous studies. The calibration analysis concludes that the model has low risk of underestimating the true credit risk when predicting bankruptcies. However, the model exhibits some risk of underestimation in distressed economies when predicting financial failures. Moreover, the analysis concludes that the logarithmic functional form in the logit model underestimate the true credit risk.

Part I - Introduction

The quest for a method that accurately predicts defaults amongst companies is old and a trustworthy model has been sought by many. Banks, bondholders, equity holders, portfolio managers, and other stakeholders of an economy would benefit from being able to detect signs of financial failure before it occurs. Researchers of credit risk first started to develop such methods in the early 1930's (Trueck & Rachev, 2009). In 1974, Robert K. Merton presented a model that viewed bonds and stocks issued by firms as contingent claims on the assets of the firm. His model has been credited for being amongst the most central models of default prediction (Lando, 2004). As markets have grown more mature and efficient, more advanced models have evolved, but the fundamentals of Merton's model remain. In particular, a default prediction model of the U.S. rating agency Moody's is partly based on that of Merton (Stein & Sobehart, 2000).

The primary objective of this thesis is to investigate how well Merton's corporate debt model performs in predicting defaults amongst public firms in Norway. In validating the model, three analyses are conducted: regression analysis, discriminatory power analysis and a calibration analysis of the probability of default quantification. The results of the two former analyses are compared to those of pure statistical models. Literature does not conclude on an optimal approach of how to estimate the volatility input in the model. Eleven different volatility candidates are applied in the model, which allows for studying the importance of the volatility parameter. To bring nuance to the study, this paper examines the models' performance in light of two different definitions of default; bankruptcies and failure of any financial promise. Winsorization of the equity returns is conducted to observe how outliers influence the probability of default. In addition, logarithmic transformation of the covariate in the logit model is performed to study the impact of different functional forms.

Research Question and Hypotheses

Three hypotheses are assessed in this paper. Each hypothesis challenges an aspect of Merton's corporate debt model. The hypotheses are meant to give a comprehensive picture of how well Merton's corporate debt model predicts defaults amongst Norwegian public firms.

The first hypothesis states that the logarithmic distance to default variable of the Merton's corporate debt model exhibits the highest explanatory power in predicting defaults when the model is based on GARCH(1,1) volatility estimates with untreated returns of equity. Previous research has shown that treating outliers of equity returns has minor effects on the model's explanatory power, while taking the logarithm of the independent variable has noteworthy impact (Loffler & Posch, 2007). Another empirical study suggests that GARCH-estimated volatility may be applied to improve the performance of the Black & Scholes model (Duan, 1995). The hypothesis will be assessed through a regression analysis by testing different versions of the model differing in estimation window for GARCH and historical equity volatility estimates.

The second hypothesis states that the model outperforms pure statistical models[1] in distinguishing defaults from non-defaults. Altman's Z-score and Ohlson's O-score are used as proxies for the performance of such models. Previous research has shown that Merton's corporate debt model has outperformed Altman's Z-score in such discrimination (Stein & Sobehart, 2000). The study was conducted on U.S. non-financial public firms, and it is interesting to see if similar conclusion follows for Norwegian public firms. The hypothesis will be assessed by a discriminatory analysis where three simple, yet powerful, techniques are applied to compare the performance of the models. Those techniques are the Cumulative Accuracy Profile and Accuracy Ratio.

The third hypothesis states that the estimated probability of default generated by Merton's corporate debt model underestimates the true probability of default. The hypothesis is inspired by research showing that the model tends to underestimate the true credit risk (Gemmill, 2002).The hypothesis will be assessed in a calibration analysis by comparing the estimated probabilities to the actual outcomes. A Vasicek one-factor model which accounts for default correlation is applied to determine the likelihood of underestimation.

Scientific Contribution and Justification of Study

The Norwegian oil sector represents a substantial portion of the country's economy, and the past years' drop in the oil price has led to lower activity level within the

[1] Statistical models have an unexpected, default-triggering event that is governed by an exogenous default-intensity process (Chen & So, 2014).

sector. While lower interest rates, weaker domestic exchange rates and expansionary fiscal policy have had redeeming effects on the Norwegian economy, there are strong signals that the credit risk amongst firms in the country has increased in the past years (The Financial Supervisory Authority of Norway, 2016). The turmoil has brought workforce cutbacks and an increased amount of covenant breaches of corporate debt issuers (Stamdata, 2016). This provides strong motivation to study the relevant topic of default prediction amongst Norwegian public firms.

Several extensions have evolved from the original version of Merton's corporate debt model. The application of stochastic interest rate, stochastic volatility and different types of debt structures are now common in the literature (Lando, 2004). However, default prediction models that have gained attention in Norway appears to be of more statistical nature, such as the SEBRA model (Bernhardsen & Larsen, 2007). Miklos and Ullsfoss have conducted an empirical analysis of the KMV Merton corporate debt model on Swedish real estate companies (Miklos & Ullsfoss, 2015). Grøstad has studied a Merton approach for the Norwegian High Yield Bond Market (Grøstad, 2013), but with focus on credit spreads instead of discrimination and calibration. As far as our literature research has shown, there is little research that explicitly analyzes Merton's corporate debt model with focus on the discriminatory power, nor the calibration power for Norwegian public firms. Hopefully, this paper represents an important contribution for academics who are interested in the use of structural credit risk models for Norwegian public firms in particular.

Only a few[2] companies listed on Oslo Børs ASA[3] are rated by internationally recognized credit rating agencies (Sundheim & Kvisvik Hårstad, 2012). Bondholders and other creditors must therefore rely on alternative ratings to anchor their investment decisions. Not recognized by ESMA standards[4], these ratings go by the name *shadow ratings*. The problems associated with maintaining adequate ratings may cause discrepancy between the firms' true credit risk and the one reflected in these ratings. Thus, in addition to providing relevant academic research,

[2] The number of listed non-financial companies with an official credit rating from Moody's, Standard & Poors or Fitch appears to have been 7 in 2012.
[3] Oslo Børs ASA offers the only regulated markets for securities trading in Norway (Oslo Børs ASA, 2016)
[4] No Norwegian rating institutions are registered in or certified by the European Securities and Markets Authority (European Securities and Markets Authority, 2016).

there may be a great potential for a Merton-approach to be applied by Norwegian financial institutions to better assess credit risk.

In our thesis, we present evidence that the distance to default variable produced by the Merton's corporate debt model does carry explanatory power on historical defaults. Moreover, we show that the model's ability to discriminate between defaulting and non-defaulting firms exceeds that of pure statistical models, in accordance with previous studies on the model's performance in other markets. Lastly, our analysis indicates that proper use of the model involves low risk of underestimation when predicting bankruptcies, however, higher risk when predicting financial failures. Underestimation seems inevitable when probabilities of default are estimated with logarithmic transformation of the covariate.

The structure of this thesis is as follows: Part I elaborates on the research question and hypotheses, motivation of study and a brief summary of the results. Part II introduces previous literature on Merton's corporate debt model. Part III summarizes the research design of the study. Part IV presents the data and part V describes the details of the methodology. In Part VI we present the findings and implications of the study. Part VII concludes on the hypotheses and research question, and discusses further research areas.

Part II - Literature Review

This section introduces previous literature on the topic of the Merton corporate debt (MCD) model. The goal of this section is to give an overview of the topic and its underlying assumptions.

Merton Bibliography

The roots of the MCD model traces back to Merton's own research and the Black & Scholes option pricing model to value corporate bonds (Merton R. C., 1974) (Merton R. C., 1973) (Black & Scholes, 1973). The original MCD model base the risk structure from a risk neutral perspective, where debt is seen as a zero-coupon bond with the possible extension of accounting for coupon-paying debt. Because of its intuitive appeal and economic reasoning, the model is seen as a key reference point to credit risk modeling (Lando, 2004). Several extensions have since been added to relax some of the strict assumptions surrounding the model.

In 1990, Merton extended his original model to account for jump-diffusions (Merton R. , 1990) which has been further developed to be more general about the specification of risk premiums (Lando, 1994). Alternatives to Merton's own jump-diffusion model can be found in articles of Zhou and Mason and Bhattacharya (Zhou, 2001) (Mason & Bhattacharya, 1981). In 1977, Geske used numerical integrals with finite-differences to price coupon bonds in terms of multivariate normal integrals (Geske, 1977). Today, this has become a standard procedure. The first paper that presents a detailed description of a continuous point of default with perpetual debt and finite horizons are the one of Black and Cox (Black & Cox, 1976). This article is studied further to introduce stochastic interest rates (Briys & Varenne, 1997). Stochastic interest rates are relatively complicated, and multiple researcher have come up with different procedures to account for the same process. First-passage time densities are especially challenging with stochastic interest rates, yet Buonocore et al. presents an integral equation to solve the problem when the transition densities of the process are known (Buonocore, Nobile, & Ricciardi, 1987). The same integral techniques are later used by Longstaff and Scwartz (Longstaff & Schwartz, 1995) which is clarified by a two-dimensional version of a numerical algorithm.

The extensions presented above are theoretically intuitive, but more complicated to apply in practice. One will have to adapt the pricing model to different patterns of coupons, covenants or call ability features that is not easy to model. The first noteworthy paper that attempts to test structural models was published in 1989 (Sarig & Warga, 1989). The researchers test a dataset with zero-coupon bonds only, and find that the MCD model performs well in portraying the term structure for bonds with different ratings. Several papers have since tested the MCD model and accounted for coupons, stochastic interest rates and compound option effects (Delianedis & Geske, 2003) (Ericsson & Reneby, 2002). Eom, Hwlwege and Huang find support for that the MCD model underestimates the true credit risk for investment grade firms (Eom, Helwege, & Huang, 2004). Arguably, the most practical approach of the MCD model was presented by Moody's KMV where the maximum-likelihood method is used to assess credit risk (Crosbie & Bohn, 2002) (Bohn, 2000) (Sobehart, Stein, Mikityanskaya, & Li, 2000). This is a hybrid model, coined as Moody's Public Firm Risk Model, that couples the MCD model with financial information and firms previous credit ratings. Their model exhibits discriminatory power that exceeds that of the simple MCD model.

Credit Risk Theory from a Structural Perspective

Credit risk is the risk that a debtor fails to meet its repayments according to a pre-determined schedule (Tung, Lai, Wong, & NG, 2010). Default may be defined as a condition when failure of repayment is met and occurs when a company cannot cover the payments with cash or proceeds from selling assets. The definition of default can therefore be modified to a condition when the market value of assets *(V)* is not sufficient to cover its debt *(F)* at maturity,

$$V < F. \tag{1}$$

If E is the market capitalization and D is the market value of the firm's liabilities, then by definition the following accounting relationship exist

$$V = D + E. \tag{2}$$

Since equity holders never lose more than they invest (ignoring time value), the lower boundary of equity is defined as $min(E) = 0$. This implies that the only way to reach the condition in Equation 1 is when

$$(\Delta V < 0 \mid E = 0) \leftrightarrow D = V < F. \tag{3}$$

In contrast, if the default condition is not met, $E >= 0$, then the debt will have the convergence, $D \rightarrow F \; as \; t \rightarrow T$, where T is the time when debt matures and t is a time before T. The probability of default (PD) can therefore be expressed as

$$PD = P(D_T < F) = P(V_T < F) \tag{4}$$

where $P(\cdot)$ is an unknown probability function. The only observable parameter in the Equation 4 is F, making the study of PD a challenging task.

Structural credit risk models study PD through the observable E and are often called option theoretic or contingent claim models (Loffler & Posch, 2007). This is because E can be seen as a residual claim that force Equation 2 to be true at any time. The option theory stems from the concept of how equity can be expressed as a call option with a strike price equal to the face value of the debt

$$E = \max(V - F, 0). \tag{5}$$

Robert C. Merton is recognized as the first to apply option theory to the problem of valuing corporate debt and rests on Black, Scholes and Merton's option theory.

Assumptions

The MCD model follows the same seven assumptions as in the Black, Scholes and Merton (BSM) option pricing model, which is arguably the most common method of valuing options (Black & Scholes, 1973) (Merton R. C., 1973). The list of assumptions are as follows

1. There are no transaction costs or taxes where all assets are tradeable.
2. Markets are perfect.
3. Short sales are possible without constraints.
4. Trading in assets takes place continuously in time.
5. The Modigliani-Miller theorem that the value of the firm is invariant to its capital structure holds.
6. There is one risk-free interest rate and every asset can be discounted at this rate.
7. The dynamics for the value of the firm V, through time can be described by a diffusion-type stochastic process with stochastic differential equation

   $$dV = \mu V dt + \sigma_v V dz \tag{6}$$

   where μ is the drift rate (expected return), σ_V^2 is the variance rate and dz is a wiener process.

Merton argues that assumption 1 to 3 can be relaxed, that assumption 5 is proved and assumption 6 is there to distinguish the risk structure from the term structure effect on pricing. Assumption 4 and 7 are critical for the MCD model. Details about assumption 7 are left out in this thesis, and we refer to Hulls book "Options, Futures and other Derivatives" for an extensive review (Hull, 2015).

The MCD Model

The process described in assumption 7 is only approximately valid when $dt \to 0$. For larger time intervals, it is common to assume that the asset is log-normally distributed. By applying Ito's Lemma on the function $\ln(V)$ reach the following process for dV[5]

$$dV = \left(\mu - \frac{\sigma_V^2}{2}\right) V dt + \sigma_V V dz \tag{7}$$

where the drift rate is $\left(\mu - \frac{\sigma_V^2}{2}\right)$. Assumption 2 can be relaxed, but if the no-arbitrage condition holds, the price of options needs to be the same regardless of risk preferences[6]. It is therefore possible to estimate the equity value from a risk-neutral perspective where every risky asset can be discounted by the risk free interest rate r_f. For simplicity, annotations reflecting current time is neglected. The BSM formula for valuing a call option is expressed as

$$E = V\Phi(d_1) - Fe^{-r_f T}\Phi(d_2) \tag{8}$$

where

$$d_1 = \frac{\ln\left(\frac{V}{F}\right) + \left(r_f + \frac{\sigma_v^2}{2}\right)T}{\sigma_v \sqrt{T}} \; and \; d_2 = d_1 - \sigma_v \sqrt{T}. \tag{9}$$

The $\Phi(\cdot)$ is the normal cumulative distribution function. Equation 8 and 9 are central in Merton's article; one can easily obtain an equation of the market value of debt, *D*, by inserting those equations into Equation 2 such that

$$D = Fe^{-rT} - \max(F - V, 0). \tag{10}$$

The equation shows that *D* is equal the present value of a risk-free bond with face value of *F* less a put option on the asset. Equation 10 is the MCD model. However, in studying PD, it is necessary to apply the model further.

One can observe *E* and *D* through the market prices of equity and bonds, but not *V* nor σ_v. However, it is possible to solve the two unknowns from the fact that $E = f(V, \sigma_V)$ and the assumption that the process of *E* is similar to that of the asset,

[5] The expression, $dln(V)$ is simplified to dV.
[6] Going long in the underlying and short a fraction of the security that is written on the underlying must give the return equal the risk free interest rate, otherwise an arbitrage condition exist.

described in assumption 7. From Ito's Lemma, one can then derive a second equation

$$\sigma_E = \frac{\sigma_V \Phi(d_1) V}{E}. \tag{11}$$

Here, σ_E is the volatility of the equity, which may be estimated from historical market prices. Together, Equation 8, 9, and 11 make it possible to solve for V and σ_V, which is the technique used in this thesis.

PD and DD

The output d_2 in Equation 9 is a quantitative measure of how many standard deviations the expected log-asset value is away from the point that triggers default. It is directly connected to PD because, $\Phi(d_2) = \Phi(V > F)$, is the probability of exercising the call option, $P(E > 0)$. The risk neutral PD can therefore be expressed as

$$1 - \Phi(d_2) = \Phi(V_T < F) \tag{12}$$

$$\Phi(-d_2) = \Phi(D_T < F). \tag{13}$$

Another key point is that from the assumption of normal distribution makes $P(\cdot)$ referred to in Equation 4 to be $\Phi(\cdot)$.

Assumption 5 needs to be neglected in order to change the d_2 in Equation 9, and conversely, the risk neutral PD in Equation 13 to resemble a real world PD. This is achieved by applying a drift rate, μ, located in Equation 7. A method that is both practical and consistent with economic theory is to estimate μ through the capital asset pricing model (CAPM) (Tung, Lai, Wong, & NG, 2010)

$$\mu = r + \beta E(R_M) \tag{14}$$

where $\beta = \frac{\sigma_{V,M}}{\sigma_M^2}$ and $E(R_M) = E(r_M) - r_f$, where M represents the Market portfolio. We choose not to present details on the CAPM, but refer to Bodie, Kane and Markus' book "Investments" for an in-depth explanation of the model. When including μ, the d_2 term in Equation 9 is often referred to as the distance to default (DD), which is just another way of expressing PD. This thesis will concentrate on the DD output, now expressed by

$$DD = \frac{\left(\ln\left(\frac{V}{F}\right) + \left(\mu - \frac{\sigma_V^2}{2}\right) T\right)}{\sigma_V \sqrt{T}}. \tag{15}$$

The DD output in Equation 15 will be used as the independent variable in the logit model that produces estimates of PD. Details of the regression are presented in Part V– Methodology.

Intuition Behind the Model

The MCD model views equity holders as those who run the company. When debt matures, they need to decide whether they want to keep the assets and pay debt holders the amount of F or abandon the assets to the debt holders. Assumed to be rational, investors will only choose the latter if $V < F$. In such case, debt holders will sell the assets directly at the market price *V*, which is the recovery amount they receive instead of the promised F.

By inserting Equation 8 and Equation 10 into Equation 2, the fundamental put-call parity will be obtained

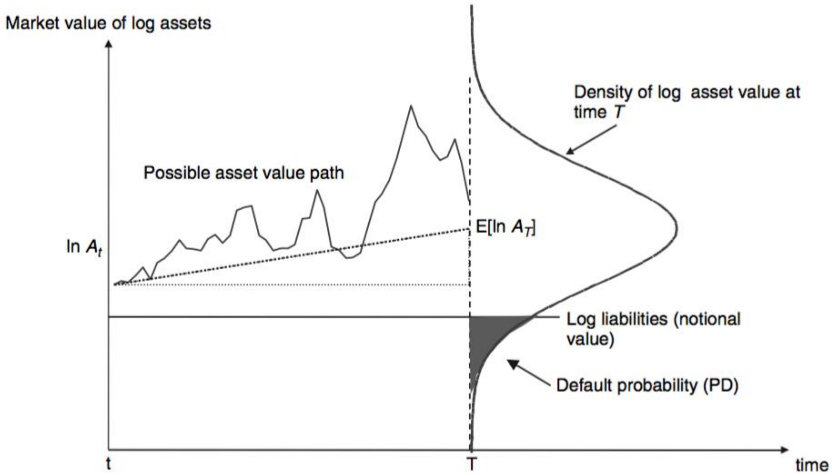
$$\max(V - F, 0) - \max(F - V, 0) = \mathrm{V} - Fe^{-r_f T}. \qquad (16)$$

Important connections between *V*, *E* and *D* in the MCD model can be pointed out from this parity relationship. If *F*, r_f, *T* and *V* is fixed, changing any other feature of the model will influence call and put options correspondingly. One result is that $\Delta D > 0|\Delta V > 0$ because the put value incorporated in Equation 10 will be less valuable. Furthermore, $\Delta D > 0|\Delta F > 0$ and $\Delta E < 0\ |\Delta F > 0$, because a higher strike price *F* reflects a higher promised future cash flow to debt holders at the cost of a lower value of the call option, *E*. It can also be seen from Equation 9 that $\Delta E > 0|\Delta r_f > 0$ and $\Delta D < 0|\Delta r_f > 0$ because the sum of the options remains unchanged. If the time to maturity, T, increases, the value of *D* will decrease since the effect of the discounting factor on *F* will dominate. Perhaps the most interesting consequence of *E* and *D* is when $\Delta\sigma_v > 0$ and $\Delta V = 0$. In this case, value will be moved from the debt holders to equity holders. From option theory and the put-call parity, the long call option will increase in value, and conversely, the short put option will decrease. Equity holders do not have any power to change σ_v, which is a reason for covenants in loan agreements that gives debt holders some level of control of the investment decision.

Summary

Figure 1[7] summarizes the concept of the MCD model. The drift rate of assets works as an estimator for the asset value path at time *T*. The asset value itself follows a process of random walk with drift. The horizontal axis symbolizes time, where *T* is the current time. The vertical axis depicts the market value of the asset. The figure illustrates that at time *T* there is a range of possible asset values and the frequency distribution located to the left in the figure illustrates the likelihood of various asset values. The most likely outcome is nearest to the starting value added to the drift rate times T. The drift rate is shown as a straight line that is increasing with time. Greater volatility represents higher probability of extreme outcomes. The horizontal line shows the logarithm of the face value of debt, which is the critical point where debt matures and triggers default.

Figure 1: Summary of the MCD model.

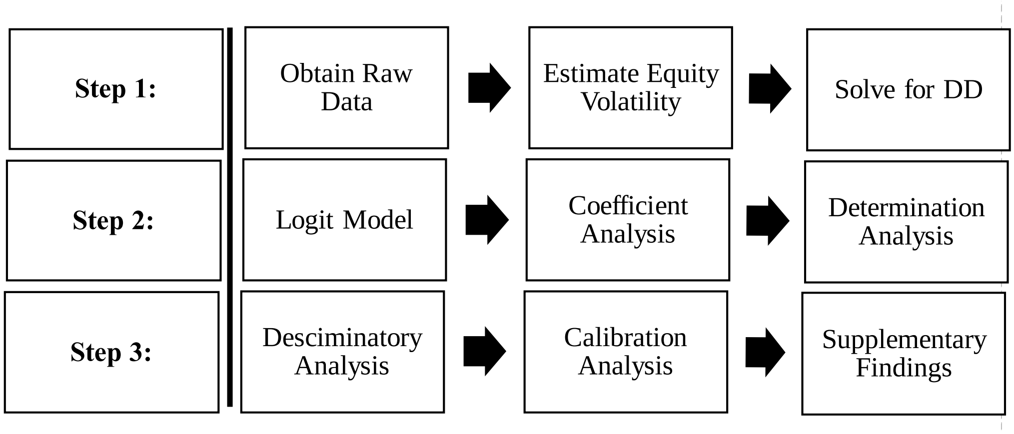


[7] The figure is obtained from Loffler & Posh (2007).

Part III – Research design

The methodology of this thesis is divided into three main parts. The first part aims to produce DD for the sample. The second is the process of model testing and the third is the course of analyzing the PD output. Figure 2 displays the research design.

Figure 2: Summary of research design.



Step 1 starts with obtaining raw data from external sources, including market cap, face value of debt, interest rates and equity prices. It is essential to have an outline of which firms that went default at which time in order to validate the model's performance in predicting default. Equity prices will be used to estimate historical and GARCH-estimated volatilities. Next, solve for asset value, asset volatility and asset drift rate. The two former parameters are solved numerically, while the drift rate is estimated through the CAPM. The final output in step 1 will be DD.

In Step 2, the DD will be used as the covariate in the logit model, similar to the scores in the Altman and Ohlson models. The MCD model is changed to a statistical model and becomes comparable to the pure statistical ones. The robustness of the estimated PDs will be analyzed through the significance levels and the models' fit. The main focus of the logistic regression analysis will be to determine whether any version of the MCD models perform better, as well as studying the effects of winsorization and logarithmic transformation of the DD variable.

Step 3 starts with an analysis of the discriminatory power. Each MCD model will be compared to the pure statistical ones in order to find out which model that discriminate best and if there is a clear indication of the MCD model's superiority. The calibration analysis will only focus on the MCD models, where the goal is to detect signs of underestimation of PD. Lastly, supplementary findings of PDs' evolvement will be discussed to complete the assessment of the research question.

Part IV – Data

The process of data gathering is twofold; finding historical defaults and non-defaults for Norwegian public firms and combine them with relevant financial data.

Financial Data

Financial data is obtained from Datastream Thomson Reuters (Datastream). Datastream offers time series for more than 3,5 million financial instruments across assets classes with up to 50 years of historical depth. They cover over 80 000 active equities and over 85 000 inactive equities for emerging and developed markets that includes prices, volumes, market capitalizations, earnings, dividends and much more. They source direct from exchanges, leading international and local suppliers and published reports (Datastream, 2016).

Daily stock prices from 1974 to 2014 are used to obtain volatility estimates. The downloaded time series includes 918 different equities and other equity instruments. Based on company names, equities have been manually filtered out which were totaled to 653 firms. Rolling holidays are accounted for by letting unpadded network days for all securities to be interpreted as holidays.

The same accounting process for holidays are done for daily values of the short term interest bearing debt, long term interest bearing debt and market capitalizations as well. One weakness of the data is that debt values are only updated quarterly. Since debt tends to change in a more continuous manner, the firms' true credit risk may not be reflected in the model's estimated PDs. Another disadvantage by using Datastream, instead of manually searching in financial reports, is that there may be riskless debt included and risky debt excluded in the downloaded data. However, we trust the provider's ability to distinguish interest-bearing from non-interest-bearing debt.

On the 26th of May 2014, a letter from Norges Bank to the Financial Supervisory Authority of Norway came to the conclusion that there is currently no realistic alternative to NIBOR[8] as a reference rate. It is further argued that the reference rate can be decomposed into a risk free rate (Norges Bank, 2014). We assume that the rate includes no risk premiums and the 3-month NIBOR is used as proxy for the

[8] NIBOR is intended to reflect the interest rate level lenders require for unsecure money market lending in NOK with delivery in two days after the trade (Oslo Stock Exchange 2014)

risk free rate. Lastly, when estimating the drift rate of the asset, daily market returns are based on the currency adjusted World MSCI index.

Default data

Default data is gathered from two sources: Brønnøysund Register Centre and Stamdata. Agreements of data confidentiality from both sources were necessary to conduct the study. The two data sources define default differently and has recorded defaults from different points in time. To maintain data consistency, the analyses of the two definitions of default are performed separately.

Brønnnøysund Register Center develops and operates a large portion of Norway's most important registers and electronic solutions (The Brønnøysund Register Centre, 2016). The original dataset of bankruptcies provided by the registry contained 94 defaults of Norwegian firms with limited liability from 1996 to 2015. Of these defaulting firms, 25 have been listed on the Oslo Børs ASA. The recorded defaults are cases where companies have formally applied for bankruptcy directly to the registry, similar to that of Chapter 7 in the US bankruptcy code (United States Court, 2016). A practical constraint in the sample is that bankruptcy typically occurr 1-3 years after the date of delisting. In order to conduct experiments, the dates of de-listing are counted as date of default. As such, market capitalization close to the de-listing date will not necessarily be able to reflect information about the expectations of default.

