BI

WISEflow
Europe/Oslo(CEST)
03 Jul 2023

Handelshøyskolen BI

GRA 19703 Master Thesis

Thesis Master of Science 100% - W

Predefinert informasjon

Startdato:	09-01-2023 09:00 CET	**Termin:**	202310
Sluttdato:	03-07-2023 12:00 CEST	**Vurderingsform:**	Norsk 6-trinns skala (A-F)
Eksamensform:	T		
Flowkode:	202310||11184||IN00||W||T		
Intern sensor:	(Anonymisert)		

Deltaker

Navn:	Marthe Carine Steffen og Ida Kristine Honnemyr

Informasjon fra deltaker

Tittel *:	Optimising Personnel Rostering Through Mixed Integer Linear Programming
Navn på veileder *:	Karim Tamssaouet

Inneholder besvarelsen konfidensielt materiale?:	Nei	**Kan besvarelsen offentliggjøres?:**	Ja

Gruppe

Gruppenavn:	(Anonymisert)
Gruppenummer:	52
Andre medlemmer i gruppen:	

Optimising Personnel Rostering Through Mixed Integer Linear Programming

A case study of the Norwegian Police´s operations centre

Master Thesis

Authors:
Ida Kristine Honnemyr
Marthe Carine Steffen

Supervisor:
Karim Tamssaouet, BI Norwegian Business School

Hand-in date:
03.07.2023

Campus:
BI Norwegian Business School, Oslo

GRA 1974 – Master Thesis

Master of Science in Business Analytics

Abstract

Personnel planning is an important component of business management. Improved personnel planning can lead to more satisfied employees and decreased personnel costs. This thesis explores how mixed integer linear programming (MILP) can assist the police´s operations centre with personnel planning, by allocating employees to predefined shifts following legally required work regulations. Further, the effects of recommended health-promoting shift patterns and employee satisfaction are explored.

The research finds that the optimisation model considering only the legally required regulations, can provide complete schedules within a satisfactory time limit. However, when implementing recommended health-promoting shift patterns and personnel preferences, the complexity of the model increases significantly. Key findings reflect that the running time and complexity of the rostering problem increases significantly when considering all regulations, preferences, and health-promoting shift patterns. Hence, for the purpose of generating high quality solutions that considers all kinds of preferences and regulations, also the less important ones, heuristics approaches should be considered to obtain improved solutions more efficiently.

Acknowledgements

We would like to thank all who have contributed to the content of this master thesis. A special thank you to our supervisor Karim Tamssaouet, Assistant Professor at BI Norwegian Business School, for his patience, advice, and guidance throughout the process.

Additionally, we highly value the insight from John Martin Dervå, the informants from the police as well as the other professors at BI Norwegian Business School involved in this project. This thesis would not have been possible without their contribution.

Finally, we want to thank our friends and family who have supported the authors through this project.

Table of Contents

1. Introduction

The police´s operations centres are amongst the priority functions of the police and require continuous staffing and availability to serve the population. The operations centres´ main responsibility is to handle emergency calls and inquiries from the population, as well as manage and coordinate the police´s operational resources both in daily operations and in the event of extraordinary incidents (Politidirektoratet, 2021, p. 1). In the event of an incident the operational centre operates as the primary contact between the police and the caller. The police must staff their organisation efficiently and thoughtfully to provide a level of service which is both satisfactory in terms of time spent on handling incidents and costs (Politidirektoratet, 2021, p.1).

Prior to 2015, the government had no requirements regarding response time for serious incidents. Response time is defined as the time it takes from the operations centres to receive a notification of an incident until the first unit from the police arrives at the scene (Politidirektoratet, 2021, p. 77). In 2022 the police were the first emergency service on scene in only 5.4% of all incidents (Brannstatistikk, 2023). Today there are requirements for response time for all extraordinary incidents, incidents where life is directly threatened and other incidents where an immediate response from the police is required (Report. St. 29t, (2019-2020), p. 48). The requirements for response time differs according to the area population where the incident happens and are divided into three groups. Settlements with less than 2000 inhabitants have a requirement of 30 minutes, for settlements with between 2000 and 19 999 inhabitants the requirement is 19 minutes, and for settlements with over 20 000 inhabitants the requirement is 11 minutes (Politidirektoratet, 2021, p. 78). The police must meet the required response time for 80% of all incidents described above. Further, the police are required to answer 95% of all telephone calls to the emergency number 112 (Report. St. 29t, (2019-2020), p. 47-48). A way to reduce the response time is to ensure that enough employees are present at the operations centres to handle the incoming emergency calls.

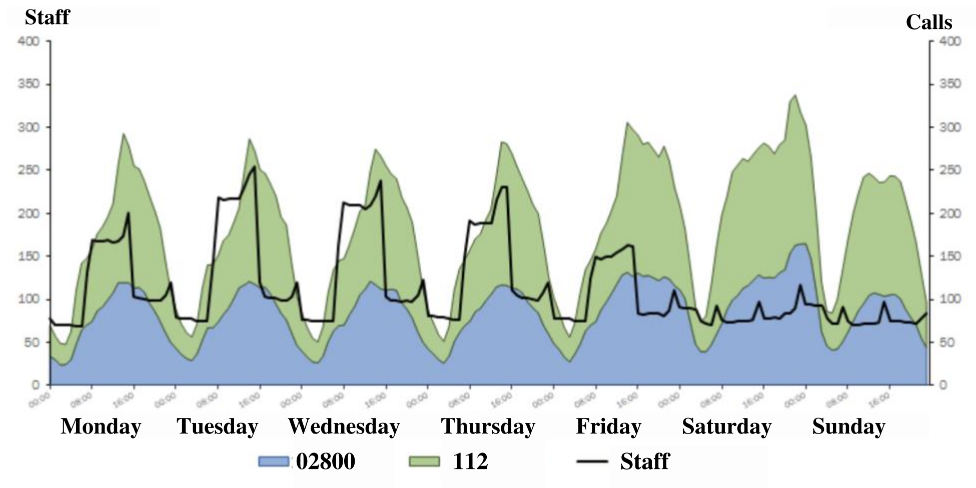

Figure 1.1 – Relationship Between Emergency Calls and Staffing
(Politidirektoratet, 2021, p. 71)

In a report published by the Norwegian police directorate (2021), they did an analysis of the capacity of the police´s operational area based on data from 2018 and 2019. Figure 1.1 illustrates the relationship between staffing level and emergency calls provided in this report. It was found that the staffing level at the operations centre during the afternoon on weekdays and evening/night on weekends is only half of the workforce compared to daytime on weekdays. Moreover, they found that the number of incidents and assignments registered increased by 34% on evenings/night on weekends compared to daytime weekdays and only decreased by 2% in the afternoon on weekdays. This is a doubling of calls per service person in the afternoon on weekdays, and a 132% increase in calls per service person during the evening/night on weekends.

The increase in calls per service person results in longer waiting times, longer response times and more missed emergency calls (Politidirektoratet, 2021, p. 70-73). A missed call is defined as a call that has been dialled for more than three seconds without being answered, and that has not been transmitted internally in the operations centre (Politidirektoratet, 2021, p. 73). Findings from the analysis show that there are 38% more missed calls during the afternoon on weekdays, and 23% more missed calls during the evening/night on weekends, compared to daytime weekdays at the operations centre. If we only look at 112 emergency calls the numbers are 25% and 20% respectively (Politidirektoratet, 2021, p. 73).

Today the personnel planning process of the operations centre relies heavily on manual procedures. This is a time-consuming process as the planners are restricted by governmental regulations, in addition to cultural and personal preferences. Additionally, frequent absenteeism and turnover complicate the planning process. Due to of insufficient planning planners rarely experience schedules to go as planned (personal communication, March 8, 2023). Making adjustments to already implemented schedules is time-consuming and a costly process (Politidirektoratet, 2022, p.3).

Decision support tools are often used in organisations to assist in creating rosters that effectively allocate the appropriate employees at appropriate times and costs, while also promoting high levels of job satisfaction among staff. Although there are many rostering software options available that gives sufficient results, they are usually specific to a certain field and cannot be used in other industries. On the other hand, software with a broader application usually provides manual editing and reporting features, with limited support for automatic roster generation (Ernst et al., 2004, p. 4). Insufficient shift planning within the police´s operations centre could have major consequences as they are responsible for handling emergency calls. To safeguard their required performance level, adequate shift management becomes crucial.

This master thesis is a contribution to a bigger collaboration with The Norwegian Police´s operations centres, which have expressed a desire for an improved decision support system to help with their shift planning process. The purpose of this thesis is to provide valuable insight to the rostering part of the system. The solution should be able to allocate employees to different shifts while accounting for the restrictions defined in the work regulations, as well as exploring how cultural and personal preferences can be integrated into the solution. The overall goal is to investigate how schedules can meet the predefined demand while maximising satisfaction amongst the employees. Considering the challenges mentioned, this paper addresses the main research question:

How does work regulations and employee preferences affect the complexity of a rostering model for the Norwegian police´s operations centre?

An elaboration of the motivation behind this thesis and the research methodology are provided in Chapter 2, followed by a presentation of theoretical background and relevant literature in Chapter 3. In Chapter 4, the problem characteristics of the model is elaborated, before presenting the base model and extended model in Chapter 5. Finally, the quality and complexity of the models are explored Chapter 6, followed by concluding remarks and research limitations in Chapter 7.

2. Motivations and Research Methodology

The purpose of this chapter is to present relevant background information, aiming to provide a deeper understanding of the thesis. First, the purpose of the project will be explained in Chapter 2.1. Thereafter, the current resource allocation and planning practice at the operation centre will be elaborated in Chapter 2.2 and 2.3, followed by an explanation of regulations and other factors limiting the rostering process in Chapter 2.4 - 2.6. Finally, the research methodology will be presented in Chapter 2.7.

2.1 The Project

This study has been initiated due to a greater initiative by the Norwegian police to improve their personnel planning. On a request from the Ministry of Justice and Public Security, the Police Directorate have planned for, and carried out, a pilot in Innlandet- and Sør-Øst Police District, where the aim was to optimise operational staffing in the light of incidents and assignments throughout the week. The pilot included the process of investigating how better personnel planning can lead to better service for the public by having more available resources when the demand for service is greater. The goal is to achive this in a cost-efficient way, while contributing to improved environment, health, and safety (EHS) for employees.

Some improvements to the planning process have already been implemented in the two pilot operation centres, but to further improve the planning process, the police have collaborated with proffessors at BI Norwegian Business School. The collaboration is a project which aims to explore this problem more in-depth and create a prototype of a decision support system (DSS) that will help the operations centres optimise their personnel planning. The DSS includes the process of forecasting demand, determining staffing requirements, shift scheduling and rostering.

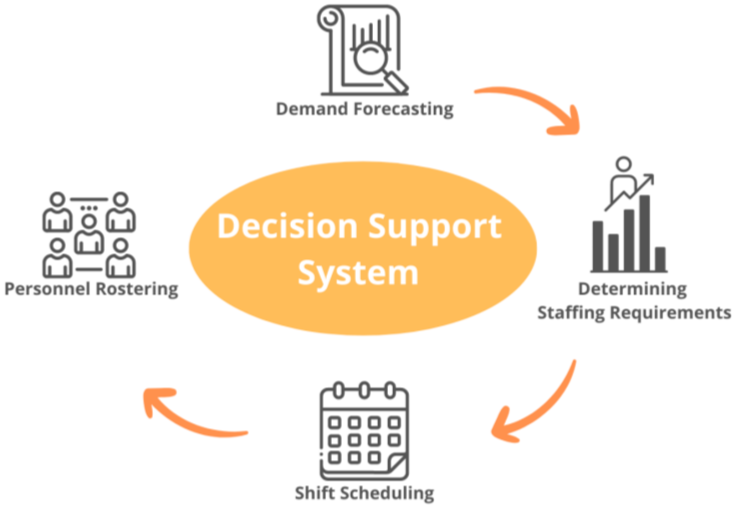

Figure 2.1 – Decision Support System

This prototype will provide the police valuable insight into how they should allocate their employees at the operations centres to create equality amongst employees and save costs. This thesis will contribute to the research of this prototype and mainly focus on the final step of the system, the personnel rostering where employees are assigned to the predefined shifts retrieved from the first three modules of the system. The focus will be to cover all legally required work regulations and investigate how emphasis on employee satisfaction can affect quality, adaptiveness, and complexity of the rostering process.

2.2 Resource Allocation

Every year the police directorate conducts a resource analysis tracking how allocated recourses are distributed and used within the police department. (Politidirektoratet, 2022, p.4). Findings from the analysis show that personnel costs are the biggest cost for the police, and accounts for 76% of the total operating costs in 2021. Personnel costs consists of the expenses related to salaries and allowances for all employees (Politidirektoratet, 2022, p. 12).

The salary costs, including overtime, was 12.6 billion NOK in 2021. This corresponds to a 6.9% increase from 2020 to 2021, which is relatively high compared to earlier years. This is a result of an increase in staffing, salary, as well as overtime expenses. Expenses related to overtime correspond to 4.4% of the total personnel costs. (Politidirektoratet, 2022, p. 13-14).

In 2015 the Norwegian government introduced a de-bureaucratisation- and efficiency reform (ABE reform). The goal was to make public administration more efficient by making all public organisations implement necessary changes to become more efficient and reduce costs. It is challenging for the police department to implement these cost-saving changes as most of their costs are related to personnel costs. However, different actions have been implemented to reduce costs, including better personnel planning. It is estimated that better personnel planning could reduce the costs related to overtime by 48 million NOK, as it would reduce adjustments made after the schedules are implemented that leads to overtime. However, they have not yet succeeded in decreasing the use of overtime with the current practice (Politidirektoratet, 2022, p.3).

The report published by the Norwegian police directorate (2021) and conversations with planners at the police indicates that the current staffing and distribution of employees do not correspond to the distribution of demand for workforce (personal communication, January 31, 2023). The most distinct remark is that the emergency preparedness decreases significantly during the weekends at the same time as number of emergency calls increases. The same trend occurs in the afternoon on weekdays, making the operations centre more sensitive to unforeseen events at these times (Politidirektoratet, 2021, p. 71).

The deviation between staffing and the demand occurs as a reaction to the strong culture within the organisation and is a common problem for planners, making it difficult to create feasible schedules (personal communication, February 6, 2023). This will be elaborated further in Chapter 2.5.

2.3 Current Planning Practice

The current planning practice at the operations centre is mainly performed manually in Excel and through TTA, a tool for resource management and staffing planning in the police (Politidirektoratet, 2021, p. 8). TTA controls the manually created schedules and makes sure they comply with all work regulations. It is an outdated staffing tool that is so complex that only a few people know how to use the system. This creates more workload on the planners, as tasks assigned to leaders are forwarded to the planners that know the system (personal communication, February 6, 2023).

Personnel planning in the operations centre is a complex and tedious process as it is challenging to find a feasible schedule that complies with all regulations, employees´ preferences and absenteeism. Schedules go through two different phases, a planning phase, and an operational phase. In the planning phase the schedules are created to follow the forecasted demand and adapt to other special events, like planned public events that might affect the workload in the specific planning period. The final schedules must be finalised and approved by the section leader and a representative of the trade union two weeks prior to implementation (personal communication, February 10, 2023). When a schedule is implemented, it enters the operational phase, where all changes must go through the department leader. A high level of absenteeism and turnover affect the complexity of the planning. In Sør-Øst Police District planners experience around 15% absenteeism in the operational phase. The work put into the planning phase is important and affects the extent of changes that must be made by leaders in the operational phase. Adjustments that are done while the schedule is in operation often leads to overtime and increased costs. If schedules are successful, they can be reused the following periods, however, as they experience a lot of absenteeism, a lot of work is put into adjusting the schedule to cover demand (personal communication, February 10, 2023).

Today the operations centres use cyclic rostering with fixed teams. The teams consist of operators, assignment managers and operations managers, reflecting their different skills, roles, and responsibilities. The teams are relatively fixed, allowing the shift pattern in the roster to be reused for many periods to come. However, there is some flexibility as individual changes must be made to meet individual needs and to cover shifts with higher demand. The teams are quite small, so to cover demand, more teams are often working together on the same shifts.

The operations centres are practising the use of teams to ensure a stable and reliable work environment for the employees. Employees that work together have a close relationship and are familiar with their roles within the team. However, in dialogues with the contacts from the police, they stated that they are open to the idea of mixing the teams to get to know each other and exchange knowledge across the teams. Studies show that if employees have worked together as a team before, they are familiar with their team members´ working styles, roles, and responsibilities, but if working together for the first time, the lack of familiarity and clarity might potentially affect the performance of the team (Schmutz et al., 2019, p. 3). However, when the teams are fixed for longer periods of time it can potentially affect the knowledge sharing across the teams, and differences in work methods and practises might occur. Knowledge sharing is essential to improve efficiency, organisational and employee performance (Khattak et al., 2020, p. 1). Fixed teams could also potentially limit the ability to adapt to personal preferences and limit the flexibility and possibility of a feasible solution. Therefore, this thesis will not explore the practice scheduling employees in fixed teams using a cyclic roster. Instead, it will investigate the practice of acyclic rostering without predefined teams.

2.4 Rules and Regulations

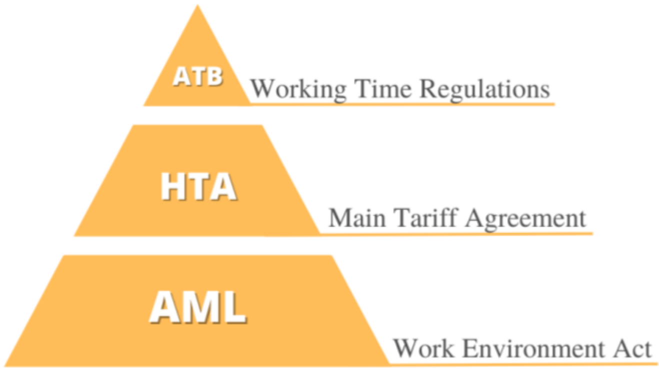

Figure 2.2 – The Regulation Hierarchy

The personnel planning in the police is constrained by rules and regulations provided by the government and internally in the police. The regulations and their relative importance are illustrated in Figure 2.2. The fundamental regulations are based on the Work Environment Act (AML), which purpose is to ensure safe employment conditions and equality of treatment among employees (AML §1-1). There are further regulations set by the Main Tariff Agreement (HTA), which contains regulations regarding wages and working conditions for employees working for the government. Further, the police have composed a set of rules and regulations that regulate working times and required rest for their employees. These are the Working Time Regulations (ATB) and are the regulations that should be followed in the case of contradictions between the regulations. In addition to the three main regulations mentioned, planners need to account for overall personnel policy and EHS instructions, a local instruction concerning environment, health, and safety guidelines at the workplace. The police also have their own risk assessment used when composing schedules which is based on the regulations mentioned above and health-promoting work patterns, created to ensure healthy working conditions. A complete overview of the various rules and regulations used in this thesis is attached in Appendix A.

2.5 Personal Preferences and Culture

In addition to the rules and regulations presented in the previous chapter, personnel planning in the police is to a high degree influenced by a strong culture and personal preferences. Work schedules affect employees´ personal lives and employees tend to have preferences for when and what type of shift they work. Since the operations centres must operate at a sufficient service level, there is often conflict between the work scheduele and employees´ preferences (personal communication, February 6, 2023).

Considering employees' preferences when creating schedules is crucial to prevent turnover. In February 2023, 14 out of 25 nurses at Oslo University Hospital resigned as a reaction to changes in the shift schedule. These changes were implemented without a proper dialogue with the employees, despite several attempts to express their preferences and needs. This created great dissatisfaction among the nurses as they felt that their voice did not matter, which eventually resulted in their resignation (Onsøien, 2023). According to planners at the police, the trade union is extremely strong in the police, and they fight for employees' rights and for the work arrangements to benefit the employees as much as possible (personal communication, January 31, 2023). To prevent turnover, the police have no choice but to obey most of the personal preferences that have been integrated into the culture over time. The most prominent example of this is the culture of not being willing to work more than every third weekend, even though regulations allow them to work every second weekend. The willingness to work weekends is generally low amongst employees and over time the practice of not working more than every third weekend has been strongly integrated into the culture, forcing the planners to obey these preferences. However, the weekends, mainly Friday and Saturday nights, are also the times with the highest workload, making this a bottleneck for planners (personal communication, February 6, 2023). Information retrieved from the police also indicate that employees experience vulnerability during the weekends because of the low staffing, however, they are not willing to work more weekends.

