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How to Determine the Optimal Procurement Plan, with the Objective of Maximizing the Total Profit Using a Mathematical Optimization Model

A case study for a large Norwegian furniture manufacturer

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June 2022

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Abstract

The Norwegian furniture industry has a strong position in Norway and produces about 30 percent of products sold in this country, employed 5,000 people (in 2018) and there are about 450 companies registered. The companies are mainly small and medium-sized, but there are some larger groups. The industry had a production value in Norway in 2018 of about NOK 8.2 billion, with an export share of around 30 percent. Just over a third of the industry is in the Møre region and is otherwise spread evenly throughout southern Norway. Among the largest furniture manufacturers in Norway are Ekornes, Stokke HÅG and Hilding Anders Norway (formerly Jensen Møbler). Ekornes received a major international boost with the seating concept Stressless in 1971. In 2019, Ekornes had sales of NOK 3.3 billion (Tronstad, 2019).

In the early 2000s, the furniture and interior design industry was undergoing a major period of restructuring. To meet international competition, the industry wants competitive framework conditions in Norway, something the industry feels they do not have today. In recent years, several companies have seen the need for an optimized procurement plan. Procurement is a vital process in many organizations. It conserves resources, raw materials, finished goods and many other items that are crucial for the success of an organization. Optimization models help manufacturers store and transport the right order quantities at the right time with minimal waste.

To conduct the research, an in-depth case study of Ekornes Beds AS current practices in form of procurement were performed. Two independent mathematical optimization models were made in the modelling language AMPL alongside an extensive phase of data collection. Our research unveiled that there are several aspects one should investigate before deciding on which optimized procurement plan is the best practice for the company. We found out that the mixed-integer linear programming approach does a good job in solving the challenges this business is currently facing with its current procurement practice. Our finished product model increased the total profit by 16% compared to their actual model. Further on, more domain knowledge is required within this field, but the overall rule cannot be ignored.

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List of Abbreviations

BoM	Bill-of-Materials
CLSP	Capacitated Lot Sizing Problem
DLSP	Discrete Lot sizing Scheduling Problem
DP	Dynamic Programming
DSI	Days Sales in Inventory
ELSP	Economic Lot Scheduling Problem
EOQ	Economic Order Quantity
FOQ	Fixed Order Quantity
JIC	Just-in-Case
JIT	Just-in-Time
LFL	Lot-for-Lot
LP	Linear Programming
LTC	Least Total Cost
LUC	Least Unit Cost
MAD	Mean Absolute Deviation
MAPE	Mean Absolute Percentage Error
MILP	Mixed Integer Linear Programming
MSE	Mean Square Error
MTO	Made-to-Order
MTS	Made-to-Stock
NOK	Norwegian Krone
POQ	Period Order Quantity

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1. Introduction

We are writing our master thesis on a real-world use case from Ekornes Beds AS, but the research question about how to determine the optimal procurement plan is relevant for most companies that deal with procurement. Schmidt et.al (2015) conducted a business survey in which German industrial enterprise representatives were questioned about the significance of the topic “Lot sizing in industrial practice” for their productions. This survey aimed to get a sense of the knowledge these users have regarding lot sizing methods and which of the approaches are being deployed in the industry. The key findings were that 60% of the companies indicated to be using a specific lot sizing method, 84% of the companies indicated to know several other lot sizing methods and around 50% of them have already implemented the base model in the study.

The study also found out that most of the responders (85%) consider the choice of lot size as crucial for a good logistic performance. During the same time (77%) of the responders do not find logistic dependencies taken into consideration in an appropriate methodology. Lot sizing problems are very generic that allow for further extensions, and they suit well in modeling a great variety of different lot-sizing problems in this thesis. Because of this, the model developed could be adjusted to work for any company.

Ekornes Beds AS are a daughter company of furniture manufacturer Ekornes, which today produces Svane beds and mattresses at their factory in Fetsund, Norway. They operate in the premium segment of beds and mattresses and their main competitors are Jensen and Wonderland. Their product offering consists of a combination of beds and mattresses made-to-order (MTO) and make-to-stock (MTS/Express). For their Express products, they promise delivery within 2 weeks. With long delivery times of either 28 or 90 days (depending on the product), it is essential that when an order for an express product comes in, the product is already in stock and for the MTO components with 90 days delivery time is already ordered.

According to their financial statement, they had inventory on hand for around 38 million NOK. This represents about a third of the yearly purchasing (order) cost and they had Days Sales in Inventory (DSI) for their stock of between 85 and 100 days. From their management team, the DSI measure should be a maximum of 90 days.

Based on the above points, procurement is of great importance for a company. Finding a method to create an optimal procurement quantity could help the company limit their storage capacity, reduce the capital binding, and stay within the DSI time goal. In this paper, mathematical programming will be applied to the research problem with the mathematical optimization tool AMPL, and later we will discuss the contributions our thesis will have for academic research and the contributions it will make for Ekornes Bed AS.

1.1 Objective of Research Area

The objective of the research area is within the field of lot-sizing and optimization. By definition, a lot is a quantity that is specified by certain products to either be sold or produced. Lot size determination in production areas is a vital component of production planning and management.

According to Belvaux & Wolsey (2000), the core effective solution to any variety of lot-sizing problems depends crucially on the development of formulations for a special problem that occurs in practice.

In this thesis, a mathematical optimization model will be implemented for a large Norwegian furniture company. As mentioned by Duda et.al (2014) in lot-sizing optimization problems, the core problem in the production planning problem consists of determining the lot size of the items to be produced or shipped during each period of a finite planning horizon.

1.2 Problem Statement

The research question which this thesis is attempting to answer is ***"How to Determine the Optimal Procurement Plan, with the Objective of Maximizing the Total Profit Using a Mathematical Optimization Model".***

As a practical application, the Mixed Integer Linear Programming (MILP) model developed will, as mentioned earlier, be applied to the real-world business case from Ekornes Bed AS. In the process of this application, we will identify the possible gap(s) between academic research in lot sizing and the real-world business case we have been given.

As part of answering the research question stated above, we will also attempt to solve the following sub-questions:

1. *Is the total cost reduced while using the optimization model compared to the current practice today?*
2. *Is the stated goal of maximum 90 days of Days Sales in Inventory the optimal solution?*
3. *Is the current method for forecasting the optimal solution or is there a better method?*

1.3 Planned Thesis Structure

The thesis is structured into 9 sections and numerous sub-sections:

(1) Introduction
An introduction of the objective of the thesis's research area and the research question along with the sub-questions that the thesis is attempting to answer.

(2) Company Presentation

An introduction to the company which the real-world case we are attempting to solve is based on, and the current practice for determining procurement plans and demand forecasting.

(3) Literature Review

The theoretical foundation for the thesis is reviewed and justification for which theoretical methods the demand forecasting and model development are based is done.

(4) Demand Forecasting

The theoretical justification and method for developing the forecasting model used to produce the future demand data on material level.

(5) Research Methodology

The research methodology used for this thesis

(6) Cost Structure

The theoretical foundation and method used to develop the cost structure are justified before the actual calculations are shown.

(7) Model Development

The problem definition and the assumption and limitations used for the model development are discussed and justified before the full mathematical models are shown.

(8) Analysis & Discussion

The output from the models will be discussed and compared with the actual numbers. Scenario and sensitivity analysis will also be conducted in this section on some of the variables.

(9) Conclusion

A summary of the findings in section 8 will be presented and how Ekornes Beds AS can adjust its current practice to make use of the model. This section will also discuss the contribution this thesis has academically and practically.

2. Company Presentation

Ekornes Beds AS is one of 3 wholly-owned subsidiaries of Ekornes ASA and was established in 1989. Today their 84 employees produce the trademark mattress and bed Svane (launched in 1937) at their 24,000 m^2 factory located in Fetsund, Norway. Below are a map showing the location of the factory, which is approximately 30 minutes away from Oslo by car.

Figure 1 : Location of Factory Facility

In Norway, their products are sold through the retailers Fagmøbler, Møbelringen, Skeidar and some independent stores. According to their financial statement for 2021, they had revenues of NOK 282 million and a net income of NOK 1.33 million.

Its parent company Ekornes ASA was established in 1934 and is the Nordic biggest furniture producer. Their headquarters are in Sykkylven in Norway, and they develop and produce furniture that is marked under the trademarks Stressless, Svane and IMG. According to their financial statement for 2021, Ekornes ASA had a net income of NOK 281.5 million.

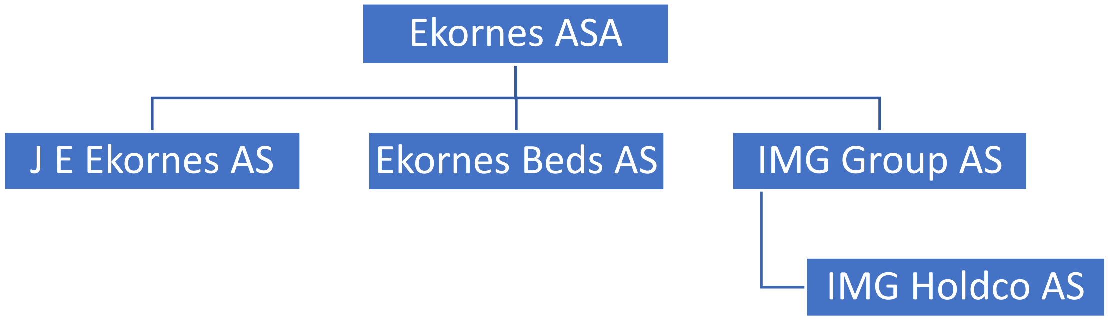

Figure 2 : Company Structure

Figure 2 below is an illustration of the company structure of Ekornes ASA with its 3 wholly-owned subsidiaries: J E Ekornes AS, Ekornes Beds AS and IMG Group AS. IMG Holdco AS is a wholly-owned subsidiary of IMG Group AS that only holds equity and debt.

The demand for Ekornes Beds AS's products, follows a consistent sales cycle, with tops occurring around January and June. Their lowest demand is between these two highs, and after June the demand is either consistent or rising for the rest of the year.

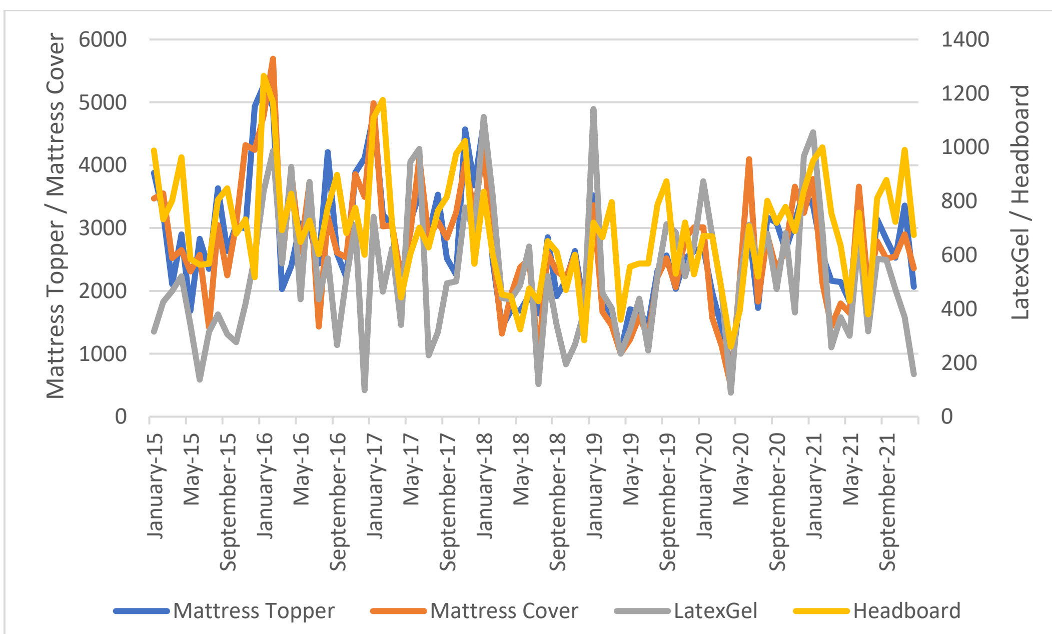

Figure 3 : Yearly Sales cycle

Out of the 1826 materials Ekornes Bed AS produces or purchases, in this thesis, we are looking at the 358 materials used to create mattress toppers plus headboards.

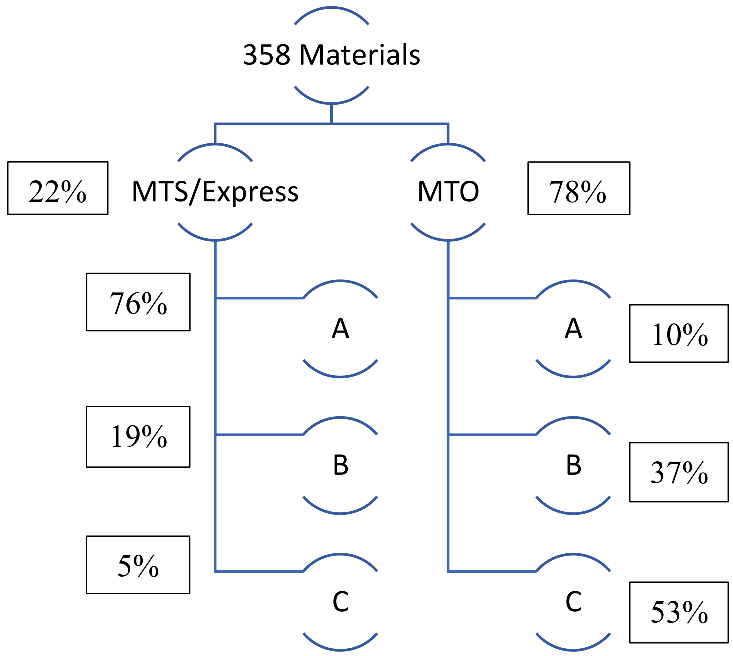

Figure 4 : Component Structure

Figure 4 is an illustration of how the 358 components are divided into Type (MTS/Express and MTO) and Importance (A, B and C). The type of MTS/Express is components in the measurements 90*200, 150*200, and 180*200 in selected textiles and colours. Everything else is classified as the type MTO. The components are divided into importance based on the demand and revenue for the components:

- A - 60% of revenue
- B – 30% of revenue
- C – 10% of revenue

Due to long replacement times (28 or 90 days, depending on the components) and the 2 weeks promise to their clients for delivery, MTS/Express components must always be in inventory while MTO A can be in inventory but could also stock out.

The factory facility they operate out of is 24,000 m^2 and is rented from their parent company Ekornes ASA with an annual rent of NOK 748,000. Compared to the estimated market price (which we will come back to in section 6.2), the warehouse rent is very low. Out of the 24,000 m^2 factory, 12,960 m^2 is used for storage and of this 4,600 m^2 is specifically used for components/products that are ready to be shipped.

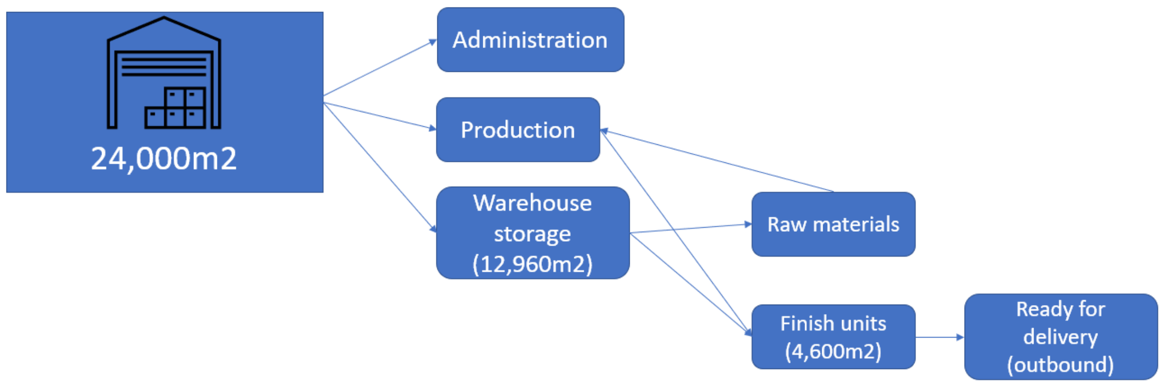

Figure 5 : Factory Structure

Figure 5 is an illustration of how the factory facility is used, it is divided into three sections: *Administration, Production, and Warehouse storage*. Combined, they have 84 employees of whom administration amongst other employees contain 5 that handle everything from customer orders to registration and 2 that work with procurement. In the production section, semi-finished components are turned into finished products. The warehouse portion of the facility is separated into one for raw materials (around 8,360 m^2) and one for finished products (around 4 600 m^2).

The figure below is an illustration of the process the 3 different groups of components (LatexGel, Mattress Cover and Headboard) go through from when they first arrive at the facility to when two of them (LatexGel and Mattress Cover) are developed into a finished product (Mattress Topper) and then shipped out with Headboards who go straight to the finished unit storage area.

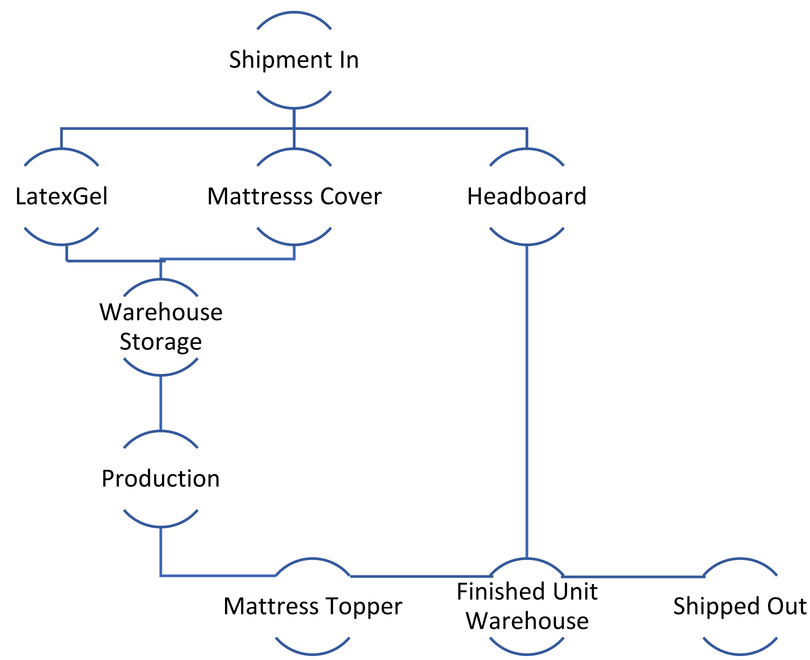

Figure 6 : Component Process from Start to End

The pictures below were taken during a factory visit. Pictures 1 to 3 are from the production part of the facility where LatexGel and Mattress Covers are combined to make Mattress Toppers. Pictures 4 and 5 are of the warehouse storage area used for semi-finished and raw materials. The last picture is from the docking area where the finished units are placed when a truck is coming to get them (and when raw materials are arriving at the facility).

Figure 7 : Pictures from the Factory Facility

2.1 Current Practice

When this thesis is attempting to determine the optimal procurement plan, it is important to look at what the company's current practice is and whether the proposed model can improve the results.

Ekornes Beds AS current practice for deciding on the purchasing plan is to do it based on historical data, review with the markets, experience and as orders come in (more details later in the section).

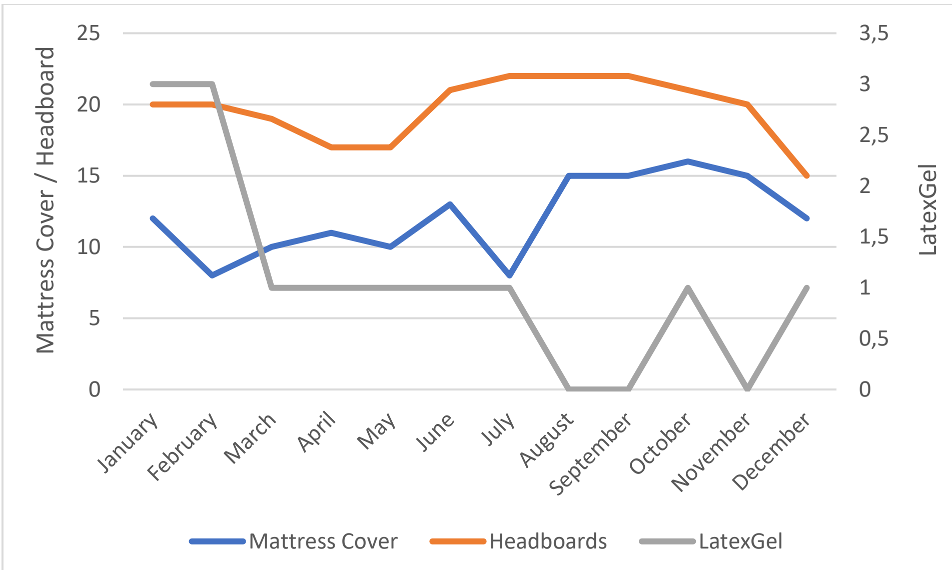

Figure 8 : Order Frequency

Figure 8 below shows the order frequency for the three material groups (LatexGel, Mattress Cover and Headboard) per month in 2021. On average, the company placed a Mattress Cover order every 2.5 days, a Headboard order every 1.5 days and a LatexGel order once a month.

