Handelshøyskolen BI

GRA 19703 Master Thesis

Thesis Master of Science 100% - W

Predefinert informasjon

Startdato:	09-01-2023 09:00 CET	**Termin:**	202310
Sluttdato:	03-07-2023 12:00 CEST	**Vurderingsform:**	Norsk 6-trinns skala (A-F)
Eksamensform:	T		
Flowkode:	202310||11184||IN00||W||T		
Intern sensor:	(Anonymisert)		

Deltaker

Navn:	Nahom Berhane Abrahamog Hans-Ole Lislegaard Giske

Informasjon fra deltaker

Tittel *:	COMMODITIES & HEDGING IN THE PRESENCE OF GEOPOLITICAL TENSION
Navn på veileder *:	Paul Ehling

Inneholder besvarelsen konfidensielt materiale?:	Nei	**Kan besvarelsen offentliggjøres?:**	Ja

Gruppe

Gruppenavn:	(Anonymisert)
Gruppenummer:	190
Andre medlemmer i gruppen:	

COMMODITIES & HEDGING IN THE PRESENCE OF GEOPOLITICAL TENSION

MASTER THESIS

By:
Nahom Berhane Abraham & Hans-Ole Giske
MSc in Business, with Major in Finance

Supervisor: Paul Ehling

Oslo, July 3, 2023

Abstract

This thesis extends on the work of Triki & Maatoug, (2021) and investigates the relationship between selected commodities and indices, in the presence of geopolitical tension, represented by the index of Caldara & Iacoviello (2022). By utilizing different multivariate regression approaches, namely the DCC-GARCH and Copula-GARCH, we examine the relationships both from an economical and statistical standpoint. The empirical results suggest that the S&P 500 correlates less with gold during periods of lowered tension, and conversely higher in the presence of higher tension. This further indicates gold as a good diversifier and safe haven for the S&P 500. We are not, however, able to find strong evidence nor patterns that allude to the same for oil and OBX, but evidence of oil working as a hedge for OBX is provided.

Acknowledgements

We thank our supervisor, Professor Paul Ehling, for insightful guidance through this thesis. We also extend our gratitude to BI Norwegian Business School for providing academic resources and access to databases and analytical tools. Lastly, we thank our friends and family for unwavering encouragement and valuable feedback for our master thesis.

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1 Introduction

Over recent decades, financial markets have witnessed substantial growth in terms of the volume and value of diverse financial instruments. This expansion, coupled with increased globalization and interdependence among various markets and assets, has amplified the demand for safe haven investments. Historically, gold has been regarded as an exceptional commodity, primarily attributed to its capacity to preserve value, particularly during periods of financial distress.

The examination of the impact of distress-inducing events on the gold market is of significant academic interest, especially after the substantial increase in gold price after the financial crisis of 2008. According to extant literature, gold is deemed a safe-haven asset, and its demand is often driven by the anticipation of substantial losses owing to systemic risk. This safe haven characteristic of gold is empirically observable in the shifts in its price during times of economic crisis. (Cai et al., 2001; Koutsoyiannis, 1983). Baur & Lucey (2010) defines safe haven as an asset that is either uncorrelated or negatively correlated with another asset or portfolio during periods of market stress or turmoil. Given the established safe-haven properties of gold, it is plausible to consider it as a hedging instrument during distressing periods. However, this concept invariably invites speculation about the potential of other assets exhibiting similar safe haven characteristics. Consequently, it underscores the need for further empirical exploration into the universe of potential safe-haven assets beyond gold. Our paper contributes to growing literature, and extends on the work of Triki & Maatoug, (2021).

This thesis concurs with prior research, presenting evidence of gold's safe haven properties vis-à-vis the S&P 500. We demonstrate an increased correlation between these assets amid geopolitical tension, as observed during the financial crisis and initial stages of the COVID-19 pandemic. Furthermore, we endeavor to incorporate these findings into an optimal portfolio to highlight the significance of a hedge for portfolio investors and the execution of monetary policy.

Additionally, we advance this research by extending the same analysis to Crude oil and the OBX index. However, we encounter difficulties in discerning a pattern that exhibits similar safe-haven characteristics. Nonetheless, we detect instances where the pair almost manifests as an ideal hedge, with correlation peaks of 0.93 and troughs of -0.86. Intriguingly, we also note the relatively high volatility of crude in comparison to the OBX index, further challenging its potential utility as a hedging instrument.

Employing a DCC-GARCH and dynamic copulas, our benchmark study from Triki & Maatoug, (2021) illuminates the time-varying correlation between gold and the S&P 500 amidst geopolitical tension. Furthermore, they uncover significant volatility spillover from the S&P 500 to gold, thus highlighting the considerable influence that the stock market exerts on gold. Contrarily, Triki & Maatoug (2021) do not find evidence of substantial spillover from the gold market to the S&P 500. In a parallel approach, we conduct a multivariate analysis using VAR, DCC- and Copula GARCH models on paired combinations of crude, gold, S&P 500, and OBX. Additionally, we construct portfolios leveraging hedge ratios obtained by the residuals extracted from the DCC-GARCH. This comprehensive analysis further extends our understanding of the complex interrelationships between these diverse markets and their potential roles in portfolio construction and risk management.

This thesis diverges from earlier studies by conducting an analysis on crude and OBX as a pair, a comparative analysis seldom investigated in extant literature. The choice of this pairing primarily emanates from the hypothesis that these two elements exhibit high correlation, given the substantial reliance of the Norwegian economy and market on oil. Consequently, this research not only enriches the academic discourse surrounding this pair but also provides unique insights into their potential interdependencies and implications for portfolio management and economic policy.

This thesis is principally driven by the desire to procure further empirical evidence corroborating the concepts of safe haven assets and hedging mechanisms. While a significant portion of existing literature concentrates on the relationship between gold and the S&P 500, other markets and commodities remain largely unexplored. Hence, our study aims to fill this research gap and broaden the understanding of safe haven characteristics and hedging possibilities across different asset classes and markets.

The structure of this paper is organized as follows: Chapter 2 offers a literature review of pertinent studies informing our research. Chapter 3 articulates our hypothesis and outlines the methodology employed in this thesis. Chapter 4 delineates our dataset and its descriptive statistics, providing a solid foundation for our empirical investigation. Subsequently, Chapter 5 validates our hypothesis through a series of empirical tests and presents an in-depth discussion of our findings. Finally, Chapter 6 encapsulates the study with a conclusion, summarizing the key insights and implications of our research.

2 Literature Review

2.1 Main supporting articles

The literature examining the relationship between commodities and stock indices in the presence of geopolitical tension is scarce. In the study proposed by Triki & Maatoug (2021) they examine the relationship between the US stock market, represented by the S&P 500, and gold prices in the presence of geopolitical tension and conflicts. The latter is represented by the Geopolitical Risk Index calculated by Dario & Iacoviello (2016). The authors carry on their investigation by implementing both an MV-GARCH model and dynamic copula, to capture the return link and spillover between the two markets. The empirical study shows that the American market, namely the S&P 500, correlates less with gold prices during periods of lowered geopolitical risk Geopolitical Risk (GPR), and more in periods characterized by higher GPR. This evidence indicates that gold is a viable diversifier and takes on safe-haven characteristics during periods of great tension. This further implies that gold could also be a viable hedging instrument against the volatility of the S&P 500 in periods of great tension.

Schwartz (1997) investigates the stochastic behavior of commodity prices in terms of their ability to price existing future contracts and their implications for valuating other assets. The author developed three models for this research. The first model is a simple one-factor model in which the spot price of commodities is assumed to follow a mean-reverting pattern. The second model includes a second stochastic factor that captures the convenience yield. This model is also assumed to follow a mean-reverting pattern. Lastly, the third model includes yet another stochastic factor, the interest rates. Using The Kalman filter methodology, the author estimates all the relevant parameters for copper, oil, and gold. The study aims to analyze the output from the models for the term structure of futures prices and for hedging contracts for future delivery. The resulting estimates conclude that the futures volatility is expected to decrease and converge to a fixed value different from zero. Furthermore, the term structure of the futures prices will eventually turn upwards and converge to a fixed growth rate even in strong backwardation.

The combination of the work of both Schwartz (1997) and Triki & Maatoug (2021) lays the ground for further research in our thesis. In the latter, we learn that gold does take on safe-haven characteristics in periods of great stress. However, the study is based on an older GPR index which utilizes an arguably worse methodology. Further, the study only investigates how gold can be used to hedge the American market with respect to historical events in which the US itself was largely involved in. Schwartz (1997) helps us understand what drives commodity prices and how these relate to futures contracts.

2.2 Contribution to literature

After discussing the two main supporting articles it becomes apparent that the work of Triki & Maatoug (2021) is somewhat narrow. Our proposed thesis intends to further extend the work of Triki & Maatoug through including new commodities and markets.

As discussed, the article largely focuses on the American market and events and then concludes gold is a safe haven. We find it necessary to not only include other commodities to optimize hedging strategy, but also look at different markets. As the American, European, and Asian markets open and close in sequences, the different markets may react differently to news as the story may develop in the meantime. Different markets may also react differently based on their individual exposure. Take for example the OSEAX index grounded in the Oslo Stock Exchange; this specific index is highly exposed to fluctuations in oil and gas price.

We also intend to include the newer version of the GPR by Dario & Iacoviello (2020), which utilizes a perceived better methodology.

2.3 Other related articles

Von Furstenberg et al. (1989) study the daily movements in four major stock indices and how it relates to gold and oil prices. Their investigation uncovered the effect both gold and oil prices have on stock price movement. Adding to these findings, Tully & Lucey (2007) utilized the asymmetric power GARCH model of Ding et al. (1993) to study the macroeconomic influence on the price of gold. In their study, they examined the relationship between both cash and futures prices of gold and economic events throughout 1983-2003, with special emphasis on the stock market crashes in 1987 and 2001. Their study revealed no significance in the relationship between the movement in gold prices and the FTSE index.

Further, more recent studies have explored the dynamic interaction between fluctuations in the price of gold and stocks. Baur & Lucey (2010) present gold as a safe haven following major shocks, particularly negative ones, in the United States, the United Kingdom, and Germany, covering three major currencies. The econometric approach in this study is based on a regression model in which gold returns are regressed on stock and bond returns and two interaction terms that test whether gold indeed serves as a safe haven if stock or bond markets fall or exhibit extreme negative returns. Baur & Lucey (2010) further confirm the safe haven nature of gold in their study but here relates it to recession in major emerging and developing countries. Their study revealed that gold showed safe haven characteristics throughout the 1979 to 2009 period for major European markets and the United States but not in selected markets in the BRIC countries, Japan, Australia, or Canada.

Several studies have highlighted the behavior of WTI crude oil prices around the start of crises. A 2017 study by Omar et al., (2017) found that oil prices increased on average as much as 12.08% due to war outbreaks and by 8.46% under serious international crisis. The data was shown to be statistically significant and a large portion of the appreciation in oil price occurred prior to day 0. Oil price inflation may have several underlying causes, including increased demand out of caution, military purchases, panic buying, or uncertainty about supply due to an increased risk of oil embargoes and a breakdown in trade. Price pressures start to show up before conflicts, which could mean that governments and armed forces are building up reserves in preparation for anticipated operations.

Lieber, (1992) reports a rise in crude oil prices during the Gulf War in an earlier study. Similarly, Rigobon & Sack, (2005) and Leigh et al. (2003) claim that the greater possibility of conflict in Iraq contributed to crude oil being more expensive. A spike in speculative demand during the time leading up to these two events may help to partially explain why fossil fuel prices increased during the Gulf crisis (Kilian & Lee, 2014; Kilian & Murphy, 2014). Their findings suggest that these earlier conclusions can be generalized in a sample of 43 wars and 64 occasions of international crisis.

El Hedi Arouri et al. (2015) studied the relationship between the price of gold and the Chinese stock market throughout the 2004-2011 period, utilizing the VAR-GARCH of Ling & McAleer (2003). Their study documents significant cross effects for volatility and return among both gold and stock prices, further strengthening the ability of the VAR-GARCH model.

Recent studies have documented the progressive importance of the financialization of commodity markets. Wen et al. (2019) studied the dynamic effects of financial factors on oil prices and concluded that the global financial crisis of 2008 strengthened the relationship between oil futures markets and financial markets. Today we are presented with more products and common trends in the financial market through integration and similar statistical properties.

Recent studies have highlighted the non-linear relationship between financial series through time-varying copulas. These studies generally unveil an increased dependence in the tail between volatility and the financial markets (Avdulaj & Barunik, 2015; Talbi et al., 2020). The dependence between two markets can be determined using time-varying copulas, however, the estimates they yield typically result in average correlations that are more volatile than the expected normal distribution of a DCC-GARCH model. It seems that the estimates produced by this model are negatively impacted by changes in the joint distribution tail. The increased frequency of volatility and geopolitical shocks in the market since September 11, 2001, can be used to explain this.

3 Hypothesis and Methodology

This thesis examines the relationship between selected commodities and indices. The literature in Chapter 2 inspires the creation and application of analysis based on the following hypothesis and methodology.

