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Abstract

We develop and estimate a DSGE model to evaluate the economic repercussions of cryptocurrency. In our model, cryptocurrency offers an alternative currency option to government currency, with endogenous supply and demand. We uncover a substitution effect between the real balances of government currency and cryptocurrency in response to technology, preferences and monetary policy shocks. We find that an increase in cryptocurrency productivity induces a rise in the relative price of government currency with respect to cryptocurrency. Since cryptocurrency and government currency are highly substitutable, the demand for the former increases whereas it drops for the latter. Our historical decomposition analysis shows that fluctuations in the cryptocurrency price are mainly driven by shocks in cryptocurrency demand, whereas changes in the real balances for government currency are mainly attributed to government currency and cryptocurrency demand shocks.

Keywords: DSGE Model, Government Currency, Cryptocurrency, Bayesian Estimation.
JEL classification: E40, E41, E51, E52.

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1 Introduction

Cryptocurrency has recently gained considerable interest from investors, central banks, and governments worldwide. There are numerous reasons for this intensified attention. Since June 2021, El Salvador has been the first country in the world to allow Bitcoin as legal tender. Currently, several advanced economies (such as Denmark, France, Germany, Iceland, Japan, Spain, the UK, and the US) allow Bitcoin to be used in transactions and have developed some form of regulation. Many large companies accept Bitcoin as a form of payment. For example, Wikipedia accepts donations in Bitcoin. Microsoft allows the use of Bitcoin to top up user accounts. PayPal users in the US can buy, sell, or hold a select few cryptocurrencies, including Bitcoin, Ethereum, Bitcoin Cash, and Litecoin.

In this paper, we develop and estimate a Dynamic Stochastic General Equilibrium (DSGE) model in order to evaluate the economic repercussions of cryptocurrency. Our model includes the demand and supply of cryptocurrency by extending and reformulating standard DSGE models with money (see, among others, Nelson, 2002, Christiano et al., 2005, Ireland, 2004) with the new sector of the economy related to cryptocurrency. Our analysis allows us to compare the responses of real money balances for government currency and cryptocurrency with several demand and supply shocks driving the economy. Moreover, we are able to evaluate the responses of the main macroeconomic fundamentals to a cryptocurrency productivity shock.

Figure 1 shows the overall cryptocurrency market capitalisation per month from June 2013 to April 2022 in billion USD.[1] By December 2017, Bitcoin, the first decentralised cryptocurrency that was created in 2008 and documented in Nakamoto (2008), had grown to a maximum of approximately

[1] For the sample period, 2013:M6-2022:M4, the series of overall cryptocurrency market capitalisation and the series of Bitcoin market capitalisation display almost the same dynamics. Indeed, their estimated correlation is above 0.98.

2,700 percent price return and, in the same year, some cryptocurrencies had achieved far higher growth than Bitcoin. In early 2018, a large sell-off of cryptocurrencies occurred. From January to February 2018, the price of Bitcoin fell by 65 percent. By the end of the first quarter of 2018, the entire cryptocurrency market fell by 54 percent, with losses in the market topping USD 500 billion. In November 2018, the total market capitalisation for Bitcoin fell below USD 100 billion for the first time since October 2017, and the price of Bitcoin fell below 5,000 USD. At the end of 2019, the price of Bitcoin was still low at around 7,200 USD. However, with the spread of the COVID-19 pandemic and the shutdown of economies around the world, the price of Bitcoin started to accelerate in its upward climb. By December 2020, the price of Bitcoin had increased by over 300 percent since the beginning of the year. In the same period, the market capitalisation of all cryptocurrencies had grown by more than 290 percent. The year ended with a Bitcoin price of approximately 29,374 USD, the highest since its creation. Bitcoin doubled its value in 2021, skyrocketing to an all-time high of over 64,000 USD in the first half of 2021 and then falling back below 30,000 USD over the summer. In November 2021, the market capitalisation of all cryptocurrencies achieved almost 3.0 USD trillion and Bitcoin hit another all-time high of over 68,000 USD. Since January 2022, the price of Bitcoin has dropped back below 35,000 USD. In April 2022, the total cryptocurrency market was valued at 2.2 USD trillion.

Cryptocurrency is a form of private-sector-issued currency and is issued in divisible units that can be easily transferred in a transaction between two parties (Nakamoto, 2008; Ethereum, 2014; Ripple, 2012). Digital currencies are intrinsically useless electronic tokens that travel through a network of computers. Advances in computer science have allowed for the creation of a decentralised system for transferring these electronic tokens from one person or firm to another. The key innovation of the cryptocurrency system is the creation of a payment system across a network of computers that does not require a trusted third party to update balances and keep track of the ownership of the virtual units. The technology behind the system is called Blockchain.[2]

The characteristics of cryptocurrency are as follows. The first characteristic relates to the fact that cryptocurrency is not based on a central authority that holds private information. On the contrary, it relies on public information, such as computation from a large number of individual distributed computers and servers that are connected to each other via the network and not by a recognised authority. Secondly, issuing of new currency and operations are validated by the network via complex pre-defined mathematical operations, an algorithm known as proof of work. This kind of network approves pre-defined, encrypted, and immutable operations, so that history cannot be changed and manipulated. The last characteristic refers to the ease of payment and management. Cryptocurrency is, by definition, computer-based and when linked to a portfolio the only requirement for transferring value or paying bills is an internet connection.

Most previous studies have analysed cryptocurrency empirically. For example, Hencic and Gourieroux (2014) applied a non-causal autoregressive model to detect the presence of bubbles in the Bitcoin/USD exchange rate. Sapuric and Kokkinaki (2014) measured the volatility of the Bitcoin exchange rate against six major currencies. More recently, Catania et al. (2018) analysed and predicted cryptocurrency volatility, whereas Catania et al. (2019) predicted the full distribution of cryptocurrency. Both Bianchi (2020) and Giudici and Pagnottoni (2020) have investigated the structural relationships between cryptocurrency and other macroeconomic and financial time-series.

However, there have only been a few theoretical studies that have modelled cryptocurrency. In this regard, Boehme et al. (2015) introduced

[2] Cryptocurrency is just one of the many applications of Blockchain.

the economics, technology and governance of Bitcoin, whereas Fernández-Villaverde and Sanches (2019) developed a model of competition among privately-issued fiduciary currencies. Garratt and Wallace (2018) and Schilling and Uhlig (2019) focused on the exchange rate of Bitcoin and its theoretical determinants. Brunnermeier and Niepelt (2019) derived a model of money and liquidity to identify the sources of seigniorage rents and liquidity bubbles in the context of cryptocurrency. As we will explain in the next section, most of these studies have assumed partial equilibrium models and did not examine the economic repercussions from the introduction of cryptocurrency to the overall economy and its different sectors.

We fill this gap by developing a Dynamic Stochastic General Equilibrium (DSGE) model where cryptocurrency is considered an alternative to government currency. Our DSGE model includes a utility function that is non-separable across consumption and real balances of government currency and cryptocurrency in household preferences. Moreover, we assume two separate demand shocks to government currency and cryptocurrency, respectively, and one cryptocurrency productivity shock. This productivity is proxied by a new series: the total quantity of cryptocurrency that is supplied to the market in the form of tokens.

We estimate our model with Bayesian techniques using monthly data from the US and the cryptomarkets for the period 2013:M6-2022:M4. To the best of our knowledge, our work is the first attempt to provide a general equilibrium model with cryptocurrency and to estimate its parameters with Bayesian techniques.

The estimated results of our DSGE model contribute to the ongoing debate concerning the nature of cryptocurrency by suggesting that cryptocurrency and government currency exhibit a high degree of substitution (Gans and Halaburda, 2019). This finding is also confirmed by the empirical evidence provided in our preliminary structural VAR (SVAR) analysis.

The impulse response analysis obtained from our DSGE model indicates that the reaction of the economy to shocks in preferences, technology and monetary policy are in line with the findings of previous literature (see, for example, Ireland, 2004 and Andrés et al., 2009). In addition, we find that in response to these "traditional" shocks,[3] cryptocurrency is highly substitutable with government currency. In terms of the transmission mechanisms of these shocks, we observe that the real balances of cryptocurrency are not the main drivers of the responses of the other macroeconomic aggregates. Moreover, our findings indicate that cryptocurrency and government currency are also substitutes in response to both government currency and cryptocurrency demand shocks.

In response to an increase in the productivity of cryptocurrency, the price of cryptocurrency becomes cheaper relative to the value of government currency. Since cryptocurrency and government currency are highly substitutable, this effect makes cryptocurrency more attractive compared to government currency. Therefore, the demand for the former increases, whereas it drops for the latter. We should note that the magnitudes of these effects on output, inflation and the nominal interest rate are much lower than in the case of preferences, technology and monetary policy shocks.

We also provide a historical decomposition analysis based on the estimated DSGE model. Firstly, our findings indicate that changes in the cryptocurrency price are mainly driven by shocks in cryptocurrency demand. This implies that when the cryptocurrency price increases, so does its demand, thereby pushing up the price even more. On the other hand, when the cryptocurrency price falls, the lower demand for cryptocurrency depresses the price even further. Secondly, our results show that government currency and cryptocurrency demand shocks play a dominant role in the variation of the real balances for government currency. Once again, from our analysis, a

[3] The term "traditional" shocks in our DSGE model refers to household preferences, technology and monetary policy shocks.

substitution effect between government currency and cryptocurrency demand can be seen. This substitution effect was particularly evident in the first half of 2020, when the M2 money supply experienced an unprecedented increase. Due to the fears of a rise in inflation, households and financial investors switched their resources towards cryptocurrency. This, in turn, was the cause of the spectacular increase in the demand for cryptocurrency.

We perform several robustness checks on the functional form of the utility function and we show that our main findings remain unchanged. Finally, we assess the role of monetary policy in the presence of shocks to cryptocurrency productivity. Our sensitivity analysis indicates that the larger the response of the monetary policy rule to a change in government currency growth, the stronger the decline in output.

The remainder of this paper is organised as follows. Section 2 reviews previous literature and provides some relevant stylised facts. Section 3 outlines the new DSGE model on which our study is based. In Section 4, we present the data used for the analysis and our Bayesian estimates. Section 5 presents the main findings of our analysis. Section 6 provides some robustness exercises. The concluding remarks are found in Section 7.

2 Previous studies and empirical evidence

In this section, we first review the relevant literature to which our study refers and, secondly, we provide some important stylised facts that corroborate our DSGE approach.

2.1 Literature review

Our paper refers to two different streams of literature. On the one hand, we contribute to studies that have developed theoretical models to analyse and describe cryptocurrency dynamics. However, these studies have focused mainly on partial equilibrium models. In our work, we develop a general equilibrium framework, introducing cryptocurrency as an alternative to government currency. On the other hand, our study also contributes to the DSGE literature that has analysed the role of government currency in the economy.

Regarding the first strand of literature and the theoretical models, Boehme et al. (2015) presented the design principles and properties of the Bitcoin platform for a non-technical audience. They reviewed the past, present and future uses of Bitcoin, identifying the risks and regulatory issues that arise as Bitcoin interacts with the conventional financial system and the real economy.

Furthermore, Fernández-Villaverde and Sanches (2019) built a model of competition among privately-issued fiduciary currencies.[4] They found that the lack of control over the total supply of money in circulation has critical implications for the stability of prices across the economy. In other words, the economy ends up in a state of hyperinflation. These authors also illustrated that in the short and medium terms, the value of digital currencies goes up and down unpredictably as a result of self-fulfilling prophecies.

Another theoretical model analysing the exchange rate between fiat currency and Bitcoin was developed by Athey et al. (2016). In particular, they argued that the Bitcoin exchange rate can be fully determined by two market fundamentals: the steady-state transaction volume of Bitcoin when used for payments and the evolution of beliefs about the likelihood that the technology will survive. Garratt and Wallace (2018) also studied the behaviour of the Bitcoin-to-Dollar exchange rate. They used the model introduced by Samuelson (1958) with identical two-period lived overlapping generations with one good per date. After exploring the problems of pinning down money prices in the one-money model, these authors expanded their

[4]More specifically, Fernández-Villaverde and Sanches (2019) extended the Lagos and Wright (2005) model by including entrepreneurs who can issue their own currencies to maximise profits following a predetermined algorithm (as in Bitcoin).

analysis to include a competing outside fiat money (Bitcoin), and they also discussed other aspects of competing cryptocurrencies.

More recently, Sockin and Xiong (2020) developed a model in which cryptocurrency has two main roles: (i) to facilitate transactions of certain goods among agents; (ii) as the fee to compensate coin miners for providing clearing services for the decentralised goods transactions on the platform. As a consequence of the first role of cryptocurrency, households face difficulties in making such transactions as a result of severe search frictions. In turn, such rigidity induced by the cryptocurrency price leads to either no or two equilibria.

Schilling and Uhlig (2019) used a model in the spirit of Samuelson (1958), assuming that there are two types of money: Bitcoin and fiat money, such as dollars. Both monies can be used for transactions. These authors found a "fundamental condition", which is a version of the exchange-rate indeterminacy result in Kareken and Wallace (1981), demonstrating that the Bitcoin price in dollar terms follows a martingale, adjusted for the pricing kernel. Schilling and Uhlig (2019) also found that there is a "speculative condition", in which the dollar price for Bitcoin is expected to rise and some agents start hoarding Bitcoin in anticipation of the price increase. Finally, Brunnermeier and Niepelt (2019) developed a dynamic and stochastic model with heterogeneous households, firms and banks, as well as the government sector. They demonstrated that a swap from public money to private money does not imply a credit crunch nor undermine financial stability.

However, most of the aforementioned theoretical studies have utilised partial equilibrium models. In our work, we develop a general equilibrium set-up. Many DSGE models have analysed the role of government currency in the economy. For example, Nelson (2002) presented empirical evidence for the US and the UK illustrating that real money-based growth matters for real economic activity. In particular, Nelson (2002) showed that the presence of the long-term nominal rate in the money demand function increases the effect of nominal money stock changes on real aggregate demand when prices are sticky.

In addition, Christiano et al. (2005) developed a model embodying nominal and real rigidities that accounts for the observed inertia in inflation and persistence in output. They included money among the variables of interest and found that the interest rate and the money growth rate move persistently in opposite directions after a monetary policy shock.

A small monetary business cycle model that contains three equations summarising the optimising behaviour of the households and firms that populate the economy was developed by Ireland (2004). This author found that, if changes in the real stock of money have a direct impact on the dynamics of output and inflation, then that impact must come simultaneously through both the IS and the Phillips curve. In the same spirit, Andrés et al. (2009) have analysed the role of money in a general equilibrium framework focusing on the US and the EU. Their findings uncovered the forward-looking nature of money demand.

Therefore, our work represents an extension of these studies, one which redefines the standard DSGE model with money by including a new sector of the economy related to cryptocurrency, thereby generating endogenous supply and demand in a general equilibrium framework.

2.2 Some stylised facts

In this section, we present an empirical analysis that has two main objectives. Firstly, we aim to unveil the relationship between the real balances for government currency and the cryptocurrency price in response to a monetary policy shock. Secondly, we focus on the shocks to cryptocurrency productivity and demand. Therefore, we estimate two SVAR models that have the following reduced form:

$$Y_t = C + \sum_{i=1}^{P} \Psi_i Y_{t-1} + \mu_t \tag{1}$$

where Y_t is a $(n \times 1)$ vector containing all n endogenous variables, C is a $(n \times 1)$ vector of constants, Ψ_i for $i = 1, ..., P$ are $(n \times n)$ matrices of parameters. P denotes the number of lags and μ_t is the vector of reduced-form innovations. We estimate the parameters of the two models using the Bayesian technique. We specify diffuse priors so that the information in the likelihood function becomes dominant. Priors give rise to a Normal-Inverse Wishart posterior with the mean and variance parameters corresponding to OLS estimates. The sample period of our SVAR analysis is the same as in our DSGE model. The number of lags corresponds to twenty.

As an identification strategy to estimate the first SVAR model, we adopt a Cholesky factorisation to recover the vector of structural shocks ϵ_t (and its variance Σ) from the reduced-form error μ_t in equation (1). The vector of variables Y_t is expressed as:

$$Y_t = [BP_t, GC_t, INT_t] \tag{2}$$

where BP_t denotes the real Bitcoin price, GC_t corresponds to the US real balances for government currency and INT_t is the US effective federal funds rate. In terms of Cholesky factorisation, we order first the Bitcoin price, then the real balances for government currency, and the last ordered variable is the effective federal funds rate. The three variables included in equation (2) are constructed as the observables in our DSGE model.

Figure 2 shows the empirical impulse responses of the real Bitcoin price, the real balances for government currency and the effective federal funds rate to a contractionary monetary policy shock. An increase in the effective federal funds rate induces a significant decrease in the real balances for government currency for the first two months. On the other hand, the response of the Bitcoin price is significantly positive in the same period. This result is also interpreted as an increase in the demand for Bitcoin.[5] Therefore, our empirical analysis indicates that real balances for government currency and cryptocurrency demand are inversely related in response to a contractionary monetary policy shock.

For the second SVAR model, we apply identification via sign restrictions. In particular, we follow the procedure described in Rubio-Ramirez et al. (2010). Our analysis is in line with Fry and Pagan (2011), who have used sign restrictions to solve the structural identification problem of a simple demand-supply model by providing sufficient information to identify the structural parameters. In order to map the economically meaningful structural shocks from the reduced form estimated shocks, we need to impose restrictions on the estimated variance-covariance matrix. In detail, the prediction error μ_t in equation (1) can be written as a linear combination of structural innovations ϵ_t:

$$\mu_t = V\epsilon_t$$

with $\epsilon_t \sim N(0, I_N)$ where I_N is an $N \times N$ identity matrix and V is a non-singular parameter matrix. The variance-covariance matrix has thus the following structure $\Sigma = VV'$. Our goal is to identify V from the symmetric matrix Σ, and to do that we need some restrictions. In line with Canova and Paustian (2011) and Furlanetto et al. (2019), these restrictions are imposed only on impact. In this second model, we include the series of the real cumulative initial coin offering (ICO_t) and the real Bitcoin price (BP_t). These two variables correspond to the same observables as in our DSGE model.

Cryptocurrency productivity shocks lead to a change in the quantity

[5] According to Athey et al., 2016, the demand for Bitcoin can be measured by Bitcoin transaction volume. In turn, the series of Bitcoin price and Bitcoin transaction volume exhibit identical behaviour over time. Indeed, the estimated correlation between the Bitcoin price and the Bitcoin transaction volume in our sample is equal to 0.96.

of cryptocurrency. In this regard, new initial coin offerings increase this quantity. A cryptocurrency productivity shock moves the quantity of cryptocurrency and the price of cryptocurrency in the opposite direction. On the other hand, cryptocurrency demand shocks are disturbances that displace the cryptocurrency demand curve and, hence, move the quantity of cryptocurrency and the price of cryptocurrency in the same direction.[6]

From Figure 3 we observe that, in response to a positive shock in cryptocurrency productivity, the real cumulative initial coin offering increases significantly on impact, whereas the real price of Bitcoin falls. The response of the latter variable remains significant and negative for the first four months after the shock. In Figure 4, we observe that a positive shock in the demand for cryptocurrency leads to an increase in both the real cumulative initial coin offering and the real Bitcoin price. The response for the former variable remains positive and significant for almost all the periods considered, whereas the response is only significantly positive for the first month for the second variable.

In summary, our empirical analysis sheds light on some important stylised facts that we will further analyse with our DSGE model. Firstly, in response to a monetary policy shock, cryptocurrency shows significant substitutability with government currency. As we will see in Section 5, our estimated DSGE model confirms this result.[7] Secondly, our SVAR analysis demonstrates that cryptocurrency productivity shocks are negatively related to the price of cryptocurrency, whereas an increase in the demand for cryptocurrency

[6] In our SVAR model, we use the real price of Bitcoin as the representative price of cryptocurrency. We made this choice due to the longer sample period that is available for the price of Bitcoin compared to other cryptocurrencies. Our assumption is plausible since, as we have described above, the correlation between the overall cryptocurrency market capitalisation and the Bitcoin market capitalisation is above 98 percent for our sample.

[7] We have also estimated a structural VAR model with industrial production, real balances for government currency and the real Bitcoin price. The results of the SVAR confirm the main transmission mechanisms predicted by our DSGE model in Section 5. We present the findings of this SVAR analysis in online Appendix C.

pushes up its price. Moreover, demand shocks seem to have a larger effect on both variables. These empirical findings are also confirmed by our estimated DSGE model in Section 5.

3 Model

3.1 Households

The representative household of the economy maximises the following expected stream of utility:

$$\max_{\{C_t, H_t, B_t, M_t^g, M_t^c\}} E\sum_{t=0}^{\infty}\beta^t A_t\left[u\left(C_t, \frac{\frac{M_t^g}{P_t}}{E_t^g}, \frac{\chi_t\frac{M_t^c}{P_t}}{E_t^c}\right) - \eta H_t\right] \tag{4}$$

where $0 < \beta < 1$ and $\eta > 0$. The budget constraint for each period is given by:

$$M_{t-1}^g + \chi_t M_{t-1}^c + T_t + B_{t-1} + W_t H_t + D_t = P_t C_t + \frac{B_t}{R_t} + M_t^g + \chi_t M_t^c \tag{5}$$

The variable $\frac{M_t^g}{P_t}$ represents the real balances for government currency, whereas $\frac{M_t^c}{P_t}$ denotes the real balances for cryptocurrency. Moreover, χ_t indicates the relative price of government currency with respect to cryptocurrency. Formally, we have that $\chi_t = P_t/P_t^c$, where P_t^c is the price of cryptocurrency.

Equation (4) shows that consumption, as well as the real balances of government currency and cryptocurrency, enter the utility function. In particular, for our benchmark model, we assume that the marginal utility of consumption is a function of the amount of real balances of government currency and cryptocurrency optimally demanded by the households. This means that consumption, government currency and cryptocurrency are non-separable in the utility function. Our approach implies that cryptocurrency is a private digital currency that is an alternative to government currency.[8]

[8] According to monetary theory, the three functions of money are medium of exchange, unit of account and store of value. Since cryptocurrency satisfies all three of these functions, it can be considered an alternative currency.

In equations (4) and (5), C_t and H_t denote household consumption and labour supply during the period t. The shocks A_t, E_t^g and E_t^c follow the autoregressive processes:

$$\ln(A_t) = \rho^a \ln(A_{t-1}) + \varepsilon_t^a \tag{6}$$

$$\ln(E_t^g) = \rho^{eg} \ln\left(E_{t-1}^g\right) + \varepsilon_t^{eg} \tag{7}$$

$$\ln(E_t^c) = \rho^{ec} \ln\left(E_{t-1}^c\right) + \varepsilon_t^{ec} \tag{8}$$

where $0 < \rho^a, \rho^{eg}, \rho^{ec} < 1$ and the zero-mean, serially uncorrelated innovations ε_t^a, ε_t^{eg} and ε_t^{ec}, are normally distributed with standard deviations σ^a, σ^{eg} and σ^{ec}. As we will illustrate below, in equilibrium, a shock to A_t translates to disturbances to the model's IS curve, whereas E_t^g and E_t^c indicate disturbances to the government money and cryptocurrency demand curves.[9]

In the budget constraint, household sources of funds include T_t, a lump-sum nominal transfer received from the monetary authority at the beginning of period t, and B_{t-1}, the value of nominal bonds that mature during period t. The household's sources of funds also include labour income, W_tH_t, where W_t denotes the nominal wage, and nominal dividend payments, D_t, received from the intermediate goods-producing firms. The household's uses of funds consist of consumption, C_t, of finished goods, purchased at the nominal price, P_t, newly-issued bonds of value $\frac{B_t}{R_t}$, where R_t denotes the gross nominal interest rate.

For the sake of convenience, going forward household real balances of government currency and cryptocurrency will be denoted by $m_t^g = \frac{M_t^g}{P_t}$ and $m_t^c = \frac{M_t^c}{P_t}$, respectively. Moreover, we denote the gross inflation during period t with $\pi_t = \frac{P_t}{P_{t-1}}$.

[9] In this regard, we would like to note that we do not assume any specific correlation between the disturbances to government currency and cryptocurrency. Therefore, our empirical results (Section 5) on the co-movement of these two key variables is not mechanically imposed but is rather an outcome of our model.

We note that our modelling approach does not include financial frictions. In this regard, an emerging and buoyant literature has focused on the financial disintermediation and instability that the adoption of central bank digital currency (CBDC) may cause (see, for example, Chiu et al., 2020; Sanches and Keister, 2021; Williamson, 2022).[10] However, these studies have exclusively considered the CBDC, while our work focuses on cryptocurrency. It is well known that there are at least four major differences between CBDC and cryptocurrency. Firstly, while cryptocurrency is decentralised and runs on its own blockchain, CBDC is controlled by the entity issuing it, which will either be the central bank or the government. Secondly, CBDC uses a permissioned blockchain network while cryptocurrency uses a permissionless one. Thirdly, the identity of CBDC users is known, while for cryptocurrency there are no data regarding their exact holdings from the population that can be assigned to "lenders" and/or "borrowers". Fourthly, the issuing authorities (such as central banks) will decide on the rules for CBDCs, while for cryptocurrency the users control the network by making consensus decisions.[11] Given these important differences between cryptocurrency and CBDC, the use of cryptocurrency will not necessarily lead to financial disintermediation as would most likely occur with the adoption of a CBDC. In the case of cryptocurrency, two possible scenarios may provoke a disruption of the financial sector. Firstly, individuals may decide to switch their savings from commercial bank deposits to cryptocurrency holdings. Secondly, initial coin offerings (ICOs), as a popular way to raise funds for products and services related to cryptocurrency, may lead non-financial firms to avoid borrowing from commercial banks. However, these aspects go beyond the objective of the current study and, therefore, we have decided to leave them

[10] More generally, a sizeable new literature has investigated the effects of the introduction of CBDCs on the macroeconomy (see, for example, Davoodalhosseini, 2018; Ferrari et al., 2022; Barrdear and Kumhof, 2022).

