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# Petro Populism\*

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## Abstract

We aim to explain petro populism — the excessive use of oil revenues to buy political support. To reap the full gains of natural resource income politicians need to remain in office over time. Hence, even a rent-seeking incumbent who prioritizes his own welfare above that of citizens, will want to provide voters with goods and services if it promotes his probability of remaining in office. While this incentive benefits citizens under the rule of rent-seekers, it also has the adverse effect of motivating benevolent policymakers to short-term overprovision of goods and services. In equilibrium politicians of all types indulge in excessive resource extraction, while voters reward policies they realize cannot be sustained over time.

**Keywords:** resource curse, political economy.

**JEL:** D72, O13, Q33

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# 1 Introduction

Much anecdotal evidence and an increasing number of careful empirical studies argue that economies rich in natural resources tend to save too little of their resource income. Estimates by the World Bank (2006) and van der Ploeg (2011) show that countries with a high share of natural resource rents in gross national income (GNI) typically have lower, and often negative, genuine saving rates.<sup>1</sup> A main explanation of this pattern is that politicians in resource-rich countries use resource revenues to secure political support and hold on to their power. Smith (2004), Cuaresma, Oberhofer and Raschky (2011), and Andersen and Aslaksen (2012) find that political leaders in oil rich countries stay longer in office. Monteiro and Ferraz (2010) find the same for municipalities with oil windfalls in Brazil. Goldberg, Wibbels and Mvukiyehe (2008) argue that in the United States officials in states with mineral wealth are able to buy public support and increase their vote share. They conclude that "politicians in resource-rich states have shown considerable skill in using mineral wealth to their advantage" (p. 495). Accounts of policy in various resource rich countries by political analysts (e.g., Parenti 2005; Looney 2007) and in the news media (e.g., Lapper 2006; Foroohar 2009) commonly refer to such policies as *petro populism*.

In this paper, we analyze and aim to explain the phenomenon of petro populism. We define it as follows:

**Definition:** *Petro populism is the economically excessive use of natural resource revenues to buy political support.*

The term of petro populism was introduced by Parenti (2005) to describe the regime and policy of Venezuela's Hugo Chávez. Parenti vividly describes how Chávez pledged *sembrar el petróleo* — to sow the oil. According to data from the IMF (2011), in Venezuela government spending as a share of GDP increased by almost 10 percentage points between 2000 and 2010, with the budget deficit averaging 1.5 percent of GDP despite a historically high oil price for much of the decade. The World Bank (2006) calculated Venezuela's genuine saving rate at the start of that decade as  $-2.7$  percent of GNI. Commentators both inside and outside of Venezuela have pointed out that Chávez's policies were overly dependent on high oil prices, and therefore unsustainable (Parenti 2005; Lapper 2006). Yet he won numerous presidential elections and national ballots over his 15 years in power.<sup>2</sup> His popularity is widely recognised as being linked to oil. *The Economist*, for instance, in their leader September 29, 2012, claims that "Had it not been for the oil boom, Mr Chávez would surely have long since become a footnote in Venezuelan

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<sup>1</sup>Genuine saving is traditional net saving (aggregate saving less capital depreciation), plus spending on education to capture change in human wealth, minus damages of stock pollutants, *minus the value of net depletion of natural resources*. This definition is taken from van der Ploeg (2011) and is based on Hamilton and Clements (1999).

<sup>2</sup>The only exception was the 2007 referendum to abolish term limits, although this was again voted over in the 2009 referendum and this time Chávez got it his way.

history.”

Other politicians commonly associated with petro populism include Mahmoud Ahmadinejad in Iran and Vladimir Putin in Russia. Looney (2007) explains how before Iran’s 2005 presidential election Ahmadinejad promised to “put the oil money on everyone’s dinner table,” and argues that it contributed greatly to him winning the election. Despite a genuine saving rate of –11.5 percent of GNI in 2000 (World Bank 2006), Iran’s government expenditures increased by 27 percent during Ahmadinejad’s first year in office, with observers arguing that his policies were designed to boost popular support. During Ahmadinejad’s first term, the head of Iran’s central bank resigned, and publicly accused the president of plundering Iran’s sovereign wealth fund (Foroohar 2009).

Under Putin, Russia’s economic policy has been compared to those of Chávez and Ahmadinejad. Foroohar (2009) refers to Putin as a “Petro-Czar” and argues that he built his popularity on oil-fueled public spending. While Russia reduced its sovereign debt from 70 percent to 10 percent of GDP during Putin’s first two presidential terms, the government simultaneously promised dramatic rises in budget spending on pensions, wages for state employees, and the military. According to Goryunov *et al.* (2013) Russia’s fiscal gap is among the largest of any developed country, despite its foreign reserves and vast energy resources.<sup>3</sup> In the aftermath of Putin’s March 2012 election victory, the American bank Citigroup calculated that the price of oil must reach and sustain \$150 per barrel for Putin to be able to fulfill his campaign promises. Other analysts of the Russian economy express concern that, even if the government can fulfill its promises, too little of the oil revenues will remain for the country’s sovereign wealth fund.<sup>4</sup> The attempts to use oil revenues to secure political support is thus seen as a cause of excessive spending.

These examples may lead to the conjecture that petro populism is confined to weakly institutionalized regimes, but we would argue otherwise. An illustrative case in point is Norway, whose oil management policy is often put forward as a success story. Yet this success has occurred against the backdrop of the right-wing populist *Progress Party* rising to 20-30 percent support in opinion polls by running on an economic platform of tax cuts *and* higher government spending. For example, Wiedswang (2011) describes the rise of the Progress Party in these terms, and writes (our translation from Norwegian):

The latest sharp increase in support of the Progress Party started in the 1990’s, almost in parallel with the growth of the Oil Fund [Norway’s sovereign wealth fund].

The party’s solution to nearly all problems has been to spend oil revenues; it became

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<sup>3</sup>Goryunov *et al.* (2013) define the fiscal gap as the difference between the present value of a government’s future expenditures and its future receipts.

<sup>4</sup>*New York Times*, March 17, 2012.

more petro populist than classical right-wing populist.<sup>5</sup>

With the 2013 election, the Progress Party was voted into national government for the first time (with the Conservative Party). Their party leader became minister of finance and responsible for the Oil Fund. Our theory makes clear, however, that petro populist policies do not even require that petro populists be in power. Rather, it can be the result of political competition from such candidates.<sup>6</sup> Snoen (2013) for instance, notes that (our translation from Norwegian):

The petroleum revenues have fostered a class of politicians that cannot say no - and petro populism has affected far more politicians than those of the Progress Party.<sup>7</sup>

A key assumption in our theory is that it takes time to reap the full financial gains of petroleum resources. Decisions about extraction rates are decisions about flow variables, and the commitment problems associated with sales of property rights to oil fields became evident with the renationalizations of petroleum ownership in the 1970s. Thus, the market price of oil fields would tend lie considerably below the present value of future oil income.<sup>8</sup> By implication, maintaining political influence *over time* is more valuable in oil abundant countries because holding political power in the future is necessary to reap the full benefits of oil revenues.

The core question of our analysis is how systematic overextraction of natural resources can stimulate popular support. Of course, one answer could be that citizens mistakenly perceive high public spending as strong performance by the government, and do not realize it might be financed by overextracting natural resources. Yet given the considerable attention to populism and excessive resource extraction in the popular press, such an explanation seems simplistic; voters are likely to be aware of these practices. We therefore propose a political economy theory of petro populism where, in equilibrium, voters are fully aware that an excessive use of oil revenues is taking place, but still reward it. To our knowledge, this is the first study that attempts to apply political economy insights to show how excessive extraction of natural resources creates popular political support.

Although the connection between natural resource income and populism is novel, our paper is related to several literatures. There is a large anecdotal literature on populism, but few formal

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<sup>5</sup>*Dagens Næringsliv*, June 10, 2011.

<sup>6</sup>Partly as a response to populist pressure, the Norwegian government implemented a fiscal rule for oil revenue spending in 2001. The rule is generally regarded as a good example for other resource-rich countries, but as argued by Harding and van der Ploeg (2013) it does not necessarily provide for sufficient public savings to cover future costs of Norway's aging population. It should also be noted that not a single *krone* was set aside in the Oil Fund until 1996, i.e., after Norway had been an oil producer for 25 years.

<sup>7</sup>*Aftenposten*, October 30, 2013.

<sup>8</sup>Today, with the exception of the United States, subsoil petroleum is public property in all countries.

models of this phenomenon. The recent paper by Acemoglu, Egorov and Sonin (2013) represents the main exception and serves as an inspiration for our study.<sup>9</sup> They study left-wing populism in a setting where a rich elite has interests that are at odds with the majority of the population, and show that even moderate politicians choose a policy to the left of the median voter as a way of signaling that they are not right-wing. A bias in terms of leftist policies is preferred by the median voter because the utility loss before the election increases the probability that the politician is not right-wing and thus yields higher expected future utility. Acemoglu, Egorov and Sonin (2013) do not discuss resource extraction, so our paper can be seen as an application and extension of their methodology to a setting where policy has dynamic effects. Another difference is that populism in their model involves lowering voters' utility before the election, while in our model populist policies entail a short-term utility gain for voters.

Our paper is also related to the equilibrium political business cycle literature, pioneered by Rogoff and Sibert (1988) and Rogoff (1990), in which good (competent) politicians might use fiscal policy before an election to signal their type to voters. However, within this tradition no papers study resource extraction as a means to finance public spending. Another difference is that in equilibrium voters in Rogoff and Sibert (1988) and Rogoff (1990) are perfectly able to discern if a politician is good or bad. Therefore, in these models, bad politicians are never reelected, whereas this may well happen in our theory.

The resource curse literature provides a third link with our paper. Existing political economy theories of the resource curse predict that increased duration of political regimes fosters a more efficient extraction path, see, e.g., Robinson, Torvik and Verdier (2006, 2014). Our theory demonstrates how the causality may run in the reverse direction, and also with an opposite sign of the correlation: a more inefficient extraction path may increase regime duration. Despite a large literature on the political economy of the resource curse,<sup>10</sup> we are not aware of other papers that investigate how the efficiency of the extraction path affects political support.

