Working Paper Series 1/11

EXCHANGE RATE RISK IN THE GOVERNMENT PENSION FUND GLOBAL

Øystein Børsum
Ministry of Finance

April 2011

Centre for Monetary Economics
BI Norwegian School of Management

ISSN 1503-3031

Exchange Rate Risk in the Government Pension Fund Global*

Øystein Børsum†

April 13, 2011

Abstract

The Government Pension Fund Global (the Fund) is an important instrument of national saving. In a national perspective, the Fund's role is to save from the current export surplus (oil and gas) to finance future purchases of goods and services produced abroad (imports). In this perspective, exchange rate risk relates to the difference between the currency allocation in the Fund and the currency composition of future imports. Exchange rate risk amounts to deviations from international purchasing power parity (PPP) in tradable goods. A literature review suggests that the evidence for PPP in the long run is considerably stronger today than commonly thought 10-15 years ago. Also, it seems justified to expect large deviations from PPP to be significantly more short-lived than previously thought. Given the Fund's long investment horizon and regular withdrawals through the fiscal policy guideline, exchange rate risk seems small. This warrants a change in the geographical allocation of the Fund. Today, more than half of the Fund's capital is invested in Europe – a certain form of "home bias." There no longer appears to be a basis for such a strong concentration of the investments. In fact, an argument can be made to invest more in countries that are farther from home, e.g. in emerging markets.

*The views expressed in this paper are those of the author, and not necessarily the official views of the Norwegian Ministry of Finance.

†Asset Management Department, Norwegian Ministry of Finance. Email: oystein.borsum@fin.dep.no

1 Introduction

The Government Pension Fund Global (the Fund) is a fiscal policy tool to support long-term management of Norway's petroleum revenues. Fiscal policy, which regulates the inflow and outflow from the Fund, is anchored in the guideline that over time the structural, non-oil budget deficit shall correspond to the expected real return on the Fund, estimated at 4 percent. The large current petroleum revenues are thus transformed into a permanent income stream. In order to protect the domestic economy from the adverse effects of large and volatile capital inflows, the Fund is invested entirely abroad.[1]

The Fund capital reflects state saving. At the same time, the Fund is an important instrument of national saving. The allocations to the Fund are approximately in line with Norway's export surplus resulting from the production of oil and gas. In a national perspective, the Fund's role is to save this surplus to finance future purchases of goods and services which are produced internationally, i.e. future imports, allowing Norway to run a permanent trade deficit after the production of oil and gas ends. Accordingly, the aim for the Fund's investment strategy is to maximise the Fund's international purchasing power subject to a moderate level of risk.

Norway's imports and exports result from the aggregation of millions of individual decisions by firms, households, and government entities. Historically – and in consistence with so-called "gravity models" of international trade (see e.g. Bergstrand, 1985) – these trade patterns have been heavily skewed towards neighbouring countries in Europe. Though integration of world trade is likely to deepen, it seems reasonable to expect locally-oriented trade patterns to persist, so that an important share of Norwegian imports will continue to come from Europe in the future. This suggests that unless the Fund is heavily invested in Europe (or in European currencies) relative to other parts of the world, its international purchasing power is subject to exchange rate risk.[2]

On this backdrop, the purpose of this paper is to review two key questions:

- How important is exchange rate risk for the Fund?
- What are the implication of exchange rate risk for the Fund's geographical allocation and currency hedging policy?

[1] Leaving this capital outflow to the Norwegian private sector, e.g. by investing domestically or distributing the petroleum revenues directly to citizens, would presumably require a very low domestic interest rate and depreciation expectations (i.e. a strong Norwegian krone) to induce the private sector to invest abroad. A further discussion of these issues can be found in e.g. NOU 2004:1 *Modernisert folketrygd*, enclosure 2: *Konsekvenser av fondering av pensjoner for pengepolitikken.*

[2] This can be thought to justify a certain form of "Equity Home Bias", i.e. a investing more closer to home. For a closer discussion of the the equity home bias, see Obstfeld and Rogoff (2000) and the references therein.

2 Exchange Rate Risk from a National Perspective

The type of exchange rate risk that I consider in this paper can be illustrated with a simple example. Suppose there are only two geographies outside Norway: Europe and America. For the sake of exposition, let Europe be represented by the eurozone, and America by the the United States, so that there are only two currencies, the euro and the U.S. dollar. A share a of an investment portfolio is initially invested in Europe; the remaining share $1-a$ is invested in the U.S. at an initial exchange rate normalised to one. Let q_E and $q_\$$ denote the nominal gross return factors on European and U.S. financial assets, respectively, and let e denote the nominal dollar-euro exchange rate (that is, the price of one dollar in terms of euros).[3] The worth of one unit of the portfolio after some time is therefore $aq_E + (1-a)eq_\$$.

Next, suppose Norway will import only European goods. The real international purchasing power of the portfolio (and the utility it provides) is then appropriately measured as the volume of European tradable goods it can buy. Using x to denote the volume, and p_E to denote the price measured in euros of a basket of European tradable goods, this equals

$$x = a\,\frac{q_E}{p_E} + (1-a)\,\frac{ep_\$}{p_E}\,\frac{q_\$}{p_\$}, \tag{1}$$

where the last part of the expression has been multiplied and divided by the price of a basket of U.S. tradable goods measured in U.S. dollars. We see that the purchasing power of the portfolio depends on three sets of variables:

1. Real returns in the world's asset markets, q_E/p_E and $q_\$/p_\$$, respectively.
2. The real dollar-euro exchange rate, $ep_\$/p_E$.
3. The portfolio's regional allocation, determined by a.

