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The Low Volatility Puzzle: Norwegian Evidence

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Abstract

In this paper we investigate the relation between idiosyncratic volatility and returns in the Norwegian stock market for the period 1981 – 2012. By utilizing the methodology developed by Ang et al. (2006), we show that the internationally documented strong performance of low volatility stocks relative to high volatility stocks is not present in Norway. Our findings are robust for exposure to size, liquidity, momentum and book-to-market effects. The results also hold for different subsamples, industry exposure, variations of methodological approach and various data filters. We conclude that there is no idiosyncratic volatility puzzle in Norway. Our results have important implications for studies seeking to explain the key drivers behind the idiosyncratic volatility puzzle in other markets, as a deeper understanding of the Norwegian market could shed new light on this literature.

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1 Introduction

One of the most commonly accepted relationships in the field of finance is that between risk and return; bearing risk can be expected to produce a reward in form of higher expected returns. The basic capital asset pricing model (CAPM) expresses risk as covariance with the market and implies that all agents should invest in the market portfolio as it yields the highest return per unit of risk (see among others Sharpe (1964), Lintner (1965a) and Mossin (1966)). Early studies discovered that the security market line for U.S. stocks is flatter than predicted by the CAPM (Black, Jensen, and Scholes 1972), and in recent years numerous studies have been conducted to explore the cross-sectional relationship between past volatility and returns. One finding is that low volatility stocks have a tendency to earn too high risk-adjusted returns, illustrated by showing that they have a significantly higher Sharpe ratio than stocks with higher volatility. This is a remarkable result; Baker, Bradley, and Wurgler (2011) consider it to be the greatest anomaly in finance since it challenges the basic notion of a risk-return trade-off.

In academic literature this phenomenon has become known as the “low volatility puzzle” and has been documented in three different versions. The three versions are highly related, but use different measures of volatility to define stocks’ return risk. The first version, early synthesised by Haugen and Heins (1975) and recently revisited by Frazzini and Pedersen (2011), is the beta puzzle. The risk measure in this version is the covariance with the market portfolio, i.e. the systematic risk as defined by the CAPM. The second version, which we will focus our main effort on, uses idiosyncratic volatility as the measure of risk.[1] Ang et al. (2006; 2009) have made major contributions regarding this version of the phenomenon and their framework is the basis for our approach. The third version of the puzzle uses total volatility as the measure of variance, aggregating the results from the other two. In addition to present empirical evidence for the phenomenon, research conducted has included controls for many factors that may explain the effect, such as the CAPM, Fama and French factors, the momentum effect and others. As the effect is robust to controlling for multiple factors,

[1] The terms idiosyncratic volatility and IVOL are used interchangeably throughout the text.

possible reasons for the over-performance of low-volatility stocks are also outlined. There are two main types of explanations for the anomaly; one set of rational explanations and one set based on behavioural finance.

As the research conducted up to date has mostly focused on U.S.- and large international markets, our main contribution is to test whether the negative relationship between past idiosyncratic volatility and returns also is present in the Norwegian stock market. This has never been done explicitly for the Norwegian market up to this date. Only two papers include the Norwegian stock market in their studies of the low volatility anomaly. The first was Ang et al. (2009) who, in their paper investigate the relationship between past idiosyncratic volatility and future returns for international developed markets, but the Norwegian results are aggregated together with 15 other developed countries. Thus they make no explicit comments regarding the Norwegian results. The second study to include Norwegian data was Baker and Haugen (2012) who test the total volatility version of the puzzle. They present evidence for a low volatility effect in the countries they examine, including Norway. Other issues we wish to address in the thesis are what exposure low volatility strategies in Norway have to systematic risk factors such as the Fama-French-, momentum- and liquidity factors.

Contrary to other studies from international markets, we do not find evidence of an idiosyncratic volatility puzzle in Norway. The relationship between idiosyncratic volatility and returns tend to be positive, and the alphas mostly improve going from the low volatility portfolios to the high volatility portfolios. Due to limited significance for the alphas in the different quintiles, it is however hard to make inferences that high idiosyncratic volatility portfolio provides better risk adjusted return. We conduct a number of robustness checks and find that our original result still holds after this.

The rest of this thesis is organized as follows; in Section 2 we review the literature on the low volatility anomaly, Section 3 describes the data, Section 4 outlines our methodological approach, in Section 5 we discuss our results and in Section 6 we present our conclusion.

2 Literature Review

The relationship between risk and return is a fundamental topic in finance, and has been extensively studied in the literature, both in theoretical- and empirical frameworks. Section 2.1 reviews the literature regarding the idiosyncratic volatility puzzle, centred around the findings of Ang et al. (2006) whose methodology we later apply on the Norwegian data. Section 2.2 discusses the beta puzzle while section 2.3 addresses the total volatility puzzle. Section 2.4 reviews proposed explanations behind the existence and persistence of the low volatility anomaly. These explanations are split into a set of rational- and a set of behavioural explanations. Since the focus of our thesis is on the IVOL puzzle, we devote more space to studies addressing this version of the puzzle specifically.

2.1 The Idiosyncratic Volatility Puzzle

The finding that high idiosyncratic volatility stocks tend to have low risk adjusted returns is as a pure anomaly since in classic asset pricing models idiosyncratic risk can be fully diversified away, and hence should be unrelated to returns. Even if we acknowledge that investors may not be perfectly diversified, the finding can still be classified as an anomaly considering the insights of Levy (1978) and Merton (1987) who propose that the relationship between idiosyncratic volatility and returns should be positive in the presence of undiversified investors.[2]

2.1.1 The Findings of Ang, Hodrick, Xing and Zhang

The recent literature finding a negative relation between idiosyncratic volatility begins with Ang et al. (2006).[3] They examine the cross-sectional relationship between idiosyncratic volatility and expected returns, where idiosyncratic volatility is defined relative to the Fama and French (1993) model. Using a one month time horizon for measuring volatility, their results show that U.S stocks with high idiosyncratic volatility have abnormally low average returns in the period 1963 to 2000. Highly significant results show that stocks in the bottom quintile of idiosyncratic volatility outperform stocks in the top quintile by 1.06%

[2] See e.g. Goetzmann and Kumar (2008) for an empirical study on undiversified investors.

[3] Other recent studies that find a negative relation between IVOL and expected returns include Jiang, Xu, and Yao (2009), Ang et al. (2009), Guo and Savickas (2010) and Chen et al. (2012).

per month. They control for a number of factors and conclude that the result cannot be explained by exposures to size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, coskewness, or dispersion in analysts' forecasts. They show that the results hold in bull and bear markets, NBER recessions and expansions, volatile and stable periods, and under different formation and holding periods as long as one year. In their 2009 article, Ang et al. extend the scope of their 2006 article and investigate whether the relation between lagged idiosyncratic volatility and future average returns found in U.S. data also exists in other markets. They find that low returns for stocks with high past idiosyncratic volatility is observed world-wide, suggesting that the results from Ang et al. (2006) is not just a country-specific nor a sample-specific effect. Stocks across 23 countries (including Norway) are sorted on past idiosyncratic volatility, and the difference in alphas between the highest- and the lowest quintile of idiosyncratic volatility stocks is a very large -1.31% per month and very significant.[4] This is after adjusting for market, size and book-to-market factors.

In addition the study investigates the degree of international comovement in returns of stocks with high idiosyncratic volatility. They find that the low returns earned by stocks with high idiosyncratic volatility commove significantly with the idiosyncratic volatility effect in the U.S., meaning that the global idiosyncratic volatility effect is captured by a simple U.S. idiosyncratic volatility factor. This suggests that broad factors may lie behind the phenomenon, implying that it would be difficult to mitigate the effect by diversification.

Ang et al. (2009) also introduces new controls on factors that might explain the anomaly. By using the U.S. data, the 2009 article investigates possible explanations for the anomaly such as trading or clientele structures, higher moments, information dissemination, and the leverage interaction story of Johnson (2004). These hypotheses are generally rejected and the article concludes that further studies are needed to investigate if there are true sources of economic risk that lies behind the phenomena causing stocks with high idiosyncratic volatility to have low expected returns.

[4] Note that Ang et al. (2009) aggregate the results from 16 of the countries including Norway. I.e. they do not conduct a country specific analysis on the Norwegian data.

2.1.2 Studies Finding a Positive Relation between IVOL and Expected Returns

A positive relation between idiosyncratic volatility and expected return is accommodated by various theoretical departures from the classical paradigm (Staumbaugh, Yu, and Yuan 2013). Theoretical explanations behind a positive relation include Levy (1978), Merton (1987), Malkiel and Xu (2002) and Jones and Rhodes-Kropf (2003) who argue that undiversified investors will demand a premium for taking idiosyncratic risk. In a more recent study, Eiling (2013) argues that the positive relation can be explained by high IVOL-stock's exposure to industry specific human capital returns. In addition, Barberis and Huang (2001) present behavioural models that give support to a positive relation between high idiosyncratic volatility stocks and expected returns. Early studies by Lintner (1965b) and Douglas (1969) document a significant positive relationship between idiosyncratic volatility and expected returns, but Miller and Scholes (1972) point out an important statistical problem with these results, i.e. that the positive skewness in individual security returns imply that stocks with high average returns also typically exhibit high idiosyncratic variance. Later studies include Tinic and West (1986) and Malkiel and Xu (2002) who find that portfolios with high idiosyncratic volatility have higher returns, but they do not include any significance levels in their results. Lehmann (1990) find that residual variance has a positive significant coefficient in cross-sectional regressions, but he also shows that his result is sensitive to different econometric specifications. A recent study in the idiosyncratic volatility literature is Fu (2009) who uses an EGARCH model to estimate expected idiosyncratic volatilities and, using those findings, show a significantly positive relation between the estimated conditional idiosyncratic volatilities and expected returns. He argues that the Ang et al. (2006) results are driven by a short term return reversal effect. However, Guo, Kassa, and Ferguson (2010) show that the results by Fu (2009) are driven by a look-ahead bias which is accidently introduced into the recursive volatility forecasts by including the month t return in the estimation of the month t EGARCH idiosyncratic volatility. When correcting for this they find no relation between EGARCH idiosyncratic volatility and returns. This conclusion is supported by Fink, Fink, and Hui (2010) who also report a look-ahead bias in Fu's (2009) results. According to Guo, Kassa, and Ferguson (2010); Fu himself acknowledges that a look-ahead bias is present in his calculations.

2.1.3 Studies Finding No Relation between IVOL and Expected Returns

In their classic study, Fama and Macbeth (1973) find no relation between idiosyncratic volatility and expected return, after mitigating the methodological issues raised by Miller and Scholes (1972). A more recent study which also finds no relation is Bali and Cakici (2008). They argue that methodological differences in previous studies have led to conflicting evidence in the literature. In particular; (i) data frequency (daily versus monthly) used to estimate idiosyncratic volatility, (ii) the weighting schemes used to compute average portfolio returns, (iii) breakpoints in sorting stocks into quintile portfolios, and (iv) different filter rules, all play a crucial role in determining significant relationship between idiosyncratic volatility and expected return. When using within-month daily data to calculate idiosyncratic volatility and using value-weighted quintile portfolios (replicating the methodology in Ang et al. (2006)), Bali and Cakici (2008) find a significant negative relationship between IVOL and returns, thus confirming the results of Ang et al. (2006). However, Bali and Cakici (2008) argue that the realized idiosyncratic volatility measure obtained from monthly data is a more accurate proxy for the expected future volatility than the daily version. When repeating their tests, they find that the relationship between monthly idiosyncratic volatility and the cross-section of expected returns is flat or very weak. The negative relationship also becomes insignificant or even positive when equal-weighted portfolios are used. This leads Bali and Cakici (2008) to conclude that the negative trade-off between risk and return does not exist.

2.2 The Beta Puzzle

The initial publications regarding the flatness of the security market line appeared in the seventies beginning with Black, Jensen, and Scholes (1972), and later Haugen and Heins (1975). Examining the 1963-1990 period, Fama and French (1992) find that the relation between market beta and average return is flat after controlling for size. A more recent study by Frazzini and Pedersen (2011) further explore the relationship between beta and returns and they find that investing in high beta assets results in a lower alpha than investing in low beta assets.[5] They

[5] Several other studies find similar results. See for instance Blitz and Van Vliet (2007) and Baker Bradley and Wurgler (2011).

argue that leverage restrictions are a key explanation behind why high beta assets seem to provide lower returns than what CAPM predicts, this will be further discussed in Section 2.4.

2.3 The Total Volatility Puzzle

Several studies who examine the relation between risk and return use total volatility as a risk measure instead of separating the risk measure into its systematic and unsystematic components. Since high (low) IVOL stocks typically have high (low) total volatility these studies are particularly relevant in relation to the IVOL puzzle. Similarly, high (low) beta stocks tend to have high (low) volatility making these studies closely connected to the beta puzzle as well.

Clarke, de Silva, and Thorley (2006) construct minimum-variance portfolios using a large set of U.S. equities, and examine the realized return statistics over several decades. They find that minimum-variance portfolios that do not rely on any expected return theory or return forecasting signal show promise in terms of adding value over the market capitalization weighted benchmark. More specifically they find that realized standard deviation is lowered by one-fourth, and risk measured by market beta is lowered by about one-third compared to the capitalization weighted benchmark. In other words the minimum-variance portfolios are capable of delivering similar or higher returns than the market portfolio at a substantially lower risk level. The authors comment that their results are consistent with the findings of Ang et al. (2006) regarding the low average returns of stocks with high idiosyncratic volatility. They also highlight that the minimum variance portfolios tend to have a value and a small size bias. But when controlling for these biases, the realized Sharpe ratios of the minimum-variance portfolios are still relatively high.

Scherer (2010) provides “a new look at minimum variance investing” and seeks to explain the variation of the excess returns of the minimum variance portfolio, relative to a capitalization weighted alternative, by using the Fama-French factors and two characteristic anomaly portfolios. The article wants to test the hypothesis that the excess returns of the minimum variance portfolio are a function of risk related factors or known anomaly portfolios. The article shows that 83% of the variation of the minimum variance portfolio can be attributed to the proposed factors/anomaly portfolios. The excess returns of the minimum variance portfolio returns are regressed on the MKT, HML, SMB and two anomaly portfolios. The first anomaly portfolio is a cash neutral long/short portfolio that is long (equal weighted) the 20% stocks with the lowest beta and short the 20% stocks with the highest beta in the S&P 1500 universe. The second anomaly portfolio is a portfolio long the 20% stocks with the lowest residual risk and short the 20% stocks with the highest residual risk.[6] Scherer (2010) finds that all the explanatory variables are highly significant and have a sign in line with expectations. The coefficient for market returns is negative, which is intuitive as low volatility portfolios are likely to underperform in bull markets. The coefficient for the factor book-to-market (HML) is positive, in line with the idea that low volatility investing is often associated with "value investing". The coefficient for the size factor (SMB) is negative, as MVP by construction will prefer large companies that tend to be more diversified (implying lower risk). The coefficient for the small beta versus large beta portfolio is positive. The last coefficient (the residual risk portfolio) is also positive. This coefficient is in line with the findings of Ang et al. (2006) and it is positive when regressed on the excess returns of the minimum variance portfolio.

