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Contagion between Real Estate and Financial Markets: A Bayesian Quantile-on-Quantile Approach[1]

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Abstract

We study contagion between Real Estate Investment Trusts (REITs) and the equity market in the U.S. over four sub-samples covering January, 2003 to December, 2017, by using Bayesian nonparametric quantile-on-quantile (QQ) regressions with heteroskedasticity. We find that the spillovers from the REITs on to the equity market has varied over time and quantiles defining the states of these two markets across the four sub-samples, thus providing evidence of shift-contagion. Further, contagion from REITs upon the stock market went up during the global financial crisis particularly, and also over the period corresponding to the European sovereign debt crisis, relative to the pre-crisis period. Our main findings are robust to alternative model specifications of the benchmark Bayesian QQ model, especially when we control for omitted variable bias using the heteroskedastic error structure. Our results have important implications for various agents in the economy namely, academics, investors and policymakers.

JEL classification: C22; G10; R30.

Keywords: Contagion; Real Estate Market; Stock Market; Quantile-on-Quantile Model; Bayesian Estimation.

[1]We would like to thank an anonymous referee for many helpful comments. However, any remaining errors are solely ours.

1 Introduction

The benefits of including real estate in mixed-asset portfolios are now well-recognized (Hoesli et al., 2004; MacKinnon and Al Zaman, 2009; Bouri et al., 2018). However, investing in real estate can be problematic due to the high unit value and illiquidity of properties. Thus, it is not surprising that the importance of the securitized real estate market, i.e., Real Estate Investment Trusts (REITs) has grown substantially during the past decades. As indicated by the Nareit (the worldwide representative voice for REITs),[2] REITs of all types collectively own more than $3 trillion in gross assets across the U.S., with stock-exchange listed REITs owning approximately $2 trillion in assets. Moreover, U.S. listed REITs have an equity market capitalization of more than $1 trillion, and more than 80 million Americans invest in REIT stocks (through their 401(k) and other investment funds). Indeed, the characteristics of REITs have overcome many of the drawbacks associated with direct real estate. Hence, an understanding of the nature of real estate stocks is crucial for investors.

In this regard, an important stream of research has examined the relationships of REITs with stocks, bonds and its underlying asset i.e., real estate (see for example, Li et al., (2015), Tsai (2015), Chiang et al., (2017), Damianov and Elsayed (2018)). More recently, the extreme events that unfolded in financial markets during the global financial and the European sovereign debt crises have strengthened the desire of researchers to better understand contagion, whereby, loosely speaking, contagion can be defined as a rapid shock spillover that increases cross-market linkages.[3] While there exists a vast literature on contagion involving bonds, stocks, currencies, and more recently hedge funds (see for example, Pericoli and Sbracia (2003), Dungey et al. (2005), Pesaran and Pick (2007), and Forbes (2012)), the literature disentangling contagion issues concerning real estate markets is limited. In this regard, few studies that test for financial contagion in

[2]See: `https://www.reit.com/nareit`.

[3]The existing literature has recognized at least three possible *theories* of contagion, i.e., through financial linkages (which in turn has three channels, i.e., information correlation, liquidity correlation, and portfolio rebalancing), trade links, and herding behaviour (Hoesli and Reka, 2015).

REITs and are worth mentioning, involves the works of Kallberg et al., (2002), Gerlach et al., (2006), Fry et al., (2010), Hoesli and Reka (2013, 2015). In general, these studies confirm the existence of contagion involving real estate markets during the Asian crisis of 1997 and the global financial crisis of 2007-2008.

We aim to extend this limited literature associated with real estate markets, by studying contagion between REITs and the equity market (S&P500) in the U.S. based on an extended sample period of daily data covering 2003 till 2017, which in turn allows us to study the impact of not only the global financial crisis, but also the European sovereign debt turmoil. But more importantly, we aim to contribute to this literature by applying quantile-on-quantile (QQ) based nonparametric regressions to study the impact of the REITs market on U.S. equities. The QQ approach allows us to trace the effect of the entire unconditional distribution of REITs on the conditional distribution of the U.S. equity market. In the process, we are able to analyze how changes in the REITs returns from its initial state of bear (lower quantiles), normal (median), or bull (upper quantiles) regimes affect the entire conditional distribution S&P500 returns, i.e., capturing various corresponding states of the equity market. As Caporin et al. (2018) show, quantile regression in the context of contagion analysis is robust to several misspecification errors, including endogeneity. This is possible in our framework, considering that REITs are a narrow sector of the aggregate economy and the S&P500 represents about 75% of total market capitalization.

Understandably, compared to copula models used to analyze extreme tail dependence, and to standard quantile regressions to study the conditional distribution of the equity markets as in Hoesli and Reka (2013), our QQ approach is more informative, as it studies contagion over all possible states associated with REITs and equity markets. In other words, our paper presents a more complete picture on stability of parameters, associating real estate and equity markets of the U.S., and hence investigates the presence of (possible) shift-contagion during the crises periods. We define shift-contagion borrowing from Caporin et al. (2018): shift-contagion is a shift in the intensity of propagation of shocks from the Real Estate market to the stock market when contrasting large (positive or negative) shocks to normal times (i.e. shocks around the median). By extending the quantile regression approach of Caporin et al. (2018) to QQ, we will get a finer analysis on the existence of shift-contagion not just along the dependent variable (the stock market) quantiles, but also across quantiles of the explanatory variable (the real estate market). In addition, and following from the discussion in Caporin et al. (2018), we evaluate the occurrence of shift-contagion by also controlling for the existence of structural breaks. If structural instability is ignored, we could mix data from different regimes, possibly associated with different levels and intensity in shift contagion. Consequently, the estimated quantiles would not be those of a specific density but are recovered from a mixture of different densities (Qu, 2008; Caporin et al., 2018). Given this, we conduct our analysis based on sub-samples of January, 2003 to July, 2007; August, 2007 to December, 2009; January, 2010 to December, 2012, and; January, 2013 to December, 2017. These break-ups also allow us to study the periods of pre-, during-, and post- the financial and sovereign debt crises.

Note that, contagion is defined as the presence of a significant increase of cross-market linkage after a shock, i.e, departure from fundamentals (Forbes and Rigobon, 2002). In light of this, to analyze contagion, one should ideally assess the connections between markets after having controlled for economic fundamentals. But with contagion associated with high-frequency data, such type of data is not available for macroeconomic variables. To control for issues such as omitted variables (latent factors) and endogeneity, we supplement our QQ approach with a Bayesian heteroskedastic version, where the conditional variance of the residuals follows a Generalized Autoregressive Conditional Heteroskedasticity (GARCH(1,1)) specification, since biases due to omitted variables and endogeneity are strictly related to heterskedasticity effects (Chen et al., 2009; Caporin, et al., 2018).

To the best of our knowledge, this is the first paper to study contagion across REITs and equity markets of the U.S. surrounding the extreme events of the global financial and European sovereign debt crises, based on a QQ approach controlling for various types of biases due to omitted variables, endogeneity and structural breaks.[4] Note that, our model can be considered as an extension of the quantile-GARCH approach of Caporin et al., (2018) to a corresponding QQ-version of the same. The remainder of the paper is organized as follows: Section2 discusses the econometric model and estimation methodologies, with Section 3 presenting the data and empirical results. Robustness analyses is performed in Section 4, and Section 5 concludes the paper.