3.1 Hypothesis

To investigate the relationship between the selected assets, we first examine the correlation and covariance of the returns. Further, we replicate four portfolios, each consisting of combinations of the two asset classes. The literature shows empirical evidence for gold as a safe haven asset, and points to increased correlation between gold and the SP 500 index during periods of tension. However, we argue that markets with greater interdependence with a specific commodity might be a more optimal instrument for hedging against geopolitical risk. The OBX index is an example of an index with strong connection to a specific commodity, namely crude oil. 9 of out the 25 firms have oil as their sole business model (*OBX-Aksjer*, 2023). In line with that reasoning our main hypothesis is the following:

H1: Oil is more effective, relative to gold, in terms of hedging OBX in periods of geopolitical tension.

It is important to mention that crude oil does not have to take on safe haven characteristics to be a more effective hedge. The only necessity is existence of significant long-term correlation. However, with a “buy and hold” strategy safe haven characteristics is necessary, meaning negative or zero correlation in times of stress.

H2: Oil takes on safe haven characteristics in relation to OBX.

3.2 Methodology

This thesis examines the relationship between selected commodities and indices mainly through three multivariate models, namely a DCC-GARCH, Copula GARCH and VAR.

3.2.1 The DCC-GARCH

The foundation of conditional correlation models is the decomposition of the conditional covariance matrix into conditional standard deviations and correlations, allowing the univariate and multivariate dynamics to be separated and the estimation process to be made easier. The covariance matrix in Bollerslev's constant conditional correlation model (CCC) of 1990 can be divided into:

$$H_t = D_t R D_t = p_{ij}\sqrt{h_{iit}, h_{jjt}},$$

Where $D_t = diag\left(\sqrt{h_{11,t}}, \dots, \sqrt{h_{nn,t}}\right)$, and R is the positive definite constant conditional correlation matrix. However, considering that the constraint of constant conditional correlation may be impracticable in practice, Engle, (2002) and Tse & Tsui, (2000) presented a class of models known as Dynamic Conditional Correlation (DCC) that permits the correlation matrix to change over time in response to motion dynamics, such that:

$$H_t = D_t R_t D_t.$$

This change implies that R_t, must be inverted at every point in time, and in turn imposes a constraint to be positive definite. The dynamic conditional correlation model models a proxy process, $\boldsymbol{Q}_t$ to achieve this constraint as

$$\boldsymbol{Q}_t = \overline{\boldsymbol{Q}} + a\,(z_t - 1z'_{t-1} - \overline{\boldsymbol{Q}}) + b(\boldsymbol{Q}_{t-1} - \overline{\boldsymbol{Q}})$$
$$= (1 - a - b)\overline{\boldsymbol{Q}} + az_{t-1} - 1z'_{t-1} + b\boldsymbol{Q}_{t-1}$$

where a and b are positive scalars, with the condition of a combined sum of less than 1 $(a + b < 1)$, in order to ensure positive definiteness and stationarity.

The volatility component is the total of the individual GARCH likelihoods in the Multivariate Normal case when no shape or skew factors are introduced into the density. These likelihoods can be jointly maximized by individually maximizing each univariate model. In other distributions, such as the multivariate Student, the presence of a shape parameter necessitates performing the estimation in a single step in order to jointly estimate the shape parameter for all models. The likelihood can be divided into two pieces for practical large-scale estimation. The separation accommodates for a large-scale estimation.

In our analysis the GPR index is used as an exogenous variable in the DCC-GARCH model. This imply that the conditional variance and covariance is given by:

$$h_{11,t} = c_{11}^2 + a_{11}^2\varepsilon_{1,t-1}^2 + \beta_{11}^2 h_{11,t-1} + k_{11}GPR_t$$
$$h_{22,t} = c_{22}^2 + a_{22}^2\varepsilon_{2,t-1}^2 + \beta_{22}^2 h_{22,t-1} + k_{22}GPR_t$$
$$h_{12,t} = p\sqrt{h_{11t}h_{22t}}$$

3.2.2 The Copula GARCH

Copulas are a viable alternate approach for simulating the structure of interdependence between financial series. Copula functions were first introduced by Sklar, (1996) as a way of joining various marginal distributions to create a single multivariate distribution. The function was later introduced to financial literature by McNeil & Frey, (2000) and Li (2000). In terms of different variations of the Copula, Breymann et al., (2003) found the Student Copula to be the best fit when investigating bivariate FX rates. However, (Malevergne & Sornette, 2001) found that the Normal Copula fits better on currency and equity pairs in general but fails to capture tail events.

The GARCH-Copula model combines the GARCH model and the copula function to model the dependence between multiple time series with time-varying volatility. First, we model each time series individually using a GARCH model. Suppose we have two time series, x and y, and we want to model their joint distribution. We start by fitting a GARCH model to each series separately. This gives us a series of residuals $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ for each series, as well as the conditional volatilities $\sigma_{x,t}$and $\sigma_{y,t}$. The residuals $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ are assumed to be standardized, and the volatilities $\sigma_{x,t}$ and $\sigma_{y,t}$ capture the time-varying volatility of each series.

Second, we transform the residuals $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ into uniform variables on the unit interval [0, 1]. This is done by applying the empirical cumulative distribution function (CDF) of the residuals to each residual.

Let F_x and F_y be the empirical CDFs of the residuals $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$, respectively. Then, the transformed residuals are:

$$u_{x,t} = F_x(\varepsilon_{x,t})$$
$$u_{y,t} = F_y(\varepsilon_{y,t})$$

Finally, we model the dependence between the transformed residuals $u_{x,t}$ and $u_{y,t}$ using a copula function. Let C be a copula function and let θ be the parameters of the copula. The joint distribution of $u_{x,t}$ and $u_{y,t}$ is given by $C(u_{x,t}, u_{y,t}; \theta)$. The copula parameters θ are typically estimated using maximum likelihood estimation. The likelihood function is given by the product of the copula density function evaluated at the data points ($u_{x,t}$ and $u_{y,t}$).

Once we have estimated the GARCH models and the copula function, we can use them to generate the joint distribution of the original time series x and y.
This is done by simulating from the estimated GARCH models to generate new residuals, transforming these residuals into uniform variables using the empirical CDFs, and then applying the inverse of the copula function to generate joint realizations of the uniform variables.

3.2.3 VAR model

Our MV-DCC GARCH model is not designed to capture and measure spillover effects. Hence, we introduce the Vector Autoregression (VAR) model with GPR as an exogenous variable. This model allows us to measure how changes in one variable impacts another variable over time.

$$Y_{1,t} = \mu_1 + \alpha_{1,1}Y_{1,t-1} + \cdots + \alpha_{1,p}Y_{1,t-p} + \beta_{1,1}Y_{2,t-1} + \cdots + \beta_{1,p}Y_{2,t-p} + GPR_{1,t} + \varepsilon_{1,t}$$

$$Y_{2,t} = \mu_2 + \alpha_{2,1}Y_{2,t-1} + \cdots + \alpha_{2,p}Y_{2,t-p} + \beta_{2,1}Y_{1,t-1} + \cdots + \beta_{2,p}Y_{1,t-p} + GPR_{2,t} + \varepsilon_{2,t}$$

Where p is the optimal number of lags based on the Akaike Information Criterion.

3.2.4 Hedge Ratios and Portfolio Construction

Hedge ratios are integral to financial risk management, serving to quantify the optimal quantity of a hedging instrument, such as futures contracts or options, required to hedge a specific exposure to an underlying asset. The primary objective of hedging is to mitigate the risk associated with price fluctuations in the underlying asset. A critical determinant of hedge ratios is volatility, which measures the degree of variation in a trading price series over time. Greater volatility signifies increased risk, necessitating a higher hedge ratio to safeguard against this risk.

Nonetheless, volatility is not a static measure; it fluctuates over time due to a myriad of factors, including shifts in market sentiment, economic events, or alterations in the fundamentals of the underlying asset. This is where the concept of conditional volatility becomes pertinent. Conditional volatility estimates provide a mechanism for modeling and predicting volatility changes over time, based on historical data.

Incorporating conditional volatility estimates in the calculation of hedge ratios allows for the adjustment of hedging strategies to accommodate anticipated changes in volatility. For instance, an anticipated increase in future volatility might warrant a higher hedge ratio to shield against this augmented risk. Conversely, an expected decrease in volatility might justify a lower hedge ratio.

In essence, the use of conditional volatility estimates in determining hedge ratios facilitates the creation of a more dynamic and adaptive hedging strategy, capable of providing enhanced protection against risk. This approach is more sophisticated than employing a static hedge ratio based on historical volatility and has the potential to yield superior risk management outcomes.

The strategy implemented entails holding a long position in one asset (hereafter referred to as asset i) and counterbalancing this with a short position in a second asset (hereafter referred to as asset j). The hedge ratio, a crucial component in this hedging strategy, between asset i and asset j is determined by the conditional volatility estimates of the two assets. The hedge ratio between asset i and j is given by:

$$\beta_{ijt} = h_{ijt}/h_{jjt}$$

The determination of optimal weights in a portfolio is predicated on the expected returns and the covariance matrix of the returns of the constituent assets. The covariance matrix quantifies the extent to which returns on disparate assets co-move. In the context of portfolio optimization, the objective is to select weights that maximize expected return for a given level of risk, or equivalently, minimize risk for a given level of expected return. This is typically accomplished using mean-variance optimization, predicated on the assumption that asset returns follow a multivariate normal distribution.

However, the assumption of constant covariance often proves unrealistic in practice. Covariances can fluctuate over time due to a variety of factors, including shifts in market conditions, economic events, or alterations in the fundamentals of the underlying assets. Incorporating conditional covariance estimates in portfolio optimization allows for the adjustment of portfolio weights to accommodate anticipated changes in the covariances between asset returns. For instance, an anticipated increase in the covariance between two assets might warrant a reduction in exposure to these assets to limit risk. Conversely, an expected decrease in covariance between two assets might justify an increase in exposure to these assets to exploit potential diversification benefits.

In essence, the use of conditional covariance estimates in portfolio optimization facilitates the creation of a more dynamic and adaptive investment strategy, capable of providing enhanced risk management and potentially improving portfolio performance. This approach is more sophisticated than employing a static covariance matrix based on historical data and has the potential to yield superior investment outcomes.

Utilizing the theory behind portfolio optimization we constructed the portfolio weights in our research the following:

$$w_{ij,t} = \frac{h_{jj,t} - h_{ij,t}}{h_{ii,t} - 2h_{ij,t} + h_{jj,t}}$$

$$w_{ij,t} \begin{cases} 0, & if\ w_{ij,t} \prec 0 \\ w_{ij,t} & if\ 0 \leq w_{ij,t} \leq 1 \\ 1 & if\ w_{ij,t} \succ 1 \end{cases}$$

Here, $w_{ij,t}$ represents the weight of the first asset in a one-dollar portfolio comprising asset i and asset j at time t. The term $h_{ij,t}$ denotes the conditional covariance between asset i and asset j at time t, while $h_{jj,t}$ denotes the conditional covariance of asset j at time t. Conversely, the weight of the second asset is represented by $1 - w_{ij,t}$.

4 Data & Databases

4.1 Data

In the realm of theoretical analysis, two primary trends emerge when examining the relationship between stock markets and gold prices. A review of the expanding literature reveals that the correlation between these two markets can manifest as either positive or negative. From an empirical standpoint, as demonstrated in Chapter 2, the relationship between gold and stock prices is not stable. First, there is a lack of consistent and definitive empirical evidence to support any particular trend (Baur & Lucey, 2010; Mensi et al., 2013; Naifar, 2012). Moreover, according to the findings of Do et al., (2009) the volatility spillover effect, which describes the influence of gold prices on stock returns, is often bidirectional in many emerging markets. However, in developed stock markets, the influence tends to be unidirectional, typically flowing from gold prices to stock markets.

In our research, similar to Triki & Maatoug, (2021) we propose the addition of the new and updated GPR index to the existing body of work exploring the relationship between gold- and crude oil prices and market indices. This index serves as an external factor, independent of the markets themselves, encompassing a range of elements such as political discord, global economic disputes, and confrontations. Such elements, along with others like warfare, conflict, and acts of terrorism, have the potential to disrupt international financial flows and impact stock markets. Geopolitical instability, conflicts, and insurgencies can create disturbances in gold and equity markets, as demonstrated in the study by Omar et al., (2017).

The Geopolitical Risk (GPR) Index is formulated based on the proportion of newspaper articles that mention geopolitical tensions. This is achieved by scanning for specific keywords associated with geopolitical risks within these articles. The revised version of the index, compared to its 2019 iteration, incorporates slight modifications in the selection of these keywords.
One key distinction of the updated index is its rebasing: the recent index is normalized to average 100 for the period from 1985 to 2019, while the historical index is rebased to average 100 for the period from 1900 to 2019.

The updated index also includes new word combinations that were absent in the previous version, such as the proximity of "foe" or "enemy" to "attack" or "offensive", and "peace" in proximity to "threats". These combinations are frequently employed in the context of escalating geopolitical risk. Consequently, events like the Korean War or the Second World War are more prominently featured in the revised version compared to the previous one, making the adverse geopolitical events of the 2000-2020 period appear less risky in comparison to their 20th-century counterparts.

While the revised index maintains a high correlation with its predecessor, it exhibits a less pronounced upward trend. This refinement in the index provides a more nuanced understanding of the evolution and impact of geopolitical risks over time.

In their study, Das et al., (2019) employed the index of Dario & Iacoviello, (2016) as a gauge for geopolitical risk. They concluded that this index is particularly specialized as it captures specific events that should have a direct influence on financial variables. Contrary to econometric models that use dummy variables to account for the impact of geopolitical risk, the GPR index is continuous, not binary. Additionally, this index is updated on a monthly basis, making it suitable for inclusion in a time series model, and continuous analysis.