Finally, our paper relates to studies of politically motivated debt accumulation, such as Persson and Svensson (1989) and Alesina and Tabellini (1990). Besides the different topic under investigation, our theory differs from these in the direction of causality between popularity and policy: in Persson and Svensson (1989) and Alesina and Tabellini (1990) future exogenous election outcomes drive current policy (debt accumulation), while in the environment we consider election outcomes are endogenously determined by policy (resource extraction).

The rest of the paper is organized as follows. In Section 2 we present our model, and in Section 3 we derive the equilibrium, discuss when petro populism applies, and what forms it take. We also discuss some comparative statics of the model. In Section 4 we conclude. The

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<sup>9</sup>Sachs (1989) analyzes a "populist cycle," where high inequality leads to policies that make all voters worse off. Populism in Sachs's model depends on shortsighted voters, whereas we have forward-looking voters.

<sup>10</sup>For surveys of the resource curse literature, see Deacon (2011), Frankel (2010), and van der Ploeg (2011).

Appendix contains lemmas and proofs of propositions.

## 2 Basic Model

In this section we describe our model of petro populism.

### 2.1 Citizens, Policies, and Politicians

We consider a two-period economy with a continuum of citizens with measure normalized to 1. Citizens' period  $t$  utility,  $U_t$ , is determined by publicly provided goods and services and a stochastic component that affects the utility of all citizens in an identical manner. For simplicity, we assume that period utility is linear:

$$U_t = G_t + z_t, \quad t = 1, 2,$$

where  $G_t$  is period  $t$  provision of goods and  $z_t$  is the random component of utility. This formulation captures the notion that voter utility may be affected by random factors outside the control of politicians. In particular, this implies that voters cannot use their utility to perfectly observe the amount of resources that the government devotes to goods provision. The stochastic component of period utility is distributed on the real line with support  $(-\infty, \infty)$ , has cumulative density function  $H(z)$ , and probability density function  $h(z)$ . Moreover, we assume that  $h(z)$  is symmetric around zero, everywhere differentiable, satisfies  $h'(z) < 0$  for all  $z > 0$ , and  $h'(z) > 0$  for all  $z < 0$ .<sup>11</sup> Note that these properties imply that the variance of  $z$  is strictly positive, and that  $h(z)$  is bounded:  $\max h(z) = h(0) \equiv \bar{h} < \infty$ .

The government extracts natural resources  $e_t \geq 0$  to finance  $G_t$  and rents  $R_t$  which are pocketed by the incumbent. The government budget constraint reads

$$G_t + R_t = f(e_t), \quad t = 1, 2, \tag{1}$$

where  $f(\cdot)$  is the natural resource revenue function. We assume that period  $t$  resource revenues increase at a diminishing rate with extraction,  $f' > 0$ ,  $f'' < 0$ . The latter property is standard and could be due to, e.g., increasing marginal costs in resource extraction. Importantly,  $f'' < 0$  implies that it takes time to reap the full revenues from natural resource extraction. We also assume that  $f(0) = 0$  and  $f''' = 0$ .<sup>12</sup> There is a given amount  $E$  of resources available, implying that the natural resource constraint is

$$e_1 + e_2 \leq E. \tag{2}$$

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<sup>11</sup>These assumptions would, for instance, be satisfied if  $z$  has a normal distribution.

<sup>12</sup>The assumption that  $f''' = 0$  is made to simplify notation, and has no fundamental bearing on our mechanisms.

There are two types of politicians in the economy; a benevolent type, denoted by  $b$ , and a rent-seeking type, denoted by  $r$ . Benevolent politicians have the same preferences as citizens. In contrast, rent-seeking politicians also care about the rents that they appropriate for themselves,<sup>13</sup> and their period utility is given by

$$U_t^r = u(R_t) + G_t, \quad t = 1, 2, \quad (3)$$

where  $u' > 0$ ,  $u'' < 0$ , and  $u'(0) > 1$ ,  $u(0) = 0$ . Note that to simplify notation we assume rent-seekers are unaffected by  $z_1$  and  $z_2$ . This assumption has no effect on our results.

Benevolent types constitute a fraction  $p$  of the pool of political candidates, while the remaining fraction  $1 - p$  are  $r$  types. Citizens are aware of this distribution, but they cannot observe a politician's type other than potentially through the actions of the incumbent. Moreover, citizens do not see the amount of rents appropriated by the politician in office and, by implication, cannot know the amount of resources left untapped for future use.

In period 1, an incumbent of type  $j = \{b, r\}$  holds office, chooses resource extraction  $e_1^j$ , and allocates the resource income between goods provision and rents. At the end of the first period, there is an election in which voters decide to either reelect the incumbent or allow a challenger of unknown type to take power. The politician with the highest number of votes has the right to decide policy after the election. The reelection probability of the incumbent, to be determined in equilibrium, is denoted by  $\Pi$ .

Before the election voters know their utility from past policies  $U_1$ , but not the exact amount  $G_1$  of past provision of goods by the government.<sup>14</sup> Hence, voters use their utility to infer the nature of period 1 policy, and thereby to form a judgment about the incumbent's type. Although voters do not immediately know the exact amount that the incumbent has spent on goods provision, they do not make systematic mistakes when estimating this amount. Moreover, our assumptions about the sign of  $h'(z)$  ensure that voters are more likely to make small rather than large errors when estimating the previous provision of public goods. The policy that is implemented is more likely to lie close to rather than distant from estimated policy; the voters' estimate is informative.

Using that  $\mathcal{E}[z_1] = \mathcal{E}[z_2] = 0$ , where  $\mathcal{E}$  is the expectations operator, and denoting by  $G_t^j$  the goods provision by a politician of type  $j = \{b, r\}$  in period  $t$ , we can express the expected

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<sup>13</sup>Note that in our model appropriating rents is not confined to politicians transferring resource income to their own bank accounts. Rather, rents include spending revenues on any purpose that the representative citizen does not care about. Examples would include enriching cronies and insiders as long as this group constitutes a negligible fraction of the electorate.

<sup>14</sup>At this point, there is a conceptual difference between Acemoglu, Egorov and Sonin (2013) populism model and our approach. In their model, voters have deterministic utility defined over policy, but voters have imperfect information about this policy. Thus, in Acemoglu, Egorov and Sonin, citizens are uncertain about their own utility when they vote. By contrast, in our model voters know their own utility, but cannot fully determine what part of it was due to implemented policy, and what part was caused by random impulses.



lifetime utility of a benevolent incumbent as

$$V^b = G_1^b + [\Pi + (1 - \Pi)p] G_2^b + (1 - \Pi)(1 - p) G_2^r. \quad (4)$$

The corresponding expected lifetime utility of a rent-seeking incumbent is given by

$$V^r = u(R_1) + G_1^b + \Pi u(R_2) + [\Pi + (1 - \Pi)(1 - p)] G_2^r + (1 - \Pi)p G_2^b. \quad (5)$$

## 2.2 Timing of Events and Equilibrium Concept

The precise timing of events is as follows:

1. The incumbent decides policy  $\{G_1, e_1, R_1\}$ .
2. Citizens observe and enjoy  $U_1 = G_1 + z_1$ , and use this to update their prior beliefs about the incumbent's type.
3. The election takes place and each citizen supports the incumbent or the opponent.
4. The politician with a majority of votes decides policy  $\{G_2, e_2, R_2\}$ .
5. Citizens observe and enjoy  $U_2 = G_2 + z_2$ .

Since we have a dynamic game of incomplete information, the beliefs of players need to be specified. As usual we allow voters to use Bayes' rule to update all relevant subjective probabilities; thus, we look for perfect Bayesian equilibria (in pure strategies). Given that we have many voters, the set of perfect Bayesian equilibria involves a large number of equilibria in which voters use weakly dominated strategies, such as voting for politicians known to be rent-seekers because a majority of other voters are doing so. To rule out such unreasonable equilibria we focus on perfect Bayesian equilibria in undominated strategies. This simply implies that citizens vote for the politician that will give them the highest expected utility should their vote turn out to be decisive.<sup>15</sup>

Throughout the analysis we make two assumptions. The first is that the initial stock of natural resources  $E$  is not too small. The second is that the derivative of the probability density function  $h(z)$  is not too high (which reduces to an assumption that the variance is not too small if  $z$  has a normal distribution). The precise assumptions are specified and explained in the Appendix. We also show in the Appendix that with these assumptions in place, all the optimization problems to follow are globally concave and they imply interior solutions for all policy variables. Finally, we show in the Appendix that equilibrium always exists, and also that it is unique.

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<sup>15</sup>We also adopt the convention that if voters are indifferent, they vote for the incumbent. This has no bearing on our results and occurs with probability zero in equilibrium.

### 3 Analysis

We next give a brief characterization of the first best situation in our model, and then solve the model by backwards induction.

#### 3.1 First-Best Solution

From the citizens' perspective the first-best solution entails zero rents,  $G_t = f(e_t)$ ,  $t = 1, 2$ , and an extraction path that solves

$$\max_{e_1, e_2} \mathcal{E} \{f(e_1) + z_1 + f(e_2) + z_2\} \quad (6)$$

subject to (2) holding with equality. Inserting  $e_2 = E - e_1$  and  $\mathcal{E}[z_1] = \mathcal{E}[z_2] = 0$ , the first-order condition reads

$$f'(e_1) = f'(E - e_1). \quad (7)$$

This optimality condition reflects the linearity of the utility function, and implies that resource revenues should be extracted smoothly over time to maximize revenues. We denote by  $(e^{fb}, G^{fb})$  the first best extraction level and the associated goods provision. Since  $f(\cdot)$  is strictly concave, (7) implies that  $e^{fb} = \frac{1}{2}E$  and  $G^{fb} = f(\frac{1}{2}E)$ .

