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Transportation strategies for dynamic lot sizing: Single or multiple modes?

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Abstract
The complexity of decision-making for companies buying transportation services has increased due to the presence of more options and pricing schedules for transportation. Many companies make transportation and inventory decisions in an uncoordinated way and select only one transportation mode, missing opportunities for logistics cost savings. The experimental study in this paper is based on a real-world decision problem faced by a Scandinavian company that distributes fast-moving consumer goods and wants to determine its transportation strategy. We propose a novel multi-mode lot-sizing model with dynamic deterministic demand to illustrate the cost impact of accurately modelling piecewise-linear transportation costs and allowing a more flexible usage of transportation modes when planning order replenishments. We compare three transportation strategies with increasing degrees of flexibility: two single mode strategies, where one strategy is more flexible than the other, and a multi-mode strategy. We conclude that managers can significantly reduce costs by increasing the flexibility of mode selection in transportation strategies.

KEYWORDS
Transportation strategy; Mode Selection; Transportation costs; Inventory Management; Lot sizing

1. Introduction

Companies procure transportation services provided via multiple modes of transportation by air, road, rail and sea, with different costs, capacities and lead times (Lu et al., 2020). According to Lapierre, Ruiz, and Soriano (2004), typical transportation modes include parcel deliveries for small shipments, less-than-truckload (LTL) for intermediate shipments and full-truckload (FTL) for large shipments. Transportation price functions are non linear for LTL mode and piecewise linear for FTL mode. According to Bausch, Brown, and Ronen (1994), even organizations with a private fleet consisting of heterogeneous vehicles have to choose among multiple modes.

In practice, shippers may source services from several logistics companies offering multiple transportation FTL modes, for example containers of various sizes, and LTL modes, and may switch from one mode to another to achieve logistics costs savings. Various modes can be related to the same transportation mean, for example sea containers with capacities of 20, 30 and 40 feet shipped by the same vessel, which are considered as various FTL modes. A contract between the shipper and the carrier, i.e. the transportation company, specifies the transportation modes, the compensation format, often negotiated based on a forecast annual freight volume, and the applied Incoterms; that is the trading terms that regulate whether the buyer or the seller carries the transfer risk and the freight costs, and from which location. The literature on supplier selection has widely studied multiple sourcing, also known as order-splitting, which occurs when several suppliers deliver a partial order, in particular when all suppliers have small capacities or are non-dominant with regards to some specific criteria (for example delivery time, price or quality) (Aissaoui, Haouari, and Hassini, 2007). However, the number of studies considering multiple transportation modes for partial orders is rather small, despite the potential advantages of cost and emission reductions, as well as the lower risk in case of disruptions (Engebrethsen and Dauzère-Pérès, 2019). Instead of using analytical planning tools, considering inventory and transportation planning decisions simultaneously, shippers often make subjective choices of transportation modes that may not be optimal (Caputo, Fratocchi, and Pelagagge, 2006). For example, order lot size decisions are based on a single predetermined transportation batch size or are taken prior to transportation mode decisions. In the inventory management literature, the optimal order size needs to be determined by minimizing the total logistics cost, including the ordering and inventory holding costs. Although the transportation costs constitute a significant part of the logistics costs, in most inventory management models, they are often omitted or oversimplified by disregarding the availability of multiple modes, discounts and transportation capacity. According to Ke, Bookbinder, and Kilgour (2014), any savings achieved through improved inventory management are overwhelmed by such inaccuracy. Büyükkaramikli, Gürler, and Alp (2014) stress that joint modelling of inventory replenishment and transportation operations have not been much elaborated in the literature.

Although it is motivated by the real-world decision problem faced by a Scandinavian company, the problem studied in this paper is relevant for many companies in different industries as an increased number of companies outsource their transportation services. The main characteristics of the modes considered in this paper are the price structure (FTL or LTL) and the capacity, which are common for most transportation services. The proposed mathematical model aims to improve the decision-making practice in many companies, and in particular the case company, where order lot sizing and transportation planning are performed in a disaggregated manner and using a single mode. Previous research stresses the need to integrate these decisions, and to address research gaps such as the consideration of a single mode and the simplified modelling of costs. We compare three transportation strategies related to mode usage by optimizing the total logistics costs for each strategy over 600 realistic problem instances. In the experimental study, the base case is built based on real data from the case company for the product demand forecast and the transportation modes with prices and capacities associated to a specific origin-destination pair. The proposed model is generic, as the considered mode capacities are offered by most logistics companies, as they correspond to the sizes of the most often used intermodal ISO containers expressed in Euro pallets. The results of our empirical analysis show that the multi-mode strategy (MM) allows significant cost reductions, up to 15% savings on total logistics costs, particularly compared to the SM strategy, but also compared to the SSM strategy, with up to 6% cost savings.

The potential cost saving from applying a multi-mode transportation strategy depends on both mode-dependent parameters, related to mode costs and capacities, and mode-independent parameters, related to the characteristics of the demands and inventory or ordering costs. The analysis of mode-dependent parameters shows that the savings are the largest when one can choose among both FTL and LTL modes, and when the cost and capacity differences are small among the modes. The analysis of mode-independent problem parameters shows that the largest savings and most frequent mode combinations are observed when inventory costs are high and the demand mean is small. Therefore, managers need to understand the impact of accepting simplified decisions (such as the use of a single mode instead of a combination of modes) on the total cost, and in which cases the saving potential is the highest to balance the expenses related to managing additional transportation modes. The main contributions of this article are:

1. A joint inventory and transportation planning model with multiple FTL and LTL modes considering typical piecewise linear transportation price structures observed on the freight market. The methodological contributions include formalizing the over-declaring practice based on the price list and applying the multiple choice formulation for LTL modes as in Croxton, Gendron, and Magnanti (2003a). In contrast to the previous research, our model finds the optimal combination of any FTL and LTL mode for any quantity without pre-processing;
2. Computational experiments considering empirical data for transportation costs and modes to test the impact of flexibility in transportation strategies on cost savings;
3. An investigation of the impact of various model parameters, both mode-dependent (that is, mode cost or capacity difference) and mode-independent (such as demand and holding costs), on cost savings and mode combination frequency, i.e how often the modes are combined when ordering;
4. Managerial recommendations on when different transportation strategies are the most relevant.

The remainder of the article is organized as follows.The decision problem and the scope of analysis is presented in 2. Sections 3 and 3.2 briefly describe the typical transportation modes and strategies used in practice, as well as the modelling methods in inventory management research. Then, a novel multi-mode inventory model, motivated by the case of a Scandinavian distribution company, is introduced in Section 5, followed by a description of the methodology for the parameter analysis of various transportation strategies based on actual data in Section 6. In Section 7 we analyse the different transportation strategies with regards to mode usage and investigate the impact of mode-dependent and mode-independent parameters on the total costs. Finally, we discuss the managerial implications of the analyzed transportation strategies in Section 8 and propose directions for future research in Section 9.

2. Problem statement and a scope of the analysis

We consider the real case of a Scandinavian distribution company that imports fast-moving consumer goods for a retail chain, and needs to decide when and how much to order from a specific supplier to a central warehouse, as well as the transportation modes among several FTL and LTL alternatives, so that the total logistics costs are minimized. The data from the company, including the transportation mode prices, the costs and the demands are used in the computational study. Motivated by the observed problem, a multi-mode dynamic lot-sizing model is proposed to analyse the following transportation strategies with various degrees of flexibility:

(1) The 'Single Mode' (SM) strategy, where only one transportation mode with the lowest unit cost is allowed in each period. The same mode is used throughout the entire planning horizon. This strategy is based on the observations of real-life practice, where companies often choose a single mode with the lowest unit cost. This practice makes it easier to manage a single mode and to include the selected mode capacity as a batch size in order planning tools. However, the mode with the lowest unit cost usually has the largest loading capacity and may potentially increase the holding costs.
(2) The 'Single Mode Shifting' (SSM) strategy, proposed by Diaby and Martel (1993) who used the pre-processing method without combining modes, where only one transportation mode is allowed in each period, but the mode can be different for each period. This strategy is motivated by the observations that the majority of logistics service providers offer multiple modes. The shippers therefore negotiate contracts for several modes with several logistics service providers to have the flexibility when the order size varies or to secure a back-up supplier. However, it can be time consuming to follow-up several transporters if the modes are provided by different freight companies during the same period, hence the shippers use only one mode per period.
(3) The 'Multi-Mode' (MM) strategy, where partial orders by each transportation mode and any mode combination are allowed in each period, (as in Jaruphongsa, Cetinkaya, and Lee (2005) and Absi et al. (2013)). This strategy is the most flexible compared to the others, allowing to combine and change modes every period.

The goal of our analysis is to understand and evaluate how the flexibility level on mode selection impacts the performance of a transportation strategy in terms of potential cost savings, and how the model parameters impact this performance. We examine both transportation mode-dependent model parameters (such as the number and types of modes, costs, and capacities) and mode-independent parameters. The suggested Mixed-Integer Linear Programming (MILP) model and modelling details of all strategies can be found in Section 5.

3. Transportation costs and modes

In this section, we focus on relevant research and practice within inventory management considering multiple transportation modes and mode-related strategies relevant for the problem discussed above. For a more general review of dynamic lot sizing research, we refer the reader to Brahimi et al. (2017), and to Mosca, Vidyarthi, and Satir (2019) that focus on integrated inventory-transportation models. Mosca, Vidyarthi, and Satir (2019) stress that integrated modelling techniques have risen in popularity, specially those that simultaneously address transportation and inventory decisions, in particular routing, transportation policy and mode selection, VMI and environmental concerns. The authors call for covering more realistic industry practices and organizational policies, incorporating more complex transportation policies and using a piecewise transportation cost structure due to the rise of LTL shipments, which is also a motivation for the problem studied in this paper.

