FOREIGN SHOCKS IN AN ESTIMATED MULTI-SECTOR MODEL

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Abstract

How are macroeconomic fluctuations in open economies affected by international business cycles? To shed some light on this question, I develop and estimate a medium scale DSGE model for a small open economy. The model incorporates i) international markets for firm-to-firm trade in production inputs, and ii) producer heterogeneity where technology and price setting constraints vary across industries. Using Bayesian techniques on Canadian and US data, I document several macroeconomic regularities in the small open economy, all attributed to international disturbances. First, foreign shocks are crucial for domestic fluctuations at all forecasting horizons. Second, productivity is the most important driver of business cycles. Investment efficiency shocks on the other hand have counterfactual implications for international spillover. Third, the relevance of foreign shocks accumulates over time. Fourth, business cycles display strong co-movement across countries, even though shocks are uncorrelated and the trade balance is countercyclical. Fifth, exchange rate pass-through to aggregate CPI inflation is moderate, while pass-through at the sector level is positively linked to the frequency of price changes. Few of these features have been accounted for in existing open economy DSGE literature, but all are consistent with reduced form evidence. The model presented here offers a structural interpretation of the results.

Keywords: *DSGE, small open economy, international business cycles, Bayesian estimation.*
JEL Classification: *C11, F41, F44.*

1 INTRODUCTION

How and to what extent do international shocks and business cycles shape macroeconomic fluctuations in open economies? These questions are fundamental in open economy macroeconomics, and of first order importance for welfare evaluation and policy making. Still, our understanding of relevant transmission channels at play, as well the sign and magnitude of spillover, is limited. The contribution of this paper is a general equilibrium model which is quantitatively consistent with the open economy dimension of macroeconomic fluctuations.

Vast empirical literature, using data from different countries and time periods, show that foreign business cycles are central determinants of macroeconomic fluctuations in open economies.[1] DSGE models – a cornerstone in modern macroeconomic theory – have a hard time accounting for this view. Perhaps the most striking example is offered by Justiniano and Preston (2010), who document how an estimated small open economy (SOE) New Keynesian model attributes virtually all business cycle fluctuations to domestic shocks. Across model specifications and estimation approaches, the model suggests that macroeconomic variables in the SOE are almost fully detached from international events. This result is not an exception, but rather the standard finding in estimated SOE-DSGE models.[2] Thus, one might ask how useful these models are for understanding the open economy dimension of data. Another aspect in which DSGE models tend to fail is that of exchange rate pass-through. Typically they generate either very high pass-through from exchange rates to domestic prices, or almost zero (and even negative) pass-through (Gopinath, Itskhoki, and Rigobon, 2010). This is problematic because the question of pass-through is essential for how monetary authorities should respond to exchange rate movements, and because DSGE models have become standard tools for policy making, evaluation, and communication in many central banks.

In this paper I revisit the role of international business cycle disturbances within a multi-sector open economy framework. To this end I develop and estimate a two-country New Keynesian model, and shed light on how macroeconomic fluctuations are determined in SOEs. Key features of the model are i) international markets for firm-to-firm trade in production inputs, and ii) producer heterogeneity where firms operate in segmented markets and face different technological constraints. These extensions to the one-sector DSGE model build on Bouakez, Cardia, and Ruge-Murcia (2009) and Bergholt and Sveen (2014), and create sectoral trade interdependence both within and across economies: First, imported intermediates represent a new cost-channel for spillover of foreign shocks. In contrast to existing models, where exchange rates only affect domestic firms indirectly via changes in demand, they also shift supply schedules in the current framework. This direct exchange rate effect on the domestic production frontier is particularly relevant for firms who compete in international markets, even more so if these markets are characterized by frequent and large price changes. Second, intersectoral firm-to-firm linkages induce

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[1] A non-exhaustive list of recent empirical studies that support this view include Kose, Otrok, and Whiteman (2003, 2008), Aastveit, Bjornland, and Thorsrud (2011), Crucini, Kose, and Otrok (2011), Mumtaz, Simonelli, and Surico (2011), and Kose, Otrok, and Prasad (2012).

[2] Recent examples (although they do not necessarily discuss it explicitly) include Adolfson, Laséen, Lindé, and Villani (2007, 2008), de Resende, Dib, and Kichian (2010), Rabanal and Tuesta (2010), Dib (2011), and Christiano, Trabandt, and Walentin (2011). Schmitt-Grohé (1998) demonstrate that also international real business cycle models fail to account for the spillover across countries.

substantial spillover to relatively non-traded industries. For example, when the price of manufactured goods deflate, e.g. due to a terms of trade improvement, the supply of domestic service firms shifts out. This is because manufactured goods are important inputs in service production. It follows that even the supply of completely non-traded firms in general will react to international shocks. These intersectoral firm-to-firm linkages are crucial as most of aggregate GDP is produced by domestic service firms. Third, feedback loops in the domestic production network accelerate initial impulses, resulting in higher order propagation effects.

While Bergholt and Sveen (2014) explain basic mechanisms in a stylized environment, I extend the setup along several dimensions to facilitate a quantitative assessment. In particular I incorporate consumption habits, sticky wages, partial price and wage indexation, capital as input in production, fixed production and variable investment adjustment costs, incomplete international asset markets, and pricing-to-market strategies by exporting firms. The modeling framework allows for an arbitrary number of industries, and nests as a special case the workhorse one-sector SOE-DSGE model (see e.g. Adolfson et al. (2007)). From a Bayesian perspective, it is therefore straight forward to make a formal evaluation of firm-to-firm trade and sector heterogeneity, and whether these features are favored by data. During estimation I make explicit distinctions between the production of raw materials (commodities), manufactured goods, and services. Input-output (I-O) data reveal substantial asymmetry between these sectors in terms of i) export and import intensity, and ii) intersectoral trade linkages. I estimate structural parameters using Bayesian techniques on 9 Canadian and 8 US time series, but restrict them to fit I-O data in both countries. I then conduct a broad evaluation of the open economy dimension of macroeconomic fluctuations in Canada (the SOE). Several important results emerge from this exercise:

First, as in wide empirical literature, foreign shocks account for substantial variation in macroeconomic variables at all forecasting horizons. Within the business cycle, they are responsible for 30-70% of the volatility in domestic GDP, consumption, investments, hours, wages, inflation, the interest rate, and the trade balance. Thus, when confronted with data, the DSGE theory presented here proposes that international disturbances play a crucial role for domestic business cycle fluctuations in Canada. This is a first, but critical pass for analyzing spillover from international markets to the SOE.

Second, while a cocktail of disturbances is responsible for macroeconomic fluctuations in the very short run, total factor productivity stands out as the most prominent type of shock over the business cycle. In the long run, domestic and foreign productivity shocks explain about 75-80% of aggregate volatility in GDP and wages, 70% of consumption volatility, and about half of the movements in inflation and interest rates. This contrasts the major role of investment efficiency shocks found in recently estimated DSGE models.[3] I show that these shocks have counterfactual implications for international synchronization patterns, implying that the likelihood based estimation procedure downplays their role when open economy data are used.

Third, in a forecasting perspective the role of foreign shocks tends to build up over time. For instance, while 22% of the one step ahead forecast error in GDP is attributed to foreign shocks, they are responsible for almost 50% at the year-on-year horizon, and 75%

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[3]See Justiniano, Primiceri, and Tambalotti (2010), Justiniano, Primiceri, and Tambalotti (2011) and Jacob and Peersman (2013).

in the long run. These numbers are well in line with VAR evidence, see e.g. Cushman and Zha (1997) and Justiniano and Preston (2010). The main reason, according to the posterior estimates, is that productivity shocks at the sector level are relatively persistent events. Since productivity is the most important foreign disturbance, the model assigns substantial domestic fluctuations to foreign shocks at longer forecasting horizons.

Fourth, estimated business cycles display strong co-movement across countries, even though none of the shocks are correlated and the trade balance is countercyclical. For instance, the contemporaneous correlation between US and Canadian GDP is about 0.82, as in the data. Importantly, high co-movement does not follow from large foreign variance shares. For instance, in a recent paper Christiano et al. (2011) define markup shocks in both import and export prices as foreign, even though these shocks only affect domestic variables. This "re-interpretation" obviously increases the role of foreign shocks, but does not help in explaining co-movement. In contrast, aggregate and disaggregate co-movement in my model comes about endogenously, due to intermediate goods trade between heterogeneous firms. However, as a result of real interest rate synchronization, the model also predicts too high correlation between consumption across countries.

Fifth, the pass-through from exchange rates to aggregate CPI inflation in the model is moderate, about 12%, and within the range of reduced form estimates by e.g. Gopinath et al. (2010) and Gopinath and Itskhoki (2010) for US-Canadian data. Typically, models with local currency pricing, a modeling choice used in this paper, predict too little pass-through. This is not the case here, because exchange rate fluctuations affect relatively non-traded firms via domestic supply chains. The empirical analysis by Goldberg and Campa (2010) suggest that these are the dominant channels for pass-through. My results also confirm Gopinath and Itskhoki (2010), who show that goods with frequent price adjustments have higher pass-through than those with relatively sticky prices.

This paper contributes to existing literature along different dimensions. First I provide an open economy model within the Smets and Wouters (2003, 2007) tradition that takes supply side heterogeneity and firm-to-firm trade explicitly into account. In doing so, I link open economy DSGE theory to literature on the interplay between inter-sectoral networks and macroeconomic volatility. Recent contributions are Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Carvalho and Gabaix (2013). They emphasize how disaggregate shocks can lead to interesting cascade effects across industries, and eventually to aggregate fluctuations. Here, I argue that similar mechanisms apply in an open economy setting, except that cascade effects also take place across countries. Second I demonstrate, using Bayesian techniques, that the estimated model actually predicts substantial spillover across countries. International business cycles in particular become key for understanding fluctuations in domestic variables. Thus, the model presented here allows us to gain understanding – within the DSGE framework – about foreign disturbances, their nature and transmission. I offer several important results along these lines. Third I argue that some shocks – which explain data well in closed economy DSGE models – have counterfactual business cycle implications in an open economy setting. By doing so, I speak to recent literature on macroeconomic shocks in estimated closed economy models (see e.g. Justiniano et al. (2010) and Christiano, Motto, and Rostagno (2014)).

The rest of the paper is organized as follows. A multi-sector SOE model is described in section 2. Section 3 presents data, calibration choices and Bayesian parameter estimates. Main empirical results are reported in section 4. In section 5 I discuss how these

results are facilitated by important transmission channels in the model. Section 6 presents results from the counterfactual model economy without intermediate trade and sector heterogeneity. Section 7 concludes.

2 THE MODEL

I establish a general equilibrium system consisting of two blocks (referred to as home and foreign), where the home block is a small scale version of its foreign counterpart. The foreign block is thought of as the rest of the world. The framework allows the relative size of the home economy to be arbitrarily large compared to the rest of the world. However, my focus is on the limiting case where the home economy has negligible influence on the world economy. General equilibrium is therefore analyzed for this special case.[4] The full non-linear model as well as a graphical overview is described in Appendix A. Here I present the log-linearized system (a first order perturbation around the zero-inflation steady state). To save space, I focus on the domestic block below.

2.1 HOUSEHOLDS

Consider a small open economy (labeled the home economy) with a measure one of symmetric households. The representative household consists of a continuum of members indexed by $h \in (0,1)$. A fixed share of the household members is working in each production sector $j \in [1, \ldots, \mathcal{J}]$ in the domestic economy. Household members consume, work and invest in order to maximize expected lifetime utility. The maximization problem is subject to a sequence of budget constraints, with revenues coming from returns on capital, a portfolio of Arrow securities, labor income, dividends from ownership of firms, returns on domestic and foreign bonds, and government transfers.[5] Optimality conditions for the representative household with respect to consumption, domestic and foreign bond holdings, capital and investment, follow below:[6]

$$\lambda_t = z_{U,t} - \frac{\sigma}{1 - \chi_C} \left(c_t - \chi_C c_{t-1} \right) \tag{1}$$

$$\lambda_t = \mathbb{E}_t \left(\lambda_{t+1} \right) + r_t - \mathbb{E}_t \left(\pi_{t+1} \right) \tag{2}$$

$$\lambda_t = \mathbb{E}_t \left(\lambda_{t+1} \right) + r_t^* - \mathbb{E}_t \left(\pi_{t+1} + \Delta e_{t+1} \right) - \epsilon_B a_t + z_{B,t} \tag{3}$$

$$q_t = -r_t + \mathbb{E}_t \left(\pi_{t+1} + \left[1 - \beta \left(1 - \delta \right) \right] r_{t+1}^k + \beta \left(1 - \delta \right) q_{t+1} \right) \tag{4}$$

$$i_t = \frac{\beta}{1 + \beta} \mathbb{E}_t \left(i_{t+1} \right) + \frac{1}{1 + \beta} i_{t-1} + \frac{1}{\epsilon_I \left(1 + \beta \right)} \left(q_t + z_{I,t} - p_{r,t}^i \right) \tag{5}$$

The first equation aligns the shadow value of the budget constraint in period t, λ_t, with the marginal utility of aggregate consumption c_t. $\sigma > 0$ and $\chi_C \in [0,1]$ govern the intertemporal elasticity of substitution and habit persistence in consumption, respectively.

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[4]See the appendix for the general setup with two arbitrarily large economies.

[5]A complete set of tradable Arrow securities within each economy renders household member h consumption equal to aggregate consumption. I therefore drop the h-subscript whenever possible.

[6]Throughout I denote variables in percentage deviations from the non-stochastic steady state. Prices are quoted in terms of consumption units. For instance, $p_{r,t}^i = \ln \left(\frac{P_t^i}{P_t} \right)$, where P_t^i is the price on aggregate investment goods and P_t is the consumer price index (CPI).

$z_{U,t}$ is a stationary shock to intertemporal preferences. Equations (2) and (3) equate the marginal utility of more consumption today with the expected present value of more future consumption, obtained by investing in domestic and foreign bonds. π_t and Δe_t are the CPI inflation rate and the nominal depreciation rate, respectively. Nominal interest rates on domestic and foreign bonds are denoted r_t and r_t^*, while a_t is the ratio of domestically held foreign bonds to steady state GDP.[7] $\epsilon_B > 0$ introduces a risk premium on foreign bonds, as in Adolfson et al. (2007, 2008) and Christiano et al. (2011). If domestic households are net borrowers, they are charged a premium on bond returns. If they are net lenders, they receive a lower return than foreign households.[8] $z_{B,t}$ denotes temporary deviations from interest rate parity, so-called risk premium shocks. The present value of one more unit of new capital, q_t, is characterized by equation (4). $r_t - E_t(\pi_{t+1})$ is the expected real return (real interest rate) foregone by not investing in bonds, while r_t^k is the rental rate on operational (existing) capital. The parameters $\beta \in (0,1)$ and $\delta \in [0,1]$ denote the time discount factor and the capital depreciation rate, respectively. Finally, equation (5) determines the optimal demand for aggregate investment goods. It effectively equates the relative price on investments $p_{r,t}^i$ with the gain of investments – the present value of capital plus the reduction in investment adjustment costs. The latter is governed by $\epsilon_I \geq 0$, as in Christiano, Eichenbaum, and Evans (2005). $z_{I,t}$ is a stationary shock to the marginal efficiency of investment, a so-called MEI shock. The optimality conditions (1)-(5) summarize intertemporal decisions for the representative household. They are augmented with a capital accumulation equation of the form

$$k_{t+1} = (1 - \delta) k_t + \delta (z_{I,t} + i_t), \tag{6}$$

where k_t is capital operational in period t.

Next I turn to sectoral allocations. c_t and i_t are composite functions of sectoral consumption and investment goods, denoted $c_{j,t}$ and $i_{j,t}$. In turn, these quantities are combinations of goods produced by domestic and foreign firms. Thus, to a first order the aggregate CPI inflation rate π_t, and the aggregate investment goods inflation rate π_t^i, are linear combinations of domestic sector prices $p_{rj,t}$:

$$\pi_t = \sum_{j=1}^{\mathcal{J}} \tilde{\zeta}_j \pi_{j,t}, \qquad \pi_t^i = \sum_{j=1}^{\mathcal{J}} \tilde{\varpi}_j \pi_{j,t}, \qquad p_{rj,t} = \tilde{\alpha}_j p_{rHj,t} + (1 - \tilde{\alpha}_j) p_{rFj,t},$$

$\pi_{j,t}$ represents the inflation rate in sector j,[9] and $p_{rHj,t}$ and $p_{rFj,t}$ are producer prices on domestically supplied and imported goods, respectively. The weights $\tilde{\xi}_j$, $\tilde{\varpi}_j$ and $\tilde{\alpha}_j$ are determined by the steady state solution of the model. Generally international trade takes place in all sectors, but the trade intensity is sector specific. Moreover, the import shares in c_t and i_t depend both on import shares in each sector and on sector weights in aggregate demand baskets. Optimal demand for consumption and investment from sector j can be written as downward sloping functions of the sector price $p_{rj,t}$:

$$c_{j,t} = -\nu_c p_{rj,t} + c_t \tag{7}$$

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[7]a_t is measured in absolute deviations from steady state.

[8]The existence of a risk premium also ensures that steady state is well-defined, see e.g. Schmitt-Grohé and Uribe (2003).

[9]Note that sectoral and aggregate CPI inflation rates are linked by the identity $\pi_{j,t} = p_{rj,t} - p_{rj,t-1} + \pi_t$.

$$i_{j,t} = -\nu_i \left(p_{rj,t} - p_{r,t}^i \right) + i_t \tag{8}$$

The parameters $\nu_c, \nu_i > 0$ denote the elasticity of substitution between sectors. The demand functions (7) and (8) show that domestic absorption of sector j goods is relatively high when the price in that sector is relatively low. Optimal demand for domestically produced consumption and investment goods, $c_{Hj,t}$ and $i_{Hj,t}$, and for imported goods $c_{Fj,t}$ and $i_{Fj,t}$, can be written as follows:

$$c_{Hj,t} = -\eta \left(p_{rHj,t} - p_{rj,t} \right) + c_{j,t} \qquad c_{Fj,t} = -\eta \left(p_{rFj,t} - p_{rj,t} \right) + c_{j,t}$$
$$i_{Hj,t} = -\eta \left(p_{rHj,t} - p_{rj,t} \right) + i_{j,t} \qquad i_{Fj,t} = -\eta \left(p_{rFj,t} - p_{rj,t} \right) + i_{j,t}$$

The elasticity of substitution between goods from different countries is denoted $\eta > 0$. Thus, households substitute their demand towards goods from countries with relatively low prices.

Next I turn to labor markets. I construct sectoral labor markets similar to that in Erceg, Henderson, and Levin (2000), but add a friction in the sense that labor cannot move freely between sectors or countries within the business cycle horizon. However, I construct the calibration scheme in such a way that real wages are equalized across sectors in the non-stochastic steady state. This removes any incentive for workers to change sector occupation over time. To fix things, consider the labor market in sector j. A competitive labor bundler buys working hours from all the household members employed in the sector, and combine these hours into an aggregate labor service. This aggregate is then sold to all the firms in sector j. Demand for each labor variety is chosen in order to maximize profits for the bundler, and is inversely related to the individual wage rate. Each period, only a fraction $1 - \theta_{wj}$ of the household members working in sector j re-optimize wages. The remaining workers index wages partially to lagged CPI inflation. The indexation parameter is denoted $\gamma_w \in [0, 1]$. A household member who is able to reset the wage in period t, will choose the optimal wage in order to maximize lifetime utility subject to the labor demand schedule coming from the labor bundler, and the Calvo constraint on wage setting. Using the relevant first order condition, one can do a series of manipulations to obtain a modified New Keynesian wage Phillips curve of the form

$$\pi_{wj,t} = \beta \mathbb{E}_t \left(\pi_{wj,t+1} \right) + \gamma_w \left(\pi_{t-1} - \beta \pi_t \right) + \kappa_{wj} \left(mrs_{j,t} - \omega_{j,t} \right). \tag{9}$$

$\pi_{wj,t}$ and $\omega_{j,t}$ denote nominal wage inflation and the real wage level in sector j. $\kappa_{wj} = \frac{(1-\theta_{wj})(1-\beta\theta_{wj})}{\theta_{wj}\left(1+\frac{1+\epsilon_w}{\epsilon_w}\varphi\right)}$, where φ determines the Frisch labor supply elasticity and ϵ_w the steady state markup over competitive wages. The marginal rate of substitution is defined as

$$mrs_{j,t} = z_{U,t} + z_{N,t} + \varphi n_{j,t} - \lambda_t, \tag{10}$$

where $z_{N,t}$ is referred to as a labor supply shock.

2.2 FIRMS

There is a continuum of profit maximizing firms in each domestic sector j, indexed by $f \in [0, 1]$. These firms use materials, labor and capital to make differentiated consumption, investment and intermediate goods, which are then supplied in domestic and foreign

markets. I set up the calibration scheme to ensure zero profit in steady state. This is consistent with a steady state free entry condition for new firms, and also prevents arbitrage opportunities of changing sectoral occupation in the long run. Output of domestic firm f in sector j is given by a Cobb-Douglas production function augmented with fixed costs. To a first order, aggregate output in domestic sector j can be written as

$$y_{j,t} = (1 + \epsilon_p) \left[z_{Aj,t} + \phi_j m_{j,t} + \psi_j n_{j,t} + (1 - \phi_j - \psi_j) k_{j,t} \right], \tag{11}$$

where $\phi_j, \psi_j, (\phi_j + \psi_j) \in (0, 1)$, and $z_{Aj,t}$ is a stationary and sector specific productivity shock.

A defining feature of the model is the presence of segmented markets for trade between firms. I follow Bouakez et al. (2009) and Bergholt and Sveen (2014), and let firms in sector j buy a composite $m_{j,t}$ of different materials produced in the different sectors. In principle, a domestic firm f in sector j needs intermediate inputs from *all* firms in *all* industries in *all* countries. Bergholt and Sveen (2014) show how this setup amplifies the interdependencies between sectors, and therefore increases the potential role for international shocks in otherwise closed sectors such as the service industry. In particular, optimal sector j demand for materials from sector l can be written

$$m_{lj,t} = -\nu_m \left(p_{rl,t} - p_{rj,t}^m \right) + m_{j,t}, \tag{12}$$

where $p_{rj,t}^m$ is the composite material price index in sector j, and $\nu_m > 0$ is the elasticity of substitution between inputs from different sectors. It is clear from equation (12) that demand for intermediate input from sector l depends on the spread between market prices in that sector and the composite price $p_{rj,t}^m$. This spread can display large fluctuations when $p_{rl,t}$ is volatile. The nominal materials price inflation in each sector writes as

$$\pi_{j,t}^m = \sum_{l=1}^{\mathcal{J}} \tilde{\zeta}_{lj} \pi_{l,t},$$

where the weights $\tilde{\zeta}_{lj}$ are determined by the steady state solution of the model. Importantly, these weights can be found from I-O matrices in each country. Optimal demand for domestically produced intermediate goods from sector l, $m_{Hlj,t}$, and for imported intermediate goods, $m_{Flj,t}$, can be written as follows:

$$m_{Hlj,t} = -\eta \left(p_{rHl,t} - p_{rl,t} \right) + m_{lj,t} \qquad m_{Flj,t} = -\eta \left(p_{rFl,t} - p_{rl,t} \right) + m_{lj,t}$$

Thus, firms substitute their demand for intermediate inputs towards countries with relatively low prices.

Price setting by domestic and foreign firms is subject to monopoly supply power and sticky prices in a way analogous to the labor market. In particular, firms set prices á la Calvo (1983) and Yun (1996). I depart from the popular assumption of producer currency pricing (PCP) (see e.g. Galí and Monacelli (2005) and Monacelli (2003)), and instead assume that firms set prices in the buyer's currency. This is typically referred to as local currency pricing (LCP). Importantly, LCP leads optimizing firms to price discriminate between markets. There are several reasons for my modeling choice. First, Gopinath et al. (2010) report that only 4% of Canadian exports to the US is priced in Canadian dollars. Second, PCP in its standard form leads to full purchasing power parity in all periods,

a phenomenon strongly rejected by data. Third, PCP implies perfect pass-through of exchange rates into domestic prices, at odds with the empirical pass-through literature.[10] Denote domestic producer prices in sector j by $p_{rHj,t}$ and $p^*_{rHj,t}$ respectively, where the first is on goods sold at home and the second on exported goods. Let $1 - \theta_{pj}$ denote the probability that a given producer is able to reset his prices. The fraction θ_{pj} of firms that is not able to re-optimize prices can index them partially to lagged producer prices. The degree of indexation is denoted $\gamma_p \in [0, 1]$. Optimality conditions with respect to inputs can be summarized in sector j by equations (13)-(14):

$$m_{j,t} - n_{j,t} = \omega_{j,t} - p^m_{rj,t} \tag{13}$$

$$k_{j,t} - m_{j,t} = p^m_{rj,t} - r^k_t \tag{14}$$

Thus, firms demand more intermediate inputs when these are cheap relative to labor and capital. The optimality conditions with respect to domestic producer prices can be used to obtain two New Keynesian Phillips curves for domestic and export prices, respectively:

$$\pi_{Hj,t} = \kappa_1 \mathbb{E}_t \left(\pi_{Hj,t+1} \right) + \kappa_2 \pi_{Hj,t-1} + \kappa_{j3} \left(rmc_{j,t} - p_{rHj,t} + z_{M,t} \right) \tag{15}$$

$$\pi^*_{Hj,t} = \kappa_1 \mathbb{E}_t \left(\pi^*_{Hj,t+1} \right) + \kappa_2 \pi^*_{Hj,t-1} + \kappa_{j3} \left(rmc_{j,t} - p^*_{rHj,t} + z_{M,t} \right) \tag{16}$$

$\pi^*_{Hj,t}$ is here the foreign currency price on export goods, while $z_{M,t}$ is referred to as a markup shock. The slope coefficients are defined as $\kappa_1 = \frac{\beta}{1+\beta\gamma_p}$, $\kappa_2 = \frac{\gamma_p}{1+\beta\gamma_p}$, and $\kappa_{j3} = \frac{(1-\theta_{pj})(1-\beta\theta_{pj})}{\theta_{pj}(1+\beta\gamma_p)}$. Real marginal costs in sector j, $rmc_{j,t}$, can be written as

$$rmc_{j,t} = -z_{Aj,t} + \phi_j p^m_{rj,t} + \psi_j \omega_{j,t} + (1 - \phi_j - \psi_j) r^k_t. \tag{17}$$

Thus, whenever prices on inputs go up, this stimulates higher producer price inflation in the domestic economy. Note for future reference that sector level terms of trade is defined as the domestic currency export-to-import price ratio, i.e. $\tau_{j,t} = p^F_{rHj,t} - p_{rFj,t}$. This completes the description of firms. Next I turn to general equilibrium and aggregation.

2.3 DOMESTIC ABSORPTION AND GDP

Here I consider the special case of the model where trade between the world economy and the SOE is neglible from the world economy's point of view. Define $x_{Hj,t}$ as total domestic absorption of domestically produced output from sector j, and $x_{Fj,t}$ as total domestic absorption of imported sector j output. These two can then be written as

$$x_{Hj,t} = -\eta \left(p_{rHj,t} - p_{rj,t} \right) + x_{j,t} \tag{18}$$

$$x_{Fj,t} = -\eta \left(p_{rFj,t} - p_{rj,t} \right) + x_{j,t}, \tag{19}$$

where aggregate domestic absorption in sector j is

$$x_{j,t} = C_{xj}c_{j,t} + I_{xj}i_{j,t} + \sum_{l=1}^{\mathcal{J}} M_{xjl}m_{jl,t} + G_{xj}g_{j,t}. \tag{20}$$

The coefficients C_{xj}, I_{xj}, M_{xjl} and G_{xj} depend on the steady state of the model. Note for future reference that a hybrid New Keynesian Phillips curve for imported sector j goods can be written as

$$\pi_{Fj,t} = \kappa_1^* \mathbb{E}_t \left(\pi_{Fj,t+1} \right) + \kappa_2^* \pi_{Fj,t-1} + \kappa_{j3}^* \left(rmc_{j,t}^* + s_t - p_{rFj,t} + z_{M,t}^* \right), \qquad (21)$$

where $\kappa_1^* = \frac{\beta}{1+\beta\gamma_p^*}$, $\kappa_2^* = \frac{\gamma_p^*}{1+\beta\gamma_p^*}$, and $\kappa_{j3}^* = \frac{\left(1-\theta_{pj}^*\right)\left(1-\beta\theta_{pj}^*\right)}{\theta_{pj}^*\left(1+\beta\gamma_p^*\right)}$. s_t is the real exchange rate between the two countries, i.e. the price of foreign aggregate consumption in terms of domestic consumption. $rmc_{j,t}^*$ represents foreign real marginal costs, and $z_{M,t}^*$ is a foreign markup shock. Similarly to domestic import absorption, one can define $x_{Hj,t}^*$ as the foreign absorption of domestically produced sector j goods:

$$x_{Hj,t}^* = -\eta \left(p_{rHj,t}^* - s_t - p_{rj,t}^* \right) + x_{j,t}^* \qquad (22)$$

$p_{rj,t}^*$ is the international sector j price level, and $x_{j,t}^*$ is world absorption of sector j goods. Market clearing then implies that $y_{j,t} = \alpha_{xj} x_{Hj,t} + \left(1-\alpha_{xj}\right) x_{Hj,t}^*$, where α_{xj} is the steady state share of domestic output that is supplied in domestically. Sector specific GDP is defined according to the expenditure approach, and can be written as

$$gdp_{j,t} = X_{yj} \left(p_{rj,t} + x_{j,t} \right) + tb_{j,t} - M_{yj} \left(p_{rj,t}^m + m_{j,t} \right), \qquad (23)$$

with the trade balance being equal to

$$tb_{j,t} = EX_{yj} \left(p_{rHj,t}^* + x_{Hj,t}^* \right) - IM_{yj} \left(p_{rFj,t} + x_{Fj,t} \right). \qquad (24)$$

The trade balance is not log-linearized, but defined relative to steady state GDP, and in absolute deviation from steady state within each sector. The great ratios X_{yj}, M_{yj}, EX_{yj} and IM_{yj}, depend on the steady state solution of the model. Finally, by aggregating across sectors we can define economy-wide GDP and trade balance as

$$gdp_t = \sum_{j=1}^{\mathcal{J}} \gamma_j^{gdp} gdp_{j,t} \qquad \text{and} \qquad tb_t = \sum_{j=1}^{\mathcal{J}} \gamma_j^{gdp} tb_{j,t}. \qquad (25)$$

The parameter γ_j^{gdp} is here defined as the steady state ratio between sectoral and aggregate GDP. From the foreign economy's point of view, their debt is in zero net supply because the home economy engages in only a negligible part of the financial assets trade. Furthermore, I assume that foreign investors do not hold financial assets in the home economy.

2.4 MONETARY AND FISCAL POLICY

The model is closed with a specification of monetary and fiscal policy. I follow previous work in the DSGE literature (see e.g. Justiniano and Preston (2010); Smets and Wouters (2007); Lubik and Schorfheide (2007)) and assume that monetary policy can be approximated by a Taylor-type rule of the form

$$r_t = \rho_r r_{t-1} + \left(1-\rho_r\right) \left(\rho_\pi \pi_t + \rho_y gdp_t + \rho_{\Delta y} \Delta gdp_t + \rho_e \Delta e_t \right) + z_{R,t}. \qquad (26)$$

ρ_r, ρ_π, ρ_y, $\rho_{\Delta y}$ and ρ_e are policy coefficients, and $z_{R,t}$ is a monetary policy shock.

The government faces a period-by-period budget constraint with Ricardian taxes and newly issued government bonds on the income side, and fiscal spending and bonds that mature in the current period on the expenditure side. Under the assumption that public debt is zero in steady state, one can then write, up to a first order approximation, public spending as fully financed by (possibly time varying) lump-sum taxes.

2.5 Exogenous disturbances

I assume that all exogenous disturbances in the model follow a univariate AR(1) representation in log-linear form:

$$\varsigma_t = \rho_\varsigma \varsigma_{t-1} + \varepsilon_{\varsigma,t}, \qquad \varepsilon_{\varsigma,t} \overset{i.i.d.}{\sim} N\left(0, \sigma_\varsigma\right) \tag{27}$$

$\varsigma_t = \left[z_{U,t}, z_{N,t}, z_{B,t}, z_{I,t}, z_{M,t}, z_{R,t}, z_{A1,t}, \ldots, z_{A\mathcal{J},t}\right]'$ is the vector of exogenous disturbances. ρ_ς and σ_ς are diagonal, and all non-zero elements in ρ_ς are bounded between zero and one. Fluctuations in the foreign economy are subject to a similar set of disturbances, except that foreign risk premium shocks are negligible due to the small economy assumption.

3 Estimation

Sector heterogeneity induces a non-symmetric equilibrium across different industries. I solve for the steady state analytically and use the solution to parameterize the log-linear approximation of the model. The steady state as well as the full block of linear difference equations of the SOE are provided in Appendix B. Several model parameters are estimated using Bayesian techniques. This approach has been popularized by e.g. Geweke (1999), Smets and Wouters (2003, 2007), and An and Schorfheide (2007). Details about the estimation procedure are relegated to Appendix C. Before discussing the results, I describe data, calibration choices, and prior distributions.

3.1 Data

To estimate the model I use quarterly aggregate and sector level time series from Canada and US. Canada is treated as the SOE, while US proxies the world economy. This country-pair has been used in a number of two-country SOE-studies, see e.g. Schmitt-Grohé (1998) and Justiniano and Preston (2010). The data covers the time period 1982Q4-2007Q4. I model 3 different sectors in each economy, referred to as the raw material sector, the goods sector, and the service sector. These are classified according to the North American Industry Classification System (NAICS). The raw material sector constitutes NAICS industries 11-21, the goods sector 22-33, and the service sector 41-56 and 71-72. These industries are exhaustive in the sense that they aggregate to privately produced GDP in both economies. A number of macroeconomic time series are used to construct quarterly data in both economies for (sector level and aggregate) GDP, private consumption expenditures, private investment, the nominal interest rate, inflation, hours, and the real exchange rate. This leaves me with a total of 17 time series used for estimation. The raw data are collected from Federal Reserve Economic Database (FRED), Statistics Canada, and Bureau of Economic Analysis.[11]

The data used for estimation are constructed as follows: Sector level GDP series, which in the raw data are observed at an annual frequency, are interpolated to obtain quarterly series using piecewise cubic Hermite interpolating polynomials. GDP, consumption

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[11]The data used for estimation is available to the public and can be downloaded from http://research.stlouisfed.org/fred2/, http://www.statcan.gc.ca/, and http://www.bea.gov/. Original variable names are listed in Table D.1 in the appendix.

and investment expenditures are all deflated by the implicit CPI deflator to make the series model consistent. Investment is calculated as the sum of private gross fixed capital formation and change in stocks. CPI inflation is constructed as the ratio between current and lagged CPI deflator. Interest rates are divided by 4 to recast them into quarterly numbers. Hours worked (per week) in Canada is divided by total number of employed persons to get weekly hours per person. This makes the variable comparable with US hours. The real exchange rate is defined as the nominal exchange rate times the ratio of US CPI to Canadian CPI. GDP, consumption, investment and hours are divided by the labor force to render the variables model consistent. All variables except for the interest rates are logged and multiplied by 100 before estimation. All variables except for interest rates are also seasonally adjusted at the source. Data are HP filtered in the benchmark estimation to remove non-stationary trends.[12]

3.2 CALIBRATION

A subset of the parameters is calibrated according to data and previous studies. In particular I calibrate all parameters that enter the steady state of the model. Great ratios are set to match the mean of observed data series, based on the assumption that this mean reflects the steady state. Calibrated parameters and their values are reported in Table 1.