#### 3.2 Period 2: Behavior of Politicians

The election winner makes the only decision in period 2: how to spend the income from remaining natural resources. Characterizing this choice is straightforward. Let an asterisk denote the equilibrium value of a designated variable, so that  $G_2^{j*}$  is the equilibrium goods provision of a type  $j = \{b, r\}$  politician in period 2. During this period, benevolent politicians devote all resource income to goods provision:

$$G_2^{b*} = f(E - e_1), \quad R_2^{b*} = 0. \quad (8)$$

In contrast, rent-seekers wish to allocate resources to rents as well as to providing goods. Maximization of (3) with respect to  $R_2$  implies that a rent-seeker will spend all available resource revenues on rents, up until the point where

$$u'(R_2^r) = 1 \Leftrightarrow R_2^r = u'^{-1}(1) \equiv \rho. \quad (9)$$

Available period 2 income,  $f(E - e_1)$ , in excess of the amount defined by equation (9) will be spent on goods provision. Lemma 1 in the Appendix establishes that, for a not too small  $E$ , period 1 incumbents of both types will always leave enough resources for period 2 choices of a  $r$  government to satisfy (9). As mentioned above, we do assume that  $E$  is sufficiently large for this to occur, and hence we will always have  $f(E - e_1) > \rho$ . It follows that

$$G_2^{r*} = f(E - e_1) - R_2^{r*}, \quad R_2^{r*} = \rho. \quad (10)$$

### 3.3 Period 1: Behavior of Voters

Having experienced  $U_1$ , each voter uses Bayes' rule to form a belief  $\tilde{p}$  about the probability that the incumbent is benevolent. Based on this updated probability, each voter decides whether to support the incumbent politician or the opposition candidate.

The incumbent is reelected if the voters' expected period 2 utility is (weakly) higher with the incumbent in office rather than with an opposition candidate. The only information voters have about the opposition candidate is that she is benevolent with probability  $p$ . From (8) and (10), it follows that the incumbent is reelected with certainty when  $\tilde{p} \geq p$ . If  $\tilde{p} < p$  the incumbent is ousted from office.

We denote voters' beliefs about spending policies of a type  $j$  politician by  $\tilde{G}_1^j$ ,  $j = \{b, r\}$ . A voter who has experienced  $U_1$  will assign the following value to the probability that the incumbent is benevolent:

$$\tilde{p} = \frac{ph(U_1 - \tilde{G}_1^b)}{ph(U_1 - \tilde{G}_1^b) + (1-p)h(U_1 - \tilde{G}_1^r)}. \quad (11)$$

Equation (11) implies that  $\tilde{p} \geq p$  if and only if  $h(U_1 - \tilde{G}_1^b) \geq h(U_1 - \tilde{G}_1^r)$ . For now assume that  $\tilde{G}_1^b > \tilde{G}_1^r$ ; voters believe that benevolent politicians provide more goods than rent-seeking politicians. (In Proposition 1 below, we show that this belief is the only correct one in equilibrium.) Since  $z$  is symmetric around zero, it follows that  $\tilde{p} \geq p$  if

$$U_1 \geq \frac{\tilde{G}_1^b + \tilde{G}_1^r}{2}. \quad (12)$$

Because  $\tilde{G}_1^b > \tilde{G}_1^r$ , equation (12) is the necessary and sufficient condition for the incumbent to be reelected.<sup>16</sup> Given (12), the probability that an incumbent is reelected after providing an amount  $G$  is

$$\begin{aligned} \Pi(G) &= \Pr\left(G + z \geq \frac{\tilde{G}_1^b + \tilde{G}_1^r}{2}\right) \\ &= 1 - H\left(\frac{\tilde{G}_1^b + \tilde{G}_1^r}{2} - G\right) \\ &= H\left(G - \frac{\tilde{G}_1^b + \tilde{G}_1^r}{2}\right), \end{aligned} \quad (13)$$

where the last equality follows from the assumption that  $h(z)$  is symmetric around zero.

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<sup>16</sup>To understand why (12) is necessary and sufficient for  $\tilde{p} \geq p$ , note that  $h(U_1 - \tilde{G}_1^b) \geq h(U_1 - \tilde{G}_1^r) \Leftrightarrow |U_1 - \tilde{G}_1^b| \geq |U_1 - \tilde{G}_1^r|$ . Because  $\tilde{G}_1^b > \tilde{G}_1^r$ , this holds always if  $U_1 - \tilde{G}_1^b \geq 0$  and requires  $U_1 \geq \frac{\tilde{G}_1^b + \tilde{G}_1^r}{2}$  if  $U_1 - \tilde{G}_1^b < 0$ .

### 3.4 Period 1: Behavior of the Incumbent

We next investigate the policy choices of each type of politician in period 1, and thereafter bring these choices together to analyze the equilibrium.

In the Appendix, we establish that the assumptions introduced in Section 2.2 are sufficient for global concavity of the period 1 problems for benevolent (Lemma 2) and rent-seeking (Lemma 3) incumbents. The crux of the assumptions is that sufficient noise in voters' utility ensures the concavity of the politicians' maximization problems. If, for example, the preference shock  $z$  has a normal distribution  $\mathbb{N}(0, \sigma^2)$ , we show in the Appendix that the period 1 problems of both types of politicians are always globally concave if  $\sigma$  is sufficiently high.

#### A Benevolent Incumbent

Denote the period 1 extraction policy of a benevolent politician by  $e_1^b$ . From the utility function (4) and the budget constraint (1), it follows directly that a benevolent incumbent will always choose zero rents and  $G_1^b = f(e_1^b)$ . By the resource constraint given in equation (2),  $b$ 's policy problem thus reduces to choosing extraction only. Using the period 2 policy of rent-seekers (10) in (4), we can formally state the problem as

$$\max_{e_1^b} \left\{ f(e_1^b) + f(E - e_1^b) - (1 - \Pi)(1 - p)\rho \right\}. \quad (14)$$

In (14), the first term inside the maximand is the incumbent's utility from consuming publicly provided goods in period 1. The next two terms together gives the expected utility from leaving resources to period 2. The benevolent incumbent enjoys all future revenues that are used to provide goods,  $f(E - e_1^b)$ , but with probability  $(1 - \Pi)(1 - p)$  the incumbent is replaced by a rent-seeker who diverts  $\rho$ .

As mentioned in Section 2.2, we establish in the Appendix that  $b$ 's optimization problem is globally concave and interior. The optimal extraction policy is characterized by the first-order condition:

$$f'(e_1^b) \left[ 1 + h \left( G_1^b - \frac{\tilde{G}_1^b + \tilde{G}_1^r}{2} \right) (1 - p)\rho \right] = f'(E - e_1^b) \quad (15)$$

where we have used that equation (13) implies  $\Pi'(G) = h \left( G - \frac{\tilde{G}_1^b + \tilde{G}_1^r}{2} \right)$ .

#### A Rent-Seeking Incumbent

Denote the period 1 extraction policy of a rent-seeker by  $e_1^r$ . By substituting from (1), (2), (8), and (10) into equation (5) and simplifying, we can express the lifetime expected utility of a rent-seeking incumbent as

$$V^r(G_1^r, e_1^r) = G_1^r + u(f(e_1^r) - G_1^r) + f(E - e_1^r) - [1 - p(1 - \Pi)]\rho + \Pi u(\rho). \quad (16)$$

The first two terms on the right hand-side of (16) give the  $r$  incumbent's utility from goods provision and rents in period 1. The three remaining terms gives the expected utility of a rent-seeker from leaving resources to period 2: The expected utility of future provision of goods is  $f(E - e_1^r) - [1 - p(1 - \Pi)]\rho$ , while the expected utility from individual rents in period 2 is  $\Pi u(\rho)$ .

The policy problem of a rent-seeking incumbent is to maximize (16) with respect to  $e_1^r$  and  $G_1^r$ . Again, we establish in the Appendix that this optimization problem is globally concave and interior. The first-order conditions are

$$u'(f(e_1^r) - G_1^r) f'(e_1^r) = f'(E - e_1^r) \quad (17)$$

and

$$u'(f(e_1^r) - G_1^r) = 1 + h \left( G_1^r - \frac{\tilde{G}_1^b + \tilde{G}_1^r}{2} \right) [u(\rho) - p\rho], \quad (18)$$

respectively. As noted in Lemma 1, the properties of  $u(\cdot)$  imply that  $u(\rho) - p\rho > 0$ .

### 3.5 Equilibrium

In a perfect Bayesian equilibrium, voters' beliefs are consistent with politicians' choices, and these choices are in turn consistent with the first order conditions given in equations (15), (17), and (18). Hence, in equilibrium,  $G_1^j = \tilde{G}_1^j = G_1^{j*}$  and  $e_1^j = \tilde{e}_1^j = e_1^{j*}$ , for  $j = \{r, b\}$ . The analysis above tells us that the period 1 equilibrium policy vector for a benevolent politician is  $\{G_1^{b*}, e_1^{b*}, 0\}$ , while it is  $\{G_1^{r*}, e_1^{r*}, f(e_1^{r*}) - G_1^{r*}\}$  for a rent-seeker.

Let us now investigate the equilibrium more closely. We first establish that in period 1 rent-seeking politicians always provide less goods than benevolent types, which validates that the criterion for reelection is equation (12) as stated earlier.

**Proposition 1** *Denote the equilibrium provision of goods of a benevolent politician in period 1 by  $G_1^{b*}$  and that of a rent-seeking politician by  $G_1^{r*}$ . Then:*

1.  $G_1^{b*} > G_1^{r*}$ , i.e., benevolent politicians always provide more goods than rent-seeking politicians;
2. The incumbent is reelected if and only if  $U_1 \geq \frac{G_1^{b*} + G_1^{r*}}{2}$ .

**Proof.** See the Appendix. ■

By equation (13), the *equilibrium* reelection probabilities of benevolent and rent-seeking politicians are

$$\Pi^{b*} = H \left( \frac{G_1^{b*} - G_1^{r*}}{2} \right)$$

and

$$\Pi^{r*} = H\left(\frac{G_1^{r*} - G_1^{b*}}{2}\right),$$

respectively. Observe that Proposition 1 and the symmetry assumption on  $h(z)$  together imply that  $\Pi^{b*} > \frac{1}{2}$  and that  $\Pi^{r*} = 1 - \Pi^{b*} < \frac{1}{2}$ . In equilibrium, a benevolent (rent-seeking) incumbent has a higher (lower) than 50 percent reelection probability, and the reelection probabilities of benevolent and rent-seeking politicians sum to one.

