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Exchange Rates and Fundamentals
A Non-Linear Relationship?

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A NON-LINEAR RELATIONSHIP?

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1. Introduction

Exchange rate economics has gone through different stages. The early theoretical models were developed mainly in the 1970s (monetary model, Dornbusch model, portfolio balance model, and others). These 'first generation' models led to testable propositions in which the changes in the exchange rate are linearly related to news in the fundamentals (money stocks, prices, output, current accounts, etc.). After intensive empirical testing it is fair to conclude that the first generation models were soundly rejected by the data, at least for the exchange rates of countries experiencing relatively low levels of inflation. Three serious anomalies of the first generation models were detected.

First, in their celebrated empirical studies Meese and Rogoff (1983), (1988) found that the random walk forecast typically outperforms a forecast based on the first generation models even when these modes have access to perfectly anticipated future fundamentals[1]. Although occasionally some researchers have claimed that their model could beat the random walk, the scientific consensus today is that the Meese and Rogoff results still stand. An important implication of this finding is that the coefficients of the fundamentals in the exchange rate equations are subject to frequent structural changes, making these equations unfit for predictive purposes. The existence of frequent structural shifts in the linear exchange rate equations has been well documented (see e.g. Frydman and Goldberg (2001)).

A second anomaly detected in the empirical literature is the following. Since the start of the floating exchange rate regime the variability of the exchange rates (both nominal and real) has increased dramatically. At the same time there is no evidence to be found that the variability of the fundamentals identified by the theoretical models has increased compared to the fixed exchange rate period (see Baxter and Stockman (1989) and Flood and Rose (1995)). This is in contradiction with the first generation models, which imply that the variability of the exchange rate can only increase when the variability of the underlying fundamental variables increases. This result has led to the view that the variability of the exchange rates is largely disconnected from the variability of the underlying fundamentals. In their recent

1 There is some evidence that when forecasting over a longer horizon, say, more than one year, fundamentals based models sometimes outperform the random walk. It should be borne in mind though, that these fundamentalist forecasts (based on perfect foresight of future fundamentals) use an information set that is much larger than the information set needed to make random walk forecasts. This also implies that the long term forecasts based on the economic models use more information than the short-term forecasts. It is therefore not really surprising that they perform better. Independent evidence on PPP also suggests that if there is a long-term mechanism driving the exchange rate, it is indeed a very long one. In this large literature on PPP it is found that it takes 3 to 4 years for half of the adjustment towards PPP to be realised after a shock. See Rogoff (1996).

paper Obstfeld and Rogoff (2000) have identified this phenomenon to be one of the six major puzzles in international macroeconomics.

A third empirical anomaly relates to the 'news' aspect of the first generation models. The rational expectations assumption underlying the first generation models implies that the exchange rates can only change at any given moment of time as a result of 'news' in the fundamentals. It is fair to conclude now that this feature of the existing models has also been rejected by the data. There is evidence that a large part of the movements of the exchange rate cannot be associated with news (see Goodhart (1989) and Goodhart & Figliuoli (1991)). More recent analysis using structural VARs comes to a similar conclusion. Unanticipated shocks in the fundamental variables explain only a small fraction of the unanticipated changes in the exchange rates. Typically over forecast horizons of up to one year, news in output, inflation, and interest rates explains less than 5% of the total unanticipated variance of the exchange rate. About 95% of the latter is attributable to the news in the exchange rate itself (De Boeck (2000), Altavilla (2000))[2].

From this evidence it is clear that the first generation models in which the exchange rate is driven by news in the fundamentals in a linear way must be called into question as a representation of the foreign exchange market.

The rejection of the first generation models of the exchange rate has led researchers into two different directions. The first one has led to what one could call the 'second generation' models, as exemplified by Obstfeld and Rogoff (1996). In these models the starting point is utility maximisation of a representative agent. These models typically lead to the conclusion that the coefficients of the reduced form equations of the first generation models do not have to be constant. These coefficients vary as a result of the underlying stochastic disturbances and of changing policy regimes.

This is an important insight. The trouble, however, is that the 'second generation' models have led to few testable propositions that would allow for their refutation. As long as these testable propositions are not formulated it is difficult to evaluate the scientific strength of these 'second generation' models.

A second direction taken by researchers in their search for an alternative to the 'first generation' models has been to introduce non-linearities into the model (see De Grauwe and Dewachter (1993), Frankel and Froot (1990), Kilian and Taylor (2001), Kurz and Motolese (2001)). These models are characterised by the existence of several agents using different information sets (e.g. chartists and fundamentalists) and/or by

[2] Again there is some evidence that over longer forecast horizons, the news in fundamentals becomes more important. It remains relatively low, however, remaining far below explaining 50% of the total variance.

the existence of transactions costs. The insight provided by these models is that they predict frequent structural breaks in linear exchange rate equations, and that they generate changes in the exchange rates that are unrelated to news about the underlying fundamentals.

In this paper we analyse the (possibly non-linear) nature of the relationship between exchange rate changes and the news in the underlying fundamentals. More specifically we test whether this relationship is subject to regime switches over time. In order to do so, we use a version of the Markov-switching autoregressive model popularised by Hamilton (1989). In addition, we perform the Markov-switching analysis both on data of low inflation and high inflation countries. This comparison between low and high inflation countries will allow us to gain additional insight about the nature of the relation between exchange rates and the fundamentals.

The rest of the paper is structured as follows. In section 2 we present the model and discuss some of its features. In section 3 we describe the estimation process, and in section 4 we present the results. Finally in section 5 we analyse the implications of our results for exchange rate modelling.

2. The model

The non-linear model we consider is derived from the Markov-switching autoregressive (MS-AR) models popularised by Hamilton (1989) as a way of characterizing expansions and contractions in empirical business cycle research. The MS-AR framework can be readily extended to various settings (see Krolzig, 1997, for an overview). However, the use of the Markov-switching model to analyse the exchange rate market is rather new[3]. Furthermore, all these applications have assumed switches in either the mean, variance or autoregressive coefficients of the models considered. In our analysis, we use the Markov-switching model to detect switches in the exogenous regressors and or intercept. Hence, our model is written as:

$$\Delta e_t = \boldsymbol{a}_{s_t} + \Delta fund_t' \boldsymbol{b}_{s_t} + \boldsymbol{e}_t \qquad \boldsymbol{e}_t \sim N\left(0, \boldsymbol{s}^2\right)$$

Where Δe_t represents the change of the exchange rate in month t relative to month t-12 and $\Delta fund_t$ the relative change in the fundamental(s) of the home country in month t relative to month t-12 compared to the US, so:

3 Examples can be found in Engel and Hamilton (1990), Engel and Hakkio (1994), Jeanne and Masson (1998) and Fratzscher (1999).

$$\Delta fund_t = \frac{fund_{\mathrm{home},t} - fund_{\mathrm{home},t-12}}{fund_{\mathrm{home},t-12}} - \frac{fund_{US,t} - fund_{US,t-12}}{fund_{US,t-12}}$$

Further, we postulate the existence of an unobserved variable (denoted s_t) that takes on the value one or two. This variable characterises the state or regime that the process is in at date t. We assume that the stochastic process generating these unobservable regimes is an ergodic, irreducible Markov chain defined by the transition probabilities[4]:

$$p_{ij} = \Pr\langle s_{t+1} = j \mid s_t = i \rangle, \sum_{j=1}^{2} p_{ij} = 1 \quad \forall i, j \in \{1,2\}$$

Hence the process for s_t is presumed to depend on past realizations of e and s only through s_t-1.

Note that an attractive feature of the model is that a variety of behaviour is allowed. No prior information regarding the dates or the sizes of the two states is required. In particular there could be asymmetries in the persistence of the two states and we do not impose that the coefficients in both states should be either significant or insignificant.

3. Estimation process

To estimate the aforementioned model, we choose to work with both monthly and quarterly data on the exchange rates and various fundamentals as gathered from the International Financial Statistics tape of the International Monetary Fund for both high and low inflation countries. For the high inflation countries, data on the home currency price for the exchange rate, the money supply, the inflation, the money market rate and the lending rate was obtained for Argentina, Bolivia, Brazil, Columbia and Ecuador. For the low inflation countries, the same data and also observations on the government bond yield and the trade balance were obtained for Germany, France, Italy, Japan, the UK and the US. See Appendix A for more details on the data.

The maximum likelihood estimates of this model can be performed by relying either on a numerical maximization technique or on the EM-Algorithm as described by Hamilton (1990) and Krolzig (1997). In this paper, both approaches were adopted whereby a Broyden, Fletcher, Goldfarb and Shanno (BFGS) routine achieved the

4 A Markov Chain is said to be ergodic if exactly one of the eigenvalues of the transition matrix is unity and all other eigenvalues are inside the unit circle. Under this condition there exists a stationary or unconditional probability distribution of the regimes. If the ergodic probabilities are strictly positive, such that all regimes have a positive unconditional probability, the process is called irreducible (Krolzig, 1997).

numerical maximization[5]. For the EM-Algorithm, standard errors were computed in the way suggested by Berndt, Hall, Hall and Hausman (1974).

As the results from estimating the model were consistent over the various methodologies and time coverages, only the monthly results as obtained by the BFGS routine are reported below, the quarterly results can be found in Appendix C. As starting values, we choose the OLS regression results for one regime and zero for the other regime. We also experimented with other starting values, but the results never changed substantially.

5 For an elaboration on the estimation techniques, see Appendix B.

4. The results

We first present the results of the univariate analysis, i.e. the analysis in which we apply the Markov switching model to univariate explanations of the exchange rate changes. In the second step we apply the model to the multivariate case.

4.1 Univariate analysis

Table 1 shows the Wald tests for the low inflation countries. As will be remembered the Wald test allows us to test for the equality of the intercepts and the slopes in the different regimes identified by the Markov switching model. We have considered three scenarios for the regime switches. In the first one we test whether there are switches in the intercept and the slope, in the second case we only allow for switches in the intercept, and in the third case we only allow for switches in the slopes.

A first conclusion from table 1 is that the model identifies significant switches in the intercept and in the slope in most cases. In particular switches in the slope are significant in all but three cases, and switches in the intercept in all but two cases.

