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Knowledge Spillovers and R&D Subsidies to New, Emerging Technologies

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Abstract

Is knowledge spillover a rationale for supporting R&D on new, emerging technologies more than R&D on other technologies? In this paper I analyze whether innovation externalities caused only by knowledge spillovers differ between technologies of different maturity. I show that R&D should not be subsidized equally across industries when the knowledge stocks differ. This is because knowledge spillovers depend on the size of the knowledge stock and the elasticity of scale in R&D production. R&D in the emerging technology should be subsidized more when the elasticity is smaller than one. However, R&D in the mature technology should be subsidized more when the elasticity is larger than one.

JEL classification: O30; O31

Keywords: Innovation policy; Knowledge spillovers; Sector-specific R&D.

1 Introduction

Is knowledge spillover a rationale for supporting R&D on new, emerging technologies more than R&D on other technologies? It is well known that the social benefits from R&D may be greater than the private benefits from R&D as knowledge spills across firms¹. These spillovers may be present in both emerging technologies and other more mature technologies. Consequently, there is reason for governments to support all R&D. However, policymakers and environmentalists often claim that R&D in new technologies requires special attention. One recurring argument is that new technologies need a pull or a push to get started since the social benefit of new knowledge is greater in new technologies than in old technologies. Nonetheless, not much is known about the role of technology maturity in relation to knowledge spillovers.

In this paper I analyze whether innovation externalities caused only by knowledge spillovers differ between technologies of different maturity. The maturity of a technology is defined here as the size of the knowledge stock. When a technology is new, the accumulated knowledge stock in that technology is small compared to other more mature technologies. I show that a difference in accumulated knowledge, *ceteris paribus*, is a rationale for differentiated R&D support. The reason is that the knowledge spillovers are related to the knowledge stocks.

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¹See Griliches (1995), Klette et al. (2000), and Hall et al. (2010) for overviews.

I develop a partial equilibrium model in the spirit of a Jones (1995)-type semi-endogenous growth model. The model has two R&D industries which deliver patents (ideas) in two different technologies: one emerging technology and one mature technology. The productivity of the R&D industries is increasing in the accumulated knowledge stock in the respective technologies. On the balanced growth path the model gives the standard result on scale effects for semi-endogenous growth models, i.e. subsidies to R&D do not influence long-run growth rates. However, innovation policy affects the growth along a transition path and thus affects the level of income (production) on the balanced growth path (Jones and Williams, 2000). On the transition path towards the balanced growth path, R&D subsidies correct for the undersupply of R&D from private firms. This correction speeds up the process of reaching the balanced growth path and gives rise to income level effects. Perez-Sebastian (2007) shows that the long-run level effects of innovation policies can be substantial in a semi-endogenous growth model. The main focus in the paper presented here is to study the relative undersupply of R&D in the two types of technology outside the balanced growth path. This relative undersupply arising from the different maturity of technologies is, to my knowledge, not studied much.

The reason maturity matters for optimal policy in this paper is that the production of new ideas depends on the accumulated stock of knowledge. When the knowledge stock increases the private firms get a productivity gain through improved output of conducting R&D. Further, this productivity gain from new ideas is declining in the size of the knowledge stock, i.e. there are decreasing returns to new ideas (Jones, 1995 and 1999). Since these spillovers to future R&D are external to private firms, R&D activity should be subsidized. The size of the spillovers depends on both how large the productivity gain from a new idea is and how many researchers take advantage of that productivity gain, i.e. the level of R&D activity in future periods. R&D activity is higher in the mature technology due to lower costs, while the productivity gain from a new idea is higher in the emerging technology. That the spillovers are dependent on the knowledge stocks implies that R&D subsidies to the two technologies should not be equal.

In this paper the growth rates of the knowledge stocks determine whether the emerging technology or the mature technology is more undersupplied in the market equilibrium. R&D in the technology that grows faster should be subsidized more since the knowledge spillovers are larger in that technology. The reason is that the spillovers and the growth rates of the knowledge stocks are determined by the same two opposing effects. First, both the growth rate and the spillovers in a technology are increasing in the knowledge stock because the labor productivity is high when the knowledge stock is large. Second, both the growth rate and the spillovers in a technology are declining in the knowledge stock because there is less productivity gain from new patents when the knowledge stock is large. The relative strength of the two opposing effects determines whether the growth rate in the mature or the emerging technology is greater.

1.1 Related literature

Low carbon emission technologies – like solar power, carbon capture and sequestration, and hydrogen cars – are typically new, emerging technologies. There is recent literature on R&D subsidies directed to clean, new technologies versus dirty, mature technologies. In particular, Acemoglu et al. (2012) analyze R&D subsidies in a model with clean and dirty technologies where the clean technology has the smaller knowledge stock, i.e. the clean technology is less mature. They find that it is optimal to support clean innovation more than dirty. However, the reason is that the subsidy

to clean technologies is used to deal with future environmental externalities. In other words, it is not differences in innovation externalities that drive the main result in Acemoglu et al. (2012). In the paper presented here, innovation externalities are analyzed in isolation of all other potential market failures, i.e. there are no environmental, or other, externalities. Thus, the only difference between the technologies stems from differences in the knowledge stocks. This keeps the analysis focused on whether knowledge spillovers are a rationale for differentiated R&D support to technologies of different maturity.

Gerlagh et al. (2009 and 2011) also study the maturity of clean technologies and public R&D support. They find that the optimal subsidy to a maturing technology falls over time. However, they find this in a model with one technology sector and thus lack the relative consideration of the undersupply between emerging and mature technologies. Further, their finding comes as a result of inefficiencies in the R&D market related to limited patent lifetime. R&D is biased towards technologies that pay back within the patent lifetime. Hence, subsidies should be larger in early periods because patent lifetime is limited, not because the technology is less developed in the beginning, i.e. a small initial knowledge stock, which is the case in this paper.

How different types of technology can be undersupplied in the market is analyzed by Hart (2008). Rather than looking at public support for technology investment, he implements optimal second-best carbon taxes. These taxes may be higher than the Pigouvian level in order to encourage investment in emissions-saving technology at the expense of general production technology. The reason is that the emissions-saving technology may be relatively more undersupplied than the other technology. However, this result is derived from an increased scarcity of the environmental good through a rising shadow price of emissions rather than the maturity of technology, which is the sole cause of the relative undersupply of technology in this paper.

In another study, Kverndokk and Rosendahl (2007) find that newly adopted technologies should be subsidized more than older technologies. Their technology externalities, however, come from learning effects, as opposed to R&D externalities in this paper. In their model the learning effects are strongest for newly adopted technologies so they have higher spillovers than older technologies. Hence, the optimal subsidies decrease over time as the learning effects diminish.

R&D externalities and subsidies to clean technologies are studied in Heggedal and Jacobsen (2011). They find that the R&D subsidies should fall over time since spillovers are larger in early periods due to decreasing returns to new ideas. However, they only study subsidies to R&D in one type of technology and do not analyze the relative undersupply of R&D in technologies with different maturity.

