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# FORECASTING CRUDE OIL FUTURES VOLATILITY

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## **ABSTRACT**

This thesis examines the accuracy of different volatility models in forecasting the volatility of West Texas Intermediate crude oil futures returns. We examine the information content of implied volatility by embedding it as an explanatory variable to GARCH and EGARCH models. The results suggest that even though implied volatility is a highly significant variable for explaining crude oil futures returns, time series models also provide some information that is not accounted for by implied volatility. We also find that the more complex EGARCH model is to be preferred when modeling crude oil futures returns, implying the existence of an asymmetry in the volatility response of futures returns to shocks. The out-of-sample tests conclude that even though implied volatility fail the rationality test, it outperforms both GARCH-type and historical volatility models. Combining time series models with implied volatility adds, on average, no significant information that is not already incorporated in implied volatility. This indirectly gives support to the hypothesis that the crude oil futures options market is informationally efficient.

# Table of Contents

<b>1</b>	<b>INTRODUCTION .....</b>	<b>1</b>
1.1	MOTIVATION AND BACKGROUND.....	1
1.2	OBJECTIVES AND SHORT DESCRIPTION.....	2
1.3	LITERATURE REVIEW .....	3
<b>2</b>	<b>DATA.....</b>	<b>6</b>
<b>3</b>	<b>AN OVERVIEW OF THE OIL MARKET.....</b>	<b>7</b>
3.1	HISTORICAL MOVEMENTS OF THE WTI CRUDE OIL PRICE .....	7
3.2	FUNDAMENTALS .....	9
3.3	ECONOMIC THEORY.....	10
	<i>Hotelling's Rule .....</i>	<i>10</i>
	<i>Cost of carry and convenience yield.....</i>	<i>11</i>
	<i>Futures market.....</i>	<i>12</i>
	<i>Seasonality .....</i>	<i>13</i>
3.4	INVESTOR BEHAVIOUR .....	14
3.5	STATISTICAL PROPERTIES .....	15
<b>4</b>	<b>ESTIMATION OF OIL PRICE VOLATILITY .....</b>	<b>18</b>
4.1	THE BEHAVIOR OF OIL PRICES.....	18
4.2	TIME SERIES VOLATILITY MODELS.....	19
	4.2.1 <i>Historical Volatility.....</i>	<i>19</i>
	4.2.2 <i>GARCH.....</i>	<i>19</i>
	4.2.3 <i>Forecasting.....</i>	<i>21</i>
	4.2.4 <i>EGARCH.....</i>	<i>21</i>
4.3	IMPLIED VOLATILITY .....	22
	4.3.1 <i>Estimation of Implied Volatility.....</i>	<i>23</i>
	4.3.2 <i>Possible Specification Errors .....</i>	<i>24</i>
<b>5</b>	<b>EMPIRICAL RESULTS.....</b>	<b>26</b>
5.1	WITHIN-SAMPLE TESTS.....	26
5.2	OUT-OF-SAMPLE TESTS .....	29
5.3	MODEL COMPARISON.....	32
<b>6</b>	<b>CONCLUSION .....</b>	<b>34</b>
	<b>BIBLIOGRAPHY .....</b>	<b>36</b>

<b>FIGURES .....</b>	<b>40</b>
<b>TABLES .....</b>	<b>44</b>
<b>APPENDIX.....</b>	<b>46</b>
JAVA CODE; FILTERING THE DATA .....	46
VISUAL BASIC CODE; IMPLIED VOLATILITY .....	50
EViews CODE .....	51
WTI CONTRACT SPECIFICATIONS.....	52
<i>Light Sweet Crude Oil Futures</i> .....	52
<i>Light Sweet Crude Oil Options</i> .....	53
<b>PRELIMINARY THESIS REPORT .....</b>	<b>56</b>

## List of Figures

Figure 1a. Nominal WTI crude oil spot price.....	40
Figure 1b. Real WTI crude oil spot price .....	40
Figure 1c. Global real WTI crude oil spot price.....	40
Figure 2a. Crude oil supply and demand .....	41
Figure 2b. Crude oil consumption split .....	41
Figure 2c. Crude oil production split.....	41
Figure 3. The price of one USD in terms of one SDR.....	42
Figure 4. Daily returns of WTI futures .....	42
Figure 5. Comparison of IV and HV as a predictor of RV.....	43
Figure 6. Comparison of GARCH and EGARCH forecasts.....	43

## List of Tables

Table 1a. Augmented Dickey-Fuller test of WTI futures prices .....	44
Table 1b. Augmented Dickey-Fuller test of WTI futures returns .....	44
Table 2. Descriptive statistics of WTI futures returns.....	44
Table 3a. Correlogram of WTI futures returns.....	45
Table 3b. Information criteria of WTI futures returns.....	45
Table 4. ARCH test for the WTI futures returns .....	45

# 1 INTRODUCTION

## 1.1 Motivation and background

From a finance perspective, the notion of volatility is undoubtedly one of the most important concepts to study. This is due to the fact that most financial decisions are based on a tradeoff between risk and return and, even though volatility is not completely interchangeable with risk, volatility is often seen upon as a rough measure of the total risk of a financial asset. In all asset-pricing theories volatility is a fundamental quantity that directly affects the value of uncertain investments. For example, in the Capital Asset Pricing Model investors are rewarded for taking on non-diversifiable risk measured by beta. Beta describes the volatility of an asset relative to the market, and so (ceteris paribus) an increase in the asset's volatility (relative to the market) should lead to a reduction in the asset's value. Moreover, volatility is important for risk managers, pricing of derivative securities, monetary policy makers, and even enters directly into international financial laws and regulations. An example of the latter is the Basel II and the new Basel III standards. However, measuring and forecasting volatility is not a trivial matter because conditional volatility is unobservable. To model this parameter, GARCH models are often used. These types of models have a good track record in providing accurate within-sample estimates for the volatility of returns, but their accuracy decreases as forecast horizon is extended in out-of-sample tests. As an alternative, option valuation models such as Black and Scholes (1973) could be used to obtain implied volatility forecast that can be interpreted as the "market's" volatility forecast. Assuming that the option market is efficient, and that the chosen option valuation model is specified correctly, all relevant information should be incorporated in the option prices, and so the realized volatility should equal the implied volatility plus a zero mean random error. This suggests that implied volatility should be a superior forecast. However, in practice the implied volatility estimates are subjected to biases, concerning model misspecification and violation of the underlying Black Scholes assumptions (e.g. bid-ask spreads and nonsynchronous prices will cause implied volatility to differ from market expectations).

The rationale for choosing to study WTI futures and WTI futures options market comes from the fact that they are traded on the same floor and it is the

most liquid commodity market in the world, averaging 1,000,000 traded contracts per day (translates into one billion barrels of oil). This mitigates the problem of nonsynchronous trading and provides us with the large amounts of data needed to obtain consistent estimates of options implied volatility and measure the forecasting accuracy for different volatility models. Furthermore, crude oil accounts for 10 percent of international trade and 4 percent of global GDP (World Economic Outlook IMF 2008), and revenues from crude oil exports accounted for more than 34 percent of Norwegian exports in 2010 (MIT Media Lab). Finally, the price fluctuations in recent years have been substantial, which in turn has a big impact on economic activity and stock market returns. This means that being able to understand the oil price movements and to generate as precise forecasts of future volatility as practicable is very important for instance financial decisions involving strategic investments in oil related assets and portfolio risk management, in particular with respect to the valuation of oil-related derivative instruments.

## **1.2 Objectives and short description**

This thesis seeks to compare the accuracy of within-sample estimates and the out-of-sample forecasting power of implied volatility (IV), GARCH, EGARCH, and historical volatility (HV) models. Our goal is to investigate whether or not the different volatility models represent unbiased forecasts of the WTI futures returns volatility, and which is the best model for predicting future volatility. The performance of IV, GARCH, EGARCH, and HV models will be compared and evaluated on the basis of the statistical significance of the regression coefficients and forecasting accuracy using Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). Implied volatility is estimated using Newton-Raphson algorithm based on Black's option pricing model. The options used are nearest to at-the-money with maturities ranging from 11 to 31 trading days. The implied volatility used in regression analysis is the average of both put and call implied volatilities on a given day. For within-sample tests we fit the data with each model just once. For out-of-sample tests, implied volatility is assumed to produce volatility forecast that is the average volatility expected to prevail over the life of the option. GARCH-type models are estimated to produce a 21-day-

ahead forecast of volatility<sup>1</sup> and the models are estimated 1859 times using a static rolling  $t+1$  day window with 2/3's of our data. The historical model is estimated based on a 21 day window of subsequently realized WTI futures returns volatility.

### **1.3 Literature review**

The most recent studies comparing the accuracy of time series and implied volatility forecasting models for crude oil futures are papers written by Day and Lewis (1993) and Duffie and Gray (1995).

In “Forecasting Futures Market Volatility” Day and Lewis (1993) compare the volatility forecasts obtained from at-the-money calls on WTI oil futures with GARCH-type models and simple historical volatility. The data consists of daily closing prices for WTI crude oil futures from November 1986 to March 1991, and the options used are two- and four-month calls with, on average, 32 and 72 trading days to expiration. For the out-of-sample tests GARCH and EGARCH models are refitted for each day using historical data from the previous 500 days' futures prices. Historical volatility is calculated using a number of trading days set equal to option maturity. In-sample tests are conducted by including IV as an exogenous variable in the conditional variance equation of the GARCH and EGARCH models. The results show that both the time series and IV models have statistically significant explanatory power for volatility forecasting. Furthermore, no evidence of asymmetry in the volatility response to futures price changes was found, and thus there is no advantage of using the relatively more complex EGARCH model. For the out-of-sample tests, IV is found to produce more accurate volatility forecast than both the GARCH-type and the HV models. The authors conclude that neither GARCH nor EGARCH contain information that is not already embedded in IV. Implied volatility is shown to be an unbiased predictor of future near-term volatility (2 months), while both GARCH and EGARCH forecasts have statistically significant biases. This is the only paper that we are aware of, in which IV has passed the rationality test. It should be noted, however, that none of the models have passed the unbiasedness test using the longer maturity of 4 months. Our approach differs from Day and Lewis' in terms of the estimation of

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<sup>1</sup> This is the average time to expiration for sample options



IV, where Day and Lewis use a binomial approach<sup>2</sup>, we are using Black's model for pricing European futures options. Another difference is related to the fact that instead of using IV from call options with constant maturity of 2 and 4 months, we are averaging call and put IV's from at-the-money nearest maturity contracts ranging from 11 to 31 trading days (averaging 21 trading days).

In "Volatility in Energy Prices" Duffie and Gray (1995) conduct a similar research as Day and Lewis (1993), but using data from various energy markets, such as crude oil, natural gas, heating oil, and electricity. They use daily closing prices from May 1988 to July 1992 and compare the performance of GARCH, EGARCH, bivariate GARCH, regime switching, past historical volatility, and Black-Scholes implied volatility forecasts with the realized volatility. The models are evaluated using the root mean squared forecast error (RMSE) expressed in terms of annualized percentage volatility. Their main findings are that the Black-Scholes IV forecasts outperform both time series and HV models for both within-sample and out-of-sample tests.

A similar study that also examine the information content of implied volatility, only for S&P 100 stock index (OEX) options, is performed by Canina and Figlewski (1993) in "The Information Content of Implied Volatility". Their dataset consists of more than 17,000 daily closing prices for S&P100 stock index call options over a four year period, from 1983 to 1987. Implied volatility is derived from a binomial model with 500 time steps<sup>3</sup> adjusted for dividends. Call options are divided into eight different strike price categories ranging from 20 basis points out-of-the money to 20 basis points in-the-money, and four different maturities ranging from 1 to 4 months. Historical volatility is computed from the preceding 60 calendar days. The regressions for the rationality tests were estimated for each strike and maturity combination, but none of them were close to passing this test. The results show that HV contains more information about future realized volatility than IV, and that there is no relation between the implied volatility and subsequently realized volatility. However, these results might be biased due to nonsynchronous trading between stocks and S&P 100 stock index options, and large transaction costs.

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<sup>2</sup> Binomial approach takes into account the value of early exercise embedded in American options

<sup>3</sup> For an option with 50 days to expiration, we will have 10 steps per day

Szakmary, Ors, Kim and Davidson (2003) in "The Predictive Power of Implied Volatility: Evidence from 35 Futures Markets" is just one of many papers that tries to take advantage of using futures and futures options, which trade on the same floor and where trading costs are much lower than for cash market transactions. Their dataset consist of daily closing prices from 35 futures options markets (from eight separate exchanges) such as equity-index (S&P 500 index), interest rates, currencies, energy, metals, agriculture, and livestock futures options. IV is calculated as the average of two calls and two puts with strike price nearest to the underlying futures, representing a time series of point estimates of IV. For historical volatility, a 30 day average is used. The authors test how well the implied volatility embedded in the option prices predict subsequently realized volatility and analyze the unbiasedness of forecasting models (IV, historical volatility, GARCH). The results indicate that for the majority of the 35 futures markets, IV is the best predictor of the subsequent realized volatility in the underlying futures (over the remaining option life). Historical volatility and GARCH models do not appear to contain information that is not already incorporated in implied volatility. These results are confirmed for options with maturity  $\leq 30$  trading days, 31-49 trading days, and  $\geq 50$  trading days to maturity. The slope coefficients for IV range from 0.351 (for sugar) to 0.759 (for crude oil, which has the highest explanatory power among all futures markets). The conclusion is that even though IV is the best predictor, it is a biased estimate of future volatility.

The main contribution of our article is to update the results from Day and Lewis (1993) by using a larger and more recent dataset. Depending on our results, we will be able to determine whether or not using a standard fixed-volatility model<sup>4</sup> is an efficient way to obtain consistent volatility forecasts. Also, it will be interesting to see whether or not using Black's model for American near-term at-the-money futures options will provide statistically significant information about future volatility that is consistent with the findings of Day and Lewis (1993) who uses a binomial model.

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<sup>4</sup> Black's model with constant mean and volatility.

## 2 DATA

Our data consists of daily closing prices of WTI Light Sweet Crude Oil futures, and associated American options on those futures. The data was provided by the Commodity Research Bureau and given as .csv files which were arranged by contracts.

In order to obtain volatility forecast implied by option prices it was necessary to obtain a time series for short-term at-the-money options (both put and call) and the underlying futures. Each option contract is held for approximately 1 month (on average 21 trading days) and rolled over to the next nearest-to-maturity contract when the options has exactly 10 trading days to expiration. Excluding close to expiration options (those with less than 10 days till expiration) reduces the problem of infrequent trading and provides us with larger information content needed to obtain reliable volatility estimates. Java programming was used to filter and arrange the data and the code is provided in the Appendix. The selected time period ranges from 01/01/1990 to 30/12/2011 (5513 trading days). We hope that the recent volatility shocks in the oil market caused by the financial crisis will provide us with a good opportunity to evaluate the speed of adjustment at which the new information is incorporated in implied volatility models relative to time-series models. The risk-free interest rate needed in Black's approximation is the one-month US Treasury-bill rate<sup>5</sup> obtained from the DataStream.

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<sup>5</sup> The one-month US Treasury-bill rate is chose as it contains close to no default risk. However, one might argue that this not an entirely realistic assumption to make, as it does not fully reflect the funding costs of an investor who might need to borrow money.

### **3 AN OVERVIEW OF THE OIL MARKET**

During 2007 and the first half of 2008 the spot price of WTI crude oil nearly doubled (from USD71/bbl to USD140/bbl), before dropping by almost 70% in the second half of 2008 (from USD140/bbl to USD45/bbl), just to surge up again more than 75% during 2009 (from USD45/bbl to USD79/bbl). What drove these changes? What defines how the market set spot and futures prices of crude oil?

This paper will explore statistical properties of the oil price in an attempt to explain and forecast price changes. Other commonly cited factors used to explain and forecast oil price movements are factors related to fundamentals (i.e. supply and demand), predictions made by economic theory (i.e. how oil prices should behave over time) and the behaviour of market participants (e.g. speculation). James D. Hamilton (2008) concludes that when trying to explain the movements of oil prices, one should consider all these factors together, as they are not necessarily mutually exclusive but may rather complement one another. Even though this study will focus mainly on statistical properties of the oil price, it is worth looking into other models to see what insights they may provide.

In the following section we will present some stylized facts about the historical development of the WTI crude oil price. Then we will discuss the aforementioned factors, before concluding on what statistical properties to focus on and justify our model choice.