2.6 Shift-Based Schedules

Since the operations centre must be operated all day every day, the use of shifts is necessary to maintain a sufficient service level. Shift arrangement involves the use of evening, night and weekend shift, and good personnel planning is essential to maintain healthy working arrangements, predictability, and prevent turnover at the operations centres (Ernst et al., 2004, p. 1). Shift based schedules that include night and weekend shifts are known to put a high pressure on employees and their health. In a study conducted on nurses they found that the risk of headaches and musculoskeletal complaints is higher after completing a night shift compared to a day shift (STAMI, 2018). Studies also show that police officers have an increased risk of several cancers, namely colon, breast, and prostate cancer. The elevated risk of cancer has been linked to night shift work (Heikkinen et al., 2018).

A study conducted by Garde et al. (2020) investigated ways of scheduling night shift work to reduce the health and safety risks. They found that schedules that reduce circadian disruption, defined as disruptions in your sleep patterns, may reduce the risk of cancer, and schedules that optimize sleep and reduce fatigue may reduce the occurrence of injuries. Hence, fewer consecutive night shifts, health-promoting shift patterns and shorter night shifts were recommended. Another study presented by Matre and Lie, recommend using clockwise shift schedules to reduce sleep and circadian rhythm disorders. That involves setting up schedules with forward rotating order of shift types, meaning employees first work dayshift, then evening shift, thereafter night shifts. The cycles of the shifts should also be as short as possible, and never more than three consecutive shifts of the same type (Matre & Lie, 2019).

2.7 Methodology and Data Collection

In this section the applied methodology of the thesis will be elaborated. First, the research design will be defined, followed by a clarification of data collection and the available data used for analysis.

2.7.1 Research Design

A combination of qualitative and quantitative research design was found appropriate for answering the research question of this thesis. While the quantitative process concerns the modelling part of the research, a qualitative approach was used to gain insight into the current practises, regulations, and employee preferences. The findings from the qualitative research were later translated and merged into the model as quantitative model constraints.

The purpose of this research is to utilise existing theory and apply it to the case of the Norwegian police´s operations centres. Even though the theoretical models from the literature are frequently used for related problems in call centres, this thesis supplements the literature by adapting it to call centres within an emergency department.

2.7.2 Data Collection

Majority of the data used in this research was provided to us by the Norwegian police. The report and models are based on real life data which includes information about every emergency call collected from the Sør-Øst and Innlandet police districts between 2018 and 2022. However, this thesis only considers the demand of the Sør-Øst police district. The information provided included call type, duration of calls, time of the call and other information related to the specific emergency calls.

Information about current practices and regulations relevant to our study was retrieved through interviews with relevant people within the organisation, as well as government reports and other legal documents referred to throughout this thesis. Additionally, a contact person from the police Directorate was available for questions and updates when needed. The qualitative data was collected through a total of four in-depth interviews with four different offices within Sør-Øst and Innlandet police district. All interviews were conducted in person and in the same manner at all offices. The representatives from the police presented information about the planning practise and specific problems regarding the process, and appropriate follow-up questions were asked if necessary. The interviews were carried out in Norwegian, to prevent misunderstandings and to get a correct perception of information and the relevant regulations. The main data collection took place between the 31st of January and the 17th of February. The qualitative data pertaining current practices and regulations showed high similarity between the two districts. However, in the cases where they differed, the information about Sør-Øst district will be employed in this thesis.

3. Literature Review

The purpose of the literature review is to enlighten the state of art overview of research done within the field of shift scheduling and rostering. First an introduction to the shift scheduling process will we provided in Chapter 3.1, followed by an elaboration of the personnel rostering process in Chapter 3.2. Thereafter, different approaches and methods for personnel rostering are explored in Chapter 3.3 and 3.4, followed by a presentation of the approaches that are typically used for call centres in Chapter 3.5 Finally, common features of personnel rostering are introduced in Chapter 3.6.

3.1 Shift Scheduling Process

Shift scheduling can be traced back to 1954 when Leslei C. Edie studied traffic delays at toll booths and was able to decrease toll collection expenses and improve the level of service (Edie, 1954, p. 107). Since then, the field of shift scheduling and rostering has gained significant importance as businesses are more service oriented and cost-conscious than before. Today it is also applied for transportation systems, emergency services, call centres and many other industries (Ernst et al., 2004, p. 1).

There are many different classifications of the shift scheduling process, Ernst et.al (2004) presents a general framework classifying shift scheduling process by dividing it into six different modules, demand modelling, days off scheduling, shift scheduling, line of work construction, task assignment and staff assignment. Another research by Örmeci et al. (2014) divided the process into three stages including staffing, shift scheduling and rostering. The decision support system in this project is based on the framework of Defraeye & Van

Nieuwenhuyse (2016) decomposing the shift scheduling process under nonstationary demand into four modules: (1) Demand forecasting, (2) Staffing requirements, (3) Shift scheduling and (4) Personnel rostering.

The purpose of *demand forecasting* is to determine the demand of employees at different times during a certain planning period, also referred to as a rostering horizon (Ernst et al., 2004, p. 5). Employees are needed to handle incidents that occur during the planning period, in the context of this report the tasks would be to handle emergency calls. The process of demand forecasting is to translate the predicted pattern of incidents into tasks which then can predict the demand for staff at different times. Staff demand can be based on three different incident categories, task-based, flexible, and shift-based demand. Task-based demand is obtained from a list of individual tasks to be performed and the demand is well known. Flexible demand is modelled using forecasting techniques as future incidents are more uncertain. Shift based demand is obtained directly from the required number of staff needed on duty during different shifts to meet service measures (Ernst et al., 2004, p. 5-6). In this project forecasting techniques have been used to model uncertain flexible demand.

Determining *staffing requirements* refers to the process of selecting the required staffing level to achieve some performance targets on the satisfaction of the forecasted demand (Defraeye & Van Nieuwenhuyse, 2016, p. 1). This is done by finding the minimal number of employees that would guarantee satisfying the forecasted demand from the previous module, for each interval of the planning period while minimising costs (Örmeci et al., 2014, p. 42).

Shift scheduling involves the process of creating shifts and determining how many employees to assign to each shift type, to cover the staffing requirements (Defraeye & Van Nieuwenhuyse, 2016, p. 1). This involves selecting from a potentially large pool of candidates, what shifts are to be worked and how many employees are needed for each shift to meet demand. For task-based demand, this process is mainly to select a good set of feasible tasks, shifts and pairings to cover all tasks. In the case of flexible demand, one also needs to consider the timing of work and lunch breaks, to satisfy workplace regulations and requirements from the company (Ernst et al, 2004, p. 6). This module also involves the process of determining how rest days are distributed between workdays for different lines of work. This process is more relevant when rostering for flexible or shift-based demand compared to task based demand (Ernst et al, 2004, p. 6).

This study will mainly focus on the last module, *personnel rostering*, which involves the process of assigning employees to predefined shifts. This will be further elaborated in the next chapter. The results of the first three modules will work as input to the rostering module.

3.2 Personnel Rostering

According to Ernst et al. (2004), personnel scheduling, also known as rostering, is the process of constructing work schedules for employees to satisfy the organisation´s demand for goods and services. It is often difficult to determine optimal schedules that minimise costs, satisfy employees and all the workplace requirements, as these problems are often complex and highly constrained (Ernst et al., 2004, p.3). Furthermore, the modelling techniques used when creating rosters are very different for the variety of industries, and rosters need to be tailored to fit the specific organisation (Ernst et al, 2004, p. 7).

There are two main approaches to solve the personnel rostering problem, according to Musliu et al. (2004). The first approach is to combine the process of creating shifts together with the rostering process of allocating employees and solve them as a single problem. The second approach is to separate these two processes into two different stages. This is done by first creating shifts and staffing levels that satisfy the forecasted demand, and thereafter allocating employees based on these predefined shifts. The first approach often provides better solutions and adaptation to regulations and preferences. However, this approach is not as good to handle large and complex problems as the second approach. Additionally, the second approach is typically easier to solve as the process is divided into smaller modules (Musliu et al., 2004, p. 51-52). In the current project for the police´s Operations Centre, the second approach is followed.

Rostering is the process of generating complete schedules for all employees for a chosen time horizon that satisfies demand, while accounting for constraints like workforce requirementsand regulations. (Örmeci et al., 2014, p. 41-43). Personnel rostering involves the process of constructing lines of work, also referred to as work schedules or roster lines. These lines are spread over the rostering horizon for each employee or team, normally fortnightly or monthly. The construction of lines depends on the type of building blocks that are used. The basic building blocks can typically be shifts, duties and stints (Ernst et al, 2004, p. 6). For example, if the basic building blocks used are shifts, then an individual's workday can be assigned any type of shift. Different constraints can limit the valid shift patterns, by for example regulations on rest time between shifts or the number of night shifts in a row. Duties can only be included once in a roster and are tasks that make up only parts of a shift or stretch over several shifts. Stints are predefined sequences of shifts and rest days that reflect workplace rules and regulations. A roster needs to consider both the rules relating to lines of work to ensure feasibility of individual lines of work, and the pattern of demand that ensures that work requirements are throughout the rostering horizon (Ernst et al, 2004, p. 6).

Further, many rostering processes involves assigning one or more tasks to be carried out during each shift. These must be associated with specific lines of work as they may require a particular employee skill or level of seniority (Ernst et al, 2004, p. 6). Finally, employees are assigned to different lines of work. This step is often performed manually as employees might select from the lines of work based on seniority or other consultative processes. If the process of employee assignment is not performed manually, it would be reasonable to consider the allocation of employees already during the construction of the lines of work. Lines of work can be assigned to individual employees, or fixed teams consisting of predefined employees. (Ernst et al, 2004, p.7).

An additional step might be necessary if the roster is going to consider future short-term schedule adjustments like changes to the schedule because of temporary permits, sick notices, or other temporary changes (Defraeye & Van Nieuwenhuyse, 2016, p. 1). As real-time schedule adjustments made by computer-based heuristics are generally more profitable than manual adjustments made by experienced managers, this could be considered (Van den Berg et al., 2013, p.373).

3.3 Personnel Rostering Approaches

The process of personnel rostering can be divided into four methodologies, centralized-, unit-, self- and preference scheduling. Burke et al. (2004) presented the first three approaches, then Bard and Purnomo (2005) presented preference scheduling as a fourth approach.

Centralized scheduling involves the process of assigning the responsibility of scheduling the entire organisation to a small team. This enables managers and other employees from scheduling, giving them more time to focus on their own responsibilities. Additionally, it leads to better utilization of resources and a decrease in planning costs (Burke et al., 2004, p. 449). In unit scheduling the planning process is moved closer to those it concerns by assigning the task of constructing schedules to the head of the departments. The advantage of this method is that it increases the ability to adapt to personal preferences and unit specific requirements as the planner have more knowledge and work more closely with the departments (Burke et al., 2004, p. 449). The current practise of the police is a combination of centralized- and unit scheduling, as each district have their own personnel planners with the responsibility of scheduling for all the departments within their own district (personal communication, January 31, 2023).

Self-scheduling is the approach where the responsibility of creating schedules is given to the employees themselves. The process is often done manually, where every employee creates an individual schedule. In the case of conflicting schedules, the employees negotiate to find a feasible agreement. This is a time-consuming approach but creates the opportunity to include personal preferences and gives the employees the ability to affect their schedules, making them more satisfied. However, this approach could lead to unfair schedules as some employees are better at negotiating than others and it increases the possibility of conflicts amongst employees (Burke et al., 2004, p. 450).

The last approach is preference scheduling, presented by Bard and Purnomo (2005). This approach integrates personal preferences into the personnel scheduling process. Bard and Purnomo (2005) solved this by getting each employee to submit a list of preferred shifts or other preferences, which then was taken into consideration by managers when creating the schedules. This approach is suitable when wanting to maximise the satisfaction of employees, as the planners can incorporate employees´ individual preferences when creating schedules, and is the approach explored in this thesis (Bard & Purnomo, 2005, p. 512).

Whether the rosters are cyclic or acyclic is another aspect that affects the personnel rostering process. In a cyclic roster the idea is to create a repeating, non-changing pattern of shifts for a predefined number of weeks. Employees preform the same patterns or line of work but have different starting times for the first shift. This type of roster is most applicable for organisations that faces repeating demand patterns and are typically easier to solve as the schedule can be reused in upcoming planning periods, making it more predictable than acyclic rosters. It also distributes the workload equally (Ernst et al, 2004, p. 8). In an acyclic roster the pattern of shifts is independent for each individual employee, and a new schedule is created for each planning period. This approach is often implemented in call centres as it is suitable when demand fluctuates, and shifts have different lengths and starting times. Additionally, acyclic rosters are often better at adapting to personal preferences and increasing flexibility (Ernst et al, 2004, p. 8). The operations centres are currently practising the use of cyclic rosters. However, as they experience a lot of absenteeism and spend a lot of time adjusting the schedules, it is interesting to explore the use of an acyclic roster. An acyclic roster would also be better at adapting to the different preferences of employees, as we assume that not all employees have the same preferences and needs. Therefore, for the purpose of this thesis, an acyclic roster was found the most suitable.

3.4 Methods for Personnel Rostering

Finding good solutions to highly constrained and complex rostering problems can be challenging. The process of figuring out the optimal solution that minimises costs, satisfies employee preferences, evenly distributes shifts across employees, and satisfies all the workplace constraints makes it even more difficult (Ernst et al., 2004, p. 1). For personnel rostering solutions to be effective, industry-specific mathematical models and algorithms must be developed (Ernst et al., 2004, p. 4). Standard industry-specific software packages are often used when rostering personnel. However, these software packages do not have the capability to model employee satisfaction, forcing planners to use considerable time and effort to adjust for this (Örmeci et al., 2014, p.42).

In a review by Ernst et al. (2004) the different applications, methods and models of staff scheduling and rostering was presented. The methods discussed in the paper are artificial intelligence, constraint programming, metaheuristics, and mathematical programming. For call centres, a mathematical programming approach often achieves the lowest cost, and Ernst et al. (2004) argues that this is the most suitable approach for call centres. Moreover, Alfares (2007) developed a queuing model that estimated the hourly staffing demand, followed by an integer programming model constructed to find the most cost-efficient employee schedule that accounted for labour requirements. Further, Wallance and Whitt (2005) used queuing methods and simulation to construct an algorithm for both routing and staffing with the purpose of minimising the total staff.

As personnel rostering models can turn out to be highly complex, heuristic algorithms can be used to find good quality solutions in reasonable computational times. (Van den Berg et al., 2013, p. 377). Burke et al. (2010) created a hybrid model by combining integer programming and variable neighbourhood search to solve highly constrained nurse rostering problems. Another study conducted by Bellanti et al. (2004) used a greedy based local search approach to solve the nurse rostering problem. Other heuristics like scatter search (Maenhout & Vanhoucke, 2006), Neural networks (Hadwan & Ayob, 2009) and greedy random adaptive search procedures (Goodman et al., 2009) have been developed when dealing with the personnel rostering problem. These create flexibility to the roster, however, heuristics does not necessary produce the optimal solution, so trade-off between computation time and solution quality may be necessary to be able to achieve a solution at all (Van den Berg et al., 2013, p. 377).

3.5 Personnel Rostering in Call Centres

In a call centre, workforce management plays an essential role. Call centres typically operate for 24 hours off the day, and experience non-stationary demand and call arrivals, varying by months, weeks, days, and hours of the day. As a result, call centres may require a higher number of shifts per day and use a different approach than usual personnel rostering (Örmeci et al., 2014, p.41). By allowing shifts to overlap it is possible to increase the number of employees working at times with high demand and prevent excess staff during periods of low demand. This also contributes to reduced salary costs as overtime and the process of hiring extra employees is avoided (Van den Berg et al., 2013, p.370).

Opposed to traditional rostering, the rostering in call centres does not involve a geographical feature, making employees more flexible as they are not limited to different locations. Further, the exact description and number of tasks to be performed are not known prior to the shift, because of the uncertainty around arrival times of calls. These factors complicate the scheduling and rostering of call centres. To obtain good, low-cost rosters that satisfy specified requirements, shift start times and shift lengths need to vary. Additionally, in certain time intervals the roster may need to over- and understaff other intervals to meet all pre-distinguished requirements. Further, as calls might not be of the same character, staff with different skills might be required (Ernst et al., 2004, p.367).

Mixed integer programming is commonly used to solve rostering problems in call centres. Al-Yakoob and Sherali (2007) created a mixed integer programming model which determined the most cost-efficient personnel schedule for a hierarchical workforce with multiple workplaces. Their model selected employees to shifts and workplaces based on specific demand requirements, employee skills and employees´ pre-stated preferences. Örmeci et al. (2014) studied the rostering problem for call centres that operate in a multi-skill environment and provide pick-and-drop services for their employees. They developed a mixed integer programming model that incorporated the skill set of agents, workforce regulations, agent preferences as well as transportation, to assist with schedule planning.

Furthermore, another study investigated the process of staffing call centres with uncertain demand and in which customers of different classes are served by agents with varying skills through a chance-constrained optimization approach (Gurvich et al., 2010). The uncertainty of arrivals and demand are common categories of call centre rostering problems. Campbell (2011) created a two-stage stochastic model which deals with these two uncertainties to schedule and allocate multiskilled workers in a workplace with different departments and demand. The two stages include the stage of assigning days off for employees over a certain planning period, and the stage of allocating available employees to satisfy realised demand.

3.6 Common Features of Rostering Problem

This section addresses the variety of characteristics that appear in personnel rostering problems.

There is often a distinction between hard constraints, which must be satisfied, and soft constraints, which may be violated. These constraints differ between organisations and contribute to the great variety of rostering problems and models (Ernst et al, 2004, p. 8). The staffing requirements and shift schedules provided in the first modules of the decision support system are inputs in the form of hard constraints in this rostering problem.

In addition to the constraints mentioned below, researchers have defined many other constraints, such as minimising waiting time or other service criteria. All constraints must be modelled to fit the specific situations and organisations. The composition and construction of these constraints is what contribute to the complexity of rostering problem (Van den Berg et al., 2013, p.376).

3.6.1 Demand

The demand coverage constraints are the most used constraints in personnel rostering problems. In most of the personnel rostering literature the demand is expressed directly through the shift, meaning that each shift has a specific demand for workforce (Örmeci et al., 2014; Musliu et al., 2004). The task is therefore to allocate enough employees to the various shifts. Demand is commonly modelled as a hard constraint to ensure that understaffing is not permitted. However, this does not mean that demand and number off assigned employees must always correspond. It is still possible to schedule excess staff to deal with unexpected demand (Van den Berg et al., 2013, p.371). In a study by Burke and Curtois (2014) where new approaches to nurse rostering was investigated, they operated with both a minimum and a maximum demand as hard constraints. Making the model more flexible while still operating within the upper and lower limits.

The demand can also be linked to specific competencies, this is commonly handled through a categorical skill or a hierarchical skill approach. The categorical skill approach involves the process of defining a set of different competencies. If an employee has a specific competency the employee can cover a demand for that competency. There is no differentiation on the level of competency, employees either have a particular competency or set of competencies, or not. The second approach appoints employees with different skill levels, and an employee with higher skill levels can perform more tasks and cover more demand than a lower-level employee (Van den Berg et al., 2013, p.369).

3.6.2 Qualifications

Majority of existing literature assumes that both customer types and employee types are homogenous, that is they all have the same skills and qualities.

However, more recent studies have incorporated both customer and employee heterogeneity into their models. Making it possible to differentiate between personal qualifications and creating a more realistic model (Defraeye & Van Nieuwenhuyse, 2016, p. 7). If personnel are heterogenous and possess different qualifications, a hard constraint can be added to ensure the presence of the required number of employees per qualification (Van den Berg et al., 2013, p.371-372).

Additionally, the necessity of a specific qualification can be modelled as a soft constraint, where people with other qualifications can take over when there is a lack of employees with the desired qualification, this would however penalise the objective value. In most research qualifications are fixed, meaning one cannot obtain new qualifications. However, these hard-constrained qualifications do not reflect reality, and one could try to incorporate the ability to obtain new qualifications for employees at a certain training cost (Van den Berg et al., 2013, p.371-372).

3.6.3 Work and Rest Restrictions

Work and rest limitations are constraints concerning the regulations considering working hours, aiming to promote good working conditions and prevent overworked and fatigued employees. These are typically defined by the government, the labour union, or organisational practices. Organisations often face regulation regarding working hours and rest time. These are often handled as hard constraints regulated by official regulations, like maximum working hours per day and week, and rest time between shifts (Van den Berg et al., 2013, p.374).

Ensuring that employees at most work one shift per day is a commonly used constraint. What types of shifts the employee work might also affect what shift they can work the following day. If an employee works a night shift, it might not be possible to work the following day because of rest restrictions. Further, there can be constraints limiting the number of consecutive working and non-working shifts per employee, ensuring enough rest-days between work periods

(Rönnberg & Larsson, 2010, p. 41). Additionally, a coverage constraint could be used to incorporate breaks. The timing of breaks is usually limited to a pre-defined window based on the employee's start- and finishing hours. One could also differentiate between lunch breaks and shorter rest breaks (Van den Berg et al., 2013, p.372).

