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Asset Allocation in European Equity Markets: A comparison of sector, factor and country investing

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MSc in Business, Major in Finance

Abstract

With the evolution of index-based investing in the past decades, we investigate three investment universes consisting of sector, factor, and country indices in the European equity market. We employ several asset allocation strategies such as 1/N, Risk-Parity, Minimum-Variance, Mean-Variance, Bayes-Stein, and Kelly Growth. We then investigate and extend the research on blended portfolios (Ghayur et. al, 2018) by introducing countries and joint blending. In the period from 2002-2021, we find that factor portfolios dominates sector and country portfolios both in expansions and recessions when short-selling is disallowed. When short-selling is allowed, country portfolios often outperform factors across several performance measures. Contrary to previous findings, we find that sector investing do not yield better performance when diversification is needed, and is overall the weakest dimension in performance.

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1 Introduction

Whether investors should invest across countries or sectors has been widely researched for a long time. For many decades, research suggested that investors should invest across countries to capture its diversification benefits (Lessard, 1974; Heston & Rouwenhorst, 1994; Cavaglia et al., 2000; Baca et al., 2000). However, Eiling et al. (2005) found that after the introduction of the single currency in the European Union (EU) in the late 90's, country and sector investing offered the same risk-return trade-off. Various papers (Cavaglia et al., 2000; Baca et al., 2000) later showed that sector investing was superior to country investing starting around the millennium in developed markets. In recent years, a relatively new investment style has become widely adopted, namely factor investing. Comparisons between these investment styles is yet to be thoroughly investigated. Bessler et al. (2021a) found that factor investing offered superior performance compared to sector investing in the U.S. capital markets. Will the same be true for the European market?

Our motivation for researching the chosen topic is to better understand how investors should allocate their capital. Should they allocate everything to sector, factor, our country indices, or will they enhance their performance by combining these investment styles? Previous studies in this field have mainly focused on the importance of country and sector investing, but little research has been done on the combination of factors. We want to expand the literature on factor combination and also attempt to better understand which factors are driving the returns from these investment styles. Lastly, the literature usually focuses on either a set of developed markets or the US market only. Only a small number of papers research the European market, and we have not seen any papers focusing on the European market for approximately 15 years. Given the clean data that MSCI reports of the European market, we believe that performing a similar analysis to the ones that have been conducted for other markets will contribute to new insights, and hopefully pave the way for further research on this topic.

Our first null hypothesis is that among the three investment dimensions, factor investing will prove to be superior to both country and sector investing over a variety of performance measures. Furthermore, our second hypothesis is that by combining the dimensions into blended portfolios, we believe that this will enable the portfolios to maintain the diversification benefits of sector investing while capturing the risk premia associated with factor investing and outperform any single dimension.

To test our hypothesis, we use some of the strategies in DeMiguel et al. (2009) like 1/N, Minimum-Variance, Mean-Variance, Bayes-Stein and Risk Parity. We supplement the strategies presented in DeMiguel et al. (2009) with the Kelly Growth strategy, adding another layer of robustness with a return-maximizing strategy. To analyze if combining investment styles produce better performance we construct blended portfolios based on Ghayur et al. (2018). The data we use to analyze sector, factor, and country indices are constructed by MSCI and retrieved from Bloomberg. We also collect data on the factors presented in Fama and French (1992; 2015), Carhart (1997), Frazzini and Pedersen (2014) from Kenneth Frenchs' website. This data will be used for multi-factor alpha regressions to determine whether the returns can be attributed to known risk factors.

The thesis is structured as follows: In section 2, 3 and 4 we present the research question, the literature review, as well as some theory on the factors used in this thesis and our hypothesis. Then, we proceed to explain the methodology in section 5. Section 6 is dedicated to describing the data and some descriptive statistics. In section 7, we progress to present the quantitative results of the analysis. Section 8 shows the result of our robustness checks. Then, we proceed to perform the factor regressions in section 9. Finally, we discuss the results and conclude in section 10 and 11.

2 Research question

The research question we will investigate is: Which investment style has the best performance; sector, factor, or country investing? And, is performance enhanced if the investment styles are combined?

3 Literature review & theory

3.1 Literature review

In 1952, Harry Markowitz published the paper *Portfolio selection* and introduced modern portfolio theory (MPT) to the asset management industry. In short, MPT, also known as mean-variance analysis, allows investors to maximize the expected return given a certain level of risk, such that they can achieve their desired risk-adjusted return. One can argue that Markowitz's work laid the foundation for further research into portfolio optimization.

Grubel (1968) was among the first to show that international diversification can improve the risk-reward characteristics of an investor's portfolio. Later, Haim and Sarnat (1970) supplemented the literature with their paper *International Diversification of Investment Portfolios*. Using a sample of 28 countries in the period 1951-1967 they show that diversification across countries can be highly beneficial for risk-adjusted returns and results suggests that security prices can be highly affected by capital flow/international trade restrictions (Haim & Sarnat, 1970).

A few years later, research on sector versus country optimization emerged, and in recent years a handful of papers have brought in factors into the comparison. However, the vast majority of research is on country and sector investing. The literature shows no clear evidence that any one of these investment styles are superior to the others over the long-term. However, the first papers that compared sector and country investing suggests that country investing is more beneficial from a diversification point of view. One of these papers are Lessard (1974). Using a sample of 30 international industry indices and 16 national market indices, Lessard show that when assessing common return elements, industry factors are not as important as country factors. This means that country factors are more important than sector factors for diversification purposes (Lessard, 1974). Solnik (1977) and Heston & Rouwenhorst (1994) strengthened the case for country factors.

Based on the literature there seems to be a shift from country to sector investing being the most beneficial in the years before year 2000. Two of the first papers to show this was Cavaglia et al. (2000) and Baca et al. (2000). The papers use data sets from 1979-1999 and 1986-1999, respectively. Both papers show that the importance of country and sector diversification is roughly the same. Cavaglia et al. (2000) further show that the last five years in their sample, sector diversification was superior to country diversification. The latest research that focus purely on the European market and compares country and sector factors is the paper *Asset Allocation in the Euro-zone: Industry or Country Based?* by Eiling et al. (2005). The paper analyzes the period 1990-2003 in the Euro-zone. They found that in the period 1992-1998, country investing outperformed sector investing. However, after the single currency was introduced, country investing and sector investing offered the same diversification benefits and risk return trade-off (Eiling et al., 2005). Ferreira and Ferreira (2006) also show that there is an increasing importance of sector diversification in the European equity markets. They argue that the reason for the shift is that there is less cross-sectional variation of interest rate movements across countries in Europe.

More recent studies such as Eiling et al. (2012), Marcelo et al. (2013) and Aspergis et al. (2014) all argue for the diminishing effect of country diversification. In Bessler et al. (2021b) the authors further strengthen the case for sector diversification using ten country indices and ten sector indices of developed countries in the period 1975 to 2020. This paper shows that sector optimization yields superior performance compared to country optimization and explains that is due to the increased correlation and integration between countries.

Factor investing first emerged in 1976 when Stephen A. Ross published the research paper *The arbitrage theory of capital asset pricing*. In short, the paper claimed that asset returns can be explained by a set of factors. Approximately a decade earlier, in 1964, William F. Sharpe published *Capital asset prices: A theory of market equilibrium under conditions of risk*. The paper presented the famous capital asset pricing model (CAPM) which Ross (1976) extends from single factor to multifactor. Fama and French (1993) presented some interesting new findings for factor investing. They identified three common factors that drive equities return and two which drive bond returns. The three equities factors are the market factor, book-to-market factor and firm size factor. In 2015 the authors extended this model to five factors with a factor for investment pattern and profitability (Fama & French, 2015). Carhart (1997) also contributed greatly to the literature by showing that performance of equity mutual funds means and risk-adjusted returns can be explained by investment expenses and common factors (Carhart, 1997).

The literature on the potential benefit of diversification through factor-based investing is quite new. However, with increasing popularity of factor investing, more research has emerged on the topic. One of the first papers to display the potential benefit of diversifying through factor investing was Asness et al. (2013). They provided evidence of strong common factor structures between value and momentum strategies. The papers Bender et al. (2010) and Hjalmarsson (2011) builds on the preliminary work of Asness et al. (2013), and strengthens the case that combining multiple factors increases the diversification benefits in factor portfolios. Hjalmarsson (2011) studies long-short portfolios constructed on factors such as value, size and past returns. The paper analyses data from Kenneth French's website [1] for the period 1951 to 2008, and the data includes all stocks on NASDAQ, AMEX and NYSE. Hjalmarsson (2011) finds that all the single-characteristic portfolios that are tested in the paper are profitable. However, the paper shows that by combining multiple characteristics, one can construct an equal-weight portfolio which dominates the single-characteristic portfolios. Hjalmarsson (2011) attributes this to low correlation between the characteristics. llmanen and Kizer (2012) compares diversification across dynamic and static factors in the US equity markets in the period 1973 to 2010 using equal weighted portfolios. They find that investors could benefit by investing across factors instead of asset classes. Their results are true for both long-short strategies and long-only strategies.

Briere and Szafarz (2018) investigates the benefits of blended portfolios between sectors and factors using mean-variance efficiency tests on non-investable data from Kenneth Frenchs' website. The data is made up of U.S. stocks listed on NYSE and is from the period 1963 to 2016. The authors conclude that there are benefits of combining the two styles and the benefit becomes especially clear in long-short portfolios during crisis periods, where sector investing outperforms factor investing. Instead of looking at non-investable data, we look at data that can be used as proxies for the investable ETF's. In Bessler et al. (2021a) the authors compare sector versus factors diversification in investable low-cost ETFs in U.S. markets and conclude that factor diversification is superior to sectors. The papers methodology is based on DeMiguel et al. (2009). They implement a variety of asset allocation models and use the Sharpe ratio as the main performance measure. They find that there are substantial Sharpe ratio differences among all the implemented asset allocation strategies. Furthermore, they suggest that further research can be done by investigating whether blending the investment styles would add value. We supplement this paper by including portfolio blending, new strategies, and compare sectors, factors, and countries in the same analysis in the European market.

[1] Website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html

3.2 Factor theory

The factor indices in this analysis are not directly investable but follows an investable ETF close enough to be seen as a proxy. For the multi-factor regression later on, we use the Fama & French 5-factor model augmented by the momentum factor, and the betting-against-beta factor. The time-series data of Fama & French factors are not directly investable and difficult to replicate (Ang et. al, 2009), which is why we follow Bessler et al. (2021a) and choose investable indices to perform our analysis. Below is an introduction of the FF factors and the BAB-factor.

Small minus big (SMB): The SMB factor is one of three factors in the Fama-French three factor model. It is often referred to as the size factor, and tries to take advantage of the fact that over the long-term, small capitalization stocks often outperform large capitalization stocks.

High minus low (HML): The second factor is the HML factor which attempts to lock in profits by going long value stocks and shorting growth stocks.

Winners minus losers (WML): This is the momentum factor. The factor tries to capture the effect that winners and losers often have a drift. The factor goes long winners and short losers.

Robust minus weak (RMW): The RMW factor focus on firms with high and low operating profitability, and assumes that firms with high and robust operating profitability will outperform firms with weak operating profitability. The RMW factor was one of two factors that was introduced in the five-factor model.

Conservative minus aggressive (CMA): CMA is the second of the two factors that was added to the five-factor model. Often referred to as the investment factor, CMA measures the differences between firm that invest lightly (conservative) versus firms that invest heavily (aggressive).

Betting against beta (BAB): This factor assumes that high beta stock are overpriced and low beta stock are underpriced due to inefficiencies in the capital asset pricing model (CAPM). Thus, it goes long low beta stocks and short high beta stocks.

4 Hypothesis

Our review of the literature tells us that the benefits of investing in countries has seen a diminishing effect over the past decades, where diversification and performance within sectors has been superior. The emerge of factor investing has proven to be dominant in harvesting risk-premia, and combination of factors and sectors has shown promising results. Based on this review and the summary statistics presented in Appendix 13.8, we form the following two null hypotheses:

1. H_0 : Factor-based portfolios will yield the overall best out-of-sample performance measured by Sharpe ratio, certainty equivalent return, and turnover.[2]
H_A : Factor-based portfolios performs worse or the same as sector and/or country-based portfolios measured by the same performance measures.

2. H_0 : A portfolio combining the dimensions into blended portfolios will outperform any single one dimension measured by the same performance measures.
H_A : Sector, factor or country investing is better than the blended portfolios we construct.

