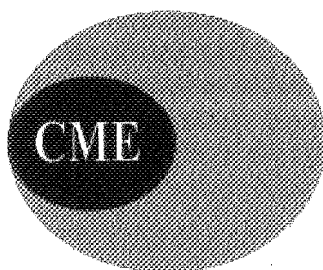


Working Paper Series 3/02

The Exchange Rate and its Fundamentals in a Complex World

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University of Leuven/CME
April 2002

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Centre for Monetary Economics

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1. Introduction

Since the start of the floating exchange rate regime in the early seventies, an increasing amount of empirical anomalies and puzzles have been uncovered that seem to contradict the existing exchange rate theories.¹

The first and foremost empirical puzzle has been called the “disconnect” puzzle (see Obstfeld and Rogoff(2000)), i.e. the exchange rate appears to be disconnected from its underlying fundamentals most of the time. It was first analysed by John Williamson who called it the ‘misalignment problem’. This puzzle was also implicit in the celebrated Meese and Rogoff studies of the early 1980s documenting that there is no stable relationship between exchange rate movements and the news in the fundamental variables. Goodhart (1989) and Goodhart and Figlioli (1991) found that most of the changes in the exchange rates occur when there is no observable news in the fundamental economic variables. This finding contradicted the theoretical models, which imply that the exchange rate can only move when there is news in the fundamentals.

Other empirical anomalies have been uncovered over the years. One is the puzzle of “excess volatility” of the exchange rate, i.e. the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables. Baxter and Stockman (1989) and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate no such increase could be detected in the volatility of the underlying economic variables. This contradicted the ‘news’ models that predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases.

A third puzzle relates to PPP and is closely related to the “disconnect puzzle”. Many researches have found that the deviations from PPP are large and sustained (Rogoff (1995), Obstfeld and Rogoff (2000), Cheung and Lai (2000)). The half-life of the PPP deviations has been estimated to be of the order of 4 to 5 years. Some researchers have found even longer half-lives (Lothian & Taylor (1998), Engel(2000), O’ Connell (1998)). Other researchers (Dumas (1992)) have stressed that the long time needed to adjust to PPP might be due to the existence of transaction costs. The transaction cost hypothesis implies a non-linearity in the adjustment process. This hypothesis has been confirmed by the empirical evidence based on time series analysis (Michael, Nobay, Peel (1997), Kilian & Taylor (2001)).

A fourth puzzle is that the distribution of the exchange rate returns does not follow a random walk. Most of the empirical findings document that the exchange rate returns have fat tails (see de Vries(2001), Lux T. (1997, 1998), Lux and Marchesi (1999, 2000)). This evidence is difficult to rationalise in existing exchange rate models, since there is little evidence of fat tails in the fundamental variables that drive the exchange rate in these models.

It is obvious that in the face of these empirical anomalies the existing exchange rate models that have been developed mostly in the 1970s need to be re-evaluated. The empirical failure of these exchange rate models has led to new attempts to model the exchange rate. These attempts have led to three different modelling approaches. The first one uses the Obstfeld–Rogoff(1996) framework of dynamic utility optimisation of a representative agent. The models that came out from this approach have a high content of intellectual excitement. However, they have not yet led to many testable propositions.

¹ We are grateful to Yin-Wong Cheung, Ronald MacDonald, Assaf Razin and Marc Flandreau for many interesting comments and suggestions. We also profited from comments made during seminars at the Nederlandsche Bank, the European Central Bank, and CESifo Munich.

A second approach starts from the analysis of the microstructure of the foreign exchange market with heterogeneous agents (see Evans and Lyons (1999), Lyons (2001)). This approach has led to new insights into the way information is aggregated and is important for the understanding of the very short-term behaviour of the exchange rate. As a result, this approach is capable of explaining some of the empirical anomalies, e.g. the excess volatility. However, it is as yet unclear whether it can solve the other empirical anomalies discussed here.

In this paper we use the insights of the microstructure literature recognising that heterogeneous agents have different beliefs about the behaviour of the exchange rate. The difference in the beliefs of agents introduces non-linear features in the dynamics of the exchange rate.² We develop a simple model of the exchange rate, which incorporates these non-linear features and we analyse their implications for the dynamics of the exchange rate. It will be shown that our simple non-linear model is capable of explaining the empirical puzzles about the exchange rate behaviour.

2. A Simple non-linear exchange rate model

In this section we briefly describe the model. In appendix we present the model in more formal detail.

The essential features of the model are the following. There are two kinds of traders, *fundamentalists* and *chartists*. The *fundamentalists* compare the market exchange rates with the fundamental rate and they forecast the future market rate to move towards the fundamental rate. In this sense they follow a *negative feedback* rule³. We make the assumption that they expect the speed with which the market rate returns to the fundamental rate to be determined by the speed of adjustment in the goods market.

As pointed out earlier, there is an increasing amount of empirical evidence indicating that the speed of adjustment in the goods market follows a non-linear dynamics, i.e. the speed with which prices adjust towards equilibrium depends positively on the size of the deviation from equilibrium. We assume that this adjustment process is quadratic in nature⁴. Fundamentalists take this non-linear dynamic adjustment into account in making their forecast.

Thus when the size of the deviation from equilibrium is large the fundamentalists expect a faster speed of adjustment towards the fundamental rate than when the size of the deviation is small. The economics behind this is that in order to profit from arbitrage opportunities in the goods market, some fixed investment must be made, e.g. trucks must be bought, planes be chartered, etc. These investments become profitable with sufficiently large deviations from the fundamental exchange rate.

The *chartists* are assumed to follow a *positive feedback* rule, i.e. they extrapolate past movements of the exchange rate into the future. The parameter of extrapolation will be seen to play an important role in the dynamics of the exchange rate.

Our choice to introduce chartists' rules of forecasting is based on empirical evidence. The evidence that Chartism is used widely to make forecasts is overwhelming (see Cheung and Chinn (1989), Taylor and Allen (1992)). Therefore, we give a prominent role to chartists in

² It should be noted that the heterogeneity of agents' expectations has been recognised as being important to explain the dynamics of asset prices, including the exchange rate (see De Long, et al. (1990), Hommes and Brock (1998), Lux and Marchesi (2000), Frankel and Froot (1986)).

³ Note that this is also the approach taken in the Dornbusch model.

⁴ See Kilian and Taylor (2001). See also De Grauwe and Grimaldi (2001) in which we showed that a quadratic specification fits the data rather well.

our model. It remains important, however, to check if the model is internally consistent. In particular, the chartists' forecasting rule must be shown to be profitable within the confines of the model. If these rules turn out to be unprofitable, they will not continue to be used. They can then also not be maintained in the model. We will therefore check whether these rules are indeed profitable.

A final feature of the model is the existence of transactions costs. We assume that transactions costs prevent goods arbitrage to function as long as the exchange rate remains within the transactions cost band. Fundamentalists take the existence of this transactions cost band when forecasting the exchange rate, i.e. when the exchange rate remains within the band they know that arbitrage does not operate. As a result, they do not expect to return to its equilibrium value.