3.1. *Transportation costs*

A fixed price is charged for an FTL shipment up to a full capacity of a vehicle or container, expressed in pallets, volume or weight units, also known as multiple-setup cost structure (Toptal, 2012). The price per unit shipped is usually the lowest for the FTL mode with the largest capacity when different FTL modes are available for the same origin-destination, assuming that the mode is fully loaded (Engebrethsen and Dauzère-Pérès, 2019).

For smaller shipments, LTL modes are preferred, where different customer orders are consolidated to fill up the shipping capacity. A piecewise linear, all-unit discount cost function, where the price discount applies to all units, with a minimum fee discouraging from sending shipments of extremely small size, is typical for a LTL mode. Over-declaring is a common practice for LTL shipments to obtain a lower price corresponding to the next rate breakpoint, when the shipment quantity is between the rate breakpoint and a so-called indifference point. An indifference point corresponds to a quantity that, when multiplied by its corresponding unit rate, is equal to the total costs charged at the next rate, and can be observed in some price intervals (Russell and Krajewski, 1991).

Many transportation companies quote prices with various rates and product class structures (depending on density and value of the shipment), which complicates the comparison of rates for shippers in a straightforward way, and thus motivates more advanced decision support (Engebrethsen and Dauzère-Pérès, 2019). Archetti, Bertazzi, and Speranza (2014) studied the classical economic inventory replenishment problem with discounts and stressed that the all-unit discount schedule applied for LTL cost modelling is known to be NP-hard. Despite the existence of freight discount schedules in practice, most inventory models simplify the transportation costs and disregard mode capacities and over-declaring. In a review of inventory models, Engebrethsen and Dauzère-Pérès (2019) provide classifications of methods for modelling multiple modes and transportation costs, stressing that realistic transportation cost structures are piecewise linear and observing several shortcomings in the existing literature:

- *Ignoring transportation costs.* According to Mendoza and Ventura (2013), an increase of almost 15% of the average monthly logistics costs can be observed when transportation costs are not considered.
- *Simplifying transportation costs* and disregarding discount schedules, for example by assuming constant unit costs (Swenseth and Godfrey (2002) reported a 37% cost difference when considering discounts), using approximation cost functions (3% higher transportation costs according to Ventura, Valdebenito, and Golany (2013)) or including freight costs into purchasing or set-up costs.
- *Handling the freight discount similar to purchasing discounts,* disregarding transportation capacity limits and the over-declaring practice leading to sub-optimal lot-sizing decisions.
- *Assuming the availability of only one transportation mode.*

The above-mentioned shortcomings make the problems simpler to model and solve, although this can lead to increased costs and suboptimal freight plans. Implicitly including transportation costs into the purchasing unit cost is not relevant if the buyer has the responsibility for transportation and can choose among various modes, according to certain trading terms (Incoterms). Therefore, the inventory policy and logistics costs are affected by the transportation price structure. In the proposed model, our goal is to overcome the shortcomings of the previous research by modelling realistic price-schedules with multiple modes and capacities and by formalizing over-declaring for LTL modes.

3.2. *Modelling multiple modes*

When multiple modes are available, there are two ways of modelling transportation costs according to Engebrethsen and Dauzère-Pérès (2019):

(1) *One cost function represents multiple modes.* The transportation modes and costs are predefined for each quantity, without the mode choice or combination of modes being a decision variable, and using one of the following methods:
   - Creation of a general cost function by pre-processing the costs and selecting for each quantity the lowest cost mode, assuming that modes cannot be combined for the same quantity. For each piecewise-linear segment of the pre-processed LTL cost function, a fixed cost and a variable cost are provided as inputs of the optimization model (Diaby and Martel, 1993) or (Croxton, Gendron, and Magnanti, 2003b), without explaining how these costs can be obtained from a real price schedule. Ignoring over-declaring leads to a sub-optimal solution.
   - A 'carload' discount schedule, where a single cost function represents only two modes, an LTL mode with a single price interval and a FTL mode, with identical capacity, as in Li, Hsu, and Xiao (2004). Freight rates from the carriers can be directly used in this approach, and it is easy to model without pre-processing.

(2) *Each transportation mode is explicitly modelled with its respective cost function and a decision variable related to the shipment quantity.* The main benefit is that lower costs can be obtained compared to the first approaches. Using this approach, the carriers' freight rates and restrictions per mode can be used in a straightforward manner in the model, allowing mode combinations as in Jaruphongsa, Çetinkaya, and Lee (2007), who considered multiple FTL modes. We apply this method in our model for multiple FTL and LTL modes, allowing the model to find the optimal combination of the modes, as an additional combinatorial optimization decision layer, and considering over-declaring.

Several papers considered multiple modes for a dynamic deterministic demand, either assuming simplified transportation costs or restricted mode usage. Diaby and Martel (1993) proposed one of the first inventory planning models for a dynamic deterministic demand with multiple modes assuming various discount schedules without modal split. This was further extended by Rizk, Martel, and Ramudhin (2006) and Rizk, Martel, and D'Amours (2006) by including multiple items and considering transportation cost as a part of the purchasing cost, so-called unit replenishment cost. These models apply a pre-processing approach for replenishment cost modelling, where general cost functions have been created and different modes could not be combined for the same shipment. Croxton, Gendron, and Magnanti (2003b) considered a merge-in-transit system with two echelons, considering four modes (a small package mode, a single LTL mode, a single FTL mode and an air mode), all pre-processed and without over-declaring, not allowing modes to be combined. The authors proposed disaggregation techniques and a cost approximation approach to improve solution times, suggesting that, in future research, model split should be further investigated. Li, Hsu, and Xiao (2004) proposed a lot-sizing model for a single item with a car-load discount schedule for one FTL mode and one LTL mode with a single price break, allowing a combination of the modes. Jaruphongsa, Cetinkaya, and Lee (2005) and Jaruphongsa, Çetinkaya, and Lee (2007) modelled two modes with an FTL-like multiple set-up structure, assuming that the capacities of the two modes are integers of each other and allowing the two modes to be combined for the same order. Palak, Ekşioğlu, and Geunes (2018) applied the model proposed by Jaruphongsa, Cetinkaya, and Lee (2005) for perishable product replenishments. Ekşioğlu (2009) extended the work of Jaruphongsa, Çetinkaya, and Lee (2007) by including more than two multiple FTL-like modes. Hammami, Frein, and Hadj-Alouane (2012) and Mogale et al. (2017) assumed constant unit costs for transportation when modelling multiple transportation modes. Choudhary and Shankar (2013) and Choudhary and Shankar (2014) considered a situation involving multiple carriers with FTL fleets, where a single carrier with a limited capacity per period should be selected for each supplier. Kopanos, Puigjaner, and Georgiadis (2012) modelled decisions on the procurement of additional FTLs from an external transportation company to be used in addition to an internal fleet that has unit transportation costs every period. Toptal, Koc, and Sabuncuoglu (2014) considered two FTL types of vehicles for deliveries from a plant to customers, not allowing order splitting among various modes. Absi et al. (2013) and Absi et al. (2016) proposed a dynamic inventory model with multiple replenishment modes, each having a fixed cost and a unit cost and carbon emission parameter for both transportation and production. In this model, the modes should be selected without violating the carbon emission constraints to satisfy the demand, while minimizing the total costs. Modes can be combined, but transportation capacity limits are not considered. When the transportation mode is modelled as a part of the replenishment mode, the supplier also needs to be selected. Akbalik and Rapine (2018) studied a single-item uncapacitated inventory problem with multi-mode replenishment and batch deliveries, where each replenishment mode has an FTL cost structure and incurs a fixed ordering cost plus a fixed cost per batch. The authors show that this problem is NP-hard even for a single period, and use dynamic programming algorithms and heuristics to solve it. Hwang and Kang (2016) proposed a two-phase algorithm for the lot-sizing problem with backlogging for stepwise transportation cost without speculative motives, considering a single FTL and LTL modes with linear unit cost available, assuming that carriers could vary over periods.Ventura et al. (2022) considered product supplier selection decision in a multi-stage supply chain with multiple FTL modes,comparing integrated approach for simultaneous determination of optimal dynamic supplier selection and inventory-transportation planning to a sequential approach.

The planning model used in our study differs from the existing inventory management research by considering multiple FTL and LTL modes, realistic discount schedules and over-declaring, by explicitly modelling the cost function for each mode.

4. Transportation strategies for using modes in inventory management

Only few studies have focused on investigating transportation strategies with regards to the factors impacting the use of the transportation modes and the costs considering realistic transportation costs. Rieksts and Ventura (2010) study an inventory problem with FTL and LTL modes available for a static demand case. They concluded that using multiple modes simultaneously can be optimal, in particular when the setup and inventory costs are dominating the other costs. Jain, Groenevelt, and Rudi (2010) and Jain, Groenevelt, and Rudi (2011) studied two freight modes, regular and express, each characterized by variable and fixed costs, for a stochastic demand case. Jain, Groenevelt, and Rudi (2011) identified more than 5% savings for the best (s, S) policy when using both modes. The authors conclude that a single mode is preferred if the ordering cost is small compared to the fixed costs of the transportation modes, as the transportation costs dominate the inventory cost savings. In the opposite case, and when the variable costs of the express mode is not too high, using both modes provides cost savings. If one mode is dominating the other, the costs of the policy combining modes is closer to a single-mode policy.

To our knowledge, no analysis of transportation strategies and the parameters impacting the mode usage has been conducted for the case of dynamic deterministic demand considering the flexibility of switching and combining multiple modes with realistic cost schedules.

The main contributions of this paper are summarized in Table 1.

