BI Norwegian Business School - campus Oslo

GRA 19703

Master Thesis

Thesis Master of Science

Multifactor Strategy Implementation in the Norwegian Equity Market

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Start: 15.01.2021 09.00

Finish: 01.07.2021 12.00

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Abstract

We study the performance of implementation methods for multifactor strategies in the Norwegian equity market. We compare the risk-adjusted performance of three different strategies implemented with equal weights, mean-variance optimized weights and factor-timed weights. During the financial crisis, the mean-variance optimization strategy performed exceptionally well with a Sharpe ratio if 0.402. The factor timing strategy underperformed during the financial crisis, but outperforms in normal times, generating a Sharpe ratio of 0.705 between March 2009 and December 2019. Moreover, the factor timing strategy is superior in the long run, although differences in risk-adjusted returns are minor. Our findings indicate that implementing factor-timed weights estimated on macroeconomic variables and moving to mean-variance optimized weights during crises may enhance the risk-adjusted returns of a multifactor strategy.

Master of Science in Business, Major in Finance
BI Norwegian Business School

Hand-in date: July 1st 2021

Acknowledgements

We extend our sincere gratitude and appreciation to Janis Berzins, for his valuable advice, guidance and support throughout the process of writing this thesis.

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1.0 Introduction and motivation

Becker & Reinganum (2018) attribute the effective era of factor investing to the infamous Fama & French (1993) three-factor model, which they refer to as the next evolutionary step in "the triumph of indexing." Following the publication of the three-factor model, the interest in factor investing has been present in both academic research and sophisticated investment strategies. In this thesis, we investigate the performance of different implementation methods for factor investing strategies, focusing on the Norwegian equity market.

To begin with, we study the presence and persistence of the well-known factors value, size, and momentum. As the Capital Asset Pricing Model (CAPM) often fails in practice (Ang, 2014), we also examine "betting against beta," a factor contradicting CAPM's intuition that the risk premium of an asset *only* depends on its beta. We find evidence for the size and betting against beta premiums in the Norwegian equity market but observe that the value and momentum effects are non-existent between 2006 and 2019. During this period, the four factors achieve Sharpe ratios[1] of 0.424, 1.043, -0.060, and -0.114, respectively. Our findings regarding size, value, and momentum essentially contradict international evidence (see for example Fama & French 2012; Lakonishok et al., 1994), but conforms with previous research on the Norwegian market conducted by Næs et al. (2009). The positive Sharpe ratio for betting against beta is in line with Frazzini & Pedersen's (2014) findings in the Norwegian equity market.

We further study the performance of three different strategies, where weights are determined using three different approaches. Firstly, we test the performance of a static, equal-weighted strategy relative to the single equity factors. Secondly, we investigate whether the equal-weighted portfolio can benefit from implementing mean-variance optimized weights. Lastly, we examine whether tactical implementation based on macroeconomic variables can enhance a mean-variance multifactor strategy.

The static equal-weighted portfolio achieves a Sharpe ratio of 0.398, meaning that it only outperforms half of the individual factors. This indicates that an investor

[1] See section 3.6 for an explanation of the Sharpe ratio

might be better off if invested in a pure size or betting against beta portfolio in terms of risk-adjusted returns. We do, however, observe a great reduction in volatility, from around 30% for three of the factors, to 15.87% for the equal-weighted strategy. Our findings of low correlations between factors and the diversification benefits arising from them conform with prior literature (e.g., Ghayur et al., 2018). However, the outperformance of size and betting against beta is somewhat more contradictory.

As a second approach, we test whether implementing dynamic mean-variance optimized weights can improve the risk-adjusted returns of the equal-weighted strategy. Mean-variance optimization represents one of the most practiced methods of choosing optimal portfolio weights and is constructed to provide the best tradeoff between risk and return (Ang, 2014). In contrast to the equal-weighted portfolio, the mean-variance strategy is dynamic in the sense that the weights may fluctuate across periods. We find that mean-variance weights yield annualized returns of 16% during the global financial crises, in line with the post-crisis critique that investors should diversify across factors rather than asset classes (Becker & Reinganum, 2018). Despite the impressive performance during the "great recession," the mean-variance strategy produces a Sharpe ratio of 0.382 between 2006-2019, thus underperforming the naïve equal-weighted strategy in the long run on a risk-adjusted basis.

Finally, we study whether the mean-variance optimization can benefit from tactical implementation based on macroeconomic variables. We use vector autoregressive models combined with Granger Causality tests to uncover relationships between five macroeconomic indicators and four equity factors. We find a positive relationship between liquidity and the size and betting against beta factors and a negative relationship between value and market volatility. We further find that the composite leading indicator, an indicator designed to predict turning points in business cycles (OECD, n.d.), is useful in predicting both momentum and betting against beta.

Due to strong cyclicality in factor returns (Zhang et al., 2009), tactically moving between different factors dependent on expected market conditions may increase risk-adjusted returns. The benefits of so-called factor timing are, however, strongly debated. For example, Asness (2016) argues that factor timing strategies have historically performed weakly. On the other hand, Arnott et al. (2016) argue that factor timing approaches can enhance performance as long as it does not compromise diversification benefits. We find that factor timing enhances risk-adjusted returns in normal times[2], generating a Sharpe ratio of 0.705 between March 2009 and December 2019. The factor timing strategy does, however, underperform the other strategies during crises, suggesting that it is challenging to predict factor returns during market turmoil. Nevertheless, the factor timing strategy outperforms all other multifactor strategies in the long run, generating a Sharpe ratio of 0.431 over the period 2006-2019.

The remainder of this paper is structured as follows: Part 2 reviews relevant literature on factor investing and implementation methods of multifactor models. Part 3 presents the theory and methodology applied in our research. Part 4 describes the data used to construct the investment strategies. In part 5, we present and discuss our findings. Part 6 concludes.

[2] We define normal times as periods where market movements are not associated with crises or market returns are abnormally high.

2.0 Literature Review

2.1 Factor Investing

Based on Markowitz's (1952) diversification and mean-variance utility principle, the Capital Asset Pricing Model was formulated as a model to explain the relationship between asset returns and systematic risk. According to the CAPM, the return of an asset i is given by

$$E(r_i) = r_f + \beta_i\big(E(r_m) - r_f\big) \tag{1}$$

where $E(r_i)$ is the expected return of asset i, r_f is the risk-free rate, β_i is the beta of asset i and $E(r_m)$ is the expected return of the market. Due to its prediction that the market portfolio is the only factor that matters and that asset risk premiums only depend on the asset's beta, the CAPM does not hold in practice (Pedersen, 2015). Nevertheless, the CAPM continues to be considered the "workhorse model of finance" (Ang, 2014), and the basic intuition of the model still holds; the underlying factors of an asset incur risk premiums as compensation for investors' losses during bad times.

Individual equity factors perform well in good times. However, they may suffer major losses during bad times, which is the reason factors accrue risk premiums (Ang, 2014). In fact, factor investing is sometimes referred to as "risk-premia investing." Although there is no broadly accepted categorization of factor risk premiums, Ang (2014) suggests separating between macroeconomic and style factors. Whereas the former captures risks *across* asset classes, the latter captures risk *within* asset classes and can consequently explain asset returns. An important distinction between the two types of factors is that, while macroeconomic factors may be difficult to trade directly, an investor can easily implement style factors.

As previously mentioned, the CAPM states that there exists only one factor; the market factor. The CAPM market risk premium is given by $E(r_m) - r_f$, i.e., the expected returns of the market in excess of the risk-free rate. However, in the past decades, several other factors have been uncovered. These factors, which cannot be explained by the CAPM, can be referred to as anomalies. Style factors, such as value, size, and momentum, and macroeconomic factors, such as economic growth and volatility, are all examples of anomalies investors could attempt to exploit. In this thesis, we will focus on the three style factors mentioned above, in addition to the betting against beta factor.

2.2 The Value Factor

The fundamental principle of value investing is to purchase undervalued stocks while selling overvalued stocks, based on a comparison between the fundamental value of a stock and its current market value. A common measure used to determine whether a stock is over- or undervalued is the book-to-market (BM) ratio, i.e., a company's book value of equity relative to its market value of equity. Using this ratio, a value investor will purchase the high BM stocks (value stocks) and sell the low BM stocks (growth stocks), expecting that value stocks will outperform growth stocks. Thus, the value factor is regularly termed high-minus-low, or HML. The ratio between an asset's return and its BM ratio is often referred to as the "value effect" (Asness et al., 2013).

The zero-net value strategy captures the potential outperformance of value stocks over growth stocks, where the difference in returns is referred to as the value premium. Historically, the value premium has proven to be robust (Ang, 2014). Fama & French (1992) argue that the premium arises because high BM companies are less profitable and relatively distressed, and the premium is thus compensation for a higher fundamental risk. Behavioral theories, on the other hand, explain the value premium through overreaction of past growth. For example, Lakonishok et al. (1994) argued that the premium stems from strategies exploiting suboptimal investor choices of overpaying for growth stocks.

Value investing represents an active contrarian strategy that allows an investor to buy low and sell high and has been highly successful both across assets and regions (Pedersen, 2015). For example, the HML factor in the Fama & French (1993) model delivered an average annual excess return of 4.6% and a standard deviation of 12.3% between 1926 and 2012, resulting in a Sharpe ratio (SR) of 0.4. Næs et al. (2009) do not, however, find a significant value premium in the Norwegian market in the period 1980-2006.

2.3 The Size Factor

The size factor was first discovered in 1981 when researchers found that returns were negatively related to size. In other words, stocks of small-cap companies tend to have higher returns than large-cap stocks (Ang, 2014). Thus, investors attempting to exploit the size effect will purchase small stocks and simultaneously sell big stocks. Consequently, the long-short size factor is referred to as small minus big (SMB).

Rational theories argue that small firms often have lower earnings and are less profitable than larger firms (Fama & French, 1996). In addition, small stocks tend to be traded less frequently than large stocks and may therefore offer a liquidity premium. On the other hand, behavioral theories suggest that small stocks are evaluated over-optimistically (Koedijk et al., 2016).

Despite size being a well-known factor used in several impactful models and theories, there has not been any significant size effect since the mid-1980s, according to Ang (2014). Some have argued that the initial discovery was a result of data mining and that the size effect is non-existent. Others argue that the size effect was indeed real, but the actions of rational investors have caused it to disappear (Ang, 2014). The latter indicates that size should be removed as a factor, as it is not considered an anomaly. Nonetheless, evidence shows that the factor can amplify the effects of other factors such as value and momentum. For example, Fama & French (1993) found that the value premium for US stocks was larger for small stocks than big. Further, researchers have found evidence for the size premium being present in the Norwegian equity market (Næs et al., 2009).

2.4 The Momentum Factor

The momentum effect refers to "the relation between an asset's return and its recent relative performance history" (Asness et al., 2013). Research conducted by Jegadeesh and Titman (1993) could reveal significant abnormal returns over a 3- to 12-month horizon for an investor selling stocks that had performed poorly and purchasing stocks that had performed well in the past. Hence, the momentum strategy is based on the phenomenon that past "winners" continue to win, and past "losers" continue to lose. The long-short strategy of purchasing recent winners and selling recent losers (winner-minus-loser, or WML) will thus capture the outperformance arising from this phenomenon.

Momentum tends to follow monetary policies and government risk during market crashes and has a high correlation with the macroeconomic environment. In fact, momentum is positively related to liquidity risk, and thus momentum strategies will drive liquidity premia (Asness et al., 2013). Furthermore, behavioral theories explain the momentum effect through reaction models. According to Pedersen (2015), stocks exhibit initial underreaction and delayed overreaction, making it possible to earn high returns from investing in the momentum factor. Despite that delayed overreaction may persistently drive stock prices upwards, the reaction models recognize that prices will revert back to fundamentals after some time (Ang, 2014).

Historically, momentum has performed better than both the value and size factors (Ang, 2014). Furthermore, it is argued that a dynamic momentum strategy can double the alpha and Sharpe ratio of a static momentum strategy (Becker & Reinganum, 2018). Nevertheless, as with other factors, momentum strategies do not always perform well and have experienced large drawdowns in certain time periods, such as in 2009.

Griffin et al. (2003) found that during the period 1982-2000, a momentum strategy in the Norwegian market generated significant monthly returns of 1.11%, driven by both a positive average return in their winner portfolio and a negative average return in their loser portfolio. Næs et al. (2009), on the other hand, found no significant momentum effect in the Norwegian equity market during their sample period 1980-2006. The authors note that although the monthly differential return between their top and bottom portfolio was 0.44% on average, the WML factor incurred large losses during 1990-1999.

2.5 The Betting Against Beta Factor

Historically, low beta stocks have been found to generate higher risk-adjusted returns compared to high beta stocks. These findings represent a risk anomaly contradicting the CAPM theory, which states that asset returns should be proportional to the asset betas (Pedersen, 2015). A consequence of the anomaly observed in the market is that the empirical security market line[3] (SML) is steeper than the true SML.