3. The model with random shocks (news)

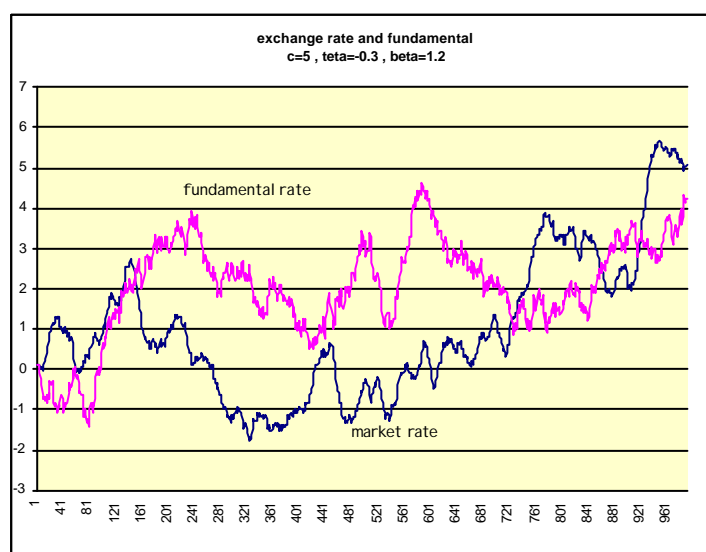
In this section we investigate the solution of the model when random shocks in the equilibrium exchange rate occur. The question we analyse in this section is how well our model mimics the empirical anomalies and puzzles identified in the introduction. We start with the 'disconnect puzzle' and we analyse how the market exchange rate behaves relative to the fundamental exchange rate in our model.

3.1 The disconnect puzzle

In figure 1 we show the two variables, for a particular combination of parameters. We show the results of a sensitivity analysis later.

We observe that the market rate can deviate from the fundamental value substantially and in a persistent way. Moreover, it appears that the exchange rate movements are often disconnected from the movements of the underlying fundamental. In fact, they often move in opposite directions. Thus, the model is able to generate an empirical regularity (the 'disconnect' puzzle) that has also been observed in reality⁵. We will return to this result later to analyse how sensitive it is to particular parameter values, like speed of adjustment and transaction costs.

Figure 1



⁵ See Obsteld and Rogoff (2000). See also De Grauwe (2000) for a survey of the empirical evidence. In De Grauwe and Vansteenkiste (2001) we present additional empirical evidence.

Closely related to the ‘disconnect’ puzzle is the empirical evidence provided by Meese and Rogoff (1983) in their celebrated empirical tests of exchange rate models. The Meese and Rogoff analysis implies that the existing exchange rate models do not forecast the frequent structural breaks in the link between the exchange rate and the fundamentals. We show that our model is capable of reproducing the Meese and Rogoff result. We implemented this idea by introducing a small change in the speed of adjustment in the goods market, θ i.e., this parameter was changed from -0.2 to -0.21 . All the other parameters were kept unchanged. We then simulated the exchange rate with the small error in θ , and compared this with the exchange rate obtained with the ‘true’ θ (-0.2). We show the result in Figure 2. These simulations can be interpreted as follows. The simulation using $\theta = -0.2$ generates the exchange rate produced by the true model. The second simulation can be considered to be the exchange rate, which is produced by a researcher who has estimated the model and made a small measurement error. This exchange rate can also be interpreted as the exchange rate forecast using the estimated model containing a small measurement error. We observe that the small error leads after some time to a very different time-path of the exchange rate, producing the appearance of large structural breaks. This result is related to the ‘sensitivity to initial conditions’ of non-linear models. It should be noted, however, that we obtain this result, even though we have a parameter combination that does not produce chaos in the deterministic part of the model.

In the next step we computed the one-period-ahead forecast error made by this researcher who uses her almost perfect estimate of the true model, and we compared this forecast error with the forecast error using the random walk. As measures of forecasts errors, we computed both the mean root squared error and the mean of the absolute deviations. The latter is a better measure of the forecast error when the exchange rate distribution is not gaussian but shows fat tails⁶.

We find that the random walk forecast outperforms the forecast made by a researcher who made a small measurement error estimating the “true” model⁷. We present some examples in table 1. The remarkable aspect of these results is that such a small measurement error is capable of producing large structural breaks and poor forecasting performance. It is therefore no surprise that when real-life econometrics is used to estimate models, these models perform so poorly in forecasting exercises when compared to the random walk forecast.

⁶ This approach was also followed by Meese and Rogoff (1983).

⁷ Note that, as in Meese and Rogoff, the researcher using a model to make forecasts has more information available than the random walk forecaster. The former has perfect knowledge of the one-period-ahead fundamental while the latter does not have that information.

Figure 2

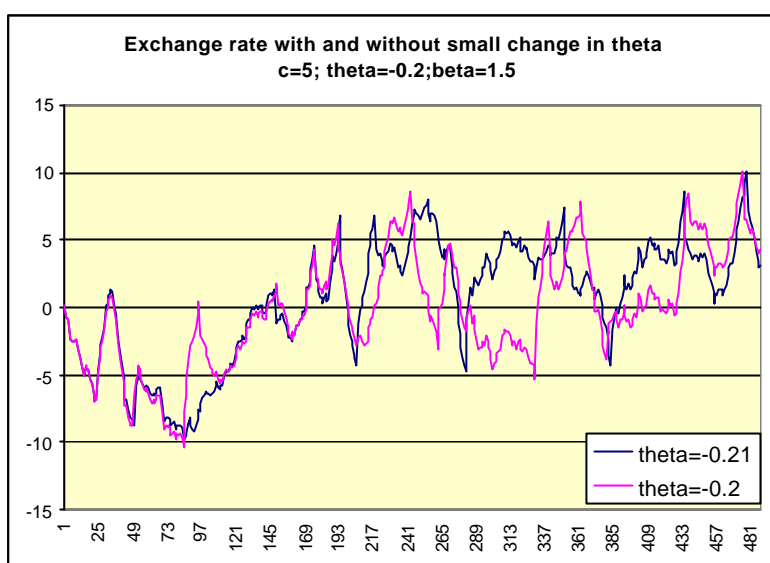


Table 1: Measures of forecast errors using an ‘econometric’ model and the random walk: 1-period ahead forecasts

	Root mean squared errors		Mean absolute deviation	
	<i>model</i>	<i>random walk</i>	<i>model</i>	<i>random walk</i>
Sample 1-1000	3.17	0.78	1.57	0.75
Sample 1001-2000	2.31	0.81	1.29	0.77
Sample 2001-3000	2.78	0.80	1.50	0.76

Another interesting aspect of these results is that the poor performance of model-based forecasts is not due to the fact that there are regime switches. Economists have been tempted to interpret the Meese and Rogoff results as reflecting changes in regime that lead to parameter instability. The problem with that explanation is that one has to assume too many regime switches to explain the parameter instability. There are just not enough regime switches around. Our model shows that one can have the appearance of frequent structural changes without regime switches. These structural changes arise from the non-linearities in the model.