If international goods markets are well integrated, a reasonable equilibrium condition is that prices of tradable goods produced in different countries are equal, once converted to a common currency, i.e. $ep_\$ \equiv p_E$. This condition is referred to as international purchasing power parity (PPP), here applied to the case of tradable goods in particular.[4]

Consider first the case when PPP holds. Eq. (1) shows that in this case, the purchasing power of the portfolio is determined by a weighted average of the real returns in the world's different asset markets. Each of these real returns will be stochastic. If international capital markets are well integrated, a reasonable equilibrium condition may be that expected (ex ante) real returns in different market are similar. Ex post returns,

[3] Since the nominal exchange rate at the time of investment is one, e is also the return factor on a dollar measured in euros.

[4] At the level of individual goods, the condition is often referred to as the "law of one price."

however, may be very different, depending on the development of each particular asset market and country in question. Imperfect correlation across countries and regions gives scope for international diversification, the benefits of which are thought to be important.[5]

To be explicit about exchange rate risk, suppose further that the price levels of different tradable goods p_E and $p_\$$ are pinned down by production costs in their respective countries of production, for instance due to competition in each country's local goods market. Then, when PPP holds, there is no real exchange rate risk.[6] In this situation, much attention should be given to international diversification in determining the portfolio's regional allocation.[7]

Clearly, PPP does not hold always and everywhere. In particular, price levels tend to be rigid, while the nominal exchange rate e is volatile, leading to significant short-run fluctuations in the real exchange rate. Eq. (1) shows that in this case, the purchasing power of the portfolio is subject to exchange rate risk. If e.g. the dollar appreciates relative to the euro in real terms, i.e. if $ep_\$/p_E$ increases, then the portfolio can buy more European goods, all other things equal. (And vice versa for a real dollar depreciation.) Exchange rate risk in this portfolio can therefore be thought of in terms of deviations from international purchasing power parity in tradable goods. Evidently, whether and in what circumstances PPP holds is key to characterising exchange rate risk for the Fund.[8] The empirical evidence for PPP will be reviewed in section 3.

In the simple example in eq. (1), only investments in the U.S. induce exchange rate risk to the Fund's purchasing power. Investments in Europe do not, because the location (currency) of investment coincides with that of the goods to be purchased. Evidently, exchange rate risk can be eliminated by not investing in the U.S. asset markets at all ($a = 1$), but it comes at the cost of reduced international diversification. This trade-off is fundamental to any unhedged international portfolio. In fact, the strategic regional

[5]See e.g. Solnik and McLeavey (2008) for a detailed account of the scope for international diversification.

[6]Strictly speaking, PPP does not specify which of the three variables will adjust to ensure $ep_\$ \equiv p_E$. One possibility is in fact that p_E adjusts to $ep_\$$, in which case there may be exchange rate risk even when PPP holds. Arguably, it seems more plausible that e adjusts to p_E and $p_\$$, as is assumed here.

[7]PPP does not insulate against unexpected inflation. As eq. (1) points out, differences in ex post real returns may arise from diverging nominal returns, but may also result from different price level developments. This aspect is especially important when it comes to international diversification in a nominal bond portfolio.

[8]To judge the importance of deviations from PPP, one must also consider the correlation between real exchange rates and real asset market returns. See Norges Bank's letter dated 11 April 2002 for an analysis of these patterns in the 1970s, 80s and 90s. Criticism may be raised as to whether it is meaningful to separate real asset returns and the real exchange rate as fundamental random variables. In particular, many of the world's largest listed firms are multinationals. As their activities are global, their revenues are internationally diversified. The quantitative importance of this proposition is not known, but it would clearly make exchange rate risk less meaningful and less transparent than suggested here. Nevertheless, Norges Bank's analysis suggests country affiliation of entreprise still has a strong bearing on shareprice performance, justifying the approach taken here.

weights of the Government Pension Fund Global can be interpreted as an attempt to balance these concerns, as they generally lie between the import composition and the market weights.[9] Import weights were a natural benchmark for the Fund's asset allocation at the onset in 1996, when the Fund was invested in government bonds only (see e.g. the discussion in Nasjonalbudsjettet 1997, the National budget for 1997). Over time, as the Fund has grown and invested in equities, weighing schemes with more emphasis on international diversification have gained ground, in particular market weights and GDP weights.[10] I return to the implications of exchange rate risk for the Fund's geographical allocation in section 4.

In principle, currency allocation and regional asset market allocation could be chosen quite independently by using a simple currency overlay portfolio (here e.g. through buying euros forward in terms of U.S. dollars).[11] However, it should be noted that throughout the history of the Fund it has been viewed as not very appropriate to separate the currency allocation from the market allocation in this way. This is in part because the long horizon of the Fund makes it difficult to determine the Fund's real currency exposure, but also because of the added transaction costs and counterparty risks that buying currency derivative contracts would entail. A discussion of the pertinence of currency hedging is included in section 4.

It is important to point out that from a national perspective, the value of the portfolio measured in Norwegian kroner is irrelevant, as it does not impact its international purchasing power.[12] However, as the Fund is fully integrated with the state budget, and the expenses on the state budget essentially relate to Norwegian kroner, there is still a question of exchange rate risk with respect to the value of the Fund in Norwegian kroner with respect to the state budget. This issue will be discussed in more detail in section 5.

