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Does it pay to be active? Norwegian mutual fund performance from 1991 to 2019

Navn:	Knut Mehl, Henrik Aunemo Reitan

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Does it pay to be active?
Norwegian mutual fund performance from 1991 to 2019

Knut Mehl
MSc in Business with Finance - QTEM

Henrik Reitan
MSc in Business with Finance - QTEM

Master's thesis
Supervisor: Bruno Gerard
Department of Finance

BI Norwegian Business School
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This thesis was written as a part of the Master of Science in Economics and Bustiness Administration at BI Norwegian Business School. Please note that either the institution nor the examiners are responsible – through the approval of the thesis – for the theories and methods used, or results and conclusion drawn in this work.

Table of Contents

Table of Contents (Appendix)

Abstract

This research paper shows that Norwegian active funds have first-order stochastically dominated Norwegian index funds for the subperiod 1991 to 2005, measured by net returns, not accounting for redemption and subscription fees. The same holds for large investors between 2006 to 2019. Our simulation studies show that the historical probability of active funds yielding a greater return than index funds is about 60% and notably above 50% for the first and most recent subperiod, respectively, for (most) investors when sorted on investment size with holding periods between 1 to 5 years. The probability is barely affected by redemption and subscription fees. Our thesis also provides further evidence that the traditional benchmark models used in mutual fund literature are sensitive to the choice of market benchmark and factor model, and therefore have severe limitations in their ability to explain whether active funds outperform index funds.

Acknowledgments

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Disclosure

While writing this thesis, Knut has worked part-time in the Corporate Banking division of DNB ASA. The research presented in this paper was done solely within the scope of the MSc degree and completely independent of Knut's part-time employment. Knut's employment in DNB ended before this thesis was submitted.

The views expressed in this paper are those of the authors and do not necessarily reflect those of DNB ASA or any other institution or organization.

0.0 Synopsis

Background: The synopsis is written as part of our Master's thesis with the hope of being published in a Norwegian newspaper. With the synopsis, we want to take part in the ongoing debate about Norwegian mutual fund performance.

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Title: Active equity funds or passive index funds? Active equity proves to be the better choice, at least if you are investing in Norway.

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Many academic studies and news articles claim that index funds yield greater returns than active funds. This is commonly justified by the lower fees associated with index funds and the prevalence of active fund managers lacking in investment capacity. However, our analysis of the data for the last 30 years of Norwegian equity mutual funds listed at the Oslo Stock Exchange suggests that investors are better off investing in active funds.

Apples and oranges

While most mutual fund studies have compared actively managed funds to benchmark indices, relatively few have compared the performance of active funds to their counterpart, index funds. In theory, an index fund strategy replicates the returns of a target benchmark index. Although the mandate is simple, numerous difficulties arise when fund managers attempt to replicate these returns in practice.

While a benchmark index represents a theoretical portfolio of securities, index fund managers must actively trade in the market to replicate the benchmark index. This distinction is not trivial, as securities cannot be traded instantly without incurring costs in the exchange. Thus, assuming that index funds consistently deliver the same results as a benchmark index return is unrealistic. For this reason, comparing index funds with benchmark indices is like comparing apples and oranges.

Active mutual funds perform well

When we compare apples to apples, the juicier fruits are active funds. Between the introduction of the first Norwegian index fund (in 1991) and 2005, the worst active funds generated net returns similar to the worst index funds, while the best active funds have yielded substantially better returns than their index fund counterparts. For all outcomes in between, active funds are seldom worse than index funds. The same holds for large investors (investing up to 300M NOK) between 2006 and 2019. After controlling for the uncertainty of historical returns through statistical tests, the data suggests that investors are better off when investing in active funds for large parts of our sample. For readers with a statistical background, this statement is based on the criterion of stochastic dominance.

What does this mean in practical terms? If you prefer more money to less, you should invest in a random active fund instead of a random index fund. This is particularly interesting because most research finds that it is extremely difficult to pick a well-performing fund. In studying historic holding periods between 1 and 5 years, we find almost exclusively that holding an active fund had a greater than 50% probability of being the best choice. In fact, the probability was closer to 60% in most cases. Although our results show that index funds have become more attractive over the sample period, active funds still appear a better investment decision for investors.

Individual investors are typically advised to choose active funds for holding periods longer than 5 years. But history shows that active funds are better also for much shorter horizons. If you are concerned with performance varying throughout the holding period or the risk of selecting a poor performing active fund, buying several active funds will reduce these risks.

Case closed?

In projecting the future, it is important to consider whether history will repeat itself. Are there any significant changes in Norwegian mutual funds, the Norwegian market, or otherwise that indicate that the future will be different from the past? Probably. Meanwhile, the data makes it hard to disagree. It shows that you are best served by putting your money in the hands of active mutual fund managers.

1.0 Introduction

In this paper, we examine whether Norwegian investors have been better off investing in domestic index funds rather than domestic active equity funds over the past 30 years. The issue has great economical and practical importance to investors and researchers alike. However, most of the existing mutual fund literature does not address this question directly. The majority of equity mutual fund research assesses fund manager skills by comparing the performance of active funds to a benchmark instead of to performance of index funds. Benchmarks are not directly investable for investors. Index funds tracking the indices are, but, due to tracking errors and fees, it is unrealistic to assume that investors can earn the benchmark return from index funds.

Renowned investors such as Warren Buffet and John Bogle claim that investors are generally better off holding index funds (Nymoen, 2020; Bogle, 2015). This view is supported by most research on American mutual funds which conclude that active funds are not able to beat the benchmark, on average, net of costs. Norwegian investors appear to be listening to the rhetoric from the American market, as the domestic market share of index funds has increased from 1.2% to 20.4% over the last 15 years. However, our examination of the domestic market indicates that mutual fund investors in Norway are worse off by trading according to conventional financial wisdom from the U.S.

The performance of domestic equity mutual funds' is frequently debated in Norwegian media. Articles written on behalf of the financial industry are generally supportive of active management. However, the sector is incentivized to pick sample periods, funds, and methodologies that favor active investment as their fees are higher than those of index funds, which raises questions about the legitimacy of the analyses. Overall, we show that the mutual fund coverage in Norwegian media has several potential sources of bias. In particular, they tend to focus on whether funds beat their chosen benchmark, rather than analyzing the funds from the investor's point of view. Additionally, their sample size is typically limited and cannot be generalized to whether investors are better off investing in passive or active funds in Norway. We review the debate and further evaluate the implications of the analyses' methodical weaknesses in Appendix A.

In this thesis, our objective is to provide a transparent and independent evaluation of whether investors are better off investing in Norwegian index funds or active funds, using an empirical approach that directly compares the returns investors could have achieved from active or index funds.

Our study considers all Norwegian mutual funds investing primarily in Norwegian equities that were offered to investors over the period 1981(91) to 2019. The data covers 99.83% of the returns generated by Norwegian equity mutual funds noted on the Oslo Stock Exchange, adjusted for survivorship and incubation bias.

In our empirical analysis, we first use traditional benchmark models to analyze the risk-adjusted returns of active and index funds. We then compare the net return distribution of the two fund types using stochastic dominance tests. To quantify the difference in returns for various holding periods, we use historical Monte Carlo simulations of the cumulative net returns.

We find that the benchmark models do not provide any clear insights on whether investors should prefer active or index funds. The net return distribution of active funds first-order stochastic dominates (FSD) the index fund distribution at a 5% significance level between 1991 and 2005 for all types of investors. The same holds for large investors (investing up to 300M NOK) between 2006 and 2019. The FSD results suggest that anyone who prefers a higher return to a lower one should prefer a random active fund, regardless of the investor's utility function (or risk appetite). Our simulations suggest that investors have had a probability of approximately 60% and notably above 50% for the two periods, respectively, for being better off by investing in active funds versus index funds for holding periods between 1 and 5 years. Although active funds still appear a better investment decision for investors, our results show that index funds have become more attractive over the sample period.

These findings contrast with the results of most research from the U.S. and conventional financial wisdom, which claim that active funds underperform passive funds net of costs. Our results suggest that in Norway, anyone investing in mutual funds of Norwegian equity between 1991 and 2005, and large investors between 2006 and 2019, would have been better served by a random active fund, regardless of their risk appetite.

The remainder of the thesis is structured as follows. In Section 2, we review existing literature and theory on mutual fund performance. In Section 3, we explain our hypothesis and methodology. In Section 4, our data is described in detail. Section 5 includes our results and discussion. Section 6 concludes our thesis.

2.0 Background and literature review

2.1 What is a mutual fund?

A mutual fund is an investment vehicle in which individual investors' savings are pooled and managed by a professional investor (Morningstar, 2019). The first equity mutual fund was formed in 1774 by a Dutch merchant named Adriaan van Ketwich (The Investment Funds Institute of Canada, 2019). More than 200 years later, in 1976, John Bogle started the first index fund, which later became known as Vanguard 500 (VFINX) (Culloton, 2011). Roughly 15 years later, Skandiabanken introduced the first index funds in Norway, Skandia Indeks Norge. Today, passive investing controls nearly half of the U.S. equity fund market (Cox, 2020), and one-fifth of the Norwegian market (see Section 4.1).

The Norwegian Fund and Asset Management Association (2019a) divides mutual funds into four categories:

1. **Equity funds**. A minimum of 80% of the capital invested in stocks. In Norway, these funds must invest in 16 or more companies.
2. **Fixed-income funds** (bond funds). Invest solely in fixed income securities, such as treasuries, corporate bonds, and municipal bonds.
3. **Balanced funds**. Funds investing in a combination of equity and fixed-income securities.
4. **Other funds**. All funds that do not fall in the first three categories. For example, hedge funds and funds that invest in derivatives.

The Norwegian Fund and Asset Management Association (2019b) also classifies equity funds according to the investment universe of the fund; (1) Norwegian equity funds, (2) Nordic equity funds, (3) European equity funds, and (4) Global equity funds. Our study focuses solely on Norwegian funds investing in domestic equities[1].

2.1.1 Actively versus passively managed equity funds

Among equity funds, investors may choose between actively or passively managed funds. An actively managed fund is a fund in which the fund manager makes decisions about how to invest to outperform a pre-defined benchmark. In general, this is achieved by attempting to predict (1) the overall market return and/or (2) which securities should outperform or underperform the market. A passively managed fund (an index fund), attempts to replicate the performance of a

1 We include foreign fund suppliers if the fund is listed on OSE and trade minimum 80% in Norwegian equities

benchmark index by holding most or all the stocks in the index. Typically, actively managed funds charge higher fees compared to passively managed funds.

In theory, an index strategy aims to exactly replicate the returns of a target benchmark. While the mandate is both well-known and simple in theory, numerous difficulties arise when fund managers attempt to do so in practice. The difference between an index fund return and its benchmark return is referred to as tracking error and arise from (a) the index fund not holding all securities in the benchmark index, (b) the index fund weighting each security different than the benchmark weights, (c) transaction costs, and (d) cash drag. The errors caused by (a) and (b) tend to decrease as the fund's assets under management increase. For index funds, (c) is smaller than for active funds, but it exists and leads to issues related to (a) and (b) for small inflows or outflows from the fund. The objective of the index funds manager thus consists in minimizing the tracking error while minimizing the costs. Index fund tracking errors are typically small and the return is close to the benchmark return, but the returns are not identical (see e.g. Frino & Gallagher, 2001) and all index funds exhibit tracking errors. Hence, assuming that investors earn the benchmark return from investing in an index fund is a good approximation but not an exact record of the returns index fund investors can achieve.

2.2 Empirical research on mutual fund performance

A large body of literature has examined the performance of mutual funds and can broadly be categorized into two groups. The first set analyzes the risk-adjusted performance of actively managed funds using benchmark models. The second set, which is scarce relative to its counterpart, examines the comparative performance of active and passive mutual funds. We review both in the following.

2.2.1 Studies of active funds' risk-adjusted performance in the U.S.

The majority of studies find that, on average, most investors are better off with the benchmark return than active fund returns, primarily due to the fees and costs associated with active funds, though some subgroups of active funds do outperform their benchmark. Unfortunately, most of these studies suggest that outperformance does not persist over time, and hence that identifying ex-ante outperformance is nearly impossible.

In an early paper, Carlson (1970) investigates a sample of U.S. mutual funds over the period 1948 to 1967 and concludes that whether mutual funds outperform the market depends largely on the choice of the time period and market proxy.

Using four different benchmark models, Grinblatt & Titman (1989) found that the risk-adjusted gross returns of some U.S. mutual funds were significantly positive between 1975 and 1984, particularly among aggressive-growth and growth funds and funds with the smallest net asset values. These funds were also characterized by higher expense ratios so that their net returns did not exhibit abnormal performance. The benefits of the funds' outperformance did not flow to the fund investors but were absorbed by the funds' costs. In a later paper, Grinblatt & Titman (1993) presented evidence that the CRSP-listed quarterly holdings of mutual fund portfolios, on average, achieved positive abnormal gross performance between 1976 and 1985. Interestingly, the performance evaluation technique uses no benchmarks, so the results cannot be attributed to benchmark inefficiencies. As in their 1989 paper, Grinblatt & Titman stress that the transaction costs and expenses associated with these funds negate the abnormal performance, leaving the net abnormal performance close to zero. Furthermore, as one would expect, not all fund managers achieved abnormal performance in the data, but the performance of those who did was, on average, persistent. The funds that did well in the first half of the sample continued to do well in the second half.

Gruber (1996) reported that the average mutual fund underperforms passive market indexes by about 65 basis points per year from 1985 to 1994 in the United States. Carhart (1997), in his study of U.S. mutual funds between 1962 and 1993, concludes that there is only slight evidence of consistently skilled (or informed) mutual fund managers. The top mutual funds manage to earn back their investment expenses and yield a positive abnormal return to investors, while the bottom-decile funds underperform by about twice their reported investment costs. Carhart states that the severe underperformance of the bottom-decile might not have practical significance as these funds are also the smallest funds (measured by the funds' assets) and because these funds may not be able to take short positions. Carhart also found that expense ratios and load fees are significantly and negatively related to performance (e.g. expense ratios reduce performance slightly more than one-for-one) and that load funds (i.e. funds with a sales fee or commission) substantially underperform no-load funds by around 80 basis points per year on average.

Using U.S. mutual fund data for 1962 to 1997, Wermers (2000) found that funds held stocks that outperformed the market by 1.3 percent annually, but that the net returns of these funds underperformed by 1.0 percent. Of the 2.3 percent discrepancy between the gross and net returns, 1.6 percent was related to costs and fees, with an almost even split between expense ratios and transaction costs. The remaining 0.7 percent was due to the lower average return of nonstock holdings (mostly cash and bonds), called "cash drag," substantially weakening the net performance of the mutual funds. According to French (2008), the typical investor would increase the average annual return by 67 basis points if the investor switched to a passive market portfolio, as compared to active funds, based on data from 1980 to 2006 in the United States.

Daniel, Grinblatt, Titman & Wermers (1997) developed and applied performance measures using characteristics-based benchmarks of the portfolios that were evaluated. Here, they isolated (1) whether fund managers could successfully time their portfolio weightings on these characteristics, referred to as “Characteristic Timing,” and (2) whether fund managers could select stocks that outperformed the average stock having the same characteristics, referred to as “Characteristic Selectivity.” Using data on over 2,500 U.S. equity funds from 1975 to 1994, they find that, on average, mutual funds do not exhibit Characteristic Timing ability, but that they, particularly aggressive-growth funds, do exhibit some Characteristic Selectivity. They estimate that Selectivity generates on average an abnormal return of about 100 basis points before costs, approximately equal to the management fees. Aggressive-growth funds which performed best in terms of Characteristics Selectivity ability, probably also generated the highest costs. In line with earlier papers, they conclude that fund managers may be able to generate excess returns before costs and fees, but not after.

Kosowski, Timmermann, Wermers & White (2006) observe that the distribution of the cross-section of mutual fund alphas is highly non-normal and propose a new bootstrap approach to evaluate the performance of U.S. open-end domestic equity mutual funds over the 1975 to 2002 period. They find that, on average, funds gross abnormal performance is not sufficient to cover their fees and expenses, and that a sizable minority of managers seem to pick stocks well enough to more than cover their costs and fees. The performance of those managers persists.

2.2.2 Studies of active funds' risk-adjusted performance in Europe and Norway

While the most widely cited studies of mutual fund performance focus on U.S. mutual funds, numerous studies focus on mutual fund performance in different national markets., although the work on each individual market is more limited.

Cuthbertson, Nitzsche & O'Sullivan (2008) studied UK equity mutual funds using data from 1975 to 2002 and concluded that investors would be better off holding the benchmark portfolio. They also found stock picking ability for somewhere between 5-10 % of the top-performing UK equity mutual funds. In contrast to evidence from the U.S., Dahlquist, Engström & Söderlind (2000) found evidence suggesting that actively managed equity funds outperformed passively managed funds in the Swedish market between 1993 and 1997. Their evidence suggests that actively managed equity funds had, on average, an alpha of 0.5% per year, net of 1.4% annual fees.

In the Norwegian market, Gjerde & Sættum (1991) concluded that active equity funds managed to beat the market between 1982 and 1984. In a more recent and extensive study, Sørensen (2009), using a survivorship-bias-free dataset from 1982 to 2008, found that Norwegian equity mutual funds are not able to deliver a positive alpha.

2.2.3 Studies comparing active and index fund performance

Despite the apparent differences between benchmark indexes and index funds, studies comparing them are rather rare. In one of the first thorough studies that performed a direct performance comparison between active and index funds, Frino and Gallagher (2001) analyzed U.S. mutual funds in both a five and eight-year sample period ending in 1999. They found that index funds earned significantly negative raw and risk-adjusted excess returns and that the margin of underperformance was roughly equal to the fund expenses. Additionally, they found that index funds outperformed active funds both in terms of raw and risk-adjusted performance for the five and eight-year period.

Fortin and Michelson (2002) compared the before and after-tax returns earned by investors for various groups of mutual funds relative to the Vanguard index funds between 1976 and 2000. They found that, on average, the Vanguard index funds outperformed actively managed funds for most equity categories both before and after-tax. However, actively managed Small Company Equity (SCE) funds significantly outperformed the index over most of the period. Although Fortin and Michelson reported evidence of index funds outperforming active funds in terms of total returns earned by investors, they did not take the risk-return trade-off into account. Also, their study did not consider either front loads or deferred loads.

Holmes (2007) examined the net returns of U.S. and international active mutual funds relative to index funds between 1995 to 2004. Holmes argued that one cannot compare all actively managed funds to a large-cap index, for instance the S&P 500, and suggested that such methodologies are comparing "apples to oranges." Consequently, she segregated the active mutual funds by fund category and style and compared the performance to the most similar index funds. For example, she compared the large-cap blend index funds with large-cap blend active funds. The results were mixed. Actively managed funds in the asset categories of mid-cap value, small-cap blend, and international mid/small-cap blend outperformed their respective index funds. However, index funds outperformed in the large-cap asset classes, the U.S. mid-cap blend, the small-cap value, and the growth asset categories. The sample sizes of several index universes were small, which could potentially impact the results of the analysis.

Crane and Crotty (2018) focused their attention on mutual funds manager skills by investigating U.S. mutual funds between 1995 and 2013. Particularly, they investigated whether the aggregate amount of skill found in actively managed funds warrants investing in active funds versus index funds by testing the distribution of alpha returns (and the t_{α} distribution) for stochastic dominance using six different benchmark models. They could not reject the null that index funds second-order stochastically dominated active mutual funds for either alpha or t_{α}, while they rejected the null that active funds second-order dominated index funds for all but two benchmark models. The economic interpretation of their study is that no risk-averse investor should choose a random active fund over a random index fund. Contrary to traditional methods that favor the investment with the most desirable mean-variance trade-off, stochastic dominance tests utilize the entire distribution and evaluate whether an investment has a higher probability of a higher return. Thus, one can determine whether the upside potential of an investment outweighs the downside.

2.3 Equilibrium accounting

Equilibrium accounting refers to a theory put forward by Sharpe (1991) and later discussed by French (2008), and Fama and French (2010). If investors are grouped into two groups and one group on average earns the market return, the other group must also earn the average market return due to simple arithmetic computations. When we group investors into active and passive and assume that passive investors earn the market return, both groups must on average earn the market return before costs. After adjusting for costs that are higher for active funds, active funds must underperform passive funds. In the words of French, "a small representative investor who switches to a passive market portfolio increases his return by the difference between the value-weight average of all investors' costs and the cost of investing passively". Generally, the theory suggests that index funds should outperform active funds.

However, with active investors such as private individuals, pension funds, banks, and insurance companies, it is possible that active mutual funds earn returns higher than the average of the market (which should equal the average of passive investors before costs) if active mutual funds as a subgroup of active investors outperform the average of the rest of active investors. If this is the case, active funds may outperform index funds even if the equilibrium accounting theory holds.

3.0 Hypothesis and empirical methods

In this study, one hypothesis is tested; although the question has been introduced previously, it is first formalized in the below.

Hypothesis: Norwegian investors have been better off investing in domestic index funds rather than domestic active equity funds.

The hypothesis is motivated by our literature review and the idea underlying equilibrium accounting. It is specific, but also general, allowing us to study it with a broad toolbox of empirical methods in the subsequent sections. The null hypothesis is that investors have been better off investing in domestic active equity funds or that there is no substantial difference between the two types.

Past studies have typically analyzed alpha performance or abnormal returns (i.e. alpha plus residual) and many of them studied gross returns (i.e. before costs and fees) (see e.g. Grinblatt & Titman (1989), Gjerde & Sættum (1991), Dahlquist, Engström & Söderlind (2000), Cuthbertson, Nitzsche & O'Sullivan (2008), and Sørensen (2009)). We study alpha and abnormal returns net of costs in what we shall refer to as "benchmark-adjusted performance."

Our thesis differs from previous research in that we focus on whether investors have been better off with active or index funds rather than whether active managers have skills. Alpha and abnormal returns are useful measures to an investor, but they do not measure the true returns earned by investors. Therefore, we add to the benchmark-adjusted performance measures by testing for stochastic dominance in the net returns of active and index funds.

The alpha returns, abnormal returns, and stochastic dominance tests use single-month returns, however, most investors hold mutual funds for periods that far exceed one month. We use Monte Carlo simulations to incorporate longer holding periods in our study, comparing active and index funds over various holding periods from 1 to 5 years; these simulations are explained in Appendix E.