[11] Accordingly, for cryptocurrency there is not an asymmetric problem that originates between entrepreneurs and their creditors.

out of our analysis.

3.2 Entrepreneurs

We assume that there is a continuum of entrepreneurs indexed by n, where $n \in [0,1]$, thereby producing cryptocurrency.[12] Each representative entrepreneur operates under perfect competition. The cost faced by entrepreneurs is assumed to be exponential with respect to the quantity of cryptocurrency produced, $-\kappa^{-\nu_t}\exp(Q_t^c)$, where Q_t^c is the amount of tokens that the entrepreneur is producing. Our assumption relates to the fact that cryptocurrency is computationally intensive. Creating cryptocurrency requires solving difficult cryptographic puzzles. Adding transactions to a digital ledger, such as the blockchain, demands verifications by algorithms. All those calculations consume a substantial amount of energy. Previous research has found that Bitcoin is more energy intensive than Norway. In this regard, processing one transaction consumes more than \$100 worth of electricity and generates more than 800 kilograms of carbon dioxide.[13]

The entrepreneur's productivity is given by an autoregressive process of order one:

$$\nu_t = \rho^\nu \nu_{t-1} + \varepsilon_t^\nu \tag{9}$$

where $\rho^\nu < 1$, and the zero-mean, serially uncorrelated innovation, ε_t^ν, is normally distributed with standard deviation σ^ν. We assume that ν_t represents the productivity shock to producing costs of cryptocurrency.

Entrepreneurs sell cryptocurrency to households at price $\frac{P_t^c}{P_t}$ or $\frac{1}{\chi_t}$. Thus, they maximise their profits with respect to Q_t^c:

$$\Pi_t = \max_{Q_t^c} \left[Q_t^c \frac{1}{\chi_t} - \kappa^{-\nu_t} \exp\left(Q_t^c\right) \right] \tag{10}$$

[12] In our model, we make the simplifying assumption that entrepreneurs are both developers of the cryptocurrency and miners who provide clearing services for transactions in the platform.

[13] For more details, see `https://www.moneysupermarket.com/gas-and-electricity/features/crypto-energy-consumption/`.

3.3 Production goods firms

We assume a continuum of monopolistically competitive firms indexed by $i \in [0,1]$ producing differentiated varieties of intermediate production goods, and a single final production goods firm combining the variety of intermediate production goods under perfect competition. During each period $t = 0, 1, 2, ...$, the representative final goods-producing firm uses $Y_t(i)$ units of each intermediate good purchased at the nominal price, $P_t(i)$, to manufacture $Y_t(i)$ units of the final goods according to the constant-returns to-scale technology described by:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{(\theta-1)}{\theta}} di \right]^{\frac{\theta}{(\theta-1)}} \tag{11}$$

where $\theta > 1$. The final goods-producing firm maximises its profits by choosing:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \tag{12}$$

which reveals that θ measures the constant price elasticity of demand for each intermediate good. Competition drives the final goods-producing firm's profits to zero in equilibrium, determining P_t as:

$$P_t = \left[\int_0^1 (P_t(i))^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{13}$$

During each period $t = 0, 1, 2, ...$, the representative intermediate goods-producing firm hires $H_t(i)$ units of labour from the representative household to manufacture $Y_t(i)$ units of intermediate good i according to the linear technology:

$$Y_t(i) = Z_t H_t(i) \tag{14}$$

where the aggregate productivity shock, Z_t, follows the autoregressive process:

$$\ln(Z_t) = \rho^z \ln(Z_{t-1}) + \varepsilon_t^z \tag{15}$$

where $0 < \rho^z < 1$, and the zero-mean, serially uncorrelated innovation, ε_t^z, is normally distributed with standard deviation σ^z. In equilibrium, this supply-side disturbance acts as a shock to the Phillips curve. Since the intermediate goods substitute imperfectly for one another in producing the final goods, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market: the firm acts as a price-setter, but must satisfy the representative final goods-producing firm's demand at its chosen price. Similar to Rotemberg (1982), the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in terms of the final goods and given by:

$$\frac{\delta}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 Y_t \tag{16}$$

with $\delta > 0$ and π measuring the gross steady-state inflation rate. This cost of price adjustment makes the intermediate goods-producing firm's problem dynamic: it chooses $P_t(i)$ for all $t = 0, 1, 2, ...$ to maximise its total market value. At the end of each period, the firm distributes its profits in the form of a nominal dividend payment, $D_t(i)$, to the representative household.

3.4 Monetary policy

We assume that the central bank sets the nominal interest rate following a modified version of the Taylor (1993) rule given by:

$$\begin{aligned} \ln\left(\frac{R_t}{R}\right) &= \rho^r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho^r)\,\rho^y \ln\left(\frac{Y_t}{Y}\right) + \\ &(1-\rho^r)\,\rho^\pi \ln\left(\frac{\pi_t}{\pi}\right) + (1-\rho^r)\,\rho^{\mu^g} \ln\left(\frac{\mu_t^g}{\mu^g}\right) + \varepsilon_t^r \end{aligned} \tag{17}$$

where:

$$\mu_t^g = \frac{\frac{M_t^g}{P_t}}{\frac{M_{t-1}^g}{P_{t-1}}} \tag{18}$$

In equation (17), ρ^r, ρ^y, ρ^π and ρ^{μ^g} are non-negative parameters, and the zero-mean, serially uncorrelated policy shock, ε_t^r, is normally distributed with the standard deviation σ^r. The monetary authority adjusts the short-term nominal interest rate in response to deviations of output and inflation from their steady-state levels, as well as government currency growth, as shown in equation (18).[14] Andrés et al. (2009) have argued that an interest-rate rule that depends on the change in real balances of government currency is motivated as part of an optimal reaction function when money growth variability appears in the central bank's loss function. As an alternative explanation, the response to money growth is justified by money's usefulness in forecasting inflation.

3.5 Equilibrium

The symmetric equilibrium of the model can be log-linearised to obtain the following set of equations:[15]

$$\begin{aligned}
\hat{y}_t &= \hat{y}_{t+1} - \omega_1 \left(\hat{r}_t - \hat{\pi}_{t+1}\right) + \omega_2 \left[\left(\hat{m}_t^g - \hat{e}_t^g\right) - \left(\hat{m}_{t+1}^g - \hat{e}_{t+1}^g\right)\right] + && (19)\\
&\quad \omega_3 \left[\left(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c\right) - \left(\hat{\chi}_{t+1} + \hat{m}_{t+1}^c - \hat{e}_{t+1}^c\right)\right] + \omega_1 \left(\hat{a}_t - \hat{a}_{t+1}\right) \\
\hat{m}_t^g &= \gamma_1 \hat{y}_t - \gamma_2 \hat{r}_t + \gamma_3 \hat{e}_t^g - \gamma_4 \hat{\chi}_t - \gamma_4 \hat{m}_t^c + \gamma_4 \hat{e}_t^c && (20)\\
\hat{m}_t^c &= \gamma_5 \hat{y}_t - \gamma_6 \hat{r}_t + \gamma_7 \hat{e}_t^c - \gamma_8 \hat{m}_t^g + \gamma_8 \hat{e}_t^g - \gamma_9 \hat{\chi}_{t+1} + \gamma_{10} \hat{\chi}_t && (21)\\
\hat{\pi}_t &= \left(\frac{\pi}{R}\right) \hat{\pi}_{t+1} + \psi \begin{bmatrix} \left(\frac{1}{\omega_1}\right) \hat{y}_t - \left(\frac{\omega_2}{\omega_1}\right) \left(\hat{m}_t^g - \hat{e}_t^g\right) \\ - \left(\frac{\omega_3}{\omega_1}\right) \left(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c\right) - \hat{z}_t \end{bmatrix} && (22)\\
\hat{\chi}_t &= \varrho \hat{\nu}_t - \rho \hat{m}_t^c && (23)\\
\hat{r}_t &= \rho^r \hat{r}_{t-1} + \left(1-\rho^r\right) \rho^y \hat{y}_t + \left(1-\rho^r\right) \rho^\pi \hat{\pi}_t + \left(1-\rho^r\right) \rho^{\mu^g} \hat{\mu}_t^g + \varepsilon_t^r && (24)
\end{aligned}$$

Equation (19) represents a log-linearised version of the Euler equation that links the household's marginal rate of intertemporal substitution to the real interest rate. When ω_2 and ω_3 are different from zero, the household's

[14] In the Taylor rule (17), we do not include a parameter accounting for changes to cryptocurrency growth because the US FED does not consider cryptocurrency when it sets up the nominal interest rate.

[15] The small letters with a hat, $\hat{x}_t$, denote the deviation of a given variable, X_t, from its steady-state value. The full derivation of the model together with the steady-state solutions are shown in online Appendix A.

utility function is non-separable across consumption and real balances of government currency and cryptocurrency. Since utility is non-separable, real balances of government currency and cryptocurrency affect the marginal rate of intertemporal substitution. Hence, additional terms involving $\hat{m}_t^g$ and $\hat{m}_t^c$ also appear in the IS curve.

Equation (20) takes the form of a money demand relationship for government currency, with income elasticity (γ_1), interest semi-elasticity (γ_2), elasticity of $\hat{m}_t^g$ with respect to government currency demand shocks (γ_3) and cross-elasticity with cryptocurrency (γ_4).[16] Moreover, equation (21) reveals the form of a money demand relationship for cryptocurrency, with income elasticity (γ_5), interest semi-elasticity (γ_6), elasticity of $\hat{m}_t^c$ with respect to cryptocurrency demand shocks (γ_7), cross-elasticity with government currency (γ_8) and elasticity of $\hat{m}_t^c$ with respect to the current (γ_{10}) and expected (γ_9) relative price of government currency with respect to cryptocurrency, respectively.[17]

Equation (22) is a forward-looking Phillips curve that also allows real balances of government currency ($\hat{m}_t^g$) and cryptocurrency ($\hat{m}_t^c$) to enter into the specification when ω_2 and ω_3 are non-zero. The non-separability in preferences across consumption and real balances of government currency and cryptocurrency implies a direct influence of the former variable on marginal cost and inflation. Therefore, real balances of government currency and cryptocurrency also appear in the Phillips curve.

Equations (19) and (22) also reveal that, wherever the real balances of government currency ($\hat{m}_t^g$) and cryptocurrency ($\hat{m}_t^c$) appear in the IS and

[16]In equation (20), we note that an increase in the demand for cryptocurrency decreases the real balances of government currency. In Section 4.4, we will show that the estimated value of the cross-elasticity of government currency demand and cryptocurrency demand is high.

[17]Equation (21) indicates that the real balances of cryptocurrency decrease when the demand for government currency rises. In Section 4.4, our estimated results will show that the value of γ_8 is above unity, indicating a strong substitution effect between cryptocurrency and government currency.

Phillips curve relationships, they are immediately followed by the money demand disturbances, $\hat{e}_t^g$ and $\hat{e}_t^c$.

Equation (23) is the log-linearised first-order condition derived from the profit maximisation problem of entrepreneurs. This expression shows that the relative price of government currency with respect to cryptocurrency is determined through the cryptocurrency supply of entrepreneurs and cryptocurrency demand by households. More specifically, an increase in productivity in the cryptocurrency sector induces an increase in the relative price of government currency with respect to cryptocurrency,[18] whereas an increase in the demand for cryptocurrency implies a fall in the price of government currency with respect to cryptocurrency.[19] Equation (23) follows the theoretical predictions by Athey et al. (2016)[20] and reflects the well-established feature of cryptocurrency that is based on a cryptographic proof-of-work system. Such a system relies on solving complex mathematical operations and generating new coins via this validation. The process is known as mining. In our model, this mechanism is interpreted as a positive productivity shock that increases the quantity of cryptocurrency, inducing a rise in the relative price of government currency with respect to cryptocurrency.

Focusing on the transmission channels of the cryptocurrency productivity shock for the economy, log-linearised equations (19)-(23) show that changes in cryptocurrency productivity, as well as in cryptocurrency demand, affect the IS and Phillips curves through the relative price of government currency with respect to cryptocurrency.

Equation (24) shows the log-linearised relation for the monetary policy

[18]This effect is due to the fall in the price of cryptocurrency that makes government currency more valuable than cryptocurrency.

[19]This effect is due to the increase in the price of cryptocurrency that makes the government currency less valuable than cryptocurrency.

[20]More specifically, Athey et al. (2016) showed that the exchange rate between Bitcoin and the US dollar is the ratio of the demand (transaction volume) and the effective supply of Bitcoin.

rule, indicating that the interest rate adjusts to output, inflation and government currency growth.

The cryptocurrency market is in equilibrium if the quantity of cryptocurrency supplied by entrepreneurs is equal to the demand for cryptocurrency by households. The goods market clearing condition implies that the output produced by production goods firms is equal to households' consumption. The model is closed by adding the log-linearised versions of the AR(1) processes for the preferences shock to consumption, the demand shocks for government currency and cryptocurrency, the cryptocurrency productivity shock and the aggregate technology shock.

4 Estimating the model

In this section, we estimate the model described in Section 3 using Bayesian techniques. In what follows, we initially describe the data used in order to estimate the model (Section 4.1), then we present the parameters of the model (Section 4.2) and the estimation process (Section 4.3). Finally, we describe the estimation results (Section 4.4).

4.1 Data

The main challenge in estimating our model is the relatively short sample for the macroeconomic series related to the cryptocurrency market due to its recent development. Accordingly, in order to have a sufficient number of observations for our estimated model, we decided to use US data at monthly frequency. Our sample period corresponds to 2013:M6-2022:M4. We use six data series in the estimation because there are six shocks in the theoretical model (see Table 1).[21]

[21] The data sources and the construction of all observed variables are reported in online Appendix B.

The six series include the industrial production index,[22] the natural log of real private consumption, the natural log of real money stock, the real Bitcoin price, the real cumulative initial coin offering (ICO) and the effective federal funds rate. All the real variables are deflated by the consumer price index (CPI). Real private consumption and real money stock are expressed in per capita terms, divided by the working-age population. Following Pfeifer (2014), we detrend each real variable separately.[23] Accordingly, the measurement equations of our model are as follows:

$$\text{Industrial Production} = \hat{y}_t \tag{25}$$

$$\text{Real Private Consumption} = \hat{a}_t \tag{26}$$

$$\text{Real Balances for Government Currency} = \hat{m}_t^g \tag{27}$$

$$\text{Real Bitcoin Price} = \hat{m}_t^c \tag{28}$$

$$\text{Real Cumulative Initial Coin Offering} = \hat{\nu}_t \tag{29}$$

$$\text{Effective Federal Funds Rate} = \hat{r}_t \tag{30}$$

Focusing on monetary variables, we follow Ireland (2004) by considering money stock M2 as an indicator that includes a broader set of financial assets held principally by households. The real Bitcoin price is obtained from the monthly average of daily data, assuming that the daily price is the average between opening and closing prices. As mentioned above, we consider the Bitcoin price to be representative of the cryptocurrency price. The ICO or initial currency offering is a type of funding that uses cryptocurrency. In an ICO, a quantity of cryptocurrency is sold in the form of tokens to buyers, in exchange for legal tender or another cryptocurrency. The tokens sold are promoted as future functional units of currency if the ICO's funding goal is

[22]In this regard, we note that in the sample period of our analysis, the series of US industrial production has a correlation of 0.88 with the series of the US monthly GDP index. Moreover, our strategy of using industrial production is in the spirit of Giannone et al. (2016) and Gelfer (2019) who use this variable in their DSGE models to enhance their analysis and predictive accuracy in now-casting and forecasting.

[23]In particular, we use the HP filter with a smoothing parameter equal to 1,600.

met and the project is launched.

4.2 Model parameters

We decided to split the parameters of the model into two groups. The first group of parameters is fixed and consistent with data at a monthly frequency. In line with Ireland (2004), we assume ω_1 equal to one, implying the same level of risk aversion as a utility function that is logarithmic in consumption. The parameter ψ is fixed equal to 0.1 following King and Watson (1996), Ireland (2000) and Ireland (2004). This value implies that the fraction of the discounted present value and future discrepancies between the target price and the actual price of production goods is equal to 10 percent. The steady-state values for the nominal interest rate and inflation are computed from the monthly data of the effective federal funds rate and natural log changes in the CPI. For our sample period, they are equal to 0.69 percent and 0.20 percent, respectively.

The second group of parameters is estimated with the Bayesian technique (Tables 2 and 3). To the best of our knowledge, our study is the first attempt to estimate a DSGE model that includes cryptocurrency. Hence, this is one of our main contributions and we rely on our judgement and the findings of previous DSGE models that consider government currency (e.g., Ireland, 2000, Ireland, 2004 and Andrés et al., 2009).

Table 2 shows the prior distributions for the endogenous parameters of our model. For the parameter indicating the output elasticity with respect to real balances of government currency (ω_2), we assume that its prior mean is in line with the range of estimates by Ireland (2004). On the other hand, we assume that the prior mean of the elasticity of output with respect to real balances of cryptocurrency (ω_3) is one fourth lower than that of government currency. We note that our assumed prior distributions for ω_2 and ω_3 include the possibility of zero values for both these parameters.

In order to set up the priors for the income elasticity of government currency demand (γ_1), the interest semi-elasticity of government currency demand (γ_2) and the elasticity of real balances of government currency with respect to government currency demand shocks (γ_3), we use values that are in line with Ireland (2004) for the US economy. Moreover, we assume a prior mean value for γ_4, such that changes in the demand for cryptocurrency affect the real balances of government currency.

Focusing on the parameters that characterise the demand relationship for cryptocurrency, we assume that γ_6 has a higher prior mean value than γ_5. Moreover, we assume that the real balances of cryptocurrency are strongly affected by exogenous changes in cryptocurrency demand, which corresponds to a large prior mean for γ_7. In addition, we believe that there is a high substitutability between cryptocurrency and government currency and assume a high prior mean value for γ_8. We also assume high prior mean values for γ_9 and γ_{10}. This implies that changes in the expected and current relative price of government currency with respect to cryptocurrency affect the real balances of cryptocurrency.

Turning to the determinants of the relative price of government currency with respect to cryptocurrency, we assume that changes in the demand for cryptocurrency play a larger role than advances in cryptocurrency productivity. Therefore, we assume a higher prior mean value for ρ than ϱ.

Regarding the parameters of the monetary policy rule, the prior for the degree of interest rate smoothing (ρ^r), the reaction coefficient of output (ρ^y), and the interest-rate response to inflation (ρ^π) are all in line with the estimates by Andrés et al. (2009). On the other hand, we assume a prior distribution for the response of the nominal interest rate to changes in government currency growth (ρ^{μ^g}), such that this parameter may also assume a negative value.

Table 3 reports the priors of the parameters related to the exogenous processes driving the economy. We set the persistence parameters of all autoregressive exogenous processes to be Beta distributed. We assume that the technology shock is more persistent than consumption preferences, cryptocurrency and government currency demand shocks. For the productivity shock to cryptocurrency, we assume that its prior mean and standard deviation correspond to 0.60 and 0.05, respectively. Finally, we use Inverse Gamma distributions for standard errors of all exogenous shocks with means equal to 0.01 and infinite degrees of freedom, which correspond to rather loose priors.

4.3 Estimation procedure

In order to approximate the posterior distribution of the parameters, we used Markov Chain Monte Carlo (MCMC) methods. Specifically, we applied the Metropolis-Hastings algorithm to generate parameter observations on which to base inference. We estimated our model using a sample of 2,000,000 posterior draws and we dropped half of them.[24] Our acceptance rate corresponds to 38 percent. In order to test the stability of the sample, we used the Brooks and Gelman (1998) diagnostics test, which compares within and between moments of multiple chains. Moreover, we performed other diagnostic tests for our estimates, such as MCMC univariate diagnostics and multivariate convergence diagnostics.[25]

We compared the prior and posterior distributions of the model parameters. For most of the parameters, we found that the prior probability density functions are wide and that the posterior distributions are different to the priors.[26] Moreover, all the parameters of our model are identified in

[24] In order to perform our estimation analysis, we used Dynare (http://www.dynare.org/).

[25] The plots for MCMC univariate and multivariate convergence diagnostics are shown in online Appendix D.

[26] We report the plots for prior and posterior density functions of all parameters in online Appendix D.

the Jacobian of steady-state and reduced-form solution matrices.

4.4 Parameter estimation

Tables 2 and 3 show the posterior means for the endogenous and exogenous parameters with their 90 percent confidence intervals.

We start by focusing on the estimated parameters of the IS curve. From Table 2, we note that the posterior distributions of ω_2 and ω_3 lie in a positive range of values and their posterior means correspond to 0.09 and 0.08, respectively. This result indicates that the utility function is non-separable between consumption and real balances of government currency and cryptocurrency. Interestingly, the estimated values ω_2 and ω_3 imply that the output response to changes in real balances of government currency is stronger compared to variations in real balances of cryptocurrency.[27] As we will see in the next section, this result has important consequences for the effects of cryptocurrency productivity shocks on the economy.

Turning to the parameters of the money demand equation for government currency, our estimated values of γ_1, γ_2 and γ_3 are in line with the ranges of estimates provided by Ireland (2004), implying that the demand shock ($\hat{e}_t^g$) has the greatest influence on the movements in the real balances of government currency. Moreover, the estimated posterior of γ_4 is well identified and indicates a cross-elasticity of roughly 0.37 between government currency demand and cryptocurrency demand.

Now we focus on the estimated parameters included in the money demand equation for cryptocurrency. From Table 2, it is possible to note that the posterior mean of γ_6 is much higher than γ_5, implying that real balances of cryptocurrency respond more to changes in the nominal interest rate than

[27] As a robustness exercise, we estimated the model by assuming a prior value of ω_2 and ω_3, both equal to zero. This assumption is in the spirit of the unconstrained model of Ireland (2004). Our estimated results (which are available upon request) are in line with those of the benchmark model and highlight a greater influence of government currency than cryptocurrency on output.

to variations in output. Moreover, we find that the posterior mean of γ_7 is much higher than its prior value. This result suggests that the demand shock ($\hat{e}_t^c$) plays a substantial role in terms of variation in the real balances of cryptocurrency. Focusing on the estimated posterior of γ_8, we observe that its mean value is close to unity. This result indicates a strong elasticity of substitution between cryptocurrency and government currency. In the next section, we are going to show that the change in government currency demand greatly affects the demand for cryptocurrency. Turning to the posterior estimates of γ_9 and γ_{10}, our estimated results confirm that changes in the expected and current relative price of government currency with respect to cryptocurrency substantially affect the real balances of cryptocurrency.

Focusing on the parameters that determine the behaviour of the relative price of government currency with respect to cryptocurrency, we find that the estimated posterior of ρ corresponds to roughly 1.29, whereas ϱ is roughly equal to 0.12. These estimates indicate that cryptocurrency demand by households plays a more important role than cryptocurrency productivity in explaining the variations in the relative price of government currency with respect to cryptocurrency.

Turning to the estimates of the monetary policy reaction function, we observe that in our sample period there is significant interest-rate smoothing. In addition, the nominal interest rate appears to react much more strongly to variations in the inflation rate than to output changes. Interestingly, our estimated parameter for the interest rate response to government currency growth (ρ^{μ^g}) has a higher value than in Andrés et al. (2009). This result is interpreted as an indication of either simple money targeting by the central bank over our sample, or as a sort of targeting of future inflation, by responding to information beyond that contained in current inflation.

Table 3 shows the posterior estimates for the exogenous processes. In general, the posteriors of these parameters are well identified. We note that technology, preferences and cryptocurrency productivity shocks are more persistent than government currency and cryptocurrency demand shocks. Finally, our posterior estimates show that shocks to government currency and cryptocurrency demand are much more volatile than the remaining shocks.

5 Main results

We start by describing the impulse response functions based on our estimated model. Secondly, we present the historical decomposition of the real Bitcoin price and the real balances for government currency.

5.1 Impulse response functions

In this section, we show the results of impulse response functions (IRFs) for the estimated model. Firstly, we focus on the "traditional" shocks to preferences, technology and monetary policy. Secondly, we analyse the shocks to the demand of households for real balances of government currency and cryptocurrency. Finally, we take into consideration the "new" shock to cryptocurrency productivity. We consider a positive one standard deviation shock for each of these exogenous processes and we set the values of the model estimated parameters equal to their mean estimates of the posterior distribution.[28]

"Traditional" shocks. Figures 5–7 present the responses of output, real balances of government currency and cryptocurrency, the relative price of government currency with respect to cryptocurrency, the inflation rate and the nominal interest rate.