In the simple example above, it was assumed that Norway would only purchase goods from Europe. While it is true that Norway tends to import much from countries in Europe, international goods markets provide great opportunities for substitution of goods from different origins. This is important to take into consideration when it comes to exchange rate risk, because as one currency appreciates, others by definition depreciate. Exchange

[9]The regional weights are, respectively, 50, 35 and 15 percent for equities and 60, 35 and 5 percent for fixed-income instruments.

[10]See e.g. Norges Bank's letter dated 11 April 2002, and Nasjonalbudsjettet 2003, the National budget for 2003, for a comprehensive discussion of the benefits and inconveniences of these alternatives.

[11]Perfect hedging would require forward contracts on real exchange rates (whereas they are usually on nominal exchange rates), as well as full knowledge of the future value of the U.S. portfolio to be hedged (which is impossible when the portfolio is dominated by equities).

[12]A possible exception to this rule would be if European exports to Norway were priced "to market," i.e. that the prices at which European tradable goods could be bought by Norwegians were determined from characteristics of the Norwegian market (e.g. Norwegians' income) rather than from characteristics of the European producers (e.g. their production cost). Free, competitive European goods markets suggests this will generally not be the case.

rate fluctuations potentially provide opportunities to buy cheap goods from countries with undervalued currencies. Consequently, it is important for the analysis of exchange rate risk to consider a multi-country (or currency) consumption basket.

Intuition may suggest that when imports come from several countries or currencies, real exchange rate risk occurs whenever the portfolio's currency allocation differs from the currency composition of future imports. (This allocation principle is often referred to as "import weights.") The appendix provides a formal set-up in which this is indeed the case. The key issue is the degree of substitution between goods produced in different countries. To see this, note that a change in the exchange rate between any two countries has two effects: On the one hand, it changes the relative value of the parts of the portfolio in these countries; on the other hand, it changes the relative price of the goods produced in these countries. For example, when e.g. the dollar-euro exchange rate appreciates, then (measured in euros) the value of the U.S. part of the portfolio increases – which is good – but so does the cost of U.S. goods – which is bad. Which of these effects dominate depends crucially on the degree of substitutability between goods. Using Cobb-Douglas preferences (elasticity of substitution equal to unity between goods of different origins), one obtains that when the portfolio's currency allocation exactly equals the fixed currency composition of consumption, the investor is indifferent to any partial change in any (nominal) exchange rate. The Cobb-Douglas assumption is obviously not the only alternative one might wish to investigate. Nevertheless, it is useful in that it provides reasonable benchmark results confirming our intuition, while exposing the assumption underpinning them.

3 International Purchasing Power Parity

International purchasing power parity (PPP) is the proposition that, once converted to a common currency, price levels should be equal across countries (in the simple example in the previous section, $ep_\$ = p_E$). In the context of tradable goods, one could imagine that arbitrage trades enforce the same price for identical goods across different locations (after adjustment for trade costs such as transportation costs and tariffs). More generally, one could expect that market competition precludes significant price differences between similar goods (e.g. European and American cars). For whichever reason, if there is a broad parity in prices across a sufficient range of individual goods, there should also be a high correlation in aggregate price levels, hence PPP holds to a good approximation.

The evidence for PPP may be stronger than many economists tend to believe. Most economists were influenced by the "PPP puzzle debate" in the 1990s. In a leading article, Rogoff (1996) pointed out that while large and volatile deviations from PPP in short-run were an incontestable fact – first and foremost due to the volatility of nominal exchange

rates – the empirical evidence to suggest that real exchange rates tended towards PPP in the long run was surprisingly weak. Moreover, the available estimates suggested that the speed of convergence was surprisingly slow.[13] The weak empirical evidence for PPP in the long run and the very slow estimated speed of convergence were deemed puzzles, because as Rogoff (1996) put it: "While few empirically literate economists take PPP seriously as a short-term proposition, most instinctively believe in some variant of purchasing power parity as an anchor for long-run real exchange rates." Nevertheless, with the empirical evidence suggesting that the best one could hope for was extremely slow mean reversion in real exchange rates (a necessary condition for PPP), disregarding real exchange rate risk on theoretical grounds seemed like wishful thinking. As Taylor and Taylor (2004) and Sarno (2008) show, however, recent advances in empirical methods have come a long way in solving these puzzles.

There are two key discoveries. First, the power of the statistical tests which were typically employed to examine the long-run stability of the real exchange rate may have low power to reject the null hypothesis of a unit root (no mean reversion) when the hypothesis is indeed false. Studies using longer time series (see e.g. Lothian and Taylor, 1996) or panels of several countries have systematically shown much stronger evidence of mean reversion. Second, exchange rate dynamics may be highly non-linear, and this could have severely biased earlier studies. The earlier models were based on a linear framework, i.e. the adjustment speed of PPP deviations from parity were assumed to be uniform for all sizes of deviations. There are good theoretical reasons for suspecting that the speed of convergence should be greater as the deviation from PPP rises. For instance, transaction costs in international arbitrage (transport costs, but also other non-pecuniary obstacles to trade) suggest that arbitrage trade is not profitable for small deviations from the law of one price. Consequently, there would be a "band of inaction" around PPP in which the rate of convergence would be very slow. But for large deviations, there would be much to gain from arbitrage trade, implying faster convergence. Estimates using non-linear models confirm these predictions: half-lives may be less than one year for the largest shocks of 40 per cent and just over two years for shocks of 5 to 10 per cent (see e.g. Taylor, Peel and Sarno, 2001). Moreover, non-linear models also strengthen the statistical evidence of mean reversion.