Blitz and van Vliet (2007) find that stocks with low historical volatility exhibit significantly higher risk adjusted returns. The volatility effect is particularly strong in a global setting, with a low versus high volatility alpha spread of 12%. In the sample used in the article (December 1985 - January 2006) the authors find alpha for portfolios ranked on beta, but this alpha is considerably less than for portfolios ranked on volatility. The volatility effect is similar in size to the value, size and momentum effect and the higher risk adjusted returns from the low volatility stocks is still present after making Fama-French adjustments and double sorts. The results are consistent with Ang et al. (2006) and compared to Clarke et al. (2006), this study find significantly lower risk and superior Sharpe ratios for U.S. minimum-variance portfolios. In a later study, Blitz, Pang, and van Vliet (2013) extend their 2007 study and tests for the low volatility effect in emerging markets and find strong evidence for its presence there as well.

[6] Residuals come from a regression of equity returns against the S&P1500 and a constant using 3 years of daily data

Baker and Haugen (2012) is the first study that conducts a country level analysis of the low volatility anomaly using Norwegian data. They sort into portfolios based on total volatility estimated over the last 24 months and look at the realized Sharpe ratio difference and the realized return difference between the high- and the low volatility portfolio as a measure of the low volatility effect. They find evidence that the low volatility anomaly exist in all testable developed- and emerging markets, including Norway. We will revisit these results in detail in section 5.1.5 as we replicate the Baker and Haugen (2012) methodology using our own dataset.

2.4 Explanations behind the Low Volatility Anomaly

There are many interesting theories and empirical studies in the literature that propose explanations behind the existence and persistence of a low volatility effect. We present these explanations below and separate them into a rational- and behavioural category.

2.4.1 Rational Explanations

Shorting Constraints: In a world where low volatility stocks outperform high volatility stocks on a risk adjusted basis, one obvious strategy would be to short the high volatility portfolio and go long the low volatility portfolio. This strategy should allow the smart money in the market to arbitrage away the observed low volatility anomaly. So why does the anomaly seem to persist? A key problem is that the high volatility portfolio is typically compromised of small stocks which are costly to trade in large quantities (Baker, Bradley, and Wurgler 2011). Another study related to shorting constraints is Staumbaugh, Yu, and Yuan (2013) who argue that high IVOL stocks are more susceptible to mispricing and that this creates the negative relation between IVOL and expected returns due to arbitrage asymmetry. Arbitrage asymmetry is the observation that short sellers wishing to exploit overpricing face more constraints than purchasers wishing to exploit underpricing. The implication is that high IVOL stocks that are overpriced tend to stay overpriced longer than high IVOL stocks that are underpriced, thus causing high IVOL stocks to have lower future returns. Short selling constraints include the risk caused by potential margin requirements due to short-run price fluctuations, and also the high tail-risk for short-sellers due to the inherent skewness in compounded returns. Another key point is that many investors groups, such as mutual funds and pension funds have investment policy restrictions that prevent them from taking short positions at all. Other studies assessing shorting constraints include Boheme et al. (2009) who find that for firms with low visibility, the relationship between IVOL and expected returns turns positive in the absence of shorting constraints. George and Hwang (2011) argue that the IVOL puzzle is driven by the low performance of high IVOL stocks that are mispriced due to low analyst coverage. They attribute the reason for the persistent mispricing to similar arguments as Staumbaugh, Yu, and Yuan (2013), i.e. short sale constraints.

Leverage Constraints: As discussed in the previous section, shorting constraints prevent investors from taking full advantage of the anomaly. But even if investors cannot short the high volatility portfolio, they should at least overweight the low volatility portfolio. In theory they could then lever this portfolio to match their risk preferences. However, investors such as mutual funds, pension funds and individuals are constrained in terms of how much leverage they can take on. This causes these investor groups to overweight risky securities instead of using leverage, to meet their expected return requirements, even though these securities have lower Sharpe ratios. Frazzini and Pedersen (2011) argue that leverage constraints are one important explanation behind the low returns on high beta assets. As leverage is central to exploit the mispricing of low beta assets, they show that the return on betting against beta is lower when funding liquidity worsens and betas are compressed towards one. Finally, a discussion regarding different types of investors (and their ability to use leverage) is provided. Here the difference between constrained investors (mutual funds and individual investors) and more unconstrained investors (LBO funds and Buffet's Berkshire Hathaway) are used to illustrate that leverage constraints have the hypothesized effects on agents' portfolio selection.

The Benchmarking Hypothesis: Baker, Bradley, and Wurgler (2011) points out that a manager who needs to beat a certain benchmark without using too much leverage has incentives to pick stocks with higher volatility to achieve this. Thus, the manager will be reluctant to overweight stocks with high alpha and low beta or underweight low alpha and high beta stocks. This finding is consistent with the average mutual fund beta of 1.10 over the last 10 years. Because of this, they argue that as long as fixed benchmark contracts remain, and the share of the market held by investment managers continue to be high, then there is no reason that the anomaly will go away anytime soon. Managers are typically disinclined to invest too much in low volatility stocks since it would increase their tracking error against the benchmark.

Mutual Funds and Cash Inflows: Karceski (2002) propose a model where fund managers are incentivized to tilt their portfolios toward high-beta stocks, thus causing these stocks to underperform relative to their CAPM equilibrium returns. His model is based on three arguments: First, mutual fund investors tend to invest more in funds that have showed recent strong performance relative to their peers.[7] Second, there are generally higher inflows of money to the mutual fund industry after a market has moved significantly upwards.[8] Thirdly, since high-beta stocks outperform in bull markets, they are excellent vehicles for attracting more money in to your fund. Simply put, being a mutual fund manager, it pays to outperform in bull markets and this creates extra demand for high volatility stocks.

Sell-Side Analyst Behaviour: Evidence suggests that sell-side analysts issue upward-biased earnings forecasts in order to please investment banking clients and senior management who are pitching for corporate deals.[9] In an empirical study, Hsu, Kudoh, and Yamada (2012) find evidence that sell-side analysts tend to inflate earnings growth forecasts more for high volatility stocks. They hypothesise that this is done because it is harder for clients to detect inflation in growth forecasts for stocks with highly volatile growth. If investors cannot adjust properly to these biased forecasts then this could push up the prices of high volatility stocks and subsequently reduce their future returns.

Corporate Information Disclosure: Jiang, Xu, and Yao (2009) examine the link between the IVOL anomaly and strategic company behaviour in information disclosure. Based on theory that firms may have an incentive to release good news and to withhold bad news about future earnings, they argue that less information disclosure generally leads to higher volatility in the form of future negative

[7] See e.g. Sirri and Tufano (1998).

[8] See e.g. Warther (1995).

[9] See e.g. Dugar and Nathan (1995).

earnings shocks. They find that high IVOL stocks tend to have poor disclosure quality and that the market does not properly adjust for this, thus causing a negative relation between high IVOL stocks and future returns.

A Priced Volatility Factor: Chen and Petkova (2012) argue that IVOL proxies for risk exposure from a missing factor in the FF-3 model. They identify the factor to be average stock variance and find that the price of this factor is negative. They explain this by investigating the amount of R&D expenditure among high IVOL stocks and find this to be significantly larger. Since firms with more R&D expenditure has been found to have more real options, they argue that high IVOL stocks are less negatively affected by increases in aggregate market variance due to their inherent real options.[10] In other words, high idiosyncratic volatility stocks command a premium because they provide a hedge for times of increasing market-wide variance. Similarly Barinov (2011) argue that aggregate volatility risk explain the IVOL discount found by Ang et al. (2006).

2.4.2 Behavioural Explanations[11]

Stocks as Lottery Tickets: Early research by Kahneman and Tversky (1979) document that individuals who are presented with a bet involving a high probability of a small loss and a low probability of a large gain, often will take the gamble. They argue that individuals' overweighting of low probabilities may contribute to the attractiveness of both insurance and gambling. Connecting this to the securities market we see that high volatility stocks are typically low priced with a small probability of multiplying in value, but a significantly higher probability of decreasing in value. In that sense, a high volatility stock resembles a lottery ticket. Baker, Bradley, and Wurgler (2011) argue that irrational investors will overpay for risky stocks and avoid low risk stocks due to behavioural biases such as individual’s preferences for lotteries. A similar argument is made by Blitz and Van Vliet (2007) who refers to Shefrin and Statman (2000)'s behavioural portfolio theory and argues that investor’s deviation from risk-averse behaviour

[10] See also Cao, Simin and Zhao (2008) who find that high IVOL stocks typically have a high level of growth options.

[11] An excellent synthesis of the academic literature that provides behavioural explanations for the low volatility anomaly is provided in Baker, Bradley and Wurgler (2011).

may cause high-risk stocks to be overpriced and low risk stocks to be underpriced. The reasoning is that investors will overpay for stocks they perceive as lottery tickets, because they would like a shot at the riches.

Several authors provide empirical evidence to support these theories, among them Kumar (2009) who find that individual investors invest disproportionately more in stocks with high idiosyncratic volatility, higher skewness and lower prices. Similarly, Boyer, Mitton, and Vorkink (2010) argue that investors might pay a premium for high IVOL stocks since it proxies for future skewness exposure. See also the studies by Barberis and Huang (2008) and Bali, Cakici, and Whitelaw (2011) who provide evidence that investors have a preference for assets with lottery-like payoffs. Such preferences contribute to the demand for high volatility stocks and could thus partly explain their anomalous low returns.

Overconfidence: A human bias that has been heavily documented within the experimental psychology literature is overconfidence. "People tend to overestimate the precision of their beliefs or forecasts, and they tend to overestimate their abilities" (Bodie, Kane, and Marcus 2011, 411). In other words; peoples' confidence in their own judgement often exceeds the accuracy of the judgement itself. This bias is particularly interesting in an investment context. Cornell (2009) argues that fundamental investors who believe they possess superior skill will want to invest in high volatility stocks because that is where they find the highest reward for security selection talent. If they overestimate their skill, the result should be overpricing of such stocks. Baker, Bradley, and Wurgler (2011) point out another important implication of the overconfidence effect; investors who disagree on stock valuation will likely stick to their own valuation because of the high confidence in their own estimates. This causes a dispersed set of views for future stock returns, which is likely even higher for stocks with very uncertain future outcomes, e.g. high volatility stocks. This can be tied to the low volatility anomaly by looking at the insights from Miller (1977) who argued that in a market with restrictions on short selling, the demand for a particular security will come from those with the most positive assessment of its returns. In other words, stock prices are set by optimists.[12] So even though short selling restrictions might be the key driver, the overconfidence among investors likely contributes to the low volatility anomaly.

The Representativeness Heuristic:[13] When estimating the probability of an event or a sample, an individual will often judge the probability by how well it represents certain salient features of the population from which it was drawn (Bar-Hillel 1984). One implication is that people commonly do not take into account the size of a sample, e.g. a small sample is considered to be just as representative of a population as a large one (Bodie, Kane, and Marcus 2011). Baker, Bradley, and Wurgler (2011) provide a great example that illustrates how the representativeness heuristic could explain the irrational preference for high volatility stocks: They consider how the quant and the layman will approach the question of defining great investments. The layman might think of companies like Microsoft and conclude that the road to riches is paved with investments in speculative technologies; after all, they seem representative of high returns based on the (small) sample the layman has seen. Thus by ignoring the high rate of failure among small, speculative investments the layman tends to overpay for risky stocks. The quant however will analyse the full sample and conclude that high risk stocks are generally a speculative investment.

2.4.3 Final Words on Causes behind the IVOL Puzzle

As seen above there are numerous different studies that propose different explanations behind the low volatility anomaly. No clear agreement exists in the literature in terms of which proposed explanation that best explain the anomaly. This issue is complicated by the variations in sample and overall methodology, thus making comparison difficult. A new study by Hou and Loh (2012) propose a methodology for evaluating a large number of explanations behind the

[12] Guo and Savickas (2010) points out that some later studies disagree with Miller's (1977) hypothesis, e.g. Doukas, Kim, and Pantzalis (2006).

[13] The representative heuristic was first described by Daniel Kahneman and Amos Tversky in the early 1970s. See e.g Tversky and Kahneman (1972)

idiosyncratic volatility anomaly.[14] By using their own proposed methodology, they argue that explanations based on investor's lottery preferences, earnings shocks and short-term return reversal show the most promise in terms of explaining the IVOL puzzle.

[14] Hou and Loh (2012) provide a comprehensive review of the current proposed explanations for the IVOL puzzle in their article.

3 Data

3.1 Return Data

We obtain daily return data for all equities traded on the Oslo Stock Exchange in the period 1980-2012. The data is downloaded from The OBI (Oslo Børs Information) Financial Database. The number of individual securities listed in a given year varies between 96 and 294 with an average of 208 securities listed per year in the overall period. The full sample consists of 872 unique securities. We require a stock to meet certain criteria related to liquidity, price and market capitalization to include it in our calculations (see detailed discussion in section 3.1.1). After applying our filters the number of securities in a given year varies between 33 and 225 with an average of 136 securities per year. Exhibit I in the appendix provides an overview of the number of securities in each sample each year, based on different sorting criteria.

3.1.1 Filtering of Sample

Not all stocks on the Oslo Stock exchange should necessarily be used in calculating representative returns for the exchange when conducting empirical asset pricing investigations (Ødegaard 2013). We therefore employ a set of filters to exclude problematic stocks from the sample. Other studies have also limited their universe of stocks by cutting the smallest stocks from the sample. Ang et al. (2009) exclude the smallest firms by eliminating the 5% of firms with the lowest market capitalization. Baker, Bradley and Wurgler (2011) also limit their sample by taking away firms with the lowest market cap. Our primary filter rules are those suggested by Bernt Arne Ødegaard in his article: ―Empirics of the Oslo Stock Exchange. Basic, descriptive, results 1980-2012." We require a stock to have a minimum of 20 trading days in a given year to enter the sample. Stocks that are seldom traded can be problematic, e.g. the observed volatility in these stocks can give a biased estimate of the intrinsic volatility. Low valued stocks (penny stocks) are also problematic since they can have very exaggerated returns. We therefore exclude stocks whose value is below NOK 10 during a year, e.g. stocks with a value above NOK 10 will be removed from the sample if their value falls below NOK 10 at any given time in a year. Similarly, we also exclude stocks whose total market value is below NOK one million during a year. Note that a stock which is excluded from the sample one year may be included in subsequent years if it fulfills the filter requirements.