Table 1
Wald test results for low inflation countries

	Switches in the intercept and slope			Switches in the intercept		Switches in the slope	
	H_0:P_{11}=1-p_{12}	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0:P_{11}=1-p_{12}	H_0: $\alpha_1 = \alpha_2$	H_0:P_{11}=1-p_{12}	H_0: $\beta_1 = \beta_2$
Changes in Inflation							
Germany	29.11	46.58	0.02	3.76	23.46	34.13	5.56
France	141.06	0.77	6.96	0.00	0.00	112.72	19.02
Italy	39.08	49.15	3.53	1.16	4.09	38.77	8.17
UK	29.56	3.28	5.27	0.07	0.12	90.87	7.46
Japan	13.81	45.69	39.77	0.93	3.89	49.60	0.54
Changes in money supply							
Germany	6.69	0.34	0.12	15.80	24.59	42.72	14.40
France	20.93	52.98	2.80	20.91	44.44	19.92	144.42
Italy	35.00	8.30	0.12	33.77	46.42	1.92	0.09
UK	35.79	36.11	1.20	39.01	42.54	1.10	5.27
Japan	5.69	9.48	2.52	3.71	19.40	11.02	0.84
Changes in government bond yield							
Germany	33.70	27.88	0.03	33.41	74.36	0.62	4.70
France	65.84	64.04	4.16	48.63	66.76	0.33	5.48
Italy	5.04	6.35	0.83	4.27	10.85	0.49	3.88
UK	5.84	2.92	5.81	23.30	18.70	92.31	88.01
Japan	4.06	1.14	0.14	5.00	5.25	14.51	5.67

Tables 2 to 4 present the estimates of the intercepts and slope coefficients obtained in the different regimes. The most remarkable result is that the slope coefficients often switch between a significant and a non-significant value, suggesting that in one regime the variable in question (inflation, money, output) has a significant effect on the exchange rate, while in the other regime its effect is not significantly different from zero. There are cases, however, where the switches are between two non-significant coefficients (this is the case for Japan and Italy, and for industrial production). It should be noted that the switch is never between two significant coefficients.

Table 2

Estimates fit to individual low inflation country data, t= 73:II to 98:11

Equation: $\frac{e_t - e_{t-12}}{e_{t-12}} = a + b_j (\frac{p_t - p_{t-12}}{p_{t-12}})$, j = 1 or 2

e represents the exchange rate of the country considered vis-à-vis the dollar and π stands for the inflation

Parameter	Germany	France	Italy	UK	Japan
a	-0.33 (0.22)	-0.11 (0.21)	0.07 (0.14)	0.15 (0.19)	-0.23** (0.10)
b$_1$	-0.64** (0.17)	0.26** (0.08)	0.01 (0.06)	0.10** (0.05)	-0.06* (0.04)
b$_2$	0.11 (0.09)	-0.81* (0.42)	0.29 (0.30)	-0.15 (0.09)	0.02 (0.07)
P$_{11}$	0.82** (0.10)	0.92** (0.02)	0.86** (0.07)	0.94** (0.05)	0.95** (0.11)
P$_{22}$	0.93** (0.05)	0.96** (0.07)	0.94** (0.05)	0.94** (0.07)	0.95** (0.07)
s^2	2.60** (0.12)	2.52** (0.11)	2.49** (0.09)	2.54** (0.10)	2.82** (0.12)

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table 3

Estimates fit to individual low inflation country data, t= 73:II to 98:11

$\frac{e_t - e_{t-12}}{e_{t-12}} = a + b_j (\frac{M_t - M_{t-12}}{M_{t-12}})$, j = 1 or 2

e represents the exchange rate of the country considered vis-à-vis the dollar and M stands for the money supply

Parameter	Germany	France	Italy	UK	Japan
a	-0.01** (0.10)	-0.41* (0.24)	0.41** (0.14)	0.49 (0.32)	-0.59** (0.19)
b$_1$	-0.12** (0.04)	24.98** (6.33)	-0.03 (0.03)	0.09* (0.05)	-0.13 (0.10)
b$_2$	0.09 (0.07)	-2.24 (2.22)	0.01 (0.03)	-0.48 (0.36)	0.07 (0.06)
P$_{11}$	0.91** (0.11)	0.85** (0.12)	0.77** (0.27)	0.85** (0.12)	0.97** (0.07)
P$_{22}$	0.92** (0.07)	0.91** (0.07)	0.83** (0.21)	0.27** (0.11)	0.75** (0.22)
s^2	2.63** (0.12)	2.52** (0.14)	2.59** (0.09)	0.14 (0.09)	2.76** (0.06)

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table 4

Estimates fit to individual low inflation country data, t= 73:II to 98:11

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a + b_j\left(\frac{GBY_t - GBY_{t-12}}{GBY_{t-12}}\right), \text{ j = 1 or 2}$$

e represents the exchange rate of the country considered vis-à-vis the dollar and GBY stands for government bond yield

Parameter	Germany	France	Italy	UK	Japan
a	-0.16 (0.41)	0.04 (0.15)	0.33** (0.15)	0.14 (0.14)	-0.24 (0.15)
b_1	-2.05** (0.82)	-1.65** (0.52)	1.66 (1.15)	3.80** (1.26)	1.60 (1.03)
b_2	-0.39 (0.65)	3.33 (2.12)	-1.46 (1.57)	0.25 (0.36)	-2.51 (1.70)
P_{11}	0.78** (0.53)	0.94** (0.07)	0.60** (0.26)	0.97** (0.03)	0.93** (0.10)
P_{22}	0.18** (0.41)	0.72** (0.39)	0.29** (0.41)	0.99** (0.01)	0.85** (0.14)
s^2	2.77** (0.39)	2.59** (0.10)	2.48** (0.12)	2.53** (0.10)	2.76** (0.11)

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

How do these results compare with the results obtained for the high inflation countries? Tables 5 to 9 give an answer to this question. In table 5 we present the Wald tests for the significance of the switches in regimes (intercepts and slopes) in the high inflation countries. The contrast with the low inflation countries is striking. We find significant switches in regimes in all countries, but these switches are never due to switches in the slope. They are caused exclusively by switches in the intercept. Thus in the high inflation countries there have been switches in the average level of inflation, but the explanatory power of the independent variables (inflation, money supply, interest rate) has remained unchanged. This result contrasts with the results of the low inflation countries in which the explanatory power of these independent variables appears to switch frequently.

In tables 6 to 9 we show the intercepts and the slopes in the different regimes for the high inflation countries. We observe that the slope coefficients are almost always significantly different from zero (although they do not always have the expected sign).

Table 5

Wald test results for high inflation countries

	Switches in the intercept and slope			Switches in the intercept		Switches in the slope	
	H_0:P_{11}=1-p_{12}	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0:P_{11}=1-p_{12}	H_0: $\alpha_1 = \alpha_2$	H_0:P_{11}=1-p_{12}	H_0: $\beta_1 = \beta_2$
Changes in Inflation							
Argentina	0.62	218.00	0.63	78.64	98.91	0.00	0.00
Bolivia	0.28	842.31	0.42	0.15	4.45	0.20	0.00
Brazil	457.51	150.26	59.66	439.19	481.25	100.70	0.00
Columbia	129.93	2.93	0.57	131.47	71.74	0.00	0.01
Ecuador	0.27	305.76	7.38	0.17	228.36	0.11	0.05
Changes in money supply							
Argentina	6.11	260.15	0.01	0.92	220.07	0.01	0.00
Bolivia	11.13	97.68	0.08	13.27	127.24	8.45	0.01
Brazil	530.80	250.01	67.13	403.51	85.03	5.51	0.00
Columbia	9.47	46.20	2.50	10.26	17.74	0.25	0.00
Ecuador	198.76	205.85	1.15	0.08	19.14	6.76	0.00
Changes in lending rate							
Argentina	-	-	-	-	-	-	-
Bolivia	51.24	17.58	0.53	128.11	18.19	0.05	0.01
Brazil	670.22	809.02	2.88	275.44	938.03	0.34	0.04
Columbia	3.10	72.58	36.12	2.67	40.94	1.63	0.00
Ecuador	0.00	406.69	3.32	0.00	46.43	2.97	2.17

Table 6

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b(\frac{p_t - p_{t-12}}{p_{t-12}})$$, j = 1 or 2

e represents the exchange rate of the country considered vis-à-vis the dollar and π stands for the inflation

Parameter	Argentina	Bolivia	Brazil	Columbia	Ecuador
a_1	160.70** (1.87)	0.64** (0.30)	26.64** (1.07)	2.59** (0.34)	53.02** (0.58)
a_2	6.01 (15.44)	0.0003** (0.00004)	3.87** (0.56)	1.14** (0.29)	4.45 (3.16)
b	-0.00002** (0.00)	0.0006** (0.0001)	0.003** (0.0004)	-0.02 (0.02)	-0.06** (0.01)
P_{11}	0.16** (0.11)	0.39** (0.18)	0.89** (0.04)	0.97** (0.02)	0.11** (0.01)
P_{22}	0.98** (0.17)	0.73** (0.15)	0.96** (0.01)	0.98** (0.01)	0.95** (0.11)
s^2	14.83** (0.11)	1.61** (0.02)	6.48** (0.31)	1.15** (0.06)	1.88** (0.08)
Period	76:1-91:1	85:2-00:11	80:12-98:1	73:1-00:11	82:5-00:1

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table 7

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b\left(\frac{M_t - M_{t-12}}{M_{t-12}}\right) \text{, j = 1 or 2}$$

e represents the exchange rate of the country considered vis-à-vis the dollar and M stands for the money supply

Parameter	Argentina	Bolivia	Brazil	Columbia	Ecuador
a_1	161.05** (9.64)	0.63** (0.30)	28.96** (0.86)	3.84** (0.35)	2.12** (0.53)
a_2	5.86 (4.06)	0.21** (0.002)	3.64** (0.39)	-0.58 (0.99)	37.89** (0.03)
b	-0.003** (0.002)	0.01** (0.001)	0.0009** (0.0008)	0.07** (0.02)	-0.07** (0.04)
P_{11}	0.16** (0.08)	0.96** (0.18)	0.78** (0.04)	0.58** (0.08)	0.97** (0.07)
P_{22}	0.98** (0.01)	0.96** (0.20)	0.97** (0.01)	0.91** (0.13)	0.05** (0.003)
s^2	14.60** (0.002)	0.25** (0.02)	5.82** (0.24)	1.50** (0.18)	2.83** (0.11)
Period	76:1-91:1	89:12-00:11	73:1-98:1	94:12-00:11	94:12-00:11

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table 8

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b\left(\frac{LR_t - LR_{t-12}}{LR_{t-12}}\right) \text{, j = 1 or 2}$$

e represents the exchange rate of the country considered vis-à-vis the dollar and LR stands for the lending rate

Parameter	Argentina	Bolivia	Brazil	Columbia	Ecuador
a_1	-	1.39** (0.05)	30.10** (0.81)	1.62** (0.19)	2.45** (0.12)
a_2	-	0.51** (0.20)	4.01** (0.38)	-3.31** (0.02)	40.68** (0.54)
b	-	-0.004 (0.008)	0.35 (0.56)	-0.05 (0.20)	-0.004 (0.008)
P_{11}	-	0.91** (0.08)	0.78** (0.04)	0.98** (0.18)	0.97** (0.01)
P_{22}	-	0.02** (0.01)	0.97** (0.01)	0.43** (0.17)	0.13** (0.56)
s^2	-	0.36** (0.02)	5.92** (0.23)	1.50** (0.19)	3.13** (0.11)
Period	-	87:1-00:11	73:1-98:1	86:1-00:11	82:5-99:11

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

We also tested for asymmetry in the regimes, i.e. we checked whether the regime the economy was in the previous period affected the current regime (see tables 10 and 11). We found that in various cases there was a significant asymmetry.