Parameters not related to the multi-sector setup are set to common values in the literature (see e.g. Smets and Wouters (2007), Adolfson et al. (2007, 2008), Justiniano and Preston (2010), and Christiano et al. (2011)). I set $\nu = 0.5$ based on recent results from Atalay (2013), who estimate sectoral substitution elasticities between 0.85 and essentially zero.[13] Finally, I follow Benigno (2009), Justiniano and Preston (2010) and Christiano et al. (2011), and set $\epsilon_B = 0.01$.

The remaining calibrated parameters are sector specific, and these deserve further attention. To parameterize sector specific steady state ratios I rely on the US and Canadian I-O matrices summarized in Appendix D. The I-O data are taken from the Structural Analysis Input Output (Total) Database constructed by OECD.[14] I define the "raw materials" sector as industries SIC01-SIC14. The "manufacturing" sector is calibrated according to industries SIC15-SIC45. The service sector constitutes the industries SIC50-SIC72.[15] The data reveal large differences across industries. For instance, while raw materials only constitute about 2% of aggregate consumption in Canada, services represent almost 70%. Still, the raw material sector produces about 16% of GDP because of its exports and large supply of intermediates. The majority of investment goods in both countries is produced by manufacturing firms. Regarding trade, Canadian export-to-GDP ratio varies from 7% in the service sector to about 102% in the manufacturing sector. These sector differences represent a key source of disaggregate heterogeneity in the model. Turning to data on materials, we see that substantial trade in intermediate goods takes place across sectors,

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[12]In an earlier version I used linearly detrended data as well as an estimated stochastic trend as suggested by Canova and Ferroni (2011) and Ferroni (2011). The main results are similar, although the identification of the stochastic trends is poor.

[13]I also tried $\nu = 1.5$, but the results remained similar (not reported).

[14]See http://www.oecd.org/trade/input-outputtables.htm for more information.

[15]The statistical agencies in Canada and US are generally using the North American Industry Classification System (NAICS) instead of the international SIC standard. However, it is straight forward to move between classification systems at this level of aggregation.

Table 1: Calibration scheme

Parameter	Description	Value		
Common				
β	Time discount factor	0.99		
σ	Intertemporal elasticity of substitution	1		
φ	Inverse elasticity of labor supply	2		
χ_N	Set to fit steady state hours equal to 1/3	18.3		
ϵ_p	Steady state mark-up, individual goods	1/7		
ϵ_w	Steady state mark-up, labor types	1/7		
ν	Elasticity of substitution, sectors	0.5		
δ	Capital depreciation rate	0.025		
ϵ_B	Risk premium elasticity	0.01		
	Small open economy			
Input shares		(1)	(2)	(3)
ϕ_j	Materials share in gross output	0.37	0.66	0.34
ψ_j	Labor share in gross output	0.12	0.21	0.32
Free parameter	*Calibrated target*		*Target*	
α_j	Steady state export share of GDP	0.67	1.02	0.07
ξ_j	Steady state share of sector consumption	0.02	0.31	0.67
ϖ_j	Steady state share of sector investment	0.02	0.85	0.13
		0.32	0.21	0.03
ζ_{lj}	Input-output matrix (row l, column j)	0.38	0.61	0.32
		0.30	0.18	0.65
	World economy			
Input shares		(1)	(2)	(3)
ϕ_j^*	Materials share in gross output	0.35	0.54	0.33
ψ_j^*	Labor share in gross output	0.10	0.22	0.29
Free parameter	*Calibrated target*		*Target*	
ξ_j^*	Steady state share of sector consumption	0.01	0.29	0.70
ϖ_j^*	Steady state share of sector investment	0.03	0.77	0.20
		0.40	0.18	0.01
ζ_{lj}^*	Input-output matrix (row l, column j)	0.33	0.58	0.28
		0.27	0.24	0.71

Note: This table presents calibrated values in the benchmark model. The sectors are (1) raw materials, (2) manufacturing, and (3) services. The two I-O matrices (at the bottom) display the fraction of total materials used in each sector that comes from each of the other sectors. Columns represent consumption (input), and rows production (output).

as illustrated by the non-zero off-diagonal elements of the I-O matrices. For instance, the service sector in Canada buys about 32% of its materials from the manufacturing sector (which trade extensively in foreign markets). This is the sense in which trade across sectors provides indirect import in the model, and thereby serves as a potential amplification mechanism for foreign shocks.

13

3.3   PRIOR DISTRIBUTIONS

The remaining parameters are estimated using Bayesian methods. I estimate a total of 30 structural parameters, 13 AR(1) coefficients, and standard deviations of 17 structural shocks. 6 measurement errors are also included, one for each of the sector level GDP series in Canada and US. I choose prior distributions in the mid range of those used by Adolfson et al. (2007), Justiniano and Preston (2010), and Christiano et al. (2011). The prior belief is that Canada and US are symmetric in terms of parameter distributions. Thus, they have the same priors on all the comparable parameters.[16] The prior of η is centered around 1. This is above estimates by Heathcote and Perri (2002), Corsetti, Dedola, and Leduc (2008), and Gust, Leduc, and Sheets (2009), but below estimates by Adolfson et al. (2007). Regarding the Calvo parameters for wages, I am not aware of any studies pointing to substantial sectoral differences in wage stickiness. Thus, θ_{wj} is centered around $0.75 \ \forall \ j$, as in e.g. Adolfson et al. (2007). The priors on sector prices are inspired by a number of microeconomic studies, who show that raw materials and manufactured goods change prices much more frequent than service goods. For instance, Bils and Klenow (2004) look at disaggregate data in the US, and find that prices on agricultural goods change more than once every quarter, while prices on non-durable and durable goods change almost every quarter. Nakamura and Steinsson (2008) on the other hand report average price durability equal to 1.31 quarters for agricultural goods, 1.56 for durables, 3.14 for non-durables, and 3.79 quarters for services. Finally, Bouakez et al. (2009) estimate price durabilities in a closed economy multi-sector model for the US, and find price durations ranging from 1.12 quarters in agriculture to 9.07 quarters in services.[17] They argue that measurement issues have created downward bias in previous estimates of price rigidity in services. I choose priors in the mid range of these estimates. In particular, I center the priors for Calvo parameters such that average price durations in raw materials, manufacturing and services are equal to 1.18, 1.25, and 5 quarters respectively. Finally, motivated by the evidence in Lubik and Schorfheide (2007) for Canada, I allow monetary authorities to respond to exchange rate fluctuations.

Priors for the seventeen structural shocks are comparable to e.g. Adolfson et al. (2007). As is standard I assume somewhat more volatile innovations to investments, labor supply and the markup in prices. Also, the priors for technology shocks in services are smaller than for other sectors. This reflects previous work, who point to much less volatility in the factor productivity of service industries.[18] Finally, I include a measurement error in each of the observation equations linking observed GDP series to those implied by the model. This is motivated by the interpolation of sector GDP data, which might introduce certain high or low frequency properties not related to the business cycle. The measurement errors are assumed to be i.i.d. with prior standard deviations centered around 0.2. This is similar to the prior measurement errors on wages used by Justiniano, Primiceri, and Tambalotti (2013).

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[16]Justiniano and Preston (2010) on the other hand scale up priors on foreign shocks to twice the size of domestic shocks. This is done to induce a more important role for international business cycles.

[17]They also report that prices in construction seem perfectly flexible (i.e. change every quarter). Construction is part of the manufacturing sector in my model, and should therefore reduce aggregate price durability in that sector.

[18]See e.g. Bouakez et al. (2009). Compared to their estimates, the priors used here are relatively conservative when it comes to sectoral differences in technology shocks.

Table 2: Priors and posterior results – Structural parameters

	Prior distribution			Posterior distribution				
	Type	Mean	St. dev.	Mode	St. Dev.	Mean	5%	95%
χ_C	B	0.500	0.100	0.606	0.055	0.588	0.464	0.712
ϵ_I	N	5.000	1.000	0.773	0.238	0.991	0.544	1.438
η	G	1.000	0.150	0.827	0.057	0.820	0.733	0.907
θ_{w1}	B	0.750	0.075	0.758	0.060	0.735	0.614	0.860
θ_{w2}	B	0.750	0.075	0.376	0.064	0.405	0.263	0.538
θ_{w3}	B	0.750	0.075	0.710	0.074	0.689	0.559	0.827
θ_{p1}	B	0.150	0.050	0.114	0.028	0.133	0.061	0.204
θ_{p2}	B	0.200	0.050	0.142	0.041	0.151	0.088	0.213
θ_{p3}	B	0.800	0.075	0.655	0.042	0.655	0.592	0.715
γ_w	B	0.500	0.150	0.295	0.096	0.342	0.150	0.527
γ_p	B	0.500	0.150	0.171	0.071	0.208	0.072	0.342
ρ_r	B	0.600	0.050	0.730	0.019	0.736	0.693	0.779
ρ_π	N	1.800	0.200	1.953	0.166	1.985	1.708	2.270
ρ_y	N	0.125	0.050	0.028	0.015	0.036	0.015	0.056
ρ_{dy}	N	0.125	0.050	0.116	0.030	0.124	0.049	0.202
ρ_e	N	0.100	0.050	0.096	0.028	0.101	0.041	0.160
χ_C^F	B	0.500	0.100	0.562	0.043	0.581	0.449	0.713
ϵ_I^*	N	5.000	1.000	2.594	0.526	2.946	1.189	4.503
θ_{w1}^*	B	0.750	0.075	0.748	0.069	0.723	0.579	0.862
θ_{w2}^*	B	0.750	0.075	0.753	0.055	0.736	0.612	0.862
θ_{w3}^*	B	0.750	0.075	0.725	0.081	0.680	0.538	0.832
θ_{p1}^*	B	0.150	0.050	0.211	0.027	0.212	0.164	0.260
θ_{p2}^*	B	0.200	0.050	0.300	0.033	0.300	0.246	0.354
θ_{p3}^*	B	0.800	0.075	0.801	0.027	0.807	0.763	0.850
γ_w^*	B	0.500	0.150	0.523	0.114	0.510	0.254	0.764
γ_p^*	B	0.500	0.150	0.870	0.050	0.850	0.762	0.941
ρ_r^*	B	0.600	0.050	0.757	0.020	0.756	0.718	0.795
ρ_π^*	N	1.800	0.200	1.697	0.128	1.706	1.482	1.937
ρ_y^*	N	0.125	0.050	0.077	0.018	0.080	0.047	0.111
ρ_{dy}^*	N	0.125	0.050	0.149	0.031	0.146	0.087	0.204
ρ_A	B	0.700	0.100	0.897	0.026	0.886	0.841	0.933
ρ_R	B	0.700	0.100	0.293	0.044	0.295	0.213	0.377
ρ_I	B	0.700	0.100	0.509	0.061	0.493	0.354	0.631
ρ_U	B	0.700	0.100	0.415	0.045	0.452	0.282	0.610
ρ_N	B	0.700	0.100	0.721	0.058	0.703	0.553	0.869
ρ_M	B	0.700	0.100	0.497	0.048	0.500	0.369	0.626
ρ_B	B	0.700	0.100	0.849	0.036	0.838	0.764	0.914
ρ_A^*	B	0.700	0.100	0.900	0.021	0.899	0.864	0.936
ρ_R^*	B	0.700	0.100	0.309	0.042	0.318	0.224	0.406
ρ_I^*	B	0.700	0.100	0.399	0.059	0.406	0.286	0.518
ρ_U^*	B	0.700	0.100	0.591	0.052	0.566	0.397	0.721
ρ_N^*	B	0.700	0.100	0.722	0.048	0.705	0.554	0.863
ρ_M^*	B	0.700	0.100	0.537	0.064	0.546	0.434	0.655

Note: B stands for Beta, N Normal, G Gamma. The two last columns report 90% posterior probability bands obtained from the MCMC simulation. See Table 3 for the marginal data density.

Table 3: Priors and posterior results – Shocks

	Prior distribution			Posterior distribution				
	Type	Mean	St. dev.	Mode	St. Dev.	Mean	5%	95%
$100\sigma_{A1}$	IG	0.20	2.00	0.093	0.280	0.245	0.042	0.419
$100\sigma_{A2}$	IG	0.50	2.00	0.708	0.184	0.727	0.381	1.062
$100\sigma_{A3}$	IG	0.20	2.00	0.896	0.107	0.898	0.723	1.080
$100\sigma_R$	IG	0.20	2.00	0.251	0.027	0.255	0.214	0.294
$100\sigma_I$	IG	0.50	2.00	2.251	0.643	2.893	1.666	4.092
$100\sigma_U$	IG	0.20	2.00	1.657	0.356	1.693	1.205	2.179
$100\sigma_N$	IG	0.50	2.00	0.235	0.174	0.479	0.116	0.905
$100\sigma_M$	IG	0.50	2.00	0.705	0.079	0.753	0.601	0.902
$100\sigma_B$	IG	0.20	2.00	0.424	0.066	0.451	0.318	0.583
$100\sigma_{A1}^*$	IG	0.20	2.00	5.494	0.380	5.600	4.861	6.335
$100\sigma_{A2}^*$	IG	0.50	2.00	1.163	0.099	1.159	0.995	1.324
$100\sigma_{A3}^*$	IG	0.20	2.00	0.608	0.070	0.616	0.501	0.723
$100\sigma_R^*$	IG	0.20	2.00	0.114	0.010	0.117	0.101	0.134
$100\sigma_I^*$	IG	0.50	2.00	5.614	1.035	6.372	2.958	9.503
$100\sigma_U^*$	IG	0.20	2.00	1.156	0.147	1.269	0.891	1.646
$100\sigma_N^*$	IG	0.50	2.00	0.235	0.327	0.507	0.117	0.850
$100\sigma_M^*$	IG	0.50	2.00	1.357	0.130	1.372	1.136	1.605
$100\sigma_{e1}$	IG	0.20	1.00	6.479	0.545	6.603	5.809	7.436
$100\sigma_{e2}$	IG	0.20	1.00	3.273	0.266	3.336	2.896	3.762
$100\sigma_{e3}$	IG	0.20	1.00	0.730	0.072	0.748	0.630	0.860
$100\sigma_{e1}^*$	IG	0.20	1.00	0.093	0.066	0.140	0.053	0.232
$100\sigma_{e2}^*$	IG	0.20	1.00	2.976	0.210	2.990	2.630	3.341
$100\sigma_{e3}^*$	IG	0.20	1.00	0.130	0.036	0.142	0.086	0.198
MDD				-2511.077		-2482.144		

Note: IG stands for Inverse Gamma 1. The two last columns report 90% posterior probability bands obtained from the MCMC simulation. The marginal data density (MDD) is estimated using i) a Laplace approximation based on the posterior mode, and ii) the modified harmonic mean estimator based on draws from the simulated Markov chains.

3.4 POSTERIOR ESTIMATES

To build the posterior distribution of the parameters I simulate 2 Random Walk Metropolis-Hastings chains with 500000 iterations per chain. The first 200000 iterations are used as burn-in. I tune the scaling factor to get an acceptance ratio of about $\frac{1}{3}$ (see Appendix C). Posterior estimates are reported in Table 2 and Table 3. Most parameters are found to be in line with those found in previous studies, with notable exceptions discussed below. First, the posterior mode and mean of investment adjustment costs are significantly smaller in both countries than what is typically found in the DSGE literature, but still higher than microeconomic estimates (see Groth and Khan (2010)). This might be due to internal propagation in the model, a point which I will turn back to later. Second, the estimated price rigidities display large differences across sectors in both countries, with service sector prices being more sticky than prices in other sectors. This is consistent with a number of microeconomic studies as discussed earlier (e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008)), and cannot be accounted for in one-sector models á la Smets and Wouters (2007) and Justiniano and Preston (2010)). Third, there is much less

16

Table 4: Aggregate and sectoral variance decomposition of foreign shocks (%)

Variable	Aggregate	Raw materials	Manufacturing	Services
GDP	74.68	63.18	66.87	82.23
Consumption	76.65	91.82	84.95	69.66
Investment	45.08	54.81	45.69	39.45
Hours	35.02	59.88	36.62	25.97
Inflation	45.71	88.89	54.44	22.10
Wage	84.32	70.94	75.97	89.43
Trade balance (% of GDP)	32.13	46.38	31.66	17.93
Intermediate inputs	–	75.52	81.61	76.75
Terms of trade	–	93.60	57.62	37.43

Note: Variance decomposition of Canadian long run volatility (shares explained by foreign shocks). A full decomposition at the sectoral level is provided in Table E.1 in the appendix.

indexation to previous prices and wages in the SOE than in US. This might have to do with the open economy dimension, as other parameters are fairly similar across countries in Table 2. Also Justiniano and Preston (2010) report less indexation in Canada compared to the US. Turning to the shock processes, we see that technology shocks are the most persistent, and that the most volatile disturbances in the model are productivity innovations in raw material sectors and marginal efficiency of investment shocks. Moreover, productivity is substantially less volatile in the service sector, in line with the results in Bouakez et al. (2009). Finally, note that data are uninformative about some parameters, in particular the volatility and persistence of labor supply shocks.

4 QUANTITATIVE RESULTS

So far I have presented a medium scale multi-sector DSGE model for a SOE. In this section I report a set of empirical results related to spillover from international business cycles and economic interdependence across countries. I focus on GDP, consumption, investment, hours, CPI inflation, real wages, the trade balance, and the policy rate. The next section investigates transmission channels and propagation mechanisms at play.

4.1 THE IMPORTANCE OF INTERNATIONAL DISTURBANCES

First I document the significance of foreign shocks for macroeconomic fluctuations in the SOE. Table 4 reports the fraction of stationary volatility in domestic variables that is attributed to foreign shocks. The first column reports the importance of foreign shocks for aggregate variables, the remaining columns report the same for sectoral variables. Risk premium shocks are labeled as domestic throughout. It is clear from the table that foreign shocks are responsible for a seizable share of the fluctuations in domestic variables. They explain about 30-75% of the volatility in aggregate GDP, consumption, investments, hours, wages, inflation, the interest, and the trade balance. Thus, when confronted with data, the model proposes that international disturbances play a crucial role for macroeconomic fluctuations in Canada.

17

Figure 1: GDP in the data and in the model with only foreign shocks
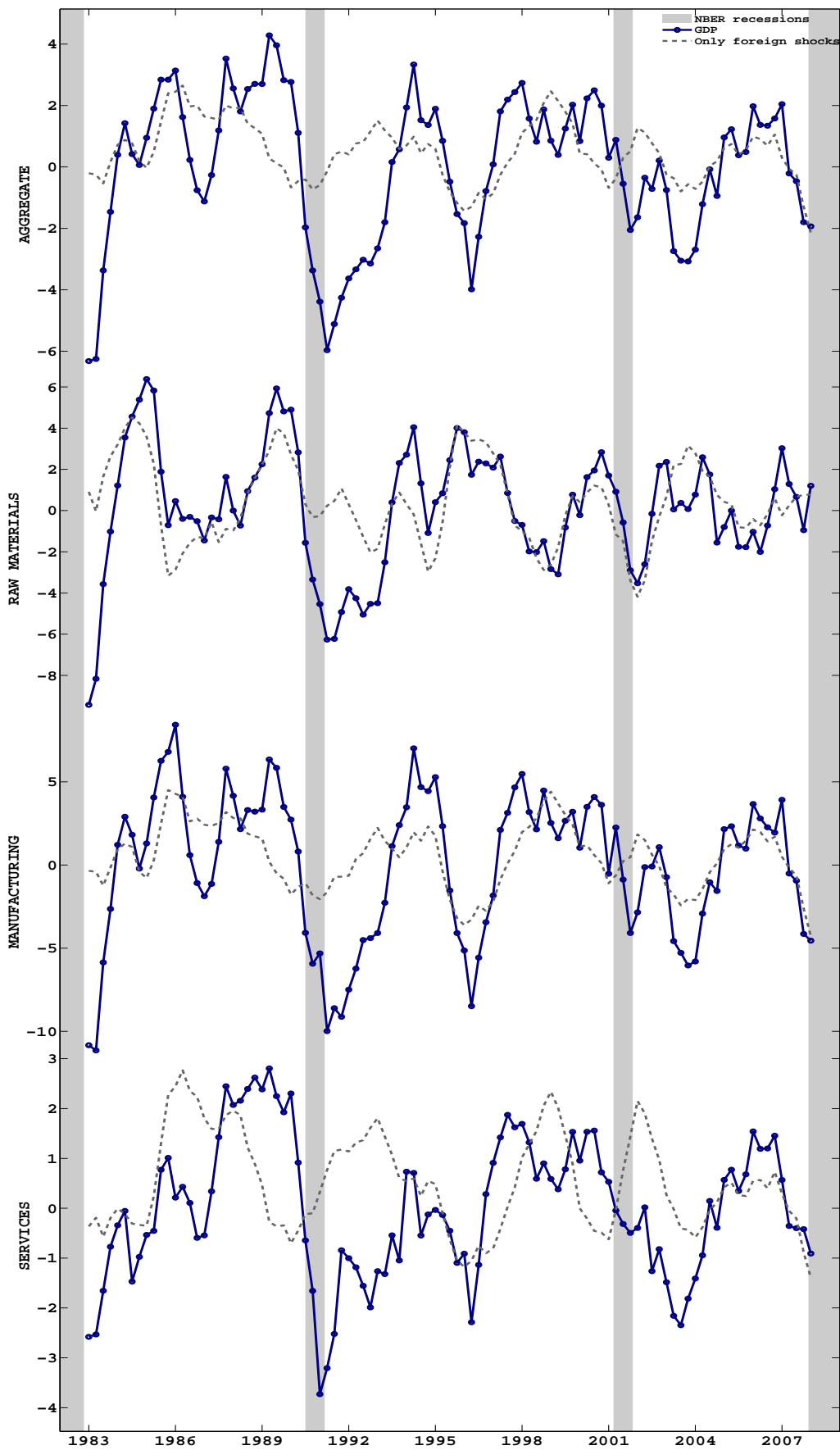
Note: GDP in data (blue) and the counterfactual GDP series when all domestic shocks are excluded (gray). Shaded bars denote US recessions as defined by NBER.

Table 5: Forecast error variance decomposition of foreign shocks (in percent)

Variable	All foreign shocks	Decomposition							
		σ^*_{A1}	σ^*_{A2}	σ^*_{A3}	σ^*_{R}	σ^*_{I}	σ^*_{U}	σ^*_{N}	σ^*_{M}
Panel A: 1 quarter horizon									
GDP	22.38	7.37	0.92	0.00	0.01	6.97	0.08	0.00	7.03
Consumption	10.71	5.29	3.21	0.43	0.08	0.47	0.11	0.00	1.12
Investment	23.36	9.12	7.57	0.79	0.08	2.62	0.26	0.00	2.91
Hours	16.90	1.31	3.49	0.46	0.01	11.22	0.39	0.00	0.03
Interest	36.08	11.05	11.23	1.66	0.76	5.17	0.52	0.00	5.70
Inflation	40.76	15.30	12.06	1.27	0.20	4.27	0.41	0.00	7.24
Wage	48.00	21.70	14.94	1.48	0.25	0.93	0.30	0.00	8.39
Trade balance	37.63	3.03	7.21	1.49	0.21	24.29	1.05	0.00	0.35
Panel B: 4 quarter horizon									
GDP	49.06	23.62	15.21	1.52	0.16	1.78	0.13	0.00	6.63
Consumption	19.53	11.12	5.56	0.77	0.05	0.62	0.18	0.00	1.22
Investment	35.60	16.42	12.39	1.17	0.04	2.69	0.34	0.00	2.54
Hours	23.91	6.72	5.98	0.65	0.13	7.68	0.23	0.00	2.51
Interest	37.04	13.06	10.12	1.66	0.37	7.18	0.64	0.00	4.02
Inflation	40.46	15.30	11.61	1.24	0.22	4.45	0.45	0.00	7.18
Wage	54.11	29.17	17.53	1.95	0.11	0.37	0.32	0.00	4.66
Trade balance	33.91	1.98	2.87	0.62	0.14	26.58	0.80	0.00	0.93
Panel C: 8 quarter horizon									
GDP	54.05	27.41	18.38	2.01	0.10	1.80	0.14	0.00	4.21
Consumption	20.16	12.11	5.23	0.76	0.03	1.00	0.14	0.00	0.89
Investment	38.81	19.40	13.56	1.13	0.03	2.53	0.27	0.00	1.88
Hours	30.91	11.19	9.28	1.03	0.12	6.63	0.24	0.00	2.43
Interest	35.78	11.98	10.46	1.47	0.36	6.41	0.59	0.00	4.51
Inflation	41.36	15.32	11.95	1.21	0.22	4.79	0.48	0.00	7.39
Wage	54.72	30.40	17.80	2.11	0.06	1.37	0.21	0.00	2.78
Trade balance	31.74	1.82	3.70	0.97	0.15	23.36	0.72	0.00	1.02
Panel D: 20 quarter horizon									
GDP	64.25	29.91	23.32	2.13	0.09	5.64	0.10	0.00	3.07
Consumption	41.45	20.32	13.78	1.34	0.05	4.31	0.09	0.00	1.55
Investment	43.81	18.18	14.06	1.01	0.04	8.69	0.23	0.00	1.59
Hours	31.73	11.19	9.47	1.00	0.12	7.35	0.23	0.00	2.36
Interest	34.33	11.61	10.12	1.42	0.34	6.00	0.56	0.00	4.28
Inflation	41.98	15.59	12.21	1.29	0.22	4.88	0.47	0.00	7.31
Wage	68.64	33.39	24.55	2.35	0.06	6.16	0.12	0.00	2.00
Trade balance	33.32	2.21	4.40	1.24	0.13	23.57	0.71	0.00	1.07
Panel E: Long run horizon									
GDP	74.68	30.08	30.93	2.30	0.08	8.83	0.10	0.00	2.36
Consumption	76.65	27.52	33.69	2.28	0.07	11.62	0.07	0.00	1.40
Investment	45.08	17.41	15.57	1.13	0.04	9.24	0.21	0.00	1.47
Hours	35.02	12.32	11.17	1.11	0.11	7.76	0.22	0.00	2.33
Interest	43.05	14.59	14.82	1.62	0.30	7.41	0.48	0.00	3.83
Inflation	45.71	16.62	14.46	1.41	0.21	5.68	0.45	0.00	6.89
Wage	84.32	32.28	36.40	2.59	0.07	11.40	0.10	0.00	1.48
Trade balance	32.13	2.06	4.44	1.27	0.12	22.54	0.65	0.00	1.05

Note: Numbers are calculated at the posterior mean. Note that when the forecasting horizon s becomes large, the contribution of a shock to the s step ahead forecast error converges to that shock's contribution to the unconditional volatility. Panel D therefore reports the share of aggregate volatility that is attributed to each shock.

As an illustration of the importance of international disturbances, Figure 1 plots quarterly Canadian GDP in the data and in the model when only foreign shocks are included. The first panel shows aggregate GDP, panels 2-4 represent sectoral GDP series. Consider first aggregate GDP. A significant share of the movements in GDP is explained by foreign shocks, and their importance rise over the sample as the initial discrepancy caused by pre-sample conditions dies out.[19] Turning to sectoral series in the figure, there is a tendency of more variation being explained by foreign disturbances in the raw material sector than manufacturing, while manufacturing seems more prone to foreign shocks than services. However, the stationary variance decomposition suggests that this is a feature of the data sample, and not the model's long run properties. Instead, it is the service sector that has the highest foreign variance share in the stationary decomposition (see Table 4). This seems counterintuitive given the sectoral differences in international trade. One possible explanation is that GDP in services is highly affected by the use of intermediate inputs. As foreign shocks are important for the long run volatility of these inputs, it might be that GDP in services inherits this property. In any case, the estimated importance of foreign shocks for the aggregate business cycle is in line with ample reduced form evidence. For instance, Kose et al. (2003) estimate a FAVAR model with separate world, region, and country specific factors. They find that the world and region factors combined explain 45-75% of Canadian GDP, consumption and investment (when the median responses are evaluated). More recent empirical studies that attribute major business cycle fluctuations to international disturbances include Kose et al. (2008), Justiniano and Preston (2010), Aastveit et al. (2011), Crucini et al. (2011), Mumtaz et al. (2011), and Kose et al. (2012).

4.2   WHAT KIND OF STRUCTURAL SHOCKS ARE THE MAIN DRIVERS?

Next I discuss what types of structural shocks that the model puts forward as main drivers of Canadian business cycles. The estimated forecast error variance decomposition (FEVD) for all foreign shocks are reported in Table 5. The FEVD of all domestic shocks is provided in Table E.2 in the appendix. It is clear from Table E.2 and Table 5 that in the very short run (1 quarter), no single shock is the major driver of the selected set of macroeconomic variables in the SOE. Rather, innovations to different variables are caused by different disturbances. For example, the one step ahead forecast error in domestic GDP is driven both by shocks to service productivity, the interest rate, and the marginal efficiency of investment (MEI). One step ahead forecast errors for consumption and investment on the other hand are mostly explained by preference and MEI shocks respectively, while the trade balance (as share of GDP) is explained well by risk premium and MEI shocks. Wages and prices are both determined by a mixture of technology shocks in raw materials and manufacturing, as well as domestic and foreign markup shocks. For the unconditional volatility in macroeconomic variables (the stationary forecast error), it seems clear that productivity shocks play a major role. Foreign and domestic technology shocks together are responsible for about 75-80% of aggregate volatility in GDP and wages, 70% of consumption volatility, and about half the movements in inflation and interest rates. Similar results are found for the US (not shown).

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[19]The figure indicates that the two largest recessions in the sample, those in 81-82 and 90-92, had little to do with international events. Foreign shocks in the first case are probably hidden in pre-sample conditions. In the latter case the recession was indeed more severe in Canada, see Voss (2009) and Cross (2011).

Figure 2: Historical variance decomposition — GDP, consumption, investment
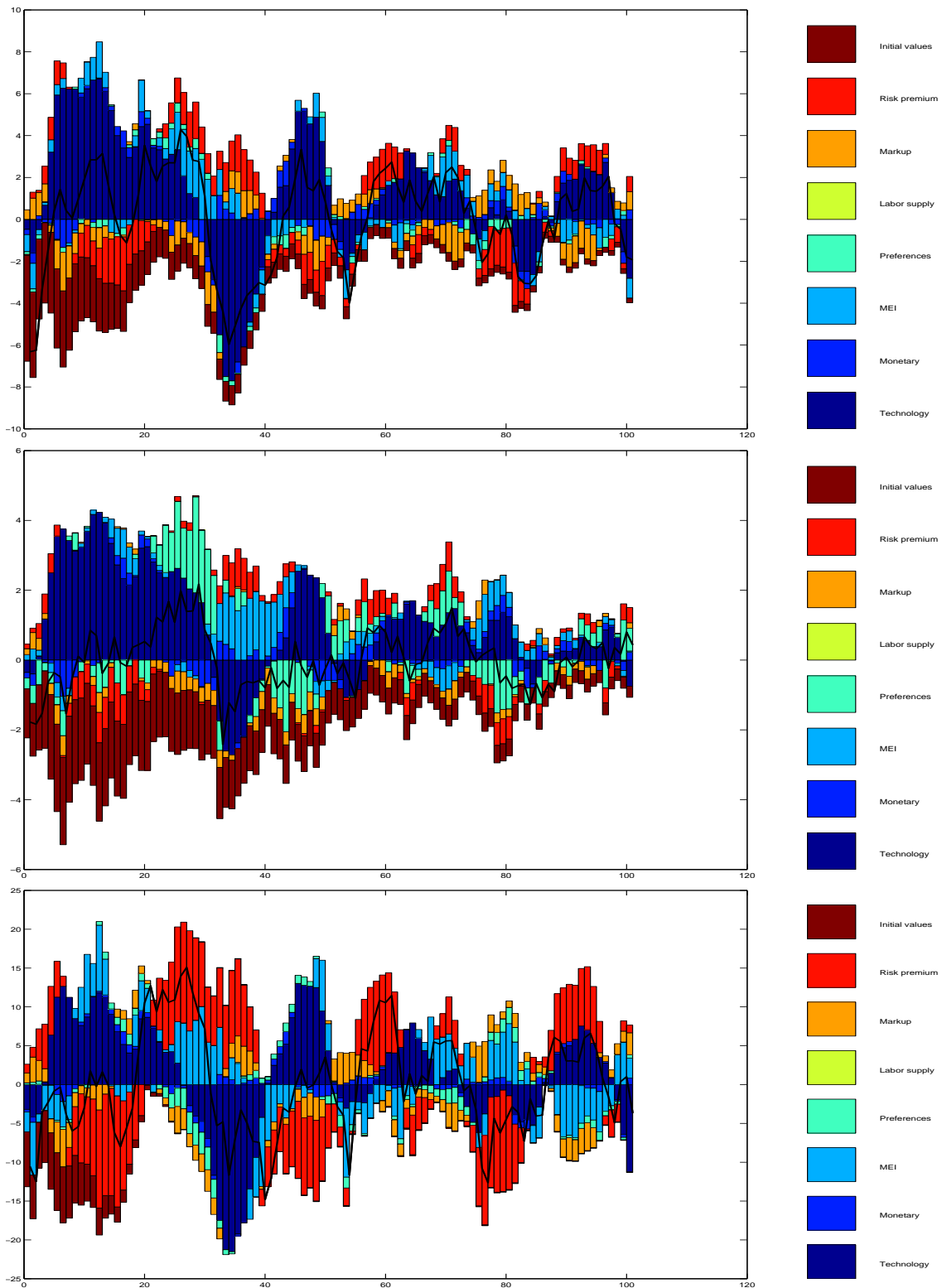
Note: Historical variance decomposition of Canadian GDP (panel 1), consumption, (panel 2) and investment (Panel 3). The black line is actual time series (HP filtered), bars represent the estimated contribution of each structural shock type. Bars sum to the black line. Initial values capture the discrepancy between observed data and (reduced form) shocks in the first period, and are attributed to pre-sample events.

The importance of productivity shocks is illustrated in Figure 2, where I plot historical variance decompositions of Canadian GDP, consumption and investment. Shocks of the same kind are added together across countries. Clearly the model attributes large macroeconomic fluctuations to technology shocks. According to the posterior estimates, they were responsible for most of the boom following the downturn in 1981-1982, and they were basically the sole cause of the severe 1990-1992 recession. Compared to most previous DSGE studies, the role of technology shocks reported here is relatively large. I investigate how these shocks transmit across countries below, and analyze why some other structural shocks play a smaller role for macroeconomic volatility in the SOE.