The second provider of defaults are Nordic Trustee's database, Stamdata. Independently owned by Nordic banks, insurance companies and security brokers, Nordic Trustee serves as a third party information agent between the issuer and the bondholder. Stamdata delivers reference data for Nordic debt securities including detailed information on bonds, certificates and structured debt securities (Stamdata, 2016). There is no legal obligation to use a trustee, but 95% of the issued volume of debt in the Norwegian market have a trustee arrangement (Grøstad, 2013). In contrast to the time horizon of the first default sample, Stamdata only contains information on bond issuers' credit events from 2007 to 2015. While these events may resemble certain aspects of Chapter 11 bankruptcies in the US bankruptcy code, resolutions from Norwegian defaults rarely take place in court (Grøstad, 2013) (United States Court, 2016). Nevertheless, the credit events we regard as defaults in the Stamdata database may be broadly defined as Chapter 11 defaults. In total, Stamdata documents 53 defaults of such definition.

The first default sample used in this thesis contains defaults of Brønnøysund Registre Center only, and spans from 1996 to 2015. It includes 25 default events and will be referred to as *the bankruptcy sample*. The second sample contains both bankruptcy events from Brønnøysund Registre Center and default events from Stamdata, spanning from 2006 to 2015. After accounting for overlapping events, the second sample amounts to 67 default events. The second default sample will be referred to as *the reorganization sample*.

Part V – Methodology

The methodology section contains three main parts. The first part describes the process of obtaining input data for the MCD model. Then, the process of testing and comparing the different versions of the model is presented. The last part presents the process of model validation.[9]

Obtaining Input Data

Returns of Equity

Estimates of equity volatility are used to obtain estimates of the value and volatility of assets. The MCD model assumes that assets are traded continuously in time. Hence, the natural logarithmic returns from daily stock prices are used and calculated as

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{17}$$

where P_t is the closing price at time *t* and P_{t-1} is the closing price the day before time *t*. Each price is adjusted for dividends and splits. The logarithmic effect on returns sometimes cause unrealistic occurrences of $r_t < -100\%$ for particular penny stocks. Penny stocks are of special interest when studying credit risk, and we do not find it feasible to exclude those time series. Instead, discrete returns are calculated for series of $r_t < -100\%$

$$\widehat{r_t} = e^{r_t} - 1. \tag{18}$$

This solution comes at the cost of violating assumption 4 in the MCD model. However, we consider that being able to studying penny stocks is worth the cost of this violation. From now, $\widehat{r_t}$ is not distinguished from r_t, but simply referred as r_t and treated as log-returns.

Data Mining

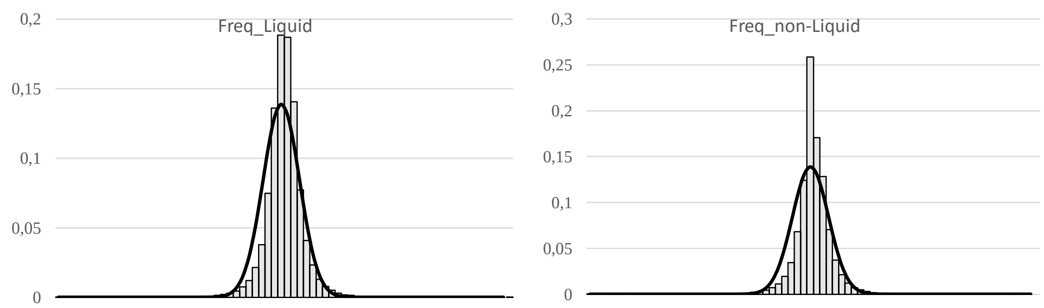
A direct consequence of assumption 7 is that $dV \sim \Phi(mean, variance)$, such that $dE \sim \Phi(mean, variance)$ because $E = f(V, \sigma_v)$ (Hull, 2015). One can therefore study the frequency distribution of equity returns to see how well the assumption holds and consequently the reliability of the models' estimated probability of default. For example, a negative skew and highly positive kurtosis of the asset

[9] All calculations are conducted in Microsoft Excel and Visual Basic for Application for Excel (VBA). Codes are to found in appendix G

return distribution will underestimate the PD by assuming $dV \sim \Phi\ (mean, variance)$.

When studying daily returns, two pitfalls arise concerning illiquidity and data errors (Koller, Goedhart, & Wessels, 2010). Firms that do not trade when they in are in fact open for exchange are problematic. Such illiquid stocks will not be excluded by the holiday procedure described in PART IV-Data and induce a downward bias in the estimates of volatility and drift rate. Continuing on the assumption that no stock can have a return of exactly zero, such particular observations are excluded as well. The effects of this filtering on the equity returns are illustrated in Figure 3, where the left figure is the filtered time series and the right figure is the unfiltered time series. The black line is a theoretical normal distribution. After filtering, the middle bin is reduced from about 25% of the observations to about 17% of the observations. This 8% reduction indicates that the Oslo Stock Exchange does suffer from illiquidity. The filtered distribution is still too peaky compared to a normal one, where the excess kurtosis for the filtered and unfiltered distribution are 176,5 and 227,48 respectively. This implies that there is a higher probability of extreme returns, but most of the observations lies close to the mean.

Figure 3: The effects of filtering zero-returns on the distribution of equity returns.



The second pitfall when studying equity returns is how observable time series of returns may include data errors, extreme events or accounting discretion. Such observations may have large influence on the PD output. In a normal distribution when using the mean and standard deviation from the filtered distribution in Figure 3; 99% of the observations should lie between -10,7% and 10,8%. The true range from the 99% percentile in our sample are -3,89% and 13,2%. It is therefore interesting to see how winsorized equity returns will affect the output from the MCD model. Winsorization is conducted on each firm separately on either (0,5%, 99,5%), (1%, 99%) or (2%, 98%) level, and the chosen levels are based on the series' four moments and percentiles. The effects of winsorization and illiquidity filtering of equity returns are quantified in Table 1. All returns are merged to one distribution to give an overview of the total effect.

Table 1: Descriptive statistics for equity returns.

Descriptive Statistics			
		Filtered	Non-Filtered
Winsorized	Avg	0,00	0,00
	Stdev_s	0,04	0,04
	Skew	0,22	0,25
	e_Kurt	17,39	19,53
	Count	766548	984170
Non-Winsorized	Avg	0,00	0,00
	Stdev_s	0,05	0,04
	Skew	2,56	2,91
	e_Kurt	176,50	227,48
	Count	766548	984170

All four samples show positive skew which is mostly caused by interchanging between discrete and log-returns. The skew of a distribution with log-returns only were in fact negative and the true skewness will lie somewhere in between, which may be interpreted as closer to zero. The most significant adjustment to the distribution occurs when the data is winsorized; the excess kurtosis on the unfiltered dataset drops from 227,48 to 19,53, and the skewness drops from 2,91 to 0,25. The dataset is still not normal and one can be critical to the assumption of normality in the asset values. Based on the large impact of winsorizing equity returns, there are of strong interest to study its effect on the PDs. It is important to remain cautious of modifying the dataset to improve the historical fit of the model, as optimized historical fit does not equal optimized prediction. Unfiltered datasets are excluded from any further analysis because there are few, if any, downsides by filtering out illiquid behaviors.

Volatility of Equity

The volatility is a measure of the dispersion of returns for a stock (Hull, 2015). To be able to obtain inputs for the MCD model, it is necessary to estimate the equity volatility. Since it is unobservable, one can only estimate its true value. John Hull argues that, in practice, the most used estimate for future equity volatility is the volatility implied by option prices observed in the market. However, options written on public firms are scarce in the Norwegian market. An alternative is to use the historical volatility, but this is backwards-looking and may not be a good estimate for the future volatility (Hull, 2015). We therefore choose to apply both historical and the GARCH (1,1)-process when estimating the equity volatility. Historical volatility estimates are calculated from daily returns, and annualized with 250 days

$$\sigma_E = \sqrt{250\left(\frac{1}{N-1}\sum_{t=1}^{N}(r_t - \bar{r})^2\right)} \tag{19}$$

where $\bar{r} = \frac{1}{N}\sum_{t=1}^{N} r_t$ and N dictates the estimation window. The window's length is not easily determined. A larger window with more data usually lead to more accuracy, but since volatility varies with time, old data may not be relevant for predicting future volatility. Hull states that a reasonable compromise is to use the most recent 90 to 180 days. Another rule of thumb is to set N equal to the number of days to which the volatility is to be applied (Hull, 2015). With no definite solution, four different rolling windows are used. That is 90 days, 180 days, 250 days and 5 years. In addition, an expanding window is used with a minimum requirement of 90 days.

The choice of applying the GARCH(1,1) is based on the model's simplicity and its theoretical appeal (Hull, 2015). The equation is

$$\sigma_t^2 = (1-\alpha-\beta)V_L + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{20}$$

where V_L is the unconditional variance and r_{t-1}^2 is the previous day's squared returns. The term σ_{t-1}^2 is the previous day's conditional variance and the parameters $(1-\alpha-\beta)$, α, β are the weights associated with each variance term. Coefficients are solved with maximum likelihood method, optimized through a Nelder Mead algorithm[10]. In order to reach a mean-reverting effect towards V_L, the model requires that $\alpha + \beta < 1$. The main difference between the GARCH(1,1) model and the equal weighted historical volatility is that the GARCH model may put more weights on recent shocks. Hence, the GARCH(1,1) may be a superior estimator if recent information is more relevant for future volatility. The rolling windows are restricted to 90, 180 and 250 days. Seeing that penny stocks may cause the GARCH volatility estimates to be too jumpy, variance targeting[11] can help to achieve robustness (Hull, 2015). In this study, we choose to test the MCD model with and without variance targeting.

[10] The Nelder-Mead algorithm minimize the value of a function by moving vertices through a set of rules (Small & Wang, 2003). In the maximum likelihood method, the value is maximized, by minimizing the negative of that value. VBA codes can be found in appendix G.

[11] Variance targeting involves setting the long-run volatility V_L equal to the sample variance.

The output for the GARCH(1,1) model is an estimation of the next day volatility. Annualizing this daily volatility estimate will violate the mean-reverting assumption of the model. However, predicting 250 days forward with $\alpha + \beta < 0$ will result in volatility estimates that are fairly close to V_L. As such, the aim of putting weight on recent shocks vanishes. The annualized, one-month estimate is used

$$E(\sigma_T) = \sqrt{250\left(V_L + (\alpha+\beta)^{\frac{250}{12}}(\sigma_t^2 - V_L)\right)}. \tag{21}$$

Due to computational constraints in calculating daily GARCH(1,1) estimates, we are forced to simplify by calculating monthly estimates and assume that volatility remains constant throughout the month. In total, six different versions of GARCH volatility estimates are obtained. Together with the five versions of historical volatility, 11 different versions of the MCD model are used to calculate different DDs. The versions are summarized in Table 2. Going forward, the versions will be referred to as their model name, as described in the first column of the table. In the table, the volatility type and estimation window for each model is described. For three of the GARCH models, the unconditional variance is fixed.

Table 2: The different versions of the MCD model.

Model Description

Model	Vola_Type	Time _Int	Comment
h90	Historical	90 days	
h180	Historical	180 days	
h250	Historical	250 days	
h5Y	Historical	5 years	
hTOT	Historical	Total	
g90*	GARCH	90 days	*Fixed unc.var
g180*	GARCH	180 days	*Fixed unc.var
g250*	GARCH	250 days	*Fixed unc.var
g90	GARCH	90 days	
g180	GARCH	180 days	
g250	GARCH	250 days	

Face Value of Debt

The face value of debt, F, is argued to be observable in the MCD model. This parameter may be extracted from financial reports, however, such reports are not updated as frequently as the debt value changes. We follow the researchers of KMV Corporation who assume that F can be estimated by

$$F = F_S + 0{,}5F_L \tag{22}$$

where F_S is current interest bearing debt and F_L is non-current interest bearing debt. The choice of using exactly half of the non-current debt is arbitrary but it has its intuitive appeal. Refinance-risk is lower during a short term perspective and the liquidation options higher as the time to maturity increases (Sobehart, Stein, Mikityanskaya, & Li, 2000). In a short-term perspective, short term debt requires a repayment of the principal, whereas long term debt requires only coupon payments to be met (Lando, 2004).

Asset Value and Asset Volatility

Daily Asset value, *V*, and asset volatility, σ_V, are obtained from the inputs specified above. The two equations-two unknowns-method is preferred in this thesis. It is solved numerically through a new Nelder Mead algorithm where the two equations are

$$E - \left(V\Phi(d_1) - Fe^{-rT}\Phi(d_2)\right) = 0 \tag{23}$$

$$\sigma_E - \frac{\Phi(d_1)\sigma_V V}{E} = 0. \tag{24}$$

Here, *E* is the market capitalization downloaded from Datastream, *F* is the face value of debt specified by Equation 22, d_1 and d_2 are specified in Equation 9. The equity volatility, σ_E, is the historical or the GARCH volatility described above.

Drift Rate

The CAPM method described in the literature review is applied to estimate the drift rate of the asset. The coefficients are estimated through the following formula

$$R = \hat{\alpha} + \hat{\beta} R_M \tag{25}$$

where $E(R) = r - r_f$, r is the return vector of the respective asset and r_f is the risk-free interest rate vector. Further, $\hat{\alpha}$ is the intercept and $\hat{\beta} = \frac{\sigma_{V,M}}{\sigma_M^2}$, the slope. $R_M = r_M - r_f$ is the risk premium vector, where r_M is the return of the market portfolio. The estimation error is minimized by using the longest period without any structural gaps to estimate the expected market returns. All returns are daily arithmetic discrete returns. The beta in Equation 25 is smoothed by the following formula

$$\beta = \frac{1}{3} + \frac{2}{3}\hat{\beta}. \tag{26}$$

When estimating the coefficients in the CAPM, the optimal length of the rolling window is five years (Koller, Goedhart, & Wessels, 2010) (Black & Scholes, 1973). Longer periods will place too much weights on old, irrelevant data. On the other hand, too frequent measurement may create errors in illiquid assets. Unfortunately, only a small fraction of the defaulted firms in our sample have been listed as long as five years. In response, daily returns with a minimum or maximum requirement of 250 days and 5 years are used, respectively. Short assessment windows increase the possibility to estimate unrealistic drift rates, such as negative ones, but such disadvantages are inevitable when studying penny stocks.

The last step is to estimate the annual drift rate last trading day each year by the following formula

$$\mu = 250\left(\ln\left(1 + r_f + \beta E(R_M)\right)\right) \tag{27}$$

where $E(R_M) = \frac{1}{5Y}\sum_{t=1}^{5Y}(r_{M,t} - r_{f,t})$, The expected market premiums are always calculated with the maximum requirement length of 5 years. The annualized drift rate calculated as continuously compounded because of assumption 4 of the MCD model specified in the literature review.

Distance to Default

The final output from the MCD model is the DD specified in Equation 15. The output DD is calculated the last trading day each year which is the predicted DD for the upcoming year. That is, the asset value, face value of debt, asset volatility and drift rate on the last trading day in December for each year from 1995 to 2014 will determine the DD for year *t+1*. This variable is the independent variable in the logit model in the subsequent stage of research.

Summary of Methodology Part 1: Obtaining Input Data

Table 3 summarizes the input variable for our study of the MCD model. The first column describes the calculated variable used in the application of the MCD model, and the second column presents the variables formula. The first volatility formula represents the annualized historical volatility, and the second represents the formula for the annualized GARCH volatility estimate. The asset volatility and asset value is solved for numerically and simultaneously. After obtaining the necessary input data for the model, each firm's distance to default (DD) is calculated for each year. The time parameter, *T*, is excluded from the formula for DD because the estimation period in our methodology is one-year ahead. Independent variable in the logit model.

Table 3: Summary of Methodology Part 1: Obtaining Input Data.

Variable	***Formula***
Volatility of Equity	$\sigma_E \sqrt{250\left(\frac{1}{N-1}\sum_{t=1}^{N}(r_t-\bar{r})^2\right)}$ $\sqrt{250\left(V_L+(\alpha+\beta)^{\frac{250}{12}}(\sigma_t^2-V_L)\right)}$
Face Value of Debt	$F_s+0{,}5F_L$
Asset Value and Asset Volatility	$\frac{E-Fe^{-rT}\Phi(d_2)}{N(d_1)}$ $\frac{\sigma_E E}{V\Phi(d_1)}$
Drift Rate	$ln\left(1+r_f+\beta E(R_M)\right)$
Distance to Default	$\frac{ln\left(\frac{V}{F}\right)+(\mu-\sigma_V^2)}{\sigma_V}$

Table 4 summarizes the final default samples we base our analyses on. The samples of defaults are constrained by data availability in the process of calculating the DD variable. With this restraint, the final samples of defaults from bankruptcy sample amounts to 11 bankruptcies amongst 1999 DDs. The second dataset of defaults consisting is restricted to 36 defaults amongst 1089 DDs.

Table 4: Summary of default samples.

Def_Bankruptcy_Sample

Year	Def	Firms	Def_rate
2015	1	133	0,75 %
2014	1	127	0,79 %
2013	2	115	1,74 %
2012	0	121	0,00 %
2011	1	120	0,83 %
2010	0	105	0,00 %
2009	3	125	2,40 %
2008	0	133	0,00 %
2007	0	109	0,00 %
2006	0	110	0,00 %
2005	0	91	0,00 %
2004	0	77	0,00 %
2003	0	67	0,00 %
2002	1	78	1,28 %
2001	2	94	2,13 %
2000	0	83	0,00 %
1999	0	74	0,00 %
1998	0	95	0,00 %
1997	0	77	0,00 %
1996	0	65	0,00 %
Total	11	1999	0,55 %

Def_Reorganization_Sample

Year	Def	Firms	Def_rate
2015	5	133	3,76 %
2014	2	127	1,57 %
2013	4	115	3,48 %
2012	4	121	3,31 %
2011	5	120	4,17 %
2010	2	105	1,90 %
2009	10	125	8,00 %
2008	3	133	2,26 %
2007	1	109	0,92 %
Total	36	1088	3,31 %

Model testing

Logistic Regression

It is reasonable to use a universal probability model to compare different models of default prediction. Linear probability models are dismissed because of the unlikeliness that the PDs are linearly dependent on a set of explanatory variables (Brooks, 2014). The logit model[12] or the probit model are examples of more sufficient models. The former assumes a logistic distribution and the latter assumes a normal distribution, but the fitted regression plots will usually be virtually indistinguishable (Brooks, 2014). Previous studies tend to weight the analysis on the logit model (Stein & Sobehart, 2000), which provides basis for using the logit model in this research paper as well.

The logistic function, Ψ, which is the cumulative logistic distribution, can be interpreted as the PD dependent on a random variable z. The binary dependent variable, y, is 0 in case of non-default and 1 in case of defaults. If $P(y = 1) = \Psi(z)$, then the formula is expressed as

$$\Psi(z) = \frac{1}{1 + e^{-z}}. \tag{28}$$

The random variable *z* is estimated thorough the linear function, $z = \boldsymbol{\beta}'\boldsymbol{x}$. Where $\boldsymbol{\beta}$ is a column vector of coefficients and $\boldsymbol{x}$ is a two dimensional set of explanatory variables. We include an intercept such that $x_1 = 1$ and so *z* will be regressed explicitly as

$$z = \beta_1 + \beta_2 DD \tag{29}$$

where DD is the distance to default series. The weights $\boldsymbol{\beta}$ are estimated through maximum likelihood method, which is calculated by the product of the likelihood function for individual observations *i*,

$$L = \prod_{i=1}^{N} L_i = \prod_{i=1}^{N} \Psi(z_i)^{y_i}\big(1 - \Psi(z_i)\big)^{1-y_i} \tag{30}$$

Equation 30 is globally concave, such that when the root of the first derivative is found, one can be sure to have found the global maximum of L. A Newton Raphson algorithm, based on Taylor approximation, is used in the optimization.

After finding the coefficients, each firm's DD for each year are inserted into Equation 29 which is the input for the final equation

[12] The logit model is also called logistic regression model.

$$PD_{i,t} = \frac{1}{1 + e^{-z_{i,t}}} \tag{31}$$

where $PD_{i,t}$ is the implied PD for the upcoming year. Mark that the coefficients are fixed, such that the PD is not binary but $0 \leq PD_{i,t} \leq 1$.

P-value

To evaluate whether a variable is helpful in explaining defaults or not, the p-value of each coefficient is assessed. The distribution of the t-statistic follows a normal distribution, and is applied to obtain the p-value,

$$p - value = 2 * \left(1 - N(|t_{stat}|)\right). \tag{32}$$

The multiplier of 2 indicates a two-tailed test, $|\cdot|$ is the absolute value, $t_{stat} = \frac{\beta_i}{SE(\beta_i)}$ and $SE(\beta_i) = \sqrt{\sigma^2_{\beta_i}}$.

McFadden R^2

The models' goodness of fit is analyzed to compare the performance of the different versions of the MCD model and the pure statistical models. The classical linear regression statistic R^2 is used to evaluate how well the sample regression function fits the data. In the logit model, this quadratic measure of variation is modified to the "pseudo-R^2" because of the non-linear property that is estimated through maximum likelihood (Loffler & Posch, 2007). Another name for the test statistic is McFadden's R^2 (R^2_{MCF}). True PDs are unobservable, and thus $\mathrm{R}^2_{\mathrm{McF}}$ cannot say anything about whether the model correctly predicted default probabilities. However, the statistics illustrate whether the model correctly predicted defaults. The formula is

$$R^2_{McF} = 1 - \frac{\ln(L)}{\ln(L_0)}. \tag{33}$$

L_0 is the log-likelihood of the function without any covariate and L log-likelihood of the function with covariate(s). R^2_{McF}has the same boundary as the R^2; $0 \leq R^2_{McF} \leq 1$, but similar general quality standards are difficult or impossible to apply (Loffler & Posch, 2007). If the model where perfectly able to classify defaults from non-defaults, it would have two different classes of firms; one high risk class, $PD \approx 100\%$ and a low risk calls, $PD \approx 0\%$. This is clearly unrealistic and one cannot hope to get $\mathrm{R}^2_{\mathrm{McF}} \approx 1$. Altman argues that a $R^2_{McF} \geq 35\%$ is achievable for credit risk models (Altman & Rijken, 2004). This is held as benchmark but with caution, seeing that our study consists of a different dataset.

Likelihood Ratio Test

The two likelihood functions used in $\mathrm{R}^2_{\mathrm{McF}}$, L_0 and L, can also be applied to statistically test the entire logit model. Here, it is called the likelihood ratio test and the formula is

$$LR = 2(\ln(L) - \ln(L_0)). \tag{34}$$

It is distributed asymptotically chi-squared with the degrees of freedom equal to number of restrictions imposed.

Functional Form

The random variable, z, in Equation 29 is linearly dependent on the value of the DD. Loffler et al. have experimented with using the natural logarithm of the DD in the regression and found that it could improve the quality of the model (Loffler & Posch, 2007). This method may be a solution to the problem of extreme values impacting coefficient estimates, which could impair the logit model. We will therefore perform similar experiments. In situations where the DD is negative, the functional form is calculated as

$$\ln(DD) = -\ln(1 - DD). \tag{35}$$

Tjur's R^2

Tue Tjur has come up with a relatively new measure of explanatory power that he calls "the coefficient of discrimination" (Tjur, 2009). Subsequent references often call the measure "Tjur's R^2" (R^2_{Tjur}). Tjur argues that $\mathrm{R}^2_{\mathrm{McF}}$ are based on ideas related to variance and quadratic variation, which are somewhat strange concepts in a universe of binary observations. The proposed alternative takes the averages of the fitted values for successes and failures

$$R^2_{Tjur} = |\overline{PD_D} - \overline{PD_{ND}}|. \tag{36}$$

Here, $\overline{PD_D}$ is the average PD for firms that defaulted and $\overline{PD_{ND}}$ is the average PD for firms that did not default. The absolute term, $|\cdot|$, is to make sure that the boundary of the measure is between 1 and 0. In contrast to $\mathrm{R}^2_{\mathrm{McF}}$, R^2_{Tjur} is not based on the value of the log-likelihood function. It is credited for its intuitive appeal and the model's ability to discriminate between defaults and non-defaults will be reflected in the statistic, and both over and under-estimation of default probability is penalized. For more details on the discrimination statistic, we refer to Tjur's own article "Coefficients of Determination in Logistic Regression Models – A new Proposal: The coefficient of Discrimination" (Tjur, 2009).

Summary of Methodology Part 2

The regression analysis is conducted on eleven different types of MCD models that only vary in terms of the equity volatility estimate. Different equity volatility estimates will influence the estimated asset value and asset volatility, which in turn will produce different distances to default. Each model is tested on two different types of default samples. The first one spans from year 1996 to 2015 where default is defined as bankruptcy. The latter from year 2007 to 2015, and consists of all credit events we define as reorganization. Each default sample is tested four different times where the sample differs in terms of winsorization, and the regression differs in terms of its functional form. Table 5 summarizes the tests applied in the section of model testing. The left column lines up the models that are tested, and the heading illustrates which samples are used in the regression analysis: "Org" indicate that the original, non-winsorized sample is used, "Ln" indicate that the independent variable is the natural logarithm of the DD, "Wins" indicates that the sample being used in the regression analysis is winsorized.

Table 5: Summary of Methodology part 2: Model testing.

	Bankruptcy_Sample				**Reorganization_Sample**			
Model	Org	Ln	Wins	Wins_ln	Org	Ln	Wins	Wins_ln
h90								
h180								
h250								
h5Y		Logit Model				Logit Model		
hTot		Likelihood Ratio Test				Likelihood Ratio Test		
g90*		P-value				P-value		
g180*		McFadden's r-squared				McFadden's r-squared		
g250*		Tjur r-squared				Tjur r-squared		
g90								
g180								
g250								

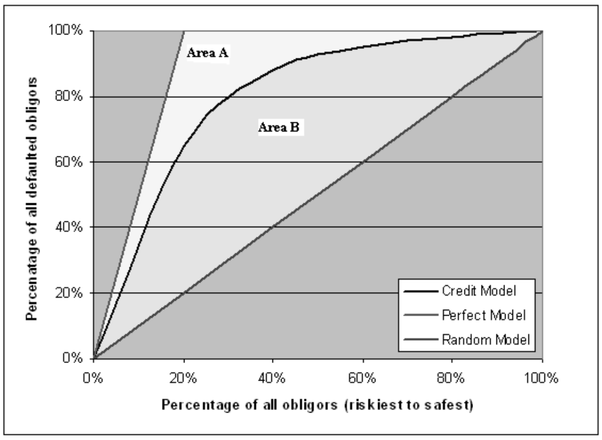
Model Validation

There are two dimensions along which credit ratings are commonly assessed: discrimination and calibration (Lando, 2004). The former dimension describes how well the model ranks firms with respect to their actual outcome, default or non-default. The latter describes to which extent the estimated PDs match the true default rates.