In the economic growth literature there are several papers that study innovation in multi-sector R&D models with symmetric equilibria (e.g. Smulders and van de Klundert, 1995 and 1997; Young, 1998; Segerstrom, 2000; Li, 2000 and 2002; Peretto and Smulders, 2002). However, the symmetric equilibria in the respective types of innovation sectors, variant expansion and/or quality improvement, imply that the papers do not study the consequences of differences in R&D productivity between firms in the same innovation sector. Nonetheless, these models may imply differences in R&D productivity between firms in different sectors. For example, Strulik (2007) shows that optimal R&D subsidies differ between firms in the quality improvement sector and firms in the variant expansion sector. There are several other papers with multi-sector R&D models that have asymmetric equilibria in the innovation sectors (e.g. Acemoglu, 2002; Smulders and Nooij, 2003; Grimaud and Rouge, 2008; Chu, 2011; Mosel, 2010). However, these papers do not study the implication that the different maturity of technologies has for the allocation of resources between R&D

industries.

Two papers with multi-sector R&D models that specifically take the maturity of technologies into account are Doi and Mino (2005) and Reis and Traca (2008)². Doi and Mino (2005) investigate equilibrium dynamics in a model of endogenous technological change with two R&D industries. They find that the relative size of the knowledge stocks matters for the allocation of resources to production of consumer goods and production of R&D. Reis and Traca (2008) analyze the implication of a leading and a laggard technology for long run growth in a model with quality improvement³. They find that intersectoral spillovers may prevent a monopolization of the market by the productivity leader and thus prevent a stagnation of growth. Neither Reis and Traca (2008) nor Doi and Mino (2005) account for decreasing returns to new ideas which is done in this paper. Further, both studies only investigate the market equilibrium and do not explore the connection between spillovers, the maturity of technologies and optimal policies.

The paper is organized as follows. An illustration of the core mechanism in the model is presented in Section 2. Section 3 sets up the model and solves for both the private market equilibrium and the socially optimal equilibrium. The relative undersupply of the technologies is analyzed in Section 4. Numerical simulations with extensions of the model are presented in Section 5, while Section 6 concludes and offers a discussion of the results.

2 Illustration

One reason why R&D in emerging and mature technologies may not be equally undersupplied is that a typical patent production function is concave in the amount of previous patents (ideas). A functional form often used in growth models is

$$\dot{A} = \nu L^\lambda A^\phi : 0 < \lambda < 1, 0 < \phi < 1, \quad (1)$$

where \dot{A} is the production of new patents, L is labor input, ν is an exogenous technology factor, λ is the output elasticity with respect to labor, ϕ is the output elasticity with respect to patents, i.e. the spillover parameter, and A is stock of patents accumulated from previous periods, i.e. the knowledge stock. The spillover parameter reflects the effect of the existing knowledge stock on the production of patents. A spillover parameter below one is supported by both theoretical studies (see e.g. d'Aspremont and Jacquemin, 1988; Jones, 1995 and 1999; De Bondt, 1997) and empirical findings (see e.g. Jones and Williams, 2000; Popp, 2002; Bottazzi and Peri, 2003; Gong et al., 2004). With a parameter below one, the model exhibits weak scale effects and the long-run growth rate of patents is dependent on the population growth rate, which are the main characteristics of semi-endogenous growth models (Jones, 2005). The patent productivity is increasing in the knowledge stock in the following way:

$$\frac{\partial \dot{A}}{\partial A} = \nu L^\lambda \phi A^{\phi-1} > 0. \quad (2)$$

This productivity gain is the source of the spillover effect. Patents from previous periods lower the cost of producing new patents. The ultimate reason is that ideas are non-rival goods, in the sense that one entity's use of an idea does not diminish

²Although few papers, to my knowledge, investigate the consequences of different maturity for R&D policy in closed economies, there are several studies of cross-country differences in technological development. Many of these studies focus on the distance to the technology frontier and differences in growth rates and income. See for example Aghion and Howitt (2006) and Madsen (2008).

³See for example Aghion et al. (2001) for more references on other leader and laggard quality improvement models.

the benefit for other entities' simultaneous use. Patent protection rights may give excludability for products based on new ideas. However, this does not prevent others from using that idea to create new ideas, i.e. "standing on the shoulders of giants". This spillover from the production of a patent to all firms that produce patents in later periods is not accounted for by the individual firms, i.e. it is an externality. The spillovers imply that the firms undersupply patent production to the market, and a policymaker should try to correct for this, e.g. by subsidizing R&D.

From (2) it is clear that patents in a mature technology can be produced with less resources than in an emerging technology, where maturity is defined by the size of A . *Ceteris paribus*, the mature technology will be allocated more labor than the emerging technology both in the private and the social (optimal) equilibrium. The amount of labor input in future periods' patent production influences the spillovers from current period production in the following way:

$$\frac{\partial \ln(\frac{\partial \dot{A}}{\partial A})}{\partial \ln L} = \lambda > 0. \quad (3)$$

In a future period with high R&D production, the benefits are greater from an increase in the stock of ideas because it lowers costs for a larger production set. In other words, the benefits are greater when a new patent spills over to more R&D firms and researchers. I find it convenient to name this effect expressed by λ the *spillover size effect*. This effect means that the spillovers are larger in the mature technology than in the emerging technology, which implies that R&D in the mature technology should be subsidized more.

On the other hand, the change in patent productivity is smaller when the knowledge stock is larger:

$$\frac{\partial \ln(\frac{\partial \dot{A}}{\partial A})}{\partial \ln A} = (\phi - 1) < 0. \quad (4)$$

The reason is that when a new patent is added to a large set of other patents this does not increase the R&D opportunities in future periods as much as an additional patent when there are few other patents. Thus, a new patent in an emerging technology sector provides a greater productivity increase than a new patent in a mature technology. I find it convenient to name this effect expressed by $\phi - 1$ the *spillover depletion effect*, which implies that R&D in the emerging technology should be subsidized more. The relative undersupply of R&D in the two technologies depends on whether the *spillover size effect* or the *spillover depletion effect* dominates.

An example of technologies where the spillover effects may be present is conventional cars with internal combustion engines and hydrogen cars with fuel cells as the energy conversion system. A lot of research has been carried out on internal combustion engines compared to fuel cells for cars. The implied large knowledge stock for internal combustion engines means that there are many ideas to build on and that new ideas can be found in many dimensions. When the R&D activity in internal combustion technology is high, a new idea may benefit many researchers in future periods, i.e. the *spillover size effect* is large. However, decreasing returns to new ideas are present. Decreasing returns do not mean that the best ideas get taken first (i.e. no fishing out), but that the benefit for future R&D is relatively small from a new idea when it is just one more idea in an already large pool of knowledge. In the fuel cell technology, the benefit for researchers from a new idea may be greater than in the internal combustion technology. The reason is that a new idea expands the future research possibilities relatively more when the knowledge stock is small, i.e. the *spillover depletion effect* is smaller in the immature technology.

3 The model

In order to analyze the relative undersupply of R&D in an emerging technology compared to R&D in a mature technology, I develop a partial equilibrium model with final goods producers, intermediate goods producers, and patents (ideas) producers.

The final goods industry manufactures output (e.g. transportation services) and is characterized by productivity increase from an expansion of the number of available capital varieties (Romer, 1990). In the intermediate goods industry, firms buy patents from one of the R&D industries. A patent gives a firm an exclusive right to produce one type of capital variety. The intermediate goods firms engage in monopolistic competition and deliver capital varieties to the manufacturer. There are two R&D industries, one emerging and one mature. These R&D industries produce new patents in their respective technology field: small scale emerging technology (e.g. hydrogen-based car engines) and large scale mature technology (e.g. internal combustion car engines). The accumulated production of patents gives rise to two different knowledge stocks, which lower the cost of patent production in the respective technologies. The maturity of a technology is defined by the amount of patents in the technology, i.e. the size of the knowledge stock.