Finally, it is worth pointing out that most studies on PPP employ consumer price indices. Eq. (1) shows that in the context of the Fund, the relevant measure of the real exchange rate uses prices of tradable goods, not consumer goods. The relevant benchmark is therefore whether PPP holds for (a broad set of) tradable goods. A general impression from individual studies is that the evidence for PPP is even stronger in this

[13]Consensus estimates suggested deviations damped out at an annual rate of roughly 15 per cent, implying half-lives in the range of 3-5 years (see Rogoff, 1996).

case, presumably because international price convergence may not take place for non-tradable goods.[14] A recent review summarising the evidence for the "law of one price" in tradable goods (Sarno and Passari, 2011) confirms this impression.

In short, a review of the literature suggests that the evidence for PPP in the long run is considerably stronger today than what was commonly thought 10-15 years ago, in particular when it comes to tradable goods. Also, and it seems justified to expect large deviations from PPP to be significantly more short-lived than previously thought.

4 Implications for Investment Strategy

The evidence for PPP laid out in the previous section are of great significance in assessing the Fund's exchange rate risk:

- The investments are made with a very long time horizon. As exchange rates tend to return to PPP levels, currency risk will be relatively smaller for long-term investors.
- The fiscal policy guideline implies that Norway will be able to make withdrawals from the Fund every single year, forever (a permanent fund). This means that Norway is less vulnerable to exchange rates at a particular time in the future, and that the average level of exchange rates over longer periods will be more important for the Fund's total international purchasing power.[15] Exchange rates can be expected, on average, to be very close to PPP levels over the Fund's long time horizon.

The new evidence on PPP seems to reduce exchange rate risk in the way that matters the most for the Fund. On these grounds, a change in the Fund's geographical allocation seems warranted. Today, more than half of the Fund's capital is invested in Europe. An important purpose of the high European proportion has been to reduce the Fund's exchange rate risk. As the Fund's exchange rate risk is less than previously assumed, and relatively small in any event, there no longer appears to be a basis for such a strong concentration of the investments in Europe. Global securities markets and production capacity are to a greater extent located in other parts of the world – and increasingly so. By any of these standards, the proportion of the Fund invested in Europe should be reduced over time, in favour of greater proportions in the rest of world.

[14] One reason for this may be the so-called "Balassa-Samuelson" effect, whereby productivity improvements in the tradable goods' sector cause real appreciation of prices of non-tradable goods. Systematic differences in productivity growth between countries would then induce a drift in the equilibrium real exchange rate measured with consumer prices – but not in the real exchange rate measured with tradable goods' prices.

[15] In principle, one needs a dynamic model of consumption and investment to evaluate the importance of year-to-year exchange rate fluctuations, but such models easily become involved and are beyond the scope of this paper. As a first approximation, one can interpret the outcome of the variables in eq. (1) as the average of outcomes over a period of some length.

The Fund's current overweight in Europe can be thought of as a certain form of "Equity Home Bias", i.e. a investing more closer to home.[16] Home bias implies that the investor does not exploit the potential for international diversification. Reduced weight on Europe should increase the Fund's international diversification.

In fact, an argument can be made for investing more *farther* from home, in order to diversify national wealth. The investments in the Fund are becoming a significant part of Norway's national wealth, along with domestic real capital and domestic human capital. In principle, the various parts of the national wealth should be managed together, even though this is difficult to implement consistently in practice. In isolation, the consideration of the Fund's role in the Norwegian economy speaks for locating the investments in countries which, broadly speaking, are different from Norway. This may indicate that Norway should increase the proportion of investments in countries located far from Norway, which have a different business structure or a different level of development. An important question for further discussion is whether these considerations justify a significantly different geographical allocation than simple weighting principles such as market weights or GDP weights. A topical issue is the extent of the Fund's investments in emerging markets.

Even if PPP is valid over time, deviations from parity may be significant both in the short and medium term. For example, rapidly growing economies often experience a prolonged strengthening of the real exchange rate. Exchange rate risk is thus not zero. In principle, attempts could be made to separate the Fund's currency distribution from the Fund's geographical distribution by means of various forms of currency hedging strategy. In practice, however, it is unclear what currency exposure a portfolio of international securities has, and what currency exposure it is desirable for the Fund to have. Moreover, currency hedging means increased costs and counterparty risk, and will increase the operational complexity of the Fund. Currency hedging must be assessed by reference to its utility value. All in all, the case for expending resources on currency hedging seems weak for the Fund.

5 Exchange Rate Risk from Other Perspectives

The analysis so far has considered the national level. Strikingly, the exchange rate risk faced by individual agents (such as individual Norwegian citizens, households, or firms) or by the Norwegian state, will tend to be very different from the exchange rate risk faced by the nation as a whole. Since utility really ought to be defined at the individual level, it is appropriate to discuss these differences and the issues they raise.

[16] For a closer discussion of the the equity home bias, see Obstfeld and Rogoff (2000) and the references therein.

Clearly, the exchange rate risks for individual agents are very heterogeneous, both in terms of actual exposure (direct and indirect exposures to different currencies and at different horizons) and in terms of risk tolerance. Compare for instance the exchange rate risk of a salmon exporter with that of a car importer, or compare that of consumers with preferences for different goods (some prefer European, other Asian produce). While every Norwegian could be said to be entitled to a share of the Fund, the Fund's currency allocation could never cater to all these heterogenous needs simultaneously. Individual agents must be assumed to hedge their own exchange rate risks as they see fit.