Discriminatory analysis

We use the Cumulative Accuracy Profile (CAP) and the Accuracy Ratio (AR) to evaluate discriminatory power. The CAP curve is a graphical illustration of a models ability to rank companies by their probability of default. To plot a CAP curve, firms are ordered by their PD from the highest to the lowest. Figure 4 illustrates a CAP curve for a hypothetical credit model. The line to the left of Area A represents an ideal model which perfectly distinguishes between defaulters and non-defaulters. The line below Area B represents a model that does not distinguish defaulters from non-defaulters. The line between Area A and Area B represents a hypothetical credit risk model (Morokoff, 2011).

Figure 4: Cumulative Accuracy Profile (CAP).



The random model is a total uninformative model that ranks PD randomly. It is expected to capture a proportional fraction of defaulters with each increment of the sorted sample, and is visualized by a straight line. The perfect model perfectly distinguishes defaulted companies from non-defaulted companies, such that the curve looks like a steep straight line until the fraction of defaulters to the total number of firms are met on the x-axis. It then turns to a horizontal line since there are no defaults left in the sample. The closer the credit model is to the perfect model, the better it is at discriminating between defaulter and non-defaulters.

The AR is a single summary measure of the CAP curve, that ranks the predictive accuracy of each credit model for both Type 1 errors and Type 2 errors. More precisely, it measures the proportion of defaulters in a sample that can be identified per increment of the risk score that is being evaluated. It has the boundary $0 \leq AR \leq 1$, where $AR \rightarrow 1$ as $Credit\ model \rightarrow Perfect\ model$. From Figure 4 one can calculate the AR by the following formula

$$AR = \frac{Area\ B}{Area\ A + Area\ B}. \tag{37}$$

Most credit risk models have an AR between 50% and 75% (Keenan & Sobehart, 1999).

A bootstrap simulation is conducted in order to reduce the sensitivity of the AR to outliers and small samples of defaults. The core idea is to re-sample from the data and re-estimate the AR analysis with the new re-sampled data. It is performed with 1000 trials, measured through a 95% confidence interval.

Comparable Models

The second hypothesis states that the MCD model outperforms pure statistical models in discriminating between defaulting and non-defaulting companies. Since it is difficult to determine a model's discriminatory power in isolation, a comparison between the models is necessary to validate their performance. Altman's Z-score and Ohlson's O-score are used as proxies for pure statistical models.

Altman's Z-score is one of the earliest examples of bankruptcy-prediction models (Altman E. I., 1968). It acts as a statistical distillation of historical data and can be used to discriminate between different level of credits. Altman concluded that a parsimonious model that best predicts bankruptcies contains the working capital, retained earnings, EBIT, market capitalization, sales and total assets. Notwithstanding the model's simplistic nature, Altman's Z-score was shown to predict bankruptcies with 70% accuracy in its 1968 experiments, and has since been subject to comparison with subsequent default prediction models (Altman E. I., 1968). The formula is as follows

$$\begin{aligned} Z = 1{,}2\left(\frac{WC}{TA}\right) + 1{,}4\left(\frac{RE}{TA}\right) + 3{,}3\left(\frac{EBIT}{TA}\right) + 0{,}6\left(\frac{MV}{TL}\right) \\ + \left(\frac{Sales}{TA}\right) \end{aligned} \tag{38}$$

where *WC*= working capital, *TA*=total assets, *RE*=retained earnings, EBIT=Earning before interest and taxes, *MV*=market value of equity and *TL*= total liabilities.

Ohlson's O-score was developed 1980 (Ohlson, 1980). In contrast with previous linear models, the model assumes that the PD is logistically distributed, which is consistent with the methodology in this paper. Ohlson concluded that the optimal model to predict bankruptcies contain nine financial ratios, primarily from companies' balance sheets. The extensive formula is

$$\begin{aligned} O = -1{,}32 - 0{,}407 \ln\left(\frac{TA}{GNP}\right) + 6{,}03\left(\frac{TL}{TA}\right) - 1{,}43\left(\frac{WC}{TA}\right) \\ + 0{,}0757\left(\frac{CL}{CA}\right) - 1{,}72X - 2{,}37\left(\frac{NI}{TA}\right) \\ - 1{,}83\left(\frac{FFO}{TL}\right) + 0{,}285Y \\ - 0{,}521\left(\frac{NI - NI_{-1}}{|NI| + |NI_{-1}|}\right). \end{aligned} \tag{39}$$

Here, TA = total asset, GNP = Gross National Product price index level, TL = total liabilities, WC = working capital, CL = current liabilities, CA = current assets, $X\begin{cases}1\ if\ TL > TA \\ 0\ otherwise\end{cases}$, NI = net income, FFO = funds from operations, $Y\begin{cases}1\ if\ NI < 0\ \ for\ last\ two\ years \\ \quad 0\ otherwise\end{cases}$.

Calibration of Rating-Specific Default Probabilities

To assess the third hypothesis in this thesis, which states that the MCD model systematically underestimates the true probability of default, the Vasicek's one factor model is applied (Vasicek, 1997). Its application is motivated by DD, but the model of dependence is Gaussian Copula. Since default rates are dependent on macroeconomic factors, it is important to account for joint default probabilities (Loffler & Posch, 2007). The Vasicek one factor model accounts for such correlations.

Default correlations are modeled through correlations in asset values and the one factor model assumes that they can be captured through the common factor Z and a unique factor ε. The asset value, V, for firm *i* can be written as $w_i Z + \sqrt{(1 - w_i^2)}\varepsilon_i$ where $0 \le w_i \le 1, \forall i$. The default correlation between form *i* and *j* is $\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$. By assuming that $Z \sim \Phi(0{,}1)$, $\varepsilon \sim \Phi(0{,}1)$, $\sigma_{\varepsilon_i,\varepsilon_j} = 0$ for $j \ne i$, $\sigma_{\varepsilon_i,Z} = 0, \forall i$ and $V \sim \Phi(0{,}1)$, the covariance term is equal to the correlation because

$$\sigma_{i,j} = cov\left(w_i Z + \sqrt{(1 - w_i^2)}\varepsilon_i, w_j Z + \sqrt{\left(1 - w_j^2\right)}\varepsilon_j\right) \tag{40}$$

$$\sigma_{i,j} = w_i w_j \sigma_Z^2 = w_i w_j = \rho_{i,j}. \tag{41}$$

In the absence of other information, the one factor model also assume that all firms have the same default probability such that every default threshold, $F_i = F, \forall i$. The correlation can therefore be simplified, $\rho_{i,j} = \rho = W^2, \forall i, j$. The joint probability of default is found by the following formula

$$\psi(F, F, \rho) \tag{42}$$

where $\psi(\cdot)$ is the bivariate standard normal distribution function. Equation 42 is solved numerically to find ρ by using the method of moments through a Nelder Mead algorithm. The two moments solved are $P(V_i < F_i)$ and $P(V_i < F_i, V_j < F_j)$.

The one factor model ignores individual bad luck, such that a single years default rate is identical to the default probability that year. The rate can be shown to be larger than the underlying default probability if the market is depreciating. The default rates are shown in Table 4 displayed in Part V – Methodology.

The Vasicek one factor model should be applied within the binomial probability distribution, $\psi(\cdot)$, but since $\psi(\cdot) \rightarrow \Phi(\cdot)$ as $observations \rightarrow \infty$, we allow ourselves to use $\Phi(\cdot)$. The formula is

$$\frac{D_t}{N_t} = \Phi\left(\frac{\Phi^{-1}(\overline{PD_t}) - \sqrt{\rho}Z_t}{\sqrt{1-\rho}}\right) \tag{43}$$

where $\frac{D_t}{N_t}$ is the default rate for year t, $\overline{PD_t}$ is the average PD for that year. The common factor, Z_t, is the only unknown value and can thus be solved,

$$Z_t = \frac{\Phi^{-1}(PD_t) - \sqrt{1-\rho}\Phi^{-1}\left(\frac{D_t}{N_t}\right)}{\sqrt{\rho}}. \tag{44}$$

The probability of observing the result of Z_t is $\Phi(Z_t)$. A one sided hypothesis test is conducted to investigate signs of underestimation. If the probability is lower than 1%, we keep the null hypothesis that the MCD model does not underestimate the true PD, $\Phi(Z_t)$. If $1\% \leq \Phi(Z_t) \leq 5\%$, we find that Z is in a “danger area”, with some probability of underestimation. Be mindful that keeping the null hypothesis is equivalent to rejecting the third research hypothesis in this thesis.

Part VI – Findings and Analysis

This section starts with presenting and analyzing the results of the regression analysis, followed by the results of the discriminatory analysis and the calibration analysis. Each analysis will assess one hypothesis of the thesis. The last part of this section is reserved to a discussion of supplementary findings, where the focus will be on how the estimated PDs evolve over time for defaulting firms.

Regression Analysis

The regression analysis will concentrate on the explanatory power of the MCD model and the pure statistical ones, with the aim of assessing the first hypothesis of the thesis. All regression results are summarized in appendix A.

The first step in evaluating hypothesis one is to determine the estimated coefficients' level of significance. All coefficients are statistically significant on the 1% level or lower, where the p-value of the intercepts are literally non-distinguishable from zero. Moreover, every version of the MCD model has $\beta_1 < 0$ and $\beta_2 < 0$. Negative coefficients were anticipated because an increased DD should lead to a lower PD; the distance to the point that triggers default increases.

Winsorization of equity returns has large impact on descriptive statistics, yet only marginal effect on the estimated coefficients. Appendix B shows that the PDs endures little changes as well, inferring that the DD values in the winsorized dataset are virtually equal those in the original dataset. On the contrary, the coefficients estimated by the logarithmic transformed regressor are lower. This stems from the how the logarithmic transformation will push the DD values toward zero, which indicates higher credit risk. To compensate for some of these effects, the coefficients are lowered, adjusting the PDs.

Overall, the DD variable does carry explanatory power because the coefficients are persistently significantly different from zero. The likelihood ratio of all regressions are also statistically significant on 1% level or better. The statistical hypothesis that the DD add nothing to the prediction of default can therefore be rejected with high confidence.

Goodness of Fit

After justifying that the DD variable does indeed carry explanatory power in predicting defaults, the next step is to study how well the variable explains defaults.

Table 6 displays the R^2_{McF} in percentage for each version of the MCD model, for both samples of default. It shows the results from the non-winsorized dataset without logarithmic transformation of the covariate DD. The highest ranked R^2_{McF} for each default sample are highlighted. The GARCH-based models are only updated monthly, while the historical estimates are updated daily. This has biased the results, but we ignore this in the analysis and discuss the models uniformly.

Table 6: Summary of McFadden's R-squared.

Logit_Results(r_mcf)

Model	Bankruptcy	Reorganization
h90	**10,0 %**	**11,3 %**
h180	9,0	10,4
h250	9,0	10,7
h5Y	7,9	9,1
hTot	7,6	8,3
g90*	8,5	9,5
g180*	5,9	6,9
g250*	5,8	7,2
g90	8,3	8,2
g180	6,6	7,7
g250	6,8	10,6

At first glance, the h90 model seems to outperform the others. It has a R^2_{McF} of 10% for the bankruptcy sample and 11,3% for the reorganization sample. In the bankruptcy sample, the difference between h90 and the MCD model with the lowest R^2_{McF}, the g250* model, is 4,2%. In the reorganization sample, the same measured difference is 4,4%.

All models, except of the g90 model has $\Delta R^2_{McF} > 0$ when observing the reorganization sample instead of the bankruptcy sample. Market prices react on many different signals concerning the future and a broader definition may allow more signals to be explained by a credit event. This might be the reason why the MCD model seems to explain failure of financial promises better than bankruptcy.

Ignoring the results for the g250, the table displays a trend that models with shorter estimation windows for volatility have higher explanatory power on bankruptcies. This is consistent with Hulls rule of thumb that 90 to 180 days, or alternatively the same length of the estimation window and forecasting period, works reasonably well when predicting future volatility (Hull, 2015). A similar trend is found amongst the historical volatility models for the reorganization sample, but not for the GARCH-based models. It is clear that g90* and g250 perform well in the reorganization sample where the g250 scores higher than g180 and g90 in terms of R^2_{McF}. Instead of striving for an economical reasoning behind this inconsistency, we conclude that the h90 model seem to be the best candidate with respect the $\mathrm{R}^2_{\mathrm{McF}}$. Nevertheless, we acknowledge that the differences between the models' fit are marginal.

Altman's argumentation that a $\mathrm{R}^2_{\mathrm{McF}}$ of 35% or more is achievable for credit risk models indicate that our results are comparatively low. While the DD variable does exhibit goodness of fit in the original dataset, it may be fruitful to study the influence of winsorization and logarithmic transformation of the DD variable.

Consequences of Data Mining

Table 7 summarizes the average increased $\mathrm{R}^2_{\mathrm{McF}}$ ($\overline{\Delta\mathrm{R}^2_{\mathrm{McF}}}$[13]) for all eleven models when winsorizing the dataset and conducting the logarithmic transformation of DD variable. The table specifies the winsorized $\mathrm{R}^2_{\mathrm{McF}}$ values less the original $\mathrm{R}^2_{\mathrm{McF}}$ values, log-DD $\mathrm{R}^2_{\mathrm{McF}}$ values less the original $\mathrm{R}^2_{\mathrm{McF}}$ values and the winsorized log-DD $\mathrm{R}^2_{\mathrm{McF}}$ values less the original $\mathrm{R}^2_{\mathrm{McF}}$ values for the bankruptcy sample and reorganization sample respectively. All figures are displayed in percentage points.

Table 7: Average change in McFadden's R-squared.

Avg_Increase(r_mcf)

	Bankruptcy	Reorganization
Wins-Org	-0,01 %	0,18 %
Ln-Org	6,52	4,44
Wins_ln-Org	7,09	5,27

The results confirm that winzorization rewards little value in predicting defaults. The $\overline{\Delta\mathrm{R}^2_{\mathrm{McF}}}$ of winsorization is of only 0,13% for the bankruptcy sample and 0,18% for the reorganization sample. Larger changes appear in the logarithmic transformation, where the $\overline{\Delta\mathrm{R}^2_{\mathrm{McF}}}$ for the bankruptcy sample and the reorganization sample is of 6,52% and 4,44% respectively. If the samples are winsorized as well, the $\overline{\Delta\mathrm{R}^2_{\mathrm{McF}}}$ is of 7,09% and 5,27%. The $\ln(DD)$ pulls the DD values downwards. This explains the differences in the $\mathrm{R}^2_{\mathrm{McF}}$ between the two sorts of data manipulation.

Table 8 below shows each model's $\mathrm{R}^2_{\mathrm{McF}}$ for the original dataset, the logarithmic transformed dataset, the winsorized dataset and the winsorized logarithmic transformed dataset. The default samples are distinguished from each other and the

[13] $\overline{\Delta\mathrm{R}^2_{\mathrm{McF}}} = \frac{1}{11}\sum_{i=1}^{11}(\mathrm{R}^2_{\mathrm{McF,Org}} - \mathrm{R}^2_{\mathrm{McF,Fixed}})$, Org symbolize the original dataset and fixed symbolize some form of data mining, like winzorized or logarithmic transformed.

highest ranked models are highlighted. Interestingly, the ranking order in terms of $\mathrm{R}^2_{\mathrm{McF}}$ values changes from Table 6.

Table 8: McFadden's R-squared for the different MCD models.

	Logit_Bankruptcy(r_mcf)				Logit_Reorganization (r_mcf)			
Model	Org	Ln	Wins	Wins_ln	Org	Ln	Wins	Wins_ln
h90	**10,0 %**	14,4 %	**10,0 %**	15,6 %	**11,3 %**	15,0 %	**11,2 %**	15,4 %
h180	9,0	14,8	9,0	13,9	10,4	13,5	10,3	13,6
h250	9,0	12,0	9,0	14,5	10,7	12,6	10,5	14,5
h5Y	7,9	**17,3**	7,9	17,1	9,1	14,4	8,6	14,6
hTot	7,6	16,3	7,6	**17,3**	8,3	14,2	8,0	13,9
g90*	8,5	15,8	8,5	14,0	9,5	**15,3**	9,0	14,2
g180*	5,9	15,7	6,0	16,5	6,9	13,7	7,2	15,2
g250*	5,8	12,7	5,9	13,2	7,2	13,0	7,9	13,4
g90	8,3	12,2	9,0	11,5	8,2	11,0	9,3	11,6
g180	6,6	13,9	6,9	15,9	7,7	12,2	9,4	15,6
g250	6,8	12,2	7,2	13,8	10,6	13,8	10,6	**15,8**

The h90 model is ranked highest in the winzorized as well as in the original dataset. In some instances, winsorization leads to $\Delta\mathrm{R}^2_{\mathrm{McF}} < 0$ which confirms the previous indications that winsorization does not provide value to the use of the MCD model. The h90 has the sixth highest $\mathrm{R}^2_{\mathrm{McF}}$ in the bankruptcy sample which is 2,9% lower than the h5Y model. This is a remarkable difference from the results in the original dataset. To explain the difference between which model seems optimal, we turn to the calculation of the $\mathrm{R}^2_{\mathrm{McF}}$. From Equation 30, one can see that $0 \leq PD_i \leq 1 \leftrightarrow 0 \leq L_i \leq 1, \forall i$, such that $\ln(L) \leq 0$. Also, $\ln(L) > \ln(L_0)$ [14], because a covariate has to endure some explanatory power. It is clear from equation 33 and the properties just presented, that $\Delta\mathrm{R}^2_{\mathrm{McF}} > 0$ when $\Delta \ln(\mathrm{L}) > 0$ and/or $\Delta \ln(L_0) < 0$. Because $\Delta L_0 = 0$ when executing the logarithmic transformation, the only alternative to get $\Delta\mathrm{R}^2_{\mathrm{McF}} > 0$ is if $\Delta \ln(L) > 0$. This is accomplished if $\Delta PD_i > 0|y_i = 1 \oplus \Delta PD_i < 0|y_i = 0$[15]. From the last validation one can study the descriptive statistics of the PDs to answer why the logarithmic transformation leads to another optimal model. Table 8 below displays descriptive statistics of the PDs for the h90, h90_ln, h5Y and h5Y_ln models in the bankruptcy sample. Similar findings for other samples are found amongst the complete sets of descriptive statistics for PDs, which is displayed in appendix B.

[14] $\ln(L_0) = N(\bar{y}\ln(\bar{y}) + (1-\bar{y})\ln(1-\bar{y}))$, where N is the total number of observations and $\bar{y}$ is the average default rate for the sample.
[15] Recall that y is a dummy variable that turns 1 if the firm defaulted and 0 if the firm did not default.

Table 9: Descriptive statistics of PD for default and non-default firms in the bankruptcy sample.

Bankruptcy Sample	h90		h90_ln		h5Y		h5Y_ln	
	Def	N_Def	Def	N_Def	Def	N_Def	Def	N_Def
Avg	0,97 %	0,46 %	0,65 %	0,18 %	0,82 %	0,46 %	0,51 %	0,22 %
Median	0,95 %	0,37 %	0,41 %	0,01 %	0,82 %	0,41 %	0,34 %	0,01 %
Std	0,3 %	1,7 %	0,7 %	2,9 %	0,19 %	1,51 %	0,50 %	3,81 %
Skew	0,66	38,77	1,56	33,78	0,30	39,37	1,33	24,95
Kurt	-0,61	1626,24	1,73	1151,11	-1,06	1701,13	1,23	632,99
Min	0,61 %	0,00 %	0,08 %	0,00 %	0,57 %	0,00 %	0,07 %	0,00 %
q1	0,74 %	0,21 %	0,16 %	0,00 %	0,66 %	0,27 %	0,13 %	0,00 %
q3	1,12 %	0,58 %	0,79 %	0,06 %	0,97 %	0,54 %	0,74 %	0,05 %
Max	1,50 %	75,10 %	2,25 %	100,0 %	1,15 %	67,99 %	1,63 %	100,0 %
Mcf	10,04 %		14,38 %		7,94 %		17,26 %	
Tjur	0,51 %		0,47 %		0,36 %		0,28 %	

The increased R^2_{McF} of the h90 model compared to the h5Y model is caused by higher PDs of the defaulters which is shown by the higher ($\overline{PD}$) and its quartiles (Q). The h90 model also displays a higher skew, implying that more PDs are shifted to the right side of the $\overline{PD}$ in the distribution. The h90 model has a lower Q1, Q2 and positive skewness for the non-defaulters, which further favors the model compared to the h5Y model. The differences in the descriptive statistics for the non-defaulters are less clear, but with only 11 defaults, such small differences will explain more of the higher R^2_{McF}.

When executing the logarithmic transformation, the resulting descriptive statistics for the defaulters favor the 90-day version even more. The $\overline{PD}$ for the non-defaulters are lower for the h90_ln model, but the Q3 and the skew is higher. This implies that more PDs are shifted to the right of the mean, which is the main explanation of how the h5Y_ln model increase R^2_{McF} further.

There are undeniably some pitfalls by valuing a model's predictive power by the R^2_{McF} (Tjur, 2009). Almost all descriptive statistics of the PD of the h90_ln model are superior to those of the h5Y_ln model, but this is not reflected in the R^2_{McF}. The g90*_ln model is ranked as the highest performed model with respect to R^2_{McF} in the reorganization sample and it is ranked higher than the h90_ln model in the bankruptcy sample. The R^2_{Tjur} values in appendix B, shows that the g90* version is superior to all other models in all samples of defaults, including the original and the manipulated series.

The implied PDs of the companies that did default are only marginally higher than those firms that did not, with a typical R^2_{Tjur} of 0,5%. This low $R^2_{Tjur}$is also reflected in the difference between the quartiles. Moreover, some extreme values of non-defaulting firms' PD push the $\overline{PD}$ upwards, further lowering R^2_{Tjur}. Such signals indicate high credit risk for firms with no upcoming default event, potentially compromising the MCD model's credibility.

The regression analysis reveals that the MCD model does carry explanatory power on historical defaults. The analysis does not conclude on whether models based on GARCH-volatility estimates are superior to those based on historical volatility. Nevertheless, the models based on a 90-day estimation window seems to be decent candidates. The results are not definite, and which volatility estimate is optimal is still up for debate. We conclude that the MCD model does carry explanatory power, yet no single version of the MCD model stand out with exceptionally good fit.

Logarithmic transformation of the DD variable is beneficial for the application of the MCD model in predicting defaults, and the model seems to work best on untreated equity returns. These findings are in accordance with the first hypothesis. However, the GARCH-based models do not systematically display extraordinary explanatory power, which contradicts the hypothesis. As such, based on the findings of the regression analysis, the first hypothesis of the thesis is rejected. These findings are indications that the use of forward-looking volatility models in the application of the MCD model is not superior to the use of backward-looking volatility models. An interpretation of this finding is that recent and non-recent information about firms' credit risk is processed uniformly by the model when predicting defaults one year ahead.

Discriminatory Analysis

This analysis will concentrate on the discriminatory power of the MCD model and the pure statistical ones, with the aim of assessing the second hypothesis of the thesis. Logarithmic transformation of the DD does not influence how the model ranks firms with respect to the estimated PDs. Moreover, seeing that winsorization is not beneficial in the application of the model, the AR and CAP analysis will solely focus on the original time series.

Accuracy Ratio

The AR is a single summary measure that ranks the predictive accuracy of each credit model for both Type 1 errors and Type 2 errors. Table 10 illustrates the AR for all models in both default samples as well as bootstrapping at the 95% confidence interval. The highest AR statistic for each sample is highlighted.

Table 10: Summary of the accuracy ratio for each MCD model.

Accuracy_Ratios

	2,5%_conf	Bankruptcy	*97,5_conf*	*2,5_conf*	Reorganization	*97,5_conf*
h90	*75,4 %*	83,2 %	*92,0 %*	*56,1 %*	72,2 %	*83,0 %*
h180	*75,6*	84,0	*93,1*	*57,1*	73,2	*83,1*
h250	*72,0*	83,6	*93,4*	*57,6*	74,0	*83,8*
h5Y	*76,7*	85,9	*93,8*	*60,3*	**76,5**	*85,4*
hTot	*76,9*	86,0	*93,8*	*58,3*	75,3	*83,9*
g90*	*78,5*	**86,8**	*93,7*	*58,5*	75,4	*85,6*
g180*	*76,9*	86,6	*94,7*	*54,9*	72,8	*83,9*
g250*	*76,6*	86,1	*94,4*	*58,2*	73,2	*83,7*
g90	*73,6*	82,1	*89,5*	*46,4*	65,7	*78,0*
g180	*72,6*	82,3	*91,4*	*55,7*	71,2	*82,7*
g250	*66,4*	79,4	*90,9*	*61,3*	73,6	*83,3*
Altman	- *91,3*	4,9	*99,9*	*15,3*	64,7	*84,4*
Ohlson	*47,8*	52,0	*77,9*	*48,2*	61,8	*74,1*

All MCD models appear to outperform the pure statistical ones in both samples of default. The AR for the Altman model is insensible because of data shortage[16] in the bankruptcy sample and are still not trustworthy in the reorganization sample. The MCD models in the bankruptcy sample range from 79% to 87% with a typical confidence interval of $\pm$ 9%. When broadening the definition of default, the AR range falls to (66%, 77%) with a typical confidence interval of $\pm$ 13%. In contrast, the pure statistical models increase their discriminatory power with higher certainty when changing the definition of default. This indicates that the pure statistical models perform better in ranking firms in terms of financial failures rather than in bankruptcies, while the MCD models work better in ranking firms in regards of bankruptcies rather than in reorganizations. Nonetheless, the discriminatory power of the MCD models always exceeds that of the pure statistical models and with high certainty uninformative rating systems, which have AR of 0%.

Since AR is influenced by number of defaulters to number of non-defaulters, it is difficult to determine exactly how high AR a good system should achieve (Loffler & Posch, 2007). Previous research of Moody's shows an AR in the range of 53% to 76% (Sobehart, Stein, Mikityanskaya, & Li, 2000). Our findings are arguably no worse, seeing that the 2.5% confidence interval in the bankruptcy sample is 75% on average, and the similar interval in the reorganization sample is 57%. These

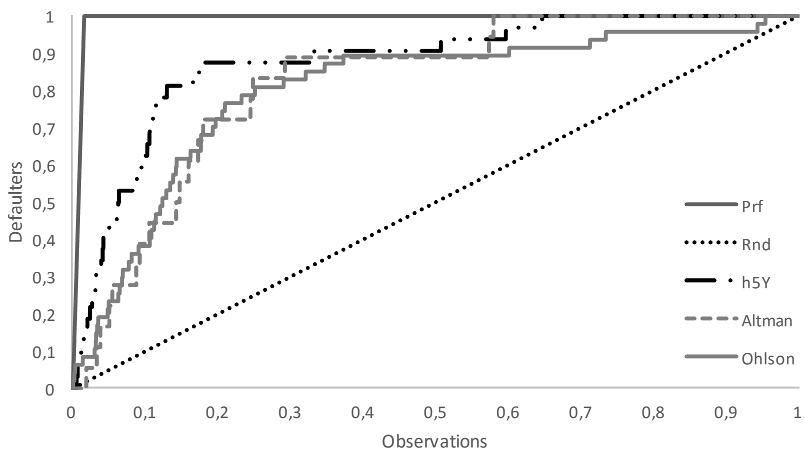
[16] The data shortage stems from Datastream's scarce coverage of "Retained Earnings" before 2002, which is included in the calculation of Altman's Z-score.

findings imply that the MCD model does, fairly successfully, distinguish defaulting from non-defaulting firms in the Norwegian market.