2 Model and Estimation Methodology

As we mentioned in the introduction, our purpose is to evaluate the impact of REITs returns on the equity market returns to uncover possible occurrences of contagion. Within the econometrics literature focusing on contagion tests, we decided to follow the recent view put forward by Caporin et al. (2018), that analyze shift-contagion by adopting a quantile regression framework.

The baseline take the form of a single index model as follows:

$$R_{SP,t} = \alpha + \beta R_{REITs,t} + \varepsilon_t \tag{1}$$

where $R_{SP,t}$ is the return of the S&P 500 index at time t and $R_{REITs,t}$ is the return of the S&P REITs index at time t. The occurrence of contagion could be addressed by evaluating the statistical significance of the β parameter across the quantiles of the variable of interest, i.e., S&P500 returns. However, this approach neglects the possible role of the location of the REITs returns across its density support. In fact, the possible impact on the equity market due to movement in REITs might could on not only the state of the equity market, but also on the phases of the real estate market. Consequently, we generalize the approach of Caporin et al. (2018), and move toward a more flexible quantile regression approach, namely a non-parametric quantile regression model (see, Koenker (2005)), also called the quantile-on-quantile model due to the work of Sim and Zhou

[4]For the link of omitted variables and heterskedasticity we refer to Caporin et al. (2018).

(2015). Within a non-parametric quantile regression, the estimation of the parameters of the model given by equation (1) corresponds to the optimization of the following criterion function over a sample of size T:

$$\min_{\alpha,\beta} \sum_{t=1}^{T} \rho_\tau \left(\varepsilon_t\right) K \left(\frac{R_{REITs,t} - \theta}{h}\right) \tag{2}$$

where $\varepsilon_t = R_{SP,t} - \alpha - \beta R_{REITs,t}$, $\rho_\tau\left(u\right) = u\left(\tau - I\left(u < 0\right)\right)$ is the usual check function adopted in quantile regressions, τ is the quantile of interest for the dependent variable, $K\left(.\right)$ is a kernel function, θ is a the unconditional quantile of the REITs returns, and h is the bandwidth. The difference between quantile-on-quantile and non-parametric quantile regression is that in the latter, the value of θ is given by a collection of pre-defined knots on the support of the conditioning variable, while in the former, the values of θ are estimated and correspond to unconditional quantiles. In our case, as we follow a quantile-on-quantile approach, we will set a collection of θ values associated with the unconditional quantiles of the REITs returns.

Parameter estimation from the previous equation lead to the evaluation of the non-linear relation between the variables when focusing on the neighbourhood of the τ-quantile for the S&P500 returns and the θ-quantile for the REITs returns. Therefore, by evaluating the variation of β over θ and τ, we are able to monitor the existence (by statistical significance) and strength (by parameter size) of the relationship between the two variables of interest.

Following Caporin et al. (2018), the adoption of quantile regression provides a flexible approach for analysing how the explanatory variable influence the location, scale, and shape of the entire response distribution. In our analyses, we take a further step, as we allow the influence of the conditioning variable to change across the distribution of the conditioning variable. However, we stress that, when the distribution of the variables of interest show evidence of different volatility properties over time, the estimation of the links across the variables (i.e., over the joint support) might be biased or at least inefficient, leading to incorrect evaluation. This is particularly relevant at extreme quantiles, where the dynamic changes might be highly influenced by volatility dynamics. Hence, we take into account the possible presence of heteroskedasticity in the variables of interest, and in this regard follow Hiemstra and Jones (1994), Koenker and Zhao (1996), and Chen et al. (2009) to allow for heteroskedasticity directly into the quantile regression.

Specifically, we follow Chen et al. (2009) by introducing heteroskedasticity characterizing the dependent variable in the criterion function adopted for the quantile-on-quantile model. To estimate this econometric framework, we resort to a Bayesian estimation. However, to simplify the computational burden, we slightly modify the criterion function, and let the kernel to interact directly with the observed quantities as follows:

$$min_{\alpha,\beta} \sum_{t=1}^{T} \left(\frac{\rho_\tau \left(R_{SP,t} K(R_{REITs,t}, \theta, h) - \bar{\alpha} - \beta R_{REITs,t} K(R_{REITs,t}, \theta, h)\right)}{\sigma_t(\tau)} + log(\sigma_t(\tau)) \right), \tag{3}$$

where $K(R_{REITs,t}, \theta, h)$ is the same kernel function adopted above,[5] and $\sigma_t(\tau)$ is the square root of residual variance computed using estimates of quantile τ of the parameters α and β together with the parameters $\delta = \{\theta_0, \theta_1, \theta_2\}$ appearing in the variance equation below:

$$\sigma_t^2(\tau) = \theta_0 + \theta_{1,} e_{t-1}^2 + \theta_2 \sigma_{t-1}^2. \tag{4}$$

The extra logarithmic term in the criterion function ensures that the parameters do not converge to infinity (see Xiao and Koenker (2009) for an alternative criterion function). We stress that the volatility and the causal effect parameters are estimated simultaneously, resulting in a vector of parameters that, similar to the baseline case, depend on both the quantile of the dependent variable, τ, and the quantile of the explanatory variable, θ. We choose a Bayesian approach to estimate the parameters, given its several advantages namely: (i) accounting for parameter uncertainty through the simultaneous inference of

[5]We repeat the basic idea: The check function is $\rho(u_t, \tau)$ with $u_t = y_{it} - \beta_{i0} - \beta_{i1} X_{it}$. The non-parametric QR minimizes $\sum_t \rho(u_t, \tau) K(X_t, \gamma)$, therefore $\rho(u_t, \tau) K(X_t, \gamma) = u_t \times I(u_t < 0) K(X_t, \gamma)$. This is equal to $\rho(u'_t(\gamma), \tau)$ with $u'_t(\gamma) = u_t K(X_t, \gamma)$. Then, we have $u'_t(\gamma) = y_{it} K(X_t, \gamma) - b0(\tau) - b1(\tau) X_t K(X_t, \gamma)$.

all model parameters; (ii) exact inferences for finite samples; (iii) efficient and flexible handling of complex model structures and non-standard parameters; and (iv) efficient and valid inference under parameter constraints.

Bayesian inference requires the specification of prior distributions. We chose weak non-informative priors to allow the data to dominate inference. As is standard , we assume a normal prior for $\Theta_\tau \sim N(\underline{\Theta}_{0,\tau}, \underline{\Sigma})$. $\underline{\Theta}_{0,\tau}$ is set equal to the frequentist estimates of model given by equation (1); and $\underline{\Sigma}$ is chosen to be a matrix with sufficiently "large" but finite numbers on the diagonal. The volatility parameters α_τ follow a jointly uniform prior, $p(\alpha_\tau) \propto I(S)$, constrained by the set S that is chosen to ensure covariance stationarity and variance positivity, as in the frequentist case. These are sufficient conditions to ensure that the conditional variance is strictly positive (in this regard, see Nelson and Cao (1992) for a discussion of sufficient and necessary conditions on GARCH coefficients). Such restrictions reduce the role of the extra logarithmic term in equation (3).