3.1 Benchmark-adjusted performance

3.1.1 Benchmark models

The appropriate benchmark model is a matter of extensive debate in the literature, which we elaborate on in Appendix B. To avoid taking a position on which model is appropriate, we use various benchmark models in our study; the single market model (CAPM) of Jensen (1968), the Fama-French three-factor model (Fama and French, 1993), the extended Fama-French-Carhart four-factor model of Carhart (1997) and the Fama-French five-factor model (Fama and French, 2015). Due to the availability of factor data for Norwegian equities, we do not control for additional systematic risk factors.

For each benchmark model, we estimate alphas and factor loadings for each fund according to the following model

$$r_{i,t} - r_{f,t} = \hat{\alpha}_i + \sum_{j=1}^{M} \hat{\beta}_{i,F_j} F_{j,t} + \hat{\epsilon}_{i,t}$$

where $r_{i,t}$ is fund i's return in month t, $r_{f,t}$ is the risk-free rate in month t, $\hat{\beta}_{i,F_j}$ is fund i's exposure to factor j, $F_{j,t}$ is the (excess) return on factor j in month t, and M is the number of factors in the model. For example, for the three-factor model, we estimate the following model for fund i

$$r_{i,t} - r_{f,t} = \hat{\alpha}_i + \hat{\beta}_{i,MKT} MKT_t + \hat{\beta}_{i,SMB} SMB_t + \hat{\beta}_{i,HML} HML_t + \hat{\epsilon}_{i,t}$$

To correct for cross-sectional correlation, we use an approach based on the work of Fama and MacBeth (1973) and estimate the model once per fund. In equation form

$$\begin{aligned}
r_{1,t} - r_{f,t} &= \hat{\alpha}_1 + \hat{\beta}_{1,F_1} F_{1,t} + \hat{\beta}_{1,F_2} F_{2,t} + \dots + \hat{\beta}_{1,F_M} F_{M,t} + \hat{\epsilon}_{1,t} \\
r_{2,t} - r_{f,t} &= \hat{\alpha}_2 + \hat{\beta}_{2,F_1} F_{1,t} + \hat{\beta}_{2,F_2} F_{2,t} + \dots + \hat{\beta}_{2,F_M} F_{M,t} + \hat{\epsilon}_{2,t} \\
&\vdots \\
r_{N,t} - r_{f,t} &= \hat{\alpha}_n + \hat{\beta}_{N,F_1} F_{1,t} + \hat{\beta}_{N,F_2} F_{2,t} + \dots + \hat{\beta}_{N,F_M} F_{M,t} + \hat{\epsilon}_{n,t}
\end{aligned}$$

for the N funds in our sample and the M factors in the benchmark model. We represent the N models in vector form to simplify the notations

$$r_t - r_{f,t} = \hat{\alpha} + \hat{\beta}_{F_1} F_{1,t} + \hat{\beta}_{F_2} F_{2,t} + \dots + \hat{\beta}_{F_M} F_{M,t} + \hat{\epsilon}_t$$

where

$$\hat{\alpha} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \vdots \\ \hat{\alpha}_N \end{bmatrix}, \qquad \hat{\beta}_{F_1} = \begin{bmatrix} \hat{\beta}_{1,F_1} \\ \hat{\beta}_{2,F_1} \\ \vdots \\ \hat{\beta}_{N,F_1} \end{bmatrix}$$

and so on.

Now that we have estimated the alpha returns of the funds, we test the significance of the alpha and abnormal return estimates across the funds. Formally, we test

$$H_0 \colon \hat{\alpha} = 0$$

and

$$H_0 \colon \hat{\alpha} + \hat{\epsilon}_t = 0$$

We test these hypotheses for both equal-weighted (EW) and value-weighted (total net asset value-weighted; TNAV-W) portfolios of the funds. We shall use two approaches for computing the test-statistic which are broadly used in the mutual fund literature. For simplicity, we illustrate the tests using the alpha term. The computations are the same for the betas.

The first approach is inspired by Fama-MacBeth (1973), later referred to as the F-M test, and tests the cross-section of the regression coefficients with an ordinary t-test

$$t_{\hat{\alpha}} = \frac{(\bar{\hat{\alpha}} - \alpha_0)}{\sigma(\hat{\alpha})/\sqrt{N}}$$

where $\alpha_0 = 0$ for the alpha returns. For the EW portfolio

$$\bar{\hat{\alpha}} = \frac{1}{N}\sum_{i=1}^{N} \hat{\alpha}_i \quad and \quad \sigma(\hat{\alpha}) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\hat{\alpha}_i - \bar{\hat{\alpha}}\right)^2}$$

For the TNAV-W portfolio, we extend the approach by allowing for different weights by using the general rules for weighted average and weighted standard deviation

$$\bar{\hat{\alpha}} = \sum_{i=1}^{N} w_i \hat{\alpha}_i \quad and \quad \sigma(\hat{\alpha}) = \sqrt{\frac{\sum_{i=1}^{N} w_i\left(\hat{\alpha}_i - \bar{\hat{\alpha}}\right)^2}{\frac{N-1}{N}\sum_{i=1}^{N} w_i}}$$

where

$$w_i = \frac{\sum_{t=1}^{T} TNAV_{i,t}}{\sum_{i=1}^{N}\sum_{t=1}^{T} TNAV_{i,t}}$$

In layman's terms, this corresponds to weighting each fund according to the sum of all observed TNAVs for the fund across all periods divided by the sum of all observed TNAVs for all funds across all periods. Summing the TNAVs allows us to adjust for the size of the fund and the number of months it has been alive during the period.

A critique of the Fama-MacBeth approach is that it does not include corrections for the fact that the alphas (and betas) are estimated (see e.g. Cochrane (2000, p. 245-250)). In other words, the estimation errors of the coefficients are not included in the computation of the overall test statistic. Cuthbertson and Nitzsche (2004, p. 227-228) discuss a test statistic that deals with the issue which we use as our second approach, later referred to as the C-N test. For the EW portfolio, it is simply an average of the cross-sectional t-statistics

$$t_{\hat{\alpha}} = \frac{1}{N}\sum_{i=1}^{N} t_{\hat{\alpha}_i}$$

where $t_{\hat{\alpha}_i}$ is the test statistic obtained from the cross-sectional regression for fund i using Newey and West (1987) corrected t-statistics.

Similar to the first approach, we extend the second approach to allow for different weights

$$t_{\hat{\alpha}} = \frac{1}{N}\sum_{i=1}^{N} w_i t_{\hat{\alpha}_i}$$

where w_i is defined in the same way for the EW and TNAV-W portfolios as in the first approach.

In addition to testing the alpha performance of the funds, we follow Carhart's (1997) approach to test the abnormal returns. Let $\widehat{AR}_t$ represent the abnormal returns from the cross-sectional regressions in period t in vector form

$$\widehat{AR}_t = \begin{bmatrix} \hat{\alpha}_1 + \hat{\epsilon}_{1,t} \\ \hat{\alpha}_2 + \hat{\epsilon}_{2,t} \\ \vdots \\ \hat{\alpha}_N + \hat{\epsilon}_{N,t} \end{bmatrix}$$

Then, we create a time-series of the weighted average of the $\widehat{AR}_{i,t}$ observations, with weights based on the funds available in month t. For the EW portfolio, we assign equal weights to each fund available in time t, while, for the TNAV-W portfolio, we weight the funds according to their relative TNAV in time t

$$w_{i,t} = \frac{TNAV_{i,t}}{\sum_{i=1}^{N} TNAV_{i,t}}$$

For T periods, we get a time-series with T observations of $\widehat{AR}_t$ which we use in a t-test for whether the abnormal returns deviate from zero

$$t_{\widehat{AR}} = \frac{(\overline{\widehat{AR}} - AR_0)}{\sigma(\widehat{AR})/\sqrt{N}}$$

where $AR_0 = 0$.

We also compare the abnormal returns generated by active and index funds, using the $\widehat{AR}_t$ estimated in the previous steps, to study whether active or index funds outperform in terms of abnormal returns. The observations are paired by month, so we use a two-sample paired t-test. It is essentially the same as a t-test on the difference between the abnormal returns for active and index funds for all months in the sample. Let $\widehat{DAR}_t = \widehat{AR}_{active,t} - \widehat{AR}_{index,t}$ be the difference in abnormal returns (DAR) in period t, then

$$t_{\widehat{DAR}_t} = \frac{\overline{\widehat{DAR}} - DAR_0}{\sigma(\widehat{DAR}_t)/\sqrt{N}} = \frac{\left(\overline{\widehat{AR}}_{active,t} - \overline{\widehat{AR}}_{index,t}\right) - DAR_0}{\sigma(\widehat{DAR}_t)/\sqrt{N}}$$

where $DAR_0 = 0$. It is essentially the same type of test as the Carhart-inspired approach, but now with two samples.

3.2 Stochastic dominance tests

When comparing the performance of active mutual funds and index funds, stochastic dominance tests measure the extent to which either distribution has higher probabilities associated with higher payoffs and lower probabilities associated with lower payoffs. In contrast to the benchmark models, the method uses the entire probability density function and not just the average effects. Furthermore, the method allows investors to rank active and index fund performance without imposing strong assumptions for the benchmarks, the test-statistic, the distribution of fund returns, or the investor's utility function. We limit our study to stochastic dominance of first and second-order.

In this section, we let F and G be the cumulative distributions (CDF) of the random variables X and Y. In our tests, we set F and G to be the active and index fund distributions, respectively, and vice versa, as we test for stochastic dominance both ways.

3.2.1 First-order stochastic dominance (FSD)

The random variable X is said to first-order stochastically dominate the random variable Y, denoted as X FSD Y, if

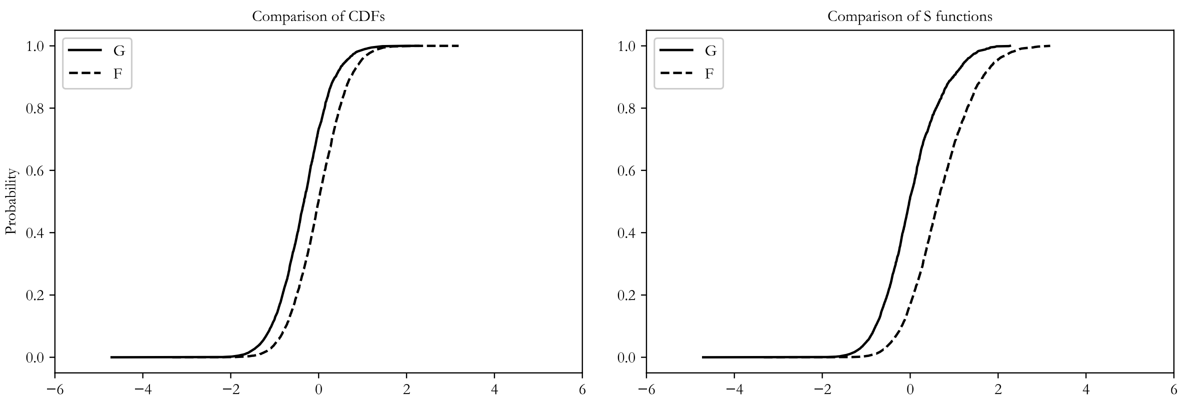
$$F(x) \leq G(x) \ \ for\ all\ x \in \mathbb{R}$$

where $F(x)$ and $G(x)$ is the CDF of X and Y, respectively (Whang, 2019, p. 1-2).

The FSD condition rests on the assumption that the investors have monotone increasing utility functions, meaning that investors prefer a higher return over a lower return, which is a reasonable assumption for mutual fund investors.

In the case of active and index funds, $\mathbb{R}$ can be replaced by all observed returns in our sample. Hence, the FSD condition implies that for any observed return, the cumulative probability of Y must be equal to or greater than the cumulative probability X for all x. It may be easier to grasp by visually observing that G lies further (or equally far) to the left than F for all possible returns (see Figure 3.1).

Figure 3.1. Illustration of FSD. The chart plots the CDFs of $X \sim N(0,1)$ and $Y \sim N(-1,1)$. The fact that the CDFs do not intersect, and that F lies further to the right of G for all values of x, demonstrate that X FSD Y. The S function is explained in Section 3.2.2.



If X and Y correspond to active and index returns, respectively, X FSD Y implies that for all x, the proportion of returns generated by active funds with returns less than or equal to a return level x, is no larger than the proportion of such returns generated by index funds. For any chosen probability, the return associated with active funds is equal to or higher than the return associated with index funds. In this case, any investor should prefer a random draw from F over a random draw from G no matter his risk appetite, assuming that the investor has an increasing utility function.

3.2.2 Second-order stochastic dominance (SSD)

The random variable X is said to second-order stochastically dominate the random variable Y, denoted as X SSD Y, if

$$\int_{-\infty}^{x} F(x)\,dx \leq \int_{-\infty}^{x} G(x)\,dx \;\; for\ all\ x \in \mathbb{R}$$

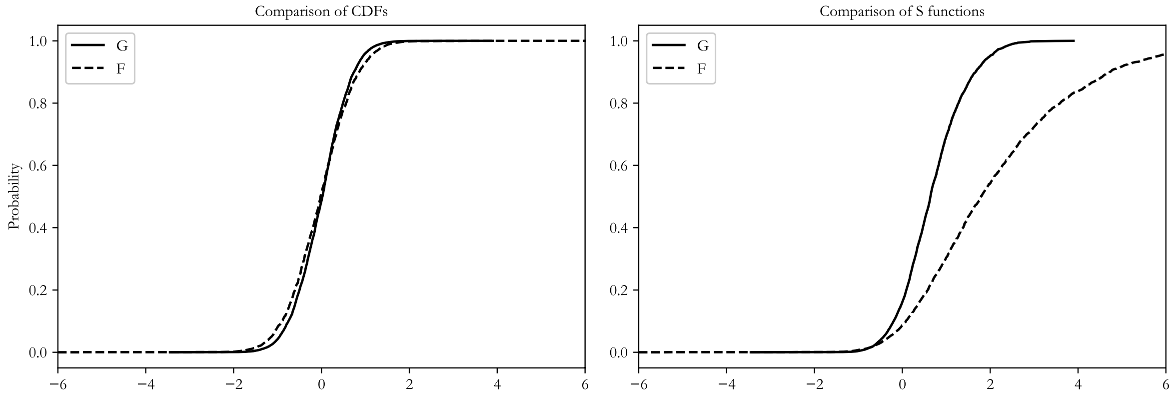
where $F(x)$ and $G(x)$ is the CDF of X and Y, respectively, and $\int_{-\infty}^{x} F(x)\,dx$ is the integral of the CDF of X, or the integral of the integral of the PDF of X, which we later refer to as the S-function (short for the "super-cumulative") (Whang, 2019, p. 1-2). As for the FSD, we can replace ℝ with the observed returns in our case.

The SSD condition also assumes monotone increasing utility functions. In addition, we now introduce the assumption of concave utility functions for investors, meaning that they are risk-averse. Empirical evidence suggests that most investors are likely to be risk averters (see e.g. Levy (1998), Borch, Hester & Tobin (1969), and Danthine & Donaldson (2015)).

For X SSD Y, the accumulated area under F must be smaller than the corresponding area under G below any value of x. If X FSD Y, then it follows that X SSD Y which can be seen visually by an example given in Figure 3.1. However, when X SSD Y, it is not necessarily the case that X FSD Y. It is easy to see visually, exemplified in Figure 3.2.

In the case of mutual fund returns, SSD differs from FSD because it introduces the assumption of risk-aversion. If X and Y correspond to active and index returns, respectively, X SSD Y implies that any risk-averse investor prefers active fund returns over index fund returns. X FSD Y implies that all investors, regardless of risk appetite, should prefer active fund returns over index fund returns.

Figure 3.2. Illustration of SSD and the relationship between FSD and SSD. The chart plots the CDFs of $X \sim N(-0.5, 3)$ and $Y \sim N(0, 1)$. The fact that the CDFs intersect demonstrates that X do not FSD Y. The S functions do not intersect, showing that X SSD Y.



3.2.3 Testing for stochastic dominance

According to Whang (2019, p. 24-26), three types of hypotheses are mainly considered in the literature

$$(1)\ H_0 : F(x) \leq G(x)\ for\ all\ x\ \ vs.\ H_1 : F(x) > G(x)\ for\ some\ x$$
$$(2)\ H_0 : F(x) \geq G(x)\ for\ some\ x\ \ vs.\ H_1 : F(x) < G(x)\ for\ all\ x$$
$$(3)\ H_0 : F(x) = G(x)\ for\ all\ x\ \ vs.\ H_1 : F(x) < G(x)\ for\ all\ x$$

To test the difference between active and index fund return distributions, we use (1) which is considered in the majority of existing tests in the literature. The tests in (1) can be classified into two groups: (A) tests comparing the CDFs at a finite number of grid points and (B) tests comparing the CDFs at all points in an interval. The latter is a full comparison of the CDFs which is why we chose a test from (B). Specifically, we use the test suggested by Barrett and Donald (2003). The test is similar to Kolmogorov-Smirnov tests and is used for studying mutual fund alpha returns by Crane and Crotty (2018). In the following, we briefly review the test. We refer to Barret and Donald for the details.

The general hypothesis for testing stochastic dominance of order j is

$$H_0^j :\ \mathcal{F}_j(x; G) \leq \mathcal{F}_j(x; F)\quad for\ all\ x \in \mathbb{R}$$

and

$$H_1^j :\ \mathcal{F}_j(x; G) > \mathcal{F}_j(x; F)\quad for\ some\ x \in \mathbb{R}$$

where F, G and x are defined above and $\mathcal{F}_j(.\,; G)$ is the integral operator that integrates G to order $j - 1$. For FSD and SSD, we have

$$\mathcal{F}_1(x; G) = G(x)$$

and

$$\mathcal{F}_2(x; G) = \int_{-\infty}^{x} G(t)\, dt = \int_{-\infty}^{x} \mathcal{F}_1(t; G)\, dt$$

respectively.

We let $\{X_i\}_{i=1}^{N}$ and $\{Y_i\}_{i=1}^{M}$ be independent random samples from the CDF's F and G, then the empirical distributions used to construct the tests can be expressed as

$$\hat{F}_N(x) = \frac{1}{N}\sum_{i=1}^{N} \mathbb{I}(X_i \leq x)$$

and

$$\hat{G}_M(x) = \frac{1}{M}\sum_{i=1}^{M} \mathbb{I}(Y_i \leq x)$$

where $\mathbb{I}$ denotes the indicator function.

Using the previously defined integral operator, we may then write the test statistic compactly as

$$\hat{S}_j = \left(\frac{NM}{N+M}\right)^{\frac{1}{2}} \sup_z \left(\mathcal{F}_j\left(x; \hat{G}_M\right) - \mathcal{F}_j\left(x; \hat{F}_M\right)\right)$$

where N and M are the sample size of F and G, respectively, sup is the supremum (i.e. the least upper bound) and one can show that

$$\mathcal{F}_j\left(x; \hat{F}_M\right) = \frac{1}{N}\sum_{i=1}^{N} \mathcal{F}_j\left(x; \mathbb{I}_{X_i}\right) = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{(j-1)!}\mathbb{I}(X_i \leq x)(x - X_i)^{j-1}$$

where $\mathbb{I}_{X_i}$ denotes the indicator function $\mathbb{I}(X_i \leq z)$. $\mathcal{F}_j\left(x; \hat{G}_M\right)$ is computed likewise. For FSD (i.e. $j = 1$), $\mathcal{F}_j\left(x; \hat{F}_M\right)$ is simply the empirical distribution

$$\mathcal{F}_1\left(x; \hat{F}_M\right) = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{(1-1)!}\mathbb{I}(X_i \leq x)(x - X_i)^{1-1} = \frac{1}{N}\sum_{i=1}^{N} \mathbb{I}(X_i \leq x) = \hat{F}_N(x)$$

We use a decision rule of the form

$$"reject\ H_0^j\ if\ \hat{S}_j > c_j"$$

where c_j is the critical value. Barret and Donald (2003) shows that the critical value must satisfy

$$P(\bar{S}_j^F > c_j) = \alpha$$

As noted by McFadden (1989), we may easily compute the critical value for FSD using

$$P(\bar{S}_j^F > c_j) = P\left(\sup_{p\in[0,1]} \mathcal{B}(p) > c\right) = \exp(-2c^2)$$

where $\mathcal{B}$ is a Brownian Bridge process as discussed later. Some common critical values are 1.073, 1.2239, and 1.5174 for the 10%, 5%, and 1% levels of significance, respectively.

For SSD (and higher order of stochastic dominance), the distribution of $\bar{S}_j^F$ will depend on F, so we may not compute the critical values in an easy manner. Barret and Donald (2003) propose to either (a) simulate the p-values using a Monte Carlo method (referred to as the multiplier method) or (b) conduct inferences using a form of the bootstrap method. We use the Monte Carlo method in our empirical study (Crane and Crotty use the bootstrap method which we report in Appendix G for robustness).

The Monte Carlo method involves the use of artificial random numbers and exploits the multiplier central limit theory to simulate a process that is identical to but (asymptotically) independent of $\mathcal{B}(F(x))$. Mathematically, the process is generated as

$$\mathfrak{B}_F^*(x; \hat{F}_N) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(\mathbb{I}(X_i \le x) - \hat{F}_N(x)\right) U_i^F$$

where $\{U_i^F\}_{i=1}^N$ denote a sequence of i.i.d. Standard Normal variables that are independent of the samples and other variables are defined in the previous.

The p-values can be obtained from

$$\hat{p}_j^F = P_U\left(\sup_x \mathcal{F}_j(x; \mathfrak{B}_F^* \circ \hat{F}_N) > \hat{S}_j\right)$$

where $\mathfrak{B}_F^* \circ \hat{F}_N$ is the process $\mathfrak{B}_F^*(x; \hat{F}_N)$ evaluated at all observed values of x, P_U is the probability function associated with the Normal random variables U_i^F, and the other variables are as defined in the previous. In our implementation, we simulate the p-value 500 times as Whang (2019, p. 222-224) does in his example code for the Barret and Donald test. The approach for computing the p-values is justified by Barret and Donald.