Table 1. Summary of contributions

Topic	Research gap	Our contribution
Transport. costs	Simplified (constant), omitted or part of ordering costs No over-declaring modelled	LTL and FTL price schedules with realistic discounts Modelling of over-declaring and discounts
Transport. modes	Uncapacitated or homogeneous capacities Combination not allowed or at most two modes Pre-processed general cost function or car-load discount schedule for multiple modes	Heterogeneous capacities Any combination allowed Each mode has own cost function
Multimode strategy and methodology	Inventory and transportation decisions dissaggregated Only two strategies analyzed (single vs. multi-mode) Only static and stochastic demand Only economic benefits of multi-mode strategy and few parameters investigated	Integrated transportation mode selection and inventory lot-sizing Three strategies compared (based on industry practice (SM, SSM, MM) Dynamic deterministic demand Both economical benefits and computational complexity investigated New combinatorial optimization decision layer for optimal mode combination added

5. Inventory model with multiple transportation modes

5.1. *Notations and formulation*

The classical dynamic deterministic lot-sizing problem assumes a time-varying and known demand over a discrete finite horizon and a single supplier to replenish the inventory. We extend this problem by assuming that several FTL and LTL transportation modes with piecewise-linear costs are available, and we need to decide the order timing and size, as well as the quantity allocated to each mode for every period. The capacity of each mode is expressed in pallets. The objective is to minimize the total ordering, holding, and transportation costs over the finite horizon without any shortages. There are no constraints on the quantity ordered in each period and backlogging is not allowed.

We assume that the buyer has to satisfy the demand without shortage on a time horizon of T periods, with the possibility of using up to M different FTL modes and up to N different LTL modes with up to J nominal price break intervals each. Ekşioğlu

(2009) suggested that the problem with various transportation lead times which are longer than a single planning period, can be transformed into a problem with zero lead times by adding in-transit inventory holding costs to the procurement costs and placing the order earlier. Therefore, in this model, if an order is placed at period t to satisfy demand d_t, then the delivery is assumed to be instantaneous. There is a fixed ordering cost S per shipment and an inventory holding cost h per unit. For each FTL mode m, there is a given capacity in pallets K_m^{FTL} per container and a freight rate per container C_m^{FTL}. For each LTL mode n, there is a freight rate r_{nj} per pallet shipped by LTL mode n within the interval j, as well as minimum and maximum prices C_n^{min} and C_n^{max}, respectively.

The decision variables for each period are the total quantity to be shipped in pallets, Q_t, the number of pallets shipped by FTL mode m denoted by X_{mt}^{FTL}, the number of FTL containers denoted by A_{mt}, and the number of pallets shipped by LTL mode n denoted by X_{nt}^{LTL}. The total costs for using the LTL mode n are denoted by $(TC)_{nt}$, whose modelling is detailed in section 3.2 and formalized in Constraint (18). The inventory level at the warehouse in period t is denoted by I_t, assuming $I_0 = 0$. We use a binary decision variable O_t to calculate the ordering costs in period t. The following notations are used in the model:

Sets:
T: Time horizon of T periods,
M: Number of different FTL modes,
N: Number of different LTL modes,
J: Number of nominal intervals for LTL modes.

Parameters:
d_t: Demand in period t, $t = 1, \ldots, T$,
h_t: Inventory holding cost per unit at the end of period t,
S: Ordering costs per shipment,
K_m^{FTL}: Container capacity in pallets per FTL mode m,
K_{max}^{FTL}: Maximum container capacity among all the FTL modes,
C_m^{FTL}: Container cost per FTL mode m,
r_{nj}: Freight rate per pallet per LTL mode n within interval j,
C_n^{min}: Minimum price for small shipments for LTL mode n,
C_n^{max}: Maximum price for LTL mode n,
K_{nj}: Quantity limit for price interval j for LTL mode n,
K_n^{max}: Maximum quantity limit for LTL mode n,
B_{nj}: Indifference breakpoints for LTL mode n and interval j.

Decision Variables:
Q_t: Total quantity in pallets to be shipped each period t,
X_{mt}^{FTL}: Quantity in pallets shipped by FTL mode m in period t,
X_{nt}^{LTL}: Quantity in pallets shipped by LTL mode n in period t,
A_{mt}: Integer number of FTL containers used in period t,
$(TC)_{nt}$: Costs for using LTL mode n in period t,
I_t: Inventory level in period t carried to period $t+1$,
O_t: Binary variable which is equal to 1 if a positive quantity is ordered in period t, and 0 otherwise,
Y_{njt}: Binary variable that ensures that, for each LTL mode n and each period t, at most one re-defined LTL interval for X_{nt}^{LTL} is chosen,
λ_{njt}: Continuous variable between 0 and 1, which can be strictly positive at most once for

each LTL mode n and period t.

The model below shows the objective function and the constraints and decision variables related to the FTL modes, while most constraints and decision variables related to the LTL modes, corresponding to Constraints (11) to (19), are formalized in Section 5.2.

$$\min S\sum_{t=1}^{T} O_t + h\sum_{t=1}^{T} I_t + \sum_{m=1}^{M}\sum_{t=1}^{T} C_m^{FTL} A_{mt} + \sum_{n=1}^{N}\sum_{t=1}^{T} (TC)_{nt} \tag{1}$$

$$Q_t = \sum_{m=1}^{M} X_{mt}^{FTL} + \sum_{n=1}^{N} X_{nt}^{LTL} \qquad t = 1, ..., T \tag{2}$$

$$I_t = Q_t - d_t + I_{t-1} \qquad t = 1, ..., T \tag{3}$$

$$Q_t \leq (\sum_{k=t}^{T} d_k - 1 + K_{max}^{FTL})O_t \qquad t = 1, ..., T \tag{4}$$

$$X_{mt}^{FTL} \leq K_m^{FTL} A_{mt} \qquad t = 1, ..., T; \qquad m = 1, ..., M \tag{5}$$

$$A_{mt} \in \mathbb{N} = \{1, 2, 3, \ldots, \infty\}, \quad O_t \in \{0, 1\} \qquad t = 1, ..., T; \quad m = 1, ..., M \tag{6}$$

$$Q_t, \quad I_t, \quad X_{mt}^{FTL}, \quad X_{nt}^{LTL} \geq 0 \qquad t = 1, ..., T; \quad m = 1, ..., M; \quad n = 1, ..., N \tag{7}$$

$$+ \text{ Constraints (11) to (19)}$$

The objective function (1) minimizes the sum of ordering costs, inventory holding costs and the transportation costs for using FTL and LTL modes. Constraint (2) specifies that the total quantity shipped to the warehouse is the sum of all shipments by LTL and FTL modes, while constraint (3) ensures the inventory balance. Note that, because of constraint (3), variable I_t could be removed by replacing it with $\sum_{k=1}^{t}(Q_t - d_t)$ if $I_0 = 0$. However, as in most lot-sizing models and because removing I_t has no impact on the resolution of the model, we keep I_t for model readability. Constraint (4) ensures that the fixed ordering cost is incurred each time there is a shipment, and also defines an upper bound for the optimal total shipped quantity Q_t. We want to define a big-M parameter in constraint $Q_t \leq MO_t$ that is as small as possible to tighten the constraint, and reduce the computational times when solving the mathematical model with a standard solver. M is defined as $\sum_{k=t}^{T} d_k - 1 + K_{max}^{FTL}$ based on the fact that having at least one unit on top of the remaining demand might create the need for utilizing the largest FTL mode. Constraint (5) states that the number of pallets shipped by FTL modes is equal to or lower than the number of pallets per FTL container, multiplied by the number of FTL containers. This implies that over-declaring is possible and FTL containers do not need to be fully filled. Constraint (6) defines the number of FTL containers as positive integer and the ordering decision variable O_t as a binary. Constraint (7) ensures that the decision variables are non-negative.

The following section details how the use of LTL modes is modelled through Constraints (11) to (19).

5.2. *Modelling LTL modes*

The LTL shipment quantity and costs can be modelled by re-defining the LTL price intervals and by calculating the indifference points in the nominal intervals. LTL prices based on an empirical example are illustrated in Table 2. A minimum charge means that, instead of paying 360 NOK for 2 pallets, the shipper has to pay 400 NOK.

Table 2. LTL prices based on an empirical example from the retail industry (Engebrethsen and Dauzère-Pérès, 2019)

Number of pallets	1-6	7-11	12-17	18-23	23-30	30 (FTL)
Price per pallet, NOK, minimum 400 NOK	180	150	130	115	107	2900 (total)

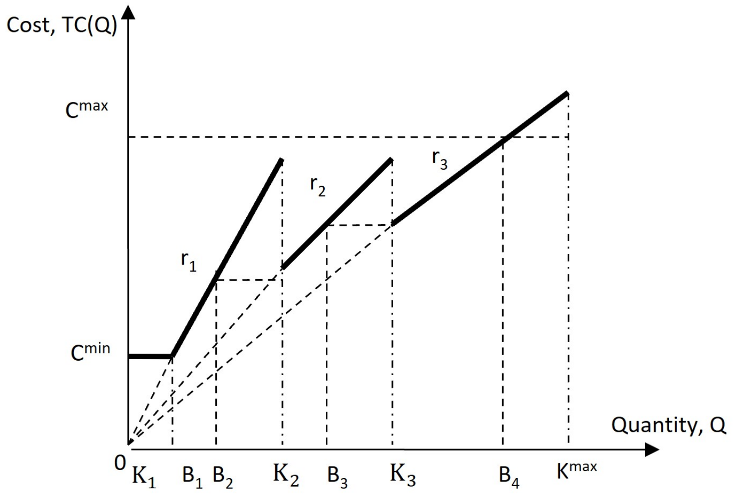

Figure 1. Caption: Cost function of LTL mode with minimum charge and three intervals. Alt Text: A graph showing transportation costs on Y axis and quantity shipped on X axis for Less Than Truckload mode with minimum charge and three price intervals

Figure 1 represents a LTL cost function with three price intervals, a minimum charge C_{min}, and a unit rate r_j, where $r_{j+1} < r_j$, applied for a quantity Q in an interval j defined by the limits K_j and K_{j+1}. For quantities within the intervals $[B_j, K_j]$, for $j \geq 2$, the total costs can be larger than the costs of shipping larger quantities at the next interval rate, as illustrated in Figure 1. This is due to the all-unit discount nature that encourages larger orders. Shippers usually over-declare the LTL shipment size