5 Methodology

In this section, we introduce the methodology and asset allocation strategies that we use to analyze our hypothesis. We follow the same methodology as DeMiguel et al. (2009), which argues that no portfolio strategy is persistently superior to 1/N. We use sector, factor, and country indices to compare and evaluate different asset allocation strategies within these dimensions, where we employ several weighting techniques and sample estimation lengths to determine the benefits of the different dimensions and asset allocation strategies. These strategies are as follows: The naive equally weighted portfolio (1/N), the risk-based models Minimum-Variance (MinVar) and Risk Parity (RP), the risk-return models Mean-Variance (MV) and Bayes-Stein (BS), and the return-maximizing Kelly Growth model (KG)[3].
The portfolio returns are calculated monthly by multiplying the corresponding

[2]In addition, we calculate the Omega ratio to incorporate all return moments and Information ratio to measure the performance relative to the MSCI Europe Index.

[3]Matlab functions to generate portfolio weights for these strategies was provided by Costas Xiouros during autumn 2021 in the course Strategic Asset Allocation (GRA6560).

weights with the returns of each component. The weights for each portfolio is calculated at the end of each month, with different rolling window sample periods. We use three different sample periods of 12, 24, and 60 months to estimate the *out-of-sample* portfolio weights. This process is then repeated throughout the data, moving the sample period one month forward for each month passed. We treat the investment dimensions separately to compare the performance, but also investigate the performance of blended portfolios (Ghayur et al., 2018).
To further increase the robustness of our results, we consider three different weight constraints in our optimization: No short sales and a maximum of 35% weight to each constituent, a maximum of 50% long and short, and a maximum of 100% long and short. We do not constrain the weights to sum to one for the strategies that do not do this by construction, which means that $1 - 1_N^T w_t$ is invested in the US 1-month T-Bill rate, which we use as the proxy for the risk-free rate. We also do not constrain the weights of the risky assets to be positive, allowing for negative positions to lever up on the risk-free rate.

Nr.	Model	Short form
1	Equally weighted	1/N
2	Risk parity	RP
3	Minimum-Variance	MinVar
4	Mean-Variance	MV
5	Kelly Growth	KG
6	Bayes-Stein shrinkage portfolio	BS

Table 1: Overview of asset allocation models

5.1 Asset allocation models

1. Naive equally weighted portfolio

The equally weighted 1/N portfolio often referred to as the naive portfolio does not involve any optimization and ignores the data. The portfolio consists of weights $w_t = 1/N$ in each of the N risky assets, and is rebalanced each month to maintain the equal weighting. The idea behind the strategy might seem counter-intuitive, as you sell the winners and buy the losers when rebalancing, but has shown to be consistently superior compared to more complex asset allocation strategies (DeMiguel et al., 2009).

2. Mean-Variance portfolio

With the groundbreaking work in Modern Portfolio Theory by Markowitz

(1952), the mean-variance model optimizes the trade-off between mean and variance of portfolio returns. To implement the strategy, we consider an investor whose preferences are determined only by the mean and variance of returns, with expected utility maximized by:

$$\max_{\mathrm{w}} U = \mathrm{w}^{\mathrm{T}} \mu_e - \frac{\gamma}{2} \mathrm{w}^{\mathrm{T}} \Sigma_t \mathrm{w} \tag{1}$$

Where U is the utility of the investor, w is the vector of portfolio weights, and μ_e the N-dimensional vector of expected excess return. γ is the risk aversion coefficient of the investor, which is set to five[4]. Finally, the term Σ is defined as the N x N variance-covariance matrix. The strategy is solved by plugging in the counterparts to μ and Σ, $\hat{\mu}$ and $\hat{\Sigma}$, corresponding to the sample mean and covariance, in the optimization solution:

$$\mathrm{w_t} = \frac{1}{\gamma} \Sigma_t^{-1} \mu \tag{2}$$

3. Minimum-Variance portfolio

The Minimum-Variance approach is the combination of assets that minimizes the variance of portfolio returns. The minimization problem is given by:

$$\min_{\mathrm{w}} \mathrm{w}^{\mathrm{T}} \Sigma_t \mathrm{w}, \quad \text{s.t.} \quad 1^{\mathrm{T}} \mathrm{w} = 1 \tag{3}$$

And the weights are given by:

$$\mathrm{w_t} = \frac{\Sigma_t^{-1} 1^T}{1^{\mathrm{T}} \Sigma_t^{-1} 1} \tag{4}$$

w_t is the vector of portfolio weights and Σ_t the covariance matrix. As only the covariance between assets is used to form the portfolio weights, the minimum-variance approach is advantageous in the sense that it does not require return estimates. These estimates are usually prone to large estimation error, which is avoided in this approach (DeMiguel et al., 2009).

4. Risk Parity

The risk parity approach is based on that each component in the portfolio contributes equally to total portfolio risk. The approach does not utilize any

[4]Bessler et. al (2017) provides some discussion regarding parameter settings in mean-variance frameworks. The findings suggests that results are in general robust to various levels of risk-aversion.

information regarding correlations between returns, and securities are weighted anti-proportional to some risk measure (volatility, variance, VAR, CVAR). We chose the naive risk parity approach, that weights securities anti-proportional to their sample variance $\hat{\sigma_i}^2$:

$$\mathrm{w_t} = \frac{\frac{1}{\hat{\sigma_i}^2}}{\sum_{i=1}^{N} \frac{1}{\hat{\sigma_i}^2}} \tag{5}$$

By construction, the strategy "over-weights" low volatility assets, and profits from the low volatility anomaly, which usually earn a higher premium per unit of volatility than higher volatility assets (Frazzini & Pedersen, 2014). As the weights are anti-proportional to the total portfolio risk, the portfolio is, by construction, constrained to sum to one.

5. Kelly Growth portfolio

Based on the 1738 paper of famous mathematician Daniel Bernoulli, John Kelly showed in 1956 that the long run growth rate G for win and lose probabilities p and q in Bernoulli trials was given by

$$G = \lim_{\mathrm{t}\to\infty} log\left(\frac{w_t}{w_0}\right)^{\frac{1}{t}} \tag{6}$$

Where t is discrete time and w_t is the wealth at time t with w_0 being the initial wealth equivalent to $\max E[logw]$. Substituting W_t into G gives $G = E[logw]$ by the law of large numbers. Maximizing the long run growth rate then becomes equivalent to maximizing the one-period expected log of wealth. The strategy has shown to be superior in the long run under the right circumstances, but can be very risky in the short run. Given a constant portfolio, in the very long run

$$\frac{1}{T}\left(r_{p,1} + ... + r_{p,T}\right) \xrightarrow{T\to\infty} \mathbb{E}(r_p) \tag{7}$$

and the problem converts into a sequence of static problems, maximizing the one-period expected log-returns. The Kelly growth portfolio is equivalent to an investor with CRRA utility where $\gamma = 1$, and the portfolio weights each period is given by:

$$\mathrm{w_t} = \arg\max_{\mathrm{w_t}} \mathbb{E}\{log\left[R_{p,t+1}\left(\mathrm{w_t}\right)\right]\} \tag{8}$$

6. Bayes-Stein shrinkage portfolio

The Bayes-Stein shrinkage portfolio was designed by James and Stein (1961) to handle estimation error when estimating portfolio expected returns by shrinking the sample mean towards a "grand mean" (Jorion, 1985). The portfolio optimization is the same as for mean-variance in equation (1), and the parameters are given by:

$$\hat{\mu}_t^{\text{bs}} = \left(1 - \hat{\phi}_t\right) \hat{\mu}_t + \hat{\phi}_t \hat{\mu}_t^{\text{min}}, \tag{9}$$

$$\phi_t = \frac{N+2}{(N+2) + M \left(\hat{\mu}_t - \mu_t^{\text{min}}\right)^\top \hat{\Sigma}_t^{-1} \left(\hat{\mu}_t - \mu_t^{\text{min}}\right)}, \tag{10}$$

$$\hat{\mu}_t^{\text{min}} \equiv \hat{\mu}_t^\top \hat{w}_t^{\text{min}} \tag{11}$$

$$\hat{\Sigma}_t = \frac{1}{M-N-2} \sum_{s=t-M+1}^{t} \left(R_s - \hat{\mu}_t\right) \left(R_s - \hat{\mu}_t\right)^\top \tag{12}$$

where $0 < \phi_t < 1$, and $\hat{\mu}_t^{\text{min}} = \hat{\mu}_t^T \hat{\text{w}}_t^{\text{min}}$ is the average excess return of the minimum-variance portfolio. In addition to shrinking the estimates of the means, the covariance matrix is also adjusted for estimation error following Jorion (1986).

5.2 Blended portfolios

To further investigate the performance of sectors, factors, and countries, we combine the dimensions using the signal blend and portfolio blend method proposed by Ghayur et. al (2018). The framework is based on exposure-matched portfolios, where assets are selected based on their exposure to factors. The idea behind the framework is to select the assets with the highest positive exposure to some factors, gaining the diversification benefits of sectors or countries while capturing the risk premia of factors.

To begin with, assets are assigned a z-score on each of the factors[5]. In both

[5]Z-scores are calculated by obtaining betas from factor regressions using the same rolling window approach for sample estimates, then subtracting the mean and dividing by the standard deviation of the most fitting distribution, which we found to be the normal distribution.

the signal and portfolio blending method, sectors and countries are ranked according to their average z-score of their exposure to factors. For the signal blending method, the top 50% quantile of assets with the highest average composite z-scores are treated as the investment opportunity set, and assigned an equal weight to the portfolio. For the portfolio blending approach, the exposure to factors are treated separately, where the top three assets with the highest exposure to each factor are treated as the investment opportunity set, and assigned an equal weight to the portfolio. If any of the assets are top three in several factors, they are assigned $\mathrm{w_i} = \frac{1}{N} * k$ weight, where k is the number of factors they are top three in exposure.
To avoid overlapping factor exposures, we remove the Multifactor index from this analysis, and are left with Size, Value, Quality, Momentum, and Volatility.

Brière and Szafarz (2020) show that sector-blended portfolios outperform pure factor ETFs using similar industries in the US. We extend this and Ghayur et. als (2018) research by including country indices and construct signal and portfolio blends jointly. The blending is applied monthly using the same sample rolling window approach as the other strategies above. The performance of the portfolios are compared to their index counterpart.

5.3 Performance measures

The objective of the thesis is to evaluate which investment style has the best performance relative to each other and to the blended portfolios. Following the methodology of DeMiguel et al. (2009), we compute three performance measures: *Out-of-sample* Sharpe ratio, the *certainty equivalent return* (CEQ), and the portfolio *turnover*. In addition, we calculate the Omega ratio to incorporate all moments of portfolio returns, and the Information ratio (IR) to measure performance relative to some benchmark.

1. Sharpe ratio
The *out-of-sample* Sharpe ratio is given by

$$\widehat{SR}_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k} \tag{13}$$

where $\hat{\mu}_k$ denotes the portfolio return of portfolio k after subtracting the risk-free rate, while $\hat{\sigma}_k$ is the standard deviation.

We also test for significance between the Sharpe ratios. DeMiguel et al. (2009) follows the methodology presented in Jobson and Korkie (1981) and then adjusting for the correction suggested in Memmel (2003). This test assumes IID returns and that the returns follow a normal distribution. For equity returns this assumption is normally violated. Therefore, we have chosen a test which is valid under more general assumptions.

Instead, we test for significance of the Sharpe ratios using the methodology presented in Opdyke (2007)[6]. The test is true under stationary and ergodic returns[7].

2. Certainty-equivalent return

The *certainty equivalent return* (CEQ) is the risk-free return an investor is willing to accept rather than investing in a risky portfolio strategy. The CEQ of each strategy k is given by:

$$C\hat{E}Q_k = \hat{\mu}_k - \frac{\gamma}{2}\hat{\sigma}_k^2 \tag{14}$$

where $\hat{\mu}_k$ and $\hat{\sigma}_k^2$ is the out-of-sample mean and variance of excess return for strategy k. As before, γ is the risk-aversion coefficient, set to five. The CEQ is closely related to risk premium, and can help an investor understand the risk they must take to increase their returns.