We extended the analysis for n-period ahead forecasts. Meese and Rogoff found that when forecasts are made over longer periods, the forecasts based on econometric models become more useful. We find the same result in our model. We computed the 5-period and 10-period ahead forecasts using the ‘econometric model’ and using the random walk. The results are presented in table 2. We observe that when the forecasting horizon increases to 10 periods the model-forecasting becomes better than the random walk. It should be stressed, however, that as in Meese and Rogoff we load the dice in favour of the model forecaster by assuming that he has perfect knowledge of the fundamental variable ten-periods in the future. No such strong informational advantage is assumed when using the random walk forecast.

Table 2: Measures of forecast errors using an ‘econometric’ model and the random walk: 5-periods and 10-periods ahead forecasts

	Root mean squared errors		Mean absolute deviation	
	model	random walk	model	random walk
<i>5-period ahead forecasts</i>				
Sample 1-1000	3.17	2.02	1.57	1.28
Sample 1001-2000	2.31	2.18	1.29	1.31
Sample 2001-3000	2.78	2.10	1.50	1.29
<i>10-period ahead forecasts</i>				
Sample 1-1000	3.18	3.48	1.57	1.68
Sample 1001-2000	2.31	3.66	1.29	1.71
Sample 2001-3000	2.78	3.63	1.50	1.71

3.2. *The disconnect puzzle: sensitivity analysis*

In this section we investigate the sensitivity of the results relating to the ‘disconnect puzzle’ to changes in the value of the *transaction costs band*. In particular, in the next figure 3 we compare the movements of the market exchange rate under two assumptions about *transaction costs*. In panel a we assume that transaction costs are zero and in panel b we assume them to be high, i.e. equal to 5.

Figure 3a

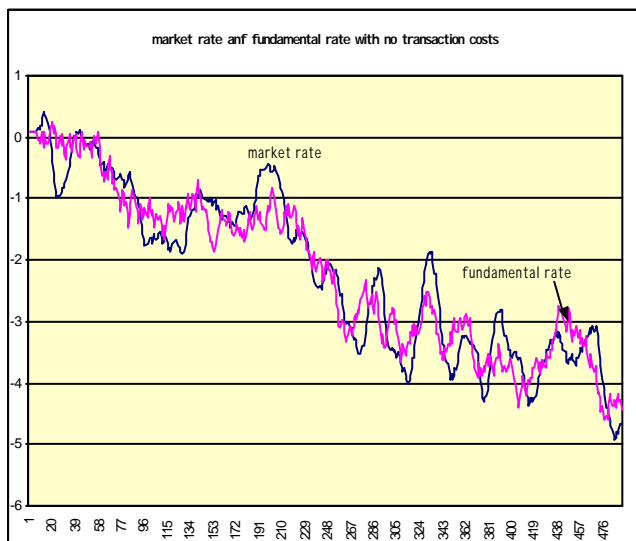
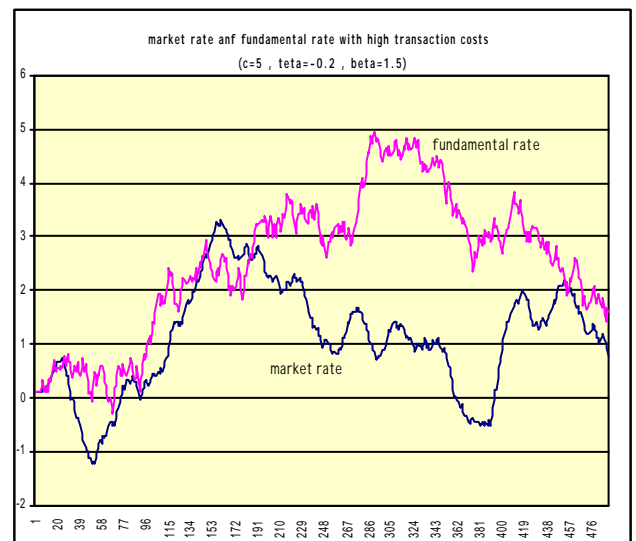


Figure 3b



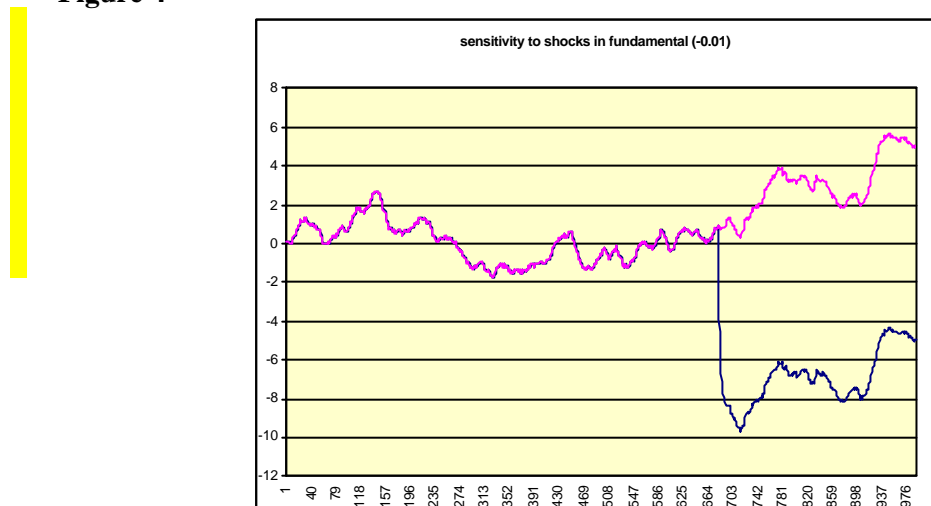
The contrast between the two panels is striking. When transaction costs are zero, the market exchange rate does not deviate substantially for a long period of time from its fundamental

value. In contrast, when transaction costs are present the deviations of the market exchange rate from its equilibrium value are large and persistent. Thus, transactions costs in the goods markets are important in explaining the disconnect puzzle (see Obstfeld&Rogoff (2001)).

Transaction costs have also other important implications for the dynamics of the exchange rate. We show this in figure 6, where we introduce a negative and permanent shock (-0.01) in the fundamental exchange rate. Thus, over time the “new” fundamental exchange rate progressively but slowly departs from the “old” one.

Figure 4 allows us to see how this accumulating small change in the equilibrium value of the exchange rate, which occurs in period 1, leads to a large jump in the exchange rate many periods later. This change has the appearance of a regime shift in spite of the fact that the change in the fundamental exchange rate is very small and continuous. This feature is much related to the existence of transaction costs, which implies that the effect of the accumulated changes in the fundamental exchange rate will be visible only when it will overcome the transaction costs band.