This order frequency indicates that Ekornes Beds AS follows a Just-In-Time (JIT) model, with orders being placed whenever they need more materials to cover demand and focus on eliminating waste (reduce inventory and obsolesce of materials) in manufacturing for Mattress Covers and Headboards. But as mentioned earlier in section 2, they divided their stock into Express/MTO and A/B/C with Express always having to be in stock when the order comes, so for these orders, Ekornes Beds AS have a Just-In-Case (JIC) approach. This approach means that the firm holds buffer stocks and is thus more flexible in their ability to meet customer demand.

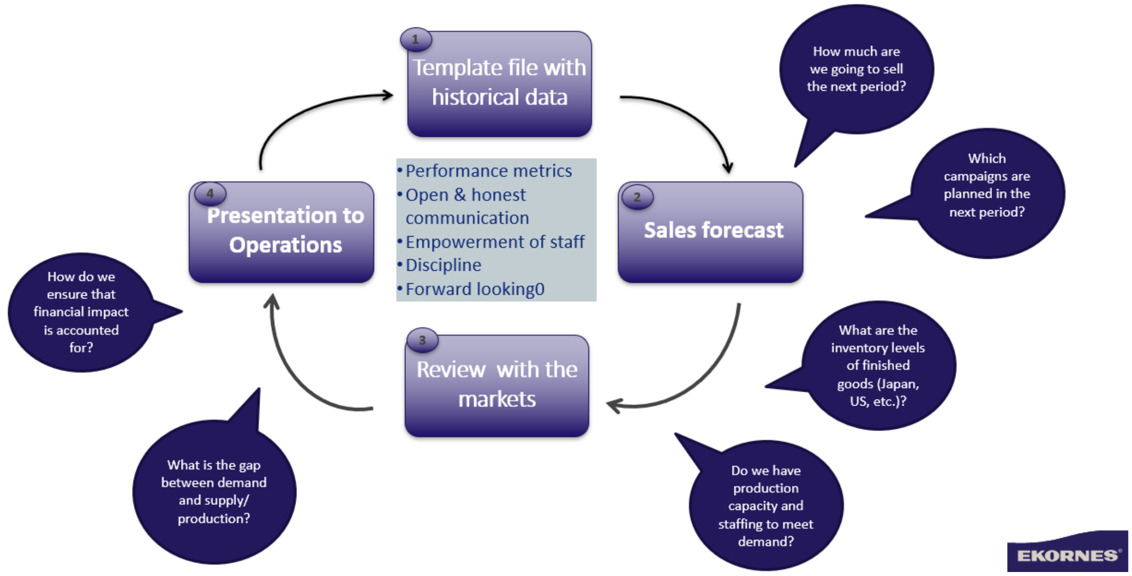

Figure 9 : Current Practice Loop

In Figure 9, the process which Ekornes Beds AS monthly uses to determine the purchasing quantities in the periods to come is shown. The starting point of the process is a template file with historical data in excel (1) which is adjusted based on a meeting with the Sales forecasting team (2). During this meeting they attempt to answer the following questions (amongst others):

- "*How much are we going to sell the next period?"*
- *"Which campaigns are planned in the next period?"*

After the sales forecasting meeting, they reach out to their customers (3) to get a sense of the expected demand in the periods to come and adjust the forecast file based on the feedback. In the next step (4), a meeting with Operations occurs and they amongst other things look at the gap between the forecasted/expected demand and supply production. Based on the four steps, a final forecast is produced and entered into their ERP system SAP, which controls when and how much of each component the company should order (shown in Figure 10).

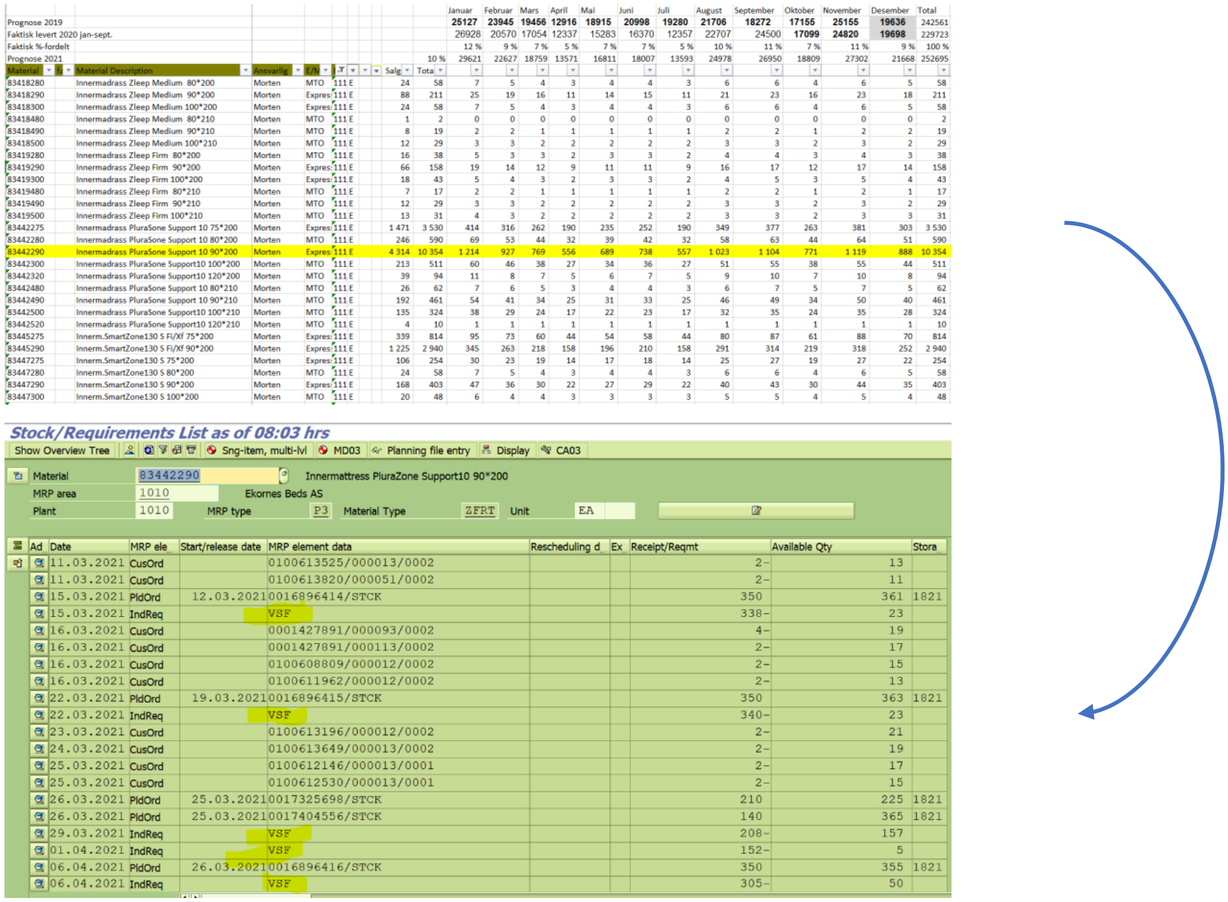

Figure 10 : From Excel to SAP

3. Literature Review

3.1 Lot Sizing: Characteristics and Attributes

Lot size determination in production areas is a vital component of production planning and management because the setup cost of switching between the production to produce a different product every period is often costly (Brahimi et.al, 2006). According to Ramya et al (2019) lot sizing is one of the most difficult problems to solve in production planning and the attributes and characteristics of the many lot-sizing models should be known to classify the complexity of lot-sizing models. Calculating affordable lot sizes is becoming increasingly crucial in industrial practice as the number of product variants grows (Schmidt et.al, 2015). In the sections below, the most common characteristics and attributes many of the lot-sizing models in production planning tend to have will be presented.

3.1.1 Period and Time-Based Characteristics and Attributes

The first set of characteristics and attributes presented are related to the planning horizon of the model developed and the time structure of the data used in the determination of the lot-sizing decisions.

3.1.1.1 Planning Horizon

The production in a planning horizon may be infinite or finite which many of these lot-sizing models are developed for. In an infinite planning horizon, the demand rate is normally constant (stationary demand) which is similar to the assumed demand in the EOQ model. In a finite time, the planning horizon for the products may vary a lot in every period (dynamic demand) (Ramya et al, 2019). In our model, the planning horizon will be finite over 48 weeks.

3.1.1.2 Time Scale

A discrete or Continuous time scale is used a lot in lot-sizing problems. Many of the lot-sizing problems are assuming a discrete time scale has small- or big-time buckets which may be non-uniform or uniform (Ramya et.al, 2019). In the standard lot-sizing problems, uniform time buckets are assumed. The lot-sizing models that are using a small-time bucket are allowing one or two products to be produced in a period. However, models assuming big-time buckets allow more than two products to be produced in the same period.

3.1.1.3 Parameter & Objectives

Lot sizing models often consist of several parameters. The most common parameters we can find in lot-sizing problems are cost parameters, production capacity, setup time, production coefficient etc. All these common parameters may or may not vary over time (Ramya et al, 2019). The general objectives of any lot-sizing problem are the minimization of several cost components such as holding cost, setup cost, lost sales cost, and backorder cost, some of which are included in our proposed model.

3.1.1.4 Cost Components

The most common cost components lot-sizing problems are inventory holding cost, setup cost, lost sales cost, and backorder cost (Ramya et al, 2019). The setup cost in any organization is different but is often incurred during the production process. Whenever a lot is produced there is a setup cost involving the retooling of the machine for example, or if the lot is ordered a setup cost can be incurred as a direct cost involving ordering specified quantities of the product which we will come back to in chapter 6.

3.1.2 Product-Based Characteristics and Attributes

This is the second set of characteristics attributes in lot sizing. According to Mungan et.al (2009) companies can no longer survive if they only focus on their businesses as induvial entities. They should instead focus on the entire supply chain of the products. That means coordinating several products and inventory restrictions.

3.1.2.1 Number of Products

Lot sizing problems are often assuming the production of a single or multiple products. Thus, if the Bill of Materials (BoM) is considered a product may have a single, - or multi-level.

Single-level products have nonintermediate sub-assemblies, it is directly produced from raw materials. Based on that, the demand for single-level products can be forecasted or known directly from customer orders since the demand for the products are independent of the demand of others. In our example, this will apply to the product group Headboard.

For Multi-level products, the raw materials of the end products must pass through several operations. The output of the first operations is the input of the other operations where the demand of the first level will depend on the demand of the second level (dependent demand). Multi-level lot-sizing problems are more difficult to solve than single-level lot-sizing problems (Ramya et al, 2019). In our example, this applies to the product groups Mattress Cover and LatexGel.

3.1.2.2 Inventory Capacity Restrictions

In lot-sizing problems, the capacity of the resources is either finite (capacitated) or infinite (uncapacitated). Resources having finite capacity are harder to solve since it is classified into both small and big bucket lot-sizing models and can be extended over time. However, for the infinite resources having no capacity constraint will mean they have no inventory, to begin with, and here the demand is assumed to be deterministic or dynamic (Ramya et al, 2019), which makes it easier to solve since it incorporates a time-dependent cost structure as addressed by Wagner and Whitin (1958). In Ekornes Beds AS case, the real-world situation is that they have some capacity restrictions in both producing the products and in warehousing them. But, since we only are looking at a small portion of the materials used and are only looking at procurement, the model is created as an uncapacitated problem.

3.1.2.3 Batch Sizing

Several products are produced using sequential series of operations. We know for a fact that losses occur at each operation, and that production planers must routinely estimate a starting batch size according to Depuy & Usher (2001). We have chosen a batch sizing approach in our model such that procurement quantities are in line with the current practice Ekornes Beds AS has.

3.2 Classification of Lot Sizing Models

Overall, lot-sizing models can be categorized into those which generate fixed order quantities repetitively or those that generate varying order quantities. Thus, this should not be confused with static or dynamic order quantities. Static order quantity, once it is computed it will stay unchanged in the planned order schedule, similar to the demand rate-oriented model/method which we will get into in the next sub-chapter (Ptak & Orlicky, 2011). While the dynamic order quantity has the discrete lot-sizing model/method as an underclass.

The classification of the lot-sizing model is summarized in Figure 11 below.

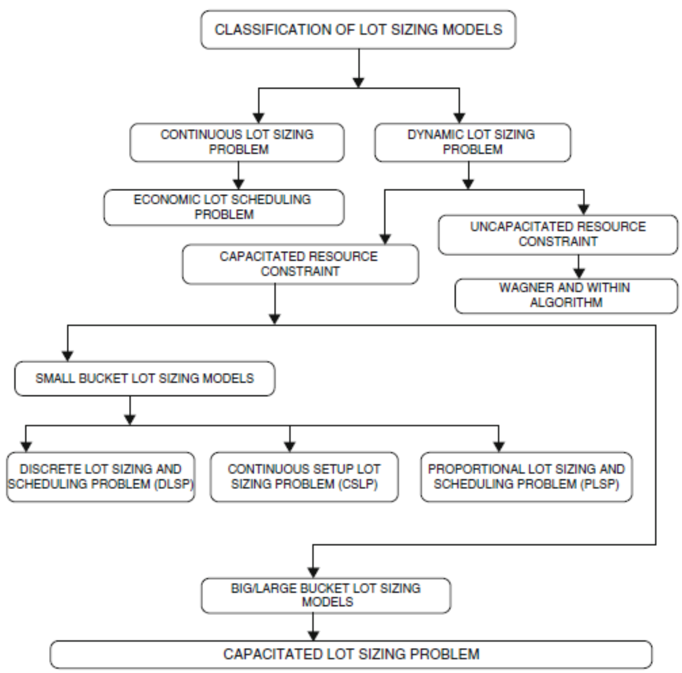

Figure 11 : Classification of the lot-sizing models

As mentioned, lot sizing models are classified on a planning and time scale attribute. Based on these attributes, the lot-sizing models can be either *continuous* or *dynamic lot-sizing problems*.

3.2.1 Continuous Lot Sizing Problem

Within continuous lot-sizing problems, the time scale is considered infinite and continuous as mentioned in chapter 3.1. In the continuous lot-sizing problem is where we can find the classical economic lot scheduling problem (ELSP) which is an extension of the EOQ model. The main objective of the ELSP model is concerning the production planning of several products on the same machine. This is due to how much and when to produce each product such that inventory holding costs and setup costs are minimized. However, the disadvantage of this model is that it assumes a stationary (constant) demand with an infinite planning horizon and deals with the objective function of minimizing the average costs (Ramya et al, 2019). Elmaghraby (1978) surveyed the ELSP model a lot by different methods for solving and found out that when capacity restrictions are involved solving the ELSP is hard.

3.2.2 Dynamic Lot Sizing Problem

The dynamic lot-sizing problem deals with single and multi-level products when demand is assumed to be time-varying and deterministic. In Figure 11 above we can see that the dynamic lot-sizing problem is further classified into uncapacitated and capacitated lot-sizing problems.

3.2.2.1 Uncapacitated Lot Sizing Problem

This single-level lot-sizing problem was addressed by Wagner and Whitin (1958) by the assumption of the demand being dynamic and deterministic. The single-level uncapacitated lot-sizing problem is where one single product is considered with a high assumed production capacity that is high enough to not be binding to the optimal solution (Brahimi et.al, 2006). A dynamic programming algorithm was provided to compute the optimal order with no starting inventory. This classification was then extended by Zangwill (1966) to allow for backorders and since the algorithm was inefficient to solve problems involving many products, several other lot sizing methods were made during the 1970s such ass Lot-for-Lot, Modified EOQ, Periodic Order Quantity (POQ) and others which we will come back to in the next chapters (Ramya et al, 2019).

Discrete Lot-Sizing Models/Methods

Discrete lot-sizing models also known as dynamic methods are lot-sizing techniques where order sizes vary. In discrete lot-sizing, the quantities that are produced will not be carried in inventory/storage for a long time without it begin sufficient to cover the future periods of demand in full (Ptak & Orlicky, 2011).

Lot-for-Lot (LFL)

The LFL technique is the most straightforward model of them all. Lot-for-lot ordering is also referred to as discrete ordering. In this model, the ordering amount is always equal to the required amount in the same period (Ptak & Orlicky, 2011). This will in return avoid any stock disruption when the ordering amount is exactly enough for the needed quantities for that given period (Ptak & Orlicky, 2011).

Period Order Quantity (POQ)

This model uses the same theory as the economic order quantity (EOQ) model, by using known future demands in the net requirements. The POQ approach counts the number of periods the order needs to be covered and not the quantity of the orders (Ptak & Orlicky, 2011). The carrying cost will be lower under the POQ approach compared to the EOQ approach, and that makes POQ more effective even though the setup cost will be the same. However, with nonuniform discontinuous demand, the effectiveness of this approach proves to be relatively low when we compare them together (Ptak & Orlicky, 2011) with the Least Unit Cost (LUC) or Least Total Cost (LTC) approach.

4. Demand Forecasting

To be able to answer the research question on what the optimal procurement plan is, the data used to plan must be the best it can be. With that in mind and to answer the sub-question of whether the current practice is the best solution or not, a demand forecast for the expected demand in 2022 (and 2021 to be able to compare with actuals) has been conducted.

4.1 Theory

In the competitive environment of today, organizations are moving towards a more demand-driven supply chain. Forecasting demand is an important element in supply chain management since it makes up the basis of future planning and causes business decisions to be taken on a better foundation (Wisner et.al, 2019). A good forecasting technique is to minimize the deviation between the actual demand at the forecasted demand. Increasing the importance of forecasting is the fact that customers are more demanding and have more options (globally) than before and being able to deliver consistently and with a shorter lead time is essential for business success.

The process of forecasting contains many steps, some of which are not objective inputs but rather human judgments. The figure below, from Silver et.al (2017) below demonstrate one version of the process.

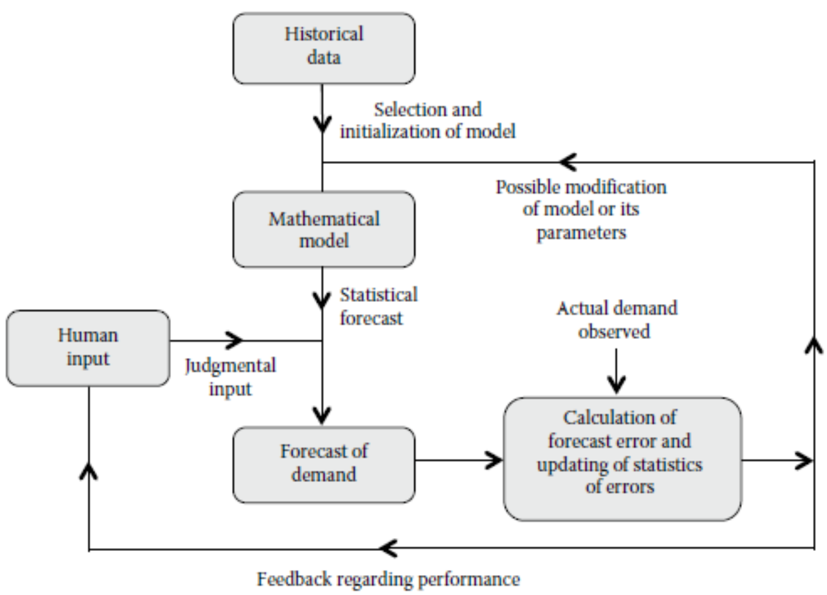

Figure 12 : Overview of Demand Forecasting

4.1.1 Mathematical Model

In demand forecast, there are many mathematical models with different forecasting techniques being used to forecast future demand. The two most well-known techniques are Time Series forecasting and Cause-and-effect forecasting. For simplicity and time constraints, the forecasting technique focused on in this thesis is time series forecasting.