Figure 5 shows the effects of a positive shock to household preferences. This is a positive demand shock that implies a higher output and an

[28]Accordingly, our strategy allows us to compare the impulse responses among the different shocks. In online Appendix F, we present the estimated impulse responses together with their confidence intervals.

increase in the price level.[29] The monetary authority responds to higher inflation by increasing the nominal interest rate, which achieves its peak after three months. Following this shock, the nominal income tends to increase. As a consequence, the level of transactions increases and this stimulates the demand for government currency on impact. These results are in line with the findings of Ireland (2004) and Andrés et al. (2009). Our impulse responses show a substitution effect between government currency demand and cryptocurrency demand on impact.[30] Therefore, the demand for cryptocurrency reduces and its price falls.[31] These effects imply an increase in the relative price of government currency with respect to cryptocurrency.

Figure 6 shows the IRFs for a positive shock to technology. As expected, in response to this shock, output increases, whereas the price level falls.[32] Accordingly, the monetary authority decreases its policy rate. The positive shock to technology raises nominal income. Therefore, the level of transactions increases which, in turn, stimulates the demand for government currency. These findings are in line with the results reported by Ireland (2004) and Andrés et al. (2009). Our impulse responses also show a substitution effect between government currency demand and cryptocurrency demand.[33] As a consequence, the demand for cryptocurrency reduces and its price falls. These effects imply an increase in the relative price of government currency with respect to cryptocurrency.

Figure 7 shows that one positive standard deviation shock to monetary

[29] We note that, on impact, the preferences shock increases output and inflation by about 0.18 percent and 0.05 percentage points, respectively.

[30] This effect is due to the large estimated value of γ_8, which indicates high substitutability between cryptocurrency and government currency.

[31] We note that the effects of this shock on the real balances for government currency and cryptocurrency are negligible when compared to those implied by technology and monetary policy shocks on the same variables.

[32] We find that one positive standard deviation shock to technology increases output, and the peak is achieved after three months and corresponds to about 0.93 percent. Inflation decreases on impact by about 0.10 percentage points and it remains negative for all the periods considered in the graph.

[33] Again, this substitution effect relates to the large estimated value of γ_8.

policy induces an increase in the nominal interest rate by 0.66 percentage points. This monetary tightening reduces output and induces a fall in the price level.[34] The increase in the nominal interest rate reduces the demand for government currency. As a consequence, the real balances for government currency decrease. These results confirm the findings of Ireland (2004) and Andrés et al. (2009). Moreover, our IRFs show a substitution effect between government currency demand and cryptocurrency demand.[35] As a consequence, the demand for cryptocurrency increases.[36] This, in turn, implies an increase in the price of cryptocurrency.[37] This effect implies the fall in the relative price of government currency with respect to cryptocurrency.

Our interesting and novel results indicate that, in response to preferences, technology and monetary policy shocks, cryptocurrency is highly substitutable with government currency. These findings confirm the first stylised fact evidenced by our SVAR analysis (Section 2), in which we observed the substitution between government currency and cryptocurrency in the case of a monetary policy shock. Focusing on the transmission mechanisms of these "traditional" shocks, we observe that the real balances of cryptocurrency are not the main drivers of the responses of the other macroeconomic aggregates. On the contrary, they simply react to changes in the demand for government currency.

Government currency demand and cryptocurrency demand shocks. Figure 8 presents the impulse responses to government currency (solid lines) and cryptocurrency demand (dashed lines) shocks.

A positive shock to government currency demand corresponds to a

[34] On impact, output decreases by 1.96 percent and inflation by 0.42 percentage points.

[35] As explained above, the large estimated value of γ_8 is responsible for this effect.

[36] We observe that the real balances of cryptocurrency increase by 0.04 percent on impact.

[37] This result is line with the empirical findings in Section 2. More specifically, our SVAR model showed that the price of cryptocurrency increases in response to a contractionary monetary policy shock.

positive aggregate demand shock that induces higher output and increases the price level. In response to this shock, the demand for government currency increases. We observe that the interest rate falls as a reaction to the negative government currency growth. This implies that households have lower returns from holding government currency. Since cryptocurrency is a valuable alternative to government currency, its demand increases. As a consequence, the price of cryptocurrency rises, leading to a fall in the relative price of government currency with respect to cryptocurrency.

Now, we focus on the effects of a positive shock to cryptocurrency demand. This is a positive shock to aggregate demand that implies higher output and higher inflation. The higher demand for cryptocurrency pushes up its price. This result is in line with the empirical predictions of Section 2.[38] Accordingly, cryptocurrency becomes more expensive than government currency. In Figure 8, this effect is represented by the fall in the relative price of government currency with respect to cryptocurrency. In this case, government currency is a more desirable option than cryptocurrency. This induces an increase in the demand for government currency. Finally, we observe that the nominal interest rate decreases in response to this shock. This is explained by the negative government currency growth as implied by the Taylor rule.[39]

To summarise, the above findings indicate cryptocurrency and government currency are substitutes in response to both government currency and cryptocurrency demand shocks.

Shock to cryptocurrency productivity. The shock to cryptocurrency productivity is presented in Figure 9. A positive shock to the

[38]In particular, our SVAR model showed that the price of cryptocurrency tends to be positive in response to an increase in the demand for cryptocurrency.

[39]We note that the effects of this shock on output, inflation and the nominal interest rate are weaker than those of the government currency demand shock. This result is explained by the lower estimated value of ω_3 compared to ω_2.

productivity of entrepreneurs producing cryptocurrency implies a fall in the price of cryptocurrency. This, in turn, induces an increase in the relative price of government currency with respect to cryptocurrency.[40] Accordingly, the demand for cryptocurrency increases.[41] Moreover, we observe a substitution effect between government currency demand and cryptocurrency demand.[42] Therefore, the demand for government currency decreases. The drop in the real balances for government currency implies a lower level of transactions and, in turn, a lower nominal income. Therefore, both output and inflation fall.[43] We note that, in terms of magnitude, the changes in both output and inflation are much lower compared to their responses in the case of "traditional" shocks.[44] Moreover, we observe that the nominal interest rate is not responsive to this shock.[45]

To sum up, the productivity shock makes the price of cryptocurrency cheaper compared to government currency. This finding is in line with the empirical impulse responses obtained from our SVAR analysis in Section 2.[46] Since cryptocurrency and government currency are highly substitutable, this effect makes cryptocurrency more attractive compared to government currency. Therefore, the demand for the former increases, whereas it drops for the latter. In terms of transmission channels, we note that the cryptocurrency productivity shock is the key driver affecting the responses of

[40] On impact, the relative price of government currency with respect to cryptocurrency increases by 0.05 percent in response to this shock.

[41] The increase in the real balances of cryptocurrency corresponds to 0.05 percent.

[42] This effect is attributable to the large estimated value of γ_8.

[43] Equations (19) and (22) show that the real balances for government currency and cryptocurrency appear in both the IS relationship and in the Phillips curve. Moreover, our estimated results indicate the larger estimated value of the output elasticity to the real balances of government currency (ω_2) compared to the output elasticity with respect to cryptocurrency (ω_3).

[44] On impact, the productivity shock induces a fall in output of only 0.008 percent, whereas the inflation rate decreases by only 0.005 percentage points.

[45] More specifically, the nominal interest rate marginally decreases in response to the productivity shock because of lower inflation.

[46] More specifically, our SVAR analysis showed that increases in the productivity of cryptocurrency tend to decrease the price of cryptocurrency.

the other macroeconomic fundamentals. However, its impact on the economy is not as strong as the “traditional” shocks presented earlier.

5.2 Historical decomposition analysis

Figure 10 shows the monthly historical decomposition for the real Bitcoin price and the real balances of government currency.

Focusing on the real Bitcoin price (top panel of Figure 10), our results provide a clear indication that the demand for Bitcoin is the main driver of the changes in its price. Therefore, the surges in the real Bitcoin price registered in December 2017, in April 2021 and in November 2021 were caused by unexpected demand shocks. Similarly, the large drops that the real Bitcoin price experienced in January 2019 and in April 2022 were driven by the same shocks. Our results extend the findings by Kristoufek (2013), which showed the importance of demand factors in the volatile nature of cryptocurrency. In other words, our results indicate that if the price of Bitcoin increases, so does its demand, pushing the price to increase even more. On the other hand, if the price of Bitcoin decreases, the lower demand for Bitcoin makes the price decline even further.

Turning to the real balances for government currency (bottom panel of Figure 10), we find that government currency and cryptocurrency demand shocks play a dominant role. Importantly, this Figure illustrates that government currency and cryptocurrency display a high degree of substitution. This can be seen by focusing on the first half of 2020. In that period, the M2 supply grew by 20 percent, from 15.33 USD trillion in January 2020 to 18.3 USD trillion at the end of July 2020.[47] In response to this increase and the higher risk of inflation, households and financial investors started to invest in Bitcoin. In turn, this helps to explain the extraordinary growth in the demand for Bitcoin that is observed in the second

[47]As is well known, the M2 supply is normally characterised by slow and steady growth.

half of 2020.[48]

6 Robustness

In this section, we start by assessing some of the assumptions of the utility function. Firstly, we distinguish the cases of non-separable and separable household preferences between consumption and real balances of cryptocurrency. Secondly, we relax the assumption for which the utility function is linear in labour. Thirdly, we consider a utility function that is non-logarithmic in consumption. In the last part of this section, we provide a sensitivity analysis on different assumptions concerning the monetary policy rule.

6.1 Separability assumption in the utility function

In Section 4, our estimated results indicated that ω_3 is different from zero. This result confirms the assumption that the utility function is non-separable between consumption and the real balances of cryptocurrency. In turn, this implies that the marginal utility of consumption is a function of the amount of these real balances optimally demanded by households. Therefore, a change in the real balances of cryptocurrency has a direct positive impact on household consumption. As explained in Section 3, the non-separability assumption introduces terms involving the real balances of cryptocurrency into the IS and the Phillips curves.[49] Hence, in equilibrium, output and inflation depend on the current and expected real balances of cryptocurrency, after accounting for cryptocurrency demand shocks.

[48] Many financial analysts have supported this idea (see, for example, `https://www.cnbc.com/2020/08/05/the-ballooning-money-supply-may-be-the-key-to-unlocking-inflation-in-the-us.html`)

[49] In equations (19) and (22), these additional terms are the shift-adjusted real balances of cryptocurrency, i.e., $\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c$, respectively.

In this section, we are going to provide a counterfactual analysis on this non-separability assumption. Practically, the effect of the real balances of cryptocurrency on aggregate demand vanishes when the parameter ω_3 is equal to zero, i.e., as long as the cross derivative between consumption and the real balances of cryptocurrency is zero in the utility function. Therefore, we simulate our model assuming that ω_3 is equal to zero. We further extend this analysis by considering the case of separability between consumption and the real balances of government currency and cryptocurrency. Thus, we also simulate our model, assuming that both ω_2 and ω_3 are equal to zero. This experiment provides a more direct comparison of our approach with the introduction of cryptocurrency and the work of Ireland (2004) and Andrés et al. (2006), which incorporates a separable utility function between private consumption and real balances of government currency.

Figure 11 shows the responses of the several macroeconomic aggregates to one standard deviation shock in cryptocurrency productivity. The solid lines represent the IRFs of the several macroeconomic aggregates for the benchmark model (with non-separable utility function), whereas the dashed lines correspond to the counterfactual model A (with separable utility function between consumption and the real balances of government currency and cryptocurrency, i.e., $\omega_2 = 0$ and $\omega_3 = 0$) and the dotted lines indicate the counterfactual model B (with separable utility function between consumption and the real balances of cryptocurrency, i.e., $\omega_3 = 0$). Overall, our results indicate that the patterns of the several macroeconomic aggregates are qualitatively the same under the three different scenarios. This means that cryptocurrency and government currency have a high degree of substitutability in response to a cryptocurrency productivity shock. Our results also show that there is a much more pronounced decrease in output for the models with separable utility functions compared to the model with the non-separable utility function.

6.2 Other assumptions about the utility function

Most of the papers that analyse monetary business cycle models have assumed that the utility function is linear in labour (see, for example, Ireland, 2004). Therefore, in our theoretical framework, we have also followed this assumption. More specifically, in equation (4), our specification implies that the labour term is linear. In what follows, we relax this assumption and consider a more general form of the utility function. In our counterfactual analysis, the utility function reads as follows:

$$E\sum_{t=0}^{\infty}\beta^t A_t \left\{ u\left(C_t, \frac{\frac{MG_t}{P_t}}{EG_t}, \frac{\varkappa_t \frac{MC_t}{P_t}}{EC_t} \right) - \frac{H_t^{1+\sigma_h}}{1+\sigma_h} \right\}$$

where σ_h represents the inverse of the elasticity of work effort with respect to the real wage. In line with Smets and Wouters (2003), Smets and Wouters (2007) and Del Negro and Schorfheide (2008), we consider a value of σ_h that corresponds to two. Figure 12 shows the impulse responses of the simulated model to one standard deviation shock in cryptocurrency productivity. The solid lines represent the IRFs of the several macroeconomic aggregates for the benchmark model, while the dashed lines correspond to the model in which we assume that labour is non-linear in the utility function. From Figure 12, it is evident that the patterns of the several impulse responses are qualitatively the same under both scenarios. As expected, we note that on impact, output falls less when labour is non-linear in the utility function.[50] This is explained by the lower elasticity of work effort with respect to the real wage that, in turn, implies a mitigated reaction in hours worked in such a case.

A second common assumption in monetary business cycle models is that the utility function is logarithmic over consumption (see, for example, Ireland, 2004 and Andrés et al., 2009). In our model, this corresponds to the case in which the parameter ω_1 is equal to one. In order to assess whether our main findings are still valid under a more general specification, we simulated

[50] On impact, the productivity shock induces a fall in output of only 0.005 percent.

our model assuming a lower value for this parameter. In particular, we have halved the value of ω_1. This implies an increase in the level of risk aversion of the representative household. We believe that such a counterfactual exercise is of particular importance because in developed economies, such as the US, cryptocurrency is more unstable as a store of value than government currency. Figure 13 shows the IRFs of the several macroeconomic aggregates to a positive cryptocurrency productivity shock for the benchmark model and for the model with non-logarithmic preferences over consumption. The solid lines represent the responses of the variables for the benchmark model, whereas the dashed lines correspond to the model in which the utility function has non-logarithmic preferences over consumption. We note that our main results remain robust in this case also. As expected, we observe that output falls less in the case of a higher level of risk aversion. This result is explained by the fact that aggregate demand responds less strongly to changes in the nominal interest rate.

Therefore, we can conclude that the main findings of our benchmark model remain robust under different specifications of the utility function.[51]

6.3 Different assumptions about the Taylor rule

In this section, we investigate the role of monetary policy in the presence of a cryptocurrency productivity shock. In particular, we provide a sensitivity analysis with three different scenarios of the Taylor rule (24). More specifically, the parameter measuring the response of the policy rate to government currency growth (ρ^{μ^g}) is assumed to be: equal to its estimated value (benchmark scenario), equal to the half (scenario 1) and to the double (scenario 2) of its estimated value in our model.[52]

[51] The authors are indebted to an anonymous reviewer for providing insightful comments and directions for additional work, which has resulted in this section.

[52] Scenario 1 implies a small weight of government currency growth in the Taylor rule, whereas scenario 2 implies a large weight of government currency growth in the Taylor rule.

Figure 14 shows the responses of the key variables of our model in cases of a cryptocurrency productivity shock.[53] The solid lines represent the impulse responses of the variables in the benchmark scenario, whereas the dashed and dotted lines show the impulse responses for the same variables in scenarios 1 and 2, respectively.

The increase in the entrepreneurs' productivity induces a fall in the relative price of cryptocurrency with respect to government currency. Due to the substitution effect, the demand for cryptocurrency increases, whereas that of government currency falls. The drop in real balances for government currency implies a lower level of transactions and, in turn, output decreases. However, from Figure 14, we note that the magnitude of this decrease is different between the three scenarios. This result clearly depends on the response of the central bank to a cryptocurrency productivity shock. When the monetary authority gives a small weight to government currency growth in the Taylor rule (scenario 1), the response of the nominal interest rate is smaller in magnitude. In turn, the decreases in output and inflation are less pronounced than in the benchmark case. On the contrary, when the weight of government currency growth in the Taylor rule is larger (scenario 2), the response of the nominal interest rate is stronger than in the benchmark case. In turn, this effect induces a larger decrease in output and inflation.

7 Conclusion

In this paper, we have developed and estimated a Dynamic Stochastic General Equilibrium (DSGE) model to evaluate the economic repercussions of cryptocurrency. Our model assumed that the representative household maximises its utility by also accounting for cryptocurrency holdings. Moreover, in our theoretical framework, we have included the entrepreneurs

[53] As above, we simulate one standard deviation increase in the cryptocurrency productivity.

who determine the supply of cryptocurrency in the economy. We estimated our model using US monthly data and we compared our empirical findings with the "state-of-the-art" models without cryptocurrency.

We provided an impulse response analysis to show the effects of preferences, technology and monetary policy shocks on the real balances of government currency, as well as on the real balances of cryptocurrency. Moreover, we evaluated the responses of the main macroeconomic fundamentals to a productivity shock for the production of cryptocurrency.

We found a strong substitution effect between the real balances of government currency and the real balances of cryptocurrency in response to technology, preferences and monetary policy shocks. Similarly, government currency and cryptocurrency show a high degree of substitution in response to shocks in the demand for government currency and cryptocurrency. We also found that a cryptocurrency productivity shock implies an increase in the relative price of government currency with respect to cryptocurrency. In response to this shock, output and inflation fall. However, the magnitude of the effects of this shock is much lower than the "traditional" shocks.

Our results provide transmission mechanisms through which fluctuations in the cryptocurrency price can spill over to the real economy. Therefore, our analysis may be helpful to policymakers who aim to understand the macroeconomic repercussions of cryptocurrency. Moreover, we show that cryptocurrency can be an alternative to government currency, especially during periods of high expected inflation. Such an aspect could be considered by central banks when (and if) they decide to issue their own digital currency.

Our analysis opens up several extensions. For example, our estimated DSGE framework could be extended to a two-country exercise, extending studies on global cryptocurrency, such as Benigno et al. (2019), or even to a heterogeneous household setup.

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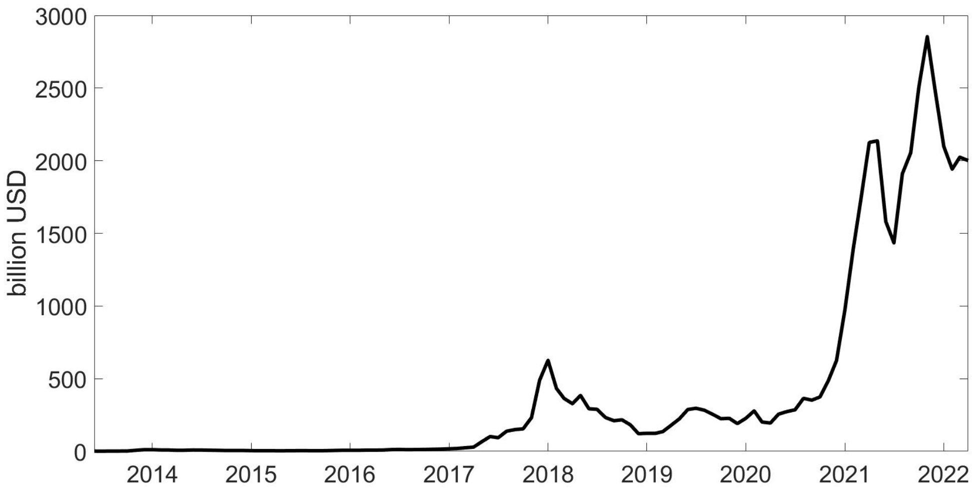

Figure 1: Overall cryptocurrency market capitalisation

Notes: Average monthly data from June 2013 to April 2022. The x-axis corresponds to the time period. *Source:* CoinGecko (`https://www.coingecko.com/`) and authors' estimates.

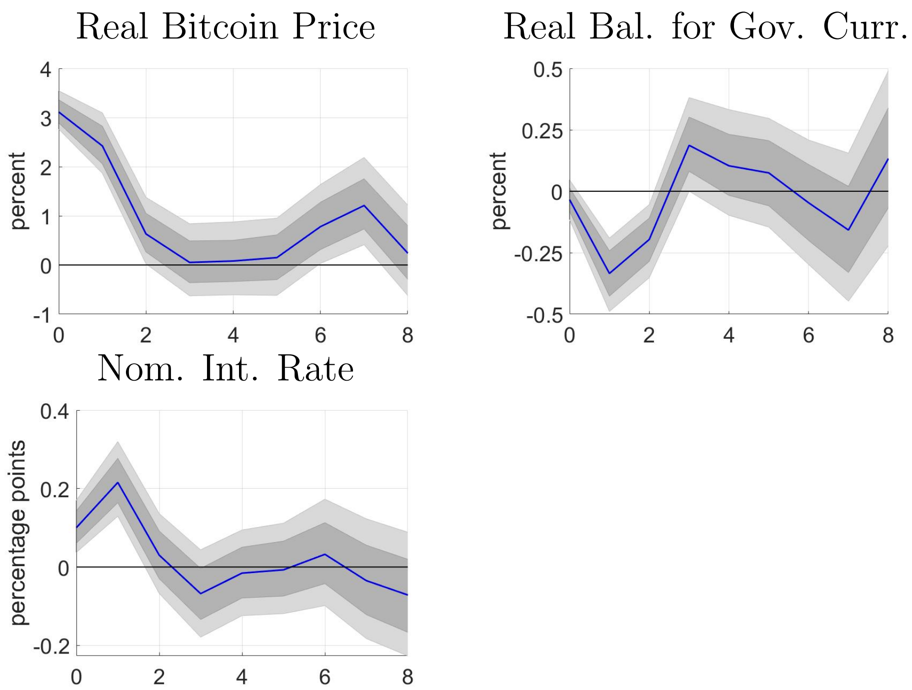

Figure 2: Responses to a contractionary monetary policy shock - IRFs obtained using the SVAR model based on the Cholesky identification as in equation (2)

Notes: Estimated one standard deviation shock to monetary policy. The nominal interest rate corresponds to the effective federal funds rate. Sample period 2013:M6 to 2022:M4. In each panel, the solid blue line represents the posterior median at each horizon, whereas the dark and light shaded areas indicate the 68*th* and 90*th* posterior probability regions, respectively, of the estimated impulse responses. Horizontal axis: months after shock.

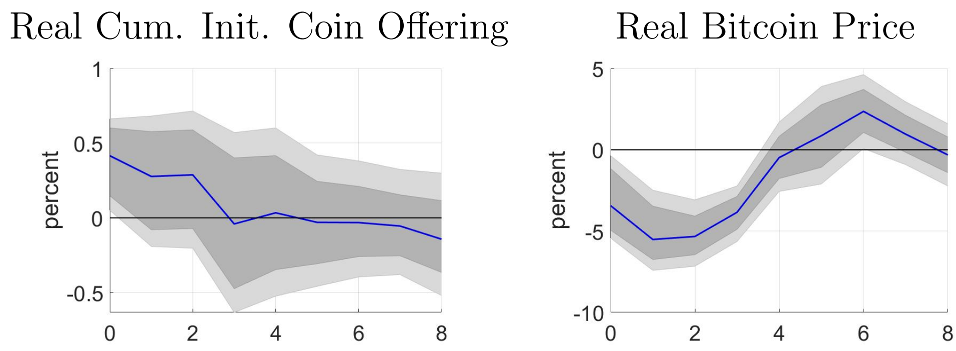

Figure 3: Responses to a positive shock in cryptocurrency productivity - IRFs obtained using the SVAR model based on sign restrictions

Notes: Estimated one standard deviation shock to cryptocurrency productivity. Sample period 2013:M6 to 2022:M4. In each panel, the solid blue line represents the posterior median at each horizon, whereas the dark and light shaded areas indicate the $68th$ and $90th$ posterior probability regions, respectively, of the estimated impulse responses. Horizontal axis: months after shock.

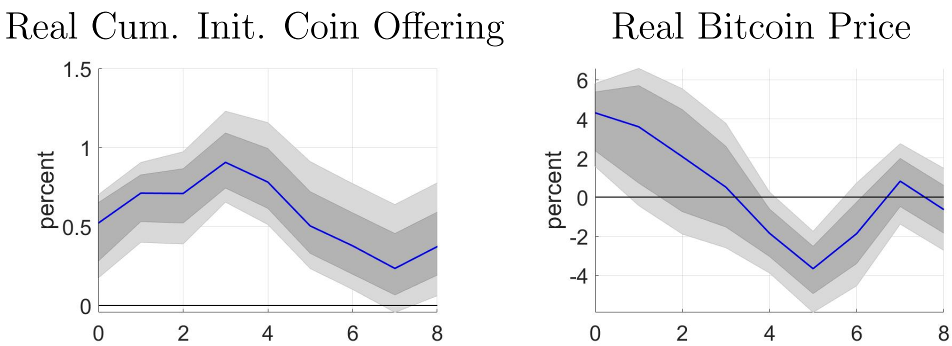

Figure 4: Responses to a positive shock in cryptocurrency demand - IRFs obtained using the SVAR model based on sign restrictions

Notes: Estimated one standard deviation shock to cryptocurrency demand. Sample period 2013:M6 to 2022:M4. In each panel, the solid blue line represents the posterior median at each horizon, whereas the dark and light shaded areas indicate the $68th$ and $90th$ posterior probability regions, respectively, of the estimated impulse responses. Horizontal axis: months after shock.