3.6.4 Personnel Preferences

Personnel preferences are often referred to as employees' requests regarding the schedule. To account for this, organisations often consider soft constraints to maximise employee satisfaction. This typically involves preferences for working or not working certain shifts or at specific times (Bard and Purnomo, 2005, p. 510). Isolated working days, as well as isolated days off are often considered undesired for employees and (Bard & Purnomo, 2005, p. 518). This can also be constraints limiting the number of night shifts or concerning weekends (Van den Berg et al., 2013, p.374). Employees tend to prefer to work complete weekends rather than partial weekends, the constraint of complete weekends is commonly used in personnel scheduling. Another weekend constraint is where the number of weekends allowed within a planning period, and the number of consecutive weekends is restricted (Van den Berg et al., 2013, p.374). A common approach is also to distribute weekend and night shifts evenly among the employees (Rönnberg & Larsson, 2010, p. 42)

However, the process of quantifying employees' preferences can be difficult, and an alternative method is to define discontent measures. This can be done by collecting employees´ individual preferences through surveys or by getting employees to assign important scores for different discontent factors and targeting fairness by distinguishing these discontent factors and limiting the total discontent factors for each employee (Örmeci et al., 2014, p.41). Another approach, introduced by Rönnberg and Larsson (2010), allow employees to state preferred hard and soft requests for work- and off-shifts. Each employee is given a set of vetoes for the hard request that must be met, whereas the soft constraints are preferences that not necessarily have to be fulfilled.

3.6.5 Fairness

Schedules that are generated only based on the motivation of minimising costs could inherent unfairness and dissatisfaction in general, as employees could be treated differently. When creating schedules, employers often want to create fairness to maintain satisfaction amongst their employees and decrease turnover rates. This can be obtained by a soft balancing constraint, trying to balance differences between employees or balancing the treatment of employee preferences (Van den Berg et al., 2013, p.374).

3.6.6 Financial Considerations

Financial constraints are often related to different costs like personnel-, overtime-, travel-, and operating costs, as well as costs depending on the day of the week or time of the day, or alternative costs like the cost of a missed call. These are often dealt with by minimising the costs of interest, for example personnel cost. By minimising personnel costs instead of number of employees the complexity and possibilities of the model increase, as the trade-off between overtime, hiring new employees or part time workers is included by assigning a relative cost to all these factors (Van den Berg et al., 2013, p.373). If the workload is not covered by the regular work hours scheduled, the organisation can introduce overtime to make the model more flexible, however, this often comes with regulations and an increase in costs. With budget limitations, the rosters might not be as flexible as the organisations desire. (Van den Berg et al., 2013, p.373).

A variety of different fairness objectives have been implemented in scheduling problems. Lin et al. (2015) maximised the number of granted employee's preferences for both desired and undesired working shifts, while other studies have included penalty costs in the objective function for employees working undesired shifts (Defraeye & Van Nieuwenhuyse, 2016, p. 17).

3.6.7 Objective

When creating a personnel roster, various stakeholders with different priorities must be considered. While the organisation aims to handle workload at the lowest expense, the operations manager may advocate for restricting the number of casual workers to maintain consistency and access to expertise. On the other hand, employees might have other preferences and needs.

Consequently, most developers create multi-objective models to balance the conflicting objectives that managers face, using penalty factors to weight their importance (Van den Berg et al., 2013, p.376). In a multi-objective problem, the goal is to find a feasible solution that represent a trade-off between the different objectives. Multi-objective problems normally offer more comprehensive and nuanced approach to decision-making compared to single-objective optimisation. Handling the multi objective problems by including penalty terms in the objective function is an efficient technique to discourage undesired events (Ernst et al, 2004, p. 19).

Objectives frequently used in personnel rostering includes minimising the under coverage of demand, minimising costs, and maximising employee satisfaction (Ernst et al, 2004, p. 4). In their mathematical model for personnel rostering in call centres, Örmeci et al. (2014) optimised three objectives simultaneously over the planning horizon: minimising operating cost and uncovered shifts, while maximising employee satisfaction. Aiming to balance the conflicting objectives according to priorities provided by managers. Wright et al. (2006) created a model where they minimised labour costs while maximising employees fulfilled shift requests.

4. Problem Characteristics and Assumptions

This chapter aim to provide an outline and description of the rostering problem. First, important characteristics of the problem are described in Chapter 4.1, followed by an elaboration of the problem´s assumptions in Chapter 4.2.

4.1 Problem Characteristics

The purpose of this thesis is to develop a personnel rostering model customized for the Norwegian police operational centres. The schedules must satisfy the predefined demand while complying with the legally required regulations defined by the government, police, and the trade union. The results are evaluated based on their ability to cover demand and simultaneously gain satisfaction amongst the employees. The model is only feasible if the critical demand is covered by enough qualified personnel and the legally required regulations are satisfied. The thesis further aims to investigate how constraints concerning soft regulations and organisational norms affect the complexity of the problem. This is explored by first creating a model that only consider the regulations that *must* be met, hereafter referred to as the base model. Thereafter, exploring how the incorporation of soft constraints affect the quality and complexity of the model. A complete overview of the work regulations used in this thesis is provided in Appendix A. The following chapter will provide more detailed information on the main components considered in this problem.

4.1.1 Demand

The demand is defined as the set of employees required to satisfy the predicted workload for each shift. In this thesis the demand is expressed directly through the shift, thus the challenge is to allocate enough employees to the various shifts. To be able to handle all 112 emergency calls, the operations centre should cover the minimum number of employees during all shifts. However, due to limited number of employees and work regulations, this is not always feasible. Hence, the model allows for understaffing as long as it covers the critical staffing level of five employees, which is the operations centre´s lowest acceptable staffing level. The model strives to allocate a total number of employees that is as close to optimal staffing level as possible, as it penalizes the deviation between the the actual- and optimal staff level in the objective function. The optimal staff level is the number of employees to be assigned per shift to cover both 112 calls and the 02800 calls transferred to the operations centre outside the regular business hours of the police switch board. It is assumed that the optimal staff also works as a safeguard to account for unexpected absence that the operations centre might experience during the operational phase. To avoid overstaffing, the optimal coverage of demand works as the maximum number of employees to be allocated to each shift.

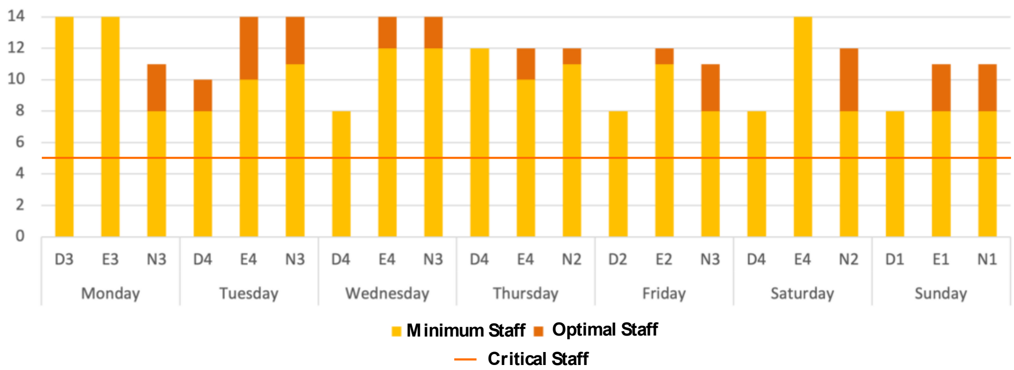

Figure 4.1 – Example of Weekly Demand

An extraction of a weekly demand is illustrated in Figure 4.1. The minimum and optimal staff levels are based on a forecasting model from outside this thesis and is adjusted for the minimum and maximum seat capacity at the operations centre, which are five and 14 respectively at Sør-Øst Police District. The critical staff level is equal to the minimum seat capacity.

4.1.2 Shifts

A shift is defined by a starting- and end time and is linked to the demand. Work shifts are divided into three different shift types reflecting which part of the day the shifts belong to. These include day-, evening- and night shifts. Night shifts are defined as shifts within the time period between 23.00 and 06.00 (AML § 10-11(1)). There is no clear definition of evening shifts. However, for the purpose of this thesis, evening shifts are defined as shifts where the majority of working hours are placed between 17.00 and 23.00, as employees get evening compensation after 17.00 (HTA § 15-3). Day shifts are defined as shifts where majority of the working hours are between 06.00 and 17.00.

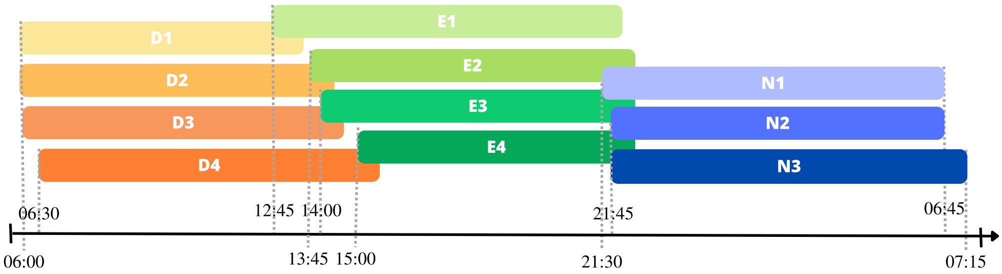

Figure 4.2 – Shift Overview

Employees must be assigned to a work shift to cover a unit of demand. Further, employees are also assigned to off shifts, indicating a day off where employees are not assigned any shifts. Employees can only be assigned to one shift per day and must meet required rest before being assigned to a new shift. As shift optimisation is beyond the scope of this thesis, the shifts used in this thesis are predefined shifts. These are not optimal shifts developed for this specific rostering problem. Every day consist of three shifts, one day-, evening- and night shift. Figure 4.2 illustrates the different shift types used in analyses in this thesis. However, different shifts could be used for the model. An overview of what shift types are used on the different days and shift durations are attached in Appendix D.

4.1.3 Qualifications

There are further requirements for employees with different work positions or roles present at the shifts. The model differentiates between three main roles, hereafter referred to as qualifications. These qualifications are operators, assignment managers and operations managers. An employee can hold one of these three types of qualifications and their qualification determine which demand the employee can cover. The highest level of qualification is operation manager, thereafter assignment manager, followed by operators. Employees with a higher degree of qualification are allowed to cover demand of employees with a lower degree of qualification, but not the other way around. That means that operation managers can also cover demand for assignment managers and operators, and assignment managers can also cover demand for operators. Operators on the other hand, can only cover demand for operators. It is assumed that all qualifications can handle calls at the operations centres. The demand for the different qualifications is fixed for all shifts but differs for the different operational centres.

4.1.4 Work

The work regulations differentiate between effective and accounted working hours. The effective working time is the exact clock hours an employee is assigned to works, while the accounted working time is the hours an employee is assigned to work included additional compensating time. When calculating accounted working time (HTA § 7(3)), every day between 20.00 and at 06.00 each worked hour accounts for 1 hour and 15 minutes, and on weekends between 06.00 and at 20.00, each hour worked accounts for 1 hour and 10 minutes. For employees at the operations centre the total effective working time must not exceed 35.5 clock hours per week on average over the planning period. (HTA § 7(3)). Accounted working hours must not exceed 48 hours per week (AML §10-4), and 37.5 hours on average per week over the planning period (ATB § 2-2(1)). A week is defined in the regulations as seven days starting from Monday 00.00 until Sunday 24.00 (ATB § 1-7(5)).

Further, the effective working time cannot exceed 10 hours within 24 hours (ATB § 2-1(2)), where a new reference period of 24 hours starts after the restriction of 11 hours of rest between worked shifts (AML § 10-8(2)) is fulfilled (ATB § 1-7(13)). An employee can also at most have nine consecutive working days (ATB § 3-1(7)). This is often regulated by the constraint ensuring the required weekly rest, however, in cases where employees have a day off in the beginning of the week and the end of the following week, an additional constraint must limit the number of consecutive working days to not exceed nine days.

Employees should work as close to their contracted hours as possible. However, total hours worked are not allowed to exceed the maximum working hours specified in the work regulations. The deviation between contractual and actual hours worked is penalized in the satisfaction score of each employee.

In the police´s own risk assessment of schedules it is stated that a work schedule should practice clockwise shift rotations. This indicates a work pattern where the employees gradually transition from day shifts, to evening shifts, to night shifts. Further, employees should not work more than three consecutive evening- or night shifts, and when transferring from night shift to another shift type, 24 hours of rest is recommended. These are recommended health-promoting shift patterns to prevent accidents and give better working conditions for employees. On request from the police and based on preferences, employees should maximum work every third weekend, if possible. When working weekends employees work either day-, evening- or night shifts for the entire weekend. The shift types an employee work on weekends should also change chronologically. Moreover, the current practise in the police is to not work partial weekends but work Friday until Sunday if they are allocated to work weekends.

4.1.5 Rest

Employees are required to get enough continuous daily and weekly rest. A daily rest period is defined as an uninterrupted period of rest between two working shifts that exceed a certain limit regulated by the work regulations. The lower limit of daily rest between shifts is 11 hours (ATB § 5-1(1)). In addition to the daily rest, it is obligated to give employees a weekly day off consisting of a continuous rest that exceeds 36 hours. The weekly rest is to preferably be allocated on a Sunday, and at least every second Sunday (HTA § 14(1)).

Further, there are regulations aiming to expand the daily and weekly rest for employees. It is preferable that employees get a continuous work-free period of 16 hours between shifts (ATB § 3-1(8)). To the greatest extent possible, an additional day off shall be given during the week, preferably combined with the weekly rest if possible (HTA § 14(2)). Finally, as far as service allows, the days off should be arranged so that employees are not assigned shifts that ends later than 18:00 the day before the day off (ATB § 3-1(11)).

4.1.6 Personnel Preferences

Employees can state their personal preferences for what shift types they prefer to work. The distribution of the preferences used in this thesis is based on information retrieved from the police, which indicate that most employees want to work more dayshifts (41%), a large proportion (38%) want to work more evenings, and a small proportion want to work more night shifts (21%).

4.1.7 Fairness

The schedules should aim to distribute the workload and gain fairness among the employees as specified in the work regulations (ATB § 7-9(9)). This suggest that all factors of fairness are taken into consideration and that employees are treated equally regarding workload and preference fulfilment. Fairness is achieved by tracking the satisfaction score for employees and distributing the satisfaction evenly. This will be explored further in Chapter 6.

4.1.8 Objective

The objective of the model is to create schedules that maximise the total satisfaction of all employees, while ensuring fairness and satisfying the forecasted demand. The schedules must satisfy the limitations caused by the predefined rules and regulations, while also considering personal preferences and organisational culture.

4.2 Problem Assumptions

In the work of creating the rostering problem some assumptions have been made. This chapter presents these assumptions, and the reasoning behind.

4.3.1 Isolated Planning Period

It could be beneficial to retrieve information about previous planning periods and pass on information to later periods to ensure an even distribution of satisfaction for all planning periods. However, for the scope of this thesis, the model is only considering an isolated planning period, where no information is retrieved from the previous period, and no information is transferred to the next.

4.3.2 Breaks

Employees are obligated minimum one rest break if the working hours exceed five and a half hours. The length of the break should be specified in employee´s work agreement (AML $ 14-6(1)). However, if the working hours exceed eight hours, the employee´s total rest during the shift should be minimum 30 minutes (AML § 10-9). The roster is not handling the allocation of breaks during shifts, as employees select the most appropriate times for breaks themselves. This is also the current practise at the operations centres.

4.3.3 No Part-Time Workers

It is assumed that there are no part-time workers at the operation centres, therefore all employees have 100% contracts. This reflects the current situation at the operations centres.

4.3.4 Overtime

The roster does not plan for overtime. According to the regulations it is not allowed to plan for overtime unless extraordinary work capacity is needed for bigger events or state visits (ATB §4-4). It is assumed that the planning periods do not involve events of this character, as extraordinary work schedules that adjust the normal working hours according to the extraordinary conditions should be made.

4.3.5 Public Holidays

According to the work regulations, special regulations apply to public holidays, like Easter and Christmas, hence individual work schedules must be provided (ATB 3-1(10)). For the case of this thesis, it is assumed that the work schedules for public holidays are created outside this model and is implemented as input to the model.

4.3.6 Equal Prioritisation

When it comes to the prioritisation of employees' preferences, it is assumed that all employees are prioritised equally, regardless of seniority or qualification.

5. Mathematical Model

To solve the rostering optimisation problem for the operations centre, different versions of Mixed Integer Linear Programming Models (MILP) are created and presented in this chapter. First, the sets are presented in Chapter 5.1, followed by the parameters in Chapter 5.2. The variables and objective function of the base model are defined in Chapter 5.3 and 5.4 respectively. The base model consists of work regulations that *must* be followed to create a roster within legal boundaries. In Chapter 5.5, the regulations are presented as constraints restricting the problem´s possible values and combinations of the variables. Finally, the model extensions are presented in Chapter 5.6. The model extensions reflect regulations that *should* preferably be followed, recommended health-promoting work patterns and employees´ preferences. In Chapter 6, the model extensions are systematically added to the base model to explore their effect on the complexity of the model.

5.1 Sets

E	Set of employees
Q	Set of qualifications
W	Set of weeks in planning period
D	Set of days in planning period
D^{Mon}	Set of Mondays
D_w^{Mon}	Set of Mondays in week w
D^{Sat}	Set of Saturdays
D^{Sun}	Set of Sundays
D^{we}	Set of weekend days in planning period
D^{wd}	Set of weekdays in planning period
S_d	Set of shifts with positive demand on day d
ST	Set of shift types (day, evening, night)
S_d^{st}	Set of shifts of shift type st with positive demand on day d

5.2 Parameters

5.2.1 General Parameters

eh_s	Number of effective working hours per shift s
ah_s^{wd}	Number of accounted working hours per shift s
ah_s^{we}	Number of accounted working hours per weekend shift s
em_s	Number of hours from end of shift s to midnight
ms_s	Number of hours from midnight to start of shift s
ch_e	Weekly contract hours for employee e
$p_{s,e}$	Preference of shift s for employee e
$eq_{e,q}$	$\begin{cases} 1, \text{ if employee e has qualification q} \\ 0, \text{ otherwise} \end{cases}$
$a_{e,d}$	$\begin{cases} 1, \text{ if employee e is available at day d} \\ 0, \text{ otherwise} \end{cases}$
rh_{s^1,s^2}	$\begin{cases} 1, \text{ if it is at least 11 rest hours from shift } s^1 \text{ to shift } s^2 \text{ the next day} \\ 0, \text{ otherwise} \end{cases}$

5.2.2 Limiting Parameters

$\underline{critical}$	Critical staff level
$\underline{min_{s,d}}$	Minimal number of employees at shift s at day d to cover all 112 calls
$\overline{opt_{s,d}}$	Optimal number of employees at shift s at day d
$\underline{rq_q}$	Minimum number of employees with qualification q required at all shifts
$\overline{ah}$	Maximum accounted hours per week
$\overline{ah^{avg}}$	Maximum weekly accounted working hours on average
$\overline{eh^{avg}}$	Maximum weekly effective working hours on average
$\overline{cd}$	Maximum consecutive working days
$\overline{we}$	Employees can maximum work every $\overline{we}$ weekend
$\underline{cr}$	Minimum hours of consecutive rest in relation to the weekly day off

5.2.3 Scaling Parameters

min^P	Minimum days an employee can be assigned to its preferred shift type
max^P	Maximum days an employee can be assigned to its preferred shift type
min^{IX}	Minimum isolated workdays an employee can be assigned to
max^{IX}	Maximum isolated workdays an employee can be assigned to
min^{IO}	Minimum isolated days off an employee can be assigned to
max^{IO}	Maximum isolated days off an employee can be assigned to
min^{CD}	Minimum deviation between contracted and assigned hours an employee can have
max^{CD}	Maximum deviation between contracted and assigned hours an employee can have
min^S	Minimum sum of all employees´ satisfaction scores
max^S	Maximum sum of all employees´ satisfaction scores
min^{SD}	Minimum total deviation between actual and optimal staff levels
max^{SD}	Maximum total deviation between actual and optimal staff levels

n^P	Scaling factor for employees' shift preferences
n^{IX}	Scaling factor for working isolated days
n^{IO}	Scaling factor for having isolated day off
n^{IW}	Scaling factor for working partial weekends (only Saturday or Sunday)
n^{CD}	Scaling factor for deviation between contracted and assigned hours
n^S	Scaling factor for sum of all employees´ satisfaction scores
n^{SD}	Scaling factor for total deviation between actual and optimal staff levels

The scaling parameters are used to make the satisfaction aspects and the variables in the multi-objective comparable, to ensure accurate weighting of the different elements´ importance. Using min-max normalisation, the values will be standardised with a common scale between 0 and 1. The normalisation formula used is given by:

$$\frac{value - min}{max - min}$$

To simplify the presentation of the mathematical model, a scaling parameter n is created for all the elements that is normalised in the satisfaction score or objective:

$$n = \frac{1}{max - min}$$

where min is the minimum possible total value that the specific variable can have within the entire planning period, whereas max is the maximum possible total value the variable can have. The scaling parameter n is later multiplied with the variable of interest minus min to ensure correct normalisation of the satisfaction aspects and objective elements.