4.3 Differences across forecasting horizons

The next key result concerns differences in the importance of foreign shocks across forecasting horizons. In fact, when comparing the first column in panels A-D in Table 5, we see that foreign variance shares are higher in the long run than in the short run. This is true for all variables except the trade balance. For instance, while 22% of the one step ahead forecast error in GDP is attributed to foreign shocks, they are responsible for more than 50% at the year-on-year horizon, and 75% in the long run. Also this is consistent with empirical evidence. Justiniano and Preston (2010) estimate a SUR model and document that foreign shocks explain 22% of Canadian GDP at the 1 quarter horizon, 44% at the 4 quarter horizon, and 76% in the long run.[20] The results obtained here closely resemble those findings. Also Cushman and Zha (1997) and Aastveit et al. (2011) find higher foreign variance shares at longer horizons.

Arguably, the increasing role of foreign shocks over time is related to two observations. Most importantly, Bayesian estimates point to relatively persistent productivity processes. As sector level productivity shocks in the US are important drivers of domestic business cycles in the model, this allows these shocks to explain a substantial share of the forecasting error at longer horizons. Second, shocks that originate in international markets will necessarily have to propagate through more economic linkages before they hit domestic markets. Arguably this therefore takes time, compared to domestic shocks.
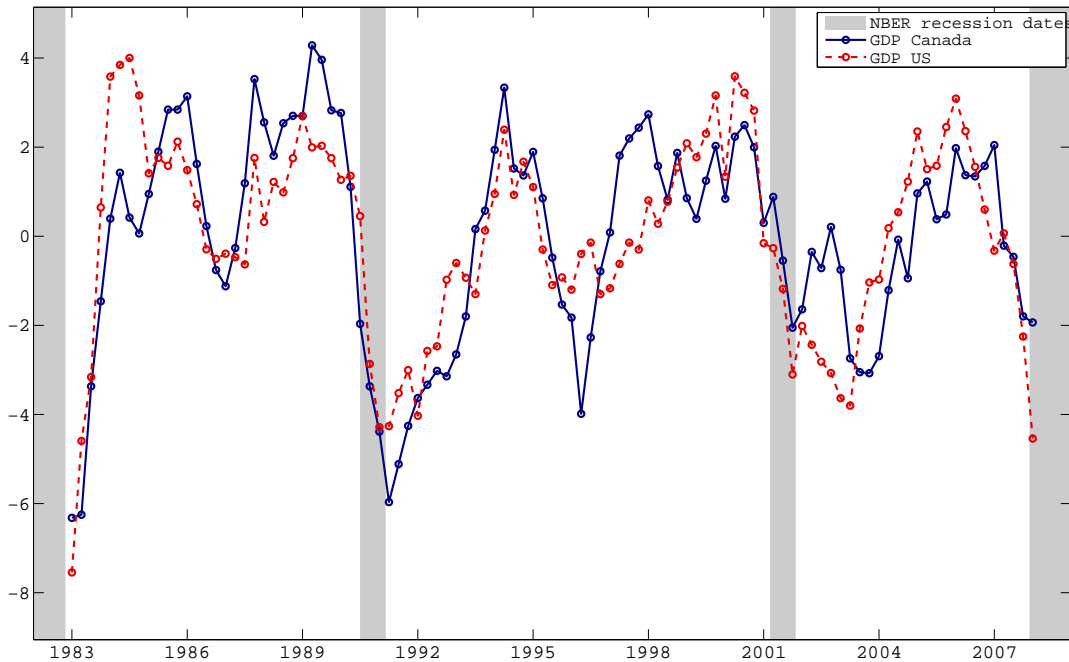
4.4 Cross-country co-movement

Co-movement in macroeconomic variables across countries is a stylized fact in open economy macroeconomics. This is true regardless of how data is detrended. An illustration is provided in Figure 3, where I plot the smoothed series of GDP in Canada and the US. However, co-movement between foreign and domestic variables in DSGE models is not guaranteed, even though substantial domestic business cycle volatility may be driven by international disturbances. As an example, Christiano et al. (2011) define markup shocks in both import and export prices as foreign. This "re-labeling" naturally increases the role of foreign shocks, but will not make any change in correlation coefficients. One can in principle still have low or even negative cross-country correlation in macroeconomic variables.[21] In fact, Eyquem and Kamber (2013) demonstrate how the widely used SOE

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[20]One should note that the 90% probability bands reported for the SUR estimates are wide; $7 - 41\%$ and $44 - 98\%$ respectively. Thus, the point estimates need not reflect the true role of foreign shocks.

[21]Think about a bi-variate VAR, $y_t = Ay_{t-1} + e_t$, with negative numbers on the off-diagonal of A.

Figure 3: Smoothed GDP in Canada and the US

Note: Canadian and US GDP, 1982Q4-2007Q4 (the same data used in estimation).

model by Galí and Monacelli (2005) generates negative correlation in GDP across countries for reasonable parameter values. Lack of cross-country co-movement is also one robust feature in the model estimated by Justiniano and Preston (2010). Data on the other hand is clearly not consistent with this finding, at least not for integrated economies such as US and Canada.

In Table 6 I report model implied cross-country correlations in key macroeconomic variables. The first column shows the international correlation in aggregate variables, second to last column report correlations across countries, but within sectors. Autocorrelation plots for selected variables are provided in Figure 4. Clearly, the model suggests substantial business cycle co-movement in the variables, both at the aggregate and the disaggregate level. For instance, the contemporaneous correlation between US and Canadian

Table 6: Aggregate and sectoral cross-country correlations (scaled by 100)

Variable	Aggregate	Raw materials	Manufacturing	Services
GDP	81.70	68.78	73.12	87.59
Consumption	83.03	92.13	89.00	77.75
Investment	49.56	62.61	50.46	41.28
Hours	45.45	67.05	48.57	31.27
Inflation	59.50	93.64	70.53	38.85
Wage	90.02	70.65	83.92	92.97
Interest rate	54.65	–	–	–

Note: Unconditional correlation coefficients for macroeconomic variables across countries. Calculated based on the posterior mean.

23

Figure 4: Autocorrelation plots
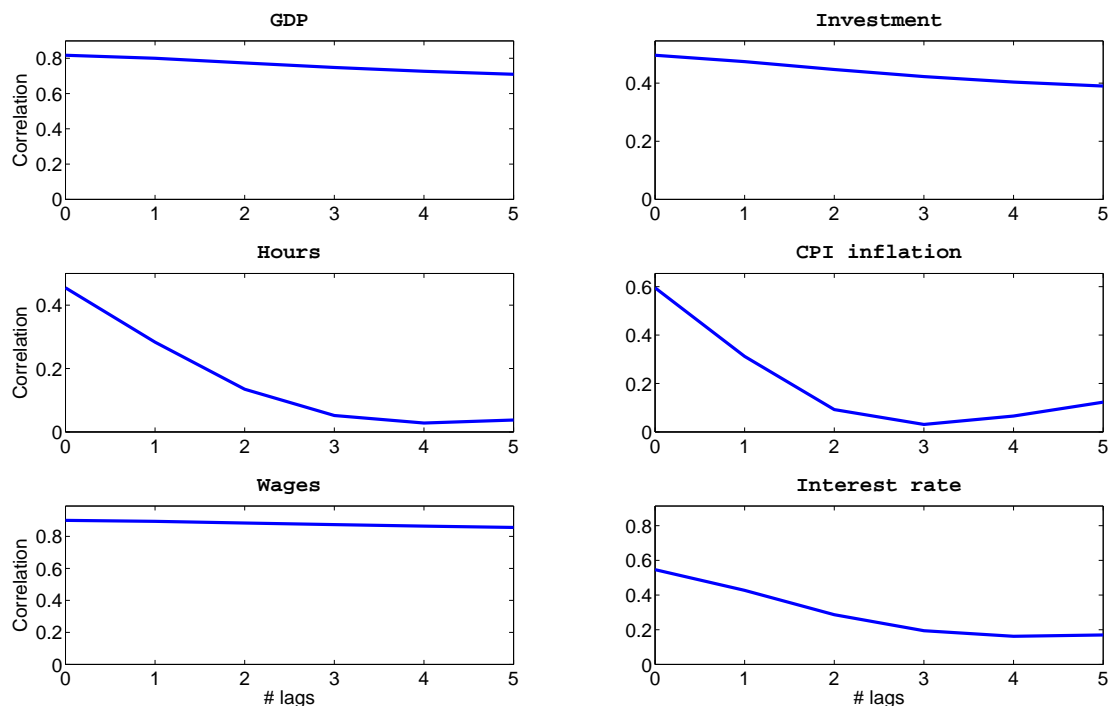
Note: Plots of $Corr(y_t, y_{t-s}^*)$ for $s = 0, \ldots, 5$ lags. The variables are $y_t = [gdp_t, i_t, n_t, \pi_t, \omega_t, r_t]$. Calculated based on the posterior mean.

GDP is about 0.82, compared to 0.78 in the data. An interesting observation is that the co-movement between same sectors across countries often is stronger than across sectors within countries (numbers not reported). This is also a key finding in the reduced form study by Voss (2009) of more disaggregate sectors in US and Canada. The author concludes that the two economies from this perspective operate as a large economic region.

Finally, note that the estimated correlation between domestic and foreign consumption turns out to be very high, with a contemporaneous correlation coefficient similar to that of GDP. In fact, the model vastly overstates the degree of international synchronization in aggregate consumption (0.83 in the model compared with 0.44 in data). This is disappointing given that trade shares in consumption are low in the current multi-sector setup (aggregate import share in consumption of about 15%). The problem can be traced back to estimated cross-country correlations in long real interest rates ($\sum_{s=t}^{\infty} (r_s - \pi_{s+1})$). Given that both the nominal interest rate and the inflation rate tend to move together over time, the consumption Euler equation imposes a close relationship between consumption paths as well. Thus, the simple period utility function used here makes it difficult to fit both the (cross-country) covariance structure of nominal price measures and consumption.

4.5 EXCHANGE RATE PASS-THROUGH

Exchange rate pass-through is defined as the response of some price measure, resulting from a change in the nominal exchange rate. The degree of exchange rate pass-through is essential for monetary authorities in open economies – it determines the relevance of

24

Table 7: Pass-through rates (scaled by 100)

Price measure	Aggregate	Raw materials	Manufacturing	Services
$Corr(\Delta e_t, \pi_t)$	16.85	20.04	32.14	-11.42
$Corr(\Delta e_t, \pi_{Fj,t})$	–	20.17	49.35	16.04
Aggregate CPI	12.39	–	–	–
Aggregate IPI	26.22	–	–	–
Sector prices	–	35.46	29.54	3.77
Imported inflation	–	65.34	54.03	7.81
PPI domestic goods	–	13.64	16.54	3.58
PPI export goods	–	31.30	36.53	79.86
ToT	–	-34.04	-17.50	72.06

Note: Pass-through rates scaled by 100, calculated using the posterior mean. CPI: Consumer price index (π_t). IPI: Investment price index (π_t^i). PPI: Producer price index ($\pi_{Hj,t}$ and $\pi_{Hj,t}^*$). ToT: Terms of trade ($\tau_{j,t}$). Calculated using the posterior mean.

exchange rate fluctuations for domestic prices and quantities, and thus the extent to which monetary policy should respond to the exchange rate. In the data, one typically finds low, but positive pass-through from exchange rate depreciations to the domestic CPI. That is, domestic currency prices seem to be partially, but not fully disconnected from exchange rate movements. New Keynesian open economy models often struggle to replicate the pass-through in data. Under the popular assumption that imports are priced in the producer's own currency, so-called producer currency pricing (PCP), pass-through rates turn out too high. If exporters instead are assumed to price goods in the buyer's currency, so-called local currency pricing (LCP), predicted pass-through rates instead become too low (and even negative in some cases). Lack of pass-through in models with LCP is problematic, given that most export goods in SOEs are priced in foreign currency.

In this section I document the degree of pass-through implied by the estimated model. A first, naive attempt to measure pass-through is to look at the unconditional correlation between the nominal exchange rate and domestic inflation. This is the approach used by e.g. Eyquem and Kamber (2013). The top row in Table 7 reports this statistic for aggregate CPI inflation as well as sectoral inflation series. The model suggests that the nominal exchange rate co-moves moderately with aggregate domestic CPI inflation, as found by Eyquem and Kamber (2013) for a set of SOEs. Co-movement is also present if one looks at correlations between the exchange rate and imported inflation (second row). However, inferring pass-through from raw correlations comes with an obvious drawback – both the exchange rate and prices are endogenously determined in general equilibrium. Thus, other forces than exchange rate innovations might drive the observed co-movement. To measure the pass-through from exogenous exchange rate fluctuations, I calculate the responses of a shock to the risk premium on foreign bonds. Arguably this shock is exogenous to model fundamentals, making it comparable to exchange rate shocks analyzed in reduced form studies. Results are shown in the third to last row in Table 7, where price responses are expressed relative to the exchange rate depreciation (i.e. $\frac{\Delta price_t}{\Delta e_t}$). On impact, a unitary shock to the risk premium causes the nominal exchange rate to depreciate by about 3.44%, while inflation rises by 0.44%. The pass-through to CPI inflation is therefore about $\frac{\Delta \pi_t}{\Delta e_t} = 12.4\%$. This is comparable to reduced form evidence from cross-

country studies. For instance, using US-Canadian data Gopinath et al. (2010) estimate an average pass-through on impact of about 12%. Campa and Goldberg (2005) and Goldberg and Campa (2010) report similar results. The numbers in Table 6 also point to large differences across price measures. Pass-through to investment price inflation is 26.2%, a high number compared to that of the CPI. Moreover, the model delivers relatively strong pass-through in sectors with frequent price changes: 35.5% in the raw materials sector and 29.5% in the manufacturing sector, compared to only 3.8% in the service sector. Gopinath and Itskhoki (2010) show that a positive relationship between pass-through and the frequency of price changes is a robust feature in the data. Sectoral differences also explain the wedge between pass-through to consumer and investment prices, as the aggregate investment basket puts much higher weight on manufactured goods. The inverse relationship between price stickiness and pass-through in the model comes about from the observation that firms who re-optimize prices frequently, have higher probability of responding to exchange rate depreciations. In the next section, I further discuss implications of the model for exchange rate pass-through.

5   TRANSMISSION CHANNELS

Previous literature has studied intermediate trade and producer heterogeneity in closed economies, and found that these features affect both the transmission of monetary policy (Bouakez et al., 2009), the co-movement between producers of durables and non-durables (Sudo, 2012), and the potential for macroeconomic fluctuations following disaggregate shocks (Acemoglu et al., 2012; Carvalho and Gabaix, 2013). Bergholt and Sveen (2014) show how the combination of intermediate trade and sector heterogeneity also can generate substantial spillover of shocks across countries. One purpose of this paper is to test whether that prediction holds when the model is confronted with data. Ex ante this is not obvious, because the model introduces a number of frictions such as habits, investment adjustment costs, and sticky wages. Several additional structural shocks are also introduced, and it is not clear whether these have the same potential for spillover as standard technology shocks.

In this section I describe the mechanisms leading to transmission of business cycle fluctuations across countries, and analyze the role of different shocks. First I point out an important feedback loop that comes about from the intersectoral linkages. It's main implication is synchronization of producer costs across sectors and countries, and thus synchronization of producer prices. This in turn leads to co-movement of domestic and foreign real variables. Shocks that are able to trigger the intersectoral feedback loop are potential sources of international business cycle synchronization. I first describe these mechanisms in more detail, and then study impulse responses to shed light on the dynamic effects of different shocks.

5.1   THE ROLE OF INTERMEDIATE TRADE AND HETEROGENEITY

To better understand how intermediate trade and sector heterogeneity change spillover from international business cycles to the SOE, I proceed in three steps. First, note that the laws of motion for domestic producer prices $p_{rHj,t}$ and $p^*_{rHj,t}$ in sector j can be written as follows, where the first equation captures dynamics for prices on goods sold at home, and

the second for prices on export goods:

$$p_{rHj,t} = \theta_{pj} \left(p_{rHj,t-1} - \pi_t + \gamma_p \pi_{Hj,t-1} \right) + \left(1 - \theta_{pj} \right) \bar{p}_{rHj,t}$$
$$p^*_{rHj,t} = \theta_{pj} \left(p^*_{rHj,t-1} + \Delta e_t - \pi_t + \gamma_p \pi^*_{Hj,t-1} \right) + \left(1 - \theta_{pj} \right) \bar{p}^*_{rHj,t} \tag{28}$$

Both prices above are quoted in domestic currency and in terms of consumption goods.[22] The two equations in (28) state that prices on domestically produced goods are linear combinations of the lagged price level (and some terms related to indexation and exchange rate changes) and the new prices set by firms who re-optimize in the current period, $\bar{p}_{rHj,t}$ and $\bar{p}^*_{rHj,t}$. If optimal new prices rise, we get inflationary pressure on the sector averages $p_{rHj,t}$ and $p^*_{rHj,t}$ as well. The second step is to note that the forward-looking nature of the dynamics described above is captured by two optimality conditions for newly set prices:

$$\bar{p}_{rHj,t} = p_{rHj,t} + \frac{1 - \beta\theta_{pj}}{1 - \beta\theta_{pj}\rho_M} z_{M,t} + \left(1 - \beta\theta_{pj} \right) \mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta\theta_{pj} \right)^s \left(rmc_{j,s} - p_{rHj,s} \right)$$
$$\bar{p}^*_{rHj,t} = p^*_{rHj,t} + \frac{1 - \beta\theta_{pj}}{1 - \beta\theta_{pj}\rho_M} z_{M,t} + \left(1 - \beta\theta_{pj} \right) \mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta\theta_{pj} \right)^s \left(rmc_{j,s} - p^*_{rHj,s} \right) \tag{29}$$

These two equations show that the profit maximizing price, from the individual firm's point of view, is a linear combination of the sector specific averages and expected current and future deviations from the optimal unconstrained price. In the limit as θ_{pj} goes to zero, the expectation sums disappear.[23] However, when $\theta_{pj} > 0$, then any innovation that increases (decreases) real marginal costs relative to it's first best causes temporary upward (downward) pressure on $\bar{p}_{rHj,t}$ and $\bar{p}^*_{rHj,t}$. This takes us to the third step, the introduction of intermediate trade and sector heterogeneity. The linearized real marginal cost in sector j can be written as follows:

$$rmc_{j,t} = -z_{Aj,t} + \phi_j \sum_{l=1}^{\mathcal{J}} \gamma_{lj} p_{rl,t} + \psi_j \omega_{j,t} + \left(1 - \phi_j - \psi_j \right) r_t^k$$
$$= -z_{Aj,t} + \phi_j \sum_{l=1}^{\mathcal{J}} \gamma_{lj} \left[\tilde{\alpha}_l p_{rHl,t} + \left(1 - \tilde{\alpha}_l \right) p_{rFl,t} \right] + \psi_j \omega_{j,t} + \left(1 - \phi_j - \psi_j \right) r_t^k \tag{30}$$

The first line shows that costs are directly affected by market prices $p_{rl,t}$ in all domestic industries $l \in \mathcal{J}$, because intermediate trade takes place across sectors. The second line demonstrates that costs depend on import prices $p_{rFl,t}$, set by firms in foreign sectors. This is true as long as the domestic absorption parameters $\tilde{\alpha}_j$ are less than one. Importantly, $p_{rFl,t}$ can be represented by a system similar to (28)-(30). Thus, shocks that affect sectoral marginal costs in the foreign economy will in principle show up in equation (30). Three important observations immediately follow from the system (28)-(30). First, intermediate trade introduces co-movement between domestic and foreign producer

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[22]That is, $p_{rHj,t} = \ln\left(\frac{P_{Hj,t}}{P_t} \right)$ and $p^*_{rHj,t} = \ln\left(\frac{\mathcal{E}_t P^*_{Hj,t}}{P_t} \right)$. See the appendix for details.

[23]Optimal prices without price setting rigidities are $p_{rHj,t} = p^*_{rHj,t} = rmc_{j,t} + z_{M,t} \; \forall \, t$.

Table 8: Intermediate trade in OECD and BRICS countries

| Country	Share	Country	Share	Country	Share
OECD:					
Australia	0.51	Greece	0.44	Portugal	0.54
Austria	0.49	Hungary	0.62	Slovakia	0.63
Belgium	0.59	Ireland	0.56	Slovenia	0.58
Bulgaria	0.57	Italy	0.54	Spain	0.52
Canada	0.50	Japan	0.49	Sweden	0.53
Czech Republic	0.63	Korea	0.59	Switzerland	0.50
Denmark	0.49	Luxembourg	0.63	Turkey	0.43
Estonia	0.60	Netherlands	0.53	United Kingdom	0.53
Finland	0.55	New Zealand	0.55	United States	0.46
France	0.51	Norway	0.47	OECD	0.54
Germany	0.50	Poland	0.55		
BRICS:					
Brazil	0.51	India	0.48	South Africa	0.50
China	0.64	Russia	0.49		

Note: Intermediate goods share of gross output (OECD data).

prices. That is, a rise (fall) in any import price $p_{rFl,t}$ directly increases (reduces) domestic producer costs $rmc_{j,t}$ (equation (30)), and hence domestic producer prices $p_{rHj,t}$ and $p^*_{rHj,t}$ via New Keynesian Phillips curves (equation (28)). Second, the model features an important feedback loop. That is, the first round rise (fall) in $p_{rHj,t}$ further increases (reduces) domestic sector j's costs, because $p_{rHj,t}$ shows up in (30). There is a similar feedback loop involving foreign producer prices and foreign marginal costs. Third, the initial impulse propagate across sectors as long as $\tilde{\alpha}_j < 1$. Thus, foreign shocks can enter the SOE through some industries, notably those with high trade intensity, and then propagate to others via intermediate trade. The latter kind of spillover is governed by the off-diagonals of the I-O matrix, and allows even relatively non-traded sectors to be affected by international disturbances.

The setup presented here nests as special cases some common approaches in the literature, including models with i) one sector ($\mathcal{J} = 1$),[24] ii) no intermediate trade ($\phi_j = 0$), and iii) no foreign trade ($\tilde{\alpha}_j = 1$). However, all these dimensions matter for the transmission of foreign shocks, the extent of international synchronization, and the degree of exchange rate pass-through. Obviously, if $\tilde{\alpha}_j = 1 \ \forall \ l \in \mathcal{J}$, then economic activity in the SOE is completely unrelated to the rest of the world. If $\phi_j = 0$, then there is one less source of co-movement in producer prices (the one described above), and hence one less mechanism that induces business cycle synchronization. If $\mathcal{J} = 1$, then the entire transmission has to take place at the aggregate level without sectoral reallocations. In contrast, the multi-sector model presented here allows industries with limited trade to be affected via cross-sectoral intermediate market linkages. Thus, even fluctuations in completely non-traded industries can in principle be driven by business cycle shocks abroad.

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[24]Or alternatively, no (ex ante) sector heterogeneity ($\phi_j = \phi$, $\gamma_{lj} = \gamma$, $\tilde{\alpha}_j = \tilde{\alpha}$).

Figure 5: Intermediate goods share (%) of total trade between Canada and the US
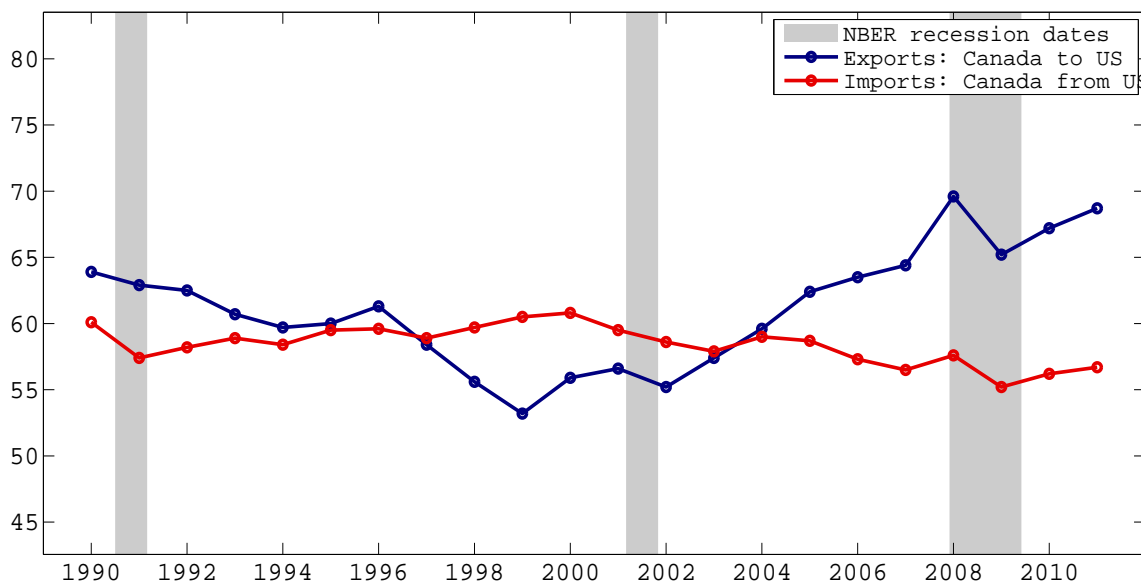
Note: Annual data, 1990-2011. OECD STAN database.

## 5.2 INTERNATIONAL EVIDENCE OF INTERMEDIATE TRADE ACTIVITY

A crucial assumption underlying this analysis is obviously the presence of intermediate goods trade, i.e. the exchange of resources between firms. Thus, one should ask how important this feature is in the data for other countries than Canada and the US. Table 8 reports the share of intermediate trade in gross output in all OECD countries where data were available, as well as in the BRICS economies. About half of gross output in most countries is sold as input to other firms.

Firm-to-firm trade is particularly relevant for the open economy dimension, as intermediate goods are more intensively traded across countries than final user goods. Figure 5 makes this point for the Canada-US country pair. About 60% of all trade between these two countries is between firms. Thus, open economy models with only final products (consumption and investment) miss out on more than half of the trade in physical goods that actually takes place. Clearly, this might give rise to mismatch between theory and reality when it comes to international spillover and synchronization. The natural question to ask next is what kind of shocks in the present model that are key drivers of prices and costs at the sector level, of flows in intermediate goods markets, and hence of business cycle fluctuations in the SOE.

5.3 THE SCOPE FOR INTERNATIONAL SYNCHRONIZATION

To better understand the role of foreign disturbances for domestic business cycles, I analyze the impulse responses of domestic variables to selected international shocks. I explain why foreign productivity shocks generate business cycle co-movement, how foreign investment shocks create a wedge between domestic and foreign investment, and argue that the multi-sector setup facilitates higher pass-through than what is typically found in models with LCP.

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Figure 6: A productivity shock in foreign manufacturing
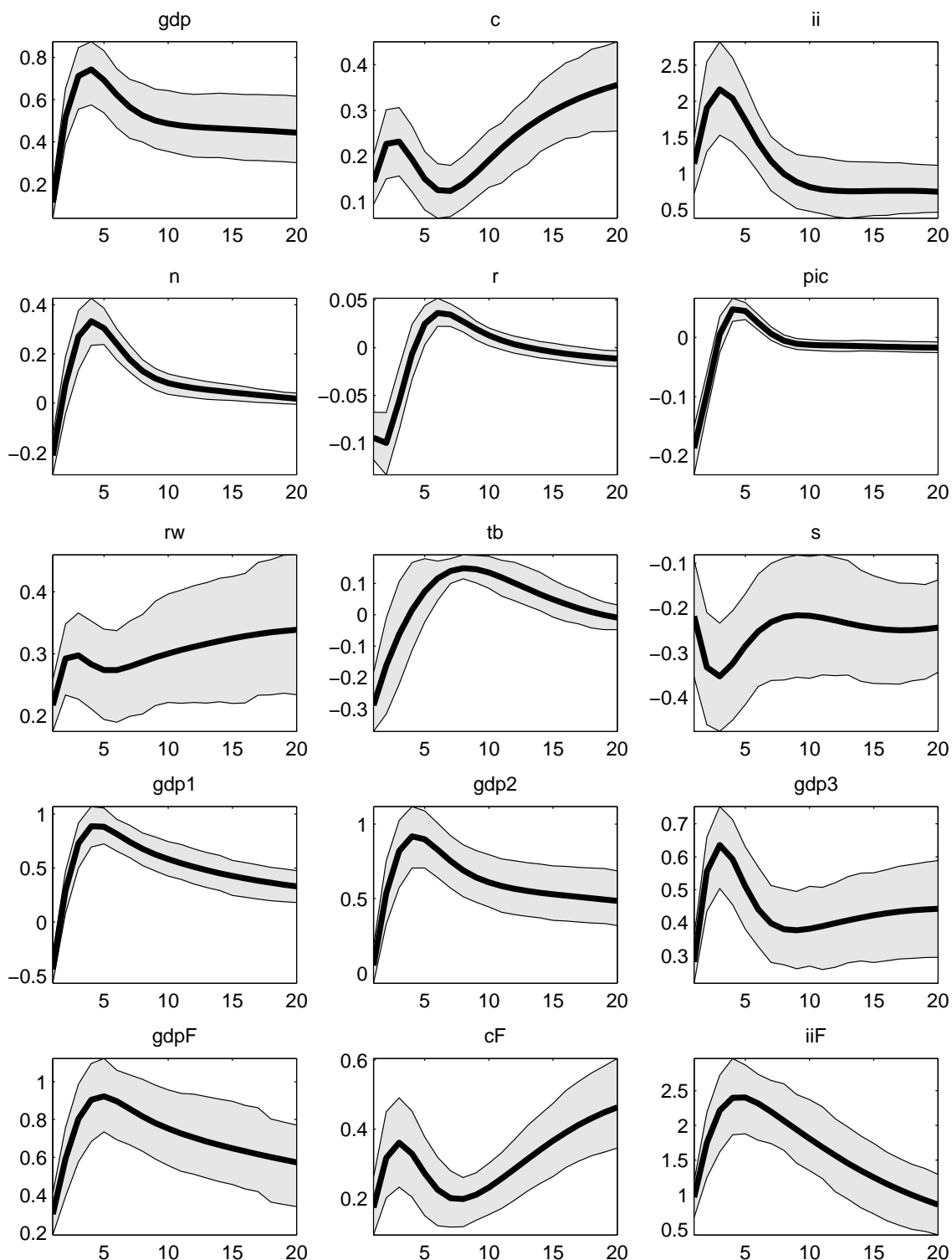
Note: Impulse responses to a productivity shock in foreign manufacturing (one standard deviation). The panels report responses of GDP (gdp), consumption (c), investment (ii), hours (n), interest rate (r), CPI inflation (pic), real wage (rw), trade balance (tb), the real exchange rate (s), and GDP in raw materials (gdp1), manufacturing (gdp2) and services (gdp3). The trade balance is measured in absolute deviations from steady state relative to GDP.

5.3.1 Dynamic effects of productivity shocks

I plot a unit standard deviation shock to productivity in the foreign manufacturing sector in Figure 6. Additional sector level IRFs in both economies are provided in Appendix E. Productivity innovations in the US manufacturing sector are essential for understanding macroeconomic fluctuations in the SOE, according to Table 5. One thing to note from Figure 6 and figures in the appendix is the striking co-movement in aggregate responses across countries in both GDP, consumption and investment. What is going on here? Consider first the foreign IRFs. As expected, higher foreign productivity raises foreign GDP, consumption, and investment. The set of frictions in the model, in particular sticky prices and monopolistic competition, also implies less working hours and a lower foreign interest rate (see Figure E.2). All these effects are well known from the textbook one-sector, closed economy model. However, cheaper manufactured goods in the foreign economy not only lead to expenditure switching towards that sector, but also to cheaper manufactured intermediates. This latter effect reduces costs and prices in the other foreign industries as well, and therefore creates the feedback loop emphasized by e.g. Acemoglu et al. (2012). Regarding spillover to the SOE, note first that lower prices on manufacturing imports induce expenditure switching in that sector towards imports. While this kind of expenditure switching helps in generating co-movement between domestic and foreign absorption of manufactured (sector j) goods, it is contractionary from the point of view of domestic sector j firms. In a one-sector world, the substitution towards imports is basically be the main spillover effect. This is why previous models find little co-movement in GDP, hours, and other supply side variables across countries.