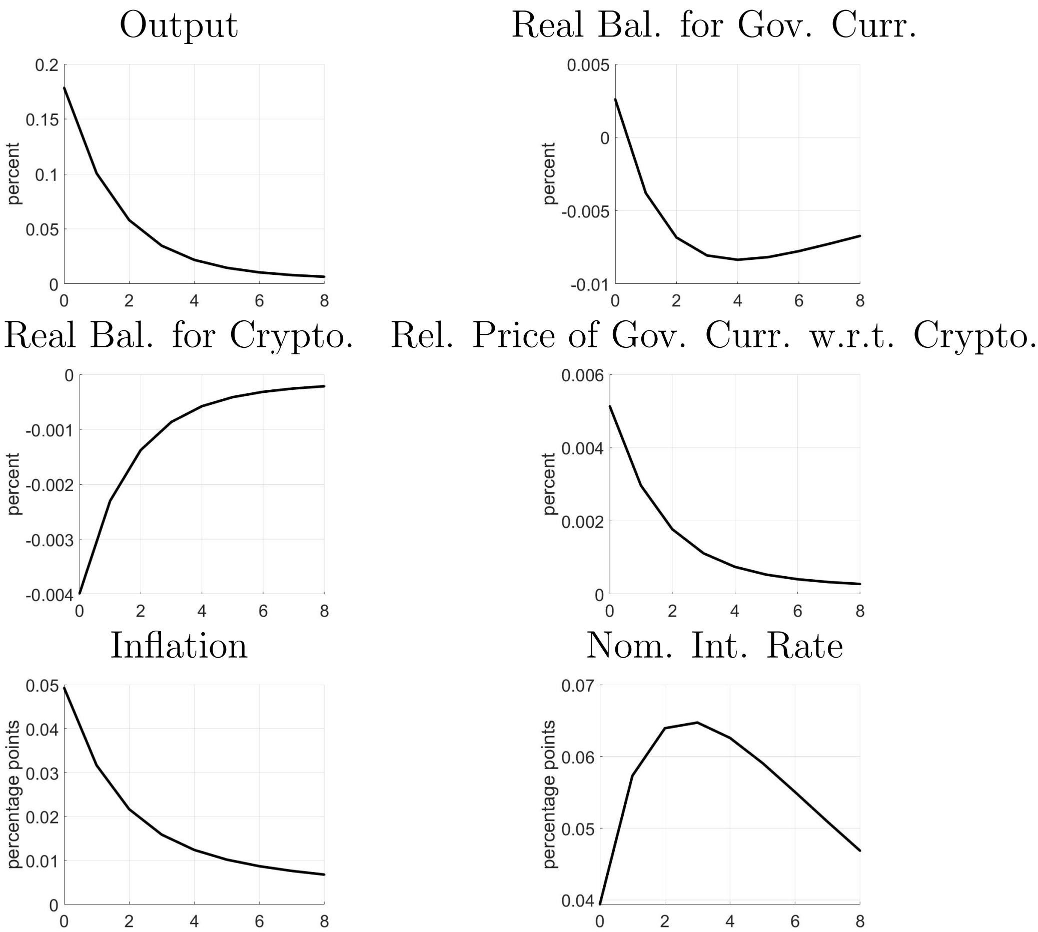

Figure 5: Responses to preferences shock - IRFs based on the baseline DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to household preferences. We set the values of the estimated parameters of the model equal to their mean estimates of the posterior distribution. Horizontal axis: months after shock.

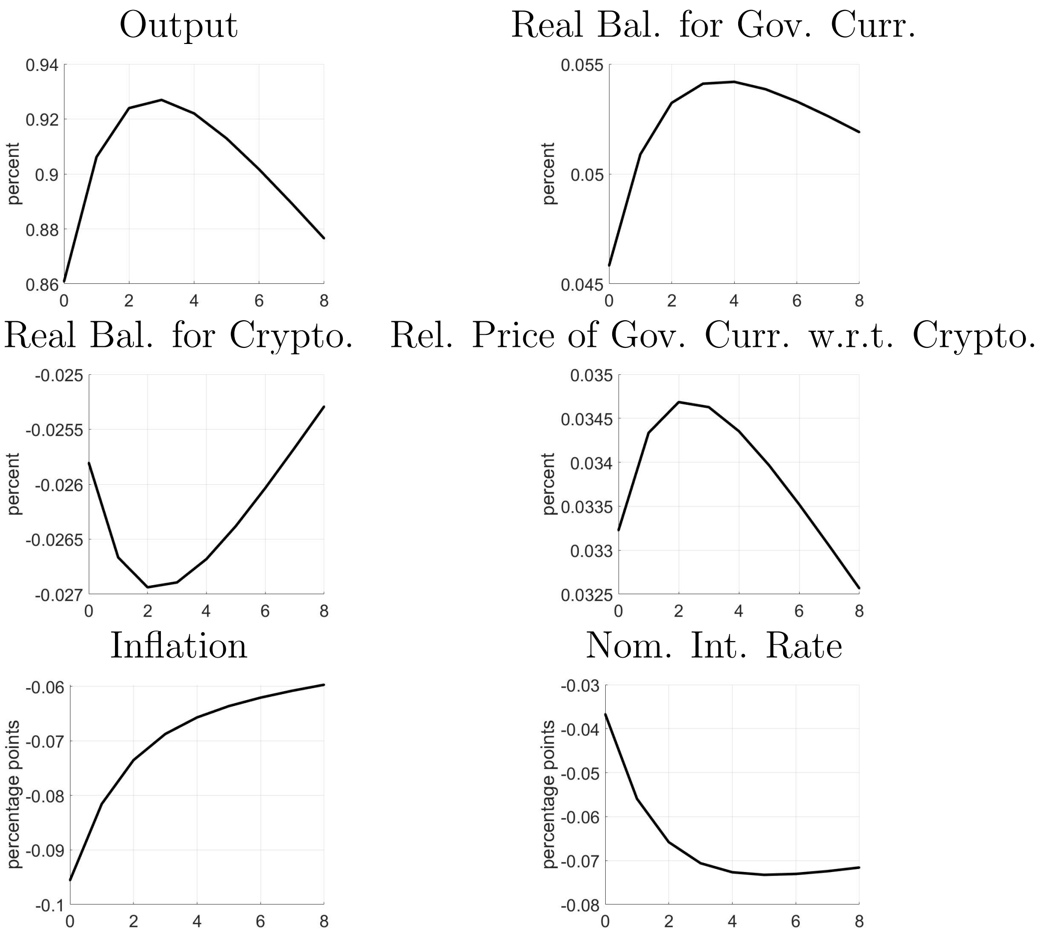

Figure 6: Responses to technology shock - IRFs based on the baseline DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to technology. We set the values of the estimated parameters of the model equal to their mean estimates of the posterior distribution. Horizontal axis: months after shock.

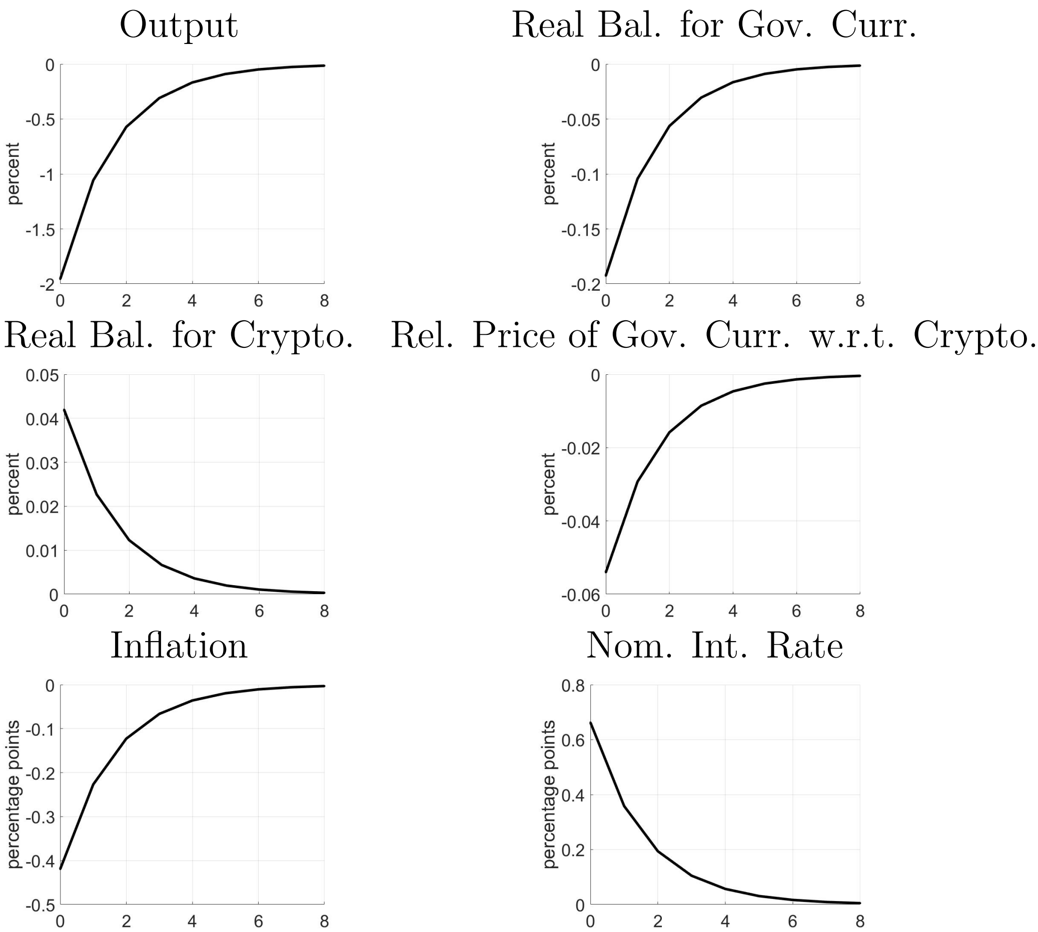

Figure 7: Responses to monetary policy shock - IRFs based on the baseline DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to monetary policy. We set the values of the estimated parameters of the model equal to their mean estimates of the posterior distribution. Horizontal axis: months after shock.

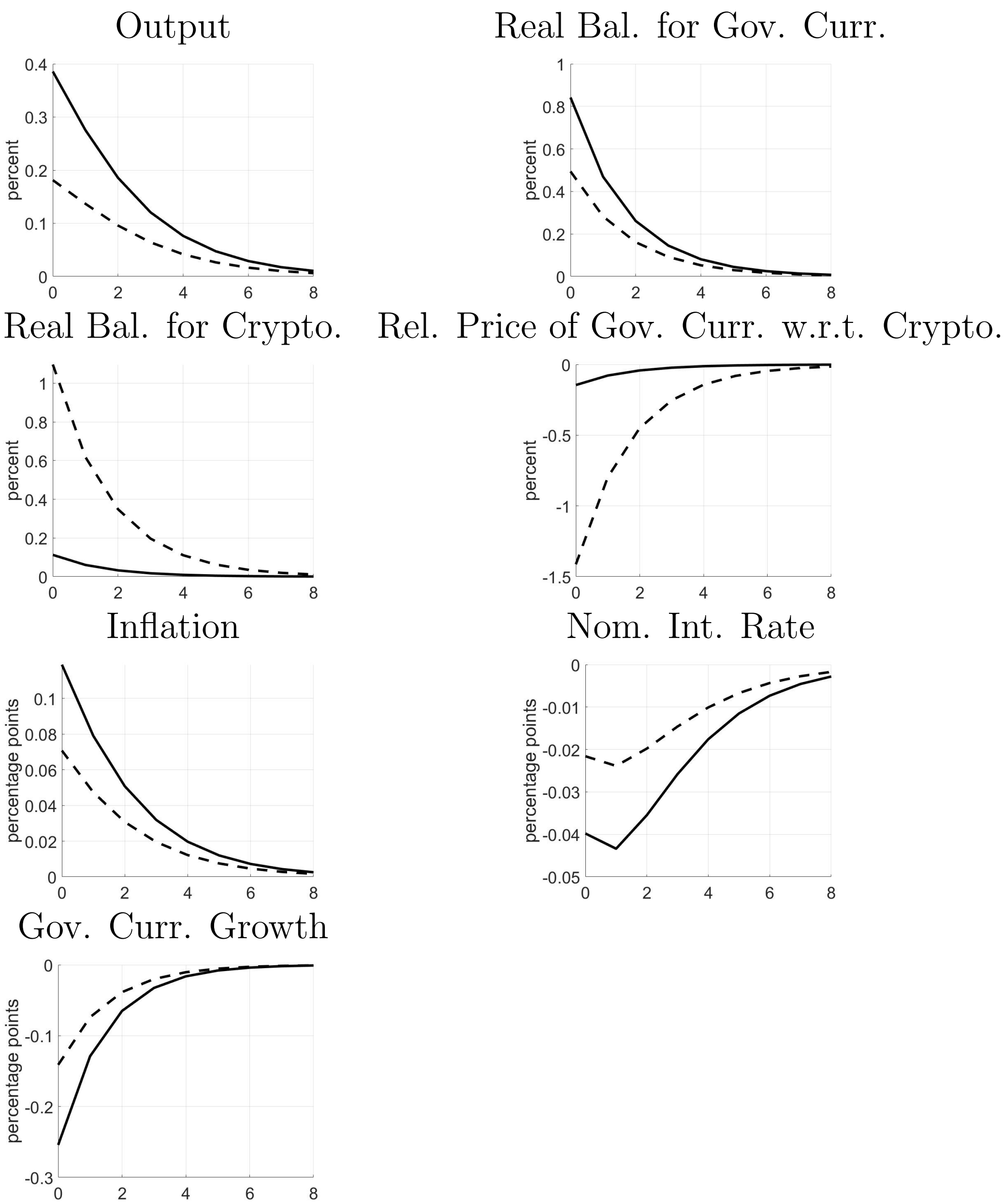

Figure 8: Responses to government currency and cryptocurrency demand shocks - IRFs based on the baseline DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to government currency demand and cryptocurrency demand, respectively. We set the values of the estimated parameters of the model equal to their mean estimates of the posterior distribution. In each panel, the solid line denotes the response to a government currency demand shock, whereas the dashed line represents the response to a cryptocurrency demand shock. Horizontal axis: months after shock.

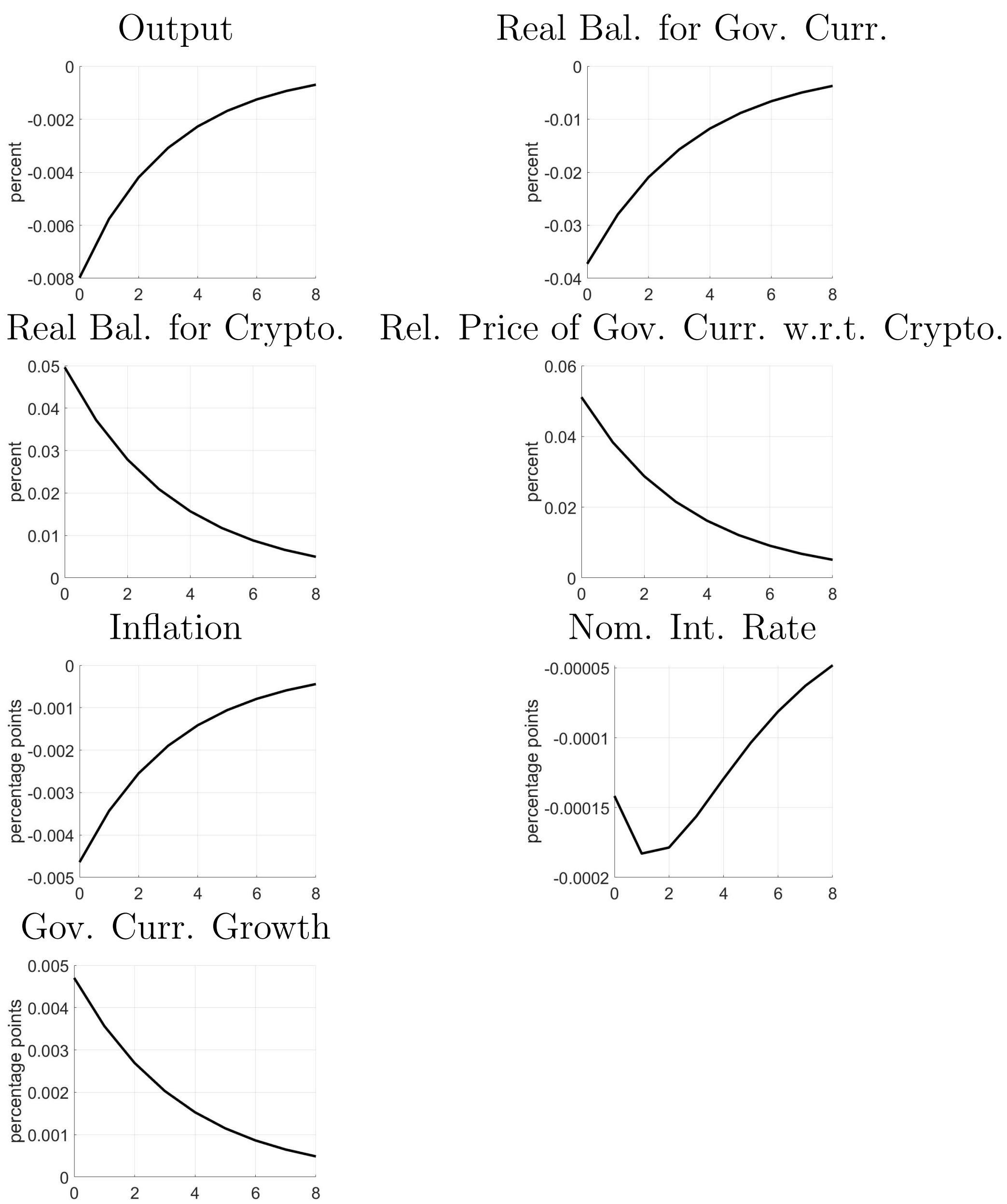

Figure 9: Responses to cryptocurrency productivity shock - IRFs based on the baseline DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to cryptocurrency productivity. We set the values of the estimated parameters of the model equal to their mean estimates of the posterior distribution. Horizontal axis: months after shock.

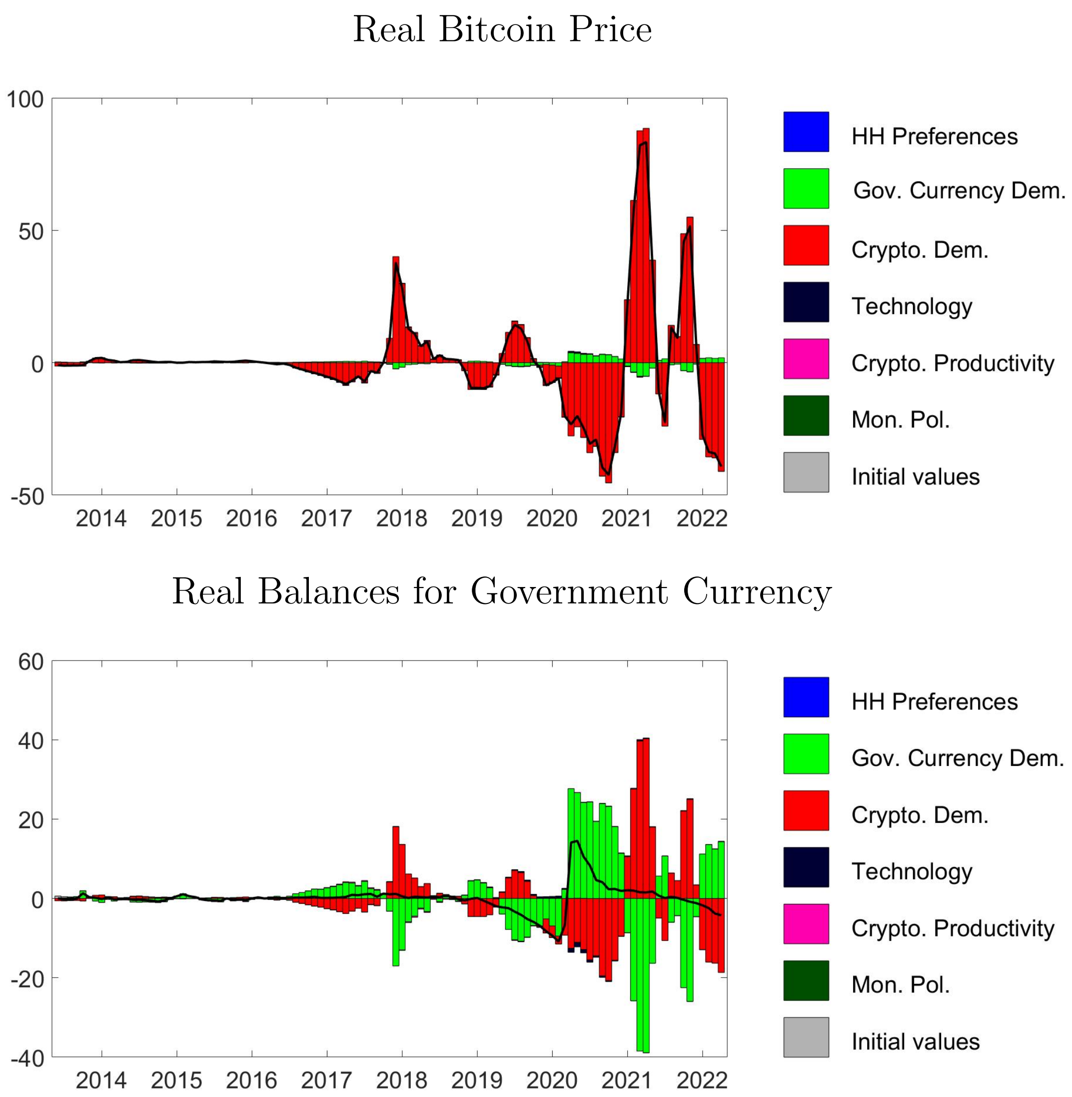

Figure 10: Historical decomposition based on the estimated DSGE model

Notes: The Figure shows the historical decomposition for the real Bitcoin price (top panel) and the real balances for government currency (bottom panel) for the sample period 2013:M6-2022:M4. The historical decomposition is calculated by using the Kalman smoother, i.e., it decomposes the historical deviations of these endogenous variables from their respective steady-state values into the contribution coming from the various shocks. In each panel, the black line represents the deviation of the endogenous variable from its steady state, whereas the bars of different colours indicate the several shocks of the model.

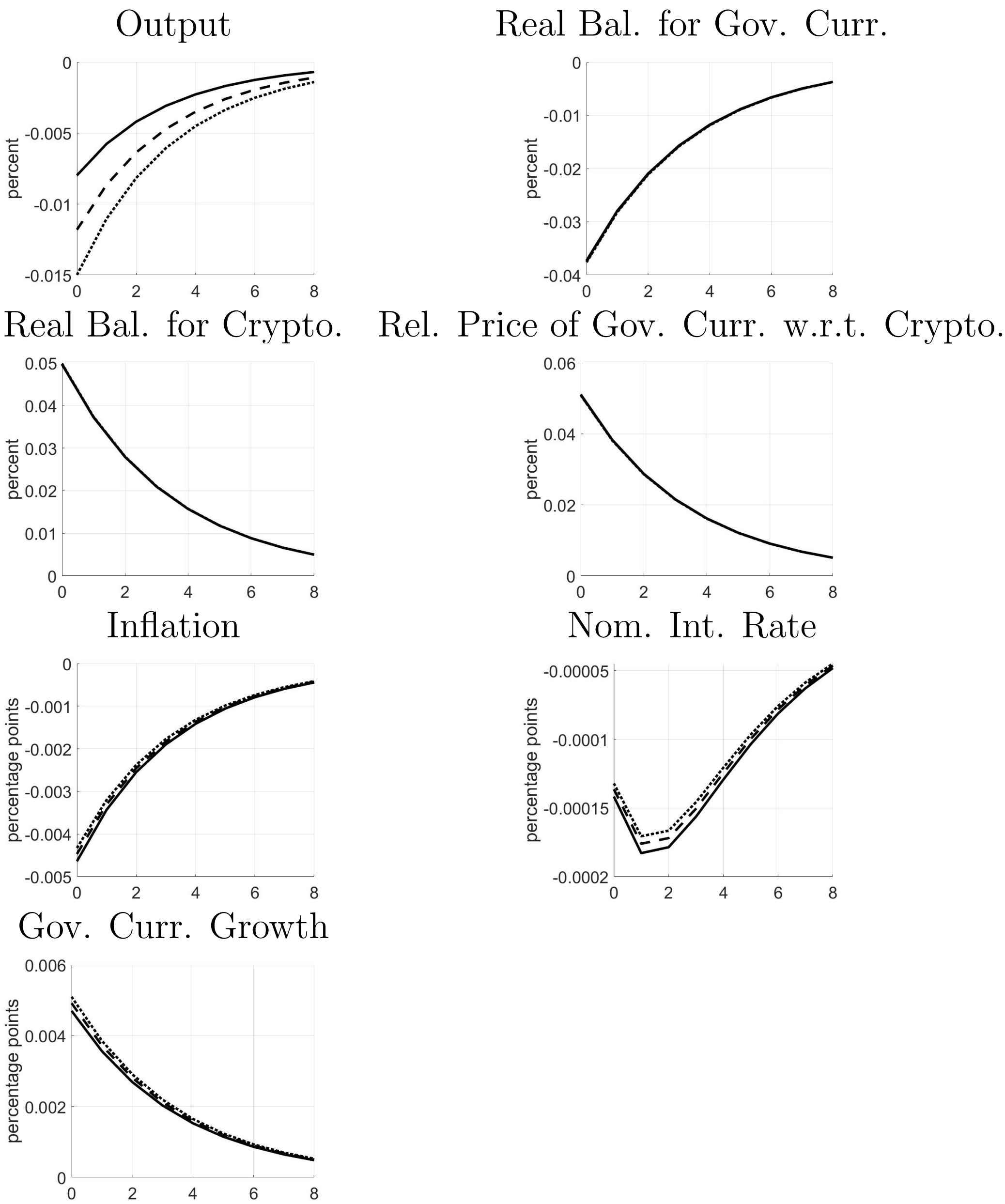

Figure 11: Non-separability vs. separability - IRFs to cryptocurrency productivity shock based on the DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to cryptocurrency productivity for the benchmark model and the models with separable utility function. In each panel, the solid line denotes the IRF of the benchmark model, whereas the dashed and dotted lines represent the responses of the counterfactual models A (where $\omega_2 = 0$ and $\omega_3 = 0$, i.e., separability between consumption and the real balances of government currency and cryptocurrency) and B (where $\omega_3 = 0$, i.e., separability between consumption and real balances of cryptocurrency), respectively. Horizontal axis: months after shock.