3.1.2 Return Computation

Returns are generated using the following algorithm for calculating the price: If close (trade) price is available, use that. Otherwise, if both bid and ask (offer) is available, use the average. If only bid or ask is available, use that. The return data are adjusted for dividends and other corporate events, like stock dividends and stock splits.[15]

3.1.3 Return Outliers and Winsorization of Daily Data

We do examine the possibility that our sample includes outliers which potentially could impact our results. An examination of the daily returns in the complete universe of 872 stocks show that 126 of these stocks have one or more observations with returns above 100% in a single day. Here we see that the filters discussed in section 3.1.1 work quite well as 99 of these 126 stocks are removed from the sample at the particular date where the return exceeds 100%. Nevertheless there are still stocks left with suspiciously high (low) return values implying that there could be spurious outliers who affect our results. In order to deal with this we perform winsorization on our (pre-filter) daily data sample each year. We winsorize at the 0.1^{th}- and 99.9^{th} percentile meaning that all returns below the 0.1^{th} percentile are set to the 0.1^{th} percentile value and similarly all return values above the 99.9^{th} percentile are set to the 99.9^{th} percentile value. As an example; in 2012 we have 62,154 daily return observations. The 62 highest return values in 2012 are set to the 99.9^{th} percentile cut off point which that year is 41.5%. And the 62 lowest return values are set to the 0.1^{th} percentile cut off point which is minus 28.6%. Examining the cut off points each year, which can be seen in Exhibit II, we see that the winsorization successfully removes the extreme return values, i.e. no daily returns in any year are now above 100% or below -51% on a single day. Our results based on daily data will be presented with winsorization in the main body of the text; however, in the appendix tables based on the original data are included. The results are not significantly affected by the winsorization, i.e. it does not change our conclusion. This indicates that when

[15] Source: OBI Financial Database 2013.

applying the filters discussed in section 3.1.1., spurious outliers in the remaining sample do not seem to be an issue. Note that the filters discussed in section 3.1.1 (that remove penny stocks, small cap stocks and stocks with limited trading days) are applied for the winsorized data set as well, but the winsorization is done before the filters are applied.

3.2 Risk Free Rate

Norwegian interest rate data is downloaded from Bernt Arne Ødegaard's homepage. In the period from 1986-2012 we use monthly NIBOR rates. The availability of suitable interest rate data pre 1986 is limited and one must use some imperfect proxies (Ødegaard 2013). From 1982-1986 the overnight NIBOR is used as an approximation for the monthly risk free rate. Before 1982 the shortest possible bond yield for treasuries in Eitrheim, Klovland, and Qvigstad (2006) is used (Ødegaard 2013). The daily risk free rate is calculated as the simple daily rate that over the number of trading days in the month compounds to the monthly rate.

3.3 Pricing Factors

Five pricing factors for the Norwegian market are obtained from Bernt Arne Ødegaard's webpage. Value weighted market returns, where end of year values at the previous yearend are used for value weighting. The Fama French factors, HML and SMB, as calculated by Fama and French (1993), and the Carhart Momentum factor, PR1YR, as calculated by Carhart (1997) are all replicated using Norwegian data. The fifth factor, a liquidity factor, is developed for the Norwegian market (see Næs, Skjeltorp, and Ødegaard (2008)). Factor data for all factors is available in the period from July 1981 to December 2012.

3.4 Industry Return

Industry return from eight different sectors is available for the period July 1981 to December 2012 in the OBI Financial Database. The sectors follow the Global Industry Classification Standard (GICS) and we use the value weighted portfolios within each industry for our regressions.[16]

[16] The industries are Energy and consumption, Material/labor, Industrials, Consumer Discretionary, Consumer Staples, Health Care/liability, Financials and Information Technology.

4 Methodology

4.1 Definition of Idiosyncratic Volatility

Consistent with Ang et al. (2006) we define idiosyncratic risk as the variance of the error term in the Fama French 3 factor model (hereafter FF-3).[17]

$$r_{i,t} - r_{f,t} = \alpha_{it} + \beta_{i,t}\left(r_{m,t} - r_{f,t}\right) + s_{i,t}SMB_t + h_{i,t}HML_t + \varepsilon_{i,t} \tag{1}$$

In equation (1) idiosyncratic volatility is defined as $Var\left(\varepsilon_{i,t}\right)$. The other factors are standard, as defined in Fama and French (1993), where $r_{i,t} - r_{f,t}$ is the excess return of stock i at time t, $r_{m,t} - r_{f,t}$ is the excess return of the market, SMB_t reflects the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks, HML_t reflects the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio.

4.2 Portfolio Estimation

During a formation period of *F* months we estimate the idiosyncratic volatility of each stock in the filtered sample based on daily return data and by the end of the period we sort the stocks from low to high idiosyncratic volatility. We then divide the stocks into quintile portfolios, compound each stock's total daily return for a holding period of *H* months, subtract the period's risk free rate and then compute value- and equally weighted portfolio excess return over the *H* months. For *H* equal to one this yields a trading strategy which provide monthly excess return figures from each portfolio over our sample period (less the first *F* months, as it only serves as formation months). For the purpose of having a robust and investable approach we demand the stocks to be listed for the full formation period and the first day of the holding period to enter the portfolios. Our focus is on the strategy where both *F* and *H* are one month, such that for example our first formation period is July 1981 and the first holding period for the portfolios is August 1981. Testing the 1/1-Strategy on the full sample yields a total of 377 monthly portfolio excess returns.

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[17] Ang et al. (2006) use simple returns and the standard deviation of the error variance.

4.3 Performance Evaluation

To assess the excess return of the five portfolios we both calculate the mean monthly return, ex-post monthly standard deviation and Sharpe Ratios, and run regressions on the excess returns to calculate, ex-post IVOL, alphas and factor loadings. Ordinary least squares regressions are run relative to the CAPM, the FF-3 model and a five factor model. Furthermore we also regress portfolios on a set of industry return portfolios. Generalized method of moments with four lags is used to correct the standard errors and robust Newey-West t-statistics are calculated for the coefficients.[18]

4.4 Robustness Checks

4.4.1 Alternative Sorting Methods

The volatility sorting is critical to obtain the right portfolios and to assess this issue we use three different approaches. Firstly, we test whether other models than the FF-3 might be more appropriate for defining idiosyncratic volatility. Equation (2) show a Fama and French regression that includes a lead and lagged beta based on Scholes and Williams (1977) which in the case of stale prices will provide more representative estimates of $Var(\varepsilon_{i,t})$.

$$r_{i,t} - r_{f,t} = \alpha_{it} + \sum_{t=-1}^{t=1} \beta_{i,t}\left(r_{m,t} - r_{f,t}\right) + s_{i,t}SMB_t + h_{i,t}HML_t + \varepsilon_{i,t} \tag{2}$$

We also computed IVOLs using the CAPM and a five factor model (including the market-, the HML-, the SMB-, the PR1YR- and a liquidity factor), but the results based on these IVOLs are very similar to those using our base model and hence are not reported. The second approach to assure the quality of the sorting is to use monthly return data when estimating $Var(\varepsilon_{i,t})$. For instance, Bali and Cakici (2008) argued that estimating IVOL based on monthly returns is a more robust method than to use daily data. The use of monthly data is adapted from the paper by Baker and Haugen (2012) and our third assessment of the sorting is therefore to replicate their methodology and sort stocks based on total volatility.

[18] The Matlab code for this regression is downloaded from the homepage of Professor John H. Cochrane (http://faculty.chicagobooth.edu/john.cochrane/).

4.4.2 Applying Different Filters

When examining other studies we have noticed that some of the common filters applied would have excluded stocks that are not filtered out by the standard method we discuss in section 3.1.1. To account for this we both test filtering out the ten percent smallest of all stocks before dividing into portfolios, and we test if an increase in the required number of trading days from 20 to 125 impact our results.

4.4.3 Testing Subsamples

As described by Ang et al. (2006); ”a possible explanation for the idiosyncratic volatility effect may be asymmetry of return distributions across business cycles.” To test this we use subsample analysis to check if our primary findings are valid. In addition to the full sample period, we therefore test three subsamples of our data. The three subsamples tested are 1990-2012, 2000-2012 and 1981-2000.

4.4.4 Other

We focus on the strategy where both *F* and *H* are one in this paper, but for robustness we have checked certain variations. Changing the formation and the holding period up to twelve months for both does not have any major impact on the results, and these types of analysis are therefore not reported

Lastly, in this paper we choose to focus on results for quintile portfolios. We examined tercile and decile results as well, but the results were very similar. We note that drawing inferences using deciles can be problematic for the Norwegian market due to the small number of stocks available, especially in the eighties.

5 Results

5.1 The 1/1-Strategy

Table 1 reports quintile portfolios that are formed every month by sorting stocks on idiosyncratic volatility calculated from daily return data for the past month relative to the Fama-French (1993) model. Portfolio 1 (5) is the portfolio containing the stocks with the lowest (highest) volatility. Panel A replicates Ang et al. (2006) and shows value-weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. The Alpha columns report Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2012.

Panel A shows that average monthly returns increase from 0.71% per month for quintile 1 to 1.69% for quintile 5. Other studies, such as Ang et al. (2006), find that the returns typically do increase going from the lowest volatility quintile to the next two or three quintiles, but that the returns then fall dramatically in the portfolio with the highest volatility. This is not observed with the Norwegian data, quite the opposite, we find consistently that the high volatility portfolio exhibits the highest returns. The FF-3 alpha for quintile 1 in the value weighted portfolio, is -0.002 per month and significant. The FF-3 alpha for quintile 5 is 0.006, but not significant. We see that the FF-3 alphas increase monotonically from the low IVOL portfolio to the high IVOL portfolio. Assessing the significance of the FF-3 alphas we see that only portfolio 1 has a significant alpha. Measuring alpha with respect to CAPM and a five-factor model, again only quintile 1 has a significant alpha expect for quintile 5 which has a significant CAPM alpha. When calculating the returns based on equally weighted portfolios all the FF-3 alphas become significant. But the difference in alphas between the different quintiles is now negligible. These results imply that we do not find evidence of the idiosyncratic volatility puzzle being present in Norway.

Panel A									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,71 %	6,41 %	0,11	52,2 %	1,11 %	0,0006	-0,004	-0,002	-0,003
							[0,00]	[0,02]	[0,00]
2	0,93 %	6,69 %	0,14	22,0 %	1,76 %	0,0010	-0,001	-0,001	-0,001
							[0,59]	[0,50]	[0,68]
3	1,35 %	8,08 %	0,17	13,5 %	2,35 %	0,0015	0,002	-0,001	0,001
							[0,41]	[0,77]	[0,63]
4	1,34 %	9,33 %	0,14	8,4 %	3,23 %	0,0035	0,002	0,000	0,002
							[0,35]	[0,89]	[0,45]
5	1,69 %	8,50 %	0,20	4,0 %	5,94 %	0,0040	0,008	0,006	0,005
							[0,01]	[0,09]	[0,17]
Panel B									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,52 %	5,02 %	0,10	52,2 %	1,11 %	0,0006	-0,003	-0,005	-0,005
							[0,04]	[0,00]	[0,00]
2	0,58 %	5,76 %	0,10	22,0 %	1,76 %	0,0007	-0,003	-0,006	-0,005
							[0,02]	[0,00]	[0,00]
3	0,80 %	6,70 %	0,12	13,5 %	2,35 %	0,0010	-0,002	-0,007	-0,006
							[0,29]	[0,00]	[0,00]
4	0,66 %	6,39 %	0,10	8,4 %	3,23 %	0,0012	-0,002	-0,007	-0,007
							[0,24]	[0,00]	[0,00]
5	0,97 %	6,44 %	0,15	4,0 %	5,94 %	0,0017	0,002	-0,004	-0,005
							[0,37]	[0,02]	[0,00]

Table 1 – Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3

Considering the number of studies that have documented the idiosyncratic volatility puzzle across global markets, we find it quite surprising that we do not see hints of the low IVOL puzzle in the Norwegian data. In light of Bali and Cakici (2008)'s arguments related to the IVOL puzzle's sensitivity to the chosen methodology we perform a number of robustness checks by making changes to our initial methodology.

5.1.1 Scholes and Williams (1977) Beta

In Exhibit III we repeat the procedure from Table 1, except that we now estimate the idiosyncratic volatility relative to a Fama-French model including one lead and one lag of the market beta (see equation (2) in section 4.4.1). The impact of the two extra factors seems to have minimal impact on the error variance, as the reported results for the five portfolios does not change significantly. This indicates that market microstructure effects, such as stale prices do not seem be a major issue for the IVOL sorting.

5.1.2 Applying Different Filters

As mentioned in the data section, we base our filtering on the suggestions by Ødegaard (2013). As noted earlier many different filters are used in existing literature, for instance many studies exclude some fraction of the stocks with the lowest market capitalization. In Exhibit IV we test to filter out 10 percent of the market (in addition to the standard filter) and we see the same pattern, i.e. the high IVOL portfolios provide higher alphas and better Sharpe Ratios.

Another example of a more restrictive filter can be found in Chen et al. (2012), who demand stocks to be traded 15 out of 20 days a month to enter the sample. In the case of the Norwegian stock market we find this to be too restrictive and instead test a filter requiring 125 trading days per year. The results are reported in Exhibit V and we see that this filter does not change our findings from Table 1 considerably.

We have further tested other variations of the filters, such as increasing the penny stock filter or increasing the market capitalization requirement, but as none of these variations change the conclusion from above, these tables are not included for the sake of brevity.

5.1.3 Testing Different Subsamples

In this section we explore different sample periods in the Norwegian data. Chen et al. (2012) points out that the approach of comparing the return differentials between extreme IVOL portfolios makes the outcome susceptible to stock sample selection. In our sample the first nine years have limited number of stocks and also less trading volume than more recent data, making the data vulnerable to market microstructure effects such as nonsynchronous trading. We therefore test excluding this period. Exhibit VI presents results from the period 1990-2012. Again the results are very similar to the main approach. The mean return for quintile 1 is 0.62% per month while quintile 5 has a mean return of 1.75% per month. The FF-3 alphas for quintile 1-3 are negative while the quintile 4 and 5 has positive FF-3 alphas, however only the quintile 1 alpha is significant. Looking at the equal weighted portfolios the FF-3 alpha for both quintile 1 and 5 is -0.005% per month, both being highly significant.

Repeating the same analysis for the post 2000 period shows very similar patterns and can be seen in Exhibit VII. In Exhibit VIII we test the period 1981-2000. In this subsample quintile 5 does not have the highest return in the value weighted panel, but the alpha pattern is similar to the one observed in the full sample. For the equal weighted panel the results are very similar. Overall it seems that testing different subsamples has limited impact on the results.

5.2 Using Monthly Returns to Calculate Idiosyncratic Volatility

As mentioned previously, Baker and Haugen (2012) are so far the only study to explicitly analyze the low volatility anomaly in Norway. They examine the 1990-2011 period and sort portfolios based on total volatility estimated over the previous 24 months. They argue that the low volatility anomaly is present in Norway based on an observed Sharpe Ratio- and return difference between the two extreme quintile portfolios of 42.7% for the Sharpe Ratio and 5.3% for the return differential (quintile 1 minus 5). In their paper version, they only report the results from the VW portfolios.

In light of their results we decided to test the idiosyncratic volatility puzzle using monthly data as well. Table 2 presents the results of the IVOL sorted portfolios that are based on monthly return data (note that except for using the previous 24 months of monthly return data, our methodology is the same as our main approach). For the value weighted portfolio the return of quintile 1 is 0.66% per month. It increases monotonically until quintile 5 where the return is 2.72% per month. Consistent with our earlier results the high volatility portfolio shows high returns, but in this table we see that the monthly standard deviation for the high volatility portfolio is also substantially higher than the other portfolios. The Sharpe Ratios of quintile 1 and 5 are both close to 0.10 per month. This marginal difference makes it hard to argue that low volatility stocks perform significantly better than high volatility stocks in Norway. Looking at the FF-3 alphas they are quite similar for quintile 1-4, (all negative between 0.002 and 0.004). Quintile 5 has a FF-3 alpha of 0.021 making it markedly higher than the others. However, except for quintile 1, none of the FF-3alphas are significant.