In contrast, the multi-sector structure presented here provides us with a rich story about additional mechanisms at work. First, lower imported inflation in the domestic manufacturing sector implies lower overall inflation in manufactured prices, relative to prices from other industries. This creates domestic substitution towards all manufactured goods, including those that are produced domestically. The sectoral substitution effect is expansionary from the point of view of domestic manufacturing firms. Second, the cheaper manufactured goods also reduce domestic firms' expenditures on intermediate goods. This is seen from equation (30). In fact, producer costs decline in all domestic industries, as also non-manufacturing producers use manufactured goods as input. Profit maximizing behavior then induces domestic firms to lower their prices as well, and overall domestic inflation declines even further. That triggers another round of cheaper intermediate goods, and another round of price reductions, and so on. The interest rate naturally declines as well. Domestic prices are not perfectly adjusted, so some of the increased productivity is materialized as lower demand for labor – a well known outcome in New Keynesian models. Thus, total hours decline in the domestic economy the first periods. Cheaper domestic goods also limit the initial expenditure switching towards imports, implying further domestic expansion in demand for consumption and investment goods. For domestic producers, substitution towards imports and cost reduction are two forces that push in opposite directions. Their relative importance in each sector depends on the sectoral trade intensity, the degree of price stickiness, and the share of intermediate inputs in production. In total, the increase in foreign manufacturing productivity generates relatively large dynamics in the SOE. Part of the reason is the high trade intensity in the manufacturing sector. Another point is that manufactured goods prices are much less sticky than service prices, implying that they react more following the shock. Also,

Figure 7: A foreign MEI shock
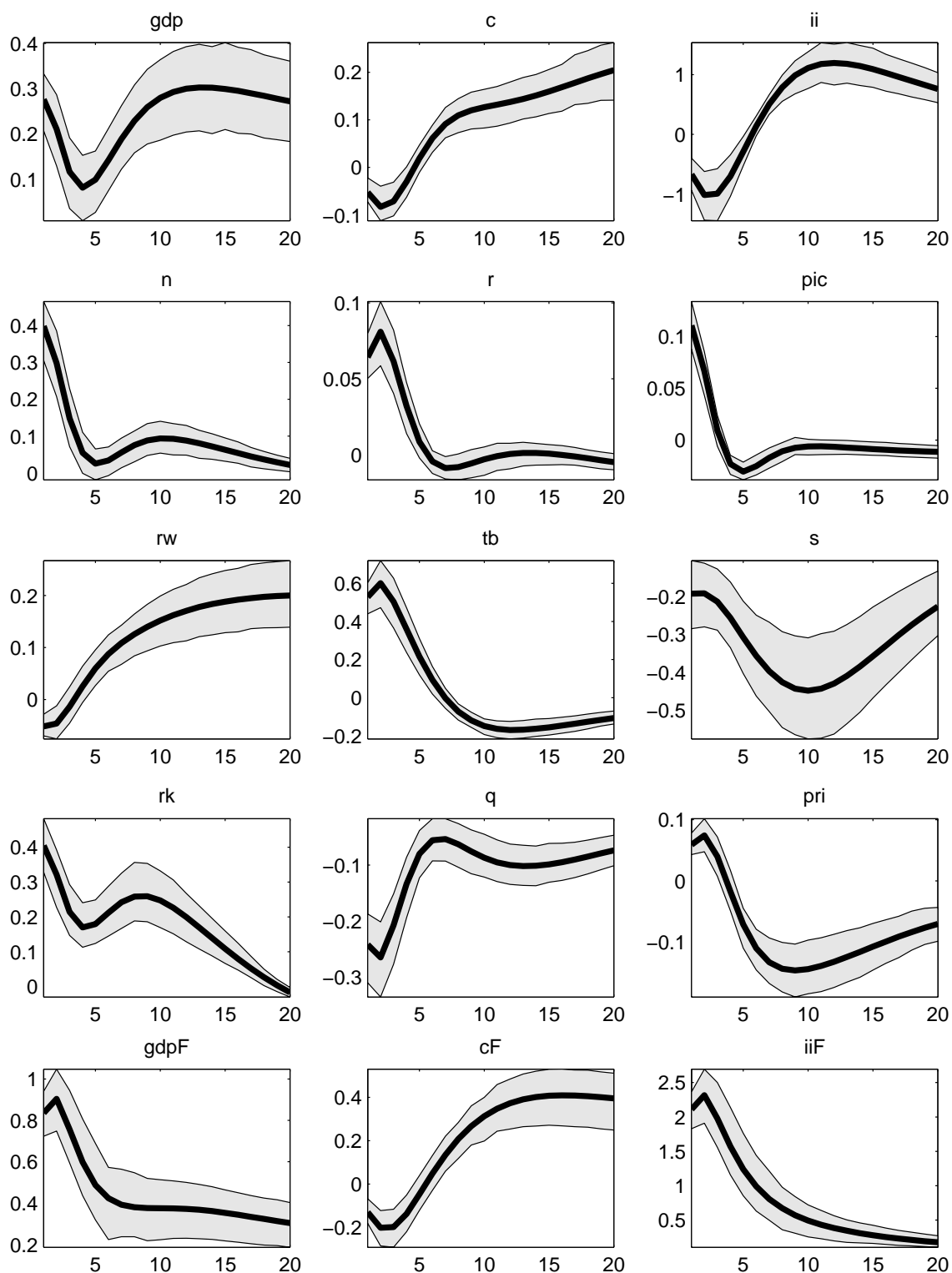
Note: Impulse responses to a one standard deviation investment efficiency shock. Panels in the fourth row report responses of capital rental rates (`rk`), the present value of installed capital (`q`), and the relative investment price (`pri`). See Figure 6 for remaining variables.

manufactured goods are important inputs in services, the largest sector in the economy.

5.3.2 DYNAMIC EFFECTS OF INVESTMENT SHOCKS

Next I describe effects of a shock to the marginal efficiency of investment (MEI) in the foreign economy. The goal is to understand why estimated DSGE models for closed economies have attributed larger macroeconomic fluctuations to this shock than what I find here. Figure 7 plots IRFs. The MEI shock temporarily increases the amount of capital transformed from each investment unit, and thereby raises the relative return to capital investments. This induces foreign households to invest more, and cut back on consumption the first periods due to resource constraints (see Furlanetto, Natvik, and Seneca (2013) for an analysis of this issue). The net effect is still a positive shift in aggregate demand, leading to upward inflation pressure. After some periods the investments start to pay off in form of capital abundance in the foreign economy, leading to a prolonged period with higher consumption demand as well.

In the SOE, the foreign MEI shock generates responses in GDP, consumption, hours, interest rate and inflation that are qualitatively similar to those in the foreign economy. That is, due to higher imported inflation, overall price level and the interest rate in the SOE increase. This reduces domestic consumption demand and makes production more expensive. Yet, high foreign investment demand is expansionary for domestic raw material and manufactured goods producers, who export investment goods intensively.

Still, the MEI shock cannot explain all international synchronization patterns – it causes strong divergence between investment in the two countries. To see why, note that domestic absorption of investment goods from sector j can be written as follows:

$$i_{j,t} = -\nu_i \left(p_{rj,t} - p_{r,t}^i \right) + i_{t-1} + \frac{1}{\epsilon_I \left(1 - \beta \rho_I \right)} z_{I,t} + \frac{1}{\epsilon_I} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(q_s - p_{r,s}^i \right) \qquad (31)$$

Equation (31) shows that $i_{j,t}$ is linked to the relative sector price $p_{rj,t}$, and via aggregate investment demand, to the expected path for real returns to investments, $\left\{ q_s - p_{r,s}^i \right\}_{s=t}^{\infty}$.[25] Intuitively, when the value of current and future capital exceeds the cost of capital accumulation, sectoral and aggregate investment demand is high. Investment co-movement across countries is therefore stimulated by synchronization of expected capital returns. However, in the case of a foreign MEI shock, higher import prices in the SOE raises $p_{r,t}^i$, while higher real interest rates reduce the present value of installed capital.[26] This lowers domestic sectoral and aggregate investment demand, and generates a wedge between domestic and foreign investment activity that is not typically seen in the data. At the same time, some of the increase in foreign demand is targeted towards domestic goods, in particular those who can be transformed into capital abroad. The result is a large improvement in trade balances, and higher aggregate GDP. Also, domestic demand towards domestic goods goes up, due to relatively cheaper home products. Taken together, the foreign MEI shock is able to generate co-movement between several domestic and foreign variables, but not between domestic and foreign investment. This latter point implies that the posterior weight on foreign MEI shocks is smaller, although they still explain important parts of several domestic variables.

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[25]Investment adjustment costs are priced into $p_{r,t}^i$. Without adjustment costs, equation (31) collapses to $q_s + z_{I,s} = p_{r,s}^i \ \forall \ s$.

[26]That the present value of installed capital drops is seen by solving the linearized optimality condition for capital forward. The result is $q_t = E_t \sum_{s=t}^{\infty} \left(\beta \left(1 - \delta \right) \right)^{s-t} \left[- \left(r_s - \pi_{s+1} \right) + \left(1 - \beta \left(1 - \delta \right) \right) r_{s+1}^k \right]$. Thus, an increase in current or future expected real interest rates reduce the value of capital.

5.3.3 RISK PREMIUM SHOCKS AND EXCHANGE RATE PASS-THROUGH

The empirical section established that exchange rate pass-through to import prices and the CPI is moderate, with large sectoral differences depending on the degree of price flexibility in each industry. Next I investigate these results in more detail. Insights can be found if we study the responses of a shock to the risk premium on foreign bonds. As argued earlier, this shock should be exogenous to model fundamentals, making it comparable to exchange rate shocks analyzed in empirical studies. Impulse responses to the risk premium shock are provided in Figure 8. On impact, a shock which lowers the risk premium causes the nominal exchange rate to depreciate, while inflation rises moderately at the aggregate level, with sectoral price responses depending on the frequency of price adjustment.

The intuition is as follows: Lower risk premium raises demand for foreign bonds, and thereby reduces the value of domestic currency in nominal terms. This is seen from the linearized no arbitrage condition, which writes as

$$r_t = r_t^* + \mathbb{E}_t \left(\Delta e_{t+1} \right) - \epsilon_B a_t + z_{B,t}.$$

Clearly, following a rise in $z_{B,t}$ there must be a contemporaneous depreciation unless the monetary authority pegs the exchange rate.[27] Due to LCP, weaker currency in the SOE then directly reduces foreign firms' export income and profits. This is seen from the marginal income of foreign exporters under LCP, which is $p_{Fj,t} - e_t$ at the sector level. These firms react by raising their export prices. The result is higher imported, and thus higher overall domestic inflation. This is what we define as exchange rate pass-through.

Two forces in the model limit the pass-through to domestic prices. First, the presence of price stickiness reduces pass-through to import prices, as only a subset of the foreign exporters can adjust their prices optimally. Indeed, when $\theta_{pj}^* \to 1$ the pass-through becomes zero. In one-sector New Keynesian models with LCP, the presence of price stickiness typically leads to less pass-through than suggested by empirical literature (Gopinath et al., 2010). Second, as households now find it more profitable to save abroad, they lower consumption and investment demand to reallocate resources towards foreign bonds. The decline in domestic absorption is seen in Figure 8. Lower consumption demand is particularly relevant for service firms, who supply most domestic consumption goods. The drop in investment demand on the other hand affects GDP in the manufacturing sector, which produce most investment goods. In fact, the aggregate decline in domestic absorption is large enough to lower GDP in these two sectors.[28] Most importantly, it puts downward pressure on both domestic producer prices and import prices, and thus limits exchange rate pass-through. Taken together, the combination of LCP and contraction in domestic absorption should lead to small or even negative pass-through rates in the SOE.

Sector heterogeneity modifies the pass-through story outlined above. As seen in Table 6, we have relatively high pass-through in the sectors with frequent price changes. In the model, this relationship comes about from the simple observation that firms who re-optimize prices frequently, have higher probability of responding optimally to the exchange rate depreciation. As the optimal sector price equates $p_{Fj,t} - e_t$ with marginal

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[27]The SOE assumption implies that foreign variables, including r_t^*, do not change.

[28]Yet, the drop in total GDP is muted, because the accumulation of foreign assets is financed by a trade surplus driven by exports of raw materials and manufactured goods.

Figure 8: A risk premium shock
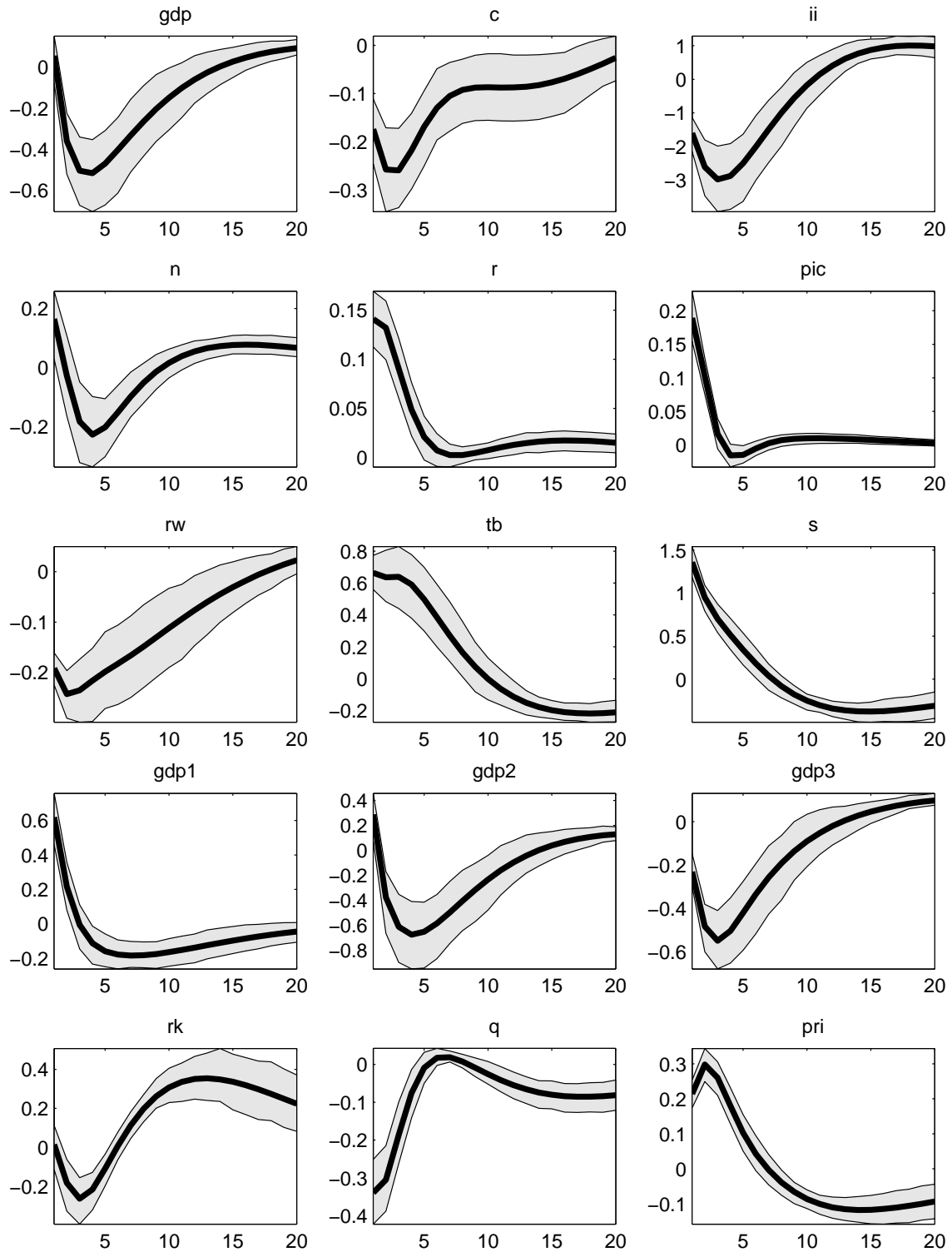
Note: Impulse responses to a one standard deviation risk premium shock. See Figure 6.

costs, $p_{Fj,t}$ will rise more aggressively when price stickiness is low. Moreover, CPI measures in raw materials and manufacturing put high weights on the import price $p_{Fj,t}$, adding to the positive pass-through in these industries. The presence of intermediate trade

Table 9: Counterfactual model – Business cycle predictions

	Panel A: $100\rho_{y,y^*}$	Panel B: 1Q	4Q	8Q	20Q	LR	Panel C: $\Delta\pi/\Delta e$	
GDP	27.4	1.89	3.11	3.52	6.22	10.33	π_t	7.86
Consumption	34.8	0.94	1.22	1.93	5.47	16.43	$\pi_{F,t}$	36.77
Investment	3.2	1.37	1.37	1.28	2.69	3.42	$\pi_{H,t}$	-2.27
Hours	9.1	1.72	2.15	2.18	2.26	2.36	$\pi_{H,t}^*$	57.55
Interest	17.6	3.73	3.08	3.04	3.00	4.02	τ_t	20.78
Inflation	14.9	2.76	2.60	2.64	2.70	3.03		
Wage	39.3	4.50	4.30	5.31	10.34	19.17		
Trade balance	–	8.14	10.03	10.17	10.21	10.16		

Note: See Table 5 and Table 7 for details.

further increases pass-through rates: Higher imported inflation drives up producer costs among those firms who import intermediate goods, and thus puts upward pressure on domestic producer prices. Again, this cost channel is particularly important for the trade intensive raw material and manufacturing firms. This explains why pass-through to both imported and domestic producer price inflation is relative high in these sectors. Moreover, cross-sectoral linkages in domestic intermediate markets allow exchange rate fluctuations to spill over to marginal costs of domestic service firms as well. Thus, both the existence of firms with relatively flexible prices, and the presence of intermediate trade channels in the model, help to increase the pass-through to domestic prices. This is consistent with the empirical work by Goldberg and Campa (2010), who find that intersectoral linkages are the most important source of exchange rate pass-through to the domestic CPI.

6 THE COUNTERFACTUAL MODEL ECONOMY

This paper analyzes the role of foreign shocks in a model with intermediate trade between heterogeneous firms. In this section I ask whether the role of international business cycles survives in a context without these extensions to the conventional open economy DSGE model. To this end, I estimate the particular version of the model when $\mathcal{J} = 1$ and $\phi = 0$. The model now becomes a fairly standard DSGE model for a small open economy.[29] Calibrated values are set as follows: First, I rescale labor and capital shares in both economies to keep the constant returns to scale assumption based on the numbers in Table 1. This gives $\psi = 0.543$. Second, I calibrate trade shares in GDP by subtracting the intermediate input share of imports in each sector, and then calculating aggregate (sector GDP weighted) import share in the economy. The resulting import share of GDP is 0.26 ($\alpha = 0.7405$). The remaining calibrated values are chosen as before. Also the prior distributions are as in the baseline model, except that price and wage stickiness have prior modes equal to 0.7, while the aggregate TFP shock has a mode equal to 0.2.

Business cycle predictions from the counterfactual model (based on posterior mean estimates) are provided in Table 9. Parameter estimates are reported in the appendix.

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[29]Obviously, one counterfactual implication of this model version is the symmetric response of all firms within countries to all kinds of business cycle shocks. Bouakez et al. (2009) analyze implications of imposing such symmetry in a closed economy setting.

Figure 9: Impulse responses to a foreign MEI shock in the counterfactual model
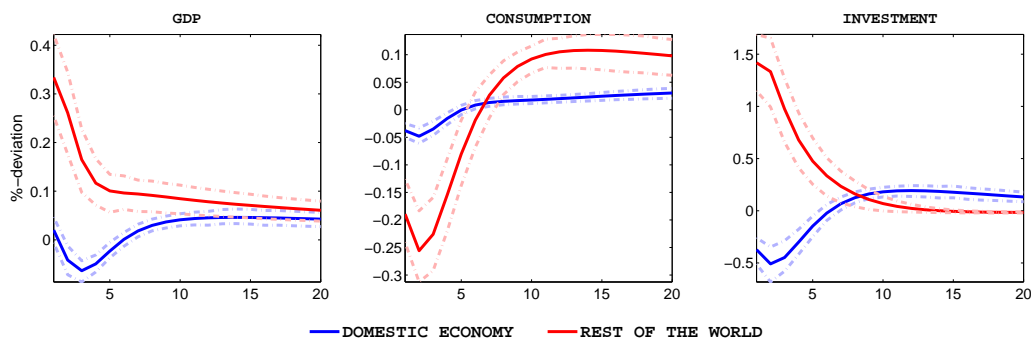
Note: See Figure 6 for details.

Consider first model implied cross-country correlations (Panel A). For all variables under consideration, they drop to less than half of those in the baseline model. The drop is particularly large for investment. Still, the degree of co-movement is higher than that found by Justiniano and Preston (2010), and international consumption synchronization actually comes fairly close to that in data. Part of difference from Justiniano and Preston (2010) is attributed to the inclusion of investment, which is abstracted from in their study. When higher foreign productivity takes down international prices, domestic investment (and capital) is stimulated by cheaper imports.

Turning to the decomposition of shocks, we see that foreign shocks become nearly irrelevant for most domestic variables within the business cycle. They explain less than 7% of the variation in all variables except wages and the trade balance within the 5-year horizon. This is bout one tenth of the shares attributed to foreign shocks in the baseline model (Table 5). In the long run, foreign shocks account for about 2-20% of the macroeconomic volatility in the SOE, far below typical estimates in the VAR literature. Estimates of the exchange rate pass-through are provided in Panel B in Table 9. The short run pass-through to CPI drops from 12.4% to 7.9% – still a fairly high number given that exporters in the model price their goods in local currency. The main reason is the estimated low degree of price stickiness, with a posterior centered around 0.5. As an illustration of the limited role for foreign shocks in the counterfactual model, Figure 9 plots the impulse responses to a foreign MEI shock. As before we get a drop in domestic consumption and investment. However, unlike before we also get a drop in domestic GDP. The intuition is straight forward: Without intermediate trade, the increase in foreign investment demand does not call for more exports of materials. Instead, the main transmission channel to domestic GDP is via higher domestic real interest rates, which lower domestic consumption and investment demand. Thus, without intermediate trade, the foreign MEI shock cannot even explain international co-movement in GDP. Finally, note that the negative effect on foreign consumption is amplified in the one-sector model, a reasonable result given that consumption and investment now are close substitutes.

7 CONCLUSIONS

I ask how and to what extent international business cycle disturbances generate macroeconomic fluctuations in small open economies. To shed some light on this question, I

construct and estimate a medium scale small open economy model with several shocks and frictions typically used in the DSGE literature. The model is embedded with i) trade between firms in intermediate goods, and ii) sectoral producer heterogeneity. These extensions to the workhorse one-sector open economy model are sufficient to reconcile DSGE theory with data along international dimensions.

When the model is fitted to Canadian and US data, a set of important empirical results emerge: First foreign shocks explain a major share of macroeconomic fluctuations in the SOE. Second, posterior estimates emphasize the role of productivity, in the sense that technology shocks, not investment efficiency fluctuations, are the major drivers of business cycles. Third, foreign shocks become increasingly important over longer forecasting horizons. Fourth, the model generates substantial business cycle synchronization even though shocks are uncorrelated. Fifth, exchange rate pass-through is moderate, with sectoral pass-through depending on the frequency of price changes. While these results are consistent with reduced form literature such as VAR and FAVAR studies, they are typically not found in the literature using open economy DGSE models.

The model presented here allows us to gain insight about the mechanisms that cause these results. An important implication of intermediate trade is that it synchronizes producer prices and costs in the cross-section of firms, both within and across borders. This helps in generating co-movement in an environment with producer heterogeneity and otherwise segmented markets. Foreign shocks in particular can enter the SOE through some industries exposed to international trade, and then propagate to others via domestic factor markets. Synchronized producer prices across sectors and countries generate substantial international co-movement in i) current and future real interest rates, which determines consumption, and ii) the expected path of capital returns, a key statistic for investment decisions. However, synchronization of real interest rates comes at the cost of too high consumption co-movement across countries. I find that foreign technology shocks are particularly well suited for international business cycle synchronization. These are also relatively persistent, an important reason why foreign shocks explain more of the forecast error in domestic variables at longer forecasting horizons. Foreign investment efficiency shocks on the other hand cause international divergence in the present value of capital. Investment is positively correlated across countries, implying that the likelihood based estimation procedure attributes a smaller role to investment efficiency shocks.

One obvious limitation with the present model is the lack of meaningful interactions between financial markets and the macroeconomy. Indeed, the recent financial crisis has demonstrated the potential importance of financial frictions for international business cycles. By know, there is a large (an growing) literature on financial frictions in closed economies, and their implications for monetary and fiscal policy. Yet, for many, if not most small open economies, the recent financial crisis was a foreign shock. Therefore, a topic for future research is the propagation of financial distress across countries, e.g. an open economy extension of the market frictions studied by Christiano et al. (2014). However, for such an analysis to make sense, one should be equipped with a model that can account for macroeconomic spillover as well. This paper offers a preliminary, but instructive step towards that end.

APPENDIX

A   THE FULL MODEL

I establish a general equilibrium system consisting of two blocks (referred to as home and foreign), where the home block is a small scale version of its foreign counterpart. The foreign block is thought of as the rest of the world. I first derive optimality conditions in a general setting where the home economy is arbitrarily large compared to the rest of the world. However, my focus is on the limiting case where the relative size of the home economy goes to zero. General equilibrium is therefore evaluated for this special case. The approach allows me to model the foreign block of the model as a closed economy version of the domestic block. To save space, I only derive the domestic block below.

A.1   ILLUSTRATIVE MODEL OVERVIEW

Figure A.1 summarizes the relevant transaction channels in the model when $\mathcal{J} = 2$. Households buy consumption and investment goods (in all domestic markets), and enjoy leisure. This is financed by labor and capital income, dividends, and transfers. Firms in each sector hire labor, capital and buy materials, to produce consumption goods, investment goods, and production goods (sold as materials to other firms). Domestic supply chains are highlighted by red arrows. The central bank stabilizes inflation.

Figure A.1: A bird's view of the model economies when $\mathcal{J} = 2$
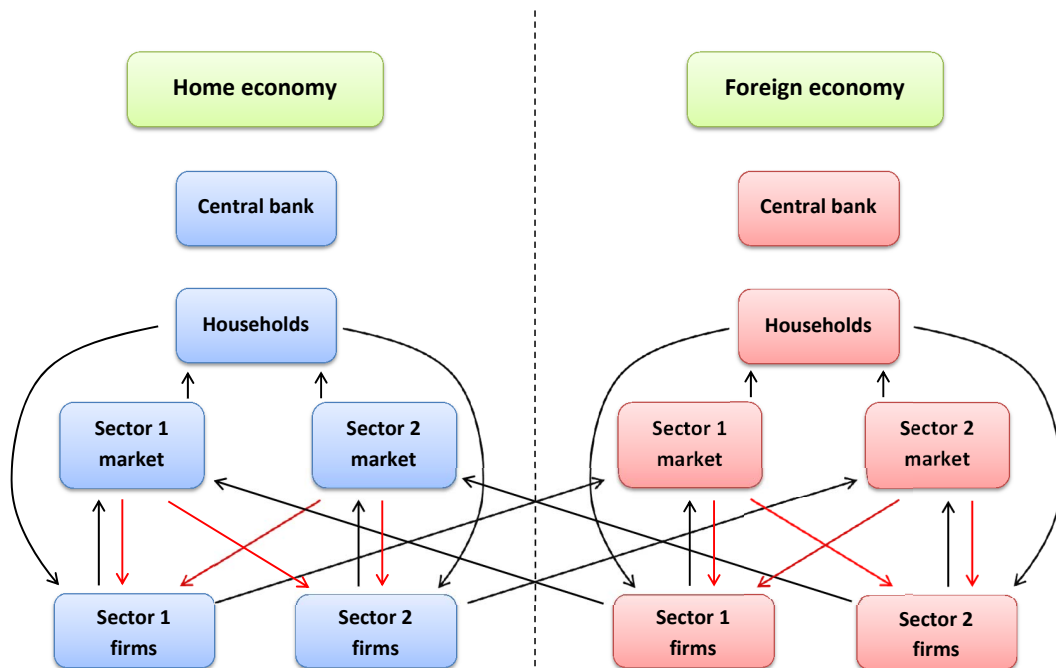
Note: Two-sector version of the model economies. The vertical line represents the country border. Arrows summarize the trade flows (quantities), and supply chain channels are highlighted in red.

A.2 THE NON-LINEAR MODEL

In this section I provide a detailed characterization of the model economy at Home.

A.2.1 HOUSEHOLDS

Household member h working in sector j maximizes lifetime utility given at time t by

$$\mathcal{U}_{j,t}(h) = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Z_{U,s} \left[U_{j,s|t-i}(h) - V_{j,s|t-i}(h) \right],$$

where $U_{j,t|t-i}(h)$ is period t utility of consumption, and $V_{j,t|t-i}(h)$ period t disutility of labor, for a member that was last able to re-optimize the wage i periods ago. $\beta \in (0,1)$ is a time discount factor. Components of period utility are specified in period t as follows:[30]

$$\begin{aligned}
U_{j,t|t-i}(h) &= \frac{(1-\chi_C)^\sigma \left(C_{t|t-i}(h) - \chi_C C_{t-1}\right)^{1-\sigma}}{1-\sigma} \\
V_{j,t|t-i}(h) &= Z_{N,t} \chi_N \frac{L_{j,t|t-i}(h)^{1+\varphi}}{1+\varphi}
\end{aligned} \tag{A.1}$$

Given wage re-optimization i periods ago, $C_{t|t-i}(h)$ denotes period t consumption while $L_{j,t|t-i}(h)$ denotes hours worked for household member h. $Z_{U,t}$ and $Z_{N,t}$ represent stationary shocks to intertemporal preferences and the labor supply, respectively. I assume the existence of a complete set of tradable Arrow securities within each economy. This makes consumption independent of the wage history, i.e. $C_{t|t-i}(h) = C_{t|t}(h) \equiv C_t(h)$. Because the representative household is of measure one, household member h consumption is also aggregate consumption ($C_t(h) = C_t$). I drop the h-subscript whenever possible from now on.

Households buy consumption goods, invest in capital, accumulate domestic and foreign bond assets, and sell of labor services to domestic firms. Maximization of lifetime utility is subject to a sequence of budget constraints. In period t the budget constraint takes the following form:

$$\begin{aligned}
P_t C_t + P_t^i I_t &+ B_{H,t+1} + \mathcal{E}_t B_{F,t+1}^* + \mathbb{E}_t \left\{ \mathcal{Z}_{t,t+1} D_{t+1} \right\} \\
&\leq D_t + W_{j,t}(h) L_{j,t}(h) + R_t^k P_t K_t + P_t \mathcal{D}_t + R_{t-1} B_{H,t} + R_{t-1}^* \Upsilon_{t-1} \mathcal{E}_t B_{F,t}^* - P_t T_t
\end{aligned} \tag{A.2}$$

Domestic households pay a premium on the return on foreign bonds given by $\Upsilon_t = \exp\left[-\epsilon_B \left(A_t - A\right)\right] Z_{B,t}$, where $A_t = \frac{\mathcal{E}_t B_{F,t+1}^*}{P_t GDP} = \mathcal{S}_t \frac{B_{F,t+1}^*}{P_t^* GDP}$ is real net foreign asset holdings as share of steady state GDP. $Z_{B,t}$ captures deviations from uncovered interest rate parity and is referred to as a risk premium shock. Investment in capital is subject to the following capital accumulation equation:

$$K_{t+1} = (1-\delta) K_t + Z_{I,t} \left[1 - F\left(\frac{I_t}{I_{t-1}}\right) \right] I_t, \tag{A.3}$$

where the adjustment cost function F satisfies $F' \geq 0$, $F'' \geq 0$, and $F(1) = F'(1) = 0$.

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[30]The term involving consumption is scaled by $(1-\chi_C)^\sigma$ to render the steady state independent of χ_C.

First I describe optimal demand schedules at the disaggregate level in the SOE. The economy consists of $\mathcal{J}$ different industries or sectors. Final consumption and investment aggregates are composites of consumption and investment goods from each of the different sectors:

$$C_t = \left[\sum_{j=1}^{\mathcal{J}} \xi_j^{\frac{1}{\nu_c}} C_{j,t}^{\frac{\nu_c-1}{\nu_c}} \right]^{\frac{\nu_c}{\nu_c-1}} \qquad\qquad I_t = \left[\sum_{j=1}^{\mathcal{J}} \varpi_j^{\frac{1}{\nu_i}} I_{j,t}^{\frac{\nu_i-1}{\nu_i}} \right]^{\frac{\nu_i}{\nu_i-1}} \qquad \text{(A.4)}$$

For given levels of consumption and investment, the optimal demand for inputs from sector j are given by the following downward sloping demand schedules:

$$C_{j,t} = \xi_j \left(\frac{P_{j,t}}{P_t} \right)^{-\nu_c} C_t \qquad\qquad I_{j,t} = \varpi_j \left(\frac{P_{j,t}}{P_t^i} \right)^{-\nu_i} I_t \qquad \text{(A.5)}$$

Corresponding consumer and investment price indexes are $P_t = \left[\sum_{j=1}^{\mathcal{J}} \xi_j P_{j,t}^{1-\nu_c} \right]^{\frac{1}{1-\nu_c}}$ and $P_t^i = \left[\sum_{j=1}^{\mathcal{J}} \varpi_j P_{j,t}^{1-\nu_i} \right]^{\frac{1}{1-\nu_i}}$. The domestic sector markets are populated by domestic and foreign suppliers. In each of these markets there is trade in private and public consumption goods, investment goods, and intermediate production goods. Demand for consumption $C_{j,t}$ and investment $I_{j,t}$ in sector j are constructed according to a nested CES-structure:

$$C_{j,t} = \left[\bar{\alpha}_j^{\frac{1}{\eta}} C_{Hj,t}^{\frac{\eta-1}{\eta}} + (1-\bar{\alpha}_j)^{\frac{1}{\eta}} C_{Fj,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad I_{j,t} = \left[\bar{\alpha}_j^{\frac{1}{\eta}} I_{Hj,t}^{\frac{\eta-1}{\eta}} + (1-\bar{\alpha}_j)^{\frac{1}{\eta}} I_{Fj,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

$$C_{Hj,t} = \left[\int_0^1 C_{Hj,t}(f)^{\frac{1}{1+\epsilon_{p,t}}} \, df \right]^{1+\epsilon_{p,t}} \qquad I_{Hj,t} = \left[\int_0^1 I_{Hj,t}(f)^{\frac{1}{1+\epsilon_{p,t}}} \, df \right]^{1+\epsilon_{p,t}}$$

$$C_{Fj,t} = \left[\int_0^1 C_{Fj,t}(f)^{\frac{1}{1+\epsilon_{p,t}^*}} \, df \right]^{1+\epsilon_{p,t}^*} \qquad I_{Fj,t} = \left[\int_0^1 I_{Fj,t}(f)^{\frac{1}{1+\epsilon_{p,t}^*}} \, df \right]^{1+\epsilon_{p,t}^*}$$

$C_{Hj,t}$ and $I_{Hj,t}$ are indexes of all the consumption and investment goods $C_{Hj,t}(f)$ and $I_{Hj,t}(f)$, made by each domestic firm $f \in [0,1]$. $C_{Fj,t}$ and $I_{Fj,t}$ are corresponding indexes of all consumption and investment goods $C_{Fj,t}(f)$ and $I_{Fj,t}(f)$, imported from each firm f in the foreign economy. $\epsilon_{p,t}$ is a time varying mark-up on domestically produced goods, while $\epsilon_{p,t}^*$ is the mark-up on imported goods. η is the substitution elasticity between goods from different countries. Sector level quantities in the foreign block, denoted $C_{j,t}^*$ and $I_{j,t}^*$ respectively, are constructed by equivalent systems. Deep production parameters however are allowed to vary across economies. $\bar{\alpha}_j$ and $\bar{\alpha}_j^*$ in particular, which measure the weights of domestic products in the production of final goods, are defined as

$$\bar{\alpha}_j = 1 - (1-\varsigma)(1-\alpha_j) \text{ and } \bar{\alpha}_j^* = 1 - \varsigma\left(1-\alpha_j^*\right).$$

The relative size of the home economy compared to the foreign block is denoted $\varsigma \in [0,1]$, while the degrees of bias toward domestic products in sector j are captured by $\alpha_j \in [0,1]$ and $\alpha_j^* \in [0,1]$.[31] For future reference, note that both $C_{j,t}$ and $I_{j,t}$ consist of both

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[31]This setup encompasses some interesting special cases, including i) complete autarky ($\alpha_j = \alpha_j^* = 1$), ii) perfectly integrated markets ($\alpha_j = \alpha_j^* = 0$), and iii) the limiting case of a small open economy ($\varsigma \to 0$).