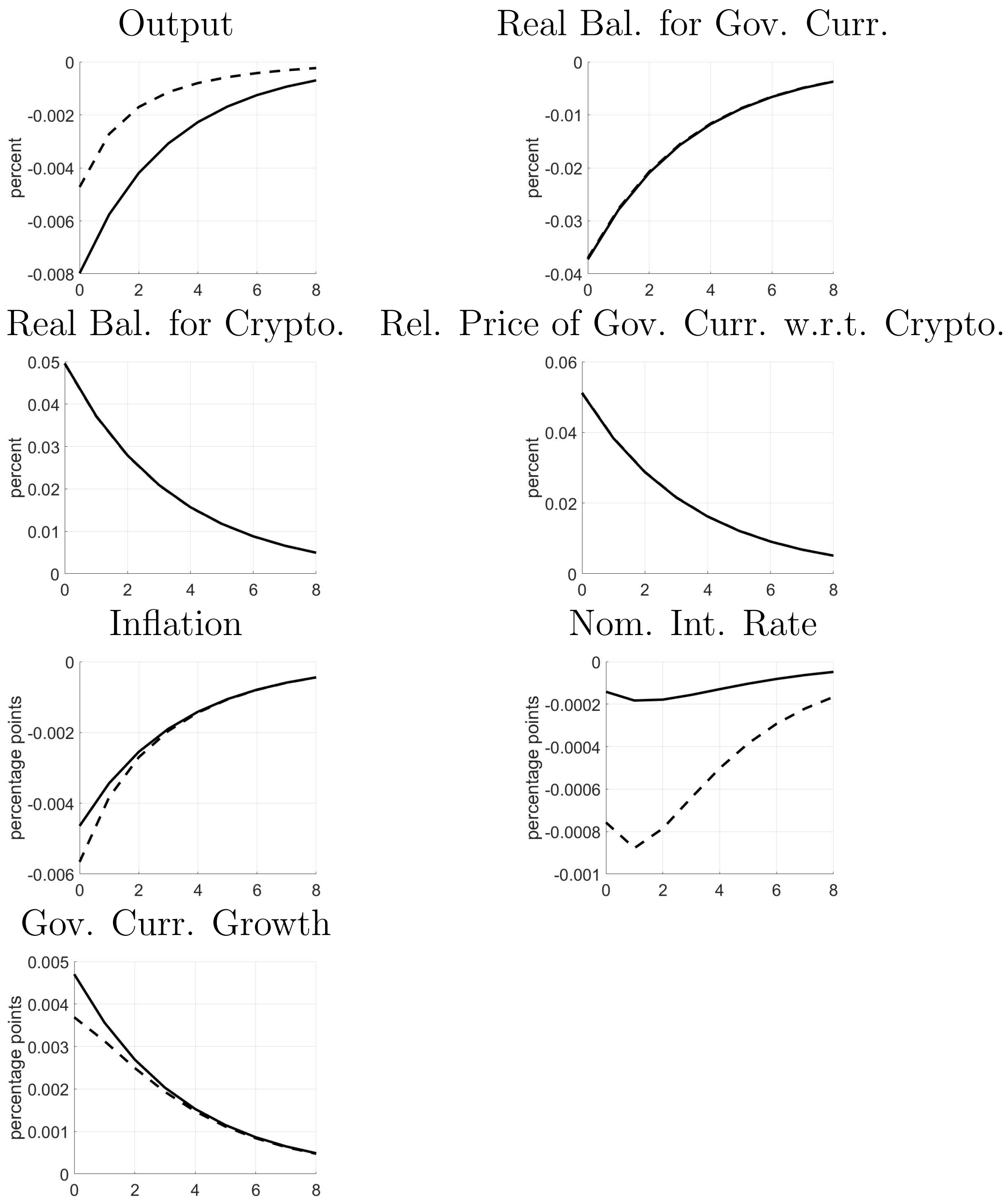

Figure 12: Non-linear labour in the utility function - IRFs to cryptocurrency productivity shock based on the DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to cryptocurrency productivity for the benchmark model and the model in which labour is non-linear in the utility function. In each panel, the solid line denotes the IRF of the benchmark model, whereas the dashed line represents the response of the counterfactual model with the utility function that is non-linear in labour. Horizontal axis: months after shock.

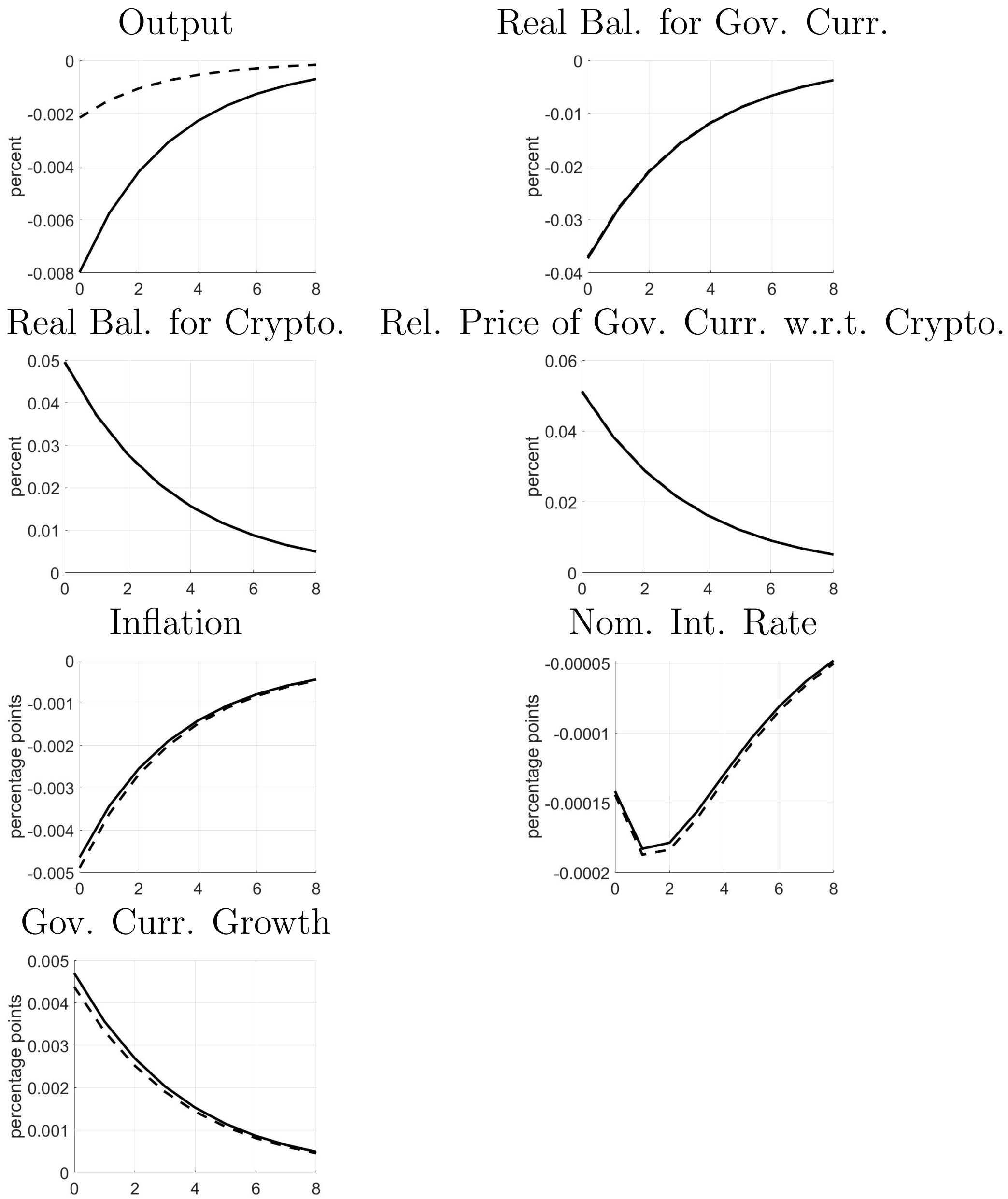

Figure 13: Non-logarithmic preferences over consumption - IRFs to cryptocurrency productivity shock based on the DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to cryptocurrency productivity for the benchmark model and the model with a higher level of risk aversion of the representative household. In each panel, the solid line denotes the IRF of the benchmark model, whereas the dashed line represents the response of the counterfactual model with higher risk aversion. Horizontal axis: months after shock.

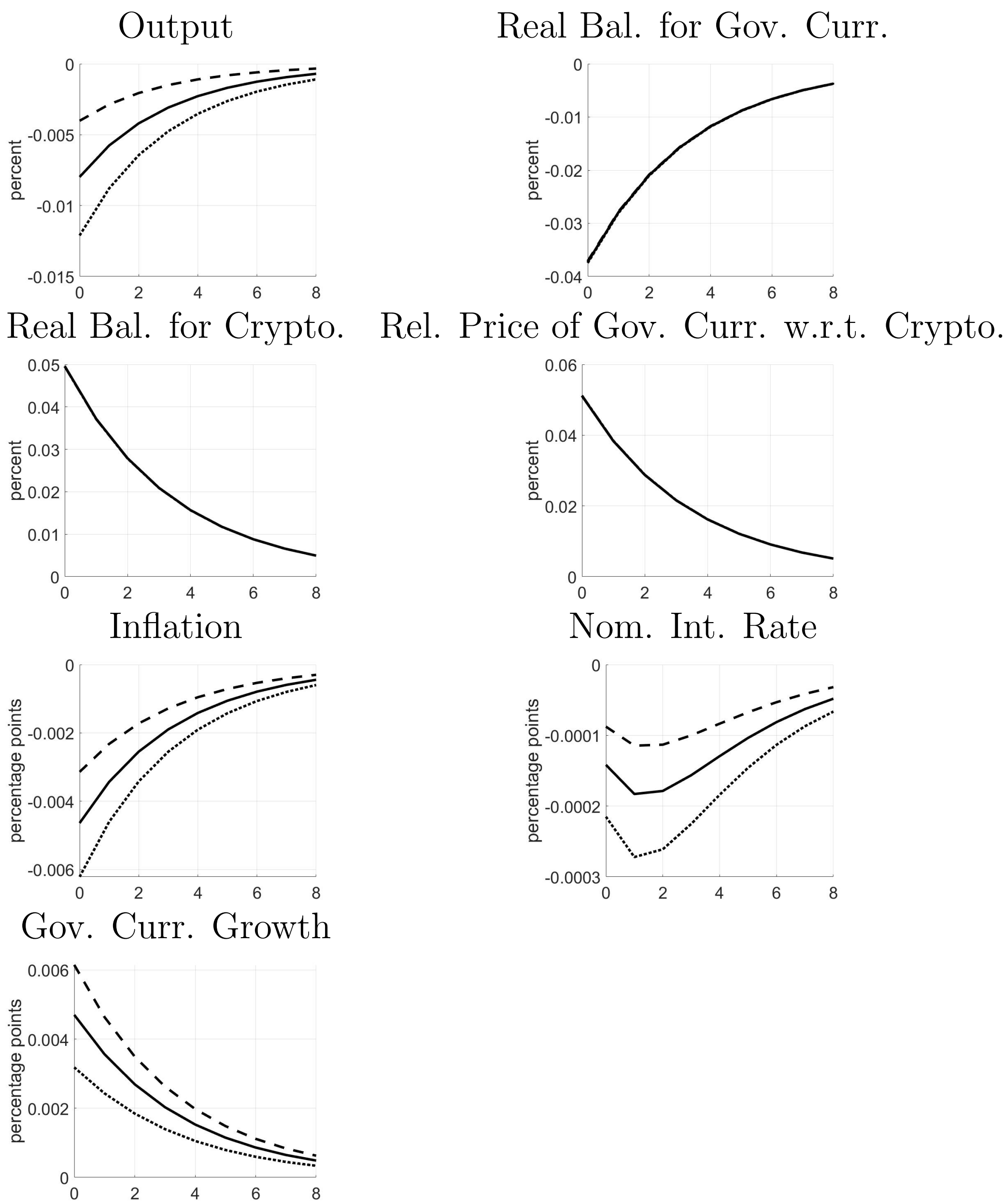

Figure 14: The role of monetary policy - IRFs to cryptocurrency productivity shock based on the DSGE model

Notes: The impulse responses are obtained from a simulated one standard deviation shock to cryptocurrency productivity for the benchmark model and the model with alternative monetary policies. In each panel, the solid line denotes the IRF of the benchmark model, whereas the dashed and dotted lines represent the responses of the model in scenarios 1 (i.e., half of the estimated value for ρ^{μ^g}) and 2 (i.e., double of the estimated value for ρ^{μ^g}), respectively. Horizontal axis: months after shock.

Table 1: Exogenous shocks and observed variables

Shocks	**Observed Variables**
Technology Shock	US Industrial Production Index
Shock to Household's Preferences	US Real Private Consumption
Shock to Household's Demand for Government Currency	US Real Balances for Government Currency
Shock to Household's Demand for Cryptocurrency	Real Bitcoin Price
Cryptocurrency Productivity Shock	Real Cumulative Initial Coin Offering (ICO)
Monetary Policy Shock	US Effective Federal Funds Rate

Notes: The Table shows the shocks and observed variables of the DSGE model. Shocks relate to equations (6), (7), (8), (9), (15) and (17) in the main text. Observed variables are described in Section 4.1. Their data sources and construction are reported in online Appendix B.

Table 2: Priors and posteriors for the endogenous parameters

Parameter	*Symbol*	*Priors*			*Posteriors*		
		Dist.	Mean	St. Dev.	Mean	Conf. Inter.	
Output El. to Real Bal. of Gov. Currency	ω_2	N	0.2000	0.1000	0.0946	0.0168	0.1654
Output El. to Real Bal. of Cryptocurrency	ω_3	N	0.0500	0.0200	0.0775	0.0487	0.1068
Income El. of Gov. Currency Demand	γ_1	G	0.0500	0.0050	0.0500	0.0418	0.0581
Interest Semi-El. of Gov. Currency Demand	γ_2	G	0.1500	0.0001	0.1500	0.1498	0.1502
El. of Real Bal. of Gov. Curr. wrt Gov. Curr. Dem. Shock	γ_3	G	0.8000	0.0100	0.8048	0.7882	0.8213
Cross El. of Gov. Cur. Dem. and Crypto. Dem.	γ_4	G	0.4000	0.0100	0.3662	0.3512	0.3819
Income El. Cryptocurrency Demand	γ_5	G	0.0150	0.0010	0.0150	0.0133	0.0166
Interest Semi-El. of Cryptocurrency Demand	γ_6	G	0.1500	0.0500	0.1511	0.0707	0.2292
El. of Real Bal. of Crypto. wrt Crypto. Dem. Shock	γ_7	G	1.5000	0.1000	2.1144	2.0096	2.2263
Cross El. of Crypto. Dem. and Gov. Cur. Dem.	γ_8	G	1.0000	0.0500	0.9991	0.9908	1.0072
El. of Real Bal. of Cry. w.r.t. Exp. Rel. Pr. btw. G. C. and Cry.	γ_9	G	0.7000	0.0050	0.7009	0.6928	0.7093
El. of Real Bal. of Cry. w.r.t. Cur. Rel. Pr. btw. G. C. and Cry.	γ_{10}	G	0.9000	0.0500	0.7702	0.6972	0.8410
El. of Rel. Price btw. Gov. Curr. and Cry. w.r.t. Cry. Sup.	ϱ	G	0.1000	0.0500	0.1150	0.0341	0.1933
El. of Rel. Price btw. Gov. Curr. and Cry. w.r.t. Cry. Dem.	ρ	G	1.3000	0.0100	1.2876	1.2711	1.3040
Interest Rate Smoothing	ρ^r	B	0.8000	0.0100	0.8161	0.8006	0.8316
Taylor Rule Coef. on Output	ρ^y	B	0.2000	0.0050	0.1907	0.1828	0.1988
Taylor Rule Coef. on Inflation	ρ^π	G	1.8000	0.0500	1.8883	1.8026	1.9741
Taylor Rule Coef. on Gov. Currency Growth	ρ^{μ^g}	N	0.4500	0.3500	2.0236	1.8179	2.2340

Notes: The Table shows the names, the acronym symbols, the prior distributions, means and standard deviations as well as the posterior means and credible intervals for the $5th$ and $95th$ percentiles of the endogenous parameters of the DSGE model. N, G and B stand for Normal, Gamma and Beta distributions, respectively.

Table 3: Priors and posteriors for the shock processes parameters

Parameter	*Symbol*	*Priors*			*Posteriors*		
		Distr.	Mean	St. Dev.	Mean	Conf. Inter.	
Household's Preference Shock Pers.	ρ^a	*B*	0.7000	0.0500	0.9149	0.9011	0.9312
Gov. Cur. Demand Shock Pers.	ρ^{eg}	*B*	0.5000	0.0100	0.5495	0.5389	0.5625
Crypto. Demand Shock Pers.	ρ^{ec}	*B*	0.6000	0.0100	0.5644	0.5529	0.5759
Technology Shock Pers.	ρ^z	*B*	0.9500	0.0100	0.9847	0.9802	0.9896
Crypto. Prod. Shock Pers.	ρ^ν	*B*	0.6000	0.0500	0.7499	0.7015	0.7991
Household's Preference Shock St. Err.	σ^a	*I-G*	0.0100	Inf	0.9841	0.8724	1.0912
Gov. Cur. Demand Shock St. Err.	σ^{eg}	*I-G*	0.0100	Inf	8.0309	6.9127	9.1251
Crypto. Demand Shock St. Err.	σ^{ec}	*I-G*	0.0100	Inf	12.4326	10.5678	14.2034
Technology Shock St. Err.	σ^z	*I-G*	0.0100	Inf	3.0989	2.7174	3.4783
Crypto. Prod. Shock St. Err.	σ^ν	*I-G*	0.0100	Inf	1.2723	1.1233	1.4187
Monetary Policy Shock St. Err.	σ^r	*I-G*	0.0100	Inf	0.0741	0.0636	0.0847

Notes: The Table shows the names, the acronym symbols, the prior distributions, means and standard deviations as well as the posterior means and credible intervals for the $5th$ and $95th$ percentiles of the exogenous parameters of the DSGE model. *I-G* and *B* stand for Inverse-Gamma and Beta distributions, respectively. Finally, "Inf" denotes infinite degrees of freedom.

Appendices to “A Bayesian DSGE Approach to Modelling Cryptocurrency”

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Contents

1 Appendix A: Model Solution

1.1 First Order Conditions

The representative household chooses C_t, H_t, B_t, M_t^g and M_t^c for all $t = 0, 1, 2, ...$ to maximize its expected utility, subject to its budget constraints. The first-order conditions for this problem can be written as follows.

The first order condition for C_t is given by:

$$\lambda_t = -A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) \frac{1}{P_t} \tag{A1}$$

$m_t^g = \frac{M_t^g}{P_t}$ and $m_t^c = \frac{M_t^c}{P_t}$, whereas λ_t is the Lagrange multiplier associated with the representative household budget constraint. Moreover, u_1 denotes the derivative of the utility function, u, with respect to its first argument.

The first order condition for H_t is given by:

$$-A_t \eta - \lambda_t w_t = 0 \tag{A2}$$

where $w_t = \frac{W_t}{P_t}$.

Combining equations (A1) and (A2) we obtain:

$$\eta = u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) w_t \tag{A3}$$

The first order condition for B_t is given by:

$$A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) = \beta R_t \left[A_{t+1} u_1 \left(C_{t+1}, \frac{m_{t+1}^g}{E_{t+1}^g}, \frac{\chi_{t+1} m_{t+1}^c}{E_{t+1}^c} \right) \right] \frac{1}{\pi_{t+1}} \tag{A4}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$.

The first order condition for M_t^g is given by:

$$R_t u_2 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) = (R_t - 1) E_t^g u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) \tag{A5}$$

where u_2 denotes the derivative of the utility function, u, with respect to its second argument.

The first order condition for M_t^c is given by:

$$R_t u_3\left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c}\right) = \left(R_t - \frac{\chi_{t+1}}{\chi_t}\right) E_t^c u_1\left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c}\right) \tag{A6}$$

where u_3 denotes the derivative of the utility function, u, with respect to its third argument.

The market clearing conditions imply that:

$$M_t^g = M_{t-1}^g + T_t$$
$$M_t^c = M_{t-1}^c$$
$$B_t = B_{t-1} = 0$$

Therefore, from the household's budget constraint we obtain that:

$$w_t H_t + d_t = C_t \tag{A7}$$

where $d_t = \frac{D_t}{P_t}$.