4.0 Data

All Norwegian mutual funds are open-end domestic equity funds (Blørstad and Bakkefjord, 2017). Open-end means that there is no set on the number of fund shares available on the market, and the shares outstanding can be redeemed or issued at any given time.

Table 4.1. Descriptive statistics for the Norwegian Mutual fund industry. The table includes the total net asset value (TNAV) for various fund types, selected market fractions, and the number of index funds. TNAV in billion NOK. ETFs are included in the figure for index funds. The data is provided by the Norwegian Fund and Asset Association (VFF) and includes only members of VFF. Note that the numbers in this figure summarize VFF's data which we do not use as our sample. Statistics for our sample are reported in the succeeding tables and may differ. The sample period is 2004 to 2019.

Year	Mutual funds investing in Norway					Mutual funds investing in Norway and abroad	
	Active funds	Index funds	ETFs	Index funds % of total	Number of index funds	Total	In Norway % of total
2004	30.0	0.4	0.0	1.2 %	2	95.5	31.76 %
2005	37.1	1.2	0.1	3.1 %	5	146.1	26.20 %
2006	48.8	2.0	0.2	3.9 %	5	207.2	24.50 %
2007	50.6	2.3	0.3	4.4 %	5	228.8	23.13 %
2008	24.1	1.4	0.4	5.5 %	6	129.5	19.68 %
2009	52.6	4.9	1.4	8.6 %	6	232.0	24.80 %
2010	69.9	7.7	1.3	9.9 %	7	292.1	26.55 %
2011	54.3	6.5	1.0	10.7 %	7	246.8	24.63 %
2012	60.4	7.7	1.1	11.3 %	7	278.3	24.46 %
2013	71.8	9.8	1.0	12.0 %	7	364.8	22.36 %
2014	72.4	12.6	1.1	14.8 %	8	406.2	20.94 %
2015	71.9	14.8	1.1	17.1 %	9	435.0	19.94 %
2016	89.5	19.4	1.1	17.8 %	9	476.3	22.86 %
2017	108.0	25.0	2.9	18.8 %	10	581.7	22.86 %
2018	102.2	26.3	2.7	20.5 %	12	557.5	23.05 %
2019	121.8	31.3	1.8	20.4 %	15	677.6	22.59 %

Table 4.1 reports descriptive statistics for the Norwegian mutual fund industry between 2004 and 2019. The data is obtained from the Norwegian Fund and Asset Association (VFF) and includes its member funds, including 96 Norwegian mutual funds in 2019. The data for Norwegian ETFs is obtained from Morningstar Direct (hereafter referred to as "MS Direct"). The assets under management ('AUM' or 'TNAV' for total net asset value) for funds investing in Norway have grown from 30.4 billion NOK in 2004 to 153.1 billion NOK in 2019. In the same period index funds' share of assets under management has increased almost consistently year over year from 1.2 percent to 20.4 percent. The rise in the relative TNAV of passive funds may reflect the increasing

popularity of index funds, in particular after the financial crisis in 2007/2008, or, the results may be impacted by the number of funds registered with VFF. Our sample includes all domestic ETFs, although they comprise a tiny share of TNAV for index funds. For instance, in 2019, ETFs represented 1.8 out of 31.8 billion NOK (5.8%) index fund assets. While the total assets under management have increased from 2004 to 2019, the fraction invested in Norwegian funds has decreased from 31.76% to 22.59%. This suggests that investors prefer the broader diversification provided by international equity funds over Norwegian funds.

4.1 Data description

4.1.1 Mutual fund returns

We obtained the mutual funds' returns from Oslo Stock Exchange Information (hereafter referred to as "OSE Information") and MS Direct. In both databases, we limited our search criteria to mutual funds that invest at least 80% in Norwegian equities. Next, we merged data from these two sources to construct a rich dataset for the Norwegian mutual fund industry. Of the 158 fund classes listed since July 1991 (Oslo Stock Exchange, personal communication, March 5th, 2020) only 3 are not included in our dataset.

We downloaded data for 114 of the funds from MS Direct and 41 of the funds from OSE Information. The 3 funds for which no data is available only represents 39 monthly returns (1 + 0 + 38), thus, our data includes 99.83% of the monthly returns for equity mutual funds listed on OSE.

We merged the databases using a combination of ISIN, ticker, and Morningstar's security ID to ensure that no funds were included in our data more than once. The data provided by OSE Information are net returns, while MS Direct provides gross and net returns. Our analyses use net returns.

In our cleaning process, we primarily focused on removing data errors while filtering out unlisted MS Direct funds. Our full sample consists of 164 equity mutual fund classes, 103 distinct funds, and 16 219 monthly fund returns. The sample includes 9 fund classes not directly listed on the OSE, but other classes of these funds have been listed. The first monthly return dates to August 1981, while the last is for December 2019. The first index return is from September 1990. A full list of the funds is reported in Appendix I.

For the analysis in the subsequent sections, we will report three time periods; the full sample and two subsamples starting from 1991 with a split before and after 2006. The rationale for the subsample split is as follows. Firstly, our analyses focus on comparing active and index funds, and index funds have no return prior to 1991. Secondly, we want to include roughly the same number of returns per subperiod due to the statistical inference of our analyses. Thirdly, the subsamples will spread the effects of crises (e.g. the dot-com bubble and the financial crisis in 2007/2008) making it possible to compare performance under similar market conditions. Lastly, we suspect that the costs of index funds have decreased considerably over our sample period. However, this is a challenging claim to document due to a lack of data on Norwegian mutual funds' costs and fees. We also suspect that index funds have become more efficient in tracking their benchmarks. The idea stems from the U.S. market, where index fund costs and fees have dropped considerably compared to previous decades (Bogle, 2019). The Investment Company Institute (ICI, 2019), which is the leading association of regulated funds globally, found that the average expense ratios dropped from 1.04% to 0.76% and 0.27% to 0.08% from 1997 to 2018 for active funds and index funds, respectively. We have tried to quantify our suspicions by searching for time-series data for the costs and fees of Norwegian mutual funds in VFF, Oslo Stock Exchange, Morningstar, Eikon, Orbis, and Bloomberg. Still, none of these sources contains useful data for the 1990s and early 2000s. We contacted various Norwegian index fund providers in search of information. Carnegie, managing Carnegie Norge Indeks from 1991 to 2016, stated that "increasing offerings of cheap ETFs from the competition was a contributing factor to the closure of our index fund" (C Worldwide Asset Management, personal communication, June 4th, 2020). Although we do not have sufficient data to document that index funds costs have dropped since 1990, this statement goes a long way in supporting our suspicion. There are only two relevant Norwegian ETFs, namely DNB OBX and XACT OBX, where the former was introduced in 2005 and the latter in 2017. As illustrated in Table 4.1, they represent a small share of index funds, accounting for 5.8% of index funds TNAV in 2019. Both ETFs are included in our sample.

4.1.2 Market return, factor returns, and risk-free rate

We use a similar approach as Sørensen (2009, p. 7-9) to construct time-series data for the market return back to 1981; we use the OSE Total Return Index (OBX) as a benchmark from 1987 to 1995 and the OSE Mutual Fund Index (OSEFX) from 1996 to 2019. Our market index differs from Sørensen's as he used the OSE All Share Index (OSEAX) prior to 1996. We decide to use OBX since it is value-weighted, whereas OSEAX is equally-weighted. From 1981 to 1987, we use the value-weighted market return published by Ødegaard (2020a) as data for the OBX is not available prior to 1987.

The OBX includes the 25 most traded securities on the OSE. The OSEFX is a capped version of the OBX, which must meet specific diversification requirements set by European Union directives for regulating investments in mutual funds. Norwegian mutual funds are required to invest in at least 16 different stocks, and the weight of any holding cannot exceed 10% of the portfolio. The OSEFX index takes these requirements into account and should, therefore, be a suitable benchmark for mutual funds (see Sørensen (2009, p. 7-9) for a more in-depth discussion), but it is not available prior to 1996. As most index funds track the OSEBX and not the OSEFX, we study the robustness of our chosen market index in Appendix D (see also the discussion in Section 5.1).

The equity risk factors for the Fama-French three-factor model (MKT, HML, and SMB) and the Carhart four-factor model (MKT, HML, SMB, and PR1YR) are computed and shared by Ødegaard based on empirics of the OSE. We employ his calculated factor variables for the period spanning from 1981 to 2019. Norwegian RMW and CMA are, to our knowledge, not publicly available. We constructed these two factors ourselves based on Kenneth French European RMW and CMA factors (see Appendix C for details).

We use monthly rates published by Sørensen as the risk-free rate in the benchmark models. The data apply two-year bond yields as an estimate of the monthly risk-free rate from 1981 to 1986, and the Norwegian Interbank Offered Rate (NIBOR) from 1987 to 2019. Ødeaard (2020b) argues that the NIBOR is the most appropriate proxy for the interest rate but applies an alternative measure prior to 1987 due to what he describes as "slightly messy data" from the period.

4.2 Summary statistics

Table 4.2 presents an overview of the number of distinct funds, fund classes, and monthly returns for active and index funds for various sample periods. Four time-periods are reported: the full sample, the two subperiods, and the period before the subsamples. For the full sample period, our data has 103 distinct equity mutual funds, whereas 17 are index funds. If we count each share class individually, the data includes 164 funds. Although fund classes are essentially the same fund, they need not generate the same return. For instance, Alfred Berg Aktiv I and II have an average monthly absolute difference of 1.09 percent in net returns per month, which is substantial seen from the perspective of the mutual fund investor. To incorporate the difference between the fund classes in our analysis, we compute a weighted average of the fund class returns based on the fund classes' monthly net asset value (NAV). If the NAV is not available for any fund classes of a fund in a given month, we use the return of the fund's oldest share class (see Appendix II for details). We have 16,219 monthly returns in the full sample period, where 2,288 are index fund returns. The split between active and index funds is presented in Figure 4.1.

Table 4.2. Summary of the return data. The table shows the distribution of funds and monthly returns over various sample periods. The sample period is reported in each panel.

Panel A: Mutual funds				
	1981 - 2019	**1981 - 1990**	**1991 - 2005**	**2006 - 2019**
Number of distinct funds	103	12	77	77
Number of distinct index funds	17	1	12	13
Number of distinct active funds	86	11	65	64
Number of distinct fund classes	164	12	96	136
Panel B: Mutual fund returns				
	1981 - 2019	**1981 - 1990**	**1991 - 2005**	**2006 - 2019**
Number of monthly returns	16 219	548	7 297	8 374
Number of monthly returns for index funds	2 288	4	914	1 370
Number of monthly returns for active funds	13 931	544	6 383	7 004
Average number of observations per month	35.1	4.8	40.5	49.8

Figure 4.1. Split between active and index funds in the return data. The plot shows the percentage of active and index funds over the full sample period. Only distinct funds are included, so a fund with multiple fund classes is counted once. The sample period is 1981 to 2019.

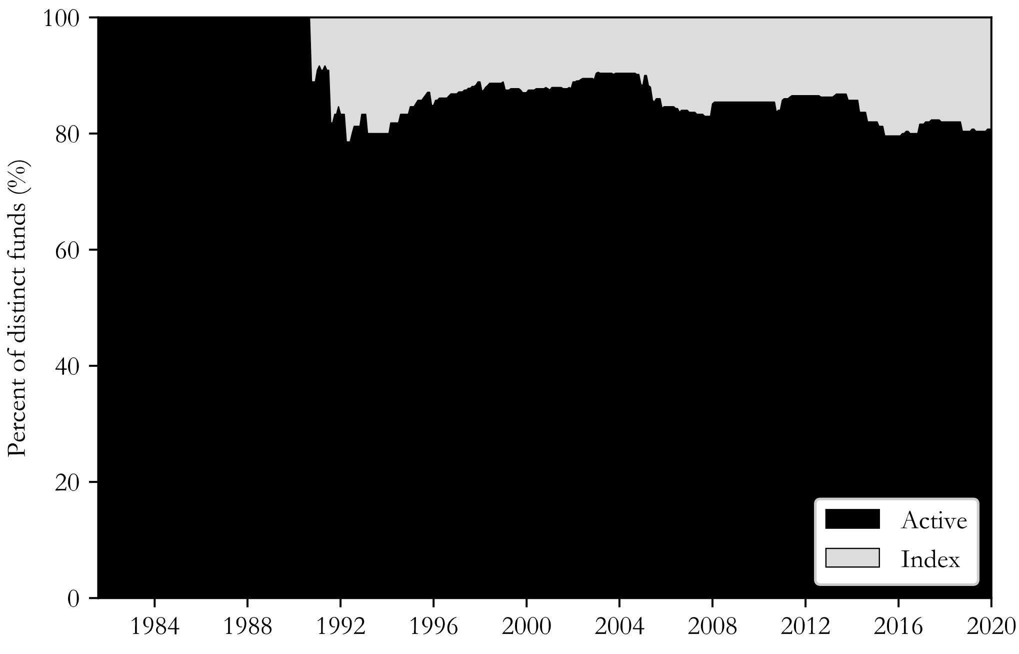


Table 4.3 presents summary statistics for equally weighted portfolios of all funds, active funds, and index funds for all sample periods. In the full sample, active funds have a higher mean and standard deviation than index funds. The index fund distribution is, somewhat surprisingly, more skewed towards negative returns than active funds as measured by the skewness, and index funds have fatter tails than active funds, as demonstrated by the difference in the kurtosis.

The highest mean return for active funds occurred between 1981 and 1990 before the first index fund was introduced in the Norwegian market. As shown in Appendix III, the 1981 to 1990 period had the greatest market return and risk-free rates. In the subsequent periods, active funds have higher mean returns, standard deviation, and skewness than index funds, while index funds have the highest kurtosis for all periods except 1991 to 2005. In all periods, active funds have higher maximum returns and lower minimum returns. We return to discussing the distributions of active and index fund returns in Section 5.2.

Table 4.4 shows descriptive statistics for the Norwegian equity factors in various periods. The SML and HML factors have the highest and the lowest cumulative return over the full sample period, respectively. Appendix V includes plots of the cumulative returns for the factors. Table 4.5 reports the correlation matrix between the factors for the 1991 to 2019 period. SMB is negatively correlated to the market with a coefficient of -0.4560. HML, PR1YR, RMW, and CMA have a comovement with the excess market return in the same direction as SMB but with correlation coefficients of -0.0018, -0.2414, -0.0093, and -0.0041, respectively. SMB is correlated with HML and PR1YR, showing correlation coefficients of -0.1369 and 0.1163, respectively, while the correlation coefficient between HML and PR1YR is -0.1262. The RMW and CMA factors are only slightly correlated to the MKT, SMB, and HML factors, which is to be expected, given how we constructed them (see Appendix C for details).

Table 4.3. Descriptive statistics for the mutual fund returns. Numbers are reported per month in percentage (e.g. 0.0100 means 0.01 %). The sample period is reported in each panel.

Panel A: All funds							
	Obs	**Mean**	**Min**	**Max**	**Std**	**Skew**	**Kurt**
1981 - 2019	16 219	0.9693	-30.0616	41.7696	6.1440	-0.7045	2.7188
1991 - 2019	15 671	0.9541	-30.0616	41.7696	6.1068	-0.7354	2.7628
1981 - 1990	548	1.4043	-26.7000	32.9179	7.1188	-0.1783	1.6173
1991 - 2005	7 297	1.1191	-28.7962	41.7696	6.7746	-0.4445	1.2780
2006 - 2019	8 374	0.8103	-30.0616	22.0966	5.4551	-1.2323	5.0247
Panel B: Active funds							
	Obs	**Mean**	**Min**	**Max**	**Std**	**Skew**	**Kurt**
1981 - 2019	13 931	0.9866	-30.0616	41.7696	6.2142	-0.6758	2.6517
1991 - 2019	13 387	0.9677	-30.0616	41.7696	6.1747	-0.7086	2.6973
1981 - 1990	544	1.4516	-26.7000	32.9179	7.1084	-0.1825	1.6510
1991 - 2005	6 383	1.1362	-28.7962	41.7696	6.8408	-0.4223	1.2802
2006 - 2019	7 004	0.8141	-30.0616	22.0966	5.4938	-1.2265	4.9478
Panel C: Index funds							
	Obs	**Mean**	**Min**	**Max**	**Std**	**Skew**	**Kurt**
1981 - 2019	2 288	0.8640	-25.4100	17.1000	5.6983	-0.9353	3.1285
1991 - 2019	2 284	0.8744	-25.4100	17.1000	5.6934	-0.9376	3.1511
1981 - 1990	4	-5.0275	-12.5300	0.1900	6.2265	-0.4800	-3.2878
1991 - 2005	914	0.9996	-24.5600	16.8936	6.2953	-0.6503	1.1398
2006 - 2019	1 370	0.7908	-25.4100	17.1000	5.2544	-1.2656	5.4632

Table 4.4. Descriptive statistics for the factor returns. The table shows statistics for the factors. Numbers are reported per month in percentage (e.g. 0.0100 means 0.01 %). Data from Bernt Ødegaard, except RMW and CMA which is constructed with Kenneth French data (see Appendix C). See Appendix III for the subperiod tables. The sample period is 1991 to 2019.

Sample 1991 - 2019						
	Mean	**Min**	**Max**	**Std**	**Skew**	**Kurt**
Rm	0.9185	-27.1659	16.5207	5.9240	-1.1272	3.8723
Rf	0.2938	0.0520	2.0740	0.2153	1.5580	6.1755
Rm-Rf	0.6247	-27.8089	16.3497	5.9638	-1.1731	3.9730
SMB	0.6418	-17.0784	22.1400	3.7894	0.0073	3.1716
HML	-0.0872	-16.6487	14.6609	4.3904	-0.3361	1.1902
PR1YR	0.9575	-16.7805	15.4272	4.5072	-0.3411	1.6158
RMW	0.4173	-4.7199	4.7749	1.5146	-0.2893	0.4689
CMA	0.2372	-5.2569	6.9349	1.6353	0.4593	1.9179

Table 4.5. Correlation matrix. The table shows correlation coefficients for relevant variables, including the excess returns of equally weighted (EW) and total net asset value-weighted (TNAV-W) portfolios of the funds. Factor data from Bernt Ødegaard, except RMW and CMA which is constructed with Kenneth French data (see Appendix C). The sample period is 1991 to 2019. See Appendix IV for the subperiod tables.

Sample 1991 - 2019								
	EW Active	**TNAVW Active**	**EW Index**	**Rm-Rf**	**SMB**	**HML**	**PR1YR**	**RMW**
EW Active	1.0000							
TNAVW Active	0.9916	1.0000						
EW Index	0.9607	0.9654	1.0000					
Rm-Rf	0.9737	0.9796	0.9796	1.0000				
SMB	-0.3409	-0.3751	-0.4519	-0.4560	1.0000			
HML	-0.0277	0.0118	0.0069	-0.0018	-0.1369	1.0000		
PR1YR	-0.2383	-0.2553	-0.2275	-0.2414	0.1163	-0.1262	1.0000	
RMW	-0.0174	-0.0197	-0.0152	-0.0093	0.0053	0.0004	0.1710	1.0000
CMA	-0.0268	-0.0106	-0.0066	-0.0041	-0.0031	0.0024	-0.0005	-0.2499

Table 4.6. Summary of minimum investment groups. The table displays the number of returns for the minimum investment groups over relevant sample periods. Each group includes all returns up to their designated minimum investment (e.g. 'Large' includes all returns for 'Medium' and 'Small'). Only active funds are included as the variable is only used for active funds. The sample period is reported in the table columns.

Investor size	**1981 - 2019**		**1981 - 1990**	
	Count	**% of All**	**Count**	**% of All**
Small	5 480	39.34 %	350	64.34 %
Medium	9 741	69.92 %	369	67.83 %
Large	11 225	80.58 %	369	67.83 %
All	13 931	100.00 %	544	100.00 %
Investor size	**1991 - 2005**		**2006 - 2019**	
	Count	**% of All**	**Count**	**% of All**
Small	2 002	31.36 %	3 128	44.66 %
Medium	3 706	58.06 %	5 666	80.90 %
Large	4 145	64.94 %	6 711	95.82 %
All	6 383	100.00 %	7 004	100.00 %

Table 4.7. Summary of the Minimum investment and TNAV variables. The table shows the variables' coverage in terms of monthly returns for various sample periods. Only active funds are included as the variables are only used for active funds. The sample period is reported in the table rows.

	All	**Minimum investment**		**Net Asset Value**	
	Count	**Count**	**% of All**	**Count**	**% of All**
1981 - 2019	13 931	11 160	80.11 %	9 056	65.01 %
1981 - 1990	544	369	67.83 %	178	32.72 %
1991 - 2005	6 383	4 145	64.94 %	2 190	34.31 %
2006 - 2019	7 004	6 646	94.89 %	6 688	95.49 %

4.3 Additional data

Parts of our study include other variables, primarily to group the funds.

To separate index and active funds and dead and alive funds, we constructed dummy variables. For the MS Direct funds, we used the index fund variable included in the database. The OSE data does not have such a variable, so we constructed the variable ourselves based on data received from VFF. Funds that are not registered members with VFF are not included in their data; for these funds, we used publicly available information to categorize them. We define alive funds as having returns in December 2019, while dead funds as having returns series ending before December 2019. The variables are reported in Appendix I (see the last return column for alive funds).

The MS Direct database includes a variable for the minimum investment on a fund class level for 114 of 123 fund classes. The OSE Information has no such variable. We analyzed publicly available information and were able to categorize 7 of the remaining 50 fund classes. Although the data is not complete, the variable covers 80.11% of the 13 931 monthly returns for active funds and lacks ~5.1% of the returns between 2006 and 2019, as reported in Table 4.7. The variable is reported in Appendix I on a fund class level.

We use the minimum investment variable as a proxy to form groups of investors based on size. In the preceding, 'Small' investors are fund classes with a minimum investment below 1 000 NOK, 'Medium' below 100 000 NOK, and 'Large' up to 300 000 000 NOK. 'All' includes all funds and differs from 'Large' as the group also includes funds without minimum investment data. The use of the minimum investment variable has two potential problems; (1) we do not have time-series data for the variable (only the 'last known value') and (2) one of our constructed groups includes the funds we do not have data for. For (1), we believe that mutual funds seldom drastically change the size of their minimum investments (although it may happen), so the variable will be relatively correct back in time, in particular for the 2006 to 2019 period. The consequence of (2) is a type of survivorship bias. Most of the funds where we lack minimum investment data are dead funds. Later, we will see that survivorship bias exists for Norwegian mutual funds, likely leading to a positive bias in the groups we construct. The 'All' group includes all funds and is not subject to the bias. Note that we do not filter index funds on minimum investment due to the limited sample size. Table 4.6 presents summary statistics for the investment groups.