The reason I employ a model with two R&D industries instead of using a model with only one R&D industry is that analyzing how optimal subsidies rise and fall along the transition path proves to be difficult. Rather than analyzing the optimal subsidies directly, I analyze the relative undersupply of R&D from two industries along the transition path in order to obtain analytically tractable results.

I make two major assumptions in order to focus on the role of maturity in the allocation of resources to R&D in different technologies. First, I assume that it does not matter for the final goods industry whether capital variants are produced by one technology or the other. The final goods industry gets the same productivity increase from a capital variant based on the emerging technology as one based on the mature technology. By making this assumption I manage to isolate the effect that the maturity of the technologies, through the knowledge stocks, has on the investment decision in the R&D industries.

The role of maturity could be studied in a more elaborate model where it would matter for the final goods production which of the two technologies are used. This would add a market size and a price (quality) effect on the demand side of the market (see Acemoglu, 2002, and Acemoglu et al., 2012). These effects would clearly matter for the allocation of R&D. However, whether they would matter for optimal R&D policy is less clear as the market size and price effect, at least in part, would be appropriated by the private firms. If other asymmetric market failures than knowledge spillovers were included in the analysis, these should be targeted by separate policies. For instance if there are emission externalities on the use of the mature technology, the optimal policy would be to set the Pigouvian tax. In this case, the environmental effect is taken care of and we are left with the question: Is there still a rationale for differentiated R&D support? The focus of this paper is to analyze the knowledge externality on the supply side of the market in isolation without being confounded by other effects. See Section 6 for a further discussion of demand side effects and knowledge spillovers.

The second major assumption is that the total allocation of resources dedicated to R&D is given. I disregard the allocation between final goods production and patent production since the focus of this paper is on the relative undersupply of the two technologies. In general, R&D is undersupplied by private firms in the model presented here, and subsidies should be given to internalize knowledge spillovers. The

undersupply of R&D depends on the difference between the social and the private rates of return from R&D, where the rates of return give the social and private allocation of resources. However, in this paper I do not study the undersupply of R&D per se, but the difference in the social and the private allocation between the two technologies. Both technologies are undersupplied, but the question I raise is whether one is more undersupplied than the other. If one technology is more undersupplied than R&D in that technology should be subsidized more than R&D in the other.

3.1 The private market equilibrium

In the private market equilibrium private firms maximize profits without taking into account the externalities arising from knowledge spillovers. In this section I derive the market allocation of resources to the two R&D industries.

3.1.1 Final goods industry

The final goods industry manufactures output with the following production function:

$$Y_t = L_{Y,t}^{1-\alpha} \int_0^{A_t} x_{i,t}^\alpha di : \alpha \in (0, 1), \quad (5)$$

where $L_{Y,t}$ is labor input, $x_{t,i}$ is input of capital variant i , and A_t is total knowledge stock. The total knowledge stock is given by $A_t = A_{e,t} + A_{m,t}$, where $A_{e,t}$ is knowledge stock in the emerging technology, e , and $A_{m,t}$ is knowledge stock in the mature technology, m . The knowledge stocks represent the amount of patents available. More patents correspond to more capital variants and increased productivity. Time, t , is suppressed in the rest of the paper where not otherwise noted. Y is sold for a numeraire price equal to 1.

A representative firm hires labor at rate w_Y and buys capital variants at price p_i , takes prices as given, and solves

$$\max_{L_Y, x_i} : L_Y^{1-\alpha} \int_0^A x_i^\alpha di - w_Y L_Y - \int_0^A p_i x_i di.$$

The maximization problem gives the following first order conditions:

$$L_y = (1 - \alpha) \frac{Y}{w_Y} \quad (6)$$

$$x_i = \left(\frac{\alpha}{p_i} \right)^{\frac{1}{1-\alpha}} \frac{(1 - \alpha)Y}{w_Y} : \forall i, \quad (7)$$

where I have substituted back for Y from (5). Equation (6) gives the demand for labor in the final goods industry and (7) gives the demand for capital variant i .

3.1.2 Intermediate goods industry

Firms in the intermediate goods industry buy one patent each from one of the R&D industries. The patent is a fixed cost for the firm and gives an exclusive right to produce a capital variant based on that patent. They transform capital goods into intermediate goods in a one to one ratio and sell to the final goods industry under monopolistic competition. The production technology (or rather the capital-conversion) is the same for all intermediate firms. There is free entry into this industry in the sense that anyone can bid for a patent and produce a capital variety. An intermediate goods firm solves the following problem:

$$\max_{x_i} : p(x_i)x_i - rx_i,$$

where r is interest rate on capital, i.e. cost of production, and $p(x_i)$ is the inverse demand for capital variety i from the final goods sector. The first order condition is

$$\frac{\partial p_i}{\partial x_i} \frac{x_i}{p_i} + 1 = \frac{r}{p_i}, \quad (8)$$

where $\frac{\partial p_i}{\partial x_i} \frac{x_i}{p_i}$ is equal to the negative inverse price elasticity from equation (7), $\alpha - 1$. The price elasticity is equal for all capital variants. Thus the price for the variants is equal for all i , $p_i = p = \frac{r}{\alpha}$, where $\frac{1}{\alpha}$ can be interpreted as a markup factor.

The equal price together with demand from equation (7) implies that the demands for all capital variants are equal, $x_i = x$, and that the instantaneous profit π is the same for all the intermediate goods firms:

$$\pi = px - rx = (1 - \alpha)px. \quad (9)$$

The instantaneous profit for all intermediate goods firms is the same since they have the same marginal costs and face the same elasticity of demand for their products.

3.1.3 R&D industries

There are two industries producing patents, one in the emerging technology, e , and one in the mature technology, m . The production of patents from the firms is given by the following production function:

$$\dot{A}_j = \bar{\nu}_j L_j = \nu L_j^\lambda A_j^\phi : j = e, m : 0 < \lambda < 1, 0 < \phi < 1, \quad (10)$$

where $\bar{\nu}_j = \nu L_j^{\lambda-1} A_j^\phi$ is the average productivity. The R&D firms take the average productivity as given. However, productivity changes over time as the current period patent production contributes to the knowledge stock. Further, productivity within a time period depends on the R&D activity due to the *stepping on toes effect* (Jones and Williams, 2000). When more researchers pursue new ideas, duplication increases and average productivity declines.

The only difference between producing patents in the two technologies follows from the knowledge stocks. The initial knowledge stock is smaller in the emerging technology than in the mature technology, i.e. $A_{e,0} < A_{m,0}$. Note that there are no inter-industry knowledge spillovers in the model. In Section 5 – the numerical simulations part of the paper – the model is extended to allow for spillovers between the industries as well as differences in the spillover parameters.