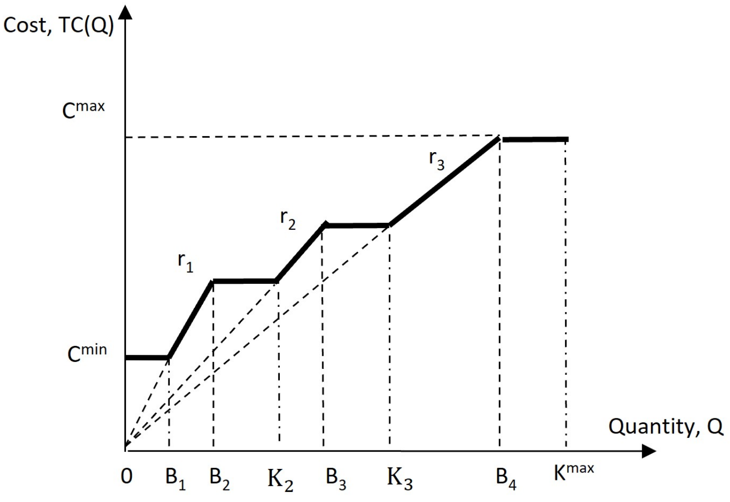

Figure 2. Caption: Modified LTL cost function with over-declaring. Alt Text: A graph showing transportation costs on Y axis and quantity shipped on X axis for a modified Less Than Truckload mode with overdeclaring

to reduce the costs, although the actual shipped quantity is lower, meaning that the cost is calculated at the next interval's rate as $TC(Q) = \min(r_jQ, r_{j+1}K_{j+1})$, when shipping Q units for $K_j \leq Q < K_{j+1}$.

In the example of Table 2, 11 pallets cost 1 650 NOK to ship, compared to shipping 12 pallets for 1 560 NOK. In practice, the shipper over-declares the shipment and pays the lowest price, termed as a 'phantom' policy (Sethi, 1984) or 'phantom freight' (Ke, Bookbinder, and Kilgour, 2014),'shipping Q but declaring K_{j+1}' (Chan et al., 2002) or the 'bumping clause', where the shipment quantity is bumped into the next interval (Çetinkaya and Bookbinder, 2003). Over-declaring is a common practice if the freight quantity is between the rate breakpoint and an indifference point B_j for interval $j \geq 2$, expressed as $B_j = \frac{r_j K_j}{r_{j-1}}$, where r_j and K_j are, respectively, the unit rate and the lower limit for the next interval j, and r_{j-1} is the unit freight rate for interval $j-1$ (Russell and Krajewski, 1991).

The LTL cost function in Figure 1 is therefore modified as shown in Figure 2 by cutting off the saw-teeth from Figure 1. Sometimes, the indifference point might be such that $B_{j+1} \leq K_j$. In this case, K_j is an anomalous or 'fictive' breakpoint (Abad, 2007), and should be dropped together with the corresponding freight rate from the schedule, as the shipment will be over-declared anyway. The nominal intervals (provided by the carrier) are redefined by increasing the number of intervals to $2J + 1$, where J is the number of nominal LTL intervals, e.g., for the example in Figure 2 with 3 nominal intervals, 7 new intervals are redefined. The minimum quantity that can be shipped within the interval j and the maximum quantity that can be shipped by LTL mode n are denoted by K_{nj} and K_n^{max}, respectively. A set of indifference breakpoints for LTL mode n is calculated as follows (a total of $J + 1$ indifference breakpoints):

$$B_{n1} = \frac{C_n^{min}}{r_{n1}} \qquad n = 1, ..., N \tag{8}$$

$$B_{nj} = \frac{r_{nj} K_{nj}}{r_{n(j-1)}} \qquad n = 1, ..., N; \quad j = 2, ..., J \tag{9}$$

$$B_{n(J+1)} = \frac{C_n^{max}}{r_{nJ}} \qquad n = 1, ..., N \tag{10}$$

The total quantity shipped by a LTL mode n in period t and its associated cost are then calculated by introducing continuous variables λ_{njt} and binary variables Y_{njt} for each re-defined interval j, and the constraints below, similar to the multiple choice formulation of (Croxton, Gendron, and Magnanti, 2003a). Variable Y_{njt} ensures that, in each period and for each LTL mode, at most one re-defined LTL interval is chosen, limiting the maximum value of X_{nt}^{LTL} in interval j as detailed in the explanation of Constraints (11) to (18).

$$\lambda_{njt} \leq Y_{njt} \qquad t = 1, ..., T; \quad j = 0, ..., 2J+1; \quad n = 1, ..., N \tag{11}$$

$$\sum_{j=1}^{2J+1} Y_{njt} \leq 1 \qquad n = 1, ..., N; \quad t = 1, ..., T \tag{12}$$

$$\lambda_{n2t} \geq \frac{Y_{n2t} B_{n1}}{B_{n2}} \qquad n = 1, ..., N; \quad t = 1, ..., T \tag{13}$$

$$\lambda_{n(2j)t} \geq \frac{Y_{n(2j)t} K_{nj}}{B_{n(j+1)}} \qquad n = 1, ..., N; \quad j = 2, ..., J; \quad t = 1, ..., T \tag{14}$$

$$\lambda_{n(2j+1)t} \geq \frac{Y_{n(2j+1)t} B_{n(j+1)}}{K_{n(j+1)}} \qquad n = 1, ..., N; \quad j = 1, ..., J-1; \quad t = 1, ..., T \tag{15}$$

$$\lambda_{n(2J+1)t} \geq \frac{Y_{n(2J+1)t} B_{n(J+1)}}{K_n^{max}} \qquad n = 1, ..., N; \quad t = 1, ..., T \tag{16}$$

$$X_{nt}^{LTL} = \lambda_{n1t} B_{n1} + \sum_{j=1}^{J-1} \lambda_{n(2j+1)t} K_{n(j+1)} + \sum_{j=1}^{J} \lambda_{n(2j)t} B_{n(j+1)} + \lambda_{n(2J+1)t} K_n^{max} \qquad n = 1, ..., N; \quad t = 1, ..., T \tag{17}$$

$$(TC)_{nt} = C_n^{min} Y_{n1t} + \sum_{j=1}^{J-1} Y_{n(2j+1)t} K_{n(j+1)} r_{n(j+1)} + Y_{n(2J+1)t} C_n^{max} + \sum_{i=1}^{J-1} \lambda_{n(2j)t} K_{n(j+1)} r_{n(j+1)} +$$

$$+\lambda_{n(2J)t}C_n^{max} \qquad t=1,...,T; \quad n=1,...,N \tag{18}$$

$$\lambda_{njt}\geq 0; Y_{njt}\in\{0,1\} \qquad n=1,...,N; \quad j=0,...,2J+1; \quad t=1,...,T \tag{19}$$

Constraints (11) and (12) ensure that, in each period and for each LTL mode, at most one re-defined LTL interval is chosen, i.e. that λ_{njt} is strictly positive at most once. Then, constraints (13) through (16) limit λ_{njt} to its minimum value in the interval j for LTL mode n. Note that, in the first interval, i.e. $j=1$, λ_{n1t} can vary from 0 to 1, i.e. X_{nt}^{LTL} can vary from 0 to B_{n1}. Then, constraint (13) corresponds to the second interval where X_{nt}^{LTL} is at least equal to B_{n1}. Constraint (14), resp. (15), limits λ_{njt} in the intervals between K_{nj} and $B_{n(j+1)}$, resp. between $B_{n(j+1)}$ and $K_{n(j+1)}$, so that X_{nt}^{LTL} is at least equal to K_{nj}, resp. $B_{n(j+1)}$. Finally, constraint (16) corresponds to the last interval where X_{nt}^{LTL} is at least equal to $B_{n(J+1)}$. This is through constraint (17) that X_{nt}^{LTL} is determined with variables λ_{njt}, and through constraint (18) that the cost of LTL mode n in period t is determined. Constraint (19) defines variables λ_{njt} as positive and variables Y_{njt} as binary. The LTL costs are modelled as in the multiple choice formulation for a piecewise linear function of Croxton, Gendron, and Magnanti (2003a), but we applied it to each mode, allowing modes to be combined. We also considered minimum price and over-declaring in the model.

5.3. *Modelling transportation strategies*

Constraint (2) ensures that the MM strategy is valid, i.e. that any mode can be chosen and combined. To model the SM strategy, constraint (2) is replaced by constraint (20) to only consider the FTL mode with the largest capacity, which is assumed to be the one with the lowest unit cost.

$$Q_t = X_{m't}^{FTL} \qquad m' \text{ such that } K_{m'}^{FTL}=K_{max}^{FTL}; \qquad t=1,...,T \tag{20}$$

To model the SSM strategy, binary variables O_t are replaced by binary variables O_{mt}^{FTL} and O_{nt}^{LTL} that indicate whether FTL mode m or LTL mode n is used in an order in period t, and constraint (21) is introduced to only allow one mode per period.

$$\sum_{m=1}^{M} O_{mt}^{FTL}+\sum_{n=1}^{N} O_{nt}^{LTL}\leq 1 \qquad t=1,...,T \tag{21}$$

Constraint (4) is replaced by the following constraints, that include the modified binary variables for ordering FTL mode m or LTL mode n, to ensure that the fixed ordering cost is incurred per shipment if required, and that define an upper bound for the optimal quantity shipped by each mode (each upper bound again corresponds to a big-M parameter as small as possible):

$$X_{mt}^{FTL}\leq (\sum_{k=t}^{T} d_k - 1 + K_{max}^{FTL})O_{mt}^{FTL} \qquad t=1,...,T; \quad m=1,...,M \tag{22}$$

$$X_{nt}^{LTL} \leq (\sum_{k=t}^{T} d_k - 1 + \max_{n=1,..N} K_n^{max}) O_{nt}^{LTL} \qquad t = 1, ..., T; \quad n = 1, ..., N \qquad (23)$$