Figure 4



3.3 “Excess volatility”

The model discussed in the previous sections is driven by exogenous news in the fundamentals and by the noise produced by the non-linear speculative dynamics embedded in the model. As a result, the non-linear dynamics is capable of producing “excess volatility” in the exchange rate, i.e. volatility that exceeds the volatility of the underlying fundamental. In this section we analyse the sources of this excess volatility. We do this by computing the noise to signal ratio in the simulated exchange rate. It gives a measure of how large the noise produced by the non-linear dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise to signal ratio for different values of the parameters of the model. We show the results in figures 5-7. We observe that the noise to signal ratio is very sensitive to the extrapolation parameter of the chartists and to a lesser degree to the transactions costs. We observe that the noise produced by the non-linear dynamics can become very large relative to the volatility of the fundamental exchange rate when the extrapolation parameter is large and when transactions costs are large. The noise to signal ratio is less sensitive to the speed of adjustment in the goods market. Overall our model is capable of generating volatility of the exchange rate that is much in excess of the volatility of the underlying fundamental (Goodhart (1989) and Goodhart and Figlioli (1991)).

Figure 5

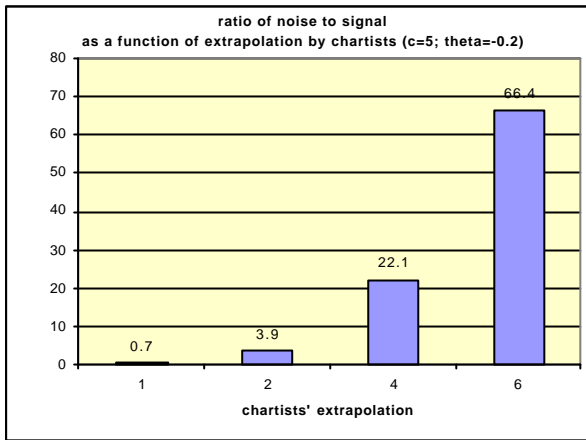


Figure 6

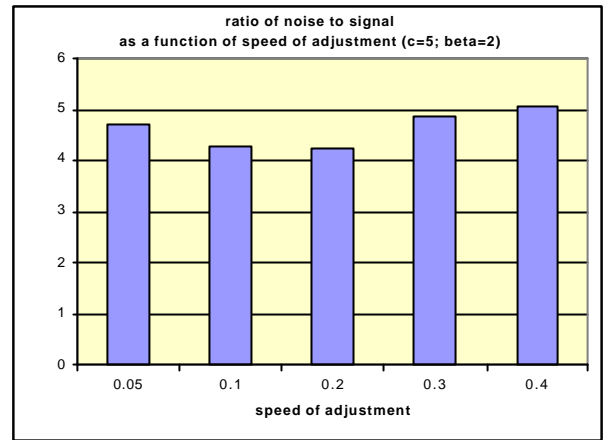
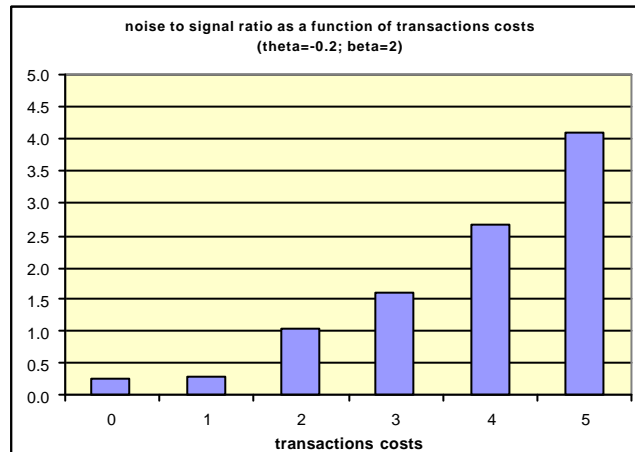


Figure 7



3.4 Fat tails

It is well known that the exchange rate changes do not follow a normal distribution. Instead it has been observed that the distribution of exchange rate changes has more density around the mean than the normal and exhibits fatter tails than the normal (see de Vries (2001)). Put differently, the exchange rate returns typically have a kurtosis exceeding 3 and a measure of fat tails (Hill index) ranging typically between 2 and 5. It implies that most of the time the exchange rate movements are relatively small but that occasionally periods of turbulence occur with relatively large exchange rate changes. However, it has been also detected that the kurtosis is reduced under time aggregation. This phenomenon has been observed for most exchange rates. We checked whether this is also the case with the simulated exchange rate changes in our model.

The model was simulated using normally distributed random disturbances (with mean = 0 and standard deviation = 1). We computed the kurtosis and the Hill index of the simulated exchange rate returns. We computed the Hill index for 5 different samples of 2000 observations. In addition, we considered three different cut-off points of the tails (2.5%, 5%, 10%). We show the results of the kurtosis in table 3 and of the Hill index in table 4. We find that for a broad range of parameter values the kurtosis exceeds 3 and the Hill index indicates the presence of fat tails. This suggests that the non-linear dynamics of the model transforms

normally distributed noise in the exchange rate into exchange rate movements with tails that are significantly fatter than the normal distribution and with more density around the mean. Thus our model mimics an important empirical regularity, i.e. that exchange rate movements are characterised by tranquil periods (occurring most of the time) and turbulent periods (occurring infrequently).

Table 3: Kurtosis index

Kurtosis of simulated exchange rate returns					
theta beta	-0.05	-0.1	-0.2	-0.3	-0.4
0.5	2.99	3.21	8.48	8.58	11.84
1	2.98	3.09	2.14	15.17	9.66
1.5	2.92	2.96	3.78	9.27	7.03
2	2.85	2.88	3.31	4.84	6.05
2.5	2.11	2.64	2.8	3.86	5.68
3	1.44	1.85	2.49	3.54	5.64

Table 4 : Measure of fat tails : the Hill index

Parameter values	Kurtosis	Median Hill index (5 samples 2000 observations)		
		2.5% tail	5% tail	10%tail
c=5, beta=0.5, theta=-0.2	8.48	2.42 (2.10 – 2.61)	2.66 (2.50 – 3.15)	2.93 (2.89 – 3.83)
c=5, beta=1, theta=-0.2	8.1	2.14 (2.08 – 2.37)	2.35 (2.21 – 2.45)	2.65 (2.57 – 2.73)
c=5, beta=1, theta=-0.3	15.17	1.54 (1.34 – 3.89)	1.62 (1.46 – 2.00)	2.14 (1.90 – 2.55)
c=5 , beta=1.5 theta=-0.3	9.27	7.36 (5.57 – 7.67)	1.82 (1.58 – 3.13)	1.70 (1.62 – 1.80)

4. Small and large shocks and the dynamics of the exchange rate

In linear models the size of the shocks does not affect the nature of the dynamics. In non-linear models things are different. The size of the shocks matters. This is also the case in our exchange rate model. In order to illustrate this, we simulated the model under two different assumptions about the variance of the shocks in the fundamental exchange rate. In the first case we assume low variance of these shocks, in the second case we assume a high variance (ten times higher). The results of our simulations are presented in figures 8-9. (The simulations shown here are representative for a wide range of parameter values).