There is free entry into the R&D industries. A representative firm solves

$$\max_{L_j} : P_j \bar{\nu}_j L_j - w_A L_j : j = e, m,$$

taking the price of the patents P_j and the wage rate w_A as given. The maximization problem gives the following first order condition in the two industries:

$$\begin{aligned} P_j \bar{\nu}_j &= w_A : j = e, m \\ &\Leftrightarrow \\ P_j \nu L_j^{\lambda-1} A_j^\phi &= w_A : j = e, m, \end{aligned} \quad (11)$$

where I have used $\bar{\nu}_j = \nu L_j^{\lambda-1} A_j^\phi$. The first order condition gives the resource allocation to R&D in the emerging and the mature technology. This condition can be interpreted as a free entry condition as firms establish in both industries until revenue equates costs: $P_j \nu L_j^{\lambda-1} A_j^\phi = w_A \Leftrightarrow P_j \nu L_j^\lambda A_j^\phi = w_A L_j \Leftrightarrow P_j \dot{A}_j = w_A L_j$.

That the intermediate goods firms have the same profits whichever technology they supply to the final goods industry implies that the price of a patent is equal in the two technologies, i.e. $P_e = P_m = P$ (see Appendix A). The equality of patent prices implies that the only source of difference in the production of ideas between the two R&D industries emanates from the knowledge stocks. The difference in knowledge stocks leads to the following proposition:

Proposition 1 *Labor allocation and patent production are always higher in the mature R&D industry than in the emerging R&D industry in the private market equilibrium.*

Proof. Rearranging (11) gives $L_j = \left(\frac{\nu P A_j^\phi}{w_A}\right)^{\frac{1}{1-\lambda}}$. Then, $A_m > A_e \Rightarrow L_m > L_e$, since $\phi > 0$ and $\lambda < 1$. This together with (10) gives $\dot{A}_m > \dot{A}_e$. ■

Proposition 1 follows from the mature technology having a larger knowledge stock than the emerging. A larger knowledge stock implies higher patent productivity for given output elasticities. Since factors other than productivity, i.e. patent price and wage rate, are equal in the two R&D industries, firms always invest more in the mature R&D industry.

The labor allocation is larger to the mature than to the emerging R&D industry, but which growth rate is larger depends on the sum of the output elasticity parameters λ and ϕ , i.e. the elasticity of scale. The relationship between the output elasticities and the growth rates of the knowledge stocks is given in the following proposition:

Proposition 2 *In the private market equilibrium, the emerging technology grows faster (slower) than the mature technology if $\lambda + \phi$ is smaller (larger) than one, while the technologies grow at the same rates if $\lambda + \phi$ is equal to one.*

Proof. The growth rate is given by $\frac{\dot{A}_j}{A_j} = \nu L_j^\lambda A_j^{\phi-1}$. This together with (11) give $\frac{\dot{A}_j}{A_j} = k A_j^{\frac{\lambda+\phi-1}{1-\lambda}}$, where $k = v\left(\frac{w_A}{P\nu}\right)^{\frac{\lambda}{1-\lambda}}$. Since $A_m > A_e$, it follows that $\frac{\dot{A}_e}{A_e} > \frac{\dot{A}_m}{A_m}$ when $\lambda + \phi < 1$, $\frac{\dot{A}_e}{A_e} < \frac{\dot{A}_m}{A_m}$ when $\lambda + \phi > 1$, and $\frac{\dot{A}_e}{A_e} = \frac{\dot{A}_m}{A_m}$ when $\lambda + \phi = 1$. ■

Proposition 2 states that whether the growth rates are increasing or decreasing in the knowledge stocks follows from the scale elasticity. There are two opposing effects from a larger knowledge stock on the growth rate. First, the labor input grows as productivity improves. This gives that the growth rate is increasing in the knowledge stock, i.e. the *spillover size effect*. Second, there is less productivity gain from new patents when the knowledge stock is large. This gives that the growth rate is decreasing in the knowledge stock, i.e. the *spillover depletion effect*. The scale elasticity determines the dominating effect.

Total labor dedicated to R&D in the economy L_A is given by assumption, i.e. $L_A = L_e + L_m$. This assumption can be understood as a division of the labor force into two separate markets; one market for R&D with a highly specialized workforce and one for other activities, L_Y . The assumption implies that when one type of R&D increases, e.g. from a subsidy, the other type of R&D is crowded out. In order to compare the market allocation with the socially optimal allocation, which is derived in the next section, I normalize L_A to one and define the allocation ratio between the two technologies $\frac{1-L_m}{L_m}$. From (11) we have that the values of the marginal products in the mature and the emerging R&D industry equate in equilibrium:

$$\begin{aligned} P\nu(L_m^p)^{\lambda-1}A_m^\phi &= P\nu(1-L_m^p)^{\lambda-1}A_e^\phi \\ &\Leftrightarrow \\ \frac{1-L_m^p}{L_m^p} &= \left(\frac{A_e}{A_m}\right)^{\frac{\phi}{1-\lambda}}, \end{aligned} \tag{12}$$

where L_m^p is the labor allocation to the mature R&D industry in the market equilibrium⁴. We see that the private market allocation ratio between the two technologies, $\frac{1-L_m^p}{L_m^p}$, is given by the knowledge stock ratio, $\frac{A_e}{A_m}$ ⁵.

The allocation ratio in equation (12) reproduces Proposition 1. This allocation ratio highlights that it is only the maturity of the technologies that matters for the private firms' allocation of labor between the two technologies.

3.2 The socially optimal equilibrium

In this section I solve a simplified social planner problem to find the socially optimal (efficient) allocation of labor between the mature and the emerging R&D industries.

The social planner maximizes output over the time period by allocating labor between the two R&D industries. Final goods production is given by $Y_t = L_{Y,t}^{1-\alpha} \int_0^{A_t} x_{ti}^\alpha di$ and total capital is given by $\int_0^{A_t} x_t dt = K_t$. The symmetry of capital goods implies that $x_i = \frac{K}{A}$ and the final goods production function can be written $Y = A^{1-\alpha} K^\alpha L_Y^{1-\alpha}$. The labor allocation to final goods production is given by assumption. Thus, maximizing output is equivalent to maximizing the total knowledge stock.

The social planner's problem is then

$$\begin{aligned} \max_{L_m} : & \int_0^\infty (A_m + A_e) e^{-rt} dt : L_m \in (0, 1) \\ \text{s.t.} \quad & \dot{A}_m = v L_m^\lambda A_m^\phi \\ & \dot{A}_e = v(1 - L_m)^\lambda A_e^\phi, \end{aligned}$$

given the initial stocks of knowledge $A_{m,0}$ and $A_{e,0}$, and the discount rate r , i.e. the interest rate. The first order condition gives the social allocation ratio between the two R&D industries (see Appendix B):

$$\frac{1 - L_m^*}{L_m^*} = \frac{\mu_e \dot{A}_e}{\mu_m \dot{A}_m}, \quad (13)$$

where L_m^* is the socially optimal allocation of labor to R&D in the mature technology, μ_m is the shadow value of patents in the mature technology, and μ_e is the shadow value of patents in the emerging technology⁶. As in the private market equilibrium equation (13) implies that the value of the marginal products equates in equilibrium. The social allocation ratio can be rewritten:

$$\frac{1 - L_m^*}{L_m^*} = \left(\frac{\mu_e}{\mu_m} \right)^{1/(1-\lambda)} \left(\frac{A_e}{A_m} \right)^{\phi/(1-\lambda)}. \quad (14)$$

In the social optimum, the allocation ratio is dependent on the relative shadow values of patents in addition to the knowledge stocks. The allocation of labor to the emerging technology can only be larger than that to the mature technology if the shadow value is larger in the emerging technology, since the knowledge stock is larger in the mature technology on a transition path.