Using these results in equations (15), (17), and (18), we can now state the optimality conditions that must hold in equilibrium. By equation (15), the equilibrium policy of benevolent politicians is characterized by

$$\frac{f'(E - e_1^{b*})}{f'(e_1^{b*})} = 1 + (1 - p)\rho h\left(\frac{G_1^{b*} - G_1^{r*}}{2}\right). \quad (19)$$

Similarly, the equilibrium policy of rent-seeking politicians is described by

$$u'(f(e_1^{r*}) - G_1^{r*})f'(e_1^{r*}) = f'(E - e_1^{r*}), \quad (20)$$

and

$$u'(f(e_1^{r*}) - G_1^{r*}) = 1 + [u(\rho) - p\rho] h\left(\frac{G_1^{b*} - G_1^{r*}}{2}\right). \quad (21)$$

In equation (21), we have used that  $h(z) = h(-z)$  because  $h$  is symmetric around  $z = 0$ .

We now turn to the existence and uniqueness of equilibrium.

**Proposition 2** *There exists a unique perfect Bayesian equilibrium (in pure strategies).*

**Proof.** See the Appendix. ■

While the proof is delegated to the Appendix, Figure 1 provides the intuition.

Mathematically, equation (19) characterizes the equilibrium policy of benevolent politicians,  $G_1^{b*}$ , when voters believe that rent-seeking politicians would choose some policy  $\tilde{G}_1^r$ , and  $\tilde{G}_1^r = G_1^{r*}$ . The proof of Proposition 2 (in the Appendix) shows that the relationship between  $G_1^{b*}$  and  $\tilde{G}_1^r$  is monotonic with a positive slope, as illustrated by the line  $G_1^{b*}(\tilde{G}_1^r)$  in Figure 1. Points  $A$  and  $C$  in Figure 1 are  $G_1^{b*}(0)$  and  $G_1^{b*}(G_1^{b*})$ , respectively. The proof of Proposition 2 shows that point  $A$  is below  $f(E)$  on the vertical axis. Equations (20) and (21) determine the rent-seeking politicians' choice  $G_1^{r*}$  when benevolent politicians are believed to pursue  $\tilde{G}_1^b$ , and  $\tilde{G}_1^b = G_1^{b*}$ . Figure 1 plots this relationship, labeled  $G_1^{r*}(\tilde{G}_1^b)$ , as downward sloping. Points  $B$  and  $D$  in Figure 1 are  $G_1^{r*}(0)$  and  $G_1^{r*}(G_1^{r*})$ , respectively. We show in the Appendix that point  $B$  is located in the interior of the horizontal dashed line in Figure 1. Then, a sufficient condition for existence of equilibrium is that point  $C$  is located to the upper-right of point  $D$ . The proof of Proposition 2 shows that this condition is always fulfilled under our assumption that the initial resource stock is not too low. A political equilibrium is at the intersection between the two curves. We

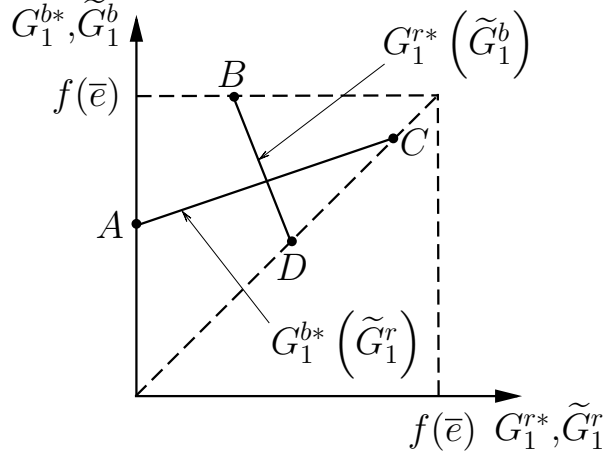


Figure 1: **Political Equilibrium.**  $G_1^{r*}(\tilde{G}_1^b)$  is rent-seekers' optimal provision of public goods in period 1 consistent with individual optimality conditions and voter beliefs,  $G_1^r = \tilde{G}_1^r = G_1^{r*}$ , for given voter beliefs about benevolent policy,  $\tilde{G}_1^b$ .  $G_1^{b*}(\tilde{G}_1^r)$  is benevolent incumbents' optimal provision of public goods in period 1 consistent with individual optimality conditions and voter beliefs,  $G_1^b = \tilde{G}_1^b = G_1^{b*}$ , for given voter beliefs about rent-seeker policy,  $\tilde{G}_1^r$ . Point A is  $G_1^{b*}(0)$ , point B is  $G_1^{r*}(0)$ , point C is  $G_1^{b*}(G_1^{r*})$  and point D is  $G_1^{r*}(G_1^{b*})$ . The dashed upward sloping curve is the 45 degree line.

also show in the Appendix that  $\frac{dG_1^{r*}}{d\tilde{G}_1^b} < 0 < \frac{dG_1^{b*}}{d\tilde{G}_1^r}$  as drawn, which implies that the equilibrium is unique.

### Overextraction

By comparing equation (19) to the first-best solution in equation (7), it is easy to see that  $e_1^{b*} > e^{fb}$ . In equilibrium, a benevolent incumbent will extract more natural resources than in the first-best situation. The reason is intuitive: A benevolent incumbent overextracts natural resources because it increases her reelection probability. Analytically, this mechanism shows up by the last term in equation (19). In this term,  $h\left(\frac{G_1^{b*} - G_1^{r*}}{2}\right)$  identifies the marginal effect of goods provision on the incumbent's reelection probability. By Proposition 1, this effect is positive. The higher reelection probability is in turn valued by the expected gain from being reelected, and this is given by  $(1-p)\rho$ : the risk that a successor is a rent-seeker times the resources that such a type would divert from the public. In a nutshell, by depleting resources to increase goods provision in period 1 above the first-best level, benevolent politicians increase their reelection probability and thereby the likelihood that future resource income will be used to finance  $G$  rather than  $R$ .

Turning to rent-seeking types, we can substitute from (21) in (20) to show that the intertem-

poral extraction path of a  $r$  type is characterized by:

$$\frac{f'(E - e_1^{r*})}{f'(e_1^{r*})} = 1 + [u(\rho) - p\rho] h \left( \frac{G_1^{b*} - G_1^{r*}}{2} \right) > 1, \quad (22)$$

from which it immediately follows that rent-seekers will also overextract in equilibrium,  $e_1^{r*} > e^{fb}$ . The incentive leading to overextraction is, as for a benevolent incumbent, to increase the reelection probability as is evident by the term  $h \left( \frac{G_1^{b*} - G_1^{r*}}{2} \right)$  in (22). The higher reelection probability is in turn valued by his expected utility gain from winning the election,  $u(\rho) - p\rho$ .

Although both type of politicians overextract natural resources, this inefficiency will be most severe with a rent-seeking incumbent;  $e_1^{r*} > e_1^{b*}$ . It is straightforward to show this results by comparing (19) and (22): Since  $f'' < 0$ ,  $e_1^{r*} > e_1^{b*}$  is equivalent to  $\frac{f'(E - e_1^{r*})}{f'(e_1^{r*})} > \frac{f'(E - e_1^{b*})}{f'(e_1^{b*})}$ . From (19) and (22) it follows that this inequality holds when  $u(\rho) > \rho$ , which is always satisfied.

A rent-seeking incumbent extracts more resources than a benevolent because  $r$  types value future political power higher than  $b$  types. To see this, note that both types have the same marginal value of future goods provision. However, the rent-seeker in addition values the future possibility of diverting public resources to personal rents. To be able to cash in rents, political power is necessary. Thus as long as a rent-seeking politician chooses to grab rents when in office, which he always does in our model, his future utility of power is higher than that of a benevolent politician. In turn, the higher value of future political power implies a stronger marginal incentive to extract in the present in order to increase the win probability. Analytically this can be seen by the fact that the term  $u(\rho) - p\rho$  in front of  $h \left( \frac{G_1^{b*} - G_1^{r*}}{2} \right)$  in (22) exceeds the term  $(1 - p)\rho$  in front of  $h \left( \frac{G_1^{b*} - G_1^{r*}}{2} \right)$  in (19). Also, for this same reason, a rent-seeking incumbent grabs less rents that he would do if reelection incentives were not a concern, which can be seen from (21) by that his marginal utility of rents exceeds unity ahead of the election.

The next proposition summarizes these results (proof in the text):

**Proposition 3** *In political equilibrium:*

1. *Benevolent and rent-seeking incumbents overextract natural resources, i.e.  $e_1^{b*}, e_1^{r*} > e^{fb}$ .*
2. *Rent-seeking incumbents extract more natural resources than benevolent incumbents, i.e.  $e_1^{r*} > e_1^{b*}$ .*

The reason for overextraction, preelection signaling, speaks directly to the phenomenon of petro populism. In the Introduction, we defined petro populism as the excessive use of resource revenues to buy political support. In our model this is exactly what both types of politicians attempt in period 1: by providing more goods than would be supplied with their ideal policy, politicians can improve their reelection prospects. Note, however, that the two kinds of politicians have contrasting underlying motivations for petro populist policies. In period 1, a



benevolent incumbent spends an excessive amount of resource revenues *to signal her true type* to voters. A rent-seeking incumbent, on the other hand, spends more on goods provision than he prefers in period 1 *to conceal his true type*. Both types of incentives lead to overextraction of natural resources.

To our knowledge, the political incentives for the excessive extraction of natural resources just proposed are new to the literature. We note that these incentives imply that incumbents increase their expected time in office by shifting extraction towards the present. This contrasts previous literature, which finds that an increase in expected time in office leads to less overextraction. In this previous literature,. political stability (i.e., a higher reelection probability) causes a more efficient extraction path, while in our theory causality runs from (in)efficiency in the extraction path to political stability.