The model is estimated using the Metropolis-within-Gibbs MCMC algorithms. Similarly to Chen et al. (2009), we combine Gibbs sampling steps with a random walk Metropolis-Hastings (MH) algorithm to draw the GARCH parameters (see also Vrontos et al. (2000) and So et al. (2005)). To speed-up the convergence and allow for an optimal mixing, we employ an adaptive MH-MCMC algorithm that combines a random walk Metropolis (RW-M) and an independent kernel (IK)MH algorithm. The reader is referred to Caporin et al. (2018) for further details on the estimation.

With the estimated classical and Bayesian models, we proceed to the evaluation of the presence of shift-contagion. On a sub-sample basis, shift-contagion is associated with a change in the coefficients linking the real estate market to the stock market across the quantile surface. The absence of shift-contagion is associated with a flat surface of coefficients estimated along the quantiles of the dependent and explanatory variables. If the surface is not flat, we would conclude that we find evidence in favor of shift-contagion. At the same time, by comparing the degree of contagion across the four sub-samples associated with during-, and post- the financial and sovereign debt crises, with the pre-crises period, we will are also able to test whether contagion, characterized by higher spillovers from REITs returns onto stock returns, did actually occur, and if this was conditional on the states, as defined by the quantiles, of the two asset markets.

3 Data and Results

3.1 Data

As indicated above, our estimations involves two variables measuring the behavior of the overall equity market and the REITs sector. In this regard, we use the S&P500 equity and S&P REITs indices daily data, which in turn are obtained from Datastream of Thomson Reuters, and converted to log-returns. Our analyses cover the entire period of 2nd of January, 2003 to 29th of December, 2017 for a total of 3776 observations. The sample size is chosen to include the US financial crisis and the following recovery period. We decided not to start before 2003 to avoid the internet bubble, wherein the role of REITs was limited.

As our purpose is to show that relationship (spillover) from the REITs sector to the equity market is time-varying, we partition the entire period into four sub-samples: the calm period arriving up to the onset of the financial crisis covering 2nd of January, 2003 to 31st of July, 2007 (1152 observations); the crisis period of 1st of August, 2007 to 31st of December, 2009 (611 observations); 4th of January, 2010 to 31st of December, 2012 (754 observations), which includes the post subprime crisis period up to the European sovereign crisis, and, finally, 2nd of January, 2013 to 29th of December, 2017 (1259 observations) to capture the period post the global financial and the European Sovereign debt crises. Note that, the starting point of the second sub-sample in August, 2007 is in line with an increase in perceived credit risk in the general economy, given that the TED spread (the difference between the interest rates on interbank loans and on short-term U.S. government debt i.e., Treasury bills) hiked up in July 2007, and reaching a record high of 4.65% in October, 2008 following the bankruptcy of Lehman Brothers on September, 2008.[6] Understandably, the end point of this second sub-sample corresponds to the official end date of the "Great Recession", and is also in line with the start of the European sovereign debt crisis, that started in the European Union since the end of 2009. Several eurozone member states (Greece, Portugal, Ireland, Spain and Cyprus) were unable to repay or refinance their government debt or to bail out over-indebted banks under their national supervision without the assistance of third parties like other eurozone countries, the European Central Bank (ECB), or the International Monetary Fund (IMF). Given that this crisis was at peak during 2010-2012, justifies our third sub-sample, with the final sub-sample starting from 2013 till the end of the data sample comprising our fourth sub-sample associated with a relative less tumultous period following the two crises.[7]

Figure 1 plots the REITs and S&P 500 price series and the corresponding transformation into log-returns. Both series have a positive trend in the first sample, almost explosive for the REITs; then a huge drop in the period of financial crisis, associated with dramatic spikes and large volatility; a moderate positive trend in the third period is observed accompanied with large instability; finally, the two series keep increasing in the fourth period, but stock prices at a rate higher than REITs. Returns in this last period indicate a lower level of volatility.

Table 1 provides the summary statistics for each of these sub-samples. What stands out is the non-normality in the distribution of the log-returns of both the variables due to negative skewness and excess kurtosis, and hence provides reasoning for a quantiles-based framework. In addition, within each of the sub-samples, the S&P REITs returns is consistently more volatile, and produces higher positive returns on average in the first and third sub-samples.

[6]See: `https://fred.stlouisfed.org/series/TEDRATE`.

[7]When we applied the Bai and Perron (2003) tests of multiple structural breaks to equation (1), we were able to detect three break dates at: 23rd July, 2007; 23rd October, 2009, and; 6th June, 2013, which in turn were quite close to our economic approach of identifying the corresponding four sub-samples. Further details on the break tests are available upon request from the authors.