Figure 8: low variance of equilibrium exchange rate

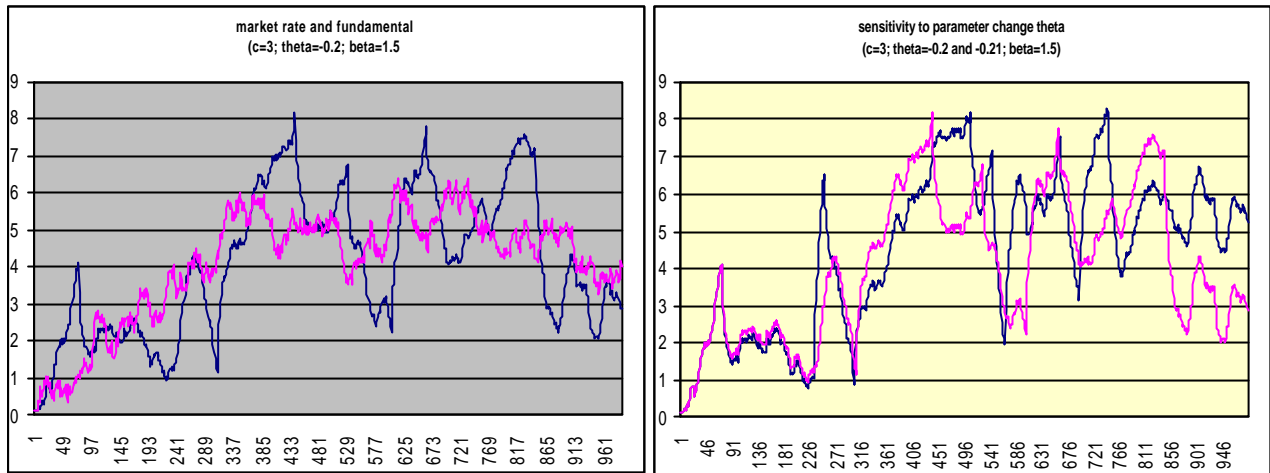
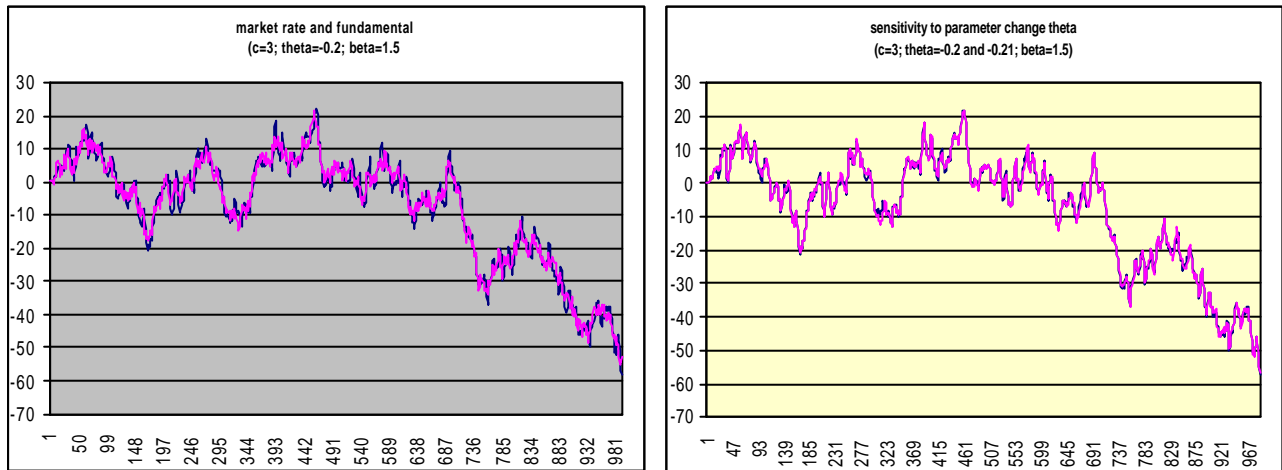


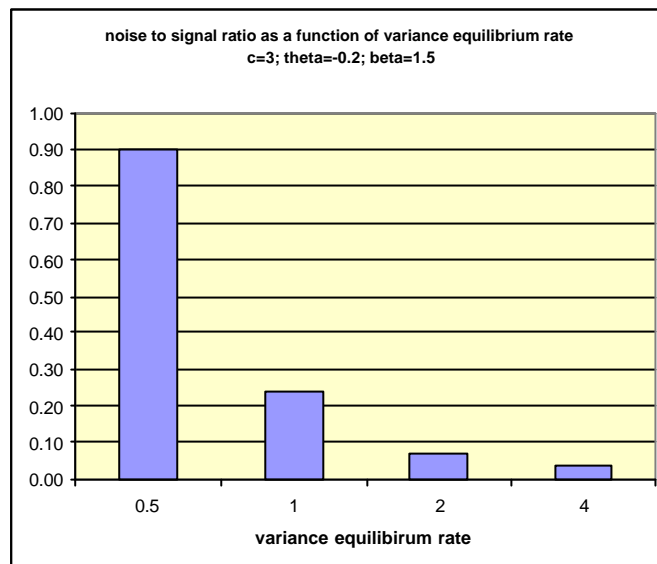
Figure 9: high variance of equilibrium exchange rate



Two conclusions follow from a comparison of the low and high variance cases. First, in the low variance case we observe sustained deviations from the equilibrium exchange rate; this is not the case when the equilibrium exchange rate is subject to large shocks (compare left-hand panels of figure 8 and 9). Second, the sensitivity to small changes in parameters is clearly visible when the variance of the exchange rate is low (see right-hand panel of figure 8). When this variance is high, no such sensitivity can be observed (right-hand panel of figure 9). It is important to stress that the transactions cost band is the same in both cases. Thus, when the shocks are small relative to the given band of transactions costs the movements of the exchange rate show more complexity than when the shocks are large.

This feature is also evident from a comparison of the noise to signal ratio for different variances of the fundamental exchange rate. We show this in figure 10. We observe that when the variance of the equilibrium exchange rate is low, a large part of the volatility of the exchange rate is produced by the noise from the non-linear dynamics. For high variance the noise is very small, implying that the exchange rate follows the fundamental rate very closely⁸.