### Overbidding

The above discussion shows that benevolent politicians respond to increased public goods provision by rent-seeking candidates by increasing their own spending. Such competitive pressure on benign, well-intentioned politicians is the central reason for equilibrium petro populism. The existence of rent-seekers motivates benevolent politicians to choose excessive extraction and spending in equilibrium. The more goods rent-seekers are willing to provide in equilibrium, the more sensitive is the benevolent candidate's reelection probability to her own provision of goods. Mechanically, this follows from the assumption that  $h(z)$  is single peaked at zero, which implies that in equilibrium  $h' \left( \frac{G_1^{b*} - G_1^{r*}}{2} \right) < 0$ . Intuitively, when (for some reason) a rent-seeking politician would increase his equilibrium provision, the two types of politicians becomes harder to distinguish, and a benevolent politician responds to this by pushing overextraction and goods provision up, since the marginal effect of provision on her popularity has become higher.

### 3.6 Comparative Statics

We now turn to two particularly interesting questions about politically determined resource extraction: how the quality of political candidates, and the quality of voter information, affect equilibrium extraction rates.

#### Extraction and the Quality of Political Candidates

When the pool of political candidates is of poor quality, in the sense that  $p$  is low, citizens can expect lower welfare in the future for a given amount of resources left after period 1. On the other hand, how the quality of political candidates affects extraction in period 1 remains an open question. The following proposition provides an answer:

**Proposition 4** *A lower quality of the pool of politicians, that is a lower  $p$ , affects period 1 extraction choices as follows:*

1. *A benevolent incumbent increases overextraction.*
2. *A rent-seeking incumbent increases overextraction if  $|h'(z)|$  is not too high for any  $z$ .*

**Proof.** See the Appendix. ■

A notable consequence of Part 1 is that benevolent incumbents are especially prone to excess resource extraction in societies where politicians in general are likely to be rent-seekers, i.e., where  $p$  is low. The intuition behind this result is as follows: when a benevolent incumbent knows that an eventual electoral loss is likely to bring a rent-seeker into office, it becomes particularly important for her to get re-elected. Therefore, a benevolent incumbent will be more willing to overprovide goods, financed by excessive resource extraction so as to gain popularity. This phenomenon is petro populism.

A rent-seeking incumbent also perceives the cost of losing the election as being higher, the higher is the probability that he will be replaced by another rent-seeker. Hence, the same force that lifted a  $b$  incumbent's extraction in Part 1, will motivate rent-seekers to increase overextraction when  $p$  is lower. However, for rent-seekers there is also another effect that pulls in the opposite direction. Because a lower  $p$  increases the equilibrium level of goods provision that voters expect from a  $b$  incumbent, the marginal effect of a rent-seeker's goods provision on her reelection probability will change. Mathematically,  $\Pi''_{G^r, G^b}$  comes into play. The sign of this derivative is determined by  $h' \left( \frac{G^b - G^r}{2} \right)$ , which is negative since  $G_1^{b*} > G_1^{r*}$ . Hence, for a rent-seeker the popularity gain from providing goods are lower when  $p$  is high. In isolation, this effect pulls toward less overextraction by the rent-seeker. When this effect is moderate, as it will be when  $|h'(z)|$  is not too high, a rent-seeker's overextraction is increasing in  $p$  due to the costs of losing to another rent-seeker. Should  $|h'(z)|$  be large, then the indirect effect might dominate and the rent-seeker might extract *less* for a higher  $p$ .

### Extraction and the Quality of Information

Our final proposition deals with voters' information about the policies being implemented. In our model, the precision, or quality, of voters' political information is conveniently summarized by the variance of  $z$ . The following proposition demonstrates how this variance affects the pre-election extraction choice:

**Proposition 5** *A higher precision of voters' signal about policy, that is a lower variance of  $z$ , affects extraction as follows:*

1. *Overextraction may increase or decrease. Benevolent and rent-seeking politicians always push overextraction in the same direction.*

2. When  $z \sim \mathbb{N}(0, \sigma^2)$ , then if the precision,  $\frac{1}{\sigma^2}$ , is sufficiently low initially, higher precision will increase overextraction

**Proof.** See the Appendix. ■

The degree to which voters can observe implemented policies affects an incumbent’s marginal incentive to provide goods. With normally distributed noise and a low initial precision, a higher  $\frac{1}{\sigma^2}$  always increases the marginal incentive of both types of politicians to undertake petro populism. The reason is that when precision increases, politicians become easier to distinguish, making the marginal effect of goods provision on their election probabilities higher. Since petro populism has a higher political payoff, overextraction increases.<sup>17</sup>

## 4 Conclusion

In many countries with abundant natural resources, politicians seem to base their popularity on unsustainable depletion and spending policies, saving too little of their resource revenues. This paper has presented a framework that can explain this phenomenon. We have shown how rational, forward-looking voters reward excessive spending, as they are more likely to reelect politicians that pursue such policies. This equilibrium behavior of voters and politicians explains the occurrence of petro populism: excessive levels of spending financed by short-term revenue streams obtained from selling non-renewable resources.

Even benevolent politicians, sharing preferences with the representative voter, choose to pursue petro populist policies. Facing political competition from rent-seeking candidates, benevolent politicians are motivated to pursue the type of “overbidding” that characterizes petro populism. Moreover, our model predicts that higher spending of resource revenues improves the incumbent’s prospects for political survival and causes lower political turnover. We have also seen that, perhaps counterintuitively, with less noisy information about the implemented policy, overextraction may actually increase.

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<sup>17</sup>The quality of information available to voters is exogenous in our model. An interesting area of future research is to allow for an endogenous precision in voters policy signal. For instance, rent-seeking politicians in resource abundant countries may have strong incentives to crack down on media freedom to more easily conceal their true type. In this respect it is interesting to note that Egorov, Guriev and Sonin (2009) find that media are less free in oil-rich economies.

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## Appendix

In this Appendix we collect technical material related to the model and the analysis. We start by precisely stating the two assumptions that we introduced in Section 2.2, here referred to as Assumption 1 and 2. We then show in Lemma 1 that when Assumption 1 holds, an incumbent of either type will always leave enough natural resources unextracted for an election winner of type  $r$  to provide a strictly positive amount of goods in period 2. Then, in Lemmas 2 and 3, we establish that under Assumption 2, the objective functions of both benevolent and rent-seeking politicians are globally concave.

In the remainder of the Appendix we give the proofs of the propositions provided in the main text. After establishing that a benevolent incumbent always provides more goods than a rent-seeking incumbent (Proposition 1), we prove existence and uniqueness of equilibrium (Proposition 2). Finally, we provide proofs of the propositions containing comparative statics.

### Assumptions

**Assumption 1.** The initial stock of natural resources is not too small, that is  $E > \underline{E}$ .

Here  $\underline{E} = \max \{ \underline{E}, \underline{E} \}$ , with  $\underline{E}$  defined in (25) and  $\underline{E}$  defined after equation (34).

**Assumption 2.** The derivative of the probability density function is not too high, that is  $\max h'(z) < \overline{h'(z)}$ .

Here  $\overline{h'(z)} = \min \{ \underline{h}'_1(z), \underline{h}'_2(z) \}$ , with  $\underline{h}'_1(z)$  defined after condition (27) and  $\underline{h}'_2(z)$  defined after condition (29). Alternatively, if  $z \sim \mathbb{N}(0, \sigma^2)$ , then Assumption 2 can be replaced by an assumption on the size of the variance  $\sigma^2$  alone:

**Assumption 2A.** The variance of the probability density function is not too low, that is  $\sigma^2 > \underline{\sigma}^2$ .

Here  $\underline{\sigma} = \max \{ \underline{\sigma}_1, \underline{\sigma}_2 \}$ , with  $\underline{\sigma}_1$  defined after condition (27) and  $\underline{\sigma}_2$  defined after condition (29).

## Lemmas

**Lemma 1** *If  $E > \underline{E}$ , then  $G_2^{r*} > 0$  and  $R_2^{r*} = \rho$ .*

### Proof

The proof consists of two parts. Part 1 assumes that the period 1 incumbent is a rent-seeker. We show that if  $E > \underline{E}$ , an incumbent of this type will always leave enough natural resources for a period 2 government of type  $r$  to choose strictly positive goods provision. Part 2 considers the case of a benevolent period 1 incumbent. We show that the condition for positive goods supply by a period 2 government of type  $r$  is weaker in this case than in Part 1, and therefore it is also satisfied if  $E > \underline{E}$ .

**Part 1: Rent-seeking incumbent** Let  $e_1^r$  be the period 1 extraction chosen by a rent-seeking incumbent. We will show that if  $E > \underline{E}$ , then  $E - e_1^r > f^{-1}(\rho)$  and hence, by equation (9), a period 2 government of type  $r$  will choose  $R_2^{r*} = \rho$  and  $G_2^{r*} > 0$  in equilibrium.

Start by assuming the opposite, namely that  $e_1^r > e_\rho$ , where  $e_\rho$  is such that  $f(E - e_\rho) = \rho$ . This implies that  $f(E - e_1^r) < \rho$ ,  $u'(f(E - e_1^r)) > 1$  and hence, by (9),  $G_2^r = 0$ ,  $dG_2^r/de_1^r = 0$ . We will now show that when  $E > \underline{E}$ , this constitutes a contradiction.