3. Turnover

Turnover is a measure of how much trading is required for each strategy, and is defined as the average sum of the absolute value of trades across the N available assets:

$$Turnover = \frac{1}{T-M}\sum_{t=1}^{T-M}\sum_{j=1}^{N}(|\hat{w}_{k,j,t+1} - \hat{w}_{k,j,t^+}|) \tag{15}$$

where $\hat{w}_{k,j,t}$ are the portfolio weights for asset j at time t for strategy k, $\hat{w}_{k,j,t^+}$ the portfolio weights *before* rebalancing at time $t+1$, and $\hat{w}_{k,j,t+1}$ the portfolio weights at $t+1$ *after* rebalancing. We report the absolute turnover for all strategies, which can be interpreted as the average percentage of wealth traded in each period. Higher levels of turnover affects the returns of the portfolio due to increased transaction costs.

[6]We use Opdykes' own developed Excel-file that uses the same methodology as described in Opdyke (2007). Opdyke refers to the file in the paper and states that it is available to be downloaded from his website, DataMineIt.com. However, when downloading from DataMineIt.com a password is needed. Therefore, we retrieved the file from https://www.researchgate.net/publication/313782495_Comparing_Sharpe_Ratios_-_Excel_Workbook_Implementation_of_Opdyke_2007.

[7]See Appendix 13.2 for a thorough explanation of the variables used in Opdyke's paper

4. Omega ratio

The Omega ratio is an alternative risk-return performance measure to the Sharpe ratio that, by construction, considers all moments of the return distribution. Devised by Keating and Shadwick (2002), the ratio evaluates the risk-reward of return distributions as the probability weighted ratio of gains versus losses relative to an investors loss threshold. The ratio is calculated as:

$$\Omega = \frac{\int_{\theta}^{\infty}(1 - F(x))dx}{\int_{-\infty}^{\theta} F(x)dx} \tag{16}$$

where F is the cumulative probability distribution function of asset returns and θ the return threshold that considers whether the return is a gain or loss. We use the average monthly return of the 1/N strategy for each dimension as the threshold in our analysis. $\Omega > 1$ indicates that the portfolio provides more gains relative to losses for the threshold θ.

5. Information ratio

The Information ratio (IR) measures the portfolio returns beyond some benchmark relative to the volatility of those returns, and is a common measure of a portfolio manager's level of ability to generate excess return to the benchmark. The ratio is calculated as:

$$Information\ Ratio\ (IR) = \frac{E(R_i - R_b)}{\sigma_{ib}} \tag{17}$$

where R_i is the portfolio return, R_b the benchmark return, and σ_{ib} the standard deviation of the difference between portfolio and benchmark returns, also known as the tracking error. We use the MSCI Europe Index as a benchmark for all asset dimensions. If $IR > 0$, the portfolio manager outperformed the benchmark.

6 Data

6.1 Description of data

Our analysis consists of 15 country indices, 10 sector indices, and 6 factor indices in the European market covering the period from November 2001 to December 2021 (241 observations for every index). Index data is collected from Bloomberg using MSCI indices. Factor-data to analyze and measure exposure to well-known risk factors are collected from Kenneth French's and AQR Capital Managements websites. From all sources we extract USD-denominated monthly return time-series based on closing prices for each month. The US 1 month T-Bill rate is used as a proxy for the risk-free rate in the analysis, and the MSCI Europe Index is used as the benchmark to calculate the Information ratio.

6.1.1 Sector data

The sector data used in this analysis consists of monthly returns of 10 sector indices formed by Morgan Stanley Capital International (MSCI) and collected from Bloomberg. These sectors are: Energy, Materials, Industrial, Consumer Discretionary, Consumer Staples, Healthcare, Financials, Information Tech, Telecom, and Utilities. The collected data span from November 2001 to December 2021. A more detailed list of all sector indices can be found in table 11 in Appendix 13.1.

6.1.2 Factor data

The factor data used in this analysis consists of monthly returns of 6 factor indices formed by Morgan Stanley Capital International (MSCI) and collected from Bloomberg. These factors are: Size (Fama & French, 1993), Value (Fama & French, 1993), Quality (Fama & French, 1993), Momentum (Carhart, 1997), Low volatility (Frazzini & Pedersen, 2014), and Multifactor. The collected data span from November 2001 to December 2021. A more detailed list of all factor indices can be found in table 12 in Appendix 13.1.

6.1.3 Country data

The country data used in this analysis consists of monthly returns of 15 country indices formed by Morgan Stanley Capital International (MSCI) and collected from Bloomberg. These countries are: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and United Kingdom. The collected data span from November 2001 to December 2021. A more detailed list of all country indices can be found in table 13 in Appendix 13.1.

6.1.4 Factor regression data

We use Kenneth French's website to retrieve factors for the European market[8] which can be used to check if the returns in the country, sector or factor indices can be explained by exposure to one or more of these factors. We download time series data on the market in excess of the risk-free rate, the small minus big factor (SMB), the high minus low factor (HML), the robust minus weak factor (RMW), the conservative minus aggressive factor (CMA), the winner minus losers factor (WML) and finally the US 1-month T-Bill rate which will be used as the risk-free rate in the rest of the paper. In addition to this, we collect data on the betting-against-beta (BAB) factor from the AQR Capital Management website.

6.2 Data summary

Table 19 in Appendix 13.8 shows the descriptive statistics and correlations of the factor indices. The factor indices exhibit stable returns across all factors. The average mean return of all factors is 0.84 % per month, with a standard deviation of 5.12%. All the factor indicies are near perfectly correlated, with the highest pair "Size" - "Multifactor" (0.97), and the lowest pair "Value" - "Momentum" (0.88). Table 20 in Appendix 13.8 shows the descriptive statistics and correlations of the country indices. The mean return varies significantly across countries, with returns ranging from 1.11% (Denmark) to 0.11% (Portugal) per month. The average mean return of all countries is 0.50% per month, with a standard deviation of 6.40%. The highest correlated pair is "France"

[8]Note that French includes Greece in his data set. Greece is not included in the MSCI indices. He does not specify which shares have been included in each market. Thus, it is reasonable to assume that there are some discrepancies between the shares used in each market for the MSCI indices and French's data.

- "Germany" (0.94), and the lowest pair "Ireland" - "Portugal" (0.62). Table 21 in Appendix 13.8 shows the descriptive statistics and correlations of the sector indices. Similar to countries, the mean return also varies significantly, with returns ranging from 0.80% (Industrial) to 0.05% (Telecom). The average mean return of all sectors is 0.49% per month, with a standard deviation of 5.99%. The highest correlated pair is "Industrial" - "Consumer Discretionary" (0.92), and the lowest pair "Information Tech" - "Energy" (0.51). From the preliminary analysis of the different statistical properties, the factor indices are superior to sectors and countries in terms of both risk and return. However, the large positive correlation between factors means that diversification benefits will be limited. While factors exhibit the highest return and lowest risk, the constituents of countries and sectors has the potential to provide a better risk-adjusted return. To provide empirical evidence of this, we employ several optimization strategies that are both dependent and independent of traditional input parameters.

7 Analysis & results

In this section, we compare the performance of asset allocation strategies for all asset dimensions and evaluate them to each other and the $1/N$ benchmark. For each strategy and each estimation period, we compute the out-of-sample Sharpe ratio (Table 2), the certainty equivalent return (Table 4), portfolio turnover (Table 5), Omega ratio (Table 6), and Information ratio (Table 7).

7.1 Sharpe ratio

Table 2 presents the annualized Sharpe ratios of all the strategies considered. Our analysis covers one naive optimization (1/N), two risk-based models (RP, MinVar), two risk-return models (MV, BS), and one return maximization model (KG). For all the strategies except 1/N, we employ three different estimation periods of moments, and three different cases of weight constraints. The first restriction of weights allows a maximum of 35% allocation to a single index, and short sales prohibited. The second case allows for a 50% maximum allocation both long and short, whereas the third case extends this even further to a maximum allocation of 100%.

Our first observation is that the general trend follows that of the 1/N strategy, where factors outperform both sectors and countries. Especially for the case where short-selling is disallowed and the weights are constrained to 35%, the relative relation between the Sharpe ratios of factors, sectors, and countries are very similar to 1/N. However, when the weight constraints are relaxed, both sectors and countries often outperforms factors as well as the 1/N strategy. Country portfolios are in some cases significantly better than factor portfolios, especially for longer estimation lengths.

The second observation we make is that the estimation window of sample estimates makes a significant difference in performance. This is consistent for almost every single strategy and all weight constraints. This is also consistent with the findings of Bessler et al. (2021a) which uses data from US factor and sector indices, however the magnitude between estimation length seems to be stronger for the European market. Our dataset from 2001-2021 contains several periods of high volatility such as the 2008 financial crisis and the 2013 oil crisis. Short periods of large changes in returns are incorporated quicker into the optimization models in shorter estimation periods, as they by construction contributes more to the estimation parameters.

Our third observation is that strategies that do not estimate means, which are usually prone to large estimation error (DeMiguel et al., 2009), consistently outperforms those who do. Minimum-Variance and Risk Parity exhibit stable performance across all estimation windows and weight constraints, and for sample estimation periods of 24 and 12 months, outperforms the 1/N strategy in every case.

The final observation that we make is that for the risk-return strategies (MV, BS) and the Kelly Growth strategy, short-selling only yields better performance for longer estimation lengths. Examining the case of 12-month estimation length, long-only portfolios yields a better Sharpe ratio than those who allow for short-selling.

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation length	S	F	C	S	F	C	S	F	C
1/N		0.27	0.52**	0.25						
Risk Parity	12	0.35*	0.59***	0.35*						
	24	0.31*	0.52**	0.28						
	60	0.17	0.31	0.10						
Minimum-Variance	12	0.36*	0.61***	0.34*	0.35*	0.62***	0.37*	0.49**	0.67***	0.33*
	24	0.34*	0.55**	0.37*	0.31*	0.61***	0.42**	0.40**	0.65***	0.49**
	60	0.23	0.37*	0.13	0.33	0.49**	0.50**	0.31	0.52**	0.55**
Mean-Variance	12	0.43**	0.68***	0.53**	0.34*	0.59***	0.55***	0.37*	0.60***	0.59***
	24	0.26	0.52**	0.31	0.33*	0.41**	0.42**	0.35*	0.35*	0.45**
	60	-0.02	-0.12	-0.07	0.29	-0.06	0.35*	0.30	0.00	0.48**
Bayes-Stein	18[1]	0.53**	0.66***	0.52***	0.53**	0.64***	0.54**	0.45**	0.66***	0.50**
	24	0.28	0.52**	0.36*	0.35*	0.43**	0.48**	0.38*	0.40*	0.57***
	60	-0.14	-0.13	-0.11	0.14	0.03	0.40*	0.10	0.10	0.55**
Kelly Growth	12	0.35*	0.58***	0.49**	0.19	0.36*	0.39**	0.26	0.45**	0.41**
	24	0.25	0.53**	0.27	0.13	0.28	0.26	0.13	0.33*	0.34*
	60	-0.06	0.06	-0.19	-0.08	-0.02	-0.14	0.07	-0.03	0.05

Table 2: Sharpe ratios

Note: This table reports the annualized Sharpe ratios of the sector, factor, and country portfolios. The first column displays the optimization technique used, and includes the naive 1/N strategy, Risk parity, Minimum-Variance, Mean-Variance, Bayes-Stein, and Kelly growth. The second column displays the estimation length used to estimate parameters for the *out-of-sample* portfolio weights. The abbreviation S denotes sectors, F denotes factors, and C denotes countries. *, **, *** indicates the significance level as portrayed in table 3 at the 10%, 5% and 1% level, respectively.

[1]: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

Table 3 reports the results from the significance tests of Sharpe ratios following Opdyke (2007). For long-only portfolios, factor portfolios are statistically significant in most configurations, as well as being significantly different from both sector and country portfolios. When the weights are relaxed to 50% long/short, the significance relative to other dimensions drops sharply compared to the long-only case. In the case where weights are relaxed to 100% long/short, country portfolios appears to have the most cases of significance, both alone and relative to other dimensions.