domestic and imported goods. However, import shares vary across sectors, so aggregate import shares in C_t and I_t depend on the sectoral weights ξ_j and ϖ_j. Cost minimizing allocations between domestic and imported products, and between single products from each country's sector j, are given in the home economy by

$$C_{Hj,t} = \bar{\alpha}_j \left(\frac{P_{Hj,t}}{P_{j,t}} \right)^{-\eta} C_{j,t}, \qquad I_{Hj,t} = \bar{\alpha}_j \left(\frac{P_{Hj,t}}{P_{j,t}} \right)^{-\eta} I_{j,t},$$

$$C_{Fj,t} = (1 - \bar{\alpha}_j) \left(\frac{P_{Fj,t}}{P_{j,t}} \right)^{-\eta} C_{j,t}, \qquad I_{Fj,t} = (1 - \bar{\alpha}_j) \left(\frac{P_{Fj,t}}{P_{j,t}} \right)^{-\eta} I_{j,t}$$

$$C_{Hj,t}(f) = \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} C_{Hj,t}, \quad I_{Hj,t}(f) = \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} I_{Hj,t},$$

$$C_{Fj,t}(f) = \left(\frac{P_{Fj,t}(f)}{P_{Fj,t}} \right)^{-\frac{1+\epsilon_{p,t}^*}{\epsilon_{p,t}^*}} C_{Fj,t}, \quad I_{Fj,t}(f) = \left(\frac{P_{Fj,t}(f)}{P_{Fj,t}} \right)^{-\frac{1+\epsilon_{p,t}^*}{\epsilon_{p,t}^*}} I_{Fj,t}.$$

$$(A.6)$$

The foreign economy allocates consumption and investment goods according to similar first order conditions. The corresponding price indexes in the SOE follow as

$$P_{j,t} = \left[\bar{\alpha}_j P_{Hj,t}^{1-\eta} + (1 - \bar{\alpha}_j) P_{Fj,t}^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

$$P_{Hj,t} = \left[\int_0^1 P_{Hj,t}(f)^{-\frac{1}{\epsilon_{p,t}}} \, df \right]^{-\epsilon_{p,t}},$$

$$P_{Fj,t} = \left[\int_0^1 P_{Fj,t}(f)^{-\frac{1}{\epsilon_{p,t}^*}} \, df \right]^{-\epsilon_{p,t}^*}.$$

Next I describe optimality conditions with respect to $C_t(h)$, $I_t(h)$, $K_{t+1}(h)$, $B_{H,t+1}(h)$, and $B_{F,t+1}^*(h)$. Let $\Lambda_t(h) \beta^t$ be the (period t) Lagrangian multiplier on equation (A.2), and $\Lambda_t(h) \beta^t \mathcal{Q}_t(h)$ be the multiplier for (A.3). The Lagrangian at time t for household member h working in sector j is stated below, where I abstract from Arrow securities and government transfers:

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} Z_{U,s} \left[\frac{(1-\chi_C)^\sigma (C_s(h) - \chi_C C_{s-1})^{1-\sigma}}{1-\sigma} - Z_{N,s} \chi_N \frac{L_{j,s|s-i}(h)^{1+\varphi}}{1+\varphi} \right] \right.$$

$$- \sum_{s=t}^{\infty} \Lambda_s(h) \beta^{s-t} \left[C_s(h) + \frac{P_s^i}{P_s} I_s(h) + \frac{B_{H,s+1}(h)}{P_s} + \frac{\mathcal{E}_s B_{F,s+1}^*(h)}{P_s} - \frac{W_{j,s|s-i}(h)}{P_s} L_{j,s|s-i}(h) \right]$$

$$+ \sum_{s=t}^{\infty} \Lambda_s(h) \beta^{s-t} \left[R_s^k K_s(h) + \mathcal{D}_s(h) + \left(R_{s-1} \frac{B_{H,s}(h)}{P_{s-1}} + R_{s-1}^* \Upsilon_{s-1} \frac{\mathcal{E}_s B_{F,s}^*(h)}{P_{s-1}} \right) \Pi_s^{-1} \right]$$

$$\left. - \sum_{s=t}^{\infty} \Lambda_s(h) \beta^{s-t} \mathcal{Q}_s(h) \left[K_{s+1}(h) - (1-\delta) K_s(h) - Z_{I,s} \left[1 - F \left(\frac{I_s(h)}{I_{s-1}(h)} \right) \right] I_s(h) \right] \right\}$$

Optimality conditions in period t with respect to consumption, domestic and foreign bond holdings, capital and investment, follow below. To ease the notation I drop the reference to household indexing h:

$$\Lambda_t = Z_{U,t} (1 - \chi_C)^\sigma (C_t - \chi_C C_{t-1})^{-\sigma} \tag{A.7}$$

$$\mathbb{E}_t\left(R_t^{-1}\right) = \beta \mathbb{E}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{-1}\right) \tag{A.8}$$

$$\mathbb{E}_t\left(R_t^{*-1}\right) = \beta \mathbb{E}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{-1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\Upsilon_t\right) \tag{A.9}$$

$$\mathcal{Q}_t = \beta \mathbb{E}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{t+1}^k + (1-\delta)\mathcal{Q}_{t+1}\right]\right) \tag{A.10}$$

$$\frac{P_t^i}{P_t} = \mathcal{Q}_t Z_{I,t}\left[1 - F\left(\frac{I_t}{I_{t-1}}\right) - F'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right]$$
$$+ \beta \mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\mathcal{Q}_{t+1}Z_{I,t+1}F'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{A.11}$$

Equation (A.7) states that maximization of lifetime implies equating the marginal utility of consumption with Λ_t, the shadow value of the budget constraint. Equation (A.8), the optimality condition for domestic bond holdings, defines the optimal intertemporal consumption path by equating the marginal utility loss from less consumption today with the marginal utility gain from more consumption next period. The stochastic discount factor is $\beta \mathbb{E}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{-1}\right)$. Equation (A.9) makes the household indifferent between domestic and foreign bond holdings at the margin. $\mathcal{Q}_t$ in (A.10) can be interpreted as the present value of an additional unit of operational capital in the next period. It is equal to the discounted sum of next period's capital returns and the next period's present value of capital net of depreciation. Finally, equation (A.11) equates the relative price on investment goods with the gain by an additional unit of capital today. One more unit of capital saves $Z_{I,t}\left[1 - F\left(\frac{I_t}{I_{t-1}}\right) - F'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right]$ units of investments. It also reduces expected adjustment costs tomorrow by $\mathbb{E}_t Z_{I,t+1}F'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$ units.

Next I move to the labor market in sector j. I construct sectoral labor markets similar to that in Erceg et al. (2000), but add a friction in the sense that workers cannot move freely between sectors. Denote the measure of household members working in sector j by $\mu_j \in (0,1)$, where $\sum_{j=1}^{\mathcal{J}}\mu_j$, the measure of workers in the economy, is normalized to unity. A competitive labor bundler buys hours from all the household members employed in the sector, and combine these hours into an aggregate labor service $N_{j,t}$. This aggregate takes the form

$$N_{j,t} = \left[\left(\frac{1}{\mu_j}\right)^{\frac{\epsilon_w}{1+\epsilon_w}}\int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j}L_{j,t}(h)^{\frac{1}{1+\epsilon_w}}dh\right]^{1+\epsilon_w}, \tag{A.12}$$

where $\bar{\mu}_j = \sum_{l=1}^{j}\mu_l$ denotes the total mass of workers employed in sectors $1,\ldots,j$. The labor bundler sells his aggregate to all the firms in sector j, charging $W_{j,t}$. He chooses demand for each labor variety to maximize profits, given by

$$W_{j,t}N_{j,t} - \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j}W_{j,t}(h)L_{j,t}(h)\,dh.$$

The optimal number of hours purchased from household member h is

$$L_{j,t}(h) = \frac{1}{\mu_j}\left(\frac{W_{j,t}(h)}{W_{j,t}}\right)^{-\frac{1+\epsilon_w}{\epsilon_w}}N_{j,t} = \left(\frac{W_{j,t}(h)}{W_{j,t}}\right)^{-\frac{1+\epsilon_w}{\epsilon_w}}L_{j,t}, \tag{A.13}$$

where $L_{j,t} = \frac{N_{j,t}}{\mu_j}$ is defined as the average effective labor hours per worker in sector j. The corresponding wage index is $W_{j,t} = \left[\frac{1}{\mu_j} \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} W_{j,t}(h)^{-\frac{1}{\epsilon_w}} dh \right]^{-\epsilon_w}$. Market clearing implies $N_{j,t} = \int_0^1 N_{j,t}(f) \, df$, where $N_{j,t}(f)$ is the amount of labor rented to firm f in sector j. Total hours worked in sector j is

$$\int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} L_{j,t}(h) \, dh = \frac{1}{\mu_j} N_{j,t} \Delta_{wj,t} = \mu_j L_{j,t},$$

where it has been used that $\Delta_{wj,t} = \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} \left(\frac{W_{j,t}(h)}{W_{j,t}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w}} dh = \mu_j$ holds up to a first order. Hours worked per person in the entire economy follows as $L_t = \sum_{j=1}^{\mathcal{J}} \mu_j L_{j,t} = N_t$. Each period, only a fraction $1 - \theta_{wj}$ of the household members working in sector j can re-optimize wages. The remaining $1 - \theta_{wj}$ household members can index wages according to the indexation rule $W_{j,t}(h) = W_{j,t-1}(h) \Pi_{t-1}^{\gamma_w}$. Let $\bar{W}_{j,t}(h)$ denote the optimal wage for a household member h that is able to re-optimize in period t. The wage in period $s > t$ for a member that was last able to re-optimize in period t is then found by backward substitution:

$$W_{j,s|t}(h) = W_{j,s-1|t}(h) \Pi_{s-1}^{\gamma_w} = \bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\gamma_w} \tag{A.14}$$

For this household member equation (A.13) can be written as

$$L_{j,s|t}(h) = \left(\frac{\bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\gamma_w}}{W_{j,s}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w}} L_{j,s}. \tag{A.15}$$

Finally, the Calvo restriction on nominal wage changes implies that

$$W_{j,s+1}(h) = \begin{cases} \bar{W}_{j,s+1}(h) & \text{with probability } 1 - \theta_{wj} \\ W_{j,s}(h) \Pi_s^{\gamma_w} & \text{with probability } \theta_{wj} \end{cases}.$$

A household member who is able to reset the wage in period t, will therefore choose the optimal wage $\bar{W}_{j,t}(h)$ to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_{wj})^{s-t} \left[-Z_{U,s} Z_{N,s} \chi_N \frac{L_{j,s|t}(h)^{1+\varphi}}{1+\varphi} + \Lambda_s \frac{W_{j,s|t}(h)}{P_s} L_{j,s|t}(h) \right]$$

subject to equations (A.14)-(A.15). The relevant first order condition for this problem is

$$0 = \mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_{wj})^{s-t} \Lambda_s \frac{L_{j,s}(h)}{P_s} \left[\bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\gamma_w} - (1 + \epsilon_w) MRS_{j,s|t}(h) P_s \right], \tag{A.16}$$

where $MRS_{j,s|t}(h) = -\frac{u_{j,L}\left(C_s, L_{j,s|t}(h)\right)}{\Lambda_s}$ is the marginal rate of substitution (between consumption and labor) in period s, given a wage last set in period t. Equation (A.16) collapses to $\Omega_{j,t}(h) = (1 + \epsilon_w) MRS_{j,t}(h)$ in the limiting case with flexible wages, where $\Omega_{j,t}(h) = \frac{W_{j,t}}{P_t}$ is the real wage. This holds for all workers, so $\Omega_{j,t}(h) = \Omega_{j,t} \, \forall \, h$ in this

case. In the more general case with nominal wage stickiness, one can combine (A.16) with the equation linking individual and aggregate marginal rates of substitution,

$$MRS_{j,s|t}(h) = \left(\frac{\bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\gamma_w}}{W_{j,s}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w}\varphi} MRS_{j,s},$$

and the law of motion for aggregate wages in sector j,

$$W_{j,t} = \left[\theta_{wj} \left(W_{j,t-1} \Pi_{t-1}^{\gamma_w} \right)^{-\frac{1}{\epsilon_w}} + (1 - \theta_{wj}) \bar{W}_{j,t}^{-\frac{1}{\epsilon_w}} \right]^{-\epsilon_w},$$

to derive the New Keynesian wage Phillips curve.

A.2.2 FIRMS

In this section I describe the domestic production process in detail. Output of domestic firm f in sector j is given by a Cobb-Douglas production function augmented with fixed costs:

$$Y_{j,t}(f) = Z_{Aj,t} M_{j,t}(f)^{\phi_j} N_{j,t}(f)^{\psi_j} K_{j,t}(f)^{1-\phi_j-\psi_j} - \Phi_j, \tag{A.17}$$

where $M_{j,t}(f)$, $N_{j,t}(f)$ and $K_{j,t}(f)$ are firm f's use of materials, labor and capital respectively. Φ_j is a fixed production cost that will be calibrated to ensure zero profit in steady state. Constant returns to scale in variable output implies $\phi_j, \psi_j, (\phi_j + \psi_j) \in (0,1)$. $Z_{Aj,t}$ is sector specific productivity.

Intermediate trade is modeled as in Bouakez et al. (2009) and Bergholt and Sveen (2014). Monopolistic firms in sector j buy a composite of different materials produced in the different sectors. The materials input aggregate in sector j is given by

$$M_{j,t} = \left[\sum_{l=1}^{\mathcal{J}} \zeta_{lj}^{\frac{1}{\nu_m}} M_{lj,t}^{\frac{\nu_m-1}{\nu_m}} \right]^{\frac{\nu_m}{\nu_m-1}}, \tag{A.18}$$

where $\sum_{l=1}^{\mathcal{J}} \zeta_{lj} = 1$ and $\zeta_{lj} \in (0,1)$. The materials are distributed such that $M_{j,t} = \int_0^1 M_{j,t}(f)\, df$. Optimal demand for materials from sector l follows as

$$M_{lj,t} = \zeta_{lj} \left(\frac{P_{l,t}}{P_{j,t}^m} \right)^{-\nu_m} M_{j,t}, \tag{A.19}$$

where $P_{j,t}^m = \left[\sum_{l=1}^{\mathcal{J}} \zeta_{lj} P_{l,t}^{1-\nu_m} \right]^{\frac{1}{1-\nu_m}}$ is the relevant price index for intermediate inputs in sector j.

A detailed sketch of the input-output matrix in the domestic economy is provided in Figure A.2 for the case $\mathcal{J} = 2$. The first column shows the total material costs in sector 1, where $M_{11,t}$ and $M_{21,t}$ are the quantities firms in this sector are buying from sectors 1 and 2, respectively. In the same way, sector 2 material costs are the sum of the elements in the second column. The first row then denotes the total value of materials sold from sector 1 to itself ($P_{1,t}M_{11,t}$) and sector 2 ($P_{1,t}M_{12,t}$), respectively. More generally, firms in sector j take as inputs the materials composite and labor service specific to that sector,

and produces output sold to i) domestic households, ii) domestic firms, and iii) foreign households and firms.

Figure A.2: The I-O matrix for domestic markets when $\mathcal{J} = 2$
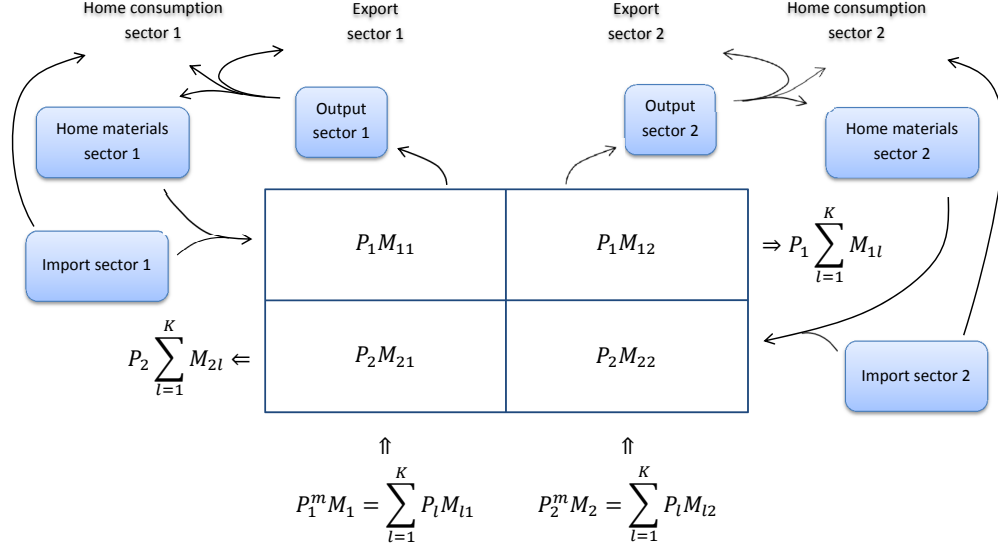
Note: Input-output matrix in a two-sector version of our model economy. Arrows summarize the trade flows.

Given the flows of intermediate goods across domestic producer markets, one can find cost minimizing allocations between domestic and imported intermediates, and between single intermediates from each country's sector j, as follows:

$$M_{Hlj,t} = \bar{\alpha}_l \left(\frac{P_{Hl,t}}{P_{l,t}} \right)^{-\eta} M_{lj,t}, \qquad M_{Flj,t} = (1 - \bar{\alpha}_l) \left(\frac{P_{Fl,t}}{P_{l,t}} \right)^{-\eta} M_{lj,t},$$

$$M_{Hlj,t}(f) = \left(\frac{P_{Hl,t}(f)}{P_{Hl,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} M_{Hlj,t}, \quad M_{Flj,t}(f) = \left(\frac{P_{Fl,t}(f)}{P_{Fl,t}} \right)^{-\frac{1+\epsilon_{p,t}^*}{\epsilon_{p,t}^*}} M_{Flj,t}.$$

Next I describe the general profit maximization problem that emerges once intermediate goods have been allocated. Price setting by domestic and foreign firms is subject to monopoly supply power and sticky prices in a way analogous to the labor market. Firms set prices á la Calvo (1983) and Yun (1996), but export goods are priced in local currency (LCP). Denote prices set by domestic producer f in sector j by $P_{Hj,t}(f)$ and $P_{Hj,t}^*(f)$ respectively, where the first is on goods sold at home and the second on exported goods. Let $1 - \theta_{pj}$ denote the probability that a given producer is able to reset his prices. The fraction θ_{pj} of firms that is not able to re-optimize prices, update them according to the indexation rules $P_{Hj,t}(f) = P_{Hj,t-1}(f)\Pi_{Hj,t-1}^{\gamma_p}$ and $P_{Hj,t}^*(f) = P_{Hj,t-1}^*(f)\Pi_{Hj,t-1}^{*\gamma_p}$, where $\Pi_{Hj,t} = \frac{P_{Hj,t}}{P_{Hj,t-1}}$ and $\Pi_{Hj,t}^* = \frac{P_{Hj,t}^*}{P_{Hj,t-1}^*}$ are gross inflation rates. Let $\bar{P}_{jH,t}(f)$ and $\bar{P}_{Hj,t}^*(f)$ denote optimal prices for a firm f that is able to re-optimize in period t. Prices for a firm that was last able to re-optimize $s - t$ periods ago are found by backward substitution:

$$P_{Hj,s|t}(f) = P_{Hj,s-1|t}(f)\Pi_{Hj,s-1}^{\gamma_p} = \bar{P}_{Hj,t}(f) \prod_{i=1}^{s-t} \Pi_{Hj,s-i}^{\gamma_p} \qquad \text{(A.20)}$$

$$P^*_{Hj,s|t}(f) = P^*_{Hj,s-1|t}(f)\Pi^{*\gamma_p}_{Hj,s-1} = \bar{P}^*_{Hj,t}(f)\prod_{i=1}^{s-t}\Pi^{*\gamma_p}_{Hj,s-i} \tag{A.21}$$

Define domestic and foreign absorption of output, produced by firm f in sector j, as follows:

$$X_{Hj,t}(f) = C_{Hj,t}(f) + I_{Hj,t}(f) + \sum_{l=1}^{\mathcal{J}} M_{Hjl,t}(f) + G_{Hj,t}(f)$$

$$\tilde{X}^*_{Hj,t}(f) = \tilde{C}^*_{Hj,t}(f) + \tilde{I}^*_{Hj,t}(f) + \sum_{l=1}^{\mathcal{J}} \tilde{M}^*_{Hj,t}(f) + \tilde{G}^*_{Hj,t}(f)$$

These quantities are in per capita terms as seen from the small open economy. The individual firm then chooses a plan $\mathcal{P}_{j,t}(f)$ for production, supply, prices, and inputs,

$$\mathcal{P}_{j,t}(f) = \begin{array}{l} \left\{Y_{j,s}(f), X_{Hj,s}(f), \tilde{X}^*_{Hj,s}(f)\right\}_{s=t}^{\infty} \\ \left\{P_{Hj,s}(f), P^*_{Hj,s}(f)\right\}_{s=t}^{\infty} \\ \left\{M_{j,s}(f), N_{j,s}(f), K_{j,s}(f)\right\}_{s=t}^{\infty} \end{array},$$

to maximize an expected discounted dividend stream given by

$$\mathbb{E}_t \sum_{s=t}^{\infty} \mathcal{Z}_{t,s} P_s \mathcal{D}_{j,s}(f),$$

where time s dividends and total costs in terms of consumption goods are given by

$$\mathcal{D}_{j,s}(f) = P_{rHj,s}(f) X_{Hj,s}(f) + P^*_{rHj,s}(f)\tilde{X}^F_{Hj,s}(f) - TC_{rj,s}(f) \text{ and}$$
$$TC_{rj,s}(f) = P^m_{rj,s} M_{j,s}(f) + \Omega_{j,s} N_{j,s}(f) + R^k_s K_{j,s}(f),$$

respectively. The stochastic discount factor is defined as $\mathcal{Z}_{t,s} = \beta^{s-t}\frac{\Lambda_s}{\Lambda_t}\frac{P_t}{P_s}$, the real price on materials as $P^m_{rj,s} = \frac{P^m_{j,s}}{P_s}$, while $P^*_{rHj,s}(f) = \frac{\mathcal{E}_s P^*_{Hj,s}(f)}{P_s}$ is the domestic currency price of exports of f-goods. $\mathcal{E}_s$ is the nominal exchange rate between the domestic and the foreign currency. Profit maximization is subject to a set of constraints:

$$X_{Hj,s}(f) + \tilde{X}^F_{Hj,s}(f) = Y_{j,s}(f)$$
$$Y_{j,s}(f) = Z_{Aj,s} M_{j,s}(f)^{\phi_j} N_{j,s}(f)^{\psi_j} K_{j,s}(f)^{1-\phi_j-\psi_j} - \Phi_j$$
$$X_{Hj,s}(f) = \left(\frac{P_{Hj,s}(f)}{P_{Hj,s}}\right)^{-\frac{1+\epsilon_{p,s}}{\epsilon_{p,s}}} X_{Hj,s}$$
$$\tilde{X}^*_{Hj,s}(f) = \left(\frac{P^*_{Hj,s}(f)}{P^*_{Hj,s}}\right)^{-\frac{1+\epsilon_{p,s}}{\epsilon_{p,s}}} \tilde{X}^*_{Hj,s}$$
$$P_{Hj,s+1}(f) = \begin{cases} \bar{P}_{Hj,s+1}(f) & \text{with probability } 1-\theta_{pj} \\ P_{Hj,s}(f)\,\Pi^{\gamma_p}_{Hj,s} & \text{with probability } \theta_{pj} \end{cases}$$
$$P^*_{Hj,s+1}(f) = \begin{cases} \bar{P}^*_{Hj,s+1}(f) & \text{with probability } 1-\theta_{pj} \\ P^*_{Hj,s}(f)\,\Pi^{*\gamma_p}_{Hj,s} & \text{with probability } \theta_{pj} \end{cases}$$

The first constraint is a market clearing condition, the second a technological constraint, and the third and fourth the demand schedules faced by firm j. Domestic and foreign absorption of domestically produced sector j goods are defined as follows:

$$X_{Hj,t} = C_{Hj,t} + I_{Hj,t} + \sum_{l=1}^{\mathcal{J}} M_{Hjl,t} + G_{Hj,t}$$

$$\tilde{X}_{Hj,t}^* = \tilde{C}_{Hj,t}^* + \tilde{I}_{Hj,t}^* + \sum_{l=1}^{\mathcal{J}} \tilde{M}_{Hj,t}^* + \tilde{G}_{Hj,t}^*$$

Optimality conditions with respect to $Y_{j,t}(f)$, $X_{Hj,t}(f)$, $\tilde{X}_{Hj,t}^*(f)$, $M_{j,t}(f)$, $N_{j,t}(f)$, and $K_{j,t}(f)$, are stated below. $\Xi_j(f)$, $MC_j(f)$, $\Gamma_j(f)$ and $\Gamma_j^*(f)$ represent the Lagrangian multipliers on the constraints.

$$\Xi_{j,t}(f) = MC_{j,t}(f) \tag{A.22}$$

$$\Gamma_{j,t}(f) = P_{Hj,t}(f) - MC_{j,t}(f) \tag{A.23}$$

$$\Gamma_{j,t}^F(f) = \mathcal{E}_t P_{Hj,t}^F(f) - MC_{j,t}(f) \tag{A.24}$$

$$MC_{j,t}(f) = \frac{P_{j,t}^m}{MPM_{j,t}(f)} \tag{A.25}$$

$$MC_{j,t}(f) = \frac{W_{j,t}}{MPL_{j,t}(f)} \tag{A.26}$$

$$MC_{j,t}(f) = \frac{R_t^k P_t}{MPK_{j,t}(f)} \tag{A.27}$$

The marginal products of material, labor and capital for firm f in sector j are denoted $MPM_{j,t}(f)$, $MPL_{j,t}(f)$, and $MPK_{j,t}(f)$ respectively. Optimality conditions (A.25)-(A.27) can be summarized by two equations determining the optimal use of relative inputs:

$$\frac{M_{j,t}(f)}{N_{j,t}(f)} = \frac{\phi_j}{\psi_j} \frac{\Omega_{j,t}}{P_{rj,t}^m}$$

$$\frac{N_{j,t}(f)}{K_{j,t}(f)} = \frac{\psi_j}{1 - \phi_j - \psi_j} \frac{R_t^k}{\Omega_{j,t}}$$

Next I state the optimality conditions with respect to $P_{Hj}(f)$ and $P_{Hj}^*(f)$:

$$0 = E_t \sum_{s=t}^{\infty} (\theta_{pj})^{s-t} \mathcal{Z}_{t,s} X_{Hj,s}(f) \left[\bar{P}_{Hj,t}(f) \prod_{i=1}^{s-t} \Pi_{Hj,s-i}^{\gamma_p} - (1 + \epsilon_{p,s}) MC_{j,s}(f) \right] \tag{A.28}$$

$$0 = E_t \sum_{s=t}^{\infty} (\theta_{pj})^{s-t} \mathcal{Z}_{t,s} X_{Hj,s}^*(f) \mathcal{E}_s \left[\bar{P}_{Hj,t}^*(f) \prod_{i=1}^{s-t} \Pi_{Hj,s-i}^{*\gamma_p} - (1 + \epsilon_{p,s}) MC_{j,s}(f) \right] \tag{A.29}$$

In the limiting case with flexible prices, these first order conditions collapse to $\frac{P_{Hj,t}}{P_t} = \frac{\mathcal{E}_t P_{Hj,t}^*}{P_t} = (1 + \epsilon_{p,t}) RMC_{j,t}$ for all firms. The law of one price holds period by period in this case. It is clear from equation (A.17) and (A.25)-(A.27) that all firms in sector j face the same marginal cost. The real marginal cost can be written as

$$RMC_{j,t} = \frac{1}{Z_{Aj,t}} \left(\frac{P_{rj,t}^m}{\phi_j} \right)^{\phi_j} \left(\frac{\Omega_{j,t}}{\psi_j} \right)^{\psi_j} \left(\frac{R_t^k}{1 - \phi_j - \psi_j} \right)^{1 - \phi_j - \psi_j}, \tag{A.30}$$

where $RMC_{j,t} = \frac{MC_{j,t}}{P_t}$ measures costs in terms of consumption goods. Finally, the staggered price setting structure combined with partial indexation implies that prices of domestically produced goods can be written as follows:

$$P_{Hj,t} = \left[\theta_{pj} \left(P_{Hj,t-1} \Pi_{Hj,t-1}^{\gamma_p} \right)^{-\frac{1}{\epsilon_{p,t}}} + (1 - \theta_{pj}) \bar{P}_{Hj,t}^{-\frac{1}{\epsilon_{p,t}}} \right]^{-\epsilon_{p,t}}$$

$$P_{Hj,t}^* = \left[\theta_{pj} \left(P_{Hj,t-1}^* \Pi_{Hj,t-1}^{*\gamma_p} \right)^{-\frac{1}{\epsilon_{p,t}}} + (1 - \theta_{pj}) \bar{P}_{Hj,t}^{*-\frac{1}{\epsilon_{p,t}}} \right]^{-\epsilon_{p,t}}$$

One can combine these with the optimality conditions for prices to derive two New Keynesian price Phillips curve for domestic goods and exports.

A.2.3 MONETARY AND FISCAL POLICY

Monetary authorities are assumed to follow an extended Taylor-rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\rho_\pi} \left(\frac{GDP_t}{GDP} \right)^{\rho_y} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{\rho_{dy}} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \right)^{\rho_e} \right]^{1-\rho_r} Z_{R,t} \quad \text{(A.31)}$$

$Z_{R,t}$ is a monetary policy shock. Fiscal authorities face a period-by-period budget constraint of the form

$$P_t^g G_t + R_{t-1} B_{H,t} = B_{H,t+1} + P_t T_t$$

I assume that public debt is zero in steady state. This implies that government spending is fully financed by lump-sum taxes up to a first order approximation. Finally I let government spending be shocked according to the process $\frac{G_t}{G} = \exp(\varepsilon_{G,t}) \left(\frac{G_{t-1}}{G} \right)^{\rho_G}$, where $\varepsilon_{G,t}$ is a public spending shock.

A.2.4 MARKET CLEARING AND AGGREGATION

Trade between the world economy and the SOE becomes neglible from the world economy's point of view when $\varsigma \to 0$. Previously I defined $\tilde{X}_{Hj,t}^*(f)$ as home firm f's export units per home capita. Similarly, let $X_{Hj,t}^*(f)$ denote home firm f's export units per foreign capita. These two are linked via the identity $\tilde{X}_{Hj,t}^*(f) = \frac{1-\varsigma}{\varsigma} X_{Hj,t}^*(f)$. When (A.6) and the relevant optimality condition for foreign import are evaluated in the limit as $\varsigma \to 0$, we get the following system of trade demand schedules in the small open economy:

$$X_{Hj,t} = \alpha_j \left(\frac{P_{Hj,t}}{P_{j,t}} \right)^{-\eta} X_{j,t}, \quad \text{(A.32)}$$

$$X_{Fj,t} = (1 - \alpha_j) \left(\frac{P_{Fj,t}}{P_{j,t}} \right)^{-\eta} X_{j,t}, \quad \text{(A.33)}$$

$$\tilde{X}_{Hj,t}^* = \frac{1-\varsigma}{\varsigma} X_{Hj,t}^* = \left(1 - \alpha_j^* \right) \left(\frac{P_{Hj,t}^*}{P_{j,t}^*} \right)^{-\eta} X_{j,t}^* \quad \text{(A.34)}$$

Here total absorption in the domestic sector market j is defined as

$$X_{j,t} = C_{j,t} + I_{j,t} + \sum_{l=1}^{\mathcal{J}} M_{jl,t} + G_{j,t} \qquad (\text{A.35})$$

Moreover, it is clear from the expressions for $X_{Fj,t}^*$ and $X_{Hj,t}^*$ in the foreign block, as well as the export demand schedule $\tilde{X}_{Fj,t}^H$, that $\varsigma \to 0$ implies

$$X_{Fj,t}^* = \bar{\alpha}_j^* \left(\frac{P_{Fj,t}^*}{P_{j,t}^F} \right)^{-\eta} X_{j,t}^* = X_{j,t}^*, \qquad (\text{A.36})$$

$$X_{Hj,t}^* = \left(1 - \bar{\alpha}_j^*\right) \left(\frac{P_{Hj,t}^*}{P_{j,t}^*} \right)^{-\eta} X_{j,t}^* = 0, \text{ and} \qquad (\text{A.37})$$

$$\tilde{X}_{Fj,t}^H = \frac{\varsigma}{1 - \varsigma} X_{Fj,t} = 0. \qquad (\text{A.38})$$

The first line uses $\lim_{\varsigma \to 0} P_{Fj,t}^F = P_{j,t}^F$. Aggregate output in sector j is

$$Y_{j,t} = \int_0^1 Y_{j,t}(f) \, df = X_{Hj,t} \Delta_{Hj,t} + \tilde{X}_{Hj,t}^F \Delta_{Hj,t}^F, \qquad (\text{A.39})$$

where the two relative price dispersion terms $\Delta_{Hj,t} = \int_0^1 \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} df$ and $\Delta_{Hj,t}^* = \int_0^1 \left(\frac{P_{Hj,t}^*(f)}{P_{Hj,t}^*} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} df$ are equal to one up to a first order. Nominal gross sales in sector j is $P_{Hj,t} X_{Hj,t} + \mathcal{E}_t P_{Hj,t}^* \tilde{X}_{Hj,t}^*$. Real value added, which is the nominal value added denominated by the CPI, can be written in three different, but model consistent ways:

$$\begin{aligned}
GDP_{j,t} &= \frac{P_{Hj,t}}{P_t} X_{Hj,t} + \frac{\mathcal{E}_t P_{Hj,t}^*}{P_t} \tilde{X}_{Hj,t}^* - \frac{P_{j,t}^m}{P_t} M_{j,t} \\
&= \Omega_{j,t} N_{j,t} + R_t^k K_{j,t} + \mathcal{D}_{j,t} \\
&= \frac{P_{j,t}}{P_t} \left(C_{j,t} + I_{j,t} + G_{j,t} \right) + TB_{j,t} + \frac{1}{P_t} \left(P_{j,t} \sum_{l=1}^{\mathcal{J}} M_{jl,t} - P_{j,t}^m M_{j,t} \right) \quad (\text{A.40})
\end{aligned}$$

The first line defines GDP in sector j according to the output approach, i.e. as the value of gross output minus the value of intermediate consumption. The second line measures GDP according to the income approach. A no arbitrage condition implies that real dividends from a portfolio of stocks in sector j, $\mathcal{D}_{j,t} = \int_0^1 \mathcal{D}_{j,t}(f) \, df$, is zero in the steady state. The last line in (A.40) uses the expenditure approach, where one calculates the integral of all domestic demand functions. The trade balance in sector j is given by

$$TB_{j,t} = \frac{\mathcal{E}_t P_{Hj,t}^*}{P_t} \tilde{X}_{Hj,t}^* - \frac{P_{Fj,t}}{P_t} X_{Fj,t}. \qquad (\text{A.41})$$

Economywide GDP is defined as $GDP_t = \sum_{j=1}^{\mathcal{J}} GDP_{j,t}$. Thus, one can aggregate the second line of equation (A.40) over all j. The result is $GDP_t = \Omega_t N_t + R_t^k K_t + \mathcal{D}_t$. A

more familiar expression is found by combining this with the representative household's budget constraint, which must hold with equality. Then we get

$$GDP_t = C_t + \frac{P_t^i}{P_t}I_t + \frac{\mathcal{E}_t B_{F,t+1}^*}{P_t} - R_{t-1}^* \Upsilon_t \frac{\mathcal{E}_t B_{F,t}^*}{P_t} + \frac{B_{H,t+1}}{P_t} - R_{t-1}\frac{B_{H,t}}{P_t} + T_t$$

$$= C_t + \frac{P_t^i}{P_t}I_t + \frac{P_t^g}{P_t}G_t + TB_t, \tag{A.42}$$

where the last line follows from the budget constraint of the government and the current account identity

$$TB_t = \frac{\mathcal{E}_t B_{F,t+1}^*}{P_t} - R_{t-1}^* \Upsilon_{t-1}\frac{\mathcal{E}_t B_{F,t}^*}{P_t}. \tag{A.43}$$

The identity simply states that positive trade balances are used to accumulate foreign assets. Another way to derive (A.42) is by summing the last line in (A.40) over all j and noting that $TB_t = \sum_{j=1}^{\mathcal{J}} TB_{j,t}$. From the foreign economy's point of view, their debt is in zero net supply because the home economy engages in only a neglible part of the financial assets trade. Furthermore, I assume that foreign investors do not hold financial assets in the home economy. Equilibrium in the foreign bonds market is finally represented by a modified uncovered interest rate parity condition, found from (A.8) and (A.9):

$$\mathbb{E}_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_{t+1}^{-1}\left(R_t - \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}R_t^*\Upsilon_t\right)\right\} = 0 \tag{A.44}$$

The foreign economy is characterized by a similar system of equations, except that trade constitutes a negligible part of economic activity. This completes the description of the model.