Panel A									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F months ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,66 %	6,30 %	0,10	44,8 %	5,07 %	0,0008	-0,004	-0,003	-0,004
							[0,00]	[0,02]	[0,00]
2	0,89 %	6,83 %	0,13	21,3 %	7,24 %	0,0012	-0,002	-0,001	-0,001
							[0,41]	[0,45]	[0,57]
3	0,74 %	7,86 %	0,09	16,4 %	9,19 %	0,0022	-0,003	-0,004	-0,002
							[0,12]	[0,08]	[0,24]
4	1,08 %	8,32 %	0,13	11,0 %	11,79 %	0,0022	-0,001	-0,003	-0,001
							[0,70]	[0,23]	[0,67]
5	2,72 %	27,62 %	0,10	6,5 %	22,31 %	0,0708	0,025	0,021	0,022
							[0,11]	[0,19]	[0,18]
Panel B									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F months ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,28 %	4,67 %	0,06	44,8 %	5,07 %	0,0006	-0,005	-0,007	-0,007
							[0,01]	[0,00]	[0,00]
2	0,31 %	5,63 %	0,06	21,3 %	7,24 %	0,0008	-0,006	-0,009	-0,008
							[0,00]	[0,00]	[0,00]
3	0,36 %	6,50 %	0,05	16,4 %	9,19 %	0,0013	-0,006	-0,010	-0,009
							[0,02]	[0,00]	[0,00]
4	0,46 %	6,91 %	0,07	11,0 %	11,79 %	0,0015	-0,005	-0,010	-0,009
							[0,01]	[0,00]	[0,00]
5	1,53 %	34,00 %	0,04	6,5 %	22,31 %	0,1119	0,020	0,009	0,010
							[0,32]	[0,66]	[0,63]

Table 2 – Portfolios Sorted by Monthly Idiosyncratic Volatility Relative to FF-3

The table reports portfolios that are sorted based on the last 24 months idiosyncratic volatility relative to the Fama-French (1993) model, calculated from monthly returns. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2012.

Assessing Panel B and the equal weighted portfolios, we observe a Sharpe Ratio for quintile 1 of 0.06 per month while quintile 5 has a Sharpe Ratio of 0.04 per month. This is more in line with Baker and Haugen's (2012) result. However we do not find a positive return differential like they do. The FF-3 alphas decrease from quintile 1 to 4, and are all negative and significant, while quintile 5 reports a positive, but insignificant FF-3 alpha. Baker and Haugen (2012) do not report any alpha values. Overall our results from using monthly data are mostly consistent with our main approach. We find it either hard to infer any relation between IVOL and returns or it seems that the high IVOL portfolio does better. In the next section we further explore the Baker and Haugen (2012) study.

5.2.1 Sorting Based on Total Volatility

Even though our paper is mainly concerned with the IVOL puzzle, we wish to replicate the methodology of Haugen and Baker (2012) using total volatility as a risk measure and test our data on the same period they do (1990-2011). Table 3 report our results were we sort based on the previous 24 months of total volatility. Contrary to Baker and Haugen (2012) we find that the high volatility portfolio has a marginally higher Sharpe Ratio than the low volatility portfolio (0.11 vs. 0.09 for VW, and 0.04 vs. 0.03 for EW). To explain what causes the differences we closely examine their methodology section to assess what we may be doing differently. They write that they sort stocks based on the last 24 months of total volatility, but they do not specify how many months of data they require from a stock to be included in a portfolio. As explained in Section 4.2, we require a stock to be listed in the entire formation period. Table 4 reports our results when we relax this requirement, i.e. we include a stock as long as it has at least two monthly return observations. Similar to Baker and Haugen (2012) we now find that the low volatility portfolios achieve the highest Sharpe Ratios for both VW and EW portfolios. For instance the VW low volatility portfolio has a Sharpe Ratio of 0.12 compared to 0.10 for the high volatility portfolio. Our results are not as strong as Baker and Haugen (2012), considering that they find a Sharpe Ratio difference of 42.7% between the two extreme quintile portfolios. We also fail to find the positive return differential they report since our high volatility portfolio consistently earns very high returns. To explain this difference we first note that their sample size of Norwegian stocks often differ from the sample we obtained from OBI (see Exhibit I).[19] Secondly, they do not specify any specific filters used on their data. It is worth mentioning that in the total sample of 21 developed countries in the Baker and Hagen (2012) study; only 4 other countries have a

[19] A spreadsheet that contains the complete research results of Baker and Haugen (2012) can be downloaded from "lowvolatilitystocks.com" or "quantitativeinvestment.com".

lower Sharpe ratio differential than Norway indicating that the low volatility effect is less present in Norway than in other countries.[20]

Panel A									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F months Std Dev	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,55 %	5,99 %	0,09	22,8 %	6,02 %	0,0014	-0,003	-0,002	-0,003
							[0,25]	[0,36]	[0,26]
2	0,49 %	6,72 %	0,07	26,4 %	8,58 %	0,0012	-0,005	-0,004	-0,004
							[0,00]	[0,03]	[0,04]
3	0,37 %	7,64 %	0,05	25,3 %	10,74 %	0,0019	-0,007	-0,007	-0,007
							[0,01]	[0,00]	[0,00]
4	0,58 %	9,00 %	0,06	16,7 %	13,65 %	0,0027	-0,006	-0,006	-0,004
							[0,04]	[0,03]	[0,12]
5	2,71 %	25,69 %	0,11	8,8 %	25,40 %	0,0594	0,025	0,021	0,022
							[0,14]	[0,22]	[0,20]
Panel B									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F months Std Dev	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,13 %	4,13 %	0,03	22,8 %	6,02 %	0,0007	-0,004	-0,007	-0,007
							[0,03]	[0,00]	[0,00]
2	0,25 %	5,40 %	0,05	26,4 %	8,58 %	0,0009	-0,005	-0,008	-0,008
							[0,02]	[0,00]	[0,00]
3	0,27 %	6,15 %	0,04	25,3 %	10,74 %	0,0011	-0,006	-0,010	-0,010
							[0,01]	[0,00]	[0,00]
4	0,28 %	7,77 %	0,04	16,7 %	13,65 %	0,0019	-0,007	-0,012	-0,011
							[0,01]	[0,00]	[0,00]
5	1,55 %	39,02 %	0,04	8,8 %	25,40 %	0,1489	0,025	0,013	0,015
							[0,36]	[0,63]	[0,58]

Table 3 – Portfolios Sorted on Total Volatility

The table reports portfolios that are sorted based on the last 24 months standard deviation, calculated from monthly returns. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month Std Dev is the average standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is January 1988 to December 2011. For stocks to be included in the portfolios we require that they are listed during the full period of 24 months prior to portfolio formation.

[20] These four countries are Japan, Italy, Austria and Ireland.

Panel A									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F months Std Dev	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,71 %	5,83 %	0,12	23,6 %	6,05 %	0,0012	-0,001	-0,001	-0,002
							[0,57]	[0,60]	[0,42]
2	0,51 %	6,79 %	0,08	26,0 %	8,64 %	0,0012	-0,005	-0,004	-0,004
							[0,00]	[0,02]	[0,04]
3	0,33 %	7,47 %	0,04	25,2 %	10,84 %	0,0016	-0,007	-0,008	-0,007
							[0,00]	[0,00]	[0,00]
4	0,66 %	8,95 %	0,07	16,1 %	13,77 %	0,0027	-0,005	-0,006	-0,004
							[0,08]	[0,05]	[0,19]
5	2,47 %	25,85 %	0,10	9,1 %	25,54 %	0,0598	0,022	0,019	0,020
							[0,18]	[0,26]	[0,25]
Panel B									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F months Std Dev	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,19 %	4,09 %	0,05	23,6 %	6,05 %	0,0006	-0,004	-0,006	-0,007
							[0,06]	[0,00]	[0,00]
2	0,26 %	5,35 %	0,05	26,0 %	8,64 %	0,0008	-0,005	-0,008	-0,008
							[0,02]	[0,00]	[0,00]
3	0,22 %	6,27 %	0,03	25,2 %	10,84 %	0,0011	-0,007	-0,011	-0,010
							[0,01]	[0,00]	[0,00]
4	0,33 %	7,88 %	0,04	16,1 %	13,77 %	0,0020	-0,007	-0,012	-0,011
							[0,01]	[0,00]	[0,00]
5	1,44 %	39,01 %	0,04	9,1 %	25,54 %	0,1488	0,023	0,012	0,014
							[0,39]	[0,65]	[0,61]

Table 4 – Portfolios Sorted on Total Volatility with Relaxed Restrictions

The table reports portfolios that are sorted based on the last 24 months standard deviation, calculated from monthly returns. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month Std Dev is the average standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is January 1988 to December 2011. For stocks to be included in the portfolios they need only to be listed a minimum of two months before the formation period.

5.3 Pricing Factor Loadings

Table 5 present the loading on various pricing factors for the lowest and highest volatility quintiles discussed in section 5.1 and 5.2. In three out of four regressions, the high volatility portfolios have positive and significant liquidity betas, while none of these betas are significant for the low volatility portfolios.

This is in line with our expectations, as the high volatility portfolios are often associated with illiquid stocks.

Panel A				
	1/1-Strategy (daily returns)		24/1-Strategy (monthly returns)	
Portfolio	1	5	1	5
Size	20,09	18,39	19,83	18,62
Alpha	**-0,003**	0,005	**-0,004**	0,022
	[0,00]	[0,17]	[0,00]	[0,18]
Beta MRK	**0,911**	**1,057**	**0,890**	**1,407**
	[0,00]	[0,00]	[0,00]	[0,00]
Beta SMB	**-0,154**	0,069	-0,135	**0,383**
	[0,00]	[0,52]	[0,05]	[0,02]
Beta HML	0,023	-0,040	0,024	-0,270
	[0,49]	[0,61]	[0,59]	[0,28]
Beta PR1YR	**0,089**	0,019	0,057	-0,329
	[0,00]	[0,82]	[0,09]	[0,06]
Beta LIQ	0,075	**0,361**	0,097	0,272
	[0,21]	[0,01]	[0,19]	[0,25]
Panel B				
	1/1-Strategy (daily returns)		24/1-Strategy (monthly returns)	
Portfolio	1	5	1	5
Size	20,09	18,39	19,83	18,62
Alpha	**-0,005**	-0,005	**-0,007**	0,010
	[0,00]	[0,63]	[0,00]	[0,63]
Beta MRK	**0,701**	**1,015**	**0,663**	**1,354**
	[0,00]	[0,00]	[0,00]	[0,00]
Beta SMB	**0,162**	**0,274**	**0,153**	**0,809**
	[0,00]	[0,00]	[0,01]	[0,00]
Beta HML	**0,103**	0,021	**0,113**	-0,138
	[0,00]	[0,54]	[0,00]	[0,54]
Beta PR1YR	0,043	**-0,127**	0,002	**-0,520**
	[0,12]	[0,02]	[0,94]	[0,02]
Beta LIQ	-0,058	**0,579**	0,039	**0,642**
	[0,27]	[0,01]	[0,51]	[0,01]

Table 5 – Pricing Factor Loadings for Quintile 1 and 5 as Reported in Table 1 and Table 2

The table reports factor loadings for the extreme portfolios presented in Table 1 and Table 2. The portfolios are regressed on a five factor model and, as before, Panel A contains the value weighted portfolios and Panel B contains the equally weighted portfolios. Size is the log of the yearly average of the portfolios average market cap. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2012. Significant coefficients are boldfaced.

Considering the market betas we also find the relation we expect; the low volatility portfolios have lower factor loadings than the high volatility portfolios. These betas are all highly significant. We note that for the 1/1-Strategy the difference between the betas is smaller than for the 24/1-Strategy. For the SMB factor we see that the high volatility portfolios consequentially have higher loadings, all significant except two. This is consistent with earlier findings reporting that the high volatility portfolios often exhibit a small cap effect, i.e. consist of many small stocks.

The HML loadings are mostly insignificant, but the pattern is that the loading is higher for the low volatility portfolios. This is again consistent with the literature, as the low volatility portfolio often consists of many value stocks. For the momentum effect the loadings for three of the high volatility portfolios have negative coefficients, two of them significant. The loadings on the low volatility portfolios are all positive, but only one is significant. This suggests that the high volatility portfolios exhibit a return reversal, which is not present among the low volatility portfolios.

In terms of summarizing the economic differences between the high- and low IVOL portfolios, the high volatility portfolio consist of more smaller and less liquid firms. The HML loadings points towards a higher concentration of growth stocks in this portfolio. On the contrary the low volatility portfolio consist of more larger firms, which likely has more diversified operations causing lower risk as well as higher liquidity. As mentioned above, the low volatility portfolios also consist of more value stocks.

5.4 Industry Exposure

In this section we examine the high and low volatility portfolios loadings on several industry factors, which are presented in Table 4. Descriptives for all industries are reported by Ødegaard (2013) and some of his tables are reprinted in Exhibit IX, these will be referred to throughout this section. The first observation is that the energy beta is highly significant for all portfolios, with consistently higher loadings on the high volatility portfolios. In the Norwegian context this could be interpreted as an oil effect on the high volatility portfolios, meaning that their high returns can partly be explained by the strong development in many small oil related Norwegian companies, particularly many volatile oil-service companies. Looking at the equally weighted industry returns the energy sector posts the second highest return of the sectors.

Panel A				
	1/1-Strategy (daily returns)		24/1-Strategy (monthly retruns)	
Portfolio	1	5	1	5
Alpha	**-0,007**	0,004	**-0,007**	0,025
	[0,00]	[0,15]	[0,00]	[0,20]
Beta Energy	**0,259**	**0,322**	**0,262**	**0,460**
	[0,00]	[0,00]	[0,00]	[0,00]
Beta Material	0,017	-0,008	0,009	0,021
	[0,45]	[0,77]	[0,59]	[0,72]
Beta Industry	**0,389**	0,097	**0,393**	-0,308
	[0,00]	[0,23]	[0,00]	[0,26]
Beta ConsDisc	-0,026	0,044	**-0,054**	0,087
	[0,20]	[0,32]	[0,01]	[0,37]
Beta ConsStapl	**0,115**	0,039	**0,096**	-0,575
	[0,00]	[0,60]	[0,00]	[0,26]
Beta Health	**0,010**	**-0,016**	**0,008**	0,000
	[0,02]	[0,00]	[0,00]	[0,98]
Beta Finance	**0,130**	**0,222**	**0,125**	0,665
	[0,00]	[0,00]	[0,00]	[0,14]
Beta IT	0,004	**0,179**	0,020	**0,645**
	[0,77]	[0,00]	[0,22]	[0,00]

Panel B				
	1/1-Strategy (daily returns)		24/1-Strategy (monthly retruns)	
Portfolio	1	5	1	5
Alpha	**-0,006**	0,000	**-0,007**	0,021
	[0,00]	[0,90]	[0,00]	[0,39]
Beta Energy	**0,160**	**0,270**	**0,131**	**0,438**
	[0,00]	[0,00]	[0,00]	[0,01]
Beta Material	0,009	**0,051**	0,029	-0,003
	[0,74]	[0,01]	[0,17]	[0,96]
Beta Industry	**0,128**	0,019	**0,090**	-0,304
	[0,00]	[0,72]	[0,00]	[0,36]
Beta ConsDisc	0,035	0,047	0,000	0,017
	[0,16]	[0,11]	[1,00]	[0,88]
Beta ConsStapl	**0,136**	-0,067	**0,107**	-0,805
	[0,00]	[0,16]	[0,00]	[0,22]
Beta Health	0,002	0,002	0,003	-0,002
	[0,74]	[0,61]	[0,53]	[0,84]
Beta Finance	**0,203**	**0,222**	**0,244**	1,084
	[0,00]	[0,00]	[0,00]	[0,07]
Beta IT	**0,027**	**0,135**	0,020	**0,345**
	[0,05]	[0,00]	[0,14]	[0,00]

Table 6 – Industry Factor Loadings for Quintile 1 and 5 as Reported in Table 1 and Table 2

The table reports industry factor loadings for the extreme portfolios presented in Table 1 and Table 2. The portfolios are regressed on eight different sectors and, as before, Panel A contains the value weighted portfolios and Panel B contains the equally weighted portfolios. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2012. Significant coefficients are boldfaced.