From (5), the objective function for the incumbent now is  $V^r = u(R_1) + G_1^r + \Pi u(R_2) + (1 - \Pi)pG_2^b$ . Substituting from (1), the first-order conditions for  $e_1^r$  and  $G_1^r$  are:

$$\begin{aligned} 0 &= u'(f(e_1^r) - G_1^r) f'(e_1^r) - [\Pi u'(f(E - e_1^r)) + (1 - \Pi)p] f'(E - e_1^r), \\ 0 &= 1 + h(\cdot) [u(f(E - e_1^r)) - pf(E - e_1^r)] - u'(f(e_1^r) - G_1^r). \end{aligned}$$

Together, these conditions imply that in the case we are now considering (where an  $r$  incumbent leaves too little resources for a successor of type  $r$  to provide goods in period 2), the extraction path is characterized by

$$\frac{f'(e_1^r)}{f'(E - e_1^r)} = \frac{p + \Pi [u'(f(E - e_1^r)) - p]}{1 + h(\cdot) [u(f(E - e_1^r)) - pf(E - e_1^r)]}. \quad (23)$$

Because  $f'' < 0$ , the highest possible  $e_1^r$  consistent with (23), denoted by  $\widehat{e}_1^r$ , is obtained when the expression on the right hand-side has its lowest value. The numerator on the right hand-side

of (23) can never be smaller than  $p$ ; this occurs if  $\Pi = 0$ . As for the denominator, we recall that  $\bar{h}$  is the upper bound on  $h(\cdot)$ . Furthermore, it is straightforward to show that the term in the square brackets of the denominator is positive and strictly decreasing in  $e_1^r$  when  $e_1^r > e_\rho$ . Moreover,  $\lim_{e_1^r \downarrow e_\rho} [u(f(E - e_1^r)) - pf(E - e_1^r)] = u(\rho) - p\rho > 0$ , where the inequality follows from the properties of  $u(\cdot)$ . The largest possible value of the denominator is thus  $1 + \bar{h}[u(\rho) - p\rho]$ . It follows that  $\hat{e}_1^r$  is implicitly given by

$$\frac{f'(\hat{e}_1^r)}{f'(E - \hat{e}_1^r)} = \frac{p}{1 + \bar{h}[u(\rho) - p\rho]}. \quad (24)$$

From (24) it follows that  $\hat{e}_1^r = \hat{e}_1^r(E)$ , with

$$\frac{d\hat{e}_1^r}{dE} = \frac{\frac{p}{1 + \bar{h}[u(\rho) - p\rho]} f''(E - \hat{e}_1^r)}{f''(\hat{e}_1^r) + \frac{p}{1 + \bar{h}[u(\rho) - p\rho]} f''(E - \hat{e}_1^r)}.$$

To save on notation we use that  $f''' = 0$  implies that  $f''(\hat{e}_1^r) = f''(E - \hat{e}_1^r)$ .<sup>18</sup> Thus,

$$\frac{d\hat{e}_1^r}{dE} = \frac{\frac{p}{1 + \bar{h}[u(\rho) - p\rho]}}{1 + \frac{p}{1 + \bar{h}[u(\rho) - p\rho]}}$$

and

$$\frac{d(E - \hat{e}_1^r)}{dE} = 1 - \frac{d\hat{e}_1^r}{dE} = \frac{1}{1 + \frac{p}{1 + \bar{h}[u(\rho) - p\rho]}} > 0.$$

As  $E - \hat{e}_1^r$  is increasing in  $E$ , while  $f^{-1}(\rho)$  is independent of  $E$ , it follows that for  $E$  sufficiently high we must have  $E - \hat{e}_1^r(E) > f^{-1}(\rho) \implies E - e_1^r > f^{-1}(\rho)$ , where the latter implication follows since  $e_1^r \leq \hat{e}_1^r$ . But this contradicts  $f(E - e_1^r) < \rho$ .

Let  $\underline{E}$  be implicitly defined by

$$\underline{E} - \hat{e}_1^r(\underline{E}) = f^{-1}(\rho). \quad (25)$$

Note that this expression uniquely determines  $\underline{E}$ . We have then shown that if  $E > \underline{E}$ , it must be the case that  $E - e_1^r > f^{-1}(\rho)$ .

**Part 2: Benevolent incumbent** Let  $e_1^b$  be the period 1 extraction chosen by a benevolent incumbent.

Assume  $e_1^b > e_\rho$ , so that  $G_2^r = 0$ . From (4), the objective function of the benevolent incumbent is then  $V^b = G_1^b + z_1 + [\Pi + (1 - \Pi)p]G_2^b + z_2$ . Upon substitution from (1), it is straightforward to show that the extraction path is characterized by

$$\frac{f'(e_1^b)}{f'(E - e_1^b)} = \frac{p + \Pi(1 - p)}{1 + h(\cdot)(1 - p)f'(E - e_1^b)}$$

<sup>18</sup>Again, note that  $f''' = 0$  just simplifies the expression, and that the proof can easily be established also when  $f''' \neq 0$ .

in this case. Using the same logic as for a rent-seeking incumbent above, we can show that highest possible  $e_1^b$  consistent with this expression, denoted by  $\hat{e}_1^b$ , is implicitly given by

$$\frac{f'(\hat{e}_1^b)}{f'(E - \hat{e}_1^b)} = \frac{p}{1 + \bar{h}(1-p)\rho}. \quad (26)$$

Equation (26) implies that  $\frac{d(E - \hat{e}_1^b)}{dE} > 0$ . Hence, the contradiction that we demonstrated above for a period 1 government of type  $r$  also applies to a benevolent period 1 incumbent. This leads to the conclusion that if  $E > f^{-1}(\rho) + \hat{e}_1^b$ , the optimal extraction policy  $e_1^b$  of a benevolent incumbent always fulfills  $E - e_1^b > f^{-1}(\rho)$ .

Finally, by comparing (24) and (26) we can easily show that  $\hat{e}_1^r > \hat{e}_1^b$ , which implies that  $\underline{E} > f^{-1}(\rho) + \hat{e}_1^b$ .

The above establishes that, independently of period 1 incumbency type, when  $E > \underline{E}$  a period 2 government of type  $r$  will always inherit enough resources to optimally choose  $G_2^r > 0$  and  $R_2^r = \rho$ . ■

**Lemma 2** *For any pair  $(\tilde{G}_1^b, \tilde{G}_1^r)$  satisfying  $\tilde{G}_1^b > \tilde{G}_1^r$ , the lifetime expected utility function of a benevolent incumbent is globally concave:  $V^{b'''}(e_1^b) < 0$ .*

**Proof.** From equation (4) with  $G_1^b = f(e_1^b)$  and  $G_2^b = f(E - e_1^b)$ , it follows that

$$V^{b'''}(e_1^b) = \left(1 + \Pi'(G_1^b)(1-p)\rho\right) f'' + \Pi''(G_1^b)(1-p)\rho f'(e_1^b)^2 + f''.$$

Next, we use that  $\Pi''(G) = h'\left(G - \frac{\tilde{G}_1^r + \tilde{G}_1^b}{2}\right)$ . Hence, a sufficient condition for  $V^{b'''}(e_1^b) < 0$  is that

$$\max h'(z) < -\frac{(2 + \Pi'(G_1^b)(1-p)\rho) f''}{(1-p)\rho f'(e_1^b)^2}. \quad (27)$$

Note that the term on the right-hand side is strictly positive for all  $(e_1^b, G_1^b)$  because  $\Pi'(G) > 0$  and  $f'' < 0$ . Let  $\underline{h}'_1(z)$  be defined as the lowest value that the right-hand side term can attain for any feasible  $(e_1^b, G_1^b)$ . Then a sufficient condition for global concavity is  $\max h'(z) < \underline{h}'_1(z)$ .

Thus under Assumption 2 the lifetime expected utility function of a benevolent incumbent is globally concave.

If  $z \sim \mathbb{N}(0, \sigma^2)$ , then  $\max h'(z) = \frac{1}{\sqrt{2\pi\sigma} \exp}$ . In this case, let  $\underline{\sigma}_1$  be defined as the  $\sigma$  that solves (27) with equality when the right-hand side of (27) is minimized with respect to  $(e_1^b, G_1^b)$ . Then a sufficient condition for global concavity is  $\sigma > \underline{\sigma}_1$ . Thus, in this case, under Assumption 2A the lifetime expected utility function of a benevolent incumbent is globally concave. ■

**Lemma 3** *For any pair  $(\tilde{G}_1^b, \tilde{G}_1^r)$  satisfying  $\tilde{G}_1^b > \tilde{G}_1^r$ , the lifetime expected utility of a rent seeking incumbent  $V^r(e_1^r, G_1^r)$  is globally concave:  $V_{ee}^{r'''}(e_1^r, G_1^r) \leq 0$ ,  $V_{GG}^{r'''}(e_1^r, G_1^r) \leq 0$ ,  $V_{ee}^{r'''}(e_1^r, G_1^r) V_{GG}^{r'''}(e_1^r, G_1^r) - V_{eG}^{r'''}(e_1^r, G_1^r)^2 \geq 0$ .*



**Proof.** From (16) we have

$$V_{ee}^{r''}(e_1^r, G_1^r) = u''(f(e_1^r) - G_1^r) f'(e_1^r)^2 + u'(f(e_1^r) - G_1^r) f'' + f'' < 0,$$

where the inequality follows directly from the properties  $u'' < 0$  and  $f'' < 0$ .

Next, (16) implies that

$$V_{GG}^{r''}(e_1^r, G_1^r) = u''(f(e_1^r) - G_1^r) + \Pi''(G_1^r) [u(\rho) - p\rho].$$

Upon using  $\Pi''(G) = h'(G - \frac{\tilde{G}_1^r + \tilde{G}_1^b}{2})$ , we thus have  $V_{GG}^{r''}(e_1^r, G_1^r) < 0$  if

$$\max h'(z) < \frac{-u''(f(e_1^r) - G_1^r)}{u(\rho) - p\rho}. \quad (28)$$

Recall from the proof of Lemma 1 that  $u(\rho) - p\rho > 0$ . Since  $u'' < 0$ , a sufficient condition for  $V_{GG}^{r''}(G_1^r) < 0$  is accordingly that  $\max h'(z)$  is low enough. As we return to below, this will always be satisfied under Assumption 2.