Finally we analysed the persistence (duration) of the regimes. The results are also shown in tables 10 and 11. For the low inflation countries (table 10) we find that the regime in which the slope is not significant usually lasts longer than the regime in which the slope is significant. In the high inflation countries we find a strong asymmetry in the persistence of the regimes whereby one is long lasting (25 to 50 months) and the other is very short in timing (1.2 to 9.1 months). More detail is obtained from the transition probabilities, which are presented in appendix D.

Table 10

Test of asymmetry in regimes for the low inflation countries (switches in the slope)

	Germany	France	Italy	UK	Japan
H_0: p_{11}=1-p_{21}					
Change in inflation	34.13	112.72	38.77	90.87	49.60
Change in money	42.72	19.92	1.92	1.10	11.02
Change in government bond yield	0.62	0.33	0.49	92.31	14.51
Expected duration (months) of state 1: $(1\text{-}p_{11})^{-1}$					
Change in inflation	5.56	12.50	7.14	16.67	20.00
Change in money	11.11	6.67	4.35	6.67	33.33
Change in government bond yield	4.55	16.67	2.50	33.33	14.29
Expected duration (months) of state 2: $(1\text{-}p_{22})^{-1}$					
Change in inflation	14.29	25.00	16.67	16.67	20.00
Change in money	12.50	11.11	7.69	3.70	4.00
Change in government bond yield	5.56	1.39	3.45	100	6.67

Table 11

Test of asymmetry in regimes for the high inflation countries (switches in the intercept)

	Argentina	Bolivia	Brazil	Columbia	Ecuador
H_0: p_{11}=1-p_{21}					
Change in inflation	78.64	0.15	439.19	131.47	0.17
Change in money	0.92	13.27	403.51	10.26	0.08
Change in lending rate	-	128.11	275.44	2.67	0.001
Expected duration (months) of state 1: $(1\text{-}p_{11})^{-1}$					
Change in inflation	1.19	1.64	9.09	33.33	1.64
Change in money	1.19	25.00	8.33	2.38	33.33
Change in lending rate	-	11.11	8.33	50.00	33.33
Expected duration (months) of state 2: $(1\text{-}p_{22})^{-1}$					
Change in inflation	50.00	3.70	25.00	50.00	3.70
Change in money	50.00	25.00	33.33	11.11	1.05
Change in lending rate	-	50.00	33.33	1.75	1.15

4.2 Multivariate analysis

In the multivariate analysis we analyse the regime switches in regression equations explaining the changes in the exchange rates by changes in relative money supplies, changes in relative inflation and changes in relative bond yields. We analyse switches in all the coefficients taken together, and then in the coefficients separately. As before we apply the analysis to low and high inflation countries.

Tables 12 and 13 present the Wald tests for the low and high inflation countries. Our results lead to broadly similar results as in the univariate case. For the low inflation countries we find many significant switches both in the intercept and in the slope coefficients. For the high inflation countries we only find switches in the intercept, but never in the slope coefficients.

Table 12

Wald test results for low inflation countries

	Switches in the intercept and slope				**Switches in the intercept**	**Switches in the slope**		
	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$
Germany	4.85	4.75	0.02	8.59	47.42	7.03	3.06	18.46
France	47.99	3.20	0.01	1.36	25.42	1.22	0.02	10.94
Italy	22.41	4.76	1.71	3.24	43.12	17.68	6.72	0.01
UK	18.80	12.90	1.10	1.31	1.13	4.30	0.002	2.27
Japan	-	-	-	-	47.48	91.34	33.78	3.35

Table 13

Wald test results for high inflation countries

	Switches in the intercept and slope				**Switches in the intercept**	**Switches in the slope**		
	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$
Argentina	105.78	288.61	12.23	14.70	-	-	-	-
Bolivia	16.91	15.32	0.51	8.42	110.42	1.10	0.71	1.46
Brazil	160.52	100.60	36.78	1.00	423.69	1.38	1.47	0.05
Columbia	40.48	15.08	1.10	5.62	52.97	0.30	1.87	0.003
Ecuador	3.93	61.12	21.26	0.27	384.87	-	-	-

Tables 14 and 15 present the estimated coefficients in the different regimes. We find again that in the case of the low inflation countries the switches mostly occur between significant and non-significant slope coefficients (with the exception of the coefficients of the relative money supplies). In the case of the high inflation countries the slope coefficients are almost always significant, and the switches only occur between the intercepts that are always significant.

Table 14

Estimates fit to individual low inflation country data, t= 73:II to 98:11

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b_j\left(\frac{p_t - p_{t-12}}{p_{t-12}}\right) + g_j\left(\frac{M_t - M_{t-12}}{M_{t-12}}\right) + d_j\left(\frac{GBY_t - GBY_{t-12}}{GBY_{t-12}}\right)$$, j = 1 or 2

Parameter	Germany	France	Italy	UK	Japan
a_1	0.66 (0.72)	1.75** (0.43)	4.07** (1.01)	-0.52** (0.24)	-
a_2	-1.10** (0.25)	-1.36** (0.37)	-0.41 (0.35)	4.26** (1.15)	-
b_1	-0.56** (0.16)	0.06 (0.11)	-0.21 (0.14)	0.11** (0.04)	-
b_2	-0.14 (0.12)	2.25* (1.18)	0.09** (0.04)	-1.13** (0.35)	-
g_1	0.00 (0.11)	-0.29* (0.16)	0.08 (0.08)	-0.15 (0.74)	-
g_2	-0.02 (0.10)	-0.17 (1.02)	-0.02 (0.02)	2.02 (1.86)	-
d_1	-4.11 (5.10)	-0.73 (0.48)	0.64 (0.52)	0.83** (0.39)	-
d_2	0.10** (0.02)	-1.74** (0.76)	-0.41 (0.28)	-0.39 (0.91)	-
P_{11}	0.89** (0.10)	0.77** (0.07)	0.68** (0.09)	0.96** (0.02)	-
P_{22}	0.94** (0.09)	0.82** (0.09)	0.95** (0.03)	0.85** (0.09)	-
s^2	2.47** (0.14)	2.09** (0.16)	2.08** (0.11)	2.28** (0.10)	-

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table 15

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b(\frac{p_t - p_{t-12}}{p_{t-12}}) + g(\frac{M_t - M_{t-12}}{M_{t-12}}) + d(\frac{GBY_t - GBY_{t-12}}{GBY_{t-12}})$$, j = 1 or 2

Parameter	Bolivia	Brazil	Columbia
a_1	0.54** (0.04)	23.30** (0.92)	2.74** (0.75)
a_2	0.10** (0.05)	4.13** (0.50)	-2.92** (1.09)
b_1	0.03** (0.01)	0.01** (0.0008)	0.11** (0.04)
g_1	-0.002** (0.001)	-0.01** (0.0006)	0.11** (0.03)
d_1	-0.002 (0.002)	1.45 (1.01)	-0.06 (0.08)
P_{11}	0.96** (0.03)	0.90** (0.04)	0.59** (0.16)
P_{22}	0.97** (0.02)	0.96** (0.02)	0.93** (0.04)
s^2	0.21** (0.01)	5.58** (0.25)	1.46** (0.04)
Period	89:12-00:11	80:12-98:1	94:12-00:11

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

5. Theoretical Issues

The results discussed in the previous section can be summarized as follows. The relation between the exchange rate and the fundamentals of low inflation countries is characterized by frequent regimes shifts. We found that the coefficients of these fundamentals change over time quite often from significant values to insignificant ones, and vice versa. This feature is absent in the exchange rate equations of high inflation countries. In those countries we find that the coefficients of the fundamentals are quite stable (only the intercept switches).

These results suggest that for the high inflation countries the linear first generation model may be the right framework for explaining the movements of these countries' exchange rates. This is not the case for the low inflation countries, whose exchange rates cannot be explained by a stable relation with underlying fundamentals.

Any explanation of these empirical results must be capable of accounting for the differences observed in the stability of the exchange rate equations between low and high inflation countries. There are two alternative explanations. The first alternative is based on the second-generation model. We claim that this explanation is unsatisfactory. The second-generation model is based on explicit utility maximization of a representative agent. In this model the structural instability of the coefficients in the exchange rate equations can be explained by shifts in the underlying stochastic structure, which may or may not be induced by changes in policy regimes. The contrasting evidence between high and low inflation countries, however, makes this explanation implausible. If anything, high inflation countries experience stronger changes in the underlying stochastic structure (mainly induced by shifts in policy regimes) than low inflation countries. And yet it is in the high inflation countries that the linear first generation model seems to be dong well while it fails for the low inflation countries.

For this reason our preferred explanation is based on non-linearities. In what follows, we outline the nature of two non-linear features that in our view are capable of explaining the unstable relation between the exchange rate and its underlying fundamentals in low inflation countries. Here we only briefly sketch the nature of these non-linearities and how these affect exchange rate models. We intend to do further research to formalise these ideas.

A first non-linearity has been stressed by Obstfeld and Rogoff (2000), who show that many of the current puzzles in international macroeconomics can be explained by transaction costs. In our case, introducing transaction costs can contribute to understanding the difference in the relationship between the exchange rate and its fundamentals for low inflation and stable in high inflation countries. To see this, consider the following setup.