Another interesting observation is that all the high volatility portfolios have positive significant loadings on the IT industry, whereas only one of the low volatility portfolios is significantly different from zero. Based on the strong historical return of the IT sector in Norway (highest both for equally and value weighted portfolios) this could partly explain the high returns we observe for the high volatility portfolios. In the next section we assess this hypothesis further.

5.5 Excluding High Return Sectors from the Data Sample

To assess the hypothesis that the strong returns of the energy and IT industries are driving the high returns of the high volatility portfolios, and thus causing the absence of the low volatility anomaly in Norway, we run the 1/1-strategy on a data sample where we exclude stocks from these industries. These results are reported in Table 7. We now see that all mean returns are lower than reported in Table 1. For the value weighted portfolios the drop is higher for the two high volatility portfolios than for the others, supporting the hypothesis that IT and energy stocks are drivers of the high return in the high volatility portfolios. Nevertheless, after excluding energy- and IT, it still appears to be no low volatility effect in the remaining stocks.

Even though the relative performance of portfolio 4 and 5 is not as strong as before the low IVOL portfolios still show limited signs of outperforming the high volatility portfolios. For the VW portfolios most of the alphas are insignificant making it hard to draw any inferences about a clear relationship between IVOL and expected returns. The same goes when looking at the Sharpe Ratios since they increase from 0.09 to 0.16 going from portfolio 1 to 3, but then fall for portfolio 4 and then increases back to 0.16 for portfolio 5. For the equal weighted portfolios, the FF-3 alphas are significant for portfolios 1 to 4, but the alphas are all very similar with no clear pattern. Portfolio 5 has a slightly better alpha than the others, but it is insignificant. The Sharpe Ratios are also quite similar for portfolio 1 to 4 while portfolio 5 has a slightly higher Sharpe Ratio.

Based on this, we cannot infer that the historical strong performance of oil and IT firms in the Norwegian stock market causes the absence of the IVOL effect in Norway.

Panel A									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,62 %	6,72 %	0,09	42,9 %	1,07 %	0,0008	-0,005	-0,004	-0,004
							[0,00]	[0,01]	[0,01]
2	0,87 %	6,80 %	0,13	28,3 %	1,70 %	0,0013	-0,001	-0,002	-0,001
							[0,46]	[0,42]	[0,76]
3	1,22 %	7,76 %	0,16	16,0 %	2,28 %	0,0020	0,002	-0,001	0,001
							[0,44]	[0,79]	[0,73]
4	0,90 %	8,15 %	0,11	9,1 %	3,20 %	0,0027	-0,001	-0,004	-0,002
							[0,67]	[0,18]	[0,42]
5	1,21 %	7,54 %	0,16	3,7 %	6,02 %	0,0035	0,005	0,001	-0,001
							[0,10]	[0,80]	[0,74]

Panel B									
Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,41 %	4,81 %	0,09	42,9 %	1,07 %	0,0006	-0,003	-0,006	-0,006
							[0,03]	[0,00]	[0,00]
2	0,61 %	5,38 %	0,11	28,3 %	1,70 %	0,0007	-0,002	-0,005	-0,004
							[0,14]	[0,00]	[0,01]
3	0,69 %	6,14 %	0,11	16,0 %	2,28 %	0,0010	-0,002	-0,007	-0,006
							[0,26]	[0,00]	[0,00]
4	0,46 %	5,83 %	0,08	9,1 %	3,20 %	0,0012	-0,003	-0,008	-0,008
							[0,13]	[0,00]	[0,00]
5	0,95 %	6,20 %	0,15	3,7 %	6,02 %	0,0019	0,003	-0,003	-0,004
							[0,26]	[0,11]	[0,03]

Table 7 – Excluding Energy and IT Companies from the Sample

6 Conclusion

According to basic finance principles, investors expect higher returns when taking on extra systematic risk. The finding that low risk stocks outperform high risk stocks is therefore a candidate for one of the greatest anomalies in finance. In this paper we focus specifically on idiosyncratic risk. According to classic financial theory, idiosyncratic risk should not be priced in the cross-section of stock returns as it can be diversified away. Various theoretical departures from this paradigm postulate a positive relation between idiosyncratic risk and expected returns, the classical example being undiversified investors who demand a premium for the unsystematic risk component of their portfolios. It is therefore very surprising that more recent studies, beginning notably with Ang et al. (2006), find a negative relation between idiosyncratic volatility and returns. This has been known as the idiosyncratic volatility puzzle. Subsequent studies have proposed multiple explanations behind this anomalous relation, and the discussion still remains open. Nevertheless, the findings in Ang et al. (2006) do suggest a profitable trading strategy.

In this paper we are the first to explicitly analyse the idiosyncratic volatility puzzle using Norwegian data. We investigate the relation between idiosyncratic volatility and excess returns in the Norwegian stock market for the period 1981 – 2012. By utilizing the methodology developed by Ang et al. (2006), we show that the internationally documented strong performance of low volatility stocks relative to high volatility stocks is not present in Norway. Quite the contrary, we tend to find that returns increase monotonically from the low volatility quintile to the high volatility quintile. A similar pattern is observed for the FF-3 alphas, but they are often insignificant. Our findings are robust for exposure to size, liquidity, momentum and book-to-market effects. The results also hold for different subsamples, variations of methodological approach and various data filters. This leads us to conclude that there is no idiosyncratic volatility puzzle in Norway, a finding which is consistent with the classic asset pricing theory. This is a surprising result considering the body of literature who find that low volatility stocks outperform high volatility stocks. Assessing the industry composition in the Norwegian stock market, we find evidence that the high volatility portfolios consist of many energy- and IT companies. These sectors have done remarkably well over the sample period in the Norwegian market, thus an oil- and IT effect may partly explain our results. We test this hypothesis by running our tests without these industries included. While we see signs that these two industries contribute to the strong returns observed in the high volatility portfolios, they do not explain the absence of a low volatility effect in Norway.

Regarding future research, our findings have implication for papers that attempts to explain the reasons behind the idiosyncratic volatility puzzle. A better understanding of the particular features of the Norwegian market might contribute to understanding the key drivers of the anomaly in other markets. Understanding more about why the Norwegian results differ from other international markets is also a relevant topic for fund managers who are increasingly utilizing low volatility strategies.

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Appendix

Exhibit 1 – Security Samples

Year	1 month *F*	10% Filter	125 TD Req	24 month *F*	Excl NRG/IT	Baker Haugen	BAØ
1981	47	42	17	n/a	43	n/a	47
1982	57	51	17	n/a	50	n/a	58
1983	86	77	46	28	75	n/a	90
1984	113	102	68	59	97	n/a	121
1985	141	128	95	87	118	n/a	149
1986	134	121	90	108	111	n/a	146
1987	122	111	88	118	100	n/a	133
1988	107	97	61	104	87	n/a	113
1989	121	110	82	97	99	n/a	138
1990	134	121	81	88	108	60	149
1991	117	105	68	96	94	60	123
1992	97	87	51	108	76	54	101
1993	109	98	77	98	87	39	114
1994	131	119	88	89	106	38	143
1995	136	123	96	95	109	36	148
1996	152	138	118	107	119	38	169
1997	175	160	132	121	131	58	199
1998	177	160	118	141	129	85	190
1999	173	156	115	154	122	85	185
2000	166	151	119	134	116	71	183
2001	134	120	89	129	96	72	143
2002	114	102	66	137	86	102	119
2003	103	92	65	112	80	98	109
2004	125	113	91	96	88	97	132
2005	144	131	110	92	96	108	166
2006	172	156	139	106	107	142	195
2007	198	181	155	119	120	175	225
2008	141	127	101	149	92	194	150
2009	117	106	85	162	81	174	120
2010	135	122	101	117	99	158	144
2011	126	113	95	107	94	150	131
2012	118	106	84	123	87	n/a	122

The table reports the average yearly number of stocks in the data sample for different filtering methods. 1 month F is for all strategies where the formation period is one month. 10% Filter is the sample size when we exclude the decile with lowest market capitalization each year. 125 TD Req is the number of stocks when stocks are required to trade 125 days a year to enter the sample. 24 month F is for all strategies where the formation period is 24 months. Excl NRG/IT is the sample size when IT and energy companies (GICS code 10 and 45) are excluded. Baker Haugen reports the yearly sample size of the 2012 paper by Baker and Haugen. The last column is adapted from Bernt Arne Ødegaard (2013) and shows the universe of Norwegian stocks after applying his filters.

Exhibit II – Winsorization Statistics

Year	# of Returns	# of Returns Edited	0,1 Percentile	99,9% Percentile
1980	22 497	22	-22,2 %	23,5 %
1981	24 726	24	-17,9 %	25,0 %
1982	27 799	27	-22,6 %	24,9 %
1983	30 822	30	-22,9 %	27,7 %
1984	34 462	34	-21,3 %	26,4 %
1985	40 254	40	-20,1 %	26,6 %
1986	41 262	41	-22,3 %	27,2 %
1987	40 686	40	-28,6 %	33,3 %
1988	37 035	37	-37,5 %	56,2 %
1989	37 148	37	-33,4 %	42,8 %
1990	39 933	39	-30,6 %	37,5 %
1991	38 723	38	-36,1 %	53,8 %
1992	38 333	38	-50,0 %	100,0 %
1993	38 160	38	-50,0 %	100,0 %
1994	42 523	42	-28,0 %	40,0 %
1995	41 618	41	-28,1 %	33,3 %
1996	44 752	44	-23,1 %	32,1 %
1997	51 152	51	-19,4 %	14,5 %
1998	60 669	60	-33,4 %	42,8 %
1999	60 132	60	-30,5 %	50,0 %
2000	56 005	56	-30,0 %	46,8 %
2001	54 981	54	-20,8 %	61,2 %
2002	52 408	52	-45,0 %	69,8 %
2003	48 328	48	-50,0 %	88,7 %
2004	46 738	46	-25,0 %	33,3 %
2005	51 623	51	-15,4 %	25,2 %
2006	56 487	56	-16,7 %	23,6 %
2007	62 787	62	-14,9 %	19,4 %
2008	68 860	68	-35,5 %	50,0 %
2009	63 848	63	-47,4 %	80,0 %
2010	61 227	61	-25,0 %	38,4 %
2011	63 548	63	-31,0 %	47,9 %
2012	62 154	62	-28,6 %	41,5 %

The table reports the yearly number of returns, the number of returns replaced in each percentile and the values these returns are replaced with.

Exhibit III – Sorting Includes Scholes-Williams (1977) Beta

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,66 %	6,33 %	0,10	52,6 %	1,10 %	0,0006	-0,004	-0,003	-0,004
							[0,00]	[0,01]	[0,00]
2	1,10 %	6,77 %	0,16	21,4 %	1,74 %	0,0009	0,000	0,000	0,001
							[0,90]	[0,96]	[0,77]
3	1,22 %	7,93 %	0,15	13,7 %	2,33 %	0,0015	0,001	-0,001	0,000
							[0,80]	[0,54]	[0,97]
4	1,43 %	9,29 %	0,15	8,4 %	3,21 %	0,0035	0,003	0,002	0,003
							[0,18]	[0,57]	[0,24]
5	1,56 %	8,41 %	0,19	4,0 %	5,94 %	0,0039	0,007	0,004	0,003
							[0,03]	[0,22]	[0,31]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,48 %	4,99 %	0,10	52,6 %	1,10 %	0,0006	-0,003	-0,006	-0,006
							[0,02]	[0,00]	[0,00]
2	0,65 %	5,72 %	0,11	21,4 %	1,74 %	0,0007	-0,003	-0,005	-0,005
							[0,06]	[0,00]	[0,00]
3	0,74 %	6,76 %	0,11	13,7 %	2,33 %	0,0010	-0,003	-0,007	-0,007
							[0,18]	[0,00]	[0,00]
4	0,75 %	6,46 %	0,12	8,4 %	3,21 %	0,0013	-0,001	-0,006	-0,006
							[0,52]	[0,00]	[0,00]
5	0,91 %	6,39 %	0,14	4,0 %	5,94 %	0,0017	0,001	-0,005	-0,005
							[0,51]	[0,01]	[0,00]

The table reports quintile portfolios that are formed every month by sorting stocks on idiosyncratic volatility calculated from daily return data relative to the Fama-French (1993) model with one lead and one lag of the market beta. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2012.

Exhibit IV – Filtering Out One Decile

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,71 %	6,41 %	0,11	52,0 %	1,11 %	0,0006	-0,003	-0,002	-0,003
							[0,00]	[0,02]	[0,00]
2	0,91 %	6,78 %	0,13	21,6 %	1,73 %	0,0010	-0,001	-0,002	-0,001
							[0,47]	[0,39]	[0,50]
3	1,34 %	8,03 %	0,17	13,0 %	2,27 %	0,0016	0,002	0,000	0,001
							[0,39]	[0,93]	[0,58]
4	1,42 %	9,23 %	0,15	8,9 %	3,07 %	0,0032	0,003	0,001	0,003
							[0,26]	[0,74]	[0,29]
5	1,68 %	8,52 %	0,20	4,5 %	5,51 %	0,0040	0,008	0,006	0,005
							[0,01]	[0,07]	[0,14]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,56 %	5,25 %	0,11	52,0 %	1,11 %	0,0006	-0,003	-0,005	-0,005
							[0,04]	[0,00]	[0,00]
2	0,55 %	5,94 %	0,09	21,6 %	1,73 %	0,0008	-0,004	-0,006	-0,006
							[0,01]	[0,00]	[0,00]
3	0,85 %	6,91 %	0,12	13,0 %	2,27 %	0,0011	-0,002	-0,007	-0,006
							[0,42]	[0,00]	[0,00]
4	0,87 %	6,66 %	0,13	8,9 %	3,07 %	0,0012	-0,001	-0,005	-0,004
							[0,67]	[0,00]	[0,01]
5	1,13 %	6,54 %	0,17	4,5 %	5,51 %	0,0018	0,004	-0,003	-0,003
							[0,12]	[0,15]	[0,05]

The table reports quintile portfolios that are formed every month by sorting stocks on idiosyncratic volatility calculated from daily return data relative to the Fama-French (1993) model. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2012 and, in addition to the standard filters, 10 percent smallest of the available stocks are filtered out before every portfolio formation.