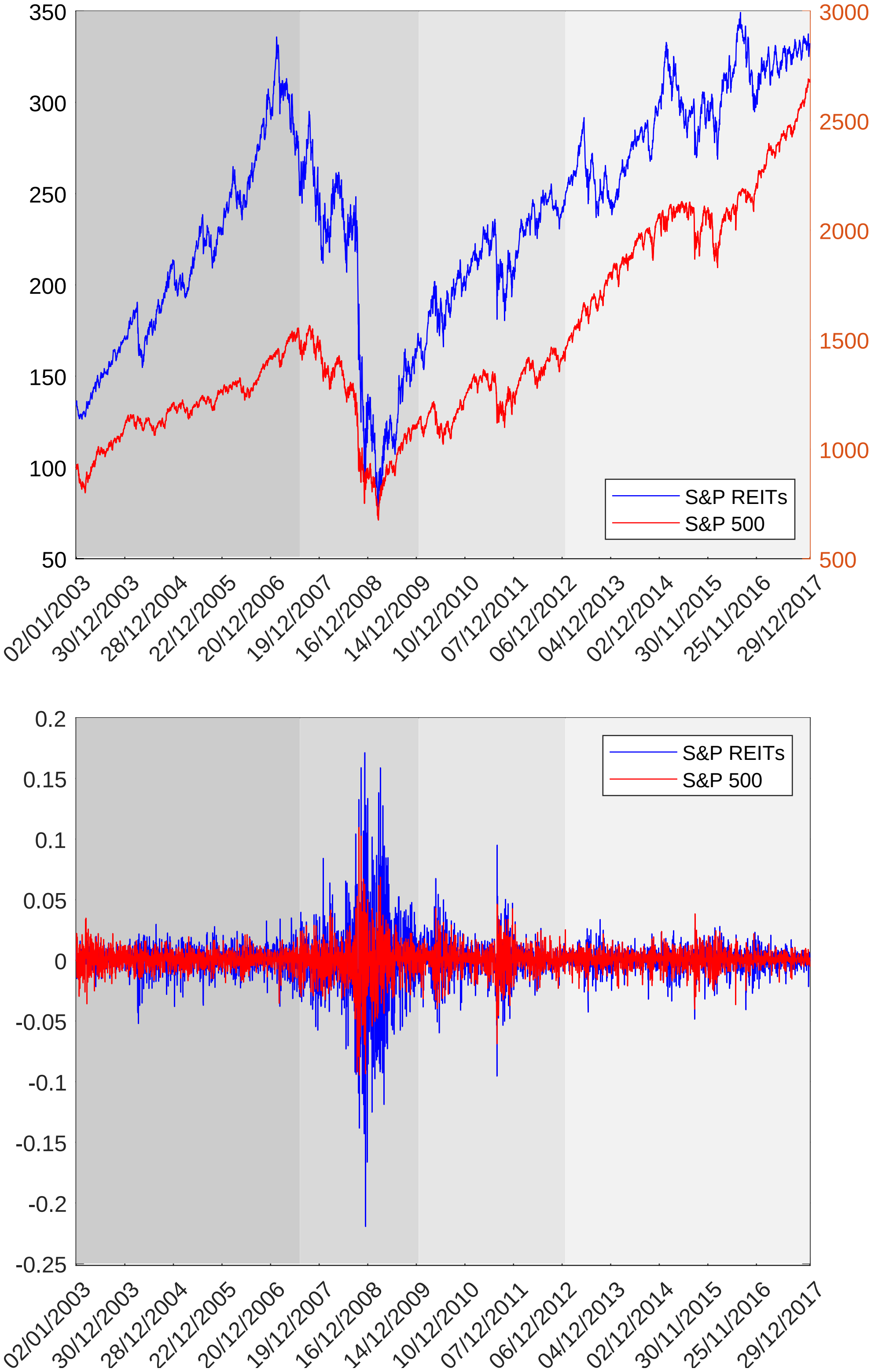

Figure 1: Data Plot of S&P REITs and S&P500 prices. Top: S&P REITs (left axis) and S&P500 (right axis) prices. Bottom: S&P REITs and S&P500 log returns. The four shaded areas indicate our sub-samples: the calm period from January 2, 2003 to July 31, 2007; the crisis period from August 1, 2007 to December 31, 2009; the post subprime crisis period up to the European sovereign crisis from January 4, 2010 to December 31, 2012; and, finally, the period post the global financial and the European sovereign debt crises from January 2, 2013 to December 29, 2017.

	02/01/2003 to 31/07/2007		01/08/2007 to 31/12/2009		04/01/2010 to 31/12/2012		02/01/2013 to 29/12/2017	
Statistic	S&P REITs	S&P 500	S&P REITs	S&P 500	S&P REITs	S&P 500	S&P REITs	S&P 500
Mean	0.001	0.000	-0.001	-0.000	0.001	0.000	0.000	0.001
Maximum	0.04	0.035	0.171	0.110	0.095	0.046	0.034	0.038
Std. Dev.	0.010	0.008	0.042	0.021	0.016	0.012	0.009	0.008
Skewness	-0.579	-0.070	-0.045	-0.133	-0.088	-0.427	-0.517	-0.410
Kurtosis	4.834	4.734	6.286	7.811	7.769	6.681	4.993	5.892
Q(1%)	-0.030	-0.018	-0.118	-0.062	-0.044	-0.033	-0.026	-0.021
Q(10%)	-0.011	-0.009	-0.045	-0.023	-0.017	-0.013	-0.010	-0.008
Q(25%)	-0.005	-0.004	-0.019	-0.009	-0.007	-0.005	-0.005	-0.003
Q(50%)	0.001	0.001	-0.001	0.001	0.001	0.001	0.001	0.001
Q(75%)	0.007	0.005	0.018	0.009	0.009	0.006	0.006	0.005
Q(90%)	0.012	0.009	0.042	0.021	0.016	0.013	0.010	0.009
Q(99%)	0.022	0.019	0.129	0.061	0.045	0.032	0.022	0.019
Jarque-Bera	225.900	145.258	275.166	590.935	715.607	448.624	264.455	474.000
Probability	0.000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000
Observations		1152		611		754		1259

Table 1: The table reports descriptive statistics of equity and real estate indexes returns, for the four sub-samples reported in the first row. We report Mean, Median, Standard Deviation, Skewness, Kurtosis, selected quantiles and the Jarque-Bera normality test with its p-value. The last row reports the size of the sub-samples.

In the second sub-sample, the mean return in the REITs sector as well as the overall equity market is understandably negative, with the former being higher in absolute terms - an indication of the origination of the crisis from the real estate sector. In the final sub-sample, the stock market yielded higher mean positive return than the S&P REITs, suggesting relatively stronger growth in the equity market in recent times. The analyses of sample quantiles highlights how the dispersion increases in the second (mostly) and third sub-samples compared to the pre-crisis period (the first sub-sample). In the last period, empirical quantiles of the returns are close to those of the pre-crisis period.

3.2 Empirical Results

We proceed with the estimation of our main model ($M0$, i.e., equation (1)) and of the robustness checks on the various sub-samples. We report here the results focusing only on the parameter β of equation (1). Given that the parameter depends on two quantiles, the estimation output takes the form of a surface plot where we include only statistically significant coefficients. Figure 1 reports the estimated coefficient surfaces in the four subsamples with our main model, while Table 1 includes a subset of the estimated coefficients for the four samples of our analysis, with Table 2 reporting the differential effects evaluated with respect to the first sub-sample, i.e., the pre-crises calm period.

Figure 1 shows that the overall pattern is similar across periods, in particular when comparing the first and the fourth sub-samples, which is understandable given that these two periods correspond to the non-crises episodes of financial markets. Considering the sign of coefficients over all sub-samples, we note that the impact of REITs return on the S&P500 return is positive across all quantiles of the two variables, but with differences in the size of the impact. In particular, the relation between quantiles is higher when both variables are in their tails. This suggests that, when the stock market is experiencing a negative and turbulent phase (given the well-established leverage effect (Black, 1976), i.e., volatility increases when the stock returns fall), the impact of negative REITs return is higher compared to the impact of positive REITs return, thus contributing to the equity market instability. An opposite, and positive, behaviour is observed on the upper tails. The size of the impact decreases when the two variables are located in opposite tails, and this is particularly evident in the second and fourth sub-samples. Overall, our results provide strong evidence of shift-contagion across the four sub-samples, and highlight also the fact that these size of the spillovers from the real estate market on to the equity market is indeed state-contingent, as characterized by differential impact across the quantiles of the two asset returns.