Time series forecasting has many different components, but the most common ones are trend, random and seasonal variations. *Trend variations* are linear, exponential, asymptotic or S-curve and they represent either the increment or decrements of movement over many years the factors that influence these trends can be cultural changes, population growth etc. *Random variations* occur due to unpredictable and unexcepted events such as wars or natural disasters. These random variations can shift the direction of the future predictions that are planned. Now at least *seasonal variations* show the seasonal trends that repeat over consistent years. These seasonal variations can be in the interval of hours, days, weeks months, years, or seasons (Wisner et al., 2019). The seasonal component is very important today as many companies including Ekornes Beds AS (as shown in chapter 2) experience a seasonal sale cycle.

As many different components could be included to improve demand forecasting, there are also different types/versions of forecasting models. Examples include Simple Moving Average, Simple Linear Regression, Exponential Smoothing and Holt-Winters Exponential Smoothing (which is an extension of the simple version) among others. For this thesis, after some testing Holt-Winters, exponential smoothing has been used as it included many components that improved the forecast in comparison with the current practice of the company.

4.1.1.1 The Simple Exponential Smoothing Forecast

In the weighted moving average forecasting technique, the demand for the net period is the current period's forecast adjusted by the difference between the current's periods actual demand and forecast (Wisner et al., 2019). Since this technique only requires two data points, less data is needed in comparison with the weighted moving average technique which requires more data points.

Simple Exponential Smoothing equation:

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

or

$$F_{t+1} = F_t + \alpha(1 - \alpha)F_t$$

Where;

F_{t+1} = forecast for period t + 1

A_t = actual demand for period t

F_t = forecast for period t

α = smoothing constant ($0 \leq \alpha \leq 1$)

Briefly looking at the smoothing constant α, the closer it is to 1, the greater emphasis the forecast has on the recent data and the lower α is the more weight will be placed on the past demand. A lower α will thus result in slower changes in the forecasted demand and vice versa. According to Wisner et al (2019), a good rule of thumb is to have α = 0.5.

The simplicity and the minimal data requirements of this technique are one of the reasons why exponential smoothing is the most widely used model for forecasting. But the simplicity has its downfalls, and more complex models could be tailored to fit the relevant case and thus provide a better demand forecast.

4.1.1.2 Holt-Winters Exponential Smoothing Forecast

As mentioned earlier, the Holt-Winter method for forecasting is an extension of simple exponential smoothing forecasting. According to Gelper et.al (2009), it is a simple technique used to smooth and forecast a time series without the necessity of fitting a parametric model.

The researchers Makridakis et.al (1998) and Kotsialos et.al (2005) have studied this forecasting technique and its performance in practice. They concluded that the technique good measurement of performance. Based on that and the components included fit this thesis case, this technique was chosen to implement for demand forecasting in this thesis.

The Holt-Winters method is sometimes referred to as double exponential smoothing as it can include both trend and seasonality (Gelper et al, 2009).

<u>The basic formulation:</u>

$$F_{t+1} = L_t + kT_t + S_{t+k-M}$$

Where;

F_{t+1} = forecast for period t + 1

L_t = the level estimate for period t

T_t = the time estimate at period t

S_t = the seasonal estimate at period t

k = the number of forecasts in the future

M = the number of seasons

The above equation is almost equivalent to the formula chapter 4.1.1.1 Simple exponential smoothing forecast with some extensions (*trend, level, and seasonal* components) added. This formula is for the basic method, it will be different if the approach corresponds to additive or multiplicative. As mentioned by Koehler et.al (2001) if the approach is additive, then the time series is the sum of its components, and the time series is the product of its components in a multiplicative model.

4.1.2 Forecast Accuracy

The goal of any demand forecasting is to have an unbiased and accurate forecast. The reason why this is the goal is that companies that do a good job of tracking forecasting errors will have a substantial improvement in their forecasting techniques and minimize the costs of lost sales, safety stock and unsatisfied customers (Wisner et.al, 2019).

There are many methods and measurements for calculating forecasting accuracy. Below 3 widely used forecasting measurements will be introduced.

<u>Common notations in the equations:</u>

$e_t = A_t - F_t =$ forecast error for period t

$A_t =$ actual demand for period t

$F_t =$ forecast for period t

$n =$ number of periods of evaluation

<u>Forecasting Accuracy Measurements:</u>

$$\boldsymbol{Mean\ absolute\ deviation}\ (\boldsymbol{MAD}) = \frac{\sum_{i=1}^{n} |e_t|}{n}$$

The MAD measurement is computationally simple and widely used in practice today because of this (Silver et.al, 2017). In this model, the vertical lines for the forecast error e_t in period *t* mean that we take the sum of the absolute value of the differences and divide them by the number of periods of evaluation, *n*.

$$\boldsymbol{Mean\ absolute\ percentage\ error\ (MAPE)} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{e_t}{A_t}\right|$$

The MAPE measurement is also widely used for estimating accuracy because it is not affected by the magnitude of very low demand values (as it is expressed as a percentage), which the MAD model is (Silver et.al, 2017). Here the forecast error e_t is divided by the actual value A_t. Then the absolute value ratio is summed for every forecast period t and divided by the number of periods of evaluation, n.

$$\boldsymbol{Mean\ square\ error\ (MSE)} = \frac{\sum_{i=1}^{n} e_t^2}{n}$$

The MSE measurement is calculated by taking the summation of the squared forecast errors e_t in period t and dividing them by the number of periods of evaluation, n (Wisner et.al, 2019). By squaring the forecast errors, it forces the errors to be positive values which in turn cause results in a larger mean squared error score (Silver et.al, 2017). Because of this, the MSE measurement will often give worse scores than other measurements.

4.2 Results

To both answer the sub-question of whether the company's current practice for demand forecasting can be improved and the main research question of determining the optimal procurement plan, forecasting is an important element to be able to match supply with demand (Wisner et al., 2019).

Historical sales data from 2015-2021 (2015-2020 for the comparison with actuals) was used to forecast the future demand (in months over a year) for the four product categories: Headboard, LatexGel, Mattress Cover and Mattress Topper. The forecast was created using Holt-Winters exponential smoothing method through the programming language Python (code attached in Appendix B).

The figure below shows the forecasting graph from one of the four categories, Headboard. Similar graphs exist for the other 3 product categories, but for this chapter, the Headboard category will be used. The forecast was done at the product category level instead of the sub-group or material level based on tests showing that there weren't enough data to get adequate forecasting accuracy on these levels.

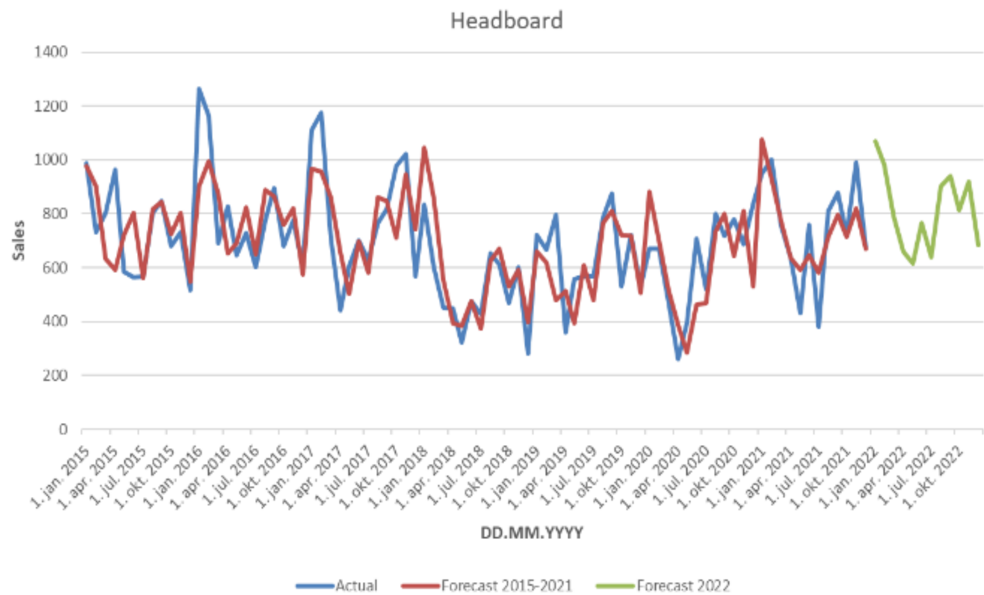

Figure 13 : Forecasting graph, - Headboard

Figure 13 represents the forecasted graph for the product group Headboard in 2022 with the blue line representing the actual demand from 2015 to 2021 and the green line being the forecasted demand for 2022. The red line can be disregarded as that forecast number (and thus its errors/accuracy) is polluted as it contains data leakage, meaning that the data used for training the red line contains information that the model is trying to forecast. The forecasting accuracy based on the MAPE for the product group Headboard is 15.9%, which is considered good accuracy. In chapter 4.2.2 we will go into more detail about the accuracy measured.

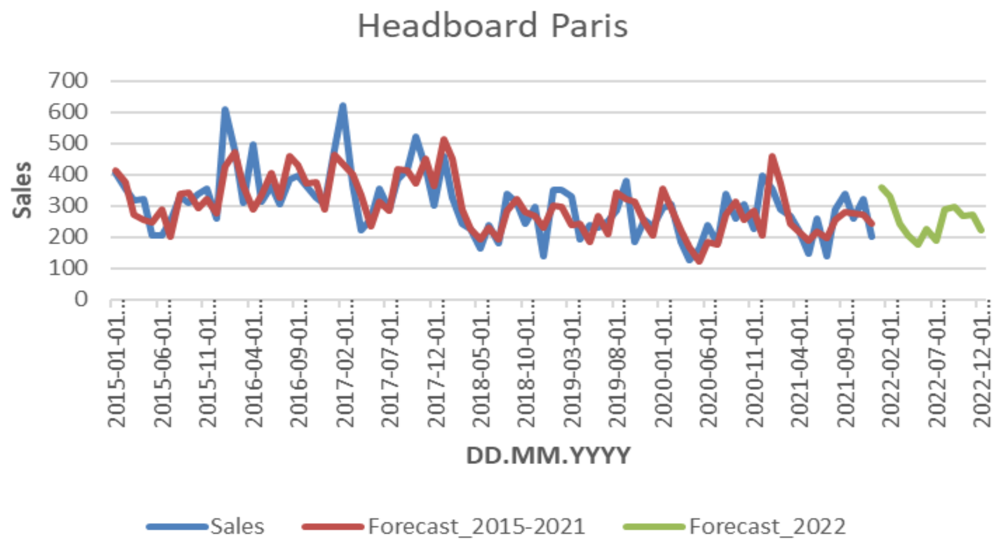

Figure 14 : Forecasting graph, - Headboard Paris sub-group level

Figure 13 shows have the same as Figure 14 but now for the sub-group Headboard Paris which is 1 of 6 sub-groups in the product category Headboard. The accuracy for this specific headboard remains good but this is not true for all the sub-groups. Some have little historical data (Headboard Paris has from 2015) as they are new materials and/or have a small monthly demand, thus ending up with very low accuracy.

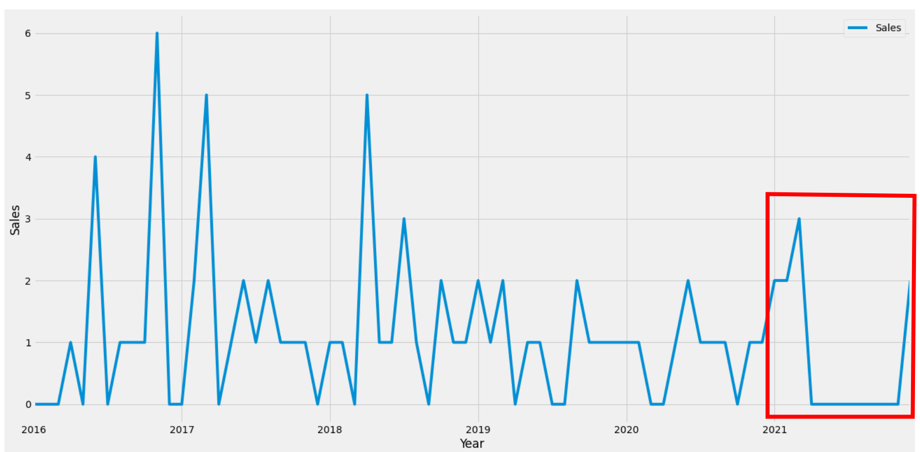

Figure 15 : Forecasting graph, - Headboard Paris material level

The problem with little data and small values is exacerbated if we look at the material level, as shown in Figure 15. The figure is for the material Paris H115 200cm Nordic Grey, and because of the low quantity of units sold each month, the forecast for 2022 (in the red box) is unreliable.

But to determine the optimal procurement plan, the demand forecast needs to be at the material level to be useful for the company. To archive this, we have taken the demand forecasted on the product group level and extrapolated the material level demand based on the demand distribution for 2021 for each group, type of product (MTO or MTS/Express) and priority (A, B or C). How the extrapolation was done is shown and discussed in chapter 4.2.3.

4.2.1 Seasonal Effects

From the figure below, there appears to be some seasonality in the demand patterns before the pandemic and after the pandemic (although enhanced demand). The red circle indicates where the pandemic started, and it coincides with the rapid increase in demand. This can be explained by the fact that Ekornes Beds AS took advantage of the situation and was able to continue to deliver and meet their customer's demands while some of their competitors were unable.

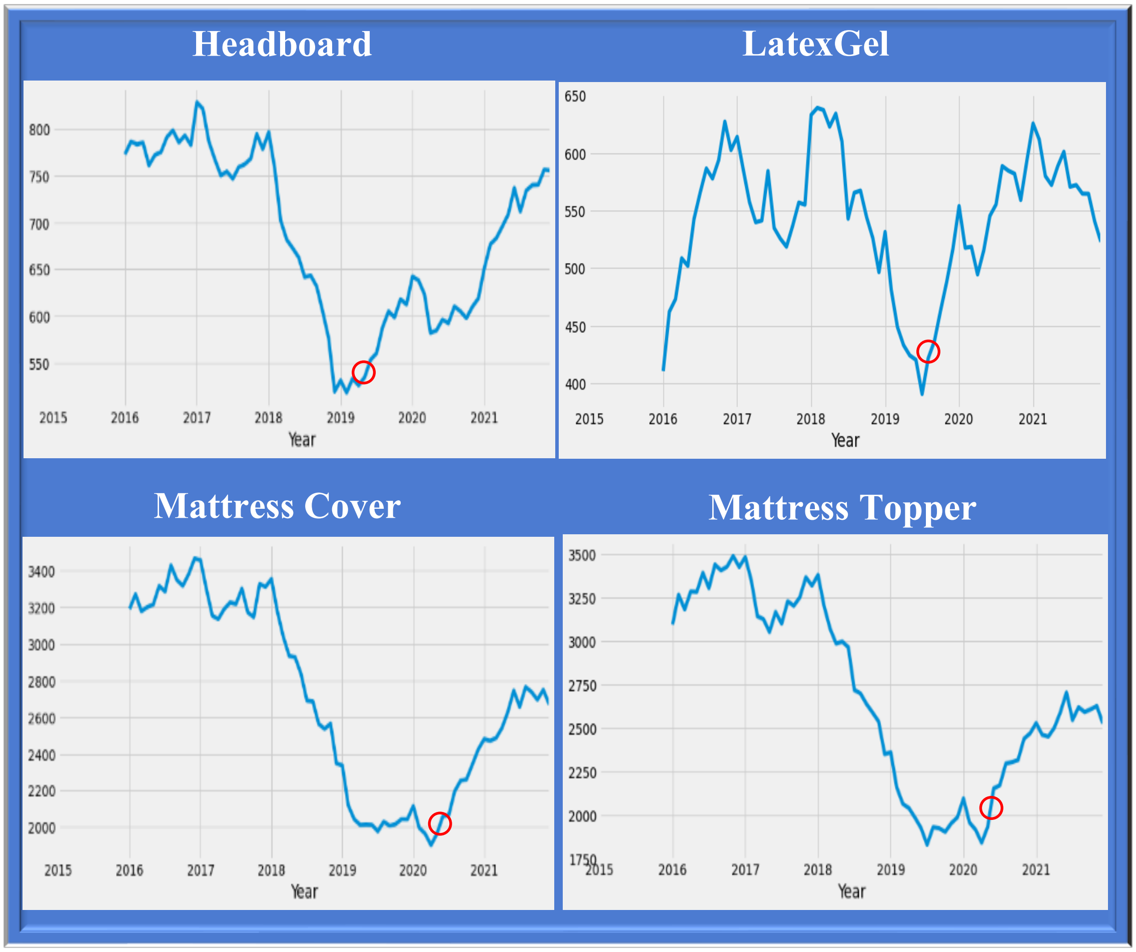

Figure 16 : Seasonality demonstration

Figure 17 below shows the different components (trend and seasonality) used in the Holt-Winters method for forecasting. The first graph shows the demand/sales of the material group Headboard from 2015 to 2021 and then the 2nd and 3rd graph reveals the trends and seasonality the python model found in the demand data. The last graph shows the residuals (the difference between the actuals and the mean value forecasted).

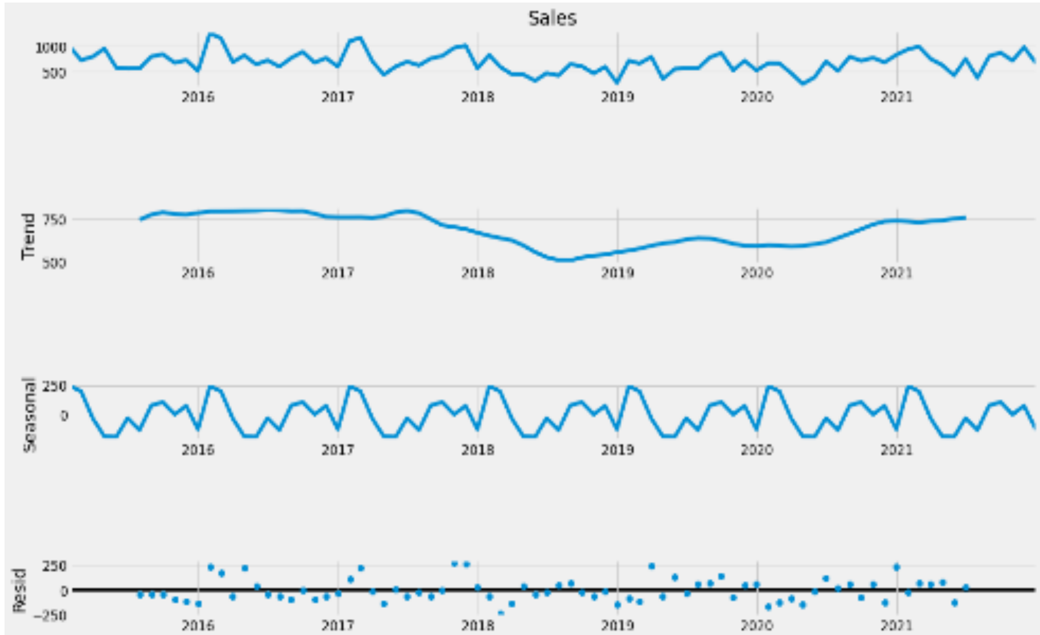

Figure 17 : Statistical analysis, - Headboard

4.2.2 Statistical Analysis

The table below shows the 4 different product category's accuracy in forecasting the demand in 2022.

	Headboard	LatexGel	Mattress Cover	Mattress Topper
MAPE	18.33%	114.13%	21.65%	17.95%
MAD	*102.52*	*356.90*	*471.72*	*430.48*
MSE	*18 755*	*154 105*	*302 556*	*331 232*

Table 1 : Forecast Accuracy Results

Out of the 4 product groups, Headboards have the highest accuracy based on two of the measures. Depending on which measure you look at, at first glance either LatexGel, Mattress Cover or Mattress Topper has the worse accuracy. But you must be cautious when looking at accuracy measures as they can all be affected by the magnitude of values. For example, if one month 4 units are sold but the forecasted demand was 2 units, that is a 50% error which looks bad but is not that many units and would not affect the bottom line significantly. 50% error could also be thousands of units and could affect the bottom line significantly.

4.2.3 Meeting Demand on Material Level

As discussed in chapter 4.1.2, forecasting on material or even sub-group level for the four product categories resulted in worse accuracy than doing it on the product group level. But as mentioned earlier, for the demand forecast to be useful in determining the optimal procurement plan, the demand forecast needs to be on the material level.

To solve this challenge, a product level matrix, shown in Figure 18, was created. Using the company's categorization of their materials into MTS/Express or MTO and A, B or C, the forecasted monthly demand at the product group level was down to the material level. This matrix is designed for Ekornes Beds AS, but if other companies follow what Silver et. al (2017) consider good practice in dividing the production situation into classes, it could be used for them too.

Headboard		Antall	January	February	March	April	May	June	July	August	September	October	November	December
Express	A	18	377	411	252	204	168	329	148	340	395	306	424	256
	B	10	57	49	38	67	40	46	30	60	71	35	64	42
	C	14	24	64	11	45	21	43	21	33	21	41	24	61
MTO	A	2	6	1	0	0	0	1	4	5	6	7	64	9
	B	10	69	47	56	43	17	26	20	68	46	30	55	32
	C	140	100	88	166	86	52	87	32	75	99	89	90	81
Express	A	9 %	59,56 %	62,27 %	48,18 %	45,84 %	56,38 %	61,84 %	58,04 %	58,52 %	61,91 %	60,24 %	58,81 %	53,22 %
	B	5 %	9,00 %	7,42 %	7,27 %	15,06 %	13,42 %	8,65 %	11,76 %	10,33 %	11,13 %	6,89 %	8,88 %	8,73 %
	C	7 %	3,79 %	9,70 %	2,10 %	10,11 %	7,05 %	8,08 %	8,24 %	5,68 %	3,29 %	8,07 %	3,33 %	12,68 %
MTO	A	1 %	0,95 %	0,15 %	0,00 %	0,00 %	0,00 %	0,19 %	1,57 %	0,86 %	0,94 %	1,38 %	8,88 %	1,87 %
	B	5 %	10,90 %	7,12 %	10,71 %	9,66 %	5,70 %	4,89 %	7,84 %	11,70 %	7,21 %	5,91 %	7,63 %	6,65 %
	C	72 %	15,80 %	13,33 %	31,74 %	19,33 %	17,45 %	16,35 %	12,55 %	12,91 %	15,52 %	17,52 %	12,48 %	16,84 %

Figure 18 : The product level matrix, - Headboard

The upper half of Figure 18 shows the number of products ("Antall") and demand for each product for the 12 different months of 2021. The lower half of the figure shows the percentage distribution of the monthly demands, which was used in combination with the number of products to extrapolate the demand per material.

$$Demand\ on\ material\ level_t = \frac{\left(\frac{\left(D_{p,month} * \frac{Sales_{p,t}}{Total\ Sales_{p,t}}\right)}{Number\ of\ materials\ in\ type\ and\ category}\right)}{4}$$

The equation below was used in the process of taking the monthly material level demand down to the material level. For each *t*, which in this case is weeks, the demand for that month on a material level was multiplied by the sales or consumption ratio of that material's categorization (MTS/Express or MTO) and prioritization (A, B or C) before being divided by the number of materials of the same categorization and prioritization. This gives the monthly demand, which is divided by 4 (justification for why 4 and not 4.33 will be chapter 7.2) to get the weekly forecasted demand on the material level.

This demand on the material level would most likely not be in whole numbers, and as all the products need to be procured and sold in whole units, we used the below equation to get the weekly demand.

$$\text{If } \sum_{i=n}^{product} Y_{product,n} \begin{cases} \geq \frac{D_{product,month}}{4} * n = 0 \\ < \frac{D_{product,month}}{4} * n = roundup\left(\frac{D_{product,month}}{4}, 0\right) \end{cases}$$

For week 1 (n=1) in a month (n=4), the number found in the previous equation were rounded up to the nearest whole number. For the subsequent weeks (n=2, n=3 and n=4), if the sum of the previous week(s) in the month was greater or equal to the weekly demand multiplied by the number of weeks (n=2, n=3 or n=4) then the demand for that week was 0. Otherwise, the demand was the rounded-up value of the weekly demand.

5. Research Methodology

In this chapter, the research methods used to develop the models are introduced. We have done as Bryman et. al. (2019) said that d academics in that we have conducted this thesis while reading the literature on topics that are relevant or reflect what is going on in organizations.

Walliman (2011) makes a mention of research as “a process that is undertaken systematically with a clear purpose, to find things out”. This definition suggests that research is based on one or several logical relationships and not just beliefs. This has motivated our research to involve several explanations of the methods used to collect the relevant data, and we will later argue why these data results are meaningful to our research question and explain any limitations that are associated with them.

5.1 Research Strategy

To answer the research question *“How to Determine the Optimal Procurement Plan, with the Objective of Maximizing the Total Profit Using a Mathematical Optimization Model”* and the accompanying sub-questions, our research strategy focuses on mainly quantitative research. Quantitative research involves the collection of numerical data (Bryman et al., 2019).

As this thesis is based on a real-world case, numerical data is needed to provide sufficient analysis and to be able to measure relevant key variables. This kind of quantitative research is usually associated with a deductive approach, where data is collected and analyzed to test a theory. Based on the findings from the test, in quantitative research, a hypothesis (or several) is deducted. This type of testing hypothesis is a characteristic of experimental research (Bryman et al., 2019).

5.1.1 Case Study Design

Since this thesis is based on a real business case from Ekornes Beds AS, case study design was chosen as the research method. The case study design is very popular and a widely used research design which can be used in different business research areas (Bryman et. al., 2019).

In our case, the business research area is a single organization that is looking to determine the optimal procurement plan to maximize their profit (and indirectly minimize their costs). To be successful in determining the optimal solution it is important to have a good knowledge of the company (current practices, company structure etc.) and the business environment they are operating in.

According to Dubois & Gadde (2002) case studies approaches have not always been recognized as a proper scientific method. This is because they are too situational specific and based on that they are not appropriate for generalization. But over the years the case study approach has been increasingly popular, and it is now a very common method in many scientific disciplines.

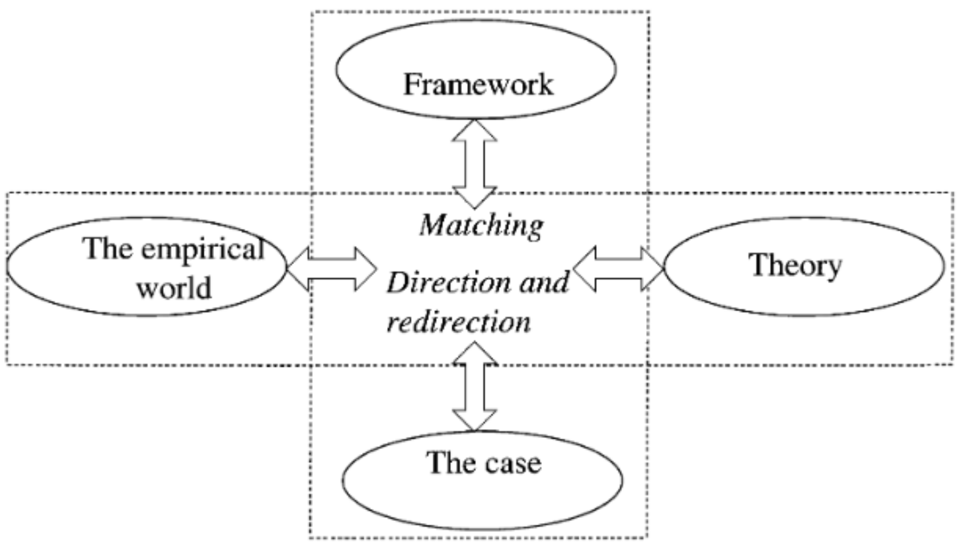

Figure 19 : Case study, - Systematic combining

Case studies provide a unique understanding of developing theory by utilizing in-depth insight into empirical research and its context as Figure 19 above illustrates (Dubois & Gadde, 2022).

The main objective of this research is to challenge theory with the empirical world we are living in. We have found many academic articles that inspired us to the creation of our final model. For example, the theory around capacitated Lot-Sizing problems that we read about in Ramya et al, (2019), the article *"An Economic Lot Size Model with Shortages and Inflation"* written by Onawumi et al, (2009), and the motivation for implementing a mathematical model is based on the survey done by Schmidt et.al (2015) in which German industrial enterprises representatives were questioned about the significance of the topic "Lot sizing in industrial practice" for their productions.

5.1.2 Chosen Model

According to Cardona-Valdés et.al (2020) companies today are immersed in a globalized market with considerable demands for different products. Dealing with capacity constraints, single and multilevel-items lot sizing problems is a strain on industrial companies today. New models are constantly being evaluated to see whether they improve the results compared with the current best practice. Lot-sizing problems have been studied broadly since in the field of operations research and production planning numerous complex problems are covered.

The core problem and many other variants of lot-sizing problems have been solved by Dynamic Programming (DP) and other lot-sizing problems that have a time-varying production capacity, multi-echelon and setup costs are for the most time dealt with Mixed Integer Linear Programming (MILP) formulations. The MILP is a very well-known optimization method that is widely used in operational research and in solving industrial symbioses. The method is optimizing a linear function subject to a finite number of linear equality and inequality constraints on variables and integers. Solvers such as CPLEX or other generic solvers are used in these types of problems to get a more efficient solved mode. Generic solvers are used a lot in production planning problems and in our proposed model these solvers will be utilized.

However, without going in-depth, there are other state-of-the-art approaches that are being used for very complex lot-sizing problems that are using polyhedral techniques that can handle a large class of problems with side constraints (Barany et.al, 1984). In our example, we have chosen dynamic programming with the MILP formulation and found out that it is sufficient in our business case.

5.2 Research Process

According to Mark et.al (2019) many of the research textbooks represent a multi-stage process that must be followed to complete and undertake your research project. The number of stages varies, but formulating and clarifying a topic, reviewing the literature, design of the research, collecting and analyzing the data are recurring often. Most research processes are described as a series of stages through which you must pass, it needs to be rational and straightforward, but unfortunately in reality this is very rarely true (Mark et.al, 2019).

To answer the research question about how to determine the optimal procurement plan, our research process consists of four parts.

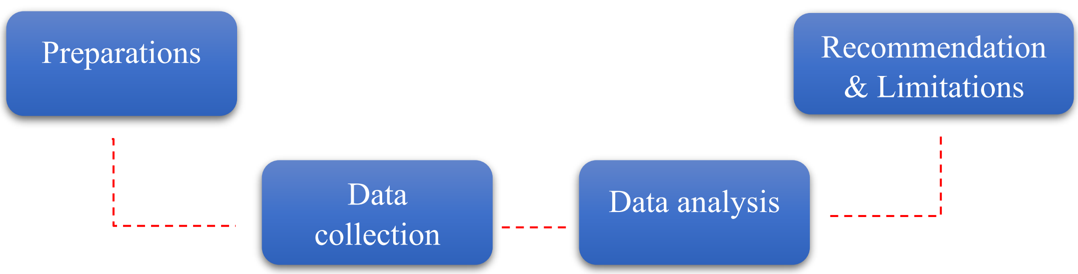

Figure 20 : Research procedures

Walliman (2011) argues that many of the everyday uses of the term "research" are not research in the true meaning of words. He first states that even though research often involves the collection of information, if it is not undertaken in a system without any clear purpose, it will not be seen as research. Then he contends that this issue is commonplace in many reports. He mentions that the data are collected from a variety of different sources and then assembled into a single document without the sources of these data listed without any explanation, in order words no interpretation of the data collected.

In the sub-chapters below, we will give explanatory information about the data we have collected, how it is collected, and how it is analyzed.

5.2.1 Preparations

This is a critical stage for any analysis and is the starting phase in our research process. To be able to achieve the results you want, careful preparation is important. This includes reading on the topic and relevant literature.

We started early by reading up on literature which is relevant to be able to answer the research questions. Reading publicly available information (such as annual reports and financial statements) and keeping continuous communication with the company was essential in getting more insight into the business environment Ekornes Beds AS operated in.

Mark et.al (2019) states that in this critical stage it is also important to keep in mind the relevant theories and the key concept in the literature continuously to identify relevant variables which must be collected.

5.2.2 Data Collection

A key point in any research is data collection, which can be quantitative data such as procurement data from ERP systems or more qualitative data from interviews and questionnaires. The qualitative data can be of structured or unstructured form. Structured forms mean that the researchers establish in advance the broad contours of what they want to find out about and then the design of the research will come accordingly (Bryman et.al, 2019).

Data collected can be broadly divided into two categories: Primary and Secondary, which we will go into detail about below.

***Primary Data*:**

Primary data were given to us directly from the company based on requests for data to the Supply Chain Manager who extracted the data from their ERP system SAP. In the table below, the different data units used in the analysis and model develop which were taken directly or indirectly (through calculations) from the primary data given.

Data unit	Data description
Number of Products	*358 different types of products.*
Material Number	The unique material number for each product.
Material Description	A detailed description of product types.
Product Group(s)	Four product groups: Headboard, LatexGel, Mattress Topper, Mattress Cover.
Product Sub-Group(s)	Detailed products groups categorization.
Total Value Stock	The purchase value of the stock balance or the sum of the purchase price of the materials used to make the final product.
Volume	Volume in m^3 for each product.
Basic Material	Types of delivery: Express (2 weeks) | MTO (Made-To-Order).
ABC	ABC categorization of products: A > B > C.
Sales Price	The sales price of the finished products.
Finished Goods/Product/Materials	A column that distinguishes the production and final goods components.
Inventory Value	Purchase cost per product: Total Value Stock/Stock.

Standard Price/Order cost	Standard price or order cost for making or purchasing different types of product/materials components.
Stock	Current balance while doing the analysis.
Stock Balance 31.12.2020	Stock balance at date 31.12.2020.
Stock Balance 31.12.2021	Stock balance at date 31.12.2021.
Product Consumption/Sales in 2021	Product consumption for production material components, and sales quantity for finished products in 2021.
Procurement Order (PO) Quantity in 2021	Sum of PO for each product in 2021.
Transportation fee in %, Inbound	11% for Mattress Cover and LatexGel. 6% for Headboard.
Transportation fee in %, Outbound	Ranging from 2% to 14% for different products. In this stage, it is only relevant for the finished products (Headboard and Mattress Topper).
Transportation Cost in NOK, Inbound	Transportation fee in % converted to NOK.
Transportation Cost in NOK, Outbound	Transportation fee in % converted to NOK.
Inventory Storage Cost in NOK, 2021	Storage cost for each product in NOK, for having them in inventory in year 2021.
Inventory Storage Cost in NOK, 2022	Storage cost for each product in NOK, for having them in inventory in year 2022.
Inventory Storage Cost in %, 2021	Percentage inventory storage cost extracted in year 2021.
Inventory Storage Cost in %, 2022	Percentage inventory storage cost extracted in year 2022.
Holding Cost per Month	Inventory holding cost per month for each product.

Table 2 : Data Collection

Secondary data:

Secondary data was collected from relevant textbooks and academic journals. In the literature review (chapter 3) a review and critique of the secondary data used in this thesis have been done.

5.2.3 Data Analysis

The next step after data collection is turning the data into value by analyzing it. This Data Analysis step consists of several elements. The application of statistical techniques to data is the most obvious element. But most often even though the data is amenable to quantitative data analysis, there are always other things going on when it is analyzed (Bryman et.al, 2019). For example, data transcripts in our example need to be uploaded into a computer software program to be analyzed. In short, the data analysis part is fundamentally about reducing the large corpus of information that is gathered into something that makes sense and can be used to create value.

For our analysis part, there are several tools that we will use to get the most out of these data. The first one is Excel which will be used for many things such as descriptive statistics and some simple graphs and tables, but it will mainly be used for sorting and filtering the primary data. AMPL is a mathematical optimization program that will also be utilized in this paper to create optimization models. The purpose of using AMPL is to optimize our objectives and set constraints which will be of use when working with a large amount of data. To create the demand forecast used in the 2021 model (for comparison with actuals) and the 2022 model, Python coding in Jupyter Notebook was used.

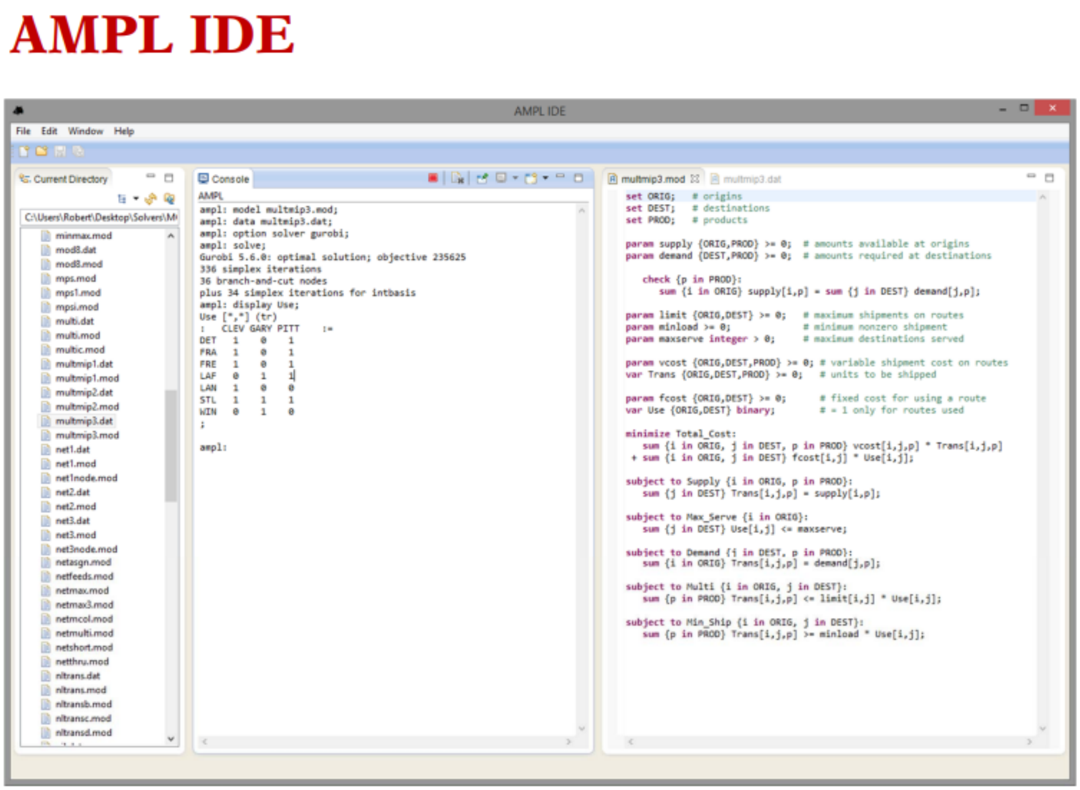

Figure 21 : AMPL IDE example

5.2.4 Recommendation & Limitations

According to Walliman (2011), research is often appearing abstract, it still influences our daily lives and creates an understanding of the world. Therefore, coming up with legitimate and reasonable recommendations is the core ending of any research. One example of a good recommendation can be the proposal of an optimization model and its solution. But before digging down to examples, the researcher often needs to construct validity.

Internal *validity* refers to the ability to measure the things it is intended to measure. In order words, if our intended research represents the reality of what you are measuring (Walliman, 2011). In this thesis, we will strive to validate our results through different distinct measurements, such as testing for reliability.

Reliability refers to consistency. For something to be valid, it must also be reliable. Walliman (2011) states that reliability is concerned with robustness. In our thesis, it is important to see if our optimization model will produce consistent findings under different times and conditions, and with different sample sizes.

Limitations

The company have over 50 different types of product categories and over 1800 different materials within these different product categories. However, we will only be focusing on 4 product categories (figure 22) that encompasses 358 unique materials. The product categories we will include are:

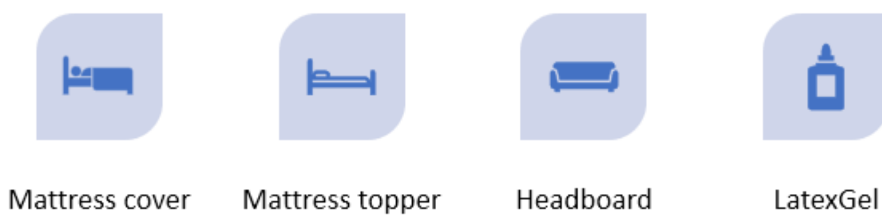

Figure 22 : Product categories

<u>Distribution of materials in Product Categories</u>

1. Mattress cover | 87 material groups
2. Mattress topper | 60 material groups
3. Headboard | 199 material groups
4. LatexGel | 12 material groups

The reasoning behind the inclusion of these four-product categories is after a discussion with our original Supervisor Atle Nordli and the Supply Chain Manager at Ekornes Beds AS. These product categories are relevant for us to include since they are often very expensive and have a comprehensive lead time and combine to make some finished products.

Our results will heavily depend on the data we can get from the company itself. We will primarily work with historical data from 2015 to 2021 when we want to determine future lot size quantities for 2022. During the demand forecasting part of our paper, we will work with historical data for the past 6 years. The reasons are that we want to be as accurate with our predictions as possible when we apply them to our models.

6. Cost Structuring in The Proposed Model

6.1 Theory

While there are no articles or textbooks written that encompass the exact cost structure of the problem in this thesis, there are sources that include it with or without modifications part of it.

6.1.1 Unit Value

The unit value, denoted by the symbol v, of an item, is for a merchant the price (including freight) paid to the supplier plus any cost incurred to make it ready for sale. This value can depend on the size of the replenishment if the firm gets a quantity discount. For producers, it is more difficult to determine the exact unit value, but it is seldom the conventional accounting value assigned in most organizations. A good starting point according to Silver et al. (2017), is the cost figure given by accounting, adjusted for obvious errors.

6.1.2 Carrying Cost

Few articles go into depth about how and why the carrying cost is calculated as it is, but from a research standpoint, it appears that there is a nearly universal agreement about how it should be calculated.

According to Silver et al. (2017) and Beranek, W (1967), the conventional method for defining the total carrying cost over T is:

$$Total\ Carrying\ Cost\ over\ T = \bar{I}vr$$

Where $\bar{I}$ is the average inventory in units, v is the unit variable value or cost and r is the cost of carrying inventory for 1 year. These costs include all costs of carrying inventory including but not limited to storage cost, running the warehouse, taxes, obsolesce of materials and opportunity cost.

Stewart (1965) talked about how one study showed that the all-industry average amount of inventory carrying cost and in-plant warehousing came out to be 5.9% of sales per year, with 2.2% being contributed from interest rates and 2.1% from warehousing.

Out of these costs, the opportunity cost is the hardest to quantify the next most attractive investment opportunity can change from day to day. Furthermore, this cost can be the one that could make up the largest portion of the carrying cost. For practical purposes, the opportunity cost is usually set at some level and will only be changed if major changes have taken place.

Obsolesce costs are incurred when an item in inventory becomes obsolete before it is either sold or used in production. An item may become obsolete due to product design changes, changes in customer demand or by not being sold within an acceptable shelf life for products with limited shelf-life. The costs include the unit value of the item plus the cost of disposal.

6.1.3 Setup Cost

The setup cost, usually denoted by the symbol A, is the fixed cost associated with replenishment and it is independent of the size of the replenishment. It includes all costs associated with ordering, such as time spent ordering, receiving the order and handling of supplier invoices.

Several of the factors mentioned earlier can be quite complicated to quantify and divide the cost between normal product and setup costs. For example, if the warehouse employee has a full-time contract with a fixed salary, should the time he spends unloading the truck be included in the setup cost or not? The answer to this question mostly depends on whether you are looking at a long-term or short-term view.

In a short-term view, Silver et al. (2017) argued that the employee's fixed salary is not a setup cost as a firm would have this cost whether a replenishment order was made or not. But in the long-term the wage should be included as the employee could be laid off, thus the decision to replenish orders more infrequently affects what the firm pays.

Shaikh et al. (2019) challenge some of the most common economic order quantity (EOQ) model assumptions, which this thesis also assumes. Despite this, the managerial implications found in the sensitivity analysis is could apply to our case. According to Shaikh et al. (2019), if the setup cost per order is high, a manager should attempt to negotiate down this cost with their supplier or reduce the cost by increasing the order size sufficiently.

6.2 Calculation

The costs in our two models (production and finished product) are made up of 4 costs: carrying-, setup-, order- and transportation costs.

6.2.1 Unit Value

Because of the large number of materials and the impracticalities of measuring exactly how much labour is added to them, for calculating unit value, only transportation- and purchase costs were used. Based on information from the company, no discount for larger replenishment orders was added to the model to reflect the reality of the case.

Both transportation- and purchase costs were provided to us by the company. Transportation costs inbound was given to us by Ekornes Beds AS as a percentage of the purchase cost and was either based on a stated percentage they got from the supplier or an estimation for those suppliers who have transport baked into their sales price. For the outbound transportation costs, they charge NOK 400 per cubic meter transported in Norway.

Material Group	Inbound	Outbound
LatexGel	11%	-
Mattress Cover	11%	-
Headboard	6%	7%
Mattress Topper	-	3%

Table 3 : Average Transportation Cost in %

The table above shows the average percentage cost of the unit value of the outbound transportation per material group and the percentage stated or estimated for the inbound.

As mentioned, we have based the unit value on the accounting numbers given to us by the company. We found no obvious errors and have chosen to proceed with these numbers. The numbers are consistent over the different materials, with unit values for similar materials being almost identical.

6.2.2 Carrying Cost

In calculating the carrying cost, we have diverged slightly from the most common theoretical method in that we have used the actual monthly inventory instead of the annual average. By doing this, we get a more accurate measurement of the carrying cost. Thus, the equation used is:

$$Carrying\ Cost\ over\ T = \sum^{t} I_t vr$$

Based on Stewart's (1965) Table 2 "Physical Distribution Costs by Function", adjusted for our case and discounting any costs that should be attributed to unit- or setup cost, we have included: Warehousing (storage cost), Interest (Opportunity Cost), Taxes and Obsolesce Cost.

With us including the Inventory Storage Cost as part of the inventory carrying cost, we have according to La Londe & Lambert (1975) only considered the warehouse costs related to storage and not those related to throughput. In our case, this is the correct decision as the storage facility is given with no decisions on whether opening or closing a facility is open for discussion.

To calculate the storage cost, we used a combination of given data and observations at the factory to estimate the total monthly storage cost.

Data	Symbol	Type	Value
Factory Size, m^2	FC_{m^2}	Given	24 000 m^2
Storage portion of Factory, %	$S_{\%}$	Given	54%
Height Finished Storage, m	FSH_m	Given	8 m
Finished Storage Proportion, m^2	FSP_{m^2}	Given	4 600 m^2
Height rest of Storage	SH_m	Estimation	2 m
Rent whole factory, NOK	$Rent$	Given	NOK 748 000

Table 4 : Inventory Storage Cost Factors

$$Inventory\ Storage\ Cost\ = \frac{\frac{Rent}{FC_{m^2} * S_{\%}}}{\frac{FSH_m * \frac{FSP_{m^2}}{FC_{m^2} * S_{\%}} + SH_m * \left(1 - \frac{FSP_{m^2}}{FC_{m^2} * S_{\%}}\right)}{48}}$$

$$Inventory\ Storage\ Cost = \frac{\frac{NOK\ 748\ 000}{24\ 000\ m^2 * 0.54}}{\frac{8\ m * \frac{4\ 600\ m^2}{24\ 000\ m^2 * 0.54} + 2\ m * \left(1 - \frac{4\ 600\ m^2}{24\ 000\ m^2 * 0.54}\right)}{48}} = 0.16$$

Based on the above data and formula, we found that the cubic meter price for storage at the factory would be 0.16 NOK per week per cubic meter for 2022. It is worth noting that the rent is low as they rent the factory from their parent company.

There is little public information on what the market price of a comparable storage facility would be, with the height and price of the warehouse often omitted from public listings and only available upon serious enquires. But the nearest City Self-Storage unit to the factory has published rates and heights and goes for between NOK 2.03 and NOK 2.79 per cubic meter per week.

In the end, we decided to use the average of the publicly available price per cubic meter, which comes out to NOK 2.41 per cubic meter per week and 1.38% pa of the ordering cost.

$$Inventory\ Storage\ \ Rate\ per\ year = \sum \frac{Inventory\ Storage\ Cost}{Inventory\ Value}$$

As mentioned earlier, it is not easy setting the opportunity cost rate and when we asked the company in question which rate, they used, they didn't know and suggested using the normal bank interest. This was the decision we decided to go with as according to the financial statements Ekornes Beds AS has a good amount of equity saved up, nearly nothing in financial costs and no investments outside the company. Thus, the company's opportunity cost was taken as the average interest rate they could achieve by instead placing the money in a savings account. Using data from banks such as BN Bank, DNB, Nordea and Sparebank 1, the average came out as 0.36% pa.

Using Stewart's (1965)'s stated average of an interest rate of 2.20% and adjusting for the United States Federal Rate at the time (4.04%) and the Norwegian Federal Rate in 2020 (0.58%), we get an interest rate that would be equal to 0.32% p.a. which is close to what the real rate found above is.

$$r = \frac{r_{stewart}}{r_{US\ Federal\ Rate,1965}} * r_{Norway\ Federal\ Rate,2020} = \frac{2.20\%}{4.04\%} * 0.58\% = 0.32\%$$

The cause of obsolete materials in the case of Ekornes Beds AS is only due to product design changes or changes in customer demand, as there is no expiry date on the materials and in theory, they could stay in storage forever. Based on accounting, this value should be 0% as the financial statements contain zero in down writing. This seems artificially low, but after talking with the company they confirmed that for the materials we were looking at, the down writing was NOK 0 in 2021.

$$\sum \frac{Down\ writing}{Inventory\ Value} = \frac{0}{5\ 857\ 852} = 0\%$$

Taxes were calculated based on the financial statement for Ekornes Bed AS for 2021, with them for the whole company paying NOK 512,000. As 19.61% of these taxes could be attributed to the components we are looking at, an annual yearly tax rate for the components being looked at 1.71% of component value.

$$Taxes = \frac{\text{Tax} * \text{Our \% of components}}{\text{Total Value Stock}} = \frac{NOK\ 512{,}000 * 19.16\%}{NOK\ 5{,}857{,}852.26} = 1.71\%\ p.a.$$

	Per Year
Inventory Storage Rate	1.38%
Obsolesce of Materials Rate	0.00%
Opportunity Cost Rate	0.36%
Taxes	1.71%

Table 5 : Holding Cost Rate

Combining the different costs, we end up with a carrying cost rate of 3.45% of the accounting value (which in this case is the ordering cost) of a component per year. Multiplying this rate (r) with the inventory level $(\bar{I})$ and unit value (v), you get the holding cost. The equation below is an illustration of how the holding cost was calculated for the LatexGel (lg) products.

$$HC_{lg} = \bar{I}_{lg} v_{lg} r \; ; \; \forall lg$$

In comparison, if we use Stewart's (1965) average numbers and adjust the interest rate to the latest numbers published from the International Monetary Fund (2021) through FRED, the Federal Reserve Bank of St. Louis, we get 4.02% pa as a carrying rate.

6.2.3 Setup Cost

When talking with representatives from the company, we were told that there were no fixed costs associated with placing an order or receiving an invoice from any of their suppliers.

Thus, our calculation of the setup costs is based on the wages of the people that work with an order (purchasing- and customer service employees) and an estimated percentage of their time spent on each of the orders handled in 2021. These employees amongst other things check whether the materials arrive when they should and everything that has to do with order registration (and processing).

Notably, as we are looking at a time horizon of a year, we have a long-term view of the setup cost and thus based on the arguments from Silver et al. (2017) included the labour cost associated with the order. The data used to calculate the hourly wage was taken from Statistics Norway's Table 11418, which shows the average monthly earnings for 407 occupations in Norway including purchasing- and customer service employees.

	Number	**Monthly Wage**
Purchasing Employee[1]	2	NOK 56 020
Customer Service Employee[2]	5	NOK 41 610

Table 6 : Income of Employees working with Orders

[1] 3323 Innkjøpere
[2] 4222 Kundesentermedarbeidere

The 7 employees' yearly salary comes out to 3,841,080 of which 19.61% can be attributed to the 358 components we are looking at distributed over the 394 orders made in 2021. This gives us an estimated setup cost of NOK 1,911.77 per order.

$$A = \frac{Total\ Wage * \%\ \text{of}\ all\ components}{Orders\ Made} = \frac{3{,}841{,}080 * 0.1961}{394} = NOK\ 1{,}911.77$$

We have here assumed that these 7 employees only do jobs associated with orders as there was no information on how much time these employees specifically used on orders versus other job tasks.

7. Model Development

7.1 Problem Definition

This thesis considers a supply chain with 1 manufacturer and 3 suppliers, each providing all materials within one material group. As mentioned in chapter 1, the materials included in this thesis are only a small part of the company's full list of materials and these 3 suppliers are a small number of all suppliers.

The company needs to order the raw material (and in some cases, finished products) to meet their demand targets at minimal cost. Each combination of the material group (LatexGel, Mattress Cover, Mattress Topper, or Headboard) and prioritization level (A, B or C), have an estimated demand at different time points which is divided equally amongst them. In addition, it has a holding cost associated with each material. For each replenishment order made to a supplier, the company incurs a setup cost. The purchase price and transportation costs are independent of the number of items ordered.

As two distinct processes are going on, one with raw materials used to produce the finished products and one with materials that are ready to be shipped to customers,

there are two models. These two models can be run independently of each other as neither takes uses inputs produced from the other model.
The first model (named production model) is for raw materials and determines the number of each material ordered so that they arrive at a certain time point, to be able to meet the demand targets at this time while minimizing the costs. The second model (named finished products) is for finished products and determines the number of each material ordered so that they arrive at a certain time point, to be able to meet the demand targets at this but now the objective is to maximize the profit.

7.2 Limitations & Assumptions

7.2.1 Assumption

The main assumption made during the development of the model was that the model would always meet the forecasted demand for each product (no stockout allowed). This holds even in the cases where it would be financial advantages not to meet demand. This approach was chosen as there are unquantifiable consequences of not meeting demand, such as loss of trust and possibly losing the customer.

It was also assumed that the order cost, and the holding cost, were constant for each material during the time horizon, that materials are always available for shipping from the supplier and would always take the stated lead-time given by the company.

7.2.2 Limitations

To be able to focus the thesis on the topic of optimization of lot sizes, due to limited space and time, a simple forecasting model based on the Holt-Winters method was used to predict future demand. This prediction was done at the product group level, and categorization (Made-To-Order or Express) and prioritization level (A, B or C) were used to extrapolate the expected demand per month for each material.

In addition, since we are only looking at a small portion of the whole material catalogue, the model does not contain any production or storage limitations. These limitations were necessary as the materials looked at share resources with materials not included in the thesis. Based on talks with the company, storage capacity is not a constraint that causes problems and thus we are confident that this limitation doesn't reduce the functionality and usefulness of the model.

The demand data from the company were monthly, which would create a model with too few details to be used by the company in practice. In combination with the necessity of the AMPL model having the parameter T (*time horizon)* as a whole number, we have chosen to assume that there are 4 weeks in a month instead of 4.33 weeks.

Due to the complexity of the models and the very long-running time without being able to state whether a solution was the absolute optimal solution, we elected to include a stop command in the code so that its maximum will run for 15 minutes. The justification for this will be stated in chapter 8.1.

7.3 Mathematical Models

Common for both models, is that we have a time horizon with *t* time points, where the period between time points is one week (with 48 weeks in a year as mentioned in chapter 7.2.1). The set of all time points is $T = \{1,2,3, \dots ,48\}$. The rest of the parameters, variables and constraints will be presented in more detail in the following sub-chapters.

7.3.1 Production Model

7.3.1.1 Parameters

Demand for each LatexGel product *lg* in period *t*	$d_{lg,t}$
Demand for each Mattress Cover product *mc* in period *t*	$d_{mc,t}$
Holding Cost	HC

Setup Cost	A
Order Cost for each LatexGel product *lg*	C_{lg}
Order Cost for each Mattress Cover product *mc*	C_{mc}
Transportation Cost as a percentage for each unit	$TransCost$
Initial Inventory for each LatexGel product *lg*	$Init_{lg}$
Initial Inventory for each Mattress Cover product *mc*	$Init_{mc}$
Minimum Purchase for each LatexGel product *lg*	$MinLot_{lg}$
Minimum Purchase for each Mattress Cover product *mc*	$MinLot_{mc}$
Batch Size for each LatexGel product *lg*	$BatchSize_{lg}$
Batch Size for each Mattress Cover product *mc*	$BatchSize_{mc}$

7.3.1.2 Variables

Inventory for each LatexGel product *lg* at end of period *t*	$I_{lg,t} \geq 0$
Inventory for each Mattress Cover product *mc* at end of period *t*	$I_{mc,t} \geq 0$
Purchase quantity for each LatexGel product *lg* in period *t*	$Y_{lg,t} \geq 0\ ;\ \epsilon\ Z$
Purchase quantity for each Mattress Cover product *mc* in period *t*	$Y_{mc,t} \geq 0\ ;\ \epsilon\ Z$
Number of batches (based on $BatchSize_{lg}$) bought of each LatexGel product *lg* in period *t*	$Batch_{lg,t}\ ;\ \epsilon\ Z$
Number of batches (based on $BatchSize_{mc}$) bought of each Mattress Cover product *mc* in period *t*	$Batch_{mc,t}\ ;\ \epsilon\ Z$
Order made for any LatexGel product in period *t, binary*	$OrderMade_t^{lg}\ ;\ \in \{0,1\}$
Order made for any Mattress Cover product in period *t, binary*	$OrderMade_t^{mc}\ ;\ \in \{0,1\}$
Total order cost of all LatexGel products sold	$COGS_{lg}$
Total order cost of all Mattress Cover products sold	$COGS_{mc}$
Average Inventory Purchase Cost LatexGel times 365	$avgInvCost_{lg}$
Average Inventory Purchase Cost Mattress Cover times 365	$avgInvCost_{mc}$

7.3.1.3 Objective Function

$$Minimize \sum_{lg}^{t=0} \left(C_{lg} * Y_{lg,t} + I_{lg,t} * HC_{lg} * C_{lg} + Y_{lg,t} * TransCost * C_{lg}\right) + \sum_{lg}^{t=1} OrderMade_t^{lg} * A$$
$$+ \sum_{mc}^{t=0} \left(C_{mc} * Y_{mc,t} + I_{mc,t} * HC_{mc} * C_{mc} + Y_{mc,t} * TransCost * C_{mc}\right) + \sum_{mc}^{t=1} OrderMade_t^{mc} * A$$

The objective of the production model was to minimize the total costs of purchasing ($C * Y$), shipping ($Y * TransCost * C$), storing ($I * HC$) the product groups LatexGel (lg) and Mattress Cover (mc) over the period T (48) and the setup cost (A) for each order made during period T (48) while adhering to all the limitations and assumptions stated in chapter 7.2.1.

7.3.1.4 Constraints

(1) $Y_{lg,t} \geq MinLot_{lg} * OrderMade_t^{LatexGel}$ $\quad \forall t \text{ and } \forall lg$

(2) $Batch_{lg} = \dfrac{Y_{lg,t}}{BatchSize_{lg}}$ $\quad \forall t \text{ and } \forall lg$

(3) $Y_{mc,t} \geq MinLot_{mc} * OrderMade_t^{Cover}$ $\quad \forall t \text{ and } \forall mc$

(4) $Batch_{mc} = \dfrac{Y_{mc,t}}{BatchSize_{mc}}$ $\quad \forall t \text{ and } \forall mc$

(5) $I_{lg,0} = Init_{lg}$ $\quad \forall lg$

(6) $I_{lg,t} = I_{lg,t-1} + Y_{lg,t} - d_{lg,t} \geq 0$ $\quad \forall t \text{ and } \forall lg$

(7) $I_{mc,0} = Init_{mc}$ $\quad \forall mc$

(8) $I_{mc,t} = I_{mc,t-1} + Y_{mc,t} - d_{mc,t} \geq 0$ $\quad \forall t \text{ and } \forall mc$

(9) if $\sum_{lg} Y_{lg,t} \begin{cases} = 0 \text{ then } OrderMade_t^{lg} = 0 \\ \neq 0 \text{ then } OrderMade_t^{lg} = 1 \end{cases}$ $\quad \forall t \text{ and } \forall lg$

(10) if $\sum_{mc} Y_{mc,t} \begin{cases} = 0 \text{ then } OrderMade_t^{mc} = 0 \\ \neq 0 \text{ then } OrderMade_t^{mc} = 1 \end{cases}$ $\quad \forall t \text{ and } \forall mc$

(11) $avgInvCost_{lg} = \sum_{lg}^{T=0} \dfrac{I_{lg,t}}{48} * C_{lg} * 365$ $\quad \forall t \text{ and } \forall lg$

(12) $$COGS_{lg} = \sum_{lg}^{T=1} C_{lg} * D_{lg,t} \qquad \forall t \text{ and } \forall lg$$

(13) $$\frac{avgInvCost_{lg}}{COGS_{lg}} \leq 90$$

(14) $$avgInvCost_{mc} = \sum_{mc}^{T=0} \frac{I_{mc,t}}{48} * C_{mc} * 365 \qquad \forall t \text{ and } \forall mc$$

(15) $$COGS_{mc} = \sum_{mc}^{T=1} C_{mc} * D_{mc,t} \qquad \forall t \text{ and } \forall mc$$

(16) $$\frac{avgInvCost_{mc}}{COGS_{mc}} \leq 90$$

(17) $$\sum_{T=1} OrderMade_{lg} \geq 12$$

(18) $$\sum_{T=1} OrderMade_{mc} \geq 12$$

7.3.1.5 Explanation of the Constraints

(1) If an order for a LatexGel is made, the min order quantity is set

(2) Forcing any LatexGel order to be in multiple of the Batch Size value

(3) If an order for a Mattress Cover is made, the min order quantity is set

(4) Forcing any Mattress Cover order to be in multiple of the Batch Size value

(5) Setting the initial inventory for each LatexGel product

(6) Inventory for each LatexGel product at the end of period t

(7) Setting the initial inventory for each Mattress Cover product

(8) Inventory for each Mattress Cover product at the end of period t

(9) Variable equal 1 if any LatexGel products are ordered at time *t*, otherwise 0

(10) Variable equal 1 if any Mattress Cover products are ordered at time *t*, otherwise 0

(11) Calculates the average inventory order cost for all LatexGel products times 365

(12) Calculates the total Cost of LatexGel products Sold during all T

(13) Restricting the number of days LatexGel products are in inventory

(14) Calculates the average inventory order cost for all Mattress Cover products times 365

(15) Calculates the total Cost of Mattress Cover products Sold during all T

(16) Restricting the number of days Mattress Cover products are in inventory

(17) Forcing the model to make at least 12 LatexGel orders every year

(18) Forcing the model to make at least 12 Mattress Cover orders every year

7.3.2 Finished Product Model

7.3.2.1 Parameters

Demand for each Mattress product *m* in period *t*	$d_{m,t}$
Demand for each Headboard product *h* in period *t*	$d_{h,t}$
Holding Cost	HC
Setup Cost	A
Order Cost for each Mattress product *m*	C_m
Order Cost for each Headboard product *h*	C_h
Transportation Cost Incoming for Headboard product *h* as %	$TransCost_h^{in}$
Transportation Cost Outgoing for each Mattress product *m*	$TransCost_m^{out}$
Transportation Cost Outgoing for each Headboard product *h*	$TransCost_h^{out}$
Sales price for each Mattress product *m*	SP_m
Sales price for each Headboard product *h*	SP_h
Initial Inventory for each Mattress product *m*	$Init_m$
Initial Inventory for each Headboard product *h*	$Init_h$
Minimum Purchase for each Mattress product *m*	$MinLot_m$
Minimum Purchase for each Headboard product *h*	$MinLot_h$
Batch Size for each Mattress product *m*	$BatchSize_m$
Batch Size for each Headboard product *h*	$BatchSize_h$

7.3.2.2 Variables

Inventory for each Mattress product *m* at the end of period *t*	$I_{m,t} \geq 0$
Inventory for each Headboard product h at the end of period *t*	$I_{h,t} \geq 0$
Consumption quantity for each Mattress product *m* in period *t*	$YC_{m,t} \geq 0\ ;\ \epsilon\ Z$
Purchase quantity for each Headboard product *h* in period *t*	$Y_{h,t} \geq 0\ ;\ \epsilon\ Z$
Number of batches (based on $BatchSize_m$) consumed of each Mattress product *m* in period *t*	$Batch_{m,t}\ ;\ \epsilon\ Z$
Number of batches (based on $BatchSize_h$) bought of each Headboard product *h* in period *t*	$Batch_{h,t}\ ;\ \epsilon\ Z$
Order made for any Headboard product in period *t, binary*	$OrderMade_t^h\ ;\ \in \{0,1\}$
Total order cost of all Headboard products sold	$COGS_h$
Average Inventory Purchase Cost Headboard times 365	$avgInvCost_h$

7.3.2.3 Objective Function

$$
\begin{aligned}
Maximize \sum_{m}^{t=1} SP_m * D_{m,t} + \sum_{h}^{t=1} SP_h * D_{h,t} \\
- \sum_{m}^{t=1} \left(C_m * D_{m,t} + I_{m,t} * HC_m * C_m + D_{m,t} * TransCost_m^{out}\right) \\
- \sum_{h}^{t=1} \left(C_h * D_{h,t} + I_{h,t} * HC_h * C_h + D_{h,t} * TransCost_h^{in} * C_h + D_{h,t} * TransCost_h^{out} * C_h\right) \\
- \sum^{t=1} OrderMade_t^h * A
\end{aligned}
$$

The objective of the Finished Product model was to maximize the profit through maximizing the income gained from meeting all demand $(SP * D)$ and minimizing the total cost of purchasing $(C * D)$, inbound shipping $\left(D * TransCost^{in} * C\right)$, outbound shipping $(D * TransCost^{out})$, storing $(I * HC)$ the product groups Headboard (h) and Mattress Topper (m) over the period T (48) and the setup cost (A) for each order made during period T (48) while adhering to all the limitations and assumptions stated in chapter 7.2.1.

7.3.2.4 Constraints

(1) $YC_{m,t} \geq MinLot_m$ $\quad \forall t \text{ and } \forall m$

(2) $Batch_m = \dfrac{Y_{m,t}}{BatchSize_m}$ $\quad \forall t \text{ and } \forall m$

(3) $Y_{h,t} \geq MinLot_h * OrderMade_t^h$ $\quad \forall t \text{ and } \forall h$

(4) $Batch_h = \dfrac{Y_{h,t}}{BatchSize_h}$ $\quad \forall t \text{ and } \forall h$

(5) $I_{m,0} = Init_m$ $\quad \forall m$

(6) $I_{m,t} = I_{m,t-1} + YC_{m,t} - d_{m,t} \geq 0$ $\quad \forall t \text{ and } \forall m$

(7) $I_{h,0} = Init_h$ $\quad \forall h$

(8) $I_{h,t} = I_{h,t-1} + Y_{h,t} - d_{h,t} \geq 0$ $\quad \forall t \text{ and } \forall h$

(9) if $\sum_h Y_{h,t} \begin{cases} = 0 \text{ then } OrderMade_t^h = 0 \\ \neq 0 \text{ then } OrderMade_t^h = 1 \end{cases}$ $\quad \forall t \text{ and } \forall h$

(10) $avgInvCost_h = \sum_h^{T=1} \dfrac{I_{h,t}}{48} * C_h * 365$ $\quad \forall t \text{ and } \forall h$

(11) $COGS_h = \sum_h^{T=1} C_h * D_{h,t}$ $\quad \forall t \text{ and } \forall h$

(12) $\dfrac{avgInvCost_h}{COGS_h} \leq 90$

(13) $\sum_{T=1} OrderMade_h \geq 24$

7.3.2.5 Explanation of the Constraints

(1) Minimum quantity of Mattress consumed in period t

(2) Forcing any Mattress order to be in multiple of the Batch Size value

(3) If an order for a Headboard is made, the min order quantity is set

(4) Forcing any Headboard order to be in multiple of the Batch Size value

(5) Setting the initial inventory for each Mattress product

(6) Inventory for each Mattress product at the end of period t

(7) Setting the initial inventory for each Headboard product

(8) Inventory for each Headboard product at the end of period t

(9) Variable equal 1 if any Headboard products are ordered at time *t*, otherwise 0

(10) Calculates the average inventory order cost for all Headboard products times 365

(11) Calculates the total Cost of Headboard products Sold during all T

(12) Restricting the number of days Headboard products are in inventory

(13) Forcing the model to make at least 12 Headboard orders every year

8. Analysis & Discussion

8.1 Computational Results

Computer	Lenovo Ideapad 530S-14IKB
Processor	Intel(R) Core (TM) i5-8250U CPU @ 1.60GHz 1.80 GHz
Memory (RAM)	8.00 GB
Operating System	Windows 10, v10.0, x86_64/win32
AMPL IDE Version	3.6.7.202106142223
Gurobi Solver Version	9.5.0

Table 7 : Computer Stats

The two models (Production and Finished) each contain many factors, data points, many restrictions and have a time horizon of 48 weeks, which caused the running time of the model to in some cases exceed the stated time limit of 15 minutes. The time limit was chosen since the model for longer periods (up to 8 hours) did not improve the results significantly.

A large cause of the longer running time is having T=48, as it drastically increases the iterations and branch-and-cut nodes the models must go through. Reducing T to monthly (T=12) would not provide enough information for the model to be implemented in the real world and did not provide any significant computational time benefits. The same applies to doing analysis daily (T=365) as the model would not produce practical or realistic output.

8.2 Comparison

In this chapter, we will compare the actual results with the forecasted results. To be able to make a fair comparison and to avoid data leakage, a 2021 version of the forecast was created that was only trained on data up to and including 2020.

	Headboard	LatexGel	Mattress Cover	Mattress Topper
Forecasted Avg. inventory (in units)	2 450	1 473	1 729	486
Actual Avg. Inventory (in units)	1 012	877	4 417	888

Table 8 : Average Inventory Forecasted 2021 Model

In the table above, the average inventory for the forecasted model and the average inventory for 2021 is shown. For the Mattress Cover and Mattress Topper product group, the forecasted model decreased the inventory significantly while for the other two products the inventory increased. The reason for this increase in inventory is related to constraint 13 in the finished product model (for Headboard) and constraint 17 in the production model (for LatexGel). Decreasing the minimum orders per year will drastically decrease the forecasted average inventory because of two reasons: both Batch Size and a minimum number of orders forcing the procurement of units that might not be demanded. Overall, the average inventory between the actual and the forecasted version was improved by 1 056 units.

Part of what increases the inventory for example Headboards is due to issues with dividing by zero in one of the constraints, the minimum Batch Size must be set at 1 which causes the model to procure at least one unit of each material for every order. This causes the minimum order quantity for headboards to go from 291 to 485 which over 12 orders (as is the standard in the default model) equal 2 328 extra units.

What this also shows, is that the stated goal of DSI being a maximum of 90 days is not the optimal solution. If we remove constraints 11 to 18 from the production model and 10 to 13 from the finished product model, the DSI ends up as the table below shows. In chapter 8.3, we will discuss this table and what it means for sub-question 2 in the problem statement.

	Headboard	**LatexGel**	**Mattress Cover**	**Mattress Topper**
DSI	177	178	187	1.76
Orders Made	1	1	1	-

Table 9 : Days in Inventory (DSI)

The very low average inventory and DSI for Mattress Topper are because it is a finished product that is being produced by Ekornes Beds AS, before being shipped out. As one of the assumptions of this thesis is that they always have materials available when they want them and no production constraints, the model produces the materials when an order needs to be shipped. The DSI for the 3 other product groups is caused by the low holding cost and no capacity restriction, which makes the company able to hold large quantities of materials at a low cost.

The table below shows the deviation between the actual and forecasted demand for 2021. In Appendix C, the individual tables for actual and forecasted demand are attached.

The formula for Errors (from chapter 4.1.1)

$e_t = A_t - F_t =$ forecast error for period t

Months	Headboard	LatexGel	Mattress Cover	Mattress Topper
1	-408	72	-378	-74
2	-67	-59	-1004	228
3	-145	-274	-516	546
4	-154	-123	-365	710
5	-333	-340	-798	147
6	-97	-71	439	1094
7	-443	-91	313	121
8	-218	-41	-113	398
9	-314	78	-121	632
10	-359	-186	-130	554
11	-124	-379	-821	578
12	-171	-528	-965	-551

Table 10 : Deviation Between Actual and Forecasted Demand 2021

From the table, our model mostly overestimates the future demand for all product groups except Mattress Topper. The cause of this is likely related to the change in demand during the pandemic in 2020 which affected the forecast significantly. If we compare it with the 2022 version of the model, the demand decreases significantly.

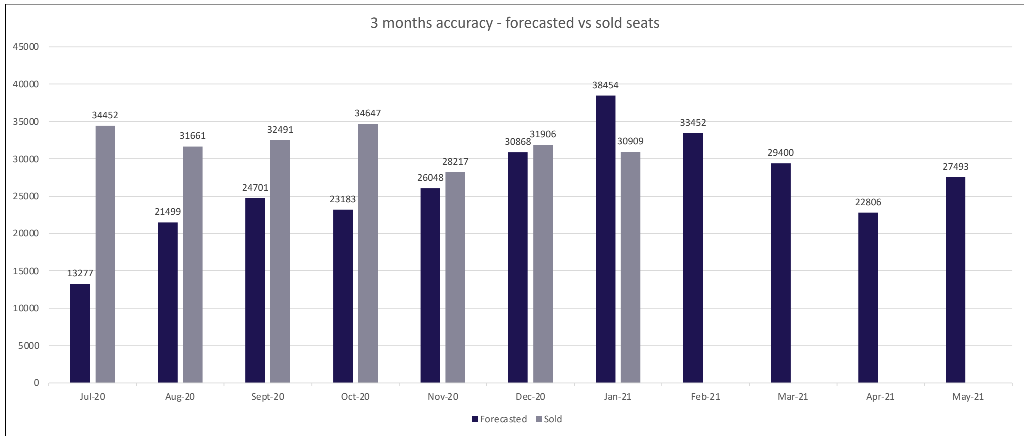

Figure 23 : 3 Months Accuracy for Ekornes AS

This figure shows Ekornes Beds AS parent company Ekornes's 3-month accuracy forecast from July 2020 to January 2021. Both companies use the same method for forecasting the demand for future periods. The figure shows that they have large gaps between the actual units sold and the forecasted number even when only forecasting 3 months ahead, while we are attempting to do a 12-month forecast. They are selling a lot more than they expected, which also could be related to the changing demand caused by the pandemic.

To answer the research question *"How to Determine the Optimal Production Procurement Plan with The Objective of Maximizing the Total Profit Using a Mathematical Optimization Model"* we need to compare the actual results and our optimization model to see how the model does in comparison with the actual numbers.

	Actual model		Optimization model	
	LatexGel	**Mattress Cover**	**LatexGel**	**Mattress Cover**
Cost	*4 049'*	*14 271'*	*5 995'*	*19 780'*
Holding Cost	*32'*	*79'*	*50'*	*59'*
Transport Cost	*445'*	*1 569'*	*659'*	*2 176*
Setup Cost	*25'*	*277'*	*23'*	*23'*
Total Cost	**4 551'**	**16 198'**	**6 727'**	**22 038'**

Table 11 : Cost comparison (in 1 000) for actual vs optimization production model

Table 11 shows a breakdown of the cost components of the production model. From the table, the actual total cost is around 20.75 million NOK, and the proposed model is around 28.77 million NOK. Thus, for the production model, the answer to sub-question 1 on whether the proposed model reduces the total costs is no. Overall, there is a 39% increase in costs which is about equally distributed between the product groups.

The worsening of the total cost in the proposed model is likely caused by the increase in demand forecasted. For example, the proposed model gives a LatexGel demand of 7 772 while the actual demand was 5 755 for a 35% increased demand. The same applies to Mattress Cover with forecasted demand at 34 057 and actual at 29 477 for a 16% increase. As mentioned earlier, the start of the Pandemic in 2020 caused a sharp increase in the demand for Ekornes Beds AS's products which significantly impacted the forecasted quantities for the 2021 model. In the 2022 model which included the historical data from 2021, the total cost was around 19 million NOK which would have been an improvement of 8%.

	Actual model		Optimization model	
	Headboard	**Mattress Topper**	**Headboard**	**Mattress Topper**
Income	*26 608'*	*94 441'*	*46 687'*	*97 241'*
Cost	*10 744'*	*30 937'*	*18 602'*	*32 604'*
Holding Cost	*56'*	*31'*	*85'*	*17'*
Transport Cost Inbound	*645'*	*0'*	*1 116'*	*-*
Transport Cost Outbound	*752'*	*928'*	*1 356'*	*1 045'*
Setup Cost	*451'*	*0'*	*23'*	*-*
Total Profit	**13 961'**	**62 545'**	**25 505'**	**63 575'**

Table 12 : Total profit (in 1 000) for actual vs optimization finished

The table above shows the breakdown of the total profit from the current practice and the proposed finished product model. Combined, the total profit Ekornes Beds AS achieved in 2021 was 76.51 million NOK and the proposed model achieved 89.08 million NOK for a total improvement of around 16%. Again, as with the Production model, the pandemic in 2020 affected the demand forecast for Headboards with a forecast of 73% above actuals which could account for part of the impressive profit increase of 83% in that product category. Mattress Toppers on the other hand had a 9% lower demand forecast than actual while still improving the profit by 2%.

Below is the figure below, we can see how the inventory value has changed through the 48 weeks. The increase in value toward the end of the period is as mentioned caused by forcing the model to do 12 orders in the default model and batch sizes being increased to 485 instead of 291 due to issues with dividing by 0.

```
ampl: display {t in 0..T} sum{p_H in P_Headboard} I_Headboard[p_H,t]*C_Headboard[p_H];
sum{p_H in P_Headboard} I_Headboard[p_H,t]*C_Headboard[p_H] [*] :=
 0 1895260    9 3370200   18 1890450   27 2454430   36 3063880   45 2232340
 1 4622960   10 3113140   19 1657320   28 2184350   37 2476400   46 2761080
 2 4232450   11 2591100   20 1470180   29 1561710   38 2153890   47 2500630
 3 3848390   12 2390540   21 3135740   30 2458940   39 1909400   48 3004570
 4 3215690   13 1853420   22 2835890   31 2101290   40 3246110
 5 2572180   14 1581290   23 2581620   32 1743630   41 2607480
 6 2196860   15 1354740   24 2281770   33 2571780   42 2233840
 7 1821540   16 2621700   25 1746710   34 2197480   43 1980860
 8 1446220   17 2123590   26 1476630   35 1852420   44 2790190
;
```

Figure 24 : Inventory Value for Headboard

8.3 Scenario Analysis

In this chapter, we analyze *What happens if we change the minimum orders made constraint?* And subsequently answer the sub-question stated in the problem statement: *"Is the stated goal of maximum 90 days of Days Sales in Inventory the optimal solution?"*.

We have done all the scenarios (for both production and finished product model) by step of 4, from the extreme of saying they do not have to make any orders (but will make at least 1) to having to make an order every week. In doing the scenario analysis, we removed constraints 13 and 16 from the Production Model and constraint 12 from the finished product model (the 90 days DSI constraint) to avoid contamination of the results.

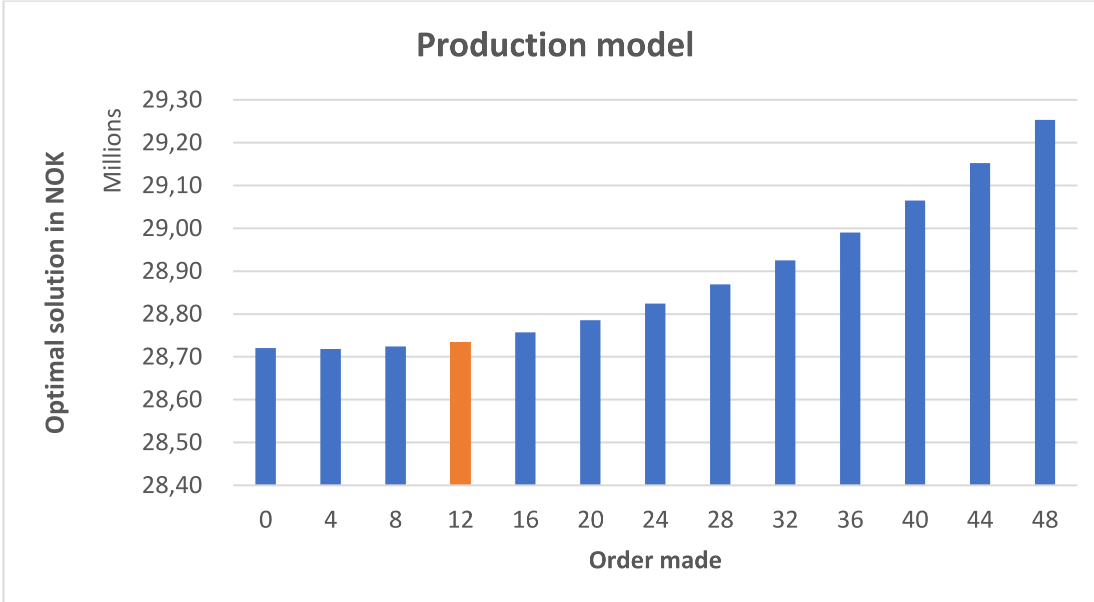

Figure 25 : Production Model, - Scenario Analysis

The figure above shows the different total costs depending on the minimum orders made. The Orange bar indicates a minimum of 12 orders, which is the default amount used for both LatexGel and Mattress Cover.