The existence of transaction costs (say as a fixed proportion of the prices of products) defines a band in which arbitrage relations, such as the PPP relation, do not hold. This is the case in both the low and high inflation countries. Now introduce exogenous shocks in the underlying fundamental values of the exchange rate. In the low inflation countries, many shocks tend to be relatively small relative to the transaction cost band (e.g. inflation shocks). Hence, arbitrage will not be profitable in these cases and will remain absent. Some shocks, however, are large relative to the transactions cost band implying that arbitrage will take place. As a consequence, the relation between exchange rates and their underlying fundamentals will be unstable. In contrast, in the high inflation countries, shocks in the fundamentals (especially nominal shocks) are always large relative to the transactions costs band, imposing strong arbitrage relations. This implies that the relation between the exchange rate and its fundamentals remains stable.

A second non-linear feature can be introduced which is capable of explaining our empirical findings. This is based on diversity of opinion (see for instance De Grauwe and Dewachter (1993), De Grauwe (1994) and Kilian and Taylor (2001). The essential ingredient of such a non-linearity is the hypothesis that economic agents use different information sets. In general, two kind of agents, 'fundamentalists' and 'chartists' (or informed traders and noise traders) can be considered. The fundamentalist is forward looking in that he computes the equilibrium (or fundamental) exchange rate to predict future exchange rate movements, while the chartist is backward looking, relying on extrapolations of past exchange rate movements for his forecasts.

The fundamentalist is uncertain about the fundamental value of the exchange rate. (This uncertainty may be due to the existence of a transaction cost band which blurs the relation between exchange rates and their fundamentals). As a result, when the exchange rate is close to its fundamental value, fundamentalists take few positions. The market is then dominated by the chartists. Conversely, as the exchange rate moves away from its fundamental value, fundamentalists move in the market again, and become more important to determine the exchange rate.

This model leads to a speculative dynamics in which the exchange rate appears to have a life of its own. This model may be appropriate for low inflation countries where there is often great uncertainty about the true equilibrium value of the exchange rate. (Note again that this uncertainty is probably linked to the existence of a transactions cost band which in low inflation countries is large relative to the size of the shocks in the fundamentals). In the high inflation, however, this uncertainty about the equilibrium value of the exchange rate is less pronounced. As a result, the market will be dominated by fundamentalist. In this case, exchange rate movements will be linked to shocks in the underlying fundamental values.

As stressed earlier, this is only a broad sketch of non-linearities in exchange rate models capable of explaining the results obtained in this paper. Further theoretical analysis will be necessary to substantiate this claim.

6. Conclusion

Characterizing the nature of the relationship between exchange rate changes and the news in its underlying fundamentals has long been an objective of empirical international macroeconomics. Although this research has contributed to our understanding of the behaviour of the exchange rates, it is also true that this empirical research has been unable to validate the existing theoretical models. In particular, the 'first generation models' of the exchange rates that were developed during the 1970s have been rejected at least when using data of the major industrial countries. The 'second generation models' based on explicit utility maximisation of agents have not produced sharp enough testable propositions allowing for their refutation by the data. As a result, they have not been confirmed nor refuted.

In this paper, we test whether the relationship between the nominal exchange rate and the news in its underlying fundamentals has non-linear features. In order to do so, we developed a Markov switching model and applied the model for a sample of low inflation and high inflation countries.

The empirical analysis shows that for the high inflation countries the first generation models appear to work well: the relationship between news in the fundamentals and the exchange rate changes is stable and always significant. This is not the case, however, for the low inflation countries, where frequent regime switches occur. This finding casts doubts about the capacity of the second-generation models to explain the facts.

We discussed two non-linear models that are capable of explaining our empirical findings. A first model is based on the existence of transaction costs; a second one starts from the existence of different types of agents using different information to forecast the future exchange rate. We conjectured that such non-linear models would be fruitful to understand the behaviour of exchange rates.

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Appendix A. Data definitions and sources

The ten countries included in the analysis are: Argentina, Bolivia, Brazil, Colombia, Ecuador, France, Germany, Italy, Japan and the UK. Information on the home currency-dollar exchange rate and six fundamentals was retrieved on a monthly and quarterly basis. More specifically, this set of fundamentals covers:

1. The inflation for the country concerned
2. The money supply for the country under scrutiny, for all countries this represents M2 except for the UK where M0 was used
3. The Money Market Rate, which is used as a measure of the short term interest rate
4. The lending rate and the long-term government bond yield which are both proxies of the long-term interest rate. The latter was however only available for the low inflation countries
5. Industrial production
6. The trade balance relative to the GDP

In table A1 below, the time period used for each separate fundamental is report for the monthly data. For industrial production and the trade balance relative to the GDP the same time periods were used. Both fundamentals were only applied for the low inflation countries, as for the high inflation countries either the data was not available or the time period covered was too short to be of any use. For the quarterly observations, the same time period was applied but then the figures were transformed to quarters rather than months.

Table A1

Time periods covered by the various fundamentals

	Fundamentals					
	Inflation	**Money supply**	**Money market rate**	**Lending rate**	**Government Bond Yield**	**Industrial Production**
Low inflation countries						
Germany	73:1-98:11	73:1-98:11	73:1-98:11	77:5-98:11	73:1-98:11	73:1-98:11
France	73:1-98:11	73:1-98:11	73:1-86:01	73:1-98:11	73:1-98:11	73:1-98:11
Italy	73:1-98:11	73:1-98:11	73:1-98:11	83:8-98:11	73:1-98:11	73:1-98:11
UK	73:1-98:11	73:1-98:11	73:1-98:11	73:1-98:11	73:1-98:11	73:1-98:11
Japan	73:1-98:11	73:1-98:11	73:1-98:11	73:1-98:11	73:1-98:11	73:1-98:11
High inflation countries						
Argentina	76:1-91:1	76:1-91:1	79:3-91:1	n.a.	n.a.	n.a.
Bolivia	85:2-00:11	89:12-00:11	95:1-00:11	87:1-00:11	n.a.	n.a.
Brazil	80:12-98:1	73:1-00:11	73:1-98:1	73:1-98:11	n.a.	n.a.
Columbia	73:1-00:11	94:12-00:11	95:3-00:10	86:1-00:11	n.a.	n.a.
Ecuador	82:5-00:1	82:5-00:1	82:5-00:1	82:5-99:11	n.a.	n.a.

Appendix B. Maximum likelihood estimation of the Markov-switching model[6]

Introduction

In this appendix, more attention is devoted to the determination of the various population parameters of the Markov-switching model. In a first part, we therefore rewrite the model in a state-space representation, which has been proven useful for the study of time series with unobservable states. Next we write down the log likelihood function that has to be optimised and we subject the EM algorithm to closer scrutiny. In the third section, the computation of the standard errors is discussed and finally in the last section, the derivation of Wald test as reported in this paper is explained.

The regime shift function and the state space representation

At this stage it is useful to define the parameter shifts more clearly by formulating the system as a single equation by introducing ‘dummy’ indicator variables:

$$I(s_t = m) = \begin{cases} 1\, if\ s_t = m \\ 0\ otherwise, \end{cases}$$

Where m = 1 or 2. Now we can collect all information about the realization of the Markov chain in the vector ξt as, whereby ξt denotes the unobserved state of the system:

$$\boldsymbol{x}_t = \begin{bmatrix} I(s_t = 1) \\ I(s_t = 2) \end{bmatrix}$$

The state space representation of the model now consists of the following set of measurement and transition equation:

1. Measurement or observation equation

$$\Delta e_t = X_t^{'} B \boldsymbol{x}_t + u_t \qquad u_t \sim N(0, \boldsymbol{s}^2),$$

$$\text{where } X_t^{'} = (1, \Delta fund_t^{'})$$

6 This section derives on Hamilton (1990) and Krolzig (1997)

and where $B = \begin{bmatrix} \boldsymbol{a}_{s_t=1} & \boldsymbol{b}_{s_t=1} \\ \boldsymbol{a}_{s_t=2} & \boldsymbol{b}_{s_t=2} \end{bmatrix}$

2. State or transition equation

$$\boldsymbol{x}_{t+1} = F\boldsymbol{x}_t + v_{t+1}$$

Maximum likelihood estimation and the EM algorithm

In order to fix the parameters of the aforementioned equation we can rely both on the classical method of maximum likelihood estimation and the EM Algorithm. Both have been applied in this paper and will be discussed in more details below.

Under the assumption that the observed variable, Δe_t, is drawn from an N(μ ,σ2) distribution, and the unobserved state is presumed to have been generated by some probability distribution, for which the unconditional probability that s_t takes on the value j is denoted by $\boldsymbol{p}_j$:

$$p\{s_t = j; \boldsymbol{q}\} = \boldsymbol{p}_j$$

where θ represents the population parameters that should be determined, so:

$$\boldsymbol{q} \equiv (\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{s}^2, \boldsymbol{p}_1, \boldsymbol{p}_2)'$$

In this case the unconditional density for Δe_t is the sum over j=1 and 2 of the density distribution functions of Δe_t given state s_t

$$f(\Delta e_t; \boldsymbol{q}) = \sum_{j=1}^{2} p(\Delta e_t, s_t = j; \boldsymbol{q}) =$$

$$\frac{\boldsymbol{p}_1}{\sqrt{2\boldsymbol{ps}}} \exp\left\{\frac{-(\Delta e_t - \boldsymbol{a}_1 - \boldsymbol{b}_1(\Delta fund_t))}{2\boldsymbol{s}^2}\right\} + \frac{\boldsymbol{p}_2}{\sqrt{2\boldsymbol{ps}}} \exp\left\{\frac{-(\Delta e_t - \boldsymbol{a}_2 - \boldsymbol{b}_2(\Delta fund_t))}{2\boldsymbol{s}^2}\right\}$$

If the regime variable s_t is distributed i.i.d. across different dates t, then the log likelihood function of the observed data can now be calculated from the above expression as:

$$L(\boldsymbol{q}) = \sum_{t=1}^{T} \log\ f(\Delta e_t ; \boldsymbol{q})$$

The maximum likelihood estimate of θ is obtained by maximizing subject to the constraint that $\boldsymbol{p}_1 + \boldsymbol{p}_2 = 1$ and $\boldsymbol{p}_j \geq 1$ for j= 1 and 2. This can be achieved using the numerical methods or using the EM algorithm. The latter approach is an iterative maximum likelihood estimation technique consisting of two steps (see Krolzig,1997):

In the expectation step (E), the unobserved states ξt are estimated by their smoothed probabilities, $\hat{\boldsymbol{x}}_{t|T}$, while in the maximization step, estimates of $\boldsymbol{l} \equiv (\boldsymbol{q}, p_{11}, p_{22})$ are obtained as a solution of the first order conditions of L(θ). In table 1 below, this algorithm is depicted in more detail. General results available for the EM algorithm indicate that the likelihood function increases in the number of iterations i. Finally, a fixed-point of this iteration schedule $\boldsymbol{l}^{(j)} = \boldsymbol{l}^{(j-1)}$ coincides with the maximum of the likelihood function.