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation length	S	F	C	S	F	C	S	F	C
1/N			**/S***/C***							
Risk Parity	12	*	***/S***/C***	*						
	24	*	**/S***/C***							
	60		S***/C***							
Minimum-Variance	12	*	***/S**/C**	*	*	***/S*	*	**	***/C*	*
	24	*	**/S**/C**	*	*	***/S*	**	**	***	**
	60		*/S*/C**			**	**		**	**
Mean-Variance	12	**	***/S**	**	*	***/S*	***	*	***	***
	24		**/S***/C**		*	**	**	*	*	**
	60				F*		*/F**	F*		**/F**
Bayes-Stein	18[1]	**	***	***	**	***	**	**	***	**
	24		**/S**/C*	*	*	**	**	*	**	***
	60						*/F*			**/S**/F**
Kelly Growth	12	*	***/S**	**/S*		*/S*	**/S**		**/S**	**/S*
	24		**/S***/C**			S*			*/S*	*/C**
	60	C**	S*/C**							

Table 3: Significance test of Sharpe ratios

Note: This table reports the Sharpe ratios significance tests obtained by performing the test from Opdyke (2007). Note that we have used Opdyke's self-developed Excel-sheet available at his website to perform the tests. *, **, *** indicates the significance level at the 10%, 5% and 1% level, respectively. Stars with letters next to them denotes whether the Sharpe ratio is significantly different to other dimensions. S denotes sectors, F denotes factors, and C denotes countries. In the cases where there are no letters in front of the star(s), this refers to the Sharpe ratio being significant by itself. In the cases where there are letters, the investment style is significant relative to the investment style it is referred to. For example, in the 1/N strategy, factor investing is significant by itself but also significant at the 1 percent level to both sector and country investing. It is important to note that we used Opdyke's self-developed excel sheet and using his method the Sharpe ratios deviated slightly from those in Table 3. However, since the discrepancies were very small, the tests should provide valid results.
[1]: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

7.2 Certainty equivalent return

Table 4 shows the certainty equivalent return (CEQ) of all the strategies and estimation windows considered. By construction, as the CEQ is given by $C\hat{E}Q_k = \hat{\mu}_k - \frac{\gamma}{2}\hat{\sigma}_k^2$, higher values of CEQ reflects stronger performance based on the relation between mean and variance of returns. Consistent with the results of our analysis of Sharpe ratios, Minimum-Variance and Risk Parity again outperforms all other strategies, including 1/N for the cases of 24 and 12 months estimation period. Comparing the performance across dimensions, factors dominates both sectors and countries. These findings differ from the performance in terms of Sharpe ratio, where factors did not significantly outperform sectors and countries under relaxed weight constraints. Another interesting observation is that the Bayes-Stein portfolios, which are essentially the same as Mean-Variance but with lower mean estimates and higher covariance estimates, produces significantly stronger results than Mean-Variance. This implies that the sensitivity to differences in variance between these strategies is larger than the differences in means, as Bayes-Stein is a more conservative strategy than Mean-Variance.

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation length	S	F	C	S	F	C	S	F	C
1/N		-0.27	0.12	-0.40						
Risk Parity	12	-0.05	0.22	-0.18						
	24	-0.12	0.13	-0.28						
	60	-0.36	-0.17	-0.62						
Minimum-Variance	12	-0.01	0.28	-0.14	-0.09	0.31	-0.13	-0.04	0.37	-0.54
	24	-0.02	0.21	-0.06	-0.09	0.30	0.02	-0.02	0.35	0.08
	60	-0.17	-0.06	-0.41	-0.05	0.14	0.15	-0.08	0.19	0.22
Mean-Variance	12	-0.11	0.37	-0.26	-1.19	-0.13	-1.46	-3.63	-0.64	-4.71
	24	-0.21	0.17	-0.31	-0.50	-0.17	-0.83	-1.81	-0.64	-3.03
	60	-0.45	-0.59	-0.70	-0.38	-0.61	-0.34	-1.35	-0.79	-1.04
Bayes-Stein	18*	0.23	0.36	0.12	0.14	0.31	0.23	-0.51	0.25	-0.10
	24	0.01	0.21	0.11	-0.13	0.03	-0.03	-0.61	-0.12	-0.48
	60	-0.39	-0.54	-0.38	-0.41	-0.42	-0.06	-0.95	-0.42	-0.18
Kelly Growth	12	-1.17	0.03	-2.68	-6.22	-2.53	-15.17	-19.62	-9.26	-38.29
	24	-1.19	-0.07	-2.99	-4.09	-2.10	-8.37	-10.73	-6.65	-19.36
	60	-2.08	-1.34	-4.42	-2.64	-2.38	-5.08	-6.08	-5.87	-8.89

Table 4: Certainty equivalent return

Note: This table reports the monthly certainty equivalent return (CEQ) for all sector, factor, and country portfolios. The abbreviation S denotes sectors, F denotes factors, and C denotes countries. The results are calculated as $C\hat{E}Q_k = \hat{\mu}_k - \frac{\gamma}{2}\hat{\sigma}_k^2$, where γ is set to five. *: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

7.3 Turnover

Table 5 reports the turnover for all strategies considered. The turnover is reported in absolute values, and can be interpreted as the average percentage of wealth traded each period. Higher levels of turnover means higher transaction costs, which negatively affects the overall portfolio return. As expected, the turnover for the optimization models are much greater than the 1/N strategy. Comparing the turnover across different optimization models, we see that the turnover is very similar across sample periods and weight constraints. Consistent with previous observations, the magnitude of turnover increases with both sample period reduction and weight constraints. The large turnover for the weight constraints -50% - 50% and -100% - 100% is expected, as the room for flexibility in optimization is larger by construction. We also observe that the relative turnover between sectors (N = *10*), factors (N = *6*), and countries (N = *15*) is consistent with the number of assets in each dimension, where again the larger the room for optimization, the higher the turnover.

Comparing the risk-based models, we observe that Risk Parity exhibits very similar levels of turnover compared to 1/N. This is however not that surprising, as the normalization that the weights sum to one using the sample variance does not produce very different weights from 1/N, especially under similar levels of volatility across assets. The turnover for the Minimum-Variance optimization is slightly lower compared to the risk-return based models, but has the single highest turnover across all strategies and dimensions with the 12 months estimation period and -100% - 100% weight being 4.74. The Bayes-Stein strategy exhibits slightly lower levels of turnover compared to Mean-Variance, implying that the shrinkage of parameter estimates has successfully reduced the turnover.

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation length	S	F	C	S	F	C	S	F	C
1/N		0.02	0.01	0.02						
Risk Parity	12	0.10	0.05	0.10						
	24	0.05	0.02	0.05						
	60	0.02	0.01	0.02						
Minimum-Variance	12	0.31	0.15	0.40	1.21	0.52	2.05	2.60	1.21	4.74
	24	0.15	0.08	0.18	0.65	0.27	1.01	1.03	0.60	1.74
	60	0.06	0.03	0.08	0.27	0.10	0.37	0.34	0.26	0.53
Mean-Variance	12	0.38	0.22	0.53	1.26	0.73	2.02	2.59	1.49	4.28
	24	0.23	0.17	0.36	0.85	0.51	1.46	1.70	1.08	2.98
	60	0.14	0.13	0.15	0.55	0.30	0.85	1.08	0.60	1.54
Bayes-Stein	18*	0.20	0.20	0.06	1.05	0.65	1.73	2.10	1.31	3.33
	24	0.19	0.16	0.18	0.86	0.54	1.44	1.62	1.05	2.68
	60	0.10	0.12	0.12	0.51	0.26	0.78	0.89	0.60	1.25
Kelly Growth	12	0.40	0.24	0.53	1.13	0.61	1.52	2.36	1.12	3.45
	24	0.29	0.14	0.40	0.87	0.35	1.09	1.91	0.77	2.49
	60	0.20	0.15	0.29	0.56	0.27	0.84	1.05	0.53	1.65

Table 5: Turnover

Note: This table reports the monthly turnover for all sector, factor, and country portfolios. The abbreviation S denotes sectors, F denotes factors, and C denotes countries. The turnover is reported in absolute values, and can be interpreted as the average percentage of wealth traded each month.

*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

7.4 Omega ratio & Information ratio

7.4.1 Omega ratio

In Table 6 we present the omega ratio for all strategies and asset classes. The threshold for each dimension is the return of the equally weighted 1/N portfolio. As with most of the other performance measures in this analysis, the estimation length plays a major role in the results. The performance for the estimation length of 60 months is generally poor across all dimensions and strategies, with some exceptions when the weights are fully relaxed. Looking at the shorter estimation periods however, we find consistently across the strategies that sector and country portfolios are significantly better than factors in generating gains above the 1/N threshold. The Kelly Growth portfolio which is very different from the other strategies, displays the consistently strongest Omega ratio across all asset classes. For a strategy that generates extreme positions and can yield both returns and losses in triple digit percentages, we find it interesting that when all moments of returns are considered, this strategy outperforms the more conservative ones.

7.4.2 Information ratio

Table 7 presents the Information ratio for all strategies and dimensions. Relative to the MSCI Europe benchmark index, all portfolios exhibits positive performance for most configurations. Similar to the findings in other performance measures, factor portfolios dominates sector and country portfolios when short-selling is disallowed. The risk-based strategies displays more stable and consistent performance compared to other strategies, where factor portfolios dominates regardless of estimation length and weight constraints.
For the other strategies however, the same pattern found in Sharpe ratios are present here. Sector portfolios close the gap in performance as weights are relaxed, while countries outperform factors in the 100%-100% configuration.

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation length	S	F	C	S	F	C	S	F	C
1/N		1.00	1.00	1.00						
Risk Parity	12	1.05	1.03	1.06						
	24	1.02	0.98	1.01						
	60	0.89	0.82	0.86						
Minimum-Variance	12	1.03	1.03	1.05	1.05	1.02	1.09	1.22	1.04	1.10
	24	1.01	0.98	1.06	1.00	1.00	1.10	1.10	0.99	1.18
	60	0.90	0.82	0.84	0.98	0.88	1.13	0.97	0.86	1.18
Mean-Variance	12	1.18	1.18	1.33	1.15	1.24	1.36	1.21	1.31	1.41
	24	0.97	0.93	1.06	1.11	0.96	1.21	1.17	1.01	1.28
	60	0.60	0.05	0.55	1.03	0.24	1.09	1.12	0.49	1.26
Bayes-Stein	18*	1.05	0.95	-2.38	1.23	1.12	1.17	1.24	1.24	1.25
	24	0.81	0.79	0.70	1.07	0.87	1.20	1.17	0.94	1.35
	60	0.28	-0.05	0.26	0.86	0.31	1.09	0.89	0.48	1.28
Kelly Growth	12	1.18	1.18	1.35	1.09	1.13	1.30	1.18	1.30	1.32
	24	1.08	1.13	1.14	1.02	1.04	1.16	1.05	1.17	1.24
	60	0.80	0.73	0.69	0.79	0.72	0.76	0.98	0.79	0.97

Table 6: Omega ratio

Note: This table reports the Omega ratio for all sector, factor, and country portfolios for the full *out-of-sample* period for each estimation length. The 1/N portfolio for each dimensions is used as the threshold θ to define gains vs. losses, and $\Omega > 1$ means that the portfolio generated more gains relative to the 1/N portfolio. The abbreviation S denotes sectors, F denotes factors, and C denotes countries.

*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation length	S	F	C	S	F	C	S	F	C
1/N		0.15	0.57	0.09						
Risk Parity	12	0.07	0.51	0.14						
	24	0.09	0.51	0.13						
	60	0.10	0.46	0.04						
Minimum-Variance	12	0.01	0.33	0.04	0.01	0.16	0.04	0.07	0.12	0.04
	24	0.03	0.32	0.11	0.01	0.16	0.05	0.04	0.12	0.07
	60	0.06	0.28	0.02	0.06	0.19	0.11	0.05	0.13	0.11
Mean-Variance	12	0.06	0.12	0.12	0.04	0.09	0.11	0.07	0.12	0.14
	24	0.00	0.07	0.03	0.04	0.05	0.07	0.06	0.05	0.10
	60	-0.05	-0.08	-0.06	0.04	-0.04	0.06	0.06	-0.02	0.11
Bayes-Stein	18*	0.01	0.08	-0.05	0.05	0.08	0.03	0.06	0.10	0.06
	24	-0.02	0.05	-0.02	0.02	0.04	0.06	0.05	0.04	0.11
	60	-0.07	-0.07	-0.06	0.01	-0.02	0.06	0.00	0.00	0.11
Kelly Growth	12	0.08	0.14	0.14	0.03	0.06	0.09	0.06	0.11	0.11
	24	0.05	0.15	0.07	0.01	0.05	0.05	0.02	0.08	0.08
	60	-0.06	0.00	-0.09	-0.04	-0.03	-0.05	0.01	-0.02	0.01

Table 7: Information ratio

Note: This table reports the Information ratio for all sector, factor, and country portfolios for the full *out-of-sample* period for each estimation length. The MSCI Europe Index is used as the benchmark to calculate the active return and tracking error. $IR > 0$ means that the portfolio outperformed the benchmark. The abbreviation S denotes sectors, F denotes factors, and C denotes countries.
*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

7.5 Risk analysis

7.5.1 Skewness

Skewness is the measure of asymmetry of a distribution relative to the normal distribution, and is together with kurtosis the two most common higher moments to consider when analyzing portfolio returns. Table 14 in Appendix 13.3 reports the skewness of all portfolios considered. The majority of the portfolios are negatively skewed, suggesting that they experience frequent small gains and a few large losses. Among all portfolios considered, we find no consistent evidence that any dimension are superior to the others.