B STEADY STATE AND LINEARIZED MODEL

In this section I provide i) the full steady state system of the model, ii) a recursive solution for the steady state, under the restrictions of unitary relative prices and balanced trade, and iii) the complete log-linearized model.

B.1 THE FULL STEADY STATE SYSTEM

Denote the steady state level of any variable without the t-subscript, e.g. the steady state level of X_t as X. The steady state equilibrium system for the small open economy follows below. The world economy is modeled as a closed economy version of the model described above, and has a similar steady state (not shown):

$$\left(\frac{C}{C^F}\right)^\sigma = \mathcal{S}$$

$$1 = \sum_{j=1}^{\mathcal{J}} \xi_j P_{rj}^{1-\nu_c}$$

$$\Omega = (1 + \epsilon_w)\chi_N C^\sigma L^\varphi$$

$$\sum_{j=1}^{\mathcal{J}} \mu_j = 1$$

$$I = \delta \sum_{j=1}^{\mathcal{J}} K_j$$

$$R^k = \mathcal{Q}\left(\beta^{-1} - (1-\delta)\right)$$

$$P_r^{i\,1-\nu_i} = \sum_{j=1}^{\mathcal{J}} \varpi_j P_{rj}^{\,1-\nu_i}$$

$$\mathcal{Q} = P_r^i$$

$$C_j = \xi_j P_{rj}^{-\nu_c} C$$

$$\sum_{j=1}^{\mathcal{J}} \xi_j = 1$$

$$I_j = \varpi_j \left(\frac{P_{rj}}{P_r^i}\right)^{-\nu_i} I$$

$$\sum_{l=1}^{\mathcal{J}} \varpi_j = 1$$

$$M_{lj} = \zeta_{lj} \left(\frac{P_{rl}}{P_{rj}^m}\right)^{-\nu_m} M_j$$

$$\sum_{l=1}^{\mathcal{J}} \zeta_{lj} = 1$$

$$P_{rj}^{m\,1-\nu_m} = \sum_{l=1}^{\mathcal{J}} \zeta_{lj} P_{rl}^{\,1-\nu_m}$$

$$L = \frac{N_j}{\mu_j}$$

$$X_j = C_j + I_j + \sum_{l=1}^{\mathcal{J}} M_{jl} + G_j$$

$$Y_j = X_{Hj} + X_{Hj}^*$$

$$P_{rHj} Y_j = P_{rj}^m M_j + \Omega N_j + R^k K_j$$

$$Y_j = M_j^{\phi_j} N_j^{\psi_j} K_j^{1-\phi_j-\psi_j} - \Phi_j$$

$$\frac{N_j}{K_j} = \frac{\psi_j}{1-\phi_j-\psi_j} \frac{R^k}{\Omega}$$

$$\frac{M_j}{N_j} = \frac{\phi_j}{\psi_j} \frac{\Omega}{P_{rj}^m}$$

$$RMC_j = \left(\frac{P_{rj}^m}{\phi_j}\right)^{\phi_j} \left(\frac{\Omega}{\psi_j}\right)^{\psi_j} \left(\frac{R^k}{1-\phi_j-\psi_j}\right)^{1-\phi_j-\psi_j}$$

$$P_{rHj} = (1+\epsilon_p) RMC_j$$

$$P_{rFj} = (1+\epsilon_p) RMC_j^F \mathcal{S}$$

$$P_{rj}^{1-\eta} = \alpha_j P_{rHj}^{1-\eta} + (1-\alpha_j) P_{rFj}^{1-\eta}$$

$$X_{Hj} = \alpha_j \left(\frac{P_{rHj}}{P_{rj}} \right)^{-\eta} X_j$$

$$X_{Fj} = (1 - \alpha_j) \left(\frac{P_{rFj}}{P_{rj}} \right)^{-\eta} X_j$$

$$X_{Hj}^* = (1 - \alpha_j) \left(\frac{P_{rHj}}{P_{rj}^F} \right)^{-\eta} \mathcal{S}^{\eta} X_j^*$$

Additional equations:

$$GDP_j = P_{rHj} Y_j - P_{rj}^m M_j$$

$$GDP = \sum_{j=1}^{\mathcal{J}} GDP_j$$

$$TB_j = P_{rHj} X_{Hj}^* - P_{rFj} X_{Fj}$$

$$TB = \sum_{j=1}^{\mathcal{J}} TB_j$$

$$\mathcal{T}_j = \frac{P_{rHj}}{P_{rFj}}$$

B.2 RECURSIVE SOLUTION OF STEADY STATE VARIABLES

Next, I derive an analytical solution for the steady state system above. I restrict the analysis to an equilibrium with balanced trade, zero public spending, and relative prices equal to unity, i.e. $P_{Hrj} = P_{Frj} = P_{rj}^* = 1$. Time preferences, markup, and capital depreciation rates are also assumed to be the same in both countries. First I solve recursively for the steady in the foreign economy. Second, I use that solution as input to find steady state in the small open economy.

B.2.1 THE FOREIGN (CLOSED) ECONOMY

We get steady state investment and material prices from $P_{rj}^* = 1$, $\sum_{j=1}^{\mathcal{J}} \varpi_j^* = 1$, and $\sum_{j=1}^{\mathcal{J}} \zeta_{lj}^* = 1$:

$$P_r^{i*} = \left(\sum_{j=1}^{\mathcal{J}} \varpi_j^* P_{rj}^{*\,1-\nu_i} \right)^{\frac{1}{1-\nu_i}} = 1$$

$$P_{rj}^{m*} = \left(\sum_{l=1}^{\mathcal{J}} \zeta_{lj}^* P_{rl}^{*\,1-\nu_m} \right)^{\frac{1}{1-\nu_m}} = 1$$

From (A.10):

$$\mathcal{Q}^* = P_r^{i*} = 1$$

From (A.9):

$$R^{k*} = \mathcal{Q}^* \left[\beta^{-1} - (1 - \delta) \right] = \beta^{-1} - (1 - \delta)$$

From $P_{rj}^* = 1$ and (A.28):

$$RMC_j^* = \frac{1}{1 + \epsilon_p}$$

Without loss of generality I normalize $C^* = 1$. Thus, steady state variables are measured in consumption units. Foreign consumption at the sector level follows:

$$C_j^* = \xi_j^* C^* = \xi_j^* \qquad \forall\, j$$

Next, I set out to derive sector level output. Note first that the large economy assumption implies $Y_j^* = X_j^* \ \forall \ j$. Second, using (the foreign economy versions of) equations (A.3), (A.5), (A.19), and (A.26), we can write $I^* = \delta \sum_{j=1}^{\mathcal{J}} K_j^*$, $I_j^* = \varpi_j^* I^*$, $K_j^* = \frac{1 - \phi_j^* - \psi_j^*}{R^{k*}} Y_j^*$, and $M_j^* = \phi_j^* Y_j^*$. Combining these expressions with the market clearing condition $Y_j^* = C_j^* + I_j^* + \sum_{l=1}^{\mathcal{J}} M_{jl}^*$, the foreign economy's production network follows in compact form as

$$\mathbf{Y}^* = \mathbf{C}^* + \mathbf{\Psi}^* \mathbf{Y}^*,$$

where $\mathbf{Y}^* = \left[Y_1^*, \ldots, Y_{\mathcal{J}}^*\right]'$ is the output vector and $\mathbf{C}^* = \left[C_1^*, \ldots, C_{\mathcal{J}}^*\right]'$ is the final consumption vector. The (j, l)'th element of the $\mathcal{J} \times \mathcal{J}$-matrix $\mathbf{\Psi}^*$ is equal to

$$\Psi_{jl}^* = \zeta_{jl}^* \phi_l^* + \varpi_j^* \delta \frac{1 - \phi_l^* - \psi_l^*}{\beta^{-1} - 1 + \delta}.$$

Standard matrix manipulation therefore gives us the following solution for gross output:

$$\mathbf{Y}^* = \tilde{\mathbf{\Psi}}^* \mathbf{C}^*,$$

with $\tilde{\mathbf{\Psi}}^* = \left[\mathbf{1} - \mathbf{\Psi}^*\right]^{-1}$ being referred to as the steady state influence matrix. $\mathbf{1} - \mathbf{\Psi}^*$ is invertible under mild conditions, at least given the high level of aggregation considered here.[32] Moreover, $\det\left(\mathbf{1} - \mathbf{\Psi}^*\right)$ is generally positive. Next, one can combine the solution for Y_j^* with (A.17), (A.19), (A.25), (A.27) to get

$$M_j^* = \phi_j^* Y_j^*$$
$$M_{lj}^* = \zeta_{lj}^* M_j^*$$
$$K_j^* = \frac{1 - \phi_j^* - \psi_j^*}{\beta^{-1} - 1 + \delta} Y_j^*$$

Aggregate capital and investment, and sector level investment demand, follow below:

$$K^* = \sum_{j=1}^{\mathcal{J}} K_j^*$$
$$I^* = \delta K^*$$
$$I_j^* = \varpi_j^* I^*$$

To derive the real wage level, I sum up budget constraints across individual households and impose the no-arbitrage condition $\Omega_j^* = \Omega^*$ for the labor market. The result is $C^* + I^* = \Omega^* L^* + R^{k*} K^*$, or

$$\Omega^* = \frac{C^* - \left(\beta^{-1} - 1\right) K^*}{L^*}.$$

Taking L^* as given, a restriction follows for the shift parameter χ_N^* from households' steady state labor supply:

$$\chi_N^* = \frac{\Omega^*}{(1 + \epsilon_w) C^{*\sigma^*} L^{*\varphi^*}}$$

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[32] A necessary and sufficient restriction for non-singularity is that none of the following are true: i) $\phi_j^* = 1$ $\forall \ j$, and ii) $\zeta_{jj}^* = \phi_j^* = 1$ for some j. For the small open economy, these expressions write i) $\alpha_j \phi_j = 1 \ \forall$ j, and ii) $\alpha_j = \zeta_{jj} = \phi_j = 1$ for some j. Proofs are available from the author upon request.

Finally, sectoral productivity level, labor input, employment and markup (over variable costs) can be found from equations (A.17), (A.26), (A.30) and the identity $L^* = \frac{N_j^*}{\mu_j^*}$:

$$Z_j^* = (1 + \epsilon_p) \left(\frac{1}{\phi_j^*}\right)^{\phi_j^*} \left(\frac{\Omega^*}{\psi_j^*}\right)^{\psi_j^*} \left(\frac{R^{k*}}{1 - \phi_j^* - \psi_j^*}\right)^{1 - \phi_j^* - \psi_j^*}$$

$$\Phi_j^* = \epsilon_p Y_j^*$$

$$N_j^* = \frac{\psi_j^*}{\Omega^*} Y_j^*$$

$$\mu_j^* = \frac{N_j^*}{L^*}$$

For completeness, note that $\sum_{j=1}^{\mathcal{J}} \mu_j^* = 1$, $\sum_{j=1}^{\mathcal{J}} N_j^* = L^*$, and that sectoral and aggregate GDP write as follows:

$$GDP_j^* = \left(1 - \phi_j^*\right) Y_j^*$$

$$GDP^* = \sum_{j=1}^{\mathcal{J}} GDP_j^* = C^* + I^*$$

This completes the foreign block. Next I derive steady state in the small open economy.

B.2.2   THE SMALL OPEN ECONOMY

From $P_{rHj} = P_{rFj} = 1$ we get

$$P_{rj} = \left[\alpha_j P_{rHj}^{1-\eta} + (1 - \alpha_j) P_{rFj}^{1-\eta}\right]^{\frac{1}{1-\eta}} = 1$$

Thus, the relative price on investment and on sector level material inputs become

$$P_r^i = \left(\sum_{j=1}^{\mathcal{J}} \varpi_j P_{rj}^{1-\nu_i}\right)^{\frac{1}{1-\nu_i}} = 1$$

$$P_{rj}^m = \left(\sum_{j=1}^{\mathcal{J}} \zeta_{lj} P_{rl}^{1-\nu_m}\right)^{\frac{1}{1-\nu_m}} = 1$$

The solutions for $\mathcal{Q}$, R^k and RMC_j are found following the procedures used in the foreign block. The real exchange rate is unity due to the assumption of unitary real import prices and $P_{rFj} = (1 + \epsilon_p) RMC_j^* \mathcal{S}$, the optimality condition for foreign firms:

$$\mathcal{S} = \frac{P_{rFj}}{(1 + \epsilon_p) RMC_j^*} = 1$$

The steady state UIP condition $\mathcal{S} = \left(\frac{C}{C^*}\right)^\sigma$ then implies that $C = C^* = 1$. Moreover, balanced steady state trade in the small open economy implies that $Y_j = X_j = C_j + I_j + \sum_{l=1}^{\mathcal{J}} M_{jl}$. Thus, C_j, Y_j, K_j, M_j, M_{lj}, K, I, I_j, Ω, χ_N, RMC_j, Z_j, N_j and μ_j are found in that order, and in the same manner as their foreign counterparts. Similarly, under the assumption of balanced trade we get sectoral and aggregate GDP as follows:

$$GDP_j = (1 - \phi_j) Y_j$$

$$GDP = \sum_{j=1}^{\mathcal{J}} GDP_j = C + I$$

Given data on export-to-GDP ratios $\frac{X^*_{Hj}}{GDP_j}$, we now also have X^*_{Hj}. The model is completed with the following trade block:

$$X_{Fj} = X^*_{Hj}$$

$$X_{Hj} = Y_j - X_{Fj}$$

$$\alpha_j = \frac{X_{Hj}}{X_j}$$

$$\alpha^*_j = 1 - \frac{X^*_{Hj}}{X^*_j}$$

B.3   LOG-LINEARIZED SYSTEM

Define small case variables as log-deviations from steady state, e.g. $x_t \equiv \ln\left(\frac{X_t}{X}\right)$. $100x_t$ can be interpreted as the percentage deviation in a neighborhood around the steady state. I do a first order approximation around a symmetric zero-inflation steady state. The log-linearized system of equations that constitutes the home block follows below, where foot script j refers to sector j.

$$c_{j,t} = -\nu_c p_{rj,t} + c_t \tag{B.1}$$

$$\lambda_t = z_{U,t} - \frac{\sigma}{1 - \chi_C}\left(c_t - \chi_C c_{t-1}\right) \tag{B.2}$$

$$\lambda_t = \mathbb{E}_t\left(\lambda_{t+1}\right) + r_t - \mathbb{E}_t\left(\pi_{t+1}\right) \tag{B.3}$$

$$\lambda_t = \mathbb{E}_t\left(\lambda_{t+1}\right) + r^*_t - \mathbb{E}_t\left(\pi_{t+1}\right) + \mathbb{E}_t\left(\Delta e_{t+1} + \upsilon_t\right) \tag{B.4}$$

$$\upsilon_t = -\epsilon_B a_t + z_{B,t} \tag{B.5}$$

$$i_{j,t} = -\nu_i\left(p_{rj,t} - p^i_{r,t}\right) + i_t \tag{B.6}$$

$$q_t = -r_t + \mathbb{E}_t\left(\pi_{t+1} + \left[1 - \beta\left(1 - \delta\right)\right]r^k_{t+1} + \beta\left(1 - \delta\right)q_{t+1}\right) \tag{B.7}$$

$$q_t = -z_{I,t} + p^i_{r,t} + \epsilon_I\left[\left(i_t - i_{t-1}\right) - \beta\mathbb{E}_t\left(i_{t+1} - i_t\right)\right] \tag{B.8}$$

$$k_{t+1} = \left(1 - \delta\right)k_t + \delta\left(z_{I,t} + i_t\right) \tag{B.9}$$

$$k_t = \sum_{j=1}^{\mathcal{J}} \frac{K_j}{K} k_{j,t} \tag{B.10}$$

$$\pi^i_t = p^i_{r,t} - p^i_{r,t-1} + \pi_t \tag{B.11}$$

$$\pi^i_t = \sum_{j=1}^{\mathcal{J}} \frac{P_{rj} I_j}{P^i_r I} \pi_{j,t} \tag{B.12}$$

$$\pi_{wj,t} = \omega_{j,t} - \omega_{j,t-1} + \pi_t \tag{B.13}$$

$$\pi_{wj,t} = \beta\mathbb{E}_t\left(\pi_{wj,t+1}\right) + \gamma_w\left(\pi_{t-1} - \beta\pi_t\right) + \kappa_{wj}\left(mrs_{j,t} - \omega_{j,t}\right) \tag{B.14}$$

$$mrs_{j,t} = z_{U,t} + z_{N,t} + \varphi n_{j,t} - \lambda_t \tag{B.15}$$

$$\pi_t = \sum_{j=1}^{\mathcal{J}} P_{rj} \frac{C_j}{C} \pi_{j,t} \tag{B.16}$$

$$\pi_{j,t} = p_{rj,t} - p_{rj,t-1} + \pi_t \tag{B.17}$$

$$\pi_{j,t} = \frac{P_{rHj}X_{Hj}}{P_{rj}X_j}\pi_{Hj,t} + \frac{P_{rFj}X_{Fj}}{P_{rj}X_j}\pi_{Fj,t} \tag{B.18}$$

$$\pi_{j,t}^m = p_{rj,t}^m - p_{rj,t-1}^m + \pi_t \tag{B.19}$$

$$\pi_{j,t}^m = \sum_{l=1}^{\mathcal{J}} \frac{P_{rl}M_{lj}}{P_{rj}^m M_j}\pi_{l,t} \tag{B.20}$$

$$\pi_{Hj,t} = p_{rHj,t} - p_{rHj,t-1} + \pi_t \tag{B.21}$$

$$\pi_{Hj,t} = \kappa_1 \mathbb{E}_t\left(\pi_{Hj,t+1}\right) + \kappa_2 \pi_{Hj,t-1} + \kappa_{j3}\left(rmc_{j,t} - p_{rHj,t} + z_{M,t}\right) \tag{B.22}$$

$$\pi_{Hj,t}^* = p_{rHj,t}^* - p_{rHj,t-1}^* + \pi_t - \Delta e_t \tag{B.23}$$

$$\pi_{Hj,t}^* = \kappa_1 \mathbb{E}_t\left(\pi_{Hj,t+1}^*\right) + \kappa_2 \pi_{Hj,t-1}^* + \kappa_{j3}\left(rmc_{j,t} - p_{rHj,t}^* + z_{M,t}\right) \tag{B.24}$$

$$rmc_{j,t} = -z_{Aj,t} + \phi_j p_{rj,t}^m + \psi_j \omega_{j,t} + \left(1 - \phi_j - \psi_j\right)r_t^k \tag{B.25}$$

$$\pi_{Fj,t} = p_{rFj,t} - p_{rFj,t-1} + \pi_t \tag{B.26}$$

$$\pi_{Fj,t} = \kappa_1 \mathbb{E}_t\left(\pi_{Fj,t+1}\right) + \kappa_2 \pi_{Fj,t-1} + \kappa_{j3}\left(rmc_{j,t}^* + s_t - p_{rFj,t} + z_{M,t}^*\right) \tag{B.27}$$

$$\tau_{j,t} = p_{rHj,t}^* - p_{rFj,t} \tag{B.28}$$

$$r_t = \rho_r r_{t-1} + \left(1 - \rho_r\right)\left(\rho_\pi \pi_t + \rho_y gdp_t + \rho_{\Delta y}\Delta gdp_t + \rho_e \Delta e_t\right) + z_{R,t} \tag{B.29}$$

$$x_{j,t} = C_{xj}c_{j,t} + I_{xj}i_{j,t} + M_{xj}m_{j,t}^x + G_{xj}g_{j,t} \tag{B.30}$$

$$m_{j,t}^x = \sum_{l=1}^{\mathcal{J}} \frac{M_{jl}}{M_j^x}m_{jl,t} \tag{B.31}$$

$$m_{lj,t} = -\nu_m\left(p_{rl,t} - p_{rj,t}^m\right) + m_{j,t} \tag{B.32}$$

$$m_{j,t} - n_{j,t} = \omega_{j,t} - p_{rj,t}^m \tag{B.33}$$

$$n_{j,t} - k_{j,t} = r_t^k - \omega_{j,t} \tag{B.34}$$

$$x_{Hj,t} = -\eta\left(p_{rHj,t} - p_{rj,t}\right) + x_{j,t} \tag{B.35}$$

$$x_{Fj,t} = -\eta\left(p_{rFj,t} - p_{rj,t}\right) + x_{j,t} \tag{B.36}$$

$$x_{Hj,t}^* = -\eta\left(p_{rHj,t}^* - s_t - p_{rj,t}^*\right) + x_{j,t}^* \tag{B.37}$$

$$y_{j,t} = \frac{X_{Hj}}{Y_j}x_{Hj,t} + \frac{\tilde{X}_{Hj}^*}{Y_j}x_{Hj,t}^* \tag{B.38}$$

$$y_{j,t} = \left(1 + \epsilon_p\right)\left[z_{Aj,t} + \phi_j m_{j,t} + \psi_j n_{j,t} + \left(1 - \phi_j - \psi_j\right)k_{j,t}\right] \tag{B.39}$$

$$gdp_{j,t} = C_{yj}\left(p_{rj,t} + c_{j,t}\right) + I_{yj}\left(p_{rj,t} + i_{j,t}\right) + G_{yj}\left(p_{rj,t} + g_{j,t}\right) + tb_{j,t} \tag{B.40}$$

$$gdp_t = \sum_{j=1}^{\mathcal{J}} \frac{GDP_j}{GDP}gdp_{j,t} \tag{B.41}$$

$$tb_{j,t} = EX_{yj}\left(p_{rHj,t}^* + x_{Hj,t}^*\right) - IM_{yj}\left(p_{rFj,t} + x_{Fj,t}\right) \tag{B.42}$$

$$tb_t = \sum_{j=1}^{\mathcal{J}} \frac{GDP_j}{GDP}tb_{j,t} \tag{B.43}$$

$$a_t = \frac{1}{\beta}a_{t-1} + tb_t + \frac{TB/GDP}{\beta - 1}\left(r_{t-1}^* + \upsilon_{t-1} + \Delta e_t - \pi_t\right) \tag{B.44}$$

$$z_{Aj,t} = \rho_A z_{Aj,t-1} + \varepsilon_{Aj,t} \tag{B.45}$$

$$z_{I,t} = \rho_I z_{I,t-1} + \varepsilon_{I,t} \tag{B.46}$$

$$z_{U,t} = \rho_U z_{U,t-1} + \varepsilon_{U,t} \tag{B.47}$$

$$z_{N,t} = \rho_N z_{N,t-1} + \varepsilon_{N,t} \tag{B.48}$$

$$z_{M,t} = \rho_M z_{M,t-1} + \varepsilon_{M,t} \tag{B.49}$$

$$z_{R,t} = \rho_R z_{R,t-1} + \varepsilon_{R,t} \tag{B.50}$$

$$z_{B,t} = \rho_B z_{B,t-1} + \varepsilon_{B,t} \tag{B.51}$$

Some price identities:

$$\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

$$\pi_{j,t} = \ln\left(\frac{P_{j,t}}{P_{j,t-1}}\right)$$

$$p_{rj,t} = \ln\left(\frac{P_{j,t}}{P_t}\right)$$

$$\pi_{Hj,t} = \ln\left(\frac{P_{Hj,t}}{P_{Hj,t-1}}\right)$$

$$p_{rHj,t} = \ln\left(\frac{P_{Hj,t}}{P_t}\right)$$

$$\pi^*_{Hj,t} = \ln\left(\frac{P^*_{Hj,t}}{P^*_{Hj,t-1}}\right)$$

$$p^*_{rHj,t} = \ln\left(\frac{\mathcal{E}_t P^*_{Hj,t}}{P_t}\right)$$

$$\pi_{Fj,t} = \ln\left(\frac{P_{Fj,t}}{P_{Fj,t-1}}\right)$$

$$p_{rFj,t} = \ln\left(\frac{P_{Fj,t}}{P_t}\right)$$

$$p^*_{rj,t} = \ln\left(\frac{P^*_{j,t}}{P^*_t}\right)$$

$$\pi^m_{j,t} = \ln\left(\frac{P^m_{j,t}}{P^m_{j,t-1}}\right)$$

$$p^m_{rj,t} = \ln\left(\frac{P^m_{j,t}}{P_t}\right)$$

$$\pi_{wj,t} = \ln\left(\frac{W_{j,t}}{W_{j,t-1}}\right)$$

$$\omega_{j,t} = \ln\left(\Omega_{j,t}\right)$$

$$\pi^i_t = \ln\left(\frac{P^i_t}{P^i_{t-1}}\right)$$

$$p^i_{r,t} = \ln\left(\frac{P^i_t}{P_t}\right)$$

$$s_t = \ln\left(\frac{\mathcal{E}_t P^*_t}{P_t}\right)$$

C   BAYESIAN ESTIMATION OF MODEL PARAMETERS

In this appendix I explain the estimation procedure in detail. A more general introduction to Bayesian estimation and Markov Chain methods can be found in e.g. Koop (2003) and Bauwens, Lubrano, and Richard (1999).

C.1   PRELIMINARIES

Before estimation I log-linearize all optimality conditions and resource constraints around the non-stochastic steady state of the model (see Appendix B). Steady state values, which naturally show up in the linearized system, are obtained using standard Newton methods. The full linear model is solved numerically for the rational expectations solution by means of a generalized Schur decomposition (see Klein (2000)). The resulting policy function is finally combined with data to form a state space representation:

$$\tilde{y}_t = \mathcal{A}\tilde{y}_{t-1} + \mathcal{B}\varepsilon_{1,t} \tag{C.1}$$

$$y_t^* = \mathcal{C}\tilde{y}_t + \varepsilon_{2,t} \tag{C.2}$$

$$\mathbb{E}\left(\varepsilon_{1,t}\right) = \mathbb{E}\left(\varepsilon_{2,t}\right) = \mathbb{E}\left(\varepsilon_{1,t}\varepsilon_{2,s}'\right) = 0 \qquad \forall\,(s,t)$$

$$\mathbb{E}\left(\varepsilon_{1,t}\varepsilon_{1,s}'\right) = \mathcal{M}\delta_{ts}$$

$$\mathbb{E}\left(\varepsilon_{2,s}\varepsilon_{2,t}'\right) = \mathcal{N}\delta_{ts}$$

$\tilde{y}_t$ denotes the time t vector of all the choice variables (the policy function), y_t^* the vector of observables of sample size T, and $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ the vectors of structural shocks and measurement errors. $\mathcal{A} = \mathcal{A}\left(\Theta\right)$ and $\mathcal{B} = \mathcal{B}\left(\Theta\right)$ are matrices with known functions of the structural parameters Θ. $\mathcal{C}$ is a selection matrix that extracts the vectors of observables from $\tilde{y}_t$. δ_{ts} is the Kronecker delta. Equations (C.1) and (C.2) are referred to as the transition and measurement equations, respectively, and are used as basic building blocks during estimation.

C.2   BAYES' RULE AND THE LIKELIHOOD FUNCTION

The purpose of Bayesian estimation is to combine prior beliefs with information from the data to characterize a posterior distribution of structural parameters Θ. Denote the prior density by $p\left(\Theta\right)$, and the likelihood function of Θ given data by $\mathcal{L}\left(\Theta|y_T^*,\dots,y_1^*\right) \equiv p\left(y_T^*,\dots,y_1^*|\Theta\right)$. Bayes theorem then allows us to write the posterior density as

$$p\left(\Theta|y_T^*,\dots,y_1^*\right) = \frac{p\left(y_T^*,\dots,y_1^*|\Theta\right)p\left(\Theta\right)}{\int p\left(y_T^*,\dots,y_1^*|\Theta\right)p\left(\Theta\right)\,d\Theta}$$

$$\propto p\left(y_T^*,\dots,y_1^*|\Theta\right)p\left(\Theta\right) \equiv \mathcal{K}\left(\Theta|y_T^*,\dots,y_1^*\right), \tag{C.3}$$

where the integral is a constant that corresponds to the marginal data density. All posterior moments of interest can be computed given $\mathcal{K}\left(\Theta|y_T^*,\dots,y_1^*\right)$, which is referred to as the posterior kernel. However, this object must be approximated numerically as no analytical solution is available for the likelihood function. To this end we consider a general likelihood function which can be factorized recursively to yield

$$p\left(y_T^*,\dots,y_1^*|\Theta\right) = p\left(y_1^*|\Theta\right)\prod_{t=2}^{T}p\left(y_t^*|y_{t-1}^*,\dots,y_1^*,\Theta\right).$$

In the case of a Normal likelihood function the log likelihood follows as

$$\ln p\left(y_T^*, \ldots, y_1^* | \Theta\right) = \ln p\left(y_1^* | \Theta\right) + \sum_{t=2}^{T} \ln p\left(y_t^* | y_{t-1}^*, \ldots, y_1^*, \Theta\right)$$

$$= -\frac{Tn}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^{T} \ln |\Sigma_t| - \frac{1}{2} \sum_{t=1}^{T} x_t^{*\prime} \Sigma_t^{-1} x_t^*, \tag{C.4}$$

where $x_t^* = y_t^* - \mathbb{E}\left(y_t^* | y_{t-1}^*, \ldots, y_1^*, \Theta\right)$ is the one step ahead prediction error of the data and $\Sigma_t = \mathbb{E}\left(x_t^* x_t^{*\prime}\right)$ is the conditional variance of the prediction error. The log likelihood can be evaluated given information about x_t^* and Σ_t.

C.3 THE KALMAN FILTER

To derive x_t^* and Σ_t we rely on the Kalman filter. Define $y_{t|t-1}^* \equiv \mathbb{E}\left(y_t^* | y_{t-1}^*, \ldots, y_1^*, \Theta\right)$, $\tilde{y}_{t|t-1} \equiv \mathbb{E}\left(\tilde{y}_t | y_{t-1}^*, \ldots, y_1^*, \Theta\right)$, and $\mathcal{P}_{t|t-1} \equiv \mathbb{E}\left[\left(\tilde{y}_{t|t} - \tilde{y}_{t|t-1}\right)\left(\tilde{y}_{t|t} - \tilde{y}_{t|t-1}\right)'\right]$. It follows from this notation and equation (C.1) that

$$\tilde{y}_{t|t-1} = \mathcal{A}\tilde{y}_{t-1|t-1} \tag{C.5}$$

$$\mathcal{P}_{t|t-1} = \mathcal{A}\mathcal{P}_{t-1|t-1}\mathcal{A}' + \mathcal{B}\mathcal{M}\mathcal{B}'. \tag{C.6}$$

These two are referred to as prediction equations in the Kalman filter. Furthermore, equation (C.2) implies $y_{t|t-1}^* = \mathcal{C}\tilde{y}_{t|t-1}$. Thus, using (C.2) we can write

$$x_t^* = y_t^* - y_{t|t-1}^* = y_t^* - \mathcal{C}\tilde{y}_{t|t-1} \tag{C.7}$$

$$\Sigma_t = \mathbb{E}\left(\left(y_t^* - y_{t|t-1}^*\right)\left(y_t^* - y_{t|t-1}^*\right)'\right) = \mathcal{C}\mathcal{P}_{t|t-1}\mathcal{C}' + \mathcal{N}. \tag{C.8}$$

The filter is completed with the time t updates $\tilde{y}_{t|t}$ and $\mathcal{P}_{t|t}$. To this end we use the identity $\tilde{y}_t = \tilde{y}_{t|t-1} + \left(\tilde{y}_t - \tilde{y}_{t|t-1}\right)$, and combine (C.2) with (C.7) to get $x_t^* = \mathcal{C}\left(\tilde{y}_t - \tilde{y}_{t|t-1}\right) + \epsilon_{2,t}$. It follows from these expressions and the definition of $\mathcal{P}_{t|t-1}$ that

$$\begin{pmatrix} x_t^* \\ \tilde{y}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ \tilde{y}_{t|t-1} \end{pmatrix}, \begin{pmatrix} \Sigma_t & \mathcal{C}\mathcal{P}_{t|t-1} \\ \mathcal{P}_{t|t-1}\mathcal{C}' & \mathcal{P}_{t|t-1} \end{pmatrix}\right).$$

The rule for the conditional Normal distribution (see e.g. Bauwens et al. (1999), Theorem A.12 p. 299) allows us to write the distribution of $\tilde{y}_t$ given x_t^* and past data as

$$\mathcal{L}\left(\tilde{y}_t | x_t^*, y_{t-1}^*, \ldots, y_1^*, \Theta\right) = N\left(\tilde{y}_{t|t-1} + \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}x_t^*, \mathcal{P}_{t|t-1} - \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}\mathcal{C}\mathcal{P}_{t|t-1}\right).$$

Note that x_t^* contains the time t information y_t^*. Thus, $\tilde{y}_{t|t}$ and $\mathcal{P}_{t|t}$ are just the two moments above:

$$\tilde{y}_{t|t} = \tilde{y}_{t|t-1} + \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}x_t^* \tag{C.9}$$

$$\mathcal{P}_{t|t} = \mathcal{P}_{t|t-1} - \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}\mathcal{C}\mathcal{P}_{t|t-1} \tag{C.10}$$

These are referred to as the updating equations. The system (C.5)-(C.6) and (C.9)-(C.10) constitute the Kalman filter. The recursive nature of the filter allows us to successively obtain x_t^* and Σ_t, where the output from (C.9) and (C.10) is used as input in (C.5) and (C.6) the next period.[33] Once the series $\{x_t^*\}_{t=1}^T$ and $\{\Sigma_t\}_{t=1}^T$ are in place, we are ready to evaluate the log likelihood function (C.4) for any given parameter vector Θ.

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[33]Starting values $\tilde{y}_{1|0}$ and $\mathcal{P}_{1|0}$ are set to the unconditional mean and variance of $\tilde{y}$, and are calculated using equation (C.1).