Frazzini & Pedersen (2014) attempt to exploit the anomaly by constructing a betting against beta (BAB) factor, which is long low-beta assets and short high-beta assets. The authors construct a market-neutral strategy by leveraging and deleveraging the low-beta and high-beta assets in order to obtain a beta of zero for the overall strategy. They further report a SR of 0.78 for US stocks and conclude that the BAB factor generates positive returns in most global stock markets. Using the MSCI Norway, they obtained a SR of 0.25. The BAB factor is similar to a low volatility factor, as low-beta stocks tend to be less risky than high-beta stocks. However, an asset's beta only represents its co-movement with the market, and a BAB strategy is therefore not equivalent to a low-volatility strategy.

Behavioral theories explain the risk anomaly by investors being too focused on tracking error rather than actual risk (Koedijk et al., 2016). Another explanation may be the "lottery ticket" effect; investors purchase volatile assets hoping to achieve extraordinary returns fast. In addition, many investors avoid taking on leverage or are restricted by leverage and short-selling constraints. These investors may buy riskier stocks to achieve higher returns, pushing the prices of high-risk stocks up, while the price on low-risk stocks is reduced as a result of low demand (Pedersen, 2015).

2.6 Multifactor models

The first multifactor model, Ross' (1976) arbitrage pricing theory (APT), was proposed as an alternative to the mean-variance CAPM. The APT relies on two key underlying assumptions. Firstly, it assumes that no arbitrage opportunity will last because asset prices will revert back to equilibrium. Secondly, there is a linear relationship between the expected return of an asset and various macroeconomic factors. Thus, the price of an asset results from these macroeconomic factors and the risk premiums they yield. However, the APT is purely theoretical and does not specify which and how many factors are appropriate (Becker & Reinganum, 2018).

[3] The security market line is a graphical representation of the return-to-beta relationship, showing the required rate of return to compensate investors for risk and the time value of money (Bodie et al., 2021).

Later, several other multifactor models have been developed, which specifies several empirically based factors to explain asset returns.

The aforementioned Fama & French (1993) three-factor model represents a major contribution to factor investing and asset pricing, as it proved that asset prices cannot be explained by market betas alone. Fama & French (2015) further developed the model by adding two additional factors, profitability and investment, resulting in a multifactor model performing even better than its predecessor.

Applying multifactor models as strategies can be powerful in understanding and managing a portfolio's risk profile, as multifactor models recognize that bad times are not restricted to only include low or negative market returns (Ang, 2014). By combining individual factors, a multifactor strategy will provide an investor with exposure to several factors simultaneously. Thus, as each individual factor defines a different set of bad times, multifactor strategies can deliver great diversification benefits. Diversification can further be enhanced by the low and sometimes negative correlation between factors. E.g., Asness et al. (2013) found that the negative correlation between value and momentum generated higher risk-adjusted returns than either did alone. Moreover, when comparing combinations of multifactor models with single factors, Vincent et al. (2018) found that the multifactor SRs were superior in almost all cases.

2.7 Mean-Variance Portfolio Optimization

In the Markowitz (1952) mean-variance (MV) portfolio theory, the optimal portfolio weights are determined as those providing the optimal trade-off between volatility (risk) and returns. The volatility is highly dependent on the correlation between the assets within a portfolio, and an investor should choose a diversified portfolio to reduce risk and increase returns (Ang, 2014). Several constrained subcategories of MV include risk-parity and minimum variance, but we restrict this section to only discuss the general framework of MV optimization. The theory assumes that investors will favor a portfolio with lower risk for the same expected return, resulting in the mean-variance efficient portfolio being the one that maximizes the Sharpe ratio.

In the MV framework, the best set of portfolios an investor can obtain, by only considering means and volatilities, is located along the mean-variance frontier, illustrated in figure 1. The top half of the frontier (bold line) is efficient, meaning that one can obtain a higher return for the same risk by moving from the bottom half to the top half of the frontier. As shown in figure 1, the portfolio located farthest to the left on the MV frontier is called the minimum variance portfolio.

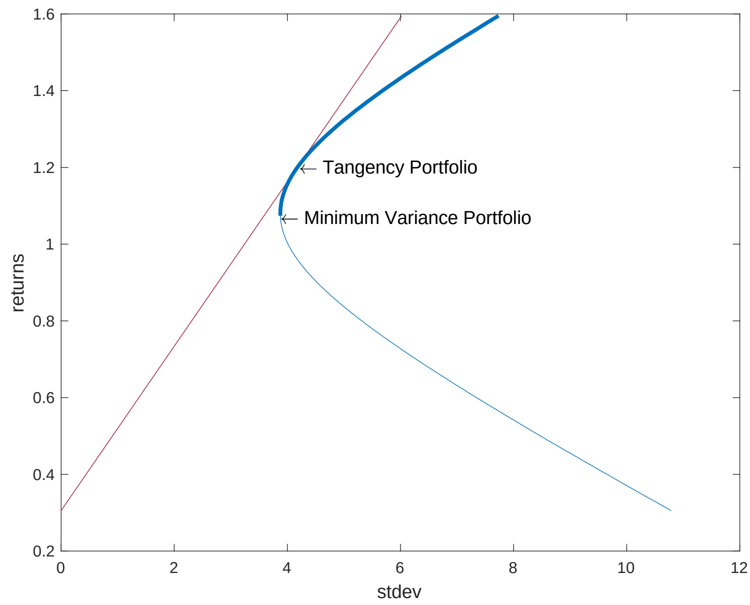

Figure 1: Illustration of Mean-Variance Frontier and Capital Allocation Line

The blue line represents the mean-variance frontier, where the bold part of the line is the efficient part of the frontier. The linear, red line illustrates the Captial Allocation Line. The figure shows that the tangency portfolio is located where the Capital Allocation Line and mean-variance frontier intersects, and the minimum variance portfolio is located at the leftmost point of the frontier.

The mean-variance efficient portfolio, or tangency portfolio, is the one that maximizes the Sharpe ratio. This portfolio is located at the tangency point where the mean-variance frontier intersects with the Capital Allocation Line (CAL). The CAL represents all risk-return combinations an investor can obtain, where its slope is equal to the Sharpe ratio.

Although the MV framework is the most practiced method used to choose optimal portfolio weights, it has been widely criticized due to its high sensitivity to small changes in inputs (Ang, 2014). Correlations and standard deviations are estimated with standard errors and expected returns estimates have even larger standard errors, representing the most problematic of the three inputs. In addition, MV optimization uses historical returns as an indicator for future returns, but there is no guarantee that past performance predicts future results. The problems associated with MV optimization can result in troublesome weights; for example, MV tends to create concentrated asset allocations, which may be risky. Jagannathan and Ma (2003) showed that using upper and lower bounds on portfolio weights can mitigate extreme positions and keeps the weights at an economically reasonable level.

2.8 Market Timing

Tactical asset allocation, or market timing, refers to a short to medium-term asset allocation strategy in which an investor reallocates portfolio weights according to current market views (Pedersen, 2015). Investors thus attempt to time the market by adjusting weights across major asset classes, such as equities and bonds, based on predictions of future returns.

Market timing can be extremely difficult to implement successfully. For example, Samuelson (1994) argues that a simple buy-and-hold portfolio outperforms market timing strategies, pointing out that only a small fraction of investors succeed in going in and out of the market. Chong & Philips (2014), on the other hand, found that a market timing strategy based on macroeconomic factors and mean-variance optimization outperformed the S&P500 with a return-to-risk ratio of 0.97 (MV) compared to 0.19 (S&P500) in the period 2006-2013. However, Zakamulin (2014) argues that market timing strategies' performance is highly overstated.

When attempting to time the market, one can choose between several approaches, but we will in this thesis focus on timing factor returns ("factor timing") based on predictions from macroeconomic variables. There exist countless variables that one might use as indicators, but we limit this thesis to emphasize variables closely linked to the state of the economy. Studying the relationships between macroeconomic variables and factors, Zhang et al. (2009) found that both value and small stocks performed well during economic expansions and when interest rates were low. Further, literature suggests that momentum profits are related to liquidity and macroeconomic risk and vary with business cycles (Ang, 2014). Frazzini & Pedersen (2015) find that the BAB factor performs poorly when funding constraints tighten, i.e., when liquidity is low.

3.0 Theory and Methodology

3.1 Theory and Hypothesis

3.1.1 Assumptions

Prior to presenting our theory and hypothesis, we find it helpful to disclose our assumptions. We assume a large investor with long horizon, as these characteristics are favorable for factor investing. Since all factors are long-short by construction, a necessary assumption is also that short-selling and leverage is allowed. We further assume that all securities can be purchased or sold at the closing price for the given day. Taxes and transaction costs are ignored.

3.1.2 Theory and Hypotheses

In this paper, we investigate whether our expectations of different multifactor implementations hold or not. Our expectation is that diversification across factors will reduce portfolio volatility and thereby yield higher risk-adjusted returns than single-factor portfolios. Further, we believe that implementation based on optimized weights will outperform a static equal-weighted portfolio. Finally, we expect that such an optimization may benefit from the use of forecasted returns rather than historical. Consequently, we construct three multifactor strategies:

(1) A static, naïve equal-weighted multifactor portfolio consisting of the four factors HML, WML, SMB, and BAB, rebalanced monthly. We refer to this strategy as "EWS."

(2) A dynamic multifactor strategy where the weights are determined using recursive[4] and rolling windows to find the mean-variance optimal portfolio. Optimal weights are determined ahead of each month based on historical data. The selection criterion for the best portfolio is the Sharpe ratio. We will refer to this strategy as "MVS."

(3) A dynamic multifactor strategy where the weights are determined in the same manner as MV but using forecasted returns based on macroeconomic variables as inputs in the mean-variance optimization. The strategy will be referred to as "FTS."

[4] Brooks (2014) defines a *recursive window* as one where "a set of time series regressions are estimated using sub-samples of increasing length. After the first model is estimated, an additional observation is added to the end of the sample so that the sample size increases by one observation." (Brooks, 2014, p. 692).

We focus on risk-adjusted returns measured by the Sharpe ratio to evaluate the performance of each factor. We formulate three hypotheses corresponding to each of the multifactor models.

Hypothesis 1: A multifactor strategy is superior to different single factor portfolios, represented by value, size, momentum and BAB, in terms of risk-adjusted returns.

$$H_0: SR_{EWS} \leq SR_{SF_i}, \quad for\ i\ HML, SMB, WML, BAB$$
$$H_A: SR_{EWS} > SR_{SF_i}$$

When combing four factors with low correlations in one strategy, it is reasonable to expect the standard deviation of the multifactor strategy to be lower than that of the individual factors. By exploiting low correlations, we expect to achieve high diversification benefits, which will result in higher risk-adjusted returns for an equal-weighted multifactor strategy than each single factor portfolio. Therefore, we expect to reject the null hypothesis.

Hypothesis 2: A dynamic mean-variance optimization will improve the risk-adjusted returns of an equal-weighted multifactor model.

$$H_0: SR_{MVS} \leq SR_{EWS}$$
$$H_A: SR_{MVS} > SR_{EWS}$$

We expect that two differences between the EW and MV strategies will affect performance. Firstly, the mean-variance optimization is constructed to find the optimal trade-off between risk and return for each period. Secondly, the weights in the MV strategy may change as a result of the time period where inputs are estimated. Since the MV takes into account historical returns and correlations, we expect the mean-variance optimized weights to yield higher risk-adjusted returns, mainly driven by a reduction in portfolio risk. Thus, we expect to reject the null hypothesis.

Hypothesis 3: A dynamic mean-variance strategy will benefit from tactical implementation with regard to risk-adjusted returns.

$$H_0: SR_{FTS} \leq SR_{MVS}$$
$$H_A: SR_{FTS} > SR_{MVS}$$

Based on empirical evidence that factors exhibit cyclicality (Zhang et al., 2009), we construct a factor timing strategy that attempts to predict factor returns. A successful factor timing strategy will allocate less to factors performing poorly, compensated by increased weights in better-performing factors. The macroeconomic variables tested for their predictive power include market liquidity and volatility, oil prices, interest rate levels, and the Composite Leading Indicator. All macroeconomic risk factors are chosen based on their close relationships to the state of the market, while the oil price is chosen due to the common assumption of its relation to the Norwegian economy (Ødegaard, 2021). As the mean-variance optimization uses past returns as a proxy for future returns, we expect the strategy to benefit from implementing tactical allocations based on forecasted returns. We thereby expect to reject the null hypothesis.

In the sections to follow, we first explain how the four individual factors are constructed. Second, we establish how the multifactor strategies are constructed, including the methods we use to predict factor returns. Lastly, we provide detailed descriptions of the performance measurements used to evaluate the multifactor strategies.