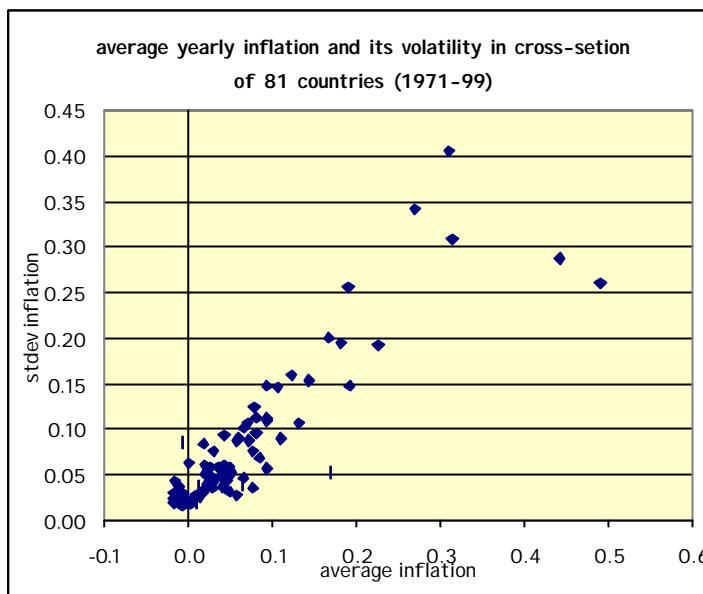
Figure 10



The intuition of this result is that when the fundamental shocks are small the exchange rate regularly switches from the dynamics inherent in the band to the one prevalent outside the band. This non-linearity produces a lot of noise and complexity in the dynamics of the exchange rate. When the shocks are large relative to transactions cost band the dynamics outside the band mostly prevails, leading to a tighter link between the exchange rate and the fundamental. This feature has also been found to hold empirically. In particular, it has been found that the PPP-relationship holds much tighter in high inflation countries than in low inflation countries (See De Grauwe and Grimaldi (2001)). Put differently, in high inflation countries the link between the exchange rate and one of its most important fundamentals is tighter than in low inflation countries. It may be argued that this is not yet evidence for the model because the latter predicts that it is a high *variance* in the fundamental that produces the tight link. However, there is also strong empirical evidence that the variance of inflation increases with its level (see Okun (1971) Fischer (1982)). This strong correlation between the level and the variance of inflation is illustrated in figure 11, which shows the average rate of inflation and its standard deviation in a cross-section of 81 countries during 1971-1999.

⁸It should be stressed that the total variability of the exchange rate in the high variance scenario is much larger than the total variability of the exchange rate in the low variance scenario. The point is that in the high variance scenario almost all of the variability of the exchange rate is explained by the (much higher) variability of the fundamental. This is not the case in the low variance scenario where a large part of the variability of the exchange rate cannot be related to the variability of the underlying fundamental.

Figure 11

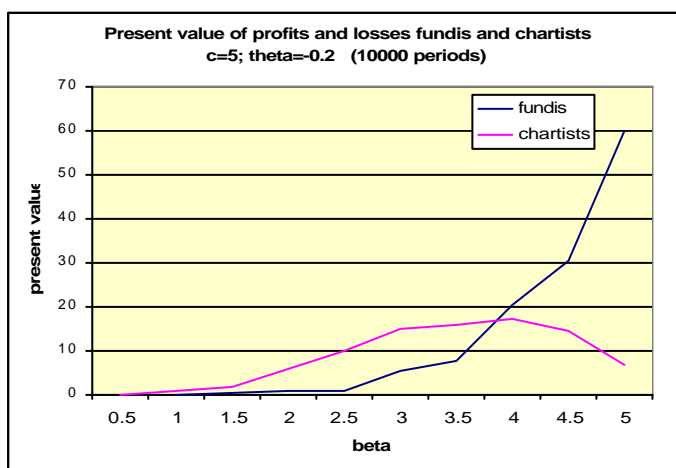


5. Is chartism profitable?

In this section we analyse how profitable forecasting based on chartism is in relation with fundamentalism. This analysis is important because particular forecasting rules will only survive if they are profitable. If chartism turns out to be unprofitable, fewer and fewer agents will use this technique, and it will disappear. In that case chartists should have no role in our model.

In order to analyse this issue we simulated the model and asked the question how the profit and loss accounts of chartists and fundamentalists evolve over time. We assumed that each of them started with an initial capital of €1. When they expect the exchange rate to increase (decrease) they buy (sell), and hold for one period. They repeat this operation each period. We calculated the net present value of these profits and losses using a discount rate of 4%. Results are shown in figure 12 where the present value of profits and losses are related to different values of beta.

Figure 12



We observe the following. First, the cumulative profits of both chartists and fundamentalists are positive.⁹ Second, for values of beta lower than 4 the chartists make profits higher than the fundamentalists. However, for high values of beta the chartists' rule loses its profitability and the fundamentalists' rule becomes much more profitable. This implies that we are unlikely to observe chartists to use large extrapolation parameter values in their forecasting.

The next step was to analyse profits and losses under two different stochastic regimes. The first one has a low variance of noise (same as in previous simulations). The second regime has a variance ten times higher. Results are shown in figures 13 and 14. We see that chartism becomes less profitable in a regime of high variance, while fundamentalism then becomes more profitable. It is worthwhile to note that this result is consistent with the results obtained in the previous section, where we showed that in a high variance regime the link between fundamentals and the market exchange rate is tighter than in the low variance regime. Thus, it is not surprising that in a high variance regime the fundamentalists' forecasting rule is relatively profitable. This result also implies that we should observe more chartists in the low variance currency markets than in the high variance markets. We leave it for further research to verify the empirical validity of this hypothesis¹⁰.

Figure 13

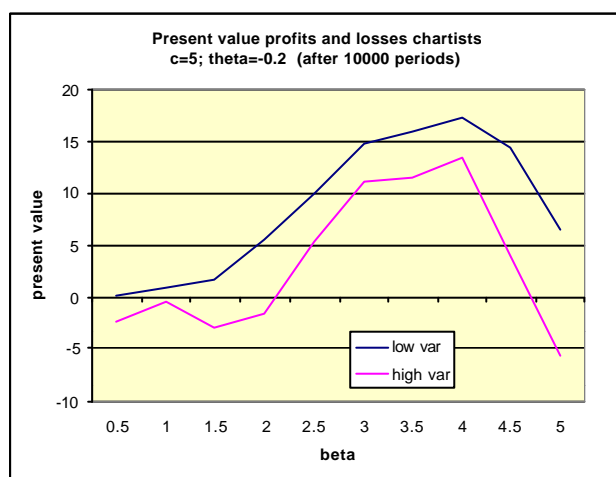
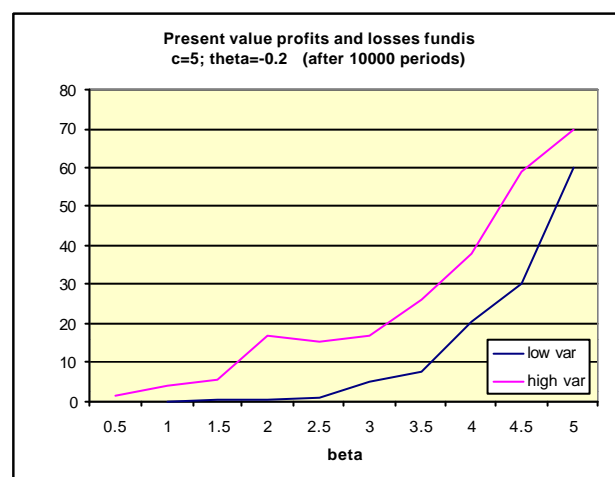


Figure 14



6. Conclusion

In this paper we analysed the workings of a simple non-linear exchange rate model in which agents hold different beliefs about the underlying model. We distinguished between chartists and fundamentalists, where the chartists apply a *positive feedback rule* and the fundamentalists a *negative feedback rule*. The non-linearities in the model originate from transactions costs and from the existence of non-linear adjustment dynamics in the goods market.