Our study utilizes four variables, namely: the S&P 500, OBX and the price of both gold and crude oil. The data used is collected on a monthly basis from 1985 to 2022, for crude, S&P 500 and gold, and from 1996 to 2022 for OBX. The data was obtained from the World Bank Commodity Price Data (*Commodity Markets*, n.d.) for gold and oil, and from Bloomberg for the S&P 500 and OBX. It's important to note that the nominal values for all variables are adjusted by the Consumer Price Index, sourced from U.S. Bureau of Labor Statistics, (1947) to derive their real-term values. Additionally, we incorporate the geopolitical risk (GPR) index, an external variable sourced from Caldara & Iacoviello, (2022) and assess its influence on the covariance between gold and stock, as well as their returns and variances.

The impact of external variables like the GPR index, economic policy uncertainty, and others on correlations has been largely overlooked in existing literature. However, the GPR index holds the potential to concurrently affect both the conditional mean and the conditional volatility of asset returns (Guidolin & Timmermann, 2008; R. F. Engle & Rangel, 2008). Indeed, the research conducted by Schneider & Troeger, (2006), Wolfers & Zitzewitz, (2009), and Choudhry et al., (2015) underscores the impact of domestic and global political factors on the attitudes and actions of investors in financial markets. As such, geopolitical events, which typically serve as unexpected external shocks to the stock market, appear to be perfect candidates for gauging investor sentiment.

4.2 Databases

Bloomberg's database serves as an exhaustive repository for financial data, offering a broad spectrum of information pertaining to both public and private entities, diverse markets, and global economies. As an internationally recognized authority in the realm of business and financial data, Bloomberg meticulously amasses and curates' data from an extensive array of sources, thereby ensuring the precision and contemporaneity of the information (*Bloomberg L.P.*, 2023)

The database furnishes comprehensive financial data for a multitude of companies across the globe, encompassing elements such as stock prices, financial statements, and pivotal performance indicators. This data is procured directly from the companies themselves, exchanges, and regulatory filings, thereby bolstering its credibility and precision.

Beyond company-specific data, Bloomberg's database also proffers data on a vast array of markets, inclusive of equities, bonds, commodities, and currencies. This encompasses both real-time and historical data, thereby facilitating the analysis of market trends and fluctuations over temporal spans.

The World Bank Commodity Price Data is an extensive source of global commodity price information. As a leading international institution, the World Bank ensures the data's reliability and timeliness by systematically gathering and curating it from a wide range of sources.(*World Bank Open Data*, 2023)

This database offers detailed price data for a vast array of commodities around the world, including energy products, metals, and agricultural goods. The data is derived directly from various global exchanges, market observations, and government agencies, which enhances its authenticity and accuracy. In essence, the World Bank Commodity Price Data is a valuable and dependable source for commodity price data, offering a broad spectrum of information on global commodities and their price movements.

4.3 Descriptive Statistics and Performance Measures

As previously indicated, our focus lies in examining the relationship between the volatility of selected commodities and stock markets. Table 1 presents the summary statistics of monthly returns for the five series under consideration. Monthly returns imply $r_t = \log\left(\frac{P_t}{P_{t-1}}\right) * 100$ for all series, including the GPR index.

Summary Statistics of Monthly Return

	Gold	SP500	GPR	Crude	OBX
Mean	0,391	0,673	0,024	0,457	0,762
Variance	12,015	19,98	536,384	91,348	35,732
Skewness	0,361	-0,972	1,86	-1,023	-1,428
Excess Kurto:	1,301	2,953	14,43	4,1	4,89
J-B Normalit	41,988	237,007	4210,342	282,701	431,854
ADF	-6,835	-6,922	-9,726	-6,917	-6,556
KPSS	0,236	0,115	0,013	0,066	0,031
PP	-368,113	-439,56	419,553	-207,432	-276,74

Table 1: Summary Statistics of Monthly Return

From table 1 we can observe the variation in the different series. Excluding GPR, we see crude boasting the highest volatility with OBX coming in second. Contrary, we see gold and S&P 500 demonstrate relatively similar variances. Although early, suggesting an easier pair to hedge. More on that in chapter 5. Further, it is worth noticing that S&P 500, crude and OBX all demonstrate negative skewness, implying a relative probability of incurring losses over the time frame under study. Conversely, gold and GPR demonstrate positive skewness. The positive excess kurtosis statistic across all series confirms that nearly all the series are leptokurtic, meaning they exhibit a more pronounced peak around the mean and heavier tails compared to a normal distribution. The substantial skewness and kurtosis values justify the application of a Skewed Generalized Error Distribution in subsequent estimations. The deviation from normality is formally substantiated by the Jarque-Bera test statistics, which dismiss the null hypothesis of a normal distribution for all the series at the 1% significance level. Figure 1 offers evidence of time-varying volatility for all four series in the studied time frame.

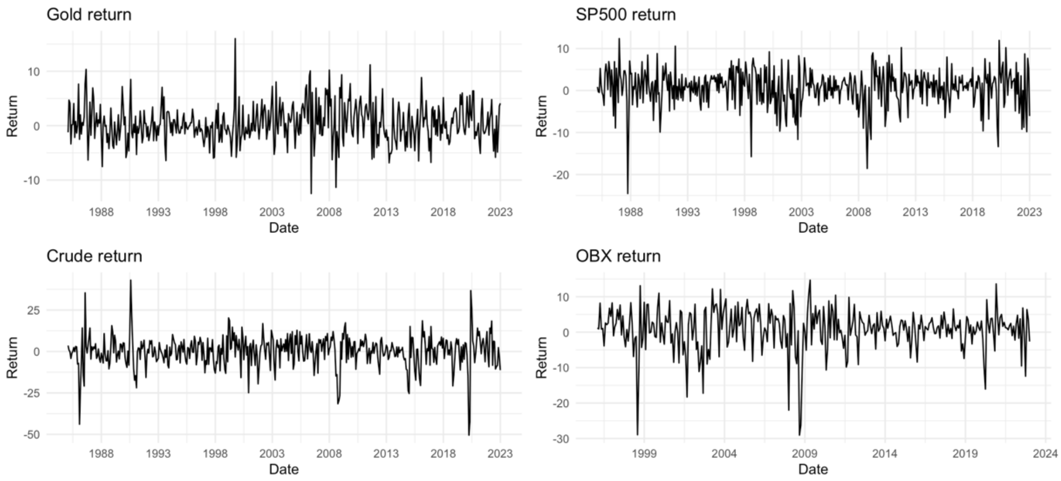


Figure 1: Time Varying Volatility

Furthermore, to probe the stationarity of the data, we employ Augmented Dickey-Fuller (ADF), Phillips and Perron (PP), and Kwiatkowski et al. (KPSS) tests to ascertain the order of integration for each series. The results from these tests confirm that all series exhibit stationarity at the 1% significance level.

5 Results & Analysis

In addition to investigating stationarity, as shown in Table 1, we test for bivariate ARCH- and normality effects using the Hacker and Hatemi-J (2005) and Jarque-Bera test respectively, with 12 lags.

Bivariate ARCH and Normality of Monthly Returns

	ARCH		Normality	
	χ^2	P-value	χ^2	P-value
Gold/SP500	51.184	0.000	1009.074	0.000
Crude/OBX	211.71	0.000	1009.074	0.000
Gold/OBX	79.269	0.000	1072.376	0.000
Crude/SP500	228.05	0.000	2498.756	0.000

Table 2: Bivariate ARCH and Normality results

The Autoregressive Conditional Heteroskedasticity (ARCH) test, first introduced by Engle (1982), serves as a statistical procedure to identify the presence of heteroskedasticity in a time series dataset. Heteroskedasticity characterizes a situation where the variability of the error term, also referred to as the model's "noise", does not remain constant over time. This phenomenon is frequently observed in financial time series, where periods of heightened volatility are often succeeded by intervals of lower volatility. The ARCH test is specifically designed to formally recognize such patterns. A significant test statistic implies that the series demonstrates an ARCH effect, indicating a time-dependent volatility.

Normality tests, including the Shapiro-Wilk and Jarque-Bera tests, are statistical procedures employed to ascertain whether a given dataset adheres to a normal distribution. These tests scrutinize the skewness and kurtosis of the data, comparing these characteristics to those inherent in a normal distribution. Within the realm of time series analysis, normality tests frequently serve to examine the distribution of residuals, or error terms, derived from a model. The assumption of normally distributed residuals underpins many statistical models, and deviations from this assumption may indicate model misspecification.

5.1 In-Sample Analysis

For the remainder of our analysis, we have divided the data into subsamples. Our in-sample period equates to January 1985, or January 1996 depending on the asset, to November 2019. Subsequently, this leaves the last 36 months for our out-of-sample. More on the latter in chapter 5.3.

Table 3 reports outputs for our four DCC-GARCH models. The GARCH coefficients (β_i) encapsulate the lagged dependence and the enduring persistence in the conditional variance equation. These coefficients signify enduring persistence in elucidating conditional variance. The coefficient β_1 is associated with the GARCH term for the commodity returns equation, whereas β_2 is linked to the GARCH term for the stock index returns equation. The estimated β_i coefficients are statistically significant at the conventional 1% for all assets except for oil.

The statistical significance of α_i, the GARCH parameter, shows that the conditional volatility at the present time is considerably influenced by previous information shocks. It measures the impact of past squared residuals on current volatility. The value of the estimated alpha coefficients looks to be lower for gold, the S&P 500, and OBX than the beta coefficients, indicating that the assets' volatility is greater effected by past shocks over a longer period than in the short term. Contrary to the other assets, crude oil has a far higher estimated alpha. These findings align with those of von Furstenberg et al., (1989). Due to the ARCH parameter (β_2) for crude oil lack of statistical significance, it is not possible to tell whether short-term or long-term volatility better captures the current volatility. The ω_1 estimate for crude oil is also statistically significant for both DCC models which suggest that the long-term average volatility is statistically significant different from zero. If we look back to table 1 we can identify the high variance of oil as a potential explanation for the high ω_1 estimate.

The parameters related to the DCC part of the model (DCC_A and DCC_B) aims to estimate the time-varying correlation between the series. Here, DCC_A is not statistically significant for any of the models, which implies that the previous period's residual does not significantly influence the current correlation. DCC_B is, on the other hand, statistically significant across all DCC-GARCH models indicating that the previous period's correlation between commodity and index affect the current period's correlation. These findings misalign with those of Triki & Maatoug, (2021) as they deem both DCC_A and DCC_B statistically significant for gold and S&P 500 as a pair.

The κ_i parameter captures the impact on volatility from the exogenous variable GPR in the variance equation. The estimated κ_i parameter is not statistically significant across both commodities and indices as documented by Triki & Maatoug, (2021). These results suggest that GPR does not influence volatility of returns in any of the commodities or indices. The same result would be true for the χ_i parameter that captures GPR's influence on the returns in the mean equations.

Table 5 in the appendix portrays the outputs from our four Copula-GARCH models. As seen, the model shows some deviation from the results of the DCC. However, the statistical significance of different parameters remains unchanged. Based on the fit and performance of the two, we from here on discard the Copulas-GARCH models and base our further analysis on the DCC-GARCH.

DCC-GARCH								
Parameters	$GOLD_1/SP500_2$		$GOLD_1/OBX_2$		$CRUDE_1/SP500_2$		$CRUDE_1/OBX_2$	
	Coef	**P-value**	**Coef**	**P-value**	**Coef**	**P-value**	**Coef**	**P-value**
μ_1	0.231	0.308	0.387	0.107	**0.967**	0.026	1.253	0.133
AR_1	**0.190**	0.000	**0.190**	0.001	**0.170**	0.003	**0.176**	0.023
χ_1	0.010	0.130	0.008	0.230	0.022	0.349	0.011	0.762
ω_1	1.025	0.410	**2.563**	0.012	**40.583**	0.000	**46.773**	0.098
α_1	**0.113**	0.027	**0.131**	0.012	**0.567**	0.000	**0.422**	0.098
β_1	**0.805**	0.000	**0.672**	0.000	0.000	1.000	0.000	1.000
κ_1	0.099	0.321	0.029	0.704	0.000	1.000	0.000	1.000
μ_2	**0.711**	0.000	**1.046**	0.005	**0.711**	0.000	**1.046**	0.005
AR_2	-0.048	0.399	**0.108**	0.092	-0.048	0.408	**0.108**	0.100
χ_2	-0.013	0.195	-0.024	0.182	-0.013	0.195	-0.024	0.182
ω_2	0.623	0.192	3.270	0.130	0.623	0.193	3.270	0.126
α_2	**0.165**	0.000	**0.147**	0.093	**0.165**	0.000	**0.147**	0.092
β_2	**0.826**	0.000	**0.769**	0.000	**0.826**	0.000	**0.769**	0.000
κ_2	0.000	1.000	0.071	0.701	0.000	1.000	0.071	0.701
DCC_A	0.000	1.000	0.000	0.998	0.037	0.206	0.017	0.218
DCC_B	**0.926**	0.000	**0.888**	0.000	**0.945**	0.000	**0.965**	0.000
Log-Likelihood	**-2483.887**		**-1871.472**		**-2894.688**		**-2142.321**	

Table 3: DDC-GARCH Output

The application of VAR models allows us to capture the interdependencies and temporal dynamics, providing an in-depth understanding of how changes in one variable can influence others over time. Moreover, by treating the GPR index as an exogenous variable, we can distinguish between variables that react to shocks and variables that act as sources of shocks. We applied the Akaike Information Criterion (AIC) to determine the optimal number of lags for each of the times series. All series had the lowest AIC at p = 1, except from the series with crude oil and the S&P 500 index when p = 2 gave the best fit. Further we computed the F-statistic for each of the VAR models and all models are statistically significant, except the models with the S&P 500 as dependent variable. The results from table 3 help us assess the return spillover effects between the two commodities and indices. The results explain the movement in the dependent variable based on one unit change in independent variable.