C.4 THE POSTERIOR DISTRIBUTION AND THE RWMH ALGORITHM

The last step of the estimation procedure is to obtain estimates of posterior moments of interest, in particular measures of central tendency and variability. For that we need to characterize the entire posterior distribution of Θ. It is analytically intractable, so Markov Chain Monte Carlo (MCMC) techniques are used for this purpose. The posterior mode, denoted Θ_m, is first found by numerical optimization of $\mathcal{K}\left(\Theta|y_T^*, \ldots, y_1^*\right)$. I use a Metropolis-Hastings type optimization routine to find the mode (see below). The variance of Θ_m is calculated from the inverse of the negative Hessian evaluated at Θ_m:

$$\Sigma_m = (E\left[H(\Theta_m)\right])^{-1} = \left(-\left[\left.\frac{\partial^2 \ln(\mathcal{K}\left(\Theta|y_T^*, \ldots, y_1^*\right))}{\partial\Theta\partial\Theta'}\right|_{\Theta=\Theta_m}\right]\right)^{-1}$$

The variance of each estimate in Θ_m is just the diagonal elements of Σ_m.

The rest of the posterior distribution is simulated using the Random Walk Metropolis-Hastings (RWMH) algorithm. The general idea, in the words of Canova (2007) is "to specify a transition kernel for a Markov Chain such that starting from some initial value and iterating a number of times, we produce a limiting distribution which is the target distribution we need to sample from". The RWMH algorithm is stated below:

1. Choose starting point $\Theta^{(0)}$ (I use the posterior mode). For $s = 1, \ldots, S$, run a loop over steps 2-4.

2. Draw a proposal $\hat{\Theta}^{(s)}$ from the jumping distribution

$$\mathcal{J}\left(\hat{\Theta}^{(s)}|\Theta^{(s-1)}\right) = N\left(\Theta^{(s-1)}, c\Sigma_m\right),$$

   where Σ_m is the covariance matrix evaluated at the posterior mode and c is a scaling factor of the covariance matrix.

3. Compute $\mathcal{K}\left(\hat{\Theta}^{(s)}|y_T^*, \ldots, y_1^*\right)$ and the acceptance ratio defined as

$$r = \frac{p\left(\hat{\Theta}^{(s)}|y_T^*, \ldots, y_1^*\right)}{p\left(\Theta^{(s-1)}|y_T^*, \ldots, y_1^*\right)} = \frac{\mathcal{K}\left(\hat{\Theta}^{(s)}|y_T^*, \ldots, y_1^*\right)}{\mathcal{K}\left(\Theta^{(s-1)}|y_T^*, \ldots, y_1^*\right)}.$$

4. Accept the proposal $\hat{\Theta}^{(s)}$ with probability $\min(r, 1)$. Set $\Theta^{(s)} = \hat{\Theta}^{(s)}$ if $\hat{\Theta}^{(s)}$ is accepted, and $\Theta^{(s)} = \Theta^{(s-1)}$ otherwise.

5. Build a histogram of the retained values of Θ. Let this be the final approximation of the posterior distribution.

Step 4 implies that we accept all draws that make us move to a more dense part of the posterior. However, we also accept some draws with lower density. The idea is to frequently visit the region of the parameter space with high probability, while at the same time visit as much as possible of the space. Common practice in the literature is to set the scaling factor c such that the acceptance ratio lies somewhere between 20% and 40%. I tune c to get an acceptance ratio around 30%. Finally, step 5 provides us with an estimate of the full posterior distribution which can be used for Bayesian inference.

D DATA

D.1 DATA SOURCES AND VARIABLE DEFINITIONS

Raw data are taken from the datasets listed below:

Table D.1: Data set/series – Raw data

Variable	Canada	United States
GDP (sector and aggregate)	CANSIM 379-0023	GDPbyInd_VA_NAICS
Consumption	CANPFCEQDSMEI	USAPFCEQDSMEI
Gross fixed capital formation	CANSIM 380-0068	GDPI
Change in stocks	CANSIM 380-0069	GDPI
Implicit CPI deflator	CANPCEDEFQISNAQ	USAPCEDEFQISNAQ
Implicit GDP deflator	CANGDPDEFQISMEI	USAGDPDEFQISMEI
Interest rate	INTGSTCAM193N	FEDFUNDS
Hours	CANSIM 383-0008	PRS85006023
Employment (females)	CANEMPFEMQDSMEI	USAEMPFEMQDSMEI
Employment (males)	CANEMPMALQDSMEI	USAEMPMALQDSMEI
Labor force	CANLFTOTQDSMEI	USALFTOTQDSMEI
Exchange rate	EXCAUS	–

Variables are defined as follows before detrending, where LF denotes the labor force (interpolation abstracted from):

- $\log\left(GDP_{j,t}\right) = \log\left(\frac{C379\text{-}0023_t}{PCEDEF_t LFTOT_t}\right) \times 100$

- $\log\left(C_t\right) = \log\left(\frac{PFCE_t}{PCEDEF_t LFTOT_t}\right) \times 100$

- $\log\left(\frac{P_t^i}{P_t}I_t\right) = \log\left(\frac{C380\text{-}0068_t + C380\text{-}0069_t}{PCEDEF_t LFTOT_t}\right) \times 100$

- $\log\left(L_t\right) = \log\left(\frac{\frac{C383\text{-}0008_t}{100}\left(EMPFEM_t + EMPMAL_t\right)}{LFTOT_t}\right) \times 100$

- $\log\left(\Pi_t\right) = \log\left(\frac{PCEDEF_t}{PCEDEF_{t-1}}\right) \times 100$

- $\log\left(R_t\right) = \frac{INTGST_t}{4}$

- $\log\left(\mathcal{S}_t\right) = \log\left(\frac{EXCAUS_t USAPCEDEF_t}{CANPCEDEF_t}\right) \times 100$

Figure D.1: Data series 1982Q4-2007Q4
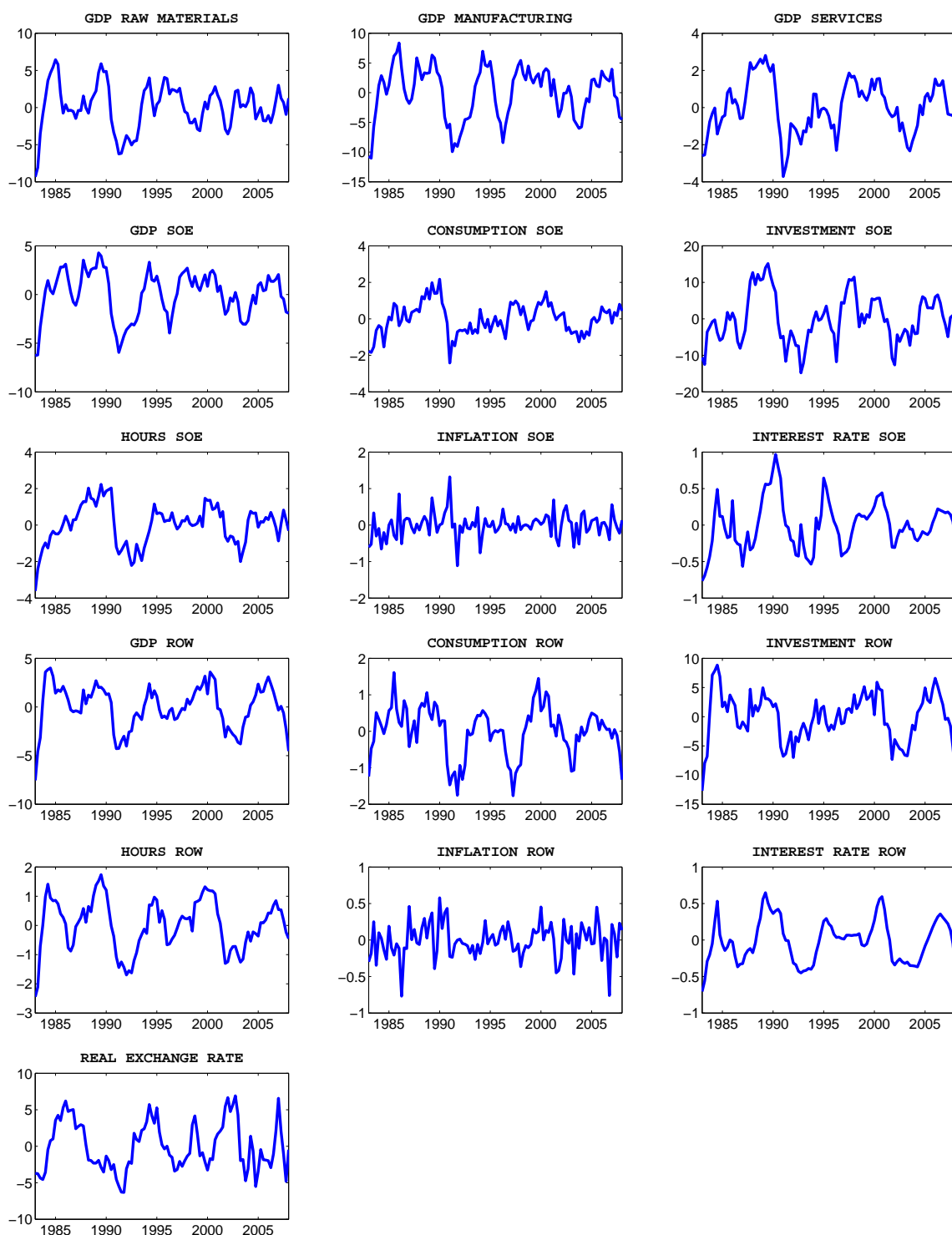
Note: Smoothed data using the Kalman smoother (%-deviations from trend). Raw data are HP filtered with $\lambda = 1600$. Model correspondence: GDP is gdp, consumption c, investment i, hours n, inflation π, interest rate r, and real exchange rate s.

D.2 TRADE FLOWS – CANADA AND UNITED STATES

I-O matrix (dollars)	1	2	3	Aggregate
1	24702,44	112979,8	6146,992	143829,2
2	29910,85	333671,2	79512,34	443094,4
3	23196,33	98881,33	161465,4	283543,1
Sum	77809,62	545532,3	247124,8	870466,7

I-O matrix (shares)	1	2	3
1	0,317473	0,2071	0,024874
2	0,384411	0,611643	0,32175
3	0,298116	0,181257	0,653376
Sum	1	1	1

Trade flows	1	2	3	Aggregate
PX	160591,9	860310,5	706434,6	1727337
PC	12777,22	174043,5	385362,2	572182,9
PI	3985,49	243172,6	37529,37	284687,4
PM	143829,2	443094,4	283543,1	870466,7
PHXH	119431,8	500414,5	655807	1275653
PFXF	41160,15	359895,9	50627,64	451683,7
PHFXHF	90680,82	331690,4	77079,76	499451
alpha	0,256303	0,418333	0,071666	0,261491
Con.share	0,022331	0,304175	0,673495	1
Output	210112,6	832104,9	732886,7	1775104
Labor cost	25313,51	176133,4	237791,4	439238,3
C/X	0,079563	0,202303	0,545503	0,331251
I/X	0,024817	0,282657	0,053125	0,164813
M/X	0,895619	0,51504	0,401372	0,503936

GDP and ratios	1	2	3	Aggregate
GDP	66283,39	389010,6	449343,7	904637,6
C/GDP	0,192767	0,447401	0,857611	0,6325
I/GDP	0,060128	0,625105	0,08352	0,314698
EXP/GDP	1,368078	0,852651	0,171539	0,552101
IMP/GDP	0,620972	0,925157	0,11267	0,499298

Cost shares	1	2	3	Aggregate
phi	0,370323	0,655605	0,337194	0,490375
psi	0,120476	0,211672	0,324459	0,247444
1-phi-psi	0,509201	0,132723	0,338348	0,262181

I-O matrix (dollars)	1	2	3	Aggregate
1	126612,2	607570,4	25125,94	759308,6
2	103401,7	1985450	686110,5	2774963
3	85331,73	878585,4	1751546	2715463
Sum	315345,7	3471606	2462783	6249735

I-O matrix (shares)	1	2	3
1	0,401503	0,175011	0,010202
2	0,3279	0,571911	0,278592
3	0,270597	0,253077	0,711206
Sum	1	1	1

Trade flows	1	2	3	Aggregate
PX	898195,1	6429781	7458578	14786554
PC	61198,95	1770631	4261054	6092884
PI	77687,51	1884188	482061,2	2443936
PM	759308,6	2774963	2715463	6249735
PHXH	640917	5049027	7398197	13088142
PFXF	257278,1	1380754	60380,93	1698413
PHFXHF	36145,77	647801,1	331612,7	1015560
alpha	0,286439	0,214744	0,008096	0,114862
Con.share	0,010044	0,290606	0,699349	1
Output	898195,1	6429781	7458578	14786554
Labor cost	88985,8	1390266	2196918	3676170
C/X	0,068135	0,27538	0,571296	0,412056
I/X	0,086493	0,293041	0,064632	0,165281
M/X	0,845372	0,43158	0,364073	0,422663

GDP and ratios	1	2	3	Aggregate
GDP	138886,5	3654818	4743115	8536820
C/GDP	0,44064	0,484465	0,898366	0,713718
I/GDP	0,55936	0,515535	0,101634	0,286282
EXP/GDP	0,260254	0,177246	0,069915	0,118962
IMP/GDP	1,852435	0,37779	0,01273	0,198951

Cost shares	1	2	3	Aggregate
phi	0,351088	0,539926	0,330195	0,422663
psi	0,099072	0,216223	0,294549	0,248616
1-phi-psi	0,54984	0,243851	0,375256	0,328721

E ADDITIONAL TABLES AND FIGURES

Table E.1: Sectoral variance decomposition of foreign shocks (%)

Variable	Sector	All foreign shocks	Decomposition							
			σ^*_{A1}	σ^*_{A2}	σ^*_{A3}	σ^*_{R}	σ^*_{I}	σ^*_{U}	σ^*_{N}	σ^*_{M}
					Long run horizon					
	1	63.18	9.88	32.17	3.05	0.18	11.59	0.30	0.00	6.00
GDP	2	66.87	33.35	22.80	1.98	0.08	6.77	0.11	0.00	1.79
	3	82.23	32.96	33.79	2.25	0.07	11.02	0.09	0.00	2.04
	1	91.82	55.72	23.68	1.78	0.05	9.41	0.09	0.00	1.09
Consumption	2	84.95	29.85	38.02	2.47	0.08	12.48	0.11	0.00	1.94
	3	69.66	24.66	30.79	2.10	0.06	10.75	0.05	0.00	1.25
	1	54.81	31.65	11.96	1.06	0.03	8.75	0.20	0.00	1.16
Investment	2	45.69	17.45	16.01	1.15	0.04	9.29	0.22	0.00	1.53
	3	39.45	15.39	12.89	1.00	0.03	8.84	0.20	0.00	1.11
	1	59.88	20.23	16.23	1.81	0.17	16.48	0.46	0.00	4.51
Hours	2	36.62	12.90	11.09	1.29	0.11	8.94	0.28	0.00	2.00
	3	25.97	11.01	7.76	0.63	0.09	4.65	0.10	0.00	1.73
	1	88.89	79.05	2.50	0.86	0.10	2.87	0.31	0.00	3.20
Inflation	2	54.44	15.70	19.91	1.48	0.39	3.49	0.49	0.00	12.99
	3	22.10	6.92	7.42	0.86	0.02	6.37	0.15	0.00	0.36
	1	70.94	17.11	40.25	3.10	0.07	8.49	0.15	0.00	1.77
Wage	2	75.97	31.76	30.98	2.34	0.06	9.40	0.09	0.00	1.34
	3	89.43	32.81	39.26	2.69	0.07	12.96	0.10	0.00	1.55
	1	46.38	5.33	8.49	1.77	0.13	21.96	0.56	0.00	8.15
Trade balance	2	31.66	1.88	4.07	1.14	0.11	23.27	0.54	0.00	0.64
	3	17.93	0.76	2.45	3.08	0.38	5.86	4.38	0.00	1.02
	1	75.52	21.39	32.98	3.14	0.17	11.02	0.21	0.00	6.62
Materials	2	81.61	46.47	23.62	1.97	0.07	7.27	0.11	0.00	2.11
	3	76.75	30.73	31.24	2.11	0.09	10.52	0.07	0.00	1.99
	1	93.60	88.85	1.09	0.48	0.01	2.40	0.07	0.00	0.70
Terms of trade	2	57.62	1.73	40.71	1.84	0.13	7.34	0.13	0.00	5.74
	3	37.43	0.80	6.29	15.21	0.54	13.36	0.10	0.00	1.13

Table E.2: Forecast error variance decomposition of domestic shocks (%)

Variable	All domestic shocks	σ_{A1}	σ_{A2}	σ_{A3}	σ_R	σ_I	σ_U	σ_N	σ_M	σ_B
					Decomposition					
Panel A: 1 quarter horizon										
GDP	77.62	0.02	10.65	23.32	15.00	23.81	4.24	0.00	0.23	0.33
Consumption	89.29	0.01	1.24	8.36	8.92	1.54	62.45	0.00	1.91	4.86
Investment	76.64	0.01	4.04	1.59	1.46	52.09	0.27	0.00	2.17	15.01
Hours	83.10	0.02	2.62	13.88	18.08	25.87	4.46	0.02	16.06	2.08
Interest	63.92	0.01	3.35	2.40	13.24	8.07	0.84	0.00	10.98	25.02
Inflation	59.24	0.02	6.92	8.14	14.64	3.79	0.46	0.00	12.94	12.33
Wage	52.00	0.02	6.87	12.10	2.35	0.08	0.14	0.08	18.65	11.69
Trade balance	62.37	0.00	0.59	1.04	0.14	19.51	1.07	0.00	2.67	37.35
Panel B: 4 quarter horizon										
GDP	50.94	0.01	6.80	18.16	4.52	12.32	1.24	0.01	0.40	7.48
Consumption	80.47	0.01	1.63	23.86	7.29	2.34	36.40	0.00	1.40	7.53
Investment	64.40	0.01	6.60	2.15	0.70	29.43	0.42	0.00	1.24	23.85
Hours	76.09	0.01	2.24	5.97	19.67	27.21	3.21	0.08	14.75	2.94
Interest	62.96	0.01	1.97	9.96	7.81	14.40	1.28	0.00	5.68	21.84
Inflation	59.54	0.02	5.11	10.14	17.82	4.14	0.47	0.00	10.04	11.80
Wage	45.89	0.02	4.00	21.38	1.23	0.33	0.08	0.11	7.17	11.58
Trade balance	66.09	0.00	1.68	1.06	0.59	19.89	0.47	0.00	2.70	39.70
Panel C: 8 quarter horizon										
GDP	45.95	0.01	7.02	18.77	2.83	8.31	0.74	0.01	0.25	7.99
Consumption	79.84	0.01	1.79	37.09	5.80	2.05	25.56	0.01	0.97	6.56
Investment	61.19	0.01	8.97	2.56	0.53	22.18	0.49	0.00	0.92	25.53
Hours	69.09	0.01	2.79	5.55	17.67	23.64	2.72	0.12	12.47	4.12
Interest	64.22	0.01	1.84	15.46	6.96	14.26	1.25	0.00	5.03	19.41
Inflation	58.64	0.02	4.97	10.42	17.30	4.20	0.46	0.00	9.79	11.49
Wage	45.28	0.02	3.99	26.20	0.67	0.84	0.06	0.08	4.04	9.37
Trade balance	68.26	0.01	2.89	1.83	0.54	17.20	0.46	0.00	2.31	43.04
Panel D: 20 quarter horizon										
GDP	35.75	0.01	6.19	15.39	1.91	5.65	0.54	0.01	0.21	5.84
Consumption	58.55	0.01	3.23	28.72	3.19	4.97	13.80	0.01	0.53	4.10
Investment	56.19	0.01	7.87	2.20	0.46	20.14	0.40	0.00	0.96	24.14
Hours	68.27	0.01	2.83	5.69	16.90	23.13	2.61	0.12	12.04	4.93
Interest	65.67	0.01	1.89	18.11	6.50	14.50	1.20	0.00	4.68	18.77
Inflation	58.02	0.02	4.93	10.46	16.90	4.35	0.45	0.00	9.56	11.35
Wage	31.36	0.01	3.61	19.47	0.32	0.90	0.11	0.04	1.90	5.00
Trade balance	66.68	0.01	2.76	1.71	0.50	17.36	0.39	0.00	2.24	41.72
Panel E: Long run horizon										
GDP	25.32	0.01	4.29	10.51	1.30	3.84	0.38	0.00	0.16	4.83
Consumption	23.35	0.00	1.82	10.20	1.11	1.95	4.87	0.00	0.24	3.16
Investment	54.92	0.01	7.52	2.17	0.43	18.67	0.37	0.00	0.88	24.87
Hours	64.98	0.01	2.97	5.56	15.76	21.74	2.45	0.11	11.24	5.12
Interest	56.95	0.01	1.95	15.63	5.51	12.48	1.04	0.00	3.97	16.36
Inflation	54.29	0.01	4.64	9.77	15.75	4.07	0.43	0.00	8.92	10.70
Wage	15.68	0.01	1.87	8.82	0.14	0.43	0.09	0.02	0.87	3.45
Trade balance	67.87	0.01	3.10	1.80	0.47	16.42	0.36	0.00	2.10	43.61

## Figure E.1: Historical variance decomposition — GDP at the sector level
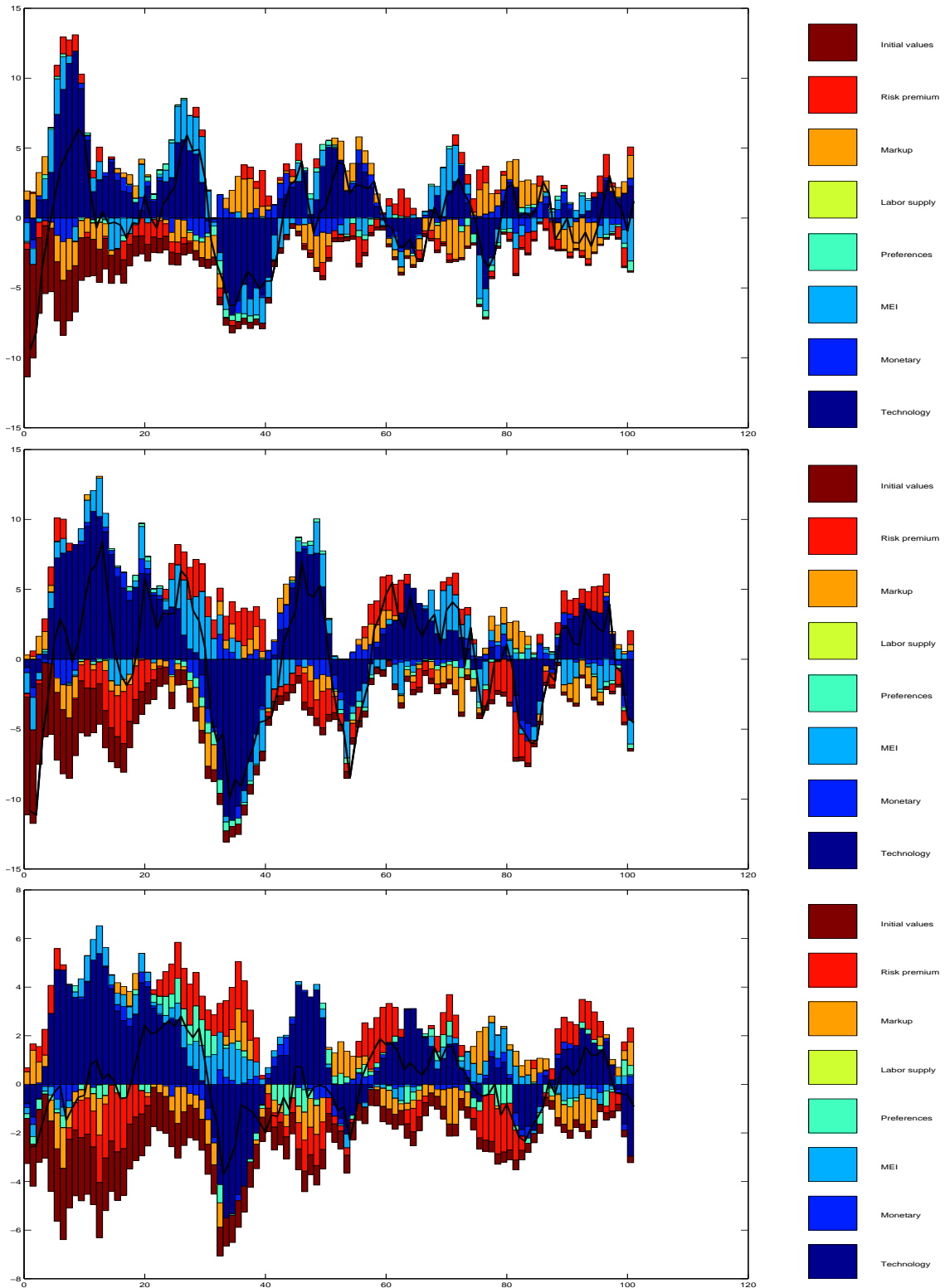
Note: Historical variance decomposition of Canadian GDP in the raw material sector (panel 1), manufacturing sector (panel 2), and service sector (Panel 3). See Figure 2.

Figure E.2: A productivity shock in the foreign manufacturing sector – Foreign IRFs
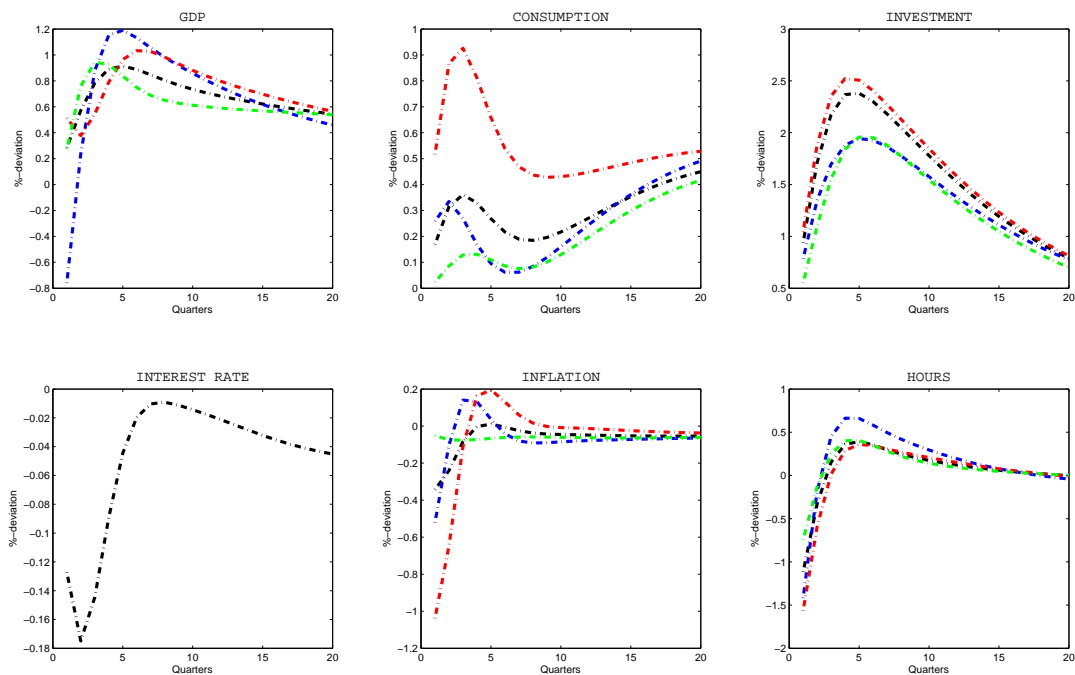
Note: Impulse response functions for foreign aggregate variables (black), and variables in the foreign raw materials sector (blue), manufactured goods sector (red), and service sector (green) respectively. All IRFs are computed at the posterior mean.

Figure E.3: A productivity shock in the foreign manufacturing sector – Domestic IRFs
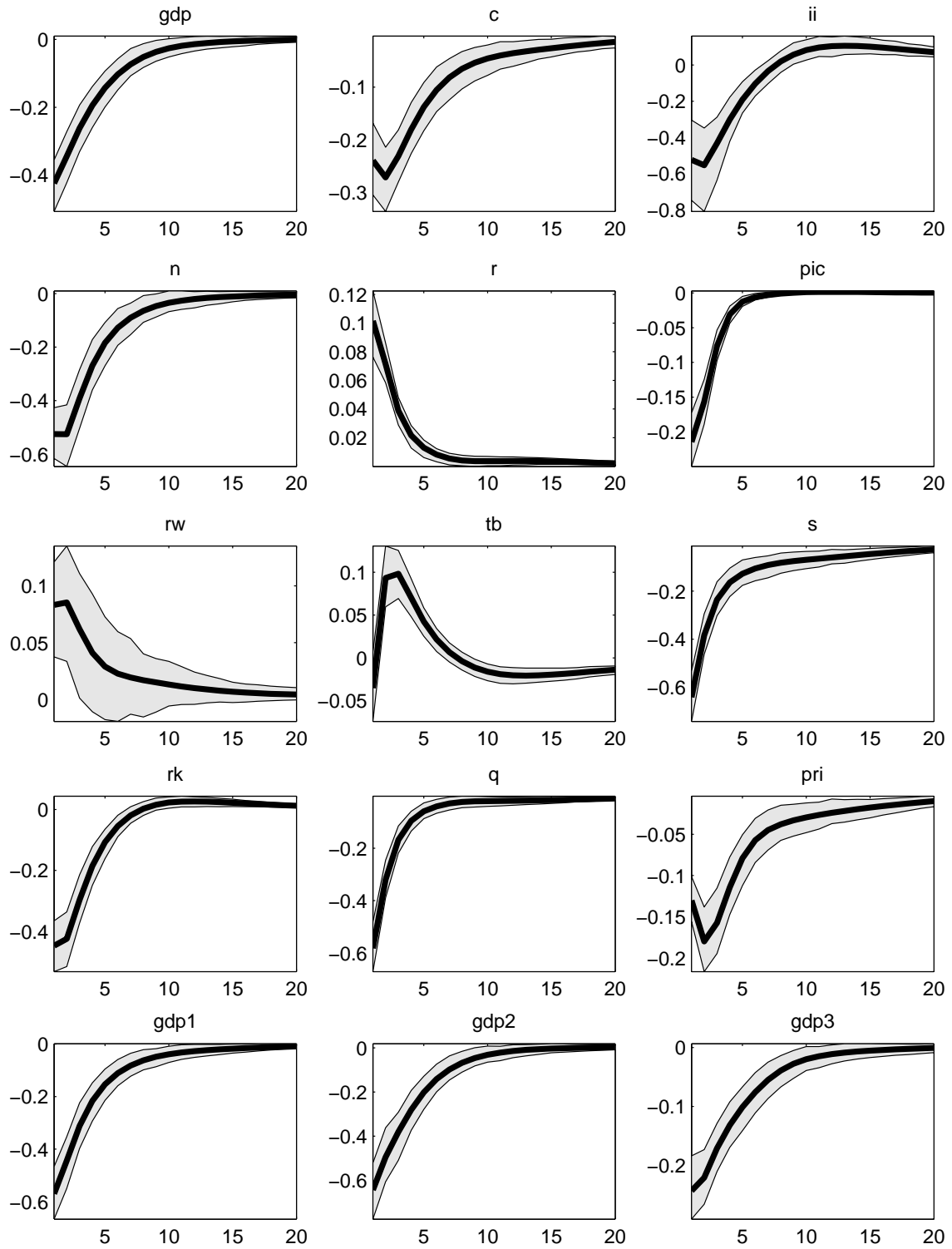

Note: See Figure E.2.