6. Computational experiments

As discussed earlier, the previous research confirms that including transportation costs and several modes in inventory models leads to cost savings and a more realistic modelling. We performed simulation experiments with the purpose of understanding the model behaviour, by testing the effects of various factors on the total costs of each transportation strategy. The goal was to identify the mode-dependent and mode-independent parameters that contribute the most to cost improvements, as well as to identify the conditions and policies under which certain strategies are superior to the others. The base case parameters in our model, such as transportation mode capacities and costs, are those of a Scandinavian distribution company for fast-moving consumer goods. The other parameters are randomly generated data, but are also close to the values observed in the case company. To control the course of the experimental study and facilitate the same comparison basis across the scenarios, several control tools are applied, such as maximum computational time and the same demand replications for each scenario. The multiple design points (scenarios) have been explored by manually changing the factor level in the input data and re-running the model. Optimal solutions for all problem instances have been obtained by using the standard solver IBM ILOG CPLEX version 12.10 with a solution time limit of 3 hours without any customization of the default parameters. The average computational time for all scenarios and strategies was 5 seconds. The SSM strategy had the longest average computational time for all scenarios, 11 seconds, followed by the MM strategy, 3 seconds, and SM strategy, 1 second. We analyse the performance of transportation strategies by comparing them pairwise:

- SM versus SSM,
- SSM versus MM,
- SM versus MM.

We believe that introducing some flexibility in the transportation strategy for the choice of transportation modes, as in the SSM strategy, leads to lower costs compared to the SM strategy. Increased flexibility - that is, when modes can be combined as in the MM strategy - should lead to further cost reduction. The model can be used to compare the potential savings of mode combination vs. costs associated with the management of additional modes or transportation suppliers. We analyse the performance measures of the strategies by calculating the cost savings (as a percentage) for each strategy pair, based on the following formula: $\frac{C_a - C_b}{C_a}$, where C_a is the total cost of strategy A, and C_b is the total cost of strategy B. For each performance output, we investigate the following:

- The impact of the mode-dependent parameters on cost savings and mode combination frequency
- The impact of the mode-independent parameters on cost savings and mode combination frequency.

Table 3. Transportation mode parameters for each scenario

Scenario	Mode cost diff.	No of FTL modes	No of LTL modes	FTL capacities	FTL costs	LTL capacities	No of intervals LTL modes	Min price LTL modes	LTL intervals in pallets	LTL costs per pallet
1	Large	4	0	11, 25, 30, 33	2596, 3850, 4080, 4191	0	-	-	-	-
2	Small	4	0	11, 25, 30, 33	2123, 4000, 4560, 4917	0	-	-	-	-
3	Large	2	2	11, 25	2596, 3850	11, 25	3, 3	450, 550	1-5, 1-9, 5-9, 9-16, 9-11, 16-25	260, 265, 250, 240, 245, 164
4	Small	2	2	11, 25	2123, 4000	11, 25	3, 3	450, 500	1-5, 1-9, 5-9, 9-16, 9-11, 16-25	235, 215, 225, 194, 213, 170
5	Large	2	2	30, 33	4080, 4191	30, 33	5, 5	440, 450	1-6, 1-7, 6-12, 7-14, 12-18, 14-21, 18-24, 21-28, 24-30, 28-33	215, 220, 204, 205, 192, 175, 174, 138, 162, 155
6	Small	2	2	30, 33	4560, 4917	30, 33	5, 5	450, 500	1-6, 1-7, 6-12, 7-14, 12-18, 14-21, 18-24, 21-28, 24-30, 28-33	220, 225, 205, 210, 202, 190, 192, 170, 172, 155

6.1. *Mode-dependent parameters*

We believe that mode-dependent parameters, such as the number and type of modes (FTL or LTL), as well as cost and capacity characteristics, impact the performance of each strategy. We generated three scenarios characterized by different types of transportation modes:

- Scenario 1: Four FTL modes,
- Scenario 2: Two FTL and two LTL modes with large capacity difference (11 and 25 europallets, corresponding to 20 and 40 ft container capacities),
- Scenario 3: Two FTL and two LTL modes with small capacity difference (30 and 33 europallets, corresponding to respectively 40 ft palletwide and 45 ft container capacities).

We also generated scenarios for two levels of mode cost differences:

- Large cost difference between unit transportation costs for each fully utilized mode (cost per pallet per mode: 236, 154, 138, 127)
- Small cost difference between unit transportation costs for each fully utilized mode (cost per pallet per mode: 193, 160, 152, 142).

The capacities, transportation costs for large cost differences and number of LTL intervals are taken from the company's contracts with transportation companies for a specific origin-destination pair. Hence, six scenarios with mode-dependent parameters are generated, which are described in Table 3.

6.2. *Mode-independent parameters*

The parameters for the first scenario, i.e. the base case, are presented in Table 4.

Table 4. Base case parameters

Demand mean, pallets	CV (Coefficient of Variation)	Ordering cost, NOK	Holding cost, NOK	Number of periods
25	0.3	750	15	12

The varying parameters for other scenarios include the mean demand, demand variation, holding costs, ordering costs, transportation costs, as well as different combinations of relations between the holding, ordering, and transportation costs. Four factor levels are generated for the parameters: high, extra high, low, extra low. The changing parameters are expressed in relation to the base case, using a coefficient showing the number by which the base case data are multiplied. For the base case the time between orders is assumed to be equal to 1, The settings of the 21 scenarios for the mode-independent parameters can be found in Table 5.

Table 5. Transportation mode characteristics for each scenario (BC-Base Case, L-Low, H-High, XH-eXtra High, XL-eXtra Low)

Scenario	Scenario description	Changing parameters compared to BC	Value of changing parameter	Cost relationships
1	Base case	BC	-	L-L-L
2	High mean demand	2BC	50	L-L-L
3	Extra high mean demand	2.5 BC	62.5	L-L-L
4	Low mean demand	1.5 BC	37,5	L-L-L
5	Extra low mean demand	0.5 BC	12,5	L-L-L
6	High CV (coefficient of variation) demand	2 BC	0.6	L-L-L
7	Extra high CV demand	2.5 BC	0.75	L-L-L
8	Low CV demand	1.5 BC	0.45	L-L-L
9	Extra low CV demand	0.5 BC	0.15	L-L-L
10	High holding cost	4 BC	60	H-L-L
11	High ordering cost	4 BC	3000	L-H-L
12	High transportation costs	2 BC	2BC	L-L-H
13	Low holding, high ordering, high transp. cost	BC, 4BC,2BC	15, 3000, 2BC	L-H-H
14	High holding, low ordering, high transp. cost	4BC, BC, 2BC	60, 750, 2BC	H-L-H
15	High holding, high ordering, low transp. cost	4BC, 4BC, BC	60, 3000, BC	H-H-L
16	Extra low holding, high ordering, high transp.	0.25BC, 4BC, 2BC	3.75, 3000, 2BC	XL-H-H
17	Extra high holding, low ordering, low transp.	8BC, BC, BC	120, 750, BC	XH-L-L
18	High holding, extra low ordering, high transp.	4BC, 0.25BC, 2BC	60, 187.5, 2BC	H-XL-H
19	Low holding, extra high ordering, low transp.	BC, 8BC, BC	15, 6000, BC	L-XH-L
20	High holding, high ordering, extra low transp.	4BC, 4BC, 0.5BC	60, 3000, 0.5BC	H-H-XL
21	Low holding, low ordering, extra high transp.	BC,BC, 3BC	15, 750, 3 BC	L-L-XH

For each of the scenarios in Table 5, five replications are generated, resulting in 105 experiments. In each replication, the demand is randomly generated under normal distribution with a mean of 25 and a coefficient of variation of 0.3. A total of 1890 experiments (3 strategies (SM, SSM and MM) with 6 mode-related scenarios per strategy and 105 mode-independent problem instances per scenario) were carried out and analysed.

7. Parameter analysis of the transportation strategies

In this section, we investigate the impact of increasing the flexibility of transportation strategies. We compare the three transportation strategies (MM, SMS and SM) in terms of costs and identify the parameters with the highest impact on potential cost savings when changing strategy. We start the analysis with mode-dependent parameters (such as mode type, capacities and cost variations) in Section 7.1, and then continue with mode-independent parameters (demand, inventory holding costs, ordering costs) in Section 7.2. We also investigate how often various modes are combined and which modes are combined in different scenarios when applying the MM strategy. Finally, we analyze in Section 7.3 the solution times and the computational results on a set of problem instances with a longer planning horizon.

7.1. *Mode-dependent parameters*

Based on the computational results, when comparing the average savings of the total logistics costs for all problem instances for both scenarios with different types of modes and cost differences, the benefits of the MM strategy compared to the SSM and SM strategies are confirmed and illustrated in Table 6 .

Table 6. Cost savings when comparing the MM strategy to the SM and SSM strategies

Scenario	**Problem instances**	**Relative savings in %**		
		SM/MM	SM/SSM	SSM/MM
Scenario 1: 4 FTL	Average all	3.1	2.6	0.6
	Average large cost dif.	1.9	1.4	0.5
	Average small cost dif.	4.4	3.8	0.6
	Maximum among all	11.9	11.6	3.3
Scenario 2: 2 FTL/2 LTL **Large capacity difference: 11, 25**	Average all	3.1	1.5	1.6
	Average large cost dif.	2.9	1.4	1.5
	Average small cost dif.	3.3	1.7	1.6
	Maximum among all	10.8	8.7	6.2
Scenario 3: 2 FTL/2 LTL **Small capacity difference: 30, 33**	Average all	4.3	3.1	1.2
	Average large cost dif.	3.6	2.4	1.2
	Average small cost dif.	4.8	3.8	1.0
	Maximum among all	14.2	12.6	5.1

On average, the savings from the SSM strategy compared to the SM strategy are between 1.4% and 3.8%. The highest average saving, 3.1%, and the maximum saving, 12.6%, are for Scenario 3. When comparing the SSM and MM strategies, the savings are the highest for Scenario 2 (6.2%) and average savings are between 0.5% and 1.6%. On average, the savings from the MM strategy compared to the SM strategy are between 1.9% and 4.8%. This means that managers can obtain significant cost savings by introducing some flexibility in the transportation strategy, that is when the mode can be shifted in each period, and even greater savings can be achieved with more flexibility by combining modes, particularly when the mode capacity difference is large.