In the VAR models with gold as the dependent variable we can see that, a one-unit lagged increase corresponds to a current increase of 0.146, significant at the 1% level. The constant term is also positive and significant at the 5% level. We also identify that an increase in GPR seem to increase returns for gold. These results could be interpreted as evidence of the persistence in gold prices, and it suggests that investors might expect some degree of stability in gold prices, which reinforces gold's status as a safe-haven asset and a good hedging instrument during times of increasing geopolitical risk. The results for crude oil are more ambiguous. The constant and GPR term is strongly rejected in both models.

Estimates VAR models

	Gold/Sp500		Sp500/Gold		Gold/OBX		OBX/Gold	
Parameters	**Estimate**	**P-value**	**Estimate**	**P-value**	**Estimate**	**P-value**	**Estimate**	**P-value**
Gold t-1	**0.146**	0.002	-0.005	0.940	**0.146**	0.009	0.083	0.361
Sp t-1	-0.020	0.573	0.017	0.717				
Obx t-1					**0.072**	0.033	**0.161**	0.004
GPR t	**0.353**	0.032	**-0.019**	0.033	0.009	0.305	**-0.032**	0.024
const	**0.014**	0.050	**0.662**	0.002	**0.340**	0.095	**0.603**	0.072
F-stat	**4.809**	0.003	1.571	0.197	**3.975**	0.008	**4.699**	0.003

	Crude/Sp500		Sp500/Crude		Crude/OBX		OBX/Crude	
Parameters	**Estimate**	**P-value**	**Estimate**	**P-value**	**Estimate**	**P-value**	**Estimate**	**P-value**
Crude t-1	**0.320**	0.000	-0.014	0.537	**0.205**	0.000	0.023	0.529
Crude t-2	**-0.104**	0.028	0.035	0.131				
Sp t-1	**0.176**	0.067	0.022	0.636				
Sp t-2	0.049	0.611	-0.057	0.227				
obx t-1					**0.491**	0.000	**0.148**	0.010
GPR t	0.020	0.289	**-0.018**	0.051	0.012	0.556	**-0.032**	0.022
const	0.021	0.962	**0.692**	0.001	-0.021	0.966	**0.639**	0.054
F-stat	**11.010**	0.000	1.678	0.138	**21.860**	0.000	**4.546**	0.003

Table 4: VAR Model Output

The most notable result from the models with OBX as at the dependent variable is the negative impact of increase in GPR. The negative impact of increase in GPR is statistically significant for both models and allude to the fact that geopolitical tension tend to induce losses in financial markets. In the models with the S&P 500 index as the dependent variable we are unable to reject the null-hypothesis in the F-test, hence we cannot trust that the estimated parameters influence the current value of the S&P 500 index.

Figure 2 portray the temporal evolution of geopolitical risk in conjunction with the correlation, thereby illustrating the suggested direct influence of geopolitical risk on the time-varying covariance. This graphical representation provides a comprehensive depiction of the dynamic interplay between geopolitical risk and the correlation, underscoring the significant role of geopolitical factors in shaping the covariance structure over time.

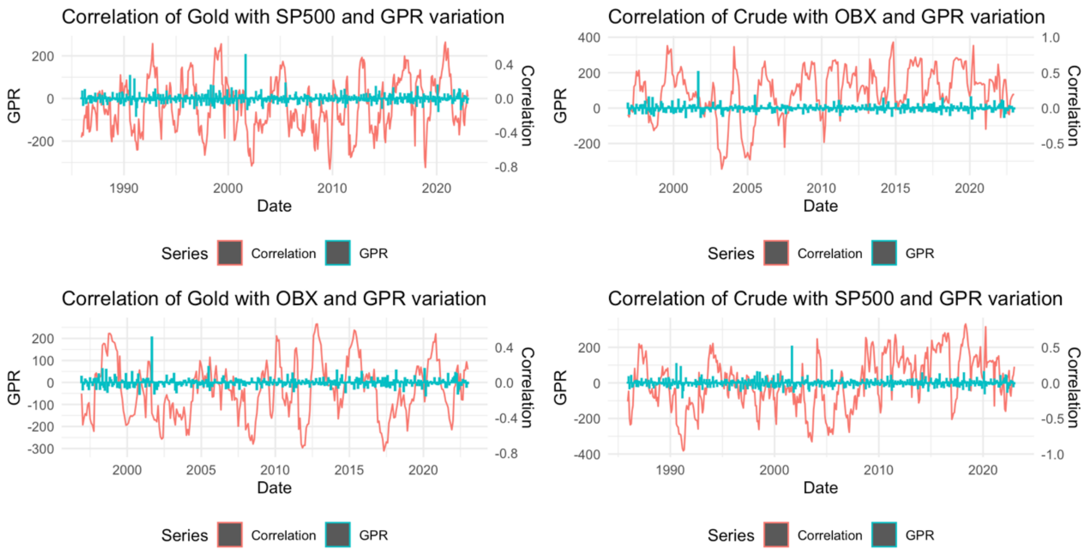


Figure 2: Correlation of pairs against GPR

5.2 Hedging and Portfolio Weights

Before computing our portfolio weights, we need to look at the hedge ratios for the different pairs. The superior estimation model, namely the Dynamic Conditional Correlation (DCC) model, is utilized to calculate the hedge ratio.

Table 4 exhibits the summary statistics for our hedge ratios. First observing the gold and S&P 500 as a pair. The mean hedge ratio for Gold/SP500 is 0.32, whilst the hedge ratio between the SP500/Gold is 0.42, somewhat deviating from the results of Triki & Maatoug, (2021). These measures suggest that in order to minimize risk a short position in gold or S&P 500 can be hedged only 32% and 42% respectively, by a long position in the opposite asset. Further we notice that the min and max value for the respective pairing are relatively close to each other, suggesting a sense of similar volatility in the underlying assets. Furthermore, Gold/OBX and OBX/Gold demonstrates similar numbers to the previous pairing. The similar numbers may point to gold's ability as a hedging instrument. This also may point towards similar volatility in the two markets.

Hedge ratio (long/short) summary statitics

	Mean	St.Dev.	Min	Max
Gold/SP500	0,326	0,075	0,157	0,676
SP500/Gold	0,428	0,149	0,223	1,078
Gold/OBX	0,359	0,0626	0,279	0,711
OBX/Gold	0,582	0,171	0,374	1,535
Crude/OBX	2,213	1,333	0,528	15,553
OBX/Crude	1,42	0,53	0,352	4,855
Crude/SP500	1,292	1,536	0,007	18,259
SP500/Crude	0,644	0,606	0,003	3,709

Table 4: Hedge Ratio Summary Statistics

Moving along to crude and OBX pairing, we see substantially larger numbers. Crude/OBX returns a mean of 2.213, indicating that a short position in OBX can be hedged 221% by a long position in crude. We also notice an enormous max value for Crude/OBX at 15.553. The enormous numbers may hint at the volatility of crude. Similarly, we see a rather substantial number for the mean of OBX/Crude, resulting at 1.42, however, a relatively smaller max value of 4.855. Again, suggesting volatility in crude.

Lastly, observing the pairing of crude and S&P 500 we see similar numbers to the previous pair. Again, we see overall higher values compared to the pairs consisting of gold. We also notice a relatively larger results when crude is used as the hedging instrument compared to the market. Consequently, these results also may suggest high volatility in crude in the studied period.

Table 5 presents the descriptive statistics for the proposed portfolio weights for the respective pairs. Observing the Gold/SP500 pair, the mean portfolio weight is 0.59, suggesting that, on average, 59% of a portfolio composed solely of this pair should be allocated to gold, with the residual 41% allocated to the S&P 500.
In contrast, the average weight for the Gold/OBX portfolio is notably higher at 0.70. This elevated average weight could be attributed to a multitude of factors. However, it does suggest that the OBX, in comparison to the S&P 500, is subject to a greater risk exposure. This could also be a reflection of the inherent characteristics of the two indices, with the OBX comprising 25 stocks, while the S&P 500 encompasses 503 (*S&P 500®*, 2023) (*OBX-Aksjer*, 2023).

Portfolio Weights summary statistics

	Mean	St.Dev.	Min	Max
Gold/SP500	0,598	0,141	0,14	1
Gold/OBX	0,702	0,119	0,35	1
Crude/OBX	0,622	0,230	0	1
Crude/SP500	0,378	0,322	0	1

Table 5: Portfolio Weights Summary Statistics

Broadly, these findings imply that to optimize risk minimization without compromising the expected return of the two portfolios, investors should maintain a higher proportion of gold relative to their exposure in the two markets. The minimum and maximum weights for the Gold/SP500 and Gold/OBX portfolios are 0.14 and 1, and 0.35 and 1, respectively. The relatively higher minimum for the Gold/OBX portfolio further suggests a greater risk exposure.

Overall, these results may indicate a high variance in weight associated with fluctuations in geopolitical tension. However, these interpretations should be considered with caution, as the portfolio weights are subject to the specific assumptions and constraints of the portfolio optimization model.

The graphical representation of the Gold/SP500 portfolio in Figure 3 reveals a peak in the weight favoring gold during the stock market crash of October 1987. A similar apex is discernible at the onset of the COVID-19 pandemic in 2020. The plot also exhibits a noteworthy pattern in the period leading up to and during the subprime crisis of 2007-2008. The weight in gold reaches its minimum just prior to the crisis, suggesting that the stock market was a favorable investment at that time. However, this weight escalates sharply, nearly reaching its peak, concurrent with the unfolding of the crisis. An overall comparable pattern is observable for the Gold/OBX portfolio, albeit with a higher minimum value of 0.35. This portfolio reaches its peak on two occasions.

Examining the Crude/OBX portfolio, the mean weight is observed to be 0.62, indicating an average allocation of 62% to crude over the period under study. In contrast, the average portfolio weight is calculated to be 0.37 for the Crude/SP500 pair. Unlike the pairs discussed previously, this suggests an allocation to the commodity that is less than 50%, further implying that, on average, the S&P 500 exhibits lower volatility relative to crude over the examined timeframe. Regarding the minimum and maximum values for the two portfolios, they are identical, with values of 0 and 1, respectively. This expanded range in minimum and maximum values suggests a greater variation in the underlying assets, and consequently, in the portfolio.

Upon examining the plots for the two pairs in Figure 3, the aforementioned variation is immediately evident. Specifically, the portfolio for Crude/OBX demonstrates abrupt transitions to extreme values throughout the latter half of the 2000s. Notably, troughs are observed around the time of the Iraq war in 2003, and during and preceding the subprime crisis of 2007-2008. Conversely, peaks are discernible immediately following or interspersed between these troughs. A peak is also noticeable in 1997, preceding a sharp decline, which reflects the burgeoning demand for crude from the Asian market prior to the ensuing Asian financial crisis. The graph corresponding to the Crude/SP500 pair exhibits similar shape changes, albeit to a greater degree. It is worth recalling our earlier discussion regarding the relative lower volatility of the S&P 500, which permits the volatility of crude to exert a more pronounced influence on the weights, as manifested in the graph.