Cumulative Accuracy Profile

The AR analysis indicates that the MCD models are superior to the pure statistical models but that there is little that distinguishes the ratio of the MCD models from each other. Appendix C illustrates CAP curves. The CAP curves confirm that the discriminatory power is approximately equal for all the MCD models. It is also clear that every MCD model outperforms the pure statistical models in the bankruptcy sample, but less so in the reorganization sample. Figure 5 displays the CAP curve of the MCD model, represented by the h5Y model, and compares it to that of the pure statistical models, the random model (Rnd) and the perfect model (Prf).

Figure 5: Cumulative Accuracy Profiles for the MCD model.



The CAP curve of the h5Y model lies mostly above the statistical models where it discriminates particularly well for the 20% riskiest firms. However, it seems like the models intersects after approximately 30% of the sorted sample is observed, while there are still defaults left in the sample. The part of firms that are left after 30% are mostly investment grade firms[17]. This could indicate that the MCD model's superiority at discriminating is less prevalent for less risky firms, which may be particularly relevant for investors that are restricted to invest in only safe assets.

The discriminatory analysis concludes that the AR for the MCD model is superior to that of the pure statistic models. Moreover, it shows that the CAP curve of the MCD model is superior to the statistical model for the bankruptcy sample, yet slightly less superior for the reorganization sample. The analysis reveals that the model benefits by broadening the definition of default, in contrast to Ohlson's O-

[17] Firms deemed as investment grade by Stamdata, i.e firms with low credit risk.

score, which appears to benefit from a narrow definition. The MCD model's ability to discriminate is not as dominant for less risky firms. Still, the MCD models ability to discriminate generally exceeds that of statistical models. In light of these findings, we conclude that the second hypothesis of the thesis holds.

Calibration Analysis

This analysis focuses on the likelihood that the MCD model systematically underestimates the true PD, with the aim to assess the third hypothesis of the thesis.

Calculated by Equation 42, the default correlation for the bankruptcy sample ρ_B is 5,5% and for the reorganization sample, ρ_R is 2,3%. Table 11 displays the test results of the Vasicek one factor model conducted on the original dataset in bankruptcy sample. All values are displayed in percentage. Because the test requires default events, years without any defaults are excluded from the table. The dark grey area reflects significant test results on the 1% level and the light grey area reflects significant test results on the 5% level. The white areas are test results that are insignificant on both 1% and 5% level, which translates to no sign of underestimation.

Table 11: Test results from the Vasicek's single factor model on the bankruptcy sample.

Vasicek_Bankruptcy

g250	Year	h90	h180	h250	h5Y	hTot	g90*	g180*	g250*	g90	g180	g250
0,184	2015	18,58	15,68	15,25	16,79	18,07	17,87	19,48	19,33	21,18	19,47	18,36
0,145	2014	11,34	12,55	12,91	17,23	16,85	13,63	15,45	15,77	13,07	14,50	14,47
0,015	2013	1,09	1,43	1,54	1,80	1,64	1,31	1,46	1,41	1,14	1,41	1,50
0,159	2011	11,80	15,43	15,33	16,83	16,81	14,18	16,03	16,28	11,84	15,16	15,94
0,106	2009	15,05	11,45	11,08	10,73	10,64	14,66	8,79	9,02	12,77	9,11	10,58
0,078	2002	10,22	8,68	8,47	6,93	6,74	7,44	6,84	7,04	9,88	7,53	7,84
0,007	2001	0,66	0,68	0,70	0,49	0,50	0,50	0,57	0,59	0,69	0,69	0,68

A general observation for both default samples is that the results for the different versions of the MCD model are non-distinguishable. The table shows signs of underestimation during the dot-com bubble in 2001. On the other hand, the models seem to predict bankruptcies quite well in the backwash of the financial crisis in 2009. One may speculate whether this is caused by how investors have learned from the previous crisis and showed more caution during this period. The danger area appearing in 2013 is somewhat unexplainable for the authors and is interpreted as a random effect. In general, the model does not seem to underestimate the true PD when it comes to predicting default in terms of bankruptcy. Table 12 illustrates quite different results for the reorganization sample.

Table 12:: Test results from the Vasicek's single factor model for the reorganization sample.

g250	***Vasicek_Reorganization***											
0,002	Year	h90	h180	h250	h5Y	hTot	g90*	g180*	g250*	g90	g180	g250
0,435	2015	0,26	0,10	0,09	0,13	0,18	0,19	0,28	0,24	0,39	0,28	0,21
0,027	2014	35,10	40,02	41,78	55,94	54,25	43,05	46,74	47,15	41,96	44,77	43,51
0,27	2013	1,49	2,55	3,11	3,28	2,66	1,84	2,22	1,89	1,39	2,05	2,73
0,005	2012	45,37	37,71	33,77	24,31	21,68	27,77	22,07	19,01	29,94	27,93	27,01
0,657	2011	0,18	0,54	0,52	0,74	0,74	0,38	0,56	0,59	0,18	0,44	0,51
0,01	2010	46,54	66,71	77,20	53,47	53,55	49,45	52,59	50,11	45,79	56,81	65,73
0,016	2009	2,37	0,95	0,72	0,51	0,48	1,76	0,28	0,57	0,94	0,41	1,02
0,443	2008	2,17	1,38	1,04	3,58	4,32	4,29	6,21	5,93	4,34	4,62	1,60
	2007	30,94	53,35	47,99	71,20	73,01	60,80	74,32	71,66	46,15	65,97	44,32

The tests performed on the reorganization sample show signs of underestimation in year 2009, 2011 and 2015. In addition, there is a risk of underestimation in year 2008 and 20013. The common trait for those years is that the default rate exceeds 2%, as illustrated in Table 4 in Part V – Methodology. On the contrary, the model produces insignificant test results in periods with default rate less than 2%. Previous research shows a median default rate of 1,8% for a period between 1976-2006 for US public firms (Altman E. , 2007). An interpretation of the above results is that the MCD model does not seem to underestimate the true PD in periods of stable economies, yet bears some risk of underestimation in periods of macroeconomic distress. That is, information about firm specific credit risk factors is more easily captured than common market factors.

Results for the logarithmic time series are shown in appendix D, and it is concluded that systematic underestimation takes place in both default samples. Appendix E shows the distribution of DDs, where it can be seen that in addition to pulling DDs towards zero, the logarithm changes the skewness from being positive to negative. All else equal, this should imply higher PDs and less likelihood of underestimation. However, the maximum likelihood method used to solve for the coefficients in the logit model responds accordingly by generating lower coefficients, resulting in lower estimated PDs[18]. This is the mechanism that causes the log-based MCD model to underestimate the PDs.

The calibration analysis shows that logarithmic transformation of the DD variable forces the PDs downwards, resulting in underestimation of the true credit risk. If the application of the MCD model was restricted to the use of the log-covariate, one could argue that the third hypothesis in this thesis would hold. Nonetheless, the

[18] Descriptive statistics for the PDs are illustrated in appendix B.

MCD model may be applied in ways that can reduce the likelihood of systematic underestimation of PD. The test results for the PDs estimated with the original DD variable show few signs of underestimation in the bankruptcy sample and indicate that the model calibrates reasonably well in normally behaving markets for the reorganization sample. With this in mind, we argue that the third hypothesis is rejected, implying that the MCD model does not systematically underestimate the true PDs for Norwegian public firms. This is in contrast to the findings of Gemmill (Gemmill, 2002). His research was conducted on low-risk, zero-coupon bonds in the U.K., which by definition will have lower volatility. The sample used in this thesis comprises firms with more nuanced debt-structures, with different levels of risk. Higher estimated volatility generates higher PDs, which may reduce the risk of underestimation.

Supplementary Findings

The above analyses conclude on the three hypotheses of the thesis, and assist in answering the paper's research question of how well the MCD model performs in predicting defaults amongst Norwegian public firms. This section discusses how PDs of defaulting firms evolve over time, as an additional finding which contributes to the discussion of the research question. Relevant graphs and tables are found in Appendix F.

As established in Part IV – Data, only 11 out of 25 defaults have sufficient data to be included in the tests of the MCD models. Of those 11 defaulters, 4 have only one PD observation and an additional 2 defaulters only have two PD observations. The few observations indicate that young public companies have higher risk of being liquidated in Norway. One explanation of this occurrence may be that the private market shows no funding interest, firms will turn to the public market to receive its capital requirement. Those companies will have higher credit risk, reflected in how banks may already have rejected their request for more debt.

Since the MCD model is unable to analyze firms with insufficient historical data, the model's applicability for the Norwegian market may be severely compromised. However, the MCD model adequately predicts bankruptcies when companies survive their first years as a public firm.

The graphs from the reorganization sample shows that most of the defaulters have enough PDs to study trends. Optimally, the firms' estimated riskiness should peak the year before a credit event occurs. However, the graphs display multiple signals of increased credit risk without any following credit event taking place. Additionally, there are signs that PDs tend to increase after such events do occur.

An interpretation of these observations is that failure to meet financial promises is more difficult to predict than bankruptcies. Since firms often fail to meet financial promises before equity holders choose to liquidate[19], the liquidation occurrence may be predicted after a series of such failures. Investors respond to financial failures by selling the stock, as they then interpret the firm as riskier. This will lower the equity, and consequently the asset value of the firm, which raises the PD. This may be one reason for why the MCD model works better in predicting defaults in the bankruptcy sample.

[19] From the MCD model theory, equity holders are viewed as those who run the company and will choose to leave all assets to debt holders when the firm goes bankrupt.

PART VII - Conclusion

We present evidence that the DD variable produced by the MCD model does carry explanatory power on historical defaults, whether default is defined as bankruptcy or reorganization of the firm. The model is tested with eleven different types of equity volatility, as well as logarithmic transformation of the covariate DD and winsorization of equity returns. The regression analysis concludes that no single version of the model exhibits exceptionally high explanatory power. It was hypothesized that GARCH-based models would exhibit superior goodness of fit, particularly those of a shorter volatility estimation window. Our findings reject this notion, as models based on GARCH and historical volatility estimates display quite similar fit. Nonetheless, models based on equity volatility with an estimation window of 90 days seem to be adequate candidates in the application of the model. The study shows that winsorizing equity returns only marginally improves explanatory power, while regressing on the logarithmic DD variable improves the fit considerably.

The study finds that the model's ability to discriminate between defaulting and non-defaulting firms exceeds that of pure statistical models, in accordance with previous studies. The analysis indicates that the MCD model's discriminatory power declines with a broader definition of default, in contrast to that of statistical models.

The results further indicate that proper use of the model involves low risk of underestimation of PDs when predicting bankruptcies, in contrast to what was hypothesized. However, the model exhibits some risk of underestimation in distressed economies when predicting financial failures. That is, information about firm specific credit risk factors is more easily captured than common market factors. The calibration analysis concludes with high certainty that the logarithm of the DD variable will underestimate the true credit risk.

We conclude that the MCD model performs adequately in predicting defaults amongst public firms in Norway. However, many defaults are unable to be predicted by the model because of insufficient market values of equity, and thus it will have difficulties in predicting default probabilities for young public firms. Our study is inspired by previous research on the MCD model's performance in predicting defaults in other markets. It provides scientific contribution by proving that market prices contain information of future defaults in Norway. We believe that financial institutions and other practitioners can apply the MCD model as a supplement to qualitative analysis, to improve their assessment of Norwegian public firms' credit risk. The PDs estimated by the MCD model only reveal parts of the risk picture of firms. The model's predictive power may be improved by extending it to a hybrid model that accounts for more relevant information concerning credit risk, as well as qualitative assessments of the respective firms.

Criticism and Further Research

One must be careful when interpreting the results from the logit model given the relatively small sample. With only 10 years of historical data in the reorganization sample, it is especially hard to evaluate the robustness of the results. Small sample sizes have made us unable to draw random samples from the population to estimate PDs and to perform out-of-sample testing. The regression analysis is therefore solely based on historical fit, which may not be optimal for future prediction.

The fact that most of the defaulters are relatively young public firms makes it difficult to study how the PDs evolve over time. Optimally, the firms' estimated riskiness should peak the year before a credit event occurs. One could conduct a transition matrix analysis to see how the models estimated credit risk change over time. With few years of historical data, such analyses are obsolete when studying annual PDs. A possible solution to overcome this constraint is to use higher frequency of PDs, such as monthly or weekly.

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Appendices

Appendix A - Regression Statistics

The tables below display the regression output for the different MCD models. The names of the models are highlighted and the details of each model are as illustrated in Table 2. In the tables below, the intercept is denoted β(1), while the coefficient for the independent variable DD is denoted β(2). “Coeff” is short for coefficient, “SE” is short for standard error of the coefficient, z-stat denotes the coefficients’ z-statisitc. Some of the p-values for the intercepts are so close to zero that excel rounds down the figure and displays it as “-“. The statistic of $R^2_{McF}$is denoted McFadR2. C&SR2 is the name for the statistics of Cox & Snell, which has the same interpretation as R^2_{McF}. However, since R^2_{McF}varies more than the Cox & Snell statistics, the latter is left out of the discussion. The amount of iterations for each regression analysis is summarized by “Iter.”. Lastly, “LR” denotes the likelihood ratio of the regression.

Non-Winsorized, Bankruptcy Sample

Logit (h90)

	β(1)	β(2)(2)
Coeff	- 4,37	- 0,29
SE()	0,35	0,07
z-stat	- 12,31	- 3,88
p-value	-	0,00
McFadR2	0,10	
C&SR2	0,01	
Iter.	9	
LR	14,07	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)(2)
Coeff	- 4,45	- 0,26
SE()	0,35	0,07
z-stat	- 12,76	- 3,86
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	12,58	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)(2)
Coeff	- 4,44	- 0,27
SE()	0,35	0,07
z-stat	- 12,63	- 3,80
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	9	
LR	12,57	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)(2)
Coeff	- 4,53	- 0,26
SE()	0,34	0,07
z-stat	- 13,21	- 3,80
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,00	
Iter.	8	
LR	11,13	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)(2)
Coeff	- 4,55	- 0,26
SE()	0,34	0,07
z-stat	- 13,38	- 3,80
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,00	
Iter.	8	
LR	10,66	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)(2)
Coeff	- 4,53	- 0,26
SE()	0,34	0,07
z-stat	- 13,32	- 3,89
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	11,95	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)(2)
Coeff	- 4,76	- 0,18
SE()	0,34	0,06
z-stat	- 13,94	- 3,12
p-value	-	0,00
McFadR2	0,06	
C&SR2	0,00	
Iter.	8	
LR	8,33	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)(2)
Coeff	- 4,78	- 0,17
SE()	0,34	0,06
z-stat	- 14,07	- 3,10
p-value	-	0,00
McFadR2	0,06	
C&SR2	0,00	
Iter.	8	
LR	8,11	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)(2)
Coeff	- 4,45	- 0,25
SE()	0,35	0,07
z-stat	- 12,57	- 3,60
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,00	
Iter.	9	
LR	11,60	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)(2)
Coeff	- 4,68	- 0,19
SE()	0,34	0,05
z-stat	- 13,87	- 3,44
p-value	-	0,00
McFadR2	0,07	
C&SR2	0,00	
Iter.	8	
LR	9,18	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)(2)
Coeff	- 4,64	- 0,20
SE()	0,35	0,06
z-stat	- 13,07	- 3,15
p-value	-	0,00
McFadR2	0,07	
C&SR2	0,00	
Iter.	8	
LR	9,46	
LR p-value	0,00	

Logarithmic Non-Winsorized, Bankruptcy Sample

Logit (h90)

	β(1)	β(2)(2)
Coeff	- 4,44	- 1,07
SE()	0,31	0,20
z-stat	- 14,33	- 5,35
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,01	
Iter.	9	
LR	20,15	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)(2)
Coeff	- 4,32	- 1,18
SE()	0,31	0,22
z-stat	- 13,75	- 5,37
p-value	-	0,00
McFadR2	0,15	
C&SR2	0,01	
Iter.	9	
LR	20,67	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)(2)
Coeff	- 4,56	- 0,90
SE()	0,31	0,18
z-stat	- 14,66	- 5,05
p-value	-	0,00
McFadR2	0,12	
C&SR2	0,01	
Iter.	9	
LR	16,78	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)(2)
Coeff	- 4,45	- 1,19
SE()	0,31	0,20
z-stat	- 14,28	- 5,96
p-value	-	0,00
McFadR2	0,17	
C&SR2	0,01	
Iter.	9	
LR	24,18	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)(2)
Coeff	- 4,49	- 1,15
SE()	0,31	0,20
z-stat	- 14,48	- 5,82
p-value	-	0,00
McFadR2	0,16	
C&SR2	0,01	
Iter.	9	
LR	22,83	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)(2)
Coeff	- 4,42	- 1,21
SE()	0,31	0,22
z-stat	- 14,25	- 5,60
p-value	-	0,00
McFadR2	0,16	
C&SR2	0,01	
Iter.	9	
LR	22,12	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)(2)
Coeff	- 4,38	- 1,23
SE()	0,31	0,22
z-stat	- 14,01	- 5,57
p-value	-	0,00
McFadR2	0,16	
C&SR2	0,01	
Iter.	9	
LR	21,99	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)(2)
Coeff	- 4,60	- 0,91
SE()	0,31	0,18
z-stat	- 14,61	- 5,07
p-value	-	0,00
McFadR2	0,13	
C&SR2	0,01	
Iter.	9	
LR	17,72	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)(2)
Coeff	- 4,44	- 1,00
SE()	0,31	0,20
z-stat	- 14,24	- 4,96
p-value	-	0,00
McFadR2	0,12	
C&SR2	0,01	
Iter.	9	
LR	17,14	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)(2)
Coeff	- 4,45	- 1,05
SE()	0,31	0,20
z-stat	- 14,34	- 5,30
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,01	
Iter.	9	
LR	19,49	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)(2)
Coeff	- 4,47	- 1,03
SE()	0,31	0,21
z-stat	- 14,44	- 4,95
p-value	-	0,00
McFadR2	0,12	
C&SR2	0,01	
Iter.	9	
LR	17,06	
LR p-value	0,00	

Winzorized, Bankruptcy Sample

Logit (h90)

	β(1)	β(2)(2)
Coeff	- 4,36	- 0,29
SE()	0,36	0,07
z-stat	- 12,26	- 3,88
p-value	-	0,00
McFadR2	0,10	
C&SR2	0,01	
Iter.	9	
LR	14,06	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)(2)
Coeff	- 4,44	- 0,26
SE()	0,35	0,07
z-stat	- 12,69	- 3,86
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	12,58	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)(2)
Coeff	- 4,43	- 0,27
SE()	0,35	0,07
z-stat	- 12,55	- 3,80
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	12,55	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)(2)
Coeff	- 4,52	- 0,26
SE()	0,34	0,07
z-stat	- 13,13	- 3,80
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,00	
Iter.	8	
LR	11,04	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)(2)
Coeff	- 4,54	- 0,26
SE()	0,34	0,07
z-stat	- 13,29	- 3,80
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,00	
Iter.	8	
LR	10,62	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)(2)
Coeff	- 4,51	- 0,26
SE()	0,34	0,07
z-stat	- 13,23	- 3,89
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,01	
Iter.	8	
LR	11,90	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)(2)
Coeff	- 4,75	- 0,18
SE()	0,34	0,06
z-stat	- 13,86	- 3,12
p-value	-	0,00
McFadR2	0,06	
C&SR2	0,00	
Iter.	8	
LR	8,35	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)(2)
Coeff	- 4,77	- 0,17
SE()	0,34	0,06
z-stat	- 13,96	- 3,10
p-value	-	0,00
McFadR2	0,06	
C&SR2	0,00	
Iter.	8	
LR	8,27	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)(2)
Coeff	- 4,42	- 0,26
SE()	0,36	0,07
z-stat	- 12,44	- 3,65
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	9	
LR	12,56	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)(2)
Coeff	- 4,73	- 0,17
SE()	0,34	0,05
z-stat	- 13,95	- 3,46
p-value	-	0,00
McFadR2	0,07	
C&SR2	0,00	
Iter.	8	
LR	9,71	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)(2)
Coeff	- 4,67	- 0,19
SE()	0,36	0,06
z-stat	- 13,06	- 3,17
p-value	-	0,00
McFadR2	0,07	
C&SR2	0,00	
Iter.	8	
LR	10,12	
LR p-value	0,00	

Logarithmic Winzorized, Bankruptcy Sample

Logit (h90)

	β(1)	β(2)(2)
Coeff	- 4,34	- 1,18
SE()	0,31	0,22
z-stat	- 13,92	- 5,44
p-value	-	0,00
McFadR2	0,16	
C&SR2	0,01	
Iter.	9	
LR	21,81	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)(2)
Coeff	- 4,37	- 1,12
SE()	0,31	0,21
z-stat	- 13,98	- 5,26
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,01	
Iter.	9	
LR	19,49	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)(2)
Coeff	- 4,35	- 1,15
SE()	0,31	0,22
z-stat	- 13,91	- 5,34
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,01	
Iter.	9	
LR	20,31	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)(2)
Coeff	- 4,42	- 1,20
SE()	0,31	0,20
z-stat	- 14,21	- 5,92
p-value	-	0,00
McFadR2	0,17	
C&SR2	0,01	
Iter.	9	
LR	23,94	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)(2)
Coeff	- 4,39	- 1,24
SE()	0,31	0,21
z-stat	- 14,10	- 5,92
p-value	-	0,00
McFadR2	0,17	
C&SR2	0,01	
Iter.	9	
LR	24,31	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)(2)
Coeff	- 4,46	- 1,07
SE()	0,32	0,20
z-stat	- 14,13	- 5,26
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,01	
Iter.	9	
LR	19,65	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)(2)
Coeff	- 4,29	- 1,32
SE()	0,32	0,24
z-stat	- 13,58	- 5,53
p-value	-	0,00
McFadR2	0,17	
C&SR2	0,01	
Iter.	9	
LR	23,12	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)(2)
Coeff	- 4,55	- 0,94
SE()	0,32	0,18
z-stat	- 14,29	- 5,08
p-value	-	0,00
McFadR2	0,13	
C&SR2	0,01	
Iter.	9	
LR	18,49	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)(2)
Coeff	- 4,53	- 0,86
SE()	0,33	0,20
z-stat	- 13,81	- 4,24
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,01	
Iter.	9	
LR	16,08	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)(2)
Coeff	- 4,32	- 1,18
SE()	0,31	0,21
z-stat	- 13,73	- 5,54
p-value	-	0,00
McFadR2	0,16	
C&SR2	0,01	
Iter.	9	
LR	22,52	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)(2)
Coeff	- 4,36	- 1,15
SE()	0,31	0,22
z-stat	- 13,95	- 5,15
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,01	
Iter.	9	
LR	19,51	
LR p-value	0,00	

Non-Winsorized, Reorganization Sample

Logit (h90)

	β(1)	β(2)(2)
Coeff	- 2,75	- 0,41
SE()	0,28	0,09
z-stat	- 9,92	- 4,80
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,02	
Iter.	8	
LR	42,99	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)(2)
Coeff	- 2,79	- 0,40
SE()	0,29	0,09
z-stat	- 9,67	- 4,53
p-value	-	0,00
McFadR2	0,10	
C&SR2	0,02	
Iter.	8	
LR	39,78	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)(2)
Coeff	- 2,75	- 0,41
SE()	0,29	0,09
z-stat	- 9,48	- 4,60
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,02	
Iter.	8	
LR	40,93	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)(2)
Coeff	- 2,99	- 0,36
SE()	0,26	0,07
z-stat	- 11,46	- 4,79
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	34,53	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)(2)
Coeff	- 3,05	- 0,34
SE()	0,26	0,07
z-stat	- 11,83	- 4,69
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,01	
Iter.	8	
LR	31,80	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)(2)
Coeff	- 3,00	- 0,36
SE()	0,26	0,07
z-stat	- 11,64	- 4,78
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,02	
Iter.	8	
LR	36,11	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)(2)
Coeff	- 3,10	- 0,31
SE()	0,28	0,08
z-stat	- 11,13	- 3,92
p-value	-	0,00
McFadR2	0,07	
C&SR2	0,01	
Iter.	8	
LR	26,25	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)(2)
Coeff	- 3,10	- 0,31
SE()	0,26	0,07
z-stat	- 11,83	- 4,26
p-value	-	0,00
McFadR2	0,07	
C&SR2	0,01	
Iter.	8	
LR	27,34	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)(2)
Coeff	- 3,01	- 0,30
SE()	0,26	0,07
z-stat	- 11,56	- 4,46
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,01	
Iter.	8	
LR	31,40	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)(2)
Coeff	- 3,12	- 0,28
SE()	0,27	0,07
z-stat	- 11,54	- 4,01
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,01	
Iter.	8	
LR	29,29	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)(2)
Coeff	- 2,72	- 0,44
SE()	0,27	0,09
z-stat	- 9,96	- 4,92
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,02	
Iter.	8	
LR	40,34	
LR p-value	0,00	

Logarithmic Non-Winzorized, Reorganization Sample

Logit (h90)

	β(1)	β(2)(2)
Coeff	- 3,13	- 1,10
SE()	0,18	0,14
z-stat	- 17,15	- 8,12
p-value	-	0,00
McFadR2	0,15	
C&SR2	0,02	
Iter.	8	
LR	57,22	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)(2)
Coeff	- 3,02	- 1,17
SE()	0,19	0,15
z-stat	- 15,79	- 7,61
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	51,49	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)(2)
Coeff	- 3,24	- 0,96
SE()	0,18	0,13
z-stat	- 17,63	- 7,52
p-value	-	0,00
McFadR2	0,13	
C&SR2	0,02	
Iter.	7	
LR	47,90	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)(2)
Coeff	- 3,14	- 1,13
SE()	0,18	0,14
z-stat	- 17,12	- 8,24
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	54,70	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)(2)
Coeff	- 3,19	- 1,11
SE()	0,18	0,13
z-stat	- 17,63	- 8,32
p-value	-	-
McFadR2	0,14	
C&SR2	0,02	
Iter.	7	
LR	54,15	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)(2)
Coeff	- 3,10	- 1,21
SE()	0,18	0,14
z-stat	- 17,04	- 8,33
p-value	-	-
McFadR2	0,15	
C&SR2	0,02	
Iter.	8	
LR	58,47	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)(2)
Coeff	- 3,07	- 1,20
SE()	0,19	0,16
z-stat	- 16,37	- 7,73
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	52,16	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)(2)
Coeff	- 3,23	- 1,01
SE()	0,19	0,14
z-stat	- 17,28	- 7,31
p-value	-	0,00
McFadR2	0,13	
C&SR2	0,02	
Iter.	7	
LR	49,50	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)(2)
Coeff	- 3,17	- 0,96
SE()	0,19	0,13
z-stat	- 17,07	- 7,15
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,02	
Iter.	7	
LR	41,81	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)(2)
Coeff	- 3,17	- 1,01
SE()	0,18	0,13
z-stat	- 17,23	- 7,54
p-value	-	0,00
McFadR2	0,12	
C&SR2	0,02	
Iter.	7	
LR	46,55	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)(2)
Coeff	- 3,17	- 1,09
SE()	0,18	0,14
z-stat	- 17,47	- 7,90
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	52,75	
LR p-value	0,00	