#### **3.1 Historical Movements of the WTI Crude Oil Price**

Between 1960 and 1973 the price of WTI crude oil remained relatively stable, increasing from about USD2.5 to USD3.5 per barrel (Figure 1a). In real terms however, prices actually decreased from about USD23 to USD18.5 per barrel (measured in 2011 USD) (Figure 1b). Price fluctuations were low, with volatility<sup>6</sup> of around 3% for both the nominal and real price series.

On October 6, 1973, a coalition of Arab states, led by Egypt and Syria, launched a surprise attack on Israel in what was later to be named the Yom Kippur war. In response to the United States' and Western Europe's support of Israel in the war,

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<sup>6</sup> Measured as annual standard deviation

the members of the Organization of Arab Petroleum Export Countries (OAPEC) decided to impose an oil embargo against the West, which caused the WTI oil price to triple to USD10 per barrel by March of 1974 in nominal terms (and USD48 per barrel in real terms).

For the next 5 years, prices remained relatively stable, increasing by about 8% annually. Then, in 1979, prices again surged in response to the Iranian revolution and the Iran-Iraq war that caused production in the two countries to plummet. Oil prices increased from about USD15 per barrel to USD39.5 by March 1980 in nominal terms (and from about USD50 to USD112 in real terms).

For the next 10 years, the oil price decreased by about 8% on average annually, dropping to about USD17 per barrel by mid 1990 in nominal terms (and USD29 per barrel in real terms), mainly caused by increased production from Saudi Arabia in early 1986 (the Saudis increased output from two million barrels per day to five million barrels per day). Then, in August 1990, prices again spiked as a consequence of the First Gulf War, and during the next couple of months the spot price of the WTI crude oil doubled from USD20.5 per barrel to USD41 per barrel in nominal terms (and from about USD32 to USD61 in real terms).

For the next 9 years the WTI oil price fluctuated within a range of about USD10 and USD25 per barrel in nominal terms (and USD15 and USD40 per barrel in real terms), which was followed by a period of strong price inflation as the WTI oil prices increased by approximately 22% per year until mid-2008 when the WTI oil price peaked at around USD140. Some of the factors explaining this appreciation were the weak dollar, the strong growth of the Asian economies and the erosion of global excess oil production capacity (loss of capacity in Iraq due to the Second Gulf War combined with increased global demand).

Over the next 6 months, the financial crisis and global recession caused the WTI oil price to decrease by approximately 78% to USD31 per barrel, before it steadily increased again to USD99 by the end of 2011.

In addition to looking at the price movements in real terms, one might argue that it would be more correct to also take into account the changes in the US Dollar against other currencies (seeing as most sales throughout the world today are denominated in USD), and create a “global real oil price”. However, the movements in such a global real oil price do not differ very much from the real oil price (Figure 1c).

### 3.2 Fundamentals

From the short summary of the development of the WTI oil price for the last 50 years, it seems evident that the volatility in oil prices to a large extent is caused by supply and demand imbalances. This implies that embedded in the spot and futures prices are predictions about future global demand, and expectations of how quickly supply can react. For example, looking at the most recent price shock (2007-08), many studies have pointed to the strong growth in demand from emerging markets, combined with a stagnating supply, as the main drivers (Figure 2a presents an overview of the supply and demand balance over the last four decades). Hicks and Kilian (2009) used revisions of professional real GDP growth forecasts as a proxy for global oil demand shocks, and showed that the price changes of 2007-08 (and the subsequent decline) was primarily caused by unexpected growth in emerging economies, whereas James D. Hamilton (2009) showed that the price run-up of 2007-08 was mainly caused by a strong growth in demand from emerging markets, in particular from China where oil consumption had been growing at a 7% compounded annual rate over the two decades leading up to the price surge. This is supported by data from the International Energy Agency (IEA) which show that consumption in emerging markets (i.e. China and other Asian countries, Latin America, Middle East, and Africa) grew by more than 4% over the period between 2004-2008 (compounded annually), while demand from OECD countries declined by 1% (Figure 2b). Furthermore, given the relatively high income elasticity of oil demand in markets characterized by rapid income growth<sup>7</sup>, and the fact that individuals in emerging markets still are consuming just a fraction of what for instance the USA and Canada are consuming<sup>8</sup>, growth in demand from emerging economies is expected to remain a determining factor of crude oil prices.

James D. Hamilton (2009) also pointed to stagnating world production as a cause of the oil shock of 2007-08. This is again supported by data from the IEA which shows that global production during the period 2004-2008 grew by only 1.1%, compared to 1.9% during the preceding four-year period. Thus, to restore

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<sup>7</sup> Gately and Huntington (2002) estimated the income elasticity of oil demand at 1.17 in countries with rapid income growth and 0.55 for OECD countries

<sup>8</sup> 2.6 vs. 24.6 barrels per person per year in 2011 according to figures from the IEA, indicating that the income elasticity should not be expected to fall significantly in the near term future.

equilibrium in a period with strong growth in demand from emerging markets, a big increase in prices was required. Also, it is important where the supply is coming from. Only 26% of the increased output between 2004 and 2008 came from non-OPEC countries, effectively increasing OPEC's share of global production from 40.1% to 41.5% (Figure 2c), and increasing its potential market power.

Another important determinant of oil prices are inventories. Low inventory levels may lead to a situation with short-term undersupply, thus spot prices exceed future prices (creating what is called a market in backwardation). Conversely, high inventory levels may lead to short term over-supply, and future prices exceeding spot prices (creating what is called a market in contango). However, while oversupply can be stored for future consumption, future production cannot be used to meet current undersupply. This may create an asymmetry in the oil price response to situations with under- or oversupply, where price reactions due to undersupply may be larger in magnitude compared to situations with oversupply.

### **3.3 Economic Theory**

#### **Hotelling's Rule**

According to Harold Hotelling (1931), the price of a non-renewable resource should increase over time at the rate of interest. This is due to the fact that an oil producer (or an owner of any exhaustible resource) has the choice between producing and consuming (i.e. selling) today, versus leaving the oil in the ground for future consumption. *Hotelling's rule* states that supply and demand will balance if, and only if, the net price<sup>9</sup> of the resource is expected to increase at the rate of interest. I.e. today (at time  $t$ ) we should expect the future price of oil (at time  $T$ ) to be equal to the present value of the spot price ( $P_t$ ) compounded by the risk-free rate ( $r$ ):

$$E_t(P_T) = P_t e^{rT}$$

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<sup>9</sup>Net price refers to the price minus any extraction costs

If for instance the crude oil price is expected to rise at a lower rate than the rate of interest, producers would be better off selling all their available resources today and investing the proceeds in for instance bonds (or some other interest bearing assets), creating an oil oversupply. Conversely, if oil prices were expected to increase faster than interest rates, then producers would be better off leaving the oil in the ground, thus creating undersupply.

However, this theory is inconsistent with the oil futures market which often displays a downward-sloping term structure (backwardation). For instance, Litzenberg and Rabinowitz (1995) estimated that between February of 1984 and April of 1992, the nine months futures price was in strong backwardation 77% of the time and in weak backwardation 94% of the time, and so it seems that Hotelling's rule does not fit observed data very well.

### **Cost of carry and convenience yield**

One explanation as to why Hotelling's rule does not fit real data very well may be due to costs and benefits that are not incorporated in the model. For instance, an investor buying an asset today may incur some storage costs (e.g. cost of storing oil in a storage tank). This can be treated as negative yield ( $u$ ), giving rise to the following equality:

$$E_t(P_T) = P_t e^{(r+u)T}$$

The interest and storage cost ( $r+u$ ) is generally referred to as the *cost of carry*. Furthermore, for some assets, investors may actually want to hold the asset physically prior to  $T$  (as inventory) regardless of the storage cost. This could, for instance, be an oil refiner who wants to hold some oil in inventories to ensure uninterrupted production. This benefit from holding physical inventories of assets is often referred to as the *convenience yield*. Denoting the convenience yield by  $y$  and the cost of carry by  $c$ , the relationship between the expected future price and today's price can be summarized as:

$$E_t(P_T) = P_t e^{(c-y)T}$$



According to this equality, the volatility observed in for instance crude oil prices should be fully explained by changes in the cost of carry and the convenience yield. However, (as will be showed later) the daily change in the crude oil price (both spot and near term futures prices) has a standard deviation of more than 2%, and it seems unlikely that the cost of carry and the convenience yield could produce movements of such magnitudes.

### **Futures market**

Instead of buying oil today and storing it, an investor may instead buy a futures contract. The relationship between the futures price ( $F_{t,T}$  – the price at time  $t$  of a futures contract expiring at time  $T$ ) and spot prices for a consumption asset can be summarized as follows (Hull 2012)<sup>10</sup>:

$$F_{t,T} = P_t e^{(c-y)T}$$

Furthermore, for assets that trade in a liquid market, the futures price is assumed to represent an unbiased expectation of future spot prices:

$$F_{t,T} = E_t(P_T)$$

However, with respect to contracts written on crude oil, Alquist and Kilian (2008) showed that oil futures prices tend to be a less accurate predictor of future spot prices than current spot prices are. In other words, a forecast of no change in oil prices performs better than futures prices in forecasting future spot prices. However, recalling that the daily change in the crude oil price (both spot and near term futures prices) has a standard deviation of more than 2%, this is not a very accurate forecast.

Another model relating futures prices with expected future spot prices is “The Theory of Normal Backwardation” introduced by Keynes (1930). He proposed that there should be a “normal backwardation” in futures markets where the

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<sup>10</sup> Strictly speaking this relationship normally only applies to forward contracts, as unexpected interest rate changes will cause forward and futures prices to differ. However, we assume here that the difference is small enough to be ignored.

expected future spot price is equal to the futures price plus a (positive or negative) risk premium ( $RP_t$ ) which represents a reward to speculators for taking on price risk from hedgers:

$$E_t(P_T) = F_{t,T} + RP_t$$

$$RP_t = E_t(P_T) - F_{t,T}$$

The sign of the risk premium will depend on whether hedgers are net long or net short in futures contracts. Keynes (1930) assumed hedgers were generally net short and speculators net long. This would for instance be the case for an oil producer wanting to hedge against price risk, thus shorting (i.e. selling) futures contracts to lock in a price today. This creates demand for long speculators (i.e. buyers of the futures contracts) who are willing to bear the price risk, implying that a long position typically should be rewarded by a futures price increase, resulting in a positive risk premium and a market in backwardation. If, on the other hand, hedgers are net long (e.g. an oil refiner wanting to lock in the cost of raw material), the risk premium will be negative resulting in a market in contango. A number of studies in recent year have found evidence of a risk premium in crude oil futures prices (e.g. Alquist and Kilian 2008). However, there are also studies that have failed to find evidence for the existence of a risk premium, e.g. Chinn, LeBlanc and Coibion (2005) who finds that futures prices are unbiased forecasts of future spot prices.

### **Seasonality**

Finally, demand for crude oil and crude oil products display clear seasonal patterns, with demand for heating oil peaking during winter and gasoline peaking during summer. These patterns are not captured by the fairly simplistic models described so far, but can be captured using time series models, for instance by including dummy variables in a regression model or by estimating Markov switching, threshold autoregressive or threshold GARCH models (Brooks 2008).

### **3.4 Investor Behaviour**

It is not only the actual producers and consumers that take part in the trade of crude oil, but also institutional investors like hedge fund managers, individual traders and speculators. They all contribute together to set the market price of crude oil, thus it might be that prices are not driven mainly by supply and demand, but rather by investment funds investing in commodities to diversify portfolio risk or by pure speculation (Tang and Xiong 2011).

For instance, modern portfolio theory states that an investor should combine a set of assets with the goal of maximizing returns for a given level of risk (as measured by the variances and covariances of the assets in the portfolio), and that the portfolio should be managed within a risk management budget. If the risk exceeds the budget, the investor should reduce risk by selling risky assets (i.e. assets with high levels of volatility and assets that are highly correlated). However, risk is often estimated based on historical data, and so all investors are using the same set of data to estimate their portfolios, and thus select the same portfolios. This leads to increased volatility and covariance between individual assets and between asset classes.

According to Michael Masters, a US hedge fund manager, asset allocation to commodity index trading strategies rose from USD13 billion to USD260 billion between 2003 and 2008, and speculative demand for crude oil futures increased by 848 million barrels (for reference, crude oil demand in China totalled 2,811 million barrels in 2008 according to IEA data). The sheer size of this type of trading implies that institutional investors and speculators have had an increasing impact on the crude oil price, and that the financialization of the crude oil trade contributed to the price run-up of 2007-08 and its subsequent collapse. In fact, according to Juvenal and Petrella (2012) speculative shocks were the second most important driver behind the oil price increase between 2004 and 2008, accounting for about 15% of the increase in the oil price during this period.

Another factor that plays an important role in setting the market price of crude oil is the price of the US Dollar (USD). Most oil sales throughout the world today are denominated in USD, and so if the USD depreciates against the domestic currency of an oil produce, the producer will want to try and regain purchasing power by increasing prices. Furthermore, with a depreciating USD, crude oil will be cheaper for non-US consumers, thus increasing demand in those countries (which in turn

may push prices up). Lastly, as the price of oil falls (as a consequence of the weakening USD), crude oil will seem like a more attractive investment, whilst for instance investments in USD will seem less attractive (which in turn may push oil prices up). Cuaresma and Breitenfellner (2008) have estimated that between 1950 and 2006 the correlation between the USD and oil prices was -0.61. If in fact causality runs from the exchange rate to the price of oil, this might be an important factor in explaining the price increase during the last decade, as the USD has been steadily depreciating since 2002 (Figure 3).

However, the size and sign of the correlation coefficient, or even the causality relationship, is not agreed upon in the literature. For instance Bénassy-Quéré, Mignon and Penot (2005) finds that causality runs from oil to the USD, and that a 10% increase in the oil price leads to a 4.3% appreciation of the USD.

### 3.5 Statistical Properties

When studying time series data it is important to investigate the issue of stationarity (in econometrics the problem of non-stationarity is referred to as unit-root). This is due to the fact that non-stationary data can produce spurious regressions (inflated t-values and  $R^2$ ), and the effects of shocks in such systems can be permanent (Szakmary, Ors, Kim, Davidson 2003). To ensure stationarity we therefore look at returns series rather than price series. In general, returns are calculated on a continually compounded basis as the natural logarithm of the period price ( $P_t$ ) less the natural logarithm of the last period price ( $P_{t-1}$ ):

$$R_t = \ln(P_t) - \ln(P_{t-1})$$

Typically, financial time series will contain one unit-root, but to ensure that our returns series is stationary, we run the augmented Dickey–Fuller (ADF) test (Dickey and Fuller 1979). The test results imply non-stationarity in price levels, but reject the null hypothesis that the WTI futures returns series contain one or more unit roots (i.e. the returns series is stationary) (Table 1a and 1b).

Descriptive statistics (Table 2) show that the average daily WTI futures returns for our sample period is 0.028% with the daily standard deviation of 2.275%, and

annualized volatility of approximately 36% (assuming 252 trading days). Jarque-Bera normality test results suggest that the null hypothesis of normality is rejected and that the sample data have non-normal properties. The distribution is left skewed and leptokurtic, i.e. fat tails and peaked around the mean. The non-normality in our data implies that the inferences we make about the regression coefficient estimates may be wrong. However, the issue of non-normality is a very frequent issue in financial time series modeling. Furthermore, looking at a plot of daily returns (Figure 4) it is clear that the data exerts time-varying volatility and volatility clustering. Tranquil periods are followed by relatively more volatile periods, where shocks to the time series seem to be persistent with large positive and negative returns being observed over a prolonged period. This can for instance be seen in 1990-91 and 2008-09 (during the First Gulf War and the financial crisis respectively). Volatility clustering can be explained by the fact that the shocks, which drive oil price changes, occur in bunches rather than being spread evenly over time. This volatility persistence can be measured by the autocorrelation in the variance, and can be tested for by using the Ljung-Box statistic. The null hypothesis under this test is no linear dependence in the data, so that any observed correlations in the data result from randomness of the sampling process. However, the test statistics for the daily WTI futures returns rejects the null hypothesis of no autocorrelation at the 5% level for all lags greater than 1, suggesting there is autocorrelation in the futures returns (Table 3a). This implies that WTI futures returns can be modeled as an ARMA process, but it is hard to precisely determine the specific order of such a model. In order to specify the appropriate model, the Akaike (AIC) or Schwartz's (SIC) information criteria can be employed. For the daily WTI futures returns, the criteria choose different models (Table 3b). AIC would select an ARMA(11,9), while the SIC selects an ARMA(0,0) model. The latter implies that the daily WTI futures returns follow more of a random walk process, i.e. no ARMA structure. However, the absolute values of the information criteria are almost identical, suggesting that none of the models provide a particular sharp description of the daily WTI futures returns, and that other models could fit the data almost as good.