The average savings from the SSM or MM strategies compared to the SM strategy are higher for the small cost difference case for all scenarios. This can be explained by the fact that, when the cost difference is large, the modes are rarely shifted or combined, and the use of a single (cheapest/largest) mode is dominating in the solution. In the small cost difference case, the mode shifting or a combination of modes is observed more often.

Table 7 presents the average total logistics costs for the various strategies. The total costs of the SSM and MM strategies are the lowest when allowing LTL modes (with the same capacities as the largest FTL modes) to be used, compared to situations where only FTL modes are available, both for the SSM and MM strategies. This can also be explained by the fact that combining or shifting to LTL modes provides lower transportation costs compared to only using the FTL modes.

As illustrated in Table 8, which details the mode combinations for the MM strategy, when LTL modes are available in addition to the FTL modes, as in Scenarios 2 and 3, the modes are combined more often. The savings from using the MM or the SSM strategies instead of the SM strategy are the largest for Scenario 3 (small LTL capacity difference), and the share of savings from shifting the mode in each period (SM/SSM)

Table 7. Total logistics costs for the SM, SSM and MM strategies

Scenario	**Total costs, NOK**	**SM**	**SSM**	**MM**
Scenario 1: 4 FTL	Average	73,198	71,309	70,939
	Aver. large cost dif.	68,299	67,390	67,052
	Aver. small cost dif.	78,097	75,228	74,827
Scenario 2: 2 FTL/2 LTL Large capacity difference: 11,25	Average	79,609	78,441	77,355
	Aver. large cost dif.	78,312	77,237	76,211
	Aver. small cost dif.	80,906	79,645	78,499
Scenario 3: 2 FTL/2 LTL Small capacity difference: 30,33	Average	73,198	70,952	70,193
	Aver. large cost dif.	68,299	66,677	65,847
	Aver. small cost dif.	78,097	75,227	74,538

is larger than it is from mode combinations (SSM/MM). However, the average savings from using the MM strategy instead of the SSM strategy are the largest for Scenario 2 (large LTL capacity difference), and the frequency of mode combinations is also the largest.

Table 8 provides insights into mode usage when an order is placed and the combination frequency, confirming that modes are more frequently combined in the scenarios with small cost difference as expected.

Table 8. Average use of various modes with MM strategy. *Mode combinations, calculated as a fraction of orders where modes are combined, i.e. more than one mode is used for the same order, compared to the total number of orders, in %. **% modes used in combinations calculated as the share of orders with combinations where the specific mode is used. ***% modes used when ordered calculated as a share of all orders that include the specific mode.

Scenario	**Cost dif.**	**Multiple modes***	**% modes used in combinations****				**% modes used when ordered*****			
			FTL1	**FTL2**	**FTL3 or LTL1**	**FTL4 or LTL2**	**FTL1**	**FTL2**	**FTL3 or LTL1**	**FTL4 or LTL2**
1: 4 FTL	Average	33	28	25	46	66	11	17	31	77
	Aver. large	30	25	31	35	65	10	19	21	83
	Aver. small	36	32	19	56	66	12	16	40	72
2: 2 FTL/2 LTL Large capacity difference: 11,25	Average	47	1	100	11	88	1	92	5	49
	Aver. large	44	0	100	22	78	0	92	10	42
	Aver. small	50	2	100	0	98	2	93	0	55
3: 2 FTL/2 LTL Small capacity difference: 30,33	Average	39	18	85	50	31	13	84	19	25
	Aver. large	36	5	89	52	35	2	91	19	24
	Aver. small	43	32	80	48	27	23	77	20	26

Table 8 illustrates that the least expensive modes (FTL3 and FTL4 in Scenario 1, and FTL2 and LTL2 in Scenarios 2 and 3) are more often used in mode combinations. In Scenario 2 with large capacity difference, the usage of the most expensive modes (FTL1 and LTL1) is very low, compared to Scenarios 2 or 3. On average, when comparing Scenario 2 and 3, the modes are combined more often when the FTL capacity difference is larger. The modes are most often combined when both LTL and FTL modes are available, as well as when the cost difference is small, while the capacity difference is large. Hence, we can conclude that the availability of multiple modes and the usage flexibility increase the potential savings for the SSM and MM strategies compared to the SM strategy for all problem instances.

The costs of the MM strategy are even lower when allowing the use of LTL modes compared to the situations when only FTL modes are available both for the SSM and MM strategies. In Scenario 2, where the mode capacity differences are large, the average savings when changing from the SSM strategy to the MM strategy are about as large as when changing from the SM strategy to the SSM strategy. For the other two scenarios, adopting the MM strategy instead of the SSM strategy provides on average between 16% to 50% of the savings obtained when shifting from the SM strategy to the SSM strategy.

Table 9 summarizes the mode-dependent parameters that impact modes combinations and have the highest cost saving potential when comparing different transportation strategies.

Table 9. Summary of parameters with the highest impact on the cost saving potential (X) for different transportation strategies

Recommended	**Mode types**		**Capacity dif.**		**Cost dif.**	
strategy	**Only FTL**	**FTL and LTL**	**Small**	**Large**	**Small**	**Large**
MM vs. SM	X	X	X		X	
MM vs. SSM		X		X		
SSM vs. SM	X	X	X		X	

7.2. *Mode-independent parameters*

The maximum savings among the average values of five replications for the 21 scenarios and the corresponding parameters can be found in Table 10.

Table 10. Summary of the highest savings among the average value of five replications for each scenario

Scenario	**Problem instances**	**Savings, SM/MM**	**Savings SM /SSM**	**Savings SSM/MM**
1: 4FTL	Average large cost dif.	3.1% 0.5BC demand mean	2.6% 0.5BC demand mean	1.1% 4BC ordering costs (H-L-L)
	Average small cost dif.	7.1 % XH-L-L	6.5 % XH-L-L	1.6 % H-H-XL
	Average all	4.9% XH-L-L	4.5% XH-L-L	1.2% L-XH-L
2: 2FTL/2LTL, capacity 11,25	Average large cost dif.	7.9 % XH-L-L	4.8 % XH-L-L	4.4 % H-H-XL
	Average small cost dif.	9.4 % XH-L-L	5.5 % XH-L-L	4.4 % H-H-XL
	Average all	8.6 % XH-L-L	5.2 % XH-L-L	4.4 % H-H-XL
3: 2FTL/2LTL, capacity 30,33	Average large cost dif.	7.0 % XH-L-L	5.2 % XH-L-L	3.3 % H-H-XL
	Average small cost dif.	9.0 % XH-L-L	7.4 % XH-L-L	3.0 % H-H-XL
	Average all	8% XH-L-L	6.3% XH-L-L	3.2 % H-H-XL

Table 10 shows that scenarios with extra high inventory holding, low ordering and low transportation costs (XH-L-L) lead to the highest average savings when comparing the SM strategy to the SSM and MM strategies. This can be explained by the fact, that when inventory holding costs are very high, orders are more frequent and their size is smaller. Therefore, the use of modes with smaller capacities (FTL) and/or LTL is preferred. The savings when comparing the SSM and MM strategies are the largest for the scenario with high inventory holding, high ordering and extra-low transportation costs (H-H-XL).

Table 11 summarizes the average results for all cases and illustrates the variation of mode-independent parameters and the impact on savings and mode combinations.

The largest savings for the SSM strategy (above the average) induced by adding flexibility to the SM strategy correspond to the following parameters:

- High and extra-high inventory holding costs compared to transportation and ordering costs,
- Low mean demand,
- Extra-low and low ordering costs compared to inventory and transportation costs,
- Extra-high and high transportation costs compared to inventory and ordering costs.

It is not surprising that, when the inventory holding costs increase compared to the ordering and transportation costs, replenishment is more frequent and modes can be shifted more often.

The largest cost savings (above the average) for switching from the SM strategy to the MM strategy correspond to the following scenarios:

Table 11. Summary of the scenario parameters and the average corresponding cost savings and combination frequencies.The shaded green and red cells indicate the highest and smallest savings among all scenarios. Mode combinations are calculated as the fraction of orders using more than one mode out of the total number of orders.