5.2.5 Weight Parameters

j_e^P	Weight factor for shift preferences per employee e
j_e^{IX}	Weight factor for working isolated days per employee e
j_e^{IO}	Weight factor for isolated days off per employee e
j_e^{IW}	Weight factor for working a partial weekend per employee e
j^{CD}	Weight factor for deviation between contracted hours and assigned hours
j^{SD}	Weight factor for deviation between optimal and actual staff levels
j^S	Weight factor for employee satisfaction

The satisfaction weight parameters indicate how the employees value and prioritise the components in the satisfaction score, while the objective weight parameters indicate how the police administration prioritises employee satisfaction compared to optimal staffing. The sum of the satisfaction weights must be equal to 1 for all employees. The sum of the objective weights must also be equal to 1.

5.3 Variables

$X_{e,s,d}$ $\begin{cases} 1, \text{if employee e is assigned to shift s on day d} \\ 0, \text{otherwise} \end{cases}$

$O_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned a day off on day d} \\ 0, \text{otherwise} \end{cases}$

$R_{e,d}$ $\begin{cases} 1, \text{if employee e has } \underline{cr} \text{ hours of continuous rest on day d} \\ 0, \text{otherwise} \end{cases}$

$OO_{e,d}$ $\begin{cases} 1, \text{if employee e has days off on both day d and d+1} \\ 0, \text{otherwise} \end{cases}$

$OS_{e,s,d}$ $\begin{cases} 1, \text{if employee e is assigned to shift s the day after a day off on day d} \\ 0, \text{otherwise} \end{cases}$

$NO_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned to a night shift the day before a day off d} \\ 0, \text{otherwise} \end{cases}$

$IX_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned to an isolated workday on day d} \\ 0, \text{otherwise} \end{cases}$

$IO_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned to an isolated day off on day d} \\ 0, \text{otherwise} \end{cases}$

$IW_{e,d}^{Sat}$ $\begin{cases} 1, \text{if employee e is assigned to a Saturday d and not Sunday} \\ 0, \text{otherwise} \end{cases}$

$IW_{e,d}^{Sun}$ $\begin{cases} 1, \text{if employee e is assigned to a Sunday d and not Saturday} \\ 0, \text{otherwise} \end{cases}$

$CR_{e,d}$ = Hours of continuous rest for employee e on day d

$SD_{s,d}$ = Deviation between actual and optimal staff on shift s on day d

CD_e = Deviation between contract- and assigned hours for employee e

SAT_e = Satisfaction score of employee e

5.4 Objective Function

$$Maximise\ \ n^S * j^S\left(\sum_{e\in E} SAT_e - min^S\right) - n^{SD} * j^{SD}\left(\sum_{s\in S_d}\sum_{d\in D} SD_{s,d} - min^{SD}\right) \quad (5.1)$$

To solve the rostering problem, a multi objective function has been formulated. The objective function aims to maximise the total satisfaction of the employees while minimising the total deviation between actual and optimal staff level. SAT_e represents the individual satisfaction scores of all employees and is based on the four satisfaction and dissatisfaction aspects that are further elaborated in Chapter 5.5.5. $SD_{s,d}$ retains the deviation between the actual number off assigned employees and the optimal staff level per shift per day, which is defined in chapter 5.5.1. The scaling parameters n^S, n^{SD}, min^S and min^{SD} normalises the objectives to values between 0 and 1, and the weight parameters j^S and j^{SD} decides the relative priority between the two objectives.

5.5 Constraints

The following chapter describes and categorises the constraints used in the base model. The constraints either reflect the legally required work regulations or defines auxiliary variables.

5.5.1 Demand

$$\sum_{e\in E} a_{e,d} * X_{e,s,d} \geq \underline{critical} \qquad \forall d \in D, s \in S_d \quad (5.2)$$

$$\sum_{e\in E} a_{e,d} * X_{e,s,d} \leq \overline{opt_{s,d}} \qquad \forall d \in D, s \in S_d \quad (5.3)$$

$$X_{e,s,d} = 0 \qquad \forall e \in E\ , d \in D, s \in S|\ S_d \quad (5.4)$$

$$\sum_{e\in E} eq_{e,q}\ * a_{e,d} * X_{e,s,d} \geq \underline{rq_q} \qquad \forall d \in D, s \in S_d, q \in Q \quad (5.5)$$

$$\overline{opt_{s,d}} - \sum_{e \in E} X_{e,s,d} = SD_{s,d} \qquad \forall\, d \in D, s \in S_d \quad (5.6)$$

If an employee is assigned to a shift, it covers one unit of demand for that shift. Constraint (5.2) ensures that enough employees cover the critical staff level on all shifts, while (5.3) makes sure that there are not assigned more employees than needed, which is the optimal staff level $\overline{opt_{s,d}}$. Further, (5.4) guarantees that no employees are assigned to shifts with no demand and (5.5) ensures that there are enough employees $\underline{rq_q}$ with qualification q during all shifts. Finally, the deviation between the optimal and actual demand coverage is stored in $SD_{s,d}$ presented in (5.5). The minimum staff level to cover all 112 emergency calls $\underline{min_{s,d}}$ is not explicitly given in the mathematical model. However, it is used to calculate the understaffing in the results, which is the deviation between the actual assigned staff level and the minimum staff level $\underline{min_{s,d}}$ per shift per day if the actual staff level is lower than the minimum staff level $\sum_{e \in E} X_{e,s,d} < \underline{min_{s,d}}$.

5.5.2 Work

$$\sum_{s \in S_d} X_{e,s,d} \leq 1 \qquad \forall e \in E, d \in D \quad (5.7)$$

$$\sum_{d=d'}^{d'+\overline{cd}} \sum_{s \in S_d} X_{e,s,d} \leq \overline{cd} \qquad \forall e \in E,\ d' \in D - \overline{cd} \quad (5.8)$$

Constraint (5.7) ensures that employees can be assigned to no more than one shift per day, while (5.8) restricts the employees to work maximum $\overline{cd}$ days consecutively.

$$\sum_{d \in D} \sum_{s \in S_d} eh_s * X_{e,s,d} \leq \overline{eh^{avg}} * W \qquad \forall e \in E \quad (5.9)$$

$$\sum_{d \in D^{wd}} \sum_{s \in S_d} ah_s^{wd} * X_{e,s,d} + \sum_{d \in D^{we}} \sum_{s \in S_d} ah_s^{we} * X_{e,s,d} \leq \overline{ah^{avg}} * W \quad \forall e \in E \quad (5.10)$$

$$\sum_{d=d_w}^{d_w+4} \sum_{s \in S_d} ah_s^{wd} * X_{e,s,d} + \sum_{d=d_w+5}^{d_w+6} \sum_{s \in S_d} ah_s^{we} * X_{e,s,d} \leq \overline{ah} \qquad (5.11)$$

$$\forall e \in E, w \in W, d_w \in D_w^{Mon}$$

Further, the employees are delimited by regulations deciding how much they can work per week and during the entire period. This is covered by the constraints (5.9) - (5.11). Constraint (5.9) calculates the employees´ effective working hours and ensures that they cannot work more than $\overline{eh^{avg}}$ hours per week on average during the entire period. Constraint (5.10) performs the same operations as in (5.9) only with the accounted hours. The accounted hours are calculated differently during the weekdays and the weekend, which is differentiated by ah_s^{wd} and ah_s^{we} accordingly. Finally, (5.11) ensures that the employees do not work more than $\overline{ah}$ accounted hours per week.

5.5.3 Rest

$$X_{e,s^1,d} + X_{e,s^2,d+1} - 1 \leq rh_{s^1,s^2} \qquad \forall e \in E\ , d \in D-1, s^1, s^2 \in S_d \qquad (5.12)$$

The required daily rest is covered by constraint (5.12), which ensures that there are enough hours between an assigned shift s^1 on day d and the assigned shift s^2 the next day (d+1).

Additionally, all employees must have at least $\underline{cr}$ minimum hours of consecutive rest associated to their weekly day off. To account for this, the continuous hours of rest per day $CR_{e,d}$ needs to be calculated, indicating the rest from the current day d, the day before (d-1) and the day after (d+1) if the continuous rest is not interrupted by an assigned shift on day d. This is calculated in (5.16) and will be further elaborated. First, there are defined three new binary variables in (5.13), (5.14) and (5.15), used to calculate the hours of continuous rest $CR_{e,d}$.

$$\sum_{s \in S_d} X_{e,s,d} + O_{e,d} = 1 \qquad \forall e \in E, d \in D \qquad (5.13)$$

Constraint (5.13) defines employees´ days off $O_{e,d}$ which is later used to add 24 hours of rest to the calculated hours of continuous rest $CR_{e,d}$ if the day before (d-1) or the current day d is a day off. A day off is defined as a full day with no assigned shifts.

$$O_{e,d} + O_{e,d+1} - 1 \leq OO_{e,d} \qquad \forall e \in E, d \in D - 1 \qquad (5.14.a)$$

$$O_{e,d} \geq OO_{e,d} \qquad (5.14.b)$$

$$O_{e,d+1} \geq OO_{e,d} \qquad (5.14.c)$$

The continuous rest is not interrupted if there is a day off on the current day d. In that case, the resting hours off the next day need to be included in the calculated hours of continuous rest $CR_{e,d}$. Constraints (5.14.a) - (5.14.c) and (5.15.a) - (5.15.c) are defined to know whether to add 24 hours if the next day is a day off, or the hours from midnight until next day´s shift begins. First, constraint (5.14.a) ensures that $OO_{e,d}$ is 1 if an employee has an additional day off on day (d+1) after a day off on the current day d. Then (5.14.b) and (5.14.c) ensure that $OO_{e,d}$ is 0 if the employee is not assigned a day off that day d or the next day (d+1).

$$O_{e,d} + X_{e,s,d+1} - 1 \leq OS_{e,s,d} \qquad \forall e \in E, d \in D - 1, s \in S_d \qquad (5.15.a)$$

$$O_{e,d} \geq OS_{e,s,d} \qquad \forall e \in E, d \in D - 1, s \in S_d \qquad (5.15.b)$$

$$X_{e,s,d+1} \geq OS_{e,s,d} \qquad \forall e \in E, d \in D - 1, s \in S_d \qquad (5.15.c)$$

Further, (5.15) provides the assigned shift the day after a day off, which is used to add the correct number of hours from midnight to shift s begins. Constraint (5.15.a) assures that $OS_{e,s,d}$ is 1 if an employee has a day off on day d and is assigned shift s the next day (d+1), while (5.15.b) and (5.15.c) make $OS_{e,s,d}$ 0 if the employee is not assigned a day off on day d or is not assigned a shift the day after (d+1).

$$O_{e,d} + \sum_{s\in S_{d-1}^{Night}} X_{e,s,d-1} - 1 \leq NO_{e,d} \qquad \forall e \in E, d \in D-1 \qquad (5.16.a)$$

$$O_{e,d} \geq NO_{e,d} \qquad \forall e \in E, d \in D-1 \qquad (5.16.b)$$

$$\sum_{s\in S_{d-1}^{Night}} X_{e,s,d-1} \leq NO_{e,d} \qquad \forall e \in E, d \in D-1 \qquad (6.16.c)$$

As night shifts "belong" to the day d the shift begins but lasts until the next day (d+1), an employee is still working on day (d+1) even if it is not assigned any new shifts that day. Hence, it should not be considered as a weekly day off with $\underline{cr}$ continuous hours of rest. Constraints (5.16.a) – (5.16.b) defines the binary variable $NO_{e,d}$ indicating whether an employee works a night shift before their day off, which is later used to eliminate the 24 hours of the "incomplete" day off. While (5.16.a) ensures that $NO_{e,d}$ is 1 if the employee e is assigned a night shift the day before (d-1) their day off on day d, (5.16.b) and (5.16.c) force it to be 0 otherwise.

$$CR_{e,d} = 24 * O_{e,d-1} + \sum_{s\in S_{d-1}} em_s * X_{e,s,d-1} \qquad (5.17.a)$$

$$+ 24 * O_{e,d} - 24 * NO_{e,d} + \sum_{s\in S_d} ms_s * X_{e,s,d} \qquad (5.17.b)$$

$$+ 24 * OO_{e,d} + \sum_{s\in S_d} ms_s * OS_{e,s,d} \qquad (5.17.c)$$

$$\forall e \in E, \forall d \in 2,..,D-1$$

$$CR_{e,1} = 24 * O_{e,1} + \sum_{s\in S_1} ms_s * X_{e,s,1} \qquad (5.17.d)$$

$$+ 24 * OO_{e,1} + \sum_{s\in S_1} ms_s * OS_{e,s,1} \qquad \forall e \in E \qquad (5.17.e)$$

$$CR_{e,D} = 24 * O_{e,D-1} + \sum_{s\in S_D} em_s * X_{e,s,D-1} \qquad (5.17.f)$$

$$+ 24 * O_{e,D} - 24 * NO_{e,D} + \sum_{s\in S_D} ms_s * X_{e,s,D} \qquad \forall e \in E \qquad (5.17.g)$$

In (5.17.a) - (5.17.c) the continuous hours of rest per day $CR_{e,d}$ is calculated for all the days except the first and final day, which are calculated in (5.17.d) (5.17.e) and (5.17.f) – (5.17.g) respectively. In (5.17.a) the resting hours from the day before (d-1) is added to the calculation. If the employee had a day off, 24 hours is added to the continuous rest calculation, but if the employee were assigned a shift, the hours from the end of shift s to midnight em_s is added. In (5.17.b) the resting hours from the current day d is added. If the employee is assigned a day off, 24 hours will be added. However, the 24 hours are eliminated if the day off is combined with a night shift ending the same day as the day off. Moreover, if the employee is assigned a shift on day d, the hours between midnight and the beginning of the assigned shift ms_s are added. As night shifts "belong" to the day that the shift begins, em_s will be negative for the night shifts, hence subtracted from the resting hours from the current day.

Finally, if the continuous rest is not already interrupted by an assigned shift on day d, the resting hours from the next day (d+1) are added in (5.17.c). If $OO_{e,d}$ is 1, the rest is not interrupted and the employee has an additional day off the next day (d+1), indicating an additional 24 hours of rest. If $OS_{e,s,d}$ is 1 the rest is not interrupted and the employee is assigned a shift the next day (d+1), hence the hours from midnight to the shift´s beginning ms_s are added. The continuous rest for the first and final day are calculated in the same way in (5.17.d) - (5.17.g).

$$\sum_{d=d'}^{d'+6} R_{e,d} \geq 1 \qquad \forall e \in E, d' \in D^{Mon} \qquad (5.18)$$

$$CR_{e,d} \geq \underline{cr} * R_{e,d} \qquad \forall e \in E, d \in D \qquad (5.19)$$

Finally, constraint (6.18) ensures that employees have at least one weekly *day off* with at least $\underline{cr}$ hours of continuous rest. The binary $R_{e,d}$ is defined in (5.19) and is equal to 1 if an employee has $\underline{cr}$ hours of $CR_{e,d}$ continuous rest on day d. Because $R_{e,d}$ must be 1 only once a week, it is not necessary to force it to be 0 in all other cases.

5.5.4 Weekend

$$\sum_{i=0}^{\overline{we}-1} \sum_{s \in S_d} X_{e,s,d+i*7} \leq 1 \qquad \forall e \in E, d \in D^{Sun} \quad (5.20)$$

Employees are entitled to a day off every $\overline{we}$ Sunday, which is covered by the constraint in (5.20). In the case of the police operations centre, this implies to every second Sunday.

5.5.5 Satisfaction

Besides striving towards the ideal staff level to handle emergency calls, the police administration is dedicated to creating better work conditions for their employees. Hence, the model tracks and maximises the employees experienced satisfaction based on five satisfaction and dissatisfaction aspects. The aspects stem from work regulations and conversations with the police administration and are presented in terms of soft constraints that are either penalized or rewarded in the calculation of the employee's satisfaction score SAT_e in (5.25). There are four new binary variables and one continuous variable defined to calculate the satisfaction score.

$$\sum_{s \in S_d} X_{e,s,d} - \sum_{s \in S_{d-1}} X_{e,s,d-1} - \sum_{s \in S_{d+1}} X_{e,s,d+1} \leq IX_{e,d} \qquad (5.21)$$

$$\forall e \in E, d \in 2,..,D-1$$

$$O_{e,d} - O_{e,d-1} - O_{e,d+1} \leq IO_{e,d} \qquad \forall e \in E, d \in 2,..,D-1 \qquad (5.22)$$

Constraint (5.21) define $IX_{e,d}$, indicating whether the employees are assigned isolated working days, while constraint (5.22) define $IO_{e,d}$ indicating whether the employees are assigned isolated days off. $IX_{e,d}$ is 1 if an employee is assigned a shift on day d, but not the day before (d-1) or after (d+1). Moreover, $IO_{e,d}$ is 1 if an employee is assigned a day off on day d, but not the day before (d-1) or after (d+1). As $IX_{e,d}$ and $IO_{e,d}$ are penalized in the calculation of the satisfaction score, they will be 0 in all other cases.

$$\sum_{s \in S_d} X_{e,s,d} - \sum_{s \in S_{d+1}} X_{e,s,d+1} = IW_{e,d}^{Sat} - IW_{e,d+1}^{Sun} \qquad \forall e \in E, d \in D^{Sat} \tag{5.23}$$

$$ch_e * W - \sum_{s \in S_{s,d}} \sum_{d \in D} * eh_s * X_{e,s,d} = CD_e \qquad \forall e \in E \tag{5.24}$$

Constraint (5.23) define $IW_{e,d}^{Sat}$ and $IW_{e,d}^{Sun}$ indicating whether the employees are assigned partial weekends. They are equal to 1 if an employee work one weekend day and not the other, and 0 otherwise as they are penalized in the satisfactions score. Lastly, (5.24) stores the deviation between the employees´ periodic contract hours and their assigned working hours.

$$SAT_e = n^P * j_e^P \left(\sum_{d \in D} \sum_{s \in S_d} p_{e,s} * X_{e,s,d} - min^P \right) \tag{5.25.a}$$

$$- n^{IX} * j_e^{IX} \left(\sum_{d \in D} IX_{e,d} - min^{IX} \right) \tag{5.25.b}$$

$$- n^{IO} * j_e^{IO} \left(\sum_{d \in D} IO_{e,d} - min^{IO} \right) \tag{5.25.c}$$

$$- n^{IW} * j_e^{IW} \left(\left(\sum_{d \in D} IW_{e,d}^{Sat} + \sum_{d \in D} IW_{e,d}^{Sun} \right) - min^{IW} \right) \tag{5.25.d}$$

$$- n^{CD} * j^{CD} * (CD_e - min^{CD}) \tag{5.25.e}$$

$$\forall e \in E$$

Finally, (5.25) calculates the individual satisfaction scores SAT_e for all employees. The first satisfaction aspect (5.25.a) rewards when employees are assigned to shifts that are consistent with their preferred shift type $p_{e,s}$. Further, (5.25.b) - (5.25.d) penalizes the cases when the employees´ are assigned to isolated workdays, isolated days off or partial weekends, while (5.25.d) penalizes the deviation between periodic contract hours and assigned work hours per employee. To be able to equally compare the satisfaction aspects, the parameters $n^P, n^{IX}, n^{IO}, n^{IW}, n^{CD}$ together with min^P, min^{IX}, min^{IO}, min^{IW}

and min^{CD}, normalises the values to be between 0 and 1. The weight parameters $j_e^P, j_e^{IX}, j_e^{IO}, j_e^{IW}, j^{CD}$ decides how different employees weight the different satisfaction aspects. The sum of the weights must be equal to 1 to ensure equal prerequisites to gain satisfaction. The weight for contract deviation j^{CD} is equally set for all employees, as this aspect is there to equalize the workload. With more dissatisfaction than satisfaction elements, "high" satisfaction scores might be negative values.

5.5.6 Variable Constraints

$X_{e,s,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D, s \in S_d$	(5.26)
$O_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.27)
$R_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.28)
$OO_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.29)
$OS_{e,s,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D, s \in S_d$	(5.30)
$NO_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.31)
$IX_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.32)
$IO_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.33)
$IW_{e,d}^{Sat}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.34)
$IW_{e,d}^{Sun}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.35)
$CR_{e,d}$	$\in \mathbb{R}^+$	$\forall e \in E, d \in D$	(5.36)
CD_e	$\in \mathbb{R}^+$	$\forall e \in E$	(5.37)
$SD_{s,d}$	$\in \mathbb{Z}^+$	$\forall d \in D, s \in S_d$	(5.38)
SAT_e	$\in \mathbb{R}$	$\forall e \in E$	(5.39)

The remaining constraints defines the nature of the variables and restricts their range. Constraints (5.26) – (5.35) are binary variables and can only be 0 or 1, (5.36) and (5.37) must be positive continuous values, (5.38) must be positive integers and (5.39) can be any continuous values.

5.6 Model Extension

In addition to the legally required regulations in the base model, there are other regulations and preferences that *should* be followed to ensure better health conditions, working environment and satisfaction among employees. The extended model is designed to explore the cultural and health promoting limitations of the problem that are not included in the base model. Implementing the extension to the base model would indicate more correct and "better" results as more regulations and preferences are considered. However, the extensions would make the model more complex than it already is.

5.6.1 Health Promoting Work Patterns

$$\sum_{s \in S_d^{Day}} X_{e,s,d} - \left(\sum_{s \in S_{d+1}^{Day}} X_{e,s,d+1} + \sum_{s \in S_{d+1}^{Eve}} X_{e,s,d+1} + O_{e,d+1} \right) \leq 0$$

$$\forall e \in E, d \in D - 1 \quad \text{(E.5.1.a)}$$

$$\sum_{s \in S_d^{Eve}} X_{e,s,d} - \left(\sum_{s \in S_{d+1}{}^{Eve}} X_{e,s,d+1} + \sum_{s \in S_{d+1}^{Night}} X_{e,s,d+1} + O_{e,d+1} \right) \leq 0$$

$$\forall e \in E, d \in D - 1 \quad \text{(E.5.1.b)}$$

$$\sum_{s \in S_d^{Night}} X_{e,s,d} - \left(\sum_{s \in S_{d+1}{}^{Night}} X_{e,s,d+1} + O_{e,d+1} \right) \leq 0$$

$$\forall e \in E, d \in D - 1 \quad \text{(E.5.1.c)}$$

Clockwise shift rotations are proved to have several performance- and life quality benefits, as the employees gradually transition to night shifts. This is ensured by the extended model constraints in (E.5.1.a) – (E.5.1.c). The former makes sure that if an employee is assigned to a day shift, it must be assigned to either a day shift, evening shift or day off the next day. Then (E.5.1.b) decides that if an employee is assigned to an evening shift, it must be assigned to either an evening shift, night shift or day off the next day. Finally, (E.5.1.c) force employees to work additional night shifts or take a day off after a period of night shifts. This makes sure that the employees get sufficient hours of rest after a period with night shifts, before starting to work other shift types.

$$\sum_{d=d'}^{d'+\overline{cs}} \sum_{s \in S_d^{Eve}} X_{e,s,d} \leq \overline{cs} \qquad \forall e \in E, d' \in D - \overline{cs} \qquad \text{(E.5.2.a)}$$

$$\sum_{d=d'}^{d'+\overline{cs}} \sum_{s \in S_d^{Night}} X_{e,s,d} \leq \overline{cs} \qquad \forall e \in E, d' \in D - \overline{cs} \qquad \text{(E.5.2.b)}$$

As recommended in the police schedule risk assessment, constraint (E.5.2.a) - (E.5.2.b) ensures that employees are assigned to maximum $\overline{cs}$ consecutive evening- and night shifts respectively.

$$SR_{e,d} \geq sr * R_{e,d}^{SR} \qquad \forall e \in E, d \in D \qquad \text{(E.5.3)}$$

$$SAT_e \mathrel{+}= n^{SR} * j^{SR} \left(\sum_{d \in D} R^{SR} - min^{SR} \right) \qquad \forall e \in E \qquad \text{(E.5.4)}$$

Although (5.12) already ensures the required daily rest of 11 hours, the regulations encourage to strive towards 16 hours of rest between shifts as *often* as possible. To cover this, (E.5.3) defines a new binary $R_{e,d}^{SR}$ which is 1 if the calculated rest between shifts $SR_{e,d}$ is greater or equal to the desired shift rest sr and 0 otherwise. The parameters n^{SR}, min^{SR} and j^{SR} normalises and weights the satisfaction aspect. The calculation of the hours of rest between shifts $SR_{e,d}$ is calculated in the following section.

$$X_{e,s,d} + O_{e,d+1} - 1 \leq SO_{e,s,d} \qquad \forall e \in E, d \in D - 1, s \in S_d \qquad \text{(E.5.5.a)}$$

$$O_{e,d+1} \geq SO_{e,s,d} \qquad \forall e \in E, d \in D - 1, s \in S_d \qquad \text{(E.5.5.b)}$$

$$X_{e,s,d} \geq OS_{e,s,d} \qquad \forall e \in E, d \in D - 1, s \in S_d \qquad \text{(E.5.5.c)}$$

$$SR_{e,d} = \sum_{s\in S_d} em_s * (X_{e,s,d} - SO_{e,s,d}) \quad \text{(E.5.6.a)}$$

$$+ \sum_{s\in S_{d+1}} ms_s * (X_{e,s,d+1} - OS_{e,s,d}) \quad \text{(E.5.6.b)}$$

First, a new binary $SO_{e,s,d}$ is defined in (E.5.5.a) – (E.5.5.c), which is 1 if the employee e is assigned shift s on day d and have a day off the next day (d+1), otherwise it is 0. It is used to adjust the hours of rest between shifts $SR_{e,d}$ when an employee is only assigned a shift on day d and not the next day (d+1). The hours of rest between shifts $SR_{e,d}$ is calculated in (E.5.6). The first element (E.5.6.a) includes the hours between the assigned shift on day d and midnight em_s if the employee is assigned work both days. $SO_{e,s,d}$ eliminates the hours between the shift on day d and midnight if the next day is a day off. (E.5.6.b) includes the hours between midnight and the assigned shift ms_s the next day (d+1) if the employee is assigned work both days. $OS_{e,s,d}$ provides the assigned shift s after (d+1) a day off on day d, hence it eliminates the added hours if the current day d is a day off.

$$O_{e,d} + \sum_{s\in S_{d-1}^{18}} X_{e,s,d-1} - 1 \leq B_{e,d}^{18} \qquad \forall e \in E, d \in 2,..D \quad \text{(E.5.7.a)}$$

$$O_{e,d} \geq B_{e,d}^{18} \qquad \forall e \in E, d \in 2,..D \quad \text{(E.5.7.b)}$$

$$\sum_{s\in S_{d-1}^{18}} X_{e,s,d-1} \geq B_{e,d}^{18} \qquad \forall e \in E, d \in 2,..D \quad \text{(E.5.7.c)}$$

$$SAT_e \mathrel{+}= n^{18} * j^{18} \left(\sum_{d\in D} B_{e,d}^{18} - min^{18} \right) \qquad \forall e \in E \quad \text{(E.5.8)}$$

For the employees to get the most out of their free time, the regulations suggest that employees are not assigned to shifts that ends later than 18:00 the day before their day off, as long as the service allows it. Hence, the soft constraint (E.5.7) – (E.5.8), rewards the occurrences where an employee is assigned a day off on day d and a shift that ends earlier than 18:00 the day before (d-1). Constraint (E.5.7.a) ensures that $B_{e,d}^{18}$ is 1 if an employee is assigned a shift that ends before

18:00 the day before an assigned day off, while (E.5.7.b) - (E.5.7.c) force it to be 0 otherwise. $B_{e,d}^{18}$ is later rewarded in SAT_e in (E.6.8). The parameters n^{18}, min^{18} and j^{18} normalises and weights the satisfaction aspect.

5.6.2 Weekend

$$\sum_{s \in S_{st,d}} X_{e,s,d} - \sum_{s \in S_{st,d+1}} X_{e,s,d+1} = 0 \quad \forall e \in E, st \in ST, d \in D^{Sat} \tag{E.5.9.a}$$

$$\sum_{s \in S_{st,d}} X_{e,s,d} - \sum_{s \in S_{st,d-1}} X_{e,s,d-1} \leq 0 \quad \forall e \in E, st \in ST, d \in D^{Sat} \tag{E.5.9.b}$$

In general, employees prefer to work as few weekends as possible, but work full weekends when they first have to. The current police practice is to work every third weekend, with a clockwise rotation of shift types per weekend. Constraints (E.5.9.a) - (E.5.9.b) ensures that the employees work full weekends of the same shift type st. While the former guarantees that employees are assigned both Saturday and Sunday if they are assigned one of them, the latter forces employees to be assigned Friday if it is assigned Saturday, but not necessarily a Saturday if it works a Friday.

$$\sum_{s \in S_{s,d}^{Day}} X_{e,s,d} - \left(\sum_{s \in S_{s,d+7*\overline{we}+m}^{Eve}} X_{e,s,d} + O_{e,d+7*\overline{we}+m} \right) \leq 0 \quad \forall e \in E, d \in D^{Sat} \tag{E.5.10.a}$$

$$\sum_{s \in S_{s,d}^{Eve}} X_{e,s,d} - \left(\sum_{s \in S_{s,d+7*\overline{we}+m}^{Night}} X_{e,s,d} + O_{e,d+7*\overline{we}+m} \right) \leq 0 \quad \forall e \in E, d \in D^{Sat} \tag{E.5.10.b}$$

$$\sum_{s \in S_{s,d}^{Night}} X_{e,s,d} - \left(\sum_{s \in S_{s,d+7*\overline{we+m}}^{Day}} X_{e,s,d} + O_{e,d+7*\overline{we}+m} \right) \leq 0 \quad \forall e \in E, d \in D^{Sat} \tag{E.5.10.c}$$

Further, (E.5.10.a) - (E.5.10.c) make sure that the employees have a clockwise rotation (day→ evening→ night→ day etc.) of shift types per every $\overline{we} + m$ weekend. The margin m is included in case an employee work fewer than every $\overline{we}$ weekend.

5.6.3 Fairness

$$\underline{SAT} \leq Sat_e \qquad \forall e \in E \qquad \text{(E.5.11)}$$

$$Maximise\ obj \ += \ n^{minS} * j^{minS}\left(\underline{SAT} - min^{minS}\right) \qquad \text{(E.5.12)}$$

$$\underline{sat} \leq Sat_e \qquad \forall e \in E \qquad \text{(E.5.13)}$$

The police emphasize equality and fairness among the employees. Even if the objective maximises the total satisfaction score of all employees, the satisfaction scores may vary a lot. Some employees might be very satisfied, while others are dissatisfied. To ensure fairness and ensure that the maximisation of the total satisfaction score does not happen at the expense of individual employees, two solutions are provided. The first solution includes a new variable $\underline{SAT}$ (E.5.11), which stores the satisfaction score of the least satisfied employee. In (E.5.12) this value is added to the already existing multi objective function. It is normalised by n^{minS}, weighted by j^{minS} and maximised. The second solution is to set a lower bound $\underline{sat}$ of the least acceptable satisfaction score per employee. This is presented in constraint (E.5.13).

6. Model Exploration

In this chapter, analyses are performed to evaluate the performance and complexity of the rostering problem of the police operations centre. First the system configurations are presented in Chapter 6.1, followed by the initial parameter values used in the model in Chapter 6.2 and an exploration of the base model and the various extensions in Chapter 6.3. Then an extended model is determined - a model that runs within a reasonable timeframe while including as many model extensions as possible. This is further compared to the results of running the full model, which includes all the possible extensions in Chapter 6.3.2. Finally, to test the models´ sensitivity to parameter changes, the base model is tested against the extended model based on scenarios in Chapter 6.4.

6.1 System Configuration

To ensure comparability, all tests are performed on the same computer. The computer´s system configurations are presented in Table 6.1.

Processor	Intel Core i5
Frequency	2 GHz
Operating System	macOS Ventura 13.3.1
Memory	16 GB
Gurobi Version	v10.0.1
Python Version	3.9.7

Table 6.1: Overview of System Configuration

The optimisation solver used to solve the MILP operations centre rostering problem is Gurobi, which is one of the leading commercial solvers on the market. They offer a free, unlimited Gurobi Optimizer license for academic use, but must be installed using the computer terminal to access the unrestricted version. The model size was too large for the restricted version installed using pip, which is a commonly used installer using Python. Gurobi support parallel computations allowing execution of multiple subproblems simultaneously to improve and expedite the solving process. This comes in handy for the more complex models of this thesis, where the optimisation solver uses eight out of eight available processors.

6.2 Parameter Values

In this section, the parameter values in the rostering modelled are presented. The parameter values presented apply to all the model analysis unless different values are explicitly given for a specific analysis.

Description	Parameter	Value
Number of employees to be assigned to shifts	E	84
Min. Number of Operation Leaders at Each Shift	$\underline{rq_{opL}}$	1
Min. Number of Assignment Leaders at Each Shift	$\underline{rq_{assL}}$	3
Number of Operations Managers	$\sum_{e \in E} eq_{e,opM}$	9
Number of Assignment Managers	$\sum_{e \in E} eq_{e,assM}$	27
Employees can maximum work no more than every $\overline{we}$ weekend	$\overline{we}$	2
Weeks in planning period	W	6
Max consecutive working days	$\overline{cd}$	9
Minimum rest hours between shifts	$\underline{r}$	11
Desired res hours between shifts	sr	16
Maximum accounted hours per week	$\overline{ah}$	48
Maximum weekly accounted working hours on average	$\overline{ah^{avg}}$	37.5
Maximum weekly effective working hours on average	$\overline{eh^{avg}}$	35.5
Contract hours for each employee e	ch_e	35.5
Minimum hours of consecutive rest associated to the weekly day off	$\underline{cr}$	36
All weight factor for employee e	j_e	0.2
Weight factor for total employee satisfaction	j^S	0.5
Weight factor for deviation between optimal and actual staff levels	j^{SD}	0.5

Table 6.2: Overview of Base Model Parameter Values

Table 6.2 presents the parameter values for the base model. The total number of employees E and the minimum number of operations- and assignment managers that must be present at all shifts are values specifically presenting the Sør-Øst Police District. The total number of operations- and assignment managers are determined to meet the criteria of the minimum number of different qualifications as the models are set to work every third weekend. However, the base model is initially set to work every second weekend, as this is the regulation that at least must be followed. The planning period is initially set to six weeks. The parameter values presented between minimum rest hours between shifts $\underline{r}$ and the minimum hours of consecutive rest in relation to the weekly day off $\underline{cr}$ stem from actual work regulations. Further, the satisfaction weight parameters $j_e^P, j_e^{IX}, j_e^{IO}, j_e^{IW}, j^{CD}$ are all set to 0.2, indicating that all employees value the different aspects equally. This is due to simplicity but is not very realistic. With one positive satisfaction aspect and four dissatisfaction aspects, the satisfaction score can range between 0.2 and -0.8 per employee. It is further assumed that the objective weight factors j^S and j^{SD} are both 0.5.

Additionally, the employees´ preferred shift types were randomly generated using Python´s Random module. With the random choice function employees were assigned a preferred shift type with the probability distribution of 41% day, 38% evening and 21% night, which is based on information from the police. Initially, it is also assumed that the employees are available every day of the whole planning period.

The extended model is assigned all the same parameters as the base model, with the exception of the maximum number of weekends. As the base model only consists of work regulations that *must* be followed to create a roster within legal boundaries, while the extended model reflects some of the regulations that beneficially *should* be followed – the base model considers working every second Sunday ($\overline{we}$ =2), while the extended model considers working full weekends every third week ($\overline{we}$ =3). Moreover, as the extended model includes more satisfaction aspects, the weight parameters $j_e^P, j_e^{IX}, j_e^{IO}, j_e^{IW}, j^{CD}, j^{SR}, j^{18}$ are all set to 0.143.

6.3 Model Extensions

This chapter aim to investigate how integrating the different model extensions considering health promoting regulations, culture and personal preferences affect the complexity and running time of the model. The goal is to create an extended model that produce reasonable results within an acceptable time limit. The extensions not included in the extended model, will be tested on a full model that includes all the constraints introduced in this thesis.

Extension	Description	Model Change
B	Base Model	(5.1) - (5.39)
E1	Health Promoting Work Patterns	(E.5.1) - (E.5.2)
E2	16 Hours of Rest	(E.5.3) - (E.5.6)
E3	End Before 18.00 before Day off	(E.5.7) - (E.5.8)
E4	Weekend Extensions	(E.5.9) - (E.5.10)
E5a	Maximise fairness	(E.5.11) - (E.5.12)
E5b	Fairness with lower bound of satisfaction	(E.5.13)

Table 6.3: Overview of Model Extensions

Table 6.3 describes the constraint extensions and their order of implementation into the model. The model extensions are systematically added to the base model to investigate how the different components affect the complexity and quality of the model. If an optimal solution is not reached after eight hours, the model is stopped. For many complex MILP it can be challenging to find the right balance between quality of the solution and the model´s running time. Often a trade-off must take place, based on the purpose of the model. In the case of this specific rostering problem, solution quality is considered important. Hence, in the assessment of whether to include the different extensions further in the extended model, the importance of the regulation or preference is a crucial factor. However, for less important regulations or preferences an acceptance criterion of an objective gap of 5% within two hours is set to determine the further implementation.

6.3.1 Extension Implementation

In the following chapter, the results of the model extension exploration are presented. Table 6.4 shows an overview of the results in terms of solution quality and running time. The '-' indicates that no solution was found within the predefined time limit of two and eight hours.. The extensions that are included in the tests are listed in the first column. Notice that not all extensions are included further in the next step. Then, Table 6.5 provides the results in terms of variable outcomes after maximum eight hours, while Figure 6.1 visualises the result development in terms of the objective gap. In addition to the solution values of staff deviation, total satisfaction score of all employees together and satisfaction score for the least satisfied employee, which are all in in the objective function at one point, the number of understaffed employees over the entire period is calculated. Notice that, since the objective only minimise the deviation between the optimal staff level and the actual staff level per shift, the result of staff deviation might be constant even if the understaffing varies. This is because the relationship between the optimal staff level and the minimum required staff level to cover all emergency calls is not constant.

	2 hours		**8 hours**		
Extensions	**Gap**	**Runtime (min)**	**Objective**	**Gap**	**Runtime (min)**
B	0.00%	11.85	0.01013	0.00%	11.85
B, E1	0.00%	40.80	0.01013	0.00%	40.80
B, E1, E2	-	120	-	-	480
B, E1, E3	-	120	-	-	480
B, E1, E4	3.00%	120	-0.06253	0.63%	480
B, E1, E4, E5a	46.1%	120	-0.04616	6.44%	480
B, E1, E4, E5b	3.09%	120	-0.06255	0.42%	480

Table 6.4: Overview of Test Results for Two and Eight Hours

Extensions	Staff Deviation	Understaffing	Total Satisfaction Score	Least Satisfied Employee
B	0	0	1.70262	-0.11448
B, E1	0	0	1.70262	-0.10477
B, E1, E2	-	-	-	-
B, E1, E3	-	-	-	-
B, E1, E4	107	73	0.10384	-0.07285
B, E1, E4, E5a	107	68	-0.16366	-0.01033
B, E1, E4, E5b	107	71	0.10244	-0.03873

Table 6.5: Overview of Test Scores for eight hours

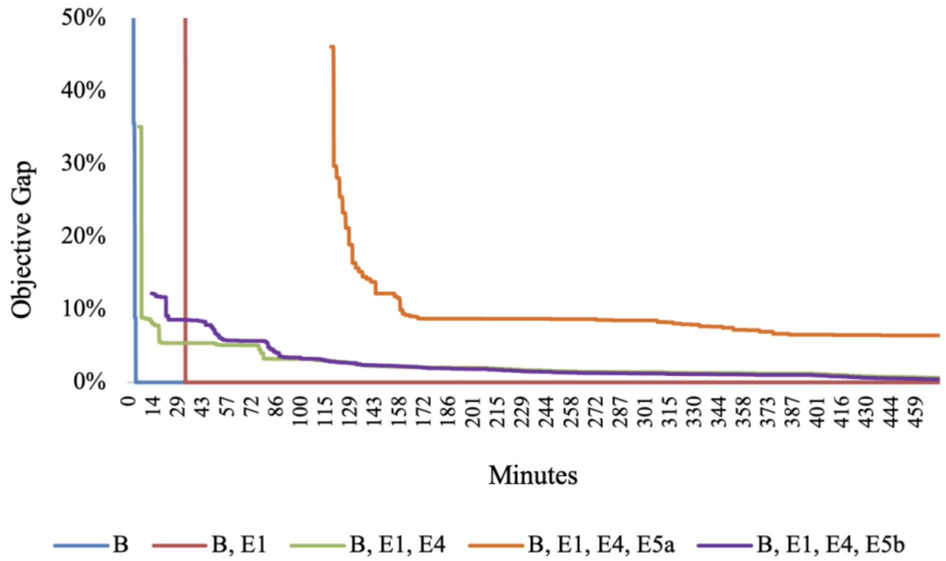

Figure 6.1 – Development of Objective Gap Per Extension

Base (B)

The base model retrieves an optimal solution within approximately 12 minutes, and successfully allocate enough employees to cover the optimal demand. It takes right under 5 minutes from the solver finds an objective gap of 35% to the optimal solution is found. The total satisfaction score for all employees together is 1.7, while the least satisfied employee has a satisfaction score of -0.114. Initially, a satisfaction score of -0.114 for the least satisfied employee might not seem bad as the satisfaction score at this point can range between 0.2 and -0.8. The satisfaction score of the most satisfied employee is 0.117. This confirms the fact that maximising the total satisfaction score alone occurs on the expense of some unfortunate employees.

Health-Promoting Shift Patterns (B, E1)

When implementing the practise of clockwise shift rotation and maximum limit of consecutive evening- and night shifts, the model has more restrictions when allocating employees to the different shifts and employees are forced to work a specific pattern. Before implementing the health promoting shift patterns employees were able to work longer periods with the shift type they preferred. Now employees are forced to change up the sequence of their shifts and the numbers on consecutive days with the same shift type is limited. After retrieving a gap of 93% after 40 minutes, the model quickly achieves an optimal solution one minute after this. The changes do not affect the objective value or the overall employee satisfaction but increased the runtime of the model by 244 %.

There is a small increase in the satisfaction score of the least satisfied employee. However, one should be careful to interpret the value of the least satisfied employee before implementing fairness restrictions. An improved satisfaction score for the least satisfied employee only reflects that individual employee and not necessarily the distribution of satisfaction scores among all employees. As the satisfaction scores can range significantly and involve outliers when the least satisfied employee is not included in the objective or the satisfaction scores does not have a lower bound, the value of the least satisfied employee does not provide much valuable insight. However, as mentioned in the literature by Matre and Lie (2019) clockwise shift patterns are proved to be health promoting. Hence, these constraints are included in the extended model.

16 Hours of Rest (B, E1, E2)

By implementing 16 hours of rest as a soft constraint, which is rewarded in the satisfaction score, the model is not able to find a feasible solution after eight hours. However, for the specific shifts used in this thesis, the constraint would most likely not have improved the solutions much. From the overview of resting hours for all shift combinations in Table 6.6, it appears that there are limited of shift combinations that fulfil 16 hours of rest. Considering that not all the shift combinations are used during the planning period, there are even fewer possible combinations than the ones highlighted in the table. As the model already ensures a minimum of 11 hours of rest, and the regulation is presented as "preferable if possible", the constraint is not considered of great importance. Hence, due to a significant increase in running time, it is not further included in the extended model. However, it will still be included in the full model.

Shift Day 2

Shift Day 1	d1	d2	d3	d4	e1	e2	e3	e4	n1	n2	n3
d1	16.5	16.5	16.5	17.0	23.25	24.25	24.5	25.5	32.0	32.25	32.25
d2	15.5	15.5	15.5	16.0	22.25	23.25	23.5	24.5	31.0	31.25	31.25
d3	15.25	15.25	15.25	15.75	22.0	23.0	23.25	24.25	30.75	31.0	31.0
d4	14.25	14.25	14.25	14.75	21.0	22.0	22.25	23.25	29.75	30.0	30.0
e1	7.75	7.75	7.75	8.25	14.5	15.5	15.75	16.75	23.25	23.5	23.5
e2	7.5	7.5	7.5	8.0	14.25	15.25	15.5	16.5	23.0	23.25	23.25
e3	7.5	7.5	7.5	8.0	14.25	15.25	15.5	16.5	23.0	23.25	23.25
e4	7.5	7.5	7.5	8.0	14.25	15.25	15.5	16.5	23.0	23.25	23.25
n1	0	0	0	0	6.0	7.0	7.25	8.25	14.75	15.0	15.0
n2	0	0	0	0	6.0	7.0	7.25	8.25	14.75	15.0	15.0
n3	0	0	0	0	5.5	6.5	6.75	7.75	14.25	14.5	14.5

Table 6.6: Hours of Rest Between Shift Combinations

End Before 18.00 (B, E1, E3)

The regulation striving to prevent employees to work longer than 18.00 the day before a day off, is handled by creating a soft constraint which is rewarded in the satisfaction score. This has similar results to the previous extension, indicating no feasible solution after eight hours. As this regulation is also presented as "preferable if possible", the constraint is not considered of great importance. Thus, as it significantly affects the runtime of the model, it is determined not to be included in the extended model. As for the last extension, this will also still be included in the full model.

Weekend Extensions (B, E1, E4)

As the weekend extensions are added, including the current weekend practice of the police operations centre, the solver finds a reasonable objective gap of 5.40% after just 30 minutes. After this, it slowly moves towards 0.63%, which is the best objective gap after eight hours. Something worth noting is that the total satisfaction score decreases by almost 1.6 points compared to the extended model that only include the health promoting work patterns (B, E1). With no isolated workdays or days-off and less contract hour deviation, this must be because the employees are assigned to work clockwise rotations of weekends, indicating shift types they do not prefer. Moreover, the practice of only working every third weekend generate a total understaffing of 73 employees during the period of six weeks. This is a weekly understaffing average of 12 employees every week that only occur during the weekend.

To decrease the extent of understaffing, the police operations centre could consider increasing the number of weekends, allowing some employees to work more weekends (e.g. young employees) or increase the staff level. The latter would probably lead to redundant staff level during the weekdays and will affect the costs. However, as this is the current police practice, the constraints are considered of great importance and is included further in the extended model. To check whether an increase in number of assigned weekends could decrease the understaffing, this is tested as a scenario later in this chapter.

Fairness (B, E1, E4, E5)

When implementing the aspect of fairness, the aim is to distribute the satisfaction factors evenly amongst the employees. This is achieved by integrating the satisfaction score of the least satisfied employee into the objective function (E5a). The model is now thriving to find the solution in which maximises both the total satisfaction score as well as the satisfaction score of the least satisfied employee. The model provides a gap of 46.1% within the predefined time limit of two hours, moves rather quickly to 10% before it slowly moves toward a gap of 6.44% after eight hours. As expected, the minimum satisfaction score is significantly higher, however, it comes at the expense of the total satisfaction score of the employees which has drastically decreased. This indicate that the satisfaction score is more evenly distributed and that the difference between the employees is now lower.

Integrating the minimum satisfaction score in the objective function increases the complexity and running time of the model. Hence, an alternative and less complex way to implement the fairness aspect, with a lower bound on the satisfaction score, is investigated. This alternative is introduced as extension

(E5b). A lower bound for the individual satisfaction scores would ensure a certain satisfaction level for all employees and prevent great differences among the co-workers. The lower bound can be further discussed, however, in this analysis it is desirable to have satisfaction scores on the "better half" of the scale. Hence, the lower bound of the satisfaction score $\underline{sat}$ is set to 70% of the highest possible satisfaction score, which reflects a score of -0.04. This resulted in a 6.01% gap within the predefined time limit of two hours, and a 0.42% gap after eight hours. Although it is a little slower in the beginning, the development of the objective gap is relatively similar to the extended version including the weekend (B, E1, E4).

Compared to the first fairness alternative, the alternative fairness extension provided "better" results faster in terms of its own objectives. The overall satisfaction is improved at the cost of a small decrease in the satisfaction score of the least satisfied employee. However, the minimum satisfaction score is better than the solution retrieved before implementing the fairness aspect. As the first fairness alternative does not pass the acceptance criteria after two hours and the second alternative provide sufficient results a lower bound on the individual satisfaction score (E5b) is included in the extended model. However, the minimum satisfaction score in the objective function (E5a) is included in the full model.

Extended Model (B, E1, E4, E5b)

To conclude, the extended model includes the base model, health promoting work patterns, the police´s current weekend practice and the fairness aspect using a lower bound of -0.04 in the satisfaction score for all employees.

6.3.2 Model Complexity

To provide even more insight about the complexity and quality of the rostering problem, a full model, including all the extensions, is executed. This also includes the extensions that were excluded from the extended model: E2, E3, E5a.

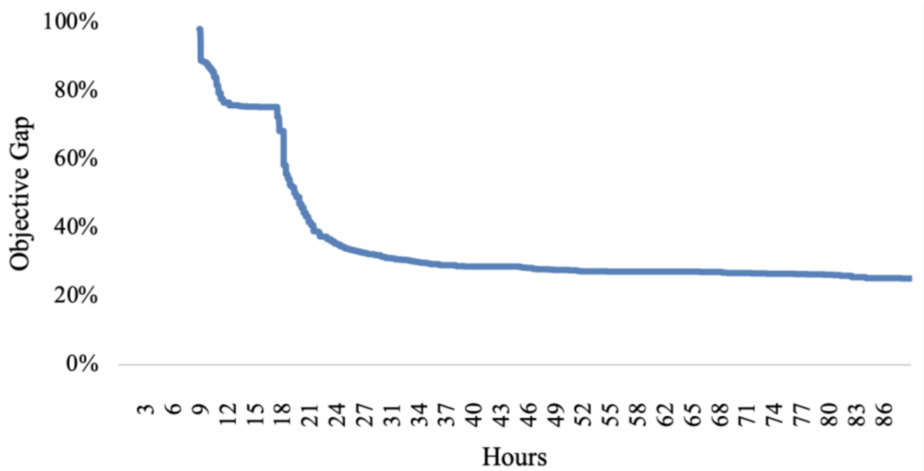

Figure 6.2 – Development of Objective Gap for the Full Model

Figure 6.2 show that the full model achieves a gap of 25.2% after four days. It first achieves an objective gap of 95% after 9 hours, then it uses 15 more hours until it is 35%, before it stabilizes and moves slowly towards 25.2%. In other words, after reaching the objective gap of 35% the model uses 81 hours to improve the results by 9.8 percentage points. Due to the scale and time limitation of this thesis, the execution of the full model was stopped after four days. However, it is not considered favourable to have a rostering model that uses several days to obtain acceptable results. Also, the planners would have to decide what is considered an acceptable result. In the end, these results emphasize the importance of investigating further improvements of the MILP formulation, or consider heuristics approaches that could be applied to obtain improved solutions more efficiently.

Number of	Base Model	Extended Model	Full Model
Utilized processors	8	8	8
Constraints	747 852	765 912	775 740
Variables	240 702	240 702	246 751

Table 6.7: Overview of Model Characteristics

As all the models are of a certain size, they all utilize eight out of eight available processors. However, the model size increase gradually as more variables and constraints are added to the problem. The base model has a total of 747 852 constraints and 240 702 variables, whereas 3 696 out of these are continuous, 123 244 are integers and 113 762 are binary variables. The final extended model shows a 2.41% increase in the number of constraints, with a total of 765 912. As the extended model do not include any new variables, the number of variables is the same as in the base model. Finally, the full model has a total of 775 740 constraints and 246 751 variables, where 3 696 out of these are continuous, 128 311 are integers and 118 440 are binary variables. This is a 3.73% increase in constraints and 2.51% increase in number of variables compared to the base model. As the base model already is considered a relative complex model, these small increases in number of constraints and variables increase the running time significantly.

6.4 Model Scenarios

This chapter evaluates the performance of the base model and the extended model by testing their sensitivity to parameter changes. The performance analyses are based on different scenarios that could occur when creating schedules at the operation centre and are described in Table 6.8.

Scenario	Description	Parameter Change
S1	Increase the number of weeks in planning period	W = 9
S2	Increase the number of weeks in planning period	W = 12
S3	Increase optimal staff level	+14%
S4	Remove Qualifications	$\underline{rq} = 0$
S5	Less available employees	- 10%
S6	Increase number of weekends (extended model only)	$\overline{we} = 2$

Table 6.8: Overview of Scenarios

The different operations centres operate with planning periods of either six, nine or twelve weeks, depending on their needs and preferences. Until now, the models have used a planning period of six weeks. Hence, planning periods of nine and twelve weeks are tested in scenario one and two respectively. Increasing the planning horizon can result in better predictability for both employees and planners, however, the schedule becomes more sensitive to short notice changes. As employees may not be aware of all their plans in time to request time off months in advance, too long planning periods can lead to a greater extent of overtime if short notice changes are made.

Further, the operation centre experience more demand during the summer months compared to the winter months. Therefore, the models will be tested and evaluated on their ability to handle a higher demand during the summer months. Based on the winter- and summer demand used in this thesis, that implies a total increase of 14.69% in optimal staff level and 11.25% increase in minimum staff level to cover all 112 emergency calls. Moreover, the models reflect the current situation at the operation centre where there must be a fixed minimum number of employees with specific qualifications present during all shifts. This limitation is removed to evaluate how the increased flexibility of the models affect the model performances.

As the models assume that all employees are always available, the effect of more unavailable employees should be evaluated. The operations centre are experiencing frequent absenteeism due to sick leaves and mandatory training. Therefore, the models will be tested on its sensitivity to a 10 % decrease in available employees. Again, the random choice function from Python´s Random module is used to randomly generate which employees are unavailable on what day, by assigning 353 $(\text{days} * \text{employees} * 0.1)$ unavailable days to random indices in the availability matrix $a_{e,d}$. Finally, to test the hypothesis about the fact that working more than every third weekend could decrease the understaffing level, the extended model is tested on the practice of working every second weekend. This is not tested on the base model, as this model only reflect the legally required regulations that *must* hold, indicating working no more than every second Sunday.

6.4.1 Results

This chapter presents the results from executing the different scenarios introduced on the base model and the extended model, followed by an interpretation of the most preeminent remarks. The time limit is set to three hours. Table 6.9 and 6.10 provides an overview of the model results within a time limit of three hours. The results of the original base- and extended model are presented in the first row, followed by the results of each scenario. Additionally, Figures 6.3 and 6.4 visualise the development of the objective gap for each of the scenarios executed on the base- and extended model respectively.

	Base Model				Extended Model		
Scenario	Objective	Gap	Runtime (min)	Change Runtime	Objective	Gap	Runtime (min)
	0.01013	0%	11.85	-	-0.0625	1.94%	180
S1	0.01176	0%	75.48	537.13%	-	-	180
S2	0.01031	0%	153.08	1192.15%	-	-	180
S3	0.02227	0%	13.68	15.47%	-0.0577	2.51%	180
S4	0.01013	0%	9.37	-20.87%	-0.0562	2.52%	180
S5	0.01013	0%	4.40	-53.78%	-0.0626	2.73%	180
S6					-0.0184	5.01%	180

Table 6.9: Overview of Model Results With 3 Hour Time Limitation

	Base Model				Extended Model			
Scenario	Staff Dev.	Under-staffed	Sat. Score	Min. Sat.	Staff Dev.	Under-staff	Sat. Score	Min. Sat.
	0	0	1.7026	- 0.101	107	73	0.1038	-0.0394
S1	0	0	1.9765	-0.089	-	-	-	-
S2	0	0	1.7314	-0.061	-	-	-	-
S3	0	0	3.7414	-0.067	142	114	1.6582	-0.0373
S4	0	0	1.7026	-0.093	107	67	-0.0168	-0.0392
S5	0	0	1.7026	-0.105	107	70	0.1022	-0.0397
S6					41	24	0.7762	-0.0375

Table 6.10: Overview of Satisfaction Results With 3 Hour Time Limitation

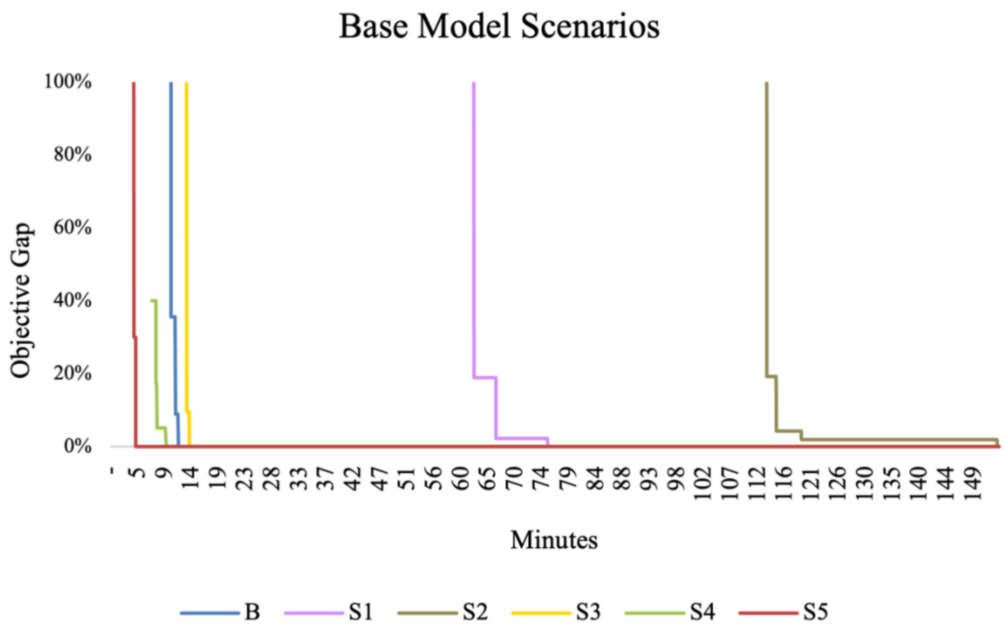

Figure 6.3 – Development of Objective Gap in Base Model

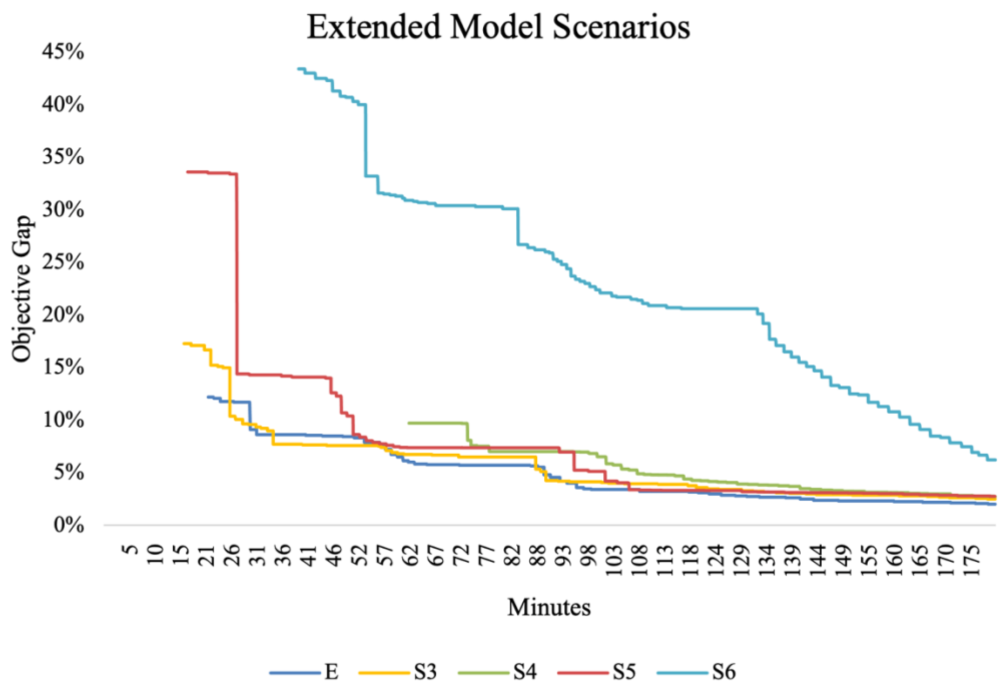

Figure 6.4 – Development of Objective Gap in Extended Model

Nine Weeks Planning Horizon (S1)

By increasing the number of weeks in the planning horizon from six to nine, the base model uses 75 minutes to find an optimal solution, which is more than five times more than the original base model. However, when the base model first finds a gap, it quickly moves to the optimal solution. This applies for all the scenarios tested on the base model. Further, there is an increase in total satisfaction score, which is probably due to coincidences in the demand for the additional three weeks. One Sunday shift with a demand out of the ordinary can alone affect the allocation pattern greatly, hence also the employees' satisfaction score. As mentioned in Chapter 6.3.1, the value of the least satisfied employee does not provide much valuable insight in the base model as the fairness aspect is not implemented. However, the base model manages to cover the optimal staff level during the whole period. As for the extended model, it is not able to find a feasible solution within the time limit.