68
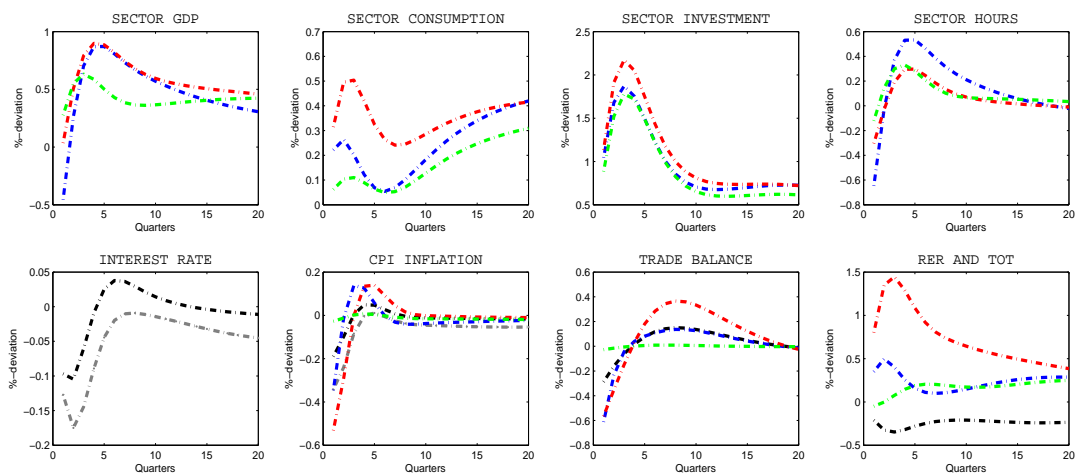
E.1 Bayesian posterior impulse responses – domestic shocks

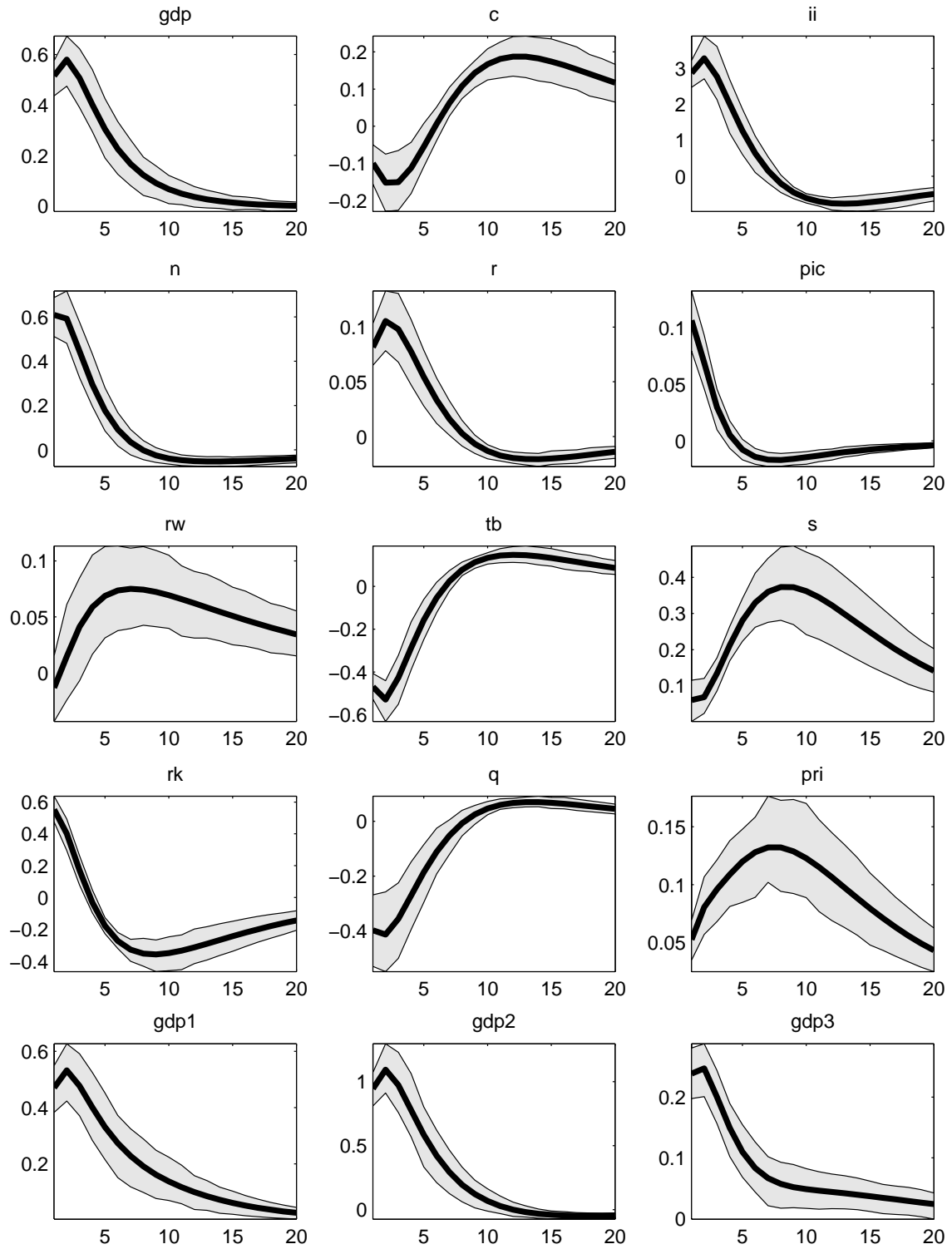Figure E.4: Productivity shock in domestic manufacturing
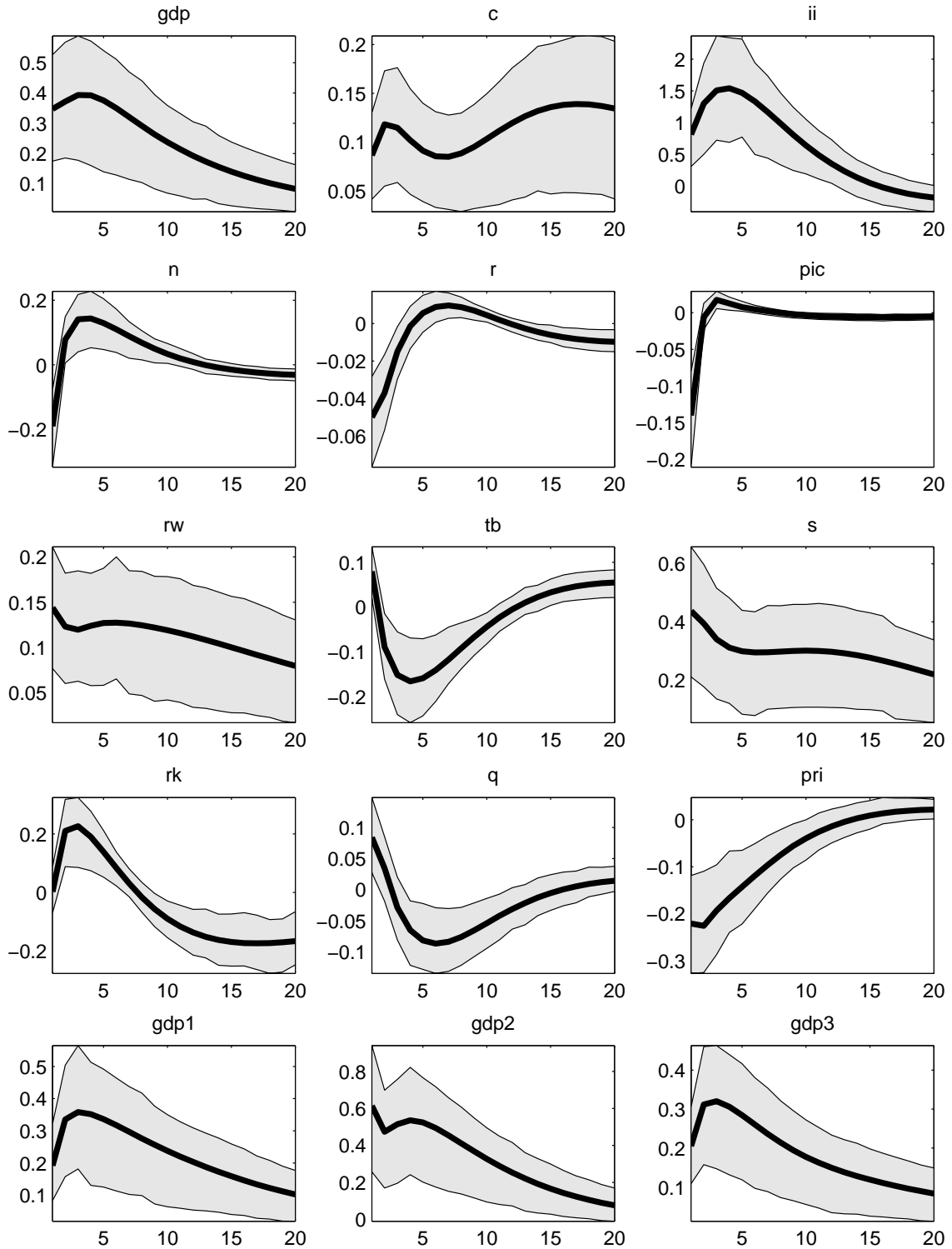
Note: See Figure 6.

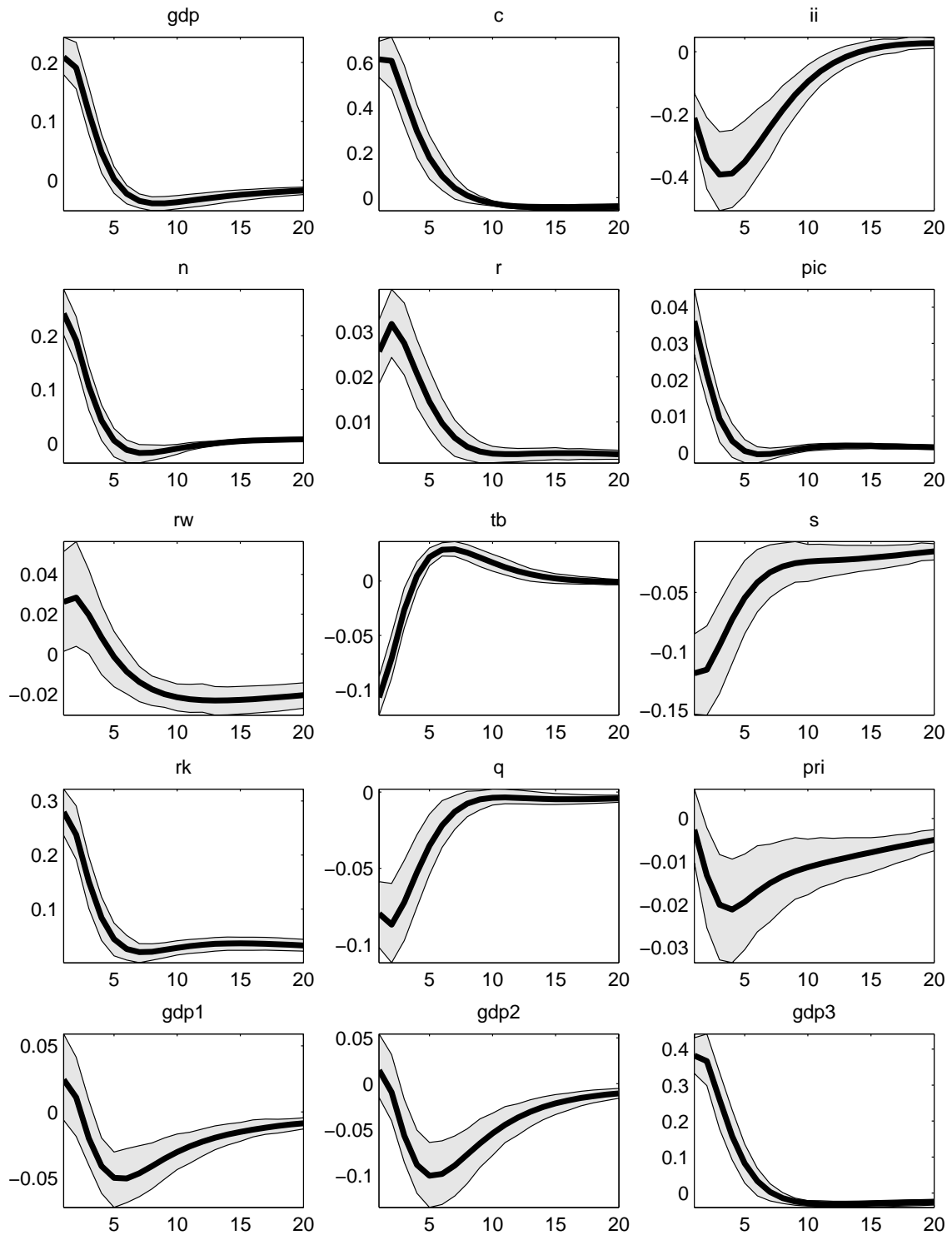Figure E.5: Domestic monetary policy shock

Note: See Figure 6.

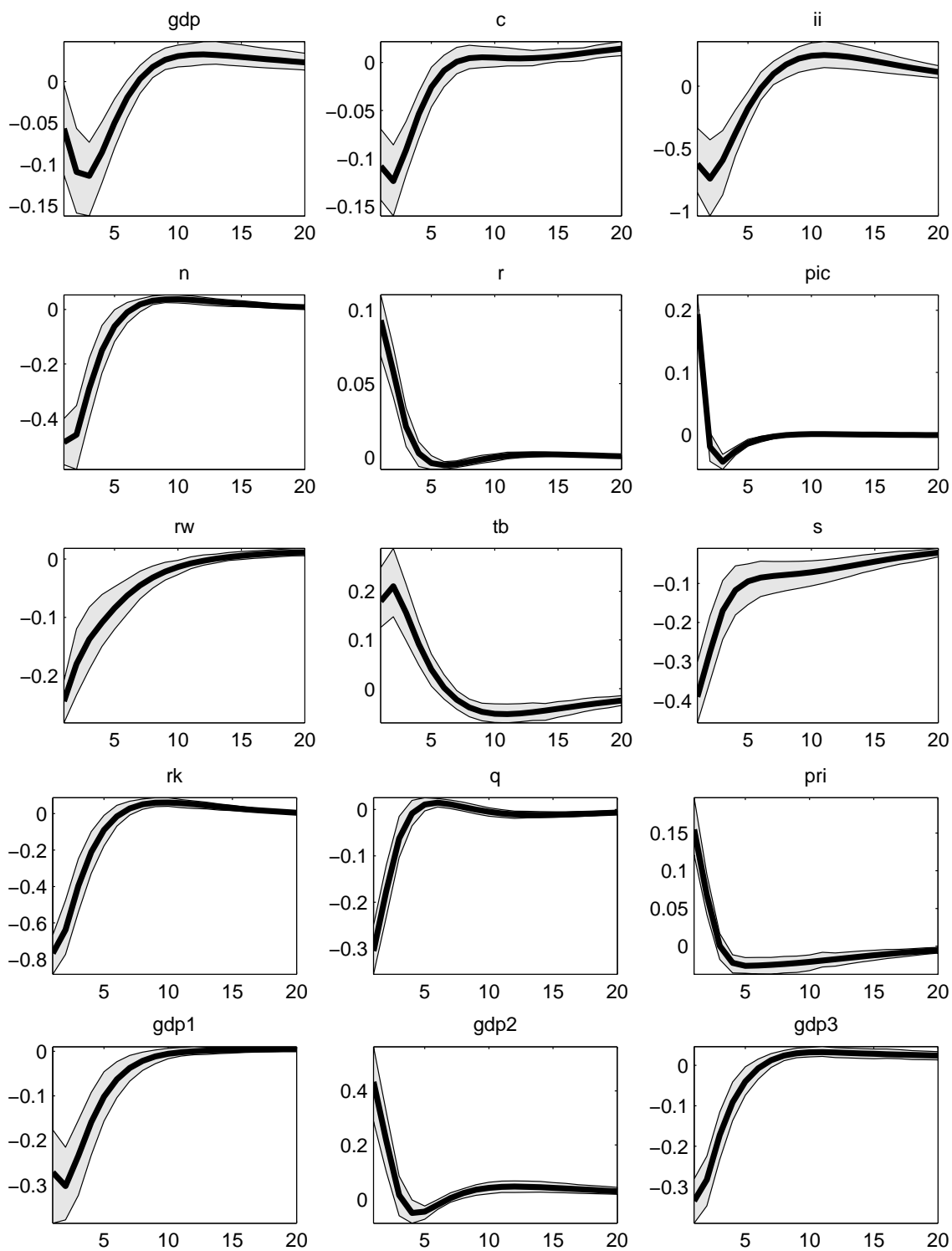Figure E.6: Domestic marginal efficiency of investment shock

Note: See Figure 6.

Figure E.7: Domestic intertemporal preference shock

Figure E.8: Domestic markup shock

Note: See Figure 6.

E.2 Posterior estimates – Counterfactual model

Table E.3: Priors and posterior results – Structural parameters

	Prior distribution			Posterior distribution				
	Type	Mean	St. dev.	Mode	St. Dev.	Mean	5%	95%
χ_C	B	0.500	0.100	0.439	0.060	0.459	0.339	0.580
ϵ_I	N	5.000	1.000	0.247	0.088	0.334	0.162	0.496
η	G	1.000	0.150	0.930	0.066	0.947	0.830	1.061
θ_w	B	0.700	0.075	0.343	0.047	0.370	0.248	0.484
θ_p	B	0.700	0.075	0.483	0.033	0.494	0.429	0.559
γ_w	B	0.500	0.150	0.331	0.118	0.388	0.148	0.614
γ_p	N	0.500	0.150	0.110	0.059	0.143	0.044	0.239
ρ_r	N	0.600	0.050	0.660	0.023	0.671	0.610	0.731
ρ_π	N	1.800	0.200	1.835	0.154	1.869	1.599	2.142
ρ_y	N	0.125	0.050	0.025	0.012	0.032	0.013	0.050
ρ_{dy}	B	0.125	0.050	0.189	0.038	0.186	0.112	0.256
ρ_e	N	0.100	0.050	0.049	0.027	0.055	0.002	0.106
χ_C^*	B	0.500	0.100	0.429	0.067	0.457	0.327	0.587
ϵ_I^*	B	5.000	1.000	0.173	0.059	0.225	0.110	0.334
θ_w^*	B	0.700	0.075	0.257	0.051	0.296	0.209	0.372
θ_p^*	B	0.700	0.075	0.512	0.033	0.526	0.467	0.587
γ_w^*	B	0.500	0.150	0.282	0.102	0.331	0.121	0.541
γ_p^*	B	0.500	0.150	0.108	0.059	0.139	0.044	0.235
ρ_r^*	B	0.600	0.050	0.672	0.028	0.679	0.623	0.737
ρ_π^*	N	1.800	0.200	1.851	0.133	1.878	1.626	2.137
ρ_y^*	N	0.125	0.050	0.019	0.012	0.025	0.010	0.041
ρ_{dy}^*	N	0.125	0.050	0.234	0.028	0.241	0.173	0.309
ρ_A	B	0.700	0.100	0.881	0.032	0.875	0.823	0.930
ρ_R	B	0.700	0.100	0.256	0.045	0.270	0.187	0.350
ρ_I	B	0.700	0.100	0.583	0.057	0.563	0.449	0.681
ρ_U	B	0.700	0.100	0.483	0.082	0.489	0.344	0.638
ρ_N	B	0.700	0.100	0.720	0.084	0.692	0.530	0.851
ρ_M	B	0.700	0.100	0.506	0.079	0.488	0.357	0.620
ρ_B	B	0.700	0.100	0.824	0.050	0.807	0.728	0.893
ρ_A^*	B	0.700	0.100	0.900	0.049	0.886	0.823	0.954
ρ_R^*	B	0.700	0.100	0.309	0.050	0.324	0.229	0.415
ρ_I^*	B	0.700	0.100	0.657	0.053	0.638	0.534	0.745
ρ_U^*	B	0.700	0.100	0.555	0.075	0.552	0.400	0.705
ρ_N^*	B	0.700	0.100	0.721	0.091	0.689	0.526	0.847
ρ_M^*	B	0.700	0.100	0.679	0.089	0.629	0.492	0.769

Note: B stands for Beta, N Normal, G Gamma. The two last columns report 90% posterior probability bands obtained from the MCMC simulation. See Table E.4 for the marginal data density.

Table E.4: Priors and posterior results – Shocks

	Prior distribution			Posterior distribution				
	Type	Mean	St. dev.	Mode	St. Dev.	Mean	5%	95%
$100\sigma_A$	IG	0.20	2.00	0.928	0.057	0.940	0.827	1.052
$100\sigma_R$	IG	0.20	2.00	0.271	0.027	0.273	0.226	0.320
$100\sigma_I$	IG	0.50	2.00	1.021	0.261	1.290	0.791	1.747
$100\sigma_U$	IG	0.20	2.00	1.085	0.133	1.185	0.894	1.475
$100\sigma_N$	IG	0.50	2.00	0.233	0.190	0.406	0.121	0.711
$100\sigma_M$	IG	0.50	2.00	1.215	0.189	1.321	1.033	1.602
$100\sigma_B$	IG	0.20	2.00	0.479	0.104	0.522	0.350	0.693
$100\sigma_A^*$	IG	0.20	2.00	0.667	0.045	0.677	0.599	0.755
$100\sigma_R^*$	IG	0.20	2.00	0.154	0.016	0.156	0.128	0.184
$100\sigma_I^*$	IG	0.50	2.00	0.645	0.124	0.767	0.521	0.998
$100\sigma_U^*$	IG	0.20	2.00	0.806	0.117	0.891	0.657	1.115
$100\sigma_N^*$	IG	0.50	2.00	0.234	0.216	0.398	0.118	0.708
$100\sigma_M^*$	IG	0.50	2.00	0.732	0.094	0.785	0.624	0.949
$100\sigma_e$	IG	0.20	1.00	1.540	0.100	1.570	1.382	1.751
$100\sigma_e^*$	IG	0.20	1.00	0.908	0.062	0.921	0.815	1.027
MDD				-1522.111		-1510.582		

Note: IG stands for Inverse Gamma 1. The two last columns report 90% posterior probability bands obtained from the MCMC simulation. The marginal data density (MDD) is estimated using i) a Laplace approximation based on the posterior mode, and ii) the modified harmonic mean estimator based on draws from the simulated Markov chains.
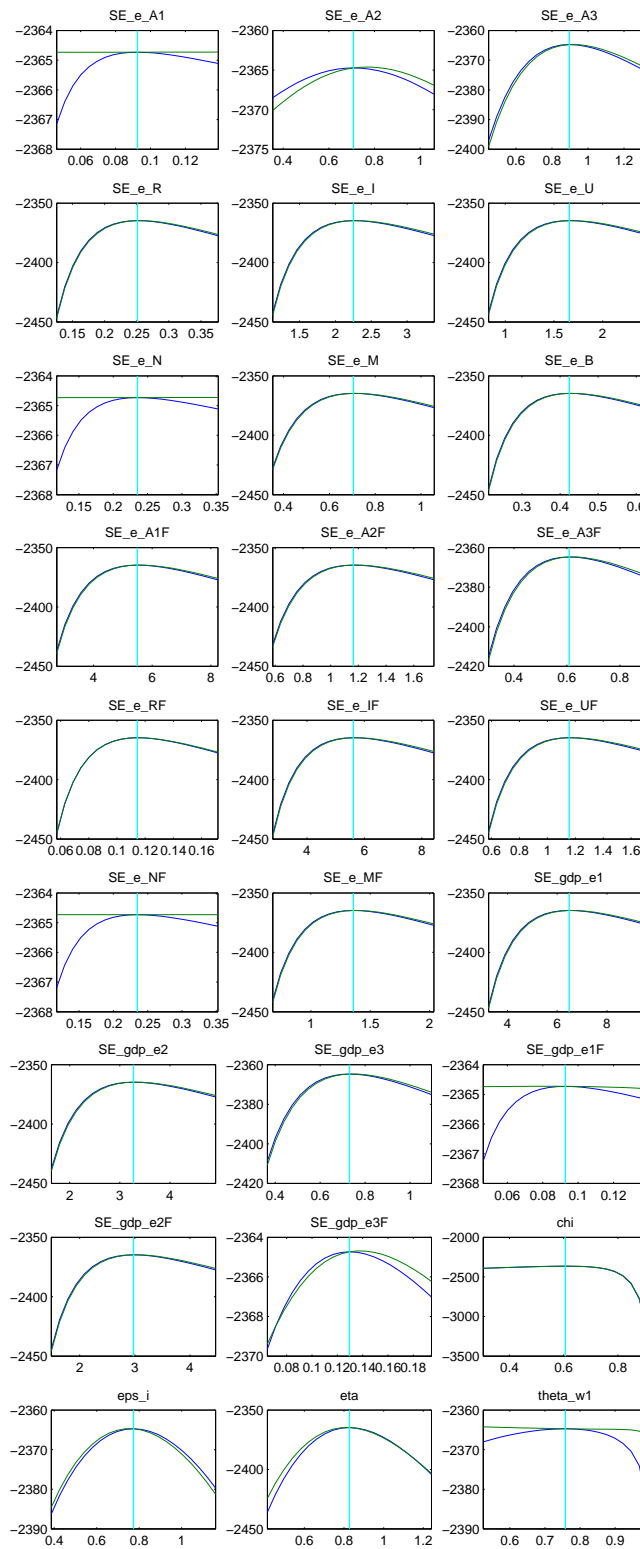
F Log posterior and log-likelihood plots
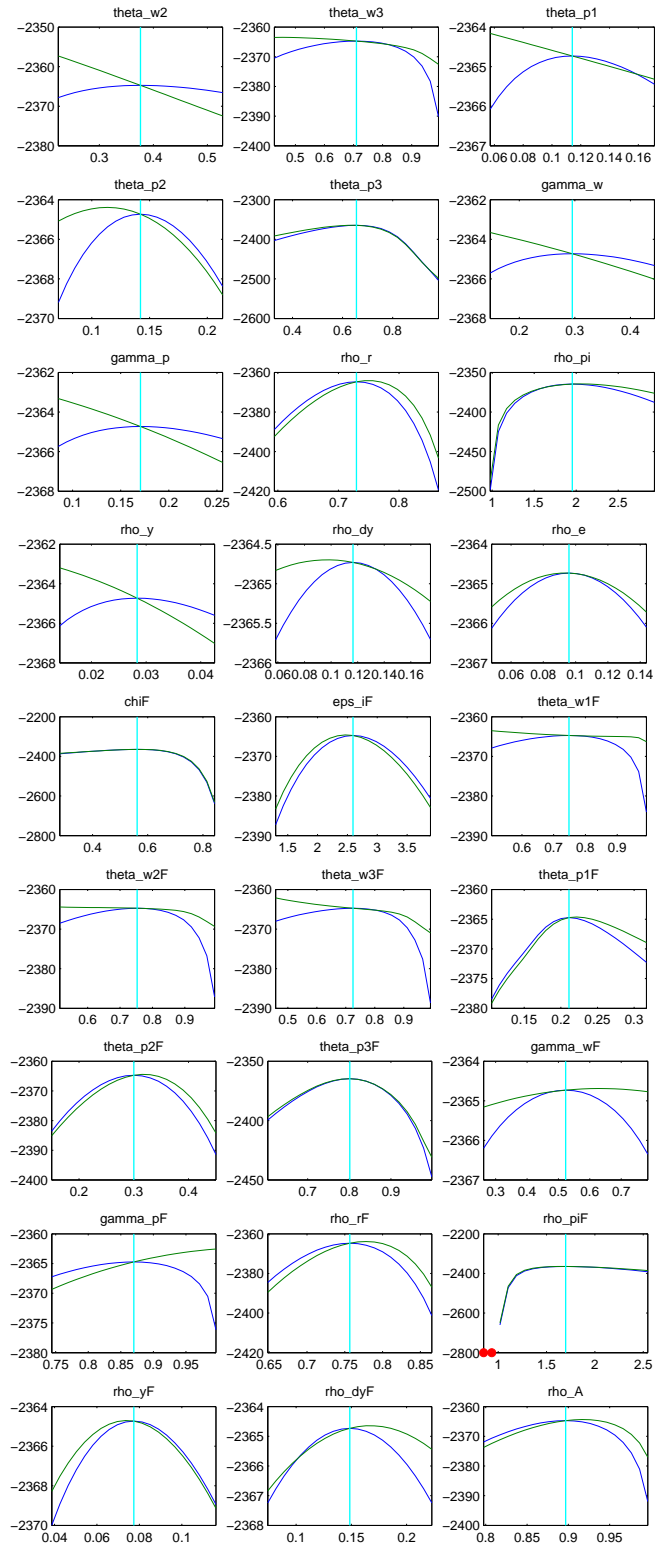

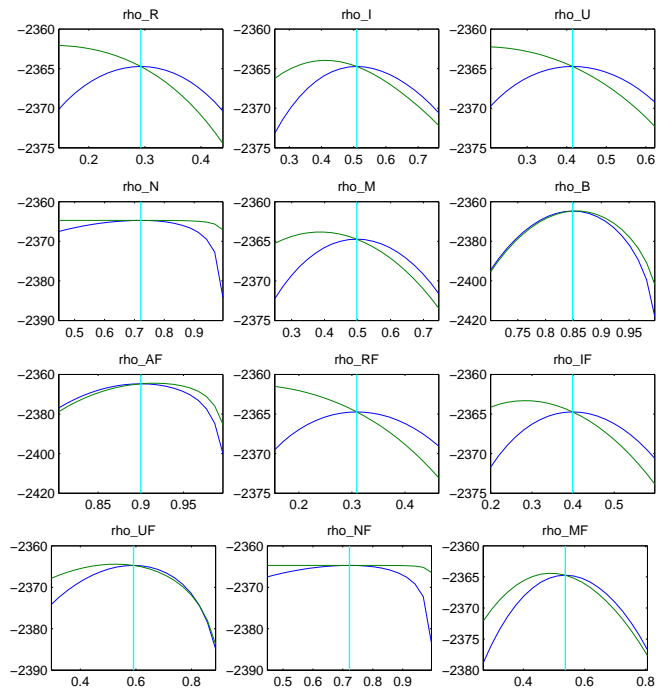Figure F.1: Check plots.

Figure F.2: Check plots.

Figure F.3: Check plots.
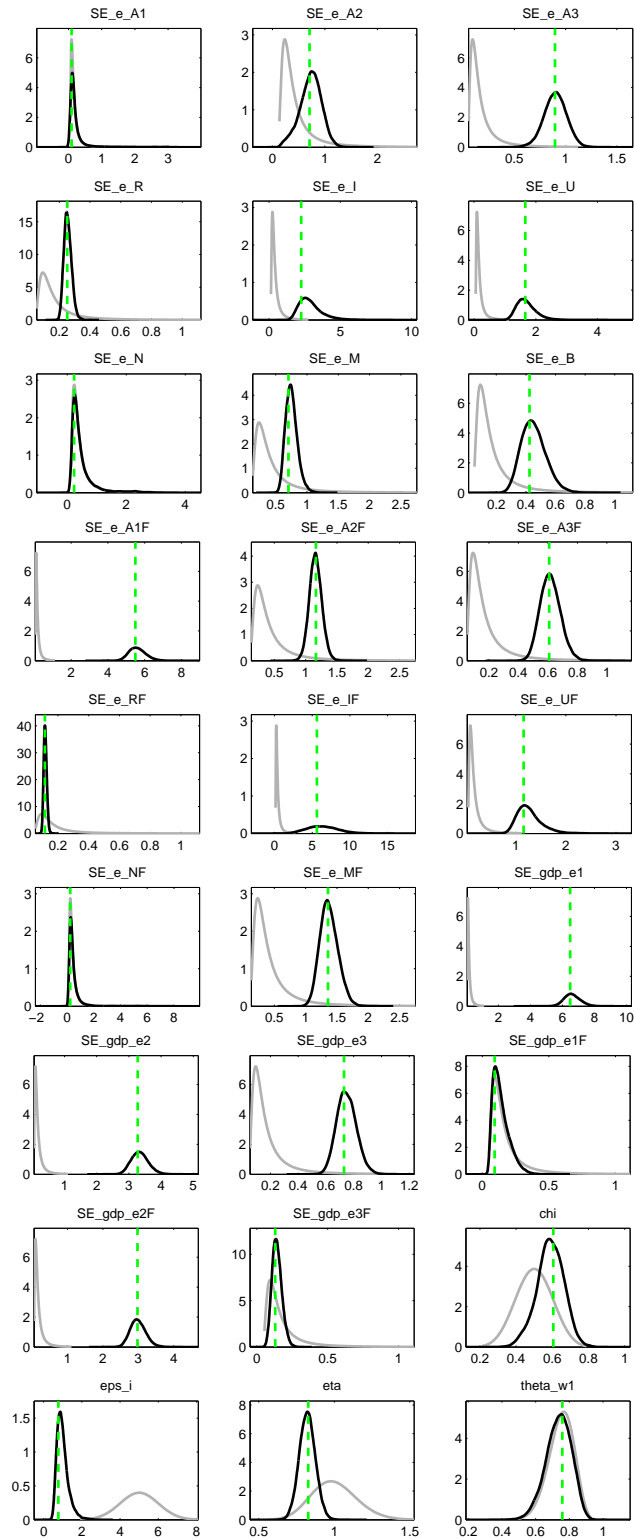
G   PRIOR AND POSTERIOR DISTRIBUTIONS

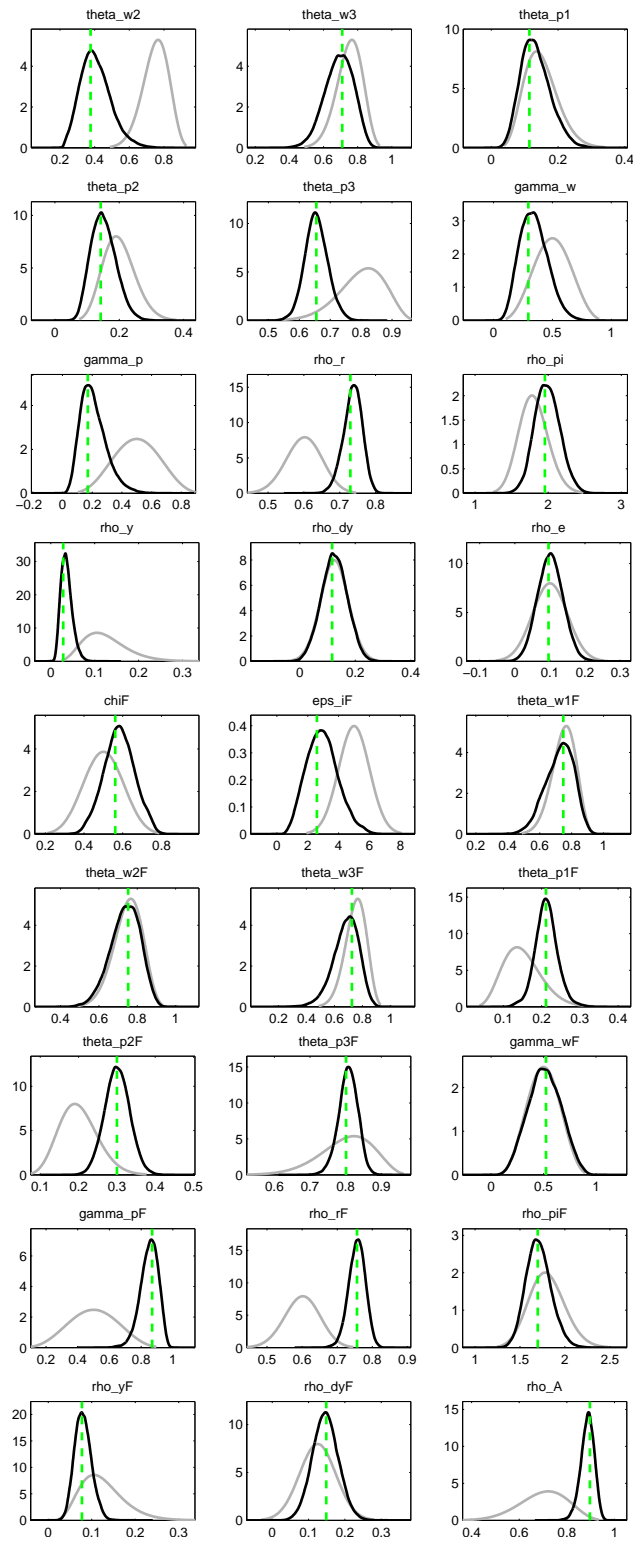Figure G.1: Priors and posteriors.

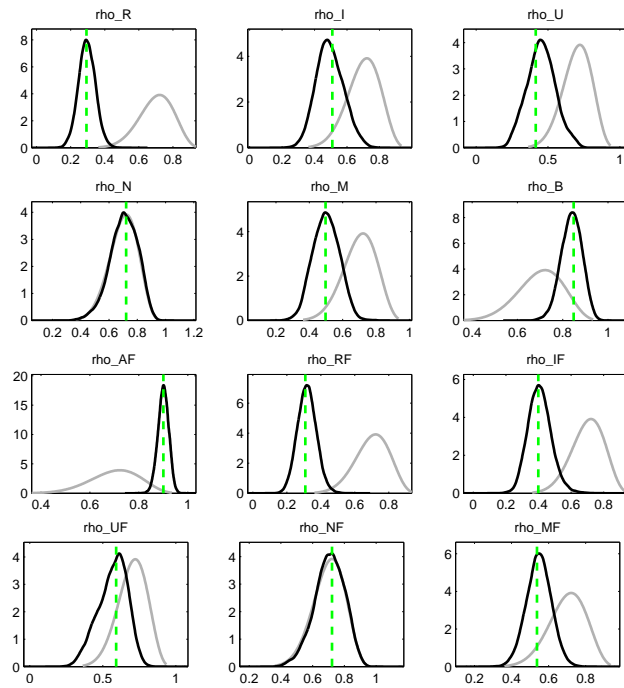Figure G.2: Priors and posteriors.

Figure G.3: Priors and posteriors.

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