Standard errors and the EM Algorithm

In order to compute the variance-covariance matrix and hence the standard errors when using the EM algorithm, we employed the way suggested by Berndt, Hall, Hall and Hausman (1974), where $s_i(\boldsymbol{q})$ represents the first derivatives of the individual log likelihood contributions, also known as scores:

$$\hat{V} = \left(\frac{1}{T} \sum_{i=1}^{T} s_i(\hat{\boldsymbol{q}}) s_i(\hat{\boldsymbol{q}})' \right)^{-1}$$

Table 1

The EM Algorithm

I. Initialization $\boldsymbol{l}^{(0)}$

II. Expectation Step

A. Filtering (forward recursion t=1, …, T) $(\Delta E_t = (\Delta e_t, \ldots, \Delta e_1)$*):*

$$p(\Delta e_t | \Delta E_t) = \sum_{s_t=1}^{2} \sum_{s_{t-1}=1}^{2} \left(p(s_t | s_{t-1}) \times p(\Delta e_t, s_t; \boldsymbol{q}) \times p(s_{t-1} | \Delta E_{t-1}) \right)$$

$$p(s_t | \Delta E_t) = \frac{\left(\sum_{s_{t-1}=1}^{2} p(s_t | s_{t-1}) \times p(\Delta e_t, s_t; \boldsymbol{q}) \times p(s_{t-1} | \Delta E_t) \right)}{p(\Delta e_t | \Delta E_t)}$$

B. Smoothing (backward recursion t=1, T-1)

$$p\left(s_{t+1}, s_t | \Delta E_{t+1}\right) = \frac{p\left(s_{t+1} | s_t\right) \times p\left(\Delta e_{t+1}, s_{t+1}; \boldsymbol{q}\right) \times p\left(s_t | \Delta E_t\right)}{p(\Delta e_t | \Delta E_t)}$$

Forward recursion for τ = t+2,… T

$$p(s_t, s_t | \Delta E_t) = \frac{\left(\sum_{s_{t-1}=1}^{2} p(s_t | s_{t-1}) \times p(\Delta e_t, s_t; \boldsymbol{q}) \times p(s_{t-1}, s_t | \Delta E_t) \right)}{p(\Delta e_t | \Delta E_t)}$$

$$p(s_t | \Delta E_T) = \sum_{s_{T=1}}^{2} p\left(s_T, s_t | \Delta E_T\right)$$

III. Maximization Step

$$\boldsymbol{s}^{2(i+1)} = T^{-1} \sum_{t=1}^{T} \sum_{j=1}^{2} (\Delta e_{t\,j} - \boldsymbol{a}_j - \Delta fund_t' \boldsymbol{b}_j)^2 \times p(s_t = j | \Delta E_t; \boldsymbol{q})$$

$$\boldsymbol{a}_j^{(i+1)} = \left(\begin{array}{l} \left[\sum_{t=1}^{T} \left[\left(\sqrt{p(s_t = j | \Delta E_T; \hat{\boldsymbol{q}})} \right) \times \left(\sqrt{p(s_t = j | \Delta E_T; \hat{\boldsymbol{q}})} \right)' \right]^{-1} \right] \times \\ \left[\sum_{t=1}^{T} \left[\left(\sqrt{p(s_t = j | \Delta E_T; \hat{\boldsymbol{q}})} \right) \times \Delta \bar{e}_t(j) \right] \right] \end{array} \right)$$

$$\hat{\boldsymbol{b}}_j^{(i+1)} = \left[\sum_{t=1}^{T} \left(\bar{X}_t(j) \times \left(\bar{X}_t(j) \right)^{-1} \right) \right]^{-1} \times \left[\sum_{t=1}^{T} \left(\bar{X}_t(j) \times \Delta \bar{e}_t(j) \right) \right]$$

where

$$\Delta \bar{e}_t(j) = \Delta e_t \sqrt{p(s_t = j | \Delta E_T; \hat{\boldsymbol{q}})}$$

$$\overline{X}_t(j) = \Delta fund_t \sqrt{p(s_t = j \middle| \Delta E_T; \hat{\boldsymbol{q}})}$$

$$p_{11}^{(i+1)} = \frac{\sum_{t=2}^{T} p(s_t = 1, s_{t-1} = 1 \middle| \Delta E_T;\, \boldsymbol{l}^{(i)})}{\sum_{t=2}^{T} p(s_{t-1} = 1 \middle| \Delta E_T;\, \boldsymbol{l}^{(i)})} \quad \text{and} \quad p_{22}^{(i+1)} = \frac{\sum_{t=2}^{T} p(s_t = 2, s_{t-1} = 2 \middle| \Delta E_T;\, \boldsymbol{l}^{(i)})}{\sum_{t=2}^{T} p(s_{t-1} = 2 \middle| \Delta E_T;\, \boldsymbol{l}^{(i)})}$$

IV. Iterate step II & III until Convergence, criterion: $\left| \boldsymbol{l}^{(i+1)} - \boldsymbol{l}^{i} \right| \leq 10^{-8}$

Wald test

There exist several ways to test hypotheses about parameters that are estimated by maximum likelihood. Here we have relied on the Wald test to check the following hypotheses:

H0: p11 = 1 – p22

H0: α1 = α2

H0: β1 = β2

For the Wald test, the test statistics for the above hypotheses are:

H0: p11 = 1 – p22: $\dfrac{\left[\hat{p}_{11} - (1 - \hat{p}_{22})\right]^2}{\left[\hat{\mathrm{var}}(\hat{p}_{11}) + \hat{\mathrm{var}}(\hat{p}_{22}) + 2\,\hat{\mathrm{cov}}(\hat{p}_{11}, \hat{p}_{22})\right]} \approx \boldsymbol{c}^2(1)$

H0: α1 = α2: $\dfrac{(\hat{\boldsymbol{a}}_1 - \hat{\boldsymbol{a}}_2)^2}{\hat{\mathrm{var}}(\hat{\boldsymbol{a}}_1) + \hat{\mathrm{var}}(\hat{\boldsymbol{a}}_2) - 2\,\hat{\mathrm{cov}}(\hat{\boldsymbol{a}}_1, \hat{\boldsymbol{a}}_2)} \approx \boldsymbol{c}^2(1)$ (same methodology for β)

Where var denotes the asymptotic variance and cov the asymptotic covariance

Appendix C: Estimation results using quarterly data

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Table C1

Wald test results for low inflation countries

Changes in Inflation							
	Switches in the intercept and slope			Switches in the intercept		Switches in the slope	
	H_0:p_{11}=1-p_{21}	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: p_{11}=1-p_{21}	H_0: $\alpha_1 = \alpha_2$	H_0: p_{11}=1-p_{21}	H_0: $\beta_1 = \beta_2$
Germany	0.00	0.27	7.13	4.22	1.91	0.03	7.70
France	34.20	1.44	0.43	8.18	1.23	2.89	25.77
Italy	0.17	2.14	8.55	0.41	0.55	0.07	6.57
UK	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Japan	0.78	50.96	7.00	0.00	0.00	2.28	80.37
Changes in money supply							
Germany	0.26	11.27	19.03	8.31	2.79	1.41	4.11
France	3.75	0.89	3.98	0.56	0.58	1.26	23.91
Italy	0.83	2.70	1.99	0.28	-12.89	21.58	1.07
UK	4.32	61.59	5.12	0.41	0.06	12.45	4.11
Japan	1.22	0.21	0.26	0.12	0.00	0.46	18.47
Changes in government bond yield							
Germany	0.32	0.16	16.87	0.00	0.00	0.26	17.12
France	18.49	10.35	4.52	6.56	16.21	n.a.	n.a.
Italy	0.50	27.77	2.02	1.24	15.86	41.73	0.02
UK	1.52	0.01	6.92	18.96	11.43	n.a.	n.a.
Japan	3.08	98.10	8.06	6.11	0.69	0.00	3.91

Table C2

Test of asymmetry in regimes for low inflation countries (switches in the slope)

	Germany	**France**	**Italy**	**UK**	**Japan**
H_0: p_{11}=1-p_{21}					
Change in inflation	0.03	2.89	0.03	n.a.	2.28
Change in money	1.41	1.26	1.41	12.45	0.46
Change in government bond yield	0.26	18.49	0.02	1.52	0.00
Expected duration (quarters) of state 1: $(1-p_{11})^{-1}$					
Change in inflation	1.92	1.09	1.12	n.a.	2.08
Change in money	1.20	1.19	1.67	1.37	2.33
Change in government bond yield	3.13	1.02	33.33	1.06	1.02
Expected duration (quarters) of state 2: $(1-p_{22})^{-1}$					
Change in inflation	1.85	1.23	1.07	n.a.	1.52
Change in money	1.37	2.5	1.18	1.10	1.11
Change in government bond yield	4.54	1.32	1.25	1.27	1.28

Table C3

Estimates fit to individual low inflation country data, t= 73:1 to 97:4

Equation: $\frac{e_t - e_{t-12}}{e_{t-12}} = a + b_j (\frac{p_t - p_{t-12}}{p_{t-12}})$, j = 1 or 2

e represents the exchange rate of the country considered vis-à-vis the dollar and π stands for the inflation

Parameter	Germany	France	Italy	UK	Japan
a	0.05 (0.66)	0.32 (0.66)	0.80 (0.73)	n.a.	-0.15 (0.59)
b_1	-0.67** (0.29)	0.47 (0.35)	0.24 (0.22)	n.a.	0.67** (0.19)
b_2	0.52 (1.48)	-1.31 (1.42)	0.02 (0.30)	n.a.	-0.28* (0.16)
P_{11}	0.52** (0.20)	0.92** (0.09)	0.89** (0.24)	n.a.	0.48** (0.11)
P_{22}	0.54** (0.23)	0.81** (0.23)	0.93** (0.22)	n.a.	0.66** (0.08)
s^2	5.83** (0.44)	5.68** (0.48)	5.90** (0.44)	n.a.	3.79** (0.35)

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table C4

Estimates fit to individual low inflation country data, t= 73:II to 98:11

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a + b_j (\frac{M_t - M_{t-12}}{M_{t-12}})$$, j = 1 or 2

e represents the exchange rate of the country considered vis-à-vis the dollar and M stands for the money supply