Winzorized, Reorganization Sample

Logit (h90)

	β(1)	β(2)
Coeff	- 2,74	- 0,41
SE()	0,28	0,09
z-stat	- 9,84	- 4,78
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,02	
Iter.	8	
LR	42,50	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)
Coeff	- 2,79	- 0,39
SE()	0,29	0,09
z-stat	- 9,54	- 4,50
p-value	-	0,00
McFadR2	0,10	
C&SR2	0,02	
Iter.	8	
LR	39,23	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)
Coeff	- 2,75	- 0,41
SE()	0,29	0,09
z-stat	- 9,38	- 4,56
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,02	
Iter.	8	
LR	40,09	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)
Coeff	- 3,01	- 0,34
SE()	0,26	0,07
z-stat	- 11,63	- 4,76
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	32,70	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)
Coeff	- 3,07	- 0,33
SE()	0,26	0,07
z-stat	- 11,94	- 4,68
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,01	
Iter.	8	
LR	30,42	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)
Coeff	- 3,01	- 0,34
SE()	0,26	0,07
z-stat	- 11,74	- 4,77
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	34,47	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)
Coeff	- 3,06	- 0,32
SE()	0,28	0,08
z-stat	- 10,87	- 3,99
p-value	-	0,00
McFadR2	0,07	
C&SR2	0,01	
Iter.	8	
LR	27,26	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)
Coeff	- 3,03	- 0,33
SE()	0,26	0,07
z-stat	- 11,49	- 4,48
p-value	-	0,00
McFadR2	0,08	
C&SR2	0,01	
Iter.	8	
LR	30,02	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)
Coeff	- 2,92	- 0,33
SE()	0,27	0,07
z-stat	- 10,99	- 4,56
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	35,34	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)
Coeff	- 2,98	- 0,32
SE()	0,29	0,08
z-stat	- 10,21	- 4,14
p-value	-	0,00
McFadR2	0,09	
C&SR2	0,01	
Iter.	8	
LR	36,36	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)
Coeff	- 2,78	- 0,40
SE()	0,28	0,08
z-stat	- 10,04	- 4,89
p-value	-	0,00
McFadR2	0,11	
C&SR2	0,02	
Iter.	8	
LR	40,92	
LR p-value	0,00	

Logarithmic Non-Winzorized, Reorganization Sample

Logit (h90)

	β(1)	β(2)
Coeff	- 3,02	- 1,20
SE()	0,19	0,15
z-stat	- 16,29	- 8,04
p-value	-	0,00
McFadR2	0,15	
C&SR2	0,02	
Iter.	8	
LR	58,70	
LR p-value	0,00	

Logit (h180)

	β(1)	β(2)
Coeff	- 3,06	- 1,12
SE()	0,19	0,14
z-stat	- 16,40	- 7,80
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	51,90	
LR p-value	0,00	

Logit (h250)

	β(1)	β(2)
Coeff	- 3,03	- 1,17
SE()	0,19	0,15
z-stat	- 16,23	- 7,96
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	55,17	
LR p-value	0,00	

Logit (h5Y)

	β(1)	β(2)
Coeff	- 3,11	- 1,15
SE()	0,18	0,14
z-stat	- 16,96	- 8,36
p-value	-	-
McFadR2	0,15	
C&SR2	0,02	
Iter.	8	
LR	55,73	
LR p-value	0,00	

Logit (hTot)

	β(1)	β(2)
Coeff	- 3,10	- 1,16
SE()	0,18	0,14
z-stat	- 16,79	- 8,16
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	53,12	
LR p-value	0,00	

Logit (g90*)

	β(1)	β(2)
Coeff	- 3,10	- 1,15
SE()	0,19	0,15
z-stat	- 16,40	- 7,63
p-value	-	0,00
McFadR2	0,14	
C&SR2	0,02	
Iter.	8	
LR	54,08	
LR p-value	0,00	

Logit (g180*)

	β(1)	β(2)
Coeff	- 2,96	- 1,32
SE()	0,19	0,17
z-stat	- 15,58	- 7,94
p-value	-	0,00
McFadR2	0,15	
C&SR2	0,02	
Iter.	8	
LR	57,87	
LR p-value	0,00	

Logit (g250*)

	β(1)	β(2)
Coeff	- 3,15	- 1,07
SE()	0,19	0,15
z-stat	- 16,34	- 7,20
p-value	-	0,00
McFadR2	0,13	
C&SR2	0,02	
Iter.	7	
LR	50,95	
LR p-value	0,00	

Logit (g90)

	β(1)	β(2)
Coeff	- 3,16	- 0,97
SE()	0,19	0,14
z-stat	- 16,55	- 6,79
p-value	-	0,00
McFadR2	0,12	
C&SR2	0,02	
Iter.	8	
LR	44,29	
LR p-value	0,00	

Logit (g180)

	β(1)	β(2)
Coeff	- 2,99	- 1,21
SE()	0,19	0,15
z-stat	- 15,73	- 8,05
p-value	-	0,00
McFadR2	0,16	
C&SR2	0,02	
Iter.	8	
LR	60,01	
LR p-value	0,00	

Logit (g250)

	β(1)	β(2)
Coeff	- 3,04	- 1,22
SE()	0,18	0,15
z-stat	- 16,51	- 8,31
p-value	-	-
McFadR2	0,16	
C&SR2	0,02	
Iter.	8	
LR	60,96	
LR p-value	0,00	

Appendix B - Descriptive Statistics of PDs

The tables below illustrate the descriptive statistics for the implied PDs for different versions of the MCD model with different samples. The models are specified above the table, with names similar to those in Table 5 in Part V - Methodology. In the table below, "Def" indicates descriptive statistics for the firms that did default and "N_Def" indicates descriptive statistics for firms that did not. "Avg" indicate average, "Std" indicate standard deviation, "Kurt" specifies kurtosis, "Min" and "Max" are short for lowest and highest observed PD, q1 and q3 are the first and third quartile and "Mcf" is short for McFadden's R-squared.

Bankrupty Sample

	h90		h90_ln		h90_wins		h90_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	1,0 %	0,5 %	0,6 %	0,2 %	1,0 %	0,5 %	0,7 %	0,2 %
Median	0,9 %	0,4 %	0,4 %	0,0 %	1,0 %	0,4 %	0,4 %	0,0 %
Std	0,3 %	1,7 %	0,7 %	2,9 %	0,3 %	1,7 %	0,9 %	2,9 %
Skew	0,66	38,77	1,56	33,78	0,71	38,77	1,59	33,64
Kurt	-0,61	1626,24	1,73	1151,11	-0,68	1625,60	1,55	1144,70
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,7 %	0,2 %	0,2 %	0,0 %	0,7 %	0,2 %	0,1 %	0,0 %
q3	1,1 %	0,6 %	0,8 %	0,1 %	1,1 %	0,6 %	0,8 %	0,1 %
Max	1,5 %	75,1 %	2,3 %	100,0 %	1,5 %	75,1 %	2,7 %	100,0 %
Mcf	10,0 %		14,4 %		10,0 %		15,6 %	
Tjur	0,5 %		0,5 %		0,5 %		0,5 %	

	h180		h180_ln		h180_wins		h180_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,9 %	0,5 %	0,6 %	0,2 %	0,9 %	0,5 %	0,6 %	0,2 %
Median	0,9 %	0,4 %	0,4 %	0,0 %	0,9 %	0,4 %	0,4 %	0,0 %
Std	0,2 %	1,6 %	0,7 %	2,9 %	0,2 %	1,6 %	0,6 %	2,9 %
Skew	0,37	38,26	1,25	33,60	0,31	38,26	1,07	33,75
Kurt	-1,14	1576,53	0,37	1142,85	-1,33	1575,53	-0,15	1149,87
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,7 %	0,2 %	0,2 %	0,0 %	0,7 %	0,2 %	0,2 %	0,0 %
q3	1,0 %	0,6 %	0,8 %	0,1 %	1,1 %	0,6 %	0,8 %	0,1 %
Max	1,3 %	70,8 %	2,0 %	100,0 %	1,2 %	70,9 %	1,7 %	100,0 %
Mcf	9,0 %		14,8 %		9,0 %		13,9 %	
Tjur	0,4 %		0,5 %		0,4 %		0,4 %	

	h250		h250_ln		h250_wins		h250_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,9 %	0,5 %	0,5 %	0,2 %	0,9 %	0,5 %	0,6 %	0,2 %
Median	0,9 %	0,4 %	0,4 %	0,0 %	0,9 %	0,4 %	0,4 %	0,0 %
Std	0,2 %	1,6 %	0,4 %	2,9 %	0,2 %	1,6 %	0,5 %	2,9 %
Skew	0,10	39,37	0,74	33,96	0,03	39,37	0,75	33,66
Kurt	-1,41	1670,05	-0,63	1159,56	-1,57	1668,98	-0,87	1145,58
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,7 %	0,2 %	0,2 %	0,0 %	0,7 %	0,2 %	0,1 %	0,0 %
q3	1,1 %	0,6 %	0,8 %	0,1 %	1,1 %	0,6 %	0,9 %	0,1 %
Max	1,3 %	72,6 %	1,3 %	100,0 %	1,2 %	72,6 %	1,5 %	100,0 %
Mcf	9,0 %		12,0 %		9,0 %		14,5 %	
Tjur	0,4 %		0,4 %		0,4 %		0,4 %	

	h5Y		h5Y_ln		h5Y_wins		h5Y_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,8 %	0,5 %	0,5 %	0,2 %	0,8 %	0,5 %	0,5 %	0,2 %
Median	0,8 %	0,4 %	0,3 %	0,0 %	0,8 %	0,4 %	0,3 %	0,0 %
Std	0,2 %	1,5 %	0,5 %	3,8 %	0,2 %	1,5 %	0,5 %	3,8 %
Skew	0,30	39,37	1,33	24,95	0,14	39,35	0,96	24,88
Kurt	-1,06	1701,13	1,23	632,99	-1,37	1700,29	-0,35	629,08
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,7 %	0,3 %	0,1 %	0,0 %	0,7 %	0,3 %	0,1 %	0,0 %
q3	1,0 %	0,5 %	0,7 %	0,1 %	1,0 %	0,5 %	0,8 %	0,1 %
Max	1,2 %	68,0 %	1,6 %	100,0 %	1,1 %	67,6 %	1,4 %	100,0 %
Mcf	7,9 %		17,3 %		7,9 %		17,1 %	
Tjur	0,4 %		0,3 %		0,4 %		0,3 %	

	hTot		hTot_ln		hTot_wins		hTot_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,8 %	0,5 %	0,5 %	0,2 %	0,8 %	0,5 %	0,5 %	0,2 %
Median	0,8 %	0,4 %	0,3 %	0,0 %	0,8 %	0,4 %	0,3 %	0,0 %
Std	0,2 %	1,5 %	0,5 %	3,8 %	0,2 %	1,5 %	0,5 %	3,9 %
Skew	0,35	38,82	1,31	25,20	0,19	38,79	1,01	24,62
Kurt	-1,02	1657,74	1,11	647,24	-1,37	1655,30	-0,26	615,12
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,7 %	0,3 %	0,1 %	0,0 %	0,6 %	0,3 %	0,1 %	0,0 %
q3	0,9 %	0,5 %	0,7 %	0,1 %	1,0 %	0,5 %	0,8 %	0,0 %
Max	1,1 %	65,8 %	1,5 %	100,0 %	1,1 %	65,5 %	1,4 %	100,0 %
Mcf	7,6 %		16,3 %		7,6 %		17,3 %	
Tjur	0,3 %		0,3 %		0,3 %		0,3 %	

	g90*		g90*_ln		g90*_wins		g90*_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,9 %	0,5 %	0,7 %	0,2 %	0,9 %	0,5 %	0,7 %	0,2 %
Median	0,8 %	0,4 %	0,4 %	0,0 %	0,8 %	0,4 %	0,4 %	0,0 %
Std	0,2 %	1,6 %	0,9 %	3,6 %	0,2 %	1,6 %	0,7 %	3,6 %
Skew	0,79	38,50	1,90	27,64	0,76	38,47	1,76	27,74
Kurt	-0,04	1633,87	3,24	768,22	-0,23	1632,02	2,68	771,82
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,7 %	0,3 %	0,2 %	0,0 %	0,7 %	0,3 %	0,2 %	0,0 %
q3	1,0 %	0,6 %	0,8 %	0,1 %	1,0 %	0,6 %	0,8 %	0,1 %
Max	1,3 %	69,7 %	2,8 %	100,0 %	1,3 %	69,4 %	2,5 %	100,0 %
Mcf	8,5 %		15,8 %		8,5 %		14,0 %	
Tjur	0,4 %		0,5 %		0,4 %		0,5 %	

	g180*		g180*_ln		g180*_wins		g180*_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,7 %	0,5 %	0,6 %	0,2 %	0,7 %	0,5 %	0,7 %	0,2 %
Median	0,7 %	0,4 %	0,4 %	0,0 %	0,7 %	0,4 %	0,4 %	0,0 %
Std	0,1 %	1,5 %	0,6 %	2,9 %	0,1 %	1,5 %	0,7 %	2,9 %
Skew	0,19	46,95	1,27	33,60	0,27	46,93	1,31	33,30
Kurt	-0,92	2249,39	0,33	1141,64	-1,12	2248,22	0,41	1127,64
Min	0,5 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,6 %	0,3 %	0,2 %	0,0 %	0,6 %	0,3 %	0,1 %	0,0 %
q3	0,8 %	0,5 %	0,8 %	0,1 %	0,8 %	0,5 %	0,9 %	0,0 %
Max	0,9 %	70,7 %	1,8 %	100,0 %	0,9 %	70,7 %	2,1 %	100,0 %
Mcf	5,9 %		15,7 %		6,0 %		16,5 %	
Tjur	0,3 %		0,5 %		0,3 %		0,5 %	

	g250*		g250*_ln		g250*_wins		g250*_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,7 %	0,5 %	0,6 %	0,2 %	0,7 %	0,5 %	0,6 %	0,2 %
Median	0,7 %	0,4 %	0,4 %	0,0 %	0,7 %	0,4 %	0,5 %	0,0 %
Std	0,1 %	1,4 %	0,4 %	3,5 %	0,1 %	1,5 %	0,5 %	3,5 %
Skew	0,11	47,16	0,83	27,78	0,11	47,15	0,88	27,74
Kurt	-1,37	2264,01	-0,57	773,66	-1,40	2263,41	-0,43	772,14
Min	0,5 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,6 %	0,3 %	0,2 %	0,0 %	0,6 %	0,3 %	0,2 %	0,0 %
q3	0,8 %	0,5 %	0,8 %	0,1 %	0,8 %	0,5 %	0,8 %	0,1 %
Max	0,9 %	69,9 %	1,3 %	100,0 %	0,9 %	70,8 %	1,5 %	100,0 %
Mcf	5,8 %		12,7 %		5,9 %		13,2 %	
Tjur	0,2 %		0,3 %		0,3 %		0,4 %	

	g90		g90_ln		g90_wins		g90_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,8 %	0,5 %	0,4 %	0,2 %	0,9 %	0,5 %	0,5 %	0,2 %
Median	0,9 %	0,4 %	0,3 %	0,0 %	0,9 %	0,4 %	0,4 %	0,0 %
Std	0,1 %	1,5 %	0,3 %	2,9 %	0,2 %	1,6 %	0,4 %	2,9 %
Skew	0,43	41,82	1,63	33,77	0,57	41,64	1,46	33,90
Kurt	0,12	1878,66	3,33	1150,22	-0,44	1861,42	2,06	1156,19
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,6 %	0,0 %	0,1 %	0,0 %
q1	0,7 %	0,2 %	0,2 %	0,0 %	0,7 %	0,2 %	0,2 %	0,0 %
q3	0,9 %	0,6 %	0,4 %	0,1 %	1,0 %	0,6 %	0,6 %	0,1 %
Max	1,1 %	69,3 %	1,0 %	100,0 %	1,3 %	73,5 %	1,4 %	100,0 %
Mcf	8,3 %		12,2 %		9,0 %		11,5 %	
Tjur	0,4 %		0,2 %		0,4 %		0,3 %	

	g180		g180_ln		g180_wins		g180_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,7 %	0,5 %	0,5 %	0,2 %	0,7 %	0,4 %	0,5 %	0,2 %
Median	0,7 %	0,4 %	0,2 %	0,0 %	0,7 %	0,4 %	0,4 %	0,0 %
Std	0,1 %	1,4 %	0,5 %	2,9 %	0,1 %	1,4 %	0,5 %	2,9 %
Skew	0,59	42,94	1,61	33,70	0,02	44,72	1,70	34,33
Kurt	-0,64	1953,42	2,26	1146,56	-0,80	2108,39	3,48	1196,44
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,5 %	0,0 %	0,0 %	0,0 %
q1	0,7 %	0,3 %	0,2 %	0,0 %	0,6 %	0,3 %	0,1 %	0,0 %
q3	0,8 %	0,6 %	0,7 %	0,1 %	0,8 %	0,5 %	0,8 %	0,0 %
Max	1,0 %	65,7 %	1,7 %	100,0 %	0,9 %	68,9 %	1,9 %	100,0 %
Mcf	6,6 %		13,9 %		6,9 %		15,9 %	
Tjur	0,3 %		0,3 %		0,3 %		0,4 %	

	g250		g250_ln		g250_wins		g250_wins_ln	
	Def	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
Avg	0,8 %	0,5 %	0,5 %	0,2 %	0,8 %	0,4 %	0,6 %	0,2 %
Median	0,7 %	0,4 %	0,3 %	0,0 %	0,8 %	0,4 %	0,4 %	0,0 %
Std	0,2 %	1,5 %	0,6 %	2,9 %	0,1 %	1,5 %	0,6 %	2,9 %
Skew	0,38	46,88	1,16	33,84	0,18	48,17	1,30	34,47
Kurt	-1,19	2246,69	-0,04	1154,01	-0,93	2372,15	0,54	1203,28
Min	0,6 %	0,0 %	0,1 %	0,0 %	0,5 %	0,0 %	0,0 %	0,0 %
q1	0,6 %	0,3 %	0,1 %	0,0 %	0,7 %	0,3 %	0,2 %	0,0 %
q3	0,9 %	0,6 %	0,8 %	0,1 %	0,8 %	0,5 %	0,7 %	0,1 %
Max	1,0 %	72,9 %	1,6 %	100,0 %	1,0 %	76,3 %	1,7 %	100,0 %
Mcf	6,8 %		12,2 %		7,2 %		13,8 %	
Tjur	0,3 %		0,4 %		0,3 %		0,4 %	

Reorganization Sample

		h90		h90_ln		h90_wins		h90_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	3,9 %	1,5 %	2,1 %	0,4 %	3,9 %	1,5 %	2,2 %	0,4 %
0,0 %	Median	3,8 %	1,1 %	1,1 %	0,0 %	3,8 %	1,1 %	1,1 %	0,0 %
2,9 %	Std	2,0 %	3,1 %	2,8 %	3,1 %	2,0 %	3,1 %	3,0 %	3,1 %
33,64	Skew	0,52	25,12	2,09	29,19	0,44	25,25	1,99	28,34
1144,70	Kurt	0,20	769,27	3,97	933,36	0,01	774,84	3,12	891,51
0,0 %	Min	0,2 %	0,0 %	0,0 %	0,0 %	0,2 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,7 %	0,5 %	0,5 %	0,0 %	2,7 %	0,5 %	0,4 %	0,0 %
0,1 %	q3	4,7 %	2,0 %	2,1 %	0,2 %	4,6 %	2,1 %	2,0 %	0,2 %
100,0 %	Max	8,8 %	99,3 %	11,5 %	100,0 %	8,2 %	99,3 %	11,5 %	100,0 %
	Mcf	11,3 %		15,0 %		11,2 %		15,4 %	
	Tjur	2,4 %		1,7 %		2,3 %		1,8 %	

		h180		h180_ln		h180_wins		h180_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	3,6 %	1,5 %	2,0 %	0,4 %	3,6 %	1,5 %	1,8 %	0,4 %
0,0 %	Median	3,4 %	1,2 %	0,9 %	0,0 %	3,5 %	1,2 %	1,0 %	0,0 %
2,9 %	Std	1,8 %	3,1 %	2,7 %	3,1 %	1,7 %	3,0 %	2,2 %	3,1 %
33,75	Skew	0,44	26,23	2,13	28,30	0,28	26,38	1,91	29,20
1149,87	Kurt	0,10	815,29	4,28	889,31	-0,10	821,72	3,49	931,56
0,0 %	Min	0,5 %	0,0 %	0,0 %	0,0 %	0,5 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,6 %	0,6 %	0,4 %	0,0 %	2,6 %	0,6 %	0,4 %	0,0 %
0,1 %	q3	4,4 %	2,1 %	2,1 %	0,2 %	4,3 %	2,1 %	1,9 %	0,2 %
100,0 %	Max	7,8 %	99,4 %	11,1 %	100,0 %	7,5 %	99,4 %	9,4 %	100,0 %
	Mcf	10,4 %		13,5 %		10,3 %		13,6 %	
	Tjur	2,1 %		1,6 %		2,1 %		1,4 %	

		h250		h250_ln		h250_wins		h250_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	3,8 %	1,5 %	1,7 %	0,4 %	3,7 %	1,5 %	1,8 %	0,4 %
0,0 %	Median	3,3 %	1,1 %	0,9 %	0,1 %	3,3 %	1,2 %	0,8 %	0,0 %
2,9 %	Std	1,9 %	3,1 %	1,9 %	3,0 %	1,8 %	3,1 %	2,2 %	3,1 %
33,66	Skew	0,46	25,90	1,71	30,53	0,29	26,10	1,79	28,57
1145,58	Kurt	-0,09	800,82	2,67	995,30	-0,35	809,50	3,12	898,44
0,0 %	Min	0,4 %	0,0 %	0,0 %	0,0 %	0,4 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,8 %	0,5 %	0,6 %	0,0 %	2,8 %	0,5 %	0,5 %	0,0 %
0,1 %	q3	4,6 %	2,0 %	2,1 %	0,3 %	4,5 %	2,0 %	2,0 %	0,2 %
100,0 %	Max	8,1 %	99,6 %	7,7 %	100,0 %	7,7 %	99,6 %	9,3 %	100,0 %
	Mcf	10,7 %		12,6 %		10,5 %		14,5 %	
	Tjur	2,2 %		1,3 %		2,1 %		1,4 %	

		h5Y		h5Y_ln		h5Y_wins		h5Y_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	3,0 %	1,5 %	1,5 %	0,4 %	2,9 %	1,5 %	1,4 %	0,4 %
0,0 %	Median	2,9 %	1,3 %	0,9 %	0,1 %	2,7 %	1,3 %	0,7 %	0,1 %
3,8 %	Std	1,1 %	3,2 %	1,6 %	4,1 %	1,1 %	3,1 %	1,6 %	4,1 %
24,88	Skew	0,17	24,40	1,94	22,89	0,23	24,66	2,09	22,83
629,08	Kurt	0,15	664,57	3,68	543,13	0,26	682,21	4,53	539,85
0,0 %	Min	0,6 %	0,0 %	0,0 %	0,0 %	0,6 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,5 %	0,8 %	0,5 %	0,0 %	2,5 %	0,8 %	0,5 %	0,0 %
0,1 %	q3	3,5 %	1,9 %	1,5 %	0,2 %	3,3 %	1,9 %	1,3 %	0,2 %
100,0 %	Max	5,7 %	98,5 %	7,1 %	100,0 %	5,5 %	98,0 %	7,3 %	100,0 %
	Mcf	9,1 %		14,4 %		8,6 %		14,6 %	
	Tjur	1,5 %		1,0 %		1,4 %		0,9 %	

		hTot		hTot_ln		hTot_wins		hTot_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	2,9 %	1,5 %	1,4 %	0,4 %	2,8 %	1,5 %	1,3 %	0,4 %
0,0 %	Median	2,7 %	1,4 %	0,8 %	0,1 %	2,6 %	1,4 %	0,6 %	0,1 %
3,9 %	Std	1,1 %	3,1 %	1,5 %	4,0 %	1,0 %	3,0 %	1,6 %	4,1 %
24,62	Skew	0,18	24,92	1,98	23,03	0,22	25,12	2,17	22,67
615,12	Kurt	0,21	689,30	3,91	549,23	0,26	704,36	4,98	533,58
0,0 %	Min	0,6 %	0,0 %	0,0 %	0,0 %	0,6 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,4 %	0,8 %	0,5 %	0,0 %	2,4 %	0,8 %	0,5 %	0,0 %
0,0 %	q3	3,4 %	1,9 %	1,5 %	0,2 %	3,2 %	1,9 %	1,3 %	0,2 %
100,0 %	Max	5,3 %	97,8 %	6,7 %	100,0 %	5,2 %	97,2 %	7,5 %	100,0 %
	Mcf	8,3 %		14,2 %		8,0 %		13,9 %	
	Tjur	1,3 %		0,9 %		1,2 %		0,9 %	

		g90*		g90*_ln		g90*_wins		g90*_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	3,2 %	1,5 %	2,1 %	0,4 %	3,1 %	1,5 %	1,8 %	0,4 %
0,0 %	Median	3,1 %	1,3 %	1,0 %	0,0 %	3,0 %	1,3 %	0,9 %	0,1 %
3,6 %	Std	1,4 %	3,2 %	2,9 %	3,7 %	1,3 %	3,1 %	2,3 %	3,7 %
27,74	Skew	0,41	24,37	2,30	25,01	0,21	24,56	2,04	25,30
771,82	Kurt	0,50	661,75	5,13	663,27	0,26	674,16	3,54	674,61
0,0 %	Min	0,5 %	0,0 %	0,0 %	0,0 %	0,5 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,5 %	0,7 %	0,4 %	0,0 %	2,4 %	0,7 %	0,4 %	0,0 %
0,1 %	q3	3,6 %	2,0 %	1,7 %	0,2 %	3,7 %	2,0 %	2,0 %	0,2 %
100,0 %	Max	6,6 %	98,6 %	12,7 %	100,0 %	5,9 %	98,1 %	9,4 %	100,0 %
	Mcf	9,5 %		15,3 %		9,0 %		14,2 %	
	Tjur	1,7 %		1,7 %		1,6 %		1,4 %	