Parameters	**Ratio between costs**	**Changing values compared to Base Case**	**Average savings in %**			**Mode combinations, in %**
			SM/MM	**SM/SSM**	**SSM/MM**	
Mean demand	L-L-L	0.5	5.0%	4.2%	0.8%	32%
	L-L-L	1	3.3%	2.2%	1.1%	40%
	L-L-L	1.5	1.8%	1.2%	0.6%	37%
	L-L-L	2	1.5%	0.9%	0.6%	32%
	L-L-L	2.5	1.4 %	0.8%	0.5%	40%
Demand CV	L-L-L	0.5	3.0%	2.2%	0.8%	33%
	L-L-L	1	3.3%	2.2%	1.1%	40%
	L-L-L	1.5	3.4%	2.5%	0.8%	41%
	L-L-L	2	4.1%	3.1%	1.1%	33%
	L-L-L	2.5	3.4%	2.5%	0.9%	32%
Inventory costs	XL-H-H	XL	3.3%	2.1%	1.2%	65%
	L-H-H	L	3.0%	1.8%	1.3%	57%
	L-L-L	BC	3.3%	2.2%	1.1%	40%
	H-L-L	H	4.8%	3.9%	1.0%	21%
	XH-L-L	XH	7.2%	5.3%	2.0%	19%
Ordering costs	H-XL-H	XL	4.2%	3.9%	0.3%	9%
	H-L-H	L	3.9%	3.5%	0.4%	16%
	L-L-L	BC	3.3%	2.2%	1.1%	40%
	L-H-L	H	2.9%	1.1%	1.8%	64%
	L-XH-L	XH	2.4%	0.9%	1.5%	75%
Transportation costs	H-H-XL	XL	4.3%	1.5%	2.9%	75%
	H-H-L	L	4.2%	1.6%	2.7%	67%
	L-L-L	BC	3.3%	2.2%	1.1%	40%
	L-L-H	H	3.2%	2.7%	0.5%	33%
	L-L-XH	XH	3.3%	3%	0.3%	31%
		Average	3.5%	2.3%	1.1%	41%
		Max	7.2%	5.3%	2.9%	75%
		Min	1.4%	0.8%	0.3%	9%

- High and extra-high inventory costs compared to transportation and ordering costs,
- Low mean demand,
- Extra-low and low ordering costs compared to inventory and transportation costs,
- Extra-low and low transportation costs compared to inventory and ordering costs.

With increasing inventory costs, the cost savings increase, but the modes are combined less frequently. When the ordering costs increase, the cost savings decrease and the mode combinations increase. When the transportation costs increase, both the cost savings and the mode combinations decrease. Hence, no consistent correlation between the cost savings and mode combinations have been observed, meaning that mode combination over few periods can generate larger cost savings than frequent mode combinations over a longer period.

When comparing the SSM strategy to the MM strategy, the highest savings are obtained for the following scenarios:

- Extra-low and low transportation costs compared to inventory and ordering costs,
- Extra-high and high ordering costs compared to inventory and transportation costs.

When transportation costs are low and ordering costs are high, replenishment is less frequent, and combining multiple modes further reduces the transportation costs.

The following scenarios provide the lowest cost savings (below average) when comparing the SM strategy to the SSM or MM strategies:

- High mean demand,
- Low holding, extra-high ordering, low transportation costs,
- Low holding, high and extra-high ordering, low transportation costs (L-XH-L)

High mean demand results in selection of the largest (and the cheapest) mode instead of the use of other modes. High ordering costs have a similar effect, since the replenishment frequency is low, resulting in larger order sizes using the largest (and the cheapest) mode.

The following scenarios provide the lowest cost savings when comparing the SSM strategy to the MM strategy:

- High holding, low ordering, high transportation costs (H-L-H),
- High holding, extra-low ordering, high transportation costs (H-XL-H),
- Low holding, low ordering and extra-high transportation costs (L-L-XH).

Low ordering costs and high transportation costs make it possible to order frequently, and the order size probably fits within the capacity of a single mode. Therefore, a multi-mode strategy overlaps or provides small savings compared to the single shifting strategy.

There are more mode combinations when the inventory or transportation costs decrease, and fewer mode combinations when the ordering costs decrease.
As a graphical illustration of this analysis, detailed figures can be found in Appendix 1.

7.3. *Numerical results with an increased number of periods*

In this section, we try to analyze the computational complexity as well as the potential cost improvements of our models despite the fact that optimal solutions were not reached.

All 1 890 problem instances for 12 periods have been solved by the standard solver IBM ILOG CPLEX version 12.10 without any customization of the default parameters in less than one minute. Table 12 gives the computational time for each strategy:

Table 12. Computational times for the problem instances with 12 periods

Computational time (seconds)	SM strategy	SSM strategy	MM strategy	All strategies
Average time	1	11	3	5
Maximum time	3	449	109	

As pointed out in previous research (Archetti, Bertazzi, and Speranza (2014), Venkatachalam and Narayanan (2016)), increasing the number of periods increases the computational complexity of the dynamic lot-sizing models with discounts. We conduct a set of experiments with 48 periods for the problem instances that generated the largest and the smallest savings in Table 11 to investigate the potential for cost savings for a longer planning horizon. In total, 120 problem instances were created for 48 periods by adding demand data for 36 periods, generated with the same parameters as for the corresponding 12 period cases. The problems were solved for each strategy within a time limit of 3 hours (10 800 seconds). Table 13 summarizes the numerical results.

The SSM strategy is the most difficult to solve, as the average solution time, the gap

Table 13. Computational results for the problem instances with 48 periods

	MM strategy	SSM strategy	SM strategy
Number of unsolved instances	55/120	90/120	18/120
Average gap between upper and lower bounds, unsolved instances (%)	0,3	1,96	0,6
Maximum gap, unsolved instances (%)	1	8,6	1,3
Solution time, all instances (seconds)	5282	9290	1996
Solution time, solved instances (seconds)	317	536	430

and the number of unsolved instances is the largest compared to the other strategies.

The average cost savings for the new problem instances with 48 periods are presented in Table 14. It can be concluded that the largest savings (shaded green) remain significant, in particular for the extra high inventory and transportation cost scenarios, also for a longer planning horizon, even though the optimal solution was not found for all problem instances within the time limit.

Table 14. Cost savings for various strategies for problem instances with 48 periods compared to the highest (shaded green) and the smallest (shaded red) cost savings for 12 periods

	Ratio between costs	Changing values compared to Base Case	Number of periods	Average savings in % SM/MM	Average savings in % SM/SSM	Average savings in % SSM/MM
Mean demand	LLL	2,5	12	1,4	0,8	0,6
			48	**0,5**	**0,2**	0,3
Inventory costs	XH-L-L	XH	12	7,2	5,3	2
			48	**6,7**	**5,0**	1,7
Transportation costs	HHXL	XL	12	4,3	1,5	2,9
			48	3,8	1,1	**2,7**
Transportation costs	LLXH	XH	12	3,3	3	0,3
			48	0,7	0,6	**0,1**
Maximum savings among 120 instances				**9,9**	**5,3**	**8,5**

8. Conclusions and managerial implications

Based on a proposed inventory planning model, we considered a practical inventory management and transportation planning problem that includes critical features such as the availability of multiple transportation modes with realistic cost functions and the flexibility of combining modes between periods and within periods. Previous research stressed the need for integrating transportation and inventory planning due to improved cost efficiency compared to disaggregated decisions (see for example, Venkatachalam and Narayanan (2016)) and due to the possibility of using heterogeneous modes compared to a single mode to reduce the costs and improve the service level (see Jain, Groenevelt, and Rudi (2010)). However, as it has been observed in the case company, the managerial decision has been done in a disaggregated manner, mostly relying on the largest FTL mode, despite the availability of various modes in the transportation contracts.

Our review of the existing research on inventory management has revealed that the observed problem with the relevant types of modes and transportation cost functions has not been studied before. The results of our empirical analysis show that significant savings can be achieved when different modes are allowed to be used in each period (SSM strategy) compared to using the same mode in all periods, and further savings (additional 16%-100% of the savings gained from adopting the SSM strategy) can be achieved when various modes are allowed to be combined for the same shipment in each period (MM strategy). For all scenarios, applying the multi-mode strategy (MM) provided significant cost reductions, particularly compared to the SM strategy (savings above 14% in some cases), but also compared to the SSM strategy (savings above 6% in some cases).

Useful insights were obtained by running the scenarios with various model parameters to analyze their impact on the total costs. The magnitude of the potential cost saving obtained by combining modes depends both on mode-dependent parameters, as well as on mode-independent parameters. The savings are particularly large when both FTL and LTL mode types are available, and when the cost and capacity difference is small among the modes. The impact of small cost difference is in line with the conclusion suggested by Jain, Groenevelt, and Rudi (2010) for a stochastic case. The mode-independent parameters, such as high inventory costs, lead to higher savings as proposed by Rieksts and Ventura (2010) for a static demand case. Our analysis shows that low mean demand, lower transportation or ordering costs also impact the increase of cost savings.

Hence, it is important for managers to consider transportation mode selection together with inventory decisions and to understand the impact of simplified decisions on the total cost, such as the use of a single mode instead of combining modes in each period or in different periods. The study reveals that shippers should increase the flexibility of their transportation strategies to achieve logistics cost savings.We also identify parameters that indicate insignificant cost savings, where the use of a single mode is enough, instead of minimizing the costs for all three strategies. Compared to previous research, we investigated more parameters impacting cost efficiency when using multiple modes as well as various transportation strategies. Table 15 summarizes the conditions for achieving cost savings when choosing the transportation strategy and for the most frequent combinations of modes depending on mode-dependent and mode-independent parameters.

Table 15. Conditions for achieving cost savings when changing the transportation strategy (X indicates high savings potential, L-low and H-high difference or ratio to other costs) and for the most frequent combination of modes

Recommended strategy	**Mode-dependent parameters**				**Mode-independent parameters**			
	Only FTL modes	**FTL and LTL modes**	**Capacity difference**	**Cost difference**	**Demand mean**	**Inventory costs**	**Order costs**	**Transport costs**
SSM vs. SM	**X**	**X**	**L**	**L**	**L**	**H**	**L**	**H**
MM vs. SSM		**X**	**H**				**H**	**L**
MM vs. SM	**X**	**X**	**L**	**L**	**L**	**H**	**L**	**L**
Combined modes		**X**	**H**	**L**		**L**	**H**	**L**

Based on the mode-independent parameters of a product, a planner can rely on the generic guidelines of Table 15 to evaluate whether a single mode strategy is sufficient, or to increase flexibility and reduce costs by using more modes, and what to expect for each mode combination. For example, if a product has a high mean demand and low inventory costs, the cost savings from considering more modes may not be significant. On the opposite, for a product with low mean demand and high inventory costs, both the SSM and the MM strategies are recommended. However, if a company already uses the SSM strategy, the increased flexibility of the MM strategy may not lead to additional savings. The cost saving conditions for mode-dependent parameters provide valuable inputs for a manager involved in the procurement of freight services, who needs to decide whether the terms for single or multiple modes offered by the providers should be negotiated, and whether the LTL mode should be included in the contract. The mode cost and capacity characteristics indicate which strategies lead to savings, and should be considered when selecting a transportation provider. For example, small capacity and cost differences between all mode types are preferable if switching from the SM strategy to the SSM strategy or the MM strategy.