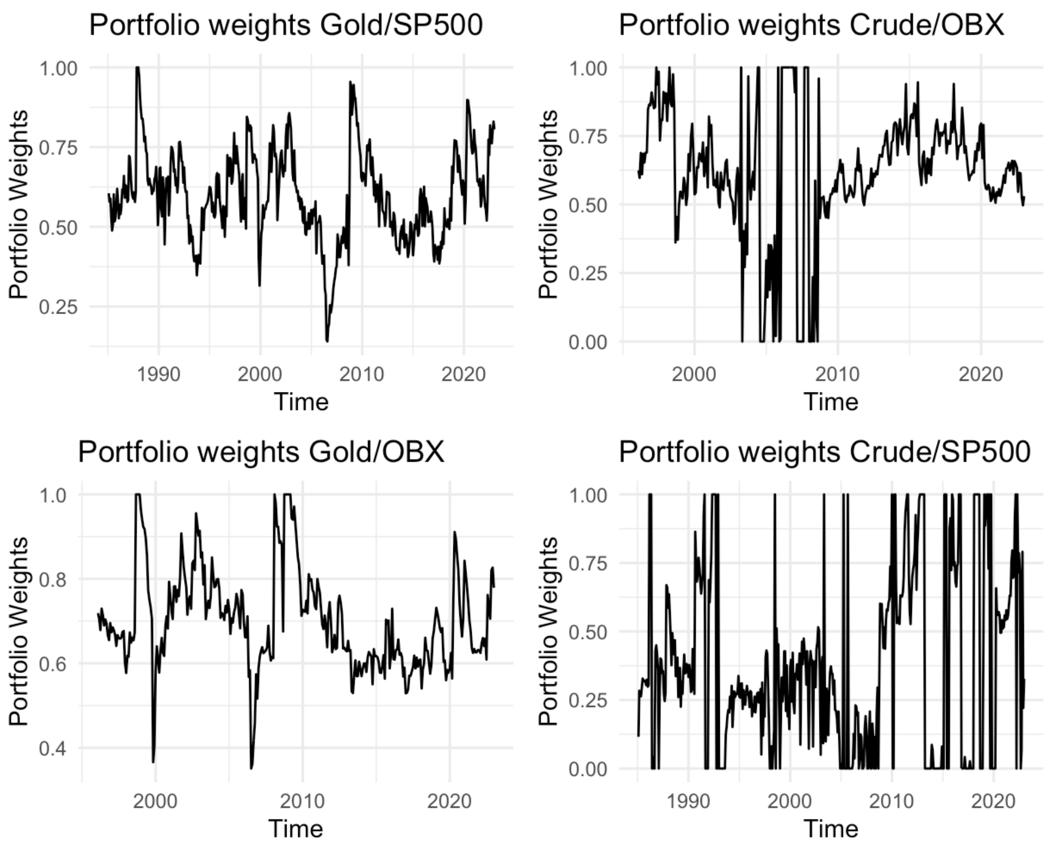


Figure 3 Portfolio Weights

5.3 Out-of-sample Performance and Analysis

Driven by a fascination with the domain of geopolitical risk, and the potential it may possess for predicting fluctuations in value, our study undertook a rigorous quantitative evaluation of geopolitical risk's (GPR) part in forecasting the conditional correlations between Gold/SP500, and Crude/OBX. We have anchored our analysis on the substantial body of empirical evidence provided by Liu et al., (2019) and Brandt & Gao (2019), thereby setting the period from January 1985 to November 2018 for the Gold/SP500 pair, and January 1996 to

November 2018 respectively as the timeframe for our in-sample estimations. This time frame yields 287 and 419 observations respectively, subsequently preserving the remaining 36 months for out-of-sample forecasts.

In their scholarly contribution concerning the precision of forecasts generated by multivariate GARCH models, Laurent et al., (2012) underscored the resilience of these models in relation to volatility predictions. Their research illuminated the effectiveness of the Engle, (2002) Dynamic Conditional Correlation (DCC) model in accurately representing the temporally fluctuating conditional correlations across diverse sampling periods, inclusive and exclusive of exogenous shocks.

Figure 4 presents the contesting out-of-sample projections. The out-of-sample timeframe commences in November 2019, coinciding with the early stages of the pandemic, and our forecast analysis' time horizon was broadened to the subsequent 36 months. Our examination delved into the effects of incorporating the Geopolitical Risk (GPR) index into the Dynamic Conditional Correlation (DCC) GARCH model within an out-of-sample context, focusing particularly on its sway over the evolution of the conditional correlations between the two pairs.

The out-of-sample forecast for the Gold/SP500 pair reveals that geopolitical risks can exert a significant, albeit transient, influence on these conditional correlations as per the unrestricted model (Correlation). Consequently, such geopolitical risks depress stock returns while bolstering safe-haven assets. Additionally, the forecast implies that after a six-month duration, the impact of the exogenous variable (GPR index) on the conditional correlations between gold and the S&P 500 diminishes. In periods of amplified geopolitical risk, investors should thus gravitate towards a portfolio rooted in safe-haven assets.

The out-of-sample forecast for the Crude/OBX pair reveals a slightly different pattern in the sense geopolitical risk exerting a significant influence. However, it does not seem to be temporary, consequently reducing the ability as a safe-haven asset. Nevertheless, the significant impact still suggests geopolitical risks depress stock returns for the pair.

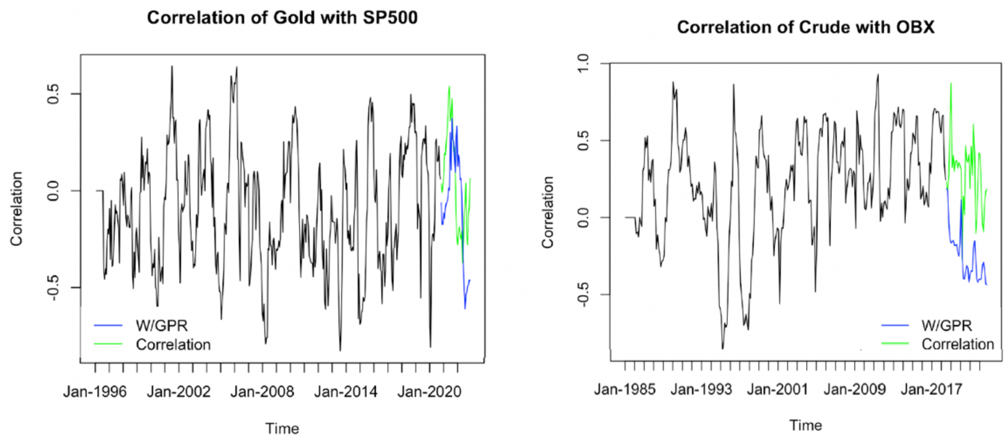


Figure 4: Out-of-Sample Forecast for Gold/S&P 500 and Crude/OBX

To assess the predictive efficacy of out-of-sample forecasts, we implement the likelihoods of the estimates stemming from both the unconstrained and constrained models; that is, models that were formulated with and without the incorporation of the GPR index for the two pairs. To facilitate this comparison, a likelihood-ratio test (LR test) was conducted. The degrees of freedom for this test were calibrated to one, indicative of the quantity of restrictions in place. The value of the likelihood-ratio statistic surpassed the 99% critical threshold. When analyzed at the 1% significance level, the restricted DCC-GARCH model was found to be less compelling compared to its unrestricted counterpart for both pairs.

A critical reflection on the specific period under forecast can illuminate the factors contributing to the deficiency in predictive efficiency. As stated, our prediction endeavor spanned from November 2019 to November 2022, a time predominantly affected by the COVID-19 pandemic. This epoch is distinctive for its aberrant market patterns, not mirrored elsewhere in the data set at our disposal. It is plausible to posit that the difficulties faced by the model in forecasting these unique patterns stem from the absence of analogous patterns within the in-sample data and the inherent characteristics of the model itself.

5.4 Limitations of Study

5.4.1 Timeframe

Choosing time frames in portfolio analysis significantly impacts study results but also introduces certain limitations, requiring careful thought. The "time-period bias", where the specific time period influences results, is a notable risk. For instance, results from a bullish market period can vastly differ from a bear market period, possibly limiting generalizability. One may argue this is evident with the OBX data (1996-2022), potentially exposed to abnormal variation and subsequent skewed inference due to fewer data points.

Different time frames can embody unique market dynamics and risks. Short-term frames may reflect market noise and volatility, while long-term ones could capture lasting market changes. This choice alters perceived risk and return attributes of the portfolio, demonstrated by the limitations imposed by OBX's fewer data points relative to other series under study.

In summary, though diverse time frames provide varied portfolio analysis perspectives, they also bring limitations. Researchers and investors must acknowledge these constraints and interpret analysis results in the chosen time frame context.

5.4.2 Assets

The OBX index's fewer stocks compared to the S&P 500 index present analytical challenges. Its restricted sample size may limit the robustness and generalizability of findings, and its limited diversification may amplify individual stock impacts, potentially skewing analysis. Additionally, fewer stocks could constrain portfolio optimization due to reduced diversification possibilities. Thus, while OBX provides insights into the Norwegian market, these limitations should be considered in portfolio analysis and interpretation.

5.4.3 Portfolio and Forecasting

The enrichment and reinforcement of our research could be achieved by a more extensive portfolio optimization. In the presented study, we relied solely on the findings from the DCC-GARCH, given its superior performance in our particular scenario. Nonetheless, it is feasible to delve further into the investigated phenomenon by incorporating more robust methodologies, including machine learning techniques or the application of extreme value theory.

Moreover, we implement a forecast to enrich our study. However, the integration of different forecasting techniques could augment the predictive capacity of the model and facilitate subsequent fine-tuning to enhance performance. Thus, while our research provides valuable insights, there is scope for extending the analytical framework to offer a more comprehensive understanding of the portfolio optimization process.

6 Conclusion

This paper extends on the work of (Triki & Maatoug, 2021) and revisits the relationship between gold and the stock market, with particular emphasis on the influence of the GPR index. Moreover, the paper endeavor to enrich this analysis by examining the relationship between crude and OBX. The authors utilize two distinct multivariate GARCH models, namely DCC and Copula GARCH, to explore correlation and volatility spillover between the commodity, market, and the GPR index treated as an exogenous variable.

The results gleaned from these models may bear significant implications for portfolio investors, especially during periods of geopolitical stress. Furthermore, this research may prove instrumental in shaping financial and investment strategies for monetary policy makers, thereby enhancing the effectiveness of policy interventions in response to market dynamics.

The DCC-GARCH model analysis revealed crucial insights into the persistence of conditional variance in both commodity and stock index returns. The GARCH coefficients β_{ii} show a statistically significant enduring persistence for all assets except oil. The significant ω_1 estimate for crude oil suggests a long-term average volatility different from zero. Interestingly, while the previous period's residual does not significantly influence the current correlation, the previous period's correlation between commodity and index does have a significant influence on the current period's correlation.

The analysis also suggests that the GPR does not significantly influence the volatility or returns of these assets, as indicated by the non-significant κ_i and χ_i parameters. This underlines the intricate dynamics and dependencies in these markets, where long-term trends and past correlations play a crucial role, while the influence of GPR appears to be limited based on this specific model.

The VAR models provide valuable insights, notably, the models show a significant persistence in gold prices, reinforcing gold's status as a safe-haven asset and a good hedging instrument during times of increasing geopolitical risk. Conversely, results for crude oil remain ambiguous. The negative impact of an increase in GPR on OBX across models suggests that geopolitical tension tends to induce losses in the OBX index.

The Crude/OBX pair demonstrates higher levels of both positive and negative correlation compared to Gold/OBX, implying that Crude/OBX is relatively more akin to a perfect hedge during periods of market stress. Nevertheless, it's crucial to note that crude exhibits a higher degree of volatility, intimating that its utility as a hedging instrument could be challenging due to its inherent volatility.

This elevated volatility might result in a more expensive hedge if investors need to frequently readjust their exposure. While the costs associated with hedging lie beyond the scope of this paper, they remain a pertinent consideration when evaluating the efficacy of a hedge. However, in a "buy and hold" scenario, the cost of hedging may become less of a factor, underscoring the need to tailor hedging strategies to specific investment approaches and risk tolerance levels. Based on the evidence presented we conclude Hypothesis 1 as inconclusive

Figure 2 presents instances of negative correlation between Crude/OBX during certain periods of market stress. However, we encounter difficulties in discerning a consistent pattern across multiple events. This observation might suggest that crude exhibits safe-haven properties to a degree. The intrinsic nature of crude and its industrial applications may influence its variability, potentially impeding crude's ability to exhibit a consistent pattern of negative correlation over time.

While one cannot conclusively reject or confirm Hypothesis 2, the empirical evidence does hint at crude's potential as a short-term hedging instrument. Nevertheless, further research is required to fully ascertain the viability and consistency of crude as a safe-haven across different market conditions and periods. Hypothesis 2 remains inconclusive.

7 Appendix

Figure 5: Time-Series Plot of Commodities

The two series show the real return of our two commodities, namely gold and crude, from January 1985 to December 20222.

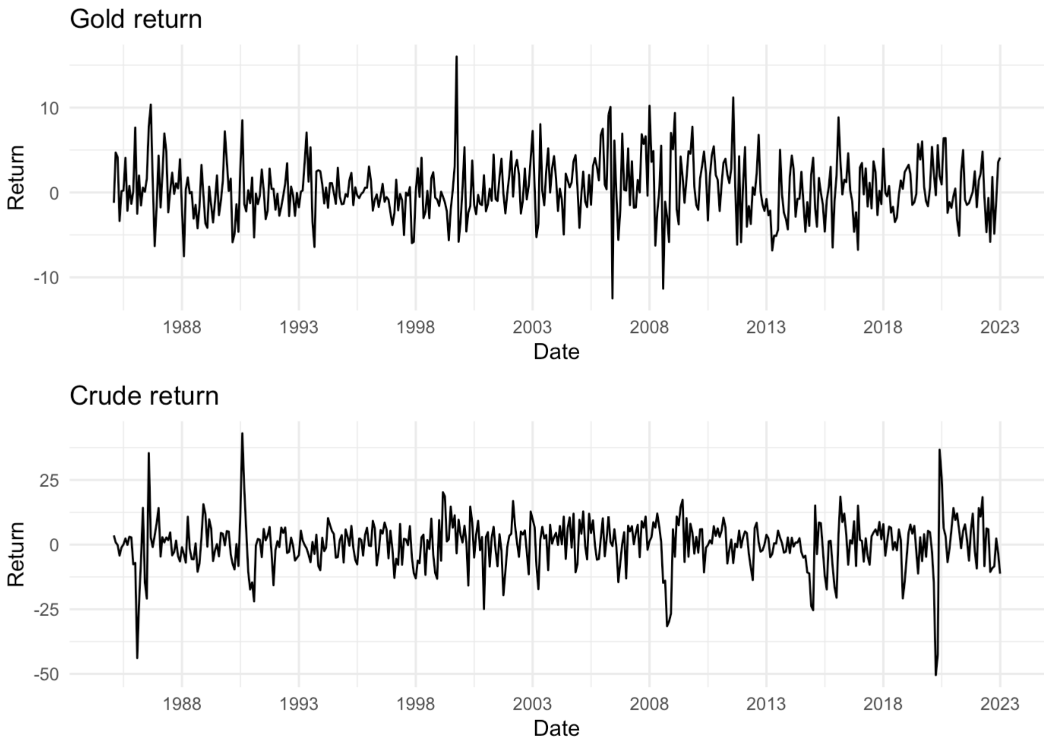


Figure 6: Time-Series Plot of Market Indices

The two series show the real return of our two indices, namely the S&P 500 and OBX. Notice a differing timeframe as the S&P 500 spans from January 1985 to December 20222, and the OBX from January 1996 to December 2022.