3.2 Individual Factor Construction

3.2.1 The Value factor – HML

HML is constructed on the basis of BM ratios of each individual company in the sample. Ødegaard (2021) gathers book values from the companies' balance sheets, and market values are calculated by multiplying the total number of stocks outstanding by the stock price at year-end. The BM ratio for company i is calculated by dividing its book value of equity for the fiscal year $t-1$ by the market value of equity at the end of December at $t-1$:

$$BM_i = \frac{BVE}{MVE} = \frac{BV(equity)}{Stocks\ outstanding \cdot Stock\ price} \tag{2}$$

Companies are ranked according to their BM ratios at the end of June and sorted into quintile portfolios. The 1st quintile consists of the firms with the lowest BM ratios, and the 5th consists of the firms with the highest BM ratios. The HML factor is then constructed as a long-short portfolio where we purchase stocks in the top quintile and sell stocks in the bottom quintile. The return from the HML portfolio at time t can thus be calculated as the difference in returns between the top and bottom quintile:

$$r_t^{HML} = r_t^{High\,BM} - r_t^{Low\,BM} \tag{3}$$

3.2.2 The Size factor – SMB

To construct the SMB factor, companies are first ranked according to equity size of individual firms. Ødegaard (2021) computes equity size as the stock price p of a company multiplied with the number of outstanding shares as of year $t-1$, or:

$$Equity\ Size_t = p_{i,t-1} \cdot No\ Shares_{i,t-1} \tag{4}$$

Equally distributing stocks into five parts, we obtain five quintile portfolios, where the top quintile consists of the largest companies, and the bottom quintile consists of the smallest companies. Contrary to HML, the size factor is constructed by taking a long position in the bottom quintile and a short position in the top quintile. Thus, the return from the SMB factor portfolio is the return from the smallest companies, less the returns from the largest companies:

$$r_t^{SMB} = r_t^{Small\ Size} - r_t^{Big\ Size} \tag{5}$$

3.2.3 The Momentum Factor – WML

Momentum is calculated using a 1-year rolling window, omitting the last month to avoid short-term reversals (Ødegaard, 2021). All stocks are divided into quintiles, where the top quintile consists of the stocks with the 20% highest returns during the last year, i.e., the winners, and the bottom quintile consists of stocks with the lowest 20% returns during the last year, i.e., the losers.

WML is constructed by taking a long position in the winner portfolio and a short position in the loser portfolio. The momentum returns capture the outperformance (or underperformance) of the winner portfolio relative to the loser portfolio:

$$r_t^{WML} = r_t^{Winners} - r_t^{Losers} \tag{6}$$

3.2.4 The Betting Against Beta factor – BAB

Our approach to construct the BAB factor follows the same methodology as Frazzini and Pedersen (2014). To improve the accuracy of covariance, we use daily returns. We estimate ex-ante betas as

$$\hat{\beta}_i^{ts} = \frac{\hat{\sigma}_i}{\hat{\sigma}_m}\hat{\rho}_{i,m} \tag{7}$$

where $\hat{\sigma}_i$ is the estimated volatility of stock i, $\hat{\sigma}_m$ is the estimated volatility of the market, and $\hat{\rho}_{i,m}$ is the estimated correlation between stock i and the market. Volatilities are estimated using one-year rolling standard deviations from one-day log returns, whereas correlations are estimated on the last 5 years using overlapping three-day log returns, calculated as $r_{i,t}^{3d} = \sum_{k=0}^{2} \ln{(1 + r_{t-k}^{i})}$. Three-day log returns are used because correlations tend to move slower than volatilities, and are affected by nonsynchronous trading (Frazzini & Pedersen, 2014). At least 120 and 750 days of non-missing data is required to estimate the volatilities and correlations, respectively.

Betas are shrinked towards the cross-sectional mean to limit the influence of outliers by applying a shrinkage factor, w_i = 0.6, as follows:

$$\hat{\beta}_i = w_i\hat{\beta}_i^{ts} + (1 - w_i)\hat{\beta}^{XS} \tag{8}$$

For simplicity, the cross-sectional mean, β^{XS}, is set to 1 as this is equal to the expected cross-sectional mean beta across all securities in the market. We use the same value as Frazzini & Pedersen (2014) of 0.6 for the shrinkage factor. This does not affect the ranking when the companies are sorted into portfolios.

Each stock is ranked in ascending order according to their corresponding beta at time t and sorted into two portfolios, low-beta and high-beta. In each portfolio, securities are weighted based on their beta value, where the smallest (largest) betas get the highest weight in the low-beta (high-beta) portfolio. The weight of stock i in the different portfolios is calculated as

$$w_{H,i} = k(z_i - \overline{z})^{+} \tag{9}$$

$$w_{L,i} = k(z_i - \overline{z})^{-} \tag{10}$$

where z_i is the rank of security i, $\bar{z}$ is the mean of all ranks in both portfolios, and k is a normalizing constant $k = 2/1_n'|z - \bar{z}|$. x^+ and x^- express the positive and negative values of a vector x, used to obtain absolute values. The portfolios are rebalanced monthly, and by construction the weights within each portfolio sum to 1.

For illustrative purposes, consider 94 betas at time t. The stocks with the lowest and highest betas will be assigned a rank of 94 and 1 respectively. $\bar{z}$ will become $\frac{n(n+1)}{2n} = \frac{94(94+1)}{2\times 94} = 47.5$, and k will be approximately 0.000905. The weight of the stock with the smallest beta in the low-beta portfolio will then be $|(0.000905 * (94 - 47.5))| = 4.21\%$, and the weight of the stock with the largest beta in the high-beta portfolio will be $|(0.000905 * (1 - 47.5))| = 4.21\%$.

The betting against beta factor is the self-financing zero-beta portfolio, where both portfolios are adjusted to have a beta of 1, going long the low-beta portfolio and short the high-beta portfolio. E.g., suppose the low-beta (high-beta) portfolio has an average beta of 0.8 (1.3). In that case, the strategy is long (short) 1.25 (0.77) in the low-beta (high-beta) portfolio. This will capture the potential outperformance of the low beta stocks relative to the high beta stocks. The return from the long-short BAB strategy is calculated as:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}\left(r_{t+1}^L - r_{f,t}\right) - \frac{1}{\beta_t^H}\left(r_{t+1}^H - r_{f,t}\right) \tag{11}$$

where $r_{t+1}^L = r_{t+1}' w_L$, $r_{t+1}^H = r_{t+1}' w_H$, $\beta_t^L = \beta_t' w_L$, and $\beta_t^H = \beta_t' w_H$.

3.3 Mean-variance optimization

For a given target mean, $\boldsymbol{\mu}$, the dynamic mean-variance optimization problem can be stated as

$$\min_{w_t} \frac{1}{2} \boldsymbol{w}_t' \boldsymbol{\Sigma}_t \boldsymbol{w}_t \tag{12}$$

where $\boldsymbol{w}_t$ is a vector of portfolio weights at time t and $\boldsymbol{\Sigma}_t$ is a covariance matrix. The optimization problem is subject to the following constraints

$$\boldsymbol{w}_t'\boldsymbol{\mu}_t = \mu^* \tag{13}$$

$$\boldsymbol{w}_t'\boldsymbol{1} = 1 \tag{14}$$

$$LB \leq w_t \leq UB \tag{15}$$

(13) states that the expected return of the portfolio should equal the target mean, and (14) states that the portfolio weights must sum to 1. The mean vector $\boldsymbol{\mu}_t$ is a 4x1 vector consisting of the historical means of the four factors. We impose upper and lower bounds to avoid extreme positions and short-selling, restricted by (15). This will also help keep diversification benefits and avoid corner portfolios.

The mean-variance optimization is performed using three different UBs of 0.35, 0.40, and 0.45, and LB = 0, where $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$ is calculated on 1-, 3- and 5-year rolling windows along with a recursive window. The optimal weights obtained from the mean-variance optimization at time t are implemented the following month.

3.4 Multivariate time series analysis

We build a 6-dimensional Vector Autoregressive (VAR) model for each of the four factors to capture the relationship between the investment factors and macroeconomic variables over time. Brooks (2014) highlights that VAR models are a-theoretical, meaning that they are not concerned with theory, and they can involve extremely many parameters. Nevertheless, VAR models have the advantage of allowing a variable to depend on more than only its own lags, and one does not need to specify which variables are endogenous or exogenous.

The optimal lag length of the macroeconomic variable within each VAR model is chosen based on the Hannan-Quinn Information Criterion (HQIC) given by

$$MHQIC = \ln\left|\hat{\Sigma}\right| + \frac{2k'}{T}\ln(\ln(T)) \tag{16}$$

where $\hat{\Sigma}$ is the variance-covariance matrix of the residuals and T is the sample size. The choice of information criterion is based on a comparison between the three most common information criteria, AIC, SBIC, and HQIC[5]. As the SBIC is

[5] See Brooks (2014) for an explanation of the AIC, SBIC and HQIC.

inefficient and the AIC is not consistent, we choose the HQIC, although no criterion is unquestionably superior to the others (Brooks, 2014).
The VAR models further undergo Granger Causality (GC) tests to determine whether the time series of the macroeconomic variables are useful for predicting the investment factors. Defining $y_{1,t}$ and $y_{2,t}$ as two different time series, GC tests the null hypothesis that lags of $y_{1,t}$ do not explain current $y_{2,t}$. For illustration, consider the VAR model

$$y_{HML,t} = \alpha_{HML,0} + \beta_{1,1} y_{HML,t-1} + \beta_{1,2} y_{liq,t-1} + \gamma_{1,1} y_{HML,t-2} + \gamma_{1,2} y_{liq,t-2} + \delta_{1,1} y_{HML,t-3} + \delta_{1,2} y_{liq,t-3}$$

To test for Granger causality, we formulate the following null hypothesis:

$$H_0\text{: Lags of } y_{liq,t} \text{ do not explain current } y_{HML,t}$$

where the implied restrictions are $\beta_{1,2} = 0$ and $\gamma_{1,2} = 0$ and $\delta_{1,2} = 0$. If the null hypothesis is rejected, one can say that liquidity "Granger-causes" HML. We perform Granger causality tests using the F-test framework for all macroeconomic variables and investment factors. Individual variables must also pass a t-test before we deem the macro factors significant and include them as predictors for future factor returns.

3.5 Tactical Implementation

With the above in place, the implementation of the factor timing strategy is as follows. Macroeconomic variables passing the significance tests are included as predictors for factor returns. We run multivariate OLS regressions with the investment factor i as dependent variable and the significant, lagged macroeconomic variables as independent variables. As an example, if liquidity is found to Granger-cause the HML factor, and the optimal lag is 3, then the OLS regression is

$$r_{HML,t} = \alpha + \beta_1 liq_{t-1} + \beta_2 liq_{t-2} + \beta_t liq_{t-3} + u_t$$

The resulting alpha and beta coefficients are further used to estimate the HML factor's return for the next period, given the lagged values of the liquidity measure.

The procedure is repeated for each factor and corresponding macroeconomic variables. For each period t, the return is estimated by multiplying the beta coefficient with the relevant macroeconomic variables, resulting in a 4x1 mean vector of estimated returns. The mean vector is then used in the mean-variance optimization, where the difference from the MVS is that the mean vector represents predicted returns, and not historical. We keep the historical covariance matrix, under the simplified assumption that $E[cov(x_t, y_t)] = cov(x_{t-1}, y_{t-1})$.

3.6 Performance measures

To evaluate the individual factors and the multifactor strategies, we use a battery of measurements to ensure robust results and a comprehensive view of their performance. Although the Sharpe ratio is the decisive measurement in our hypotheses, we also consider M^2, Information ratio, skewness, and kurtosis to make the overall assessment.

Sharpe Ratio

One of the most widely used measures of risk-adjusted return, the Sharpe ratio, is a measure of the reward per unit of risk, being the reason why it is often referred to as the risk-reward ratio. The SR is given by

$$SR = \frac{E(r_P - r_f)}{\sigma(r_P - r_f)} \tag{18}$$

where r_P is the return of the portfolio, r_f is the risk-free rate, so that $(r_P - r_f)$ is the strategy's return in excess of the risk-free rate and $\sigma(r_P - r_f)$ is the volatility of the excess return.

Naturally, the higher the SR, the better, as investors prefer high returns and low risk. Although rare, a SR above 1 is highly favorable, as this indicates that a strategy generates excess returns relative to its volatility. A SR between 0 and 1, on the other hand, indicates that a strategy's return is less than the risk taken. Negative SRs occur when a strategy yields negative excess returns.

The SR suffers from several limitations, many of which are related to the use of volatility as a measure of risk. A notable drawback of the risk-reward ratio is that it does not consider the direction of volatility. Further, the SR assumes normally distributed returns, which may not be the case. Despite these drawbacks, we find the SR to be a useful tool which can give valuable insight about the risk and return characteristics of a strategy.