Parameter	Germany	France	Italy	UK	Japan
a	-0.47 (0.75)	0.46 (1.01)	1.08 (0.67)	0.42 (0.65)	-0.95 (0.57)
b_1	0.32* (0.19)	0.22* (0.12)	-0.06 (0.13)	-0.36* (0.20)	0.25 (0.38)
b_2	-0.51 (0.39)	-1.11** (0.31)	1.17* (0.90)	0.16 (0.18)	0.26** (0.08)
P_{11}	0.83** (0.21)	0.84** (0.08)	0.98** (0.12)	0.73** (0.13)	0.43** (0.85)
P_{22}	0.73** (0.33)	0.40** (0.20)	0.80** (0.15)	0.91** (0.11)	0.10** (0.12)
s^2	5.64** (0.55)	4.69** (0.49)	5.51** (0.45)	5.41** (0.43)	5.85** (0.41)

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table C5

Estimates fit to individual low inflation country data, t= 73:II to 98:11

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a + b_j \left(\frac{GBY_t - GBY_{t-12}}{GBY_{t-12}}\right), \text{ j = 1 or 2}$$

e represents the exchange rate of the country considered vis-à-vis the dollar and GBY stands for government bond yield

Parameter	Germany	France	Italy	UK	Japan
a_1	-0.22 (0.67)	-0.32 (0.59)	12.82** (2.46)	-0.13 (0.63)	-0.74 (0.61)
a_2	-	6.13** (1.93)	0.30 (0.63)	2.87 (2.00)	-
b_1	-0.27 (0.83)	-0.41** (0.09)	0.41 (0.33)	-0.04 (0.28)	-0.05 (0.14)
b_2	-0.26* (0.15)	0.27 (0.42)	-0.06 (0.07)	-0.07 (0.29)	-0.04 (0.11)
P_{11}	0.32 (8.35)	0.98** (0.02)	0.23** (0.21)	0.94** (0.35)	0.98** (0.26)
P_{22}	0.22 (9.11)	0.76** (0.16)	0.93** (0.05)	0.79** (0.45)	0.78** (0.52)
s^2	5.95** (0.45)	5.00** (0.39)	4.81** (0.43)	5.52** (0.45)	6.08** (0.46)

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table C5

Wald test results for high inflation countries

Changes in Inflation							
	Switches in the intercept and slope			Switches in the intercept		Switches in the slope	
	H_0:P_{11}=1-p_{12}	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0:P_{11}=1-p_{12}	H_0: $\alpha_1 = \alpha_2$	H_0:P_{11}=1-p_{12}	H_0: $\beta_1 = \beta_2$
Argentina	0.00	1592.91	487.87	0.00	1848.37	14.52	0.00
Bolivia	15.75	51.81	31.57	0.00	694.47	3.94	0.21
Brazil	53.67	7.20	30.80	40.95	90.84	14.50	0.00
Columbia	111.36	4.36	0.00	0.12	25.87	469.93	0.02
Ecuador	-	-	-	-	-	-	-
Changes in money supply							
Argentina	0.00	2032.84	1904.29	0.00	1392.00	12.04	0.01
Bolivia	0.00	126.34	17.29	0.00	409.10	14.50	0.30
Brazil	105.88	22.16	46.95	50.28	162.76	0.70	0.00
Columbia	5.67	0.06	1.49	5.14	56.85	48.05	42.14
Ecuador	-	-	-	-	-	-	-
Changes in lending rate							
Argentina	-	-	-	-	-	-	-
Bolivia	-	-	-	-	-	-	-
Brazil	38.85	249.94	23.39	33.87	168.95	35.84	0.00
Columbia	7.09	16.39	3.80	0.02	10.31	0.00	0.00
Ecuador	-	-	-	-	-	-	-

Table C6

Test of asymmetry in regimes for high inflation countries (switches in the intercept)

	Argentina	Bolivia	Brazil	Columbia	Ecuador
H_o: p_{11}=1-p_{21}					
Change in inflation	0.00	0.00	40.95	0.12	-
Change in money	0.00	0.00	50.28	5.14	-
Change in lending rate	-		33.87	0.02	-
Expected duration (quarters) of state 1: $(1-p_{11})^{-1}$					
Change in inflation	1.01	100	1.22	1.59	-
Change in money	1.01	100	1.23	1.02	-
Change in lending rate	-		1.27	1.05	-
Expected duration (quarters) of state 2: $(1-p_{22})^{-1}$					
Change in inflation	100	1.01	1.05	1.03	-
Change in money	100	1.01	1.04	1.39	-
Change in lending rate	-		1.04	6.67	-

Table C7

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b\left(\frac{p_t - p_{t-12}}{p_{t-12}}\right) \text{, j = 1 or 2}$$

e represents the exchange rate of the country considered vis-à-vis the dollar and π stands for the inflation

Parameter	Argentina	Bolivia	Brazil	Columbia	Ecuador
a_1	27.49**	16.64**	132.45**	3.41**	-
	(4.48)	(7.18)	(12.59)	(0.81)	
a_2	1548.57**	1397.16**	24.23**	3.53**	-
	(36.35)	(53.74)	(6.16)	(0.34)	
b	0.003	0.004	0.01**	0.04	-
	(0.003)	(0.003)	(0.004)	(0.59)	
P_{11}	0.99**	0.99**	0.82**	0.63	-
	(0.01)	(0.01)	(0.11)	(1.74)	
P_{22}	0.01**	0.11**	0.95**	0.97*	-
	(0.001)	(0.02)	(0.03)	(0.59)	
s^2	42.27**	66.50**	34.45**	3.81**	-
	(2.67)	(3.46)	(3.07)	(0.66)	
Period	73:2-97:4	73:2-97:4	81:2-97:4	73:2-99:4	-

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table C8

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b(\frac{M_t - M_{t-12}}{M_{t-12}})$$, j = 1 or 2

e represents the exchange rate of the country considered vis-à-vis the dollar and M stands for the money supply

Parameter	Argentina	Bolivia	Brazil	Columbia	Ecuador
a_1	25.55** (3.61)	15.93** (7.02)	131.93** (9.22)	5.77** (0.64)	-
a_2	1546.23** (41.05)	1370.24** (68.72)	17.67** (3.71)	13.26** (1.06)	-
b	0.01** (0.004)	0.01** (0.004)	0.004** (0.002)	-0.13** (0.03)	-
P_{11}	0.99** (0.01)	0.99** (0.01)	0.81** (0.10)	0.98** (0.02)	-
P_{22}	0.01** (0.004)	0.01** (0.004)	0.96** (0.02)	0.72** (0.31)	-
s^2	41.52** (3.06)	66.18** (4.39)	29.44** (2.34)	1.60** (0.17)	-
Period	73:2-97:4	73:2-97:4	73:2-97:4	73:2-97:4	-

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table C9

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b(\frac{LR_t - LR_{t-12}}{LR_{t-12}})$$, j = 1 or 2

e represents the exchange rate of the country considered vis-à-vis the dollar and LR stands for the lending rate

Parameter	Argentina	Bolivia	Brazil	Columbia	Ecuador
a_1	-	-	136.65** (9.25)	4.89** (0.58)	-
a_2	-	-	19.47** (3.95)	-5.85** (0.00)	-
b	-	-	0.002 (0.004)	0.03 (0.58)	-
P_{11}	-	-	0.79** (0.12)	0.95** (0.58)	-
P_{22}	-	-	0.96** (0.03)	0.85** (0.39)	-
s^2	-	-	29.69** (2.69)	2.80** (0.58)	-
Period	-	-	73:2-97:4	86:1-97:4	-

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

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Table C10

Wald test results for low inflation countries

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b\left(\frac{p_t - p_{t-12}}{p_{t-12}}\right) + g\left(\frac{M_t - M_{t-12}}{M_{t-12}}\right) + d\left(\frac{GBY_t - GBY_{t-12}}{GBY_{t-12}}\right)$$, j = 1 or 2

	Switches in the intercept and slope				Switches in the intercept	Switches in the slope		
	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$
Germany	-1.11	-0.25	0.42	4.13	1.76	39.24	42.67	7.62
France	9.92	6.49	9.22	9.92	34.19	3.90	13.82	10.43
Italy	4.19	19.74	11.01	4.19	21.69	1.31	12.96	8.93
UK	35.03	1.43	1.99	0.03	31.66	4.82	6.22	5.13
Japan	-1.45	-0.03	0.00	0.00	13.50	5.20	2.83	5.15

Table C11

Test of asymmetry in regimes (switches in the slope)

Germany	France	Italy	UK	Japan
H_0: $p_{11}=1-p_{21}$				
0.80	4.35	116.57	1.81	0.66
Expected duration (quarters) of state 1: $(1-p_{11})^{-1}$				
4.17	4.35	14.29	4.35	3.33
Expected duration (quarters) of state 2: $(1-p_{22})^{-1}$				
1.61	1.18	33.33	6.67	1.10

Table C12

Estimates fit to individual low inflation country data, t= 73:II to 98:11

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a + b_j\left(\frac{p_t - p_{t-12}}{p_{t-12}}\right) + g_j\left(\frac{M_t - M_{t-12}}{M_{t-12}}\right) + d_j\left(\frac{GBY_t - GBY_{t-12}}{GBY_{t-12}}\right)$$, j = 1 or 2

Parameter	Germany	France	Italy	UK	Japan
a_1	0.06 (0.60)	0.84 (0.97)	-1.47 (1.21)	0.25 (0.67)	-1.63** (0.70)
b_1	0.98** (0.22)	0.80 (1.29)	0.84 (0.68)	0.51* (0.29)	-0.71** (0.35)
b_2	-1.69 (1.29)	-1.47** (0.72)	0.10 (0.23)	-0.17** (0.001)	0.32 (0.28)
g_1	0.17* (0.09)	0.35** (0.12)	-0.68** (0.26)	-0.36 (0.23)	0.39** (0.12)
g_2	0.19 (0.16)	-0.68 (1.24)	0.43 (0.33)	0.12 (0.33)	0.04 (0.16)
d_1	-0.18** (0.09)	-0.15 (0.27)	0.33** (0.15)	-0.13** (0.03)	0.07 (0.13)
d_2	0.31**	0.32**	-0.31**	0.03	-0.49**

	(0.14)	(0.15)	(0.16)	(0.30)	(0.19)
P11	0.76** (0.08)	0.77** (0.10)	0.93** (0.07)	0.67** (0.25)	0.70** (0.15)
P22	0.16** (0.04)	0.15 (0.28)	0.97** (0.03)	0.85** (0.32)	0.54** (0.22)
s^2	3.57** (0.33)	4.63** (0.40)	5.13** (0.47)	5.16** (0.54)	5.01** (0.51)