7.5.2 Kurtosis

Kurtosis describes the shape of a probability distribution, and measures how thin or fat the tails of the distribution are. Kurtosis is a relative measure to the normal distribution, and can give an indication on how often extreme values will occur. Table 15 in Appendix 13.4 reports the kurtosis of all portfolios considered. For long-only portfolios, sector portfolios seems to have a lower risk for black swan events. There is no consistent relationship between the dimensions for other configurations, but longer estimation lengths seems to generate returns with higher kurtosis. For conservative portfolios, most distributions seems to be platykurtic with thinner tails and thus less likely to experience extreme events. Strategies that estimate means exhibit evidence of leptokurtic distributions, implying a higher risk of extreme events.

7.5.3 Maximum Drawdown

Maximum drawdown (MDD) is a measure of downside risk that measures the fall from some peak value to the lowest through value in a return-series. The MDD is given by:

$$MDD = \frac{Trough\ value - Peak\ value}{Peak\ value} \tag{18}$$

Table 16 in Appendix 13.5 presents the maximum drawdown of all portfolios considered. The results show that country portfolios experience the highest drawdowns, while sector and factor portfolios are on similar levels. When short-selling is allowed, strategies that estimate means experience significantly higher drawdowns, while for long-only portfolios the results remain similar among the strategies.

7.5.4 Value-at-risk

Value-at-risk (VaR) measures the financial loss within a time period given a certain probability. Table 17 in Appendix 13.6 reports the VaR for all portfolios considered. The results do not display any evidence that any one of the investment dimensions show consistently lower or higher VaR relative to the other dimensions.

7.6 Blended portfolios

Table 8 displays the results from the sector- and country-blended portfolios. The portfolios are compared to their ”pure” dimension index, which is the average of the sectors, factors, and countries considered. Looking at the Sharpe ratios, we find similarities to the asset allocation strategies from before, where the estimation length yields significantly stronger results for the 24 and 12-month length. They are also significantly poorer than the asset allocation strategies in table 2. For the portfolio blending, sector-blending seems to have a slightly higher performance than both the country and combined portfolios. This is the same case for the signal blend method, but with the combined portfolio being somewhat better than the country-blending. Compared to pure sector and country indices, the blended portfolios with a 12-month estimation length is slightly better performing on most performance measures. None of the blended portfolios outperforms the pure factor index in terms of Sharpe ratio, nor any other performance measure.

The findings of Brière and Szafarz (2020) indicate that blended portfolios could enhance the performance of factor investing with lower trading costs and better diversification. From their empirical results, the sector-blended portfolios outperform their pure factor ETF counterpart on almost all performance measures. Using the equivalent European indices, we find that this is not the case for our analysis. The pure factor index dominates all of the blended portfolios in all performance measures.

Examining the portfolio turnover, we find that signal-blended portfolios are very similar to the pure indices. Given that both of these use a naive 1/N optimization, these findings are as expected. The portfolio-blended portfolios however, are not constrained in the same way. Assets with top exposure to several factors receive $\mathrm{w_i} = \frac{1}{N} * k$ weight, which means that these portfolios can have higher individual weights, and different number of assets each period. Hence, this strategy is more trading intensive, which is reflected in the turnover.

Looking at the Omega- and Information ratio, we find similar results relative to the other performance measures. All portfolio combinations for the 12-month estimation length deliver a stronger Omega ratio than both pure sector and country indices. For the Information ratio however, pure indices yields significantly stronger performance. Investigating this further, we find that the reason for this is the very low tracking error these portfolios have compared to the MSCI Europe Index benchmark.

Relative to the pure sector and country indices, the 12-month estimation length portfolios performs slightly better in terms of Sharpe ratio. As the portfolios are created using a naive optimization, the blending seems to have partially captured some of the risk-premia associated with factor investing. The sector-blended portfolios has harvested this better than the country-blended ones, which is likely due to diversification. Country indices contains securities that overlap different sectors while sectors, by construction, will only contain securities to that specific sector and therefore offers better diversification opportunities. Sectors also captures the majority of the investment universe, while countries are only made up of the companies specific to that area.

Strategy	Estimation length	Sharpe ratio	CEQ	Turnover	Omega ratio	Information ratio
PB - Combined	12	0.28	-0.36%	0.499	0.97	0.04
	24	0.21	-0.48%	0.275	0.91	0.01
	60	0.05	-0.86%	0.140	0.77	-0.05
PB - Sector	12	0.32	-0.20%	0.291	0.98	0.10
	24	0.28	-0.26%	0.175	0.95	0.11
	60	0.14	-0.56%	0.094	0.82	0.09
PB - Country	12	0.29	-0.37%	0.403	0.98	0.05
	24	0.22	-0.51%	0.230	0.93	0.02
	60	0.02	-0.98%	0.125	0.75	-0.08
SB - Combined	12	0.31	-0.31%	0.022	1.00	0.09
	24	0.23	-0.47%	0.021	0.93	0.03
	60	0.10	-0.80%	0.021	0.82	0.02
SB - Sector	12	0.35	-0.19%	0.022	1.02	0.12
	24	0.20	-0.54%	0.019	0.91	-0.01
	60	0.16	-0.69%	0.019	0.88	0.10
SB - Country	12	0.28	-0.41%	0.022	0.98	0.04
	24	0.20	-0.52%	0.022	0.91	0.00
	60	0.08	-0.82%	0.022	0.80	0.00
Pure Sector Index		0.27	-0.26%	0.023	0.93	0.15
Pure Factor Index		0.52	0.12%	0.009	1.14	0.57
Pure Country Index		0.25	-0.40%	0.023	0.94	0.09

Table 8: Blended portfolios

Note: This table reports the annualized Sharpe ratio, the certainty equivalent return (CEQ), portfolio turnover, Omega ratio, and Information ratio for the portfolio- and signal-blended portfolios. The first column denotes the blending strategy, where portfolio blending is denoted as PB and signal blending as SB. We use the average of the 1/N returns across the dimensions as the threshold for the Omega ratio, and the MSCI Europe Index remains the benchmark for the Information ratio.

8 Robustness check

In this section, we investigate the robustness of our results by dividing the full sample into sub-periods based on macroeconomic business cycles. We compute the annualized Sharpe ratios for each dimension using the same strategies as before with portfolios that are short-sale restricted, and those who allow for short-selling.

8.1 Time-varying performance

After analyzing the full period of returns, we now investigate the performance in different sub-periods. The data is split into four different sub-periods, which includes two periods of economic expansion and two periods of economic recession. As an indicator for macroeconomic cycles, we follow the US National Bureau of Economic Researchs (NBER) reference dates for US Business Cycle Expansions and Contractions. Based on this, the full sample is divided into four sub-periods. The first sub-period from January 2002 to April 2007 features the period of strong economic growth in especially housing prices in the US. The second sub-period from May 2007 to July 2009 features the global financial crisis. The third sub-period from August 2009 to December 2020 contains the recovery of the global economy following the financial crisis, and the last sub-period from March 2020 to August 2020 contains the recession caused by the outbreak of Covid-19.

8.1.1 Long-only portfolios

In panel A of table 9, we present the annualized Sharpe ratios for each strategy and dimension in the four sub-periods using a 12-month estimation length and the long-only weight constraint. The results from the first sub-period shows that factors clearly outperformed sectors and countries. Performance is similar across portfolio strategies, and the strong positive Sharpe ratio tells us that this sub-period is the main driver for the overall performance in the full sample.

During the financial crisis, all dimensions yield similar results. From the results of Brière and Szafarz (2018), where the diversification benefits of sector investing produced better performance during crisis periods than factors for long-only portfolios, we find that this is not the case for our results. The lower

correlation between sectors does not provide any substantial protection against crisis periods as previous literature has suggested, at least for the European market.
The third sub-period that features the expansion following the financial crisis produce more mixed results. The naive and risk-based strategies exhibits Sharpe ratios very similar to the full data sample, while risk-return strategies perform better for country investing.
The final sub-period in our analysis covers the Covid-19 recession of 2020. The results show that across the dimensions, sector investing was the biggest loser during this period. The Financial and Industrial sector were the main drivers of these losses, while Healthcare and Consumer Staples were barely effected. From the results of the financial crisis in sub-period two, sectors does yet again fail to provide any protection against crisis periods that has been found in previous literature. Looking at the different portfolio strategies, we find some more interesting results. First of all, the risk-based strategies that have displayed very similar levels of performance during previous analyses in this thesis, now differ strongly. Optimizing your portfolio using the Minimum-Variance approach and investing in European country indices would have yielded an annualized Sharpe ratio of 1.15 during the Covid-19 recession, while investing in sector indices would have yielded a Sharpe ratio of nearly zero. These results are also similar for the Mean-Variance approach, where the Sharpe ratio for sectors is as bad as -0.41. This pattern is consistent for almost every strategy considered, and suggests that the diversification benefits of sectors is limited in the European index market.

8.1.2 Portfolios with short-selling

Panel B of table 9 displays the results of portfolios that allow for up to 50% weight both long and short. Similarly to the case of long-only portfolios, factor investing dominates sectors and countries in the first sub-period. However, we observe that the performance is substantially lower when short-selling is allowed. With all factor indices being near perfectly correlated, short-selling does not seem to provide any benefits during periods of strong economic growth.
During the financial crisis, the results differ strongly from the long-only case. The Minimum-Variance strategy enhances the losses from the long-only case, while the risk-return strategies exhibits a positive performance during the crisis period. Allowing for short-selling has increased the performance of these strategies significantly, where all of the portfolios regardless of dimension yield positive Sharpe ratios. This also reveals some performance differences between the dimensions, where sector and country portfolios now deliver better perfor-

mance than factor portfolios.

For the sub-period after the financial crisis, the results are quite similar to the long-only portfolios, with a slight increase in performance across the strategies and dimensions. Moving on to the final sub-period, we find some extreme results for the risk-return strategies. Given the short span of the period, we disregard some of the magnitude of these results and rather consider the relative relation between them. In line with the findings of previous literature (Briere & Szafarz, 2017; Bessler et al., 2021a), sector investing now outperforms factor investing during a crisis period. This also applies to country investing, that has performed even better during both recessions covered in this analysis. We can therefore conclude that when short-selling is allowed in recessions, both sector and country portfolios provide stronger performance than factor portfolios.

		NBER Sub-periods			
Panel A: Long-only					
Period		01/2002-04/2007	05/2007-07/2009	08/2009-12/2020	03/2020-07/2020
Strategy	Dimension	Expansion	Recession	Expansion	Recession
1/N	Sector	1.49	-0.63	0.28	0.18
	Factor	2.20	-0.65	0.52	0.47
	Country	1.72	-0.77	0.26	0.27
Risk Parity	Sector	1.70	-0.69	0.30	0.27
	Factor	2.22	-0.68	0.54	0.60
	Country	1.87	-0.75	0.28	0.45
Minimum-Variance	Sector	1.76	-0.70	0.26	0.02
	Factor	2.25	-0.76	0.61	0.83
	Country	1.96	-0.89	0.29	1.15
Mean-Variance	Sector	1.45	-0.13	0.07	-0.41
	Factor	2.19	-0.34	0.20	0.16
	Country	1.56	-0.50	0.22	1.30
Bayes-Stein*	Sector	1.65	-0.08	0.20	-0.69
	Factor	2.09	-0.57	0.23	1.68
	Country	1.79	-0.72	0.22	1.87
Kelly Growth	Sector	1.62	-0.48	-0.04	0.27
	Factor	2.38	-0.47	0.09	-0.63
	Country	1.82	-0.50	0.07	0.56
Panel B: 50%-50% Long-short					
Minimum-Variance	Sector	1.86	-0.83	0.36	0.68
	Factor	2.16	-0.98	0.72	0.28
	Country	1.48	-1.01	0.28	0.90
Mean-Variance	Sector	0.89	0.36	0.23	7.11
	Factor	1.82	0.45	0.08	1.01
	Country	0.93	0.66	0.34	3.93
Bayes-Stein*	Sector	1.47	0.50	0.39	2.31
	Factor	1.84	0.14	0.31	4.28
	Country	0.96	0.54	0.28	2.58
Kelly Growth	Sector	1.02	0.32	-0.25	1.18
	Factor	1.72	0.40	-0.23	-0.86
	Country	1.36	0.61	-0.08	1.14

Table 9: Sharpe ratio sub-period analysis

Note: This table reports the annualized Sharpe ratios for all sector, factor, and country portfolios in the four sub-periods considered, using the 12-month sample estimation length. Panel A displays the results for the long-only portfolios with a weight constraint of 0-35, and Panel B displays the results for the portfolios with a 50-50 weight constraint.
*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

9 Multi-factor regression

9.1 Regression results

To investigate what drives the returns of the different portfolios, we run a multi-factor regression using the Fama & French 5-factor model (Fama & French, 1993), the momentum factor (Carhart, 1997), and the betting-against-beta factor (Frazzini & Pedersen, 2014). Table 10 presents the results for all portfolios using the 12-month estimation length. Among all strategies and dimensions, we find that almost none of the portfolios produce positive alphas. In addition, almost none of the alphas are statistically significant, implying that almost all the variation in returns can be attributed to the known risk-factors. These findings are as expected, as the indices used in this analysis are passive funds that do not focus on generating abnormal returns.