M^2

Modigliani-squared (M^2) focuses on total volatility as a measure of risk, and is given by

$$M_P^2 = \left(\frac{\sigma_{BM}}{\sigma_P}\right) \cdot e_P + r_f \tag{19}$$

where e_P is the average excess return of a portfolio *(*$e_P = r_P - r_f$). M^2 measures the risk-adjusted return of a portfolio P relative to a benchmark by scaling P to have the same volatility as the benchmark (Modigliani & Modigliani, 1997). One could argue that the M^2 measure is an improved version of the SR, as it is easier to interpret the differential returns between two portfolios rather than a dimensionless number, which the SR could be described as.

Information Ratio

In contrast to the SR, which uses the risk-free rate to measure excess returns, the Information ratio (IR) measures performance against a specific benchmark, e.g., the market portfolio. Consequently, the ratio focuses on the abnormal return an investment strategy generates (Pedersen, 2015). The IR is given by

$$IR = \frac{r_P - r_{BM}}{\sigma(r_P - r_{BM})} \tag{20}$$

where r_{BM} and σ_{BM} are the returns and standard deviation of the benchmark, denoted BM, respectively. Since the IR uses tracking error as denominator, $\sigma(r_P - r_{BM})$, one can interpret the IR as a measure of the excess returns of a strategy per unit of tracking error (Pedersen, 2015).

A great advantage associated with the IR is that one can measure a strategy's performance relative to any benchmark. This makes it possible to evaluate a strategy more accurately by measuring against strategies with similar levels of risk, active management, et cetera. It is essential to choose a benchmark that is appropriate in order to obtain accurate results. Hence, one could argue that one of the greatest advantages of the IR in fact represents a weakness as well, as choosing the wrong benchmark may generate inaccurate or unreliable results.

Skewness and Kurtosis

As standard deviations assume normal distributions, skewness can be a beneficial supplement as it considers the asymmetry of a distribution (Brooks, 2014). If a return distribution is positively skewed, most of the returns are located at the left-hand side of the distribution, while the right-hand tail is long. A negatively skewed distribution will have the opposite characteristics. A skewness of zero indicates that the distribution is symmetric, equal to the skewness of the normal distribution. Defining y_i as the observations of a series, the skewness can be measured as

$$skewness = \frac{\frac{1}{N-1}\sum(y_i - \bar{y})^3}{(\sigma^2)^{\frac{3}{2}}} \tag{21}$$

Kurtosis is a measure of the heaviness of the tails of a distribution and the peak of the mean of the series (Brooks, 2014). As the kurtosis of the normal distribution is 3, excess kurtosis can be calculated as the kurtosis minus 3:

$$Excess\ kurtosis = \frac{\frac{1}{N-1}\sum(y_i - \bar{y})^4}{(\sigma^2)^2} - 3 \tag{22}$$

A return distribution with excess kurtosis indicates outliers, i.e., that one may occasionally experience extreme returns. The two measures are often used together when evaluating a return series. Negative skewness and excess kurtosis represent an undesirable combination of an investment strategy, as it indicates that one sometimes may experience extreme returns, especially on the downside (Pedersen, 2015).

3.6.1 Benchmark

The Oslo Stock Exchange Allshare Index (OSEAX) will be used as a common benchmark when computing the IR and M^2, to measure all strategies on an equal basis. In addition, we compare each strategy to the relevant benchmark according to each of the hypotheses stated in section 3.1.2. Firstly, the static equal-weighted portfolio will be compared to the four individual factors, HML, WML, SMB, and BAB. Secondly, we measure the performance of the mean-variance optimized strategy against the equal-weighted portfolio. Lastly, the dynamic factor timing strategy will be measured using the mean-variance strategy as a benchmark.

4.0 Data

4.1 Data collection

To obtain the information needed to construct portfolios and strategies, we have collected data from various sources. The majority of the data is gathered from Oslo Børs Informasjon (OBI) through Bernt Arne Ødegaard, and some indicators are collected from OECD and Bloomberg. We provide a complete list of the variables, data frequency, and sources in **Appendix 1**. The in-sample period reaches from Jan 1985-Dec 2005, and the out-of-sample period reaches from Jan 2006-Dec 2019. To increase robustness in results, the VAR models in the in-sample tests are built using different lengths of periods, determined by data availability.

4.1.1 Return Data and Risk-Free Rate

Historical return data for individual stocks consists of daily, discrete returns for 897 individual companies listed on the Oslo Stock Exchange (OSE) during our sample period. The sample covers the period between January 31st, 1980, and December 31st, 2019, resulting in a time frame of 40 years. To avoid survivorship bias, all companies listed at any time during this period are included in the data set. The average number of listed securities during the period is 187, varying between 46 and 272 stocks over the whole sample.

Monthly risk-free rates are estimated using the monthly Norwegian Interbank Offered Rate (NIBOR) as an approximation. Since NIBOR is only available after 1986, the overnight NIBOR is used between 1982 and 1986, and the two-year bond yield is used from 1980 to 1982. Daily risk-free rates are forward-looking 1-day interest rates based on overnight estimates. Missing observations are calculated using spline interpolation, as portrayed in figure 2.

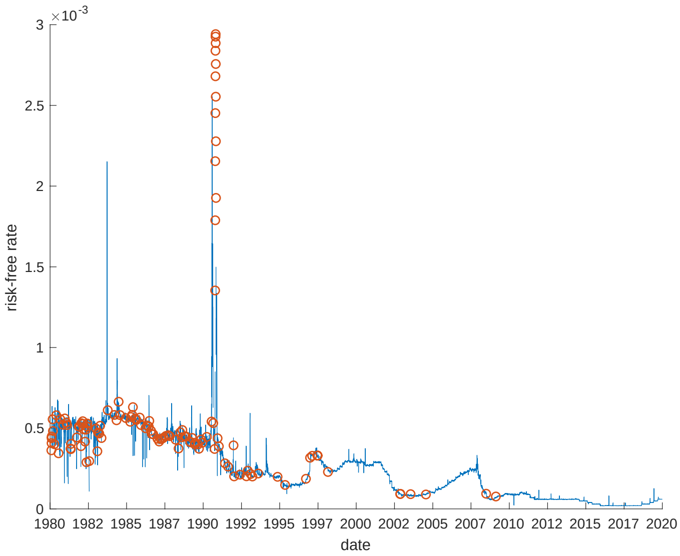

Figure 2: Daily Risk-Free Rates 1990-2019

This figure illustrates the development of daily risk-free rates over the whole sample. Risk-free rates are not annualized and not in percent (y-axis). We use spline interpolation to fill missing observations. The blue line represents non-missing data of daily risk-free rates, while red circles represent the interpolation points.

4.1.2 Cross-sectional portfolios

We collect return data consisting of three sets of cross-sectional portfolios, where each set contains ten portfolios sorted by a factor parameter. Cross-sectional portfolios corresponding to the HML factor are sorted by BM ratios, whereas SMB and WML portfolios are sorted by firm equity size and past returns, respectively. All sets of cross-sectional portfolios consist of equal-weighted, monthly returns of stocks listed on OSE between 1980-2019, except HML, which starts in January 1981. We note that we merge decile portfolios pairwise to obtain quintile portfolios before constructing the size, value, and momentum factors. Cross-sectional portfolios for HML, WML, and SMB are constructed by Bernt Arne Ødegaard, and we construct the BAB factor ourselves.

4.1.3 Macroeconomic Indicators

We collect data for five macroeconomic indicators from various sources (**Appendix 1**). The oil price indicator is represented by the logarithmic change in monthly Brent CO1 closing prices collected from Bloomberg, spanning from June 1988 to December 2019:

$$\Delta oilprice_t = \ln\left(\frac{oilprice_t}{oilprice_{t-1}}\right) \tag{23}$$

Monthly market volatility is calculated using value-weighted daily market returns from January 1980 to December 2019. Volatility is measured as the annualized standard deviation of market returns over the past 30 days. The market volatility indicator is further calculated as the natural logarithmic change in volatility from month $t-1$ to month t:

$$\Delta vol_t = \ln\left(\frac{\sigma_{M,t}}{\sigma_{M,t-1}}\right) \tag{24}$$

Market liquidity is measured as the natural logarithmic change in monthly turnover on OSE from January 1980 to December 2019:

$$\Delta liq_t = \ln\left(\frac{Turnover_t}{Turnover_{t-1}}\right) \tag{25}$$

With the risk-free rate indicator, we attempt to capture whether there is a rising or declining interest rate environment. The indicator is calculated as the natural logarithm of the change in monthly risk-free rates:

$$\Delta r_{f,t} = \ln\left(\frac{r_{f,t}}{r_{f,t-1}}\right) \tag{26}$$

The Composite Leading Indicator (CLI) is an indicator constructed by OECD to provide early signals of turning points in business cycles (OECD, n.d.). The CLI is composed of several components which may differ across countries. OECD's composite leading indicator for Norway is based on exports to the UK, share prices, CPI All Items, and three measures related to manufacturing (OECD, 2021). CLI data is published monthly, but there is a two-month lag between the reference data and publication date (OECD, n.d.). A drawback associated with the CLI is that the measure is amplitude-adjusted to have a long-term mean of 100, which has an effect backward.

5.0 Analysis and Discussion

5.1 In-Sample Analysis

In the sections to follow, we present and analyze the results from the in-sample tests. All results cover the period 1990-2005 to compare the results accurately, as the rolling windows require up to 5 years of historical returns. We note that the interest rate levels are exceptionally high during the first years of the in-sample period, resulting in large differences in returns and excess returns. We refer to figure 2 in section 4.1.1 for an illustration of risk-free rates over the full sample.

5.1.1 Individual Factor Performance

From the in-sample tests, it is clear that the SMB and BAB factors outperform HML and WML in terms of risk-adjusted returns, as shown in table 1. Moreover, three out of the four factors outperform the OSEAX.

Table 1: In-Sample Performance of Equal-Weighted Strategy and Individual Factors

The numbers presented are calculated as annualized averages based on monthly data. Skewness and excess kurtosis are not annualized. Returns are calculated using arithmetic means. The measures are computed for the period 1990-2005 in order to compare the performance of all factors and strategies over the same time period. M2 and Information ratios (IR) is calculated using the OSEAX as benchmark.

	Ret	ExRet	StDev	SR	M2	IR	Skew	ExKurt
OSEAX	12.97%	6.49%	21.28%	0.305	12.97%	N/A	-0.574	0.644
HML	15.83%	9.35%	52.31%	0.179	10.28%	0.049	0.020	3.547
WML	-10.61%	-17.09%	39.44%	-0.433	-2.74%	-0.502	-0.403	0.803
SMB	33.91%	27.43%	43.02%	0.638	20.05%	0.367	0.036	0.521
BAB	18.07%	11.59%	21.13%	0.549	18.15%	0.171	0.770	4.321
EWS	14.30%	7.82%	21.60%	0.362	14.19%	0.037	0.001	1.287

Despite its high standard deviation, table 1 shows that the SMB factor achieves the highest SR, IR, and M^2 of all individual factors, driven by its remarkable returns. The significant positive returns contradict Ang's (2014) claims that the size effect has been insignificant since the mid-1980s. However, they are in accordance with the findings of Næs et al. (2009), which also could report a significant size premium in the Norwegian market. The size factor further has the shape which most closely resembles a normal distribution but with heavier tails skewed towards positive returns.

The BAB factor achieves a SR of 0.549, driven by both high returns and low risk. This is exceptionally higher than that of Frazzini & Pedersen (2014), which could report a SR of 0.25 in the Norwegian market. The difference can, however, have several explanations; (1) we construct the BAB factor using all stocks listed on the OSE rather than only the MSCI Norway; (2) we denominate returns in NOK rather than USD; (3) we use a different sample period. The low standard deviation of the BAB factor could be explained through its similarity to a low volatility strategy, as it is long low-risk stocks and short high-risk stocks in terms of market beta.

HML represents the most volatile of the four factors, generating an extreme standard deviation of 52.31%. Although one could argue that the returns are satisfying, the SR becomes disappointingly low. The high variations in returns associated with the HML are also visible in figure 3. This could confirm the argument of Fama & French (1992), namely that high BM companies are distressed, which may cause volatile stock returns. However, the results do not dismiss the behavioral explanations as they could indicate possible extreme returns in growth stocks due to overreactions.