The Fund is fully integrated with the state budget. Most of the expenditures on the budget are valued in Norwegian kroner (e.g. pension payments, health care costs, etc.). From a budget perspective, fluctuations in the international value of the Norwegian krone could induce significant exchange rate risk. This type of exchange rate risk is qualitatively different from the relevant exchange rate risk at the national level. Fluctuations in the international value of the Norwegian krone do not impact the international purchasing power of the Fund, and are therefore irrelevant at the national level. Nevertheless, a natural question is whether the exchange rate risk in the state budget could be avoided or moderated.

An attempt at hedging the Fund's value in Norwegian krone must be expected to have many of the same effects as investing the Fund domestically. If the State bought Norwegian kroner forward, the sellers of these contracts in the private sector would seek to unwind part of the exchange rate risk they entail.[17] This would increase the demand for assets denominated in Norwegian kroner. Given the size of the Fund relative to the Norwegian capital market, equilibrium could then presumably only be attained through a very low domestic interest rates and an expected krone depreciation (i.e. a strong Norwegian krone), in order to induce the private sector to hold foreign assets rather than Norwegian assets.[18] The Fund plays an important role in stabilising the Norwegian economy. By re-investing most of the foreign currency received from the sale of oil and gas, the Fund protects the domestic economy from large and volatile capital inflows, and underpins monetary policy. Hedging its value in terms of Norwegian kroner would prevent the Fund from performing this role, and is therefore not desirable.

Thus far, the exchange rate risk in the state budget seems to have been manageable for the government, in part thanks to a provision in the guideline saying that the impact

[17]The main part of Norway's future trade deficit will have to originate from the private sector, not from the public sector (which essentially uses domestic inputs such as labour). In principle, therefore, it would be appropriate that the private sector build up a large position in foreign currency, rather than the public sector as is currently the case with the Fund. When this is not the case, one must assume it is because agents in the private sector do not perceive this exchange rate risk as relevant at the invidual level, i.e. that they are already optimising.

[18]See e.g. the discussion in NOU 2004:1 *Modernisert folketrygd*, enclosure 2: *Konsekvenser av fondering av pensjoner for pengepolitikken*.

on fiscal policy of any large changes in the value of the Fund ought to be smoothed over time. But as the Fund continues to grow, eventually financing some 10-20 per cent of the total budget, shocks to the international value of the Norwegian krone may become more problematic. Presumably, an important element in dealing with this risk would be to accept larger year-to-year deviations from the 4 percent path given in the fiscal policy guideline, but this is only possible if the guideline is seen as a credible long-term anchor for fiscal policy.

6 Conclusions

This paper reviews exchange rate risk for the Fund from a national perspective, and in doing so, points to changes in the Fund's regional and currency allocation. In particular, the Fund's current concentration of investments in Europe does not seem warranted. Global securities markets and production capacity are to a greater extent located in other parts of the world – and increasingly so. Over time, the proportion of the Fund invested in Europe should be reduced in favour of greater proportions in the rest of world.

Reducing the proportion of investments in Europe would reduce the Fund's current "home bias." An argument can be made for investing more farther from home, in order to diversify national wealth. An important question for further discussion is whether such considerations justify a significantly different geographical allocation than simple weighting principles such as market weights or GDP weights. A topical issue is the extent of the Fund's investments in emerging markets.

References

[1] Bergstrand, Jeffrey H. 1985. "The Gravity Equation in International Trade: Some Microeconomic Foundations and Empirical Evidence." *The Review of Economics and Statistics*, 67 (3): 474-481.

[2] Finansdepartementet. 1996. *Nasjonalbudsjettet 1997.*

[3] Finansdepartementet. 2002. *Nasjonalbudsjettet 2003.*

[4] Lothian, J., and M. Taylor. 1996. "Real Exchange Rate Behaviour: the Recent Float from the Perspective of the Past Two Centuries." *Journal of Political Economy*, 104 (3): 488-510.

[5] Norges Bank. 2002. "En vurdering av Petroleumsfondets regionsvekter." Letter to the Norwegian Ministry of Finance 11 April 2002.

[6] Norges offentlige utredninger. 2004. *NOU 2004:1 Modernisert folketrygd.*

[7] Obsfeld, Maurice, and Kenneth Rogoff. 2000. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" In Bernanke, Ben, and Kenneth Rogoff. *NBER Macroeconomics Annual 2000*, 15: 339-390. The MIT Press.

[8] Rogoff, Kenneth. 1996. "The Purchasing Power Parity Puzzle." *Journal of Economic Literature*, 34 (3): 647-668.

[9] Sarno, Lucio. 2008. "Purchasing Power Parity." The New Palgrave Dictionary of Economics. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan. http://www.dictionaryofeconomics.com

[10] Sarno, Lucio, and Evgenia Passari. 2011. "Purchasing Power Parity in Tradable Goods." Report to the Norwegian Ministry of Finance.

[11] Solnik, Bruno, and Dennis McLeavey. 2008. *Global investments.* Pearson Education.

[12] Taylor, Mark P., David A. Peel, and Lucio Sarno. 2001. "Nonlinear Mean-Reversion in Real Exchange Rates: Towards a Solution to the Purchasing Power Parity Puzzle." *International Economic Review*, 42 (2): 1015-42.

[13] Taylor, Alan M., and Mark P. Taylor. 2004. "The Purchasing Power Parity Debate." *Journal of Economic Perspectives*, 18 (4): 135-158.