The reason some operations centres operate with nine weeks is to plan for a cycle of three clockwise weekends and to be able to reuse the weekend shifts in the following periods. It could be beneficial to use a planning horizon of six weeks if the practise of removing clockwise weekends was implemented, as employees could potentially be more satisfied working more of their desired shift types during weekends. Alternatively, information about employees´ last assigned weekend from the previous period could work as an input in the model, so that the model could still schedule clockwise weekends while only planning for six weeks. The use of nine weeks can be predictable; however, might find it hard to plan nine weeks ahead (more if deadline to request time off is considered), and the planners experience a lot of requests for changes after the schedule is implemented. Hence, the use of six weeks should be considered.

Twelve Weeks Planning Horizon (S2)

When planning for twelve weeks the base model used significantly more time to find an optimal solution. The objective and total satisfaction score are higher than the original base model with a planning period of six weeks, but lower than the base model with a nine-week planning period. Again, this is most likely due to randomness in the demand of the additional weeks. The extended model is not able to retrieve a feasible solution within the time limit.

Increased Demand (S3)

When testing the base model on the forecasted summer demand it takes two minutes longer to find an optimal solution compared to the winter demand.

However, the base model manages to allocate enough employees to satisfy the optimal demand for each shift. The results show that the total satisfaction is more than doubled compared to the original base model. This is a result of lower deviation between contractual hours and worked hours for employees, in addition to more people are granted their preferred shifts, as employees are assigned to more shifts in general.

On the other hand, the extended model achieves an objective gap of 2.51% after three hours, which is only 0.57 percentage points lower than what the original extended model with the winter demand reaches after three hours. It uses 17 minutes to find a feasible solution with a 16% objective gap, before it relatively quickly improves to 7%, followed by a slow transition towards 2.51%. Because the original extended model limit employees to work no more than every third weekend, there is a staff deviation of 107 employees and an understaffing level of 73. As this is then combined with a higher demand, the staff deviation and understaffing level increases to 142 employees 114 respectively, which all take place during the weekend. This is a weekly average of 24 and 19 employees. Considering this in context of the number of emergency calls that the 19 employees is supposed to cover, that now must wait in line, the police should consider appropriate measures. In other words, the extended model and its weekend practice is very much sensitive to an increase in demand in terms of understaffing.

Removing Qualifications (S4)

By removing the qualification aspect of the problem, the base model retrieves an optimal solution 20.87 % quicker than the initial model. The total satisfaction stays the same as in the original base model, indicating that the qualifications do not limit the model solution. As for the extended model it takes a little more time to find an objective gap with a feasible solution. First after 65 minutes the extended model can find an objective gap of 9.87%, before it gradually moves towards 2.52% after three hours. The staff deviation is the same as in the original extended model, however, the total satisfaction score is lower. This is as expected as the employees with higher qualifications generally achieve higher satisfaction scores compared to the operators in the original extended model, because they work more hours to always cover the minimum required number of each qualification. Additionally, there are generally few occurrences of isolated workdays and days off. So, to cover the minimum presence of the different qualifications, these employees are “restricted” from having lower satisfaction scores. As the qualifications are removed, the satisfaction scores are distributed more equally, avoiding that some employees achieve higher scores at the expense of others.

Decrease in Employee´s Availability (S5)

When decreasing the number of available employees by 10%, both the objective and the total satisfaction score of the base model remains the same. However, the runtime decreases by over 50%. The decrease of available employees lowers the symmetry of the model, generating less symmetrically equivalent solutions to choose from. The unavailability has a positive effect on the model´s running time but would normally affect the planning process negatively when conducted manually. However, an improved running time is not the case for the extended model. This might be due to the extended model´s restricted work patterns in combination with unavailability, which makes it harder to allocate employees. Also, for the extended model, both the objective, satisfaction score and minimum satisfaction do not change much from the original extended model.

Increase in Number of Weekends (S6)

In the case where the weekend practice in the extended model reduces the number of weekends, from working no more than every third weekend to no more than every second weekend, the model uses a little longer time to achieve an acceptable gap. The model first achieves a gap of 43% after 39 minutes, before it evenly moves towards an objective gap of 5.01% after three hours. This is slightly higher than what the original extended model achieves after three hours. More importantly, the level of understaffing decreases to 24 employees over the entire period. The understaffing still takes place in the weekend, but now with a lower weekly average of four employees per week. This is significantly lower than the weekly average of 12 employees in the original extended model, that occur when working no more than every third weekend. Further, the increase in total satisfaction score, from 0.1038 to 0.7762, is a result of employees working more weekends of their preferred shift type. As increasing the number of assigned weekends also indicate that the cycle of clockwise weekend rotation is completed earlier, the employees are likely to make it through more cycles than when working no more than three weekends. Which also implies working more of their preferred shift type.

7. Conclusion

This thesis has explored how a roster can be developed for an operations centre at the police. It has investigated the effects of implementing personnel preferences and health promoting regulations on the quality and complexity of the model. The model size and complexity increase gradually as more variables and constraints are added to the rostering problem. However, the various constraints affect the complexity of the model differently. A model that only accounts for the regulations that are regulated by the law is much more flexible and adaptable to different scenarios than the extended and full model that also consider health-promoting patterns, soft regulations, and preferences of employees.

Findings show that when creating a roster for the operations centres, there is a trade-off between runtime and quality of the model. To create a model which takes all aspects of work regulations, health-promoting work patterns, and preferences into account, it comes at the expense of a satisfactory runtime. An extended model including all the legally required regulations, together with additional preferences and the preferred regulations of importance, can provide acceptable solutions within a reasonable time limit. However, for the purpose of generating high quality solutions that considers all kinds of preferences and regulations, also the less important ones, heuristics approaches could be applied to obtain improved solutions more efficiently.

In terms of running time, the models are all sensitive to the number of weeks in the planning period. However, the extended model is more sensitive than the base model. The most prominent bottleneck of the problem´s staff level, is the allocation of employees during the weekend, as high demand combined with weekend restrictions leads to understaffing. Moreover, the regulations and current weekend practice define how weekends should be allocated, without considering employees´ shift preferences. The sensitivity that the extended model show associated with changes in the maximum number of assigned weekends is so profound that the police should investigate necessary measures. To mention some, they could increase the staff level, allow some employees to work more weekends (e.g. young employees) or provide greater incentives to work weekends. By hiring part-time workers, they could assist in covering the demand on weekends.

7.1 Research Limitations and Future Research

The limitations of this study offer opportunities for investigations in future studies. For instance, studying and collecting data from more operations centres would make the model more adaptable to the various of practises at the different operations centres. Some aspects of the thesis might be influenced by the situation at the specific operations centre studied and might not apply for other operations centres.

Further, the collecting of relevant data was constricted by the time limitations of the thesis. As the shift preferences was created by randomly distributing the preferences according to an approximate distribution given by the police, it would be beneficial to obtain detailed data on every individual employee´s preferences, priorities, and work arrangement. This would create a more realistic and tailored model to the specific operations centres.

An interest of investigation could be, if not restricted by the complexity of the problem, to explore the practise of combining the process of creating shifts together with the rostering process as described in the literature. Such approach is more adaptable to regulations and limitations, and often provide better solutions. A next step could also be to integrate the regulations concerning public holidays directly into the model. That way the allocation of workdays during the holidays would be optimised relatively to the rest of the planning period. Moreover, the model could be modified to consider short-term schedule adjustments during the operational phase, to find the most optimal replacements to cover short-notice absence.

The full model was not capable to provide optimal solutions within a satisfactory time limit, when including all aspects of regulations and preferences. Based on the runtime of the model it would be beneficial to further investigate how the MILP could be improved or heuristics approaches could be applied to obtain sufficient solutions more efficiently. We did not find space and time to include this in this thesis, but it appears to be an aspect of relevance and importance when developing serviceable rosters.

References

Alfares, H.K. (2007) Operator staffing and scheduling for an IT-help call centre. *European Journal of Industrial Engineering.* 1(4) 414–430.

Al-Yakoob, S. M & Sherali, H.D. (2007). Multiple shift scheduling of hierarchical workforce with multiple work centers. *Informatica* 18(3) 325–342.

Bard, J. F. and H. W. Purnomo (2005). "Preference scheduling for nurses using column generation". In: *European Journal of Operational Research* 164.2, pp. 510–534.https://doi.org/10.1016/j.ejor.2003.06.046

Bellanti, F., Carello, G., Della, Croce F. & Tadei, R. (2004) A greedy-based neighborhood search approach to a nurse rostering problem, *European Journal of Operational Research* 153(1) 28–40 https://doi.org/10.1016/S0377-2217(03)00096-1.

Brannstatistikk (2023) *Hvor ofte var de ulike nødetatene først på hendelsesstedet?* https://www.brannstatistikk.no/brusui/search?searchId=A49E270C-9F6C-4B29-9F76BF78D64BC6D3&type=SEARCH_DEFINITION

Burke, E. K., De Causmaecker, P., Vanden, B. G. and Landeghem, H. V. (2004). *The State Of The Art Nurse Rostering*. Tech. rep., pp. 441–499. https://doi.org/10.1023/B:JOSH.0000046076.75950.0b

Burke, E.K., Li, J.P., Qu, R. (2010) A hybrid model of integer programming and variable neighbourhood search for highly-constrained nurse rostering problems, *European Journal of Operational Research* 203 (1) 484–493. https://doi.org/10.1016/j.ejor.2009.07.036

Burke, E. K. and T. Curtois (2014). "New approaches to nurse rostering benchmark in- stances". In: *European Journal of Operational Research* 237.1, pp. 71–81. https://doi.org/10.1016/j.ejor.2014.01.039

Campbell, G.M. (2011) A two-stage stochastic program for scheduling and allocating cross-trained workers, *Journal of the Operational Research Society* 62 (6) 1038–1047

Defraeye, M. & Van Nieuwenhuyse, I. (2016) Staffing and scheduling under nonstationary demand for service: A literature review. *Omega.* 58(1) 4-25 https://doi.org/10.1016/j.omega.2015.04.002

Edie L. (1954) Traffic delays at toll booths, *Journal Operations Research Society of America* 2(2), 107–138. https://doi.org/10.1287/opre.2.2.107

Ernst, A. T., Jiang, H., Krishnamoorthy, M. and Sier, D. (2004). Staff scheduling and rostering: A review of applications, methods and models. *European Journal of Operational Research* 153(1), 3–27. https://doi.org/10.1016/S0377-2217(03)00095-X

Garde, A. H., Begtrup, L., Bjorvatn, B., Bonde, J.P., Hansen J., Hansen, Å.M., Härmä, M., Jensen, M.A., Kecklund, G., Kolstad, H.A., Larsen, A.D., Lie, J.A., Moreno, C.R.C., Nabe-Nielsen, K. & Sallinen, M. (2020) How to schedule night shift work in order to reduce health and safety risks. *Scandinavian Journal of Work, Environment & Health* 46(6) 557-569 https://doi.org/10.5271/sjweh.3920

Goodman, M. D., Dowsland, K. A. & Thompson, J.M. (2009) A grasp knapsack hybrid for a nurse-scheduling problem. *Journal of Heuristics* 15(4) 351–379. https://doi.org/10.1007/s10732-007-9066-7

Gurvich, I., Luedtke, J., & Tezcan, T. (2010). Staffing Call Centers with Uncertain Demand Forecasts: A Chance-Constrained Optimization Approach. *Management Science*, 56(7), 1093–1115. http://www.jstor.org/stable/40785245

Hadwan, M. & Ayob, M.B (2009) An exploration study of nurse rostering practice at hospital Universiti Kebangsaan Malaysia. *Institute of Electrical and Electronics Engineers.* 107–114. https://doi.org/10.1109/DMO.2009.5341903

Heikkinen, S., Demers, P. A, Hansen, J., Jakobsen, J., Kjaerheim, K., Lynge, E., Martinsen, J. I., Mehlum, I. S., Pitkäniemi, J., Selander, J., Torfadóttir, J., Weiderpass, E. & Pukkala E. (2022) Incidence of cancer among Nordic police officers. *International Journal of Cancer.* 152(6), 1124-1136. https://doi.org/10.1002/ijc.34311

Khattak, P., Shah, M. W. & Shah, M. H (2020) Impact of Knowledge Sharing and Teamwork on Team Performance with the Moderating Role of Supervisor Support. *British Journal of Research.* 7(2) 1-8 https://doi.org/10.36648/2394-3718.7.2.57

Maenhout, B.,Vanhoucke, M. (2006) New computational results for the nurse scheduling problem: a scatter search algorithm*, Evolutionary Computation in Combinatorial Optimization* 3906 (1), 2006, 159–170. https://doi.org/10.1007/11730095_14

Matre, D. & Lie J. A. S. (2019). Råd om arbeidstid med hensyn til helse og sikkerhet. *STAMI* https://stami.no/rad-om-arbeidstid-med-hensyn-til-helse-og-sikkerhet/

Musliu, N., A. Schaerf, and W. Slany (2004). “Local search for shift design”. In: *European Journal of Operational Research*. Vol. 153. 1, pp. 51–64. https://doi.org/10.1016/S0377-2217(03)00098-5

Örmeci, E.L, Salman, F.S. & Yücel, E. (2014) Staff rostering in call centers providing employee transportation. *Omega*, 43(1), 41-53 https://doi.org/10.1016/j.omega.2013.06.003

Onsøien, O. G. (2023). Har advart ledelsen i årevis: -Nå har de gjort endringer uten god dialog, og da sier de ansatte opp. *Dagens Medisin,* https://www.dagensmedisin.no/har-advart-ledelsen-i-arevis-na-har-de-gjort-endringer-uten-god-dialog-og-da-sier-de-ansatte-opp/534435

Politidirektoratet. (2021) *Kapasitetsundersøkelse av politioperativt område.*https://www.politiet.no/globalassets/dokumenter/pod/rapporter/rapport kapasitetsundersokelsen-politioperativt-omrade.pdf?_t_id=3RP4f9mMaGK3jFGbKaJdCw%3d%3d&_t_uuid=aLlcnFHiQeqfzgepi22IXg&_t_q=kapasitetsundersøkelse+politiopertaivt&_t_ta gs=language:no%2csiteid:b78013b9-3a34-4dad-8a4f-cc70168f7f39&_t_hit.id=Politiet_Web_Models_Media_GenericMedia/86e6c330-e417-46f1-bfae-c2a4dcddf957&_t_hit.pos=1

Politidirektoratet. (2022) *Ressursanalyse for 2021 Utgifter og bemanning i politiet* https://www.politiet.no/globalassets/04-aktuelt-tall-og

fakta/bemanning-ressurser-og dekningsgrad/ressursanalyser/ressursanalyse-2021.pdf
Report. St. 29 (2019-2020). The police report - *a police force for the future*. Justis- og beredskapsdepartement https://www.regjeringen.no/no/dokumenter/meld.-st.-29-20192020/id2715224/?ch=1
Rönnberg, E. and Larsson, T. (2010). Automating the self-scheduling process of nurses in Swedish healthcare: A pilot study. *Health Care Management Science* 13(1) 35– 53. https://doi.org/10.1007/s10729-009-9107-x
Schmutz, J. B., Meier, L. L. & Manser, T. (2019) How effective is teamwork really? The relationship between teamwork and performance in healthcare teams: a systematic review and meta analysis. *BMJ Open* 9(9) 1-16 https://doi.org/10.1136/bmjopen-2018 028280
STAMI (2018) *Skiftarbeid og helseplager.* Statens arbeidsmiljøinstitutt https://stami.no/prosjekt/skiftarbeid-og-helseplager/
Van den Berg, J., Beliën, J., De Bruecke, P., Demeulemeester, E. & De Boeck,L.(2013) Personnel scheduling: A literature review. European Journal of Operational Research 226(3) 367-385. https://doi.org/10.1016/j.ejor.2012.11.029
Wallace, R.B. & Whitt, W. (2005). A staffing algorithm for call centers with skill-based routing. *Manufacturing & Service Operations Management* 7(4) 276–294. https://doi.org/10.1287/msom.1050.0086
Work Environment Act. (2006). Lov om arbeidsmiljø, arbeidstid og stillingsvern mv. (LOV-2022-12-20-99). Lovdata. https://lovdata.no/dokument/NLE/lov/2005-06-17-62#KAPITTEL_1
Wright, P. D., Bretthauer, K. M. and Côte M. J. (2006). Reexamining the nurse scheduling problem: Staffing ratios and nursing shortages" *Decision Sciences* 37(1) 39–70. https://doi.org/10.1111/j.1540-5414.2006.00109.x

Appendices

Appendix A – List of Regulations

Rest

- The employee must have at least 11 consecutive hours off work during 24 hours. The work-free period must be placed between two main work periods.
  (ATB § 5-1(1) Daily and weekly rest)
- The employee must have a continuous non-work period (weekday off) of at least 36 hours during a week, and so that it always includes a full calendar day. This free time should preferably be taken on Sunday and at least every other Sunday.
  (HTA § 14(1) Weekly Free Time)
- In addition to the weekly day off, to the greatest extent possible, an additional day off shall be granted which shall extend over an entire calendar day. As far as possible, this day off must be placed in connection with the weekly day off.
  (HTA § 14(2) Weekly Free Time)
- Between each ordinary work period, efforts should be made to ensure that as often as possible there is a continuous work-free period of at least 16 hours.
  (ATB § 3-1(8) Preparation of Work Plan)
- As far as the service allows, the free time must be arranged so that it begins at 18:00 the day before UF or F. The last part of the week off period can be completed in the following week.
  (ATB § 3-1(11) Preparation of Work Plan)

Working Hours

- The regular/normal working time is extended to 10 hours within 24 hours. The same is valid if the employee works more than 3 hours in the time period between 21:00 to 06:00
  (ATB § 2-1(2) Regular Working Time)

- For work that is carried out mainly at night, for shift and rotation work that is carried out regularly on Sundays and weekends, and for working time arrangements which result in the individual employee having to work at least every third Sunday, the total effective working time must not exceed 35.5 clock hours per week on average. (HTA § 7(3) Working Hours)
- For normal working hours all days between 20.00 and at 06.00 each worked hour is counted as 1 hour and 15 minutes.
  (HTA § 7(3) Working Hours)
- For the normal working hours on Sundays and weekends between 06.00 and at 20.00, each hour worked is counted as 1 hour and 10 minutes. (HTA § 7(3) Working Hours)
- Normal working hours must not exceed 48 hours per seven days. (AML §10-4. Normal Working Hours)
- The normal working hours shall not exceed 37.5 hours on average per week, incl. Converted/recalculated time.
  (ATB § 2-2(1). Calculation of Average Working Time)
- A work schedule can be set up with a maximum of 9 working days in a row.
  (ATB § 3-1(7) Preparation of Work Plan)

The Police´s Risk Assessment of Schedules

- Schedule clockwise shifts patterns (day, evening night)
- Amount of evening- and night shifts (recommended maximum 3 consecutive evening- and night shifts)
- Sufficient amount of rest between night shift and a new shift type (recommended minimum 24 hours)

Appendix B - List of Figures

Appendix C - List of Tables

Appendix D – Shift Overview

Day	Shift	Start Time	End Time	Effective Hours	Accounted Hours
Monday	d3	06:00	14:45	8.75	8.75
	e3	14:00	22:30	8.50	9.13
	n3	21:45	07:15	9.50	11.31
Tuesday	d4	06:30	15:45	9.25	9.25
	e4	15:00	22:30	7.50	8.13
	n3	21:45	07:15	9.50	11.31
Wednesday	d4	06:30	15:45	9.25	9.25
	e4	15:00	22:30	7.50	8.13
	n3	21:45	07:15	9.50	11.31
Thursday	d4	06:30	15:45	9.25	9.25
	e4	15:00	22:30	7.50	8.13
	n2	21:45	06:45	9.00	10.81
Friday	d2	06:00	14:30	8.50	8.50
	e2	13:45	22:30	8.75	9.38
	n3	21:45	07:15	9.50	11.31
Saturday	d4	06:30	15:45	9.25	10.79
	e4	15:00	22:30	7.50	8.96
	n2	21:45	06:45	9.00	10.81
Sunday	d1	06:00	13:30	7.50	8.71
	e1	12:45	22:15	9.50	11.27
	n1	21:30	06:45	9.25	11.13

Appendix E – Mathematical Model

Sets

E Set of employees

Q Set of qualifications

W	Set of weeks in planning period
D	Set of days in planning period
D^{Mon}	Set of Mondays
D_w^{Mon}	Set of Mondays in week w
D^{Sat}	Set of Saturdays
D^{Sun}	Set of Sundays
D^{we}	Set of weekend days in planning period
D^{wd}	Set of weekdays in planning period
S_d	Set of shifts with positive demand on day d
ST	Set of shift types (day, evening, night)
S_d^{st}	Set of shifts of shift type st with positive demand on day d