⁴The externality from the duplication effect does not matter for the allocation ratio between the R&D industries. If the firms account for duplication efforts, the first order condition is given by $P_j \nu \lambda L_j^{\lambda-1} A_j^\phi = w_A$, which gives the same allocation ratio as in the main text.

⁵There is no investment in the emerging technology if there are constant returns to labor, $\lambda = 1$ (i.e. no stepping on toes), since the marginal product of labor in R&D production is always greater for the mature technology in this case.

⁶This simple, partial maximization problem gives the same allocation rule between the two R&D industries as a full social planner problem where the resource allocation between final goods and R&D production is not given, see Appendix C.

4 The relative undersupply of the technologies

The relative undersupply of R&D in the two technologies is found by comparing the private allocation of labor with the social allocation. By combining equation (12) with equation (14) it is clear that the difference between the private and the social allocations follows the shadow values:

$$\frac{(1 - L_m^*)/L_m^*}{(1 - L_m^p)/L_m^p} = \left(\frac{\mu_e}{\mu_m}\right)^{1/(1-\lambda)}. \quad (15)$$

In the market equilibrium, the maturity of the technologies matters for the allocation of labor because this influences the productivity in the R&D industries. In the social equilibrium, the maturity of the technologies has an additional intertemporal effect because the maturity also matters for the spillovers that reduce costs of producing patents in later periods. The following proposition states the relationship between the shadow values and the relative allocation of labor in the private and the social equilibrium:

Proposition 3 *If the shadow values of patents equate, $\mu_m = \mu_e$, the private market allocation is the same as the socially efficient allocation. However, if the shadow value of patents is larger for one of the technologies, the private market equilibrium undersupplies R&D in that technology more than R&D in the other technology.*

When the market equilibrium undersupplies one type of R&D more, it is socially efficient to subsidize that type of R&D more. This social efficiency argument is stated in the following corollary:

Corollary 4 *If the shadow value of patents is larger for one of the technologies, the government should subsidize R&D in that technology more than R&D in the other technology.*

4.1 Which technology is more undersupplied?

In this section I calculate expressions for the shadow values and derive the condition that determines which of the two technologies is more undersupplied in the private market equilibrium.

The evaluation of the shadow values follows from the co-state equations (see Appendix B):

$$\dot{\mu}_j = \mu_j r - \mu_j \phi \frac{\dot{A}_j}{A_j} - 1 : j = m, e. \quad (16)$$

Together with the transversality conditions, (16) can be solved to find the expressions for the shadow values (see Appendix D for calculations):

$$\mu_j = A_j^{-\phi} e^{rt} \int_t^\infty [A_j(z)]^\phi e^{-rz} dz : j = m, e. \quad (17)$$

From (17) we see that a shadow value is a function of the current knowledge stock and the discounted knowledge stocks of all future periods.

On a balanced growth path the shadow values of patents are equal in the two technologies (see Appendix D). Hence, R&D subsidies should not be diversified on a balanced growth path. In a semi-endogenous growth model like the one presented in this paper it is a well-known result that subsidies to R&D do not affect the long run-growth rate. However, subsidies do affect the growth rates along the transition

path and, thus, affect the long-run level of patents and income (Jones, 1999). When the economy starts off with different knowledge stocks in the two technologies the economy is on a transition path, and along this path the shadow values may vary.

The relative shadow value of patents is given by

$$\frac{\mu_e}{\mu_m} = \frac{A_e^{-\phi} \int_t^\infty [A_e(z)]^\phi e^{-rz} dz}{A_m^{-\phi} \int_t^\infty [A_m(z)]^\phi e^{-rz} dz}. \quad (18)$$

Equation (18) implies that the shadow value is greater in whichever technology the knowledge stock grows faster in the social optimum. The next natural step would be to analyze the difference in these growth rates on the transition path. However, such an analysis proves to be difficult due to the complexity of the dynamic system. Instead I show which technology is more undersupplied by analyzing the effect on the Hamiltonian of deviating from the allocation ratio in the market equilibrium.

Consider the admissible solution $[A_e, A_m, L_m^p]$ to the planner's maximization problem, where we can think of L_m^p as a *constrained* maximum for the planner. If the *constrained* maximum is implemented in each period, we can calculate the knowledge stocks and the shadow values by using the solutions from the market equilibrium. By analyzing perturbations of L_m^p along this path, I show in Appendix E that which technology is more undersupplied in the private market allocation is given by

$$\frac{A_m^{-\phi} \int_t^\infty [A_m(z)]^\phi e^{-rz} dz}{A_e^{-\phi} \int_t^\infty [A_e(z)]^\phi e^{-rz} dz} - 1 \geq 0. \quad (19)$$

If the left hand side of (19) is negative (positive), then the social allocation to the emerging R&D industry is larger (smaller) than the private market allocation. Further, the sign of the left hand side of (19) depends on the growth rates of the knowledge stocks in the market equilibrium. This relationship between the growth rates and the undersupply leads to the following proposition:

Proposition 5 *The emerging technology is more (less) undersupplied than the mature technology if the growth rate of A_e is larger (smaller) than the growth rate of A_m in the private market equilibrium.*

Proof. $sign\left[\frac{A_m^{-\phi} \int_t^\infty [A_m(z)]^\phi e^{-rz} dz}{A_e^{-\phi} \int_t^\infty [A_e(z)]^\phi e^{-rz} dz} - 1\right] = sign\left[\frac{\int_t^\infty \left(\frac{A_m(z)}{A_m(t)}\right)^\phi e^{-rz} dz}{\int_t^\infty \left(\frac{A_e(z)}{A_e(t)}\right)^\phi e^{-rz} dz} - 1\right]$ which is negative (positive) if $\frac{\dot{A}_e}{A_e}$ is larger (smaller) than $\frac{\dot{A}_m}{A_m}$. In the *constrained* maximum the growth rates of A_e and A_m follow from L_m^p . ■

Proposition 5 states that the technology that grows faster in the private equilibrium is more undersupplied. The reason is that the knowledge spillovers are larger in the technology with the higher growth rate. The spillovers and the growth rates of the knowledge stocks are determined by the same opposing effects: the *spillover size effect* and the *spillover depletion effect*. First, both the growth rate and the spillovers in a technology are increasing in the knowledge stock because the labor input is large due to high labor productivity when the knowledge stock is large. Second, both the growth rate and the spillovers in a technology are declining in the knowledge stock because there is less productivity gain from new patents when the knowledge stock is large. Whether the technology with the small knowledge stock or the technology with the large knowledge stock grows faster is determined by the relative strength of the two opposing spillover effects.

Further, from Proposition 2 we have that the growth rates of the knowledge stocks in the market equilibrium depend on the output elasticity parameters λ and ϕ . Proposition 2 together with Proposition 5 lead to the following corollary:

Corollary 6 *The emerging technology is more (less) undersupplied than the mature technology on a transition path if $\lambda + \phi$ is smaller (larger) than one.*

Corollary 6 states that the relative undersupply of the technologies is determined by the elasticity of scale. The reason is that the elasticity of scale determines the growth rates of the knowledge stocks in the private equilibrium. When $\lambda > 1 - \phi$ the *spillover size effect* is larger than the *spillover depletion effect* and the market outcome gives a larger undersupply of the mature technology compared with the emerging technology. In this case it is optimal to subsidize the mature R&D industry more than the emerging R&D industry. When $\lambda < 1 - \phi$ the *spillover depletion effect* dominates and it is optimal to subsidize the emerging R&D industry more than the mature R&D industry.