The impact of logistics cost savings for a company depends on the ratio of the logistics costs to total costs (or, for example, purchasing costs). The transportation industry is characterized by low margins, leaving little room for price reduction during contract negotiations with carriers. However, shippers can achieve additional savings by better planning to ensure optimal utilization of vehicles and by implementing more flexible transportation strategies. The proposed approach can be useful to estimate the total demand per mode as an input to contract negotiations with the carrier, contractual volume commitment and to support collaborative forecasting activities with the carriers.

This approach can also be used for evaluating international trading terms (Incoterms), when buyers have several delivery options and corresponding purchasing prices, such as FOB condition (free delivery to the nearest harbour) at price A versus ExWorks (the buyer arranges and pays for delivery from the supplier's facilities) at price B. The trading terms between the buyer and the supplier define which party has responsibility for transportation, and they therefore impact the cost components included in the purchasing price from the supplier (Carter and Ferrin (1996)). For some trading terms, the supplier is responsible for product transportation; in such cases, transportation costs are included in the purchasing price. The purchasing price can be lower if the buyer arranges and pays for transportation, which means the trading terms can have a significant impact on the replenishment order size and the buyer's ability to optimize its purchasing costs.

The model can be used to compare the potential savings of mode combination vs. costs associated with the management of additional modes or transportation suppliers. In some situations, combining or shifting multiple modes can be a more time-consuming task compared to a single mode strategy, because of more complex transportation ordering procedures, the handling of multiple service suppliers, additional investment in more advanced planning tools, etc. For such cases, the model could be adjusted by including a mode-specific fixed cost or a penalty when changing modes. In order to decide on a long-term transportation strategy, a company should properly assess the costs and the benefits of allowing a more flexible use of modes, in particular for the situations where the potential savings can be high. In some cases, as the conditions presented in our study, the use of a single mode can be enough, instead of minimizing the costs for all three strategies due to insignificant cost savings.

9. Future research directions

The implementation of the proposed model has been complicated by the fact that the company used a commercial ERP-system, and that modifications of the existing information system would be both time-consuming and costly. However, a stand-alone decision tool as an add-on to the existing system could be an alternative for the case company.

Based on our findings, we have identified several areas for future research:

- Developing efficient tools for comparison of freight rates to pre-select the alternative modes based on searching engines for price quotes integrated with optimization tools to determine inventory replenishment plans and to compare transportation strategies would greatly help to support managerial decisions.

- The computational complexity of the proposed model will increase with a larger number of alternative modes, price intervals, time periods and the related number of decision variables. Therefore, the impact of mode dependent parameters motivates the development of efficient solution approaches,such as LP-relaxations and approximation algorithms as for example proposed by Croxton, Gendron, and Magnanti (2003b) or Archetti, Bertazzi, and Speranza (2014).
- Numerous options exist for the pallet type and container type. The packaging options may depend on the compatibility with the transportation modes, warehousing equipment, and compatibility with customer equipment or shelf design. In addition, managers may choose to send un-palletized goods in order to increase the container utilization and to postpone palletizing operations until the goods are received. The decision of where to perform the palletizing operations is a trade-off between the increased utilization rate of containers and the handling cost for palletizing at origin and destination. Hence, managers may need to evaluate the packaging decisions together with the selection of the transportation modes.
- Another decision problem arises when the cargo vehicles become larger and consist of several modules or containers of different sizes (multi-modular trucks). An example of such a problem is the optimal combination of modules for a vehicle and the allocation of the optimal shipment quantity.
- Although environmental concerns have become increasingly important, little attention has been paid to environmental aspects in operational planning, handling and packaging issues. Shippers increasingly demand environmentally friendly transportation alternatives to reduce the carbon footprint, which has required both managers and researchers to incorporate environmental criteria or constraints in planning decisions.
- The model can be extended to include other pricing models, mode-depending ordering costs, with contracts and discounts based on the monthly or yearly total volumes and minimum commitment.
- Other model extensions may include the consideration of multiple items and time-varying mode costs, as well as mode availability. Various operational restrictions on shipping multiple items together or separately could also be studied.
- The consideration of stochastic parameters, such as demand and lead time, is another area of future research.
- It would also be interesting to investigate how the discount policy and pricing decisions from the product and transportation suppliers impact the mode selection decision.

10. Data Availability Statement

The data that support the findings of this study are available on request from the authors. The data are not publicly available due to confidentiality restrictions of the case company.

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Appendix 1

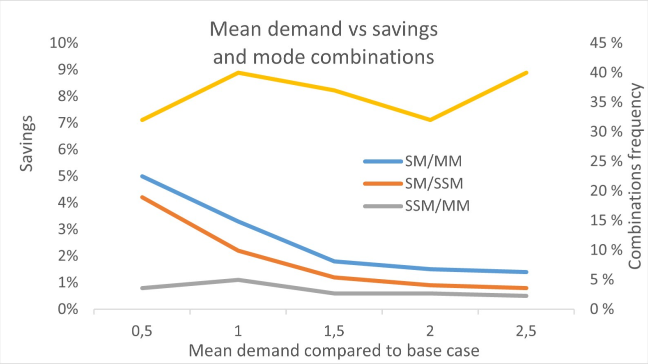

Figure 3. Caption: Demand Mean versus Cost savings and Mode combinations frequencies. Alt Text: A graph showing Demand Mean vs. Savings and Mode combinations frequencies

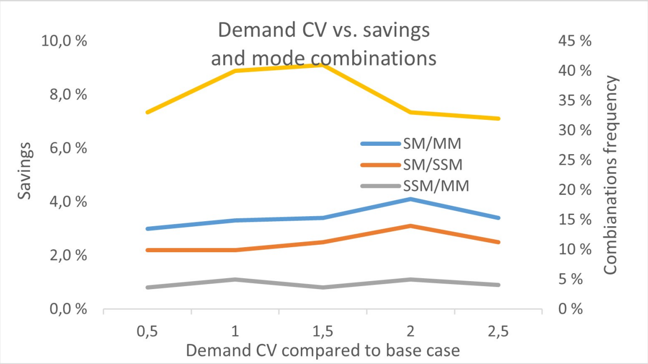

Figure 4. Caption: Demand CV versus Cost savings and Mode combinations frequencies. Alt Text: A graph showing Demand CV versus Cost savings and Mode combinations frequencies

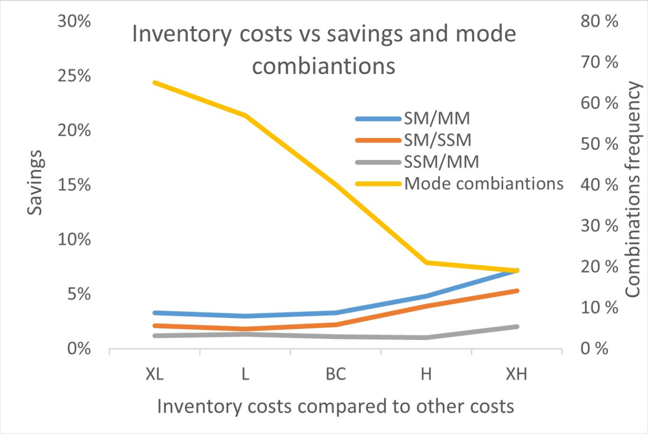

Figure 5. Caption: Inventory costs versus Cost savings and Mode combinations frequencies. Alt Text: A graph showing Inventory costs versus Cost savings and Mode combinations frequencies

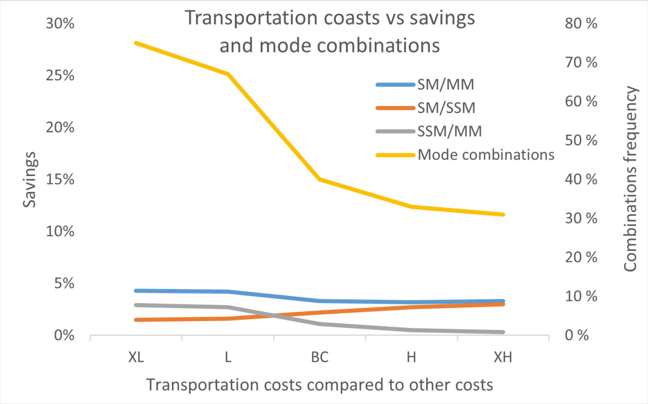

Figure 6. Caption: Transportation costs versus Cost savings and Mode combinations frequencies. Alt Text: A graph showing Transportation costs versus Cost savings and Mode combinations frequencies

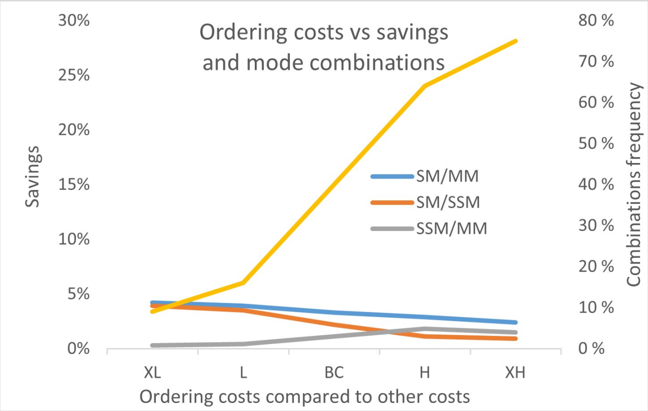

Figure 7. Caption: Ordering costs versus Cost savings and Mode combinations frequencies. Alt Text: A graph showing Ordering costs versus Cost savings and Mode combinations frequencies