The representative entrepreneur chooses Q_t^c for all $t = 0, 1, 2, ...$ to maximize its profit given by:

$$\Pi_t = \max_{Q_t^c}\left[Q_t^c \frac{1}{\chi_t} - \kappa^{-\nu_t} \exp\left(Q_t^c\right)\right]$$

The first-order condition for this problem is:

$$\chi_t = \frac{1}{\kappa^{-\nu_t} \exp\left(Q_t^c\right)}$$

As explained in the main text, the cryptocurrency market is in equilibrium if the quantity of cryptocurrency supplied by entrepreneurs is equal to the demand of cryptocurrency by households: $Q_t^c = m_t^c$. Therefore, the last expression can be re-written as:

$$\chi_t = \frac{1}{\kappa^{-\nu_t} \exp\left(m_t^c\right)} \tag{A8}$$

The representative intermediate goods-producing firm chooses $P_t\left(i\right)$ for all $t = 0, 1, 2, ...$ to maximize its total market value, given by:

$$E\sum_{t=0}^{\infty} \beta A_t u_1\left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c}\right)\left[\frac{D_t\left(i\right)}{P_t}\right] \tag{A9}$$

where $\beta A_t u_1\left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c}\right)$ measures the marginal utility value to the representative household of an additional dollar in profits received during period t. Moreover:

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta} Y_t - \left[\frac{P_t(i)}{P_t}\right]^{-\theta}\left(\frac{w_t Y_t}{Z_t}\right) - \frac{\delta}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 Y_t \quad \text{(A10)}$$

for all $t = 0, 1, 2, ...$

The expression (A10) for the firm's real dividend payment incorporates the linear production function along with the requirement that the firm supply output on demand; it also shows how the cost of price adjustment subtracts from profits. The first-order conditions for this problem are:

$$\begin{aligned} 0 = {} & (1-\theta)\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\left(\frac{Y_t}{P_t}\right) + \\ & \theta\left[\frac{P_t(i)}{P_t}\right]^{-\theta-1}\left(\frac{Y_t w_t}{Z_t P_t}\right) - \delta\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]\left[\frac{Y_t}{\pi P_{t-1}(i)}\right] + \qquad \text{(A11)} \\ & \beta\delta E\left\{\left[\frac{A_{t+1}u_1\left(C_{t+1}, \frac{m_{t+1}^g}{E_{t+1}^g}, \frac{\chi_{t+1}m_{t+1}^c}{E_{t+1}^c}\right)}{A_t u_1\left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c}\right)}\right]\left[\frac{P_{t+1}(i)}{\pi P_t(i)} - 1\right]\left[\frac{Y_{t+1}P_{t+1}(i)}{\pi P_t(i)^2}\right]\right\} \end{aligned}$$

for all $t = 0, 1, 2, ...$

In a symmetric equilibrium:

$$Y_t(i) = Y_t$$

$$H_t(i) = H_t$$

$$P_t(i) = P_t$$

$$D_t(i) = D_t$$

and:

$$Y_t = Z_t H_t$$

for all $i \in [0,1]$ and $t = 0, 1, 2, ...$ equations (A7) and (A10) can be combined to derive the economy's aggregate resource constraint:

$$Y_t = C_t + \frac{\delta}{2}\left[\frac{\pi_t}{\pi} - 1\right]^2 Y_t \quad \text{(A12)}$$

Combining equations (A3) and (A11) we obtain:

$$\theta - 1 = \theta \left[\frac{\eta}{Z_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right)} \right] - \delta \left(\frac{\pi_t}{\pi} - 1 \right) \left(\frac{\pi_t}{\pi} \right) + \tag{A13}$$

$$\beta \delta E \left\{ \left[\frac{A_{t+1} u_1 \left(C_{t+1}, \frac{m_{t+1}^g}{E_{t+1}^g}, \frac{\chi_{t+1} m_{t+1}^c}{E_{t+1}^c} \right)}{A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right)} \right] \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{\pi_{t+1}}{\pi} \right) \right\}$$

1.2 Steady State Relations

In the absence of shocks, the economy converges to a steady state, in which:

$$Y_t = Y$$
$$C_t = C$$
$$m_t^g = m^g$$
$$\chi_t = \chi$$
$$m_t^c = m^c$$
$$\pi_t = \pi$$
$$R_t = R$$

From equation (A4) we have that:

$$R = \frac{\pi}{\beta} \tag{A14}$$

From equation (A12) we have that:

$$Y = C \tag{A15}$$

From equation (A5) we have that:

$$R_t u_2\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) = (R-1)\, E^g u_1\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) \tag{A16}$$

From equation (A6) we have that:

$$R_t u_3\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) = (R-1)\, E^c u_1\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) \tag{A17}$$

From equation (A8) we have that:

$$\chi = \frac{1}{\kappa^{-\nu} \exp\left(m^c\right)} \tag{A18}$$

From equation (A13) we have that:

$$(\theta - 1)\, Z u_1\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) = \theta\eta \tag{A19}$$

1.3 Log-linearized Equations

We denote:

$$\hat{y}_t = \ln\left(\frac{Y_t}{Y}\right)$$
$$\hat{c}_t = \ln\left(\frac{C_t}{C}\right)$$
$$\hat{m}_t^g = \ln\left(\frac{m_t^g}{m^g}\right)$$
$$\hat{\chi}_t = \ln\left(\frac{\chi_t}{\chi}\right)$$
$$\hat{m}_t^c = \ln\left(\frac{m_t^c}{m^c}\right)$$
$$\hat{\pi}_t = \ln\left(\frac{\pi_t}{\pi}\right)$$
$$\hat{r}_t = \ln\left(\frac{R_t}{R}\right)$$
$$\hat{a}_t = \ln\left(\frac{A_t}{A}\right)$$
$$\hat{e}_t^g = \ln\left(\frac{E_t^g}{E^g}\right)$$
$$\hat{e}_t^c = \ln\left(\frac{E_t^c}{E^c}\right)$$
$$\hat{\xi}_t = \ln\left(\frac{\xi_t}{\xi}\right)$$
$$\hat{\nu}_t = \ln\left(\frac{\nu_t}{\nu}\right)$$
$$\hat{z}_t = \ln\left(\frac{Z_t}{Z}\right)$$
$$\hat{\mu}_t^g = \ln\left(\frac{\mu_t^g}{\mu^g}\right)$$

The first-order Taylor approximation to equation (A12) gives:

$$\hat{y}_t = \hat{c}_t \tag{A20}$$

The first-order Taylor approximation to equation (A4) gives:

$$\hat{y}_t = \hat{y}_{t+1} - \omega_1 \left(\hat{r}_t - \hat{\pi}_{t+1}\right) + \omega_2 \left[\left(\hat{m}_t^g - \hat{e}_t^g\right) - \left(\hat{m}_{t+1}^g - \hat{e}_{t+1}^g\right)\right] + \omega_3 \left[\left(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c\right) - \left(\hat{\chi}_{t+1} + \hat{m}_{t+1}^c - \hat{e}_{t+1}^c\right)\right] + \omega_1 \left(\hat{a}_t - \hat{a}_{t+1}\right) \tag{A21}$$

where:

$$\omega_1 = -\frac{u_1\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{Y u_{11}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} \tag{A22}$$

$$\omega_2 = -\frac{\frac{m^g}{E^g} u_{12}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{Y u_{11}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} \tag{A23}$$

$$\omega_3 = -\frac{\frac{\chi m^c}{E^c} u_{13}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{Y u_{11}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} \tag{A24}$$

The first-order Taylor approximation to equation (A5) gives:

$$\hat{m}_t^g = \gamma_1 \hat{y}_t - \gamma_2 \hat{r}_t + \gamma_3 \hat{e}_t^g - \gamma_4 \hat{\chi}_t - \gamma_4 \hat{m}_t^c + \gamma_4 \hat{e}_t^c \tag{A25}$$

where:

$$\gamma_1 = \left(R - 1 + \frac{Y R \omega_2}{m^g}\right)\left(\frac{\gamma_2}{\omega_1}\right) \tag{A26}$$

$$\gamma_2 = \frac{R}{(R-1)\frac{m^g}{E^g}} \left[\frac{u_2\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)}{(R-1) E^g u_{12}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right) - R u_{22}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)}\right] \tag{A27}$$

$$\gamma_3 = 1 - (R-1)\gamma_2 \tag{A28}$$

$$\gamma_4 = \frac{\frac{\chi m^c}{E^c}}{\frac{m^g}{E^g}} \left[\begin{array}{c} \frac{u_{23}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)}{u_{22}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right) - \left(\frac{(R-1)}{R} E^g\right) u_{12}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)} - \\ \frac{u_{13}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)}{\left(\frac{R}{(R-1)} \frac{1}{E^g}\right) u_{22}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right) - u_{12}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)} \end{array}\right] \tag{A29}$$

The first-order Taylor approximation to equation (A6) gives:

$$\hat{m}_t^c = \gamma_5 \hat{y}_t - \gamma_6 \hat{r}_t + \gamma_7 \hat{e}_t^c - \gamma_8 \hat{m}_t^g + \gamma_8 \hat{e}_t^g - \gamma_9 \hat{\chi}_{t+1} + \gamma_{10} \hat{\chi}_t \tag{A30}$$

where:

$$\gamma_5 = \left(R - 1 + \frac{YR\omega_3}{\chi m^c}\right)\left(\frac{\gamma_6}{\omega_1}\right) \tag{A31}$$

$$\gamma_6 = \frac{R}{(R-1)\frac{\chi m^c}{E^c}}\left[\frac{u_3\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{(R-1)E^c u_{13}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) - Ru_{33}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}\right] \tag{A32}$$

$$\gamma_7 = 1 - (R-1)\gamma_6\chi m^c \tag{A33}$$

$$\gamma_8 = \frac{\frac{m^g}{E^g}}{\frac{\chi m^c}{E^c}}\left[\begin{array}{c} \frac{u_{32}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{u_{33}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) - \left(\frac{(R-1)}{R}E^c\right)u_{13}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} - \\ \frac{u_{12}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{\left(\frac{R}{(R-1)}\frac{1}{E^c}\right)u_{33}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right) - u_{13}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} \end{array}\right] \tag{A34}$$

$$\gamma_9 = -\gamma_6 \tag{A35}$$

$$\gamma_{10} = -1 - \gamma_6 \tag{A36}$$

Since in steady-state $P = 1$, the log-linearized expression for (A8) is given by:

$$\hat{\chi}_t = \varrho\hat{\nu}_t - \rho\hat{m}_t^c \tag{A37}$$

where $\varrho = \nu \ln(\kappa)$ and $\rho = m^c$.

The first-order Taylor approximation to equation (A13) gives:

$$\hat{\pi}_t = \left(\frac{\pi}{R}\right)\hat{\pi}_{t+1} + \psi\left[\begin{array}{c} \left(\frac{1}{\omega_1}\right)\hat{y}_t - \left(\frac{\omega_2}{\omega_1}\right)(\hat{m}_t^g - \hat{e}_t^g) \\ -\left(\frac{\omega_3}{\omega_1}\right)(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c) - \hat{z}_t \end{array}\right] \tag{A38}$$

where:

$$\psi = \frac{(\theta - 1)}{\delta} \tag{A39}$$

The log-linearized version of the NKPC for the model with labour that is non-linear in the utility function (Section 6.2 in the main text) is given by:

$$\hat{\pi}_t = \left(\frac{\pi}{R}\right)\hat{\pi}_{t+1} + \psi\left[\hat{y}_t\left(\eta + \frac{1}{\omega_1}\right) - \frac{\omega_2}{\omega_1}(\hat{m}_t^g - \hat{e}_t^g) - \frac{\omega_3}{\omega_1}(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c) - \hat{z}_t(1+\eta)\right] \tag{A40}$$

Finally, the log-linearization of the Taylor rule (16) in the main text gives:

$$\hat{r}_t = \rho^r\hat{r}_{t-1} + (1-\rho^r)\rho^y\hat{y}_t + (1-\rho^r)\rho^\pi\hat{\pi}_t + (1-\rho^r)\rho^{\mu^g}\hat{\mu}_t^g + \varepsilon_t^r \tag{A41}$$

2 Appendix B: Data Construction and Sources

As we described in the main body of the paper, the data is monthly and the model is estimated for the sample period 2013:M6-2022:M4. Here, we provide the sources and construction methods of the observed series. Unless otherwise noted, all original series are seasonally adjusted.

US Industrial Production Index. The US industrial production index, index 2017=100, is taken from the Federal Reserve Bank of St. Louis (code INDPRO in Federal Reserve Economic Data, link: https://fred.stlouisfed.org).

US Real Private Consumption. It is obtained from the series of personal consumption expenditures, billions of Dollars, and it is taken from the Federal Reserve Bank of St. Louis (code PCE in Federal Reserve Economic Data, link: https://fred.stlouisfed.org). The original series is deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: https://fred.stlouisfed.org), divided by the civilian employment level, thousands of persons (code CE16OV in Federal Reserve Economic Data, link: https://fred.stlouisfed.org) and expressed in log terms.

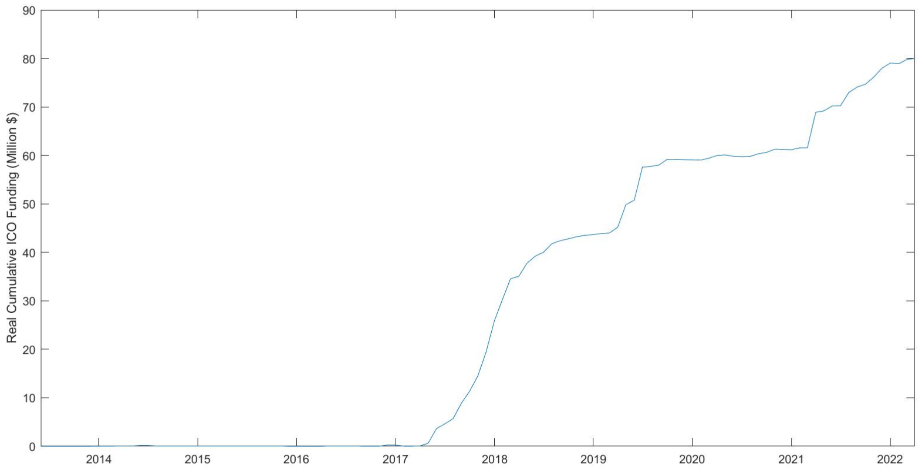
US Real Balances of Government Currency. It is obtained from the series of real M2 money stock, billions of Dollars, and it is taken from the Federal Reserve Bank of St. Louis (code M2REAL in Federal Reserve Economic Data, link: https://fred.stlouisfed.org). The original series is divided by the civilian employment level, thousands of persons (code CE16OV in Federal Reserve Economic Data, link: https://fred.stlouisfed.org) and expressed in log terms.

Real Bitcoin Price. It is obtained as the average of the series of opening and closing prices and it is taken from CoinMarketCap (link: https://coinmarketcap.com). The monthly series is obtained as average from daily data and is deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: https://fred.stlouisfed.org).

Real cumulative ICO funding. It is obtained from the series of cumulative initial coin offering (ICO) funding (ended ICO), expressed in Dollars, and it is taken from ICODROPS.com (link: https://icodrops.com/). The monthly series is obtained as average from daily data and is deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: https://fred.stlouisfed.org). This data series is shown in the figure below.

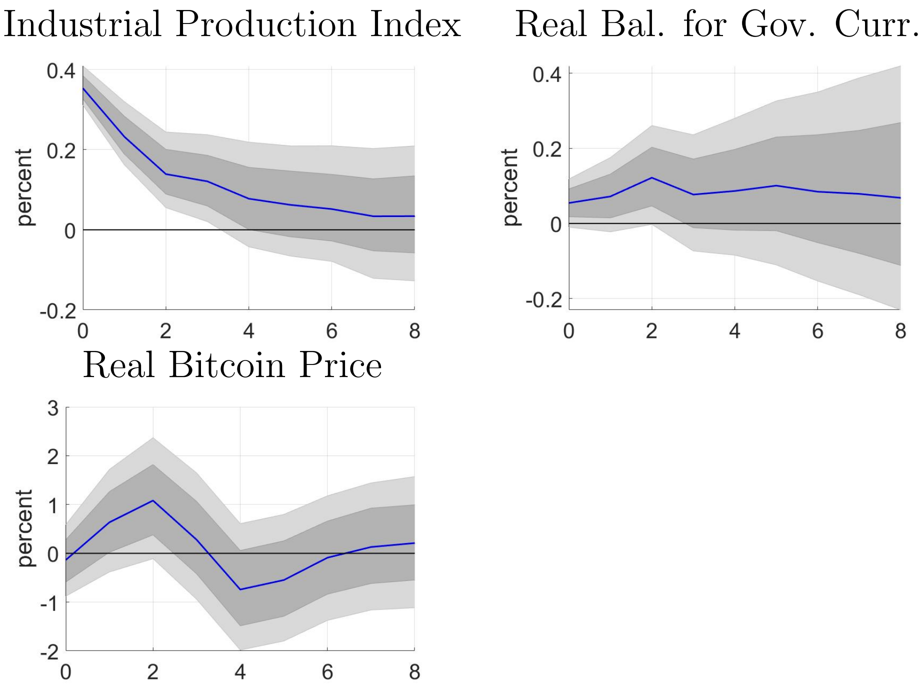
US Nominal Interest Rate. The US nominal interest rate is the series of effective Federal funds rate, %, and it is taken from the Federal Reserve Bank of St. Louis (code FEDFUNDS in Federal Reserve Economic Data, link: https://fred.stlouisfed.org).

2.1 Real cumulative ICO funding



Source: ICODROPS.com (`https://icodrops.com/`) and authors' estimates as described in Section 2. *Notes:* Data sample from June 2013 to April 2022. The x-axis corresponds to the time period.

3 Appendix C: SVAR Analysis - Responses to a Positive Technology Shock

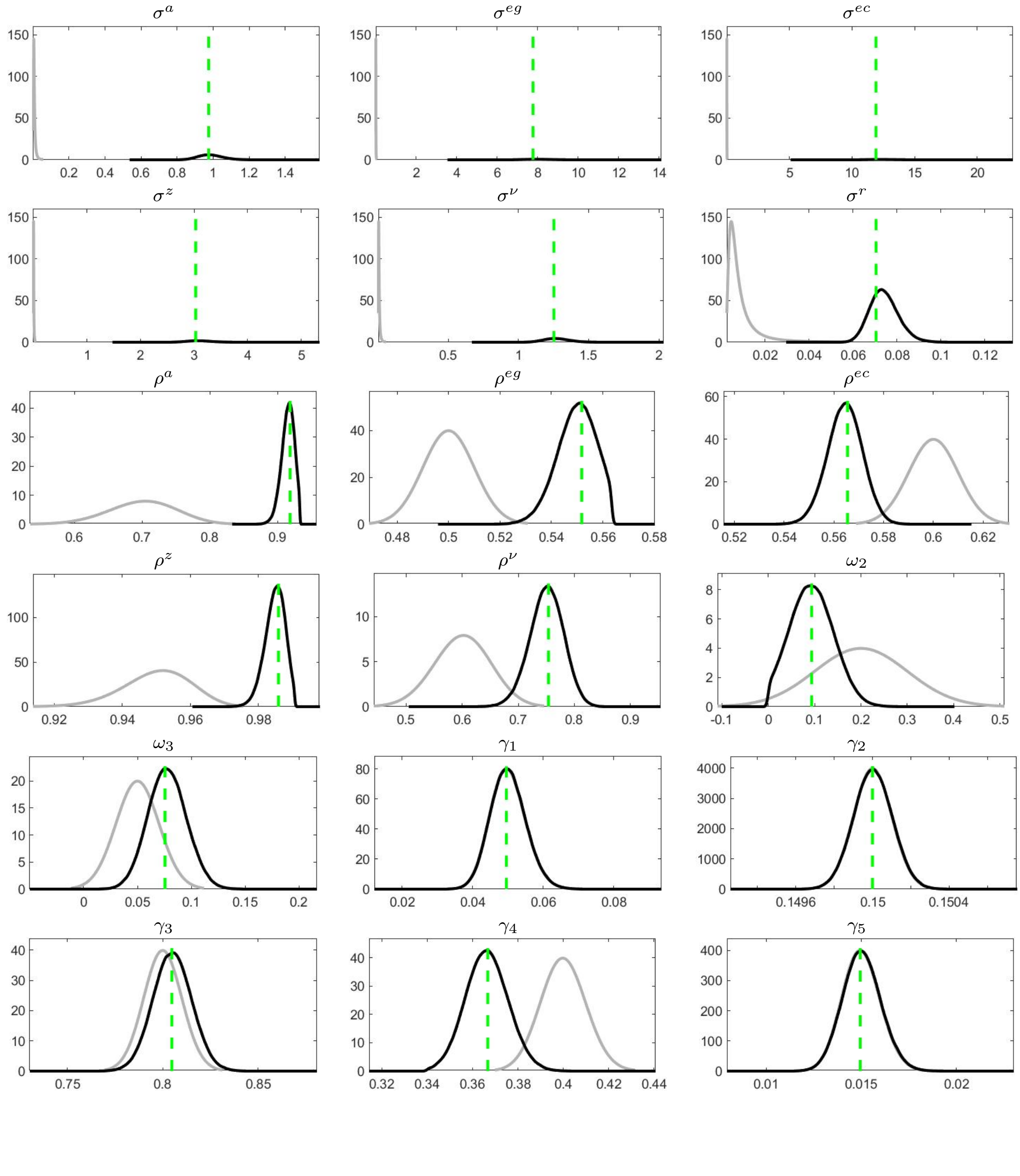


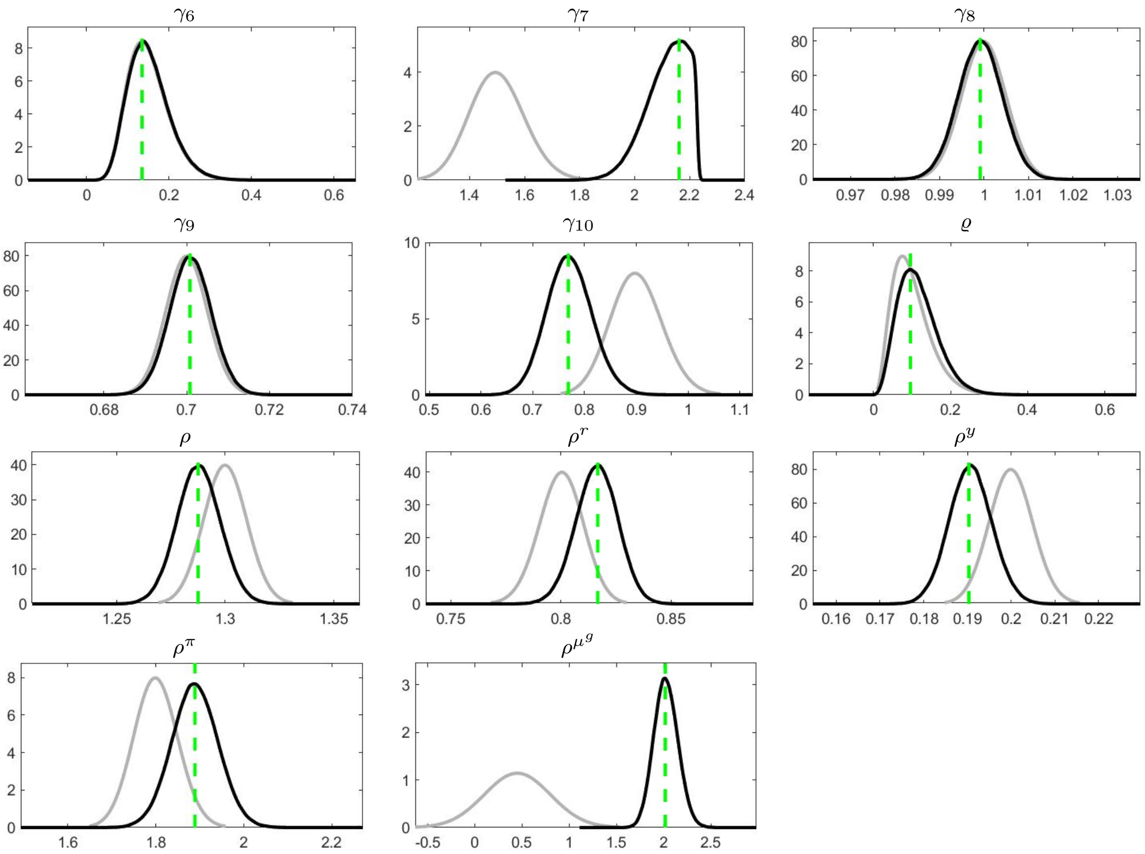
Notes: Estimated impulse responses to an increase in industrial production for the SVAR model related to the following equation: $Y_t = [IP_t, GC_t, BP_t]$, where IP_t denotes the US industrial production index, GC_t corresponds to the US real balances for government currency and BP_t is the real Bitcoin price. We assume a Cholesky ordering that has industrial production as the first variable and then the real balances for government currency and the real Bitcoin price, respectively. The three variables are constructed as the observables in our DSGE model. Sample period 2013:M6 to 2019:M12. In each panel, the solid blue line represents the posterior median at each horizon, whereas the dark and light shaded areas indicate the $68th$ and $90th$ posterior probability regions, respectively, of the estimated impulse responses. Horizontal axis: months after shock.

4 Appendix D: Diagnostic Tests

4.1 Prior and Posterior Distributions

In the graphs below, the gray lines represent the prior distributions while the black lines correspond to the posterior distributions.