Our main results can be summarised as follows. First, the simple non-linear structure of the model is capable of generating a very complex exchange rate dynamics. This implies that

⁹ We assume implicitly that the total gains (losses) of these forecasters must have a counterpart loss (gain) borne by other agents (e.g. exporters and importers) outside the model. We assume that the gains and losses of the forecasters are small in a macroeconomic sense so that spread over many agents in the rest of the economy they do not affect the latter's behaviour.

¹⁰ Recently Cheung, et al (2002) found such evidence.

small shocks in the equilibrium exchange rate lead to very different time-paths of the exchange rate.

Second, our model is capable of explaining most empirical puzzles associated with exchange rate movements. The first puzzle is that the market exchange rate can deviate substantially and for relatively long periods of time from its fundamental value (“disconnect puzzle”). We showed that such disconnections are a natural outcome of the non-linear dynamics in our simple model. There is no need to invoke exogenous events and special factors to explain why exchange rates deviate from their fundamental values. It should also be noted that our model generates these disconnections even in the absence of deterministic chaos. In other words we do not need to invoke chaos to explain disconnections.

Another empirical puzzle observed in exchange rate economics is the frequent occurrence of “regime shifts”, i.e. structural breaks in the relation between the exchange rate and the fundamentals. This phenomenon has first been noted in the celebrated studies of Meese and Rogoff (1982)¹¹. It is now customary to explain these structural breaks by changes in the policy regime. Our model provides an alternative explanation. The non-linear dynamics embedded in the model produces endogenous regime shifts that change the link between the exchange rate and its fundamentals. These structural breaks can be triggered by very small changes in parameters, or by small errors in the estimates of these parameters by agents who forecast the future exchange rate. Thus, in a non-linear world, structural breaks in the link between the exchange rate and its fundamentals occur naturally even when no changes occur in the policy regime.

Third, we found that our simple non-linear dynamic model can generate “excess volatility” in the exchange rate. The size of this excess volatility crucially depends on the degree of extrapolation applied by chartists and on the size of the transactions cost band.

Fourth, the model is also capable of generating another empirical regularity, i.e. the existence of fat tails in the distribution of the exchange rate returns. Thus the model mimicks an empirical phenomenon that has been widely observed in exchange rate economics, i.e. that tranquil periods (which occur most of the time) alternate with turbulent periods (which occur infrequently).

Fifth, we found that the size of shocks to the underlying fundamental exchange rate matters for the dynamics of the exchange rate. More specifically, we found that when these shocks are small relative to the size of the transactions cost band, the phenomena just described will tend to be prevalent. That is, in regimes of low shocks relative to the transactions cost band, the exchange rate movements are complex, and can even be chaotic. In such a regime exchange rates deviate substantially from the underlying fundamentals and frequent structural breaks in the link between the fundamentals and the exchange rate are observed. The latter occur in the absence of changes in the policy regime.

Finally, we checked whether the forecasting rules used by chartists and fundamentalists are profitable. We found that for a broad range of parameter values both rules are profitable.

Some implications of these findings are the following. The exchange rates of the major currencies are subject to relatively small shocks in the underlying fundamentals (e.g. inflation differentials are almost zero). Compared to these shocks the transactions costs can be said to be relatively large (see Obstfeld & Rogoff (2000) on this), i.e. a large part of goods and services are non-traded (or difficult to trade) because the cost of shipping them across borders is quite high. Thus the regime confronted by the exchange rates of the major industrialised countries comes close to the regime we have identified to be the one producing complexity,

¹¹ For more recent evidence see De Grauwe and Vansteenkiste (2001).

speculative noise, and structural breaks between exchange rates and underlying fundamentals. Put differently, the movements of the exchange rates of the industrialised countries are likely to be clouded by a non-linear speculative dynamics that makes it difficult if not impossible to explain this or that movement of these exchange rates. In contrast the exchange rates of high inflation countries experience large shocks in the fundamentals. As a result, the movements of the exchange rates of these countries can be explained much better by movements in underlying fundamentals (e.g. inflation differentials).

The results of our paper make it easier to understand why it will remain difficult, if not impossible to find (fundamental) logic in the movements of the exchange rates of major currencies that are subject to relatively low nominal disturbances. However, our inability to understand why, say, the dollar moved up against the euro during the 1999-2000 does not prevent analysts from developing exotic theories explaining these movements. Probably this has to do with the fact that the human mind abhors the emptiness created by its inability to understand. It is no surprise therefore that new explanations based on fundamentals are created, and will continue to be created for each and every new turn of the exchange rate.

Appendix : The theoretical model

In this appendix we present the exchange rate model that was used to perform the simulations reported in this paper. We start with a well-known model of the exchange rate, which is often used in the literature. We then introduce heterogeneous agents who use this model as a benchmark to define their beliefs about the future exchange rate. We start from the determination of the exchange rate as follows:

$$s_t = f_t + \mathbf{a}[E_t s_{t+1} - s_t] \quad (1)$$

where f_t represents the fundamentals in period t , s_t is the exchange rate in period t , s_{t+1} is the exchange rate in period $t+1$, E is the expectations operator. Underlying the fundamental one could specify a whole model of the economy, e.g. a monetary model, or a more elaborate one like the Obstfeld-Rogoff new open economy macro model (Obstfeld and Rogoff(1996)). We leave this for further research. Here we concentrate on the simplest possible exchange rate modelling. For the sake of simplicity, we assume that the fundamentals are determined exogenously¹².

Equation (1) can be rewritten as follows:

$$s_t = \frac{1}{1+\mathbf{a}} f_t + \frac{\mathbf{a}}{1+\mathbf{a}} [E_t s_{t+1}] \quad (2)$$

We use this model to define the fundamental equilibrium exchange rate. This is the rational expectations solution of equation (2). It will be used as a benchmark against which the beliefs of different agents are measured.

In the absence of bubbles the fundamental solution to (2) is given by

$$s_t^* = \frac{1}{1+\mathbf{a}} \sum_{i=0}^{\infty} \left(\frac{\mathbf{a}}{1+\mathbf{a}} \right)^i E_t f_{t+i} \quad (3)$$

For the sake of simplicity we will assume that f_t follows a random walk process without drift. We then find the following fundamental solution of the exchange rate :

$$s_t^* = f_t \quad (4)$$

We now introduce the assumption that the agents have heterogeneous beliefs and we classify them according to their beliefs. Let us assume that there are N_h individuals of type h belief (where $\mathbf{S}N_h = N$). We can then characterize the beliefs of type h agents as follows¹³:

$$E_{h,t} s_{t+1} = s_t^* + g_h(s_{t-1}, s_{t-2}, \dots) \quad (5)$$

where $E_{h,t}$ represents the expectations operator of type h agent at time t . Thus agents' beliefs can be classified depending on how they view the process by which the market price will grope towards the fundamental exchange rate s_t^* . They all use information on past exchange rates to forecast these future developments.