The MS Direct database also includes a variable for the total net asset value (TNAV) on a fund class level. Some funds report TNAV less frequently than monthly. For these funds, we linearly interpolated the TNAV between the two closest data points. The returns in the OSE Information are calculated from the funds TNAV, but we do not have access to the TNAV data in the OSE Information. The TNAV variable could therefore potentially be somewhat problematic (interpolated to be 65.01% of the 13 931 active fund returns in our sample), although the data from 2006 to 2019 (with returns primarily from MS Direct) includes 95.49% of the monthly returns. In the analyses where we use the TNAV variable, the OSE Information funds are excluded, leading to a potential bias. Due to the availability of data and that index funds are assumed to be more similar than active funds, we only use the TNAV variable in the analysis of active funds (i.e., we always weigh the index funds equally). Table 4.7 reports the number of fund months for the TNAV variable.

4.4 Potential sources of bias

4.4.1 Survivorship bias

Survivorship bias describes the bias that may occur in mutual funds data because funds that disappear tend to do so due to poor performance. Consequently, mutual fund data that is not corrected for survivorship bias will be positively skewed compared to the true distribution of fund returns, discussed in detail by Brown, Goetzmann, Ibbotson & Ross (1992) and Elton, Gruber & Blake (1996) among others.

To avoid overestimating funds' performance and potentially inferring incorrect statistical results, the data must be adjusted for survivorship bias. To illustrate the bias, we form two portfolios; one that includes all funds that have existed at some point between 1981 and 2019 but are now closed (i.e. dead), the other only including funds that are alive at the end of the sample. Table 4.8 presents summary statistics for the two portfolios over the entire sample and the subperiods used in previous sections. The average return of an equal-weighted portfolio of alive funds is higher than dead funds for all periods. Our sample includes 51 dead funds in total, which is far from trivial compared to the 52 funds alive today. Figure 4.2 illustrates the proportion of dead and alive funds throughout our sample period. Among the funds that were alive in the early 2000s, more than half are now dead.

The significance of survivorship bias becomes evident when we plot the equally weighted cumulative return of the dead and alive funds, as seen in Figure 4.3. On average, the portfolio of funds alive today outperforms the portfolio of all funds by ~0.24% per month over the full sample period, which translates to 2.93% per year. Sørensen (2009, p. 11) estimated the survivorship bias to be 0.84% annualized for Norwegian mutual funds between 1982 and 2008 and stated that his finding was comparable to the 0.7% Dahlquist et al. (2000) found when they studied Swedish mutual funds between 1993 and 1997. Without a more profound analysis, it appears that the survivorship bias may have grown in magnitude over the years since Sørensen's study.

In Appendix A**,** we examine an analysis by Alfred Berg published in Finansavisen in April 2017. As the study only included funds that had been alive for five years during the sample period, the results are likely to be biased by a form for survivorship bias.

In this thesis, we include all funds that have been alive at some point in the sample period, eliminating survivorship bias from our results.

4.4.2 Incubation bias

Funds (and fund classes) typically open to the public only if the returns turn out to be attractive. Incubation bias arises because the pre-release returns are included in mutual fund databases (Fama & French, 2010). Evans (2010) suggests filtering out a fund's returns prior to when the fund received a ticker symbol from NASDAQ (for U.S. mutual funds). We use a similar approach to Evans, but instead, we filter out the returns prior to the fund's listing date at the OSE. The listing dates were retrieved from the Oslo Stock Exchange (personal communication, March 5th, 2020).

4.4.3 Birth bias

Birth bias occurs when a fund existing in country A is introduced in country B and the historical returns transfer from country A to country B (Norwegian Consumer Agency, 2018a). The returns for the funds we include from the MS Direct database are filtered using a variable for the inception date of the particular fund class so that no funds have historical data prior to their inception. To our knowledge there are no funds in the OSE Information data that have traded in another country, meaning that our results should not be skewed by birth bias.

Table 4.8. Summary table for survivorship bias. Returns are reported per month in percentage (e.g. 0.0100 means 0.01 %). The counts are the number of fund classes, not distinct funds. Note that the average total count only equals the average dead count plus the average alive count when there are alive and dead fund returns in every month of the period, which is not the case in the first three years and the last year of the sample; for 1981 to 2019, the first 25 months with returns for dead funds have no funds alive today and the last 4 months have no dead fund returns.

	Average return			**Average number of funds per month**			**Total number of funds**		
	Alive	**Dead**	**All**	**Alive**	**Dead**	**All**	**Alive**	**Dead**	**All**
1981 - 2019	1.0616	0.8210	0.9693	22.9	13.6	35.1	52	51	103
1981 - 1990	1.6752	1.2547	1.4043	2.2	3.1	4.8	4	8	12
1991 - 2005	1.4413	0.8536	1.1191	18.3	22.2	40.5	32	45	77
2006 - 2019	0.8508	0.6693	0.8103	38.7	11.5	49.8	52	25	77

Figure 4.2. Split between dead and alive funds in the return data. The table shows the percentage of funds that are dead and alive today over the full sample period. The chart includes all fund classes, not only distinct funds. The sample period is August 1983 to 2019.

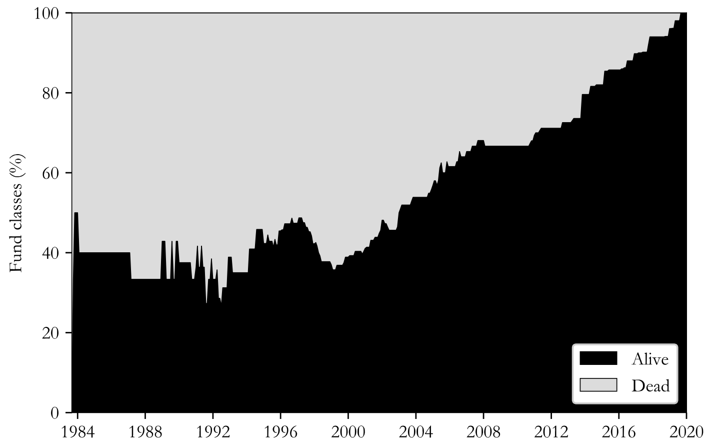
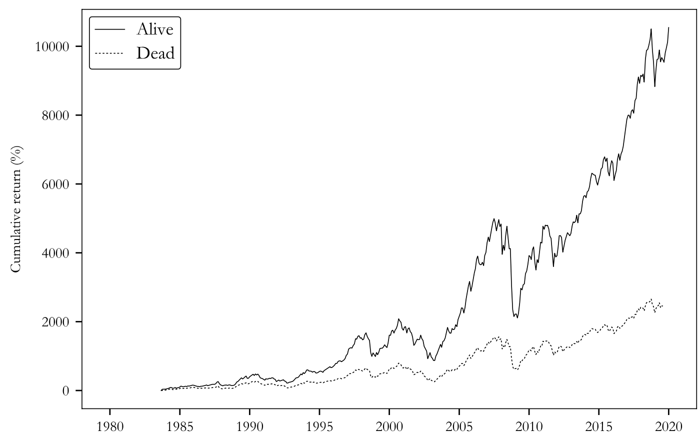


Figure 4.3. Cumulative returns for dead and alive funds. The chart starts from the first month with returns on both alive and dead funds; August 1983.



5.0 Results and analysis

We examine the benchmark-adjusted performance of active funds and index funds in Section 5.1. Section 5.2 investigates the net return performance of active and index funds through the lens of stochastic dominance. To quantify the difference in cumulative returns between active and index funds, we use Monte Carlo simulations in Section 5.3. We use the whole sample from 1981 to 2019 when we study active funds exclusively, while we use data for 1991 to 2019 when we include index funds, as there were no Norwegian index funds prior to 1991.

5.1 Benchmark-adjusted performance

5.1.1 Benchmark-adjusted performance for active funds

In Table 5.1, the benchmark models for all active mutual fund returns in our sample are presented. The intercept (alpha) in Table 5.1 indicates whether the active mutual fund sector, on average, has produced risk-adjusted returns. The far-left column indicates whether the industry has generated abnormal returns (alpha + residual). The results are reported for both EW and TNAV-W portfolios of active funds.

There is not much agreement between the various factor models, the two alpha tests for the same model, nor across the two types of weighting. The test statistics of Fama-MacBeth (F-B), which is most commonly used in previous literature, suggest that the alpha over the entire sample period for the equal-weighted portfolio based on the Fama-French three-factor model (-0.1230), the Carhart four-factor model (-0.1027), and the Fama-French five-factor model (-0.0976) is negative and significantly different from zero. By the same token, the F-B test suggests that the alpha of the TNAV-W portfolio based on CAPM is positive and significant (0.0850) and negative but insignificant for the other three models. The difference between the EW and TNAV-W performance may be due to a form of survivorship bias as most of the TNAV funds are alive today. The test statistic inspired by Cuthbertson and Nitzsche (C-N), which takes the uncertainty of the fund-level alpha estimates into account, does not reject the null for any factor models at any conventional significance level. These results suggest that we should be careful in concluding whether the active mutual fund sector has produced positive or negative alpha returns on average. The abnormal returns (AR) are not statistically significant for any factor model nor portfolio weight.

In previous studies on Norwegian mutual funds, Gjerde and Sættem (1991) found insignificant alpha based on the CAPM model between 1982 and 1984. Sørensen (2009) identified positive alphas from 1982 to 2008 for the same benchmark models except for the five-factor model, although none statistically significant. Our results for the equally-weighted portfolio using the F-B test statistic are consistent with most studies on U.S. mutual funds (e.g., Jensen (1968), Grinblatt & Titman (1993), and Malkiel (1995)) in that we've found negative risk-adjusted performance of active funds relative to the benchmark indices.

Appendix VI exhibits a more detailed analysis of active manager skills with risk-adjusted returns for various minimum investment boundaries and subperiods. Broadly speaking, the results suggest a significant negative alpha in the 1991 to 2005 period and a positive but insignificant alpha in the 2006 to 2019 period (see the 'All' investor group). The CAPM alphas are broadly positive in all time periods for the four different groups of investors. Results are mixed for the various groups of investors, portfolio weights, and subsamples when employing the Fama-French three-factor model, the Carhart four-factor model, and the Fama-French five-factor model. The results are most concise for the TNAV-W portfolio from 1991 to 2005, where we find a negative alpha on a 5% significance level for most investment groups based on the F-M test, with exceptions for the CAPM, the five-factor model and one instance for the three-factor model. For this period, the results suggest that the alpha returns mostly increase with investor size (i.e. from 'Small' to 'Large') across the models, in particular for the TNAV-W portfolios, but not when applying the five-factor model. For example, the TNAV-W portfolio available to small investors (i.e., minimum investment below 1 000 NOK) based on the Fama-French three-factor has a 4.72 basis-point monthly disadvantage to medium investors (i.e., below 1 000 000 NOK), on average. Annually this implies that the small investor runs short of 56.79 basis-points risk-adjusted return relative to the medium investor. For the four-factor model, the deviation is 41.48 basis points per annum. These results are expected since some large investors, e.g. institutional investors, receive a trading volume discount. Still, the advantage of larger investors seems to disappear when we shift focus to the 2006 to 2019 period. Here, the alpha deviations between the investor sizes exhibit no obvious systematic patterns. Note also that the C-N test statistic suggests we should not reject the null for any portfolio weight, factor model nor sample period.

Table 5.1. Active fund performance. The table shows abnormal returns, alphas, factor loadings, and adjusted R^2 for an equally weighted (EW) and a total net asset value-weighted (TNAV-W) portfolio of active funds for the Jensen CAPM model, the Fama-French three-factor model, the Carhart four-factor model, and the Fama-French five-factor model. Numbers per month. Alphas and abnormal returns (AR) are reported in percentage (e.g. 0.0100 = 0.01%) and p-values are in parentheses. The p-value for AR is based on the Carhart technique, while the top p-value for the coefficients is based on Fama-MacBeth (F-B) and the lower p-value is based on the average of the fund-level coefficient's t-stats as proposed by Cuthbertson and Nitzsche (C-B) (see Section 3.1.1 for details). Newey and West (1987) corrected standard errors are used for the latter. The null hypothesis for the MKT beta is $\beta_{MKT} = 1$. Observations (N) are reported in terms of the number of funds and the number of returns. The sample period is 1981 to 2019, except for RMW and CMA factors which only are available from 1991.

Panel A: Sample 1981 - 2019 (EW)

	AR	α	β_{MKT}	β_{SMB}	β_{HML}	β_{PR1YR}	β_{RMW}	β_{CMA}	adj R^2	N
CAPM	0.0882	0.0289	0.9400						0.85	78 (13 836)
	(0.33)	(0.34)	(0.00)							
		(0.71)	(0.05)							
Fama-French 3f	-0.0846	-0.1230	0.9818	0.1717	-0.0173				0.87	78 (13 836)
	(0.31)	(0.00)	(0.16)	(0.00)	(0.19)					
		(0.67)	(0.45)	(0.00)	(0.81)					
Carhart	-0.0692	-0.1027	0.9787	0.1695	-0.0188	-0.0148			0.87	78 (13 836)
	(0.41)	(0.00)	(0.09)	(0.00)	(0.14)	(0.10)				
		(0.70)	(0.46)	(0.00)	(0.79)	(0.85)				
Fama-French 5f	0.0108	-0.0976	0.9873	0.1773	-0.0207		-0.0624	-0.0605	0.88	78 (13 347)
	(0.85)	(0.00)	(0.32)	(0.00)	(0.08)		(0.00)	(0.04)		
		(0.79)	(0.50)	(0.00)	(0.80)		(0.60)	(0.79)		

Panel B: Sample 1981 - 2019 (TNAV-W)

	AR	α	β_{MKT}	β_{SMB}	β_{HML}	β_{PR1YR}	β_{RMW}	β_{CMA}	adj R^2	N
CAPM	-0.0278	0.0850	0.9259						0.90	51 (9 001)
	(0.75)	(0.00)	(0.00)							
		(0.41)	(0.00)							
Fama-French 3f	-0.1478	-0.0215	0.9729	0.1348	0.0261				0.91	51 (9 001)
	(0.08)	(0.27)	(0.01)	(0.00)	(0.00)					
		(0.95)	(0.24)	(0.00)	(0.48)					
Carhart	-0.1313	-0.0080	0.9699	0.1330	0.0250	-0.0088			0.91	51 (9 001)
	(0.12)	(0.62)	(0.00)	(0.00)	(0.00)	(0.25)				
		(1.00)	(0.21)	(0.00)	(0.49)	(0.84)				
Fama-French 5f	0.0021	-0.0150	0.9768	0.1355	0.0221		-0.0152	-0.0427	0.91	51 (8 835)
	(0.97)	(0.47)	(0.02)	(0.00)	(0.00)		(0.25)	(0.05)		
		(0.96)	(0.26)	(0.00)	(0.50)		(0.80)	(0.84)		

5.1.2 Benchmark-adjusted performance for index funds

Analogously to the preceding tests of active mutual funds relative to the market index in Table 5.1, Table 5.2 reports the estimates of equally-weighted index funds for the full sample and the subperiods. All performance models except the three- and four-factor model for the most recent subperiod indicate that passive index funds on average have generated a negative risk-adjusted return. However, the results are not significant for either of the test statistics across the models. While we cannot conclude based on this finding, the increased alpha in the most recent period harmonizes well with our hypothesized decrease in index fund costs and increased index fund efficiency discussed in Section 4.1.1.

The fact that the alphas are not significantly different from zero is surprising given the nature of expense ratios and tracking errors of index funds. It is in contrast to previous studies on index fund risk-adjusted performance from the U.S. (e.g. Larsen & Resnick (1998), Frino & Gallagher (2001), and Crane & Crotty (2018)). The finding could potentially be explained by the tracking error of index funds, which, on average, may have led index funds to generate higher before-cost returns than the benchmark they are tracking. The tracking error could therefore in theory cancel out the costs of index funds. Another potential explanation is tied to the choice of benchmark return. As discussed in Section 4.1.2, we use a combination of OBX and OSEFX as our market index. The latter is adjusted to meet specific diversification requirements that are not necessarily comparable with the benchmark tracked by index funds in our sample. Consequently, the monthly returns of index funds will deviate. For example, index funds tracking alternative market indices such as the OSEBX, which do not have constraints on individual stock weights, will outperform the OSEFX when the largest stocks do well. In particular, Equinor constitutes 25% of the OSEBX (Oslo Stock Exchange, 2020), a considerably larger share than the 10% cap for individual securities imposed in the OSEFX.

In Appendix D, we study the sensitivity of our chosen benchmark, estimating the factor models with five alternative market indices for index funds. The results suggest that the alpha of index funds is particularly sensitive to the choice of the benchmark index. The alpha changes from negative to significantly positive in the CAPM model for the F-B t-statistic when we replace our market index with OBX or OSEBX. The latter is even significant for the C-N t-statistic. The direction and significance of the index alpha do not change when we control for Fama-French and Carhart's additional factors, so the difference between the market indices seems to be robust across the models. Roll (1977) criticized the CAPM and argued against using the CAPM proxy as a benchmark for performance since it presupposed complete knowledge of the true market portfolio's composition. Roll's argument may explain why we find that index funds are not tracking the market portfolio more closely. Still, it may also be due to the costs and tracking error associated with managing an index fund. More importantly, as the theory defines the true market portfolio as unknown, we cannot know which market benchmark is the optimal choice. And, as Appendix D demonstrates, the choice of market benchmark does affect our results.

Tied to the benchmark index choice, we stress the poor ability of the performance models to explain the total variation of returns (formally, the R_2), which is 95% across the models for the full sample, as reported in Table 5.2. Between 1991 to 2005, the explained variance is somewhat lower (94 to 95%), while it is slightly higher (95 to 96%) for the 2006 to 2019 period. Appendix D shows that the R-squared increases to 97 to 98 % if we use OBX or OSEBX as the market benchmark for the full sample. As a comparison, Frino & Gallagher (2001) found the variation of returns between U.S. index funds and the S&P 500 index between 1991 and 1998 to be 99.9 percent. The inability of CAPM to more accurately explain the variation of index fund returns in our sample is striking, considering the objective of these funds.

5.1.3 Comparing the benchmark-adjusted performance for active and index funds

Even though active mutual funds and index funds demonstrate various risk-adjusted performance - contingent on the factor models, portfolio weights, and periods - we have not yet addressed the relative performance between the two. Table 5.3 presents the abnormal performance for both fund types and the p-values associated with a two-sample paired t-test on the difference in performance. Ultimately, the test statistics suggest whether the abnormal returns of active funds are significantly greater than index funds, or vice versa. Disregarding the results of an EW portfolio for medium- and large investors from CAPM, which suggests that active returns were significantly greater in the first subperiod at a 5% level, we cannot conclude that the abnormal returns are distinguishable. This is irrefutably driven by the high monthly variance of the abnormal returns. Since the abnormal return is the sum of alpha and the residuals, and the alpha is constant through all months for a fund, the variance stems from the latter. While the factors included in our models are among the most recognized in literature, there are other factors not included that could better explain the excess returns of the funds. As illustrated for the EW portfolio in Appendix VII, the abnormal returns have a wide dispersion that appears random within +/- 2% per month. These characteristics make it challenging to conclude with significance from the joint t-test. The results also notably suggest that the abnormal returns of active funds are greater in the first subperiod,

Table 5.2. Index fund performance. The table is explained in Table 5.1. The sample period is reported in each panel.

	AR	α	β_{MKT}	β_{SMB}	β_{HML}	β_{PR1YR}	β_{RMW}	β_{CMA}	adj R_2	N
Panel A: Sample 1991 - 2019 (EW)										
CAPM	-0.0179	-0.0226	0.9591						0.95	16 (2 268)
	(0.78)	(0.37)	(0.00)							
		(0.80)	(0.06)							
Fama-French 3f	-0.0046	-0.0076	0.9565	-0.0166	0.0457				0.95	16 (2 268)
	(0.94)	(0.84)	(0.00)	(0.32)	(0.00)					
		(1.00)	(0.06)	(0.56)	(0.12)					
Carhart	-0.0073	-0.0049	0.9550	-0.0203	0.0466	0.0075			0.95	16 (2 268)
	(0.91)	(0.88)	(0.00)	(0.24)	(0.00)	(0.40)				
		(0.99)	(0.06)	(0.50)	(0.11)	(0.74)				
Fama-French 5f	-0.0123	-0.0202	0.9559	-0.0157	0.0488		0.0345	0.0056	0.95	16 (2 268)
	(0.00)	(0.50)	(0.00)	(0.38)	(0.00)		(0.27)	(0.80)		
		(0.81)	(0.06)	(0.57)	(0.10)		(0.49)	(0.93)		
Panel B: Sample 1991 - 2005 (EW)										
	AR	**α**	β_{MKT}	β_{SMB}	β_{HML}	β_{PR1YR}	β_{RMW}	β_{CMA}	**adj R_2**	**N**
CAPM	-0.0523	-0.0232	0.9604						0.94	7 (876)
	(0.60)	(0.62)	(0.09)							
		(0.84)	(0.12)							
Fama-French 3f	-0.0746	-0.0380	0.9679	0.0191	0.0377				0.95	7 (876)
	(0.45)	(0.61)	(0.12)	(0.48)	(0.02)					
		(0.81)	(0.15)	(0.70)	(0.19)					
Carhart	-0.0636	-0.0354	0.9691	0.0196	0.0389	0.0053			0.95	7 (876)
	(0.53)	(0.62)	(0.08)	(0.47)	(0.01)	(0.65)				
		(0.82)	(0.17)	(0.69)	(0.18)	(0.94)				
Fama-French 5f	-0.0385	-0.0118	0.9667	0.0222	0.0383		-0.0442	-0.0220	0.95	7 (876)
	(0.00)	(0.85)	(0.10)	(0.45)	(0.01)		(0.37)	(0.26)		
		(0.90)	(0.14)	(0.64)	(0.20)		(0.77)	(0.79)		
Panel C: Sample 2006 - 2019 (EW)										
	AR	**α**	β_{MKT}	β_{SMB}	β_{HML}	β_{PR1YR}	β_{RMW}	β_{CMA}	**adj R_2**	**N**
CAPM	0.0154	-0.0103	0.9568						0.95	11 (1 331)
	(0.83)	(0.72)	(0.00)							
		(0.85)	(0.07)							
Fama-French 3f	0.0602	0.0281	0.9361	-0.0628	0.0464				0.96	11 (1 331)
	(0.38)	(0.43)	(0.00)	(0.01)	(0.01)					
		(0.81)	(0.03)	(0.13)	(0.16)					
Carhart	0.0450	0.0274	0.9353	-0.0671	0.0476	0.0112			0.96	11 (1 331)
	(0.50)	(0.22)	(0.00)	(0.01)	(0.00)	(0.36)				
		(0.84)	(0.03)	(0.11)	(0.13)	(0.64)				
Fama-French 5f	0.0057	-0.0176	0.9365	-0.0642	0.0541		0.0895	0.0424	0.96	11 (1 331)
	(0.00)	(0.51)	(0.00)	(0.01)	(0.00)		(0.00)	(0.21)		
		(0.79)	(0.02)	(0.11)	(0.10)		(0.21)	(0.62)		

Table 5.3. Comparing active and index fund abnormal returns. The table reports the p-values associated with a two-sample paired t-test of the abnormal returns (see Section 3.1.1 for details). Abnormal returns (AR) are reported in percentage (e.g. 0.0100 = 0.01%) and p-values are in parentheses per month. Index fund abnormal returns are EW for both EW and TNAV-W (they differ since the last observation of the returns is discarded when start-of-month TNAV is used for active funds). The sample period is reported in each panel.