Differences across sub-samples emerge when considering the differentials with respect to the first sub-sample. We note that, during the subprime crisis, the REITs impact suffer a general decrease except at the extreme quantiles of both densities, where we note a relevant increase of the beta coefficients. The general decline of the impact, barring the extreme quantiles, seems counter intuitive in line with the reasoning that contagion rises during episodes of crises. However, for the third sub-period associated with the European sovereign debt crisis, we do find overwhelming evidence of contagion, as the spillover is found to increase relative to the calm period associated with financial markets during the first sub-sample. Some evidence of stronger impact of REITs returns on stock returns in the final sub-sample relative to the first sub-period is also observed at the lower and upper tails of the equity market, associated with corresponding entire and upper quantiles respectively, of the REITs returns. So overall, while we see strong evidence of increased spillover from the REITs sector to the equity market during the European sovereign debt crisis, the same cannot be said so convincingly about the sub-sample associated with the global financial crisis, which is known to have had its roots in the real estate market. This somewhat surprising result needs more in-depth analysis, and for this we now turn our attention to the model which accounts for the heteroskedasticity that is present in financial returns, and in the process accommodate for the possible omitted variables bias. We believe that to reach correct inferences, this is important especially when analyzing the second sub-sample which was associated with a much deeper all round crisis affecting the various sectors of the economy beyond the financial markets, and in turn, requires

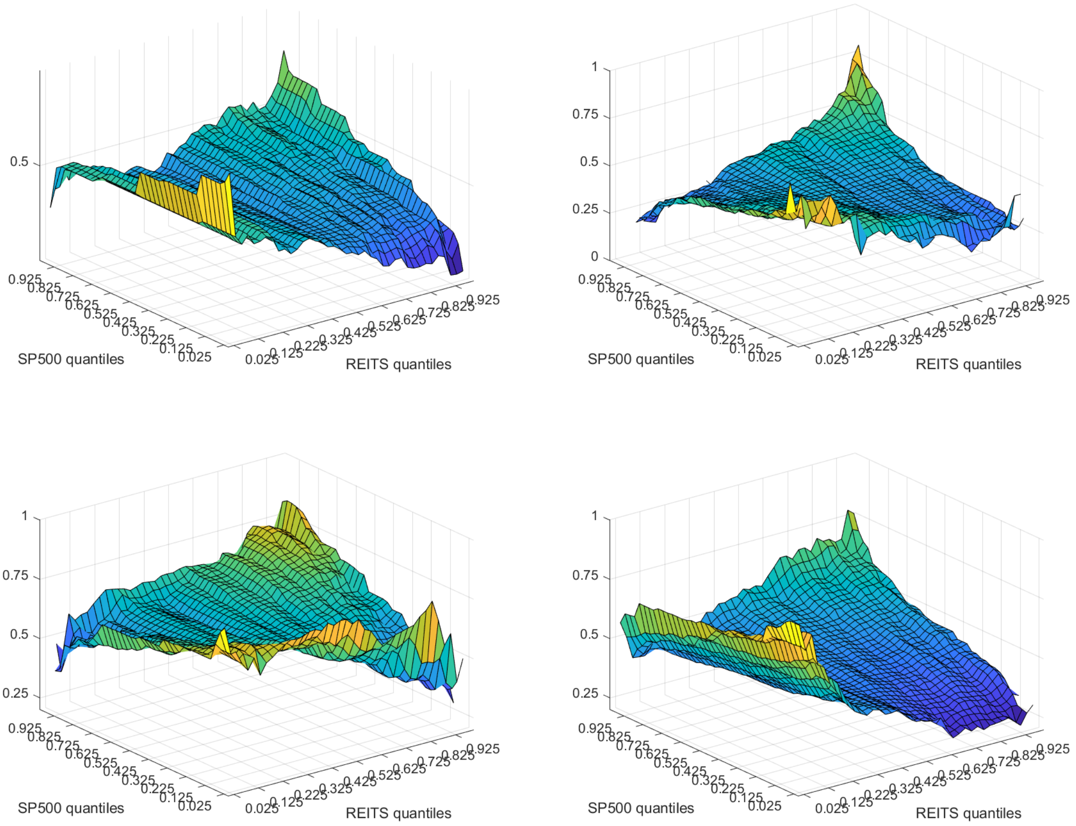

Figure 2: β parameter surface over quantiles of the REITs and S&P returns. The figure reports only statistically significant coefficients estimated from the four sub-samples: January 2003 to July 2007 in the upper left corner, while August 2008 to December 2009 is in the upper right corner; in the lower panels, on the left, the sample January 2010 to December 2012, and on the right January 2013 to December 2017.

to incorporate in the modeling approach the feedbacks of the negative impact of other economic decisions on to the financial markets.

To provide robust inference on shift-contagion within sub-samples, and to check, if connectedness between the markets do increase during crises periods, we move to the analysis of coefficient surfaces, coefficient values and differentials using the GARCH-based QQ model as outlined above in equations (3) and (4). Allowing for heretoskedasticity with Bayesian inference mainly increases tail effects, in particular when stock and REITs returns are in different phases, as shown in Figure 3 and Table 4. Interestingly, the effect is quite stable or mitigated,[8] barring the tails, across the samples (in line with the

[8] Alternative to the Bayesian QQ model, we had also estimated the benchmark model with

Quantiles of the REITs return	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05	0.521	0.469	0.426	0.391	0.343	0.765	0.578	0.392	0.260	*0.157*
0.25	0.514	0.473	0.442	0.434	0.419	0.583	0.502	0.410	0.357	0.264
0.50	0.514	0.473	0.449	0.455	0.471	0.492	0.433	0.393	0.368	0.346
0.75	0.513	0.470	0.453	0.480	0.507	0.392	0.354	0.409	0.447	0.470
0.95	0.500	0.461	0.462	0.516	0.541	*0.197*	0.251	0.366	0.347	0.750
	2010-2012					2013-2017				
0.05	0.767	0.756	0.706	0.616	0.370	0.799	0.505	0.364	0.284	0.282
0.25	0.713	0.661	0.572	0.475	0.505	0.773	0.468	0.404	0.367	0.405
0.50	0.638	0.595	0.577	0.598	0.581	0.702	0.422	0.438	0.429	0.459
0.75	0.552	0.539	0.580	0.668	0.721	0.598	0.418	0.487	0.508	0.532
0.95	0.364	0.557	0.608	0.660	0.829	0.511	0.422	0.538	0.630	0.711

Table 2: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples. Coefficients are all statistically significant at the 1% level apart the two coefficients in italics in the sample 2007-2009.

Quantiles of the REITs return	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05						0.244	0.109	-0.034	-0.131	-0.187
0.25						0.069	0.028	-0.032	-0.077	-0.155
0.50						-0.022	-0.040	-0.056	-0.087	-0.126
0.75						-0.122	-0.115	-0.044	-0.033	-0.038
0.95						-0.304	-0.211	-0.095	-0.169	0.209
	2010-2012					2013-2017				
0.05	0.246	0.287	0.279	0.225	0.027	0.278	0.036	-0.062	-0.107	-0.061
0.25	0.199	0.188	0.129	0.041	0.086	0.259	-0.005	-0.038	-0.067	-0.014
0.50	0.124	0.122	0.128	0.143	0.109	0.188	-0.051	-0.011	-0.025	-0.012
0.75	0.038	0.069	0.127	0.188	0.214	0.085	-0.052	0.034	0.028	0.024
0.95	-0.137	0.095	0.146	0.145	0.287	0.010	-0.039	0.076	0.115	0.170

Table 3: Estimated differentials in the REITs return impact on S&P500 return for selected quantiles (S&P500 return quantiles over columns, REITs quantiles over rows) with respect to the impact observed in the first sample.