7 Appendix: Multi-Currency Consumption Basket

7.1 Set-up

Let $x_{i,j}$ and $p_{i,j}$ denote the quantity and price of good i,j, where $i = 1, 2, ..., I$ denotes the type of product (i.e. cars, or machinery), and $j = 1, 2, ..., J$ denotes country or currency area of origin (hereafter simply called country). Let z_j and q_j denote the quantity and price of financial assets in country j. Let e_j denote the exchange rate vis-à-vis country 1, i.e. e_j is the price of one unit of currency of country j in the currency of country 1.

Consider an investor with Cobb-Douglas preferences, denoted

$$U\big(\{x_{i,j}\}_{i=1,...,I,j=1,...,J}\big) = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{i,j}\, log(x_{i,j}), \tag{2}$$

where $\alpha_{i,j}$ are weights, i.e. $\sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{i,j} = 1$. We shall be concerned with this investor's consumption problem, which amounts to maximising eq. (2) subject to the budget constraint

$$\sum_{i=1}^{I}\sum_{j=1}^{J} e_j\, p_{i,j}\, x_{i,j} = \sum_{j=1}^{J} e_j\, q_j\, z_j, \tag{3}$$

which is formulated in terms of the currency of country 1.

I shall make use of a particular version of what can be called the "law of one price" (LOP), stating that goods of the same type i but with different origin j have the same price when converted into a common currency, i.e.

$$e_j\, p_{i,j} = p_{i,1},\ \forall i, j. \tag{4}$$

This "law" could be justified on grounds of pure arbitrage (identical products sold in different countries have same price when converted into a common currency), coupled with competition in every country (similar products with different origins have the same price in each country).

7.2 Consumption Allocation

The consumption problem is a concave program. Hence, the first-order conditions of the corresponding Lagrangian function are both necessary and sufficient for optimality. Using λ to denote the Lagrangian multiplier, the first-order conditions are

$$\frac{1}{\lambda} = \frac{1}{\alpha_{i,j}}\, e_j\, p_{i,j}\, x_{i,j},\ \forall i, j.$$

For any specific good of type i' and origin j', we thus obtain

$$\frac{e_j\, p_{i,j}\, x_{i,j}}{e_{j'}\, p_{i',j'}\, x_{i',j'}} = \frac{\alpha_{i,j}}{\alpha_{i',j'}},\ \forall i, j.$$

The expenditure share of any good of type i' originating from country j' is then $\alpha_{i',j'}$, since

$$\frac{\sum_{i=1}^{I}\sum_{j=1}^{J} e_j\, p_{i,j}\, x_{i,j}}{e_{j'} p_{i',j'} x_{i',j'}} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{i,j}}{\alpha_{i',j'}} = \frac{1}{\alpha_{i',j'}},$$

and the expenditure share of country j' is $\alpha_{j'} := \sum_{i=1}^{I}\alpha_{i,j'}$. In optimum, total expenditures equal total resources. Using A to denote the value of the international portfolio $\{z_j\}_{j=1,2,...,J}$ measured in terms of the currency of country 1 (i.e. the right-hand side of eq. (3)), the consumption allocation is therefore

$$x_{i,j} = \frac{\alpha_{i,j}}{e_j\, p_{i,j}}\, A,\ \forall i, j,$$

and so the investor's utility in optimum is

$$U = \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{i,j}\, log\Big(\frac{\alpha_{i,j}}{e_j\, p_{i,j}}\, A\Big). \tag{5}$$

7.3 Law of One Price Holds

The investor's utility is not directly measurable, while the return (or value) on the portfolio is. I am therefore interested in analysing how utility changes as a function of the return on the portfolio. In particular, the aim is to identify conditions under which the return also measures changes in utility. The LOP is essential in this respect. In fact, when the LOP holds, the return on the international portfolio measured in *any* currency measures (a monotonic transformation of) changes in the investor's utility. To see this, note that when the LOP holds, utility in optimum becomes $U = \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{i,j}\, log\Big(\frac{\alpha_{i,j}}{p_{i,1}}\, A\Big)$, whence

$$\begin{aligned} \frac{dU}{dA} &= \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{i,j}\,\frac{1}{A} \\ \Rightarrow \frac{dU}{\frac{dA}{A}} &= 1. \end{aligned} \tag{6}$$

Moreover, as the choice of numeraire currency (country 1) is arbitrary, this means that in which currency returns are measured is of no importance when the LOP holds (either base currency is equally good for measuring changes in the investor's utility).

7.4 Law of One Price Does Not Hold

When the LOP does not hold, a partial change in an exchange rate e_j has two effects:

1. It changes the value $e_j\, q_j\, z_j$ of the part of the portfolio which is in country j (measured in terms of the currency of country 1).

2. It changes the cost $\{e_j\, p_{i,j}\}_{i=1,...,I}$ of each good from country j (measured in terms of the currency of country 1).