		g180*		g180*_ln		g180*_wins		g180*_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	2,9 %	1,5 %	1,7 %	0,4 %	3,0 %	1,5 %	1,8 %	0,4 %
0,0 %	Median	2,8 %	1,4 %	0,8 %	0,1 %	2,9 %	1,4 %	0,8 %	0,0 %
2,9 %	Std	1,1 %	2,6 %	2,4 %	3,1 %	1,2 %	2,7 %	2,5 %	3,2 %
33,30	Skew	0,11	29,87	2,21	28,68	-0,04	29,43	2,00	26,68
1127,64	Kurt	0,58	1034,39	4,48	903,72	0,26	1002,89	3,15	805,91
0,0 %	Min	0,5 %	0,0 %	0,0 %	0,0 %	0,5 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,5 %	0,8 %	0,5 %	0,0 %	2,5 %	0,8 %	0,4 %	0,0 %
0,0 %	q3	3,3 %	2,0 %	1,6 %	0,2 %	3,6 %	2,0 %	2,0 %	0,2 %
100,0 %	Max	5,4 %	99,9 %	10,1 %	100,0 %	5,3 %	99,9 %	9,6 %	100,0 %
	Mcf	6,9 %		13,7 %		7,2 %		15,2 %	
	Tjur	1,3 %		1,4 %		1,4 %		1,5 %	

		g250*		g250*_ln		g250*_wins		g250*_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	2,9 %	1,5 %	1,5 %	0,4 %	3,1 %	1,5 %	1,7 %	0,4 %
0,0 %	Median	2,8 %	1,4 %	0,9 %	0,1 %	2,9 %	1,4 %	0,9 %	0,1 %
3,5 %	Std	1,0 %	2,6 %	1,6 %	3,7 %	1,2 %	2,8 %	1,9 %	3,7 %
27,74	Skew	-0,04	27,81	1,62	25,65	0,13	26,34	1,68	25,13
772,14	Kurt	0,02	939,59	2,17	686,65	0,09	834,62	1,97	664,85
0,0 %	Min	0,7 %	0,0 %	0,0 %	0,0 %	0,6 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,5 %	0,8 %	0,6 %	0,0 %	2,6 %	0,8 %	0,6 %	0,0 %
0,1 %	q3	3,4 %	2,0 %	1,8 %	0,3 %	3,6 %	2,0 %	1,8 %	0,3 %
100,0 %	Max	5,1 %	99,9 %	6,4 %	100,0 %	5,5 %	100,0 %	7,1 %	100,0 %
	Mcf	7,2 %		13,0 %		7,9 %		13,4 %	
	Tjur	1,4 %		1,1 %		1,6 %		1,3 %	

		g90		g90_ln		g90_wins		g90_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	3,0 %	1,5 %	1,6 %	0,4 %	3,3 %	1,5 %	1,8 %	0,4 %
0,0 %	Median	3,1 %	1,3 %	1,0 %	0,1 %	3,2 %	1,2 %	1,0 %	0,1 %
2,9 %	Std	1,3 %	2,6 %	2,0 %	3,1 %	1,5 %	2,8 %	2,1 %	3,1 %
33,90	Skew	0,14	27,28	2,54	29,96	0,10	26,47	2,12	29,86
1156,19	Kurt	0,44	903,41	6,84	969,33	0,20	846,67	4,78	964,19
0,0 %	Min	0,3 %	0,0 %	0,0 %	0,0 %	0,2 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,2 %	0,7 %	0,3 %	0,0 %	2,5 %	0,6 %	0,5 %	0,0 %
0,1 %	q3	3,6 %	2,1 %	1,7 %	0,3 %	3,9 %	2,1 %	1,8 %	0,3 %
100,0 %	Max	6,2 %	96,9 %	9,2 %	100,0 %	6,8 %	98,4 %	9,4 %	100,0 %
	Mcf	8,2 %		11,0 %		9,3 %		11,6 %	
	Tjur	1,4 %		1,2 %		1,8 %		1,3 %	

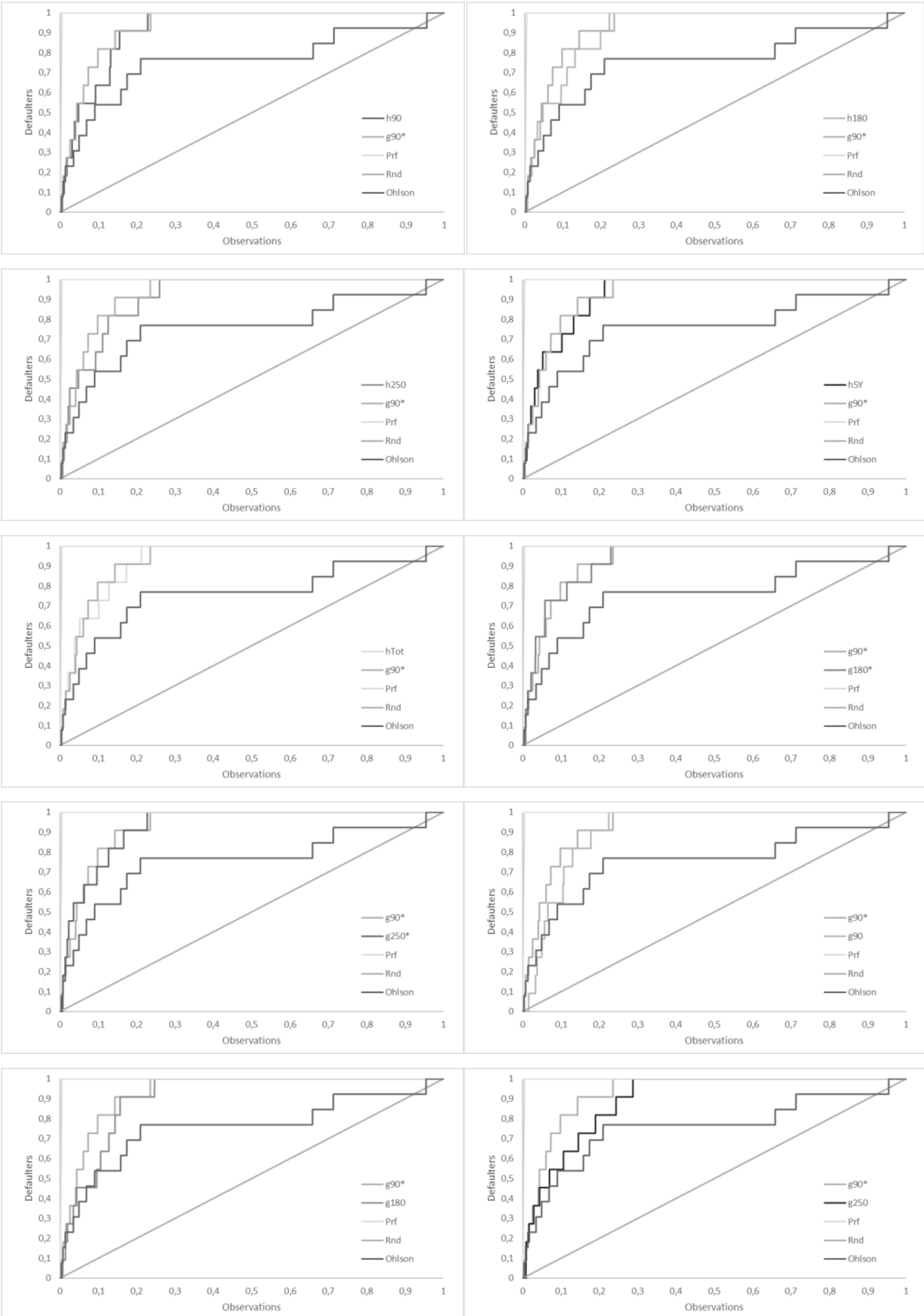
		g180		g180_ln		g180_wins		g180_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	2,9 %	1,5 %	1,7 %	0,4 %	3,3 %	1,5 %	1,8 %	0,4 %
0,0 %	Median	3,0 %	1,3 %	1,1 %	0,1 %	3,2 %	1,1 %	1,0 %	0,0 %
2,9 %	Std	1,1 %	2,8 %	2,1 %	3,1 %	1,2 %	2,9 %	2,3 %	3,1 %
34,33	Skew	-0,09	28,91	2,23	29,49	-0,06	28,59	2,33	27,83
1196,44	Kurt	0,65	942,10	4,96	944,46	0,63	933,62	5,74	870,98
0,0 %	Min	0,4 %	0,0 %	0,0 %	0,0 %	0,4 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,4 %	0,8 %	0,5 %	0,0 %	2,8 %	0,6 %	0,5 %	0,0 %
0,0 %	q3	3,4 %	2,0 %	1,8 %	0,3 %	3,8 %	1,9 %	2,0 %	0,1 %
100,0 %	Max	5,3 %	99,2 %	8,8 %	100,0 %	6,0 %	99,9 %	10,5 %	100,0 %
	Mcf	7,7 %		12,2 %		9,4 %		15,6 %	
	Tjur	1,4 %		1,3 %		1,8 %		1,4 %	

		g250		g250_ln		g250_wins		g250_wins_ln	
N_Def		**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**	**Def**	**N_Def**
0,2 %	Avg	3,9 %	1,5 %	1,9 %	0,4 %	3,7 %	1,4 %	1,9 %	0,4 %
0,0 %	Median	3,7 %	1,1 %	1,1 %	0,1 %	3,6 %	1,0 %	1,0 %	0,0 %
2,9 %	Std	2,0 %	2,8 %	2,5 %	3,0 %	1,7 %	2,8 %	2,4 %	3,1 %
34,47	Skew	0,75	24,52	2,06	29,98	0,33	25,86	1,85	28,29
1203,28	Kurt	0,43	798,06	3,57	969,67	-0,10	835,20	2,92	896,56
0,0 %	Min	0,3 %	0,0 %	0,0 %	0,0 %	0,2 %	0,0 %	0,0 %	0,0 %
0,0 %	q1	2,8 %	0,5 %	0,6 %	0,0 %	2,7 %	0,5 %	0,4 %	0,0 %
0,1 %	q3	4,6 %	2,1 %	1,9 %	0,3 %	4,6 %	2,0 %	2,1 %	0,2 %
100,0 %	Max	8,8 %	100,0 %	9,8 %	100,0 %	7,5 %	100,0 %	9,8 %	100,0 %
	Mcf	10,6 %		13,8 %		10,6 %		15,8 %	
	Tjur	2,4 %		1,6 %		2,3 %		1,5 %	

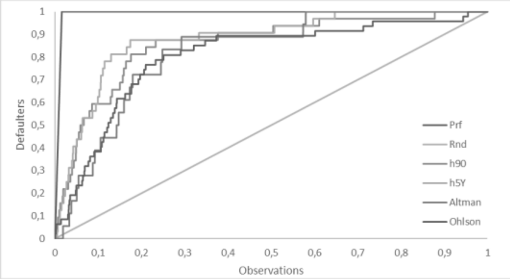
Appendix C - CAP Curves

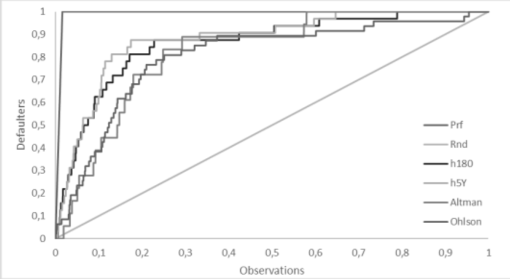
The tables below display the different CAP curves for the different versions of the MCD models, as well as the curves for Altman's Z-score and Ohlson's O-score. Every CAP curve of the MCD model is compared to the best performing model in terms of AR, the random model and the perfect model with respect to its default sample. "Prf" signifies the perfect model and "Rnd" signifies the random model.

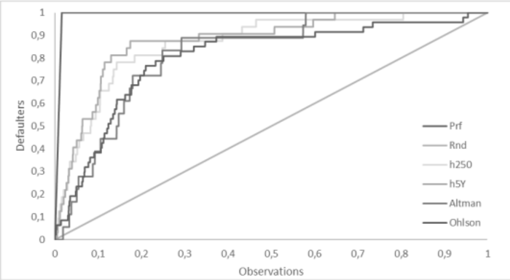
Bankruptcy Sample

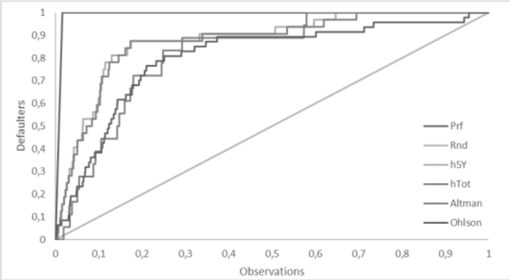


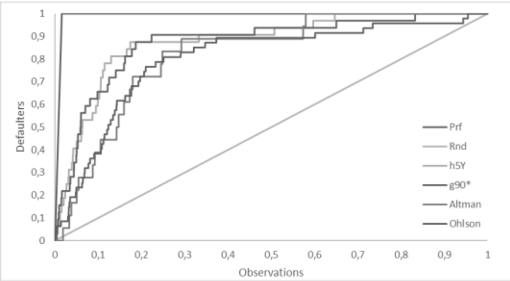
Reorganization Sample

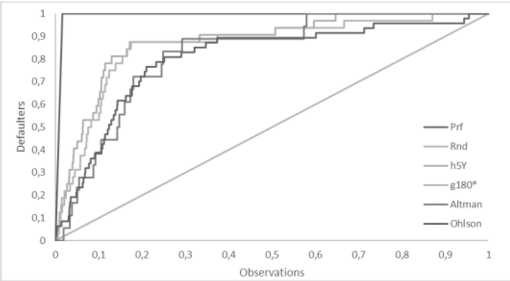


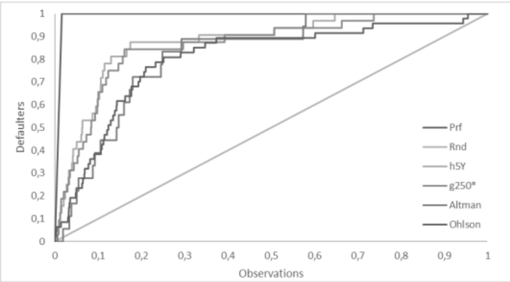


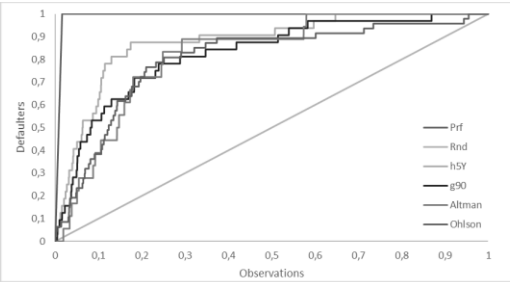


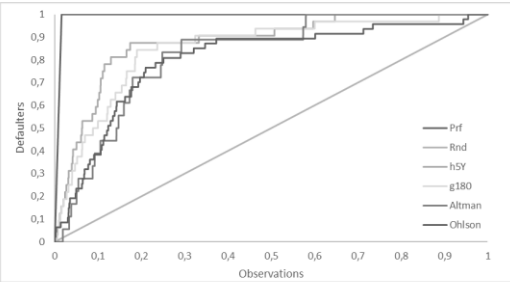


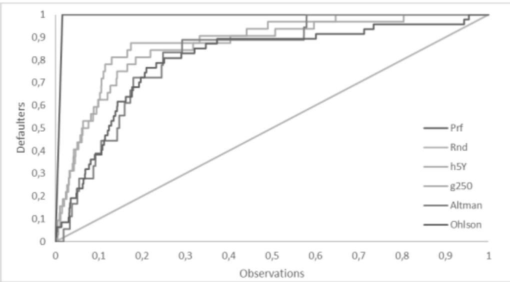












Appendix D – Vasicek One Factor Model

Vasicek_Bankruptcy_Ln

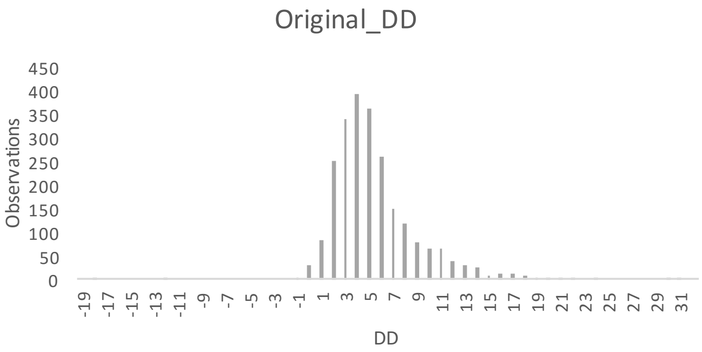
g250	Year	h90	h180	h250	h5Y	hTot	g90*	g180*	g250*	g90	g180	g250
0,004	2015	0,51	0,24	0,16	0,08	0,06	0,19	0,36	0,28	0,57	0,45	0,37
0,001	2014	0,03	0,04	0,09	0,04	0,03	0,03	0,03	0,07	0,05	0,06	0,13
5E-05	2013	0,00	0,02	0,01	0,00	0,00	0,01	0,01	0,00	0,00	0,01	0,01
7E-04	2011	0,02	0,03	0,08	0,04	0,04	0,02	0,03	0,10	0,02	0,04	0,07
0,058	2009	7,14	5,88	5,53	21,72	21,73	25,01	5,32	23,76	6,50	5,83	5,81
6E-04	2002	0,08	0,06	0,08	0,02	0,02	0,03	0,02	0,05	0,10	0,05	0,06
7E-06	2001	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

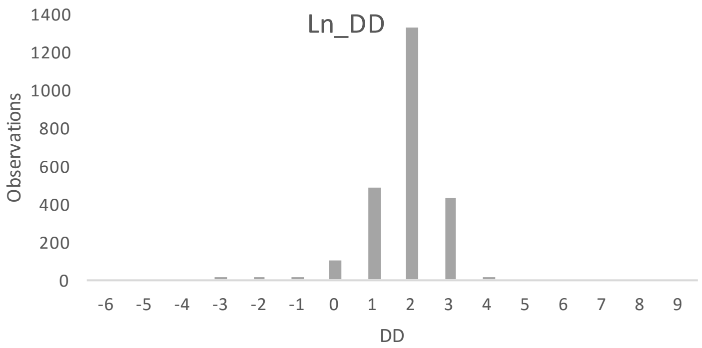
g250	***Vasicek_Reorganization_Ln***											
5E-10	Year	h90	h180	h250	h5Y	hTot	g90*	g180*	g250*	g90	g180	g250
1E-04	2015	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
1E-07	2014	0,00	0,00	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,01	0,01
0,007	2013	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
3E-11	2012	1,72	1,15	0,74	0,39	0,37	0,86	0,51	0,61	1,30	0,84	0,65
1E-04	2011	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
1E-06	2010	0,00	0,02	0,19	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,01
8E-14	2009	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,01	0,00	0,00	0,00
4E-09	2008	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	2007	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Appendix E - DD Distribution

Below follow descriptive statistics of the DD-values and the logarithmic transformed DD-values. The h90 and the h90_ln model are chosen to represent the effect. In the descriptive statistics table, "Avg": the average, "stdev": the sample standard deviation, "Skew": skewness, "Kurt": Excess kurtosis, "min" and "max" is the minimum and maximum DD value, "q1" and "q3" is the first and thirds quartiles and "b(1) and b(2) is the intercept and coefficient value for the logit model. The graphs below are the distribution of the DD values.

Bankruptcy_Sample		
	h90	h90_ln
Avg	4,99	1,37
Median	4,21	1,44
stdev	3,58	0,77
skew	1,45	-1,14
kurt	5,37	3,91
min	-18,96	-3,58
q1	2,68	0,99
q3	6,30	1,84
max	30,10	3,40
b(1)	-4,37	-0,29
b(2)	-4,44	-1,07





Appendix F - PD Evolvement

Appendix F shows how PDs of defaulters evolve over time. The years of default for each respective defaulting firm is displayed in the tables, while the firms' PD evolvement is displayed in the graphs below.

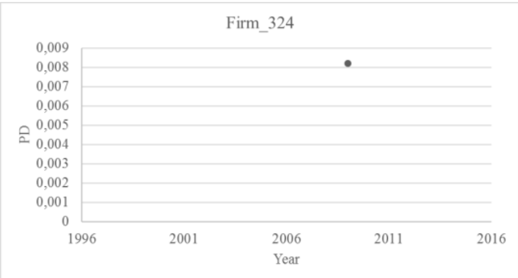
Bankruptcy_Sample

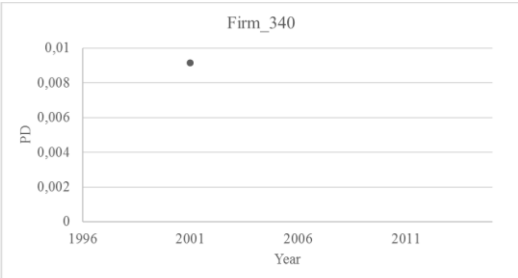
Firm	#Def	Year
404	1	2015
378	1	2014
623	1	2013
743	1	2013
431	1	2011
324	1	2009
603	1	2009
642	1	2009
488	1	2002
340	1	2001
421	1	2001
	11	

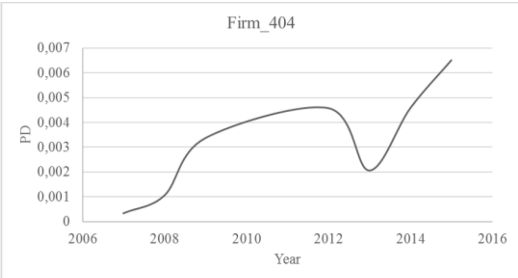
Default_Sample

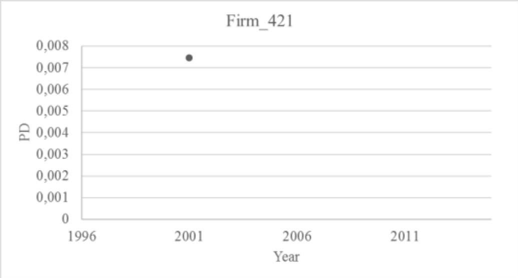
Firm	#Def	Year
64	1	2010
65	1	2006
98	1	2011
114	2	2008,2009
130	1	2010
137	1	2014
139	2	2012,2014
153	1	2014
166	2	2008,2010
170	1	2010
199	1	2008
324	1	2008
378	3	2011,2013,2014
384	1	2011
404	1	2014
415	3	2008,2011,2013
431	1	2010
473	2	2008,2012
550	1	2008
603	1	2008
623	2	2009,2012
642	1	2008
692	2	2007,2008
693	1	2007
729	1	2007
743	1	2012
Sum_def:	36	

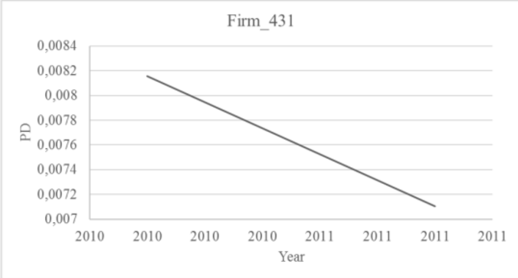
Bankruptcy Sample

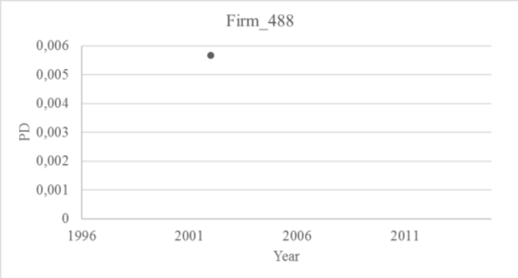


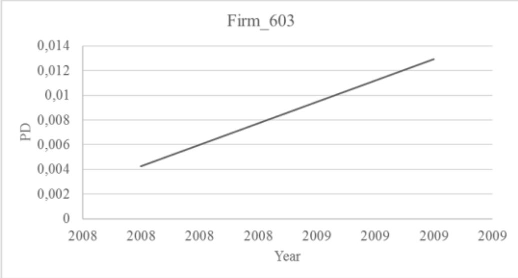


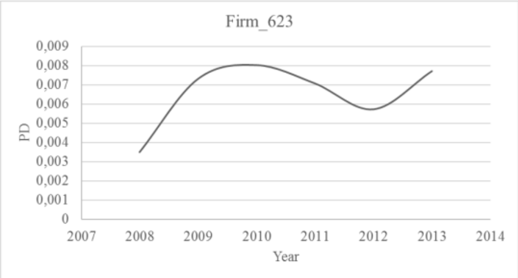


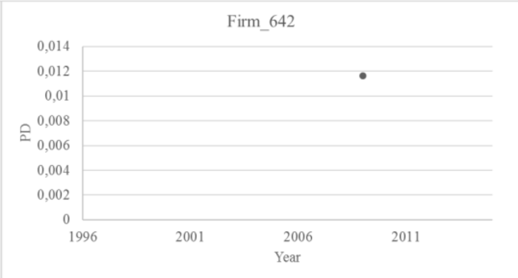


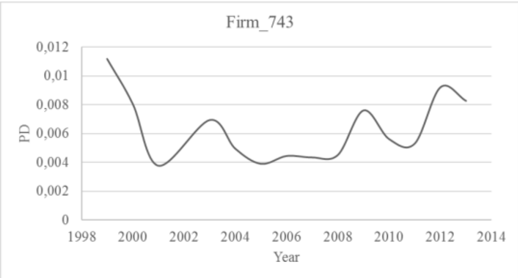




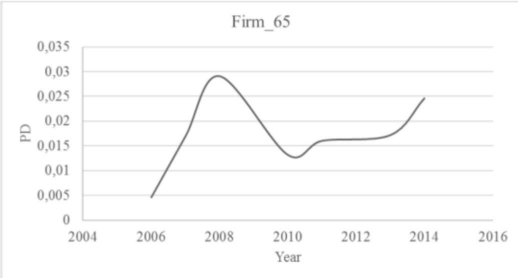


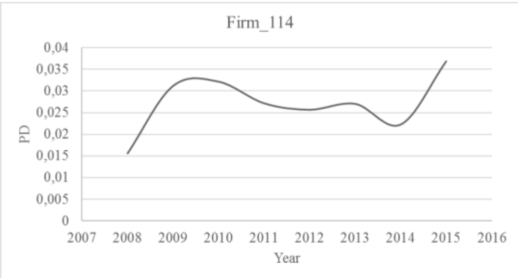






Reorganization Sample





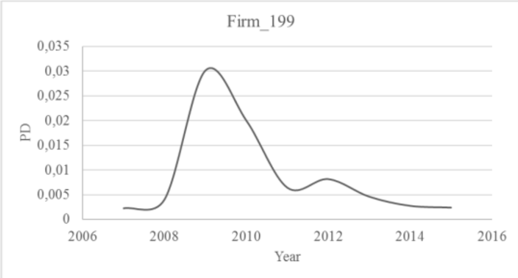

Firm_137
PD
0,045 0,04 0,035 0,03 0,025 0,02 0,015 0,01 0,005 0
2004 2006 2008 2010 2012 2014 2016
Year