findings of Forbes and Rigobon (2002)), especially for the last three sub-samples over August, 2007 to December, 2017, but evidence of shift-contagion is evident across all the four sub-samples, and in particular under the calm period across all quantiles. More importantly, when we analyze the differences across the three latter sub-samples relative to the first sub-sample in Table 5, consistent with the literature of contagion, we now find stronger impact of the REITs returns on the equity market during the second sub-sample associated with the global financial crisis virtually across all quantiles. Moreover, consistent with the results from the homoskedastic model in Table 3, higher strength of the spillover effect from the REITs sector is concentrated around the tails, not only for the second sub-sample, but also during the European sovereign debt crisis and the final sub-sample, which though in general was stable, did also depicted some degree of fluctuations during the oil price decline over 2014-2016, and of course the "Brexit" referendum in June of 2016. This might signal that, during periods of uncertainty, characterized mostly by daily volatile movements in the financial markets, the equity market experienced stronger spillover from the REITs sector onto the equity under both bear- and bull-states. With financial markets, particularly equities (overall or sector-based) being vulnerable, this is understandable, as tail-risk spillovers are likely to be stronger, since investors could be carrying out faster re-allocation in the portfolios between riskier assets and those that are considered safe-havens, especially during extreme movements.

Note that, unlike in the homeskedastic results reported in Table 3, we now under heteroskedasticity, i.e., in Table 5, find slightly weaker evidence of contagion in the sub-sample associated with the European sovereign debt crisis, relative to the first-sub-sample particularly around the moderately lower quantiles of the two asset returns. But this finding should actually make more sense than those observed under homoscedasticity, since

GARCH(1,1)-filtered stock and REITs returns. In general, and somewhat opposite to the Bayesian model, the coefficients were found to increase under the GARCH(1,1)-filtered model, relative to the benchmark one. We however, believe that the results from the Bayesian QQ approach to account for heteroskedasticity directly in the error structure, is more robust relative to the GARCH(1,1)-filtered approach. This is because, the former accounts for not only linear and the heteroskedastic effects simultaneously, but also possible differences across quantiles in the heteroskedastic behavior. Complete details of the results based on GARCH(1,1)-filtered data is available upon request from the authors.

during this sub-period both the markets were showing signs of recovery when compared to the massive downturns witnessed during the second sub-sample. Given that the GARCH-based QQ model captures the high volatility in the data in the second and in the third sub-samples as large shocks, we tend to obtain more accurate description of the contagion relationship between the REITs and S&P500 returns especially during crises, and in the process highlights the importance of accounting for heteroskedasticity to obtain reliable results.

Furthermore, when moving to the post crises period, i.e., the last sub-sample, we find that evidence of stronger impact of REITs on the equity market compared to the first sub-sample, is now relatively more profound across the quantiles in the heteroskedastic case (see Table 5) than under the homoskedastic error structure (see Table 3), which tended to provide a more mixed picture. Concentrating on the GARCH-based QQ results, the heightened spillover from the real estate sector on to the stock markets is understandable in the sense that the real estate sector leading up to the crisis was doing so well that investors were reluctant to move funds out of the REITs investments into the overall equity market. But in the wake of the two back-to-back crises associated with the real estate sector and sovereign bond markets, agents were probably more confident about the performance of the regular equity market, and hence, spillovers from the REITs sector is found to be stronger in the last sub-sample than the first one. The above line of reasoning associated with the shift in importance of equities relative to REITs seems to hold water, as the average returns across the first and last sub-samples show higher returns for the REITs in the former and lower in the latter relative to the S&P500 returns.

Overall, we find evidence in favor of shift-contagion across all the four sub-samples, suggesting that the size of the spillovers from the real estate sector to the equity market is contingent on the state of these two assets, with the impact being stronger particularly under extreme market conditions, i.e., at the tails of the distributions of the REITs and stock returns. Moreover, during crises the connectedness of these two markets, irrespective of their initial states, increases relative to a period of calm and stability. In essence,

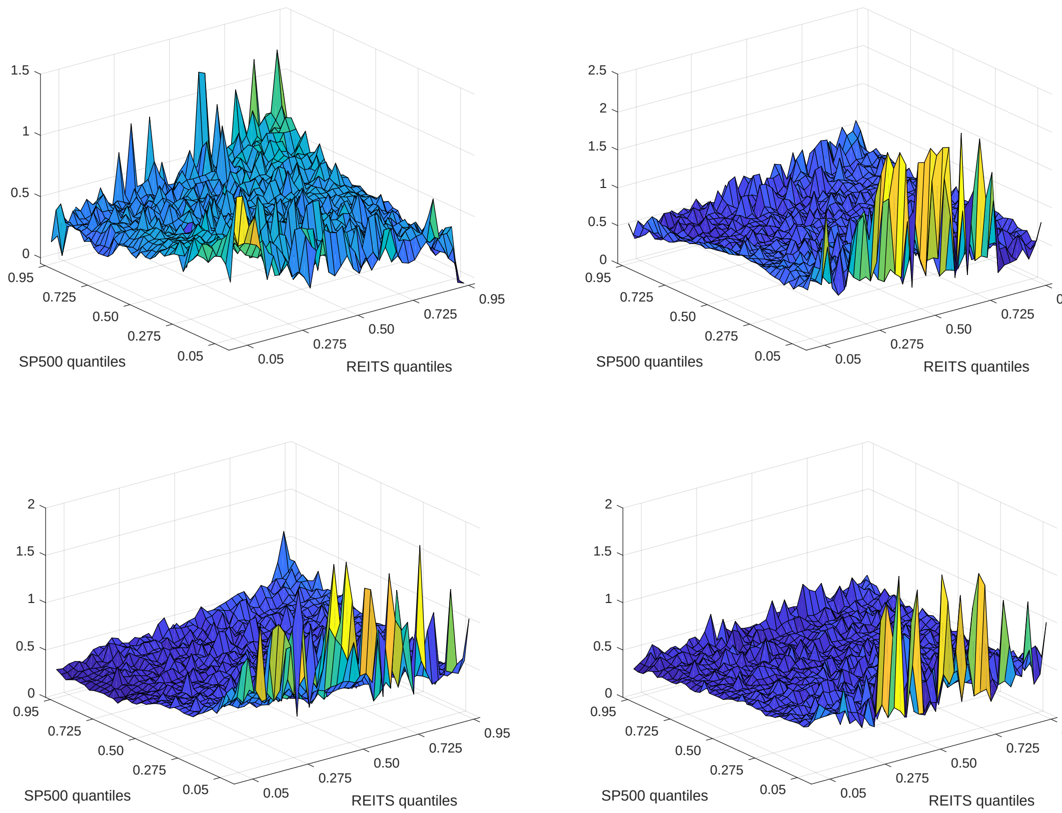

Figure 3: β parameter surface over quantiles of the REITs and S&P500 returns. The figure reports only statistically significant coefficients estimated from the four sub-samples based on the Bayesian heteroskedastic model.

these results imply that, from the perspective of an investor, irrespective of the general macroecomic scenario, i.e., whether the U.S. economy is in crisis or not, diversification benefits are not possible, especially under bearish and bullish-situations of both real estate and equity markets.