5 Extensions with numerical simulations

In this section I use numerical simulations to analyze extensions to the baseline model. First I run the baseline model and show to what degree the elasticity of scale determines the difference between the social and the private allocation to the two R&D industries. Then I show the effect of including knowledge spillovers between R&D industries. Last I show the effect of letting the spillover parameter differ across the industries.

5.1 Numerical procedure

The simulation model is programmed as a discrete time model over 150 periods. I assume that patents produced in one time period are not included in the current period's knowledge stock. Knowledge accumulates according to $A_{j,t+1} = A_{j,t} + vL_{j,t}^\lambda A_{j,t}^\phi$: $j = m, e$. The initial knowledge stocks are arbitrarily chosen so that $A_{m,0} > A_{e,0}$. Total resources devoted to R&D are given; $1 = L_e + L_m$. The model solves the firms' maximization problem by setting L_m^p in each period. Further, the model solves the social planner's maximization problem by setting L_m^* for all periods simultaneously.

5.2 The baseline model

The difference between the social and the private allocation of labor to the mature R&D industry $L_m^* - L_m^p$ for different parameter values is given in Figures 1a and 1b:

Figure 1a: Figure 1b

Figure 1a shows the difference between the allocations when $\lambda + \phi > 1$, while Figure 1b shows the difference when $\lambda + \phi < 1$. Time periods are on the horizontal axis. A positive (negative) number on the vertical axis indicates that the mature (emerging) industry should be subsidized more in that time period. Corollary 6 is reproduced in all simulations. The elasticity of scale determines whether the social or the private allocation to the mature R&D industry is larger; $L_m^* - L_m^p$ is positive when $\lambda + \phi > 1$, and $L_m^* - L_m^p$ is negative when $\lambda + \phi < 1$. Further, we see that the social allocation approaches the private allocation in the long run.

The elasticity of scale determines which of the two technologies is more undersupplied. However, when the elasticity parameters are low we see from Figure 1b that the difference between the social and the private allocation is small. The reason is that the knowledge spillovers are smaller when the elasticities are low. This small difference in spillovers might indicate that there is no case for subsidizing the emerging R&D industry more than the mature when we account for potential costs of admin-

illustrating a differentiated R&D policy, e.g. costs of determining which technology is emerging and which is mature.

Note that the initial knowledge stocks are arbitrarily chosen in the model simulations. Different choices of initial knowledge stocks that either change the relative size of the stocks or only their absolute sizes do not change the main result. Further, the interest (discount) rate is also arbitrarily chosen. A higher interest rate narrows the gap between the social allocation and the private allocation as the social planner gives less weight to late periods when the interest rate is high. However, a change in the interest rate does not change the relative undersupply of the two technologies, and the main result holds⁷.

5.3 Model with knowledge spillovers between the industries

Inter-industry knowledge spillovers may reduce the difference in externalities from R&D in the two technologies. Hence, the rationale for a differentiated R&D policy may diminish. To allow for inter-industry knowledge spillovers I include both of the knowledge stocks in the R&D production functions in the following way:

$$\dot{A}_j = \nu L_j^\lambda (A_j + \gamma A_{-j})^\phi : j = e, m, \quad (20)$$

where $\gamma \in (0, 1)$ is the inter-industry spillover parameter. A high parameter value implies large inter-industry spillovers, while a low value implies small inter-industry spillovers. Figures 2a and 2b give the difference between the social and the private allocation of labor when there are knowledge spillovers between the two R&D industries:

Figure 2a : Figure 2b

Figure 2a shows the difference between the allocations when $\lambda + \phi > 1$, while Figure 2b shows the difference when $\lambda + \phi < 1$. From the figures we see that an increase of the inter-industry spillover parameter reduces the difference between the social and the private allocation. If there are complete knowledge spillovers between the industries, i.e. $\gamma = 1$, there is no reason for the social planner to have a different allocation ratio between the R&D industries than the private firms. However, if the inter-industry spillovers are incomplete, i.e. $\gamma < 1$, the relative undersupply of the technologies follows from the elasticity of scale in the R&D production function.

5.4 Model with different spillover parameters across the industries

In the baseline model there are asymmetric spillovers between the industries as a consequence of differences in the knowledge stocks. Nonetheless, there may be an additional and more direct channel for asymmetric spillovers. There are several studies that analyze differences in intra-industry spillovers due to differences in spillover parameters or R&D production functions across industries (see e.g. De Bondt and Henriques, 1995; Atallah, 2005). In this section I show simulation results where the spillover parameter is allowed to differ across the two R&D industries. Figures 3a and 3b give the difference between the social and the private allocations of labor when ϕ may vary across industries:

Figure 3a : Figure 3b

Figure 3a shows the difference between the allocations when $\lambda + \phi > 1$, while Figure 3b shows the difference when $\lambda + \phi < 1$. In both figures there is a baseline where

⁷Tables from sensitivity analysis can be provided by the author upon request.

$\phi = 0.5$ in both industries. We see that if the spillover parameter is increased in one industry, the undersupply of R&D in that industry increases. Further, we see in Figure 3a that $L_m^* - L_m^p$ is negative when the spillover parameter is largest in the emerging industry, even though the scale elasticity is larger than one. Similarly, we see in Figure 3b that $L_m^* - L_m^p$ is positive when the spillover parameter is largest in the mature industry, even though the scale elasticity is smaller than one.

In the figures, parameters are chosen so that the effect of different spillover parameters can overturn the result on R&D support stemming from differences in the knowledge stocks. Nonetheless, it is not surprising that differences in the spillover parameters may change this result. By increasing the spillover parameter in one R&D industry, knowledge spillovers are directly increased in that industry. Thus, R&D should be supported more in that industry. A difference in the spillover parameters may pull knowledge spillovers in the same or the opposite direction as a difference in the knowledge stocks. The sum, then, determines the asymmetry of knowledge spillovers between technologies.

6 Discussion and conclusion

An important policy question is whether R&D in new, emerging technologies should be subsidized more than R&D in other more mature technologies. In this paper I analyze if innovation externalities caused by knowledge spillovers may warrant a differentiation of R&D policy towards technologies of different maturity. I show that the governmental support for R&D in emerging and mature technologies should not be equal. The reason is that R&D in the two technologies is not equally undersupplied in the market due to differences in their knowledge stocks. Both the incentives for firms to engage in R&D and the knowledge spillovers from R&D to future periods change when the knowledge stock grows. Hence, the maturity of the technologies matters when policymakers are to give socially efficient subsidies to different R&D industries.

In this paper the elasticity of scale in the R&D production function determines whether an emerging technology or a mature technology is more undersupplied in the market. The reason is that the output elasticities determine the knowledge spillovers through two opposing effects. First, the spillovers are increasing in the knowledge stock because the labor input is large in late periods due to high labor productivity. Second, the spillovers are declining in the knowledge stock because there is less productivity gain from new patents when the technology is mature.