All portfolios and asset classes loads significantly on the market factor, implying the market to be the main driver of returns. This is not surprising given that we use indices of the main sectors, countries, and factors in Europe, which is likely to contain overlapping securities with the market. For the size (SMB) and value (HML) factor, the loadings are in general low to negative across the strategies. Variations between dimensions for these factors are dependent on the type of strategy. The Minimum-Variance strategy has negative and statistically significant loadings for both, while the risk-return strategies has no clear pattern. The profitability (RMW) and investment (CMA) factors exhibits no clear pattern among the strategies and dimensions, and are not generally significant except for the Mean-Variance strategy.
Examining the results of the momentum (MOM) loadings, we find large and significant loadings among all dimensions and strategies. Not surprisingly, the strategies that account for means in their estimation loads significantly higher than the Minimum-Variance strategy. As means are one of the two moments that derive these portfolios, positive recent performance determines part of their existence in the portfolio and will therefore align well with momentum. The loading increases as weights are relaxed, and the relation between dimen-

sions remains similar across this, with sectors and countries loading slightly more than factors. The betting-against-beta (BAB) factor seems to be the least impactful factor in the regression, with low and insignificant loadings for all strategies and dimensions.

To conclude, we find that most of the variation in returns of the portfolios considered can be explained by the market (Mkt-RF) and the momentum (MOM) factor.

Strategy	Weight	Dimension	Alpha	Mkt-RF	SMB	HML	RMW	CMA	MOM	BAB
MinVar	0-35	Sector	-0.003***	0.853***	-0.324***	-0.187**	0.252**	0.365***	0.030	0.121***
		Factor	-0.000	0.910***	-0.119***	-0.096***	0.073	-0.001	0.085***	0.076***
		Country	-0.004***	0.960***	-0.009	-0.137*	0.022	-0.029	0.079**	0.069*
	50-50	Sector	-0.001	0.757***	-0.314**	-0.329**	0.456**	0.333	-0.047	0.153*
		Factor	-0.001	0.874***	-0.247***	-0.361***	0.061	0.089	0.198***	0.183***
		Country	-0.001*	0.829***	-0.203	-0.088	0.172	-0.163	0.341***	0.136
	100-100	Sector	0.004	0.735***	-0.388*	-0.576**	0.434	0.540*	-0.035	0.092
		Factor	-0.000	0.776***	-0.415***	-0.517***	-0.050	0.230*	0.223***	0.257***
		Country	-0.005	0.862***	-0.246	-0.311	0.032	-0.138	0.133	0.265*
MV	0-35	Sector	-0.004	0.920***	-0.020	0.139	0.274	0.771***	0.701***	-0.094
		Factor	-0.001	0.745***	-0.037	0.285*	0.273	0.493**	0.605***	-0.035
		Country	-0.002	0.855***	0.253	0.123	0.038	0.627**	0.654***	-0.127
	50-50	Sector	-0.011**	0.651***	0.550	-0.078	-0.377	-0.060	1.775***	-0.063
		Factor	-0.007*	0.664***	0.045	0.513*	0.408	0.846**	1.103***	0.0162
		Country	-0.001	0.491***	0.123	-0.901**	-1.365**	0.492	2.074***	0.045
	100-100	Sector	-0.011	0.974***	-0.101	-0.590	-0.726	-0.484	2.769***	-0.214
		Factor	-0.006	0.709***	0.230	0.149	0.515	1.260***	1.217***	0.096
		Country	0.009	0.536**	-0.165	-2.032***	-1.996**	1.673**	3.142***	-0.008
BS*	0-35	Sector	-0.001	0.248***	0.030	-0.063	0.034	0.233**	0.141***	-0.034
		Factor	0.000	0.344***	0.038	-0.091	-0.038	0.261*	0.188***	-0.035
		Country	0.000	0.061***	0.041**	-0.025	-0.021	0.008	0.034***	0.000
	50-50	Sector	-0.002	0.226***	0.197	-0.424*	-0.427	0.012	0.986***	-0.035
		Factor	-0.002	0.460***	0.027	-0.241	-0.043	0.190	0.366***	0.195**
		Country	0.001	0.080	-0.076	-0.254	-0.130	0.136	0.421***	0.103
	100-100	Sector	-0.001	0.240**	0.313	-0.661*	-0.914**	-0.284	1.405***	0.005
		Factor	-0.001	0.532***	0.010	-0.592**	-0.069	0.286	0.365***	0.260**
		Country	0.000	-0.008	-0.202	-0.111	-0.083	0.081	0.537***	0.278*
KG	0-35	Sector	-0.016***	1.794***	-0.338*	0.715***	0.489	0.348	1.140***	0.041
		Factor	-0.009***	1.373***	-0.283**	0.574***	0.390*	0.190	0.770***	0.229**
		Country	-0.014**	2.340***	0.135	0.920**	0.082	0.674	1.729***	-0.024
	50-50	Sector	-0.039***	1.312***	-0.108	1.888***	0.972	0.173	3.032***	0.440*
		Factor	-0.022***	1.451***	-0.515*	1.300***	0.745	0.769	1.820***	0.669***
		Country	-0.034***	1.296***	0.258	1.198	-0.603	1.282	4.535***	0.601
	100-100	Sector	-0.050***	1.939***	0.203	1.700*	-0.086	0.443	5.187***	0.422
		Factor	-0.033***	2.345***	-0.362	2.068***	1.137	1.705*	3.329***	0.908**
		Country	-0.043**	2.142***	0.327	0.273	-2.618	0.996	6.990***	0.749

Table 10: Multi-factor regression

Note: This table reports the results from the factor regressions of the monthly excess returns of the sector, factor, and country portfolios using a 12-month sample estimation window. "Alpha" is the abnormal return, "Mkt-RF" is the market risk premium, "SMB" is the size factor, "HML" is the value factor, "RMW" is the profitability factor, "CMA" is the investment factor, "MOM" is the momentum factor, and "BAB" is the betting-against-beta factor. *, **, *** indicates the significance level at the 10%, 5% and 1% level, respectively. *: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

We use robust standard errors in the analysis to deal with the issue of heteroscedasticity. The regressions was also checked for multicollinearity and non-normality, and we find that both assumptions are violated. However, in our case there are no suitable solutions to deal with multicollinearity. Therefore, the multicollinearity issue is ignored with the understanding that this will make the coefficient estimates less reliable and might be overfitted, and the model estimates might be highly sensitive to minor changes in the data. Furthermore, non-normality will not affect our coefficient results to a large degree as we use a large sample size.

10 Discussion

10.1 Main results

From the main analysis of this thesis, we have covered the the performance of European sector, factor, and country indices using several optimization strategies, estimation lengths, and weight constraints. Our hypothesis stated that factor investing would yield the overall best performance among the dimensions. We find that with a short estimation period of 12 months and the most relaxed weight configuration (max 100% long/short), both sector and country portfolios outperform factors under several optimization strategies. Contrary to several previous findings regarding the diminishing effect of country investing due to e.g. increased integration of capital markets across borders (Bessler et al., 2021b), our results show that country investing consistently outperforms sector investing under most configurations and constraints, and outperforms factor investing in certain cases.
The performance of sectors has also given some interesting results. In the subperiod analysis from table 9, sectors did not provide any protection against the financial crisis of 2008 compared to factors. Unlike countries, sectors are more susceptible to spill-over effects within industries. The rise of new industries and technology creates rapid growth that also affects other industries, while countries only benefit partially from this effect. This also has implications for crisis periods, where industry shocks does not seem to spill over substantially to countries.

Diving deeper into the portfolio optimization models in this thesis, our results display mixed results across performance measures. In terms of Sharpe ratio, the performance is relatively similar across configurations. Interestingly, all of the long-only portfolios in the 12-month estimation length outperforms the naive 1/N strategy in every dimension. These findings dispute the results from DeMiguel et. al (2009), which argues that no strategy is consistently better than 1/N. Given that our analysis covers three different types of optimizations (risk minimizing, risk-return, return maximizing) that all produce better Sharpe ratios, the argument that no portfolio is consistently better than 1/N might lack some robustness for European indices.

10.2 Blending

The sector- and country-blended portfolios analyzed in this thesis is an extension of the work of Ghayur et. al (2018) and Brière and Szafarz (2020), where we introduce country-blending and joint blending of sectors and countries. The findings in table 8 tells us that these extension does not provide any improvements to their research on blending. Comparing our results for sector-blending with previous research, we find inconsistencies in the performance of European and US ETFs. We find several reasons why this might be the case. Firstly, European factor ETFs dominates sector and country ETFs. This is not the case for US ETFs, where the performance between sectors and factors is quite similar (Brière & Szafarz, 2020; Bessler et al., 2021a; Bessler et al., 2021b). Previous research has only reported the performance between sector-blended portfolios and pure factor ETFs, not the performance compared to sector ETFs. Without this relative relationship, we are unable to infer whether the magnitude of our findings on sector-blended performance has any impact. For sector-blending to thrive as an investment style, we find it fundamental that it needs to outperform pure sector ETFs, or investors would simply be better off investing in these.

Secondly, the factor exposure matching in our analysis covers five factors, while previous research covers only value and momentum (Ghayur et al., 2018; Briére & Szafarz, 2020). Much like Modern Portfolio Theory where increasing the number of assets reduces volatility, the inclusion of more factors when determining composite Z-scores from exposure matching might dilute the "important" factors, making the composition of assets in the portfolio very different from matching with two factors. The combination of value and momentum is known to generate comprehensive return premia (Asness et al., 2013), whereas other combinations of factor relationships might not represent the best way to incorporate blending. Whether blending is an anomaly specific to value and momentum, or if there is rationale for other factors to better represent exposure matching remains a question for further research.

Lastly, all previously research on blending is performed using a naive 1/N weighting of the assets with relevant exposure. Applying mean-variance frameworks to the method of blending seems difficult due to estimation and restrictions of the covariance matrix as assets are continuously included and excluded from the portfolio. However, other estimation techniques as utility-based allocation or methods that does not rely on the common moments of portfolio theory might have some rationale for further research. In addition, the naive optimization does not account for the magnitude of the factor exposures and treats them equally. Constructing a weighted average optimization of the top factor exposures to investigate whether the magnitude of the exposures are relevant is also a question that deserves further investigation.

11 Conclusion

In this thesis, we investigate and compare the performance of European sector, factor, and country indices through several asset allocation strategies and configurations. We extend the work of Bessler et. al (2021a; 2021b) by comparing the dimensions jointly for the European market. Then, we extend the work of Ghayur et al. (2018) and Brière and Szafarz (2020) by adding countries to the method of portfolio and signal blending, as well as considering both sectors and countries jointly. We use indices that works as proxies for ETFs to make our results feasible in the real world.

In our investment period from November 2002 to December 2021, we find that factor portfolios dominates sector and country portfolios when short-selling is not allowed. Consistently among the optimization strategies considered, factor portfolios provide Sharpe ratios that are statistically significantly different from sector and country portfolios. In our analysis of sub-periods for long-only portfolios, factor portfolios yields stable and consistent performance both in expansions and recessions. Factor portfolios provide the strongest CEQ, has the lowest amount of turnover among the investment dimensions and portfolio strategies, and the best Omega and Information ratio. In our risk analysis of higher moments and tail-risk, we find that factors does not exhibit higher levels of risk compared to sectors and countries, implying that its strong performance can not be attributed to increased levels of risk.