*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

Table C13

Wald test results for high inflation countries

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b\left(\frac{p_t - p_{t-12}}{p_{t-12}}\right) + g\left(\frac{M_t - M_{t-12}}{M_{t-12}}\right) + d\left(\frac{GBY_t - GBY_{t-12}}{GBY_{t-12}}\right)$$, j = 1 or 2

	Switches in the intercept and slope				Switches in the intercept	Switches in the slope		
	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$	H_0: $\alpha_1 = \alpha_2$	H_0: $\beta_1 = \beta_2$	H_0: $\gamma_1 = \gamma_2$	H_0: $\delta_1 = \delta_2$
Bolivia	1.77	8.02	12.61	1.77	12.45	0.00	1.22	0.64
Brazil	407.80	38.71	15.97	407.80	564.55	2.43	079	1.41
Columbia								

Table C14

Test of asymmetry in regimes (switches in the intercept)

Bolivia	**Brazil**	**Columbia**
Ho: p11=1-p21		
169.05	0.001	
Expected duration (months) of state 1: $(1\text{-}p11)^{-1}$		
10	1.03	
Expected duration (months) of state 2: $(1\text{-}p22)^{-1}$		
25	33.33	

Table C15

Estimates fit to individual high inflation country data

$$\frac{e_t - e_{t-12}}{e_{t-12}} = a_j + b\left(\frac{p_t - p_{t-12}}{p_{t-12}}\right) + g\left(\frac{M_t - M_{t-12}}{M_{t-12}}\right) + d\left(\frac{LR_t - LR_{t-12}}{LR_{t-12}}\right)$$, j = 1 or 2

Parameter	Bolivia	Brazil	Columbia
a_1	2.00** (0.26)	16.56** (4.02)	
a_2	0.54 (0.40)	-180.78** (9.08)	
b_1	0.08** (0.03)	0.14** (0.01)	
g_1	0.02** (0.01)	-0.09** (0.01)	
d_1	-0.004 (0.02)	0.26** (0.01)	

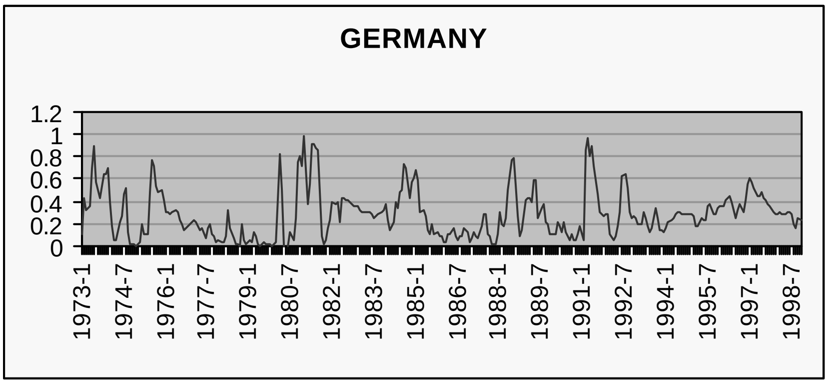
P_{11}	0.90** (0.04)	0.97** (0.04)	
P_{22}	0.96** (0.04)	0.03 (0.67)	
s^2	1.43** (0.09)	20.05** (1.27)	
Period	87:2-00:4	81:2-97:4	

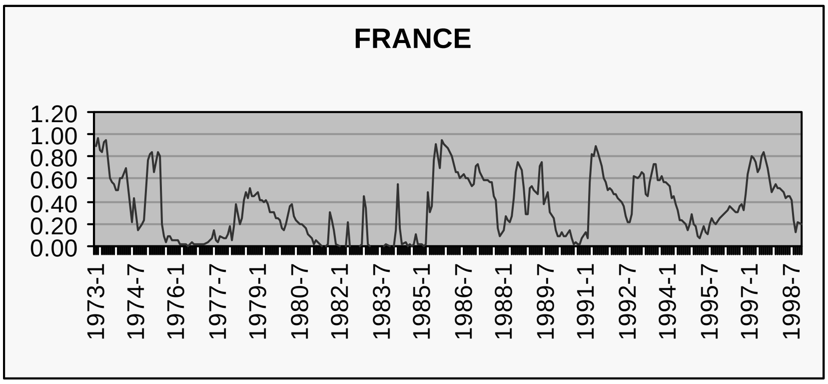
*Note: standard errors are in parentheses, * denotes significance at a 10% level, ** at a 5% level*

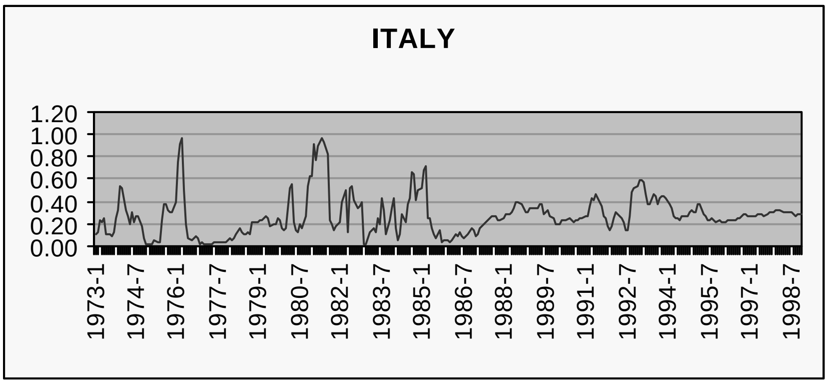
Appendix D: The transition probabilities for the estimated equations using monthly data

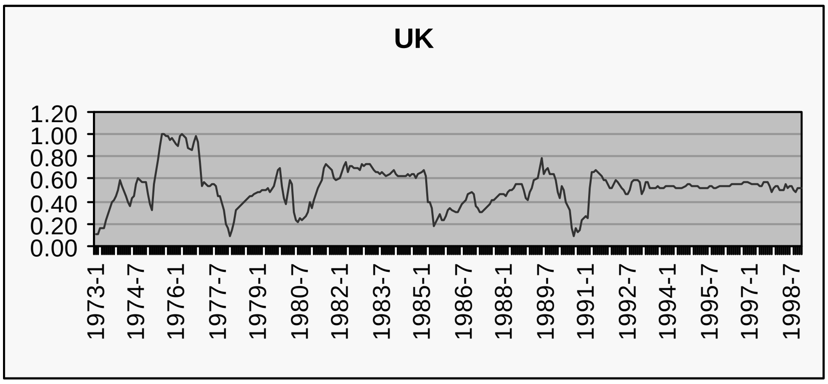
Figure D1

The smoothed probability that the economy is in state 1, table 2









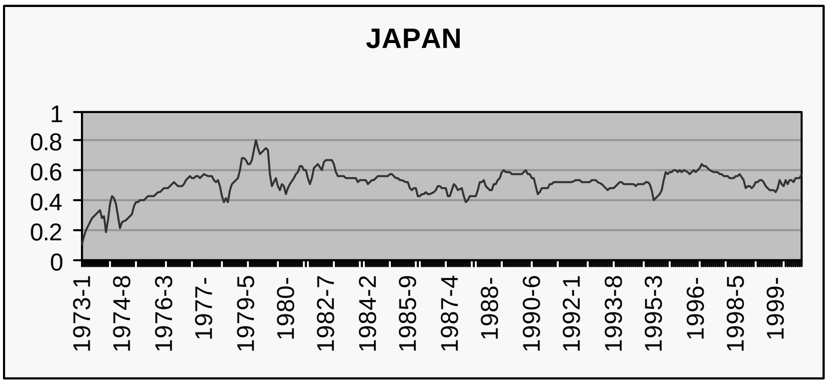
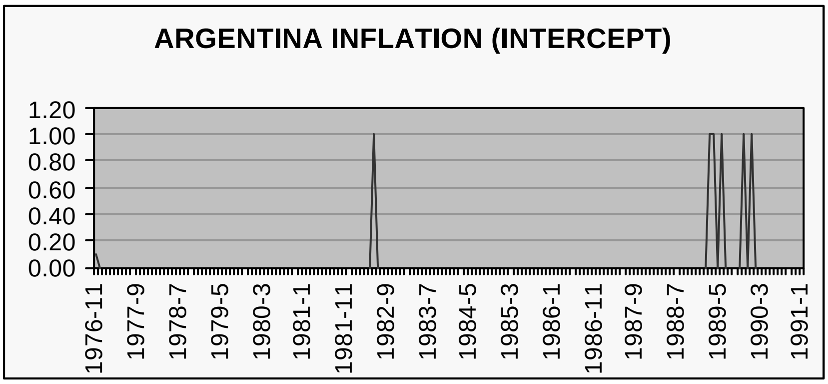
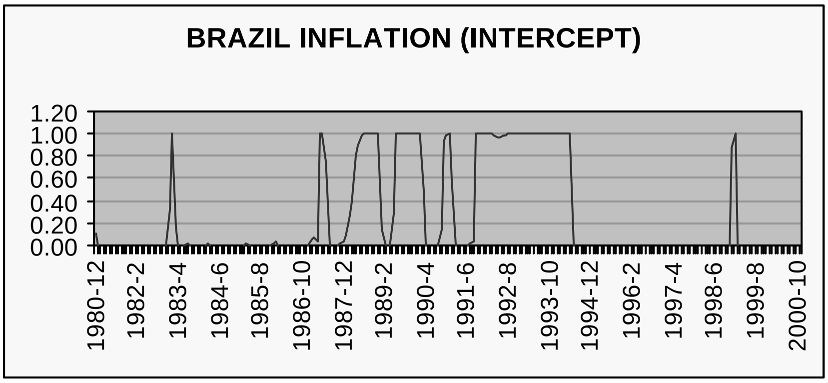
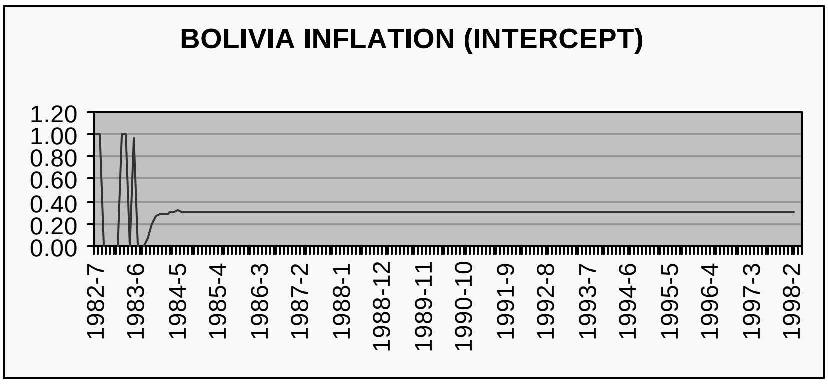