I show that the emerging technology is more undersupplied and should be subsidized more than the mature technology when the elasticity of scale is smaller than one. However, when the elasticity of scale is larger than one the mature R&D industry should be subsidized more. There are some studies that estimate the parameters in the aggregate R&D production function. Both Porter and Stern (2000) and Pessoa (2005) find a scale elasticity larger than one. In another study, Gong et al. (2004) find a scale elasticity smaller than one, though the results are not significant. Another approach is Jones and Williams (2000), where the calibrated ranges of output elasticities all give an elasticity of scale larger than one. In sum, these studies indicate that the scale elasticity is larger than one. In this case, it is not a valid argument to support R&D in new (clean) technologies more than other R&D on the basis that new (clean) technologies are less mature.

However, there are several caveats to this conclusion. First, the empirical literature on output elasticities in the R&D production function is not very well developed. Further research is needed to establish significant ranges for the output elasticities. Moreover, in a recent empirical study Dechezlepretre et al. (2013) find that spillovers

are larger in clean than dirty technologies. The driving force behind the result may be that clean technologies are newer technologies than dirty. This points in the direction of an elasticity of scale larger than one. However, their estimation of the difference in spillovers across industries may be confounded with other effects than technology maturity. For instance, the output elasticity parameters may not be the same across R&D industries. As shown in Section 5.4, different spillover parameters across R&D industries give an additional effect on the asymmetry of knowledge spillovers between technologies.

Second, the result is calculated under an assumption of symmetric technologies from the demand side. It may be natural to think that new technologies are of better quality while there is a larger market for the mature technology. The price (quality) effect and the market size effect will then influence the direction of technological change (Acemoglu, 2002; Acemoglu et al., 2012). However, these effects are only relevant for optimal R&D policy if they affect the R&D firms' ability to appropriate the value of the innovation. The implicit assumption in this paper is that the surplus appropriability problem is equal for the two technologies. If this were not the case, the appropriability problem would be a separate rationale for a differentiated R&D policy, in addition to the knowledge spillovers analyzed in this paper. In a model with asymmetric technology demand, a new technology with higher quality (lower price) would induce a positive price effect and direct R&D towards that sector. On the other hand, if the other, mature technology has a large market, this would pull R&D in the direction of mature technology. The relative strength of these effects would depend on substitution elasticities between the technologies in the final goods production and on the relative size of the knowledge stocks, i.e. the maturity. In other words, the pull from the demand side would partly be driven by the maturity of the technologies. However, it is not clear from the literature on directed technical change what effect the direction of R&D has on optimal R&D policy. In a future research project it might be interesting to further analyze the link between the direction of technical change and the surplus appropriation problem.

Third, related to the demand side of the technology markets and the appropriability problem, this paper assumes that there are no externalities from the use of the technologies. This may be a too strong assumption for analyzing real-world innovation policies. For instance, emerging technologies may be clean and environmental friendly while mature technologies may be dirty with emissions. If these emissions are not internalized, it would *ceteris paribus* be optimal to subsidize emerging R&D more than mature. However, this would be the second-best solution. The first-best solution with more than one policy tool would be to target each market failure by separate policies. The optimal environmental policy would be to set the Pigouvian tax on emissions, and, thus, provide efficient demand for the clean technology. In this case, the remaining innovation market failure would be the one stemming from knowledge spillovers, if the surplus appropriation problem is symmetric between technologies. It may be, though, that there is a correspondence between the tax level of emissions, the demand for a technology, and the surplus appropriation problem. This is a venue for future research.

Fourth, this paper assumes that there are no other externalities on the supply side of technology markets than those related to knowledge spillovers. This may also be a too strong assumption for analyzing real-world innovation policies. For instance, there is recent literature showing that financial frictions hamper innovation (see e.g. Brown et al., 2012). Financially constrained firms may invest too little in innovation compared to the socially optimal level. This problem is maybe more relevant for R&D firms in new, emerging industries than for firms in mature

industries with deeper pockets. In this case, it would be optimal to support R&D on emerging technologies more, if it is not viable to target the financial frictions directly. The interaction between financial frictions and innovation externalities is interesting to explore further. Similarly, the relationship between the relative undersupply of technologies and project risk—where investments in new technologies may be more risky—is another interesting venue for future research (see e.g. Matsumura, 2003; Atallah, 2014). This paper, however, sheds light on whether knowledge spillovers, in isolation from other effects, can be a rationale for a differentiated R&D policy for technologies of different maturity.

Appendix

Appendix A - The equality of patent prices

Free entry ensures that the price of a patent is equal to the present discounted value of the profits for an intermediate goods firm, $P_j = PDV_j : j \in e, m$. The instantaneous profit for an intermediate goods firms is given by:

$$\pi_i = (1 - \alpha)\alpha L_Y^{1-\alpha} x_i^\alpha, \quad (21)$$

where (7) is inserted in (9). Integrating (21) on both sides over the total knowledge stock A gives

$$\begin{aligned} A\pi_i &= (1 - \alpha)\alpha L_Y^{1-\alpha} \int_0^A x_i^\alpha di \\ &\iff \\ \pi &= (1 - \alpha)\alpha \frac{Y}{A}, \end{aligned} \quad (22)$$

where I have inserted from (5). Then PDV is equal across all intermediate goods firms and can be written as

$$PDV_t = \int_t^\infty e^{-rz} (1 - \alpha)\alpha \frac{Y(z)}{A(z)} dz. \quad (23)$$

Appendix B - The (simplified) socially optimal equilibrium

The autonomous Hamiltonian H is given from the simple, partial social planner problem:

$$H(A_e, A_m, \mu_e, \mu_m, L_m) = A_m + A_e + \mu_m v L_m^\lambda A_m^\phi + \mu_e v (1 - L_m)^\lambda A_e^\phi,$$

The necessary conditions are given by the first order condition

$$\frac{\partial H(\cdot)}{\partial L_m} : \mu_m \lambda v L_m^{\lambda-1} A_m^\phi - \mu_e \lambda v (1 - L_m)^{\lambda-1} A_e^\phi = 0, \quad (24)$$

the development of the shadow values from the co-state equations

$$\dot{\mu}_m = \mu_m r - \mu_m \phi v L_m^\lambda A_m^{\phi-1} - 1 \quad (25)$$

$$\dot{\mu}_e = \mu_e r - \mu_e \phi v (1 - L_m)^\lambda A_e^{\phi-1} - 1, \quad (26)$$

and the transversality conditions

$$\begin{aligned} \lim_{t \rightarrow \infty} \mu_m e^{-rt} A_m &= 0 \\ \lim_{t \rightarrow \infty} \mu_e e^{-rt} A_e &= 0. \end{aligned}$$

The sufficient conditions are satisfied by the necessary conditions together with that the Hamiltonian is concave in $[L_m, A_m, A_e]$ (Mangasarian's theorem).

I rearrange (24) to get the social allocation ratio

$$\frac{1 - L_m^*}{L_m^*} = \left(\frac{\mu_e}{\mu_m}\right)^{1/(1-\lambda)} \left(\frac{A_e}{A_m}\right)^{\phi/(1-\lambda)}. \quad (27)$$

Inserting \dot{A}_m and \dot{A}_e in equation (27) the social allocation ratio is given by

$$\frac{1 - L_m^*}{L_m^*} = \frac{\mu_e \dot{A}_e}{\mu_m \dot{A}_m}.$$

Appendix C - The full socially optimal equilibrium

In the full social planner problem, resources are allocated to final goods production as well as to R&D in the emerging and the mature technology, i.e. $L_t = L_{Y,t} + L_{e,t} + L_{m,t}$. In per capita terms we have $k = \frac{K}{L}$ and $y = \frac{Y}{L}$, and the final goods production can be written $y = A^{1-\alpha} k^\alpha (1 - s_m - s_e)^{1-\alpha}$, where $s_m = \frac{L_m}{L}$ and $s_e = \frac{L_e}{L}$.