Panel A: Sample 1991 - 2005

Model	Investor size	EW				TNAV-W			
		Active	Index	Highest	p-val	Active	Index	Highest	p-val
CAPM	Small	0.3061	-0.0523	Active	(0.04)	0.1133	-0.0730	Active	(0.15)
	Medium	0.3457	-0.0523	Active	(0.01)	0.1780	-0.0730	Active	(0.07)
	Large	0.3375	-0.0523	Active	(0.01)	0.1796	-0.0730	Active	(0.06)
	All	0.2442	-0.0523	Active	(0.04)	0.1795	-0.0730	Active	(0.06)
Fama-French 3f	Small	0.0321	-0.0746	Active	(0.47)	-0.0587	-0.0951	Active	(0.77)
	Medium	0.0927	-0.0746	Active	(0.22)	-0.0165	-0.0951	Active	(0.55)
	Large	0.0934	-0.0746	Active	(0.22)	-0.0122	-0.0951	Active	(0.52)
	All	-0.0054	-0.0746	Active	(0.59)	-0.0123	-0.0951	Active	(0.52)
Carhart	Small	0.0206	-0.0636	Active	(0.57)	-0.0518	-0.0878	Active	(0.77)
	Medium	0.0804	-0.0636	Active	(0.30)	-0.0204	-0.0878	Active	(0.60)
	Large	0.0822	-0.0636	Active	(0.29)	-0.0177	-0.0878	Active	(0.58)
	All	-0.0149	-0.0636	Active	(0.70)	-0.0178	-0.0878	Active	(0.58)
Fama-French 5f	Small	0.0751	-0.0385	Active	(0.44)	0.0157	-0.0587	Active	(0.55)
	Medium	0.1078	-0.0385	Active	(0.29)	0.0252	-0.0587	Active	(0.52)
	Large	0.1068	-0.0385	Active	(0.28)	0.0278	-0.0587	Active	(0.50)
	All	0.0066	-0.0385	Active	(0.72)	0.0277	-0.0587	Active	(0.50)

Panel B: Sample 2006 - 2019

Model	Investor size	EW				TNAV-W			
		Active	Index	Highest	p-val	Active	Index	Highest	p-val
CAPM	Small	0.0892	0.0154	Active	(0.48)	0.0150	0.0030	Active	(0.90)
	Medium	0.0776	0.0154	Active	(0.53)	-0.0145	0.0030	Index	(0.87)
	Large	0.0802	0.0154	Active	(0.49)	0.0032	0.0030	Active	(1.00)
	All	0.0743	0.0154	Active	(0.53)	0.0031	0.0030	Active	(1.00)
Fama-French 3f	Small	0.0140	0.0602	Index	(0.61)	-0.0528	0.0502	Index	(0.21)
	Medium	0.0121	0.0602	Index	(0.58)	-0.0858	0.0502	Index	(0.13)
	Large	0.0187	0.0602	Index	(0.61)	-0.0433	0.0502	Index	(0.22)
	All	0.0124	0.0602	Index	(0.56)	-0.0435	0.0502	Index	(0.22)
Carhart	Small	0.0440	0.0450	Index	(0.99)	-0.0252	0.0359	Index	(0.45)
	Medium	0.0399	0.0450	Index	(0.95)	-0.0484	0.0359	Index	(0.34)
	Large	0.0482	0.0450	Active	(0.97)	-0.0073	0.0359	Index	(0.57)
	All	0.0423	0.0450	Index	(0.97)	-0.0075	0.0359	Index	(0.57)
Fama-French 5f	Small	0.0228	0.0057	Active	(0.85)	-0.0431	-0.0051	Index	(0.63)
	Medium	0.0270	0.0057	Active	(0.80)	-0.0613	-0.0051	Index	(0.52)
	Large	0.0291	0.0057	Active	(0.77)	-0.0363	-0.0051	Index	(0.68)
	All	0.0229	0.0057	Active	(0.83)	-0.0363	-0.0051	Index	(0.68)

while it broadly shifts towards index funds in the most recent period. However, the difference is not significant using any conventional significance levels. Appendix D studies the sensitivity of the chosen benchmark and finds no significant differences between the abnormal returns of active and index funds for either of the five alternative market indices.

In the results presented so far, there has not been much agreement. For active funds, the test statistics of F-B suggest both positive and negative alphas, while we reject the null hypothesis when we include the uncertainty of the fund-level alpha estimations with the C-N statistics. For index funds, neither the F-B nor the C-N statistics allow us to reject the null, but the direction and significance of the findings shift when we apply alternative benchmark indices. Lehman and Modest (1985) early emphasized the sensitivity of mutual fund performance evaluation to benchmark choice, which is also evident in our sample. Cremers, Petajisto, and Zitzewitz (2012) find that alphas using traditional benchmark models are downwards biased, and the magnitude of the biases may differ for active and passive funds, suggesting that traditional benchmark models may be misleading. With the sensitivity of a) market benchmark, b) factor model, c) test statistics, and d) portfolio weights, we refrain from concluding whether investors are better off investing in Norwegian index funds or active funds based on the benchmark-adjusted performance. Consequently, we look elsewhere in the preceding for an answer.

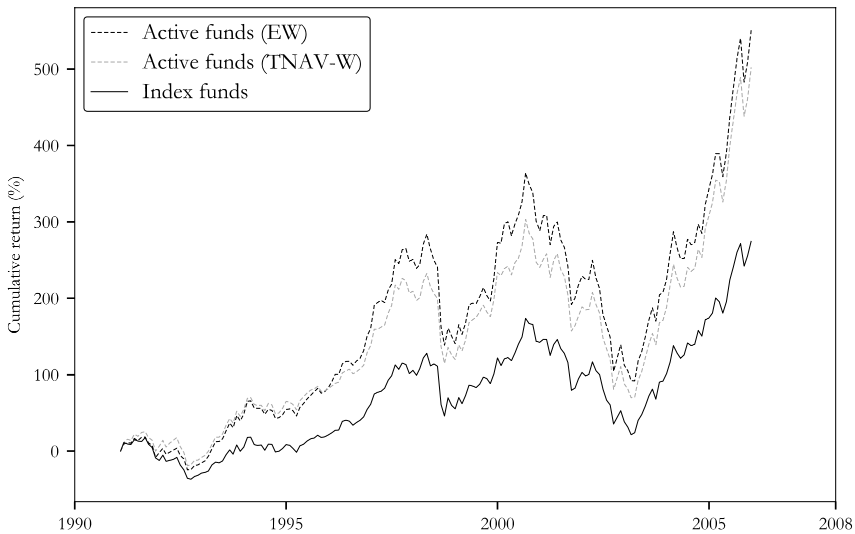
5.2 Net return performance

5.2.1 A simple comparison

From an investor perspective, the performance of active and index funds relative to the market index does not necessarily provide guidance on which fund type should be preferred over the other. Alpha returns do measure whether the fund types manage to outperform after adjusting for their factor exposure, but, at the end of the day, investors earn the net returns, not the alpha returns. This section thus compares the investment performance of active and index funds as measured by investors' net returns.

In Figure 5.1, the fund types are compared in terms of their cumulative net returns. The figure splits the returns of index funds and active funds (for the latter, both equally and value-weighted portfolios) into two subperiods. From 1991 to 2005, active funds yielded roughly twice the return of index funds, and the cumulative return of active funds was greater at all points in time from the first introduction of index funds. This observation could be explained by active mutual fund managers overweighting small-capitalization stocks as illustrated by the active fund exposure to the SMB factor in Appendix VI compared to the close-to-zero SMB exposure of index funds reported in Table 5.2. As we highlighted in Section 4.2, small-capitalization stocks did particularly well in the first period. This could also be explained by index funds having greater fund costs and lower efficiency in the first subperiod and by the small sample of index funds (and returns) for the majority of the sample period. In the most recent subperiod, the differences between active and index funds appear notably lesser than in the first subperiod. The investment alternatives seem remarkably correlated both in bull and bear market conditions, judged by their comovement pre- and post-financial crisis.

Figure 5.1. Comparing active and index fund performance (cumulative average returns). The plots include the cumulative returns for an EW and a TNAV-W portfolio of active funds and an EW portfolio of index funds. The sample period is from 1991 to 2005 and 2006 to 2019.



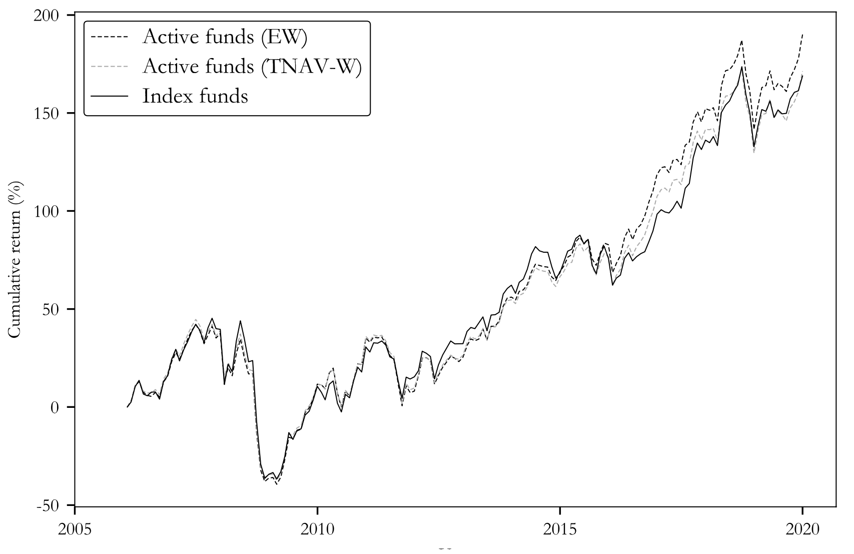


Figure 5.2. Comparing active and index fund performance (net returns). The plot includes all fund classes, not only distinct funds, and is not limited to an EW nor TNAV-W portfolio. The sample period is 1981 to 2019.

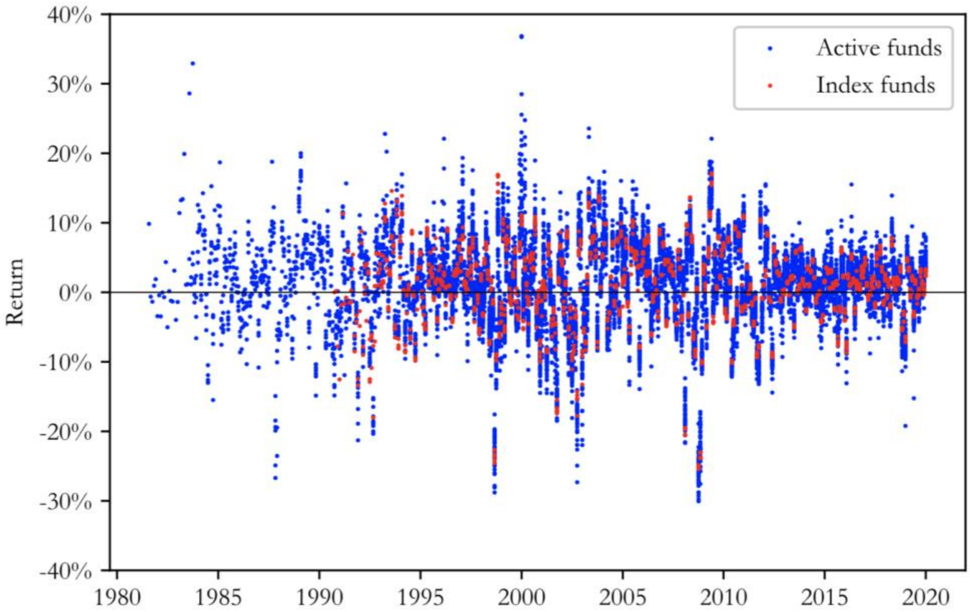


Figure 5.2 illustrates the net returns of passive index funds and active mutual funds. Visually, active funds appear more volatile in comparison to their counterpart, which is to be expected given their higher idiosyncratic risk. Also notable is the greater net return volatility relative to the abnormal return volatility, which is not surprising given the factors in the aforementioned performance models that ought to explain volatility by capturing the risk exposure (see Appendix VII for plots of the abnormal returns).

5.2.2 Stochastic dominance

In this section, the distributions of the net returns are analyzed, not solely looking at the average effects. Figure 5.3 plots the cumulative distribution function (CDF) of the net returns for index funds and active mutual funds. In the grey zones, the CDF of active mutual funds is equal or to the left of index funds, and, therefore, have a higher probability of lower returns. As the grey zones do not persist throughout the distribution, the distributions cross, and the condition for first-order stochastic dominance is violated. The above holds for both subperiods.

Figure 5.3. Comparing the net return distributions of active and index funds. The top chart illustrates the PDFs, the middle chart illustrates the CDFs, and the lower chart illustrates the S- functions (the integral of the CDFs). Grey areas indicate that index fund returns are preferred over active fund returns for the corresponding return interval. The opposite is true for the white areas. The sample period is from 1991 to 2005 and 2006 to 2019 for the charts on the left and right sides, respectively.

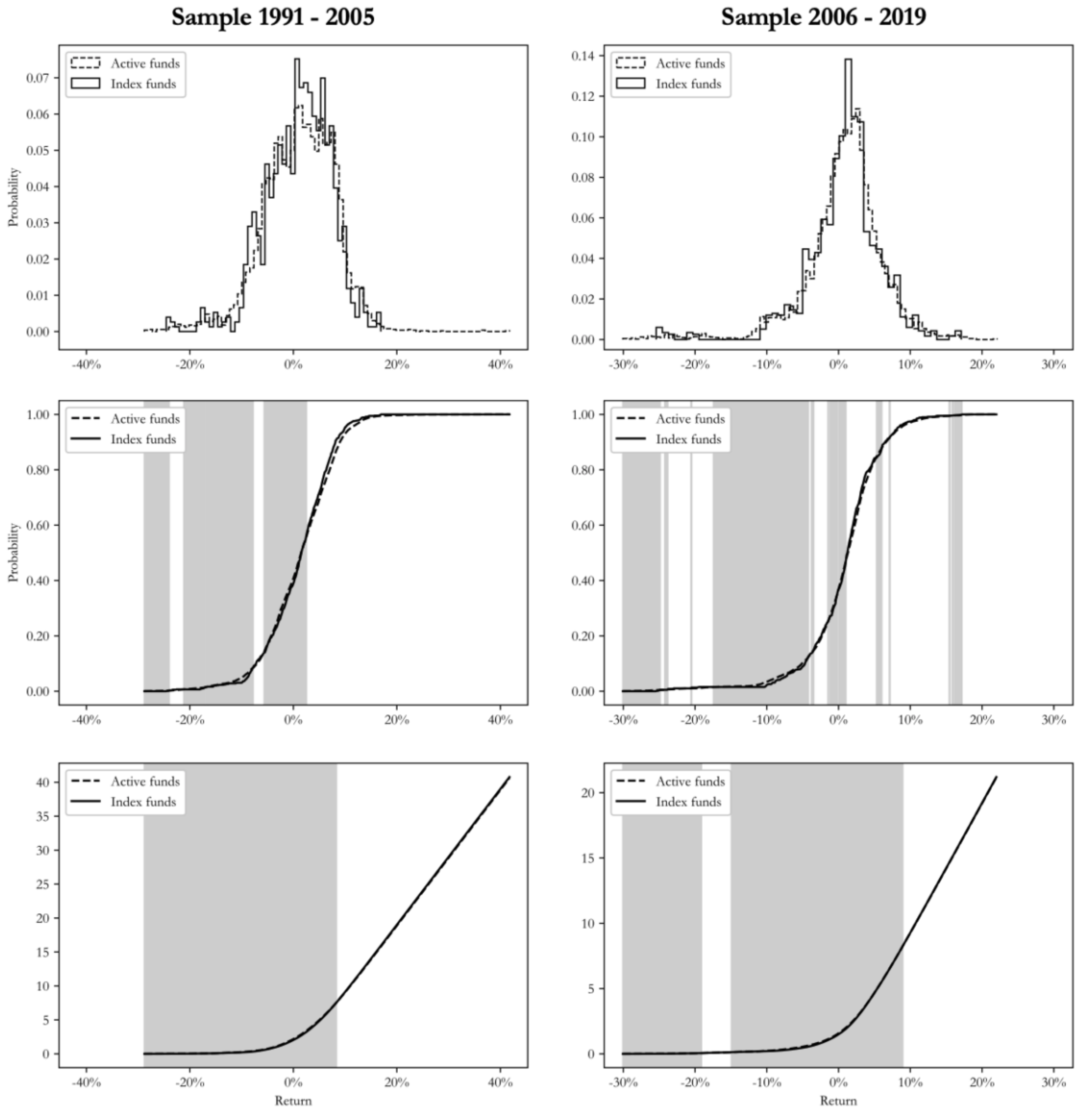


Figure 5.3 also plots the S-functions of index funds and active mutual funds, which is the integral of their respective CDFs. If, for instance, the S-function of active mutual funds is to the left of index funds at some point in the distribution, it means that the active mutual funds have a greater integral of the CDF at that point. Similar to the criterion of first-order dominance, a distribution is preferred if the S-function is equal or to the right at all points. As neither grey nor white zones persist throughout the distribution for the entire sample, the condition of second-order dominance is also violated for both subsamples. The differences between the distributions seem modest (the graphs are barely separable by visual inspection), which is somewhat surprising given the characteristics of the fund types. As our sample represents all Norwegian mutual fund returns between 1991 and 2019, we could argue that the sample represents the "true" distribution, bound to that time period. Economically, this implies that we cannot generalize, ex-post, whether risk-averse investors should prefer active or passive funds, as the criterion for second-order stochastic dominance is violated. The intuition behind this statement is that the superior performance of some active funds in the right tail does not compensate for the poor performance of other funds in the left tail.

However, the aforementioned is not sufficient to formalize whether investors, risk-averse or not, should prefer either alternative at present. From a historical perspective, our sample and time period are confined, and the distribution in our sample will likely deviate from the distribution in periods to come. For instance, while we can sensibly expect that active and index funds will behave in a similar manner in the future, it is conceivable that some fund characteristics may change. An example is increased competition that would entail a greater marginal cost reduction of active funds as the index fund costs are already closer to zero. In other words, as the empirical distribution is extracted from a sample of the past, which need not be the true distribution of returns, we must perform a statistical test to determine whether we can draw inferences on which fund type is preferred based on our sample.

Crotty and Crane (2018) tested for stochastic dominance and followed the procedure described by Barret and Donald (2003). We follow a similar approach to test for stochastic dominance; as the critical values depend on our sample, we simulate the p-values as described in Section 3.2.3. In Appendix VIII, we report the results of stochastic dominance tests of first and second-order on the alphas and t(α) for all the factor models which is the same approach Crotty and Crane used. Analogous to the benchmark-adjusted performance in Section 5.1, the results are sensitive to the choice of factor model. For instance, we reject the null that the index alpha and t(α) FSD active on a 5% significance level for CAPM between 2006 and 2019, but not for any other factor model nor subperiod. None of the factor models reject that index SSD active. We should be careful in interpreting the results as the tests lack power; there are only 7 and 11 index observations in the two subperiods, respectively. Considering that the evidence for alpha and t(α) tests are weak and the tests lack power, that the alphas of traditional benchmark models are downward biased as found by Cremers, Petajisto and Zitzewitz (2012), that the magnitude of the biases may differ between active and index funds and that we want to focus on the investor perspective, we instead focus on the net return distributions in the below. Stochastic dominance tests on raw distributions are not a new concept in statistics and are, for instance, applied by Barret and Donald for the distribution of income and Cho, Linton, and Whang (2007) for stock returns to study Monday effects.

Table 5.4 reports the results for first-order stochastic dominance (FSD). We cannot reject the null that active funds FSD index for any investor group at any conventional significance level. We can, however, reject the null that index FSD active on a 5% significance level for all investment groups in the first subperiod. In the second subperiod, we can only reject the null for large investors. Since we reject that index FSD active but not the other way around, we, in fact, find evidence that active funds first-order stochastically dominate index funds in the mentioned cases. Since FSD is a sufficient condition for second-order stochastic dominance (SSD), SSD follows from this result (see Appendix G for the SSD results and a discussion of the robustness of our test). Active funds appear to be a better choice than index funds in terms of their net return probability density function in the first subperiod and for large investors in the second subperiod.