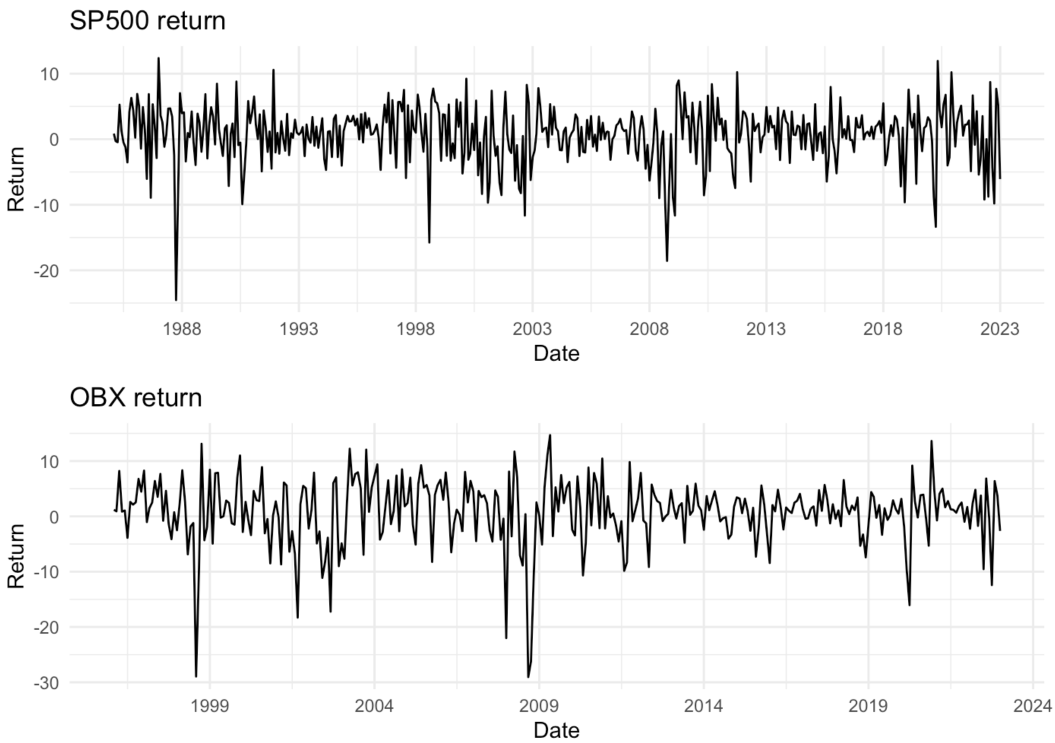


Figure 7: Time-Series Plot of GPR Index

This figure shows the index value of the GPR at different times in the period January 1985 to December 2022.

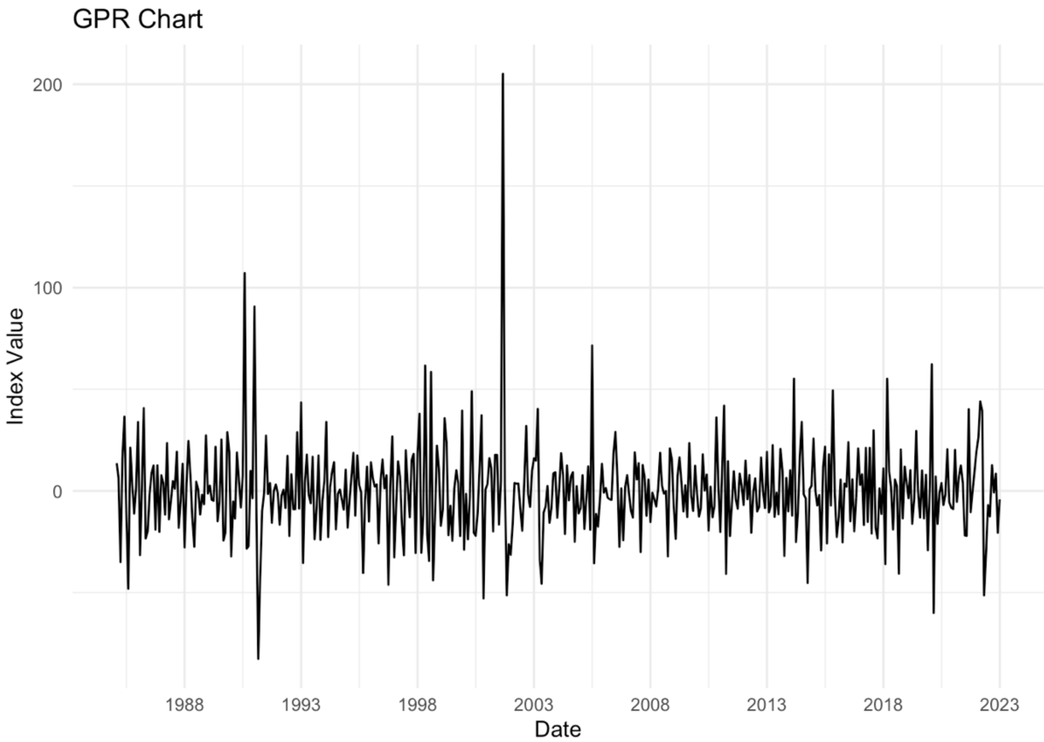


Figure 8: Rolling Rank Correlation for Gold and S&P 500

The figure shows the 6-month rolling rank correlation between gold and the S&P 500 from June 1985 to December 2022.

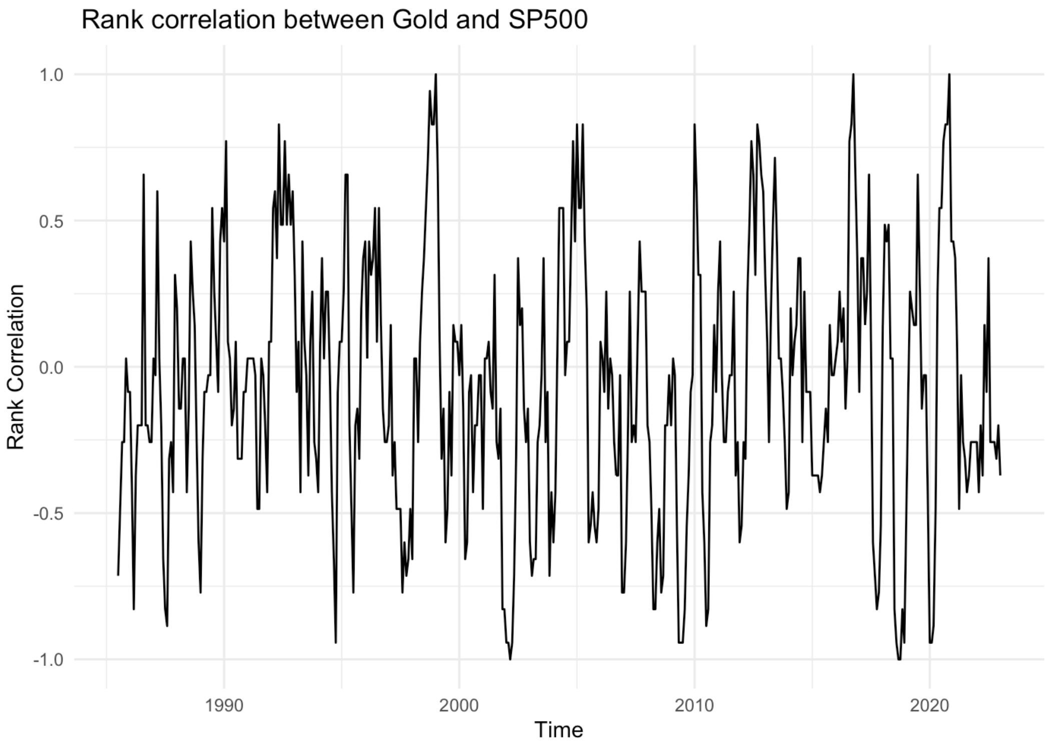


Figure 9: Rolling Rank Correlation for Crude and OBX

The figure shows the 6-month rolling rank correlation between Crude and the OBX from June 1996 to December 2022.

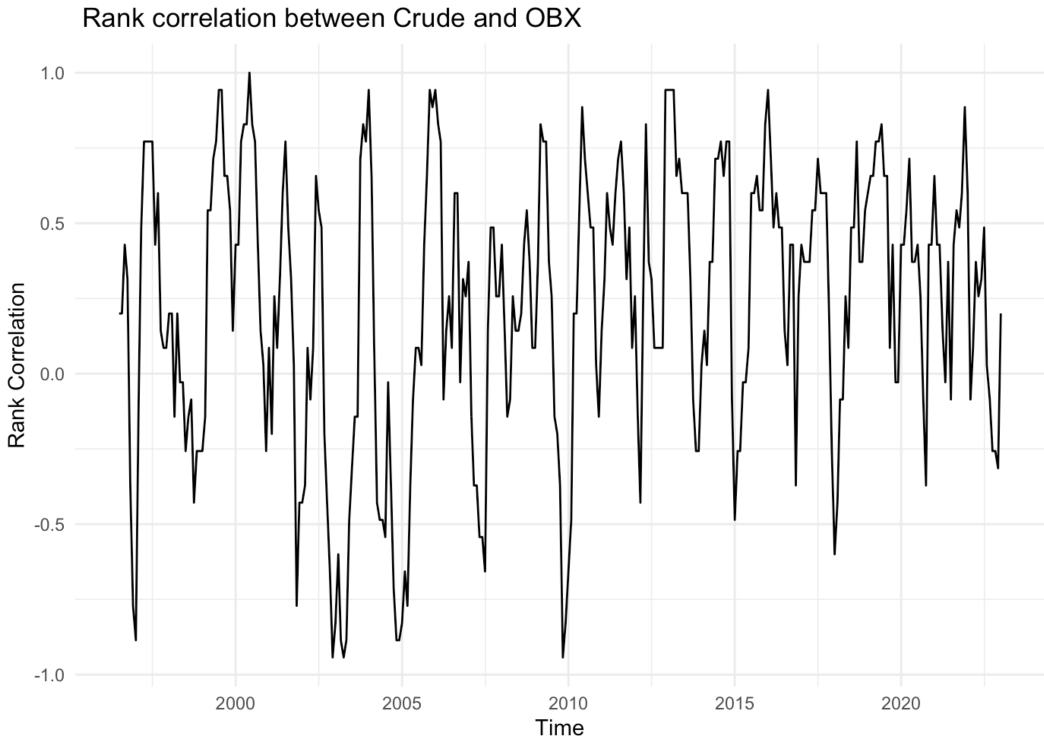


Figure 10: Rolling Rank Correlation for Gold and OBX

The figure shows the 6-month rolling rank correlation between gold and the OBX from June 1996 to December 2022.

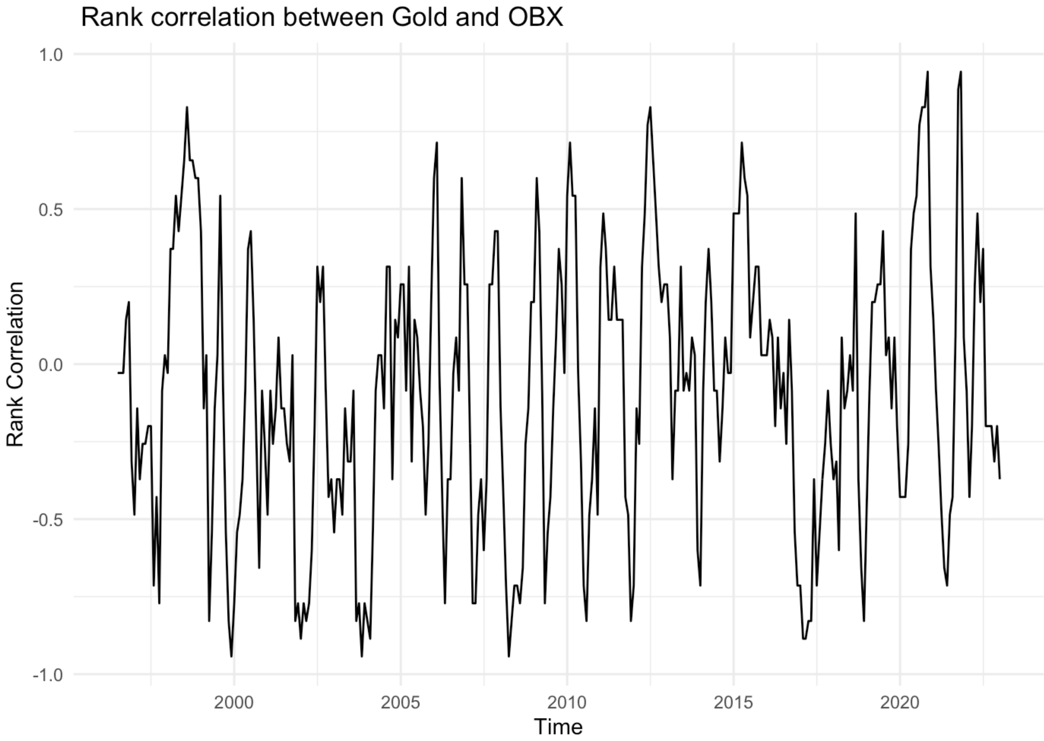


Figure 11: Rolling Rank Correlation of Crude and S&P 500

The figure shows the 6-month rolling rank correlation between crude and the S&P 500 from June 1985 to December 2022.