We next use  $V_{eG}^{r''}(e_1^r, G_1^r) = -u''(f(e_1^r) - G_1^r) f'(e_1^r)$  to calculate

$$\begin{aligned} & V_{ee}^{r''}(e_1^r, G_1^r) V_{GG}^{r''}(e_1^r, G_1^r) - V_{eG}^{r''}(e_1^r, G_1^r)^2 \\ &= u''(R_1) [u'(R_1) f'' + f''] \\ & \quad + \Pi''(G_1^r) [u(\rho) - p\rho] \left[ u''(R_1) f'(e_1^r)^2 + u'(R_1) f'' + f'' \right], \end{aligned}$$

where we have used that  $R_1 = f(e_1^r) - G_1^r$  to simplify the notation. This expression implies that if

$$\max h'(z) < \left( \frac{-u''(R_1)}{u(\rho) - p\rho} \right) \left( \frac{u'(R_1) f'' + f''}{u'(R_1) f'' + f'' + u''(R_1) f'(e_1^r)^2} \right), \quad (29)$$

then  $V_{ee}^{r''}(e_1^r, G_1^r) V_{GG}^{r''}(e_1^r, G_1^r) - V_{eG}^{r''}(e_1^r, G_1^r)^2 > 0$ . The signs imposed on the derivatives of  $f$  and  $u$ , together with  $u(\rho) - p\rho > 0$ , implies that the right hand-side of (29) is strictly positive. Let  $\underline{h}'_2(z)$  be defined as the lowest value that the right-hand side term can attain for any feasible  $(e_1^r, R_1)$ . Then a sufficient condition for  $V_{ee}^{r''}(e_1^r, G_1^r) V_{GG}^{r''}(e_1^r, G_1^r) - V_{eG}^{r''}(e_1^r, G_1^r)^2 > 0$  is  $\max h'(z) < \underline{h}'_2(z)$ , which always holds under Assumption 2. If  $z \sim \mathbb{N}(0, \sigma^2)$ , then again  $\max h'(z) = \frac{1}{\sqrt{2\pi\sigma \exp}}$ . In this case, let  $\underline{\sigma}_2$  be defined as the  $\sigma$  that solves (29) with equality when the right-hand side is minimized with respect to  $(e_1^r, R_1)$ . Then a sufficient condition for  $V_{ee}^{r''}(e_1^r, G_1^r) V_{GG}^{r''}(e_1^r, G_1^r) - V_{eG}^{r''}(e_1^r, G_1^r)^2 > 0$  is  $\sigma \geq \underline{\sigma}_2$ , which is always satisfied under Assumption 2A.

Finally note that the first term on the right-hand side of (29) is identical to the right-hand side of (28), while the last term in (29) is smaller than one. It follows that when  $V_{ee}^{r''}(e_1^r, G_1^r) V_{GG}^{r''}(e_1^r, G_1^r) - V_{eG}^{r''}(e_1^r, G_1^r)^2 > 0$ ,  $\max h'(z) < \underline{h}'_2(z)$  is a sufficient condition for  $V_{GG}^{r''}(G_1^r) < 0$ .

The above establishes that under Assumption 2, alternatively Assumption 2A if  $z \sim \mathbb{N}(0, \sigma^2)$ , the lifetime expected utility of a rent seeking incumbent is globally concave. ■

## Proofs of Propositions

### Proof of Proposition 1

**Part 1** Impose the equilibrium conditions  $\tilde{G}_1^b = G_1^{b*}$  and  $\tilde{G}_1^r = G_1^{r*}$ . There are three possibilities:  $G_1^{b*} = G_1^{r*}$ ,  $G_1^{b*} < G_1^{r*}$ ,  $G_1^{b*} > G_1^{r*}$ . If  $G_1^{b*} = G_1^{r*}$ , then (11) implies  $\tilde{p} = p$ , and thus, by assumption, the incumbent is reelected. The benevolent incumbent then chooses  $G_1^{b*} = G^{fb}$ . According to (17) and (18), the rent-seeker chooses  $e_1^{r*} = e^{fb}$  and  $R_1^* = \rho$ . This implies  $G_1^{r*} < G^{fb}$ , which contradicts  $G_1^{b*} = G_1^{r*}$ .

Consider next the case where  $G_1^{b*} < G_1^{r*}$ . Note first that when  $G_1^{b*} \neq G_1^{r*}$ , then  $\tilde{p} \geq p$  if and only if

$$h(U_1 - G_1^{b*}) \geq h(U_1 - G_1^{r*}).$$

When  $G_1^{b*} < G_1^{r*}$ , this condition simplifies to  $U_1 \leq [G_1^{b*} + G_1^{r*}] / 2$ . The probability of reelection when  $G_1^{b*} < G_1^{r*}$  is therefore given by

$$\Pi(G) = \Pr\left(G + z \leq \frac{G_1^{b*} + G_1^{r*}}{2}\right) = H\left(\frac{G_1^{b*} + G_1^{r*}}{2} - G\right),$$

which implies  $\Pi'(G) = -h\left(\frac{G_1^{b*} + G_1^{r*}}{2} - G\right) < 0$ . Moreover, in equilibrium we have that  $\Pi'(G_1^{b*}) = -h\left(\frac{G_1^{b*} - G_1^{r*}}{2}\right) = \Pi'(G_1^{r*})$  due to the symmetry of  $h(z)$ .

Next, we note that equations (17) and (18) together imply that, in equilibrium,

$$1 + \Pi'(G_1^{r*})[u(\rho) - p\rho] = \frac{f'(E - e_1^{r*})}{f'(e_1^{r*})}. \quad (30)$$

Furthermore, since  $G_1^{b*} < G_1^{r*}$  implies that  $e_1^{b*} < e_1^{r*}$ , it follows from (30) and (15) that

$$\Pi'(G_1^{r*})[u(\rho) - p\rho] > \Pi'(G_1^{b*})(1 - p)\rho. \quad (31)$$

Using that  $\Pi'(G_1^{b*}) = \Pi'(G_1^{r*}) < 0$  in (31), yields

$$u(\rho) - p\rho < (1 - p)\rho.$$

Hence,  $G_1^{b*} < G_1^{r*}$  requires that  $u(\rho) < \rho$ . But as explained in Lemma 1, the properties of  $u$  (specifically  $u'(0) > 1$  and  $u'' < 0$ ) imply that  $u(\rho) > \rho$ . Hence,  $G_1^{b*} < G_1^{r*}$  is not an equilibrium.

**Part 2** By part 1, the only remaining possibility is  $G_1^{b*} > G_1^{r*}$ . For this case, the statement in part 2 is proved in the main text. ■

### Proof of Proposition 2

We now show that the equilibrium exists, and also that it is unique.

**Preliminaries**  $G_1^{b*}$  given by (19) is strictly positive. This implies that should  $\tilde{G}_1^r = 0$  (which will never occur in equilibrium), then  $b$  will choose  $G_1^{b*} > 0$ , such as at point  $A$  in Figure

1. Moreover,  $G_1^{r*}$  implied by equations (20) and (21) is strictly positive. This implies that if  $\tilde{G}_1^b = f(E)$  (which will never occur in equilibrium), then  $r$  will choose  $G_1^{r*} > 0$ , such as at point  $B$  in Figure 1.

Denote by  $G_1^C$  the goods provision that is consistent with equation (19) and fulfills  $G_1^{b*} = \tilde{G}_1^r$ . This corresponds to point  $C$  in Figure 1. Conversely, denote by  $G_1^D$  the goods provision that is consistent with conditions (20) and (21) and that satisfies  $G_1^{r*} = \tilde{G}_1^b$ . This would be point  $D$  in Figure 1.

From Lemmas 2 and 3 we know that the objective functions are globally concave for both incumbent types. Hence, the functions  $G_1^{b*}(\tilde{G}_1^r)$  and  $G_1^{r*}(\tilde{G}_1^b)$  are both continuous.

**Existence** The discussion above implies that a sufficient condition for existence of equilibrium is  $G_1^C > G_1^D$ . In terms of Figure 1, this condition is that point  $C$  is located to the upper-right of point  $D$ .

Let  $e_1^{bC}$  be the  $e_1^{b*}$  that corresponds to  $G_1^C$ , and let  $e_1^{rD}$  be the  $e_1^{r*}$  that corresponds to  $G_1^D$ . Equation (19) with  $G_1^{b*} = \tilde{G}_1^r = G_1^{r*} = G_1^C$  then reads

$$\frac{f'(E - e_1^{bC})}{f'(e_1^{bC})} = 1 + (1 - p)\rho h(0). \quad (32)$$

It follows that if the resource stock  $E$  increases, then (simplifying the expression using that  $f''' = 0$ )

$$\frac{de_1^{bC}}{dE} = \frac{1}{2 + (1 - p)\rho h(0)}. \quad (33)$$

Similarly, combining (20) and (21) with  $G_1^{r*} = \tilde{G}_1^b = G_1^{b*} = G_1^D$  yields

$$\frac{f'(E - e_1^{rD})}{f'(e_1^{rD})} = 1 + [u(\rho) - p\rho] h(0).$$

It follows that if the resource stock  $E$  increases, then

$$\frac{de_1^{rD}}{dE} = \frac{1}{2 + [u(\rho) - p\rho] h(0)}. \quad (34)$$

We note from (33) and (34) that  $\frac{de_1^{bC}}{dE}$  and  $\frac{de_1^{rD}}{dE}$  are both constant, and that  $\frac{de_1^{bC}}{dE} > \frac{de_1^{rD}}{dE}$  since  $[u(\rho) - p\rho] > (1 - p)\rho$ . For a sufficiently high  $E$ , we thus always have  $e_1^{bC} > e_1^{rD}$ . Let  $e_1^{bD}$  be the  $e_1^{b*}$  that corresponds to  $G_1^D$ . Since a rent-seeking incumbent always allocates a strictly positive amount of resource income to rents, we have that  $e_1^{rD} > e_1^{bD}$ . The last two inequalities immediately imply that  $e_1^{bC} > e_1^{bD}$ . Since a benevolent incumbent spends all resource income on goods provision, it follows that  $G_1^C > G_1^D$ .

This shows that, for a sufficiently high resource stock  $E$ , point  $C$  in Figure 1 is always located to the upper-right of point  $D$ . Let  $\underline{E}$  be defined as the  $E$  such that  $e_1^{bC} > e_1^{rD}$  (and note that  $\underline{E} = 0$  may be sufficient). Then, under Assumption 1 the equilibrium always exists.