¹² This means that we assume implicitly that there is no feedback from the exchange rate to the goods market. We leave it for further research to explore this possibility.

¹³ See Brock and Hommes (1998) for such a formulation

The market expectation can then be written as follows:

$$E_t s_{t+1} = \sum_{h=1}^H n_h E_{h,t} [s_{t+1}] = s_t^* + \sum_{h=1}^H n_h g_h(s_{t-1}, s_{t-2}, \dots) \quad (6)$$

Note that $n_h = N_h/N$, so that n_h can be interpreted as the weight of agents of type h in the market.

The realised market rate in period $t+1$ equals the market forecast made at time t plus some white noise error (i.e. the news that could not be predicted at time t):

$$s_{t+1} = s_t^* + \sum_{h=1}^H n_h g_h(s_{t-1}, s_{t-2}, \dots) + \mathbf{e}_{t+1} \quad (7)$$

In the previous discussion the nature of the beliefs of agents was specified in very general terms. We further simplify the model by assuming that there are only two types of agents in the foreign exchange market, which we will call *fundamentalists* and *chartists*¹⁴.

The *fundamentalists* base their forecasts on a rule like in equation (5), i.e. they compare the past market exchange rates with the fundamental rate and they forecast the future market rate to move towards the fundamental rate. In this sense they follow a *negative feedback* rule¹⁵. We will make the additional assumption that they expect the speed with which the market rate returns to the fundamental rate to be determined by the speed of adjustment in the goods market.

As pointed out earlier, there is an increasing amount of empirical evidence indicating that the speed of adjustment in the goods market follows a non-linear dynamics, i.e. the speed with which prices adjust towards equilibrium depends positively on the size of the deviation from equilibrium. We will assume that this adjustment process is quadratic in nature¹⁶. Fundamentalists take this non-linear dynamic adjustment into account in making their forecast. This leads us to specify the following rule for the fundamentalists:

$$E_{f,t}(\Delta s_{t+1}) = -\mathbf{y}(s_{t-1} - s_{t-1}^*) \quad (8)$$

where $E_{f,t}$ is the forecast made in period t by fundamentalists and \mathbf{y} is a function of the size of the deviation from the fundamental variable. We assume the following simple specification

$$\mathbf{y} = \mathbf{q} \left| s_{t-1} - s_{t-1}^* \right|$$

where $\mathbf{q} > 0$

Thus when the size of the deviation from equilibrium is large the fundamentalists expect a faster speed of adjustment towards the fundamental rate than when the size of the deviation is small. The economics behind this non-linear specification is that in order to profit from

¹⁴ This way of modelling the foreign exchange market was first proposed by Frankel and Froot (1988). It was further extended by De Long et al. (1990) and De Grauwe et al. (1993) and more recently Kilian and Taylor (2001). For evidence about the use of chartism see Taylor and Allen (1992).

¹⁵ Note that this is also the approach taken in the Dornbusch model.

¹⁶ See Kilian and Taylor (2001). See also De Grauwe and Grimaldi (2001) in which we showed that a quadratic specification fits the data rather well.

arbitrage opportunities in the goods market, some fixed investment must be made, e.g. trucks must be bought, planes be chartered, etc. These investments become profitable with sufficiently large deviations from the fundamental exchange rate. Note that we do not model the goods market explicitly but we assume that in order to form their expectations about the exchange rate, the fundamentalists take into account the dynamics of the goods market and the speed of adjustment of goods prices.

The *chartists* are assumed to follow a *positive feedback* rule, i.e. they extrapolate past movements of the exchange rate into the future. Their forecast is written as:

$$E_{c,t}(\Delta s_{t+1}) = \mathbf{b} \sum_{i=1}^T \mathbf{a}_i \Delta s_{t-i} \quad (9)$$

where $E_{c,t}$ is the forecast made by chartists using information up to time t ; Δs_t is the change in exchange rate.

As can be seen, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β . Note that in contrast to the general rule as given by equation (5) (and also in contrast to fundamentalists) they do not take into account information concerning the fundamental exchange rate. In this sense they can be considered to be pure *noise traders*¹⁷. It is important to check if the model is internally consistent. In particular, the chartists' forecasting rule must be shown to be profitable within the confines of the model. If these rules turn out to be unprofitable, they will not continue to be used. They can then also not be maintained in the model. We will therefore check whether these rules are indeed profitable. In a similar logic as in equation (7) the market exchange rate can now be written as

$$\Delta s_{t+1} = -n_{fi} \mathbf{q} \left(s_{t-1} - s_{t-1}^* \right) \left(s_{t-1} - s_{t-1}^* \right) + n_{ct} \mathbf{b} \sum_{i=1}^T \mathbf{a}_i \Delta s_{t-i} + \mathbf{e}_{t+1}$$

In the simulations of the model we assume that the weights n_{fi} and n_{ct} are constant. We set them equal to 0.5, not because we think this is realistic but to see how far the simplest possible model goes in explaining the exchange rate dynamics. In our future research we will allow the weights given to fundamentalists and chartists to react endogenously to the profitability of these forecasting rules.

As stressed in the empirical and theoretical literature, transaction costs are important to explain the dynamics of the adjustment. Therefore, we will develop a version of the previous model represented by equations (1)-(9) in which the transaction costs play a role. We take the view that if transaction costs exist, the fundamentalists will take this information into account. Therefore, if the exchange rate is within the transaction costs band the fundamentalists will behave differently than if the exchange rate moves outside the transaction costs band.

Consider the first case, when the exchange rate deviations from the equilibrium value are smaller than the transaction costs, then the fundamentalists know that arbitrage in the goods market does not apply. As a result, they expect the changes in the exchange rate to follow a white noise process ϵ_t . The best they can do is to forecast no change. More formally,

¹⁷ See De Long et al. (1990)

when $\left|s_{t-1} - s_{t-1}^*\right| < C$, then $E_t^F(\Delta s_{t+1}) = 0$.

In the second case, when the exchange rate deviation from its fundamental value is larger than the transaction costs C (assumed to be of the ‘iceberg’ type). Then the fundamentalists follow the same forecasting rule as in equation (8). More formally,

when $\left|s_{t-1} - s_{t-1}^*\right| > C$ holds, then equation (8) applies.

This formulation implies that when the exchange rate moves outside the transaction costs band, market inefficiencies other than transaction costs continue to play a role. As a result, these inefficiencies prevent the exchange rate from adjusting instantaneously. In our model these inefficiencies are captured by the fact that the speed of adjustment in the goods market is not infinite (equation (8)).

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