The dominance of factor portfolios diminishes when short-selling is allowed, where we find that country portfolios provides slightly stronger performance. Country portfolios provide statistically significant Sharpe ratios that for some strategies are also statistically different from factor portfolios. Country portfolios achieve stronger Omega ratios and similar Information ratios to factor portfolios, but has a significantly higher turnover. Our null hypothesis was that factor portfolios would yield the best out-of-sample performance. We find that this is only true when short-selling is disallowed. When weights are relaxed and short-selling is allowed, country portfolios provide the overall best performance.

From the results of the portfolio and signal blending methods, we find that our extension of including country-blending and joint blending does not provide improvements compared to the previously known sector-blending. Compared to their pure index counterpart, both sector- and country-blending provides higher Sharpe ratios. Our second null hypothesis was that a portfolio combining the dimensions into blended portfolios would outperform any single one dimensions. We find that some blended portfolios enhance the performance of their pure index counterpart, but factor indices still remain the overall best performer. Building on the research of Brière & Szafars (2020) that only reports the performance of blended portfolios to a pure factor ETF, we cannot infer whether the blended portfolios in our thesis captures the risk premia associated with factors while maintaining the diversification benefits of sectors, as the US counterparts of these ETFs are different in performance.

From our analysis of the multi-factor regression, we find that almost none of the optimization strategies and dimensions provide statistically significant positive multi-factor alphas. Therefore, we conclude that almost all of the variation in returns is attributable to known risk-factors, namely the Fama & French 5-factor model augmented by the momentum factor, and the betting-against-beta factor. For all portfolios, we find that the market factor and the momentum factor are the main drivers of return variation. This is expected, as our investment universe of indices are likely to cover the main constituents of the European market portfolio, and that portfolios that consider mean estimates in their construction align well with momentum.

Comparing the performance of sector and country portfolios, our results show inconsistencies compared to Bessler et al. (2021b). From their results, sector portfolios yielded the best performance in periods of expansion while country portfolios performed better during recessions when including transactions costs. We find that country portfolios are superior to sectors regardless of business cycles, with several statistically different Sharpe ratios. The performance and correlation structure of sector and country data used in Bessler et al. (2021b) is similar to the data in this thesis, and implies that the performance between European sector and country indices is different from the largest developed economies.

Our thesis provides new empirical evidence on the relative performance of different asset allocation strategies in sector, factor, and country indices. For long-only portfolios, factor investing consistently outperforms sector and country portfolios. When short-selling is allowed, country portfolios tighten the gap in performance and outperform factor portfolios in several cases. The results of blending in this thesis provides empirical evidence of the performance compared to all pure index counterparts, and will hopefully provide valuable insights for further research.

12 Bibliography

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13 Appendix

13.1 Overview of data

Sector	Bloomberg Name
Energy	MSCI Europe Energy Sector Index
Materials	MSCI Europe Materials Sector Index
Industrial	MSCI Europe Industrials Index
Consumer Discretionary	MSCI Europe Consumer Discretionary Index
Consumer Staples	MSCI Europe Consumer Staples Index
Healthcare	MSCI Europe Health Care Index
Financials	MSCI Europe Financials Index
Information Tech	MSCI Europe Information Technology Index
Telecom	MSCI Europe Telecom Service Industry Group Index
Utilites	MSCI Europe Utilities Sector Index

Table 11: Summary of sector data

Factor	Bloomberg Name
Size	MSCI MID CAP EQUAL WEIGHTED Net EUR Index
Value	MSCI EUROPE ENHANCED VALUE Net EUR Index
Quality	MSCI EUROPE SECTOR NEUTRAL QUALITY Net EUR Index
Momentum	MSCI EUROPE MOMENTUM Net EUR Index
Volatility	MSCI EUROPE VOLATILITY Optimized in Euro NETR Euro
Multifactor	MSCI EUROPE DIVERSIFIED MULTI-FACTOR EUR NETR

Table 12: Summary of factor data

Country	Bloomberg Name
Finland	MSCI Finland Index
Norway	MSCI Norway Index
Germany	MSCI Germany Index
Denmark	MSCI Denmark Index
Spain	MSCI Spain Index
France	MSCI France Index
Switzerland	MSCI Switzerland Index
Sweden	MSCI Sweden index
Italy	MSCI Italy Index
Netherlands	MSCI Netherlands Index
Austria	MSCI Austria Index
Ireland	MSCI Ireland Index
Belgium	MSCI Belgium Index
United Kingdom	MSCI United Kingdom Index
Portugal	MSCI Portugal Index

Table 13: Summary of country data

13.2 Significance test of Sharpe ratios

Opdyke presents both a single-sample test and a two-sample test. The variables for the single-sample test are as follows:

$$\sqrt{T}(\widehat{SR} - SR)^a \sim N\left(0, 1 + \frac{SR^2}{4}\left[\frac{\mu_4}{\sigma^4} - 1\right] - SR\frac{\mu_3}{\sigma^3}\right) \tag{19}$$

$$SE(\widehat{SR}) = \sqrt{\left[1 + \frac{SR^2}{4}\left(\frac{\mu_4}{\sigma^4} - 1\right) - SR\frac{\mu_3}{\sigma^3}\Big/ T\right]} \tag{20}$$

$$\widehat{SE(\widehat{SR})} = \sqrt{\left[1 + \frac{S\hat{R}^2}{4}\left(\frac{\hat{\mu}_4}{\hat{\sigma^4}} - 1\right) - S\hat{R}\frac{\hat{\mu}_3}{\hat{\sigma^3}}\Big/ (T-1\right]} \tag{21}$$

$$\widehat{SR} \pm z_{crit}\ x\ \widehat{SE(\widehat{SR})} \tag{22}$$

Equation (15), (16), (17) and (18) denotes the distribution of $\hat{SR}$, the standard error of $\widehat{SR}$, the estimated standard error and the confidence bounds of $\widehat{SR}$, respectively.

The variables used in the two-sample statistic are:

$$H0: \ SR_a \leq SR_b \ versus \ HA: \ SR_a > SR_b \tag{23}$$

$$\hat{SR}_{diff} = (\widehat{SR}_a - \widehat{SR}_b) - (SR_a - SR_b) \tag{24}$$

$$Var(\widehat{SR}_{diff}) = Var(\widehat{SR}_a + Var(\widehat{SR})) - 2Cov(\widehat{SR}_a, \widehat{SR}_b) \tag{25}$$

$$\sqrt{T}(\widehat{SR}_{diff})^a \sim N(0, Var_{diff}), \ where \tag{26}$$

$$\begin{aligned} Var_{diff} = 1 + \frac{SR_a^2}{4}\left[\frac{\mu_{4a}}{\sigma_a^4} - 1\right] - SR_a\frac{\mu_{3a}}{\sigma_a^3} + 1 + \frac{SR_b^2}{4}\left[\frac{\mu_{4b}}{\sigma_b^4} - 1\right] \\ -SR_b\frac{\mu_{3b}}{\sigma_b^3} - 2\left[\rho_{a,b} + \frac{SR_aSR_b}{4}\left[\frac{\mu_{2a,2b}}{\sigma_a^2\sigma_b^2}\right] - \frac{1}{2}SR_a\frac{\mu_{1b,2a}}{\sigma_b\sigma_a^2} - \frac{1}{2}SR_b\frac{\mu_{1a,2b}}{\sigma_a\sigma_b^2}\right] \end{aligned} \tag{27}$$

The joint second central moment of the joint distribution of a and b is

$$\begin{aligned} \mu_{2a,2b} &= E\left[(a - E(a))^)(b - E(b))^2)\right], \\ \mu_{1a,2b} &= E\left[(a - E(a))(b - E(b))^2)\right], \\ \mu_{1b,2a} &= E\left[(b - E(b))(a - E(a))^2)\right] \end{aligned} \tag{28}$$

13.3 Skewness

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation window	S	F	C	S	F	C	S	F	C
1/N		-0.43	-0.61	-0.67						
Risk parity	12	-0.42	-0.61	-0.58						
	24	-0.46	-0.67	-0.66						
	60	-0.45	-0.58	-0.59						
Minimum-Variance	12	-0.32	-0.72	-0.59	-0.24	-0.79	-0.22	-0.53	-0.69	-0.11
	24	-0.43	-0.75	-0.66	-0.07	-0.91	-0.18	-0.16	-0.72	-0.24
	60	-0.34	-0.69	-0.58	-0.28	-0.87	-0.10	-0.31	-0.74	-0.13
Mean-Variance	12	-0.24	0.13	-0.02	0.16	0.78	0.25	-0.18	0.38	0.29
	24	-0.17	-0.08	-0.45	-0.33	-0.02	-0.26	-0.41	-0.14	-0.31
	60	-2.35	-2.46	-2.92	-0.77	-2.26	-0.65	-0.79	-1.79	-0.45
Bayes-Stein	18*	0.32	0.72	2.38	-0.27	1.03	-0.25	-0.53	1.13	-0.22
	24	-0.40	-0.04	0.16	-0.37	-0.07	-0.14	-0.69	-0.10	0.17
	60	-2.65	-2.43	-2.61	-1.01	-2.09	-0.41	-1.04	-1.46	-0.43
Kelly Growth	12	-0.60	-0.32	-0.27	0.47	0.24	1.20	0.74	0.42	0.83
	24	-0.35	-0.33	-0.49	-0.42	-0.15	-0.25	-0.46	-0.23	-0.05
	60	-1.55	-1.56	-2.65	-1.58	-2.18	-2.47	-1.55	-2.28	-1.84

Table 14: Skewness

Note: This table reports the skewness in returns for all sector, factor, and country portfolios for the full *out-of-sample* period for each estimation length. The abbreviation S denotes sectors, F denotes factors, and C denotes countries.

*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

13.4 Kurtosis

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation window	S	F	C	S	F	C	S	F	C
1/N		1.53	1.75	2.12						
Risk parity	12	1.14	1.71	1.65						
	24	1.09	1.77	1.79						
	60	1.09	1.51	1.63						
Minimum-Variance	12	0.64	1.86	1.14	0.45	2.46	0.12	1.32	1.80	0.32
	24	0.53	1.94	1.25	0.13	3.22	0.25	0.59	1.92	0.41
	60	0.49	1.72	1.09	-0.04	2.82	0.63	0.08	1.76	0.28
Mean-Variance	12	1.74	1.81	3.04	2.76	4.50	1.57	1.51	2.06	0.97
	24	1.63	2.69	5.35	0.48	2.06	1.43	0.53	2.94	1.11
	60	11.25	11.42	15.81	1.74	9.84	3.55	1.36	6.82	2.26
Bayes-Stein	18*	4.28	3.14	16.19	1.26	4.77	2.49	1.72	5.33	3.08
	24	3.77	3.18	5.55	1.03	2.46	1.41	2.34	2.13	1.81
	60	13.32	11.09	15.91	2.35	8.27	2.79	2.20	5.26	2.07
Kelly Growth	12	2.76	1.44	2.40	5.99	3.51	10.40	7.81	5.14	6.77
	24	1.25	0.71	2.37	2.91	2.53	2.66	1.72	2.77	2.27
	60	6.07	6.27	12.09	6.48	11.18	11.89	6.72	10.52	10.21

Table 15: Kurtosis

Note: This table reports the kurtosis in returns for all sector, factor, and country portfolios for the full *out-of-sample* period for each estimation length. The abbreviation S denotes sectors, F denotes factors, and C denotes countries.
*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