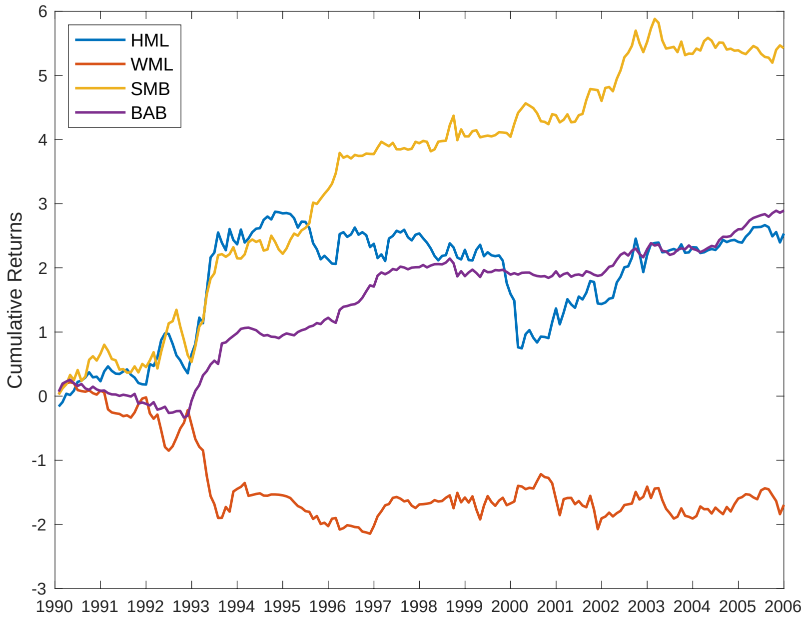

Figure 3: Cumulative Returns of Individual Factors (1990-2005)

We report monthly returns for the value, momentum, size, and betting against beta factors during the in-sample period. The vertical axis shows cumulative returns and is not in percent.

The momentum factor is inferior in all performance measurements, except for standard deviation. This is also the only factor achieving a negative average return over the sample period. Consequently, WML achieves a negative SR, M^2, and IR. These findings challenge Ang's (2014) statement that momentum globally has outperformed both value and size historically but conform with the results of Næs et al. (2009). When examining the factor further, we find that when expanding the sample period to start in 1980, the return for the whole period is 6.6% (**Appendix 2**), but the momentum effect seems to disappear. The results indicate that Norwegian stock prices revert back to fundamental values during or directly after the $12 - 1$ month estimation period, rather than continuing the past trend.

We find it essential to discuss the low correlations in our investment universe. As can be observed in table 2, the correlations are fairly low between all factors. The highest correlation can be found between value and size, indicating that, more often than not, small companies have higher BM ratios. We observe negative correlations between WML and the other factors, in line with prior literature. The negative correlation between value and momentum is clearly observed in figure 3, as the returns of the factors tend to go in opposite directions. These findings are further in accordance with those of Asness et al. (2013).

Table 2: In-Sample Correlation Matrix

The table shows correlations between the four individual factors during the period 1985-2005. Correlations are calculated using monthly returns.

	HML	WML	SMB	BAB
HML	1			
WML	-0.258	1		
SMB	0.348	-0.214	1	
BAB	0.243	-0.061	0.358	1

5.1.2 Static Equal-Weighted Multifactor Strategy Performance

When comparing the EWS to the individual factors, the general trend is that the multifactor strategy outperforms HML and WML but underperforms SMB and BAB. We observe from table 1 that the standard deviation has been drastically reduced compared to three of the factors, suggesting that combining the factors creates diversification benefits. Considering the low and negative correlations between factors, the reduction in portfolio risk is not surprising, confirming our initial expectations and in line with the findings of Ilmanen & Kizer (2012).

The risk-adjusted performance measures, on the other hand, suggest that an investor will benefit more from holding a pure SMB or BAB portfolio rather than combining all four factors. This can be explained by our findings that the momentum effect seems to disappear after some time, in addition to the factor's negative returns, especially in the first few years.

5.1.3 Dynamic Mean-Variance Strategy

Table 3: In-Sample Mean-Variance Optimization Results

The table shows performance measurements for 12 in-sample mean-variance strategies optimized on different lengths of rolling windows and with different upper bounds. All strategies are restricted by a lower bound of 0. The first column defines the length of the rolling windows and the value of the upper bound, where "rec" denotes a recursive window. M2 and Information ratios (IR) is calculated using the OSEAX as benchmark.

	Ret	ExRet	Stdev	SR	M^2	IR	Skew	ExKurt
1yr, 35%	18.02%	7.34%	21.87%	0.336	13.62%	0.024	-0.013	3.742
1yr, 40%	13.68%	7.20%	22.12%	0.326	13.40%	0.020	0.010	1.063
1yr, 45%	14.36%	7.88%	23.33%	0.338	13.67%	0.039	-0.021	1.757
3yr, 35%	18.02%	11.55%	24.51%	0.471	16.50%	0.143	0.992	3.742
3yr, 40%	19.99%	13.51%	26.45%	0.511	17.35%	0.192	1.026	3.939
3yr, 45%	19.32%	12.84%	27.25%	0.471	16.50%	0.171	1.176	4.983
5yr, 35%	16.29%	9.81%	20.56%	0.477	16.63%	0.098	0.168	1.174
5yr, 40%	15.65%	9.17%	21.20%	0.433	15.69%	0.079	0.151	1.394
5yr, 45%	16.97%	10.49%	22.24%	0.471	16.51%	0.116	0.252	2.019
Rec, 35%	15.38%	8.90%	20.89%	0.426	15.55%	0.070	0.136	1.452
Rec, 40%	13.79%	7.31%	20.59%	0.355	14.04%	0.025	0.168	1.449
Rec, 45%	10.65%	4.17%	20.58%	0.202	10.79%	-0.069	-0.009	1.351

The first takeaway from the mean-variance strategies displayed in table 3 is the risk-adjusted returns. Most outperform the EWS, and all but one outperforms the OSEAX, primarily due to higher returns. A possible explanation for their outperformance relative to the EWS may be that the factors themselves exhibit momentum. Given that a mean-variance optimization relies on past performance, the mean-variance strategies will benefit from momentum in factor returns, as the strategy typically will allocate more to factors that have performed well in the past.

We observe from table 3 that the MV optimization is quite sensitive to the window lengths of inputs, i.e., covariances and returns, are based on. The three recursive windows all achieve low standard deviations compared to the rolling windows. An explanation could be that since the estimation period keeps increasing, the inputs will become less affected by outliers, resulting in more stable weighing schemes. The reduction of risk is further reflected in lower returns.

Across all upper bounds, the 3-year rolling windows outperform their competitors regarding the risk-adjusted measures SR, M^2, and IR. Further, it can be observed that imposing a 40% UB generates the best overall performance. This strategy has the highest returns but also the highest standard deviation. Still, the strategy is superior to all others in terms of risk-adjusted returns, achieving a SR of 0.511. Contrary to the other window lengths, the 3-year rolling windows have positive skewness around 1, in addition to high kurtosis, suggesting that these strategies tend to experience extreme returns, often positive. Based on these findings, the 3-year rolling window with UB = 40% will be used in our out-of-sample analysis.

We note that the MVS can only allocate weights across four factors, and by using relatively conservative upper bounds, the difference in returns relative to the EWS is limited. However, as shown in figure 4, the MVS sometimes allocates zero weight to one factor. In fact, between 1997-2003, HML is almost completely excluded. The strategy assigns large weights to SMB and BAB over the whole period and less to HML and WML. Still, the MVS has surprisingly large weights in the momentum factor, which may be a consequence of the negative correlations between WML and the other factors.

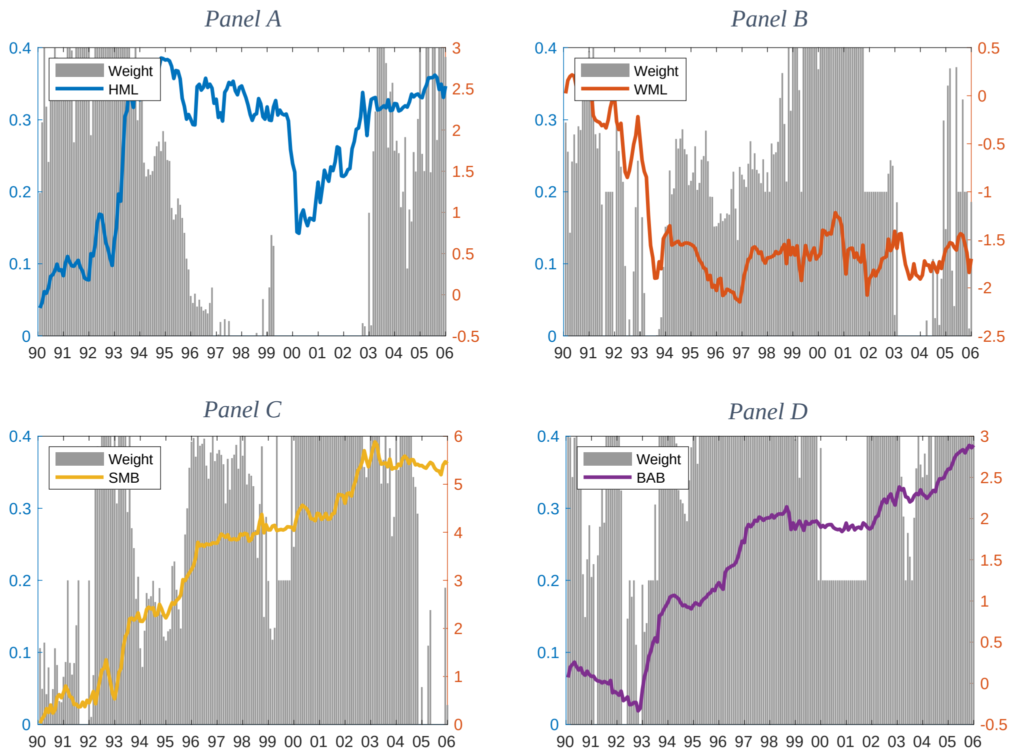

Figure 4: In-Sample Factor Weights for the MVS

Figure 4 illustrates weights assigned to the individual factors for the in-sample mean-variance strategy each month along with the respective factor returns. Panel A corresponds to HML, panel B to WML, panel C to SMB, and panel D to BAB. The axes on the left-hand sides refer to factor weights, and the axes on the right-hand sides correspond to cumulative returns. Notice that the limits on the right-hand side axes differ and that cumulative returns for WML are negative.

5.1.4 Indicator Predictions

The optimal lag length for each indicator based on the Hannan-Quinn Information Criterion is summarized in table 4. We observe that the optimal lag length for oil price is zero, ruling this indicator out as a predictor since including it will cause the

VAR model to collapse. This indicates that the change in oil price is not useful in predicting factor returns in the Norwegian equity market.

Table 4: Optimal Lag Lengths Based on HQIC

The table displays the optimal lag length (months) of each macroeconomic indicator related to the individual factors. Note that since the Composite Leading Indicator is published with a two-month lag, we exclude the first lag in the VAR models.

	HML	WML	SMB	BAB
Composite Leading Indicator	4	5	4	4
Liquidity	1	1	1	4
Oil Price	0	0	0	0
Risk-Free Rate	2	2	4	3
Volatility	2	2	2	2

Testing the significance of indicators using both t-tests and f-tests indicates that only CLI, volatility, and liquidity are useful for predicting factor returns. In contrast to Zhang et al. (2009), we do not find a significant relationship between the change in risk-free rates and any of the factors. On a 10% significance level, we obtain four regression models from the Granger causality tests used to forecast returns. We report the t-statistics in parentheses, where variables are significant when $|t\ stat| > t\ crit\ val\ 1.645$. For an illustration of the presumed causal relationships between significant indicators and factor returns, we refer to **Appendix 3**.

We find significant evidence that the HML factor is negatively related to lags of market volatility (f-stat 3.44 ≥ 1.62 f critical value). These findings may again support the rational explanations of the value premium, namely that high BM companies are unprofitable and distressed (Fama & French, 1992). If market volatility increases, returns are expected to decrease, reflecting the fundamental risk associated with these companies. We do, however, find an insignificant α but keep the intersect to avoid affecting the slope of the regression line. We conclude that when market volatility increases, we will reduce our position in the HML factor.

$$E(r_{HML,t}) = \underbrace{\alpha}_{(0.36)} + \underbrace{\beta_1}_{(-1.85)} vol_{t-1} \tag{27}$$

A notable result from regressing WML on previous lags of the CLI is that the beta coefficients change signs between each lag, as can be observed in the t-statistics in (28). The Granger-causal relationship between the momentum factor and CLI may be more easily observed in **Appendix 3B**. We further observe a significant negative alpha and a significant model (f-stat 1.82 ≥ 1.62 f critical value). In contrast to prior literature (Ang, 2014), we do not find a significant relationship between momentum and liquidity.

$$E(r_{WML,t}) = \underbrace{\alpha}_{(-1.91)} + \underbrace{\beta_1}_{(-1.56)} CLI_{t-2} + \underbrace{\beta_2}_{(1.67)} CLI_{t-3} + \underbrace{\beta_3}_{(-1.72)} CLI_{t-4} + \underbrace{\beta_4}_{(1.70)} CLI_{t-5} \quad (28)$$

Our findings suggest that the size premium is higher in the period following an increase in market liquidity (f-stat 5.01 ≥ 1.55 f critical value). This is not unexpected, as the SMB factor is long small companies, which may be less profitable (Fama & French, 1996), but also since small stocks tend to be traded less frequently. This strengthens the argument that the size factor offers a liquidity premium. We further observe a significant monthly alpha of 0.025, suggesting that, all else equal, the return of the factor is positive if the change in market liquidity is zero.

$$E(r_{SMB,t}) = \underbrace{\alpha}_{(3.57)} + \underbrace{\beta_1}_{(2.24)} liq_{t-1} \quad (29)$$

BAB represents the only factor that is significantly related to two of the macroeconomic indicators (f-stat 4.66 ≥ 1.60 f critical value). We observe the same trend in changing signs for the CLI as we did for the momentum. However, the signs are positive for lags 2 and 4, whereas these signs were negative in the WML regression. We further find that BAB is positively related to liquidity, which may indicate that the factor returns are compensation for liquidity risk.

$$E(r_{BAB,t}) = \underbrace{\alpha}_{(-1.08)} + \underbrace{\beta_1}_{(2.85)} CLI_{t-2} + \underbrace{\beta_2}_{(-2.52)} CLI_{t-3} + \underbrace{\beta_3}_{(2.18)} CLI_{t-4} + \underbrace{\beta_4}_{(1.72)} liq_{t-2} + \underbrace{\beta_5}_{(2.13)} liq_{t-3} \quad (30)$$

The resulting regressions from the VAR models and Granger causality tests are further used to estimate returns in the factor timing strategies. For each factor at time t, the significant beta coefficients are multiplied with the corresponding variables, resulting in a 4x1 mean vector of expected return each month. For illustrative purposes, consider the following coefficients for HML_t: α = 0.0005 and β_1 = - 0.03. The expected return of HML_{t+1} given a logarithmic change in volatility of -0.3317 will be

$$E(r_{HML,t+1}) = 0.0005 - 0.03(-0.3317) = 1.05\%$$

5.1.5 Dynamic Factor Timing Strategy

The mean vectors are constructed on the same window lengths as in the mean-variance optimizations. To better capture the effect of factor timing vs. mean-variance, we retain the upper bound of 40% from the best performing MVS. The results from the factor timing (FT) strategies are summarized in table 5.