The time-varying volatility is often referred to in statistics as heteroscedasticity and can be analyzed by testing for the presence of ARCH effects using the

Lagrange Multiplier (LM) proposed by Engle (1982). The test can be thought of as a test for autocorrelation in the squared residuals. The null hypothesis is “no ARCH”, meaning that all  $q$  lags of the squared residuals have coefficient values that are not statistically different from zero. In order to test for ARCH effects in the daily WTI futures returns, we first need to specify a mean equation. Seeing as the information criteria did not provide any clear model, we choose to assume that the daily WTI futures returns follow a process similar to a random walk:

$$R_t = \mu + \varepsilon_t \quad \text{where} \quad E(\varepsilon_t) = 0, \quad \text{Var}(\varepsilon_t) = \sigma_t^2$$

The test is significant at 1% level for both F-version of the test and LM- statistic. This implies the presence of ARCH effects in the daily WTI futures returns (Table 4). The explanation as to why ARCH effects are observed in the daily WTI futures returns can be related to the fundamentals. The volatility in oil prices is to a large extent caused by supply and demand imbalances, arising from geopolitical events, and changes in the global economic environment. Events like the Yom Kippur war, the Iranian revolution and the Iran-Iraq war, the First Gulf war, the strong growth of the Asian economies and the financial crisis were all events that affected the oil price dramatically.

Furthermore, due to the possible asymmetry in the oil price response to situations with under- or oversupply, where price reactions due to undersupply may be larger in magnitude compared to situations with oversupply, it could be hypothesised that an asymmetric model for the conditional variance would be a better fit for the WTI futures returns. Based on these arguments, we will explore both a GARCH and an EGARCH models in the next sections.

## 4 ESTIMATION OF OIL PRICE VOLATILITY

### 4.1 The Behavior of Oil Prices

In an efficient market, crude oil futures price returns can be modeled as a random walk with no drift, plus a random innovation term representing unpredictable market events.

$$R_t = \ln\left(\frac{f_t}{f_{t-1}}\right) = \mu + \varepsilon_t \quad E(\varepsilon_t) = 0, \quad \text{Var}(\varepsilon_t) = \sigma_t^2 \quad (1)$$

where  $f_t$  is a WTI futures option price at time  $t$ . Holbrook Working (1962) showed that randomness is to be expected if markets are efficient. The efficient market hypothesis is based on the assumption that there is no correlation in the error's,  $\varepsilon_t$ , and therefore past price movements give no information about future price movements. In deriving Black's option pricing formula that models price movements over very short time horizon there is a need to extend the random walk model to continuous time.

$$\ln\left(\frac{f_{(t+dt),k}}{f_{t,k}}\right) = \mu_k dt + \sigma_k z \sqrt{dt} \quad , \text{where}$$

$\mu_k$  = sample mean logarithmic return

$dt$  = infinitesimal change in time

$\mu_k dt$  = the 'drift' term

$\sigma_k$  = standard deviation of the  $k^{\text{th}}$  nearby WTI contract's return

$z$  = standard normal random variable with mean =0, var =1

$\sigma_k z \sqrt{dt}$  = random shock or innovation term<sup>11</sup>

This model produce continuously compounded returns that follow a lognormal distribution. If we assume constant volatility, the variance produced by the option pricing model over a finite time period is given by  $\sigma^2 T$ , and standard deviation by  $\sigma \sqrt{T}$ .

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<sup>11</sup> Where a stochastic process also known as Brownian motion represents the innovation term.

## 4.2 Time Series Volatility Models

### 4.2.1 Historical Volatility

One way to estimate future volatility is to assume that the recent realized volatility will continue in the future. Historical volatility is obtained from time series of past oil futures prices and is a measure of price variation over time. The underlying assumptions are that log-prices are normally distributed and volatility is constant over the estimation period and the forecast period. Given that the oil futures returns follow a Brownian motion from equation (1), the historical volatility for the  $k^{\text{th}}$  nearby WTI futures contract,  $\hat{\sigma}_k$ , is given by:

$$\hat{\sigma}_k^2 = \frac{1}{N-1} \sum_{i=1}^N [R_{i,k} - \bar{R}_k]^2 \quad \text{where } \bar{R}_k = \sum_{i=1}^N \frac{R_{i,k}}{N}$$

or

$$\hat{\sigma}_k^2 = \frac{1}{N} \sum_{i=0}^{N-1} \varepsilon_{t-i}^2$$

The window length  $N$  and how much of historical data to include in the calculations of historical volatility is not clear. We choose to use 1 month, i.e.,  $N = 21$  trading days, following the Energy Information Administration (2009). This is also the average time to maturity of our sample futures options.

### 4.2.2 GARCH

The Generalized Autoregressive Conditional Heteroscedasticity model developed by Bollerslev (1986) and Taylor (1986) is very appealing when modeling financial data because it can capture both volatility clustering and unconditional return distributions with heavy tails, which are typical features of commodity returns (Claessen and Mittnik 2002). The model for returns is given by eq. (1), which is a constant mean model<sup>12</sup>. The GARCH specification asserts that the best predictor of the one-period ahead WTI crude oil futures conditional variance,  $h_t^2$ , is a weighted average of the long-run average variance,  $\alpha_0$

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<sup>12</sup> Following the approach to Szakmary, Ors, Kim, Davidson (2003), and given the indistinct description of the daily futures returns resulting from the AIC and SIC information criteria analysis (presented in section 3.5).



(unconditional variance), the last period's shock to the return generating process, the innovation term  $\varepsilon_{t-1}^2$  (ARCH term), and the conditional variance from the previous lag,  $h_{t-1}^2$  (GARCH term) (Engle, 2001). To generate the GARCH conditional variance series, we estimate the following GARCH(1,1)<sup>13</sup> model with daily data for each contract:

$$R_t = \mu + \varepsilon_t; \quad \varepsilon_t \sim N(0, h_t^2); \quad h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \quad (4)$$

or

$$h_t^2 = \alpha_0 + \alpha_1 (R_t - \mu)^2 + \beta_1 h_{t-1}^2$$

where  $\alpha_0 > 0$ ,  $\alpha_1 > 0$ , and  $\beta_1 > 0$ . To estimate GARCH models, we use eViews 7 where maximum likelihood estimation with the Marquardt optimization algorithm<sup>14</sup> is used. By looking at  $\alpha_1$  and  $\beta_1$  we can evaluate how the volatility of returns evolve over time. If  $\beta_1$  is high (i.e. close to 1) most of conditional variance is explained by the historical volatility, meaning that there is a high carry-over effect of past volatility to future volatility. To insure stationarity in the variance, the sum of parameters ( $\alpha_1 + \beta$ ) should be less than 1, and when this is the case the unconditional variance or the long-run average variance is given by  $\frac{\alpha_0}{1-\alpha_1-\beta}$ .

A potential disadvantage of the GARCH model is that the impact of current return  $R_t$  on the conditional volatility is squared, meaning that if there is a major shock in oil markets in one day, this shock could have a sustained and major impact on forecasted volatility. One also need a large number of data points to produce a robust estimation (this is not a problem in our case), and the model is not designed for multi-step ahead forecasting.

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<sup>13</sup> In most cases it is enough to use 1 lag for conditional variance and 1 lag for innovation term to capture the volatility clustering in the data (Brooks 2008).

<sup>14</sup> Provides a numerical solution to the problem of minimizing a nonlinear function (default optimization in eViews)

### 4.2.3 Forecasting

The GARCH model produce a one-day-ahead forecast of volatility  $h_{t+1}^2$ , and can be easily extended to volatility forecast of k periods.

$$\begin{aligned}
 E[h_{t+1}^2] &= \alpha_0 + \alpha_1 E[\varepsilon_t^2] + \beta_1 h_t^2 \\
 &= \alpha_0 + (\alpha_1 + \beta_1) h_t^2 \\
 &\quad \vdots \\
 E[h_{t+k}^2] &= \alpha_0 \sum_{s=0}^{k-1} (\alpha_1 + \beta_1)^s + (\alpha_1 + \beta_1)^k \sigma_t^2
 \end{aligned}$$

However, when forecasting more than a few periods ahead the forecast will converge to the long run variance and will not be able to incorporate any new information from the disturbance term (Figlewski 2004). For out-of-sample forecasting, we will use an average-step-ahead forecast of variance per day over the remaining life of the option. In our case this will be a 21-day-ahead forecast, which is the average of sample options time to expiration. The rolling sample used is of constant size of 3676 (exactly 2/3 of our data) and we are moving one step at a time, meaning that we are removing the oldest observation and adding a new t+1 observation. This will generate 1838 estimates of GARCH model that are used to generate 1859 21-day-ahead GARCH volatility forecasts. A similar approach is also used for the EGARCH model.

### 4.2.4 EGARCH

The Exponential Generalized Autoregressive Conditional Heteroscedasticity model proposed by Nelson (1991) introduces logarithmic transformation of volatility to allow for correlation between futures returns and volatility changes. The EGARCH(1,1) model is given by:

$$\ln(h_t^2) = \alpha_0 + \beta_1 \ln(h_{t-1}^2) + \theta \psi_{t-1} + \gamma \left( |\psi_{t-1}| - \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \right) \quad (5)$$

where  $\psi_{t-1} = \frac{\varepsilon_{t-1}}{h_{t-1}}$ . Unlike the GARCH model, EGARCH parameter values are unrestricted. EGARCH specifies the conditional variance equation as a function of conditional variance of returns from previous lag,  $h_{t-1}^2$ , the last period's innovation term,  $\varepsilon_{t-1}$  that has been standardized to have unit variance,  $\psi_{t-1}$  (which is the ratio

of the former two parameters), and the deviation of the absolute value of  $\psi_{t-1}$  from the mean absolute value,  $\left(\frac{2}{\pi}\right)^{1/2}$ . If negative shocks to the oil market causes volatility to rise by more than a positive shocks of the same magnitude, such asymmetries should be captured by *psi* ( $\psi$ ), which when bigger than 0 increases the variance and vice versa. In contrast, GARCH model enforce a symmetric response for both positive and negative shocks.

### 4.3 Implied Volatility

An alternative model to obtain a volatility forecast is to use implied volatility. Implied volatility is the level of volatility that, when inserted to an option pricing model, will give us a theoretical value of option that is equal to the current market price of that option. Given our dataset, and the risk-free rate, we can extract a volatility forecast for crude oil futures implied by options on those futures. If financial markets are informationally efficient, implied volatilities should incorporate all available information from historical returns, current market conditions and anticipated future events. Implied volatility is therefore perceived as the market expectation of future volatility, and we expect it to be superior in forecasting future volatility compared to backward-looking time series forecasts (from GARCH, EGARCH or HV models).

In our thesis we will use Black's model introduced in 1976 to derive implied volatilities. This is an extension of Black-Scholes-Merton (B-S-M) stock option valuation model that was introduced in 1973, and represented innovative breakthrough in the investigation of risk and randomness in financial markets. Figlewski (1989) notes that B-S-M model has had a big impact on the real world security trading, and that 'all' market participants are aware of this model and use it in their decision-making. Unfortunately crude oil options are American-style options and using Black's model thus might introduce a small upward bias in the estimated volatility (caused by not including the value of early exercise). However Jorion (1995) notes that such biases are generally very small for short-term at-the-money options and much less than typical bid-ask spreads when quoted in terms of volatility. The comparison made by the Energy Information Administration (2009) between the current prices of American and European-style options on WTI futures shows that the value of early exercise has little to no value at all.

### 4.3.1 Estimation of Implied Volatility

The Black's formula for futures options is based on an arbitrage strategy that involves hedging the option against the underlying, and constantly adjusting this hedge position as price changes and times passes. Black's model assumes that the price for the underlying futures follows a logarithmic diffusion process (described in the beginning of section 4) but with constant mean and volatility.

$$C_{\tau,k} = e^{-r_k \tau_k} [f_{t,k} N(d_1) - x_k N(d_2)]$$

$$P_{\tau,k} = e^{-r_k \tau_k} [x_k N(-d_2) - f_{t,k} N(-d_1)]$$

$$d_1 = \frac{\ln\left(\frac{f_{t,k}}{x_k}\right) + \left(\frac{\sigma_k^2 \tau_k}{2}\right)}{\sigma_k \sqrt{\tau_k}}, \quad d_2 = d_1 - \sigma_k \sqrt{\tau_k}$$

$f_{t,k}$  = observed  $k^{\text{th}}$  nearby WTI futures contract's value at time  $t$ ,  $k=1,2,\dots,n$

$x_k$  = strike price corresponding to an option written on the  $k^{\text{th}}$  nearby futures

$\sigma_k^2$  = variance of the returns on the  $k^{\text{th}}$  nearby WTI futures contract

$\sigma_k$  = volatility

$\tau_k$  = time to expiration of the  $k^{\text{th}}$  nearby option contract (as a percent of a 252-day trading year)

The volatility input,  $\sigma_k$ , is the average volatility that is expected to prevail over the life of the option (Stein 1989). We will ignore storage costs, since over small time horizons they have a relatively small effect on volatility (Duffie and Gray 2004). The options used to derive IVs are at-the-money options with maturity ranging from 10 to 31 trading days to expiration, averaging a time horizon of about 21 trading days. Options that are close to expiration (i.e. options with less than 10 days till expiration) are traded less frequently and thus contain less information. This procedure should result in implied volatility estimates with the smallest possible bias. The implied volatility for each at-the-money (ATM) futures option is estimated by using a Newton-Raphson iteration algorithm. This is a linear approximation technique for solving numerical equations and can be used to estimate the implied volatility from the observed market price and the theoretical price given by Black's formula.

$$\sigma_{n+1} = \sigma_n - \frac{V_{mkt} - V_{Black}(\sigma_n)}{\frac{\partial V_{Black}(\sigma_n)}{\partial \sigma}}$$

$\sigma_n$  represents an initial guess of the volatility and  $V_{Black}(\sigma_n)$  is the theoretical option value based on the initial volatility guess,  $V_{mkt}$  is observed option market

price.  $\frac{\partial V_{Black}(\sigma_n)}{\partial \sigma}$  is the options Vega ( $v$ ), which is the options sensitivity to changes in volatility. We iterate until our estimate of implied volatility converges to within 0.00001. This operation is performed in Excel using Visual Basic programming. The function sub procedure is given in the Appendix. In the rest of the paper we will refer to IV as the average of the IV derived from put and call on a given trading day.

#### **4.3.2 Possible Specification Errors**

Black's model treat volatility as a know parameter, where the obtained implied volatility is expected to vary randomly over time. There is inconsistency in using a fixed volatility model (nonstochastic) to derive IV from options prices that follows a stochastic volatility process (Figlewski 2004). Converting an option price to implied volatility also introduce some errors due to bid-ask spreads. Because closing prices can represent a bid price, an ask price, or an intermediate price. When dealing with the crude oil returns we have to take into account the following issues concerning the underlying assumptions in our option pricing model:

- 1) Constant volatility: As we have discussed earlier, time series data displays time-varying volatility, and optimal forecasting should take this into account.
- 2) No transaction costs: In general transaction costs are very small for futures contracts and therefore makes it easier for arbitrageurs to exploit mispricing in the market by performing arbitrage between options and their underlying. Many researchers argue that such ease of performing arbitrage is positively related to information content of implied volatility (Figlewski 2004).
- 3) No serial correlation: The price movements in WTI crude oil futures are not perfectly uncorrelated and our data exhibits volatility clustering.
- 4) Normality of returns: A leptokurtic right (left) tail of oil futures returns will give the associated call (put) option a higher probability of exercising than from a normal distribution. This higher probability leads to a higher price and a higher IV (Poon and Granger 2003), i.e. IV tend to be higher for deep in the money or deep out of the money options than for those that are near or at the money.

In order to account for the time varying volatility it is possible to use a stochastic pricing model that treats volatility as a random variable so that innovations in volatility and returns are uncorrelated. But such models involve difficult and time-consuming numerical simulations, and introduce additional parameter estimates that add additional sources of error. Finally the Black's model for short-term and at-the money (ATM) options is very close to linear in the average volatility and generates estimates that are almost identical to those produced by stochastic volatility models (Jorion 1995, Fleming 1998). The effect of time varying volatility and non-normality of returns is also less pronounced when using near-term ATM options (Szakmary 2003). ATM options are also the most liquid ones, and both WTI futures, and options on those futures, are traded on the same floor, so we do not have the problem of different closing times, as is the case for stocks and options. The drawback of using ATM options is that they introduce some estimation errors associated with daily changes of current ATM option. Figlewski (2004) argue that IV is not always a good predictor for future market volatility since market prices are influenced by many factors that are not incorporated in option pricing models. Such as geopolitical risks, liquidity constraint, and bid-ask spreads. Jorion (1995) shows that IV may be a better predictor for some asset classes such as foreign exchange and crude oil than for others such as equity markets.

Finally, from the option trader's perspective, there is a possibility for violation of the no arbitrage assumption. If a trader knows the true volatility, but the market option price differ from the theoretical value, theoretically this trader should set infinitely large hedged positions (including the option and underlying) while rebalancing frequently over option's lifetime to gain from this mispricing. In practice no trader would do that as they cannot be certain if their predictions about volatility are correct. Moreover, there are transaction costs and large risks that arise from rebalancing. This means that there is room for relative mispricing that can affect implied volatility estimates, and that traders might have a different perspective on volatility compared to academic researchers. What they are interested in is the current volatility that can be used for current assessments of the underlying asset and hedging positions, not the average volatility over the remaining option life (Figlewski 2004).

## 5 EMPIRICAL RESULTS

### 5.1 Within-Sample Tests

Within-sample tests are tests that use the same data for both model estimation and forecasting. This means that the accuracy of the forecasting models is biased toward time series models that uses the entire sample for estimation (compared to IV that is the market's expectation of future volatility). The within-sample information content of implied volatilities can be examined by adding IV as an additional explanatory variable to GARCH(1,1) and EGARCH(1,1) models:

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 + \delta \sigma_{IVt-1}^2 \quad (2)$$

and

$$\ln(h_t^2) = \alpha_0 + \beta_1 \ln(h_{t-1}^2) + \theta \psi_{t-1} + \gamma \left( |\psi_{t-1}| - \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \right) + \delta \ln(\sigma_{IVt-1}^2) \quad (3)$$

The coefficient  $\delta$  measures how much of the incremental information implied volatilities contribute to the model, or how misspecified the volatility model is when IV is not included. The null hypothesis is that implied volatilities contain no additional information to that contained in the historical time series of WTI futures returns.

$$H_0: \delta = 0$$

We will also investigate whether GARCH and EGARCH models contain information that is not already included in implied volatilities. This is achieved by setting a restrictions on equation (2) ( $\alpha_0$  and  $\beta_1$  are set to zero) and equation (3) ( $\theta$ ,  $\gamma$ , and  $\beta_1$  are set to zero), and examining the statistical significance of the remaining coefficient estimates. The restricted models are given by:

$$h_t^2 = \alpha_0 + \delta \sigma_{IVt-1}^2 \quad (6)$$

and

$$\ln(h_t^2) = \alpha_0 + \delta \ln(\sigma_{IVt-1}^2) \quad (7)$$

The results for the tests of the information content of IV's relative to time series models are presented below. For both GARCH(1,1) and EGARCH(1,1) the estimate of the implied volatility coefficient is positive and significantly greater than zero (at 5% level for GARCH and 1% level for EGARCH). The information

criteria given by Akaike and Schwarz<sup>15</sup> show that equations (2) and (3) are better models than the pure time series models given by equation (4) and (5). This is also supported by the likelihood ratio tests for the unrestricted models [eq. (2) and (3)] against the restricted models [eq. (4) and (5)]. We reject the null hypothesis (at both 5% and 1% significance level) that IV adds no incremental information to time series models.

GARCH(1.1) FOR DAILY RETURNS ON CRUDE OIL FUTURES CONTRACTS

Var. specification	$\alpha_0 * 10^{-6}$	$\alpha_1$	$\beta_1$	$\delta$	Log L	$\chi^2$
(2)	-1.037 (-0.195)	0.01478 (1.073)	0.5447 (3.298)	0.4 (2.558)	13848.03	
(4)	3.721 (4.087)	0.067 (6.701)	0.9278 (112.22)		13708.89	278.28
(6)	4.244 (0.341)			0.895 (21.67)	13841.77	12.52

\*t-stats are presented in parentheses and the standard errors used in t-stats are computed using the robust method of Bollerslev and Wooldridge [1988]

\*\*  $LR = -2[L_R - L_U] \sim \chi^2$  where the statistics for eq. (3) and (5) are respectively distributed with one and two degrees of freedom

Both ARCH and GARCH terms in eq. (4) are statistically significant but the main contribution to the conditional variance comes from the recent volatility in crude oil futures ( $\beta_1$ ). When IV is added as an exogenous variable both the constant term and the ARCH term loses its significance and the contribution of the innovation to oil the futures market becomes negligible.

EGARCH(1.1) FOR DAILY RETURNS ON CRUDE OIL FUTURES CONTRACTS

Var. specification	$\alpha_0$	$\beta_1$	$\theta$	$\gamma$	$\delta$	Log L	$\chi^2$
(3)	-0.0224 (-0.155)	0.619 (5.38)	-0.0778 (-3.317)	0.0417 (1.177)	0.3873 (3.14)	13858.82	
(5)	-0.158 (-6.047)	0.9921 (304.09)	-0.0232 (-2.212)	0.1262 (7.153)		13719.69	278.26
(7)	-0.1253 (-0.369)				0.9947 (22.922)	13841.83	33.98

\*t-stats are presented in parentheses and the standard errors used in t-stats are computed using the robust method of Bollerslev and Wooldridge [1988]

\*\*  $LR = -2[L_R - L_U] \sim \chi^2$  where the statistics for eq. (3) and (5) are respectively distributed with one and three degrees of freedom

For the EGARCH specification the  $\theta$  coefficient is negative and statistically significant, which indicates that there is an asymmetry in the volatility response of the futures returns to shocks. Information criteria are also higher for these types of

<sup>15</sup> Measures the relative goodness of fit for a model by comparing the residual sum of squares (RSS) and adding penalties for the loss of degrees of freedom caused by adding extra parameters. The smaller the value of IC the better the model fit.



models and therefore EGARCH(1,1) may be more useful in forecasting volatility in crude oil market. Like for the GARCH results, when IV is added as an exogenous variable the transformed ARCH term in eq. (3) becomes insignificant. Eq. (6) and (7) are nested versions of eq. (2) and (3). The parameter estimates from these nested GARCH and EGARCH models can be used to infer whether IV is an unbiased estimate of future volatility under the assumption that the market is informationally efficient and option pricing model is specified correctly. If this is the case  $\alpha_0$  and  $\delta$  will be close to zero and one respectively. Deviation from those values is evidence of bias and inefficiency in the forecasts (Canina and Figlewski 1993).

$$H_0: \alpha_0 = 0 \text{ and } \delta = 1$$

For both nested models the constant term is relatively close to zero but not statistically significant. The implied volatility coefficient estimate,  $\delta$ , is 0.895 and 0.995 for eq. (6) and (7) respectively, and both show high significance. The test statistics of joint null hypothesis using the Wald's test is 15.19 for eq. (5) and 5.22 for eq. (6). The chi-squared critical value with 2 degrees of freedom at 5% significance level is 5.991. This implies that we cannot reject the null hypothesis for the nested EGARCH model and that IV's provide unbiased forecasts. In case of the nested GARCH we reject the null hypothesis, which implies biased estimates of IV. This bias could be caused by parameter restrictions imposed by the GARCH model.

We conclude that implied volatility has statistically significant within-sample explanatory power and that it contains information that is not included in the time series models. However the regression results also provide evidence that GARCH-type models contain information that is not included in the futures option prices. Given the statistical properties of EGARCH, this model performs better than GARCH when modeling the WTI crude oil volatility.

## 5.2 Out-of-Sample Tests

One limitation of the within-sample tests is that they assume a constant structure of the underlying financial market. Another implication is that GARCH-type models produce one-day-ahead volatility forecasts while IV over the remaining lifetime of the option. This means that we have a maturity mismatch between the forecast horizon for time series models and implied volatility model. For out-of-sample tests we can control for this problem by estimation a rolling 21-day-ahead forecast for GARCH-type models, which is the average time to maturity of our sample options. The out-of-sample tests are performed by regressing the realized volatility of WTI futures against the forecasts of alternative models using the ordinary least squares (OLS)

$$\sigma_{Ht+1}^2 = b_0 + b_1\sigma_{Ft}^2 + \eta_{t+1}$$

where  $\sigma_{Ht+1}^2$  is the realized volatility for the subsequent period, calculated as the average variance of daily returns for the remaining days until expiration of the option (following Day and Lewis 1993, Szakmary, Ors, Kim and Davidson 2003). The OLS procedure will produce consistent regression estimates even with correlated residuals, but the estimated coefficients standard errors will be biased (Figlewski 2004). In order to avoid the problem of autocorrelation of residuals and heteroscedasticity, the Newey-West estimator for the coefficient covariance matrix is used. Forecasting models are evaluated on the basis of the parameter  $b_1$ . If the forecast contain information that is useful in predicting future volatility, the  $b_1$  coefficient should be significantly greater than zero. If the forecast of volatility is unbiased the estimate of  $b_0$  should be approximately zero and  $b_1$  close to one (Day and Lewis 1993). The predictive ability of different forecasting models can be measured using the average forecast error (ME), the root mean-squared error (RMSE), and the mean absolute forecast error (MAE). As noted by Lamourex and Lastrapes (1993), the information content represented by  $R^2$  in the regression tests does not translate directly into forecast accuracy.

$$ME = \frac{1}{N} \sum_{i=1}^N (\sigma_{Ht+1}^2 - \sigma_{At+1}^2)$$

$$RMSE = \left[ \frac{1}{N} \sum_{i=1}^N (\sigma_{Ht+1}^2 - \sigma_{At+1}^2)^2 \right]^{1/2}$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |\sigma_{Ht+1}^2 - \sigma_{At+1}^2|$$

where  $\sigma_{At+1}^2 = b_0 + b_1 \sigma_{Ft}^2$

The results show that based on simple model information criteria such as  $R^2$ , the implied volatility forecasts have the most explanatory power ( $R^2 = 0.61$ ). All the volatility models on average overstate the realized volatility (RV), however IV seems to produce the closest fit. The relation between RV and the different forecasting models for period 2006-2011 is shown in Figures 4 and 5. A volatility risk premium could cause the implied volatility to overstate the market's volatility expectation, and in turn, overstate future volatility (Fleming 1998). Whether different forecast models are rational predictors of future volatility can be inferred by testing for unbiasedness of the model. The joint null hypothesis is that  $b_0 = 0$  and  $b_1 = 1$ . Looking at chi-square statistics from Wald's test for coefficient restrictions, the null hypothesis is rejected for all models but EGARCH. This indicates that GARCH, IV and historical volatility models have statistically significant biases.

The failure of IV as a rational forecast of the realized volatility of WTI futures might be caused by irrational investor behavior. Such a theory would imply that crude oil traders systematically ignore readily available information, which in turns make market expectation of future volatility a poor predictor of true conditional volatility. We decide to disregard this theory based on broad empirical research that supports the market efficiency theory, meaning that on average investors do make good use of the available information when pricing securities in different markets. It would be strange if that weren't the case for crude oil markets. A more realistic explanation, would be a violation of one or several of Black's model assumptions, or that using Black's model for pricing American options is not an adequate approach for deriving reasonable implied volatility forecasts. For the EGARCH model the null hypothesis is not rejected and this might be due to asymmetries in the volatility response to shocks in crude oil markets. As we have discussed earlier, EGARCH specification take such

asymmetries into account. When comparing with the GARCH model, EGARCH have higher explanatory power and from a statistical perspective it is a better model when dealing with crude oil volatility. Historical volatility performs the worst<sup>16</sup>. Despite the evidence of forecast bias, the regression results suggest that all models contain some extent information regarding future volatility.

OUT-OF-SAMPLE PREDICTIVE POWER OF ALTERNATIVE VOLATILITY FORECASTS

$\sigma_{Ft}^2$	$b_0 * 10^{-3}$	$b_1$	$R^2$	$\chi^2$
Historic Vol	0.177 (3.804)	0.6511 (6.265)	0.469	14.51*
GARCH	0.133 (2.409)	0.7026 (6.135)	0.482	6.75*
EGARCH	0.0039 (0.736)	0.868 (7.89)	0.531	2.67
Implied Volatility	-0.0012 (0.276)	0.913 (10.356)	0.612	11.03*

\* Significant at the 1% level

The bias of forecasting models can be corrected for by fitting the past values of  $b_0$  and  $b_1$  and use these parameter estimates to adjust the out-of-sample forecasts of volatility. However, Day and Lewis (1993) show that such bias correction does not work as intended since the regression parameter estimates are not constant and the bias itself also varies over time.

COMPARISON OF THE ACCURACY OF VOLATILITY FORECASTS

Forecast	RMSE	MAE
Historic	0.000511	0.000289
GARCH	0.000505	0.000284
EGARCH	0.00048	0.000274
Implied	0.000437	0.000241

The out-of-sample comparisons of accuracy of forecasts show that, implied volatility provides the most accurate forecasts for future WTI volatility. Ranking from the most accurate to least accurate, we have IV, EGARCH, GARCH, and historical volatility respectively. The reason for not including ME is that the positive and negative deviations would cancel each other out and result in artificially low measures of forecast errors. The other two criteria avoid that by using squared values of deviation (RMSE) and absolute values (MAE).

<sup>16</sup> A test using a 30- and 60-day sample period has also been performed yielding similar results.

### 5.3 Model Comparison

The relative content of information, and whether one forecast contain information that is different from another, can be evaluated by running the following regression using the OLS.

$$\sigma_{Ht+1}^2 = b_0 + b_1\sigma_{IVt}^2 + b_2\sigma_{Gt}^2 + b_3\sigma_{Et}^2 + b_4\sigma_{Nt}^2 + \eta_{t+1}$$

If a forecast of volatility contains information that is useful in predicting future volatility, the regression coefficients should be significant and greater than zero.

FORECAST COMPARISON OF THE RELATIVE INFORMATION CONTENT FOR OUT-OF-SAMPLE FORECASTS OF VOLATILITY

Forecast Comparison	$b_0 * 10^{-5}$	$b_1$	$b_2$	$b_3$	$b_4$	$R^2$
IV, GARCH	-1.772 (-0.392)	1.041 (6.52)	-0.122 (-1.164)			0.6146
IV, GARCH, Historic vol.	-4.675 (-0.93)	1.119 (6.58)	0.237 (0.935)		-0.405 (-1.504)	0.6219
IV, EGARCH	-0.949 (-0.221)	0.999 (5.46)		-0.094 (-0.595)		0.6129
IV, EGARCH, Historic vol.	-7.102 (-1.213)	1.014 (5.769)		0.415 (1.604)	-0.43 (-2.137)	0.6245
IV, Historic vol.	-3.583 (-0.73)	1.145 (6.254)			-0.206 (-1.7)	0.6196
GARCH, Historic vol.	13.89 (2.298)		0.566 (1.352)		0.131 (0.358)	0.4828
EGARCH, Historic vol.	-1.578 (-0.272)			1.338 (5.929)	-0.386 (-2.709)	0.5404
GARCH, EGARCH	-6.882 (-1.401)		-1.188 (-3.254)	2.251 (2.022)		0.562

Notes: The forecasting sample ranges from 09/13/2004 to 12/30/2011, including 1837 trading days. Historical volatility as mentioned before is an average over 21 trading days. The Newey-West covariance estimator that is consistent in the presence of both heteroskedasticity and autocorrelation is used.

The results show that IV is highly significant in all cases and that neither GARCH, EGARCH nor historical volatility adds much explanatory power to IV forecasts. None of these coefficients are significant when IV is present as explanatory variable and the improvement to  $R^2$  is very small (at most 0.013 comparing forecast from just IV to combined forecasts). This indicates that implied volatility subsume the information that is contained in the time series volatility forecasts. This is consistent with the efficient market hypothesis. When GARCH and EGARCH are combined, both models are significant, but only the

EGARCH model have regression coefficient greater than zero. For combination of implied volatilities and other forecasts, the combination of IV, EGARCH, and historic volatility produce the most accurate forecasts. Even though these differences are very small it seems that EGARCH is more accurate than GARCH, both in terms of single and combined forecasts.

COMPARSION OF THE ACCURACY OF COMBINED VOLATILITY FORECASTS

Forecast	RMSE	MAE
IV, GARCH	0.0004355	0.0002403
IV, GARCH, Historic	0.0004314	0.0002401
IV, EGARCH	0.0004365	0.000241
IV, EGARCH, Historic	0.0004299	0.0002392
IV, Historic	0.0004327	0.0002396

The out-of-sample results suggest that implied volatilities provide better forecasts than GARCH, EGARCH and historical volatility models. The results give support to the theory that option prices incorporate all available information. This is in contrast to results of Canina and Figlewski (1993), who argue that IV from S&P100 options has no correlation with future volatility. Evidence from a variety of studies<sup>17</sup> point out that IV is positively related to the ease of performing the arbitrage trade related to complexity of the hedge position, transaction costs and rebalancing risk. As IV did not pass the test of forecast rationality, this is indirectly in contradiction to the conclusion that IV is the best available predictor of future volatility. However, in order for implied volatility to be an efficient volatility forecast we have to eliminate biases caused by Black's model assumption and secondly, investors have to behave rational when using the available market information in decision making process.

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<sup>17</sup> Canina and Figlewski (1993), Jorion (1995), Fleming (1998)

## 6 CONCLUSION

We have looked at implied volatility as an informationally efficient forecast of the volatility that will prevail in the underlying futures throughout the option's lifetime. This means that the price of an option is based on option pricing models that incorporate the market expectation of future volatility, so that implied volatility is the market's true expected volatility. Secondly, investors are assumed to be rational when evaluating the available information, so that the volatility implied by the market is the correct conditional expected value of the future volatility. We have examined the relative ability of implied volatilities and generalized autoregressive conditional heteroskedasticity models to predict the near-term future volatility. The test results show that the implied volatility from WTI crude oil futures options has significant explanatory power for both within- and out-of-sample tests and that it is a better predictor of volatility than time series models. Combining implied volatility with GARCH, EGARCH and historical models does not add much explanatory power to predictions, and the improvement of  $R^2$  is negligible. This supports the efficient market hypothesis and the academic view that implied volatility incorporates all available information in the market. As a result, implied volatility may be useful as a real-time measure of expected WTI crude oil volatility or may be helpful in predicting expected WTI futures returns. The strength of implied volatility is that it can adapt more quickly to price shocks in the oil market and that it takes into account many factors (indirectly accounted for by investors' expectations) that are not incorporated in time series models based on historical data. When it comes to rationality tests of implied volatility, the results are similar to those of Canina and Figlewski (1993), Jorion (1995), Fleming (1998), and Szakmary, Ors, Kim and Davidson (2003). The tests result in rejection of the null hypothesis that implied volatility is an unbiased predictor of future volatility (estimates for the constant and the slope coefficients are different from 0 and 1 respectively). The only study that finds IV as a rational predictor of future near-term (2 months) volatility is Day and Lewis (1993). Such bias may either suggest misspecification of the volatility process in the option pricing model and/or the existence of early exercise opportunities. The use of Black's model for pricing of near-term at-the-money American options seems to introduce a small bias, but such early exercise premium is shown to have

on average just 2-5% of option value (Fleming 1998). It is also shown that the Rubinstein (1994) binomial approach that incorporates such early exercise possibility is less reliable for out-of-sample valuation and hedging purposes than Black/Scholes model (Dumas, Fleming and Whaley 1998). Despite this bias, implied volatility forecasts are superior to those produced by time series models, and our results support the empirical use of the implied volatility as a proxy for conditional volatility.

In order to improve our results it might be helpful to look at the options with constant time to expiration in order to avoid daily variations in implied volatilities on the same underlying. This approach should produce more consistent test results and it will be much easier to match maturities of different forecasting models. It will be also possible to take into account the term structure of volatility<sup>18</sup> and obtain the same t-step-ahead forecasts for all volatility models.

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<sup>18</sup> The return horizon is not exactly matched with the life of the option.



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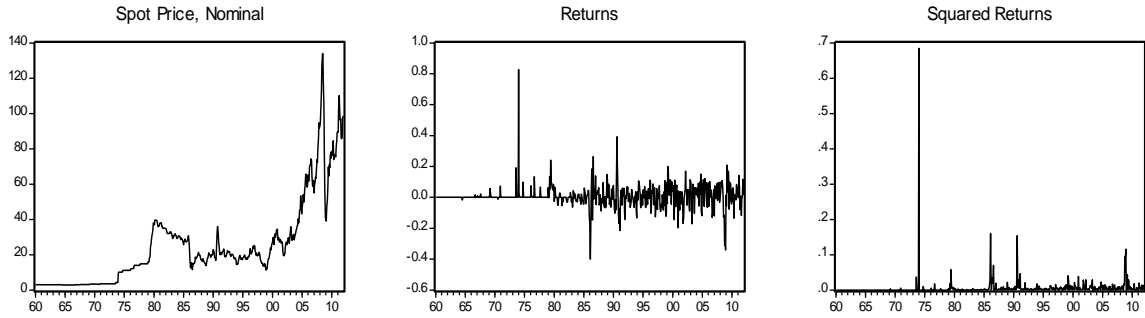
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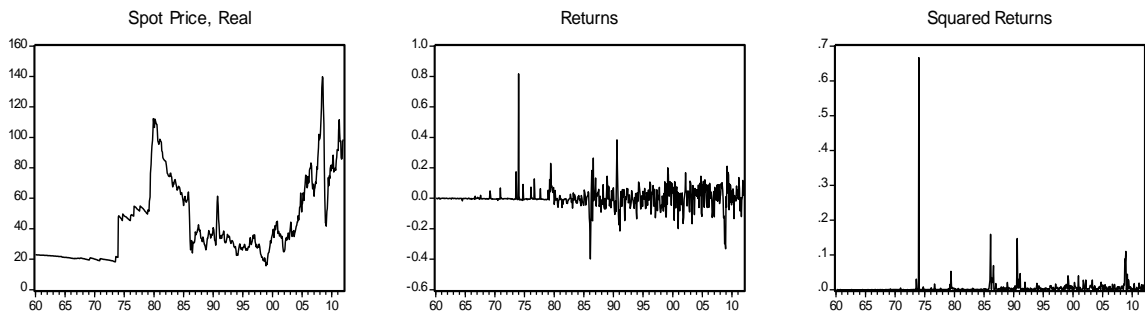
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## Figures

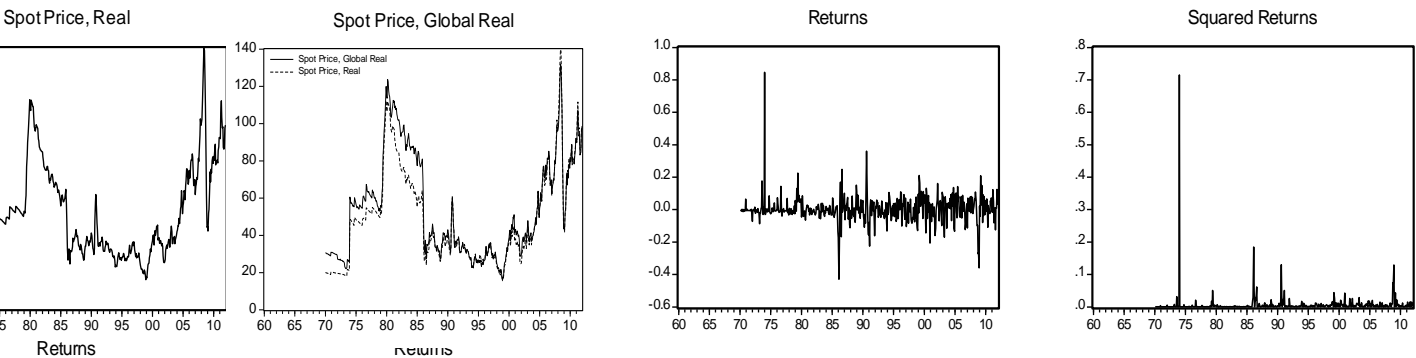
**Figure 1a. Nominal WTI crude oil spot price**



**Figure 1b. Real WTI crude oil spot price**



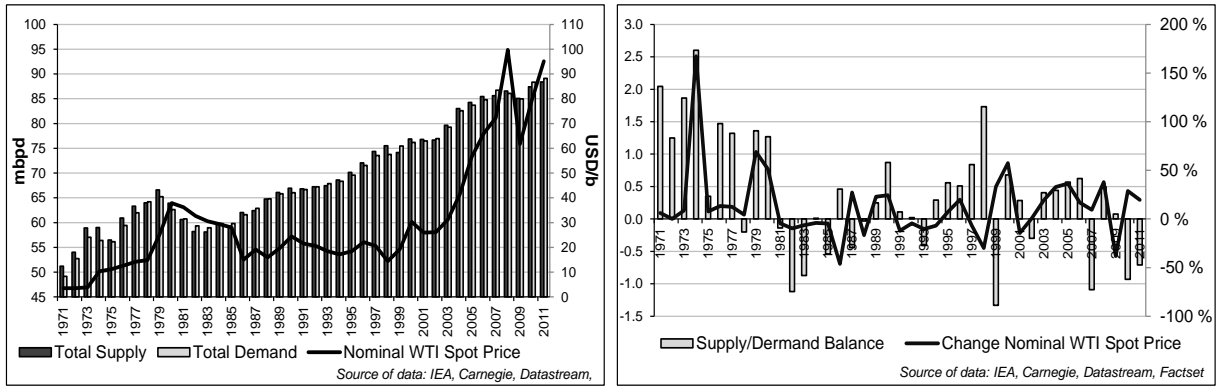
**Figure 1c. Global real WTI crude oil spot price**



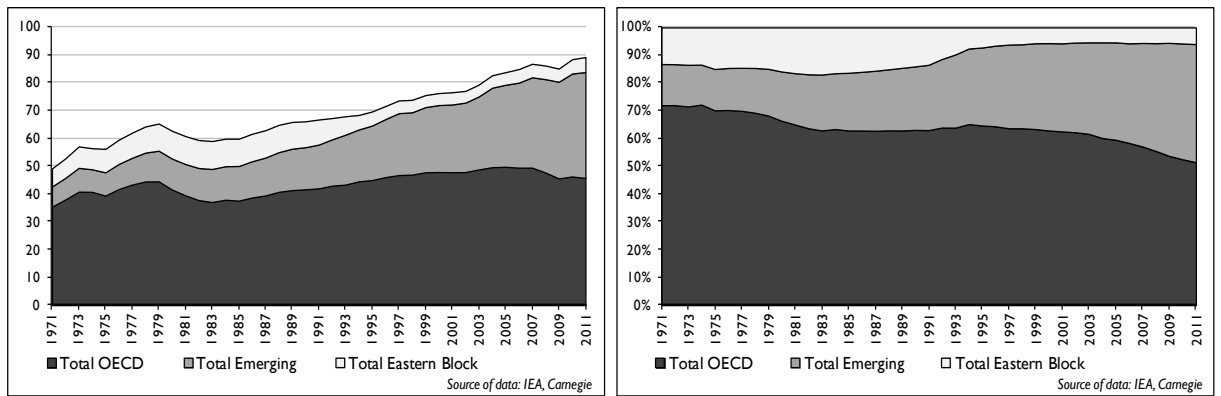
*Note:* The real spot price is calculated as the monthly average spot WTI crude oil price divided by the ratio of the US consumer price index for all urban consumers (CPI-U) for the previous month to the US CPI-U in December 2011.

*Note:* The global real spot price is calculated as the monthly average spot WTI crude oil price divided by the ratio of the G7 headline CPI (NADJ) for the previous month to the G7 headline CPI (NADJ) in December 2011, divided by the ratio of the US Dollar (USD) price of the Special Drawing Rights (SDR) for the previous month to the USD price of the SDR in December 2011.

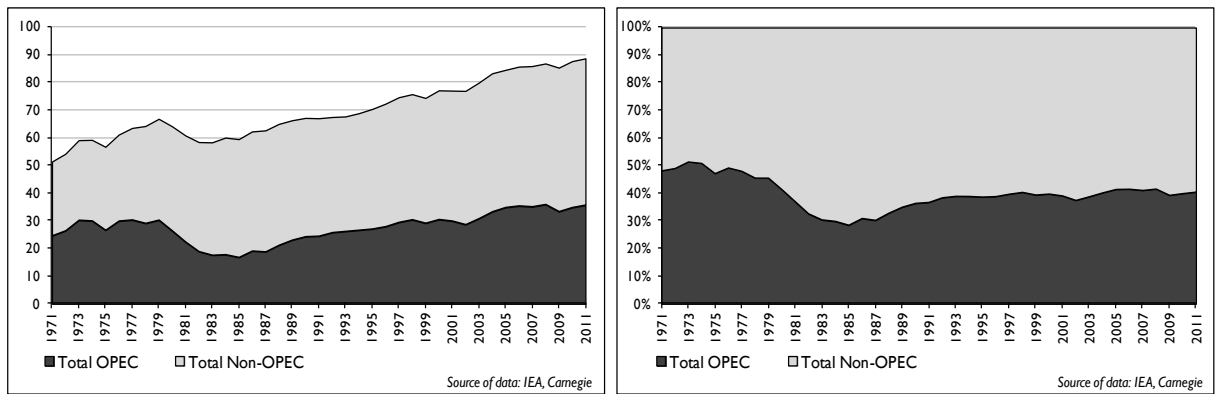
**Figure 2a. Crude oil supply and demand**



**Figure 2b. Crude oil consumption split**



**Figure 2c. Crude oil production split**

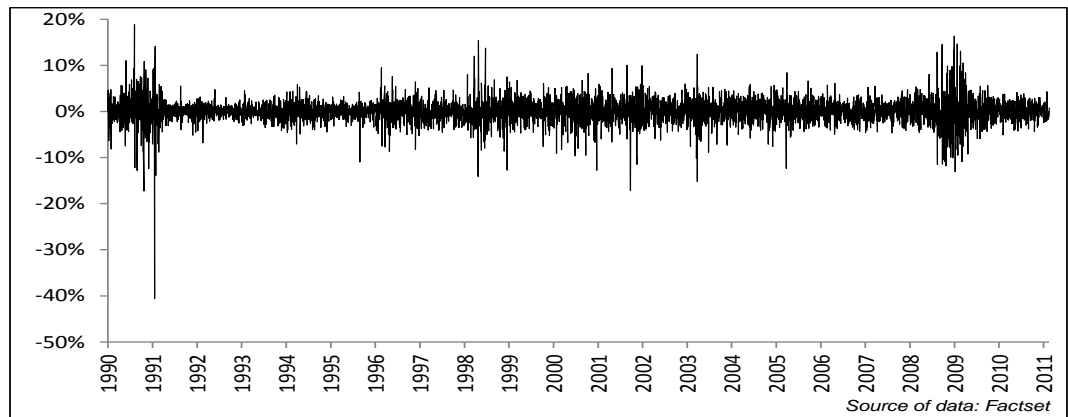


**Figure 3. The price of one US Dollar in terms of one Special Drawing Rights (SDR)**

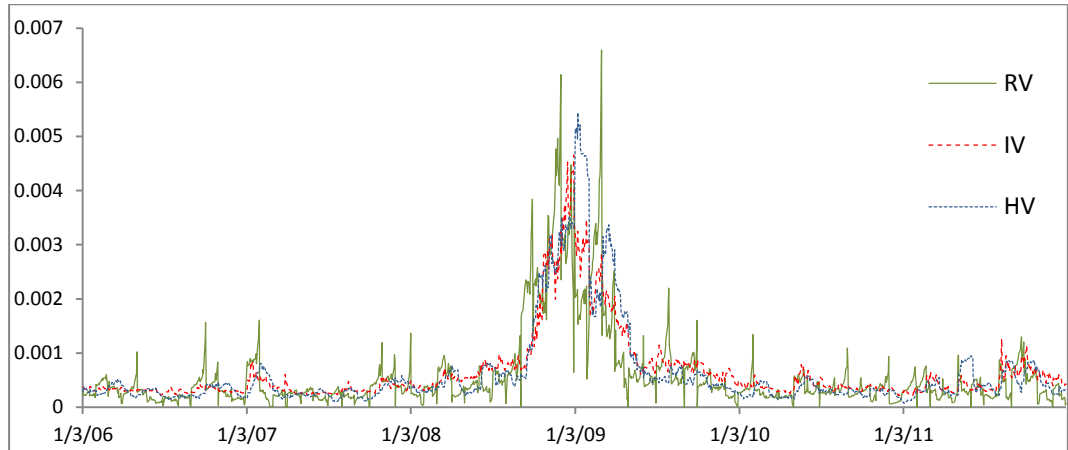


Note: A falling graph represents a weakening dollar.