4 Robustness analyses

In order to verify the impact of the model specification on the surface of the β parameters (its value depend on both τ and θ) we consider as robustness checks, several alternative specifications. We consider several possible cases applied on the Bayesian QQ model accounting for heteroskedasticity, as we consider this particular model to be more general and robust. We describe here the various models starting with the baseline specification

Quantiles of the REITs return	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05	0.793	0.543	0.428	0.315	0.311	0.767	0.764	0.645	0.505	0.324
0.25	0.540	0.515	0.387	0.314	0.298	0.589	0.912	0.591	0.478	0.493
0.50	0.507	0.607	0.473	0.387	0.336	0.592	0.683	0.586	0.636	0.440
0.75	0.451	0.297	0.437	0.549	0.969	0.737	0.629	0.616	0.677	0.613
0.95	0.083	0.187	0.302	0.320	0.113	0.504	0.673	0.635	0.680	0.734
	2010-2012					2013-2017				
0.05	0.813	0.467	0.369	0.301	0.324	0.579	0.466	0.444	0.405	0.397
0.25	0.842	0.454	0.398	0.348	0.453	0.602	0.658	0.393	0.564	0.467
0.50	0.647	0.489	0.417	0.430	0.312	0.645	0.402	0.448	0.439	0.429
0.75	0.492	0.455	0.458	0.555	0.453	0.680	0.562	0.430	0.507	0.458
0.95	0.517	0.453	0.536	0.662	0.794	0.686	0.445	0.443	0.481	0.629

Table 4: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity. The value 0 is not included in the 1% credible interval for all coefficients.

Quantiles of the REITs return	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05						-0.025	0.221	0.217	0.191	0.014
0.25						0.049	0.397	0.204	0.164	0.195
0.50						0.084	0.077	0.113	0.249	0.104
0.75						0.286	0.333	0.178	0.128	-0.355
0.95						0.421	0.486	0.333	0.360	0.621
	2010-2012					2013-2017				
0.05	0.020	-0.076	-0.059	-0.014	0.013	-0.214	-0.077	0.016	0.090	0.087
0.25	0.302	-0.061	0.011	0.034	0.154	0.062	0.143	0.006	0.251	0.168
0.50	0.140	-0.118	-0.056	0.042	-0.024	0.138	-0.205	-0.025	0.052	0.092
0.75	0.041	0.158	0.021	0.006	-0.516	0.229	0.265	-0.007	-0.042	-0.511
0.95	0.435	0.267	0.234	0.342	0.681	0.604	0.258	0.141	0.162	0.516

Table 5: Estimated differentials in the REITs return impact on S&P500 return for selected quantiles (S&P quantiles over columns, REITs quantiles over rows) with quantile regression with heteroskedasticity and with respect to the impact observed in the first sample.

that, for comparison purposes, which we report in the first line below:

$$\begin{aligned}
M0 \rightarrow R_{SP,t} &= \alpha + \beta R_{REIT,t} + \varepsilon_t && (5)\\
M1 \rightarrow R_{SP,t} &= \alpha + \beta R_{REIT,t} + \delta R^2_{REIT,t} + \varepsilon_t && (6)\\
M2 \rightarrow R_{SP,t} &= \alpha + \beta R_{REIT,t} + \gamma_1 R_{SP,t-1} + \gamma_2 R_{REIT,t-1} + \varepsilon_t && (7)\\
M3 \rightarrow R_{SP,t} &= \alpha + \beta R_{REIT,t} + \gamma_1 R_{SP,t-1} + \gamma_2 R_{REIT,t-1} + \delta_1 R^2_{REIT,t} \\
&\quad + \delta_2 R^2_{REIT,t-1} + \varepsilon_t && (8)\\
M4 \rightarrow R_{SP,t} &= \alpha + \beta R_{REIT,t} + \delta R^2_{REIT,t} + \phi R^2_{SP,t-1} + \varepsilon_t && (9)\\
M5 \rightarrow R_{SP,t} &= \alpha + \beta R_{REIT,t} + \gamma_1 R_{SP,t-1} + \gamma_2 R_{REIT,t-1} + \delta_1 R^2_{REIT,t} \\
&\quad + \delta_2 R^2_{REIT,t-1} + \phi R^2_{SP,t-1} + \varepsilon_t. && (10)
\end{aligned}$$

The first generalization ($M1$; equation (6)) controls for the non-linearity, at each single quantile, in the impact of the REITs return on the S&P500 return. Different from $M1$, $M2$ (equation (7)) takes into account lagged effects of both the dependent and explanatory variables. Specification $M3$ (equation (8)) combines the elements put forward in $M1$ and $M2$. In case $M4$ (equation (9)), we extend the baseline model by including a component that proxy heteroskedasticity effects of the dependent variable at quantiles.[9] Finally, $M5$ (equation (10)) combines all the possible effects, i.e., taking $M3$ and $M4$ together. Note that, in the latter cases we control for additional heteroskedastic effects not captured by standard GARCH models.

Tables A2 to A6 in the The Appendix of the paper contain the estimated β coefficients over selected quantiles of the S&P500 and the REITs returns for models $M1$ to $M5$ estimated using the Bayesian approach. Overall, by changing the model specifications, the impact on the size and surface (i.e., pattern of behavior) of the coefficients are

[9]If we do have heteroskedasticity in the S&P 500 return, with the conditional variance depending on its past values, the quantile of the S&P500 return depend on lagged conditional variances. We proxy the latter by lagged squared returns.

limited. However, as depicted earlier under $M0$ with heteroskedastic error structure, we find evidence of shift-contagion for each of the sub-sample, and also observe an increased spillover from the REITs market to the equity market during periods of crises. Thus, we find robust confirmation of the results obtained in the previous section based on simpler version of the models considered here.

5 Concluding Remarks

In this paper, we study contagion between REITs and the equity market in the U.S. based on an (extended) sample period of daily data covering 2003 till 2017, which in turn allows us to study the impact of not only the global financial crisis, but also the European sovereign debt turmoil. We apply quantile-on-quantile (QQ) nonparametric regressions to study the impact of the REITs market on U.S. equities. Realizing that, if structural breaks are ignored, we could mix data from different regimes, we conduct our analysis based on sub-samples of January, 2003 to July, 2007; August, 2007 to December, 2009; January, 2010 to December, 2012, and; January, 2013 to December, 2017. The adoption of the previous sub-samples also allow us to study the periods of pre-, during-, and post-financial and sovereign debt crises. Moreover, to control for possible omitted variable bias in high-frequency contagion analysis between securitized real estate and equity markets, we control for heteroskedasticity by relying on a GARCH-based Bayesian QQ model to obtain reliable and robust inferences. We find that the spillovers from the REITs on to the equity market has varied over time and states (i.e., quantiles) across the four sub-samples, and hence is indicative of shift-contagion. In addition, evidence of contagion, in other words increased impact of REITs returns on S&P500 returns is observed during episodes of crises, i.e., the second and third subsample, relative to calmer periods, i.e., the first sub-sample. Finally, our results were found to be robust to various model specifications, which extended the Bayesian QQ-GARCH model.