Table 5: In-Sample Factor Timing Strategies Results

This table shows the in-sample results from four different factor timing strategies. Column 1 describes the length of the rolling windows, and "rec" denotes a recursive window. All strategies are restricted by UB = 40% and LB = 0. All numbers are calculated as annualized averages based on monthly data, except skewness and excess kurtosis. M2 and Information ratios (IR) is calculated using the OSEAX as benchmark.

	Ret	ExRet	Stdev	SR	M^2	IR	Skew	ExKurt
1yr	19.68%	13.20%	24.58%	0.537	17.90%	0.184	0.473	3.449
3yr	15.83%	9.35%	25.08%	0.373	14.41%	0.078	0.484	2.943
5yr	14.05%	7.57%	22.31%	0.340	13.70%	0.030	-0.392	2.170
Rec	16.19%	9.71%	26.63%	0.365	14.24%	0.084	0.123	1.614

Comparing the results from the tactical implementation with the MV strategies, we observe that the FT performance is also quite reliant on window lengths. Prior to the tactical implementations, our expectations were that all window lengths would outperform their MV counterparts, but we observe that this is only true in half of the cases. With regard to risk-adjusted returns, the shortest and longest window lengths result in the best performing factor timing strategies, and both outperform their MV equivalents. We observe the same trend in returns. The volatilities are generally higher for the FT strategies, which may be due to more drastic changes in weights compared to the MVS. The best performing factor timing strategy is the 1-year rolling window, as it generates the highest risk-reward ratio of 0.537, along with the highest returns, M^2, and IR. When studying the best performing FT and MV strategies, the FTS generates lower returns and standard deviation, ultimately resulting in higher risk-adjusted returns.

We observe from figure 5 that the FTS manages to time the factor returns fairly well. From Panel A and B, we see that the strategy accomplishes to avoid the most drastic downturns and are often fully invested (40%) in the factors when they perform exceptionally well. As SMB and BAB returns are less volatile, the timing ability is more difficult to spot. We point out that if the strategy decides to exclude one factor at time t, e.g., due to predictions of extremely negative returns, the strategy is forced to take a position in all other factors as a result of (14). This may lead to the FTS taking positions in factors yielding negative returns, still, not *as* negative as the worst-performing factor. The same logic concerns the MVS.

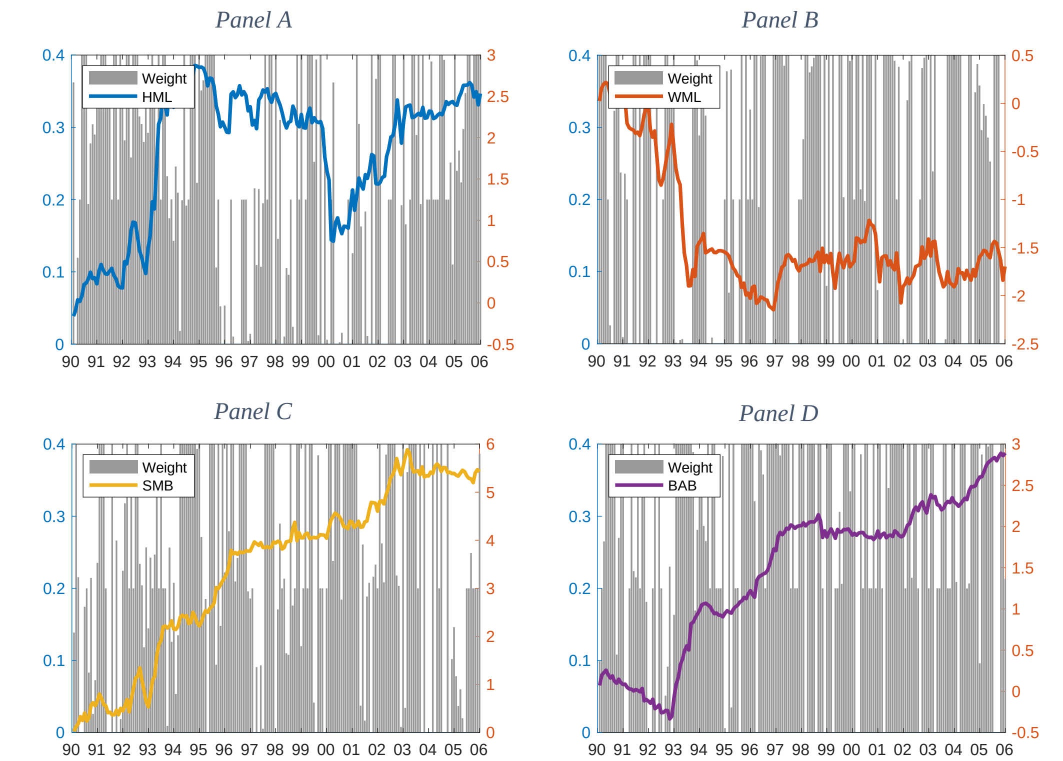

Figure 5: In-Sample Factor Weights for the FTS

Figure 5 illustrates weights assigned to the individual factors for the in-sample factor timing strategy each month along with the respective factor returns. Panel A corresponds to HML, panel B to WML, panel C to SMB, and panel D to BAB. The axes on the left-hand sides refer to factor weights, and the axes on the right-hand sides correspond to cumulative returns. Notice that the limits on the right-hand side axes differ and that cumulative returns for WML are negative.

5.1.6 Subsample Comparison

Table 6: In-Sample Subsamples Sharpe Ratios

This table shows the annualized Sharpe ratios for three multifactor strategies and the OSEAX during four subsamples during the in-sample period 1990-2005.

	OSEAX	EWS	MVS	FTS
1990-1993	-0.154	0.714	0.713	1.191
1994-1997	0.991	0.837	0.976	1.039
1998-2001	-0.201	-0.482	-0.270	-0.583
2002-2005	0.941	0.620	0.769	0.553

Table 6 demonstrates great variations in SRs over different subsamples for all strategies and the OSEAX. All have negative SRs during 1998-2001, and returns are also highly volatile during this subsample, as illustrated in figure 6(C). We further observe that the FTS outperforms during the first two subsamples, generating SRs above 1, but underperforms in the two last. A possible explanation could be that the strategy predicts returns quite well in normal market conditions but becomes more inaccurate in volatile markets. The OSEAX outperforms the multifactor strategies during the last two subsamples, in line with the rationale that factors incur risk premiums as compensation for losses during bad times (Ang, 2014).

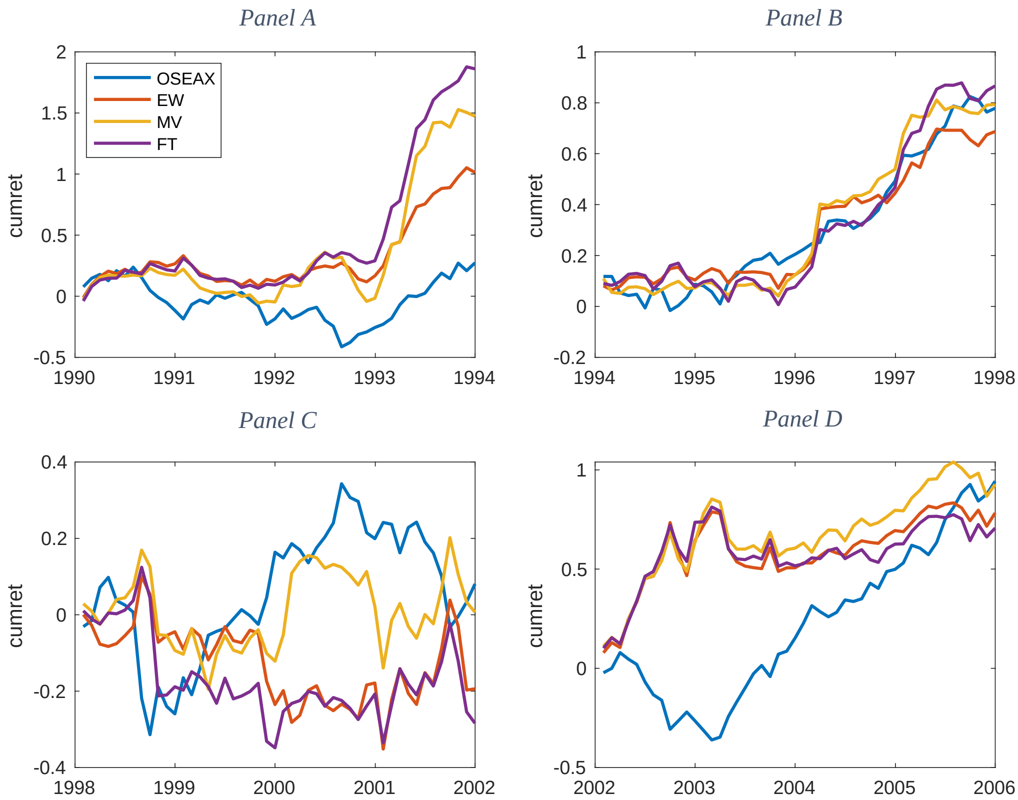

Figure 6: Cumulative Returns for Subsamples in the In-Sample Period

We report cumulative returns for the OSEAX, EWS, MVS, and FTS for four subsamples of the in-sample period. Notice that the limits on the y-axes differ across subplots. Returns are not in percent.

5.2 Out-of-Sample Analysis

Table 7: Out-of-Sample Performance

This table shows the out-of-sample results of the individual factors, the three optimal strategies based on the in-sample results, and the benchmark. All numbers are calculated as annualized averages based on monthly data, except skewness and excess kurtosis. M2 and Information ratios (IR) is calculated using the OSEAX as benchmark.

	Ret	ExRet	Stdev	SR	M^2	IR	Skew	ExKurt
OSEAX	8.93%	6.68%	18.23%	0.367	8.93%	N/A	-1.275	2.855
HML	0.61%	-1.64%	27.19%	-0.060	1.15%	-0.217	-0.316	1.138
WML	-1.35%	-3.60%	31.57%	-0.114	0.17%	-0.284	-0.574	1.080
SMB	16.79%	14.55%	34.32%	0.424	9.98%	0.161	0.187	1.244
BAB	18.21%	15.96%	15.31%	1.043	21.26%	0.349	-0.664	1.965
EWS	8.57%	6.32%	15.87%	0.398	9.51%	-0.012	-0.196	0.568
MVS	8.89%	6.64%	17.40%	0.382	9.21%	-0.001	-0.147	0.546
FTS	9.63%	7.38%	17.13%	0.431	10.11%	0.023	0.238	1.638

From the out-of-sample results displayed in table 7, we observe several differences compared to our in-sample tests. HML and SMB returns are much lower during our out-of-sample period, whereas the momentum returns are not as negative as previously. The differences in returns and excess returns are not as great as they were in-sample, as the risk-free rate is much lower after 2005. We further observe a reduction in standard deviations across all factors, especially HML. The presence of negative skewness for three of the four factors may be a result of the global financial crisis in 2008.

On a risk-adjusted basis, it is evident that the BAB factor is superior, generating a SR of 1.043 in addition to an M^2 of 21.26%. We believe that these findings could result from leverage requirements or that BAB is a relatively new factor and is therefore not priced in. HML and WML both achieve negative SRs caused by negative excess returns. Although lower than in-sample, the SR of SMB is satisfying. We find evidence of the size and BAB premiums in the Norwegian market, as both anomalies persist through time. Although the HML factor's in-sample returns were satisfactory, the out-of-sample results suggest that the value premium has disappeared. Lastly, we have not found evidence for the momentum effect out-of-sample, which could be explained by the same reasoning as in-sample.

Our out-of-sample findings regarding the equal-weighted portfolio are relatively similar to the in-sample results. The EWS still outperforms HML and WML but underperforms SMB and BAB in terms of risk-adjusted returns. We still observe a great reduction in portfolio volatility, again due to the low and negative correlations between factors observed in table 8.

Table 8: Out-of-Sample Correlation Matrix

The table shows correlations between the four individual factors during the period 2006-2019. Correlations are calculated using monthly returns.

	HML	**WML**	**SMB**	**BAB**
HML	1			
WML	-0.309	1		
SMB	0.567	-0.179	1	
BAB	0.077	0.198	0.452	1

Implementing MV optimized weights seems to increase both returns and volatility relative to the EW portfolio, ultimately resulting in a lower SR of 0.382. This indicates that an investor is better off by holding a naïve equal-weighted portfolio instead of attempting to optimize portfolio weights from a risk-return perspective. Still, the differences in returns and standard deviations are minor, making it difficult to conclude whether one strategy is better than the other.

Out-of-sample, the dynamic factor timing strategy continues to outperform the other strategies in addition to the OSEAX benchmark, as it generates both the highest returns and risk-adjusted returns. We achieve a slightly positive IR of 0.023

and an M^2 of 10.11%. Although the differences are not too great, we observe that the MV optimization indeed benefits from tactical implementation. Given that the only difference between MVS and FTS is the method of how we measure returns, our results indicate that forecasting returns on macroeconomic indicators represent a better proxy for future returns than historical data.

Figures 7 and 8 illustrate the mean-variance and factor timing strategies' weights for each month in the individual factors, compared to the factor returns. It is evident that the MVS tends to hold a position in an individual factor for longer periods than the FTS does. Unsurprisingly, the weights in the MVS are more affected by previous drops and boosts in returns. Consequently, the MVS sometimes miss out on the rapid recoveries, such as the HML in 2017 (figure 7(A)). The weights in the factor timing strategy, on the other hand, fluctuate more between months. However, we observe that the FTS often manages to reduce positions in factors before they experience major declines. Generally, the FTS seems to time factor returns fairly accurate. We also note that the MVS maximizes the allocation to BAB for almost the entire period, most likely due to the favorable risk-return profile throughout the whole period.

Figure 7: Out-of-Sample Factor Weights for the MVS

The figure illustrates weights assigned to the individual factors for the out-of-sample mean-variance strategy each month along with the respective factor returns. The axes on the left-hand sides refer to factor weights, and the axes on the right-hand sides correspond to cumulative returns. Notice that the limits on the right-hand side axes differ.

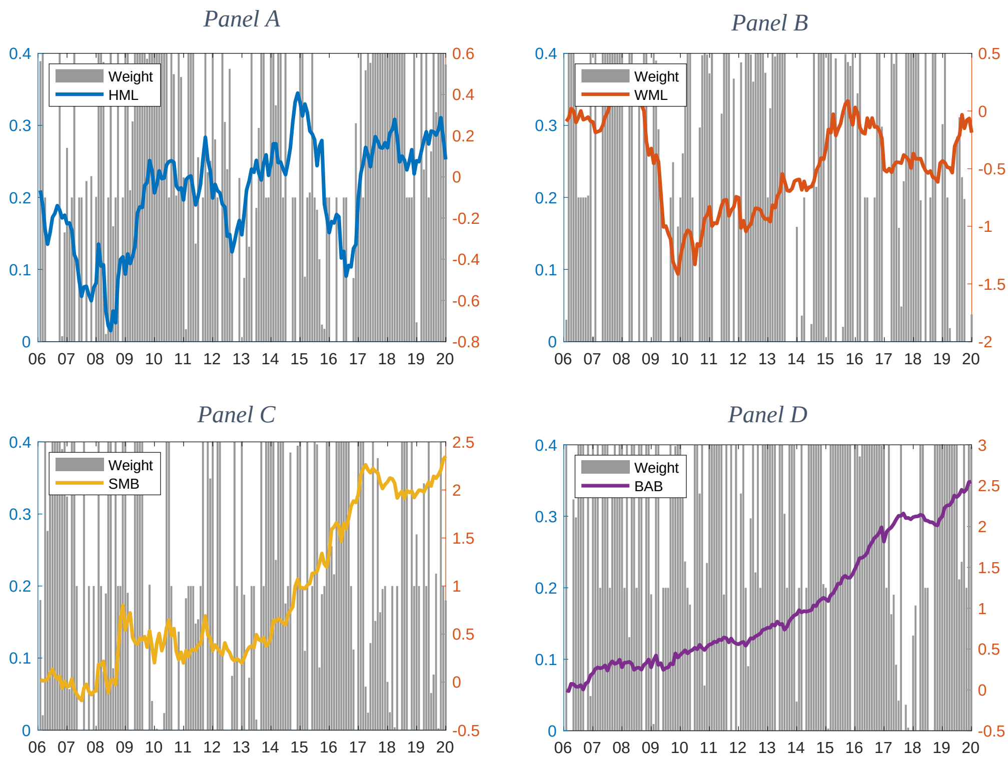

Figure 8: Out-of-Sample Factor Weights for the FTS

The figure illustrates weights assigned to the individual factors for the out-of-sample factor timing strategy each month along with the respective factor returns. Panel A corresponds to HML, panel B to WML, panel C to SMB, and panel D to BAB. The axes on the left-hand sides refer to factor weights, and the axes on the right-hand sides correspond to cumulative returns. Notice that the limits on the right-hand side axes differ.

5.2.1 Subsample Comparison

Table 9: Subsample Comparison of Multifactor Strategies and the OSEAX

Table 9 displays the returns and Sharpe ratios of the three strategies in addition the OSEAX. The Financial Crisis period displayed in the table covers the period between January 2008 and March 2009. The post-crisis period starts in March 2009 and continues throughout 2019. All returns and ratios are annualized. Notice that we report returns, and Sharpe ratios are computed using excess returns.

	OSEAX		EWS		MVS		FTS	
	Return	**SR**	**Return**	**SR**	**Return**	**SR**	**Return**	**SR**
01.2006 - 06.2009	0.79%	-0.123	-2.98%	-0.406	0.01%	-0.227	-1.01%	-0.253
07.2009 - 12.2012	12.44%	0.618	1.16%	-0.070	0.43%	-0.093	2.81%	0.038
01.2013 - 06.2016	9.39%	0.702	22.58%	1.259	22.98%	1.301	21.57%	1.200
07.2016 - 12.2019	13.10%	1.191	13.51%	1.044	12.12%	0.815	15.15%	1.058
Financial crisis	*-58.28%*	*-1.660*	*9.83%*	*0.167*	*16.00%*	*0.402*	*5.46%*	*-0.005*
Post-crisis	*13.76%*	*0.885*	*10.29%*	*0.567*	*10.48%*	*0.521*	*12.51%*	*0.705*

A notable result from table 9 is that the multifactor strategies outperform the OSEAX during the global financial crisis (GFC). The multifactor strategies yield positive returns throughout the crisis, where the MVS generates extraordinarily high returns of 16%. The FTS generates positive returns, but due to high interest rate levels throughout 2008, the strategy obtains a negative SR. Our findings support Ilmanen & Kizer's (2012) argument that investors should shift their focus to diversification across factors rather than asset classes, but conflicts with our in-sample results. Studying the performance after the crisis in figure 9(A) and (B), however, we observe that the OSEAX quickly recovers after a sharp drop in 2008, resulting in a high, positive SR. In contrast, the EWS and MVS achieve negative SRs. The underperformance of the FTS during the GFC is in accordance with our in-sample findings, strengthening the argument that it is difficult to predict factor returns in market turmoil.

Another interesting observation can be observed in figure 9(B), which shows that when the OSEAX drops significantly, multifactor strategies are peaking. We assume the volatile returns during the second subsample period are related to the European debt crisis. Less distinct examples of the multifactor strategies' lagged returns relative to the OSEAX can be observed mid-2008 (Figure 9(A)) and late 2019 (Figure 9(D)). The results may indicate that the OSEAX can be a useful indicator for future factor returns.

Our conclusion from the out-of-sample analysis is that the FTS underperforms the other multifactor strategies during crises but outperforms during normal times. Moreover, the risk-adjusted returns during normal times more than offsets the negative SR in bad times, resulting in overall higher risk-adjusted returns. During crisis times, multifactor strategies perform better than the OSEAX, but worse right after, possibly due to delayed responses in factor returns.

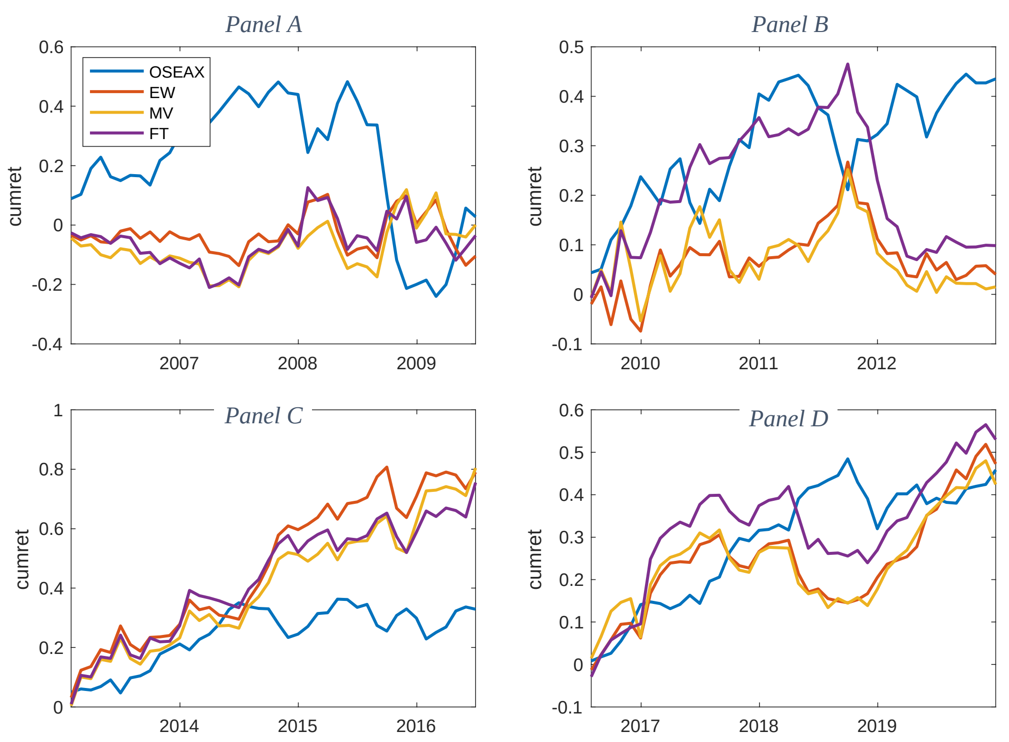

Figure 9: Cumulative Returns for Subsamples in the Out-of-Sample Period

We report cumulative returns for the OSEAX, EWS, MVS, and FTS for four subsamples during the out-of-sample period. Notice that the limits on the y-axes differ across subplots. Returns are not in percent.

5.3 Robustness Tests

We test whether there is a statistically significant difference in performance, measured by the SR, by running heteroskedasticity and autocorrelation consistent inference tests. Strategies are measured against their relevant benchmarks. Results are summarized in table 10.

Table 10: HAC Inference Results

Table 10 shows the results obtained in the HAC inference tests. We report standard errors (SE) and p-values for the differences between Sharpe ratios. P-values below 0.10 indicate that there is a statistically significant difference between the SRs of two portfolios. The relevant strategies/factors compared in each test are described as column names.

	EWS, HML	EWS, WML	EWS, SMB	EWS, BAB	MVS, EWS	FTS, MVS
HAC SE	0.0757	0.0898	0.0498	0.0767	0.0382	0.0462
HAC p-value	0.0803	0.0997	0.8820	0.0153	0.8999	0.7577

We find a statistically significant difference between the EWS and three individual factors. The differences in SRs are in favor of the EWS when measured against the value and momentum factors. BAB has a significantly higher SR than the EWS. Thus, our null hypothesis that $SR_{EWS} \leq SR_{SF_i}$ is rejected for HML and WML but is not rejected for SMB and BAB.