Intuitively, when e.g. some country's exchange rate appreciates, the value of the portfolio in that country increases (which is good), but so does the cost of goods from that country (which is bad). Utility can either increase or decrease, depending on both portfolio shares as well as consumption shares. In fact, differentiating eq. (5) with respect to the exchange rate of a specific country j' yields

$$\begin{aligned}
\frac{\partial U}{\partial e_{j'}} &= \sum_{i=1}^{I}\sum_{j=1}^{j'-1}\sum_{j=j'+1}^{J} \alpha_{i,j}\,\frac{1}{A}\underbrace{\frac{\partial}{\partial e_{j'}}A(e_{j'})}_{=q_{j'}\,z_{j'}} + \sum_{i=1}^{I}\alpha_{i,j'}\frac{e_{j'}}{A}\underbrace{\frac{\partial}{\partial e_{j'}}\frac{A(e_{j'})}{e_{j'}}}_{=\frac{e_{j'}\,q_{j'}\,z_{j'}-A}{e_{j'}^2}} \\
&= (1-\alpha_{j'})\frac{q_{j'}\,z_{j'}}{A} + \alpha_{j'}\frac{q_{j'}\,z_{j'}}{A} - \alpha_{j'}\frac{1}{e_{j'}} \\
\Rightarrow \frac{\frac{\partial U}{\partial e_{j'}}}{e_{j'}} &= \underbrace{\frac{e_{j'}\,q_{j'}\,z_{j'}}{A}}_{\text{portfolio share}} - \underbrace{\alpha_{j'}}_{\text{expenditure share}}. \qquad (7)
\end{aligned}$$

Eq. (6) shows that when the LOP holds, the return on the international portfolio measured in the currency of country 1 is an equivalent measure of the change in investor utility. Eq. (7) shows that this does not hold in general, as

$$\frac{\partial A}{A} = \frac{e_{j'}\,q_{j'}\,z_{j'}}{A}\frac{\partial e_{j'}}{e_{j'}}$$

fails to take into account the impact of the exchange rate on the cost of goods from country j' (measured by $\alpha_{j'}$).

Eq. (7) can be interpreted as an immunisation result. If for country j' the portfolio share $\frac{e_{j'}\,q_{j'}\,z_{j'}}{A}$ equals the expenditure share $\alpha_{j'}$, then the investor's utility is unaffected by a (partial) change in that country's exchange rate $e_{j'}$. By equating portfolio shares to expenditure shares for all countries $j = 1, ..., J$, the investor becomes immune to all (partial) exchange rate changes.

7.5 Measuring Portfolio Returns in a Currency Basket

Let a currency basket be defined by fixed amounts y_j of the currencies of countries $j = 1, ..., J$. For example, the Special Drawing Right of the International Monetary Fund is a particular basket consisting of $y_1 = 0.4230$ euros, $y_2 = 12.1$ JPY, $y_3 = 0.111$ GBP, and $y_4 = 0.66$ USD. Let $B := \sum_{j=1}^{J} e_j \, y_j$ denote the value of the currency basket in terms of the currency of country 1, implying basket weights denoted $\gamma_j := \frac{e_j \, y_j}{B}$.[19]

Recalling the results in eqs. (6) and (7), I now aim to find a measure for portfolio returns which holds independently of the LOP. This turns out to be a currency basket with weights equal to the investor's expenditure share for each country $j = 1, ..., J$. To see this, first note that the value of the portfolio measured in terms of the currency basket is

$$\frac{A}{B} = \frac{\sum_{j=1}^{J} e_j \, q_j \, z_j}{\sum_{j=1}^{J} e_j \, y_j}.$$

The impact on the value of the portfolio measured in the basket from a change in the currency of a specific country j' is

$$\begin{aligned} \frac{\partial\left(\frac{A}{B}\right)}{\partial e_{j'}} &= \frac{\partial}{\partial e_{j'}} \frac{\sum_{j=1}^{J} e_j \, q_j \, z_j}{\sum_{j=1}^{J} e_j \, y_j} \\ &= \frac{q_{j'} \, z_{j'} \, B - A \, y_{j'}}{B^2} \\ \Rightarrow \frac{\frac{\partial\left(\frac{A}{B}\right)}{\frac{A}{B}}}{\frac{\partial e_{j'}}{e_{j'}}} &= \underbrace{\frac{e_{j'} \, q_{j'} \, z_{j'}}{A}}_{\text{portfolio share}} - \underbrace{\frac{e_{j'} \, y_{j'}}{B}}_{\text{basket share}} . \end{aligned} \tag{8}$$

Choosing currency amounts y_j so that $\gamma_j = \frac{e_j \, y_j}{B}$ equals α_j for every $j = 1, ..., J$, we can combine eq. (8) with eq. (7) to show that

$$\frac{\partial U}{\frac{\partial\left(\frac{A}{B}\right)}{\frac{A}{B}}} = \frac{\partial U}{\frac{\partial e_{j'}}{e_{j'}}} \left(\frac{\frac{\partial\left(\frac{A}{B}\right)}{\frac{A}{B}}}{\frac{\partial e_{j'}}{e_{j'}}} \right)^{-1} = 1, \tag{9}$$

which recalls the result shown in eq. (6).

[19] Keeping the currency amounts y_j fixed means that the weights γ_j depend on e_j. Of course, the currency amounts can be rebalanced in every period to bring the weights back to desired levels.