Discounted utility is given by $U_t = \int_t^\infty L_s u(c_s) e^{-r(s-t)} ds$, where $L_t = L_0 e^{nt}$, n is the population growth rate, and $u(c_t)$ is the instant utility of consumption per capita $c = \frac{C}{L}$. The final goods are converted into consumption goods or capital goods in a one-to-one ratio so that capital per capita grows according to $\dot{k} = y - c - (n + \delta)k$, where δ is the depreciation rate of capital.

The appropriate social planner problem is

$$\begin{aligned} \max_{c, s_m, s_e} &: \int_t^\infty L_0 u(c) e^{-\bar{\rho}t} dt \\ \text{s.t.} & \dot{k} = y - c - (n + \delta)k \\ & \dot{A}_m = \nu s_m^\lambda L^\lambda A_m^\phi \\ & \dot{A}_e = \nu s_e^\lambda L^\lambda A_e^\phi, \end{aligned}$$

given $L_0, K_0, A_{m,0}, A_{e,0}$, where $\bar{\rho} = r - n$. From the first order conditions of this problem it is readily shown that

$$\frac{s_e^*}{s_m^*} = \frac{\mu_e \dot{A}_e}{\mu_m \dot{A}_m},$$

which basically is the same resource allocation rule as in the main text, equation (13), where $s_e^* = 1 - s_m^*$ when I disregard the allocation to final goods production, and $s_m^* = L_m^*$ when $L_e + L_m = 1$.

Appendix D - Calculation of the shadow values

I rewrite the equations given by (16):

$$\dot{\mu}_j + \mu_j f_j(t) = -1 : j = m, e, \quad (28)$$

where $f_j(t) = \phi \frac{\dot{A}_j}{A_j} - r$. I suppress $j = m, e$ in the following and define $F(t) = \int f(t) dt = \int \phi \frac{\dot{A}}{A} dt + \int r dt = \phi \ln A + \int r dt = \phi \ln A - rt$. I multiply both sides of (28) by $e^{F(t)}$ and derive with respect to time to get

$$\begin{aligned} \frac{\partial}{\partial t} (\mu e^{F(t)}) &= -e^{F(t)} \\ &\implies \\ \mu e^{F(t)} &= \int_\infty^t -e^{F(z)} dz + C, \end{aligned} \quad (29)$$

where C is a constant. Inserting for $e^{F(t)} = A^\phi e^{-rt}$, (29) can be written

$$\mu = A^{-\phi} e^{rt} \left(C - \int_{\infty}^t A^\phi e^{-rz} dz \right). \quad (30)$$

We can find the value of C by using the balanced growth rate and the transversality condition. First, I expand (30) by using $\int A^\phi e^{-rt} dt = -\frac{1}{r} A^\phi e^{-rt} + \int \frac{\phi}{r} \frac{\dot{A}}{A} A^\phi e^{-rt} dt$:

$$\mu = A^{-\phi} e^{rt} C + \frac{1}{r} - A^{-\phi} e^{rt} \int_{\infty}^t \frac{\phi}{r} \frac{\dot{A}}{A} A^\phi e^{-rz} dz. \quad (31)$$

It is well known that in a Jones-type model the growth rate on the balanced growth path (bgp) is given by $g_A = \frac{\lambda n}{(1-\phi)}$. In the current model $n = 0$ and $\lim_{t \rightarrow \infty} \frac{\dot{A}}{A} = 0$, i.e. the bgp is reached asymptotically and the growth rate is $\frac{\dot{A}}{A} = 0$. Using this implies that on the bgp, (31) simplifies to

$$\mu = A^{-\phi} e^{rt} C + \frac{1}{r}. \quad (32)$$

Lastly I utilize the transversality condition, $\lim_{t \rightarrow \infty} \mu e^{-rt} A = 0$. By substituting for $\mu = A^{-\phi} e^{rt} C + \frac{1}{r}$ in the transversality condition I get

$$\lim_{t \rightarrow \infty} A^{1-\phi} C + \frac{e^{-rt} A}{r} = 0, \quad (33)$$

which is only valid for $C = 0$. Inserting for $C = 0$ in (32) I get that on bgp the shadow values of patents are equal for both technologies $j = m, e$ and is given by

$$\mu = \frac{1}{r}. \quad (34)$$

I get the shadow values outside the balanced growth path by inserting $C = 0$ in equation (30):

$$\mu = A^{-\phi} e^{rt} \int_t^{\infty} A^\phi e^{-rz} dz. \quad (35)$$

Appendix E - Effects on the Hamiltonian of perturbing L_m^p

Following Seierstad and Sydsæter (1987, pp. 221), let τ be a point in time $\tau \in [t, \infty)$. Then I define a perturbation of L_m^p as a replacement of L_m^p by the constant \bar{L}_m on some interval $E(s) = [\tau, \tau + s)$. Further, the value of the Hamiltonian for a perturbation of L_m^p to the right of τ is defined as $W(s)$. If $s = 0$, then we are *at* time τ where L_m^p is *not* perturbed and $W(0)$ is given by $H(A_e, A_m, \mu_e, \mu_m, L_m^p)$, where $H(\cdot)$ is the Hamiltonian (see Appendix B). Then the social gain of perturbing from L_m^p for any τ is given by

$$\begin{aligned} \frac{dW(0)}{ds} &= H(A_e, A_m, \mu_e, \mu_m, \bar{L}_m) - H(A_e, A_m, \mu_e, \mu_m, L_m^p) \geq 0 \\ &\Rightarrow \\ \mu_m (\bar{L}_m)^\lambda A_m^\phi + \mu_e (1 - \bar{L}_m)^\lambda A_e^\phi - \mu_m (L_m^p)^\lambda A_m^\phi - \mu_e (1 - L_m^p)^\lambda A_e^\phi &\geq 0, \end{aligned} \quad (36)$$

where A_e, A_m, μ_e and μ_m follow from L_m^p . If (36) has a derivative with respect to \bar{L}_m in the neighborhood of L_m^p that is positive (negative), it follows that $\frac{dW(0)}{ds}$ is positive (negative) if \bar{L}_m is slightly larger than L_m^p . Thus, it is optimal to increase (decrease)

L_m^p with a small integer if the derivative is positive (negative). The derivative of (36) with respect to \bar{L}_m in the neighborhood of L_m^p is given by

$$\mu_m \lambda (L_m^p)^{\lambda-1} A_m^\phi - \mu_e \lambda (1 - L_m^p)^{\lambda-1} A_e^\phi \geq 0. \quad (37)$$

I use equations (12) and (18) to insert for L_m^p , μ_e and μ_m in (37):

$$\frac{A_m^{-\phi} \int_t^\infty [A_m(z)]^\phi e^{-rz} dz}{A_e^{-\phi} \int_t^\infty [A_e(z)]^\phi e^{-rz} dz} - 1 \geq 0. \quad (38)$$

If the left hand side of (38) is negative (positive), then the derivative of (36) is negative (positive) and it follows that the social allocation to the emerging R&D industry is larger (smaller) than the private allocation.

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Figures to be set into the main text:





