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Title: A study of Illiquidity Noise on International Sovereign Bonds

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Abstract:

This research aims to measure illiquidity noise in the international sovereign bond market, by analyzing the level of noise in our selected countries during normal times and especially during crises. The research will also focus on identifying potential arbitrage opportunities in the bond market by studying the informativeness of the noise in the price of bonds, in relation to the expected shape of sovereign yield curves. The results indicate that there is a positive relationship between our noise measure and the occurrence of crises, indicating a clear association with market illiquidity during these challenging periods. However, our analysis also uncovers multiple periods of substantial noise in our international sovereign bonds which implies more periods of illiquidity and arbitrage opportunities.

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Contents

1.0 Introduction

Our objective is to conduct an analysis in which we develop an illiquidity noise measure for the international sovereign bond market. The inspiration for this analysis stems from the research paper "Noise as Information for Illiquidity" published by (Hu et al. JF, 2013). In this paper, a noise measure was developed specifically for the US Treasury market to identify periods of illiquidity. These noise peaks align with periods of market crises characterized by a significant decrease in market liquidity. We intend to investigate whether similar events occurred in the international sovereign bond market by applying a comparable approach to construct an illiquidity noise measure. By doing so, we seek to uncover patterns of illiquidity and examine whether they align with historical market crises. The level of liquidity in the financial market is closely connected to arbitrage capital. So therefore during normal times investment banks and hedge funds will have richly with arbitrage capital to supply liquidity in the market. This leads to eliminating most of the arbitrage opportunities in the market because assets are then being traded at prices closer to fundamental value. However in times of market crisis, this is not the case, then the liquidity in the market dries up and the willingness to deploy the market diminishes. Further, this limits the arbitrage capital available and assets can therefore be traded at prices in difference of their fundamental value. Thus, temporary price deviations, or noise in prices, being a key symptom of shortage in arbitrage capital, contains important information about the amount of liquidity in the aggregate market. We, therefore, want to make an analysis of the noise in the price of the sovereign bond market and study its informativeness of overall market illiquidity.

This line of research is important because the “illiquidity noise” measure could help investors to identify potential arbitrage opportunities in the bond market. The concept of potential arbitrage opportunities is when the Sovereign yield curves are not in their expected shapes. So therefore we can identify “illiquidity noise” on the bonds, leading to arbitrage opportunities between the actual curves and the expected curves. Another reason why this theme is relevant for investigating is for analyzing what macroeconomic variables that drive the periods of differences, between the expected and the actual shape of the curves. This research on illiquidity noise is also important for implicating foreign funding conditions and investment opportunities.

The “illiquidity noise” measure on the sovereign bond market is interesting for investors as they could simultaneously buy and sell an asset in different markets, and take advantage of the price differences. This price difference could generate virtually risk-free profit. Therefore this subject could be important for finding a new investment opportunity with low risk.

From our data analysis, we expect to see if it is possible using illiquidity noise as an arbitrage opportunity for investors. This means in practice that investors can use this in crisis and take advantage of risk-free earnings from this illiquidity measure. Since we are especially looking at emerging markets we expect to see a difference in the noise between the bonds denominated in domestic and foreign currencies which is USD in this case. Because of the different risks associated with local currencies of emerging markets and their exposure to the global economy. While for example, U.S. government bonds are less risky, emerging markets bonds are riskier, this means in general that you may not get the principal interest back at the ending term. Another potential noise which is related to the investments in these bonds is the currencies that can deviate a lot from market to market, especially in emerging markets. Where the economy is not so well established and can be more exposed to changes in interest rates and other economic factors.

Another implication is that we would look at how currencies affect illiquidity noise. For example, when we have debt that is denominated in two different currencies, like the Colombian bond in pesos and USD, we expect the debt in the domestic currency (Pesos) to be more sensitive to changes in the domestic interest rate. While the debt dominated in the foreign currency (USD), would be more sensitive to changes in the foreign interest rate. Additionally, movements in the exchange rate between the two countries could also affect the relative value of the debt dominated in the different currencies. So the debt dominated in the domestic currency may be more noisy, due to added sensitivity to domestic rates and exchange rate movements.

Our results indicate that there is a linkage between our noise measure and when there is illiquidity in the market. This is supported by substantial noise in our measure during periods of crisis. Furthermore, we find deviations in noise between domestic and foreign-denominated debt.

The remaining sections of our thesis are structured as follows: In Section 2, we provide a comprehensive review of the relevant literature that forms the foundation for studying illiquidity and constructing our noise measure, highlighting its implications. Section 3 is dedicated to presenting the dataset we have utilized for our research. The methodology employed in our analysis is outlined in Section 4. Moving on to Section 5, we present the results of our study, followed by a detailed discussion in Section 6. Finally, in Section 7, we present a concluding summary of our findings.

2.0 Literature review

The seminal work by, (Hu et al. JF, 2013) analyzes the noise in the price of U.S. Treasuries and examines its informativeness as a measure of overall market illiquidity. They do this by using a measure of market-wide liquidity that is based on the relationship between the amount of arbitrage capital in the market and the amount of noise seen in U.S. Treasury bonds. When there is a shortage of arbitrage capital, yields can deviate more from the curve, leading to more noise in prices. Their noise measure can be used to identify instances of liquidity crises across the financial market, providing more information than current liquidity proxies.

(Du & Schreger, 2016) propose a new method to measure sovereign credit risk, called the "local currency credit spread." This is calculated by determining the difference between the yield on local currency bonds and a synthetic local currency risk-free rate, which is constructed using cross-currency swaps. They discovered that these spreads are substantial and positive. Additionally, they found that compared to credit spreads on debt denominated in foreign currencies, local currency credit spreads have lower average values, lower correlations between countries, and are less impacted by global risk factors. They also observed that global risk aversion and liquidity have a greater impact on the variation of these credit spread differentials than macroeconomic fundamentals.

From (Du et al, 2018) deviations from the covered interest rate parity (CIP) condition provide large, persistent, and systematic opportunities for arbitrage in one of the world's largest asset markets. These deviations for major currencies are not caused by credit risk or transaction costs, and they are particularly pronounced for forward contracts that appear on banks' balance sheets at the end of the quarter. This suggests that banking regulation has a causal effect on asset prices. Additionally, these deviations are significantly correlated with other fixed-income spreads and nominal interest rates.

(Augustin et al, 2022) explores the behavior of covered interest rate parity in the foreign exchange market. CIRP is focusing on the term structure of CIRP violations. The authors of the paper use a large dataset of currency forward contracts and interest rate data to study CIRP violations across different maturities. They find that CIRP violations are more likely to occur at longer maturities and that they tend to be positively correlated with changes in the slope of the yield curve. They also find that CIRP violations are more likely to occur when interest rate differentials are large and when currencies are more volatile.

Additionally, the illiquidity measure has been adapted to other asset classes. (Hofmann & Homburg, 2018) study that fitting errors of equity-option-implied volatility surfaces are informative about intermediary frictions. They do this by creating a "volatility noise", which indicates that there is a strong link between volatility noise and the constraints on intermediary equity and debt.

2.1 Liquidity

Liquid markets are highly valued due to the numerous advantages they provide, such as enhanced allocation and information efficiency. The concept of liquidity is multifaceted. When market participants can buy significant quantities of a financial asset quickly without negatively impacting its price, they consider it to be liquid. In essence, liquid financial assets are characterized by low transaction costs, ease of trading and prompt settlement, and minimal impact on market prices when large trades occur. (Sarr, A., & Lybek, T, 2002). However, Bakker states *"that there is no single unambiguous, theoretically correct or universally accepted definition of liquidity"* (Bakker, 1996). Furthermore, the significance of certain characteristics of liquid markets can evolve over time. For example, in periods of stability, the perception of liquidity in an asset may mainly revolve around transaction costs. However, in times of stress and significant shifts in fundamentals, the timely determination of prices and the ability to adapt to a new equilibrium become increasingly crucial (Sarr, A., & Lybek, T, 2002).

Among the different bond markets, the market for government securities is widely regarded as the most liquid. Government securities hold a distinct position as collateral and serve as benchmarks for pricing other securities. They are also considered a safe haven due to their limited credit risk and the substantial outstanding amounts (Sarr, A., & Lybek, T, 2002).

2.2 Liquidity in emerging markets

As our analysis focuses on government bonds for emerging market countries, it is important to understand the liquidity mechanisms in those markets.

As noted in (Lesmond, D.A., 2005, p. 411-452) in emerging markets, it is common to observe the absence of insider trading laws or a lack of specific enforcement of such rules. This situation increases the information risk faced by market makers, leading to wider spreads. Additionally, political risk can significantly impact the liquidity of emerging markets. When political institutions fail to control corruption, provide stable governance through popular support, or protect against investment expropriation, the available capital for the market and market makers is reduced, resulting in higher trading costs due to a shallower market depth. It is conjectured that differences across various markets and changes over time in the enforcement of legal rules and political risk would contribute to liquidity effects that vary across different market segments.

Chang and Velasco (Chang, R, & Velasco, A. 1999, p. 11-58.) explain some of the liquidity crises in emerging markets saying it's due to "*corruption and cronyism, lack of transparency and imperfect democracy, misguided investments subsidies and loan guarantees, external deficits that are too large, fixed exchange rates that are maintained for too long, poor financial regulation, excessive borrowing abroad* ".

The emphasis on illiquidity is inherent to emerging markets due to their restricted access to global capital markets. In mature economies, when fractional-reserve banks encounter a liquidity issue, they often have the option to obtain emergency funds from the world capital markets as long as they remain solvent. However, this is rarely the case for emerging economies. In times of stability, a private bank in Bangkok or Mexico City may receive numerous international loan offers, but when faced with a bank run by depositors, they are unlikely to receive any such assistance. (Chang, R, & Velasco, A. 1999, p. 11-58.)

3.0 Data

The data we use is the sovereign bond yield curves for emerging markets countries used in (Du et al. JF, 2016). The data is obtained via Bloomberg Fair Value (BFV), where we used a spreadsheet from (Du et al. JF, 2016) with Bloomberg tickers for BFV curves, interest rate swaps, and the CCS used in the construction of our LC credit spread. The curves are par yield curves estimated by Bloomberg on actively traded bonds using piecewise linear Zero-Coupon bonds. This database contains closing bids and asks prices which we used to extract bond data from different sovereign bonds, such as data for Colombia's sovereign bonds. We obtained monthly data with maturity from one to ten years, with a sample period that begins in January 2000 and ends in March 2023.

For the countries without national data or BFV curves, and for ensuring the reliability of the existing BFV curves, we are estimating zero-coupon yield curves, by using individual bond data. After that, we collect the data from Bloomberg by performing an exhaustive search for the available yields on active and matured sovereign bonds for our countries.

We have chosen to focus our analysis on four emerging markets countries: Colombia, Mexico, Peru, and Turkey. Emerging markets are characterized by a set of distinct features that differentiate them from developed economies. These markets encompass a group of countries with rapidly growing economies, transitioning from low-income to middle-income status. Key characteristics of emerging markets include high economic growth potential, often driven by factors such as industrialization, urbanization, and a growing middle class. These markets are characterized by relatively young and dynamic populations, abundant natural resources, and favorable demographic trends. However, they also face challenges such as political and regulatory uncertainties, volatile financial markets, limited institutional development, and higher levels of economic and financial risk compared to developed economies (Investopedia, 2022).

All four countries we are analyzing can be associated with more volatility and risk than for example G10 countries, due to uncertainties related to politics and economic factors as outlined above. During our analysis period from early 2000 until 2023, all of these countries have experienced both periods of economic setbacks and periods of economic growth.

3.1 Summary statistics of the actual yield curves

The table of summary statistics provides valuable insights into the data of the four countries we are analyzing and highlights the dynamic across the different countries, currencies, and maturities we are looking at.

From **Table 1 in appendix,** we can start looking at the average yields, where we can observe variations across countries and maturities. For example, Colombia's average yields for bonds issued in USD start at 3,09% for a 1-year maturity and increase gradually with longer maturities, reaching 6,16% for a 10-year maturity. Turkey exhibits higher average yields across the board, with USD-denominated bonds starting at 4,36% for a 1-year maturity and climbing to 7,25% for a 10-year maturity. However, it is worth noting that it is the opposite for Turkey bonds issued in local currencies, this starts at 12,14% for 1-year maturity and ends up with 11,74% for the 10-year bond.

Secondly, by examining the standard deviations, we can assess the volatility and risk associated with these yields. Generally, higher standard deviations indicate greater yield variations. In this case, Colombia's USD-denominated bonds exhibit relatively moderate volatility, with standard deviations ranging from 2,22% for a 1-year maturity to 2,93% for a 9-year maturity, as seen in **Table 1**. On the other hand, Turkey's TRY-denominated bonds display higher volatility, with standard deviations ranging from 3,85% for a 10-year maturity to 4,81% for a 1-year maturity. These variations suggest that there are some fluctuations in the yields for our 4 countries.

4.0 Methodology

4.1 Hypothesis

To answer our research question formulated in the introduction, we have defined one main hypothesis: *During normal times there should be a small deviation, but during fluctuations in markets there is an increase in noise. The motivation for this hypothesis is to investigate if we could see higher peaks in the noise measure during crises or special events. We also expect to observe deviations between local currency debt and USD-dominated debt during market fluctuations.*

4.2 Modeling the yield curve

To be able to fit a yield curve on the data from the different bonds, we plot the data into R. Further, we use the Nelson-Siegel-Svensson model (Nelson & Siegel, 1987, p. 473-489), which is a parametric approach to modeling the yield curve of a bond. It's mostly used in fixed-income analysis to forecast interest rates and estimate the value of a bond portfolio. We obtained the code for the Nelson-Siegel-Svensson model through a website, however, we had to perform some modifications in order to make it suitable for our data and analysis. (Kiandlee, 2022)

The Nelson-Siegel-Svensson (Nelson & Siegel, 1987, p. 473-489) is a non-linear least square problem with 6 parameters with some inequality constraints, which we can see in the equation under

$$
\begin{aligned}
y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} \right) \\
+ \beta_3 \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} - e^{-\tau\lambda_1} \right) \\
+ \beta_4 \left(\frac{1 - e^{-\tau\lambda_2}}{\tau\lambda_2} - e^{-\tau\lambda_2} \right)
\end{aligned}
$$

$$
\begin{aligned}
\beta_1 > 0, \beta_1 + \beta_2 > 0 \\
\lambda_1 > \lambda_2 > 0
\end{aligned}
\tag{1}
$$

Where $y(\tau)$ is the yield curve at time to maturity τ, β_1 is the long-term level of the yield curve, β_2 is the coefficient for the first exponential term, and β_3 is the coefficient for the second exponential term, β_4 has an impact on the fourth component of the overall shape of the yield curve. The parameters λ_1 and λ_2 are known as the time constants of the model and control the shape of the curve. With the parameters λ_1 and λ_2, we have the ability to manipulate the steepness or slope of the decay for each component by tuning the values of λ_1 and λ_2. Decreasing the values of λ_1 and λ_2 would cause a faster decay, resulting in a steeper yield curve. Conversely, increasing the values of λ_1 and λ_2 would lead to a slower decay, resulting in a flatter yield curve. To effectively model nominal interest rates, it is crucial to ensure that the parameter set satisfies certain conditions. Specifically, the following conditions must be met: $\beta_1 > 0$, $\beta_1 + \beta_2 > 0$, $\lambda_1 > 0$, and $\lambda_2 > 0$. These conditions ensure the validity and appropriateness of the parameters for accurately modeling nominal interest rates.

When fitting our data into the estimation process, we use yields of government bonds to back out the parameters. The yield is the actual yield at the closing point on the last trading day for each month in a year. When running the data through the estimation process, gives us the parameters β_1, β_2, β_3, β_4, λ_1, λ_2, and RMSE for each trading day τ in our dataset across all maturities. Further, we use this parameter to fill in the equation shown above. By doing so we are able to calculate the implied yield for every trading day in our data sample at all maturities, which ranks from 1 to 10 years. These computations we perform on the government bonds for yields in local currencies and foreign currency US dollar. This gives us two different yield curves for each country we are analyzing. By doing so we are able to identify liquidity crises in both currencies and compare specific events to each other.

4.3 Summary statistic of the implied yield curves

The table presents summary statistics for the model-implied yield curve, which has been estimated using the Nelson-Siegel-Svensson model.

Upon examining **Table 2 in the appendix** with the implied yield curve, we can identify disparities between its calculated values and the actual yield curve for certain bonds and maturities. Notably, there are instances where the actual yield curve exhibits higher average yields, while in other cases, the model-implied yield curve displays higher averages. Such variations are to be expected, considering that the model-implied curve is based on the actual yield curve and incorporates inherent variations.

If we first look at the average yields, we can observe a consistent pattern where higher maturities tend to yield higher returns. This relationship holds true across different countries and currencies. For instance, let's consider the Peru bond issued in PEN. The average yield starts at 3,76% for the 1-year bond and progressively increases to 5,92% for the 10-year bond, as we can see in **Table 2**. Similar patterns can be observed for bonds issued in other countries, such as Colombia, Turkey, and Mexico.

Turning our attention to standard deviations, we notice that the pattern differs from the average yields. Standard deviations reflect the degree of yield variability and can provide insights into the level of risk associated with each bond. Interestingly, there appear to be more fluctuations in standard deviations across different maturities for some of the bonds. This suggests that the volatility of yields is not strictly aligned with the maturity duration. To illustrate this, let's consider the Turkey bond issued in USD. From **Table 2** we can see the standard deviations increase from 2,52% for the 1-year bond to 2,77% for the 7-year bond but then decrease slightly to 2,69% for the 10-year bond. This contrasting pattern implies varying levels of risk at different stages of the maturity spectrum.

4.4 Noise measure

The noise measure is a metric designed to detect periods of market illiquidity. To create this measure, we utilize the actual yield curve and the implied-yield curve presented earlier. The equation below illustrates the process, where t represents the date and b_t represents the vectors of model parameters derived from the data. N_t denote government bond which we repeat for all of our countries Turkey, Peru, Colombia, and Mexico with maturities ranging from 1 to 10 years, which we calculate for each country and currency issued. Where currency issued will be both domestic and foreign currency-issued debt. For each N_t we designate y_t^i as the observed market yield on that specific day τ and $y^i(b_t)$ as the yield implied by the model. To capture the dispersion in yields around the fitted yield curve, we construct the noise measure by calculating the root mean squared error between the market yields and the model-implied yields (Hu et al. JF, 2013).

$$\boldsymbol{Noise}_{i,t} = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\left[y_t^i - y^i(b_t)\right]^2} \tag{2}$$

In constructing the noise measure, we have chosen to focus on bonds with maturities ranging from 1 to 10 years. This selection is based on the understanding that the information content of short-maturity bonds is relatively limited when it comes to the availability of arbitrage capital. The shorter end of the yield curve is known to contain higher levels of noise compared to other parts of the curve due to increased volatility caused by fluctuations in demand and supply. Conversely, bonds with maturities exceeding 10 years have the potential to introduce unnecessary time-series noise to our measure, particularly during periods of limited supply. Therefore, we exclude bonds with longer maturities from our analysis. The noise is measured in basis points (bps), where 1 bps is equal to 0,01 percent (Hu et al. JF, 2013).

5.0 RESULTS AND DISCUSSION:

5.1 Colombian Government Bond

The Colombian economy has since the extensive reform initiatives from the early 1990s experienced moderate economic growth. However, the growth observed during the time can be described as slower and more unpredictable compared to the period before 1980. Furthermore, the Colombian economy has undergone a substantial restructuring, marked by increased integration into the global economy, a growing reliance on oil and mining exports, and a rising significance of mining and services sectors in the country's GDP (Ocampo, J. A. 2015, p 3-33).

This is the important background to the time period where we are analyzing the Colombian government bonds from the early 2000s to late 2022. This implies that going into the early 2000s the Colombian economy was on the rise due to new reforms implemented, however, the economy was still subject to great volatility and uncertainty.

By examining the noise measure for the Colombian government bond, both in US Dollars and Colombian Pesos, we gain valuable insights into the fluctuation of liquidity in the government bond market during periods of temporary crises and normal market conditions, both domestically and globally. This analysis provides an intriguing perspective on the dynamics of the market and its response to various economic circumstances.

Figure 2 in the appendix shows that the noise in the USD extends back to the early 2000s, whereas, in COP, the timeline only goes back to 2004. This is due to Colombia not issuing debt in their own currency prior to 2004. This disparity does not impact the final outcome, but it does mean we cannot compare the noise from 2004 to 2000.

If we examine the time period from 2000 to 2004, we observe notable spikes in the noise measure. In **Figure 2** we can see that in the early 2000s, the noise rose to 5,6 bps, and there was another spike in the summer of 2002, reaching 8,3 bps. Since this period only includes data from government bonds issued in USD, it is reasonable to assume that these incidents are linked to the aftermath of the dot-com bubble and that similar prominent noise would have been recorded in the local currency market.

During the early 2000s, the financial markets experienced the impact of the dot-com bubble, which particularly affected the American market. This effect is evident in our noise measure, which peaks at 5,6 bps during this period, lasting until 2001. This is not surprising, considering the liquidity crisis that occurred in the market during that time.

In the summer of 2002, the noise spiked again, reaching 8,6 bps from **Figure 2**. This incident was associated with the economic decline caused by the Enron scandal, as well as the bankruptcy of numerous internet companies in the aftermath of the dot-com crash. Additionally, multiple accounting scandals raised concerns about the overall health of the economy, contributing to the prominent noise levels observed.

Going forward to the next period leading up to the financial crisis in 2008 we can observe some interesting differences in the noise between USD and COP from **Figure 9**. During 2005 and especially in 2006 we can observe substantial noise in the Colombian government bond issued in local currency, however, this noise is not replicated in USD. Some of the explanations for this can be seen in **Figure 15,** where we can observe a clear gap between domestic interest rates and foreign interest rates around the period of 2005-2006.

During the financial crisis of 2008, there was a lot of noise in both USD and COP which is natural due to the international impact the financial collapse in US had on the rest of the world, including Colombia. After this, there is only one event of substantial noise above 2 bps during mid-2013, before moving into 2020-2022 and the Covid crisis. In this period from 2020 until 2022, we observe great fluctuations in the noise ranging from 4,64 bps at its highest and down to 0,15 bps at its lowest.

Analyzing the overall noise of the Colombian government bond, we discover that the noise in COP has a mean of 1,10 bps during the period, with a standard deviation of 0,9 from **Figure 1**. Notably, all the incidents we examined were at least 3 standard deviations away from the average. In contrast, for the bond issued in USD, we observe a slightly higher mean of 1,19 bps and a standard deviation of 1,17 from **Figure 2**. These findings are somewhat expected due to the significant impact of the financial crises in the US during these periods. Consequently, the bond issued in US Dollar experienced greater noise compared to the one issued in domestic currency.

5.2 Mexican Government bond:

Mexico experienced a period of economic instability and recessions leading up to the early 2000s, which had a significant impact on their overall economy. Our analysis of the Mexican government bond starts in mid-2000 for the bond issued in USD and in 2003 for the bond issued in local currency. This distinction arises because Mexico had not issued a domestic government bond before 2003.

Comparing the descriptive statistics of the noise measure in both currencies, we observe a slight disparity. The average noise for the bond issued in MXP (Mexican Pesos) is 0,86 bps, while for the bond issued in USD, it is 0,91 bps. This indicated that, on average, the noise is slightly higher for the USD-denominated bond. Furthermore, the USD bond exhibits a higher standard deviation of 0,66 bps, suggesting greater volatility in the noise measure. However, from **Figure 10** in the appendix, we can observe a strong correlation in noise between the noise in the bonds. The reason for this is due to the strong ties between the US and Mexican economies, given Mexico's heavy dependence on the United States as an export market and the significant role that exports play in its overall economic performance, the country is highly vulnerable to fluctuations in the U.S. economy (Villarreal, M. A. 2010).

Between the years 2000 and 2003, several significant events occurred. Firstly, there was the dot-com bubble, a period of extreme speculation and rapid growth in the technology sector. This led to substantial volatility in the global financial market. These developments had implications for the market liquidity, and this is revealed by our noise measure which peaks at nearly 3 (bps) two times during that particular period, from **Figure 6.** However, we had expected to see more noise during 2002, as we did in the Colombian bond. This might indicate that the Mexican economy was less exposed to the world economy and especially the US at that point in time.

The next period following up to the financial crisis of 2008 does not contain any major peaks in the noise measure, except on 30.04.2004 when the noise hits 3,49 bps in the USD-denominated bond from **Figure 5.** Which is almost 2 standard deviations above the noise in MXP-denominated bonds. There are no clear events linked up to this date to explain why this occurred. However, it's reasonable to link this with the instability of the Mexican economy and with the uncertainty of the global financial markets, which potentially could lead to illiquidity in the market (Villarreal, M. A. 2010).

During the years 2008-2009, the noise level reached its peak for both the MXP and USD-denominated bonds as seen in **Figure 10**, which was to be expected given the occurrence of the financial crisis during that period. This event significantly reduced liquidity in the market, leading to high noise levels in the government bond market. Furthermore, there have been multiple peaks recorded around 3 bps from 2016 until the present day. This phenomenon can be explained, at least in part, by the impact of the Covid-19 pandemic, which nearly resulted in a new financial crisis.

5.3 Turkey government bond:

In the aftermath of the economic crisis in 2001, Turkey has been challenged by political uncertainty and a weakened government. Turkey has been through many political crises which have affected its economy and led to a high degree of economic uncertainty. In recent years 2018-2023, Turkey has been through an economic crisis driven by high inflation.

When analyzing the noise measure for the Turkey government bond in both US Dollar and Turkish Lira, we used data from early 2000 until the beginning of 2023. However, our dataset only includes data from 2010 to 2023 for bonds denominated in Turkish Lira, as Turkey government bonds were only issued in USD before 2010. By examining the noise measure for the Turkey government bond, we can observe relatively large fluctuations. On average, the noise for bonds issued in TRY (Turkish Lira) stands at 3,05 bps, whereas for USD-denominated bonds it is 1,69 bps from **Figure 7 & 8.** This average indicates that the noise on average is significantly higher for bonds denominated in Turkish Lira. Moreover, the TRY bond also exhibits a higher standard deviation of 3,25 bps, indicating higher volatility in the noise measure.

Data prior to 2010 is insufficient for bonds issued in TRY. However, starting from the last quarter of 2010, we observe a significant spike in the noise measure, reaching over 14 bps from **Figure 7**. One explanation could be the aftermath and consequences of the Financial crisis which occurred at the end of 2007. The period from the middle of 2018 to the beginning of 2023 also contains major peaks in the noise measure. In the middle of 2018, the noise reached 16,79 bps. The reason behind these major peaks could be that the Turkish model of economic growth collapsed in 2018. Turkey's economy suffered a liquidity crisis, characterized by high inflation, plunging currency, and high levels of investments financed by rising foreign debt (Bloomberg 2018). From **Figure 7** we can observe that on 31.08.2022 the bond registered its highest noise at 17,03 bps. The reason for this could be that the Turkish economy was still suffering from high inflation and a weak currency. Additionally, during the period 2020-2022, the covid-19 pandemic could also be a source of the high observed noise since the pandemic nearly resulted in a financial crisis.

For the bond issued in USD, we can observe that the period from 2000-2003 contains a high degree of noise. Especially in 2002 and the start of 2003, with

11,97 bps and 13,12 bps from **Figure 8**. High noise is expected for this period because several events occurred. The dot-com bubble from 1995-2001, could explain the high noise for 2001. The noise is also high for 2003 because the recession was still lasting through 2003.

During the end of 2008, and the start of 2009, the noise level reached its peak for the bond issued in 2008. This was expected given the occurrence of the financial crisis during that time period. This crisis reduced the market liquidity and led to high noise in the bond market.

If we observe the noise for the end of 2018, the noise level is 6,71 bps from **Figure 8**. This high degree of noise was expected because 2018 was the worst for stocks in 10 years and investors feared a central bank ready to tighten money policy (CNBC, 2021). There have also been multiple peaks in noise levels from 2020-2022, this could be explained by the covid-19 pandemic, which lead to economic uncertainty.

5.4 Peru Government bond:

When comparing the descriptive statistics of the noise measure for both currencies, we can observe that the average noise for the bond issued in PEN is 0,78 bps, and for the bond issued in USD, 0,81 bps from **Figure 11**. This can indicate that on average the noise is slightly higher for the bond issued in USD. Comparing the standard deviation, the PEN bond exhibits 0,68 and 0,78 for the USD-issued bond. This means that the bond issued in USD suggests higher volatility in the noise measure.

For the bond issued in PEN, we can see that the noise measure contains some major peaks around the financial crisis in 2008. The max peak is at 28.11.2008, with 4,78 bps from **Figure 3**. This is expected since the financial crisis led to rescission in the world economy.

Surprisingly, the noise was not higher in 2020 with 1,83 bps, with the covid-19 pandemic in mind. For 31.08.2021 the noise peaks up to 2,58, but there are no clear events linked up to this peak. There is also a major peak on 29.04.2022, with 3,42 bps. The same for 29.04.2022, here there are no clear events either, but the consequences of the covid-19 pandemic could be a reason.

For the bond issued in USD, we can clearly see the impact of the financial crisis on the noise measure with 6,41 bps from **Figure 4**. This is the maximum peak observed in the noise measure for Peru in USD. This is not unexpected, due to the impact of the financial crisis. There is also a major peak on 12.08.2018 with 3,24 bps. This peak is also expected due to the decline of the US stock market and a central bank ready to tighten up the monetary policy.

5.5 Robustness test

In order to test the reliability of our analysis we perform a robustness test on the Nelson-Siegel-Svensson (NSS) parameters λ_1 and λ_2. Which involves assessing the stability and sensitivity of the model's results when these parameters are varied. The purpose is to evaluate the reliability of the model and understand how changes in these parameters affect the yield curve.

λ_1 and λ_2 are as stated earlier in the thesis "*Known as the time constants of the model and control the shape of the curve. With the parameters, we have the ability to manipulate the steepness or slope of the decay for each component by tuning the values of λ_1 and λ_2. Decreasing the values of λ_1 and λ_2 would cause a faster decay, resulting in a steeper yield curve. Conversely, increasing the values of λ_1 and λ_2 would lead to a slower decay, resulting in a flatter yield curve*".

Choosing the optimal values for λ_1 and λ_2 can be challenging. In our study, we initially set λ_1 = 0.0609 and λ_2 = 0.01 as the fixed parameters for our noise measure. To perform a robustness check, we conducted three additional tests while keeping all other parameters constant. In the first test, we set λ_1 = 0.29

and λ_2 = 0.01, in the second test we set λ_1 = 0.15 and λ_2 = 0.05 and in the third test, we set the parameters at λ_1 = 0.08 and λ_2 = 0.025. By conducting these tests, we generated three alternative yield curves for the Colombian government bond in COP. To assess their accuracy, we compared these alternative implied yield curves with the actual yield curve, calculating the root mean square error (RMSE) in each case. Consequently, we obtained three alternative noise measures for the Colombian government bond.

When using λ_1 = 0.29 and λ_2 = 0.01 as parameter values, our noise measure exhibited a correlation of 99.93% with the actual yield curve. Similarly, in the second and third tests, where we used λ_1 = 0.15 and λ_2 = 0.05, as well as λ_1 = 0.08 and λ_2 = 0.025 respectively, the correlations with our noise measure remained at 99.92% for both. These results indicate that the alternative noise measures closely align with our original noise measure. It highlights the strong association between the alternative noise measures and our initial measure, showcasing the robustness of our noise measure to variations in the fixed parameters λ_1 and λ_2.

6.0 Discussion

Based on the results we have provided we can identify multiple interesting points and events where the Noise measure picks up illiquidity in the market. As discussed earlier in the thesis (Chang, R, & Velasco, A. 1999, p. 11-58.) give multiple reasons as to why liquidity crises occur in emerging markets. However, it is difficult to give an exact reason why many of the spikes in noise happen throughout the different bond markets we have analyzed. While some of the incidents exhibit clear associations with well-known global events, such as the financial crisis of 2008, leading to prominent noise in all of our analyzed bonds, other spikes may require further examination to uncover their underlying drivers.

At the same time (Du & Schreger, 2016) found *“that compared to credit spreads on debt denominated in foreign currencies, local currency credit spreads have lower average values, lower correlation between countries, and are less impacted by global risk factors. They also observed that global risk aversion and liquidity have a greater impact on the variation of these credit spread differentials than macroeconomic fundamentals"* (Du & Schreger, 2016). In relation to our findings, this provides an interesting perspective that relates to our observations on the average noise and standard deviation of government bonds denominated in local currencies versus foreign currencies.

Their research suggests that local currency credit spreads, which can be seen as an indicator of sovereign risk, tend to have lower average values compared to spreads on debt denominated in foreign currencies. This finding aligns with our analysis, as we observed lower average noise levels for government bonds issued in the local currencies for Colombia, Mexico, and Peru compared to bonds denominated in USD, however, this was not the case for Turkey.

Their observation that global risk aversion and liquidity have a greater impact on the variation of credit spread differentials aligns with our findings of higher standard deviations in noise measures for USD-denominated bonds. This implies that global market conditions and risk factors can contribute to increased volatility and fluctuations in liquidity for foreign currency-denominated bonds, reflecting their sensitivity to external shocks and market sentiment.

Another interesting perspective on this is by looking at the research done by (Augustin et al. 2022), which suggests that *"CIRP violations are more likely to occur at longer maturities and that they tend to be positively correlated with changes in the slope of the yield curve. They also find that CIRP violations are more likely to occur when interest rate differentials are large and when currencies are more volatile"*. They identify that there are arbitrage opportunities during post-crises periods, due to deviations from the CIP. They argue that CIP violations have happened systematically for G10 currencies since the financial crisis in 2008 and led to significant arbitrage opportunities. This implies that multiple of our findings where there are great deviations between the noise in domestic and foreign currency can give arbitrage opportunities for investors.

6.1 Comparison between (Hu et al. JF, 2013) measure and our Noise measure

(Hu et al, JF. 2013) has developed another liquidity measure that utilizes a noise measure to identify periods of illiquidity in the market. However, unlike our measure, (Hu et al, JF. 2013) applied this approach specifically to the US Treasury market, resulting in different outcomes compared to our analysis. (Hu et al, JF. 2013) noise measure is constructed using a similar methodology to ours, employing a daily aggregate of cross-sectional pricing errors. We obtained this data from the website of Jun Pan (Saif.stju.edu, 2021).

By calculating the correlation between the monthly changes in our noise measure for the four different government bonds issued in USD and Pans noise measure for US Treasury notes, we find the following correlations: 39,4% for the Colombian government bond, 54,1% for Peru, 39% for Turkey, and 48,3% for Mexico. This suggests a positive relationship between Jun Pan's noise measure and our noise measure.

When comparing Jun Pans noise measure to our own, we observe that, on average, Pans measure is significantly higher, with an average of 2,57 bps and a standard deviation of 2,41 during the period from December 31, 2001, to December 31, 2021, from **Figure 13**. In contrast, our noise measure for the Colombian bond has an average noise of 1,19 bps and a standard deviation of 1,17. Similarly, our noise measure for Turkey shows a higher average of 1,69 bps and a standard deviation of 2,04, indicating slightly elevated levels compared to Pan's measure.

However, it is worth noting that there are specific events in 2002 and 2003 where the noise spikes to 11,97 bps on June 28, 2002, and 13,12 bps on March 31, 2003, for the Turkish bond from **Figure 13**. These levels represent 5,03 and 5,60 standard deviations above the mean, respectively. In contrast, for US Treasuries, the noise on those specific days deviates by only 0,50 and -0,06 standard deviations from the mean. This demonstrates that our noise measure exhibits more significant spikes on certain trading days.

In contrast, when we specifically examine the date October 31, 2008, which marked the onset of the global financial crisis, we find that our noise measure for Turkey was 7,28 standard deviations above the mean, while for Colombia, it was 5,64 standard deviations above the mean. Jun Pan's noise measure, on the other hand, deviated by 6,19 standard deviations from the mean, which is lower than our noise measure for Turkey. However, this discrepancy can be explained by the overall lower average and standard deviation in our noise measure for Turkey.

If we examine the relationship between Jun Pans noise measure with our noise observed in the debt issued in local currencies, we observe a much lower correlation and also a negative correlation for Peru and Turkey at -6,5% and -12,8%, respectively from **Figure 14**. Conversely, Colombia and Mexico display positive correlations of 12% and 0,03%. These findings indicate a relatively weak correlation between the noise in the US treasury market and the noise in local currency debt for the four countries, which we can also see in the graph below.

This comparison between our noise measure and Jun Pan's noise measure provides valuable insights, as it demonstrates that our measure effectively captures the same liquidity crises. However, these are instances where our noise measure exhibits more prominent spikes compared to Jun Pan’s measure, especially for the noise on the debt issued in local currencies. This disparity is to be expected, given that the noise measure is applied to different types of bonds issued by different countries.

7.0 Conclusion

In this master's thesis, our research builds upon the methodology developed by (Hu et al. JF, 2013), providing us with a framework to construct our noise measure. By applying this measure to our bond data from emerging markets countries, our hypothesis anticipated the presence of significant peaks in noise during fluctuations in the financial markets and especially during events of financial crises. We also expected to observe deviations between local currency debt and USD-denominated debt during these periods, along with greater fluctuations in the local currency debt due to the economic volatility of emerging markets. The figures (1-15) and corresponding results consistently support and validate these expectations. During the global financial crisis, for example, the noise measure for USD-denominated debt exhibited a more pronounced spike compared to local debt. However, it is worth noting that the noise in local currency debt displayed several instances of prominent spikes, even when the USD-denominated debt behaved relatively normally without substantial noise.

By providing these results we find a strong relationship between our noise measure and illiquidity in the market during crises. These are the same results that (Hu et al, JF.2013) discovered in their paper. In addition to this, we also observed noise in local debt during times of uncertainty, which also are in line with the liquidity of emerging markets explained by (Chang, R, & Velasco, A. (1999), p. 11-58.). Furthermore, research on the deviations in noise between the debt issued in local and domestic currencies can be really interesting for investors, to potentially take advantage of the arbitrage opportunity.

8.0 Bibliography

Augustin, P., Chernov, M., Schmid, L., & Song, D. (2022). The term structure of covered interest rate parity violations.

Baker, Kent. H. 1996. In Trading Location and liquidity- An Analysis of US dealer and Agency Markets for Common Stocks. Blackwell Pubs.

Bloomberg. (2018). How Turkey created a debt crisis:https://www.bloomberg.com/news/features/2018-12-09/how-turkey-created-a-debt-crisis

Cayon, E., Sarmiento-Sabogal, J., & Shukla, R. (2016). The effects of the global financial crisis on the Colombian local currency bonds prices: An event study. Journal of Economic Studies.

Chang, R., & Velasco, A. (1999). Liquidity crises in emerging markets: theory and policy. NBER macroeconomics Annual, 14, 11-58.

Chiquiar, D., & Ramos-Francia, M. (2009). Competitiveness and growth of the Mexican economy (No. 2009-11). Working Papers.

CNBC. (2021).Why this month echoes December 2018 when stocks plummeted, according to top strategist. https://www.cnbc.com/2021/12/08/why-this-month-echoes-december-2018-when-stocks-plummeted.html

Coşkun, Y., Seven, Ü., Ertuğrul, H. M., & Ulussever, T. (2017). Capital market and economic growth nexus: Evidence from Turkey. Central Bank Review, 17(1), 19-29.

Du, W., & Schreger, J. (2016). Local currency sovereign risk. The Journal of Finance, 71(3), 1027-1070.

Du, W., Tepper, A., & Verdelhan, A. (2018). Deviations from covered interest rate parity. The Journal of Finance, 73(3), 915-957.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007), "The U.S. Treasury Yield Curve: 1961 to the Present," Journal of Monetary Economics 54-8, 2291–2304.

GX Hu, J Pan, J Wang Noise as information for illiquidity The Journal of Finance 68 (6), 2341-2382

Hofmann, M., & Uhrig-Homburg, M. (2018). Volatility noise. Available at SSRN 3130045.

Investopedia. (2022). Emerging Market Economy Definition: Examples an How they work(https://www.investopedia.com/terms/e/emergingmarketeconomy.asp)

JUNGUITO, R., & RINCÓN, H. (2007). La política fiscal en el siglo xx en Colombia, en ROBINSON. James, and URRUTIA, Miguel."Desarrollo de la economía colombiana en el siglo XX: un análisis cuantitativo." Bogotá: Fondo de Cultura Económica y Banco de la República.

Kiandlee. July 04.2022. "SH Fintech modeling"https://kiandlee.blogspot.com/2022/07/nelson-siegel-svensson-yield-curve.html?fbclid=IwAR1gCz_8rOgzcjWqwHNrZEq1GVjYQu2UHjoD0c3mrBhuzqu1EKNwHdRyFo4)

Lesmond, D. A. (2005). Liquidity of emerging markets. Journal of financial economics, 77(2), 411-452

Nelson, C. R., & Siegel, A. F. (1987). Parsimonious modeling of yield curves. Journal of Business, 473-489

Ocampo, J. A. (2015). Performance and challenges of the Colombian economy. Colombia's political economy at the outset of the twenty-first century, from Uribe to Santos and beyond, 3-33.

Orihuela, J. C., & Echenique, V. G. (2019). Volatile and spatially varied: the geographically differentiated economic outcomes of resource-based development in Peru, 2001–2015. The Extractive Industries and Society, 6(4), 1143-1155.

Pan, Jun website. 2021 "Download the noise measure"https://en.saif.sjtu.edu.cn/junpan/

Perez-Reyna, D., & Osorio-Rodríguez, D. (1960). The Case of Colombia. A Monetary and Fiscal History of Latin America, 2017.

Praet, P., & Herzberg, V. (2008). Market liquidity and banking liquidity: linkages, vulnerabilities and the role of disclosure. Banque de France financial stability review, 95-109.

Sahinoz, S., & Erdogan Cosar, E. (2018). Economic policy uncertainty and economic activity in Turkey. Applied Economics Letters, 25(21), 1517-1520.

Sarr, A., & Lybek, T. (2002). Measuring liquidity in financial markets.

Sekine, A. (2022). Estimating the Yield Curve Using the Nelson-Siegel Model: Evidence from Daily Yield Data. Public Policy Review, 18(1), 1-14.

Svensson, L.E. (1995), "Estimating Forward Interest Rates with the Extended Nelson and Siegel Method," Sveriges Riksbank Quarterly Review 3, 13–26.

Tovar, C. E. (2005). International government debt denominated in local currency: recent developments in Latin America. BIS Quarterly Review, December.

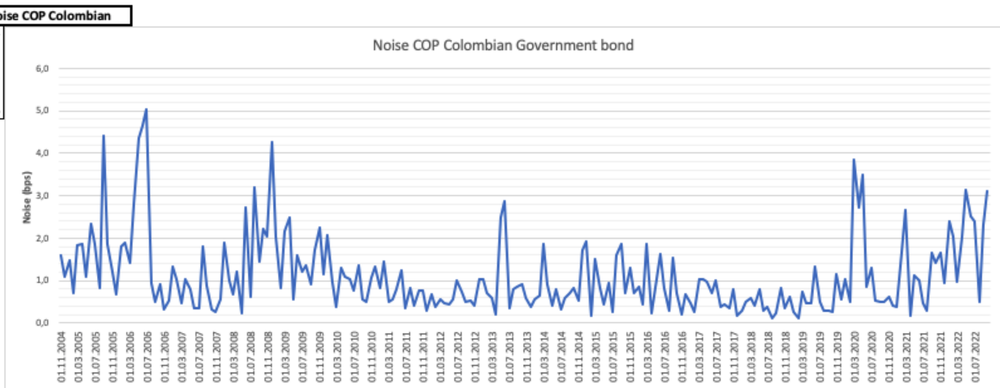
Villarreal, M. A. (2010, September). The Mexican economy after the global financial crisis. Library of Congress Washington DC Congressional Research Service

9.0 Appendix

9.1 FIGURE SECTION

Figure 1: Noise measure for COP Colombian government bond:

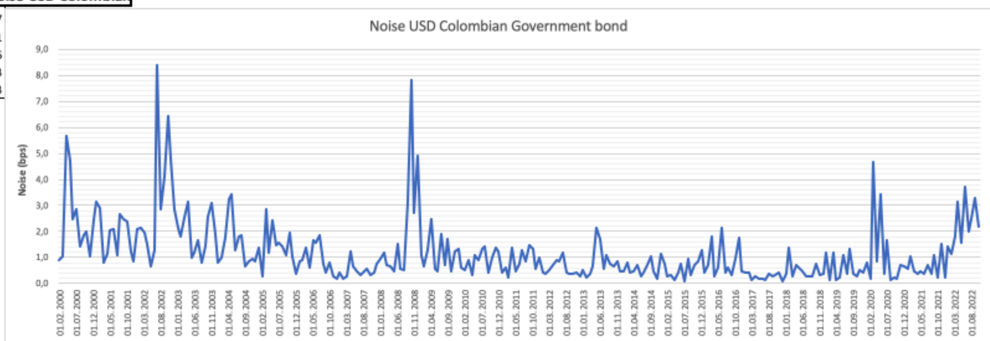
Descriptiv statistic Noise COP Colombian	
Mean	1,1082187
Min	0,111341
Std error	0,9006481
Max	5,0289555
Median	0,8228504



1 bps = 0,01%

Figure 2: Noise measure for USD Colombian Government bond:

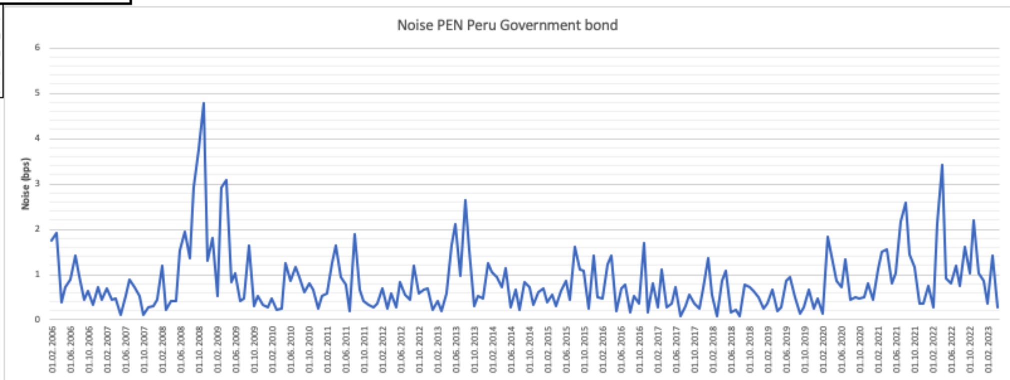
Descriptiv statistic Noise USD Colombian	
Mean	1,1987
Min	0,0981
Std error	1,1746
Max	8,3783
Median	0,8313



1 bps = 0,01%

Figure 3: Noise measure for PEN Peru Government bond:

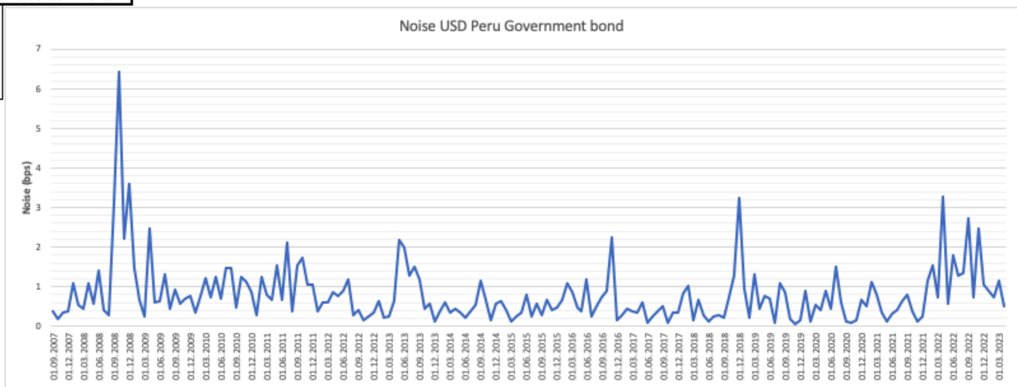
Descriptiv statistic Noise PEN Peru	
Mean	0,78764
Min	0,07810
Std error	0,67959
Max	4,78893
Median	0,58861



1 bps = 0,01%

Figure 4: Noise measure for USD Peru Government bond:

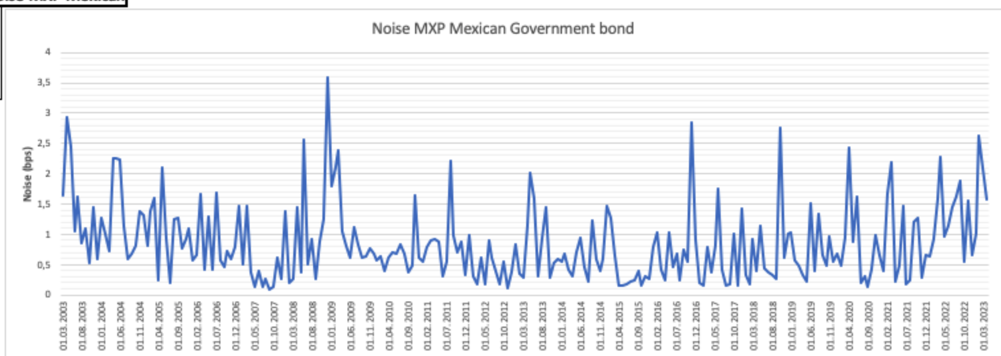
Descriptiv statistic Noise USD Peru	
Mean	0,81158
Min	0,09413
Std error	0,77851
Max	6,41207
Median	0,60216



1 bps = 0,01%

Figure 5: Noise measure for MXP Mexican Government bond:

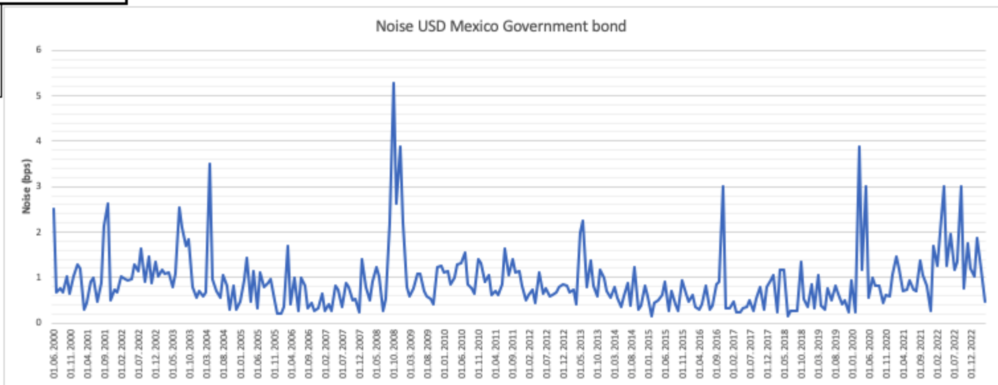
Descriptiv statistic Noise MXP Mexican	
Mean	0,8632823
Min	0,0885105
Std error	0,6337639
Max	3,5851356
Median	0,6756807



1 bps = 0,01%

Figure 6: Noise measure for USD Mexico Government bond:

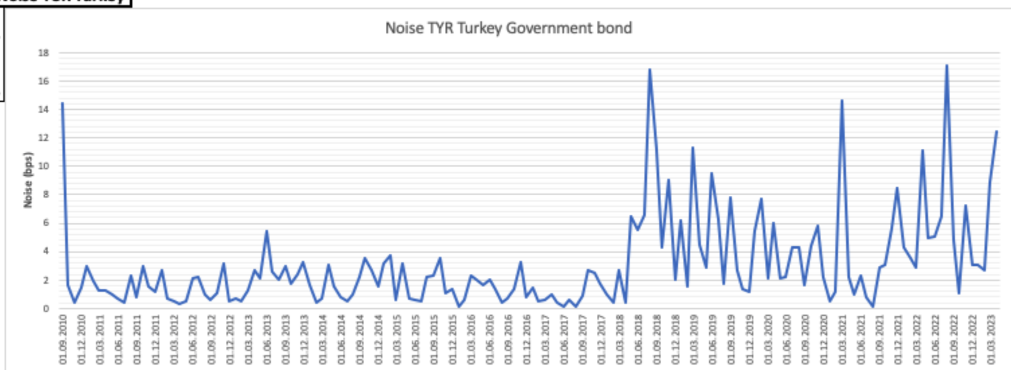
Descriptive statistic Noise USD Mexico	
Mean	0,9177115
Min	0,149135
Std error	0,668199
Max	5,26569
Median	0,7887941



1 bps = 0,01%

Figure 7: Noise measure for TRY Turkey Government bond:

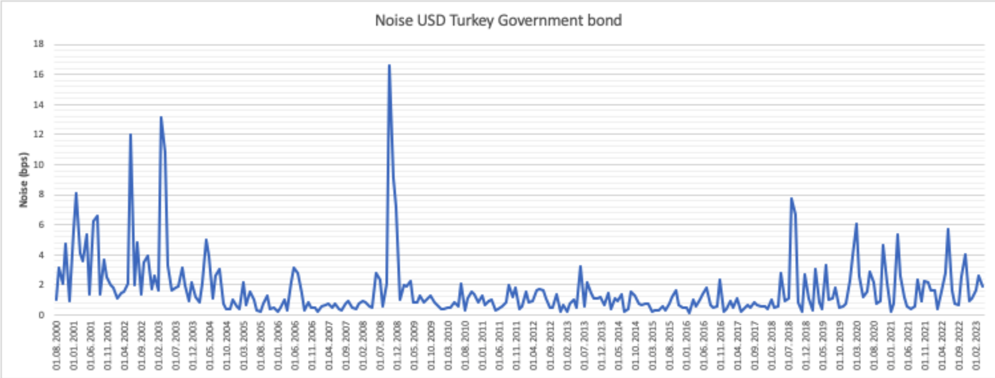
Descriptive statistic Noise TUR Turkey	
Mean	3,0534425
Min	0,1011914
Std error	3,2507265
Max	17,037095
Median	2,1012056



1 bps = 0,01%

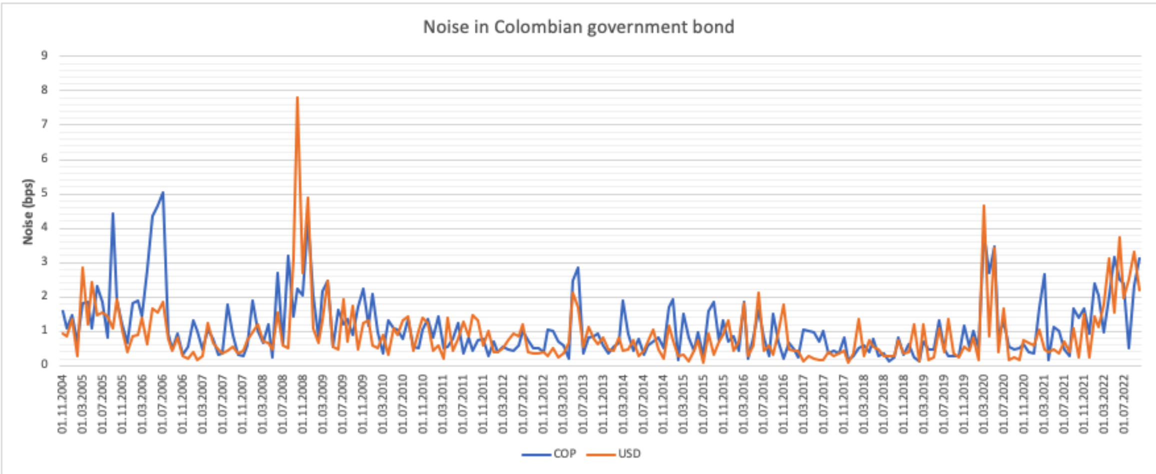
Figure 8: Noise measure for USD Turkey Government bond:

Descriptiv statistic Noise USD Turkey	
Mean	1,6916583
Min	0,1477748
Std error	2,0470299
Max	16,556568
Median	1,0380256