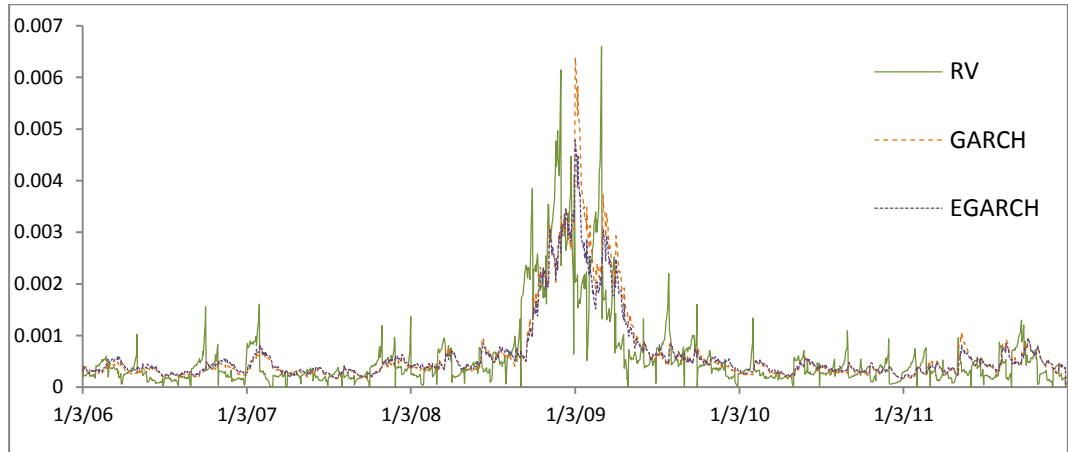
**Figure 4. Daily returns of WTI futures**



**Figure 5. Comparison of IV and HV as a predictor of RV**



**Figure 6. Comparison of GARCH and EGARCH forecasts**





## Tables

**Table 1a. Augmented Dickey-Fuller test of WTI futures prices**

Null Hypothesis: WTI has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=32)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-0.80679	0.816665
Test critical values: 1% level	-3.43136	
5% level	-2.86187	
10% level	-2.56699	

\*MacKinnon (1996) one-sided p-values.

**Table 1b. Augmented Dickey-Fuller test of WTI futures returns**

Null Hypothesis: LOG\_WTI has a unit root

Exogenous: Constant

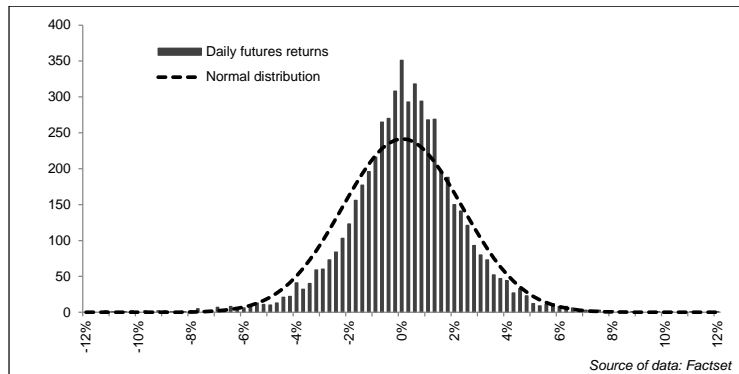
Lag Length: 0 (Automatic based on SIC, MAXLAG=32)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-74.2542	0.0001
Test critical values: 1% level	-3.43136	
5% level	-2.86187	
10% level	-2.56699	

\*MacKinnon (1996) one-sided p-values.

**Table 2. Descriptive statistics of WTI futures returns**

WTI futures returns descriptives	
Mean	0.00028
Median	0.00058
Maximum	0.21909
Minimum	-0.38407
Std. Dev.	0.02275
Skewness	-0.87511
Kurtosis	21.82277
Jarque-Bera	82088.6
Probability	0.00000
Observations	5513



**Table 3a. Correlogram of WTI futures returns**

Lag	AC	PAC	Q-Stat	Prob
1	-3.6E-05	-3.6E-05	7.0E-06	0.9979
2	-0.0335	-0.0335	6.1973	0.0451
3	-0.0200	-0.0200	8.4040	0.0384
4	-0.0118	-0.0130	9.1748	0.0569
5	-0.0256	-0.0270	12.7906	0.0254
6	-0.0295	-0.0309	17.6050	0.0073
7	0.0051	0.0027	17.7474	0.0132
8	-0.0270	-0.0304	21.7616	0.0054
9	-0.0065	-0.0083	21.9928	0.0089
10	0.0151	0.0118	23.2515	0.0099
11	0.0071	0.0040	23.5338	0.0148
12	0.0161	0.0153	24.9607	0.0150
13	0.0234	0.0231	27.9810	0.0091
14	0.0466	0.0465	39.9860	0.0003
15	0.0093	0.0127	40.4618	0.0004
16	0.0207	0.0262	42.8428	0.0003
17	-0.0265	-0.0223	46.7155	0.0001
18	-0.0264	-0.0205	50.5724	0.0001
19	0.0001	0.0037	50.5725	0.0001
20	0.0129	0.0148	51.4956	0.0001

**Table 3b. Information criteria of WTI futures returns**

Akaike's Information Criterion						
ar/ma	7	8	9	10	11	12
8	-4.7313	-4.7334	-4.7331	-4.7337	-4.7325	-4.7322
9	-4.7327	-4.7338	-4.7328	-4.7337	-4.7320	-4.7326
10	-4.7337	-4.7335	-4.7335	-4.7342	-4.7346	-4.7330
11	-4.7318	-4.7320	<b>-4.7348</b>	-4.7345	-4.7339	-4.7342
12	-4.7318	-4.7339	-4.7337	-4.7333	-4.7343	
13	-4.7319	-4.7329	-4.7330	-4.7331		
14	-4.7337	-4.7337	-4.7342			
15	-4.7321	-4.7326				
16	-4.7324					

*\*table uncomplete due to restrictions in Eviews, allowing maximum 23 terms in the ARMA model*

Schwarz's Information Criterion						
ar/ma	0	1	2	3	4	5
0	<b>-4.7267</b>	-4.7252	-4.7248	-4.7237	-4.7224	-4.7215
1	-4.7256	-4.7255	-4.7249	-4.7234	-4.7219	-4.7209
2	-4.7252	-4.7253	-4.7242	-4.7235	-4.7235	-4.7225
3	-4.7240	-4.7238	-4.7223	-4.7213	-4.7212	-4.7209
4	-4.7225	-4.7222	-4.7206	-4.7221	-4.7197	-4.7196
5	-4.7223	-4.7213	-4.7216	-4.7197	-4.7202	-4.7171

**Table 4. ARCH test for the WTI futures returns**

F-statistic	29.7201	Probability	0.0000
Obs*R-squared	144.8504	Probability	0.0000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.0004	0.0000	10.8308	0.0000
RESID^2(-1)	0.0495	0.0135	3.6696	0.0002
RESID^2(-2)	0.0472	0.0135	3.4988	0.0005
RESID^2(-3)	0.1218	0.0134	9.0916	0.0000
RESID^2(-4)	0.0458	0.0135	3.3980	0.0007
RESID^2(-5)	0.0197	0.0135	1.4585	0.1448

R-squared	0.0263	Mean dependent var	0.0005
Adjusted R-squared	0.0254	S.D. dependent var	0.0024
S.E. of regression	0.0023	Akaike info criterion	-9.2822
Sum squared resid	0.0299	Schwarz criterion	-9.2749
Log likelihood	25569.0	F-statistic	29.7201
Durbin-Watson stat	2.0002	Prob(F-statistic)	0.0000

## APPENDIX

### Java code; filtering the data

```

/**
 * An instance of type class represents option contracts
 */
class Option {
    static File dir = new File("C:\\futures\\cl_opt"); // reads inn option files (.csv)
    public String ticker ; // option ticker
    public SortedMap<Date, Double> close ; // map-object that collects dates and close
                                                values on given contract

    public int strike ; // option's strike
    public boolean pc ; // true=>put, false=>call

    /**
     * Reads all option contracts for a given underlying future file
     *
     * @param fut
     * @return Options data is collected in one array. Each contract represents one element in
     * the array.
     * @throws IOException
     * @throws ParseException
     */
    public static List<Option> loadOptions(Future fut) throws IOException, ParseException {
        List<Option> options = new ArrayList<Option>();
        for(File file : dir.listFiles()) { // for each file in option folder
            if(!file.getName().contains(fut.ticker)) {
                continue ; // ignores all option contracts that are not assign to
                            a given future contract
            }
            BufferedReader br = new BufferedReader(new FileReader(file)) ;
            String l;
            Option opt = new Option() ;
            opt.close = new TreeMap<Date, Double>();
            while(null != (l=br.readLine())) {
                // for hver linje i fila
                String[] cols = l.split(",") ; // creates a string-array from
                                                comma separated columns

                if(opt.ticker == null) {
                    opt.ticker = cols[0] ;
                    opt.pc = opt.ticker.endsWith("P") ;
                    opt.strike = Integer.parseInt(opt.ticker.substring(7,11)) ;
                    // CL1999X1800P
                }
                Date d = Future.df.parse(cols[1]) ;
                // date is assigned to a second column
                opt.close.put(d, Double.parseDouble(cols[2])) ;
                // close value is assigned to a third column
            }
            br.close() ;
            options.add(opt) ;
        }
        System.out.println("Options loaded for " + fut.ticker + ": " + options.size());
        return options ;
    }
}

```

```
/**
 * Specifying output format
 *
 */
public class Future {
public static SimpleDateFormat df = new SimpleDateFormat("MM/dd/yyyy") ;
public String ticker ;
public List<Option> options ;
public SortedMap<Date, Double> close ;

    /**
     * Selects at-the-money option contracts for a given date
     *
     * @param d
     * @param pc Put/call
     * @return
     */
    public Option pickNearestOption(Date d, boolean pc) {
        Option nearest = null ;
        double mindiff = Double.MAX_VALUE ; // min difference
        double futclose = close.get(d) ; // close value for a given future contract on a
            given day
        for(Option opt : options) { // for each option contract on the underlying future
            if(pc != opt.pc) continue ; // correct put/call
            Double val = opt.close.get(d) ; // close value for option
            if(val != null) {
                double strike;
                if(futclose > 80.0 && opt.strike < 2000) {
                    // need to adjust if crude -> 200 usd
                    strike = opt.strike / 10.0 ; // corrects the strike value
                }
                else {
                    strike = opt.strike / 100.0 ;
                }
                double diff = Math.abs(futclose - strike) ; // diff between fut.
                    close & opt. X
                if(diff < mindiff) {
                    mindiff = diff ;
                    nearest = opt ; // saves the nearest to expiration opt.
                }
            }
        }
        return nearest ; // returns ATM options
    }

    /**
     * Filter option contracts so that time to maturity is > 10 trading days
     *
     * @return
     */
    public Date getLastOptionDate() {
        List<Date> dates = new ArrayList<Date>() ;
        for(Option opt : options) {
            for(Date td : opt.close.keySet()) {
                boolean had_date = false ;
                for(Date d : dates) {
                    if(d.equals(td)) {
                        had_date = true;
                        break ;
                    }
                }
            }
        }
    }
}
```

```

        }
        if(!had_date) {
            dates.add(new Date(td.getTime()));
        }
    }
}
Collections.sort(dates);
return dates.get(dates.size() - 11);
}

/**
 * Reads inn all future contracts and sorts them after dates
 *
 * @return
 * @throws IOException
 * @throws ParseException
 */
public static List<Future> loadFutures() throws IOException, ParseException {
    File dir = new File("C:\\futures\\cl_fut");
    List<Future> futures = new ArrayList<Future>();
    for(File f : dir.listFiles()) { // for each file in the folder
        BufferedReader br = new BufferedReader(new FileReader(f));
        String l;
        Future fut = new Future();
        fut.close = new TreeMap<Date, Double>();
        while(null != (l = br.readLine())) { // for each line in a file
            String[] cols = l.split(","); // comma separated columns
            if(fut.ticker == null) fut.ticker = cols[0]; // ticker = column 1
            Date d = df.parse(cols[1]); // dato = column 2
            fut.close.put(d, Double.parseDouble(cols[5]));
            // close = column 6
        }
        br.close();
        fut.options = Option.loadOptions(fut);
        futures.add(fut);
    }
    Collections.sort(futures, new Comparator<Future>() { // sorts after ticker (ie.
                                                            dates)
        @Override
        public int compare(Future o1, Future o2) {
            return o1.ticker.compareTo(o2.ticker);
        }
    });
    return futures;
}

static public List<Date> getSortedDateList(Set<Date> dateSet) {
    List<Date> list = new ArrayList<Date>();
    for(Date d : dateSet) {
        list.add(d);
    }
    Collections.sort(list);
    return list;
}
}

```

```

/**
 * Gather all the data from futures and options files to one file where each row represents: date,
 * close price for a given future contract, close price for short-term ATM call, close price for short-
 * term ATM put
 *
 * @param args
 * @throws IOException
 * @throws ParseException
 */
public static void main(String[] args) throws IOException, ParseException {
    System.out.println("Load all futures.");
    List<Future> futures = loadFutures();
    System.out.println("Finished loading, list size=" + futures.size());
    File outFile = new File("C:\\futures\\rune.csv"); // output file
    BufferedWriter bw = new BufferedWriter(new FileWriter(outFile));
    Date next = null;
    for(Future fut : futures) {
        System.out.println("Processing " + fut.ticker);
        List<Date> dateList = getSortedDateList(fut.close.keySet());
        Date last = fut.getLastOptionDate();
        // rolling over to the next future contract
        if(next == null) next = fut.close.firstKey();
        int i=0;
        while(true) {
            Date d = dateList.get(i++);
            if(d.before(next)) continue;
            if(d.after(last)) {
                next = d;
                break;
            }
            double close = fut.close.get(d);
            Option call = fut.pickNearestOption(d, false);
            Option put = fut.pickNearestOption(d, true);
            if(call != null && put != null) {
                StringBuilder col = new StringBuilder();
                // date,future,future close,call ticker,call close,put ticker,put close

                col.append(df.format(d) + ","); // date
                col.append(fut.ticker + ","); // future
                col.append(close + ","); // close
                col.append(call.ticker + "," + call.close.get(d) + ","); // call
                col.append(put.ticker + "," + put.close.get(d) + "\r\n"); // put
                bw.write(col.toString());
            }
            else {
                System.out.println(fut.ticker + " missing date " + df.format(d)
                    + " - put="+(put!=null)+ " call="+(call!=null));
            }
        }
        bw.flush();
    }
    bw.close();
}

```

## Visual Basic Code; Implied Volatility

This code is based on DerivaGem software provided by John C. Hull as a part of Options, Futures, and Other Derivatives 2012.

```

Function BlacksOption(CallorPut, F, X, v, r, T)
    Dim d1 As Double, d2 As Double, nd1 As Double, nd2 As Double
    Dim nnd1 As Double, nnd2 As Double

    d1 = (Log(F / X) + (0.5 * v ^ 2) * T) / (v * Sqr(T))
    d2 = (Log(F / X) - (0.5 * v ^ 2) * T) / (v * Sqr(T))
    nd1 = Application.NormSDist(d1)
    nd2 = Application.NormSDist(d2)
    nnd1 = Application.NormSDist(-d1)
    nnd2 = Application.NormSDist(-d2)

    If CallorPut = "Call" Or CallorPut = "call" Then
        BlacksOption = Exp(-r * T) * (F * nd1 - X * nd2)
    Else
        BlacksOption = Exp(-r * T) * (X * nnd2 - F * nnd1)
    End If
End Function

Function ImpliedVolatility(CallorPut, F, X, r, T, OptionValue, guess)
    Dim epsilon As Double, dVol As Double, vol_1 As Double
    Dim i As Integer, maxIter As Integer, Value_1 As Double, vol_2 As Double
    Dim Value_2 As Double, dx As Double

    dVol = 1e-05
    epsilon = 1e-05
    maxIter = 100
    vol_1 = guess
    i = 1
    Do
        Value_1 = BlacksOption(CallorPut, F, X, vol_1, r, T)
        vol_2 = vol_1 - dVol
        Value_2 = BlacksOption(CallorPut, F, X, vol_2, r, T)
        dx = (Value_2 - Value_1) / dVol
        If Abs(dx) < epsilon Or i = maxIter Then Exit Do
        vol_1 = vol_1 - (OptionValue - Value_1) / dx
        i = i + 1
    Loop
    ImpliedVolatility = vol_1
End Function

```

## eViews code

The presented code produce a 21-day-ahead static rolling sample forecast for GARCH model. Similar approach is also used for the EGARCH model.

```
' set window size
!window =3655
' get size of workfile
!length = @obsrange
' declare equation for estimation
equation eq1
' set step size
!step = 1
' calculate number of rolls
!nrolls = @floor((!length-!window)/!step)

'matrix to store coefficient estimates
matrix(4,!nrolls) coefmat 'where the number of coefficients is 4.
series fcast 'series to store forecast estimates
series fcastse '
series fcastvar

%start = "@first"
%end = "@last"
'variable keeping track of how many rolls we've done
!j = 0
' move sample !step obs at a time
for !i = 1 to !length-!window+1-!step step !step
  !j = !j +1
  %first = @otod(@dtoo(%start)+!i-1)
  %last = @otod(@dtoo(%start)+!i+!window-2)
  smpl {%first} {%last}

      colplace(coefmat,eq1.@coefs,!j) 'store coefficients

' 21-period-ahead forecast
%21pers = @otod(@dtoo(%start)+!i+!window-1) 'start point
%21pere = @otod(@dtoo(%start)+!i+!window+20) 'end point

' set smpl for forecasting period
smpl {%21pers} {%21pere}

eq1.fit(f=na) g_f1 g_se g_var

' store forecasts vars
fcast = g_f1
fcastse = g_se
fcastvar = g_var
next

smpl@all
```



## WTI contract specifications

### Light Sweet Crude Oil Futures

Code	CL
Venue	CME Globex, CME ClearPort, Open Outcry (New York)
Hours (All Times are New York Time/ET)	<b>CME Globex and CME Clearport:</b> Sunday – Friday 6:00 p.m. – 5:15 p.m. (5:00 p.m. – 4:15 p.m. Chicago Time/CT) with a 45-minute break each day beginning at 5:15 p.m. (4:15 p.m. CT) <b>Open Outcry:</b> Monday – Friday 9:00 a.m. – 2:30 p.m. (8:00 a.m. – 1:30 p.m. CT)
Contract Unit	1,000 barrels
Pricing Quotation	U.S. dollars and cents per barrel
Minimum Fluctuation	\$0.01 per barrel
Termination of Trading	Trading in the current delivery month shall cease on the third business day prior to the twenty-fifth calendar day of the month preceding the delivery month. If the twenty-fifth calendar day of the month is a non-business day, trading shall cease on the third business day prior to the last business day preceding the twenty-fifth calendar day. In the event that the official Exchange holiday schedule changes subsequent to the listing of a Crude Oil futures, the originally listed expiration date shall remain in effect. In the event that the originally listed expiration day is declared a holiday, expiration will move to the business day immediately prior.
Listed Contracts	Year and the next five years; in addition, the June and December contract months are listed beyond the sixth year. Additional months will be added on an annual basis after the December contract expires, so that an additional June and December contract would be added nine years forward, and the consecutive months in the sixth calendar year will be filled in. Additionally, trading can be executed at an average differential to the previous day's settlement prices for periods of two to 30 consecutive months in a single transaction. These calendar strips are executed during open outcry trading hours.
Settlement Type	Physical
Trading at Settlement (TAS)	Trading at settlement is available for spot (except on the last trading day), 2nd, 3rd and 7th months and subject to the existing TAS rules. Trading in all TAS products will cease daily at 2:30 p.m. Eastern Time. The TAS products will trade off of a "Base Price" of 0 to create a differential (plus or minus 10 ticks) versus settlement in the underlying product on a 1 to 1 basis. A trade done at the Base Price of 0 will correspond to a "traditional" TAS trade which will clear exactly at the final settlement price of the day.
Delivery	(A) Physical delivery shall be made F.O.B. at any pipeline or storage facility in Cushing, Oklahoma, with pipeline access to TEPPCO, Cushing storage or Equilon Pipeline Company LLC, Cushing Storage. Delivery shall be made in accordance with all applicable Federal executive orders and all applicable Federal, State and local laws and regulations. (B) At buyer's option, such delivery shall be made by any of the following methods: (1) By interfacility transfer ("pumpover") into a designated pipeline or storage facility with access to seller's incoming pipeline or storage facility. (2) By in-tank transfer of title to the buyer without physical movement of product; if the facility used by the seller allows such transfer, or by in-line transfer or book-out if the seller agrees to such transfer.
Grade and Quality Specifications	Physical delivery shall consist of light, sweet crude oil that meets the requirements of the TEPPCO domestic common stream designation, with maximum sulfur content of 0.42% or less using ASTM D-4294, and gravity between 37 and 42 degrees API using ASTM D-287. The complete specifications can be viewed in the CME Rulebook under section 200.12.
Rulebook Chapter	200
Exchange Rule	These contracts are listed with, and subject to, the rules and regulations of NYMEX.

## Light Sweet Crude Oil Options

Contract Name	Light Sweet Crude Oil Option
Underlying Futures	Light Sweet Crude Oil Futures (CL)
Product Symbol	LO
Venue	CME Globex, CME ClearPort, Open Outcry (New York)
Hours (All Times are New York Time/ET)	<b>CME Globex and CME Clearport:</b> Sunday – Friday 6:00 p.m. – 5:15 p.m. (5:00 p.m. – 4:15 p.m. Chicago Time/CT) with a 45-minute break each day beginning at 5:15 p.m. (4:15 p.m. CT) <b>Open Outcry:</b> Monday – Friday 9:00 a.m. – 2:30 p.m. (8:00 a.m. – 1:30 p.m. CT)
Contract Unit	A Light Sweet Crude Oil Put (Call) option traded on the Exchange represents an option to assume a short (long) position in the underlying Light Sweet Crude Oil futures traded on the Exchange.
Price Quotation	U.S. dollars and cents per barrel
Option Style	American
Minimum Fluctuation	\$0.01 per barrel
Expiration of Trading	Trading ends three business days before the termination of trading in the underlying futures contract.
Listed Contracts	Crude oil options are listed nine years forward using the following listing schedule: consecutive months are listed for the current year and the next five years; in addition, the June and December contract months are listed beyond the sixth year. Additional months will be added on an annual basis after the December contract expires, so that an additional June and December contract would be added nine years forward, and the consecutive months in the sixth calendar year will be filled in.
Strike Prices	Twenty strike prices in increments of \$0.50 per barrel above and below the at-the-money strike price, and the next 10 strike prices in increments of \$2.50 above the highest and below the lowest existing strike prices for a total of at least 61 strike prices. The at-the-money strike price is nearest to the previous day's close of the underlying futures contract. Strike price boundaries are adjusted according to the futures price movements.
Settlement Type	Exercise into Futures
Rulebook Chapter	310
Exchange Rule	These contracts are listed with, and subject to, the rules and regulations of NYMEX.





BI Norwegian Business School

-Preliminary Thesis Report-

# Forecasting Oil Futures Volatility

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# Table of Contents

<b>1 INTRODUCTION</b> .....	<b>1</b>
1.1 MOTIVATION AND BACKGROUND.....	1
1.2 THESIS OBJECTIVES .....	2
<b>2 LITERATURE REVIEW</b> .....	<b>4</b>
2.1 COMPARING THE PERFORMANCE OF ALTERNATIVE MODELS .....	4
2.2 INFORMATION CONTENT .....	4
<b>3 THEORETICAL FOUNDATION</b> .....	<b>5</b>
3.1 HISTORICAL VOLATILITY .....	5
3.2 GARCH.....	5
3.3 EGARCH.....	6
3.4 IMPLIED VOLATILITY.....	7
3.5 MEASUREMENT ERRORS .....	9
<b>4 DATA</b> .....	<b>10</b>
<b>5 METHODOLOGY</b> .....	<b>10</b>
5.1 HISTORICAL VOLATILITY.....	10
5.2 GARCH AND EGARCH.....	11
5.3 BINOMIAL PRICING OF AMERICAN OPTIONS .....	12
5.4 THE CBOE VIX APPROACH .....	12
5.5 FORECAST EVALUATION AND HYPOTHESIS TESTING .....	13
<b>6 THESIS PROGRESSION</b> .....	<b>15</b>
<b>7 BIBLIOGRAPHY</b> .....	<b>16</b>

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# 1 INTRODUCTION

## 1.1 Motivation and Background

From a finance perspective, the notion of volatility is perhaps the most important concept to consider. This is due to the fact that most financial decisions are based on a tradeoff between risk and return, and volatility is often seen upon as a rough measure of the total risk of a financial asset (although this perception is somewhat imprecise). Volatility is a fundamental concept for risk managers (e.g. in order to assess the financial risk of a firm's positions), it is an essential variable in the pricing of derivative securities that might be used for hedging purposes, and an important input for monetary policy makers. Consequently, being able to efficiently forecast volatility in various financial markets has been a 'Holy Grail' for many theoretical and empirical researchers over the past couple of decades.

Volatility is often calculated as the sample standard deviation, or more precise, as the square root of the unconditional variance of a set of period returns. Furthermore, assuming returns are best described as a random process (i.e. as white noise process), tomorrow's volatility is often forecasted to be equal to today's volatility, or it is forecasted using some historical average. However, there are a number of well documented features about financial market volatility that such simple linear models are unable to capture. These include the tendency for leptokurtic distributions of risky asset returns (i.e. "fat tails"), volatility clustering, asymmetry, mean reversion and co movements of volatilities across assets and financial markets (Poon and Granger 2003). The ARCH (Auto Regressive Conditional Heteroscedasticity) and GARCH (General ARCH) models were designed to deal with these kinds of issues. These models use time series data on returns to model conditional variance. A popular alternative is implied volatility, where one calculates the volatility implied by option prices. Once the market has produced prices for options, it is possible to back out the volatility given by these prices. This is often interpreted as the market expectation of future volatility, and thus should be superior in forecasting volatility. Another strength of implied volatility is that it can adapt more quickly to changing market conditions. On the other hand, one might argue that the volatility estimate implied by option prices might be flawed due to model misspecifications and/or the assumptions underlying the option pricing model that are in contrast to what is observed in actual financial markets.

## 1.2 Thesis Objectives

*"Prediction is very difficult, especially if it's about the future."*

-Nils Bohr, Nobel laureate in Physics-

In our thesis we wish to compare the predictive ability of two types of approaches that can be used to forecast volatility of an underlying asset: GARCH-type models and the implied volatility from option prices. This decision is based on previous empirical results, where Akgiray (1989) finds that the GARCH model is superior to ARCH, exponentially weighted moving average and historical mean models for forecasting monthly US stock index volatility. These results are also supported by findings to West and Cho (1995), where they estimate one-step-ahead forecasts of the dollar exchange volatility. The main objective of this paper is to examine the forecasting performance of GARCH and implied volatility models in predicting the volatility of Light Sweet Crude Oil (WTI) futures. We chose the crude oil futures market for the following reasons:

- 1) Crude oil is perhaps the world's most important commodity. It constitutes 10 percent of international trade, and 4 percent of global GDP (World Economic Outlook IMF 2008).
- 2) The futures market for WTI crude oil are highly liquid and provide us with large amount of data needed to measure the accuracy of different forecasting models.
- 3) We eliminate the trading mismatch problem because both futures and options are traded on the same exchange.
- 4) From a Norwegian perspective, the development of crude oil prices is of great importance.

Based on the market efficiency theory, if the options and underlying asset markets are informational efficient, an econometric model using past data should not have significant explanatory power for future volatilities. On other hand if there is additional information that can be retrieved from econometric models based on past data, it should be possible to derive a profitable trading strategy. This study seeks to investigate whether this can be the case for the WTI future price.



We will use daily observations of the WTI crude oil future price to construct one month volatility forecasts, and compare the performance of each model with the actual realized volatility for that period.

As far as we are aware, the only studies in the academic literature comparing volatility forecasts for crude oil futures from implied volatility and GARCH-type models is the ones performed by Day and Lewis (1993) and Agnolucci (2009).

Implied volatility will be obtained using either a binomial pricing technique or the CBOE VIX approach for estimating volatility for crude oil futures. The predictive power of the different models is assessed using statistical criteria such as Mean Squared Errors (MSE) and Mean Absolute Errors (MEA), and the regression-based approach based on the significance of coefficient estimates.

## 2 LITERATURE REVIEW

### 2.1 Comparing the performance of alternative models

Since there exist a number of different approaches when trying to forecast volatility, it is necessary to limit our research to some specific models. Akgiray (1989) finds that GARCH consistently outperforms ARCH, Exponentially Weighted Moving Average (EWMA) and Historical Volatility (HIS) models in all subperiods and under all evaluation measures. Figlewski (1997) finds that GARCH is superior for short horizon forecasting only. The study of West & Cho (1995) find no clear results, as model performance is dependent on several factors such as error measurement method (MAE or MSE), sampling scheme (rolling or recursive sample), different time periods and different assets. Furthermore, the standard GARCH model have some important drawbacks pointed out by Nelson (1991), such as the non-negativity parameter restrictions that may be violated (logically, volatility can never be negative, but under an unrestricted GARCH model this might be the case) and an enforced symmetric response of volatility to positive and negative shocks (it could be argued that a negative shock yields a bigger change in volatility than a positive shock does). For this reason we will also estimate an asymmetric model, namely the exponential GARCH (EGARCH) model presented by Nelson (1991)

### 2.2 Information content

When examining the forecasting power it is important to note the difference between within-sample and out-of-sample forecasting. Within-sample tests may be biased toward favoring GARCH, since the GARCH approach is fitted over the entire sample period. The out-of-sample tests should provide more reliable results as noted by Pagan & Schwert (1990). The study of Day & Lewis (1993) show that implied volatilities from crude oil futures options provide a better volatility forecast than either GARCH or historical volatility models for out-of-sample forecasting. Even though there is evidence that GARCH models for volatility contain information that is not incorporated by implied volatility, they do not add much explanatory power to near-term volatility predictions. These findings are supported by study of Agnolucci (2009) that shows that there is information in implied volatility that is not delivered by the GARCH model (unfortunately all estimators were biased).

### 3 THEORETICAL FOUNDATION

#### 3.1 Historical volatility

The simplest model for estimating and forecasting volatility is the historical estimate. This simply involves calculating the unconditional sample variance of returns over some historical period as:

$$\sigma^2 = \frac{1}{N-1} \sum_{t=1}^N [R_t - E(R)]^2$$

where  $\sigma^2$  is the sample variance,  $N$  is the number of observations,  $R_t$  is the return of observation  $t$  and  $E(R)$  is the mean return. Usually, the mean return is set to zero (Figlewski (1997) showed that this increases the volatility forecast accuracy), so that the sample variance is simply calculated as the average squared returns over the sample period. The standard deviation (and consequently the volatility estimate) is calculated as the square root of the variance, and becomes the volatility forecast for all future periods. As explained earlier, assuming constant volatility is an unrealistic notion, at least when it comes to financial time series, but the historical volatility is still useful as a benchmark for comparing the forecasting ability of more complex non-linear models.

#### 3.2 GARCH

There are numerous different types of non-linear models intended to deal with the features of financial time-series data that linear models cannot capture (e.g. fat tails, volatility clustering etc.). We have chosen to estimate a GARCH model because of its popularity for modeling and forecasting volatility. The GARCH model was developed by Bollerslev (1986) and builds on the ARCH model postulated by Engle (1982). Instead of estimating the unconditional variance  $\sigma^2$ , the GARCH model estimates the conditional variance  $\sigma_t^2$  (from now on referred to as  $h_t^2$ ) conditioned on its own previous lags:

$$h_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}^2$$

The conditional variance  $h_t$  can be interpreted as a weighted function of a long-term average value (dependent on  $\alpha_0$ ), volatility during the previous period(s),  $\alpha_i \varepsilon_{t-i}^2$ , and the fitted variance from the previous period(s),  $\beta_j h_{t-j}^2$  (Brooks 2008). In general, one lag for each variable is sufficient in order to capture the fat tailed returns distribution and volatility clustering, giving rise to the GARCH(1,1) model given by:

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$$

Some drawbacks of the GARCH model include possible breaches of the so-called non-negativity constraints that require non-negative conditional variance at any point in time and the symmetric change in volatility due to positive and negative shocks (Brooks 2008). The non-negativity condition can be met by placing artificial constraints on the model coefficients, forcing them to be positive, but an asymmetric model cannot be created using the standard GARCH model. This has given rise to the EGARCH model.

### 3.3 EGARCH

Since the lagged error in the standard GARCH model is squared, the sign of the shock is “lost”. This means that a positive and a negative shock to the time series yield a symmetric change in volatility, but it could be argued that negative shocks lead to a larger change in volatility compared to a positive change. This feature was captured by the Exponential GARCH (EGARCH) model introduced by Nelson (1991), which specifies the conditional variance in logarithmic form:

$$\ln(h_t^2) = \alpha_0 + \beta_1 \ln(h_{t-1}^2) + \alpha_1 \left( \theta \psi_{t-1} + \gamma \left( |\psi_{t-1}| - \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \right) \right)$$

The logarithmic form of the EGARCH means that there is no need to impose non-negativity constraints on the model coefficients, and those asymmetries are allowed.

### 3.4 Implied Volatility

When it comes to obtaining implied volatility there are a numerous approaches depending on the option pricing model. The most popular one is Black-Scholes-type models. The motivation behind this is that the Black-Scholes formula provides a “correct” price for the option that is not influenced by the market price of risk (Joshi 2003), both from a mathematical perspective and from an economic theory perspective (the derivation of the formula is based upon an arbitrage argument).

Implied volatility for futures options is calculated by interpolating the Black’s formula where volatility is the only unknown. But since the WTI futures options are American-style options and there is no closed-form solution to pricing American options, it is necessary to estimate volatility using a binomial pricing technique. Another possible approach is to use an approximation to American futures option developed by Barone-Adesi & Whaley in 1987 or the newly developed CBOE Volatility Index (VIX) for crude oil futures.

The binomial options pricing model was first proposed by Cox, Ross and Rubinstein 1979. The great advantage of the binomial model over Black-Scholes model is that it can accurately price American options. There are two approaches to using the binomial model, the risk-less hedge approach and the risk-neutral approach. Either approach will yield the same answer, but the underlying approach differs. In a risk-neutral world we have two assumptions that simplify the pricing of derivatives:

- 1) The expected return on a stock is the risk-free rate.
- 2) The discount rate used to expected payoff on an option is the risk-free rate.

This means that investors risk preferences are unimportant because as investors become more risk averse, stock prices decline, but the formulas relating to option prices to stock prices remain the same. Therefore one should be able to value options assuming any set of risk preferences and get the same answer. From now on we will concentrate us on risk-neutral approach and when discussing binomial option pricing we are referring to the risk-neutral approach. One of the difficulties encountered in implementing the binomial model is the need to specify the stock

price process in a binomial tree. The common approach is the one proposed by Cox, Ross and Rubinstein (CRR) where binomial price process is constructed by using the volatility,  $\sigma$ , to estimate up (u) and down (d) price movements. The underlying assumption about stock price is that it follows a continuous-time geometric Brownian motion process given by (Jabbour, Kramin, Young 2001):

$$dS = \mu S dt + \sigma S dz$$

where  $\mu$  and  $\sigma$  are constant parameters. From Ito's lemma we can derive the process followed by  $\ln S$  when  $S$  follows the process in equation above. The model of stock price behavior is given by a lognormal distribution:

$$\ln S_t - \ln S_0 \sim \phi[(\mu - \sigma^2/2)t, \sigma^2 t]$$

In risk-neutral world all derivative assets generate only risk-free returns, meaning that investors risk preference and the required rate of return on stock  $\mu$  are irrelevant. We simply replace  $\mu$  by  $r$  (risk-free asset).

$$\ln S_t \sim \phi[\ln S_0 + (r - \sigma^2/2)t, \sigma^2 t]$$

The continuous compounded rate of return (R) realized between time 0 and t is

$$S_t = S_0 e^{Rt} \quad \text{so that} \quad R = \frac{1}{t} \ln \left( \frac{S_t}{S_0} \right)$$

In binomial model the stock price can either move up or down with risk-neutral probability  $p$  and  $(1-p)$ . In 1979 Cox, Ross and Rubinstein proposed the following system:

$$E \left[ \ln \frac{S_{\Delta t}}{S_0} \right] = p \ln(u) + (1-p) \ln(d) = \mu \Delta t$$

$$\text{Var} \left[ \ln \frac{S_{\Delta t}}{S_0} \right] = p(1-p) \left[ \ln \left( \frac{u}{d} \right) \right]^2 = \sigma^2 \Delta t$$

The exact solution proposed by CRR:

$$u = e^{\sigma \sqrt{\Delta t}}, \quad d = e^{-\sigma \sqrt{\Delta t}}$$

$$p = \frac{1}{2} + \frac{1}{2} \frac{\mu}{\sigma} \sqrt{\Delta t}$$

Although the following probability formula is actually applied:

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

### 3.5 Measurement errors

If  $\Delta t$  (time step in binomial tree)  $> \frac{\sigma^2}{r^2}$ , the CRR model will give us negative probabilities as

$$p = \frac{e^{r\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}} > 1 \quad \text{and } 1 - p < 0$$

As a consequence the volatility at any node of binomial tree is downward biased unless  $\Delta t$  is sufficiently small (Jabbour, Kramin, Young 2001). Implied volatility is the market's expectation of volatility over the life of an option and calculated volatilities from an options pricing model should give us the same volatilities for all options expiring on the same date. However the lognormal property and assumption about constant volatility is in contrast to what is observed in actual financial markets where returns are non-normal and the volatility is changing over time. The non-normality aspect of financial market is manifested by skewness, excess kurtosis, and the volatility smile for implied volatilities calculated from Black-Scholes model. This means that also implied volatilities derived from binomial model vary depending on the strike price of options and often there is a persistent smile pattern that can affect calculated volatility.

In some situations investors risk preference may be in contradiction with the risk neutral valuation applied by option pricing model. It may be that investors are willing to pay a higher than fair price because of the upside potential or because of the fear of significant portfolio losses. Such behavior could cause the market price of option being higher than the one predicted by Black-Scholes or binomial approach, translating into higher implied volatility. Another implication that can affect volatility calculations is the problem of infrequent trading that can lead to misvaluation of the index level. The possible solution to these issues is to use nearest-to-the money options when computing implied volatilities. Empirical findings presented above show that using nearest-to-the money options increase the precision of the implied volatility estimator and reduce observation errors. At last the transaction price of options is subjected to bid-ask spread, which introduce another uncertainty because the computed volatility can contain noise that is attributed to jumps of bid-ask spread.

Given the above-mentioned arguments we turn our attention to the Crude Oil Volatility Index (VIX) (code CVF) that can be used as a direct measure of implied volatility or benchmark for our calculations of implied volatility. The VIX approach will be discussed in more details in 'Methodology' section.

## 4 DATA

In our research we will look at the Light Sweet Crude Oil (WTI) futures traded on CBOE. These contracts are the most liquid crude oil contracts in the world ([www.cmegroup.com](http://www.cmegroup.com)). WTI stands for West Texas Intermediate (also known as Texas light sweet) and it is used as a benchmark in oil pricing.

The core data for implied volatility consists of daily observations of the WTI Crude Oil futures options on the Chicago Board Options Exchange (CBOE). But we could also use the CBOE Volatility Index (VIX) as direct measure of implied volatility. Unfortunately, the VIX was introduced first in 2008, which means that we have to calculate implied volatility for necessary period of time before 2008. On the up side there is a database of option prices necessary to compute the VIX dating back to 1990 ([www.cboe.com](http://www.cboe.com)). The first VIX index was introduced in 1993 and was designed to measure the market's expectation of the 30-day implied volatility from at-the-money S&P 100 option prices. It soon became a broadly used benchmark for stock market volatility and is referred to as a "fear index" (CNN/Money). In 2003 the VIX was updated and a new method for deriving expected volatility was introduced. The calculation procedure of VIX index will be explained in a later section.

## 5 METHODOLOGY

### 5.1 Historical Volatility

Historical volatility as mentioned above is calculated as the square root of the average squared returns over the sample period. Returns are calculated on a continually compounded basis as the natural logarithm of the period price less the natural logarithm of the last period price:

$$R_t = \ln(P_t) - \ln(P_{t-1})$$

This will represent our one month volatility forecast. The estimate will then be recalculated every day with a rolling constant sample size.

Furthermore, assuming future price movements are characterized by a "random walk", log prices can be modeled as:



$$\ln(P_t) = \mu + \ln(P_{t-1}) + \varepsilon_t$$

Assuming log-prices are normally distributed (with mean  $\mu$  and constant variance  $\sigma^2$ ) this will yield a log-normal price distribution, guaranteeing that prices will never be negative.

## 5.2 GARCH and EGARCH

Before estimating a GARCH-type model, one first needs to test for “ARCH effects” in the residuals to make sure that this class of models is appropriate for the data. This is done by regressing the squared residuals from a linear model (the conditional mean equation, e.g. an ARMA(1,1) model) on a constant and  $q$  lags:

$$h_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$$

The null hypothesis is that all  $q$  lags have coefficients that are not statistically significantly different from 0 (i.e. the test is a F-test, following a F-distribution). If the null hypothesis is rejected, i.e. ARCH effects are identified, a GARCH(1,1) model will be estimated as:

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$$

and EGARCH:

$$\ln(h_t^2) = \alpha_0 + \beta_1 \ln(h_{t-1}^2) + \alpha_1 \left( \theta \psi_{t-1} + \gamma \left( |\psi_{t-1}| - \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \right) \right)$$

As with the historical volatility estimate, both the GARCH and the EGARCH will be estimated using a rolling constant sample size.

As mentioned earlier, GARCH models are non-linear models, and as a consequence, OLS cannot be used to estimate the coefficients. Instead, maximum likelihood is used for estimation. Both the GARCH and EGARCH models will be estimated using “EViews” (a statistical analysis package).

### 5.3 Binominal pricing of American options

The option valuation procedure begins by dividing the time to expiration of each option into  $nT = N$  intervals.  $T$  is the number of trading days to expiration, and  $n$  is the number of changes in the futures price each day. Given current futures price  $F_0$  there are  $N+1$  possible values for the future price at the option expiration (Day and Lewis 1993):

$$F_{jN} = u^j F_0 \quad \text{with} \quad u = e^{\sigma\sqrt{\Delta t}} \quad (1)$$

where  $F_{jN}$  is the future price at expiration and  $j$  is the number of upticks minus the number of downticks that have occurred in the futures price. The value of an option at each node in the binomial tree is given by

$$C_{jt} = \max(\hat{C}_{jt}, F_{jt} - X) \quad (2)$$

$$\hat{C}_{jt} = e^{-r\Delta t} [pC_{j+1,t+1} + (1-p)C_{j-1,t+1}] \quad (3)$$

$$p = \frac{1-u}{u-1/u} \quad (4)$$

The implied volatility for each at-the-money futures option is estimated by using the Newton-Raphson algorithm:

$$\sigma_{It}(t) = \sigma_0(t) + \frac{C(\sigma) - C(\sigma_0)}{\partial C(\sigma_0) / \partial \sigma} \quad (5)$$

where  $C(\sigma_0)$  represents the value of the futures option for an underlying futures volatility of  $\sigma_0$ , and  $C(\sigma)$  represents the current market price.  $C(\sigma_0)$  is computed numerically using the binomial tree. Given the estimate of  $\sigma_0(t)$  we iterate using equation (2) until our estimate of  $\sigma_{It}(t)$  converges to within 0.0001.

### 5.4 The CBOE VIX approach

VIX measures 30-day expected volatility and contain near- and next-term put and call options. What is meant by near-term, is that they must have at least one week to expiration. When they are less than one week from expiration there is a rollover to the next month contract. The value of VIX is derived from the prices of both at-the-money and out-of-the-money calls and puts. The generalized formula used in the VIX calculation is:

$$\sigma^2 = \frac{2}{T} \sum \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2 \quad (6)$$

where

$\sigma$	VIX/100
F	Forward index level derived from index option prices
$K_0$	First strike below the forward index level, F
$K_i$	Strike price of $i^{\text{th}}$ out-of-the money option; a call if $K_i > K_0$ and a put if $K_i < K_0$ ; both put and call if $K_i = K_0$
$\Delta K_i$	$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$
r	risk-free rate
$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike $K_i$

VIX provide a more precise and robust method to measure expected market volatility. It is more robust because it pools information from options across a wide range of strike prices rather than using just at-the-money options. This should considerable reduce the volatility skew problem. Another advantage is that it derives the market expectation of volatility directly from option prices rather than an algorithm for backing out implied volatilities from an option-pricing model.

## 5.5 Forecast evaluation and hypothesis testing

The relative predictive power of the alternative forecasting models can be measured by estimating the following regression

$$\sigma_{Ht}^2 = b_0 + b_1 \sigma_{Ft}^2 + \eta_{t+1} \quad (7)$$

where  $\sigma_{Ht}^2$  represents the actual realized volatility,  $\sigma_{Ft}^2$  is a forecast of future volatility based on the information available at the end of period t, and  $\eta_{t+1}$  is the forecast error. If the forecasts of volatility are unbiased, the estimate of  $b_0$  will be approximately 0, and the estimate of  $b_1$  will be close to 1. In order to evaluate which model produce the best forecast of volatility it is necessary to use some statistical evaluation measure. There are numerous alternative models but the most popular ones are Mean Square Error (MSE) and Mean Absolute Error (MAE). It is also necessary to look at the parameter significance and explanatory power expressed by  $R^2$ .

The aim of this paper is to measure the incremental information contributed by implied volatility to changes in the conditional variance obtained using GARCH models. In other words we wish to test whether GARCH and EGARCH forecast of conditional volatility contain information that is not impounded in implied volatilities. This can be achieved by examining the following regressions (notice different notions, used by Day & Lewis 1993)

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 + \delta \sigma_{t-1}^2 \quad (8)$$

$$\ln(h_t^2) = \alpha_0 + \beta_1 \ln(h_{t-1}^2) + \alpha_1 \left( \theta \psi_{t-1} + \gamma \left( |\psi_{t-1}| - \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \right) \right) + \delta \ln(\sigma_{t-1}^2) \quad (9)$$

The null hypothesis  $H_0: \delta = 0$

Regression (8) represents the unrestricted GARCH model where implied volatility ( $\sigma_{t-1}^2$ ) is included as an exogenous explanatory variable. Regression (9) is the unrestricted EGARCH model. However if either historical volatility or GARCH forecasts contain information that is not incorporated in implied volatility, we want to find a new model that could incorporate that information with the forecast based on implied volatilities. The idea is to take implied volatility with each model alone, and implied volatility with different combinations of alternative models. The resulting parameter estimates are then used to generate a series of adjusted out-of-sample forecasts for several alternative combinations with highest statistical significance. In comparison of the relative information content for out-of-sample forecasts, the following regression will be used

$$\sigma_{Ht+1}^2 = b_0 + b_1 \sigma_{It}^2 + b_2 \sigma_{Gt}^2 + b_3 \sigma_{Et}^2 + b_4 \sigma_{Nt}^2 + \eta_{t+1} \quad (10)$$

where  $\sigma_{It}^2$  represents implied volatility at time t,  $\sigma_{Gt}^2$  is the step-ahead GARCH(1,1) forecast,  $\sigma_{Et}^2$  is the step-ahead EGARCH-AR(1) forecast, and  $\sigma_{Nt}^2$  is the historical volatility forecast over previous N days. If the forecasting models contain incremental information in predicting future volatility, the OLS regression coefficients should be significantly greater than zero.

## 6 THESIS PROGRESSION

*"I have seen the future and it is very much like the present, only longer."*

-Kehlog Albran, *The Profit*-

Our next objective is to collect all the necessary data needed for estimation of GARCH and IV models. Future prices for the WTI crude oil are readily available on Datastream. The WTI futures options prices can be obtained from CBOE, but we are not sure yet how to proceed on obtaining this data.

When all data is in place we will analyze the market specifics for the oil market (e.g. market drivers as supply and demand, and the history of the trade in physical crude oil and crude oil futures) and study the statistical features of obtained data. The next step will be to estimate the GARCH and EGARCH model, and get estimates for implied volatility. In order to draw some conclusions from alternative model estimations we need to study in more depth the assumptions underpinning univariate time series modeling and forecasting.

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