Exhibit V – Requiring 125 Trading Days

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,66 %	6,58 %	0,10	49,5 %	1,09 %	0,0007	-0,004	-0,003	-0,004
							[0,00]	[0,01]	[0,00]
2	0,85 %	6,90 %	0,12	21,2 %	1,64 %	0,0012	-0,002	-0,001	-0,001
							[0,28]	[0,54]	[0,75]
3	1,45 %	7,74 %	0,19	13,6 %	2,09 %	0,0016	0,003	0,002	0,003
							[0,10]	[0,29]	[0,16]
4	1,05 %	9,61 %	0,11	9,9 %	2,70 %	0,0032	-0,002	-0,004	-0,001
							[0,52]	[0,15]	[0,65]
5	1,32 %	9,87 %	0,13	5,9 %	4,33 %	0,0045	0,003	0,000	0,002
							[0,42]	[0,96]	[0,60]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,49 %	5,58 %	0,09	49,5 %	1,09 %	0,0007	-0,004	-0,005	-0,005
							[0,00]	[0,00]	[0,00]
2	0,54 %	6,18 %	0,09	21,2 %	1,64 %	0,0009	-0,004	-0,006	-0,005
							[0,01]	[0,00]	[0,00]
3	0,93 %	7,31 %	0,13	13,6 %	2,09 %	0,0013	-0,002	-0,005	-0,004
							[0,41]	[0,00]	[0,01]
4	0,42 %	8,12 %	0,05	9,9 %	2,70 %	0,0018	-0,007	-0,012	-0,010
							[0,00]	[0,00]	[0,00]
5	0,70 %	8,87 %	0,08	5,9 %	4,33 %	0,0031	-0,003	-0,009	-0,008
							[0,29]	[0,00]	[0,00]

The table reports quintile portfolios that are formed every month by sorting stocks on idiosyncratic volatility calculated from daily return data relative to the Fama-French (1993) model. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2012 and, in addition to the standard filters, stocks that have less than 125 trading days per year are filtered out.

Exhibit VI – Subsample 1990 – 2012

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,62 %	6,01 %	0,10	52,3 %	1,11 %	0,0006	-0,004	-0,002	-0,003
							[0,00]	[0,04]	[0,01]
2	0,92 %	6,88 %	0,13	21,9 %	1,77 %	0,0011	-0,001	-0,002	-0,001
							[0,62]	[0,52]	[0,62]
3	1,17 %	8,23 %	0,14	13,5 %	2,35 %	0,0016	0,000	-0,001	0,000
							[1,00]	[0,60]	[0,95]
4	1,45 %	9,38 %	0,15	8,4 %	3,23 %	0,0038	0,004	0,002	0,003
							[0,20]	[0,50]	[0,28]
5	1,75 %	8,65 %	0,20	4,0 %	6,06 %	0,0040	0,009	0,006	0,006
							[0,02]	[0,11]	[0,15]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,40 %	4,70 %	0,08	52,3 %	1,11 %	0,0006	-0,003	-0,005	-0,005
							[0,03]	[0,00]	[0,00]
2	0,51 %	5,74 %	0,09	21,9 %	1,77 %	0,0007	-0,004	-0,006	-0,006
							[0,02]	[0,00]	[0,00]
3	0,74 %	6,68 %	0,11	13,5 %	2,35 %	0,0010	-0,002	-0,007	-0,006
							[0,34]	[0,00]	[0,00]
4	0,74 %	6,02 %	0,12	8,4 %	3,23 %	0,0011	-0,001	-0,005	-0,005
							[0,69]	[0,00]	[0,01]
5	0,95 %	6,22 %	0,15	4,0 %	6,06 %	0,0014	0,002	-0,005	-0,005
							[0,44]	[0,01]	[0,00]

The table reports quintile portfolios that are formed every month by sorting stocks on idiosyncratic volatility calculated from daily return data relative to the Fama-French (1993) model. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is January 1990 to December 2012.

Exhibit VII – Subsample 2000 – 2012

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,67 %	5,95 %	0,11	53,5 %	1,10 %	0,0005	-0,003	-0,003	-0,004
							[0,03]	[0,04]	[0,01]
2	1,08 %	7,44 %	0,14	21,5 %	1,74 %	0,0011	0,000	-0,001	0,000
							[0,92]	[0,81]	[0,93]
3	1,27 %	8,37 %	0,15	13,1 %	2,29 %	0,0015	0,001	0,000	0,002
							[0,81]	[0,88]	[0,45]
4	1,35 %	8,90 %	0,15	8,0 %	3,11 %	0,0028	0,002	0,001	0,004
							[0,51]	[0,80]	[0,26]
5	1,98 %	8,54 %	0,23	3,8 %	5,70 %	0,0035	0,010	0,009	0,010
							[0,06]	[0,09]	[0,06]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,42 %	4,35 %	0,10	53,5 %	1,10 %	0,0004	-0,003	-0,005	-0,005
							[0,11]	[0,01]	[0,01]
2	0,62 %	5,51 %	0,11	21,5 %	1,74 %	0,0005	-0,003	-0,005	-0,004
							[0,18]	[0,02]	[0,07]
3	0,93 %	6,28 %	0,15	13,1 %	2,29 %	0,0009	0,000	-0,003	-0,001
							[0,95]	[0,28]	[0,56]
4	0,73 %	5,82 %	0,13	8,0 %	3,11 %	0,0009	-0,001	-0,004	-0,003
							[0,68]	[0,06]	[0,21]
5	0,81 %	5,27 %	0,15	3,8 %	5,70 %	0,0009	0,001	-0,002	-0,002
							[0,66]	[0,28]	[0,37]

The table reports quintile portfolios that are formed every month by sorting stocks on idiosyncratic volatility calculated from daily return data relative to the Fama-French (1993) model. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is January 2000 to December 2012.

Exhibit VIII – Subsample 1981 – 2000

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,76 %	6,73 %	0,11	46,6 %	1,12 %	0,0007	-0,004	-0,002	-0,002
							[0,01]	[0,20]	[0,11]
2	0,85 %	6,12 %	0,14	23,8 %	1,77 %	0,0008	-0,001	-0,003	-0,003
							[0,46]	[0,16]	[0,18]
3	1,39 %	7,91 %	0,18	15,1 %	2,38 %	0,0014	0,002	-0,002	-0,001
							[0,46]	[0,43]	[0,72]
4	1,35 %	9,66 %	0,14	9,8 %	3,31 %	0,0039	0,002	-0,001	0,000
							[0,47]	[0,80]	[0,92]
5	1,47 %	8,50 %	0,17	4,6 %	6,11 %	0,0043	0,007	0,002	-0,001
							[0,12]	[0,66]	[0,82]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,61 %	5,46 %	0,11	46,6 %	1,12 %	0,0007	-0,003	-0,005	-0,006
							[0,17]	[0,00]	[0,00]
2	0,57 %	5,94 %	0,10	23,8 %	1,77 %	0,0009	-0,004	-0,007	-0,007
							[0,07]	[0,00]	[0,00]
3	0,68 %	6,99 %	0,10	15,1 %	2,38 %	0,0010	-0,004	-0,010	-0,010
							[0,18]	[0,00]	[0,00]
4	0,60 %	6,78 %	0,09	9,8 %	3,31 %	0,0013	-0,003	-0,010	-0,010
							[0,21]	[0,00]	[0,00]
5	1,06 %	7,15 %	0,15	4,6 %	6,11 %	0,0023	0,003	-0,006	-0,008
							[0,48]	[0,02]	[0,00]

The table reports quintile portfolios that are formed every month by sorting stocks on idiosyncratic volatility calculated from daily return data relative to the Fama-French (1993) model. The portfolios are rebalanced every month. Portfolio 1 (5) is the portfolio of the stocks with the lowest (highest) volatility. Panel A present value weighted portfolios, whereas in Panel B equally weighted portfolios are presented. The columns Mean and Ex-Post Std Dev are measured monthly and the Sharpe Ratio is the Mean divided by the Ex-Post Std Dev, as both figures are based on excess return. Market Share reports the share of the total market value represented by the stocks in the quintile and is calculated as the average market capitalization each year across the stocks in each quintile and then averaged over all years in the sample. F month ISD is the average idiosyncratic standard deviation of the quintile's formation month and is calculated by averaging the standard deviations each month across the stocks in each portfolio and then average over all the months in the sample. Ex-Post IVOL refer to the portfolios realized idiosyncratic volatility. The Alpha columns reports Jensen's Alpha with respect to different factor models. P-values based on robust Newey and West (1987) t-statistics are reported in square brackets. The sample period is July 1981 to December 2000.

Exhibit IX – Industry Descriptives (Adapted from Ødegaard (2013))

Industry	# of firms	% of value	Avg VW Ret	Std Dev (VW)	Avg EW Ret	Std Dev (EW)
Energy	30.7	24.0	1.87	8.14	2.21	9.45
Material	10.8	6.4	1.49	12.11	1.67	11.86
Industry	50.2	29.7	1.75	7.54	1.69	6.22
ConsDisc	16.7	5.8	2.31	10.50	1.63	7.25
ConsStapl	8.6	7.4	2.13	7.56	1.92	6.66
Health	5.5	5.4	2.81	22.95	1.97	11.76
Finance	37.7	16.4	1.51	6.98	1.21	5.06
IT	23.5	5.4	3.15	13.69	2.40	11.03

Exhibit X – Tables/Exhibits Calculated with Non-Winsorized Data

Table 1

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,74 %	6,43 %	0,11	52,2 %	1,11 %	0,0006	-0,003	-0,002	-0,003
							[0,00]	[0,03]	[0,00]
2	0,93 %	6,70 %	0,14	22,0 %	1,76 %	0,0010	-0,001	-0,001	-0,001
							[0,59]	[0,50]	[0,68]
3	1,37 %	8,11 %	0,17	13,5 %	2,35 %	0,0016	0,002	0,000	0,001
							[0,37]	[0,83]	[0,58]
4	1,38 %	9,36 %	0,15	8,4 %	3,23 %	0,0035	0,003	0,001	0,002
							[0,28]	[0,79]	[0,38]
5	1,61 %	8,91 %	0,18	4,0 %	6,10 %	0,0046	0,008	0,005	0,004
							[0,03]	[0,15]	[0,28]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,54 %	5,14 %	0,10	52,2 %	1,11 %	0,0007	-0,003	-0,005	-0,005
							[0,07]	[0,00]	[0,00]
2	0,58 %	5,78 %	0,10	22,0 %	1,76 %	0,0007	-0,003	-0,006	-0,005
							[0,02]	[0,00]	[0,00]
3	0,81 %	6,70 %	0,12	13,5 %	2,35 %	0,0010	-0,002	-0,007	-0,006
							[0,33]	[0,00]	[0,00]
4	0,67 %	6,43 %	0,10	8,4 %	3,23 %	0,0012	-0,002	-0,007	-0,007
							[0,25]	[0,00]	[0,00]
5	1,01 %	6,66 %	0,15	4,0 %	6,10 %	0,0019	0,002	-0,004	-0,005
							[0,33]	[0,02]	[0,00]

Table 5

Panel A				
	1/1-Strategy (daily returns)		24/1-Strategy (monthly returns)	
Portfolio	1	5	1	5
Size	20,09	18,39	19,83	18,62
Alpha	**-0,003**	0,004	**-0,004**	0,022
	[0,00]	[0,28]	[0,00]	[0,18]
Beta MRK	**0,910**	**1,090**	**0,890**	**1,407**
	[0,00]	[0,00]	[0,00]	[0,00]
Beta SMB	**-0,146**	0,029	-0,135	**0,383**
	[0,00]	[0,81]	[0,05]	[0,02]
Beta HML	0,019	-0,028	0,024	-0,270
	[0,56]	[0,73]	[0,59]	[0,28]
Beta PR1YR	**0,098**	0,017	0,057	-0,329
	[0,00]	[0,85]	[0,09]	[0,06]
Beta LIQ	0,060	**0,424**	0,097	0,272
	[0,32]	[0,00]	[0,19]	[0,25]

Panel B				
	1/1-Strategy (daily returns)		24/1-Strategy (monthly returns)	
Portfolio	1	5	1	5
Size	20,09	18,39	19,83	18,62
Alpha	**-0,005**	-0,005	**-0,007**	0,010
	[0,00]	[0,63]	[0,00]	[0,63]
Beta MRK	**0,703**	**1,048**	**0,663**	**1,354**
	[0,00]	[0,00]	[0,00]	[0,00]
Beta SMB	**0,171**	**0,269**	**0,153**	**0,809**
	[0,00]	[0,00]	[0,01]	[0,00]
Beta HML	**0,100**	0,022	**0,113**	-0,138
	[0,00]	[0,54]	[0,00]	[0,54]
Beta PR1YR	0,054	**-0,142**	0,002	**-0,520**
	[0,08]	[0,02]	[0,94]	[0,02]
Beta LIQ	-0,073	**0,615**	0,039	**0,642**
	[0,21]	[0,01]	[0,51]	[0,01]

Table 6

Panel A

	1/1-Strategy (daily returns)		24/1-Strategy (monthly retruns)	
Portfolio	1	5	1	5
Alpha	**-0,007**	0,003	**-0,007**	0,025
	[0,00]	[0,28]	[0,00]	[0,20]
Beta Energy	**0,262**	**0,315**	**0,262**	**0,460**
	[0,00]	[0,00]	[0,00]	[0,00]
Beta Material	0,014	-0,018	0,009	0,021
	[0,54]	[0,55]	[0,59]	[0,72]
Beta Industry	**0,379**	0,112	**0,393**	-0,308
	[0,00]	[0,18]	[0,00]	[0,26]
Beta ConsDisc	-0,011	0,045	**-0,054**	0,087
	[0,68]	[0,35]	[0,01]	[0,37]
Beta ConsStapl	**0,114**	0,040	**0,096**	-0,575
	[0,00]	[0,61]	[0,00]	[0,26]
Beta Health	**0,009**	**-0,015**	**0,008**	0,000
	[0,04]	[0,00]	[0,00]	[0,98]
Beta Finance	**0,127**	**0,238**	**0,125**	0,665
	[0,00]	[0,00]	[0,00]	[0,14]
Beta IT	0,003	**0,182**	0,020	**0,645**
	[0,81]	[0,00]	[0,22]	[0,00]