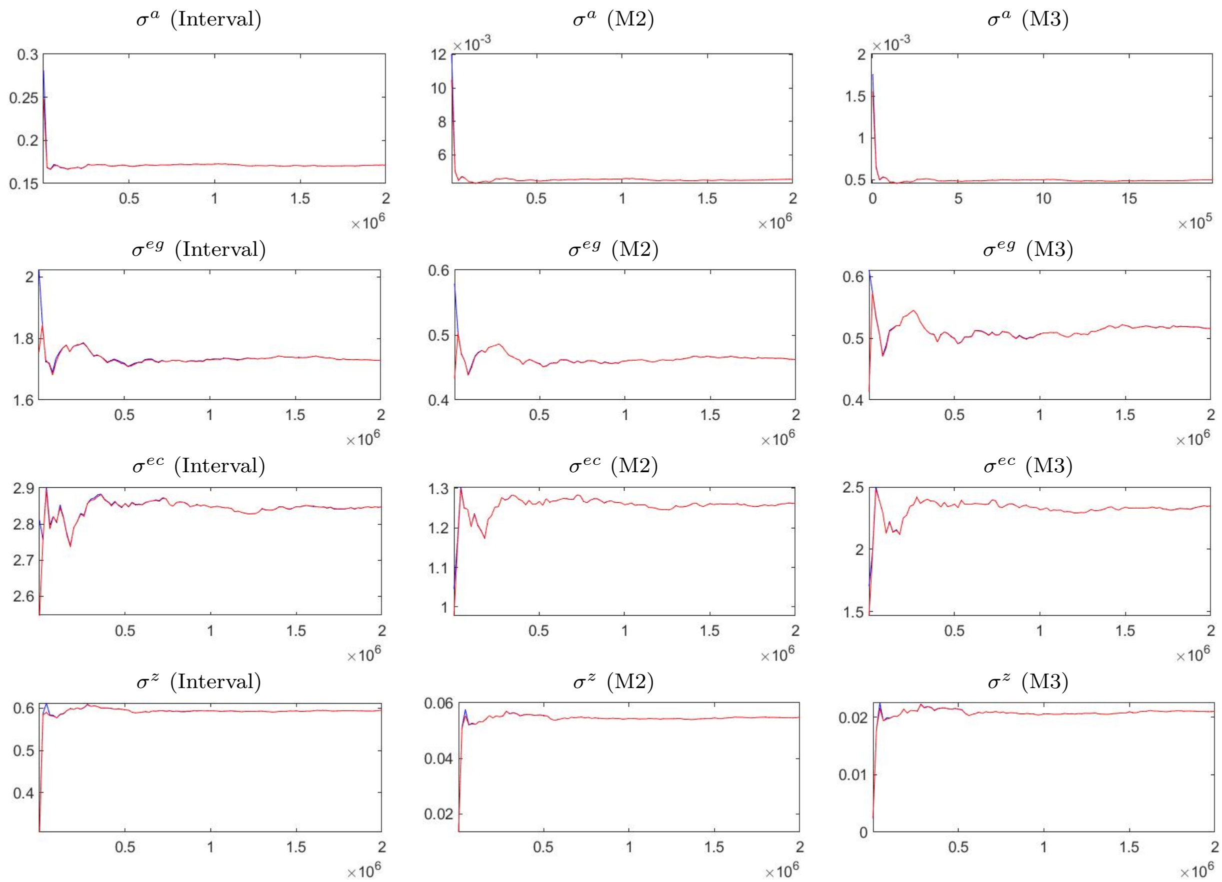

σ^a
σ^{eg}
σ^{ec}
σ^z
σ^ν
σ^r
ρ^a
ρ^{eg}
ρ^{ec}
ρ^z
ρ^ν
ω_2
ω_3
γ_1
γ_2
γ_3
γ_4
γ_5

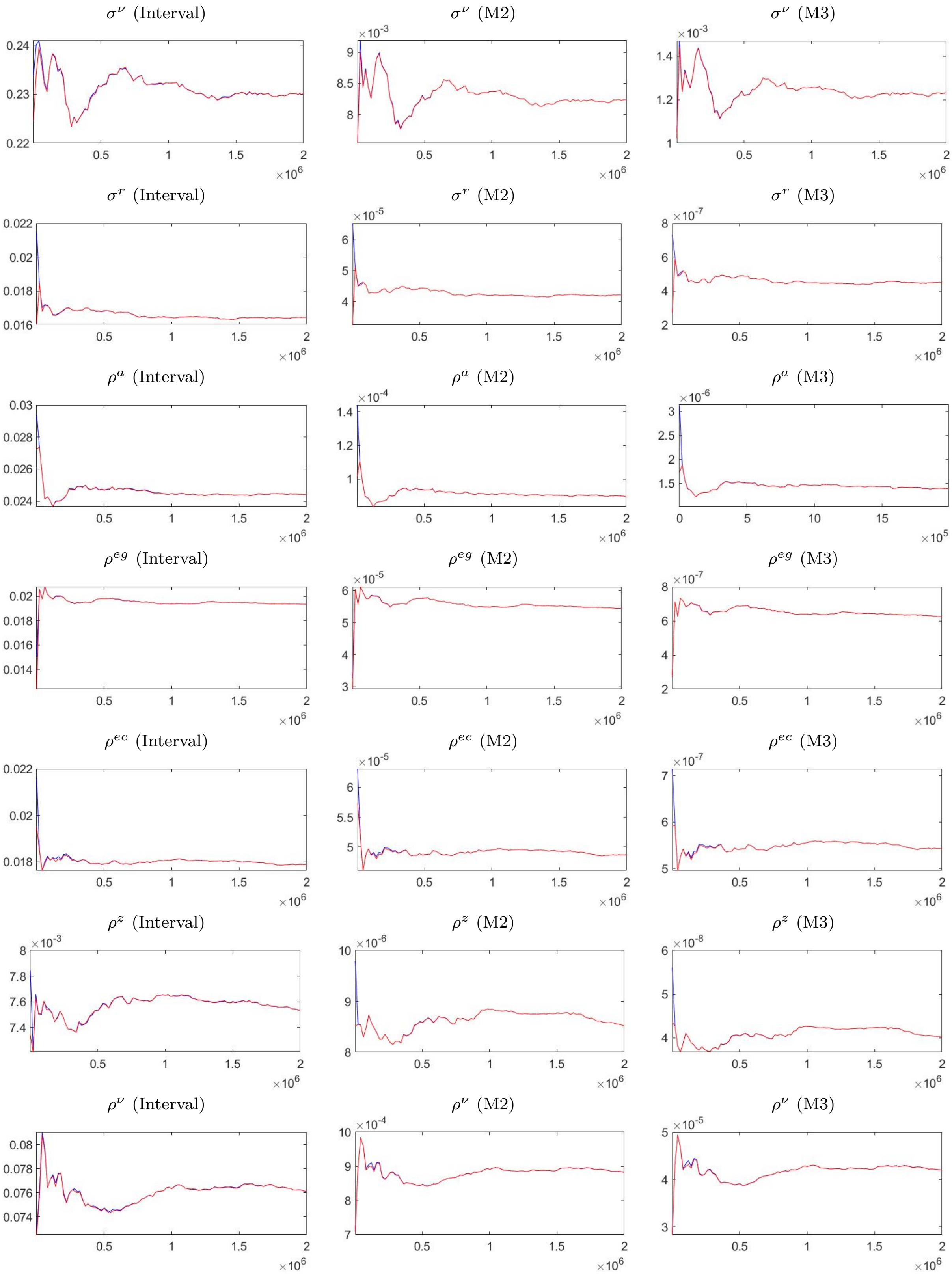


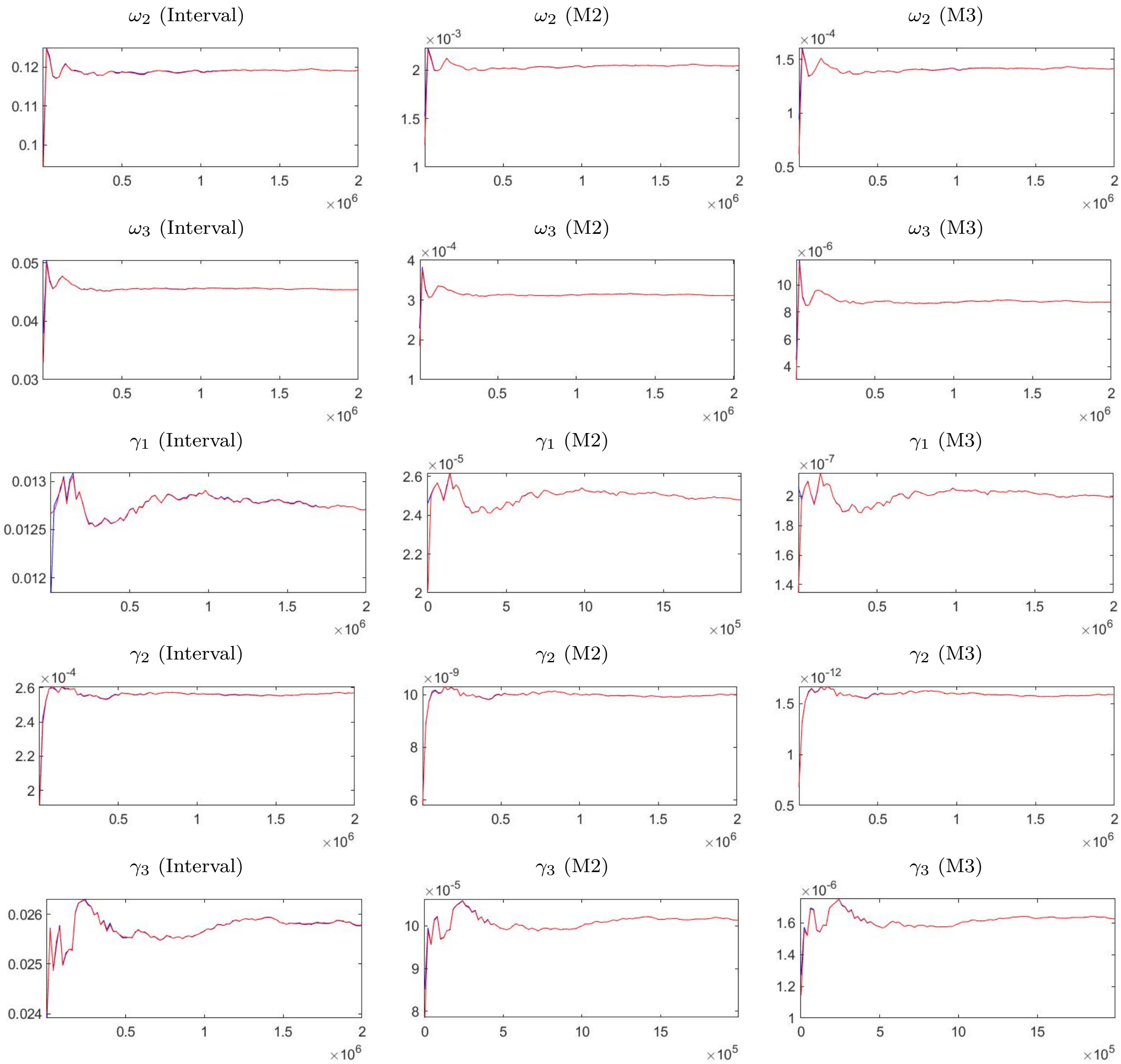
γ_6
γ_7
γ_8
γ_9
γ_{10}
ϱ
ρ
ρ^r
ρ^y
ρ^π
ρ^{μ^g}

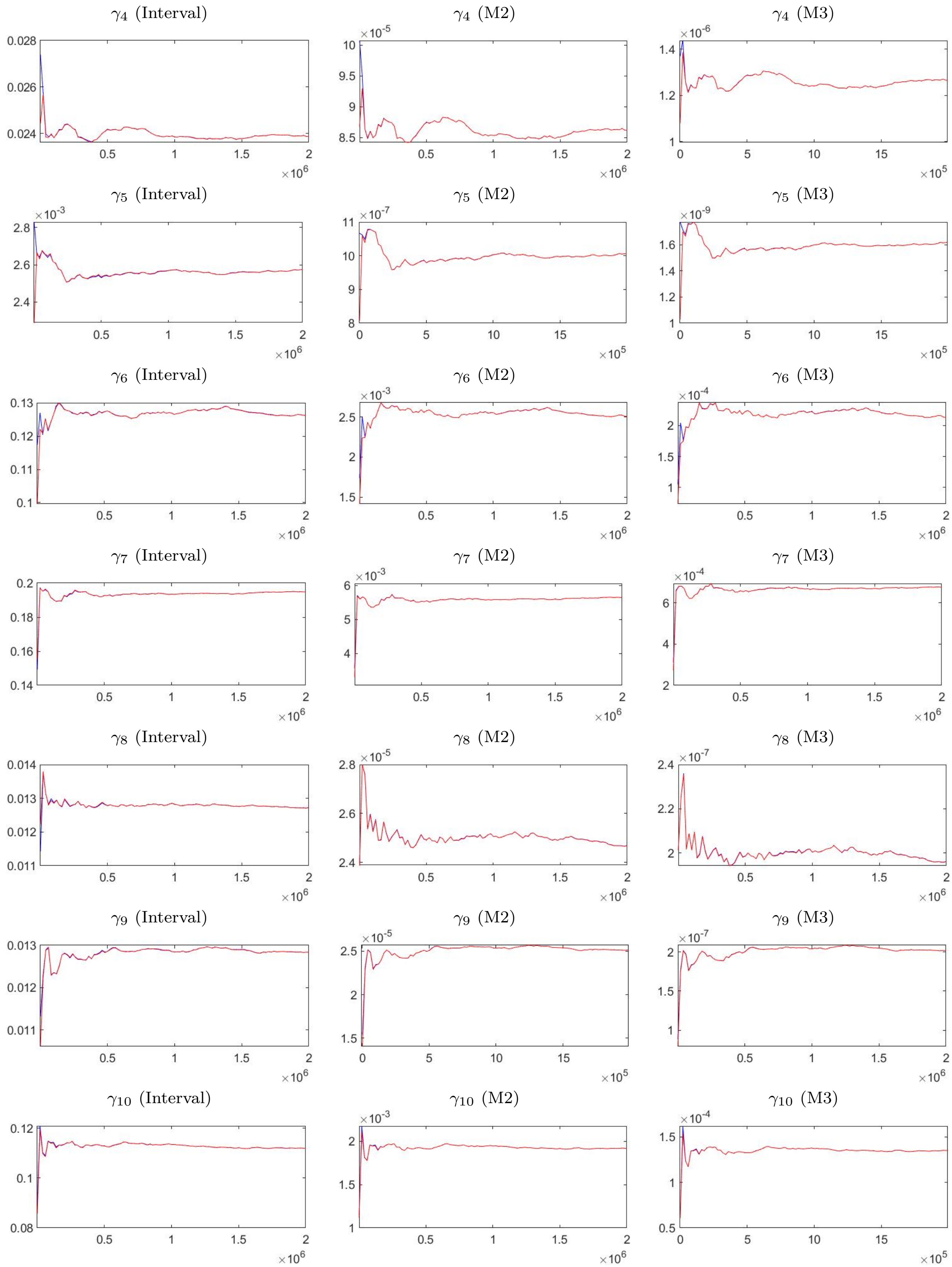
4.2 Monte Carlo Markov Chain Univariate Diagnostics

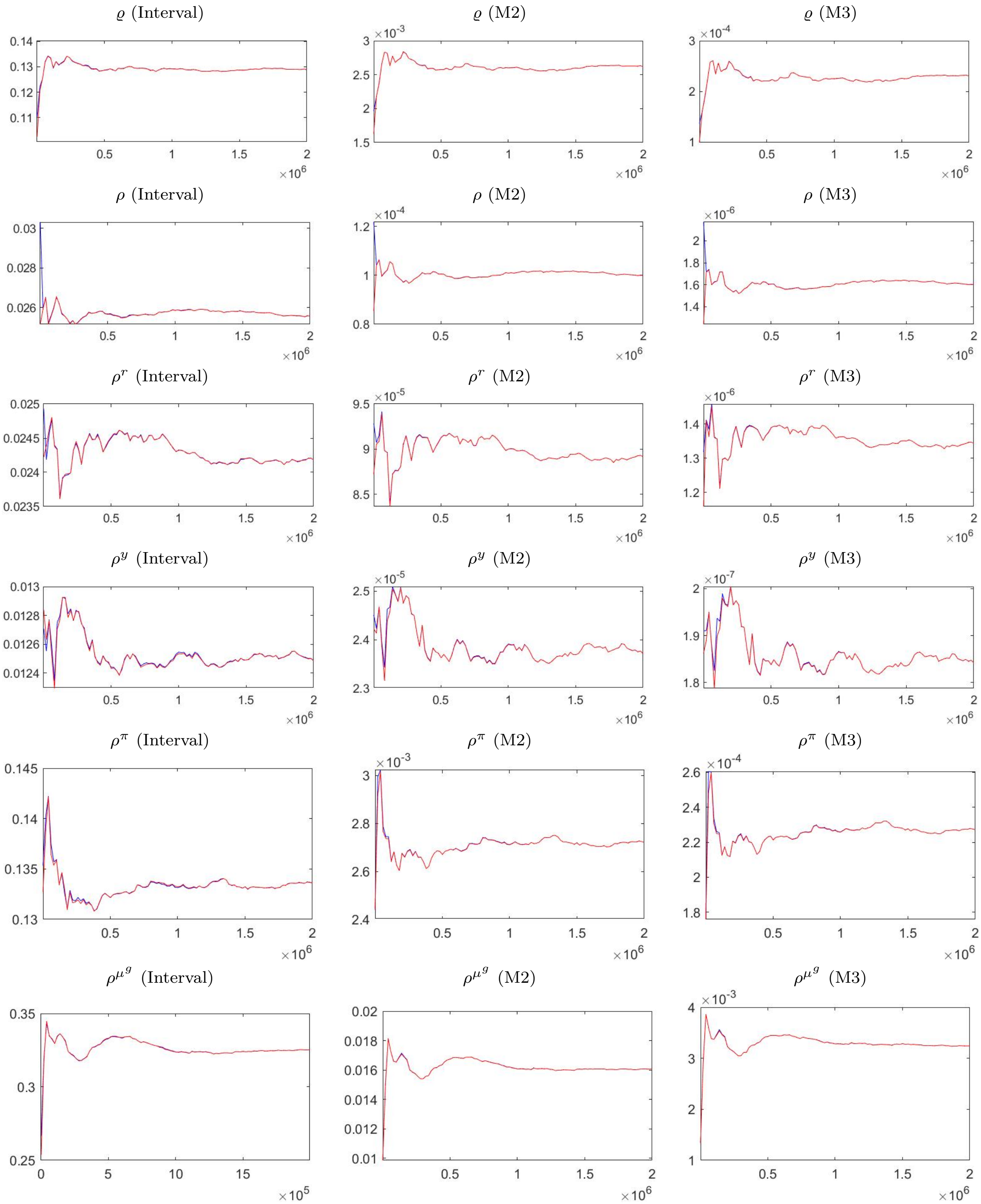
In the graphs below, the first column with the label "Interval" shows the Brooks and Gelman (1998) convergence diagnostics for the 80% interval. The blue line represents the 80% interval range based on the pooled draws from all sequences, whereas the red line indicates the mean interval based on the draws of the individual sequences. The second and the third column with labels "M2" and "M3" denote an estimate of the same statistics for the second and third central moments.




σ^{ν} (Interval)
σ^{ν} (M2)
σ^{ν} (M3)
σ^{r} (Interval)
σ^{r} (M2)
σ^{r} (M3)
ρ^{a} (Interval)
ρ^{a} (M2)
ρ^{a} (M3)
ρ^{eg} (Interval)
ρ^{eg} (M2)
ρ^{eg} (M3)
ρ^{ec} (Interval)
ρ^{ec} (M2)
ρ^{ec} (M3)
ρ^{z} (Interval)
ρ^{z} (M2)
ρ^{z} (M3)
ρ^{ν} (Interval)
ρ^{ν} (M2)
ρ^{ν} (M3)

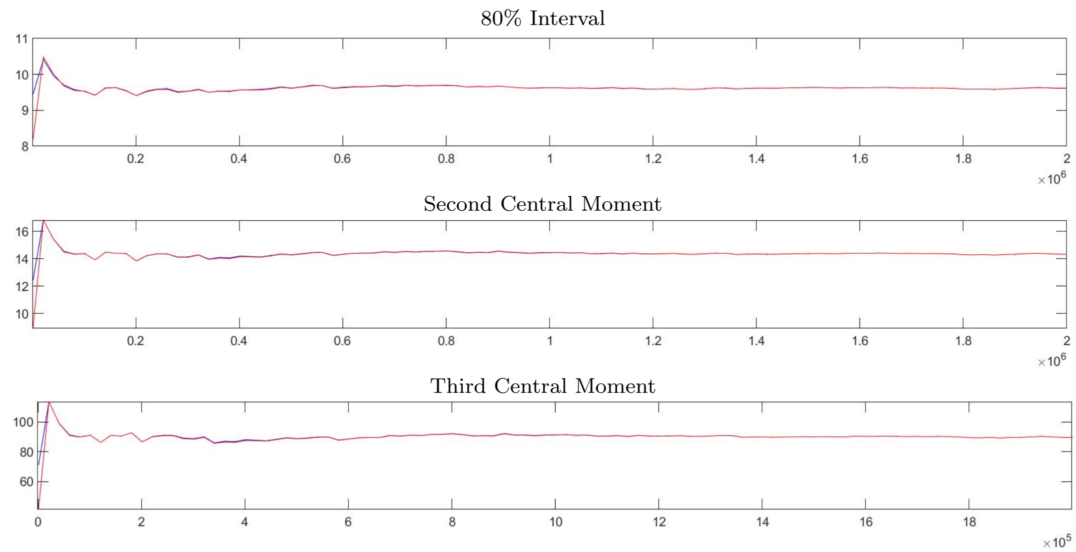

ω_2 (Interval)
ω_2 (M2)
ω_2 (M3)
ω_3 (Interval)
ω_3 (M2)
ω_3 (M3)
γ_1 (Interval)
γ_1 (M2)
γ_1 (M3)
γ_2 (Interval)
γ_2 (M2)
γ_2 (M3)
γ_3 (Interval)
γ_3 (M2)
γ_3 (M3)


γ_4 (Interval)
γ_4 (M2)
γ_4 (M3)
γ_5 (Interval)
γ_5 (M2)
γ_5 (M3)
γ_6 (Interval)
γ_6 (M2)
γ_6 (M3)
γ_7 (Interval)
γ_7 (M2)
γ_7 (M3)
γ_8 (Interval)
γ_8 (M2)
γ_8 (M3)
γ_9 (Interval)
γ_9 (M2)
γ_9 (M3)
γ_{10} (Interval)
γ_{10} (M2)
γ_{10} (M3)


ϱ (Interval)
ϱ (M2)
ϱ (M3)
×10^-3
×10^-4
×10^6
ρ (Interval)
ρ (M2)
ρ (M3)
×10^-4
×10^-6
ρ^r (Interval)
ρ^r (M2)
ρ^r (M3)
×10^-5
×10^-6
ρ^y (Interval)
ρ^y (M2)
ρ^y (M3)
×10^-5
×10^-7
ρ^π (Interval)
ρ^π (M2)
ρ^π (M3)
×10^-3
×10^-4
ρ^{μ^g} (Interval)
ρ^{μ^g} (M2)
ρ^{μ^g} (M3)
×10^-3
×10^5

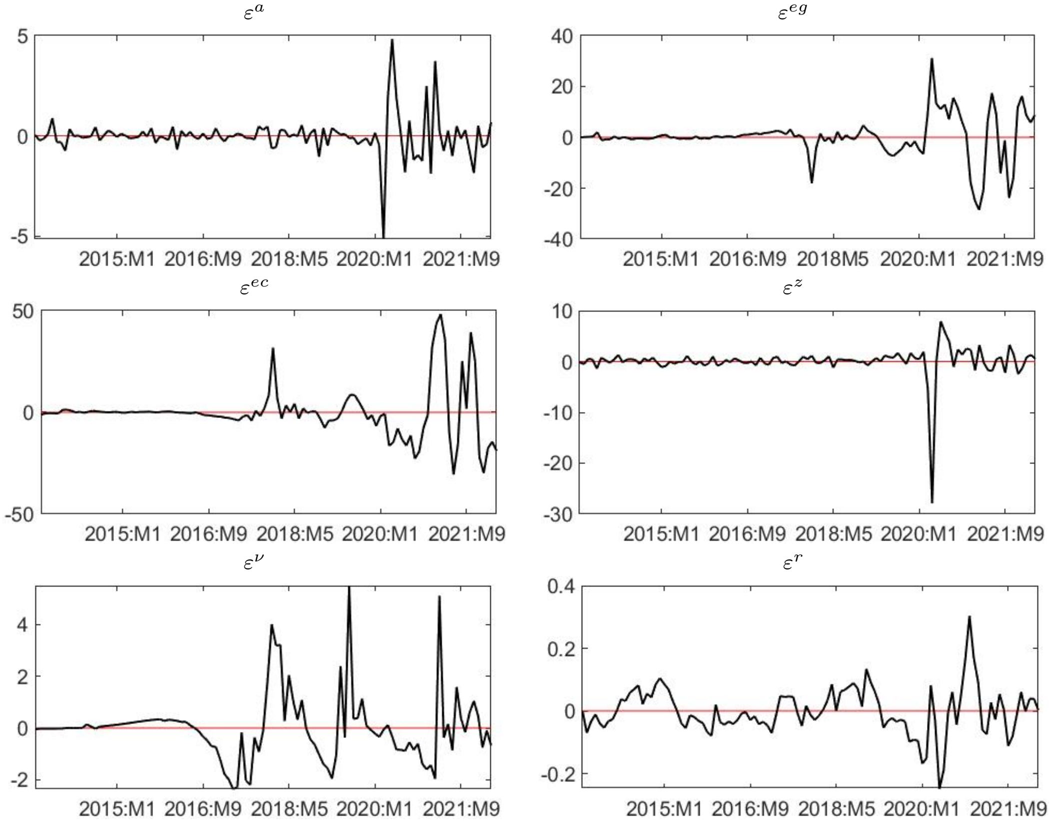
4.3 Multivariate Convergence Diagnostics

In the graphs below, the diagnostics is based on the range of the posterior likelihood function. The posterior kernel is used to aggregate the parameters.



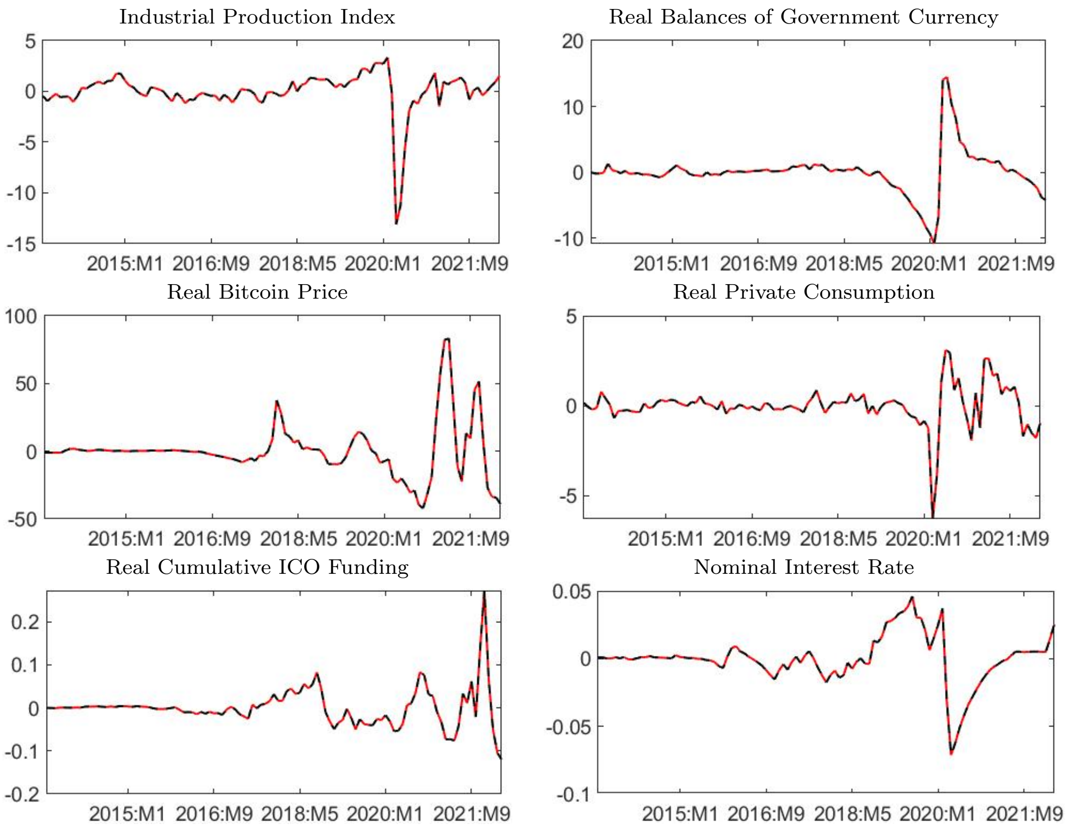
4.4 Smoothed Shocks

In the graphs below, the black lines represent the estimates of the smoothed structural shocks derived from the Kalman smoother.