In practical terms, these results have supreme importance. The FSD results suggest that anyone who prefers a higher return to a lower one should prefer a random active fund, regardless of the investor's utility function (or risk appetite). We need not establish the assumption of risk-aversion, but simply that more is preferred to less, which is a sensible assumption for any (partial) rational investor. Looking at previous studies on mutual funds, one would expect active funds to be over-represented in both sides of the distribution. Surprisingly, active funds in our sample show only marginally weaker performance in the far-left tail, while providing greater or equal probabilities of higher returns in the rest of the distribution. Graphically, this is most striking in the far-right tail. The interpretation of the stochastic test is thus that the slight underperformance in the left tail, and other parts of the distribution, does not occur with a 95% confidence in the true distribution of Norwegian mutual funds. Hence, investors can expect greater returns from holding a random active fund than a random index fund regardless of their appetite for risk, and should, therefore, always do so in the mentioned cases. Note that the results do not suggest that all investors have been better off with active funds. We return to this discussion in the simulation studies in Section 5.3, where we quantify what the first-order stochastic dominance has meant for investors.

Table 5.4. Testing for stochastic dominance. The tables report the p-values associated with the null hypothesis described in the columns. For instance, the top left p-value of 0.0140 is the p-value for the null hypothesis "index funds first-order stochastic dominates active funds". The p-values are simulated 500 times (see Section 3.2.3 for details). Appendix G includes results for SSD and a discussion of the robustness of our implementation. The sample period is reported in each panel.

Panel A: Sample 1991 - 2005		
Investor size	**First-order stochastic dominance**	
	Index FSD Active	**Active FSD Index**
Small	0.0140	0.4860
Medium	0.0140	0.5480
Large	0.0140	0.5820
All	0.0500	0.2920
Panel B: Sample 2006 - 2019		
Investor size	**First-order stochastic dominance**	
	Index FSD Active	**Active FSD Index**
Small	0.1040	0.5440
Medium	0.0900	0.5460
Large	0.0420	0.4740
All	0.0580	0.4840

Note also that the distribution of the respective fund types does not incorporate redemption and subscription fees, which is difficult to take into account for single-month returns. The implication could be an upwards bias in the favor of active investment as active funds normally have higher redemption and subscription fees. We revisit these costs in the simulation studies in Section 5.3.

In the mutual fund literature, most studies do not include a stochastic dominance test. After Crane and Crotty's 2018 paper, it may become more widespread, but until then we have few mutual fund studies to compare our results with. Crane and Crotty focus their attention on mutual fund manager skills, which is why they test the distribution of the alpha returns (and the t_{α} distributions). Based on U.S. mutual funds between 1995 and 2013, they conclude that no risk-averse investor should prefer a random active fund over a random index fund in terms of alpha returns. In comparison, we study the returns earned by investors and find evidence of the opposite, but the differing focuses make the results difficult, if not impossible, to compare. In Appendix VIII, we report test results for the alpha returns and the t(α) and find evidence for active funds FSD index funds for the CAPM in the second subperiod.

5.3 Cumulative net return performance

Based on the criteria of stochastic dominance, any investor and large investors (in the two subperiods, respectively) should always prefer a random draw from the Norwegian active fund distribution to a random draw from the index fund distribution. Next, through simulations, we will quantify the financial implications of holding active funds in favor of index funds. Investors tend to hold funds over several months, and we, therefore, simulate the cumulative returns of holding periods from 1 to 5 year(s) rather than solely focusing on single-month returns which we have done so far. A 5-year holding period is the typical minimum holding period communicated by active funds and media for when investors should choose active funds over index funds. We argue that if active management provides superior returns for a 5-year period, they will also do so for longer horizons, which is why the simulations are limited to a 5-year window (this is also driven by data considerations; a 6-year holding period 'discards' the last 6 years of data, and so on for longer horizons).

In the first simulation, referred to as "Simulation A", we simulate the cumulative returns of an EW and TNAV-W portfolio to compare the performance of active and index funds. In the second simulation, referred to as "Simulation B", we take a similar approach, but now drawing a random active and a random index fund at each starting month t and comparing them pairwise. We elaborate on our simulation methodology in Appendix E and explain the charts in Appendix F.

Table 5.5 presents a summary of Simulation A. The table reports the probability of a positive difference in cumulative returns (i.e., the cumulative return generated by the active fund portfolio minus the cumulative return generated by the index fund portfolio), summary statistics for the difference in cumulative returns, and the difference in the intra-holding period Sharpe ratios. For the EW portfolio, the probability of active returns being greater than index returns is more than 50% for all but two holding periods (and investment boundaries), and consistently above 60% for the first subperiod. The two exceptions are both in the most recent subperiod, for the All investor group over a 12 month holding period and for small investors over a 24 month holding period. The mean return difference was ~5.0% per year for the first subperiod and mostly between 0.5% and 1.0% per year in the second subperiod, with an annualized mean difference in the Sharpe ratios of around 5.0% and 2.5% for the two periods, respectively, for *all* holding periods. For the TNAV-

W portfolio, the results are less consistent and tend to favor index funds. The TNAV-W result may suggest that the larger funds (i.e. the most popular funds) are the worst performers, but it may also be due to the quality of the NAV data that may introduce biases in the results (see Section 4.3 for a discussion of the NAV data).

It is striking that active funds tend to provide superior returns and a consistently greater Sharpe ratio even for holding periods below 5 years, on average, which contradicts the rhetoric of the Norwegian mutual fund industry and the media. The examination of different investment boundaries shows that there are distinct differences between the groups. While we have established that smaller investors may have had a disadvantage to medium and large investors in terms of alpha returns in the first subperiod, the differences become clear when comparing the various holding periods in our simulations. Smaller investors tended to have a lower probability, mean return, and Sharpe ratio compared to larger investors for the EW portfolios.

It may be hard to interpret the difference in cumulative returns in Table 5.5, which is why we plot the difference for a holding period of 5 years in Figure 5.4 (Appendix IX includes the shorter holding periods). It becomes apparent that the cumulative returns of the active fund portfolios are far greater than the index fund portfolios in most months, while when the opposite is true, the active funds' cumulative returns are only slightly below those of index funds. Practically, it means that when active funds "succeed," they generate returns substantially superior to index funds, and, when they "fail," the returns are only slightly worse. The above harmonizes well with the tests for stochastic dominance and confirms Norwegian active mutual fund managers' abilities to consistently beat their passive counterpart, on average.

Table 5.5. Summary of EW and TNAV-W portfolios' cumulative returns for various holding periods. The table shows summary statistics for Simulation A based on the pairwise difference in cumulative net returns between active and index funds for holding periods of 12, 24, 36, 48, and 60 months. The probability of active returns being higher than index returns is the fraction of the pairwise observations where active has a higher cumulative net return. Sharpe is the average difference of the Sharpe ratios of the intra-holding period returns. Numbers are reported in percentages (e.g. 0.0100 = 0.01%) as annualized (geometric) means, medians, standard deviations and Sharpe ratios. Each panel includes EW and TNAV-W for active funds compared to an EW portfolio of index funds. The number of cumulative return observations range from 120 (108) for 60 months to 168 (156) for 12 months for the 1991 to 2005 (2006 to 2019) period. The sample period is reported in each panel.

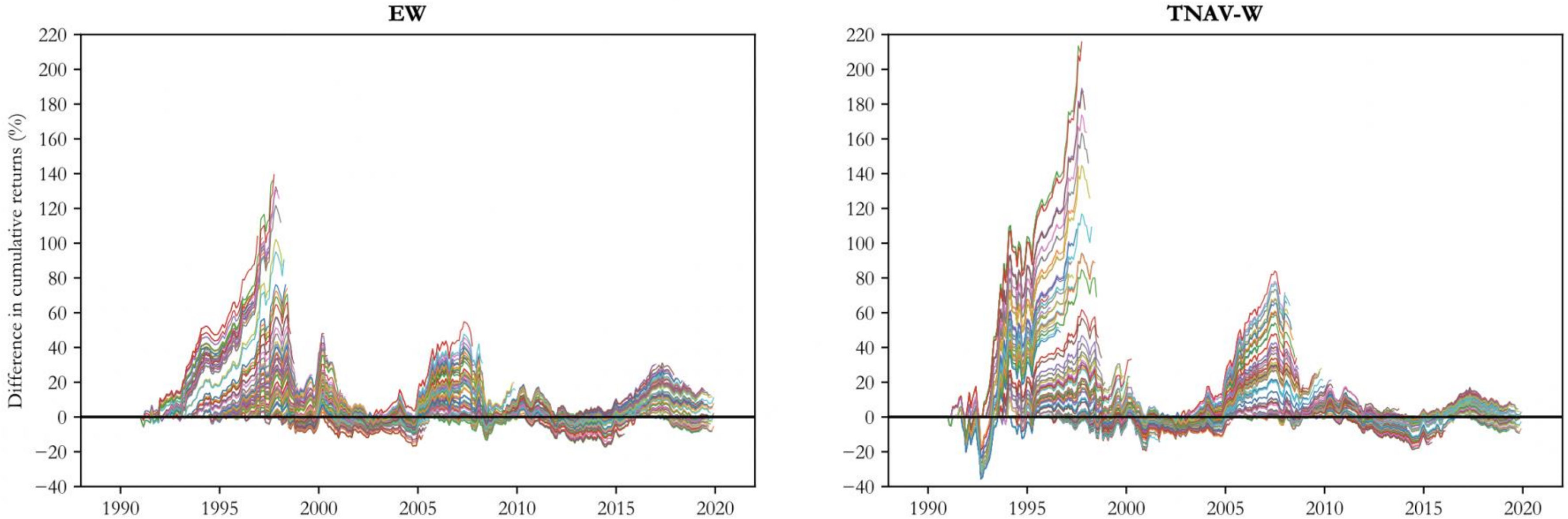
Panel A: Sample 1991 - 2005

Holding period (months)	Investor size	EW					TNAVW				
		Probability of active > index	Difference in cumulative returns			Sharpe	Probability of active > index	Difference in cumulative returns			Sharpe
			Mean	Median	Std			Mean	Median	Std	
12	Small	68.45 %	5.03 %	5.64 %	7.61 %	5.53 %	52.38 %	1.98 %	0.85 %	7.04 %	2.30 %
	Medium	70.83 %	5.77 %	6.05 %	7.77 %	6.54 %	58.33 %	3.33 %	2.05 %	7.00 %	4.01 %
	Large	72.62 %	5.40 %	5.39 %	7.20 %	6.43 %	57.74 %	3.31 %	2.13 %	6.88 %	4.11 %
	All	68.45 %	4.32 %	4.65 %	7.20 %	4.63 %	57.74 %	3.31 %	2.13 %	6.88 %	4.11 %
24	Small	74.36 %	4.93 %	6.00 %	7.66 %	5.28 %	44.87 %	1.42 %	-0.61 %	8.56 %	1.28 %
	Medium	73.72 %	5.73 %	5.74 %	8.73 %	6.26 %	54.49 %	2.76 %	0.30 %	8.42 %	2.74 %
	Large	78.21 %	5.38 %	4.95 %	8.28 %	6.04 %	55.77 %	2.77 %	0.40 %	8.21 %	2.82 %
	All	68.59 %	4.17 %	3.53 %	8.21 %	4.31 %	55.77 %	2.77 %	0.39 %	8.21 %	2.82 %
36	Small	77.08 %	4.98 %	3.66 %	9.70 %	5.58 %	38.89 %	1.09 %	-1.07 %	9.62 %	1.21 %
	Medium	81.94 %	5.64 %	3.81 %	10.47 %	6.30 %	52.78 %	2.15 %	0.18 %	9.49 %	2.40 %
	Large	83.33 %	5.29 %	3.04 %	9.93 %	6.00 %	54.86 %	2.24 %	0.21 %	9.28 %	2.50 %
	All	65.28 %	3.98 %	1.55 %	9.77 %	4.29 %	54.86 %	2.24 %	0.21 %	9.28 %	2.50 %
48	Small	78.03 %	5.16 %	4.48 %	12.25 %	5.47 %	34.85 %	0.60 %	-1.44 %	9.17 %	1.16 %
	Medium	86.36 %	5.79 %	3.85 %	13.16 %	6.04 %	43.94 %	1.59 %	-0.28 %	8.82 %	2.24 %
	Large	83.33 %	5.39 %	3.48 %	12.39 %	5.72 %	46.21 %	1.72 %	-0.33 %	8.68 %	2.34 %
	All	63.64 %	3.96 %	1.87 %	12.07 %	4.01 %	46.21 %	1.72 %	-0.33 %	8.68 %	2.34 %
60	Small	77.50 %	5.84 %	4.39 %	16.28 %	5.04 %	29.17 %	0.30 %	-1.47 %	9.63 %	0.65 %
	Medium	88.33 %	6.60 %	4.18 %	18.52 %	5.61 %	42.50 %	1.55 %	-0.39 %	9.47 %	1.75 %
	Large	89.17 %	6.08 %	3.55 %	17.34 %	5.26 %	43.33 %	1.68 %	-0.31 %	9.46 %	1.85 %
	All	65.83 %	4.51 %	1.72 %	16.26 %	3.57 %	43.33 %	1.68 %	-0.31 %	9.46 %	1.85 %

Table 5.5 (Continued).

		Panel B: Sample 2006 - 2019									
		EW					**TNAVW**				
Holding period (months)	**Investor size**	**Probability of active > index**	**Difference in cumulative returns**			**Sharpe**	**Probability of active > index**	**Difference in cumulative returns**			**Sharpe**
			Mean	**Median**	**Std**			**Mean**	**Median**	**Std**	
12	Small	50.64 %	0.40 %	0.05 %	5.44 %	2.96 %	45.51 %	-0.11 %	-0.36 %	4.76 %	2.31 %
	Medium	51.28 %	0.48 %	0.29 %	4.87 %	3.26 %	42.95 %	-0.46 %	-0.70 %	4.80 %	2.29 %
	Large	50.64 %	0.65 %	0.07 %	4.21 %	3.46 %	45.51 %	-0.06 %	-0.61 %	3.86 %	2.15 %
	All	48.08 %	0.52 %	-0.20 %	4.24 %	3.21 %	45.51 %	-0.06 %	-0.61 %	3.86 %	2.15 %
24	Small	45.14 %	0.31 %	-0.31 %	5.49 %	2.19 %	39.58 %	-0.18 %	-0.61 %	4.81 %	1.15 %
	Medium	50.69 %	0.49 %	0.08 %	5.15 %	2.49 %	42.36 %	-0.48 %	-0.70 %	5.31 %	1.01 %
	Large	54.86 %	0.70 %	0.25 %	4.49 %	2.60 %	45.83 %	-0.05 %	-0.22 %	4.10 %	1.01 %
	All	54.17 %	0.60 %	0.28 %	4.52 %	2.39 %	45.83 %	-0.05 %	-0.22 %	4.11 %	1.01 %
36	Small	59.09 %	0.51 %	0.48 %	5.07 %	2.70 %	49.24 %	0.01 %	-0.05 %	4.39 %	1.47 %
	Medium	60.61 %	0.72 %	0.65 %	4.88 %	2.94 %	43.18 %	-0.30 %	-0.43 %	5.70 %	1.33 %
	Large	65.15 %	0.93 %	0.87 %	4.30 %	2.92 %	56.06 %	0.11 %	0.22 %	3.97 %	1.24 %
	All	63.64 %	0.84 %	0.83 %	4.27 %	2.73 %	56.06 %	0.11 %	0.22 %	3.97 %	1.23 %
48	Small	55.83 %	0.66 %	0.52 %	5.38 %	2.99 %	47.50 %	0.08 %	-0.13 %	4.72 %	1.64 %
	Medium	61.67 %	0.93 %	0.83 %	5.05 %	3.22 %	46.67 %	-0.23 %	-0.20 %	6.15 %	1.53 %
	Large	70.00 %	1.17 %	0.81 %	4.39 %	3.18 %	54.17 %	0.20 %	0.24 %	3.99 %	1.48 %
	All	67.50 %	1.06 %	0.83 %	4.32 %	2.99 %	54.17 %	0.20 %	0.25 %	3.99 %	1.48 %
60	Small	50.00 %	0.51 %	0.01 %	5.14 %	2.45 %	41.67 %	-0.07 %	-0.45 %	4.63 %	1.28 %
	Medium	56.48 %	0.88 %	0.33 %	4.90 %	2.79 %	38.89 %	-0.38 %	-0.99 %	5.97 %	1.18 %
	Large	66.67 %	1.21 %	0.82 %	4.31 %	2.90 %	50.00 %	0.12 %	-0.06 %	3.79 %	1.27 %
	All	64.81 %	1.09 %	0.65 %	4.22 %	2.72 %	50.00 %	0.12 %	-0.05 %	3.79 %	1.27 %

Figure 5.4. Plot of the difference between EW and TNAV-W portfolios' cumulative returns for a holding period of 60 months (5 years) for all months. The charts show the pairwise difference between the cumulative return from holding active funds versus index funds in Simulation A. The left chart shows the equally weighted (EW) portfolios, while the right chart weighs the funds according to their monthly TNAV. All cumulative returns are plotted, i.e. one line for each month as the starting month for the holding period. The sample period is 1991 to 2019. The chart is carefully explained in Appendix F and other holding periods are reported in Appendix IX.



We now divert from the assumption that the investor holds either a portfolio of active or passive funds. In the market, we assume that investors typically pick one or a few mutual fund(s). Thus, by adjusting the simulation to incorporate the assumption, namely that investors choose between a random active fund and a random index fund, we ought to capture the wide variety of returns that the investors have experienced during our sample period. The simulation described thus provides a robustness check for the results and a more detailed discussion of the differences. Table 5.6 presents a summary of Simulation B. Again, the probability of active funds generating greater cumulative returns than index funds is higher for all holding periods and investment boundaries (the only exception is small investors over a 24 month holding period between 2005 and 2019). The performance of active funds is most dominant from 1991 to 2005, where the majority of holding periods and investment boundaries favor active investment with more than 60% probability and an annual cumulative return difference ranging between 3.32% and 7.88%. In the most recent subperiod, the performance is less concise, with probabilities closer to (but still above) 50%. The annualized mean difference in cumulative returns is still positive, between 0.63% and 1.28%, for *all* holding periods. The Sharpe ratio difference suggests that active funds have yielded a greater intra-holding period return per unit of risk across all periods, ranging from 1.50% to 4.84% in the first sample period and 0.81% to 1.99% in the last, on average, per year. The t-tests (reported to the right of the Sharpe ratios) show that we reject that the index funds have a significantly higher intra-holding period return than active funds at a rate notably higher than the 5% (which is expected by chance at the 5% significance level we use). In the first subperiod, the rejection rate is above 10% across all holding periods and, in the second period, it varies between 5.94% and 9.55%. When testing the other way around, the rejection rates mostly fall much below 5%, providing additional evidence that active funds have been preferable over index funds. It is also somewhat surprising that the probability of generating excess returns does not necessarily increase in tandem with the holding period, as often claimed by active mutual fund managers, although this appears to be the trend.

Figure 5.5 illustrates the results presented in Table 5.6 for a holding period of 5 years. As in the previous simulation, the cumulative performance of active funds is greater than index funds throughout the sample, on average, as illustrated by the thick black line. Contrasting the return dispersion of mutual fund portfolios, we now observe that picking the “wrong” active fund(s) can generate a considerable loss relative to choosing an index fund. For instance, picking the worst active fund prior to the financial crisis would have yielded a net negative cumulative return of approximately 140% (i.e. the randomly drawn index fund generated a 140% higher cumulative return than the randomly paired active fund), which is not by any means trivial. Similarly, the best pick of an active fund generated a gain of more than 500%. The results suggest that investors could mitigate the potential downside through diversification, instead of picking a single mutual fund at random - an insight that may help fund investors to gauge the risk and return tradeoff. Appendix X includes the same chart for holding periods between 12 and 48 months and demonstrates that the time-varying probability of active being best has consistently been above 50% for all holding periods.

The analysis presented in Table 5.6 and Figure 5.5 does not include redemption and subscription fees. We analyze the effect of introducing such costs in Appendix H (with the best data on fees available to us) and find that these fees affect the results marginally in favor of index funds, but not enough to change the conclusion; the overall results of the simulations do unquestionably favor active funds to index funds in Norway.

Table 5.6. Summary of simulated portfolios' cumulative net returns for various holding periods. The table shows summary statistics for Simulation B based on the difference in cumulative net returns between randomly paired active and index funds for holding periods of 12, 24, 36, 48, and 60 months. Each month is simulated 100 times (126 000 simulations in total). The probability of active returns being higher than index returns is the fraction of the pairwise simulations where active has a higher cumulative net return. Numbers are reported in percentages (e.g. 0.0100 = 0.01%) as annualized (geometric) means, medians, standard deviations and Sharpe ratios. Sharpe is the average difference of the Sharpe ratios of the intra-holding period returns. Index < Active and Active > Index reports the fraction of the t-test of the simulated intra-holding period net return difference rejected at a 5% level with the column name as the null hypothesis. The sample period is reported in each panel. See Appendix E for further details of the simulation procedure and the summary statistics.