Panel B

	1/1-Strategy (daily returns)		24/1-Strategy (monthly retruns)	
Portfolio	1	5	1	5
Alpha	**-0,006**	0,000	**-0,007**	0,021
	[0,00]	[0,90]	[0,00]	[0,39]
Beta Energy	**0,164**	**0,277**	**0,131**	**0,438**
	[0,00]	[0,00]	[0,00]	[0,01]
Beta Material	0,008	**0,043**	0,029	-0,003
	[0,79]	[0,04]	[0,17]	[0,96]
Beta Industry	**0,115**	0,021	**0,090**	-0,304
	[0,00]	[0,69]	[0,00]	[0,36]
Beta ConsDisc	0,055	0,051	0,000	0,017
	[0,11]	[0,11]	[1,00]	[0,88]
Beta ConsStapl	**0,134**	-0,051	**0,107**	-0,805
	[0,00]	[0,27]	[0,00]	[0,22]
Beta Health	0,001	0,002	0,003	-0,002
	[0,89]	[0,70]	[0,53]	[0,84]
Beta Finance	**0,197**	**0,231**	**0,244**	1,084
	[0,00]	[0,00]	[0,00]	[0,07]
Beta IT	0,025	**0,131**	0,020	**0,345**
	[0,07]	[0,00]	[0,14]	[0,00]

Exhibit III

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,69 %	6,35 %	0,11	52,6 %	1,10 %	0,0006	-0,004	-0,003	-0,003
							[0,00]	[0,01]	[0,00]
2	1,10 %	6,78 %	0,16	21,4 %	1,74 %	0,0009	0,000	0,000	0,001
							[0,90]	[0,96]	[0,77]
3	1,23 %	7,95 %	0,15	13,7 %	2,33 %	0,0015	0,001	-0,001	0,000
							[0,76]	[0,56]	[0,99]
4	1,46 %	9,31 %	0,16	8,4 %	3,21 %	0,0036	0,004	0,002	0,003
							[0,14]	[0,49]	[0,20]
5	1,47 %	8,80 %	0,17	4,0 %	6,10 %	0,0045	0,006	0,003	0,003
							[0,07]	[0,34]	[0,47]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,50 %	5,11 %	0,10	52,6 %	1,10 %	0,0007	-0,003	-0,005	-0,005
							[0,04]	[0,00]	[0,00]
2	0,65 %	5,72 %	0,11	21,4 %	1,74 %	0,0007	-0,003	-0,005	-0,005
							[0,05]	[0,00]	[0,00]
3	0,75 %	6,78 %	0,11	13,7 %	2,33 %	0,0010	-0,003	-0,007	-0,007
							[0,19]	[0,00]	[0,00]
4	0,77 %	6,49 %	0,12	8,4 %	3,21 %	0,0013	-0,001	-0,006	-0,006
							[0,56]	[0,00]	[0,00]
5	0,94 %	6,62 %	0,14	4,0 %	6,10 %	0,0018	0,002	-0,005	-0,005
							[0,47]	[0,01]	[0,00]

Exhibit IV

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,74 %	6,43 %	0,12	52,0 %	1,11 %	0,0006	-0,003	-0,002	-0,003
							[0,00]	[0,04]	[0,01]
2	0,91 %	6,79 %	0,13	21,6 %	1,73 %	0,0010	-0,001	-0,002	-0,001
							[0,47]	[0,40]	[0,50]
3	1,35 %	8,05 %	0,17	13,0 %	2,27 %	0,0016	0,002	0,000	0,001
							[0,36]	[0,95]	[0,56]
4	1,44 %	9,22 %	0,16	8,9 %	3,07 %	0,0032	0,003	0,001	0,003
							[0,21]	[0,66]	[0,24]
5	1,59 %	8,90 %	0,18	4,5 %	5,65 %	0,0045	0,007	0,005	0,004
							[0,02]	[0,13]	[0,25]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,57 %	5,37 %	0,11	52,0 %	1,11 %	0,0007	-0,003	-0,005	-0,005
							[0,07]	[0,00]	[0,00]
2	0,55 %	5,96 %	0,09	21,6 %	1,73 %	0,0008	-0,004	-0,006	-0,006
							[0,01]	[0,00]	[0,00]
3	0,86 %	6,93 %	0,12	13,0 %	2,27 %	0,0011	-0,002	-0,007	-0,006
							[0,44]	[0,00]	[0,00]
4	0,89 %	6,67 %	0,13	8,9 %	3,07 %	0,0012	-0,001	-0,005	-0,004
							[0,75]	[0,00]	[0,01]
5	1,13 %	6,75 %	0,17	4,5 %	5,65 %	0,0019	0,003	-0,003	-0,004
							[0,13]	[0,13]	[0,04]

Exhibit V

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,69 %	6,60 %	0,10	49,5 %	1,09 %	0,0007	-0,004	-0,003	-0,004
							[0,00]	[0,03]	[0,00]
2	0,85 %	6,91 %	0,12	21,2 %	1,64 %	0,0012	-0,002	-0,001	-0,001
							[0,28]	[0,53]	[0,74]
3	1,45 %	7,75 %	0,19	13,6 %	2,09 %	0,0016	0,003	0,002	0,003
							[0,10]	[0,29]	[0,16]
4	1,07 %	9,63 %	0,11	9,9 %	2,70 %	0,0033	-0,001	-0,004	-0,001
							[0,59]	[0,19]	[0,73]
5	1,34 %	9,93 %	0,13	5,9 %	4,39 %	0,0046	0,003	0,000	0,002
							[0,41]	[0,94]	[0,60]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,52 %	5,73 %	0,09	49,5 %	1,09 %	0,0008	-0,004	-0,005	-0,005
							[0,01]	[0,00]	[0,00]
2	0,53 %	6,22 %	0,08	21,2 %	1,64 %	0,0009	-0,004	-0,006	-0,006
							[0,00]	[0,00]	[0,00]
3	0,93 %	7,33 %	0,13	13,6 %	2,09 %	0,0013	-0,002	-0,005	-0,004
							[0,41]	[0,00]	[0,01]
4	0,43 %	8,14 %	0,05	9,9 %	2,70 %	0,0018	-0,007	-0,012	-0,010
							[0,00]	[0,00]	[0,00]
5	0,70 %	8,95 %	0,08	5,9 %	4,39 %	0,0032	-0,003	-0,009	-0,008
							[0,29]	[0,00]	[0,00]

Exhibit VI

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,66 %	6,03 %	0,11	52,3 %	1,11 %	0,0006	-0,003	-0,002	-0,002
							[0,01]	[0,09]	[0,03]
2	0,92 %	6,89 %	0,13	21,9 %	1,77 %	0,0011	-0,001	-0,001	-0,001
							[0,62]	[0,53]	[0,62]
3	1,18 %	8,23 %	0,14	13,5 %	2,35 %	0,0016	0,000	-0,001	0,000
							[0,98]	[0,61]	[0,97]
4	1,49 %	9,36 %	0,16	8,4 %	3,23 %	0,0038	0,004	0,002	0,003
							[0,16]	[0,43]	[0,23]
5	1,73 %	8,86 %	0,20	4,0 %	6,22 %	0,0042	0,008	0,006	0,005
							[0,04]	[0,14]	[0,20]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,42 %	4,86 %	0,09	52,3 %	1,11 %	0,0007	-0,003	-0,004	-0,004
							[0,08]	[0,01]	[0,01]
2	0,50 %	5,76 %	0,09	21,9 %	1,77 %	0,0007	-0,004	-0,006	-0,006
							[0,02]	[0,00]	[0,00]
3	0,74 %	6,68 %	0,11	13,5 %	2,35 %	0,0010	-0,002	-0,007	-0,006
							[0,36]	[0,00]	[0,01]
4	0,75 %	6,04 %	0,12	8,4 %	3,23 %	0,0011	-0,001	-0,005	-0,005
							[0,74]	[0,01]	[0,01]
5	1,01 %	6,36 %	0,16	4,0 %	6,22 %	0,0014	0,003	-0,005	-0,005
							[0,37]	[0,02]	[0,01]

Exhibit VII

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,67 %	5,95 %	0,11	53,5 %	1,10 %	0,0005	-0,003	-0,003	-0,004
							[0,03]	[0,03]	[0,01]
2	1,07 %	7,45 %	0,14	21,5 %	1,74 %	0,0011	0,000	-0,001	0,000
							[0,90]	[0,80]	[0,92]
3	1,27 %	8,36 %	0,15	13,1 %	2,29 %	0,0015	0,001	0,001	0,003
							[0,79]	[0,87]	[0,44]
4	1,36 %	8,90 %	0,15	8,0 %	3,11 %	0,0028	0,003	0,001	0,004
							[0,50]	[0,78]	[0,24]
5	1,93 %	8,89 %	0,22	3,8 %	5,82 %	0,0038	0,010	0,008	0,008
							[0,09]	[0,16]	[0,11]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,38 %	4,43 %	0,09	53,5 %	1,10 %	0,0004	-0,003	-0,005	-0,005
							[0,10]	[0,01]	[0,01]
2	0,60 %	5,55 %	0,11	21,5 %	1,74 %	0,0005	-0,003	-0,005	-0,004
							[0,15]	[0,01]	[0,05]
3	0,93 %	6,28 %	0,15	13,1 %	2,29 %	0,0009	0,000	-0,003	-0,001
							[0,97]	[0,30]	[0,58]
4	0,72 %	5,86 %	0,12	8,0 %	3,11 %	0,0009	-0,001	-0,004	-0,003
							[0,65]	[0,05]	[0,21]
5	0,71 %	5,41 %	0,13	3,8 %	5,82 %	0,0010	0,000	-0,004	-0,003
							[0,94]	[0,09]	[0,12]

Exhibit VIII

Panel A

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,81 %	6,76 %	0,12	46,6 %	1,12 %	0,0007	-0,003	-0,001	-0,002
							[0,02]	[0,36]	[0,21]
2	0,86 %	6,13 %	0,14	23,8 %	1,77 %	0,0008	-0,001	-0,003	-0,003
							[0,47]	[0,16]	[0,19]
3	1,41 %	7,96 %	0,18	15,1 %	2,38 %	0,0015	0,002	-0,002	-0,001
							[0,41]	[0,49]	[0,79]
4	1,40 %	9,71 %	0,14	9,8 %	3,31 %	0,0040	0,003	0,000	0,000
							[0,38]	[0,92]	[0,96]
5	1,37 %	8,95 %	0,15	4,6 %	6,30 %	0,0051	0,006	0,001	-0,002
							[0,18]	[0,79]	[0,68]

Panel B

Rank	Mean	Ex-Post Std Dev	Sharpe Ratio	Market Share	F month ISD	Ex-Post IVOL	CAPM Alpha	FF-3 Alpha	5-F Alpha
1	0,66 %	5,60 %	0,12	46,6 %	1,12 %	0,0008	-0,002	-0,005	-0,005
							[0,30]	[0,01]	[0,01]
2	0,57 %	5,95 %	0,10	23,8 %	1,77 %	0,0009	-0,004	-0,007	-0,007
							[0,07]	[0,00]	[0,00]
3	0,70 %	6,99 %	0,10	15,1 %	2,38 %	0,0010	-0,004	-0,010	-0,010
							[0,21]	[0,00]	[0,00]
4	0,62 %	6,83 %	0,09	9,8 %	3,31 %	0,0014	-0,003	-0,009	-0,010
							[0,24]	[0,00]	[0,00]
5	1,19 %	7,42 %	0,16	4,6 %	6,30 %	0,0025	0,004	-0,005	-0,007
							[0,30]	[0,07]	[0,00]

Preliminary Thesis Report

Kenneth Østnes
Håkon Hafskjær

BI Norwegian Business School
Preliminary Thesis Report

Working Title:
- Low Volatility Strategies in Norway -

Supervisor:
Bruno Gerard

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Introduction

One of the most commonly accepted relationships in financial markets is the correlation between risk and (expected) return. The basic capital asset pricing model (CAPM) expresses risk as covariance with the market and furthermore outlines that all agents will invest in the portfolio which gives the highest return per unit of risk, see among others Sharpe (1964), Lintner (1965) and Mossin (1966). It was early discovered that the security market line for U.S. stocks is flatter than predicted by the CAPM (Black, Jensen, and Scholes 1972), and in recent years additional studies have been conducted to explore the relationship between past volatility (a measure for risk) and returns. One finding is that low-volatility stocks have a tendency to earn too high risk-adjusted returns, illustrated by showing that they have a significantly higher Sharpe ratio than stocks with higher volatility. Important contributions regarding this phenomenon are made by Ang et al. (2006), Clarke, de Silva, and Thorley (2006), Blitz and van Vliet (2007), Ang et al. (2009), Scherer (2010), Baker, Bradley, and Wurgler (2011), Blitz and van Vliet (2011) and Frazzini and Pedersen (2011), and will be covered in greater detail later.

In addition to present empirical evidence for the phenomenon, these studies also control for many possible factors that one initially would think could diminish the effect, such as the CAPM, Fama and French factors, the momentum effect, and others. As the effect is still present after controlling for multiple factors, possible reasons for the over-performance of low-volatility stocks are also outlined. The finding that idiosyncratic volatility appears to be negatively related to relative returns and the finding of a similar relationship between total volatility and relative returns, are questioning the fundamentals of financial theory and asset pricing models. The latter relationship opposes the commonly accepted link between risk and return. The finding that high idiosyncratic volatility stocks tends to have low risk adjusted returns, is as a pure anomaly since idiosyncratic risk is diversified away in the classic asset pricing models. Even if we acknowledge that investors may not be perfectly diversified, the finding can still be classified as an anomaly considering the insights of Merton (1987) and Levy (1978) who propose that the relationship between idiosyncratic volatility and return should be positive in the presence of undiversified investors.

As the research conducted up to date is mostly focused on U.S. and international markets, our main contribution is to test whether the negative relationship between past idiosyncratic volatility and relative returns also is present in the Norwegian stock market. Furthermore we want to outline some of the possible explanations for why this holds (or does not hold) for Norway. The last objective of this paper is to search for and device trading strategies that exploits the high risk-adjusted return of low-volatility stocks, and test if any such strategies could be applicable in practice.

To address this issue in the Norwegian stock market we use daily/monthly data from Oslo Stock Exchange from 1980 until 2011/2012. The approach we use is similar to that of Ang et al. (2006) and the first step is to find the volatility of all stocks for the first five years (estimation period) and group the stocks into four portfolios based on their volatility.[1] After this period we compare the return of the different portfolios every year (and rebalance the portfolios), to find if there is a significant difference between risk-adjusted returns of the low-volatility portfolio and the returns of the high-volatility portfolio.

As we have not conducted any analysis yet, we do not present any results in this preliminary report. However, our hypothesis is that the low-volatility high return phenomenon is also present in Norway, as it is highly significant in other markets. Considering this we hope to use most of our efforts on analysing the reasons and device trading strategies. The rest of this preliminary report is organized as following; firstly, we review recent literature on the subject, secondly, we outline the methodological approach we use in this study, and finally, we provide a brief introduction of the data we are going to use.

[1] This is only an example of our approach, the number of years in the estimation period and the number of portfolios might be different. See discussion in the methodology section.