Zero orders mean that the model decides by its settings what the order frequency should be in the procurement plan. From the figure, you can see that the optimal minimum order constraint for the model is at 4 orders per year or once per quarter which stays around the 90 days DSI goal from the management without having it as a constraint. But in comparison with what they do today, the industry standard and realism, this is not realistic frequency at it would put extreme pressure on getting the forecast correct and leave no flexibility if any unexpected situations occur.

The shape of Figure 25 makes a lot of sense in that a higher-ordered frequency results in a higher total cost. With higher-order frequency, the company will incur more setup costs and potentially be left with more units in storage at the end of the period as there are a minimum number of units of each of the products that are being ordered.

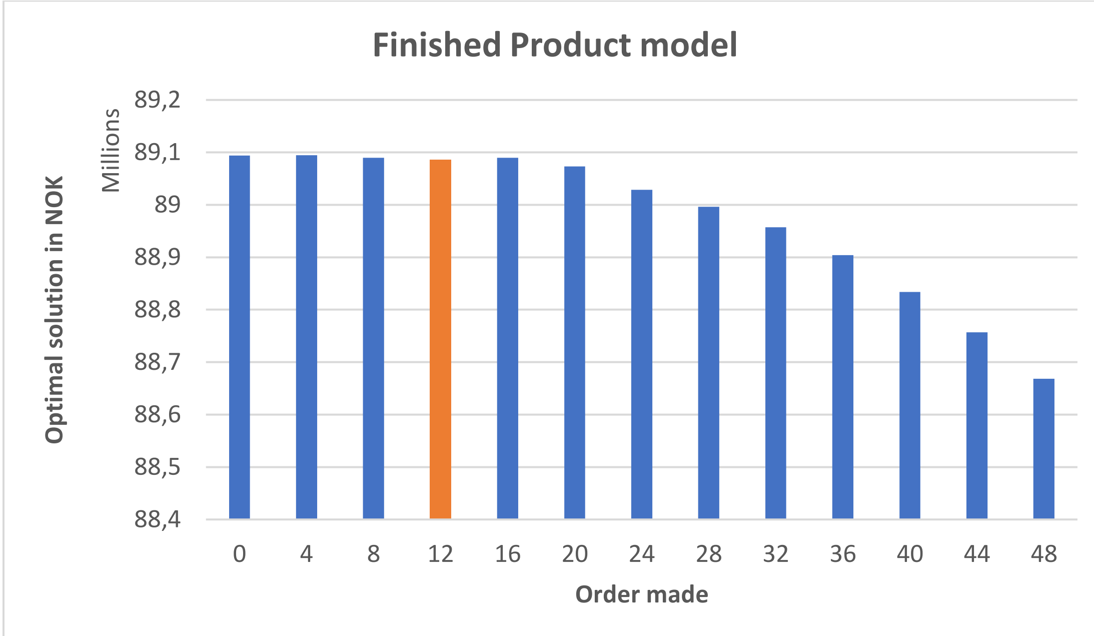

Figure 26 : Finished Product Model, - Scenario Analysis

Moving onto Figure 26, we are now looking at the maximizing of profit for the Finished Product Model. With Orange still representing the minimum 12 orders from the default model and the 90 days DSI restriction having been removed.

The profit in the models when the order frequency is low is very close, but the optimal solution is found when the minimum order frequency is set at 4. Again these 4 orders a year would likely make the company stay within the management goal of 90 days DSI even without it being a constraint. The downward trend as the order frequency increases is logical as it is the same reason for the increased cost in the production model. With higher-order frequency, the costs increase while the income remains the same as the model per the assumptions always meets the demand.

To help answer the sub-question two in the problem statement: *"Is the stated goal of maximum 90 days of Days Sales in Inventory the optimal solution?"*, we also looked at what happened if we instead of removing the 90 days DSI restriction, removed constraint 9 and 10 in the production model and constraint 9 in the finished product model (the minimum orders made constraints). The results were conclusive with the finds from the minimum orders scenarios, with costs of 28.72 million NOK in the production model and profit of 89.10 million NOK in the finished product model, this is almost equal to the 4 orders a year.

So combined with the findings from Table 9 in chapter 8.2, we can conclude that the 90 days DSI if the only goal is to minimize costs (production model) and maximize profit (finished product model), then the 90 days goal for DSI is not the optimal solution. But if you want a more realistic model with order frequency being more realistic, then the 90 days goal for DSI does not matter as doing more frequent orders would automatically reduce the DSI below the 90 days threshold if you are not overordering and being left with a large inventory at the end of the time horizon.

8.4 Sensitivity Analysis

In the sub-chapters below, we are going to do a sensitivity analysis on two of the cost factors used in the development of the model. For both analyses, we have chosen to show only the finished product model as the findings for both models are similar (as was also shown in chapter 8.3).

8.4.1 Setup Cost

The setup cost calculated in chapter 6.2.3 is a theoretical assumption as the company were unable to give us any exact or estimation number for it. Thus, to see how sensitive the model was to changes in setup cost we conducted a sensitivity analysis from a wide range of values. The lowest value we tested was 1 912 (in orange) which is the setup cost used in the default model and the highest was 49 912. All the sensitivity analyses were run with the default settings (minimum orders = 12).

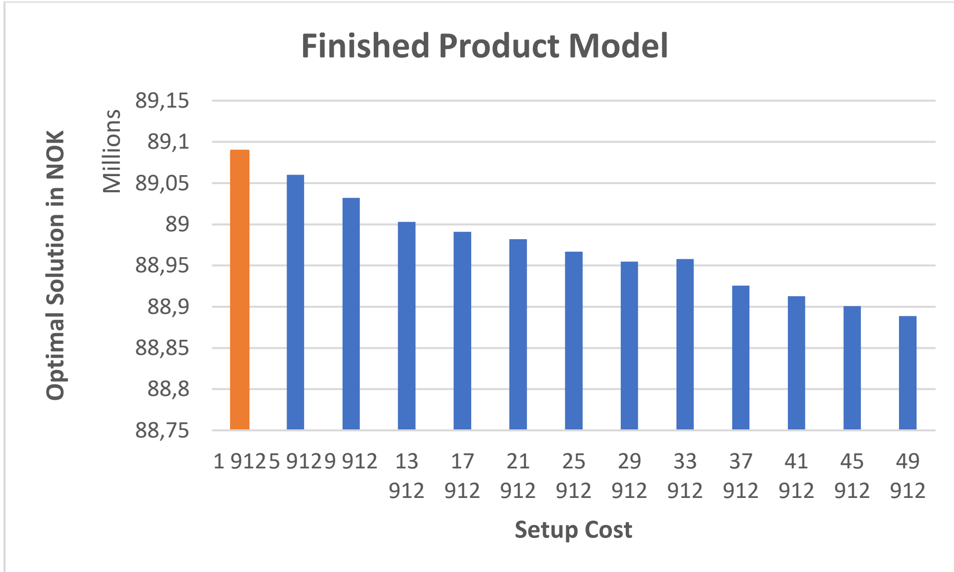

Figure 27 : Finished Product Model - Sensitivity Analysis for Setup Cost

The findings as seen in Figure 27, were that the model was not very sensitive to the level of setup cost. Increasing the setup cost 26 times the default amount only worsens the profit in the finished product model by 0.23% or around 200 000 NOK. Thus, while the setup cost used is theoretical, it does not have any significant impact on the profitability of the finished product model and presumably on the total cost of the production model.

8.4.2 Holding Cost

In this thesis, the holding rate consists of 4 factors (Taxes, cost of obsolete materials, opportunity cost and cost of warehousing). All these factors are susceptible to change and are uncertain when set. Conducting a sensitivity analysis on the parameter will give an overview of how changes would affect the profitability of the finished product model and as an extension the total cost of the production model. For each run of the code, we increased the yearly holding rate from 3.45% by 2% until we reached 26.45%.

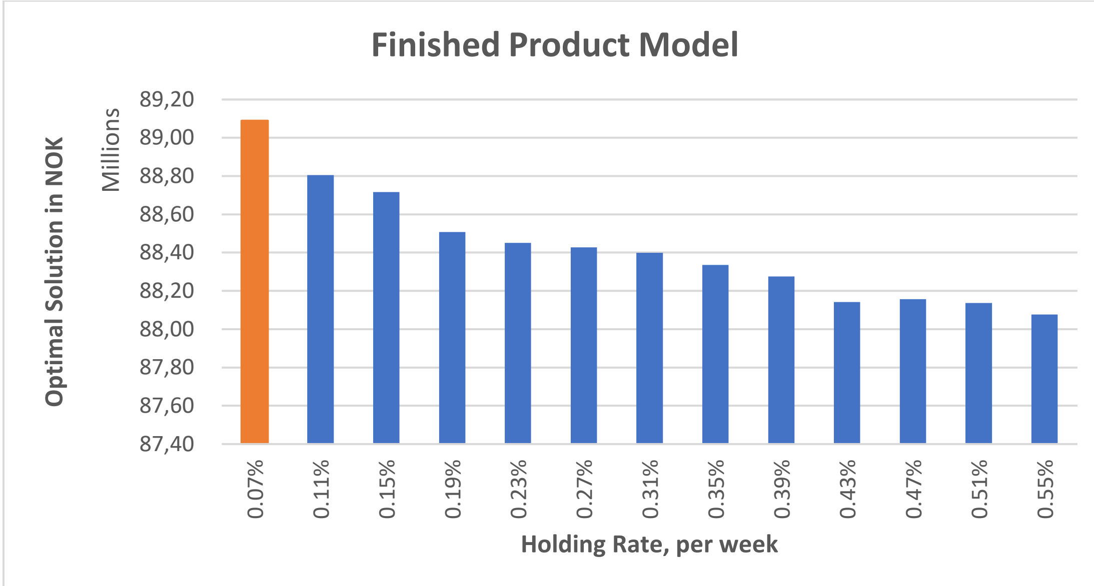

Figure 28 : Finished Product Model - Sensitivity Analysis for Holding Rate

Figure 28 looks very similar to Figure 27 but because the Holding Rate is a cost factor affecting more than just one part of the objective function, the effect is larger. Going up a 0.55% weekly holding rate (or 26.45% per year) would reduce the profitability of the model by only 1.14% or around 1 million NOK. Most of the loss of profit occurs in the first 6% yearly increase in holding rate (from 0.07% to 0.19%) with it combined accounting for 0.65% of the profit loss. The single most costly increase in the holding rate was from 3.45% yearly to 5.45%, which caused the profitability to fall by 0.32%. The relatively large loss of profitability in the beginning in comparison with the end is related to the percentage change in holding rate being larger in the beginning than at the end.

Overall, I would say that the model is somewhat sensitive to changes in the holding rate but the loss in profitability is not significant enough to have a large impact on the overall profitability of the finished product model.

9. Conclusion

This thesis aimed to develop two mathematical optimization models in AMPL to determine the optimal procurement plan, and to compare the results with the current practice in Ekornes Beds AS. As a result of this research, a Mixed Integer Linear Programming (MILP) approach was conducted together with Holt-Winters Exponential Smoothing forecasting to determine the optimal lot sizes for the procurement plan for the company.

In the finished product model, we obtained a considerable improvement of 16% from the optimization model in terms of total profit compared to their actual numbers, with the product category Headboard having the biggest improvement in total profit. Within the production model, we found an increment in the total cost for both the product category LatexGel and Mattress Cover probably caused by the effect of the start of the Pandemic. Coming back to sub-question one, we can conclude that the total cost was not reduced using the optimization model compared to the current practice, but with better forecasts the costs were reduced as proven by the 2022 model.

Afterwards, a scenario and sensitivity analyses were performed. The motivation behind the scenario analysis was to see how our model would perform when different order frequency was conducted from the default settings and how the 90 days of maximum inventory restrictions would impact the optimal solutions. In the sensitivity analysis, the objective was to vary the setup cost and the holding rate independently with the same goal as the scenario analyses to see the changes in the optimal solution.

The key findings from the scenario analysis were that the higher the order-made frequency we have in the finished product model, the larger decrease we will be seen in total profit. For the production model, the higher the order made frequency is the more the total cost will in order-made found out that the 90 days of DSI (Days Sales in Inventory) is not the optimal solution if the only goal is to minimize cost (production model) and maximize profit (finished product model). More realistically if Ekornes Beds AS want to practice the 90 DSI limitation then executing more orders would not reduce the DSI below the 90 days threshold if they do not overorder and are left with a large inventory at the end of the time horizon as explained in chapter 8.3.

From the sensitivity analysis, we found out that increasing the setup cost 26 times from the default settings worsens the profit for the finished product model only by 0.23% or around 200 000 NOK. From the holding rate changes, we found out that going up from 0.07% to 0.55% in holding rate weekly will only reduce the profitability of the model only by 1.14% or around 1 million NOK. So, overall, the model is not very sensitive to the change in those two cost factors.

9.1 Contribution

In this sub-chapter, we will talk about the contribution this thesis could have on academic research and to Ekornes Beds AS.

9.1.1 Contributions to Academic research

Writing about economic lot-sizing models by using mathematical optimization approaches with the mixed-integer linear programming method is a challenging task because of the limited research that has been done in real business cases, especially those operating in the furniture market. Our thesis will have the following contributions to further academic research:

1. The thesis uses real data from the company that applies to their current practice, with a practical model combined with theoretical components put into practice.
2. Reviewing relevant academic literature, we found a gap between current literature and the actual business. Although our company base some of their decision on gut feeling, there are very few articles that discuss the interaction human inputs will have in this kind of decision-making in production.
3. This thesis will contribute to mapping out academic research and pointing out potential topics for further studies.

9.1.2 Contributions to Ekornes Beds AS

This thesis could be used by Ekornes Beds AS as a tool and motivation to modify their current practice. While it does not give a definite answer in determining the optimal procurement plan for them due to the limitations and assumptions, it would work well as a tool that nudges in the right direction and could make the people working with the demand forecasting open to new inputs. For example, based on the limitations and assumptions used, the number of orders was drastically reduced from 3 to 5 orders a week for some of the product groups. In addition, even though the model created in this thesis is specific to 4 product groups it is easily adjustable and could be used for any of Ekornes' product groups and even other firms than them.

The Thesis also highlighted the importance of demand forecasting, while the 2021 model overestimated the demand quite a bit for some product groups (due to the 2020 pandemic start) the 2022 model appears to be much more on track. Using the demand forecasts as part of the decision-making in the current practice could reduce unnecessary costs and increase revenue while ensuring fewer stockout situations.

Implementation

A possible way that could make the results and findings from this thesis more practical and reasonable is to use SAP together with AMPL. Below is a very short guide on one way to make that possible.

1. Identify a specific optimization problem.
2. Make a spreadsheet in excel and connect it to SAP via the recoding function in SAP.
3. Connect the excel file with AMPL with the run file that is attached in the appendix sections E and H.
4. Use the mathematical model (in appendix D and G) as an inspiration to develop a specific optimization problem in your factory.
5. Find the right solver to use either CPLEX or Gurobi.
6. Run the model.

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Appendix

Appendix A: Capacitated Lot Sizing Problem

Capacitated Lot Sizing Problem

The capacity restrictions that exist in capacitated lot-sizing models make this very hard to solve by using dynamic programming techniques. As shown in figure 1 we can see that this lot sizing problem is classified into small and big bucket lot sizing models.

In ***Small Bucket Lot Sizing Models***, the sequencing and the lot sizing are made simultaneously and at most two setups are taking place in a time period (Ramya et al, 2019). There are many different small buckets models introduced in the literature such as *Discrete Lot sizing and Scheduling Problem* (DLSP), *Continuous Setup Lot sizing Problem* (CSLP), and *Proportional Lot sizing and Scheduling Problem* (PLSP).

In ***Big Bucket Lot Sizing Models,*** we can find the famous capacitated lot-sizing problem (CLSP), it is only included in the presence of setup costs. The CLSP is referred to as the model with the optimal production plan for multiple items with a sequence-independent setup cost without any setup time and having a constraint for capacity on a single machine. However, when setup cost and setup time are assumed in the CLSP only then several items can be produced using one resource with a capacity limit (Ramya et al, 2019).

Below is the basic mathematical formulation with its assumptions for the CLSP model with setup times and setup costs:

Notations

i: Products

t: Periods

Parameters

a_i: Production coefficient / the capacity consumption to produce one unit of product i

$b_{i,t}$: Large number, not limiting feasible production quantities of product i in period t

C_t: Available capacity in period t

$d_{i,t}$: Demand for product i in period t

$h_{i,t}$: Holding cost per unit of product i in period t

SC_i: Setup cost for product i

ST_i: Setup time for product i

Variables

$I_{i,t}$: Inventory of product i at the end of period t

$X_{i,t}$: Production quantity of product i in period t

$Y_{i,t}$: Binary decision variable which takes the value 1, if a setup for the product i is performed in period t; 0 otherwise

Formulation

Objective Function

Minimize

$$\sum_{i=1}^{N}\sum_{t=1}^{T} SC_i Y_{i,t} + \sum_{j=1}^{N}\sum_{t=1}^{T} h_{i,t} I_{i,t} \tag{1.1}$$

subject to the following:

$$I_{i,t-1} + X_{i,t} = d_{i,t} + I_{i,t} \qquad \forall i \text{ and } \forall t. \tag{1.2}$$

$$\sum_{i=1}^{N} a_i X_{i,t} + \sum_{i=1}^{N} ST_i Y_{i,t} \leq C_t \qquad \forall t. \tag{1.3}$$

$$X_{i,t} \leq b_{i,t} Y_{i,t} \qquad \forall i \text{ and } \forall t. \tag{1.4}$$

$$X_{i,t} \geq 0,\ I_{i,t} \geq 0 \text{ and } I_{i,0} = 0 \qquad \forall i \text{ and } \forall t. \tag{1.5}$$

$$Y_{i,t} \in \{0,1\} \qquad \forall i \text{ and } \forall t. \tag{1.6}$$

This is an example of a mathematical formulation where the CLSP is addressed with setup cost and setup times (Ramya et al, 2019).

Model Assumptions

- The products have finite capacity.
- The products are made up of a single level.
- Deterministic demand for the products.
- Finite-time horizon with discrete-time unit.
- When a product is produced the resources have to be set up for the product in the same period.

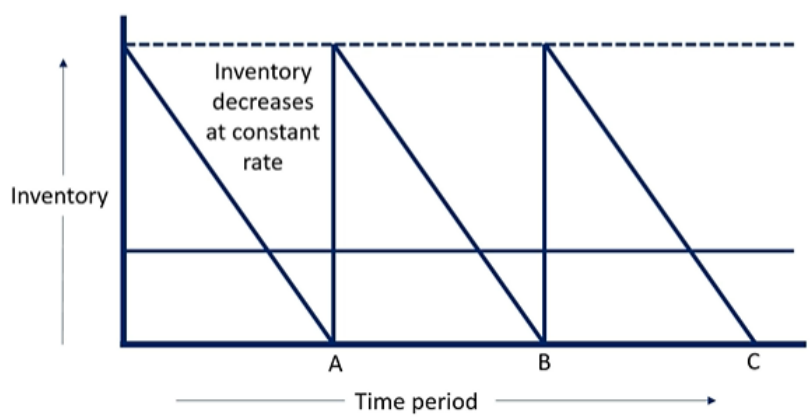
(Ramya et.al, 2019).

<u>Demand Rate-Oriented Models/Methods</u>

Demand rate-oriented models also called static methods generate order quantities that are equal to the net requirements in the planning periods (order quantities). Below we will introduce two of the demand-rate-oriented models.

Fixed Order Quantity (FOQ) Economic Order Quantity (EOQ)

The FOQ model in practice is used only for a limited selected item. According to Ptak & Orlicky (2011), this method applies to items with ordering cost that is sufficiently high to rule out by ordering in net requirements quantities period by period. The order quantity is a predetermined fixed quantity. The advantage of this model is that it is very easily understood, but one downside is that it does not do a good job of minimizing the cost that is involved in the model (Chapman et.al, 2017).



Economic Order Quantity (EOQ) is one of the most researched models and it is used a lot in inventory and production management. It was first introduced by Ford W. Harris in 1915 and is considered one of the oldest lot-sizing techniques in the field of production planning. The EOQ model is another type of static model that is used to determine the frequency and volume of orders that are required to satisfy a level of demand that is given while at the same time helping companies minimise the cost per order and holding cost (Ptak & Orlicky, 2011) while assuming a constant rate of demand per period that is known in the future rather than historical demand (Drechsel, 2010).