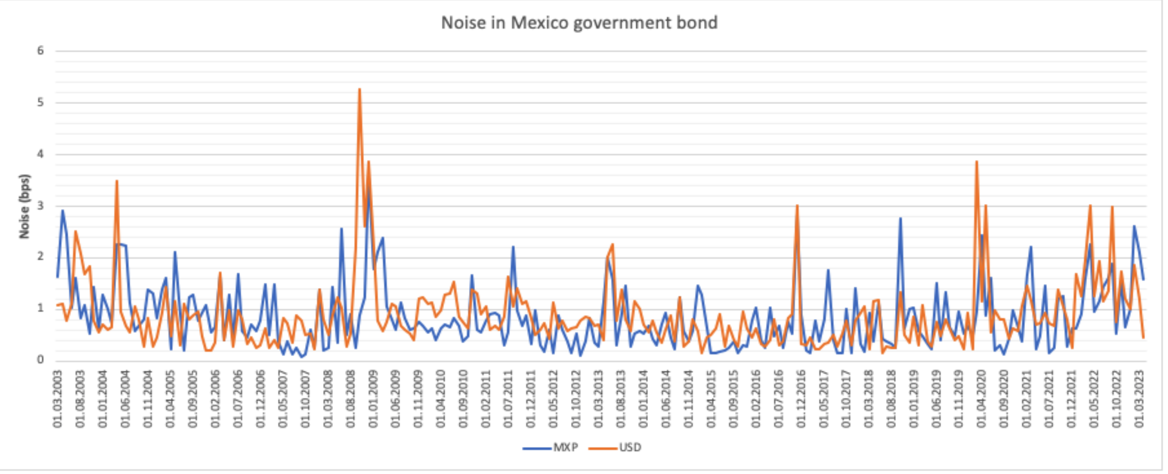
1 bps = 0,01%

Figure 9: Noise in USD and COP Colombian Government bond:



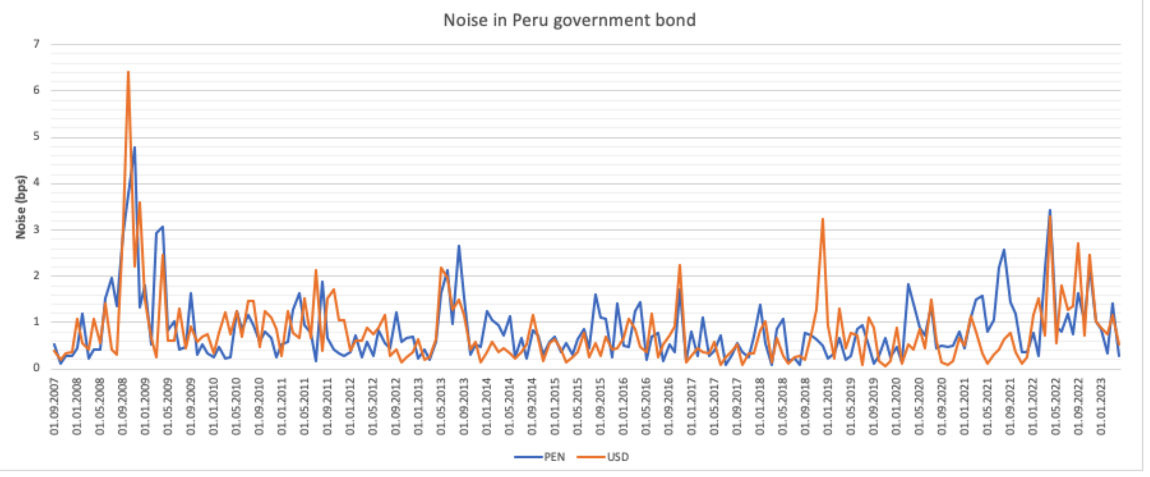
1 bps = 0,01%

Figure 10: Noise in USD and MXP Mexico Government bond:



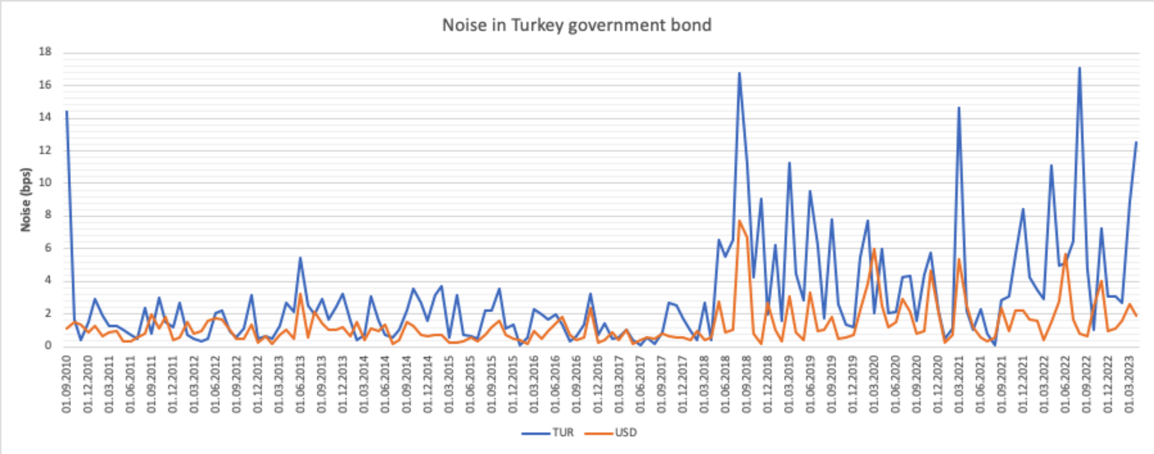
1 bps = 0,01%

Figure 11: Noise in USD and PEN Peru Government bond:



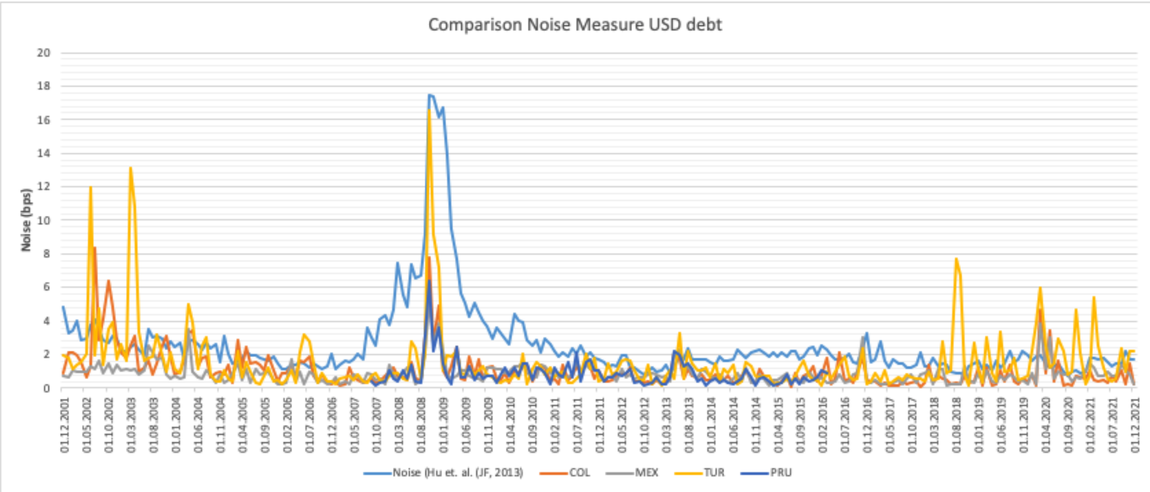
1 bps = 0,01%

Figure 12: Noise in USD and TRY Turkey Government bond:



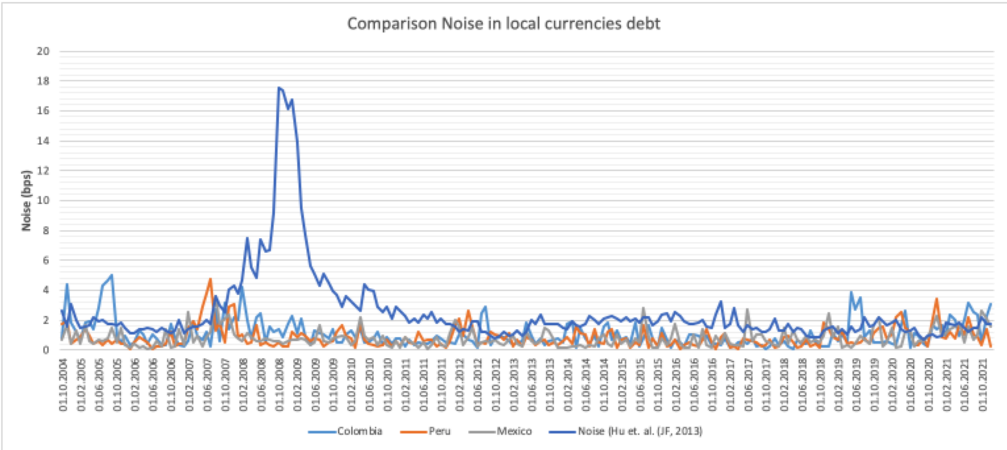
1 bps = 0,01%

Figure 13: (Hu et al, JF, 2013) Noise measure comparison:



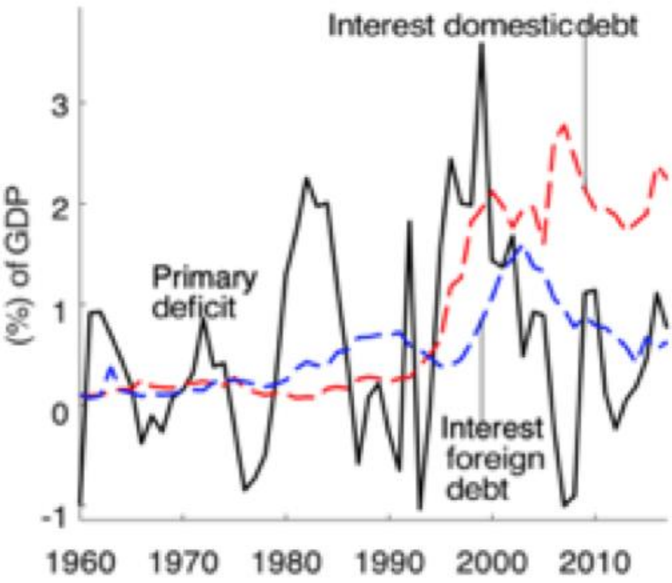
1 bps = 0,01%

Figure 14: (Hu et al. JF, 2013) Noise measure comparison with local currencies debt:



1 bps = 0,01%

Figure 15: Comparison Colombian domestic and foreign interest on debt:



9.2 Table section

Table 1: Summary statistic Actual yield:

		Average yield %								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	3,0998	3,6508	4,3000	4,7654	5,1470	5,7005	5,8831	6,0253	6,1647
	COP	6,0641	6,6390	7,1367	7,5133	7,8265	8,2438	8,3572	8,3165	8,1476
Turkey	USD	4,3608	5,0712	5,5952	6,0616	6,3809	6,8775	7,0243	7,1492	7,2565
	TUR	12,1462	12,0957	12,0767	12,0215	11,9300	11,7794	11,7841	11,7692	11,7404
Mexico	USD	2,5081	2,8743	3,3202	3,6952	4,0498	4,5598	4,7251	4,8519	4,9766
	MXP	6,2292	6,4215	6,6161	6,7896	6,9396	7,2226	7,3200	7,4007	7,4677
Peru	USD	2,2072	2,4182	2,6380	2,9252	3,1710	3,5517	3,7108	3,8590	4,0077
	PEN	3,8036	4,0550	4,3574	4,6555	4,9351	5,4126	5,6069	5,7659	5,9026

		Standard deviation								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	2,2249	2,4571	2,7399	2,8272	2,8694	2,9106	2,9357	2,9327	2,9000
	COP	2,2719	2,3300	2,4046	2,4686	2,4889	2,4771	2,4329	2,5479	2,7071
Turkey	USD	2,5081	2,6152	2,6877	2,6995	2,6991	2,7726	2,7601	2,7276	2,6799
	TUR	4,8173	4,7007	4,5780	4,4606	4,3232	4,0697	3,9940	3,8996	3,8530
Mexico	USD	1,7749	1,7381	1,7270	1,7375	1,7387	1,7259	1,7044	1,6422	1,5995
	MXP	1,9862	1,7926	1,6239	1,5189	1,4286	1,3816	1,3482	1,3160	1,3013
Peru	USD	1,5191	1,4199	1,3464	1,3211	1,3240	1,3064	1,2939	1,2813	1,2681
	PEN	1,5314	1,5446	1,5385	1,4709	1,3967	1,2497	1,1838	1,1385	1,0926

		Lower 25% precentile								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	1,3039	1,8003	2,1775	2,5903	2,9575	3,4083	3,5958	3,7510	3,8757
	COP	4,5595	4,9663	5,4350	5,7429	6,0150	6,3853	6,5225	6,6053	6,6462
Turkey	USD	2,4205	3,0283	3,5067	3,9395	4,2687	4,8146	5,0163	5,1809	5,3397
	TUR	8,6819	8,9223	9,0499	9,0770	9,0932	9,1034	9,1630	9,1630	9,1653
Mexico	USD	1,0754	1,4214	1,9681	2,2597	2,6328	3,1547	3,3757	3,5130	3,6279
	MXP	4,5224	4,7401	4,9614	5,2119	5,4813	5,8563	5,9898	6,1318	6,2343
Peru	USD	1,0384	1,3953	1,7532	2,1148	2,3137	2,5820	2,7687	2,9650	3,1065
	PEN	2,7677	3,0716	3,4918	3,8637	4,2252	4,7753	5,0347	5,2022	5,3650

		Upper 75% precentile								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	4,7266	5,0382	5,5274	6,0008	6,5773	7,3863	7,5171	7,7441	7,9146
	COP	7,7933	8,5627	9,0449	9,2526	9,6369	9,7214	9,7013	9,5415	9,1880
Turkey	USD	5,3906	6,1387	6,5759	7,0481	7,2495	7,5234	7,6111	7,7599	7,8566
	TUR	14,4717	14,1010	13,6328	13,3447	13,2250	13,0761	13,0871	12,9811	12,9368
Mexico	USD	3,5960	4,1758	4,5715	4,9039	5,1842	5,4668	5,5441	5,7060	5,7911
	MXP	7,7368	7,7361	7,7866	7,8096	7,8894	8,1367	8,2442	8,2762	8,3098
Peru	USD	2,8654	2,9659	3,1143	3,4127	3,9003	4,3552	4,5371	4,6466	4,7526
	PEN	4,8855	5,1812	5,4308	5,7020	5,9156	6,1112	6,1664	6,2779	6,3921

Table 2: Summary statistic implied yield:

		Average yield %								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	3,0727	3,7169	4,2640	4,7250	5,1098	5,6834	5,8863	6,0411	6,1530
	COP	5,8204	6,3459	6,7839	7,1447	7,4372	7,8478	7,9789	8,0679	8,1194
Turkey	USD	4,3915	5,0381	5,5745	6,0152	6,3725	6,8783	7,0442	7,1619	7,2374
	TUR	12,0751	12,0067	11,9461	11,8924	11,8450	11,7667	11,7347	11,7068	11,6826
Mexico	USD	2,4904	2,9396	3,3415	3,6980	4,0111	4,2828	4,5152	4,7103	4,8702
	MXP	6,1956	6,4126	6,6087	6,7846	6,9415	7,2018	7,3071	7,3971	7,4721
Peru	USD	2,1733	2,4271	2,6738	2,9098	3,1327	3,5321	3,7065	3,8635	4,0028
	PEN	3,7634	4,0802	4,3793	4,6596	4,9203	5,3808	5,5813	5,7621	5,9235

		Standard deviation								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	2,2646	2,4509	2,6269	2,7697	2,8727	2,9657	2,9641	2,9371	2,8898
	COP	2,1178	2,0840	2,0823	2,0872	2,0858	2,0464	2,0079	1,9592	1,9030
Turkey	USD	2,5235	2,5606	2,6261	2,6910	2,7413	2,7796	2,7675	2,7376	2,6929
	TUR	4,9084	4,7204	4,5752	4,4546	4,3470	4,1471	4,0507	3,9568	3,8675
Mexico	USD	1,7474	1,7081	1,7122	1,7300	1,7444	1,7469	1,7337	1,7040	1,6586
	MXP	1,9503	1,7568	1,6193	1,5223	1,4525	1,3645	1,3375	1,3190	1,3085
Peru	USD	1,4930	1,4024	1,3540	1,3284	1,3142	1,2982	1,2913	1,2836	1,2752
	PEN	1,5497	1,5259	1,4926	1,4481	1,3939	1,2666	1,1995	1,1337	1,0722

		Lower 25% precentile								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	1,2734	1,8796	2,2586	2,5725	2,9109	3,4095	3,5927	3,7599	3,8874
	COP	4,5232	4,9670	5,3986	5,7269	5,9569	6,3545	6,5037	6,6304	6,7406
Turkey	USD	2,4351	3,0186	3,5184	3,9252	4,2332	4,8098	5,0359	5,1682	5,2975
	TUR	8,7559	8,8855	8,9439	9,0487	9,0607	9,1019	9,1366	9,1605	9,1612
Mexico	USD	1,0240	1,5613	1,9826	2,2984	2,6033	2,8813	3,1186	3,3399	3,5242
	MXP	4,5075	4,7500	4,9450	5,1800	5,4175	5,8450	5,9900	6,1175	6,2375
Peru	USD	1,0272	1,4077	1,8040	2,0706	2,2851	2,5708	2,7806	2,9506	3,1008
	PEN	2,7344	3,1598	3,5012	3,8759	4,2108	4,7915	5,0191	5,2123	5,3914

		Upper 75% precentile								
Maturity		1	2	3	4	5	7	8	9	10
Country	Issued in									
Colombia	USD	4,6125	5,1400	5,4932	5,9995	6,4945	7,1919	7,4854	7,7402	7,8512
	COP	6,8461	7,4732	7,8775	8,2564	8,6435	9,1562	9,3006	9,2883	9,2347
Turkey	USD	5,4047	6,1017	6,5996	6,9380	7,2140	7,5560	7,6317	7,7333	7,8230
	TUR	14,2182	13,7251	13,4491	13,2753	13,0157	13,0737	13,0071	12,9585	12,9230
Mexico	USD	3,6945	4,2936	4,6275	4,9081	5,1320	5,3005	5,4549	5,5966	5,7181
	MXP	7,6700	7,6800	7,7700	7,8225	7,8850	8,0600	8,2275	8,2725	8,3325
Peru	USD	2,7755	2,9222	3,0630	3,3415	3,7569	4,3101	4,5266	4,5910	4,6825
	PEN	4,9160	5,2123	5,4227	5,6856	5,8873	6,1146	6,1968	6,2926	6,3837

9.3 Equation section

1 Equation: Nelson Siegel Svensson:

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} \right) + \beta_3 \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} - e^{-\tau\lambda_1} \right) + \beta_4 \left(\frac{1 - e^{-\tau\lambda_2}}{\tau\lambda_2} - e^{-\tau\lambda_2} \right)$$

$$\beta_1 > 0, \beta_1 + \beta_2 > 0$$

$$\lambda_1 > \lambda_2 > 0$$

2 Equation: Noise measure:

$$\boldsymbol{Noise}_{i,t} = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left[y_t^i - y^i(b_t) \right]^2}$$