Working Paper Series

1/06	**Isachsen, Arne Jon** Greenspan går fra borde, og Erna vil ha Euro
2/06	**Beine, Michel, Paul De Grauwe and Marianna Grimaldi** The impact of FX Central Bank Intervention in a Noise Trading Framework
3/06	**Isachsen, Arne Jon** Canada Goose
4/06	**Isachsen, Arne Jon** Like før det smeller? Om globale ubalanser
5/06	**Qvigstad, Jan Fredrik** When does an interest rate path ”look good”? Criteria for an appropriate future interest rate path
6/06	**Isachsen, Arne Jon** Verdens rikeste land
7/06	**Steigum, Erling** Den Keynesianske revolusjonen 70 år etter: Et tilbakeblikk
8/06	**Isachsen, Arne Jon** Elleve i Kina – Ved én av dem
1/07	**Isachsen, Arne Jon** Globalisering
2/07	**Houg, Kjetil, Steinar Juel og Frank Jullum** Økonomiske paradokser
3/07	**Isachsen, Arne Jon** Folk på vandring
4/07	**Isachsen, Arne Jon** Hvor går verden? Noen utvalgte krefter og trender
5/07	**Isachsen, Arne Jon** Utenrikspolitiske refleksjoner
6/07	**Bottelier, Pieter and Gail Fosler** Can China’s growth trajectory be sustained?
7/07	**Isachsen, Arne Jon** Uro
1/08	**Houg, Kjetil** A note on the concept of risk
2/08	**Isachsen, Arne Jon** Bolig- og finanskrisen i Amerika
3/08	**Isachsen, Arne Jon** Kjøp og salg av risiko
4/08	**Husum, Hans Olav** Hvor kommer pengene fra? Kilder til meravkastning i Statens Pensjonsfond Utland

CME
Centre for Monetary Economics, Norwegian School of Management BI
Office Manager: Siv Bjercke
Address: Nydalsveien 37, N-0484 Oslo, Norway
Phone +47 46 41 07 91. Fax +47 46 41 08 01.
E-mail: cme@bi.no http://www.cme.no
ISSN 1503-3031

Working Paper Series

5/08	**Steigum, Erling** Befolkningsaldring, pensjonsreformer og realøkonomi
6/08	**Isachsen, Arne Jon** Om finanskrisen i USA
7/08	**Isachsen, Arne Jon** Kommer Amerika tilbake?
1/09	**Isachsen, Arne Jon and Ole C. Sand** China's revival
2/09	**Isachsen, Arne Jon** Finanskrisen – Den menneskelige faktor
3/09	**Chand, Sheetal K.** The IMF, the credit crunch and Iceland: A new fiscal saga?
4/09	**Liikanen, Erkki** Finland, EMU and the euro
5/09	**Isachsen, Arne Jon** Kina viser muskler
6/09	**Isachsen, Arne Jon** En dag med China Daily
7/09	**Isachsen, Arne Jon** Scener fra en finanskrise
1/10	**Isachsen, Arne Jon** Finanskrisen – Konsekvenser for USA
2/10	**Isachsen, Arne Jon** Korrupsjon i Kina – Kommet for å bli?
3/10	**Isachsen, Arne Jon** Om spekulasjon i valutamarkedet
4/10	**Steigum, Erling** Norsk økonomi etter 1980 – fra krise til suksess
5/10	**Isachsen, Arne Jon** Fortellinger om samfunnet
6/10	**Isachsen, Arne Jon** Hva Kina vil
7/10	**Isachsen, Arne Jon** Kinas makt
1/11	**Børsum, Øystein** Exchange Rate Risk in the Government Pension Fund Global

CME
Centre for Monetary Economics, Norwegian School of Management BI
Office Manager: Siv Bjercke
Address: Nydalsveien 37, N-0484 Oslo, Norway
Phone +47 46 41 07 91. Fax +47 46 41 08 01.
E-mail: cme@bi.no http://www.cme.no
ISSN 1503-3031

Norges Bank Watch Report Series

No 1/2000	**Hamilton, Carl , Øystein Thøgersen, Marianne Andreassen og Harald Magnus Andreassen** Norsk pengepolitikk, Norges Banks rolle og bankens gjennomføring av pengepolitikken i 1999 og første del av 2000
No 2/2001	**Andreassen, Harald Magnus, Paul De Grauwe, Haakon Solheim and Øystein Thøgersen** A review of inflation targeting, the Norwegian monetary regime and its institutional arrangements and Norges Bank's actual monetary policy and communication
No 3/2002	**Svensson, Lars E. O., Kjetil Houg, Haakon O. Aa. Solheim and Erling Steigum** An Independent Review of Monetary Policy and Institutions in Norway
No 4/2003	**Ekeli, Thomas, Anne Kari Haug, Kjetil Houg and Erling Steigum** An Independent Review of Monetary Policy in Norway.
No 5/2004	**Bjørnland, Hilde C., Thomas Ekeli, Petra M. Geraats and Kai Leitemo** An Independent Review of Monetary Policymaking in Norway.
No 6/2005	**Dørum, Øystein, Steinar Holden and Arne Jon Isachsen** An Independent Review of Monetary Policymaking in Norway.
No 7/2006	**Dørum, Øystein and Steinar Holden** An Independent Review of Monetary Policymaking in Norway.
No 8/2007	**Goodfriend Marvin, Mork Knut Anton and Söderström Ulf** An Independent Review of Monetary Policymaking in Norway.
No 9/2008	**Juel Steinar, Molnar Krisztina and Røed Knut** An Independent Review of Monetary Policymaking in Norway.
No 10/2009	**Bergman Michael, Juel Steinar and Steigum Erling** Monetary Policy and the Financial Turmoil.
No 11/2010	**Bjørnland Hilde C., Clarida Richard, Holvik Elisabeth and Steigum Erling** An Independent Evaluation of Monetary Policy in Norway.
No 12/2011	**Bjørnland Hilde C. and Wilhelmsen Bjørn Roger** An Independent Evaluation of Monetary Policy in Norway.

CME
Centre for Monetary Economics, Norwegian School of Management BI
Office Manager: Siv Bjercke
Address: Nydalsveien 37, N-0484 Oslo, Norway
Phone +47 46 41 07 91. Fax +47 46 41 08 01.
E-mail: cme@bi.no http://www.cme.no
ISSN 1503-3031