We do not find a significant difference between the different multifactor models, and hence do not reject the null hypotheses $SR_{MVS} \leq SR_{EWS}$ and $SR_{FTS} \leq SR_{MVS}$. We recognize three main aspects which may explain why the results are statistically insignificant. Firstly, we use only four factors with a maximum weight restriction of 40%. This limits the extent to which the strategies can differ. Secondly, we only examine five macroeconomic indicators, of which three are significant on a 10% level. Third, the indicator predictions are only estimated in-sample, and the indicators may lose (or gain) their predictive powers out-of-sample or across subsamples. Hence, expanding the investment universe, including additional indicators, increasing the significance level, and continually testing the forecasting ability of indicators, could have a positive effect on the differences between strategies.

6.0 Conclusion

We study the performance of multifactor models in Norway by comparing three different implementation approaches. We find evidence for the betting against beta and size premiums in the Norwegian equity market, as the two factors generate Sharpe ratios of 0.424 and 1.043 from 2006-2019, respectively. The value effect seems to disappear after 2005, and the momentum effect is absent during our whole sample period, both generating slightly negative Sharpe ratios of -0.060 and -0.114 out-of-sample.

Implementing an equal-weighted, multifactor strategy shows that combining single factors generates diversification benefits. We find a substantial reduction in volatility when combining the different factors. The equal-weighted strategy achieves a standard deviation of 15.87%, compared to three of the four single factor volatilities of around 30%. The strategy underperforms the 15.31% standard deviation of betting against beta. Still, the equal-weighted strategy only outperforms value and momentum regarding risk-adjusted returns, generating a SR of 0.398.

Implementing mean-variance optimized weights does not seem to have any significant effect out-of-sample, despite the mean-variance strategy's great outperformance in-sample. The mean-variance strategy achieves higher returns than the equal-weighted strategy, but due to higher risk, the Sharpe ratio of 0.382 is lower. However, we do find that the mean-variance strategy performs better than all other strategies during crises, yielding a 16% return during the global financial crisis.

We find that three macroeconomic indicators are useful in explaining factor returns. The results suggest a negative relation between value and lagged values of change in market volatility. Further, size and betting against beta perform better when market liquidity is increasing. Lastly, we find that the composite leading indicator helps predict future returns of both betting against beta and momentum.

Our analysis has shown that factor timing with a restricted number of factors and variables results in higher risk-adjusted returns, however not statistically significant. The factor timing strategy achieves a Sharpe ratio of 0.431 and a differential return of 0.90% annually relative to the mean-variance strategy. Factor timing is revealed to perform poorly in volatile markets. However, the factor timing strategy outperforms the equal-weighted and mean-variance strategies slightly over the whole time period, especially in normal market conditions. Our findings thus indicate that implementing factor-timed weights estimated by macroeconomic variables and moving to mean-variance optimized weights during crises may enhance the risk-adjusted returns of a multifactor strategy.

We believe that the strategy can be enhanced by including additional factors to avoid enforced allocations to specific factors and still achieve diversification benefits. Further, an increased number of significant indicators may lead to more accurate predictions of factor returns. Recommended further research would thus be to include additional factors in the multifactor strategies. This could better capture the effects of the implementation methods by allowing the mean-variance optimizations to choose in a larger investment universe and possibly deviate more from an equal-weighted portfolio. It may also be beneficial to consider the OSEAX as a predictor of factor returns, given that the multifactor strategies tend to successively follow the index. Lastly, including transaction costs may help evaluate the true performance of the strategies.

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8.0 Appendix

Appendix 1

Table 11: Data, Frequencies, Start Dates, and Sources

This table describes the data we have collected, along with the dates of the first observation, frequencies and sources. All data ends December 31st 2019.

Data	Start Date	Frequency	Source
Individual Stock Returns	Jan 1980	Daily	OBI
Market Returns (VW)	Jan 1980	Daily	OBI
Benchmark (OSEAX)	Jan 1983	Monthly	OBI
Risk-free rate	Jan 1980	Daily	OBI
Risk-free rate	Jan 1980	Monthly	OBI
BM returns	Jan 1981	Monthly	OBI
Size returns	Jan 1980	Monthly	OBI
Momentum returns	Jan 1980	Monthly	OBI
CLI indicator	Jan 1980	Monthly	OECD
Turnover	Jan 1980	Monthly	OBI data
Oil Price	Jan 1988	Monthly	Bloomberg

Appendix 2

In-sample factor returns (1980-2005). Due to data availability and requirements of non-missing data for construction, the start date of the factors and the OSEAX may deviate.

Table 12: Extended In-Sample Factor Returns (1980-2005)

This table shows the in-sample returns of each individual factor HML, WML, SMB and BAB, along with the OSEAX. We also report the date of the first observation for each factor

	Start Date	Ret	Ex.Ret	Std	SR
OSEAX	01.1983	17.24%	8.72%	22.22%	0.392
HML	01.1981	15.34%	6.49%	52.33%	0.124
WML	01.1980	6.65%	-2.24%	44.37%	-0.051
SMB	01.1980	42.74%	33.85%	45.17%	0.749
BAB	01.1985	19.29%	11.18%	22.08%	0.506

Appendix 3

This appendix illustrates the presumed causal relationships between individual factor returns and their corresponding macroeconomic indicators.

***Appendix 3A** – HML returns and logarithmic change in market volatility.*

Panel A *Panel B*

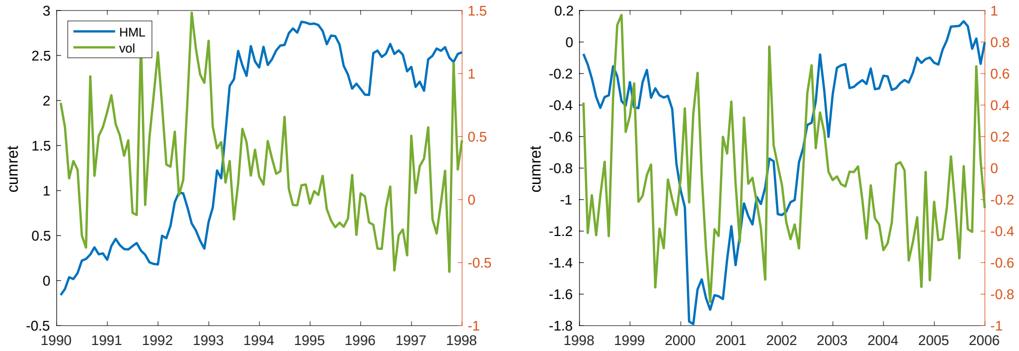


Figure 10: HML Returns and Volatility Indicator

The vertical axis on the left-hand side of both panels represent the cumulative returns of HML. The vertical axis on the right-hand side represents the change in market volatility. Panel A shows the period 1990-1998 and panel B shows the period 1998-2006.

***Appendix 3B** – WML returns and CLI.*

Panel A *Panel B*

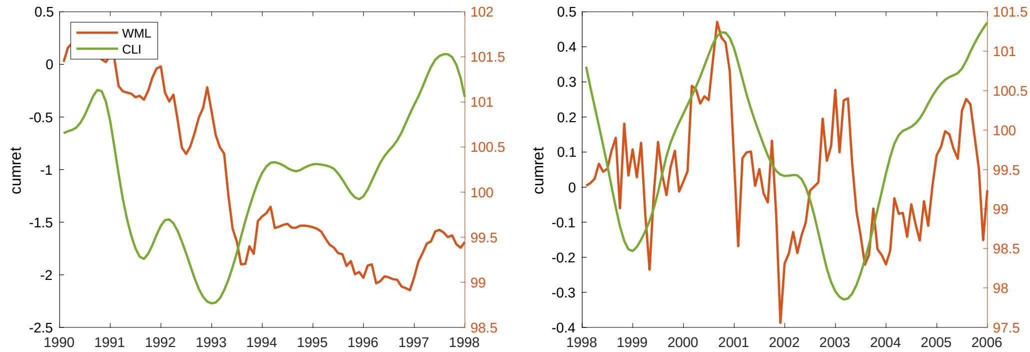


Figure 11: WML Returns and CLI

The vertical axis on the left-hand side represent cumulative returns of the momentum factor. The vertical axis on the right-hand side represents the level of the CLI. Panel A shows the period 1990-1998 and panel B shows the period 1998-2006.

***Appendix 3C** – SMB returns and logarithmic change in market liquidity.*

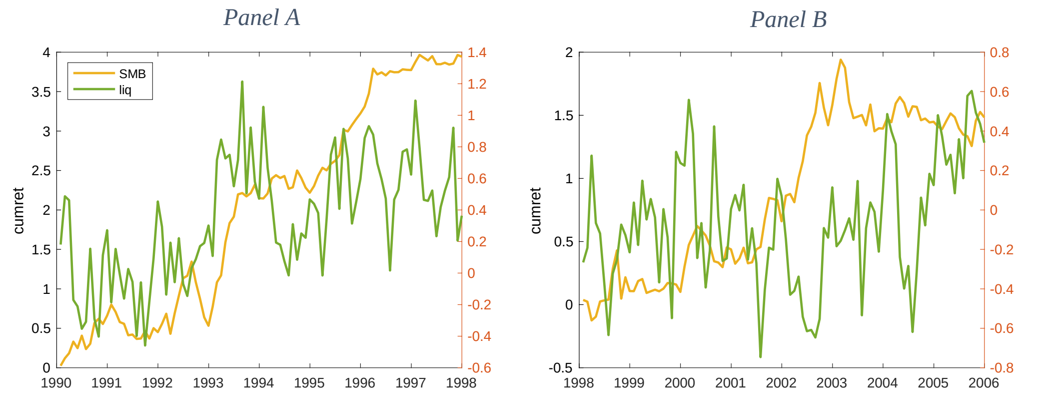


Figure 12: SMB Returns and Liquidity

The vertical axis on the left-hand side represent cumulative returns of the size factor. The vertical axis on the right-hand side represents the change in liquidity. Panel A shows the period 1990-1998 and panel B shows the period 1998-2006.

***Appendix 3D** – BAB returns and CLI.*

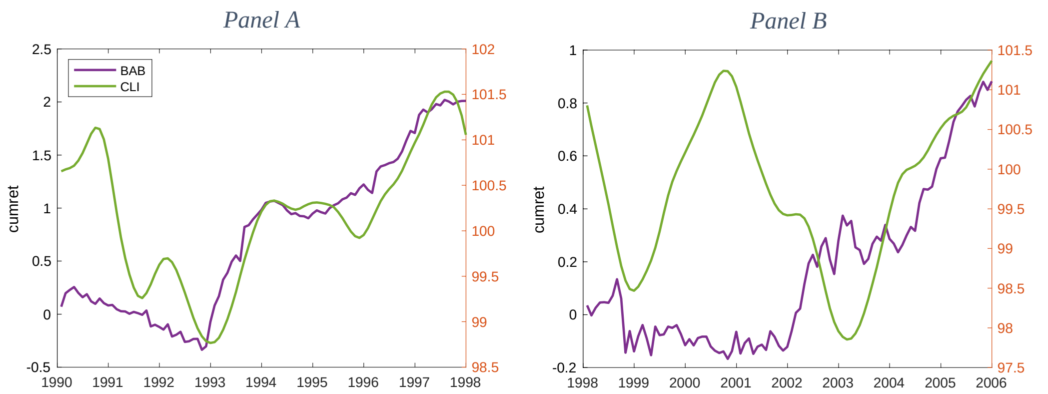
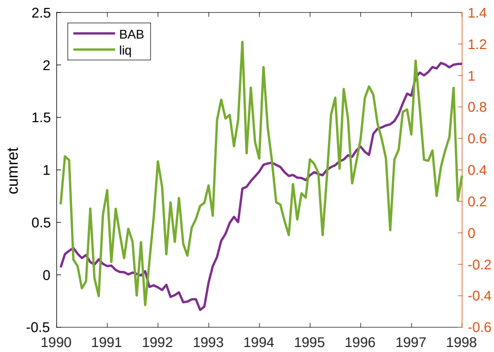


Figure 13: BAB Returns and CLI

The vertical axis on the left-hand side represent cumulative returns of the BAB factor. The vertical axis on the right-hand side represents the level of the CLI. Panel A shows the period 1990-1998 and panel B shows the period 1998-2006.

Appendix 3E *– BAB returns and logarithmic change in market liquidity.*

Panel A



Panel B

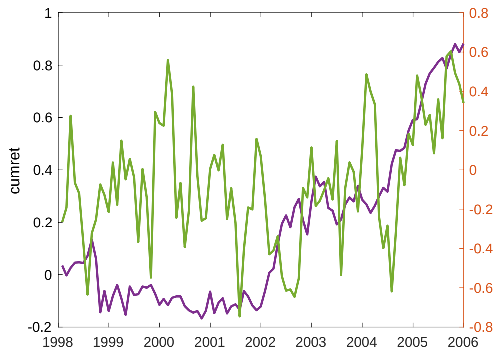


Figure 14: BAB Returns and Liquidity

The vertical axis on the left-hand side represent cumulative returns of the BAB factor. The vertical axis on the right-hand side represents the change in liquidity. Panel A shows the period 1990-1998 and panel B shows the period 1998-2006.