Firm_139
PD
0,07 0,06 0,05 0,04 0,03 0,02 0,01 0
2007 2008 2009 2010 2011 2012 2013 2014 2015
Year


Firm_153
PD
0,035 0,03 0,025 0,02 0,015 0,01 0,005 0
2009 2010 2011 2012 2013 2014 2015
Year


Firm_166
PD
0,06 0,05 0,04 0,03 0,02 0,01 0
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014
Year


Firm_170
PD
0,045 0,04 0,035 0,03 0,025 0,02 0,015 0,01 0,005 0
2005 2006 2007 2008 2009 2010 2011 2012
Year


Firm_199
PD
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2006 2008 2010 2012 2014 2016
Year


Firm_378
PD
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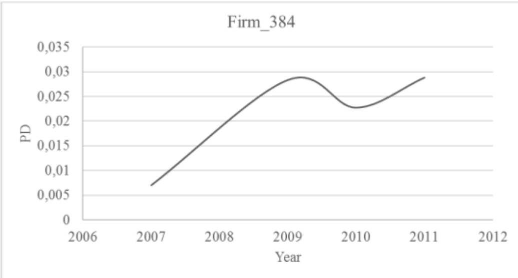
2009 2010 2011 2012 2013 2014 2015
Year


Firm_384
PD
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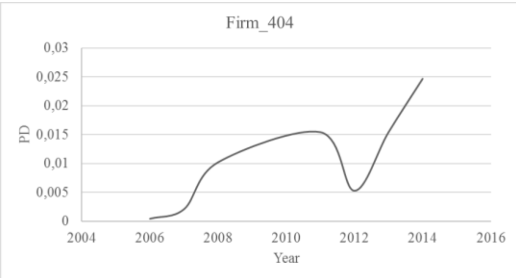
2006 2007 2008 2009 2010 2011 2012
Year


Firm_404
PD
0,03 0,025 0,02 0,015 0,01 0,005 0
2004 2006 2008 2010 2012 2014 2016
Year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PD
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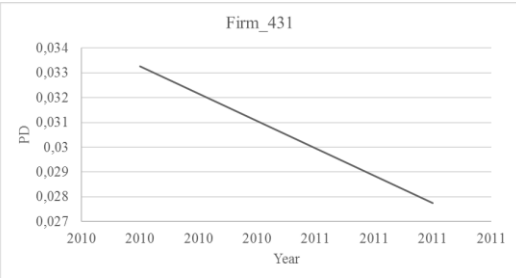
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Year


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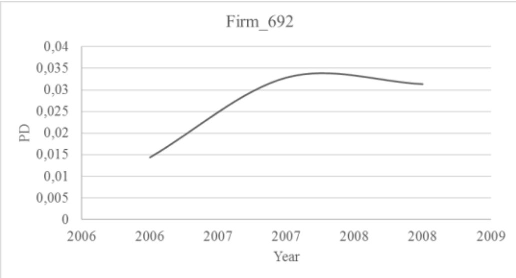
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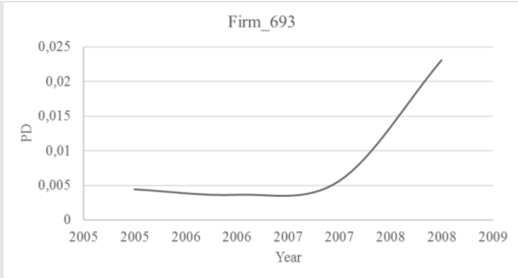
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Appendix G - VBA Codes

The VBA codes applied in this study are displayed in the following link.

https://drive.google.com/open?id=0B3G7PewLNHcXSl8wV3JBRnp0TU0

Appendix G – Preliminary Study

ID number: **0930885**

ID number: **0906174**

Preliminary Master's Thesis

Assessing Credit Risk in Norwegian High Yield Companies

Hand-in date:
15.01.2016

BI Norwegian Business School

Examination code and name: 19002 - Master Thesis

Programme:
Master of Science in Business with Major in Finance

Supervisor: Janis Berzins

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Prologue

In the backwash of the fall in the oil price, multiple Norwegian High Yield bonds have experienced a substantial increase in spreads, which in theory could an increase in credit risk. Our preliminary research indicates that about 20% of Norwegian Gross Domestic Product stems from the petroleum industry, and thus the shockwaves from the fall in the oil price may increase credit risk on a national basis. In our study, we will attempt to quantify credit risk amongst defaulted Norwegian High Yield companies that have been listed on the Oslo Stock Exchange. In particular, we will study following research question:

> *How well does Merton's extended corporate debt model perform in predicting defaults amongst Norwegian High Yield companies?*

This preliminary analysis is split in six parts. Part one provides a summary of the motivation for our area of study. Part two presents a review of existing literature on the topic at hand. Part three provides some theoretical basis for our study. Part four outlines our methodology. Part five presents our data and resources. Lastly, part six contains a brief summary of the main challenges we expect to encounter in our research.

Part I - Motivation for study

Our motivation for studying credit risk[1] amongst Norwegian high yield (HY) firms can be summarized in two key points: firstly, it is anchored in the country's dependency on the petroleum sector and the notable increase in credit spreads amongst HY firms. Secondly, our motivation is connected to the challenge of applying official rating agencies' methodology in assessing credit risk in Norwegian markets. While default risk amongst Norwegian High Yield has been studied with statistical models, no one, to our knowledge, has attempted to apply structural models to do the same. The following section summarizes the basis for our motivation.

The development of Norwegian dependence on the petroleum industry

Since the 1970's, the country of Norway has experienced an exponential growth in its GDP. Data on Norwegian GDP is displayed in Figure 1 below, and illustrates the development country's mainland GDP and total GDP. The difference between the two figures in GDP reflects contribution from the petroleum industry and shipping (OECD 2011). GDP contribution from the petroleum and shipping industry represented on average 14,5% of total GDP from 1978Q1 to 1997Q4. The industries' contribution has since increased to 20,7% on average from 1998Q1 until 2015Q3, which gives an indication of Norway's increasing dependence on oil & gas and shipping services.

[1] Credit risk as the risk that the obligor does not meet its payment on time (Lu 2008)

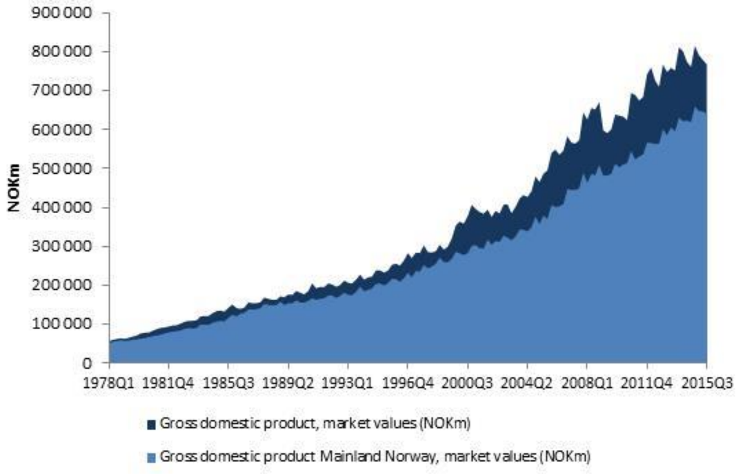

Figure 1 - Total, Mainland and petro-related GDP in Norway (Statistics Norway 2016)

Norway's dependence on petroleum and corporate debt

Figure 2 below illustrate the extent of how different industries' deliveries are dependent on the petroleum industry.

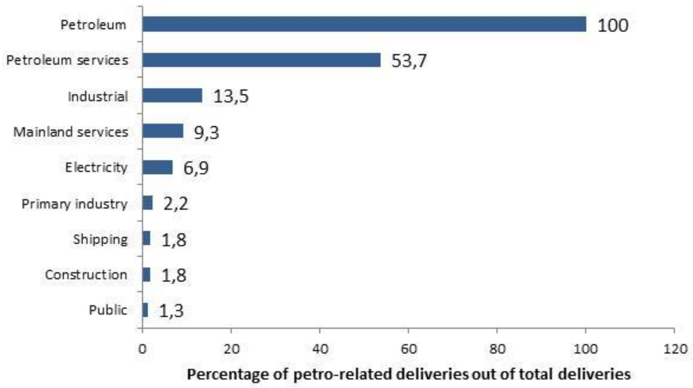

Figure 2 - Percentage of deliveries to the petroleum industry out of the industry's total deliveries. Source: (Prestmo, Strøm og Midsem 2015)

Services relating to extraction of petroleum represent 54% of the category's GDP, while the equivalent figure is 13,5% and 9,3% for industrial and mainland services respectively (Prestmo, Strøm og Midsem 2015). Evidently, multiple industries are dependent on the petroleum industry. Data from Norsk Tillitsmann displayed in Figure 3 shows that petroleum related HY industries rank at the top of outstanding corporate debt (Norsk Tillitsmann - Stamdata 2015). Furthermore, Norwegian HY companies have significant outstanding debt – 13% more than Norwegian corporate investment grade (IG)[2] companies, as shown in Figure 4.

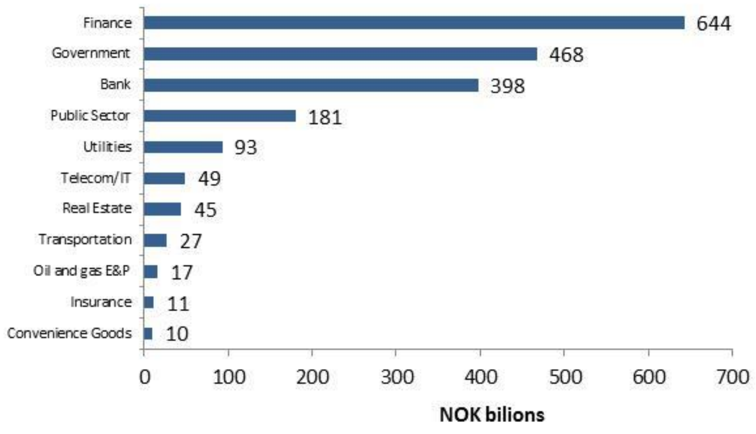

Figure 3 - Outstanding HY bonds by industry
Source: Stamdata

Figure 4 - Outstanding HY debt and corporate IG debt
Source: Stamdata

To avoid banks taking excessive debt, regulations such as BASEL I and II have enforced more focus on credit risk amongst banks on international basis (Bank for International Settlements 2016). It is likely that future regulations will strengthen the focus even more. As such, understanding of default probabilities of HY bond issuers is a hot topic.

Increase in credit risk amongst HY companies

Bond yield spreads is the terminology on the spread between bond yield to maturity and a risk free rate. These credit spreads are assumed to compensate bondholders for credit risk (Hull, Options, Futures and Other Derivatives 2015).

[2] Corporate IG = Total IG less financials and government IG bonds

Multiple researchers have proven that default risk alone does not fully explain credit spreads, a problem referred to as 'The Credit Spread Puzzle' (Amato og Remolana 2003). Attempting to explain this puzzle is beyond the scope of our paper, though in regards to the motivation for our research question, we allow ourselves to view bond yield spreads as indications of credit risk.

With the drop of the oil price during the fall of 2014, petroleum exposed industries in Norway have experienced turbulence. Figure 5 shows five selected Norwegian high yield (HY) bonds related to petroleum and shipping that have experienced a drastic increase in credit spreads in the backwash of the fall in the oil price. The correlation between these two time series, the average spread and the WTI price, is -0.92. In light of the petroleum industry's impact on Norwegian GDP, and the aforementioned increase in spreads, it may be fruitful to study credit risk amongst Norwegian High Yield companies.

Figure 5 - Average spreads over LIBOR on HY bonds for Aker ASA, Color Group, DOF Subsea, Seadrill and Stolt-Nilsen and spot prices for Western Texas Intermediate (WTI).
Source: Danske Bank Markets and Yahoo Finance

Assessing Credit risk in Norway

Researchers of credit risk have proposed numerous ways to estimate probabilities of default, some of which will be discussed in our section of literature review.

"Nationally recognized statistical rating organization" (NRSRO), is the name given to official credit rating agencies. These agencies are organizations that provide an opinion on the creditworthiness of a company or a financial instrument, in the form of a credit rating (U.S Securities and Exchange

Commission 2016). Three major U.S rating agencies are Moody's Investors Services (Moody's), Standard & Poor's (S&P) and Fitch Investor Services (Fitch). The agencies' rating methodology is frequently conveyed in their own reports, guiding investors and other financial institutions in understanding rating procedures. Norway does not have NRSRO institutions, and obtaining an NRSRO rating is often uneconomical (Stein, Keenan og Sobehart 2000). Norwegian financial institutions may provide their own rating of companies of interest, referred to as shadow ratings (DNB Markets 2014). Optimally, these shadow ratings should reflect default risk in the company, and explain the increase in spreads as illustrated in Figure 5. However, Fitch notes that the difficulty in maintaining up-to-date and adequate shadow ratings may cause discrepancy between the firms' true credit risk and the one reflected in the shadow ratings (Fitch Ratings 2015).

In their book 'Rating Based Modelling of Credit Risk', Trueck and Rachev acknowledges credit rating agencies' role during the financial crisis. Specifically, they explains how these institutions failed to provide adequate ratings for different derivative products (Trueck and Rachev 2009). With the interpreted rise in credit risk amongst Norwegian HY companies, appropriate assessment of credit risk is of dire need. Without it, default probabilities may be underestimated, and debt holders could face significant losses. This puts further basis for our research question.

We argue that there is a strong indication that Norway's dependence on the petroleum industry has indeed increased throughout the past twenty years. The petroleum sector's effects on the country's industries are substantial, and shockwaves from the petroleum sector's turmoil may affect large parts of the economy. We argue that all holders of high yield debt – banks, financial institutions, credit portfolio managers and private holders of corporate debt – will have an interest in a study of defaults with Merton's corporate debt model. This puts strong basis for our research question, as previously stated;

> *How well does Merton's extended corporate debt model perform in predicting defaults amongst Norwegian High Yield companies?*

Part II - Literature review

Credit valuation and default prediction has gained much attention in global markets throughout the past decades (Trueck and Rachev 2009). However, equivalent research on Norwegian HY companies is limited. The following section provides a brief introduction to previous literature on this field of research.

Two classes of credit risk models

The first published paper on credit risk date as far back as 1932, when Fitzpatrick presented his findings on financial multiples as signals of imminent bankruptcy (Fitzpatrick 1932). Since then, research on credit risk has grown steadily. Throughout the past forty years, two paths of modelling credit risk have been primarily studied; structural models and reduced form models. Structural models commonly use a theoretical option pricing approach, and apply equity prices to solve for default probabilities. Reduced form models use a statistical approach, and apply historical debt prices and default risk premiums to solve for default probabilities (Saunders and Allen 2002). Which of the two classes of models perform better is highly debated. (Hull, Options, Futures and Other Derivatives 2015) (Jarrow og Protter, Structural versus Reduced Models 2004).

Structural models

In 1974, Robert K. Merton proposed a model of valuing the debt of distressed companies – that is, companies with a significant probability of default – through option pricing theory (K. R. Merton 1974). This was an extension of both Black and Scholes' and his own article of the general option pricing model (Black og Scholes 1973) (R. K. Merton 1973) . With the model, Merton defined debt as a function of five parameters. Specifically,

$$D = f(V, PV(B), \sigma_V, r, t)$$

where *D* is the market value of the company's debt, PV(B) is the present value of the face value of debt, r is risk free rate of interest, t is time and V and σ_V are the market of the company's assets and its volatility, both of which are unobservable parameters. The intuition behind Merton's corporate debt model (Hereafter: the MCD model) approach is that equity can be seen as a call option on the firm's assets, where face value of debt is viewed as a strike price. When the debt matures, the equity holders may either repay the debt and keeps the assets, or relinquish the firm, leaving the debt holders with compromised asset value. The choice of repaying the debt (exercising the call option) depends on whether the market value of the firm's assets is above that of its debt (Sobehart og Keenan 1999).

Extensions to the MCD model have since been developed by researchers such as Longstaff & Schwartz (1995) and Zhou (1997), who introduced the following concepts to Merton's corporate debt model:

- Jumps
- Stochastic interest rates
- Discrete coupons

Traditional and reduced form models

Empirical research on statistical models' ability to predict defaults began in 1966, with Beaver's '*Financial Ratios as Predictors of Failure*'. This has been recognized as the birth of traditional models to predict defaults. Although credit research had been in focus since the 1930's, no previous research provided empirical evidence of financial ratios having predictive power in regards to corporate default. Beaver assessed the likelihood ratio of financial multiples such as cash-flow to total-debt, and determined that signs of default could be statistically significant even five years in advance (Beaver 1966).

Shortly after, in 1968, Altman developed the now familiar Z-score. The score was a statistical output from a linear multivariate model, and proved helpful in ranking firm's creditworthiness.

The application of logistic regression to estimate models to predict defaults was first introduced by Ohlson (1980). He, too, tested the predictive power of financial multiples, of which he acquired from financial statements. This was the first time the output of a prediction model could be interpreted directly as a probability of default (Ohlson 1980).

The application of reduced form models to address credit risk traces its roots back to 1994. This year, Fons illustrated the relationship between default probabilities, recovery rates for bonds and credit spreads (Fons 1994) (Trueck and Rachev

2009). He concluded that spreads for differently rated bonds should lie within certain intervals, and that discrepancies to the rule could be explained by a liquidity premium required by investors; an early reference to what should later be called ‘the credit spread puzzle” (Amato og Remolana 2003). Fons introduced the ability to incorporate other exogenous variables than financial figures into models to determine probabilities of default, such as corporate credit ratings and recovery rates[3].

Extensions to Fons reduced model have since been introduced. In 1997, Jarrow et al. provided a Markow model for the term structure of credit risk that could be used to price corporate debt (Jarrow, Lando and Turnbull, A Markow Model for the Term Structure of Credit Risk Spreads 1997). Other extensions include Duffie and Singleton, who extended the model to also applying the term structure of corporate debt (Duffie og Singleton 1999).

Hybrid models

Which of the types of models perform better is heavily debated. Trueck explains that, due to their superior performance, the types of models that have gained researchers’ attention the past decades are structural models and reduced form models (Trueck and Rachev 2009). Jarrow and Protter (2004) argue that choice between structural models and reduced form models is a question about the availability of information about the true value and volatility of firms’ assets. It is generally understood that the market is unable to observe the true values of assets (Saunders and Allen 2002). The authors claim that reduced form models are superior, because these models do not use asset values in their calculations. In contrast, Stein, Keenan and Sobehart shows that Moody’s Public Firm model, a hybrid extension of Merton’s option pricing model, outperforms other models when measuring default prediction.

The discussion on which model performs better has contributed to the experimentation of hybrid models – models that combines structural and reduced form models. Sobehart and Stein were amongst the first to introduce a general hybrid model to the field of credit risk studies, which proved to perform

[3] The left-over capital for debt holders post default as percentage of face value (Jarrow og Protter, Structural versus Reduced Models 2004).

particularly well at predicting defaults (Sobehart og Stein, Rating methodology - Moody's Public Firm Risk Model: A Hybrid Approach To Modeling Short Term Default Risk 2000). Their studies concluded that by combining a variation of the MCD model with financial multiples and ratings, they were able to predict defaults with surprisingly high accuracy.

Credit risk measurement of Norwegian High Yield firms

Moving towards previous research of default risk amongst Norwegian HY firms, the range of studies is shortened. Bernhardsen (2001) studied the predictive power of multiple financial ratios in regards to defaults of Norwegian companies. He constructed the 'SEBRA-model', which incorporates multiples such as equity to assets, trade accounts payable to assets and liquid assets less short-term debt as a percentage of operating revenues. The model, which would be classified as a reduced form model, proved to perform well at predicting defaults (Nordahl Grøstad 2013).

Although reduced form models have been used to quantify credit risk amongst Norwegian companies, the application of structural and hybrid models have gained little attention. The reason is most certainly the lack of data; Merton's option pricing model requires time series of share prices and sufficient amount of listed defaulters (Nordahl Grøstad 2013).

Part III -Theory

Merton's corporate debt model is at the core of our research paper. This section provides an overview of the theoretical background the model and its extensions. We have put focus on the development of the model. This makes it easier to enlighten the model's assumptions; these are central in regards to the model's strengths and weaknesses, and how applicable the model is in practice.

Risk neutrality and interest rate

If one categorizes risk behavior into risk seeking, risk neutral and risk averse, it is generally accepted that investors fall into the latter. This implies that they require additional returns to compensate for any additional risk. In contrast, a risk neutral investor would require the same rate of return for every investment opportunity. The concept of risk neutral investors is not realistic. However, in regards of valuing derivatives, a risk neutral approach has shown to be appropriate for all attitudes towards risk (Hull, Options, Futures and Other Derivatives 2015). Another main characteristic of a risk neutral approach is that the rate can be used to discount an expected pay-off.

Central in financial research is the risk free rate. In a risk neutral world, investors are assumed to require a return and discount at the same rate – the risk free rate. It is the rate of interest one earns on one's investments without taking any risk. The practical use of the risk free interest is not all that easy. The true risk free rate is in fact unobservable, and there is no clear answer as to which proxy one should use for the risk free interest rate. Natural candidates for such proxies are government bonds in countries with high creditworthiness, and thus common proxies for the risk free rate are yields of Bunds, T-bills or Gilts. However, derivative dealers argue that some government bonds may have a slightly lower yields than the risk free rate (Hull, Options, Futures and Other Derivatives 2015). They argue that the interbank-offered rates in banks with high creditworthiness or overnight index swap could be superior proxies. The choice of risk free rate proxies bears high importance, as only tiny differences in returns may have sever effect on financial valuation.

Stock movements

Modeling credit risk is, as illustrated in literature review, dependent on the application of equity prices. Therefore, to understand Merton's corporate debt model, it is crucial to understand how stock price movements are modeled.

The following section provides a brief summary of the theoretical basis for market efficiency, and its connection to credit risk modeling.

Market efficiency and Markov Property

One heavy researched topic in finance has been that of market efficiency. In academics, market efficiency has been categorized into three forms: week, semi-strong and strong market efficiency. Bodie, Kane and Marcus conclude that markets are generally competitive enough that investment opportunities with superior returns are quickly diminished by demand (Bodie, Kane og Marcus 2014). This is in line with academics' assumption that arbitrage[4] opportunities are rare and immediately taken by arbitrageurs, which supports the hypothesis that markets are at least weakly efficient.

Given the assumption of markets being efficient (at least weakly), market values can be said to follow a stochastic process[5]. A Markov process is stochastic, meaning that only the current value of a variable is relevant for predicting future movements, and past movements are irrelevant. Hence, the probability distribution of the future value is independent from past values. Hull claims that there is little evidence supporting long term abnormal returns from technical analysis[6]. Stock prices are usually assumed to follow a Markov process.

Weiner Process

A stochastic process can be classified as discrete or continuous in time and variable. From here on, we assume continuity in every variable.

---

[4]An arbitrage opportunity is defined as a trading strategy that takes advantage of two securities that are mispriced relative to each other.

[5] A stochastic process is the process in which the future value is uncertain (Hull, Options, Futures and Other Derivatives 2015).

[6] Studying past prices of stocks.

A Wiener process or Brownian motion is a type of Markov process where the mean change is 0 and the variance is 1. During a small change in time dt, the change in a Wiener process dz will be

$$dz = \epsilon\sqrt{dt}$$

Equation 1

where $P(dz_i | dz_j) = P(dz_i)$ and $\epsilon \sim N(0,1)$. A large change in time from t=0 to t=T can be written as $T = N * dt$, where N is number of time intervals, such that a large change in Z from time 0 to time T, can be written as

$$Z_T - Z_0 = \sum_{i=1}^{N} \epsilon_i * \sqrt{dt}$$

Since ϵ_i is independent from each other, $Z \sim N(0, T)$ because $\epsilon_i \sim N(0,1)$.

Generalized Wiener processes

A generalized Wiener process is a stochastic process. It has a mean change per unit time called *drift rate*. The name of the variance change per unit time is *variance rate*. In contrast to the basic Wiener process (Equation 1), the generalized Wiener process can be written as

$$dx = adt + bdz$$

where $dx \sim N(adt, b^2dt)$, a is the drift rate, and b is the variance rate. Hence, the expected value of x from time 0 to time t is[7]

$$E[x_t] = x_0 + at$$

Itô's Lemma

A stochastic process where the parameters in the generalized Wiener process are functions of its variable and time can be defined as Itô process. The mathematical expression for an Itô process is

$$dx = a(x,t)dt + b(x,t)dz$$

[7] bdz vanishes because E $[dz] = 0$.

Note that x is still a Markov process. This is because x is a function of time from time 0 to time t; it is still independent from past periods. Also, the process assumes that the parameters will stay constant in subsequent periods. From now, every drift rate and variance rate is written as constants but still assumed to be a function of the variable and time. In 1951, Itô proved that if a variable follows an Itô process, then $G = h(x)$ will follow the following process, known as *Itô's Lemma*:

$$dG = \left(\Delta a + \Theta + \frac{1}{2}\Gamma b^2\right)dt + \Delta b dz$$

Here, $\Delta = \frac{\partial G}{\partial x}, \Theta = \frac{\partial G}{\partial x}, \Gamma = \frac{\partial \partial G}{\partial x^2}$ and $dG \sim N\left(\left(\Delta a + \Theta + \frac{\Gamma b^2}{2}\right)dt, \Delta^2 b^2 dt\right)$

In financial markets, it is usually assumed that stock prices will follow a stochastic process. Yet, unlike a generalized Wiener process, where drift and variance rate are assumed to be constant, $\frac{dS}{S}$ is assumed to be constant and independent from the stock price. Hence, the stock price of time dtcan be modeled as

$$\frac{dS}{S} = \mu dt + \sigma dz$$

Equation 2

where S is the stock price, μ is the expected return (drift rate) and σ^2 is the variance rate. To better reflect the stock price movements (geometrical calculations), it is common to assume that such prices will follow a lognormal distribution. Because Equation 2 is an Itô process and in light of Itô's Lemma, the function $G = h(S)$ will have the properties

$$G = h(S) = \ln(S)$$
$$dG = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dz$$

Equation 3

where $\ln(S) \sim N\left(\ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right)$ and by integrating with respect to time,

$$G = \ln(S) = \ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T + \sigma Z,$$

$$S = s_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma Z}$$

As can be seen from Equation 3, the drift rate in lognormal stock prices are adjusted downward compared to Equation 2. This is a well known property in geometric means compared to arithmetic means (Hull, Options, Futures and Other Derivatives 2015).