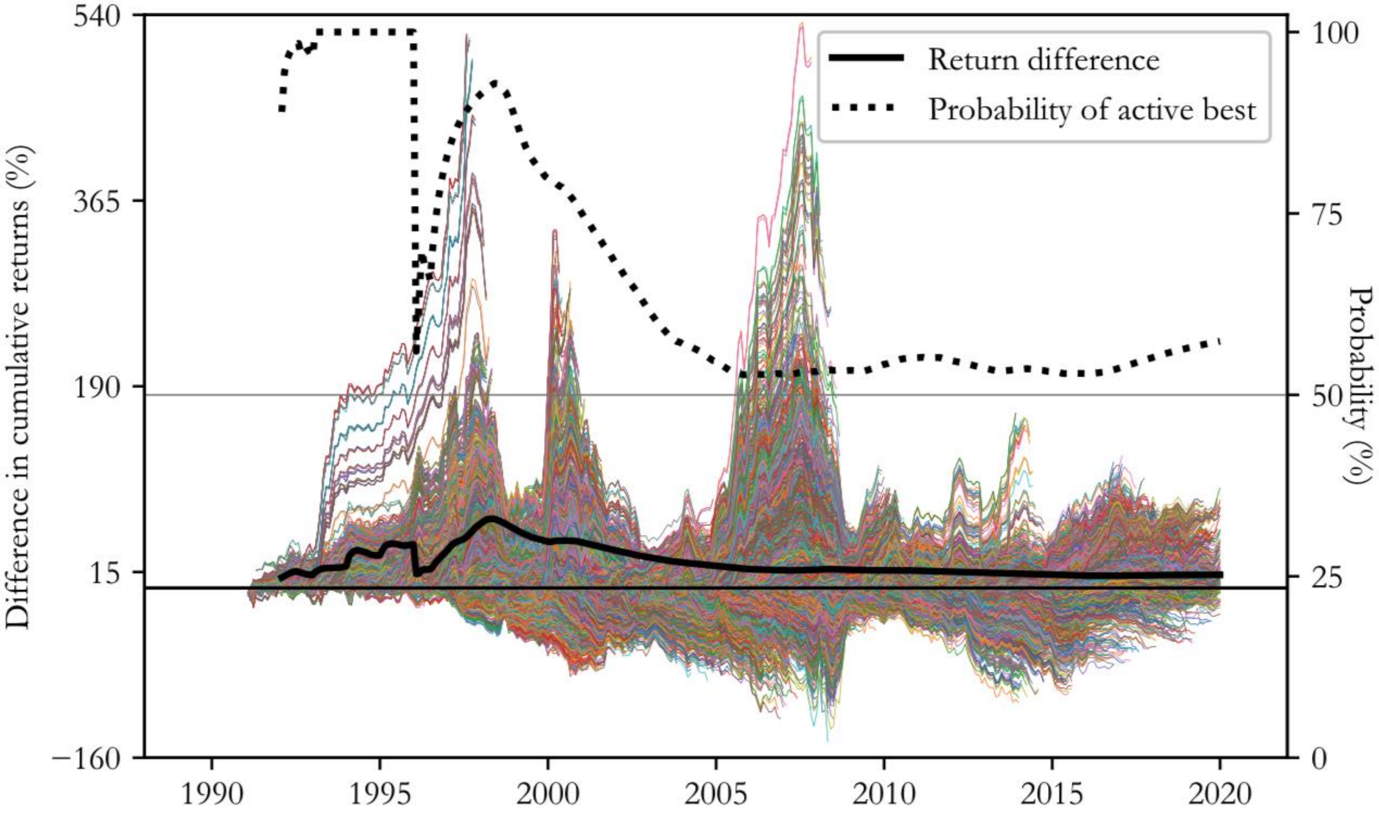
Panel A: Sample 1991 - 2005

Holding period (months)	Investor size	Probability of active > index	Difference in cumulative returns			Intra-holding period		
			Mean	Median	Std	Sharpe	Index < Active	Index > Active
12	Small	64.21 %	6.41 %	3.79 %	15.96 %	3.72 %	2.12 %	10.08 %
	Medium	66.51 %	7.88 %	4.11 %	19.79 %	4.71 %	1.39 %	10.63 %
	Large	67.26 %	7.38 %	4.14 %	18.07 %	4.74 %	1.61 %	10.71 %
	All	63.24 %	5.61 %	3.08 %	16.62 %	3.17 %	3.01 %	10.82 %
24	Small	64.01 %	5.51 %	3.38 %	16.40 %	4.07 %	2.51 %	10.60 %
	Medium	66.60 %	6.18 %	3.97 %	17.75 %	4.60 %	1.42 %	11.94 %
	Large	67.26 %	6.26 %	4.19 %	17.68 %	4.84 %	1.58 %	12.01 %
	All	63.24 %	4.40 %	2.96 %	16.18 %	3.09 %	3.21 %	11.47 %
36	Small	60.69 %	4.92 %	2.94 %	18.03 %	4.26 %	2.02 %	12.51 %
	Medium	65.15 %	5.73 %	3.72 %	19.64 %	4.69 %	1.26 %	13.77 %
	Large	64.37 %	5.51 %	3.47 %	19.33 %	4.57 %	1.37 %	13.85 %
	All	58.88 %	3.79 %	2.14 %	17.90 %	2.99 %	3.89 %	13.32 %
48	Small	60.63 %	4.88 %	1.99 %	19.93 %	4.13 %	1.94 %	13.63 %
	Medium	64.71 %	5.34 %	2.65 %	21.11 %	4.33 %	1.17 %	15.75 %
	Large	64.79 %	5.34 %	2.56 %	21.52 %	4.34 %	1.05 %	15.75 %
	All	57.27 %	3.44 %	1.35 %	19.72 %	2.66 %	4.74 %	14.27 %
60	Small	58.98 %	5.11 %	1.63 %	27.57 %	3.14 %	1.77 %	12.56 %
	Medium	64.02 %	5.68 %	2.07 %	30.16 %	3.35 %	0.71 %	13.38 %
	Large	62.77 %	5.48 %	1.97 %	29.48 %	3.21 %	0.88 %	13.60 %
	All	53.28 %	3.32 %	0.54 %	26.14 %	1.50 %	5.37 %	11.25 %

Table 5.6 (Continued).

Panel B: Sample 2006 - 2019								
Holding period (months)	**Investor size**	**Probability of active > index**	**Difference in cumulative returns**			**Intra-holding period**		
			Mean	**Median**	**Std**	**Sharpe**	**Index < Active**	**Index > Active**
12	Small	51.44 %	0.89 %	0.23 %	9.68 %	1.98 %	5.44 %	6.15 %
	Medium	51.25 %	0.75 %	0.20 %	8.84 %	1.63 %	5.12 %	6.10 %
	Large	52.61 %	0.84 %	0.41 %	8.51 %	1.99 %	4.37 %	6.37 %
	All	51.33 %	0.63 %	0.23 %	8.66 %	1.35 %	4.76 %	5.94 %
24	Small	49.46 %	0.63 %	-0.08 %	9.74 %	0.84 %	4.59 %	6.54 %
	Medium	50.88 %	0.71 %	0.11 %	9.11 %	0.93 %	4.15 %	6.91 %
	Large	52.72 %	0.87 %	0.34 %	8.84 %	1.12 %	3.90 %	7.79 %
	All	52.53 %	0.72 %	0.28 %	8.88 %	0.90 %	3.89 %	7.19 %
36	Small	50.70 %	0.82 %	0.10 %	10.79 %	1.29 %	3.70 %	7.94 %
	Medium	54.42 %	0.93 %	0.52 %	9.96 %	1.50 %	3.21 %	8.49 %
	Large	56.89 %	1.08 %	0.70 %	9.64 %	1.53 %	2.73 %	9.55 %
	All	56.88 %	1.06 %	0.68 %	9.80 %	1.50 %	2.88 %	9.21 %
48	Small	54.55 %	1.03 %	0.45 %	11.81 %	1.53 %	3.92 %	7.87 %
	Medium	57.81 %	1.11 %	0.75 %	10.90 %	1.66 %	3.37 %	8.08 %
	Large	60.54 %	1.25 %	0.96 %	10.39 %	1.72 %	2.91 %	9.12 %
	All	59.18 %	1.17 %	0.82 %	10.38 %	1.65 %	2.78 %	9.08 %
60	Small	53.24 %	0.86 %	0.31 %	13.35 %	0.81 %	4.88 %	7.75 %
	Medium	57.71 %	0.95 %	0.67 %	11.86 %	1.17 %	3.47 %	7.88 %
	Large	62.20 %	1.28 %	0.98 %	11.50 %	1.44 %	2.96 %	8.81 %
	All	60.23 %	1.18 %	0.91 %	11.39 %	1.31 %	3.18 %	8.37 %

Figure 5.5. Plot of the difference in simulated cumulative returns for a holding period of 60 months (5 years) for all months. The chart shows the pairwise difference between the cumulative return from holding active funds versus index funds for the simulated portfolios in Simulation B. All cumulative returns are plotted, i.e. one line for each month as a starting month for the holding period. The return difference (on the left-hand y-axis) and the probability of active outperforming index (on the right-hand y-axis) are plotted for the last month of the holding period, with a corresponding horizontal line for 0% cumulative return difference and 50% probability. The sample period is 1991 to 2019. The chart is carefully explained in Appendix F and other holding periods are reported in Appendix X.



5.4 Discussion

While the risk-adjusted performance of traditional benchmark models is inconclusive, our analysis of net returns provides unequivocal evidence in favor of active funds. In light of the weaknesses of benchmark models - primarily in the choice of market return but also in the choice of test statistic - and the fact that investors earn the net return and not the alpha return, we argue that the analyses of net returns are sufficient to conclude that investors have been better off investing in active Norwegian equity funds. The criterion of first-order stochastic dominance is particularly strong evidence in favor of active funds for all investors between 1991 and 2005 and large investors between 2006 to 2019.

The results are surprising and may conflict with established financial theory. Equilibrium accounting argues that when we divide investors into two groups; active mutual funds must generate the market return before costs, as passive index funds do so by definition. Due to higher costs, we expect active funds to underperform net of costs. This does not hold in our sample. Possible explanations include index funds deviating from the defined market return due to costs, tracking of dissimilar benchmarks, or regular tracking error. Alternatively, active mutual funds do not represent all active capital, which suggests that we cannot use simple arithmetic and separate investors solely as active or index funds. Nikolaisen and Skaldehaug (2018) investigated the performance of various investment groups at the Oslo Stock Exchange between 2003 and 2017 and their results were later published in Dagens Næringsliv. They found that private investors together with central and local governments underperform the market average, while private companies, mutual funds, and foreign investors outperform the market. Their findings may well explain the strong performance of active mutual funds in Norway. The results in our thesis conflict with the efficient market hypothesis that proposes all security prices to fully reflect available information. Again, considering the higher costs of active investment, it should not be possible for active mutual funds to beat their passive alternative, on average. One possible explanation for the prominent results of active investing is that active managers have an information advantage in the Norwegian market due to limited analysts following Norwegian stocks, which conflicts with the semi-strong and strong form of market efficiency. Some claim (see, for instance, Erikstad (2017) and Finansavisen (2019)) that it is easier for active managers to achieve superior returns by investing in "ignored" securities not included in the index. If these securities are eventually incorporated in the index, they will benefit from index fund inflows and increased analyst coverage.

Another possible explanation for the performance of active investing originates from the characteristics of the Norwegian market. Since Norway is a small and open economy dependent on the export of oil and gas, the cross-section of returns in the financial market may be more correlated than what is normal in larger economies. Therefore, it can be challenging for active mutual funds to deviate substantially from the market (and indirectly index funds). While active investors can do well by picking well-performing securities or predicting where the general market is going, they can never "fall flat" since the different industries (and the index funds) will generally co-move in the same direction. We emphasize that this is mostly speculation on the Norwegian anomaly and that further research is required to nuance the discussion, but it may explain why our results differ from previous studies in the U.S..

The superior net returns of active mutual funds need not stem from outstanding active management, but rather from the poor performance of passive index funds. We speculate that the fees of passive investment have significantly declined since their introduction in 1991 and that index fund efficiency has increased. Therefore, one could argue that Norwegian index funds have been "prematurely" compared to the active alternative for parts of the sample. We address this issue by comparing two subperiods; however, we cannot know with certainty whether the index funds have matured in the most recent period. For instance, we find evidence of FSD for all investors in the first subperiod but only for large investors in the latter, implying that index funds have become more attractive. Appendix X and Figure 5.5 also show a tendency of index funds becoming more attractive over the sample period, as the time-varying difference in cumulative returns and probability of a higher return from active funds have decreased. Noting that the average annual fee of index funds are only 26.1 basis points at present (The Finance Portal, 2020), that an index fund is offered at no cost (the Nordnet Indeksfond Norge), and that mutual fund costs and fees have been reduced in the aftermath of the DNB Norge case (for example the Sbanken, KLP, and DNB funds (Revfem, 2020)), we find it possible that active fund costs will decline relative to index fund costs in the future. This makes it natural to question whether active funds will outperform even more in the future.

While our results favor active investment, they align with the findings of the financial service industry, independent agencies, and several articles in the press. Our initial suspicion of their methodology, and the incentives of the financial service industry to promote active management, does, however, not change. Again, we refer to Appendix A where we highlight some of their methodical weaknesses, however broadly speaking, our research supports their conclusions.

6.0 Conclusion and further research

This paper aims to provide an independent analysis of whether investors are better off investing in Norwegian index funds or active funds. Using data free of survivorship bias, we find that anyone who prefers a higher return to a smaller one should hold active funds based on the criterion of first-order stochastic dominance and data from 1991 to 2005. The same holds for large investors between 2006 and 2019. Through simulations, we find that the probability of active funds being a better choice than index funds is notably above 50% for (most) investors for holding periods of 1 to 5 years, which is shorter time horizons than what is typically communicated in media and by financial advisors. We also find that smaller investors seem to have a systematic disadvantage to larger investors in that they are somewhere around 5 percentage points less likely to beat the index funds for holding periods up to 5 years, on average. Even though active funds still appear a better investment decision for investors, index funds' attractiveness improves over the sample period. While the notion of Warren Buffet and John Bogle to prefer passive investment is generally accepted in academic circles, we provide evidence of a Norwegian anomaly. It appears that the Norwegian mutual fund industry's rhetoric supporting active management does bear truth. The data makes it hard to disagree.

Finally, we suggest two topics for further research that could enrich the understanding of mutual fund performance, in particular the comparison of active and index funds. First, it would be interesting to extend tests, taking the entire probability density function of net returns into account, to other countries, focusing on the investor perspective. As highlighted in Section 5.2.2, few mutual fund studies include stochastic dominance tests, and doing so could enhance the understanding of risk and return dynamics. It would nuance the discussion on whether the Norwegian mutual fund industry is indeed an anomaly. Secondly, it would be interesting to examine explanatory factors for why we observe that active funds outperform index funds in Norway. Some factors might relate to particular points in time when active and index outperform (for instance the relationship to economic cycles) and whether any specific conditions in the Norwegian mutual fund market are sufficiently 'unique' to explain why U.S. research tends to find contrary results. These subjects remain open questions for future research.

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Appendix: Supplementary tables and figures

Appendix I

List of fund classes. The table reports the fund classes included in our study sorted by the names used in the databases. The index funds are grouped at the top of the table. Several identifiers are listed in the table. We use the SecID (a Morningstar identifier which is unique for each fund class) to retrieve data from the MS Direct and the ticker to retrieve data from the OSE Information. The counts are the number of observations for the variable.

Fund class name	Index fund	Class listed on OSE	Identificators			Return data				Additional data	
			ISIN	Ticker	SecID	Source	First	Last	Count	TNAV count	Minimum investment
ABN AMRO Indeks	Yes	Yes	NO0008000296	AI-INDEX	F0GBR04OWM	MS Direct	2000/04	2004/11	178	-	-
Alfred Berg Indeks Classic	Yes	Yes	NO0010700891	AI-INDXC	F00000SQ3R	MS Direct	1993/02	2007/11	70	-	25 000
Alfred Berg Indeks I	Yes	Yes	NO0010242233	AI-INDXP	F0GBR05PSR	MS Direct	1992/06	2001/11	182	-	-
Avanse OBX Indeks	Yes	Yes	NO0008000346	AF-XOBX	-	OSE Information	1990/12	1999/09	154	-	-
Carnegie Norge Indeks	Yes	Yes	NO0008001468	CA-INDEX	F0GBR04OVS	MS Direct	1996/01	2019/12	304	205	300
DIX Norway Restr NOK	Yes	No	DK0060955425	-	F000010PG0	MS Direct	1997/10	2012/09	20	118	50 000
DIX Norway Restr NOK W	Yes	No	DK0060608461	-	F00000WANF	MS Direct	1990/12	2019/12	56	205	300
DNB Norge Indeks A	Yes	Yes	NO0010582976	DK-NORIX	F00000JORR	MS Direct	2000/01	2019/12	112	205	300
DNB Norge Indeks N	Yes	Yes	NO0010827678	DK-NOINN	-	No data	-	-	0	70	25 000
DNB OBX	Yes	Yes	NO0010257801	DK-OBX	-	OSE Information	2004/11	2019/12	69	181	10 000 000
F-OBX	Yes	Yes	NO0008001708	AI-OBX2	-	OSE Information	1995/07	1999/09	47	-	-
Handelsbanken Norge Index	Yes	Yes	SE0011309525	HB-HNORI	F000010RP1	MS Direct	1998/01	2014/03	16	136	10 000 000
Handelsbanken Norge Index A9	Yes	Yes	SE0011309533	HB-HANO9	F000010RP2	MS Direct	1991/01	2019/12	16	205	5 000
KLP AksjeNorge Indeks	Yes	Yes	NO0010285042	KL-AKNIN	F0GBR060Z6	MS Direct	2002/04	2014/03	171	136	5 000
KLP AksjeNorge Indeks II	Yes	Yes	NO0010455694	KL-ANIII	F000002489	MS Direct	2014/05	2019/12	135	54	10 000 000
Nordea Norw Eq Mark Fund	Yes	Yes	NO0010325855	KF-NOEQM	-	OSE Information	1996/01	2001/11	171	-	-
Nordnet Superfondet Norge	Yes	Yes	SE0005993110	EO-NORDN	F00000TH8U	MS Direct	2011/01	2019/12	66	108	1 000
PLUSS Index (Fondsforvaltning)	Yes	Yes	NO0010606098	FO-INDEX	F0GBR04NJ0	MS Direct	2010/12	2019/12	290	109	10 000 000
Skandia Indeks Norge	Yes	Yes	-	SK-INDX	-	OSE Information	2013/03	2019/12	147	82	200 000 000
Storebrand Indeks - Norge	Yes	Yes	NO0010704265	SP-INDNO	F00000SVJF	MS Direct	2010/12	2019/12	69	109	100 000 000
WarrenWicklund Indeks+	Yes	Yes	NO0008002268	SU-INDEK	-	OSE Information	2016/03	2019/12	81	46	300 000 000
XACT OBX	Yes	Yes	SE0009723026	HF-OBX	-	OSE Information	2014/09	2019/12	176	38	1 000
ABIF Norge ++	No	Yes	NO0010089576	AI-NORS2	-	OSE Information	2014/09	2019/12	56	38	10 000 000
Alfred Berg Aksjef Norge	No	Yes	NO0008000270	AI-AKSJN	-	OSE Information	2015/02	2019/12	114	38	1 000 000
Alfred Berg Aksjespar	No	Yes	NO0008000114	AB-AKSPR	-	OSE Information	1992/03	2004/12	106	-	-
Alfred Berg Aktiv	No	Yes	NO0010089444	AI-AKTIV	F0GBR04NE5	MS Direct	2002/05	2005/05	288	-	-
Alfred Berg Aktiv II	No	Yes	NO0010105497	GA-KAPIT	F0GBR04NHC	MS Direct	1995/08	2019/12	180	293	1 000
Alfred Berg Gambak	No	Yes	NO0010105489	GA-GAMB	F0GBR04NHA	MS Direct	2002/05	2019/12	349	88	200 000 000
Alfred Berg Humanfond	No	Yes	NO0010032055	BF-HUMAN	F0GBR04P1G	MS Direct	1991/07	2016/10	240	256	1 000
Alfred Berg N. Pensjon	No	Yes	NO0008000742	AB-NOPEN	-	OSE Information	2005/11	2007/07	51	-	-

Appendix I (Continued, part 2 of 5)

Fund class name	Index fund	Class listed on OSE	Identificators			Return data				Additional data	
			ISIN	Ticker	SecID	Source	First	Last	Count	TNAV count	Minimum investment
Alfred Berg Norge +_gml	No	Yes	NO0010089519	AI-NORGS	F0GBR04NEL	MS Direct	2000/05	2019/12	195	205	3 000 000
Alfred Berg Norge Classic	No	Yes	NO0010089402	AI-NORG	F0GBR04NEJ	MS Direct	2006/12	2019/12	330	157	3 000 000
Alfred Berg Norge Etisk	No	Yes	NO0010138373	BF-NORGE	F0GBR04NH9	MS Direct	1994/02	2019/12	144	205	1 000
Alfred Berg Norge Inst	No	Yes	NO0010704422	AI-NORGI	F00000T8SM	MS Direct	1994/02	2019/12	68	205	50 000
Alfred Berg Vekst	No	Yes	NO0008000288	AI-VEKST	-	OSE Information	1994/02	2019/12	71	205	1 000
Arctic Norwegian Equities Class A	No	Yes	IE00B449S282	AC-NWECA	F00000LOWI	MS Direct	1994/07	2019/12	108	205	100
Arctic Norwegian Equities Class B	No	Yes	IE00B42BX430	AC-NEQCB	F00000LKRF	MS Direct	1997/11	2013/09	109	130	1 000
Arctic Norwegian Equities Class D	No	Yes	IE00B8P0P059	AC-NWECD	F00000PXNQ	MS Direct	1995/09	2019/08	82	-	-
Arctic Norwegian Equities Class I	No	Yes	IE00B41SY863	AC-NWECI	F00000LKIG	MS Direct	1982/02	2014/02	109	260	100
Arctic Norwegian Equities Share Class E	No	Yes	IE00BD8RS102	AC-NWECE	F00000WT8Z	MS Direct	1991/01	2014/09	46	267	1 000 000
Arctic Norwegian Value Creation A NOK	No	Yes	IE00BNGMYN11	AC-ANECR	F00000Y4U4	MS Direct	1981/11	2014/02	64	182	100
Arctic Norwegian Value Creation B NOK	No	Yes	IE00BNGMYG44	SA-SNOCA	F00000U5XR	MS Direct	1996/03	2019/07	64	247	2 500 000
Arctic Norwegian Value Creation C NOK	No	Yes	IE00BSS80K14	SA-SNOCC	F00000V91N	MS Direct	2019/08	2019/12	59	5	100
Arctic Norwegian Value Creation D	No	No	IE00BZ7PX706	-	F00000Y8IA	MS Direct	2019/08	2019/12	36	5	2 500 000
Atlas Norge	No	Yes	NO0010241508	NR-NORGE	-	OSE Information	2002/12	2019/12	262	205	10 000 000
Banco Norge	No	Yes	-	BF-NORG	-	OSE Information	2010/09	2019/12	37	112	100
C WorldWide Norge	No	Yes	NO0008001476	CA-AKSJE	F0GBR04OVU	MS Direct	0/	0/	293	-	-
C WorldWide Norge III	No	Yes	NO0010040231	CA-CWWNT	F0GBR06K3S	MS Direct	2019/11	2019/12	212	2	100
Danske Invest Aktiv Formuesf. A	No	Yes	NO0010286594	FF-AKFOR	-	OSE Information	2018/12	2019/12	21	13	100
Danske Invest Norge Aksj. Inst 1	No	Yes	NO0010047228	FF-NOIII	F0GBR04OZP	MS Direct	1996/05	2019/08	236	248	100
Danske Invest Norge Aksj. Inst 2	No	Yes	NO0010340748	FF-NOAI2	F0000007MS	MS Direct	2002/01	2019/08	157	212	2 500 000
Danske Invest Norge I	No	Yes	NO0008000577	FF-NORGE	F0GBR04HJW	MS Direct	2019/09	2019/12	311	4	100 000
Danske Invest Norge II	No	Yes	NO0008000460	FF-NORII	F0GBR04HJY	MS Direct	2019/09	2019/12	311	4	2 500 000
Danske Invest Norge Vekst	No	Yes	NO0008000486	FF-VEKST	F0GBR04HJX	MS Direct	1994/07	2019/12	311	270	10 000 000
Delphi Norge	No	Yes	NO0010039688	DF-NORGE	F0GBR04HEH	MS Direct	2019/11	2019/12	306	2	100
Delphi Vekst	No	Yes	NO0010039704	DF-VEKST	F0GBR04HEI	MS Direct	2018/12	2019/12	191	13	100
DNB Norge	No	Yes	NO0010338064	DK-PBNOR	-	OSE Information	2005/03	2010/11	288	-	-
DNB Norge (Avanse I)	No	Yes	NO0003603607	DK-NORGE	F0GBR04NG4	MS Direct	1989/12	2002/11	383	-	-
DNB Norge (Avanse II)	No	Yes	NO0008000627	DK-NORII	F0GBR04OT6	MS Direct	2001/04	2019/12	285	225	100
DNB Norge (I)	No	Yes	NO0005259705	DI-RINV	F0GBR04NH2	MS Direct	2019/11	2019/12	388	2	100