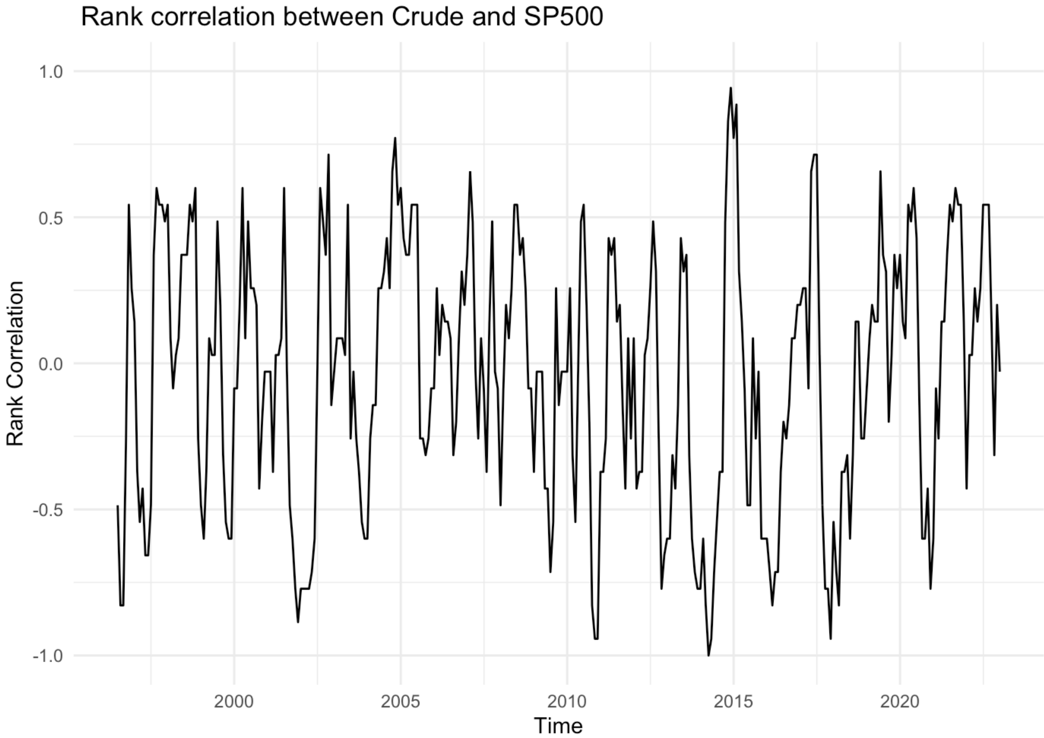


Table 6: Copulas-GARCH Outputs

The table shows our output from the Copulas-GARCH. The estimations are done on 419 or 287 observations depending on if the OBX is included in the pair or not. In all estimations the GPR is utilized as an exogenous variable.

Copulas-GARCH

Parameters	$GOLD_1/SP500_2$		$GOLD_1/OBX_2$		$CRUDE_1/SP500_2$		$CRUDE_1/OBX_2$	
	Coef	P-value	Coef	P-value	Coef	P-value	Coef	P-value
μ_1	0.231	0.307	0.387	0.107	**0.967**	0.025	1.253	0.132
AR_1	**0.190**	0.000	**0.190**	0.001	**0.169**	0.002	**0.176**	0.022
χ_1	0.010	0.129	0.008	0.227	0.021	0.348	0.010	0.762
ω_1	1.025	0.410	**2.563**	0.011	**40.585**	0.000	**46.770**	0.097
α_1	**0.113**	0.026	**0.131**	0.012	**0.566**	0.000	**0.422**	0.098
β_1	**0.805**	0.000	**0.671**	0.000	0.000	0.999	0.000	1.000
κ_1	0.099	0.321	0.028	0.704	0.000	1.000	0.000	1.000
μ_2	**0.711**	0.000	**1.045**	0.005	**0.711**	0.000	**1.045**	0.005
AR_2	-0.048	0.400	**0.107**	0.091	-0.048	0.413	**0.107**	0.101
χ_2	-0.013	0.195	-0.023	0.184	-0.013	0.197	-0.023	0.181
ω_2	0.623	0.192	3.270	0.129	0.623	0.191	3.270	0.124
α_2	**0.165**	0.000	**0.146**	0.092	**0.165**	0.000	**0.146**	0.099
β_2	**0.825**	0.000	**0.768**	0.000	**0.825**	0.000	**0.768**	0.000
κ_2	0.000	1.000	0.071	0.701	0.000	1.000	0.071	0.704
DCC_A	0.000	0.990	0.000	0.998	0.028	0.487	0.015	0.285
DCC_B	**0.966**	0.000	**0.986**	0.000	**0.958**	0.000	**0.966**	0.000
Log-Likelihood	**-2483.816**		**-1870.897**		**-2893.338**		**-2142.67**	

Figure 12: Out of Sample Forecast for Gold and OBX

The figure shows our forecast for the Gold/OBX pair from November 2019 until November 2022 (36 observations). The green line represents the unrestricted model, whilst the blue represents the restricted.

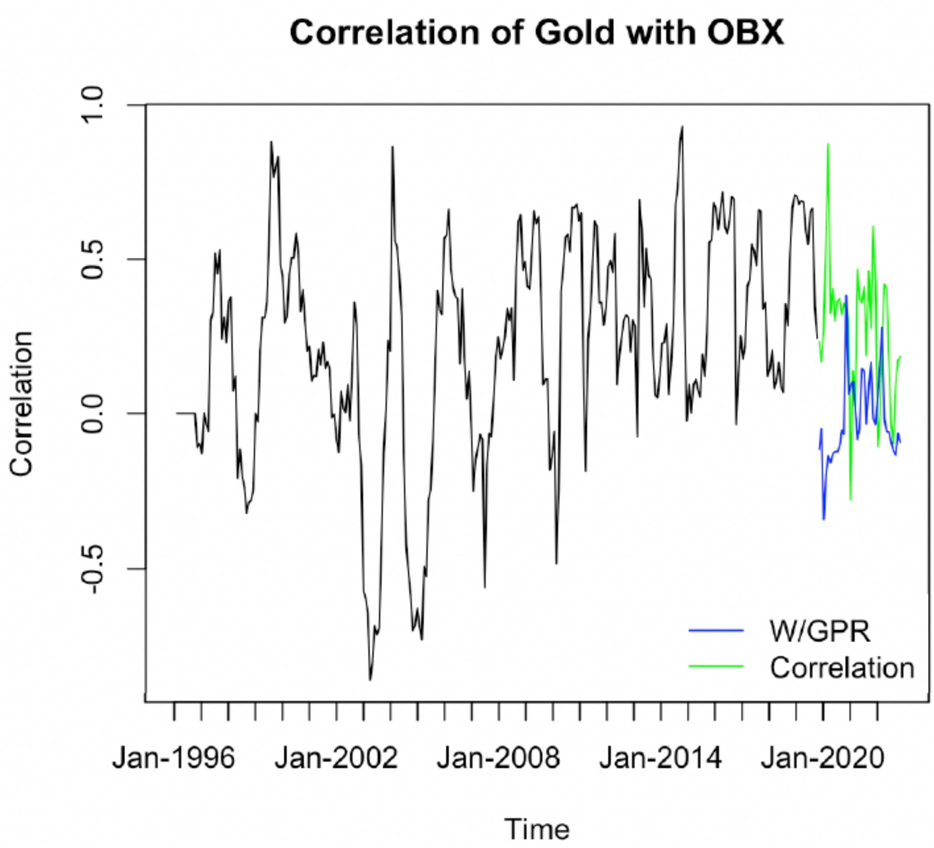


Figure 13: Out of Sample Forecast for Crude and S&P 500

The figure shows our forecast for the Crude/S&P 500 pair from November 2019 until November 2022 (36 observations). The green line represents the unrestricted model, whilst the blue represents the restricted.

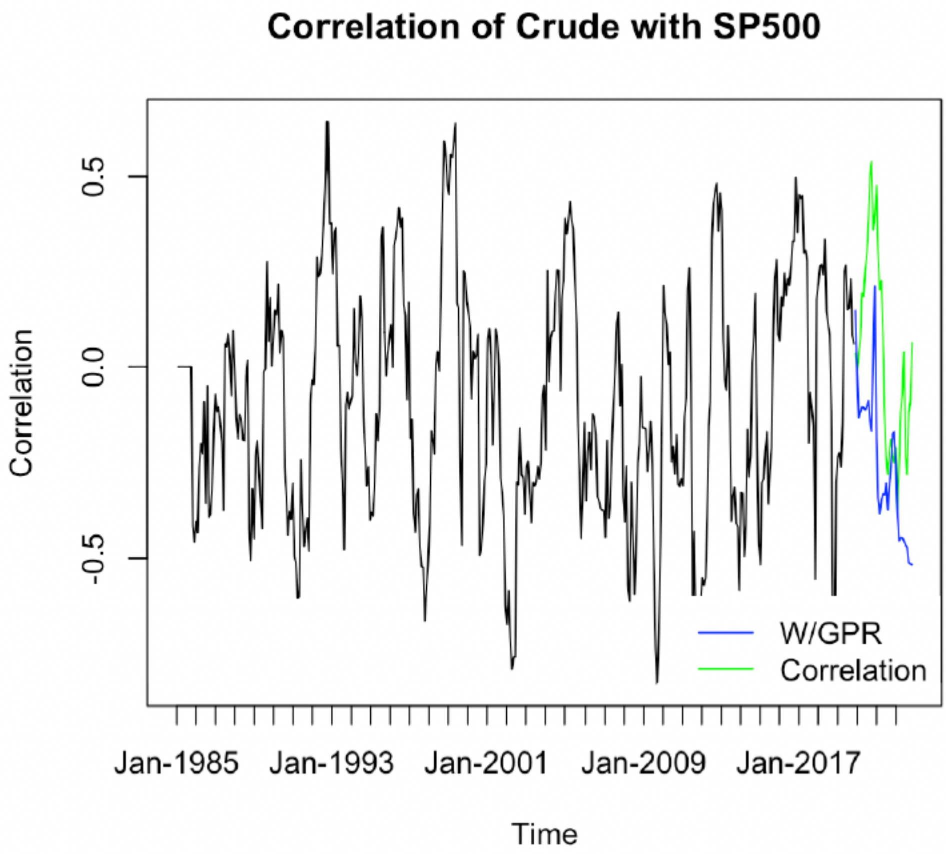


Figure 14: Hedge Ratio Gold/S&P 500

The figure shows our calculated hedge ratios for the Gold/SP500 pair based on our DCC-GARCH estimates. The figure spans from January 1985 to December 2022.

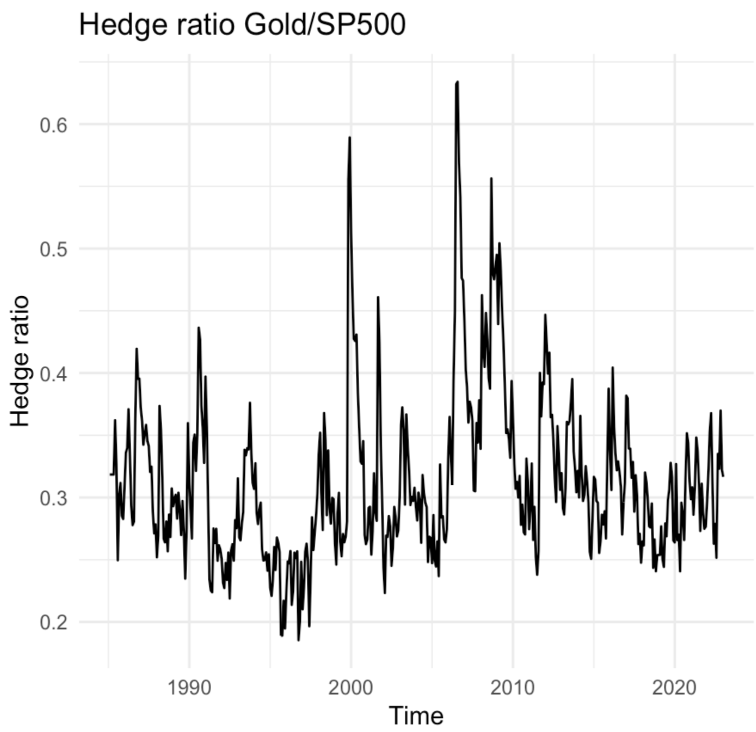


Figure 15: Hedge Ratio Crude/OBX

The figure shows our calculated hedge ratios for the Crude/OBX pair based on our DCC-GARCH estimates. The figure spans from January 1996 to December 2022.