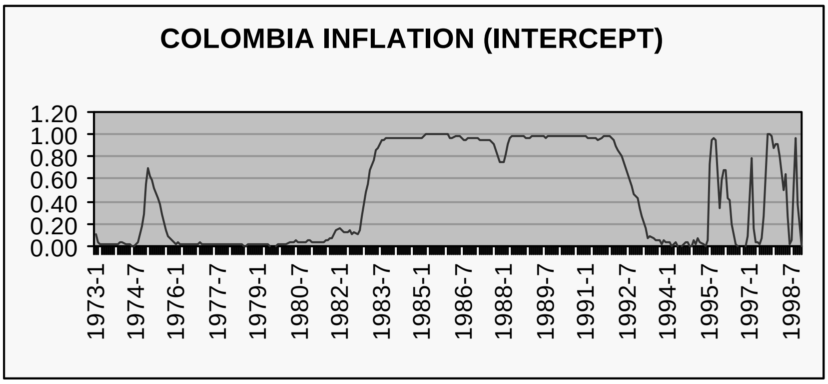
Figure D2

The smoothed probability that the economy is in state 1, table 6









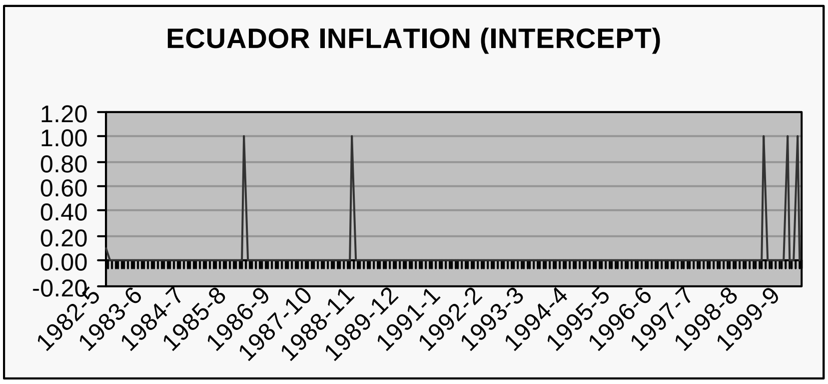
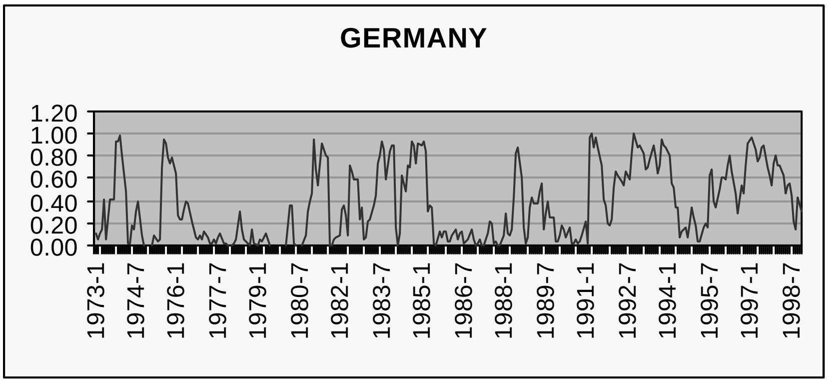
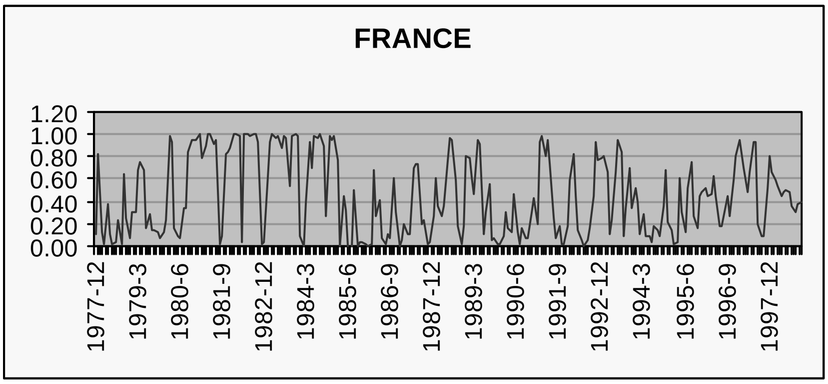
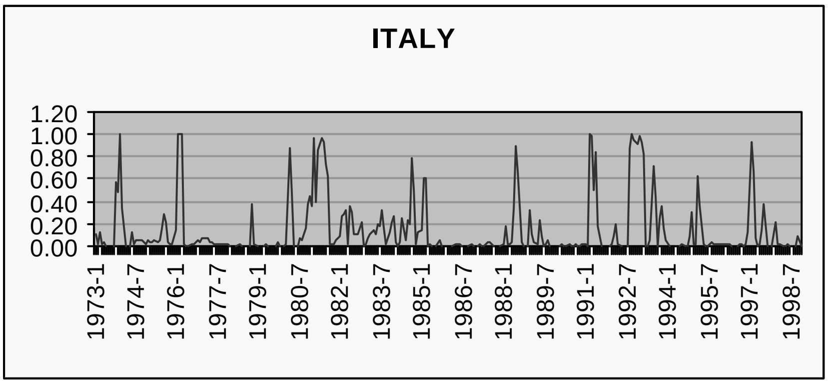


Figure D3

The smoothed probability that the economy is in state 1, table 14







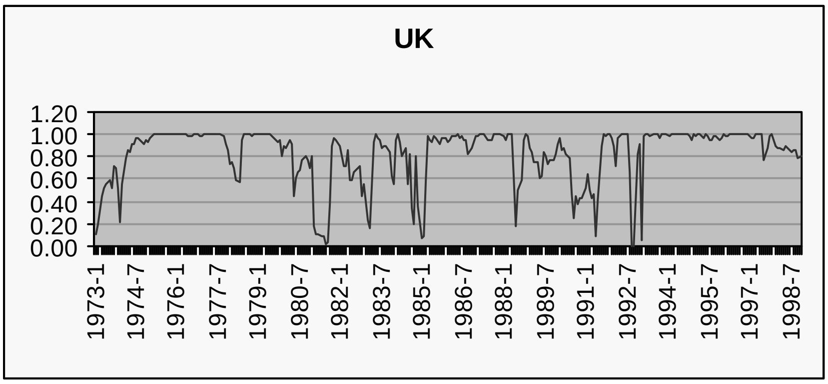
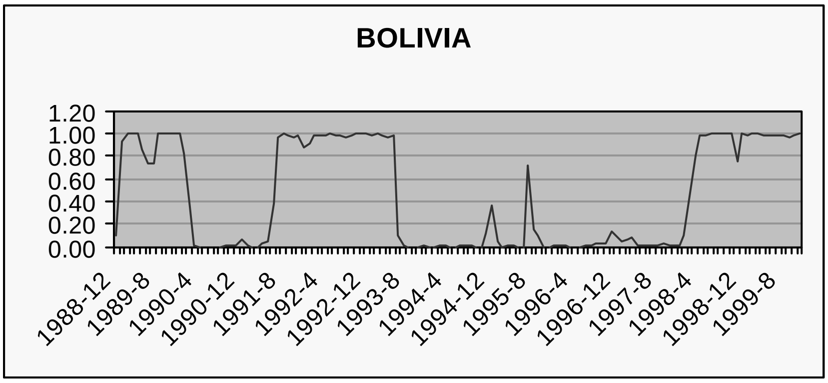
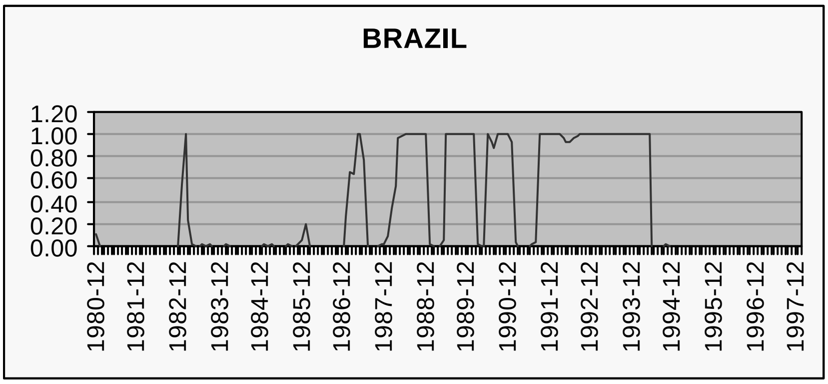
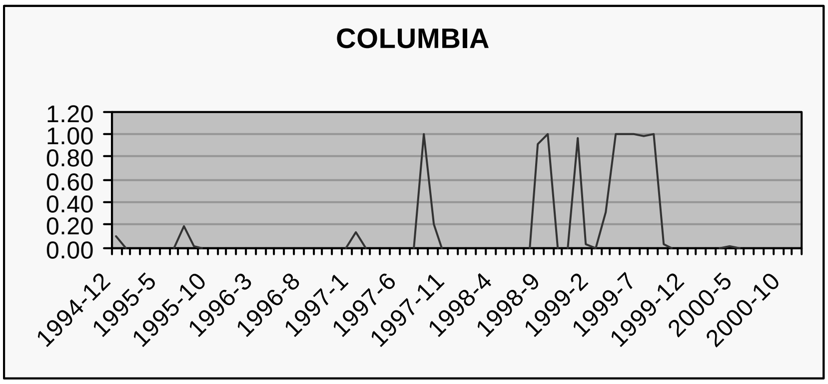


Figure D4

The smoothed probability that the economy is in state 1, table 15




BRAZIL
1.20
1.00
0.80
0.60
0.40
0.20
0.00
1980-12
1981-12
1982-12
1983-12
1984-12
1985-12
1986-12
1987-12
1988-12
1989-12
1990-12
1991-12
1992-12
1993-12
1994-12
1995-12
1996-12
1997-12


COLUMBIA
1.20
1.00
0.80
0.60
0.40
0.20
0.00
1994-12
1995-5
1995-10
1996-3
1996-8
1997-1
1997-6
1997-11
1998-4
1998-9
1999-2
1999-7
1999-12
2000-5
2000-10