Below is the formula for calculating Economic Order Quantity (EOQ) or just Q:

$$\text{EOQ (Q)} = \sqrt{\frac{2 \times A \times D}{r \times v}}.$$

Appendix B: Python code

This is the python code used to create the forecasted demand for the Product Group Headboard. The code for the three other product groups is almost identical, so to save space we have elected not to include it in the appendix.

```python
#!/usr/bin/env python
# coding: utf-8

# In[1]:


import warnings
import itertools
import pandas as pd
import numpy as np
import statsmodels.api as sm
from random import gauss
from pandas.plotting import autocorrelation_plot
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from random import random
from statsmodels.tsa.holtwinters import SimpleExpSmoothing, ExponentialSmoothing, Holt

import matplotlib.pyplot as plt
plt.style.use('fivethirtyeight')

import warnings
warnings.simplefilter(action='ignore', category= FutureWarning)


# In[2]:


# Import data

df=pd.read_excel("Forecast Hodegavl.xlsx")
df["Year"] = pd.to_datetime(df["Year"])
df.set_index("Year", inplace = True)


# In[12]:



alpha = 0.5
df.plot(figsize=(20,10)).set_ylabel('Sales')
fit1 = ExponentialSmoothing(df, seasonal_periods=12, trend='add', seasonal='add')
fit1 = fit1.fit(smoothing_level=0.5,use_boxcox=True)
fit1.fittedvalues.plot(color='red')
fit1.forecast(12).rename("Forecast").plot(color='red', legend=True)
plt.title('Headboard')
plt.show()
```

```python
# In[4]:

df["Forecast_2015-2021"] = fit1.fittedvalues

# In[5]:
decomp = sm.tsa.seasonal_decompose(df["Sales"], period=12)
figure = decomp.plot()
figure.set_size_inches(18.5, 10.5, forward=True)
plt.show()

# In[6]:
from pandas.tseries.offsets import DateOffset
future_dates=[df.index[-1]+ DateOffset(months=x)for x in range(0,13)]

# In[7]:
future_datest_df=pd.DataFrame(index=future_dates[1:],columns=df.columns)

# In[8]:
future_df=pd.concat([df,future_datest_df])

# In[9]:
future_df['Forecast_2022'] = fit1.forecast(12)

# In[10]:

future_df

# In[11]:
file_name = "headboard_exp_smooth.xlsx"
future_df.to_excel(file_name)

# In[11]:
df_mean = df.rolling(window = 13).mean()

# In[12]:
plt.figure(figsize=(10, 5))
df_mean.Sales.plot()
plt.show()
```

Appendix C: Demand tables used 2021 comparison

Actual demand for 2021

Months	Headboard	LatexGel	Mattress Cover	Mattress Topper
1	950	1056	3782	3359
2	1000	693	2129	2572
3	755	258	1434	2160
4	635	369	1800	2141
5	430	300	1646	1833
6	758	670	3656	3346
7	380	317	2039	1797
8	811	586	2789	3140
9	878	582	2513	2820
10	723	472	2548	2528
11	990	369	2900	3357
12	674	158	2362	2065

Forecasted demand for 2021 in the Proposed Model

Months	Headboard	LatexGel	Mattress Cover	Mattress Topper
1	1358	984	4160	3433
2	1067	752	3133	2344
3	900	532	1950	1614
4	789	492	2165	1431
5	763	640	2444	1686
6	855	741	3217	2252
7	823	408	1726	1676
8	1029	627	2902	2742
9	1192	504	2634	2188
10	1082	658	2678	1974
11	1114	748	3721	2779
12	845	686	3327	2616

Appendix D: AMPL Model Code – Production Model

```
reset;
option solver gurobi;

# Set #
set P_LatexGel ordered; # Product Type LatexGel
set P_Trekk ordered; # Product Type Trekk

# Parameters #
param T; # Time Horizon in Weeks

param D_LatexGel{P_LatexGel,  1..T}; # Demand LatexGel
param D_Trekk{P_Trekk,1..T}; # Demand Mattress Cover

param HC; # Holding Cost

param A; # Setup Cost
param C_LatexGel{P_LatexGel}; # Order Cost LatexGel
param C_Trekk{P_Trekk}; # Order Cost LatexGel

param TransCost; #Transportation Cost

param Init_LatexGel{P_LatexGel}; #Initial Inventory - LatexGel
param Init_Trekk{P_Trekk}; #Initial Inventory - Mattress Cover

param MinLot_LatexGel{P_LatexGel}; # Minimum Purchase lot size LatexGel
param MinLot_Trekk{P_Trekk}; # Minimum Purchase lot size Mattress Cover

param BatchSize_LatexGel{P_LatexGel}; # Rounding Value LatexGel
param BatchSize_Trekk{P_Trekk}; # Rounding Value Mattress Cover

# Variables #
var I_LatexGel{P_LatexGel,0..T} >=0 integer; # Inventory Level at end of T - LatexGel
var I_Trekk{P_Trekk,0..T} >=0 integer; # Inventory Level at end of T - Mattress Cover
var Y_LatexGel{P_LatexGel,1..T} >=0; # Purchase quantity LatexGel
var Y_Trekk{P_Trekk,1..T} >=0; # Purchase quantity Mattress Cover

var Batch_LatexGel{P_LatexGel, 1..T} integer; # Rounding Value LatexGel
var Batch_Trekk{P_Trekk, 1..T} integer; # Rounding Value Mattress Cover

# Binary variables #
var OrderMade_LatexGel {1..T} binary;
var OrderMade_Trekk {1..T} binary;
```

```
# DSI variables #
var COGS_Trekk;
var avgInvCost_Trekk;

var COGS_LatexGel;
var avgInvCost_LatexGel;

# Objective Function #
minimize TotalCost:
    sum{p_LG in P_LatexGel, t in 1..T} C_LatexGel[p_LG]*D_LatexGel[p_LG,t]
        +sum{p_LG in P_LatexGel, t in 1..T} I_LatexGel[p_LG,t]*HC*C_LatexGel[p_LG]
        +sum{t in 1..T} OrderMade_LatexGel[t]*A
        +sum{p_LG in P_LatexGel, t in 1..T} D_LatexGel[p_LG,t]*TransCost*C_LatexGel[p_LG]
        +sum{p_T in P_Trekk, t in 1..T} C_Trekk[p_T]*D_Trekk[p_T,t]
        +sum{p_T in P_Trekk, t in 1..T} I_Trekk[p_T,t]*HC*C_Trekk[p_T]
        +sum{t in 1..T} OrderMade_Trekk[t]*A
        +sum{p_T in P_Trekk, t in 1..T} D_Trekk[p_T,t]*TransCost*C_Trekk[p_T];

# Constraints #
# Purchase Level - LatexGel #
s.t. MinLotSize_LAtexGel {p_LG in P_LatexGel, t in 1..T}:
    Y_LatexGel[p_LG,t] >= MinLot_LatexGel[p_LG]*OrderMade_LatexGel[t];

s.t. RoundValues_LatexGel {p_LG in P_LatexGel, t in 1..T}:
    Batch_LatexGel[p_LG,t] = Y_LatexGel[p_LG,t]/BatchSize_LatexGel[p_LG];

# Purchase Level - Trekk #
s.t. MinLotSize_Trekk {p_T in P_Trekk,t in 1..T}:
    Y_Trekk[p_T,t] >= MinLot_Trekk[p_T]*OrderMade_Trekk[t];

s.t. RoundValues_Trekk {p_T in P_Trekk, t in 1..T}:
    Batch_Trekk[p_T,t] = Y_Trekk[p_T,t]/BatchSize_Trekk[p_T];

# Inventory - LatexGel #
s.t. InventoryInitial_LatexGel {p_LG in P_LatexGel}:
    I_LatexGel[p_LG,0] = Init_LatexGel[p_LG];

s.t. Inventory_LatexGel {p_LG in P_LatexGel,t in 1..T}:
    I_LatexGel[p_LG,t] = I_LatexGel[p_LG,t-1] + Y_LatexGel[p_LG,t]- D_LatexGel[p_LG,t];
```

```
# Inventory - Trekk #
s.t. InventoryInitial_Trekk {p_T in P_Trekk}:
    I_Trekk[p_T,0] = Init_Trekk[p_T];

s.t. Inventory_Trekk {p_T in P_Trekk, t in 1..T}:
    I_Trekk[p_T,t] = I_Trekk[p_T,t-1] + Y_Trekk[p_T,t]-D_Trekk[p_T,t];

# Costs - LatexGel #
s.t. OrderPlaced_LatexGel {t in 1..T}:
    OrderMade_LatexGel[t] = 0 ==> sum{p_LG in P_LatexGel} Y_LatexGel[p_LG,t] = 0
    else OrderMade_LatexGel[t] = 1;

# Costs - Trekk #
s.t. OrderPlaced_Trekk {t in 1..T}:
    OrderMade_Trekk[t] = 0 ==> sum{p_T in P_Trekk} Y_Trekk[p_T,t] = 0
    else OrderMade_Trekk[t] = 1;

# DSI Trekk #
s.t. InventoryAverage_Trekk:
    avgInvCost_Trekk = sum{p_T in P_Trekk,t in 0..T}
    ((((I_Trekk[p_T,t])/48)*C_Trekk[p_T])*365);

s.t. CostOfGoodSold_Trekk:
    COGS_Trekk = sum{p_T in P_Trekk, t in 1..T} C_Trekk[p_T]*D_Trekk[p_T,t];

s.t. DSI_Trekk:
    avgInvCost_Trekk/COGS_Trekk <= 90;

# DSI LatexGel #
s.t. InventoryAverage_LatexGel:
    avgInvCost_LatexGel = sum{p_LG in P_LatexGel,t in 0..T}
    ((((I_LatexGel[p_LG,t])/48)*C_LatexGel[p_LG])*365);

s.t. CostOfGoodSold_LatexGel:
    COGS_LatexGel = sum{p_LG in P_LatexGel, t in 1..T} C_LatexGel[p_LG]*D_LatexGel[p_LG,t];

s.t. DSI_LatexGel:
    avgInvCost_LatexGel/COGS_LatexGel <= 90;

# Minimum Orders Made #
s.t. MinimumOrdresTrekk:
    sum{t in 1..T} OrderMade_Trekk[t] >= 12;
```

```
data;
# Parameters #
param T = 48;
param A := 1911.77;
param TransCost:= 0.11;
param HC  := 0.0007188;
```

Appendix E: AMPL Run Code – Production Model

```
load amplxl.dll;
model ProductionModel.mod;

#LatexGel tables import from excel
table productL IN "amplxl" "ProductionModel.xlsx" "ProdL": P_LatexGel <- [P_LatexGel],
C_LatexGel,BatchSize_LatexGel,Init_LatexGel, MinLot_LatexGel;
table demandL IN "amplxl" "ProductionModel.xlsx" "DemandL" "2D": [P_LatexGel,T], D_LatexGel;

#Trekk tables from excel
table productT IN "amplxl" "ProductionModel.xlsx" "ProdT": P_Trekk <- [P_Trekk],
C_Trekk,BatchSize_Trekk,Init_Trekk, MinLot_Trekk;
table demandT IN "amplxl" "ProductionModel.xlsx" "DemandT" "2D": [P_Trekk,T], D_Trekk;


# LatexGel Tables
read table productL;
read table demandL;

# Trekk Tables
read table productT;
read table demandT;

option gurobi_options 'timelim 900';
solve;
display _solve_elapsed_time;



# Results Table for LatexGel
table LatexGelRES OUT "amplxl" "LatexGelRES.xlsx" "Results":
 [P_LatexGel,T],{p in P_LatexGel, t in 1..T} (Y_LatexGel[p,t],I_LatexGel[p,t],D_LatexGel[p,t],
 BatchSize_LatexGel[p],Batch_LatexGel[p,t],OrderMade_LatexGel[t]);

# Export LatexGel Tabless to Excel
write table LatexGelRES;

# Results Table for Trekk
table TrekkRES OUT "amplxl" "TrekkRES.xlsx" "Results":
 [P_Trekk,T],{p in P_Trekk, t in 1..T} (Y_Trekk[p,t],I_Trekk[p,t],D_Trekk[p,t],
 BatchSize_Trekk[p],Batch_Trekk[p,t],OrderMade_Trekk[t]);

# Export Mattress Cover Tables to Excel
write table TrekkRES;
```

Appendix G: AMPL Model Code – Finished Product Model

```
reset;
option solver gurobi;

# Set #
set P_Mattress ordered; # Product Type Mattress
set P_Headboard ordered; # Product Type Headboard

# Parameters #
param T; # Time Horizon in weeks

param D_Mattress{P_Mattress,1..T}; # Demand Mattress
param D_Headboard{P_Headboard,1..T}; # Demand Headboard

param HC; #Holding Cost

param TransCostO_Mattress{P_Mattress}; # Transport Cost Outbound Mattress

param TransCostI_Headboard{P_Headboard}; # Transport Cost Inbound Headboard
param TransCostO_Headboard{P_Headboard}; # Transport Cost Outbound Headboard

param Sp_Mattress{P_Mattress}; # Sales Price Mattress
param Sp_Headboard{P_Headboard}; # Saless Price Headboard

param A; # Setup Cost
param C_Mattress{P_Mattress}; # Order Cost Mattress
param C_Headboard{P_Headboard}; # Order Cost Headboard

param Init_Mattress{P_Mattress}; # Initial Inventory Mattress
param Init_Headboard{P_Headboard}; # Initial Inventory Headboard

param MinLot_Mattress{P_Mattress}; # Minimum lot size Mattress
param MinLot_Headboard{P_Headboard}; # Minimum Purchase lot Headboard

param BatchSize_Mattress{P_Mattress}; # Batch Sizes Mattress
param BatchSize_Headboard{P_Headboard}; # Batch Sizes Headboard

# Variables #
var I_Mattress{P_Mattress,0..T} >=0 integer; # Inventory at end of T - Mattress
var I_Headboard{P_Headboard,0..T} >=0 integer; # Inventory at end of T - Headboard
var YC_Mattress{P_Mattress,1..T} >=0; # Consumption amount Mattress
var Y_Headboard{P_Headboard,1..T} >=0; # Purchase amount Headboard
```

```
var Batch_Mattress{P_Mattress, 1..T} integer; # Number of batches Mattress
var Batch_Headboard{P_Headboard, 1..T} integer; # Number of batches Headboard

# DSI Variables #
var COGS_Headboard;
var avgInvCost_Headboard;

# Binary variables #
var OrderMade_Headboard {1..T} binary;

# Objective Function #
maximize TotalCost:
        sum{p_M in P_Mattress, t in 1..T} Sp_Mattress[p_M]*D_Mattress[p_M,t]
        +sum{p_H in P_Headboard, t in 1..T} Sp_Headboard[p_H]*D_Headboard[p_H,t]
        -sum{p_M in P_Mattress, t in 1..T} C_Mattress[p_M]*D_Mattress[p_M,t]
        -sum{p_M in P_Mattress, t in 1..T} I_Mattress[p_M,t]*HC*C_Mattress[p_M]
        -sum{p_M in P_Mattress, t in 1..T} D_Mattress[p_M,t]*TransCostO_Mattress[p_M]
        -sum{p_H in P_Headboard, t in 1..T} C_Headboard[p_H]*D_Headboard[p_H,t]
        -sum{p_H in P_Headboard, t in 1..T} I_Headboard[p_H,t]*HC*C_Headboard[p_H]
        -sum{t in 1..T} OrderMade_Headboard[t]*A
        -sum{p_H in P_Headboard, t in 1..T} D_Headboard[p_H,t]*TransCostI_Headboard[p_H]*C_Headboard[p_H]
        -sum{p_H in P_Headboard, t in 1..T} D_Headboard[p_H,t]*TransCostO_Headboard[p_H];

# Constraints #
# Purchase Level - Mattress Toppper
s.t. MinLotSize_Mattress {p_M in P_Mattress, t in 1..T}:
    YC_Mattress[p_M,t] >= MinLot_Mattress[p_M];

s.t. RoundValues_Mattress {p_M in P_Mattress, t in 1..T}:
    Batch_Mattress[p_M,t] = YC_Mattress[p_M,t]/BatchSize_Mattress[p_M];

# Purchase Level - Headboard #
s.t. MinLotSize_Headboard {p_H in P_Headboard,t in 1..T}:
    Y_Headboard[p_H,t] >= MinLot_Headboard[p_H]*OrderMade_Headboard[t];

s.t. RoundValues_Headboard {p_H in P_Headboard, t in 1..T}:
    Batch_Headboard[p_H,t] = (Y_Headboard[p_H,t]/BatchSize_Headboard[p_H]);

# Inventory - Mattress Toppper #
s.t. InventoryInitial_Mattress {p_M in P_Mattress}:
    I_Mattress[p_M,0] = Init_Mattress[p_M];
```

```
s.t. Inventory_Mattress {p_M in P_Mattress,t in 1..T}:
    I_Mattress[p_M,t] = I_Mattress[p_M,t-1] + YC_Mattress[p_M,t]- D_Mattress[p_M,t];

# Inventory - Headboard #
s.t. InventoryInitial_Headboard {p_H in P_Headboard}:
    I_Headboard[p_H,0] = Init_Headboard[p_H];

s.t. Inventory_Headboard {p_H in P_Headboard, t in 1..T}:
    I_Headboard[p_H,t] = I_Headboard[p_H,t-1] + Y_Headboard[p_H,t]-D_Headboard[p_H,t];

# Costs - Headboard #
s.t. OrderPlaced_Headboard {t in 1..T}:
    OrderMade_Headboard[t] = 0 ==> sum{p_H in P_Headboard} Y_Headboard[p_H,t] = 0
    else OrderMade_Headboard[t] = 1;

# DSI - Headboard;
s.t. InventoryAverage_Headboard:
    avgInvCost_Headboard = sum{p_H in P_Headboard, t in 1..T}
    ((((I_Headboard[p_H,t])/48)*C_Headboard[p_H])*365);

s.t. CostOfGoodsSold_Headboard:
    COGS_Headboard = sum{p_H in P_Headboard, t in 1..T} C_Headboard[p_H]*D_Headboard[p_H,t];

s.t. DSI_Headboard:
    avgInvCost_Headboard/COGS_Headboard <= 90;

# Minimum Orders Made #
s.t. MinimumOrdresHeadboard:
    sum{t in 1..T} OrderMade_Headboard[t] >= 12;

data;
# Parameters #
param T = 48;
param A := 1911.77;
param HC := 0.0007188;
```

Appendix H: AMPL Run Code – Finished Product Model

```
reset;
load amplxl.dll;
model FinishUnitModel.mod;

# Mattress tables import from excel
table productM IN "amplxl" "FinishUnitModel.xlsx" "ProdO": P_Mattress <- [P_Mattress],
Sp_Mattress,TransCostO_Mattress,C_Mattress,BatchSize_Mattress,
Init_Mattress, MinLot_Mattress;
table demandM IN "amplxl" "FinishUnitModel.xlsx" "DemandO" "2D": [P_Mattress,T], D_Mattress;

#Headboard tables from excel
table productH IN "amplxl" "FinishUnitModel.xlsx" "ProdH": P_Headboard <- [P_Headboard],
Sp_Headboard,TransCostI_Headboard,TransCostO_Headboard,C_Headboard,BatchSize_Headboard,
Init_Headboard, MinLot_Headboard;
table demandH IN "amplxl" "FinishUnitModel.xlsx" "DemandH" "2D": [P_Headboard,T], D_Headboard;


# Mattress - Tables
read table productM;
read table demandM;

# Headboard - Tables
read table productH;
read table demandH;

option gurobi_options 'timelim 900';
solve;
display _solve_elapsed_time;

# Results Tables for mattress topper
table MattressTopperRES OUT "amplxl" "mattressRES.xlsx" "Results":
 [P_Mattress,T],{p in P_Mattress, t in 1..T} (YC_Mattress[p,t]
 ,I_Mattress[p,t],D_Mattress[p,t],
 BatchSize_Mattress[p],Batch_Mattress[p,t]);

# Export Table to Excel - Mattress
write table MattressTopperRES;

# Results Table for Headboard
table HeadboardRES OUT "amplxl" "headboardRES.xlsx" "Results":
 [P_Headboard,T],{p in P_Headboard, t in 1..T} (Y_Headboard[p,t]
 ,I_Headboard[p,t],D_Headboard[p,t],
 BatchSize_Headboard[p],Batch_Headboard[p,t],OrderMade_Headboard[t]);

# Export Tables to Excel - Headboard
write table HeadboardRES;
```