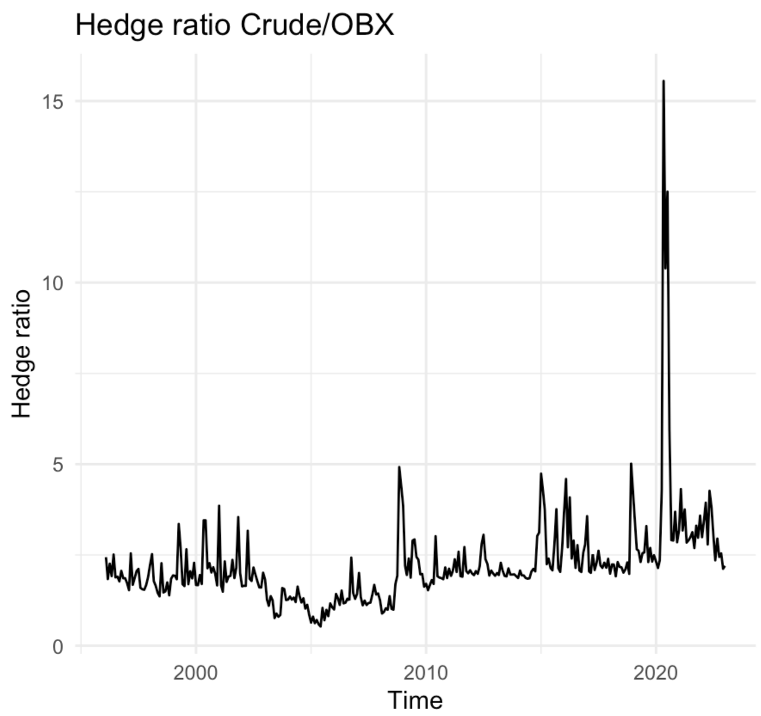


Figure 16: Hedge Ratio S&P 500/Gold

The figure shows our calculated hedge ratios for the S&P 500/Gold pair based on our DCC-GARCH estimates. The figure spans from January 1985 to December 2022.

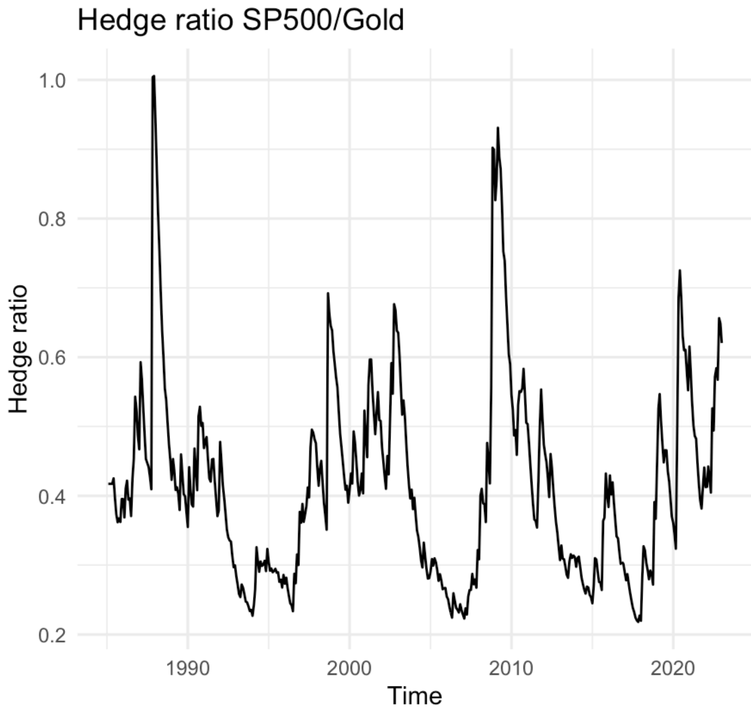


Figure 17: Hedge Ratio OBX/Crude

The figure shows our calculated hedge ratios for the OBX/Crude pair based on our DCC-GARCH estimates. The figure spans from January 1996 to December 2022.

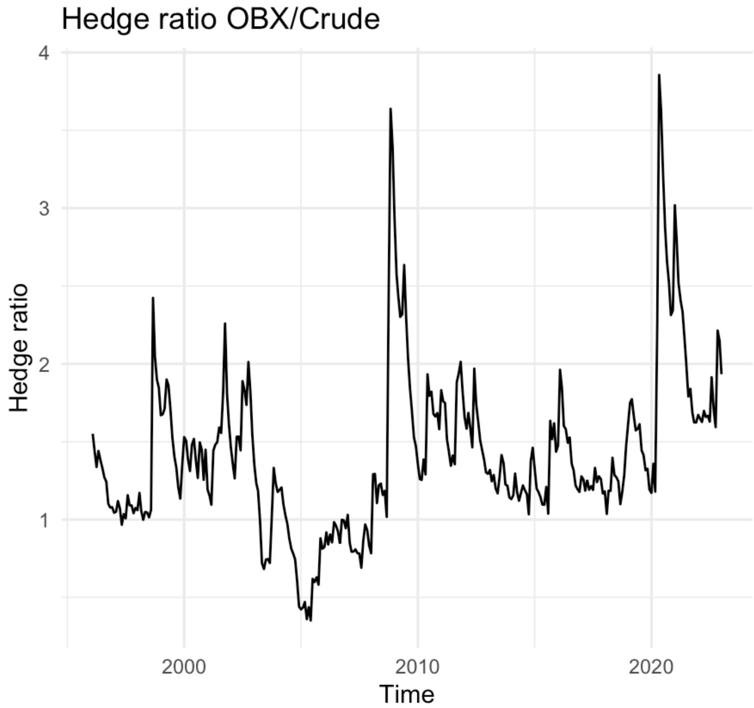


Figure 18: Hedge Ratio Gold/OBX

The figure shows our calculated hedge ratios for the Gold/OBX pair based on our DCC-GARCH estimates. The figure spans from January 1996 to December 2022.

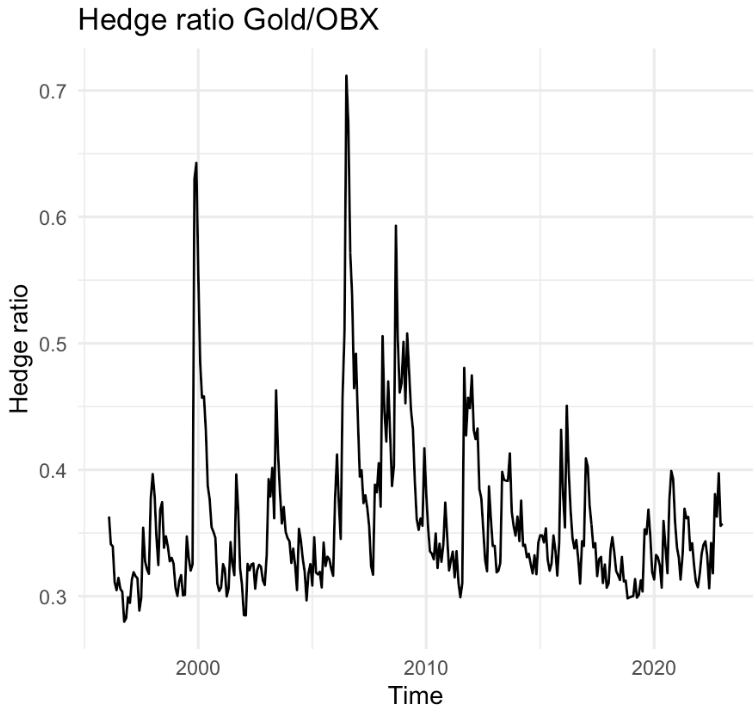


Figure 19: Hedge Ratio OBX/Gold

The figure shows our calculated hedge ratios for the OBX/Gold pair based on our DCC-GARCH estimates. The figure spans from January 1996 to December 2022.

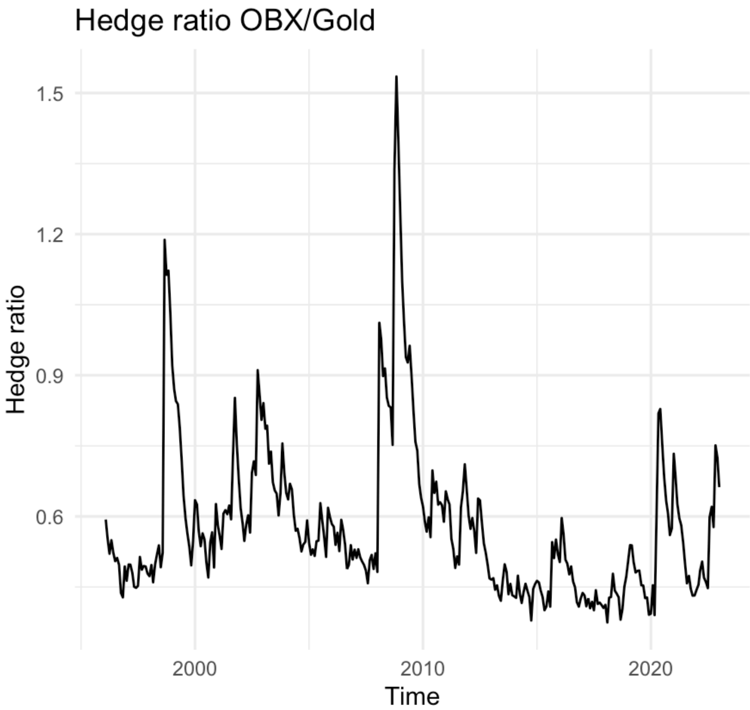


Figure 20: Hedge Ratio Crude/S&P 500

The figure shows our calculated hedge ratios for the Crude/S&P 500 pair based on our DCC-GARCH estimates. The figure spans from January 1985 to December 2022.

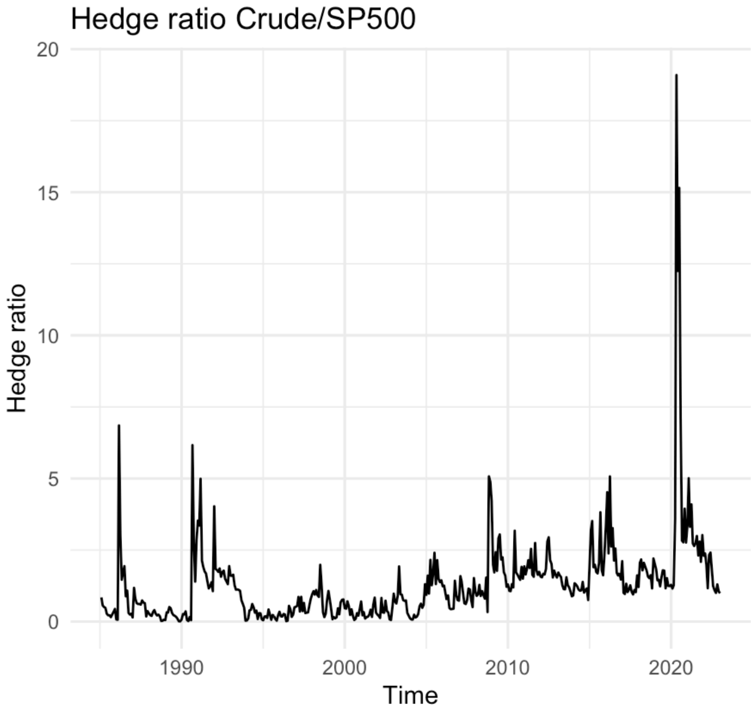
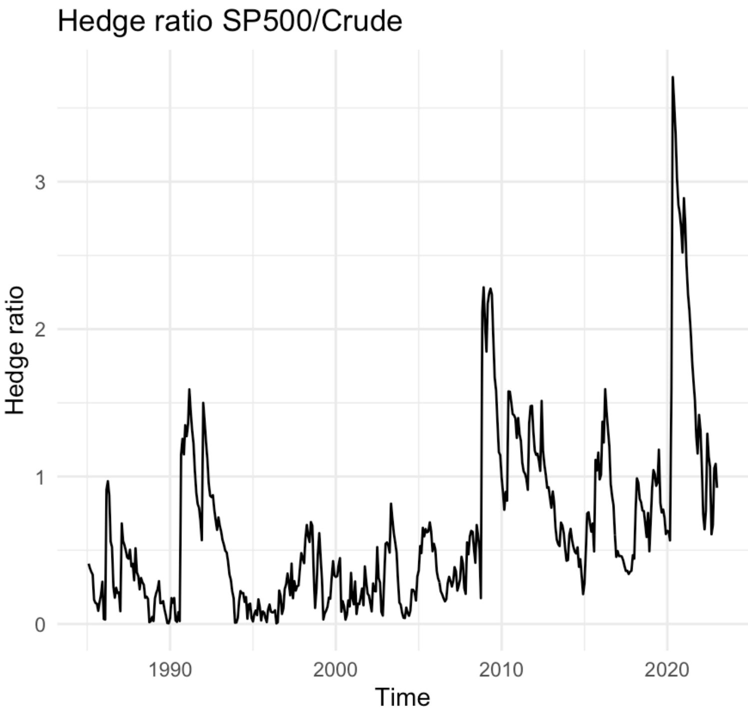


Figure 21: Hedge Ratio S&P 500/Crude

The figure shows our calculated hedge ratios for the SP500/Crude pair based on our DCC-GARCH estimates. The figure spans from January 1985 to December 2022.



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