General Parameters

eh_s	Number of effective working hours per shift s
ah_s^{wd}	Number of accounted working hours per shift s
ah_s^{we}	Number of accounted working hours per weekend shift s
em_s	Number of hours from end of shift s to midnight
ms_s	Number of hours from midnight to start of shift s
ch_e	Weekly contract hours for employee e
$p_{s,e}$	Preference of shift s for employee e
$eq_{e,q}$	$\begin{cases} 1, \text{ if employee e has qualification q} \\ 0, \text{ otherwise} \end{cases}$
$a_{e,d}$	$\begin{cases} 1, \text{ if employee e is available at day d} \\ 0, \text{ otherwise} \end{cases}$
rh_{s^1,s^2}	$\begin{cases} 1, \text{ if it is at least 11 rest hours from shift } s^1 \text{ to shift } s^2 \text{ the next day} \\ 0, \text{ otherwise} \end{cases}$

Limiting Parameters

$\underline{critical}$	Critical staff level
$\overline{opt_{s,d}}$	Optimal number of employees at shift s at day d
$\underline{rq_q}$	Minimum number of employees with qualification q required at all shifts

$\overline{ah}$	Maximum accounted hours per week
$\overline{ah^{avg}}$	Maximum weekly accounted working hours on average
$\overline{eh^{avg}}$	Maximum weekly effective working hours on average
$\overline{cd}$	Maximum consecutive working days
$\overline{we}$	Employees can maximum work every $\overline{we}$ weekend
$\underline{cr}$	Minimum hours of consecutive rest in relation to the weekly day off

Scaling Parameters

min^P	Minimum days an employee can be assigned to its preferred shift type
max^P	Maximum days an employee can be assigned to its preferred shift type
min^{IX}	Minimum isolated workdays an employee can be assigned to
max^{IX}	Maximum isolated workdays an employee can be assigned to
min^{IO}	Minimum isolated days off an employee can be assigned to
max^{IO}	Maximum isolated days off an employee can be assigned to
min^{CD}	Minimum deviation between contracted and assigned hours an employee can have
max^{CD}	Maximum deviation between contracted and assigned hours an employee can have
min^S	Minimum sum of all employees´ satisfaction scores
max^S	Maximum sum of all employees´ satisfaction scores
min^{SD}	Minimum total deviation between actual and optimal staff levels
max^{SD}	Maximum total deviation between actual and optimal staff levels

n^P	Scaling factor for employees' shift preferences
n^{IX}	Scaling factor for working isolated days
n^{IO}	Scaling factor for having isolated day off
n^{IW}	Scaling factor for working partial weekends (only Saturday or Sunday)
n^{CD}	Scaling factor for deviation between contracted and assigned hours
n^S	Scaling factor for sum of all employees´ satisfaction scores
n^{SD}	Scaling factor for deviation between actual and optimal staff levels

Weight Parameters

j_e^P	Weight factor for shift preferences per employee e

j_e^{IX}	Weight factor for working isolated days per employee e
j_e^{IO}	Weight factor for isolated days off per employee e
j_e^{IW}	Weight factor for working a partial weekend per employee e
j^{CD}	Weight factor for deviation between contracted hours and assigned hours
j^{SD}	Weight factor for deviation between optimal and actual staff levels
j^{S}	Weight factor for employee satisfaction

Variables

$X_{e,s,d}$ $\begin{cases} 1, \text{if employee e is assigned to shift s on day d} \\ 0, \text{otherwise} \end{cases}$

$O_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned a day off on day d} \\ 0, \text{otherwise} \end{cases}$

$R_{e,d}$ $\begin{cases} 1, \text{if employee e has } \underline{cr} \text{ hours of continuous rest on day d} \\ 0, \text{otherwise} \end{cases}$

$OO_{e,d}$ $\begin{cases} 1, \text{if employee e has days off on both day d and d+1} \\ 0, \text{otherwise} \end{cases}$

$OS_{e,s,d}$ $\begin{cases} 1, \text{if employee e is assigned to shift s the day after a day off on day d} \\ 0, \text{otherwise} \end{cases}$

$NO_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned to a night shift the day before a day off d} \\ 0, \text{otherwise} \end{cases}$

$IX_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned to an isolated workday on day d} \\ 0, \text{otherwise} \end{cases}$

$IO_{e,d}$ $\begin{cases} 1, \text{if employee e is assigned to an isolated day off on day d} \\ 0, \text{otherwise} \end{cases}$

$IW_{e,d}^{Sat}$ $\begin{cases} 1, \text{if employee e is assigned to a Saturday d and not Sunday} \\ 0, \text{otherwise} \end{cases}$

$IW_{e,d}^{Sun}$ $\begin{cases} 1, \text{if employee e is assigned to a Sunday d and not Saturday} \\ 0, \text{otherwise} \end{cases}$

$CR_{e,d}$ = Hours of continuous rest for employee e on day d

$SD_{s,d}$ = Deviation between actual and optimal staff on shift s on day d

CD_e = Deviation between contract- and assigned hours for employee e

SAT_e = Satisfaction score of employee e

Objective Function

$$Maximise\ \ n^S * j^S \left(\sum_{e \in E} SAT_e - min^S \right) - n^{SD} * j^{SD} \left(\sum_{s \in S_d} \sum_{d \in D} SD_{s,d} - min^{SD} \right) \quad (5.1)$$

Constraints

Demand

$$\sum_{e \in E} a_{e,d} * X_{e,s,d} \geq \underline{critical} \qquad \forall s \in S_d\ , d \in D \quad (5.2)$$

$$\sum_{e \in E} a_{e,d} * X_{e,s,d} \leq \overline{opt_{s,d}} \qquad \forall s \in S_d\ , d \in D \quad (5.3)$$

$$X_{e,s,d} = 0 \qquad \forall e \in E, s \in S|\ S_d\ , d \in D, \quad (5.4)$$

$$\sum_{e \in E} eq_{e,q}\ * a_{e,d} * X_{e,s,d} \geq \underline{rq_q} \qquad \forall s \in S_d\ , d \in D\ , q \in Q \quad (5.5)$$

$$\overline{opt_{s,d}} - \sum_{e \in E} X_{e,s,d} = SD_{s,d} \qquad \forall s \in S_d\ , d \in D \quad (5.6)$$

Work

$$\sum_{s \in S_d} X_{e,s,d} \leq 1 \qquad \forall e \in E, d \in D \quad (5.7)$$

$$\sum_{s \in S_d} \sum_{d}^{d+\overline{cd}} X_{e,s,d} \leq \overline{cd} \qquad \forall e \in E,\ d \in d, .., D - \overline{cd} \quad (5.8)$$

$$\sum_{d\in D}\sum_{s\in S_d} eh_s * X_{e,s,d} \leq \overline{eh^{avg}} * W \qquad \forall e \in E \quad (5.9)$$

$$\sum_{d\in D^{wd}}\sum_{s\in S_d} ah_s^{wd} * X_{e,s,d} + \sum_{d\in D^{we}}\sum_{s\in S_d} ah_s^{we} * X_{e,s,d} \leq \overline{ah^{avg}} * W \quad \forall e \in E \quad (5.10)$$

$$\sum_{d=d_w}^{d_w+4}\sum_{s\in S_d} ah_s^{wd} * X_{e,s,d} + \sum_{d=d_w+5}^{d_w+6}\sum_{s\in S_d} ah_s^{we} * X_{e,s,d} \leq \overline{ah} \qquad (5.11)$$

$$\forall e \in E, w \in W, d_w \in D_w^{Mon}$$

Rest

$$X_{e,s^1,d} + X_{e,s^2,d+1} - 1 \leq rh_{s^1,s^2} \qquad \forall e \in E,\ s^1, s^2 \in S_d, d \in D-1 \quad (5.12)$$

$$\sum_{s\in \mathrm{S_d}} X_{e,s,d} + O_{e,d} = 1 \qquad \forall e \in E, d \in D \quad (5.13)$$

$$O_{e,d} + O_{e,d+1} - 1 \leq OO_{e,d} \qquad \forall e \in E, d\ \in 1, .., D-1 \quad (5.14.a)$$

$$O_{e,d} \geq OO_{e,d} \qquad (5.14.b)$$

$$O_{e,d+1} \geq OO_{e,d} \qquad (5.14.c)$$

$$O_{e,d} + X_{e,s,d+1} - 1 \leq OS_{e,s,d} \qquad \forall e \in E, s \in S_d\ , d \in D-1 \quad (5.15.a)$$

$$O_{e,d} \geq OS_{e,s,d} \qquad \forall e \in E, s \in S_d\ , d \in D-1 \quad (5.15.b)$$

$$X_{e,s,d+1} \geq OS_{e,s,d} \qquad \forall e \in E, s \in S_d\ , d \in D-1 \quad (5.15.c)$$

$$O_{e,d} + \sum_{s\in S_{d-1}^{Night}} X_{e,s,d-1} - 1 \leq NO_{e,d} \qquad \forall e \in E, d \in D-1 \quad (5.16.a)$$

$$O_{e,d} \geq NO_{e,d} \qquad \forall e \in E, d \in D-1 \quad (5.16.b)$$

$$\sum_{s \in S_{d-1}^{Night}} X_{e,s,d-1} \leq NO_{e,d} \qquad \forall e \in E, d \in D-1 \qquad (6.16.c)$$

$$CR_{e,d} = 24 * O_{e,d-1} + \sum_{s \in S_{d-1}} em_s * X_{e,s,d-1} \qquad (5.17.a)$$

$$+ 24 * O_{e,d} - 24 * NO_{e,d} + \sum_{s \in S_d} ms_s * X_{e,s,d} \qquad (5.17.b)$$

$$+ 24 * OO_{e,d} + \sum_{s \in S_d} ms_s * OS_{e,s,d} \qquad (5.17.c)$$

$$\forall e \in E, \forall d \in 2, .., D-1$$

$$CR_{e,1} = 24 * O_{e,1} + \sum_{s \in S_1} ms_s * X_{e,s,1} \qquad (5.17.d)$$

$$+ 24 * OO_{e,1} + \sum_{s \in S_1} ms_s * OS_{e,s,1} \qquad \forall e \in E \qquad (5.17.e)$$

$$CR_{e,D} = 24 * O_{e,D-1} + \sum_{s \in S_D} em_s * X_{e,s,D-1} \qquad (5.17.f)$$

$$+ 24 * O_{e,D} - 24 * NO_{e,D} + \sum_{s \in S_D} ms_s * X_{e,s,D} \qquad \forall e \in E \qquad (5.17.g)$$

$$\sum_{d=d'}^{d'+6} R_{e,d} \geq 1 \qquad \forall e \in E, d' \in D^{Mon} \qquad (5.18)$$

$$CR_{e,d} \geq \underline{cr} * R_{e,d} \qquad \forall e \in E, d \in D \qquad (5.19)$$

Weekend

$$\sum_{i=0}^{\overline{we}-1} \sum_{s \in S_d} X_{e,s,d+i*7} \leq 1 \qquad \forall e \in E, d \in D^{Sun} \qquad (5.20)$$

Satisfaction

$$\sum_{s\in S_d} X_{e,s,d} - \sum_{s\in S_{d-1}} X_{e,s,d-1} - \sum_{s\in S_{d+1}} X_{e,s,d+1} \leq IX_{e,d} \qquad (5.21)$$

$$\forall e \in E, d \in 2,..,D-1$$

$$O_{e,d} - O_{e,d-1} - O_{e,d+1} \leq IO_{e,d} \qquad \forall e \in E, d \in 2,..,D-1 \qquad (5.22)$$

$$\sum_{s\in S_d} X_{e,s,d} - \sum_{s\in S_{d+1}} X_{e,s,d+1} = IW_{e,d}^{Sat} - IW_{e,d+1}^{Sun} \qquad \forall e \in E, d \in D^{Sat} \qquad (5.23)$$

$$ch_e * W - \sum_{s\in S_{s,d}} \sum_{d\in D} * eh_s * X_{e,s,d} = CD_e \qquad \forall e \in E \qquad (5.24)$$

$$SAT_e = \ n^P * j_e^P \left(\sum_{d\in D} \sum_{s\in S_d} p_{e,s} * X_{e,s,d} \ - min^P \right) \qquad (5.25.a)$$

$$- n^{IX} * j_e^{IX} \left(\sum_{d\in D} IX_{e,d} - min^{IX} \right) \qquad (5.25.b)$$

$$- n^{IO} * j_e^{IO} \left(\sum_{d\in D} IO_{e,d} \ - min^{IO} \right) \qquad (5.25.c)$$

$$- n^{IW} * j_e^{IW} \left(\left(\sum_{d\in D} IW_{e,d}^{Sat} + \sum_{d\in D} IW_{e,d}^{Sun} \right) - min^{IW} \right) \qquad (5.25.d)$$

$$- n^{CD} * j^{CD} * (CD_e \ - min^{CD}) \qquad (5.25.e)$$

$$\forall e \in E$$

Variable Constraints

$X_{e,s,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D, s \in S_d$	(5.26)
$O_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.27)
$R_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.28)
$OO_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.29)
$OS_{e,s,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D, s \in S_d$	(5.30)

$NO_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.31)
$IX_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.32)
$IO_{e,d}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.33)
$IW_{e,d}^{Sat}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.34)
$IW_{e,d}^{Sun}$	$\in \{0,1\}$	$\forall e \in E, d \in D$	(5.35)
$CR_{e,d}$	$\in \mathbb{R}^+$	$\forall e \in E, d \in D$	(5.36)
CD_e	$\in \mathbb{R}^+$	$\forall e \in E$	(5.37)
$SD_{s,d}$	$\in \mathbb{Z}^+$	$\forall d \in D, s \in S_d$	(5.38)
SAT_e	$\in \mathbb{R}$	$\forall e \in E$	(5.39)

Model Extension

Sets

S_d^{18} Set of shifts that ends before 18:00 with a positive demand on day d

Parameter

$\overline{cs}$ Maximum consecutive evening- and night shifts.

sr Desired rest between shifts

n^{OB18} Scaling factor for being off before 18:00 the day before a day off

n^{R16} Scaling factor for getting more than 16 hours rest

n^{minS} Scaling factor for minimum satisfaction

j^{R16} Weight factor for getting more than 16 hours rest

j^{OB18} Weight factor for being off before 18:00 the day before a day off

j^{minS} Weight for minimum satisfaction

$\underline{sat}$ Least acceptable satisfaction score per employee

Variables

$SR_{e,d}$ Calculation of resting hours between assigned shifts on day d and (d+1)

$R_{e,d}^{SR}$ $\left\{\right.$ 1, if the rest between the assigned shifts on day d and (d+1) is greater or equal to the desired shift rest sr. 0, otherwise

$SO_{e,s,d}$ $\left\{\right.$ 1, if employee e is assigned shift s on day d and have a day off the next day (d+1), 0 otherwise

$B^{18}_{e,d}$	$\begin{cases} 1, \text{ if employee e is off before 18:00 the day before day off on day d} \\ 0, \text{ otherwise} \end{cases}$
$\underline{SAT}$	Satisfaction score of the least satisfied employee

Health Promoting Work Patterns

$$\sum_{s\in S_d^{Day}} X_{e,s,d} - \left(\sum_{s\in S_{d+1}^{Day}} X_{e,s,d+1} + \sum_{s\in S_{d+1}^{Eve}} X_{e,s,d+1} + O_{e,d+1} \right) \leq 0$$

$$\forall e \in E, d \in D-1 \quad \text{(E.5.1.a)}$$

$$\sum_{s\in S_d^{Eve}} X_{e,s,d} - \left(\sum_{s\in S_{d+1}{}^{Eve}} X_{e,s,d+1} + \sum_{s\in S_{d+1}^{Night}} X_{e,s,d+1} + O_{e,d+1} \right) \leq 0$$

$$\forall e \in E, d \in D-1 \quad \text{(E.5.1.b)}$$

$$\sum_{s\in S_d^{Night}} X_{e,s,d} - \left(\sum_{s\in S_{d+1}{}^{Night}} X_{e,s,d+1} + O_{e,d+1} \right) \leq 0$$

$$\forall e \in E, d \in D-1 \quad \text{(E.5.1.c)}$$

$$\sum_{d=d'}^{d'+\overline{cs}} \sum_{s\in S_d^{Eve}} X_{e,s,d} \leq \overline{cs} \qquad \forall e \in E, d' \in D-\overline{cs} \quad \text{(E.5.2.a)}$$

$$\sum_{d=d'}^{d'+\overline{cs}} \sum_{s\in S_d^{Night}} X_{e,s,d} \leq \overline{cs} \qquad \forall e \in E, d' \in D-\overline{cs} \quad \text{(E.5.2.b)}$$

$$SR_{e,d} \geq sr * R^{SR}_{e,d} \qquad \forall e \in E, d \in D \quad \text{(E.5.3)}$$

$$SAT_e \mathrel{+}= n^{SR} * j^{SR} \sum_{d\in D} R^{SR} \qquad \forall e \in E \quad \text{(E.5.4)}$$

$$X_{e,s,d} + O_{e,d+1} - 1 \le SO_{e,s,d} \qquad \forall e \in E, s \in S_d \,, d \in D - 1 \quad \text{(E.5.5.a)}$$

$$O_{e,d+1} \ge SO_{e,s,d} \qquad \forall e \in E, s \in S_d \,, d \in D - 1 \quad \text{(E.5.5.b)}$$

$$X_{e,s,d} \ge OS_{e,s,d} \qquad \forall e \in E, s \in S_d \,, d \in D - 1 \quad \text{(E.5.5.c)}$$

$$SR_{e,d} = \sum_{s \in S_d} em_s * (X_{e,s,d} - SO_{e,s,d}) \quad \text{(E.5.6.a)}$$
$$+ \sum_{s \in S_{d+1}} ms_s * (X_{e,s,d+1} - OS_{e,s,d}) \quad \text{(E.5.6.a)}$$

$$O_{e,d} + \sum_{s \in S^{18}_{d-1}} X_{e,s,d-1} - 1 \le B^{18}_{e,d} \qquad \forall e \in E, d \in 2, .. D \quad \text{(E.5.7.a)}$$

$$O_{e,d} \ge B^{18}_{e,d} \qquad \forall e \in E, d \in 2, .. D \quad \text{(E.5.7.b)}$$

$$\sum_{s \in S^{18}_{d-1}} X_{e,s,d-1} \ge B^{18}_{e,d} \qquad \forall e \in E, d \in 2, .. D \quad \text{(E.5.7.c)}$$

$$SAT_e \mathrel{+}= n^{18} * j^{18} \sum_{d \in D} B^{18}_{e,d} \qquad \forall e \in E \quad \text{(E.5.8)}$$

Weekend

$$\sum_{s \in S_{st,d}} X_{e,s,d} - \sum_{s \in S_{st,d+1}} X_{e,s,d+1} = 0 \qquad \forall e \in E, st \in ST, d \in D^{Sat} \quad \text{(E.5.9.a)}$$

$$\sum_{s \in S_{st,d}} X_{e,s,d} - \sum_{s \in S_{st,d-1}} X_{e,s,d-1} \le 0 \qquad \forall e \in E, st \in ST, d \in D^{Sat} \quad \text{(E.5.9.b)}$$

$$\sum_{s \in S^{Day}_{s,d}} X_{e,s,d} - \left(\sum_{s \in S^{Eve}_{s,d+7*\overline{we}+m}} X_{e,s,d} + O_{e,d+7*\overline{we}+m} \right) \le 0 \qquad \forall e \in E, d \in D^{Sat} \quad \text{(E.5.10.a)}$$

$$\sum_{s \in S^{Eve}_{s,d}} X_{e,s,d} - \left(\sum_{s \in S^{Night}_{s,d+7*\overline{we}+m}} X_{e,s,d} + O_{e,d+7*\overline{we}+m} \right) \le 0 \qquad \forall e \in E, d \in D^{Sat} \quad \text{(E.5.10.b)}$$

$$\sum_{s \in S_{s,d}^{Night}} X_{e,s,d} - \left(\sum_{s \in S_{s,d+7*\overline{we+m}}^{Day}} X_{e,s,d} + O_{e,d+7*\overline{we}+m} \right) \leq 0 \quad \forall e \in E, d \in D^{Sat} \quad \text{(E.5.10.c)}$$

Fairness

$$\underline{SAT} \leq Sat_e \qquad \forall e \in E \qquad \text{(E.5.11)}$$

$$Maximise\ obj \ \ += \ \ n^{minS} * j^{minS}\left(\underline{SAT} - min^{minS}\right) \qquad \text{(E.5.12)}$$

$$\underline{sat} \leq Sat_e \qquad \forall e \in E \qquad \text{(E.5.13)}$$