Literature Review

As the relationship between risk and return is a critical area in finance, this subject has been heavily studied. As introduced earlier the initial publications regarding the flatness of the security market line appeared in the seventies, but for the purpose of this paper we focus on some of the more recent contributions[2]. The reason for this is that they lay the fundament for our thesis and we want to apply their methodological frameworks, particularly that of Ang et al. (2006)

Ang et al. (2006) examine the pricing of aggregate volatility risk in the cross-section of stock returns. The first goal of their paper is to investigate how the stochastic volatility of the market is priced in the cross-section of expected stock returns. I.e. they want to estimate whether volatility is a priced risk factor and estimate the price of aggregate volatility risk. With regards to this goal, they find that stocks with high sensitivities to innovations in aggregate volatility have low average returns. They estimate a cross-sectional price of volatility of approximately -1% per annum, this estimate is robust when controlling for size, value, momentum and liquidity effects. The result is significant, but due to small sample size and the small size of the negative risk premium, a potential Peso problem[3] cannot be ruled out.

The second goal of their paper is to examine the cross-sectional relationship between idiosyncratic volatility and expected returns, where idiosyncratic volatility is defined relative to the Fama and French (1993) model. The results show that stocks with high idiosyncratic volatility have "abysmally" low average returns. They control for a number of factors and conclude that the result cannot be explained by exposures to size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, coskewness, or dispersion in analysts' forecasts. They show that the results hold in bull and bear markets, NBER recessions and expansions, volatile and stable periods, and under different formation and holding periods as long as one year. It can therefore be said that the cross-sectional

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[2] There are several other papers on this subject and some of them will naturally be added as we familiarize ourselves more with both the data and the subject in general.

[3] A Peso Problem in this case refers to the fact that the article's result would have been different if the sample used had experienced one more volatility spike (such as the Peso crisis).

expected return patterns found by sorting on idiosyncratic volatility presents something of a puzzle.

In their 2009 article Ang et al. extends the scope of their 2006 article and investigate whether the relation between lagged idiosyncratic volatility and future average returns found in U.S. data also exists in other markets. They find that low returns for stocks with high past idiosyncratic volatility is observed world-wide, suggesting that the results from Ang et al. (2006) is not just a country-specific nor a sample-specific effect. Stocks across 23 countries are sorted on past idiosyncratic volatility, and the difference in alphas between the highest- and the lowest quintile of idiosyncratic volatility stocks is a very large -1.31% per month and very significant. This is after adjusting for market, size and book-to-market factors.

In addition the study investigates the degree of international comovement in returns of stocks with high idiosyncratic volatility. They find that the low returns earned by stocks with high idiosyncratic volatility commove significantly with the idiosyncratic volatility effect in the U.S. meaning that the global idiosyncratic volatility effect is captured by a simple U.S. idiosyncratic volatility factor.

Ang et al. (2009) also introduces new controls on factors that might explain the anomaly. By using the U.S. data, the 2009 article investigates possible explanations for the anomaly such as trading or clientele structures, higher moments, information dissemination, and the leverage interaction story of Johnson (2004). These hypotheses are generally rejected and the article concludes that further studies are needed to investigate if there are true sources of economic risk that lies behind the phenomena causing stocks with high idiosyncratic volatility to low expected returns.

In contrast to Ang et al. (2006; 2009), Fu (2009) show that idiosyncratic volatilities are time-varying and that the one-month lagged value is not a good proxy for the expected value. Therefore the findings of Ang et al. (2006; 2009) should not be used to imply the relation between idiosyncratic risk and expected return. Fu (2009) uses an EGARCH to estimate expected idiosyncratic volatilities and, using those findings, show a significantly positive relation between the estimated conditional idiosyncratic volatilities and expected returns. He further suggests that Ang et al.'s (2006; 2009) findings are largely explained by the return reversal of a subset of small stocks with high idiosyncratic volatilities.

Another study that indicate potential for low-volatility strategies is Clarke, de Silva, and Thorley (2006) who construct minimum-variance portfolios using a large set of U.S. equities, and examine the realized return statistics over several decades. They find that minimum-variance portfolios that do not rely on any expected return theory or return forecasting signal show promise in terms of adding value over the market-capitalization weighted benchmark. More specifically they find that realized standard deviation is lowered by one-fourth, and risk measured by market beta is lowered by about one-third compared to the capitalization weighted benchmark. In other words the minimum-variance portfolios are capable of delivering similar or higher returns than the market portfolio at a substantially lower risk level. The authors comment that their results are consistent with the findings of Ang et al. (2006) regarding the low average returns of stocks with high idiosyncratic volatility. They also highlight that the minimum variance portfolios tend to have a value and a small size bias. But when controlling for these biases, the realized Sharpe ratios of the minimum-variance portfolios are still relatively high.

Blitz and van Vliet (2007) find that stocks with a low historical volatility exhibit significantly higher risk adjusted returns. The volatility effect is particularly strong in a global setting, with a low versus high volatility alpha spread of 12%. In the sample used in the article (December 1985 - January 2006) the authors find alpha for portfolios ranked on beta, but this alpha is considerably less than for portfolios ranked on volatility. The volatility effect is similar in size to the value, size and momentum effect and the higher risk adjusted returns from the low volatility stocks is still present after making Fama-French adjustments and double sorts. The results are consistent with Ang et al. (2006) and compared to Clarke et al. (2006), this study find significantly lower risk and superior Sharpe ratios for U.S. minimum-variance portfolios, but they note that they are using an easier approach than the Clarke et al. (2006) study. After showing the significance of the volatility effect Blitz and van Vliet (2007) offer several possible explanations for the phenomenon. The difficulty of applying the amount of leverage needed in order to arbitrage away the effect may explain why it is there. Another reason is that asset managers have an incentive to tilt towards high beta stocks since this can be a simple way of generating higher than average returns. At least when assuming they cannot apply enough leverage on a portfolio of low volatility stocks. Lastly, the article offers a behavioural explanation by referring to Shefrin and Statman (2000). Investor's deviation from risk-averse behaviour may cause high-risk stocks to be overpriced and low risk stocks to be underpriced. The reasoning is that investors will overpay for stocks they perceive as lottery tickets, because they would like a shot at the riches.

Scherer (2010) provides "a new look at minimum variance investing" and seeks to explain the variation of the excess returns of the minimum variance portfolio, relative to a capitalization weighted alternative, by using the Fama-French factors and two characteristic anomaly portfolios. In other words, the article want to test the hypothesis that the excess returns of the minimum variance portfolio are a function of risk related factor portfolios.

The article show that 83% of the variation of the variation of the minimum variance portfolio can be attributed to the proposed factors/anomaly portfolios. All variables are regressed on excess returns of the minimum variable portfolio, and they are all highly significant and have a sign in line with expectations. The coefficient for market returns is negative, which is intuitive as low volatility portfolios are likely to underperform in bull markets. The coefficient for the factor book-to-market (HML) is positive, in line with the idea that low volatility investing is often associated with "value investing". The coefficient for the size factor (SMB) is negative, as MVP by construction will prefer small companies that tends to be more diversified (implying lower risk). The coefficient for the small beta versus large beta portfolio (the first of the characteristic anomaly portfolios) is positive. The last coefficient, a portfolio representing the residual risk anomaly, is also positive. This coefficient is in line with the findings of Ang et al. (2006) and it is positive when regressed on the excess returns of the minimum variance portfolio.

In a relatively new study, Frazzini and Pedersen (2011) further explore the relationship between beta and returns. Firstly, they show that investing in high beta assets gives a low alpha. Furthermore, as leverage is central to exploit the mispricing of low beta assets, they prove that the return on betting against beta is lower when funding liquidity worsens and betas are compressed towards one.

Finally, a discussion regarding different types of investors (and their ability to use leverage) is provided. Here the difference between constrained investors (mutual funds and individual investors) and more unconstrained investors (LBO funds and Buffet's Berkshire Hathaway) are used to illustrate that leverage constraints have the hypothesized effects on agents' portfolio selection.

In terms of explaining the success of low-volatility and low-beta stock portfolios, behavioural finance might be able to shed light on potential drivers behind the anomaly. Baker, Bradley, and Wurgler (2011) argue that irrational investors overpay for risky stocks and avoid low risk stocks due to behavioural biases such as individual's preferences for lotteries, representativeness and overconfidence. The mispricing that occurs from such behaviour should in theory be erased by the "smart money" but the authors argue that professional investment managers are not able to fully exploit this apparent mispricing, due to several constraints. For instance one would think that shorting the high volatility quintile portfolio makes sense, but this portfolio is typically compromised of small stocks which are costly to trade in large quantities. On the other end, the professional money managers should at least overweight the low volatility quintile, but this is likely limited by benchmarking. I.e. a manager that needs to beat a certain benchmark without using too much leverage has incentives to pick stocks with higher volatility to achieve this. Thus, the manager will be reluctant to overweight stocks with high alpha and low beta or underweight low alpha and high beta stocks. This finding is consistent with the average mutual fund beta of 1.10 over the last 10 years. Because of this, they argue that as long as fixed benchmark contracts remain, and the share of the market held by investment managers continue to be high, then there is no reason that the anomaly will go away anytime soon.

Blitz and van Vliet (2011) address the issue on how to measure performance of investment managers who have adapted a low-volatility strategy. This is relevant for our thesis since we will attempt to device and test such strategies. In addition, using a proper benchmark to evaluate performance is an important issue, considering the insights by Baker, Bradley, and Wurgler (2011) who argue that investment managers often are not incentivized to pursue profitable low-volatility strategies because their performance are compared to indexes which make them pick more volatile stocks to boost returns.

Blitz and van Vliet (2011) argue that the most robust approach is to benchmark low volatility strategies against the capitalization-weighted market portfolio, using risk-adjusted performance metrics such as the Sharpe ratio, or Jensen's Alpha. This approach recognizes that the goal of low-volatility investing is to achieve a superior risk/return relationship compared to a passive investment in the capitalization-weighted market index. The authors also propose that the use of certain, arguably intuitive benchmarks are not particularly useful. E.g. the Markowitz' minimum-variance portfolio, approximations to the theoretical minimum-variance portfolio and the MSCI Minimum Volatility Index.

Methodology

We draw on the method used by Ang et al. (2006) where idiosyncratic risk is defined as the standard deviation of the error term in the Fama French 3 factor model (hereafter FF-3), see equation (1).

$$r_{i,t} - r_{f,t} = \alpha_{it} + \beta_{i,t}\left(r_{m,t} - r_{f,t}\right) + s_{i,t}SMB_t + h_{i,t}HML_t + \varepsilon_{i,t} \tag{1}$$

Where $\left(r_{i,t} - r_{f,t}\right)$ is the excess return of stock i at time t, SMB_t reflects the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks, HML_t reflects the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio.

Our initial plan is to estimate the idiosyncratic volatility for our universe of stocks for the first five years of our sample, i.e. our estimation period is 5 years. We then rank the stocks based on their historical idiosyncratic volatility and form four equal-weighted portfolios. Each year we measure returns and rebalance the portfolios. We then look at the average yearly return and the standard deviation for the four portfolios, compute the Sharpe-ratios and test the hypothesis that the portfolio of low-volatility stocks has a higher relative return by comparing it to the portfolio of high-volatility stocks.

To assess the robustness of our results we regress the portfolio returns on both the CAPM and FF-3 factors, the purpose is to compute the difference between the alpha of the high volatility portfolio less the low volatility portfolio to see if idiosyncratic volatility is captured by these factors. Furthermore, we also want to control for a number of cross-sectional effects introduced by earlier studies, but understand that this might be somewhat constrained by data issues.

Another way of evaluating the robustness would be to use different approaches related to the data sample used. To deal with this we want to run all regressions and tests within sub-samples, test multiple estimation periods (both related to time horizon and data frequency[4]) and vary how often we rebalance the portfolios.

After testing our hypothesis stating that portfolios of low volatility assets generate risk adjusted superior returns in the Norwegian market, we will proceed with two

[4] See section about data for an extended discussion.

steps. First, we wish to explain the observed results and secondly, device trading strategies that can exploit our findings. With respect to explaining our observed result, we anticipate that this will be a challenging task, especially considering that one former study label the phenomenon we investigate as one of the greatest anomalies in finance (Baker, Bradley, and Wurgler 2011). We will here draw on the current literature and consider whether proposed explanations fit in a Norwegian context.

In terms of devising trading strategies that exploit our findings, this depends on the actual result, but if we assume that we find results that low volatility assets has earned risk adjusted superior returns in Norway, there are several issues we would like to investigate. Particularly, we are interested in how transaction costs affect the profitability of a simple strategy of going long the low-volatility portfolio, while simultaneously shorting the high-volatility portfolio (which is highly related to the decision on how often to rebalance the portfolios). Further we will look at leverage constraints, which is important in terms of how easily managers can achieve their target returns using a low volatility strategy.

One way of designing a trading strategy would be to create a low volatility index for the Norwegian stock market, based on the methodology used by Standard & Poor’s when creating the S&P500 Low Volatility Index. As indicated in Blitz and van Vliet (2011), such an index is not necessarily appropriate for benchmarking purposes, but may rather work as an implementable low volatility strategy. Using the methodology applied by Standard & Poor’s, stocks are weighted relative to the inverse of their corresponding volatility ,with the least volatile stocks receiving the highest weight (Standard & Poor's 2012).

Data

We will obtain price data for the Norwegian stock market from professor Janis Berzins at BI Norwegian Business School.[5] We have not had the opportunity to meet with him prior to the delivery date for the preliminary thesis. A more detailed and descriptive version of the data will therefore be given in later versions of the thesis. So far we have used Datastream to assess the availability of price data for the Norwegian stock market. Datastream provides price data on 659 stocks for the OSE. In this sample, many of the small stocks are very illiquid with large bid-ask spreads and must be removed from the sample. This is done since the observed volatility in these stocks can give a biased estimate of the intrinsic volatility. Other studies have also limited their universe of stocks by cutting the smallest stocks from the sample. Ang et al. (2009) exclude the smallest firms by eliminating the 5% of firms with the lowest market cap. Baker, Bradley and Wurgler (2011) also limit their sample by taking away firms with the lowest market cap. With respect to the Norwegian stock market, we note that removing too many small stocks might make the sample impractically small in terms of creating multiple portfolios with different volatility levels.

Monthly data for the Oslo Stock Exchange is available from 1980 and daily data is available from 1990. This has important implications for us with respect to our sample size. If we use monthly data, we need a longer estimation period in order to create portfolios based on their historical volatility. Using monthly data, we likely need a three to five-year estimation period, something that will cut our already short sample by roughly one-sixth. Using daily data, we can shorten the estimation period, but unfortunately daily data is not available as far back as monthly data is.

Current studies of the high volatility anomaly have so far been conducted with data from large equity markets, such as the U.S. and G7 countries. Due to the smaller sample size in the Norwegian stock market, obtaining strong results might be an issue.

[5] Janis Berzins is responsible for Norwegian stock market data at BI Norwegian Business School.

We use the three factor model by Fama French (1992) in order to calculate idiosyncratic volatility. The Fama and French factors for the Norwegian market are obtained from Bernt Arne Ødegaard's webpage.[6] His data contains HML and SMB portfolios (both equal and value weighted) dating back to 1980.

We use the U.S. T-bill rate as a proxy for the risk free rate, which we obtain from Datastream. Ang et al. (2009) and Frazzini and Pedersen (2011) study both local currency and U.S. denominated returns, but compute excess return using the one-month U.S. T-bill rate.

[6] Bernt Arne Ødegaard is a professor at the University of Stavanger in Stavanger, Norway.

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