In sum, besides the implication for investors, associated with lack of diversification opportunities, discussed above, our results have two other implications: First, from an econometric point of view, we highlight that the role of heteroskedasticity, aiming to control for misspecification due to possibile omitted variables, should not be ignored. If neglected, we are likely to get unreliable estimates of the contagion effect leading to inaccurate inferences. Second, with stronger evidence of spillover at lower quantiles in general, a policy-maker worried about contagion of especially the negative shocks, which in turn can deepen economic crises given the leading role of asset prices (Stock and Watson, 2003), should aim to revive the economy based on expansionary (monetary and fiscal) policies, but with the understanding that the strength of the intervention needs to contingent on the state of these two markets.

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Appendix

Quantiles of the REITs return		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
		2003-2007					2007-2009				
	0.05	0.793	0.599	0.360	0.271	0.192	0.750	0.775	0.796	0.664	0.364
	0.25	0.580	0.625	0.413	0.300	0.287	0.624	0.663	0.579	0.446	0.339
	0.50	0.487	0.458	0.457	0.334	0.203	0.708	0.690	0.588	0.518	0.415
	0.75	0.257	0.413	0.436	0.659	0.705	0.510	0.659	0.660	0.733	0.632
	0.95	*-0.086*	0.050	0.341	0.368	*0.107*	0.755	0.658	0.701	0.652	0.749
		2010-2012					2013-2017				
	0.05	1.069	0.513	0.338	0.210	0.298	0.486	0.468	0.323	0.225	0.245
	0.25	0.904	0.488	0.365	0.336	0.130	0.431	0.653	0.378	0.354	0.364
	0.50	0.479	0.444	0.477	0.425	0.298	0.635	0.499	0.477	0.495	0.512
	0.75	0.492	0.443	0.594	0.601	0.785	0.403	0.394	0.460	0.585	1.055
	0.95	0.138	0.608	0.737	0.877	1.393	0.353	0.281	0.499	0.720	1.068

Table A1: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M1$. A number in *italics* indicates that the value 0 is included in the 1% credible interval.

Quantiles of the REITs return	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05	0.735	0.608	0.462	0.371	0.435	0.750	0.715	0.634	0.596	0.352
0.25	0.454	0.475	0.428	0.429	0.338	0.649	0.832	0.577	0.545	0.615
0.50	0.422	0.421	0.390	0.346	0.252	0.695	0.714	0.559	0.502	0.463
0.75	0.406	0.360	0.417	0.397	0.522	0.686	0.493	0.595	0.678	0.870
0.95	0.199	0.301	0.306	0.268	-0.056	0.590	0.699	0.625	0.733	0.708
	2010-2012					2013-2017				
0.05	0.768	0.508	0.348	0.300	0.331	0.518	0.480	0.443	0.365	0.442
0.25	0.687	0.480	0.407	0.429	0.332	0.505	0.507	0.428	0.408	0.467
0.50	0.672	0.532	0.437	0.488	0.431	0.401	0.516	0.412	0.363	0.408
0.75	0.547	0.387	0.474	0.512	0.478	0.614	0.395	0.611	0.418	0.518
0.95	0.394	0.465	0.551	0.610	0.792	0.537	0.437	0.440	0.501	0.834

Table A2: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M2$.

Quantiles of the REITs return	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05	0.603	0.480	0.394	0.294	0.386	0.679	0.793	0.742	0.636	0.637
0.25	0.594	0.480	0.450	0.355	0.249	1.224	0.628	0.552	0.495	0.425
0.50	0.556	0.400	0.387	0.375	0.222	0.767	0.636	0.701	0.711	0.447
0.75	0.343	0.375	0.430	0.476	0.620	0.608	0.611	0.667	0.718	0.780
0.95	0.471	0.354	0.369	0.241	0.068	0.650	0.816	0.698	0.740	0.604
	2010-2012					2013-2017				
0.05	0.969	0.492	0.343	0.263	0.301	0.493	0.454	0.349	0.199	0.190
0.25	0.803	0.516	0.366	0.368	0.172	0.560	0.513	0.414	0.349	0.448
0.50	0.714	0.630	0.417	0.417	0.437	0.467	0.403	0.436	0.393	0.505
0.75	0.437	0.352	0.520	0.622	0.688	0.537	0.429	0.462	0.541	0.720
0.95	0.196	0.694	0.689	0.819	0.921	0.237	0.353	0.418	0.693	1.117

Table A3: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M3$.

Quantiles of the REITs return		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
		2003-2007					2007-2009				
	0.05	0.635	0.633	0.388	0.308	0.139	0.707	0.746	0.755	0.643	0.310
	0.25	0.562	0.587	0.499	0.352	0.370	0.847	0.657	0.562	0.453	0.420
	0.50	0.461	0.435	0.390	0.452	0.343	0.656	0.678	0.799	0.631	0.414
	0.75	0.334	0.317	0.524	0.732	1.088	0.420	0.524	0.763	0.645	0.696
	0.95	*-0.110*	*0.013*	0.197	0.087	0.551	0.663	0.748	0.681	0.650	0.791
		2010-2012					2013-2017				
	0.05	0.945	0.463	0.324	0.219	0.258	0.557	0.449	0.320	0.204	0.313
	0.25	0.866	0.551	0.331	0.332	0.216	0.787	0.458	0.429	0.553	0.471
	0.50	0.507	0.434	0.437	0.419	0.431	0.621	0.471	0.488	0.410	0.546
	0.75	0.403	0.411	0.564	0.617	0.772	0.454	0.392	0.582	0.564	0.870
	0.95	0.212	0.609	0.713	0.797	1.211	0.344	0.286	0.492	0.681	1.064

Table A4: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M4$. A number in *italics* indicates that the value 0 is included in the 1% credible interval.

Quantiles of the REITs return		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
		2003-2007					2007-2009				
	0.05	0.640	0.487	0.471	0.284	0.385	0.638	0.710	0.735	0.606	0.226
	0.25	0.545	0.517	0.428	0.486	0.363	1.068	0.662	0.575	0.422	0.355
	0.50	0.501	0.421	0.406	0.384	0.225	0.451	0.705	0.560	0.622	0.494
	0.75	0.419	0.340	0.561	0.477	0.401	0.448	0.559	0.596	0.785	0.708
	0.95	0.510	0.433	0.349	0.133	0.161	0.782	0.811	0.732	0.714	0.591
		2010-2012					2013-2017				
	0.05	0.948	0.520	0.350	0.282	0.245	0.618	0.448	0.380	0.224	0.218
	0.25	0.857	0.461	0.349	0.301	0.187	0.655	0.454	0.428	0.341	0.536
	0.50	0.427	0.389	0.385	0.396	0.307	0.667	0.440	0.443	0.425	0.542
	0.75	0.470	0.414	0.499	0.568	0.769	0.534	0.441	0.426	0.559	0.708
	0.95	0.206	0.606	0.729	0.745	1.065	0.287	0.313	0.397	0.681	1.036

Table A5: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M5$.