Market Price of Risk

Recall our previous mention that investors are generally considered to be risk averse, such that they require compensation for bearing risk. Investor's price on such risk is termed the market price of risk. Valuation of credit risk requires some understanding of how the market prices such risk. In light of the assumption that asset values follow the same process as stock prices, equation **Error! Reference source not found.** can alternatively be expressed as

$$\frac{dV}{V} = mdt + qdz$$

Equation 4

where $dV$is the change of asset values in the time interval dt, and m and q is the drift and variance rate. The generalized Wiener process is dz. Mark that the parameters m and q are assumed to only be dependent on the asset V and the time t. Then, two securities that are also only dependent on V and t will have the process

$$\frac{dx_i}{x_i} = \mu_i dt + \sigma_i dz \, for \; i = 1{,}2$$

Equation 5

The uncertainty term dz is the same for both securities as well as for the asset itself. This is because those three processes are based upon the same "shocks". The securities are based upon the same risk, and thus it is possible to make a riskless portfolio Π by going long $\sigma_2 x_2$ in x_1 and short $\sigma_1 x_1$ in x_2:

$$\Pi = \sigma_2 x_2 x_1 - \sigma_1 x_1 x_2$$

Equation 6

such that

$$\mathrm{d}\Pi = \sigma_2 x_2 dx_1 - \sigma_1 x_1 dx_2$$

Equation 7

and by substituting for x_1 and x_2 from Equation 5 into Equation 7, we have

$$\mathrm{d}\Pi = (\mu_1 \sigma_2 x_2 x_1 - \mu_2 \sigma_1 x_1 x_2)dt$$

Equation 8

Here the uncertain term reflected in the Wiener process is eliminated. Hence, given no-arbitrage assumption, dΠ gives the risk-free rate in return

$$\mathrm{d}\Pi = \mathrm{r}\Pi\mathrm{dt}$$

Equation 9

where r is the risk free rate in the time interval dt. And by substituting Equation 5 into Equation 6 and then substituting Equation 6 and Equation 8 into Equation 9

$$\frac{(\mu_1 - r)}{\sigma_1} = \frac{(\mu_2 - r)}{\sigma_2} = \frac{\mu - r}{\sigma} = \lambda$$

Equation 10

λ may be interpreted as the market price of risk of the asset V, that is dependent on both V and t. In other words, it is the trade-off between risk and return. Equation 10 shows that in general, for the no arbitrage condition to hold, every security with the same dependence on risk factors needs to have the same price, λ. The above equation may also be expressed as

$$\mu = r + \lambda\sigma$$

Therefore, in general, the expected return from any security with multiple risk factors may be modelled as

$$\frac{dx}{x} = (r + \sum_{i=1}^{N} \lambda_i \sigma_i)dt + \sigma dz$$

Equation 11

The intuition behind Equation 11 is that the expected percentage change in x in time interval dt is the risk free rate plus the sum of the market prices times the quantities of risks. This relationship is crucial in the following theoretical concepts.

Martingales

Some applications of credit risk models relate to option pricing theory, where option prices are heavily impacted by the concept of risk neutrality. Interest rates in such models are often assumed to be constant, whereas in the real world, they are in fact stochastic processes (Hull, Options, Futures and Other Derivatives 2015). To fully understand risk neutral valuation with stochastic interest rates it is important to understand martingales and measures. A martingale can be defined as a stochastic process with a drift rate equal to zero.

One aspect of this measure is particularly relevant for option pricing theory; the equivalent martingale measure E^Q. This concept is most easily understood by thinking of two securities, α and β, that bear the same source of undertainty and follow an Itô process as in Equation 2. Further, imagine that one measures the price $Y = \frac{\alpha}{\beta}$. Here, Y will be the relative price of α with respect to β. Therefore, β is the unit of measure and is referred to as the numeraire. By setting $\lambda = \sigma_\beta$, Y is a martingale for every security α. This way, the processes of the two securities can from be written similar to Equation 11, as

$$\frac{d\alpha}{\alpha} = \left(r + \sigma_\alpha \sigma_\beta\right)dt + \sigma_\alpha dz$$

$$\frac{d\beta}{\beta} = \left(r + \sigma_\beta^2\right)dt + \sigma_\beta dz$$

Similar to stock prices, we assume that α and β are lognormally distributed, with the processes

$$dln(\alpha) = \left(r + \sigma_\alpha \sigma_\beta - \frac{\sigma_\alpha^2}{2}\right)dt + \sigma_\alpha dz$$

$$dln(\beta) = \left(r + \frac{\sigma_\beta^2}{2}\right)dt + \sigma_\beta dz$$

By using Itô's Lemma again on $\frac{\alpha}{\beta}$ from the process of $\ln\left(\frac{\alpha}{\beta}\right)$;

$$d\left(\frac{\alpha}{\beta}\right) = \left(\sigma_\alpha - \sigma_\beta\right)\frac{\alpha}{\beta}dz$$

This process is referred to an equivalent martingale, since there is no drift rate in the process. If there was a drift rate, then an arbitrage opportunity exists and one could short the security which returns would be worse and buy the superior. To conclude, it is possible to write the price of α at time 0 as

$$\alpha_0 = \beta_0 E_\beta[\frac{\alpha_T}{\beta_T}]$$

Equation 12

This state of nature is referred to as a forward neutral risk world. Due to the no-arbitrage argument, $E_\beta[\frac{\alpha_T}{\beta_T}]$ will converge to 1 such that the return of α equals that of β.

In regards to risk neutral valuation, $\lambda = 0$, which implies that Equation 11 will be

$$dx = rxdt + \sigma xdz$$

Equation 13

This state is referred to as the *traditional risk neutral world*. Another state of a risk neutral world could be defined by setting the money market account as the numeraire, β. The process of β is

$$d\beta = r\beta dt$$

Equation 14

As can be seen from Equation 14, the change in β in the time interval dt is equal to the risk free rate times β. The Wiener process is eliminated because $\sigma_\beta = 0$. Further, from the equivalent martingale result from Equation 12, security α will have the same return as the money market account β. Equation 14 shows that this is the risk free rate r. An equivalent martingale result is normally distributed such that $\beta_T = e^{\int_0^T rdt}$. This implies that by setting $\beta_0 = 1$, the security α_0 can be valued as

$$\alpha_0 = \beta_0 E^Q\left[\frac{\alpha_T}{\beta_T}\right] = E^Q\left(e^{-\int_0^T rdt} f_T\right) = E^Q(e^{-\bar{r}T} f_T)$$

Here, Q is the equivalent martingale measure and $\bar{r}$ is the average risk free rate from time 0 to time T. The crucial point from this proof is that in a risk neutral state, where there is a martingale process, the expected return and discount factor on every security will be the risk free rate. Forward, we are referring to the Q-measures that are based on the theory of martingales.

Black-Scholes-Merton Model

As briefly summarized in the literature section, the Merton corporate debt model stems from Black-Scholes-Merton option pricing model (herafter: BSM-model). The BSM-model assume that its underlying follows an Itô process, with similar structure as Equation 2. The model assumes access to risk-free investments, with the following process

$$dB = rBdt$$

where r is the risk free interest rate, dB is the change in price of the risk free security of time dt. Note how the uncertainty, reflected through the Wiener process dz, is non-existent in the process.

Recall that in a risk-neutral world, the expectation from every security, both risky and risk-free, will be r. Therefore, as in Equation 13,

$$dS = rSdt + \sigma Sdz$$

Recall that S is assumed to be lognormally distributed, and that every function based upon an Itô process will have the properties of Equation 2. If Π is a function of S and time, then the following relationship can be expressed from time 0 to time T as

$$\frac{\Pi_{0,\mathrm{S}}}{B_0} = E^Q[h(\frac{S_T}{B_T})]$$

E reflects the risk neutral expectations with a Q-martingale measure, where B is the *numeraire*. The present value of a future risk-free investment can be written as

$$B_0 = B_T e^{-rT}$$

such that

$$\Pi_{0,\mathrm{S}} = E^Q[h(S_T)]e^{-rT}$$

By having equation x and equation x in mind we can write

$$Z_T - Z_0 = \sum_{i=1}^{N} \epsilon_i * \sqrt{dt} = Y\sqrt{T}$$

where $Y = \sum_{i=1}^{N} \epsilon_i$ and $Y \sim N(0,T)$. This implies that

$$G = h(S_T)$$

$$h(S_T) = h\left(s_0 e^{\left(r-\frac{\sigma^2}{2}\right)T+\sigma Z}\right)$$

$$h\left(s_0 e^{\left(r-\frac{\sigma^2}{2}\right)T+\sigma Z}\right) = h\left(s_0 e^{\hat{r}T+\sigma Y\sqrt{T}}\right)$$

where $\hat{r} = \left(r - \frac{\sigma^2}{2}\right)$. Every term in the equation above is assumed to be constant, except Y is assumed to be constant. Therefore, G can be interpreted as a function of Y: $\xi(Y)$

$$\xi(Y) = \xi(s_0 e^{\hat{r}T+\sigma Y\sqrt{T}})$$

Since Y is standard normally distributed, equation x can be written as

$$E^Q[h(S_T)] = \int_{-\infty}^{+\infty} \xi(Y)dQ(Y) = \int_{-\infty}^{+\infty} \xi(y)f(y)dy$$

Equation 15

where

$$f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}$$

Equation 16

It is from here that the option pricing becomes specific. If we assume that ξ(y) is a European call option on the underlying S, where the strike price is K

$$\xi(y) = \max(S_T - K, 0)$$

where, $\xi(y) = 0 \; when \; y \leq \frac{\ln\left(\frac{K}{S_T}\right) - \hat{r}T}{\sigma\sqrt{T}}$[8]

The critical point where $\xi(y)$ turns to zero can be denoted as

---

[8] $s_0 e^{\hat{r}T+\sigma Y\sqrt{T}} - K = \ln(S_0) + \hat{r}T + \sigma Y\sqrt{T} - \ln(K)$

$$y^* = \frac{\ln\left(\frac{K}{S_T}\right) - \hat{r}T}{\sigma\sqrt{T}}$$

Then, Equation 15 can be written as

$$E^Q[h(S_T)] = \int_{-\infty}^{y^*} \xi(Y)f(y)dy + \int_{y^*}^{+\infty} \xi(Y)f(y)dy = \int_{y^*}^{+\infty} \xi(Y)f(y)dy$$

By some algebraic manipulations this can be written as

$$E^Q[h(S_T)] = S_T e^{rT} \int_{y^*}^{+\infty} e^{-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}} f(y)dy - K\int_{y^*}^{+\infty} f(y)dy$$

Equation 17

The second integral in Equation 17 can be rewritten due to symmetry in a standard normal variable, such that

$$\int_{y^*}^{+\infty} f(y)dy = \int_{-\infty}^{-y^*} f(y)dy = Q(Y \leq -y^*) = N(-y^*)$$

where $-y^* = \frac{\ln\left(\frac{S_T}{K}\right) + \hat{r}T}{\sigma\sqrt{T}}$

By substituting equation Equation 16 into Equation 17, the first integral can be written as

$$\int_{y^*}^{+\infty} e^{-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}} f(y)dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-y^*} e^{-\frac{1}{2}\left(y - \sigma\sqrt{T}\right)^2} = Q\left(Y - \sigma\sqrt{T} \leq -y^*\right)$$

$$= Q\left(Y \leq -y^* + \sigma\sqrt{T}\right) = N(-y^* + \sigma\sqrt{T})$$

In the BSM-model: $N(-y^*) = N(d_2)$ and $N\left(-y^* + \sigma\sqrt{T}\right) = N(d_1)$

To conclude the BSM-meodel, when $\xi(y)$ has the boundaries of the a european call option, we have that

$$\Pi_{0,S} = E^Q[h(S_T)]e^{-rT} = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

Equation 18

Merton Corporate Debt model

In a very simple case, where a company's assets are financed through equity and zero-coupon bonds, the payoff structure for each type of investment may be described as

$$E = \max(V - F, 0)$$
$$D = PV(F) - \max(F - V, 0)$$

Here, E represents equity value, D represents the debt value, $PV(F)$ is the present value of the face value and V is the market value of the assets. In other words, equity value can be seen as a call option on the assets at a strike price equal F and debt value can be interpreted as its present value less the put price on the assets at the same strike price. It is assumed that the assets will follow an Itô process as described in Equation 2. In a risk neutral state, asset values are assumed to have the process

$$dV = rVdt + \sigma V dz$$

Such that from the BSM-model Equation 18, the equity value as a call option may be expressed as

$$E = VN(d_1) - Fe^{-rT}N(d_2)$$

Equation 19

where, $d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(r + \frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}$ and $d_2 = d_1 - \sigma_V\sqrt{T}$

The catch here is that the market value and the volatility of the assets are unobservable. However, by observing equity prices of publicly traded companies, it is possible to calculate V and σ_V applying Itô's Lemma on Equation 19, so that

$$\sigma_E E = N(d_1)\sigma_E V$$

Equation 20

Together Equation 19 and Equation 20 make a set of equations with two unknowns, V and σ_V, and thus the market value of corporate debt may be calculated

$$D = V - E$$

Equation 21

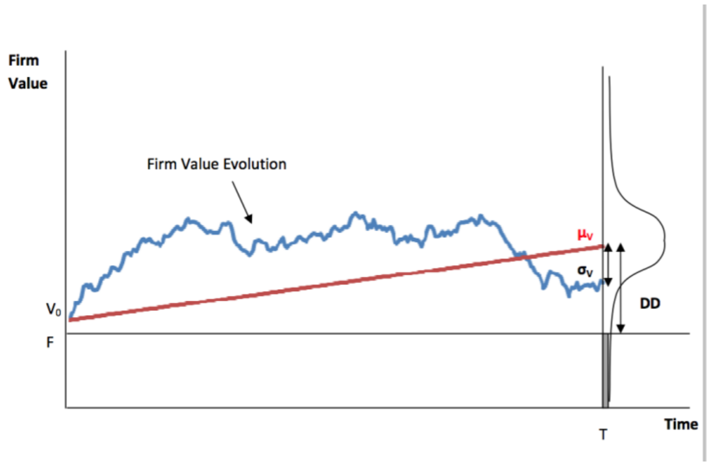

Figure 6 - Graphical illustration of Merton's corporate debt model

Figure 6 summarizes the MCD model from time 0 to time T. The blue line reflects how a hypothetical firm's assets may evolve over time, while the red line reflects the expected increase in the asset value as time evolves. Realistically, the actual asset movement will not move linearly as the market expects. The difference between the red and the blue line is the volatility of the asset. In the figure,

$$\mathrm{DD} = \mathrm{d}_2 = d_1 - \sigma_V\sqrt{T} = \frac{\ln\left(\frac{V_0}{F}\right) + \left(r - \frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}$$

DD in Figure 6 is the «distance to default» that gives the distance between the expected value of the firm's assets and the set point of default. It is standardized by dividing the distance by the estimated volatility of the assets from time 0 to T. The model assumes that the asset values are normally distributed around its mean. From Equation 21, the market value of debt can now be expressed as

$$D = De^{-rT} - (Fe^{-rT}N(-d_2) - V_o N(-d_1))$$

where

$$N(-d_i) = 1 - N(d_i) for\ i = 1{,}2$$

$N(-d_2)$, the grey area below F in Figure 6, is the risk neutral probability of default. Merton's option pricing theory assumes that every sort of risk is reflected into market price of the assets. This implies that the risk neutral probabilities will apply to the real world (Hull, Options, Futures and Other Derivatives 2015).

Extensions to Merton's corporate debt model

While having strong theoretical basis, MCD model does meet multiple challenges when applied in practice. Changes in interest rates and asset prices or coupon obligations throughout time are all aspects of corporate debt that require modifications to the model. In our thesis, we will investigate and attempt to apply three particular extensions to the model:

- The extension of modeling stochastic interest rates
- The extension of modeling jumps in asset prices
- The extension of modeling discrete coupon payments

The rather portentous proofs we have presented above is all the works of Kiyosi Itô, Robert Brown, Myron Scholes, Fischer Black and Robert Merton. The proofs themselves bear little relevance for this thesis, yet the assumptions they carry does affect the applicability of our methodology. Specifically, in order to use extensions of Merton's model, it is practical to refer to which parts of the theoretical basis are applied.

Part IV – Methodology

Existing literature on methods to assess credit risk has indeed flourished throughout the past two decades, and structural and reduced form models are central in the field of research. In this study, we attempt to construct a default prediction model by combining structural and reduced form models; a so called hybrid model. The goal is to build a hybrid model that outperforms both Merton's initial model as well as acknowledged reduced form models in predicting defaults amongst Norwegian HY companies. To our knowledge, such a study has not yet been conducted for the Norwegian market. Given the particular credit risk present in the Norwegian petroleum industry, an up-to-date default prediction model with respectable accuracy would be valuable.

Regression model

The complexity of credit risk is non-linear, and thus standard linear regression models will not suffice as tools to model default probabilities (Saunders and Allen 2002). This causes linear models such as Altman's Z-score to be less reliable. A more suitable approach to modeling credit risk is by a logistic regression model, or simply, the logit model (Woolridge 2009). This is a type of binary response model with the form:

$$P(y = 1|\boldsymbol{x}) = G(\beta_0 + \boldsymbol{x\beta})$$

Here $\boldsymbol{x}$ is a set of independent explanatory variables and $\boldsymbol{\beta}$ is the coefficients that best fit the condition

$$y\begin{cases} 1, \text{if } \beta_0 + \boldsymbol{x\beta} + \varepsilon > 0 \\ 0, \text{otherwise} \end{cases}$$

and ε is the residual which is assumed to be independent from $\boldsymbol{x}$ with symmetrical distribution of zero. In the logit model, the cumulative distribution function (CDF) G for standard logic random variables has the function:

$$G(z) = \frac{1}{1 + e^{-z}}$$

Equation 22

Here, $0 < G(z) < 1$ for $z = \mathbb{R}$. As $z \to \infty$ then $G(z) \to 1$ and as $z \to -\infty$ then $G(z) \to 0$. Further, $P(y_i = 1|\boldsymbol{x})$ may be interpreted as the probability of an observed company (the dependent variable) defaults, taking the value of 1. The suggested model should include variables such as Merton's implied probability of default $N(-d_2)$, leverage financials, interest coverage financials, operational financials, and macro economic variables.

Due to the model's non-linearity, it cannot be estimated using Ordinary Least Squares (OLS). A more appropriate method to estimate the parameters is maximum likelihood (ML). Another consequence of the non-linearity of the logit model is stricter rules of interpreting the estimated coefficients. While one is still able to determine the direction of a variable's impact by observing the estimated coefficient, exact determination of each of the coefficients' impact is only observable through derivation of $G(z)$ with respect to each variable.

Measuring model performance

We will proceed by comparing the estimated probability of default of the population with the actual defaults. Estimates of the models can be logged by categorizing them into four groups;

- true positive (TP) – Predicting high default rate in actual default event
- true negative (TN) – Predicting low default rate in actual non-default event
- false positive (FP) – Predicting high default rate in actual non-default event
- false negative (FN) – Predicting low default rate in actual default event

Figure 6 presents each error's consequences bondholders.

		Reality	
		Default event (Positive)	Non-default event (Negative)
Model prediction	High default rate	Correct assessment (True positive)	Premature selling of bonds at suboptimal prices, opportunity costs and transaction fees (False positive)
	Low default rate	Lost principle due to default, costs of recovery (False negative)	Correct assessment (True negative)

Figure 7 - Bondholders consequences of model prediction

The above categorization of estimates is crucial to our selected method of studying model precision. Inspired by Moody's Investors Service (Sobehart og Stein, Rating methodology - Moody's Public Firm Risk Model: A Hybrid Approach To Modeling Short Term Default Risk 2000) and Fitch Solutions (Fitch Solutions 2007), we aim to study each model's accuracy by applying two particular performance measurements: Cumulative Accuracy Profile (CAP) and Accuracy Ratio (AR).

Cumulative Accuracy Profile

The first performance measurement, CAP, is a visual tool which offers qualitative assessment of performance by plotting different models' precision and comparing it with a perfect model and one based on random guesses. Figure 8 illustrates the accuracy of a random model, an ideal model and an estimated model.

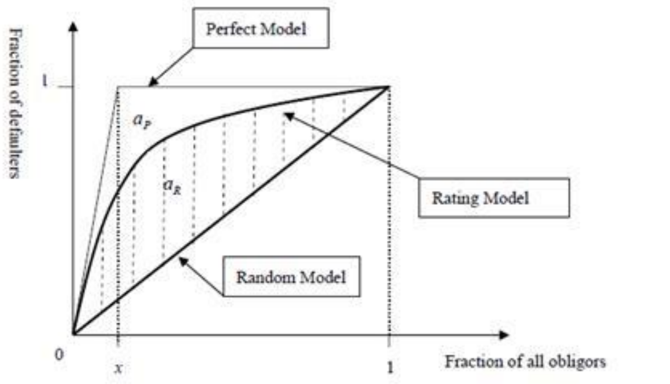

Figure 8 - Hypothetical CAP curves for perfect, random and estimated model.
Source: Quantitative Finance Research Centre, UTS (2006)

$$Fraction\ of\ defaulters = \frac{tp}{tp + fn}$$

Equation 23

$$Fration\ of\ obligors = \frac{tp + fp}{N}$$

$$x = \frac{tp + fn}{N}$$

The fraction of defaulters can be interpreted as the amount of correctly predicted defaulters as a fraction of the total amount of actual defaulters. *Fraction of obligors* can be interpreted as total amount of default predictions as a fraction of the sample size. X is the actual amount of defaulters as a fraction of the sample size. Moreover;

- The perfect model (PM) represent how precise the estimated model could possible be. It sorts out defaulters and non-defaulters with perfect accuracy. In the figure, X would mark the changing point where the observations turn from defaulters to non-defaulters, from left to right.
- The random model (RM) is the worst kind of model one could possible estimate. It can be interpreted as a model that randomly picks defaulters and non-defaulters with equal probability.
- The estimated model (EM) represents the performance of the estimated model.

Accuracy Ratio

In quantifying predictive power, one application of Figure 8 is the measurement of accuracy ratio (AR). This figure is a linear transformation that is based on integrals of integrals' CAP curves. Given the denotations in the figure, AR can be expressed as

$$AR = \frac{\alpha_R}{\alpha_P}$$

As such, the closer a model's AR is to 1, the higher its predictive power.

Recall how the logistic regression $G(z)$ from Equation 22 provides an output between 0 and 1. At which point from 0 to 1 is it appropriate to assume that a company is likely to default? This threshold is up to the researchers to define, and is referred to as *the cut-off*, C. It has a major impact on the estimated fraction of defaulters (Equation 23) and thus how well the model performs. By assuming that rating systems produces continuous rating scores it is possible find AR as a function of C (Engelmann, Hayden og Tasche 2003).

From a randomly perspective, it is possible to draw in three different ways:

- Draw randomly from the total sample of obligors N, denoted as S_T
- Randomly drawn from the total sample of defaulters N_D, denoted as S_D

- Drawn randomly from the total sample of non-defaulters N_{ND}, denoted as S_{ND}.

In this case, it is therefore possible to find α_R from the the cumulative distribution function

$$P(S_T < C) = \frac{N_D P(S_D < C) + N_{ND} P(S_{ND} < C)}{N_D + N_{ND}}$$

since the area under the “worst case model” is 0,5, it is possible to find α_R by

$$\alpha_R = \int_0^1 P(S_D < C) dP(S_T < C) - 0{,}5$$

Thus α_P may be expressed as

$$\alpha_P = \frac{N_{ND}}{2N}$$

To conclude, AR is a tool to quantify the CAP curves in Figure 8. It is an analytical tool that allows the researchers to rank models by their precision with a single statistic; even models that produce an output that is difficult to compare, such as Altman’s Z-score.

Part V - Data

Default data

Our study will primarily be based on data on default events throughout the period of 2007-2015. The data is provided by Nordic Trustee through the group's database *Stamdata*. Independently owned by Nordic banks, insurance companies and security brokers, Nordic Trustee serves as a third party information agent assisting bond investors. Specifically, the group ensures that debt issuers comply with scheduled payments and covenants. In cases of breaches of loan agreements, Nordic Trustee may proceed with legal actions.

The group's subsidiary and primary source of information, *Stamdata*, offers unparalleled information on bonds; including up-to-date figures and loan documents for all structured debt securities issued by government, municipals, banks or corporate borrowers. Amongst Stamdata's products is information on corporate defaults and recovery rates. This product offers an overview over all recorded credit events in the Nordic region; events where companies have filed for bankruptcy, failed to pay instalments, applied for restructuring or relaxations of bank covenants.

The data we currently possess consists of approximately 100 companies which have experienced a credit event in the period of 2007-2015. Of these, 35 companies have been listed and experienced a default where bondholders' face value was compromised. This is not a particularly large sample, which is may be one of the reasons why a similar study has not been conducted on the Norwegian HY marked already. Sobehart, Keenan and Stein (2000) explains that, while out-of-sample and out-of-time test of models' performance would be the optimal way of testing and comparing default prediction models, such amounts of default data is rarely available (Sobehart, Keenan og Stein, Benchmarking Quantitative Default Risk Models 2000). Previous researchers of credit risk with experience with our type of methodology claim that a sample of 50 defaulted companies will suffice in order to obtain general statistical findings (Engelmann, Hayden og Tasche 2003). Hence, amongst the challenges of our study is the pursuit of sufficient data.

Accounting information and share price data

In addition to credit event data on defaulted companies, we will need share price data on both defaulted and non-defaulted companies. This is necessary for the application of the extended MCD model. Furthermore, historical accounting information will be necessary in order to construct financial variables that may be used alone or together with the extended MCD model. Such information is usually available either on Bloomberg, of which we are familiar with, or the companies' financial statements. All computations and modelling will be performed with Excel and R.

Even though our primary source of default information will stem from Stamdata, we will investigate the database of BI Finance Faculty, which contains information on listed companies from 1994 and onwards.

Part VI – Epilogue

We are thankful for having acquired access to data from Danske Bank Markets on credit spreads, bond flows and volumes. Furthermore, we are grateful for Stamdata's contribution of default and recovery statistic, as well as substantial data on the Norwegian bond market in general. Although the data we have acquired so far is of high quality and detrimental value to our research, we fear that the empirical quality of our final findings may suffer from a small sample size. As such, one of the main challenges of our thesis will be to obtain more data on historical defaults of listed companies. Access to the internal database of BI's Finance Faculty may assist us in our search for sufficient data.

Another challenge of our thesis is the amount of assumptions of which the MCD model is based upon. While extensions to the model will help, the practical application of the model may be thwarted by unrealistic assumptions.

We expect that the learning outcome of our Master's Thesis is significant, and hopefully a respectable conclusion to the education we have attained at BI Norwegian Business School.

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