**Uniqueness** We now show that the equilibrium is unique. We first show that the relationship  $G_1^{b*}(\tilde{G}_1^r)$  is monotone and increasing over the relevant range where  $G_1^{b*} > \tilde{G}_1^r$ . Differentiating (19) with respect to  $e_1^{b*}$  and  $\tilde{G}_1^r$  yields

$$\frac{de_1^{b*}}{d\tilde{G}_1^r} = \frac{f'(e_1^{b*})(1-p)\rho h' \left( \frac{G_1^{b*} - \tilde{G}_1^r}{2} \right)}{2\Omega f'' + f'(e_1^{b*})^2(1-p)\rho h' \left( \frac{G_1^{b*} - \tilde{G}_1^r}{2} \right)},$$

where  $\Omega \equiv 2 + (1-p)h \left( \frac{G_1^{b*} - G_1^{r*}}{2} \right) \rho$ . Since  $f'' < 0$ , this expression implies that  $\frac{de_1^{b*}}{d\tilde{G}_1^r} > 0$  if  $h' \left( \frac{G_1^{b*} - \tilde{G}_1^r}{2} \right) < 0$ . Since  $h'(z) < 0$  for all  $z > 0$  and since  $G_1^{b*} > \tilde{G}_1^r$  in equilibrium, it follows that  $\frac{de_1^{b*}}{d\tilde{G}_1^r} > 0$ . Because  $G_1^{b*} = f(e_1^{b*})$ , it follows that  $\frac{dG_1^{b*}}{d\tilde{G}_1^r} > 0$  over the relevant range  $G_1^{b*} > \tilde{G}_1^r$ .

We next show that the relationship  $G_1^{r*}(\tilde{G}_1^b)$  is monotone and decreasing over the relevant range where  $\tilde{G}_1^b > G_1^{r*}$ . By differentiating (20) we obtain

$$de_1^{r*} = \frac{u''(R_1^*)f'(e_1^{r*})}{u''(R_1^*)f'(e_1^{r*})^2 + u'(R_1^*)f'' + f''} dG_1^{r*},$$

where again we have used  $R_1^* = f(e_1^{r*}) - G_1^{r*}$  to simplify the notation. By differentiating (21) we obtain

$$u''(R_1^*) [f'(e_1^{r*})de_1^{r*} - dG_1^{r*}] = [u(\rho) - p\rho] h' \left( \frac{\tilde{G}_1^b - G_1^{r*}}{2} \right) [d\tilde{G}_1^b - dG_1^{r*}].$$

Combining the two last expressions yields

$$\frac{dG_1^{r*}}{d\tilde{G}_1^b} = \Theta^{-1} \left[ [u(\rho) - p\rho] h' \left( \frac{\tilde{G}_1^b - G_1^{r*}}{2} \right) [u''(R_1^*)f'(e_1^{r*})^2 + u'(R_1^*)f'' + f''] \right], \quad (35)$$

where

$$\begin{aligned} \Theta &= -u''(R_1^*) [u'(R_1^*)f'' + f''] \\ &\quad + [u(\rho) - p\rho] h' \left( \frac{\tilde{G}_1^b - G_1^{r*}}{2} \right) [u''(R_1^*)f'(e_1^{r*})^2 + u'(R_1^*)f'' + f'']. \end{aligned}$$

The term inside the big square bracket of (35) is positive over the relevant range  $\tilde{G}_1^b > G_1^{r*}$ , since  $h'(z) < 0$  for all  $z > 0$ . Moreover, condition (29) in Lemma 3 implies that  $\Theta < 0$ . Hence,  $\frac{dG_1^{r*}}{d\tilde{G}_1^b} < 0$  when  $\tilde{G}_1^b > G_1^{r*}$ .

We have thus shown that  $\frac{dG_1^{r*}}{d\tilde{G}_1^b} < 0 < \frac{dG_1^{b*}}{d\tilde{G}_1^r}$  which implies that the equilibrium is unique, as stated in the proposition. ■

### Proof of Proposition 4

Inserting for  $G_1^{b*} = f(e_1^{b*})$ , equations (19), (20) and (21) are three equations in the three endogenous variables  $e_1^{b*}$ ,  $e_1^{r*}$  and  $G_1^{r*}$ . Let  $\sigma^2$  denote the variance of the distribution of  $z$ . To find how overextraction responds to changes in the exogenous variables  $p$  and  $\sigma^2$ , we write the three first order equations in differential form, which yields:

$$m_1 de_1^{b*} + 0 de_1^{r*} + m_2 dG_1^{r*} = n_1 dp + n_2 h'_{\sigma^2} d\sigma^2, \quad (36)$$

$$0 de_1^{b*} + m_3 de_1^{r*} + m_4 dG_1^{r*} = 0 dp + 0 h'_{\sigma^2} d\sigma^2, \quad (37)$$

$$m_5 de_1^{b*} + m_4 de_1^{r*} + m_6 dG_1^{r*} = n_3 dp + n_4 h'_{\sigma^2} d\sigma^2, \quad (38)$$

where  $h'_{\sigma^2}$  denotes the derivative of the (equilibrium) probability density function  $h$  with respect to  $\sigma^2$ , and where

$$m_1 = f'' \left( 2 + (1-p)\rho h \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) \right) + f'(e_1^{b*})^2 \frac{(1-p)\rho}{2} h' \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) < 0,$$

$$m_2 = -f'(e_1^{b*}) \frac{(1-p)\rho}{2} h' \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) > 0,$$

$$m_3 = u''(f(e_1^{r*}) - G_1^{r*}) f'(e_1^{r*})^2 + u'(f(e_1^{r*}) - G_1^{r*}) f'' + f'' < 0,$$

$$m_4 = -u''(f(e_1^{r*}) - G_1^{r*}) f'(e_1^{r*}) > 0,$$

$$m_5 = \frac{[u(\rho) - p\rho]}{2} h' \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) f'(e_1^{b*}) < 0,$$

$$m_6 = u''(f(e_1^{r*}) - G_1^{r*}) - \frac{[u(\rho) - p\rho]}{2} h' \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) < 0,$$

$$n_1 = f'(e_1^{b*}) \rho h \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) > 0,$$

$$n_2 = -f'(e_1^{b*}) (1-p)\rho < 0,$$

$$n_3 = \rho h \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) > 0,$$

$$n_4 = -[u(\rho) - p\rho] < 0.$$

Note that  $m_1$  is equivalent to  $V^{b''}(e_1^b)$  from Lemma 2, with the only difference that  $e_1^b$  is evaluated at equilibrium  $e_1^b = e_1^{b*}$ . Thus,  $m_1 = V^{b''}(e_1^{b*})$ . In the same way  $m_3 = V_{ee}^{r''}(e_1^{r*}, G_1^{r*})$  and  $m_6 = V_{GG}^{r''}(e_1^{r*}, G_1^{r*})$  from Lemma 3. Since by Lemma 3  $V_{GG}^{r''}(e_1^r, G) < 0$ , it follows that  $m_6 < 0$  as stated. Moreover, note that  $m_3 m_6 - (m_4)^2 = V_{ee}^{r''}(e_1^{r*}, G_1^{r*}) V_{GG}^{r''}(e_1^{r*}, G_1^{r*}) - V_{eG}^{r''}(e_1^{r*}, G_1^{r*})^2$ ,

and thus Lemma 3 also implies  $m_3m_6 - (m_4)^2 > 0$ . It is then straightforward to solve the system (36), (37) and (38) by Cramers rule. Defining  $D \equiv m_1 [m_3m_6 - (m_4)^2] - m_2m_3m_5 < 0$ , it follows that

$$\frac{de_1^{b*}}{-dp} = \frac{-1}{D} ([m_3m_6 - (m_4)^2] n_1 - m_2m_3n_3) > 0,$$

which proves part 1 of the proposition.

To see part 2 we find

$$\frac{de_1^{r*}}{-dp} = \frac{-1}{D} m_4 (m_5n_1 - m_1n_3).$$

After inserting for  $m_1$ ,  $n_3$ ,  $m_5$  and  $n_1$  from above, we can show that  $m_5n_1 - m_1n_3 > 0$  if

$$-f'' \left( 2 + (1-p)\rho h \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) \right) > -h' \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) \left[ \frac{u(\rho) - \rho}{2} \right] f'(e_1^{b*})^2$$

This inequality holds provided that  $|h'(z)|$  is not too high. Based on our assumptions, however, it cannot be ruled out that this inequality does not hold, and thus the proposition follows. ■

## Proof of Proposition 5

By using Cramers rule on (36), (37) and (38), we obtain

$$\begin{aligned} \frac{de_1^{b*}}{-d\sigma^2} &= \frac{h'_{\sigma^2}}{D} (-[m_3m_6 - (m_4)^2] n_2 + m_2m_3n_4), \\ \frac{de_1^{r*}}{-d\sigma^2} &= \frac{h'_{\sigma^2}}{D} m_4 (m_1n_4 - m_5n_2). \end{aligned}$$

From the definitions of  $m_1$  to  $n_4$  above, it follows that  $(-[m_3m_6 - (m_4)^2] n_2 + m_2m_3n_4) > 0$  and  $m_4 (m_1n_4 - m_5n_2) > 0$ . To confirm the latter inequality, note that  $m_4 > 0$ , and insert for  $m_1$ ,  $n_4$ ,  $m_5$ , and  $n_2$  into  $(m_1n_4 - m_5n_2)$ , in order to obtain the following condition for it to be positive:

$$f'' \left( 2 + (1-p)\rho h \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) \right) < 0,$$

This inequality is always satisfied. Thus the sign of  $\frac{de_1^{b*}}{-d\sigma^2}$  and  $\frac{de_1^{r*}}{-d\sigma^2}$  is always the same, and (since  $D$  is negative) is the opposite of the sign of  $h'_{\sigma^2}$ . Since in general  $h'_{\sigma^2}$  cannot be signed, part 1 of the proposition follows.

To see part 2, note that when  $z \sim \mathbb{N}(0, \sigma^2)$  then

$$h \left( \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left( -\frac{f(e_1^{b*}) - G_1^{r*}}{4\sigma^2} \right),$$

with

$$h'_{\sigma^2} = \frac{1}{2\sigma^5\sqrt{2\pi}} \exp \left( -\frac{f(e_1^{b*}) - G_1^{r*}}{4\sigma^2} \right) \left( -\sigma^2 + \frac{f(e_1^{b*}) - G_1^{r*}}{2} \right).$$

Since  $f(e_1^{b*}) - G_1^{r*}$  is bounded, it follows that  $h'_{\sigma^2} < 0$  for a sufficiently high  $\sigma^2$ . Thus when this is the case,  $\frac{de_1^{b*}}{-d\sigma^2}$  and  $\frac{de_1^{r*}}{-d\sigma^2}$  are both positive, and the proposition follows. ■

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