13.5 Maximum Drawdown

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation window	S	F	C	S	F	C	S	F	C
1/N		-57.68	-58.06	-65.00						
Risk parity	12	-52.44	-57.12	-66.68						
	24	-52.97	-57.13	-67.45						
	60	-54.25	-57.26	-64.28						
Minimum-Variance	12	-47.75	-55.95	-63.30	-47.83	-54.74	-57.55	-43.11	-51.09	-59.14
	24	-47.29	-55.86	-61.02	-42.95	-54.46	-58.52	-30.38	-46.97	-50.63
	60	-44.97	-56.31	-59.27	-42.01	-54.94	-53.45	-40.85	-49.30	-53.64
Mean-Variance	12	-40.36	-38.85	-57.01	-72.06	-59.58	-72.98	-84.08	-60.63	-89.25
	24	-54.94	-42.68	-61.18	-57.91	-61.75	-58.25	-73.92	-73.14	-86.00
	60	-63.87	-69.06	-69.74	-58.11	-74.44	-41.81	-70.37	-81.17	-50.19
Bayes-Stein	18*	-19.34	-26.87	-5.03	-29.57	-29,15	-42.17	-48.25	-31.34	-63.29
	24	-34.30	-34.95	-24.66	-33.61	-48.50	-52.38	-45.81	-53.98	-74.92
	60	-55.04	-66.07	-56.26	-57.60	-69.95	-32.25	-68.52	-70.57	-42.81
Kelly Growth	12	-80.75	-53.65	-90.69	-99.15	-94.86	-100.00	-101.69	-99.75	-118.35
	24	-79.73	-62.48	-96.46	-97.51	-88.55	-99.76	-99.97	-99.46	-140.76
	60	-90.58	-83.08	-98.87	-94.07	-93.95	-99.37	-99.69	-99.83	-115.94

Table 16: Maximum drawdown %

Note: This table reports the maximum drawdown for all sector, factor, and country portfolios for the full *out-of-sample* period and each estimation length. The abbreviation S denotes sectors, F denotes factors, and C denotes countries.
*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

13.6 Value-at-risk

		Weight constraints								
		0% - 35%			-50% - 50%			-100% - 100%		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Strategy	Estimation window	S	F	C	S	F	C	S	F	C
1/N		-9.04	-8.86	-9.82						
Risk parity	12	-7.61	-8.04	-8.75						
	24	-8.11	-8.31	-9.16						
	60	-8.70	-8.72	-10.06						
Minimum-Variance	12	-7.00	-7.64	-7.82	-7.39	-7.05	-8.66	-8.50	-6.10	-11.09
	24	-6.94	-7.67	-8.44	-6.91	-6.91	-7.74	-6.87	-6.19	-7.44
	60	-7.00	-8.49	-7.92	-7.10	-7.35	-6.93	-7.20	-6.62	-7.48
Mean-Variance	12	-9.72	-6.90	-9.68	-13.55	-11.13	-16.51	-21.06	-14.20	-26.15
	24	-8.02	-6.45	-9.85	-11.78	-7.56	-14.37	-17.69	-10.81	-21.29
	60	-5.37	-5.57	-5.29	-11.53	-7.42	-8.56	-19.33	-8.73	-13.44
Bayes-Stein	18*	-3.82	-4.96	-0.84	-7.05	-5.67	-5.48	-12.12	-7.35	-8.36
	24	-4.83	-5.21	-3.23	-8.41	-7.57	-8.28	-11.59	-8.00	-10.84
	60	-4.19	-5.25	-4.02	-9.36	-7.07	-6.89	-15.77	-8.90	-11.09
Kelly Growth	12	-13.08	-9.18	-19.06	-24.55	-16.76	-37.04	-39.82	-27.90	-61.76
	24	-12.85	-9.25	-19.96	-21.97	-16.22	-31.10	-35.38	-26.49	-47.62
	60	-13.65	-12.00	-14.12	-13.56	-13.34	-16.08	-24.14	-24.60	-23.05

Table 17: Value-at-risk (95% confidence interval) - Monthly

Note: This table reports the average monthly Value-at-risk at the 95% confidence interval for all sector, factor, and country portfolios for each estimation length. The abbreviation S denotes sectors, F denotes factors, and C denotes countries.

*: 12 month estimation length resulted in a singular covariance matrix after the adjustments from equation 12, where the inverse does not exist.

13.7 List of abbreviations

FI = Finland
NO = Norway
DE = Germany
DK = Denmark
ES = Spain
FR = France
CH = Switzerland
SE = Sweden
IT = Italy
NL = Netherlands
AT = Austria
IE = Ireland
BE = Belgium
UK = United Kingdom
PT = Portugal
MDD = Maximum drawdown
EU = European Union
MPT = Modern portfolio theory
MSCI = Morgan Stanley Capital International
SMB = Small minus big
HML = High minus low
RMW = Robust minus weak
CMA = Conservative minus aggressive
WML = Winners minus losers
BAB = Betting against beta
CAPM = Capital asset pricing model
SR = Sharpe ratio
CEQ = Certainty-equivalent return
IR = Information ratio
ETF = Exchange traded fund

13.8 Summary statistics

Panel A: Descriptive Statistics	Size	Value	Quality	Momentum	Low risk	Multifactor
Mean (%)	0.83	0.78	0.80	0.96	0.76	0.95
Annualized Mean (%)	9.90	9.36	9.54	11.47	9.15	11.35
Median (%)	1.16	1.69	1.17	1.35	1.19	1.31
Max (%)	19.95	23.79	14.46	14.70	10.98	14.21
Min (%)	-26.18	-21.88	-19.95	-18.52	-16.98	-24.22
Std. dev. (%)	5.80	6.01	4.83	4.70	4.17	5.23
Annualized Std. dev. (%)	20.10	20.84	16.74	16.28	14.45	18.11
Skewness	-0.61	-0.40	-0.54	-0.57	-0.68	-0.72
Kurtosis	2.61	1.92	1.46	1.37	1.38	2.39
Annualized Sharpe ratio	0.43	0.39	0.50	0.63	0.55	0.56
Jarque-Bera t-stat	78.97	40.88	31.64	30.20	35.95	74.43
JB p-value	0	0	0	0	0	0
Alpha (%)	0.37***	0.30***	0.41***	0.60***	0.44***	0.53***
Observations	241	241	241	241	241	241
Panel B: Correlations						
	Size	Value	Quality	Momentum	Low risk	Multifactor
Size	1.00	0.96	0.95	0.90	0.93	0.97
Value		1.00	0.95	0.88	0.92	0.96
Quality			1.00	0.93	0.96	0.96
Momentum				1.00	0.92	0.93
Low risk					1.00	0.95
Multifactor						1.00

Table 18: Descriptive Statistics and Correlations - Factors

Panel A: Descriptive Statistics

	FI	NO	DE	DK	ES	FR	CH	SE	IT	NL	AT	IE	BE	UK	PT
Mean (%)	0.36	0.73	0.56	1.11	0.38	0.50	0.66	0.77	0.24	0.67	0.71	0.19	0.35	0.21	0.11
Annualized Mean (%)	4.37	8.72	6.69	13.27	4.56	6.06	7.93	9.29	2.89	8.10	8.57	2.25	4.25	2.55	1.30
Median (%)	0.41	0.95	0.94	1.83	0.93	0.81	1.24	0.90	0.49	1.12	1.51	1.04	1.03	0.50	0.29
Max (%)	23.83	20.36	22.54	18.33	29.40	22.87	11.62	22.33	26.68	14.66	32.00	19.01	22.62	16.33	14.16
Min (%)	-23.49	-33.32	-24.36	-25.51	-25.35	-22.27	-12.27	-26.60	-23.46	-25.01	-37.20	-25.75	-36.42	-19.06	-26.10
Std. dev. (%)	7.23	7.50	6.60	5.59	7.15	5.95	4.32	6.53	6.96	5.80	8.14	6.54	6.44	4.87	6.31
Annualized Std. dev. (%)	25.04	25.99	22.87	19.35	24.78	20.60	14.96	22.63	24.12	20.08	28.21	22.65	22.31	16.86	21.86
Skewness	-0.22	-0.67	-0.49	-0.73	-0.10	-0.38	-0.56	-0.27	-0.24	-0.76	-0.74	-0.85	-1.11	-0.39	-0.56
Kurtosis	1.23	2.46	1.66	2.62	2.06	1.65	0.68	1.91	1.24	1.99	3.91	1.80	5.33	1.76	1.25
Annualized Sharpe ratio	0.13	0.29	0.24	0.63	0.14	0.24	0.45	0.36	0.07	0.35	0.26	0.05	0.14	0.08	0.01
Jarque-Bera t-stat	15.76	74.78	35.14	86.07	40.23	30.97	16.68	37.14	16.50	60.38	167.17	58.77	319.96	34.88	26.64
JB p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Alpha (%)	-0.11	0.21	0.05	0.73***	-0.13	0.04	0.35***	0.30	-0.27	0.23*	0.15	-0.22	-0.11	-0.16*	-0.31
Observations	241	241	241	241	241	241	241	241	241	241	241	241	241	241	241

Panel B: Correlations

	FI	NO	DE	DK	ES	FR	CH	SE	IT	NL	AT	IE	BE	UK	PT
FI	1.00	0.68	0.78	0.72	0.70	0.80	0.70	0.79	0.73	0.76	0.64	0.63	0.70	0.73	0.66
NO		1.00	0.78	0.76	0.72	0.82	0.72	0.80	0.76	0.81	0.80	0.66	0.79	0.86	0.71
DE			1.00	0.77	0.83	0.94	0.84	0.88	0.86	0.90	0.78	0.73	0.82	0.87	0.75
DK				1.00	0.68	0.78	0.76	0.81	0.72	0.81	0.71	0.69	0.77	0.76	0.72
ES					1.00	0.90	0.75	0.77	0.91	0.83	0.77	0.65	0.77	0.82	0.79
FR						1.00	0.85	0.88	0.92	0.93	0.84	0.75	0.86	0.91	0.80
CH							1.00	0.80	0.77	0.84	0.73	0.66	0.79	0.82	0.72
SE								1.00	0.79	0.87	0.73	0.72	0.79	0.84	0.70
IT									1.00	0.85	0.81	0.68	0.78	0.85	0.80
NL										1.00	0.77	0.74	0.87	0.87	0.78
AT											1.00	0.69	0.78	0.83	0.73
IE												1.00	0.72	0.74	0.62
BE													1.00	0.85	0.74
UK														1.00	0.75
PT															1.00

Table 19: Descriptive Statistics and Correlations - Countries

Panel A: Descriptive Statistics

	Energy	Materials	Industrial	Cons. Discretionary	Cons. Staples	Healthcare	Financials	Information Tech	Telecom	Utilities
Mean (%)	0.24	0.76	0.80	0.67	0.65	0.54	0.20	0.59	0.05	0.45
Annualized Mean (%)	2.92	9.13	9.61	8.01	7.77	6.48	2.42	7.06	0.55	5.34
Median (%)	0.25	0.98	1.60	0.88	1.02	0.65	1.11	1.27	-0.27	0.69
Max (%)	36.41	20.27	19.02	19.47	12.88	11.52	31.13	38.57	21.88	14.01
Min (%)	-17.77	-28.35	-29.35	-18.94	-15.30	-12.39	-31.08	-23.34	-17.97	-19.91
Std. dev. (%)	6.51	6.86	6.20	6.06	4.08	4.10	7.58	7.51	5.61	5.41
Annualized Std. dev. (%)	22.54	23.75	21.46	21.00	14.13	14.19	26.26	26.00	19.44	18.75
Skewness	0.55	-0.65	-0.89	-0.13	-0.46	-0.25	-0.15	0.12	0.06	-0.51
Kurtosis	3.59	2.19	3.17	0.85	0.76	0.30	2.86	3.21	1.16	0.83
Annualized Sharpe ratio	0.08	0.34	0.39	0.33	0.47	0.37	0.05	0.23	-0.03	0.22
Jarque-Bera t-stat	134.22	61.88	126.80	7.18	13.45	3.32	78.10	98.01	12.64	16.56
JB p-value	0.00	0.00	0.00	0.03	0.00	0.19	0.00	0.00	0.00	0.00
Alpha (%)	-0.16	0.26	0.32***	0.21	0.38**	0.29*	-0.37**	0.09	-0.31	0.08
Observations	241	241	241	241	241	241	241	241	241	241

Panel B: Correlations

	Energy	Materials	Industrial	Cons. Discretionary	Cons. Staples	Healthcare	Financials	Information Tech	Telecom	Utilities
Energy	1.00	0.74	0.68	0.65	0.59	0.52	0.67	0.51	0.56	0.64
Materials		1.00	0.90	0.85	0.68	0.63	0.81	0.71	0.60	0.71
Industrial			1.00	0.92	0.74	0.66	0.89	0.82	0.70	0.77
Cons. Discretionary				1.00	0.71	0.65	0.88	0.80	0.68	0.70
Cons. Staples					1.00	0.76	0.64	0.57	0.61	0.75
Healthcare						1.00	0.62	0.58	0.59	0.67
Financials							1.00	0.76	0.70	0.75
Information Tech								1.00	0.68	0.62
Telecom									1.00	0.69
Utilities										1.00

Table 20: Descriptive Statistics and Correlations - Sectors