4.5 Historical and Smoothed Variables

In the graphs below, the dotted black lines indicate the observed data whereas the red lines indicate the estimates of the smoothed variables derived from the Kalman smoother.



5 Appendix E: Variance Decomposition Analysis

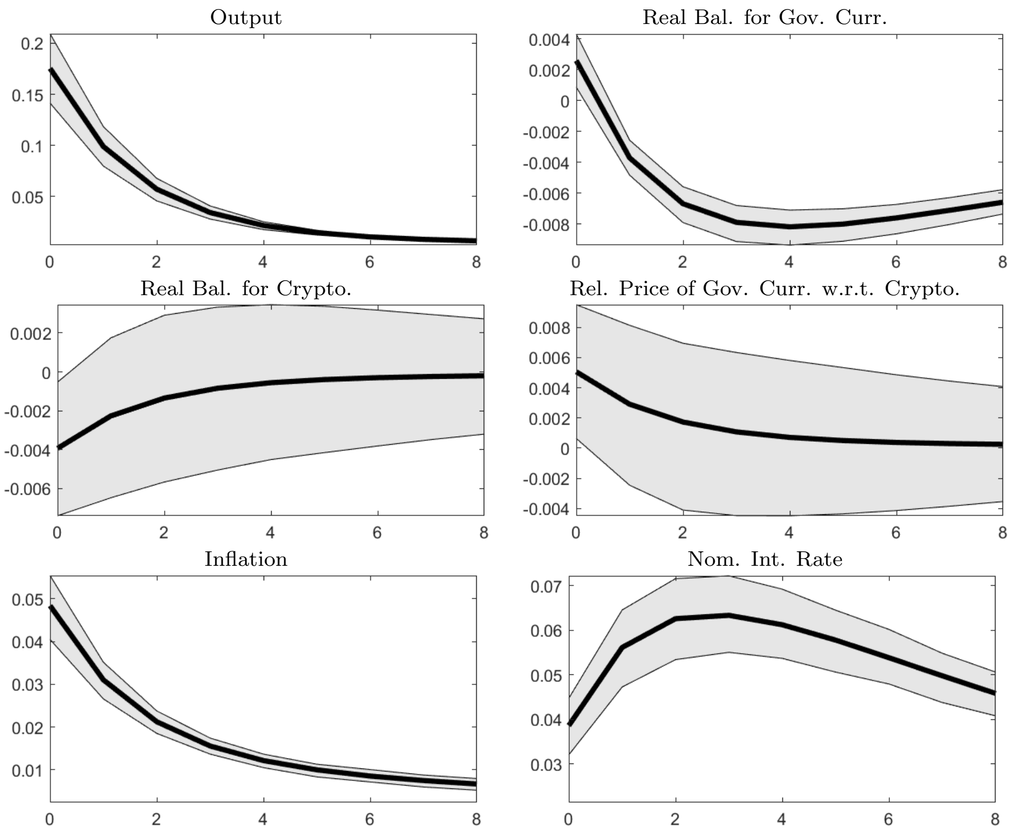
Table 1 shows the importance of each shock in terms of fluctuations in the key endogenous variables of the model. In particular, the forecast error variance decomposition, which we show for 1, 5, 12 and 30 periods ahead, is based on the simulation of the estimated model (10,000 iterations). Our simulation results are detrended using the HP filter with a smoothing parameter equal to 1,600.

In Table 1, we observe that technology and monetary policy shocks explain most of the variations in output, inflation and nominal interest rate. In particular, the contributions of technology and monetary policy shocks on output, inflation and nominal interest rate changes are above 90% for 1, 5, 12 and 30 periods ahead. Our results also show that government currency and cryptocurrency demand shocks contribute to most of the variations in the real balances of government currency. Over the periods considered, the former shock accounts more than the latter (almost 70% and 30%, respectively). Moreover, we find that the shock to cryptocurrency demand accounts for the bulk of the variations in the real balances of cryptocurrency and the nominal exchange rate for 1, 5, 12 and 30 periods ahead.

Table 1: Forecast Error Variance Decomposition (%)

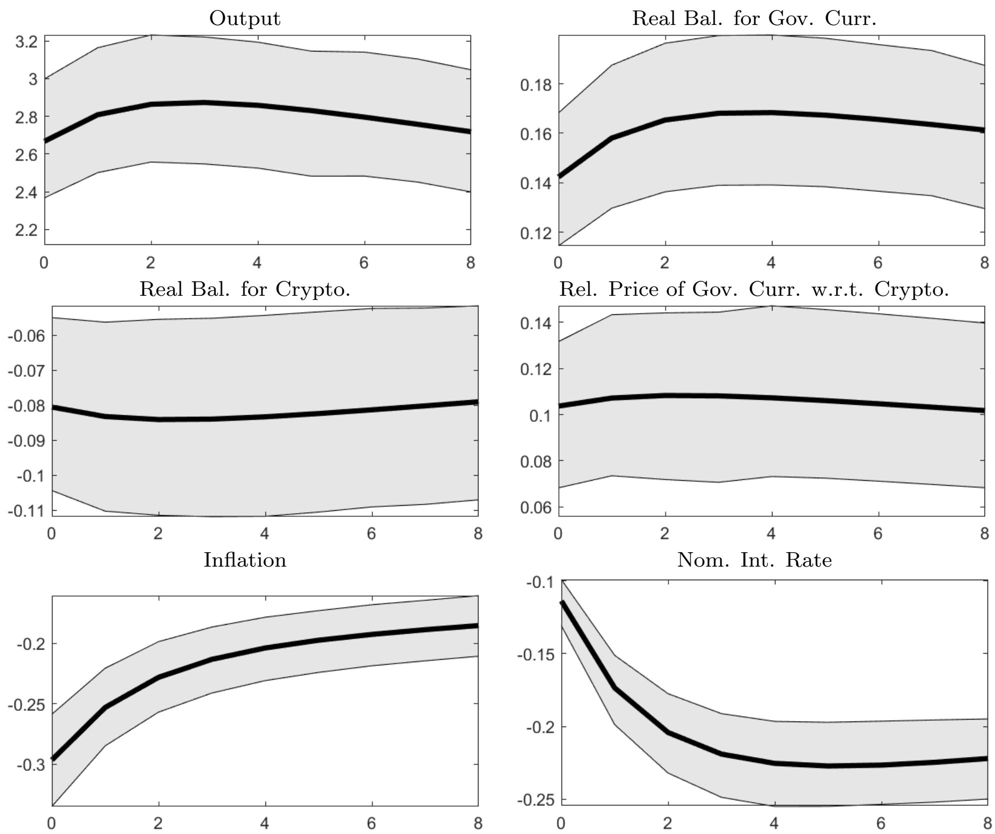
Period 1						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	0.67	1.18	0.35	0.00	0.00	0.00
σ^{eg}	3.11	6.84	0.36	71.34	1.04	1.04
σ^{ec}	0.69	2.42	0.10	24.57	98.56	98.63
σ^z	15.50	4.42	0.30	0.21	0.05	0.05
σ^ν	0.00	0.01	0.00	0.14	0.20	0.13
σ^r	80.03	85.13	98.89	3.74	0.14	0.14
Period 5						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	0.47	1.37	2.57	0.01	0.00	0.00
σ^{eg}	2.82	7.71	0.86	70.44	1.00	1.00
σ^{ec}	0.67	2.76	0.26	24.81	98.37	98.47
σ^z	41.61	9.57	2.88	0.92	0.20	0.20
σ^ν	0.00	0.01	0.00	0.21	0.30	0.19
σ^r	54.43	78.58	93.43	3.60	0.14	0.14
Period 12						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	0.31	1.37	4.61	0.04	0.00	0.00
σ^{eg}	1.85	7.18	0.83	69.51	0.99	0.99
σ^{ec}	0.44	2.57	0.25	24.51	98.11	98.22
σ^z	62.07	16.19	7.64	2.18	0.45	0.45
σ^ν	0.00	0.01	0.00	0.21	0.31	0.20
σ^r	35.33	72.67	86.67	3.55	0.14	0.14
Period 30						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	0.19	1.25	5.02	0.04	0.00	0.00
σ^{eg}	1.14	6.36	0.75	67.96	0.99	0.99
σ^{ec}	0.27	2.28	0.23	23.96	97.68	97.79
σ^z	76.64	25.71	15.10	4.35	0.88	0.88
σ^ν	0.00	0.01	0.00	0.21	0.31	0.20
σ^r	21.76	64.39	78.89	3.47	0.14	0.14

6 Appendix G: Estimated Impulse Response Functions



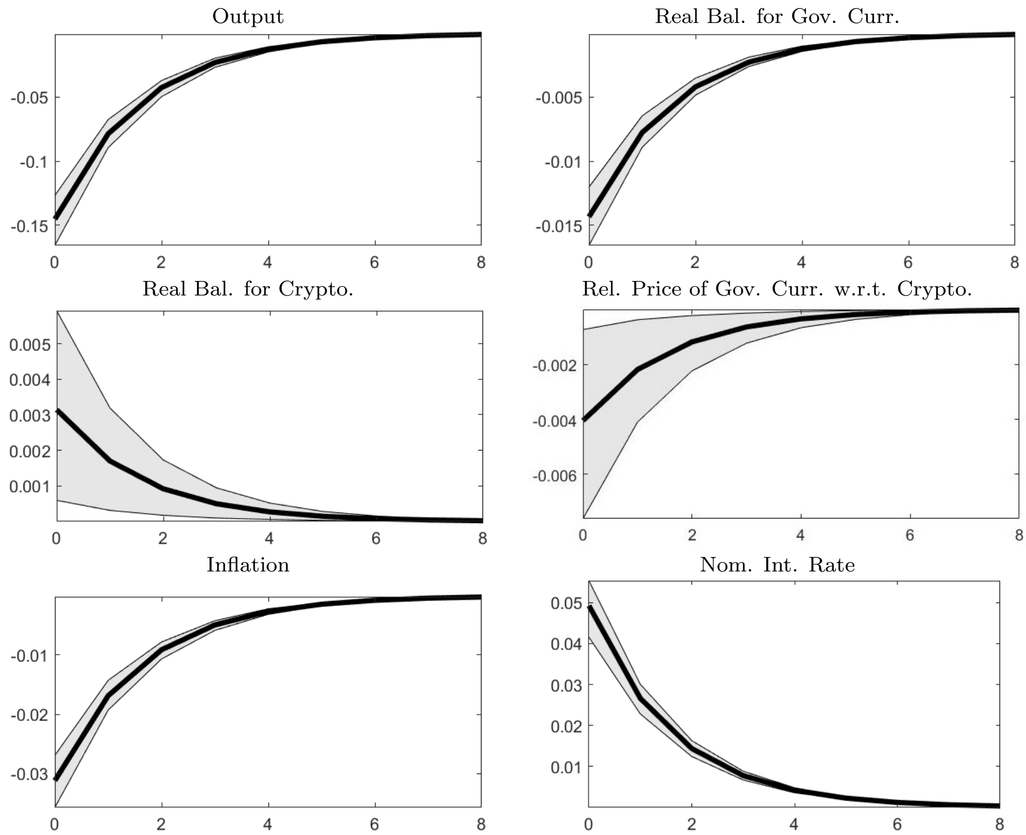
Estimated IRFs of the baseline DSGE model: Preferences shock

Notes: Estimated one-standard-deviation shock to household preferences. Sample period 2013:M6 to 2022:M4. In each panel, the solid black line represents the mean impulse response, whereas the shaded area indicates the $90th$ posterior probability region. Vertical axis: percent for all variables except for inflation and nominal interest rate that are expressed as percentage points. Horizontal axis: months after shock.



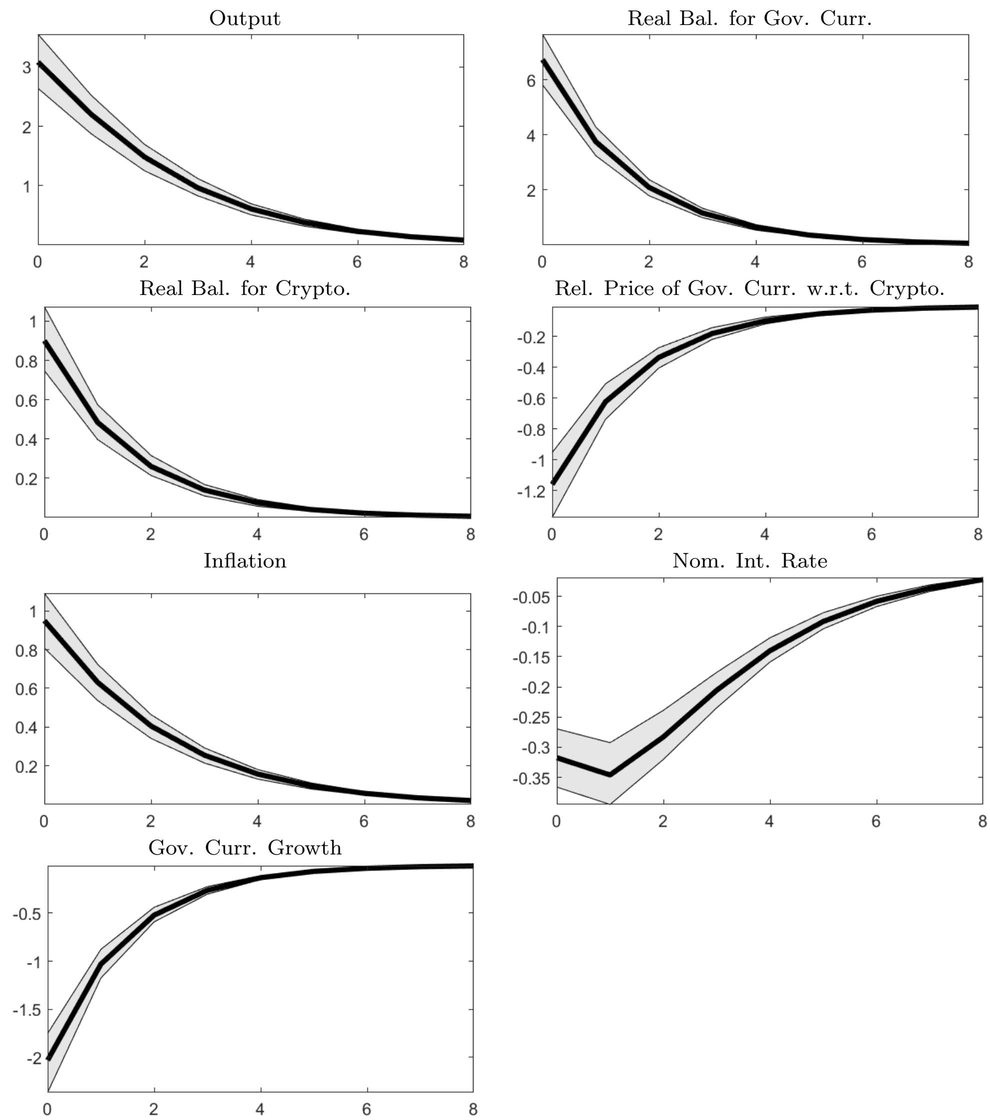
Estimated IRFs of the baseline DSGE model: Technology shock

Notes: Estimated one-standard-deviation shock to technology. Sample period 2013:M6 to 2022:M4. In each panel, the solid black line represents the mean impulse response, whereas the shaded area indicates the 90*th* posterior probability region. Vertical axis: percent for all variables except for inflation and nominal interest rate that are expressed as percentage points. Horizontal axis: months after shock.



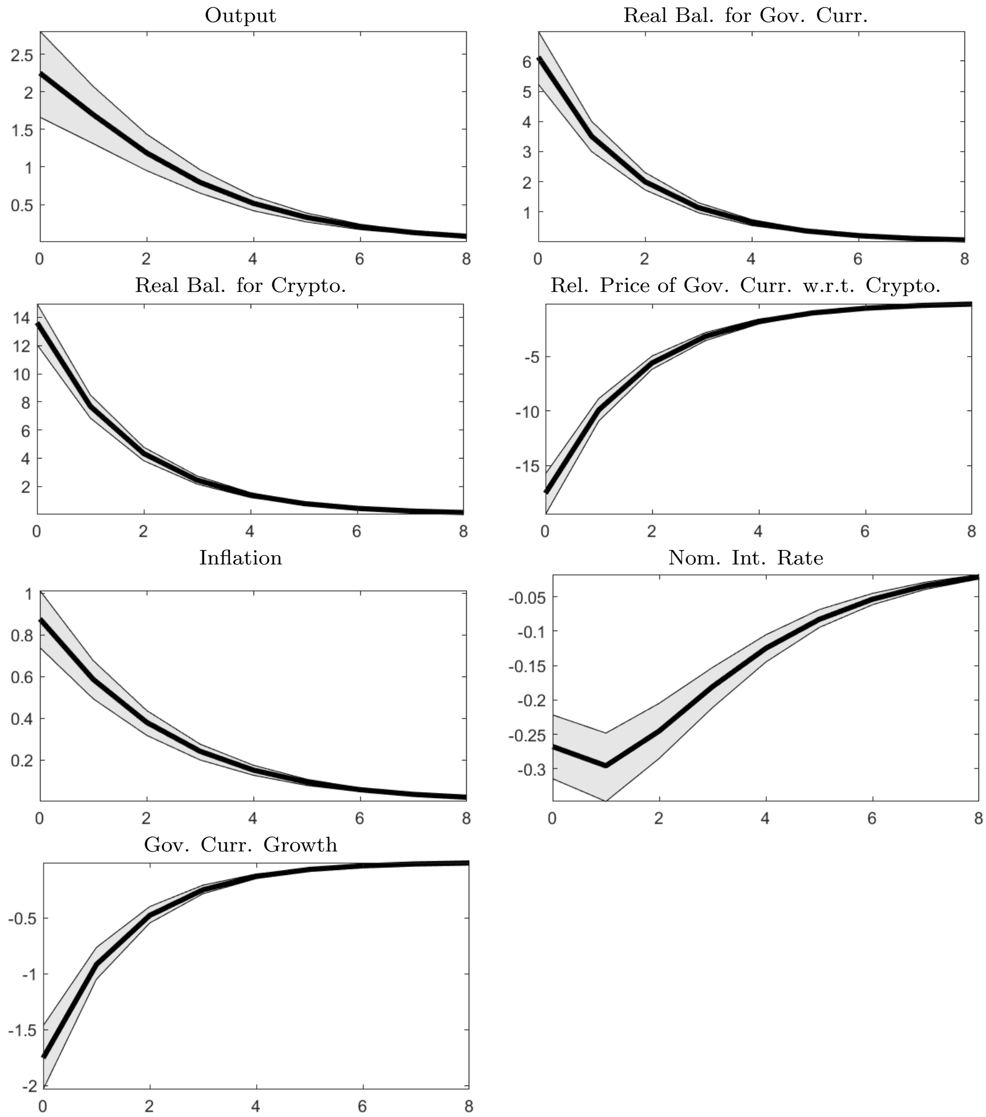
Estimated IRFs of the baseline DSGE model: Monetary policy shock

Notes: Estimated one-standard-deviation shock to monetary policy. Sample period 2013:M6 to 2022:M4. In each panel, the solid black line represents the mean impulse response, whereas the shaded area indicates the $90th$ posterior probability region. Vertical axis: percent for all variables except for inflation and nominal interest rate that are expressed as percentage points. Horizontal axis: months after shock.



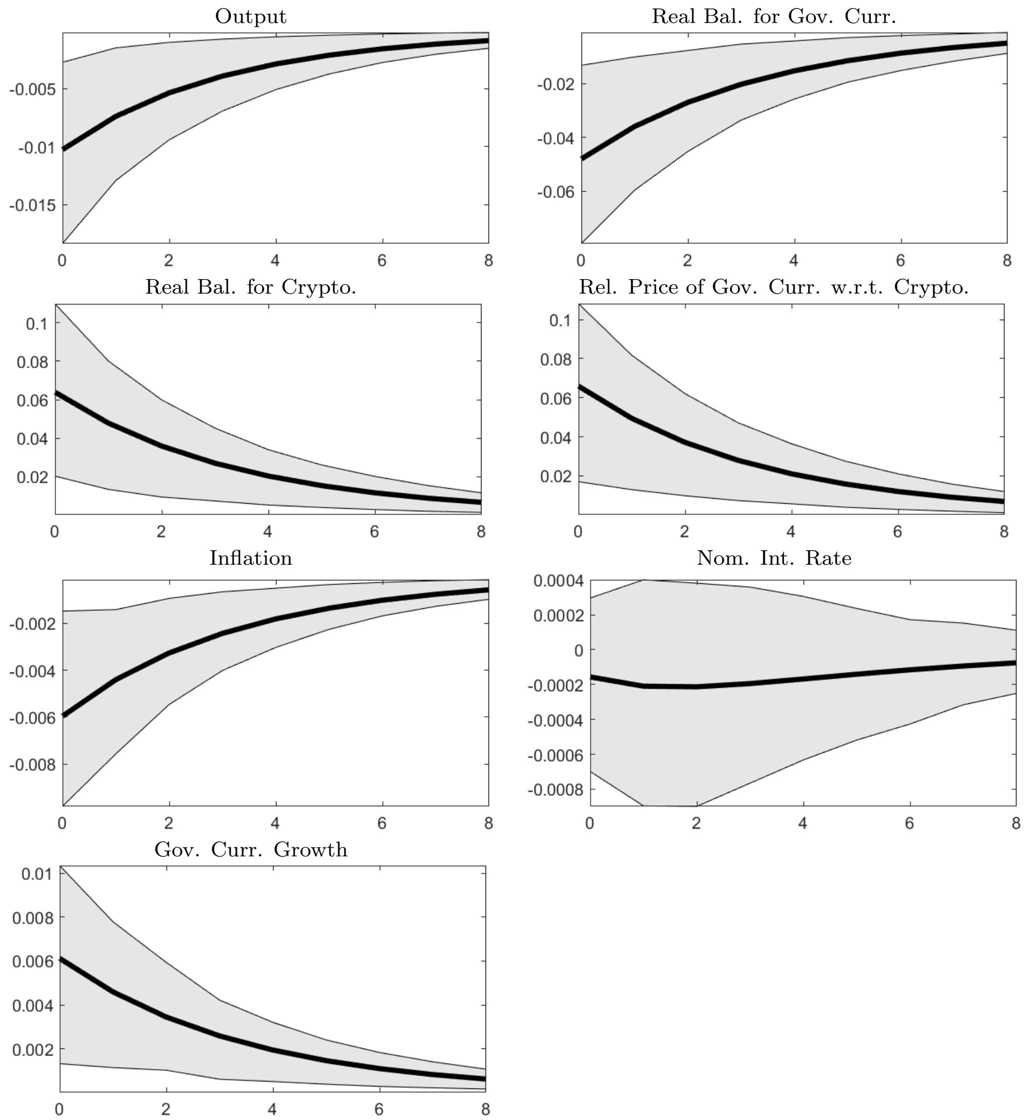
Estimated IRFs of the baseline DSGE model: Government currency demand shock

Notes: Estimated one-standard-deviation shock to government currency demand. Sample period 2013:M6 to 2022:M4. In each panel, the solid black line represents the mean impulse response, whereas the shaded area indicates the $90th$ posterior probability region. Vertical axis: percent for all variables except for inflation, nominal interest rate and government currency growth that are expressed as percentage points. Horizontal axis: months after shock.



Estimated IRFs of the baseline DSGE model: Cryptocurrency demand shock

Notes: Estimated one-standard-deviation shock to cryptocurrency demand. Sample period 2013:M6 to 2022:M4. In each panel, the solid black line represents the mean impulse response, whereas the shaded area indicates the 90*th* posterior probability region. Vertical axis: percent for all variables except for inflation, nominal interest rate and government currency growth that are expressed as percentage points. Horizontal axis: months after shock.



Estimated IRFs of the baseline DSGE model: Cryptocurrency productivity shock

Notes: Estimated one-standard-deviation shock to cryptocurrency productivity. Sample period 2013:M6 to 2022:M4. In each panel, the solid black line represents the mean impulse response, whereas the shaded area indicates the $90th$ posterior probability region. Vertical axis: percent for all variables except for inflation, nominal interest rate and government currency growth that are expressed as percentage points. Horizontal axis: months after shock.

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