Appendix I (Continued, part 3 of 5)

Fund class name	Index fund	Class listed on OSE	Identificators			Return data				Additional data	
			ISIN	Ticker	SecID	Source	First	Last	Count	TNAV count	Minimum investment
DNB Norge (III)	No	Yes	NO0010336944	DK-NORG3	F0GBR04NGW	MS Direct	2018/12	2019/12	281	13	100
DNB Norge A	No	Yes	NO0010819915	DK-NOIVA	F0000110TF	MS Direct	2003/10	2019/12	5	175	100
DNB Norge C	No	Yes	NO0010849607	DK-NOIVC	F000013HRI	MS Direct	1998/05	2013/09	5	130	-
DNB Norge D	No	Yes	NO0010337686	DK-NORIV	F0GBR04O87	MS Direct	1998/01	2001/11	205	-	-
DNB Norge N	No	Yes	NO0010819931	DK-NORGN	F0000110TH	MS Direct	2018/01	2019/12	2	24	100 000
DNB Norge R	No	Yes	NO0010819964	DK-NOIVR	F0000110TI	MS Direct	2010/10	2019/12	13	111	100 000
DNB Norge Selektiv	No	Yes	NO0010336951	DK-NSEL1	F0GBR04NGC	MS Direct	2018/12	2019/12	280	13	1 000
DNB Norge Selektiv (II)	No	Yes	NO0010337694	DK-NSEL2	F0GBR04L63	MS Direct	2017/03	2018/11	212	21	1 000
DNB Norge Selektiv A	No	Yes	NO0010819972	DK-NOSEA	F0000110TK	MS Direct	1998/02	2000/09	4	-	-
DNB Norge Selektiv C	No	Yes	NO0010849615	DK-NOSEC	F000013HRJ	MS Direct	2000/10	2004/08	4	-	-
DNB Norge Selektiv E	No	Yes	NO0008000007	DK-NSEL3	F0GBR04OYL	MS Direct	2003/01	2019/12	306	204	10 000
DNB Norge Selektiv N	No	Yes	NO0010819998	DK-NOSEN	F0000110TM	MS Direct	2019/10	2019/12	2	3	10 000
DNB Norge Selektiv R	No	Yes	NO0010820004	DK-NOSER	F0000110TN	MS Direct	2011/04	2019/12	13	100	100
DnB Real-Vekst	No	Yes	-	DI-RVKST	-	OSE Information	2013/02	2019/12	156	83	100
DNB SMB A	No	Yes	NO0010337819	DI-SMB	F0GBR04P36	MS Direct	2000/01	2001/06	225	-	-
DNB SMB N	No	Yes	NO0010801897	DK-SMBN	F00000ZFFY	MS Direct	0/	0/	2	-	-
DNB SMB R	No	Yes	NO0010801905	VI-DSMBR	F00000ZFFZ	MS Direct	1987/02	1999/08	13	-	-
Eika Norge	No	Yes	NO0010199086	EK-NORGE	F0GBR04HET	MS Direct	1992/04	2000/10	195	-	-
Eika SMB	No	Yes	NO0008001369	NF-PLUSS	F0GBR04P23	MS Direct	1998/12	2006/04	185	-	-
FIRST Aksjer Norge	No	No	IE00B40VT695	-	F000002JVY	MS Direct	1998/12	2006/11	110	-	-
FIRST Aksjer Norge Kl.I	No	No	IE00B4N94214	-	F0000045O4	MS Direct	2017/09	2019/12	93	28	1 000
FIRST Aksjer Norge Kl.III	No	No	IE00B4M3MK87	-	F000002O5I	MS Direct	2018/04	2019/12	101	21	100 000 000
FIRST Generator A	No	Yes	NO0010812258	FT-GENEA	F00000ZZIG	MS Direct	2018/09	2019/12	24	16	-
FIRST Generator S	No	Yes	NO0010584105	FT-GNRTR	F00000JXV8	MS Direct	2018/09	2019/12	111	16	10 000 000
FIRST Norge Fokus	No	Yes	NO0010835507	FT-NOFOK	F000011K02	MS Direct	2001/01	2019/12	13	205	1 000
FIRST Norge Verdi	No	Yes	NO0010775521	FT-NORGD	F00000YMC5	MS Direct	1997/11	2000/11	21	-	-
First Norway Delta Kl.IV (LAMP)	No	No	IE00B45L6G31	-	F000002O5H	MS Direct	1995/09	2003/08	56	-	-
Fokus Barnespar	No	Yes	NO0008001666	FF-BARNE	-	OSE Information	1999/04	2019/12	32	203	3 000
Fondsfinans Aktiv II	No	Yes	NO0010058191	FK-AKTI2	-	OSE Information	2005/10	2019/12	47	171	10 000 000
Fondsfinans Norge	No	Yes	NO0010165764	FK-SPAR	F0GBR04LLU	MS Direct	2008/10	2019/12	204	135	3 000
Fondsfinans Utbytte	No	Yes	NO0010860349	FK-UTBYTTE	F00001463B	MS Direct	2006/06	2016/05	3	120	300
FORTE Norge	No	Yes	NO0010601271	FV-NORGE	F00000M7ZX	MS Direct	2013/03	2019/12	105	82	300
FORTE Trønder	No	Yes	NO0010665441	FV-TRNDR	F00000PTAO	MS Direct	2018/07	2019/12	83	18	10 000 000
GAMBAK Oppkjøp	No	Yes	-	GA-OPPKJ	-	OSE Information	1996/09	2013/09	18	130	1 000
Gjensidige Aksje Norge Kl R	No	Yes	NO0010657927	GF-ANKLR	-	No data	-	-	0	466	100
GJENSIDIGE AksjeSpar	No	Yes	NO0008001153	GF-AKSJE	-	OSE Information	1999/01	2002/10	151	-	-
GJENSIDIGE Invest	No	Yes	NO0008000338	GF-INVES	-	OSE Information	1995/02	2019/12	103	298	1 000 000
Globus Aktiv	No	Yes	NO0008002276	SU-AKTIV	-	OSE Information	1999/01	2005/11	87	-	-

Appendix I (Continued, part 4 of 5)

Fund class name	Index fund	Class listed on OSE	Identificators			Return data				Additional data	
			ISIN	Ticker	SecID	Source	First	Last	Count	TNAV count	Minimum investment
Globus Norge	No	Yes	NO0008080801	SU-GLNO	-	OSE Information	2000/08	2006/04	103	-	-
Globus Norge II	No	Yes	NO0008002284	SU-NORGE	-	OSE Information	2011/05	2019/12	94	104	500 000
Handelsbanken Norge	No	Yes	SE0009696750	HF-NORGE	F00000YQR3	MS Direct	1996/03	2019/12	28	286	100
Handelsbanken Norge	No	No	NO0008000700	-	F0GBR04UQT	MS Direct	2005/10	2019/12	269	-	-
Handelsbanken Norge A10	No	Yes	SE0010920553	HB-HNORG	F0000108FZ	MS Direct	1997/06	2015/01	21	212	100
Holberg Norge A	No	Yes	NO0010073224	HO-NORGE	F0GBR04P2Q	MS Direct	1997/07	2003/03	228	-	-
K-IPA Aksjefond	No	Yes	-	KF-IPA	-	OSE Information	1981/03	2015/01	37	407	100
KLP Aksjeinvest	No	Yes	-	KL-AKSJE	-	OSE Information	2014/07	2019/12	96	66	100
KLP AksjeNorge	No	Yes	NO0010272388	KL-AKSNO	F0GBR04GPC	MS Direct	2015/11	2019/12	249	50	10 000 000
Landkreditt Norge	No	Yes	NO0010279011	IS-NORGE	F0GBR06KYQ	MS Direct	2015/11	2019/12	120	50	1 000 000
Landkreditt Utbytte A	No	Yes	NO0010662836	IS-UTBYT	F00000PLTL	MS Direct	1992/07	2019/12	82	312	3 000
Landkreditt Utbytte I	No	Yes	NO0010820632	IS-UTBYI	F000010PBW	MS Direct	2015/11	2019/12	18	50	-
NB-Aksjefond	No	Yes	NO0008001302	NF-AKSJE	F0GBR04HKA	MS Direct	2004/06	2015/10	205	59	10 000 000
Nordea Avkastning	No	Yes	NO0010325699	KF-AVKAS	F0GBR04NI0	MS Direct	1993/02	2006/06	466	-	-
Nordea Barnespar	No	Yes	-	KF-BARNE	-	OSE Information	2002/10	2019/12	46	205	500
Nordea Kapital	No	Yes	NO0010325715	KF-KAP	F0GBR04NIO	MS Direct	2006/01	2019/12	299	166	500
Nordea Kapital II	No	Yes	-	KF-KAPIT	-	OSE Information	2015/08	2019/12	83	53	20 000 000
Nordea Kapital III	No	Yes	-	KF-KAIII	-	OSE Information	2015/08	2019/12	69	53	50 000 000
Nordea Norge Pluss	No	Yes	NO0010605637	KF-NOPLS	F00000MEAO	MS Direct	2001/10	2019/12	104	205	100 000 000
Nordea Norge Verdi	No	Yes	NO0010325731	KF-AKPEN	F0GBR04NHK	MS Direct	1988/12	2019/12	286	205	2 000
Nordea SMB	No	Yes	NO0010325749	KF-SMB	F0GBR04NK1	MS Direct	2013/11	2019/12	212	73	10 000 000
Nordea SMB II	No	Yes	-	KF-SMBII	-	OSE Information	2013/11	2019/12	69	73	50 000 000
Nordea Vekst	No	Yes	NO0010325707	KF-VEKST	F0GBR04NJZ	MS Direct	1997/01	2019/12	407	166	50 000
ODIN Norge A	No	Yes	NO0010748197	OD-NORGA	F00000WH1O	MS Direct	1995/11	2019/12	50	166	50 000
ODIN Norge B	No	Yes	NO0010748205	OD-NORGB	F00000WH1P	MS Direct	1995/11	2019/12	50	166	50 000
ODIN Norge C	No	Yes	NO0008000379	OD-NORGE	F0GBR04UQF	MS Direct	1997/04	2005/03	330	-	-
ODIN Norge D	No	Yes	NO0010748213	OD-NORGD	F00000WH1Q	MS Direct	1997/09	2007/03	50	-	-
ODIN Norge II	No	Yes	NO0010220122	OD-NORII	F0GBR06K6I	MS Direct	2002/02	2006/06	137	-	-
Orkla Finans 30	No	Yes	-	OR-FIN30	-	OSE Information	2016/02	2019/12	161	47	300
Pareto Aksje Norge A	No	Yes	NO0010160575	PO-AKTNY	F0GBR04OMP	MS Direct	1997/08	2003/01	207	-	-
Pareto Aksje Norge B	No	Yes	NO0010297898	PO-VERDI	F0GBR069SV	MS Direct	2016/04	2019/12	168	45	300
Pareto Aksje Norge C	No	Yes	NO0010740590	PO-AKNOC	F00000W240	MS Direct	1994/12	2002/11	53	-	-
Pareto Aksje Norge D	No	Yes	NO0010740608	PO-AKNOD	F00000W241	MS Direct	1990/09	2002/11	53	-	-
Pareto Aksje Norge I	No	Yes	NO0010110968	PO-AKTIV	F0GBR04NJK	MS Direct	1994/12	2002/11	219	-	-
Pareto Investment Fund A	No	Yes	NO0010040496	OR-INVF	F0GBR04OY7	MS Direct	2019/02	2019/12	365	11	500
Pareto Investment Fund B	No	Yes	NO0010694771	OR-INVFB	F00000R6QY	MS Direct	2019/02	2019/12	74	11	500
Pareto Investment Fund C	No	Yes	NO0010694789	OR-INVFC	F00000R6QZ	MS Direct	2019/02	2019/12	74	11	2 000 000
PLUSS Aksje (Fondsforvaltning)	No	Yes	NO0010606072	FO-AKSJE	F0GBR04NJC	MS Direct	2019/02	2019/12	276	11	10 000 000
PLUSS Markedsverdi (Fondsforvaltning)	No	Yes	NO0010606080	FO-INDX	F0GBR04NJ2	MS Direct	1996/08	2019/12	290	205	10 000 000
Postbanken Aksjevekst	No	Yes	-	PV-VEKST	-	OSE Information	1981/07	2001/03	96	-	-

Appendix I (Continued, part 5 of 5)

Fund class name	Index fund	Class listed on OSE	Identificators			Return data				Additional data	
			ISIN	Ticker	SecID	Source	First	Last	Count	TNAV count	Minimum investment
RF Aksjefond	No	Yes	NO0008001344	NF-RFAKS	-	OSE Information	2014/04	2019/12	115	69	100
RF-Plussfond	No	Yes	NO0010127061	NF-RFPLU	-	OSE Information	1983/10	2019/12	53	205	100
Sbanken Framgang Sammen	No	Yes	NO0010754146	AI-SKAFS	F00000WSA9	MS Direct	2002/07	2005/12	47	-	-
SEB Norge LU	No	Yes	LU0075057017	SE-NORGE	-	OSE Information	2019/05	2019/12	66	8	10 000 000
SEB Norway Focus	No	Yes	LU1330103273	SE-NOFOC	F00000WVI7	MS Direct	2017/05	2019/12	45	32	100
SEB Norway Focus Fund HNWC NOK	No	No	LU1330103356	-	F00000WVI8	MS Direct	2000/05	2019/12	45	205	100 000 000
SEB Norway Focus Fund IC NOK	No	No	LU1330103430	-	F00000WVI9	MS Direct	2011/01	2014/01	45	37	100 000 000
Skandia Horisont	No	Yes	-	SK-HORIS	-	OSE Information	2001/01	2019/03	96	196	100 000
Skandia SMB Norge	No	Yes	-	SK-SMB	-	OSE Information	1992/11	2019/12	96	205	100
SR-Bank Norge A	No	Yes	NO0010814411	SR-NORGA	F000011OBA	MS Direct	1998/01	2019/12	11	205	100
SR-Bank Norge B	No	Yes	NO0010814429	SR-NORGB	F000011OBB	MS Direct	2018/04	2019/12	11	21	100
SR-Bank Norge C	No	Yes	NO0010814437	SR-NORGC	F000011OBC	MS Direct	1998/05	2013/09	11	127	300
SR-Bank Norge D	No	Yes	NO0010814486	SR-NORGD	F000011OBD	MS Direct	1998/04	2001/10	11	-	-
Storebrand Aksje Innland	No	Yes	NO0008000940	SP-INNLA	F0GBR04OTU	MS Direct	1998/03	2019/12	281	-	-
Storebrand AksjeSpar_gml	No	Yes	-	SP-AKSJ	-	OSE Information	0/	0/	237	-	-
Storebrand Norge	No	Yes	NO0008000783	SP-NORGE	F0GBR04OSS	MS Direct	1999/01	2005/09	435	-	-
Storebrand Norge A	No	Yes	NO0010147366	SP-NORGA	-	OSE Information	2005/05	2019/12	42	-	-
Storebrand Norge B	No	Yes	NO0010849151	SP-NORGB	F000013IC3	MS Direct	2017/01	2019/12	8	36	200 000 000
Storebrand Norge Fossilfri	No	Yes	NO0010788284	SP-STNOP	F00000YWBC	MS Direct	2018/05	2019/12	32	19	500
Storebrand Norge H	No	No	NO0010289895	-	F0GBR06JN9	MS Direct	2015/05	2019/12	103	56	100
Storebrand Norge I	No	Yes	NO0010044621	SP-NORGI	F0GBR04OUD	MS Direct	2016/07	2019/12	236	42	5 000 000
Storebrand Norge Institusjon	No	Yes	NO0010592330	SP-NOINS	F00000LN7H	MS Direct	2008/01	2017/02	37	57	100 000
Storebrand Optima Norge	No	Yes	NO0010080815	SP-OPTIM	F0GBR04HKF	MS Direct	2009/06	2017/02	219	57	1 000
Storebrand Vekst	No	Yes	NO0008000841	SP-VEKST	F0GBR04HH6	MS Direct	2008/10	2017/02	326	57	10 000 000
Storebrand Verdi A	No	Yes	NO0008000999	SP-VERDI	F0GBR04OUW	MS Direct	2007/12	2012/07	264	2	40 000 000
Storebrand Verdi N	No	Yes	NO0010817836	SP-STVEN	F0000109B3	MS Direct	1995/04	2017/08	21	177	1 000
Terra Norge	No	Yes	NO0008001849	TF-NORGE	F0GBR04OX5	MS Direct	2016/04	2019/12	185	45	1 000 000
Terra Vekst_gml	No	Yes	-	TF-VEKSG	-	OSE Information	2016/04	2019/12	43	45	10 000 000
VÅR Aksjefond	No	Yes	-	OD-VÅRAK	-	No data	-	-	0	103	200 000 000

Appendix II

List of unique fund classes and main fund classes. The table shows the fund classes for funds that have more than one class and its main fund class. As described in Section 4.0, we use the net asset value (NAV) weighted return in month m of a fund with available NAV data for the fund classes in month m, otherwise, we use the fund's main fund class.

Fund class name	Class of fund	Main class	Comment
Alfred Berg Aktiv	Alfred Berg Aktiv	Yes	
Alfred Berg Aktiv II	Alfred Berg Aktiv	No	Not a share class by definition, but .9997 correlated returns, same fund manager, and almost identical information about the funds.
Alfred Berg Indeks Classic	Alfred Berg Indeks I	No	
Alfred Berg Indeks I	Alfred Berg Indeks I	Yes	
Alfred Berg Norge +_gml	Alfred Berg Norge Classic	No	Not a share class by definition, but .9999 correlated returns, same fund manager, and almost identical information about the funds.
Alfred Berg Norge Classic	Alfred Berg Norge Classic	Yes	
Alfred Berg Norge Inst	Alfred Berg Norge Classic	No	
Arctic Norwegian Equities Class A	Arctic Norwegian Equities Class A	Yes	
Arctic Norwegian Equities Class B	Arctic Norwegian Equities Class A	No	
Arctic Norwegian Equities Class D	Arctic Norwegian Equities Class A	No	
Arctic Norwegian Equities Class I	Arctic Norwegian Equities Class A	No	
Arctic Norwegian Equities Share Class E	Arctic Norwegian Equities Class A	No	
Arctic Norwegian Value Creation A NOK	Arctic Norwegian Value Creation A NOK	Yes	
Arctic Norwegian Value Creation B NOK	Arctic Norwegian Value Creation A NOK	No	
Arctic Norwegian Value Creation C NOK	Arctic Norwegian Value Creation A NOK	No	
Arctic Norwegian Value Creation D	Arctic Norwegian Value Creation A NOK	No	
C WorldWide Norge	C WorldWide Norge	Yes	
C WorldWide Norge III	C WorldWide Norge	No	Not a share class by definition, but .9998 correlated returns, same fund manager, and almost identical information about the funds.
Danske Invest Norge Aksj. Inst 1	Danske Invest Norge Aksj. Inst 1	Yes	
Danske Invest Norge Aksj. Inst 2	Danske Invest Norge Aksj. Inst 1	No	Not a share class by definition, but .9989 correlated returns, same fund manager, and almost identical information about the funds.
Danske Invest Norge I	Danske Invest Norge I	Yes	
Danske Invest Norge II	Danske Invest Norge I	No	Not a share class by definition, but .9999 correlated returns, same fund manager, and almost identical information about the funds.
DIX Norway Restr NOK	DIX Norway Restr NOK W	No	
DIX Norway Restr NOK W	DIX Norway Restr NOK W	Yes	
DNB Norge	DNB Norge (I)	No	
DNB Norge (Avanse I)	DNB Norge (I)	No	
DNB Norge (Avanse II)	DNB Norge (I)	No	
DNB Norge (I)	DNB Norge (I)	Yes	
DNB Norge (III)	DNB Norge (I)	No	
DNB Norge A	DNB Norge (I)	No	
DNB Norge C	DNB Norge (I)	No	
DNB Norge D	DNB Norge (I)	No	
DNB Norge N	DNB Norge (I)	No	
DNB Norge R	DNB Norge (I)	No	
DNB Norge Selektiv	DNB Norge Selektiv E	No	
DNB Norge Selektiv (II)	DNB Norge Selektiv E	No	
DNB Norge Selektiv A	DNB Norge Selektiv E	No	
DNB Norge Selektiv C	DNB Norge Selektiv E	No	
DNB Norge Selektiv E	DNB Norge Selektiv E	Yes	

Appendix II (Continued, part 2 of 2)

DNB Norge Selektiv N	DNB Norge Selektiv E	No	
DNB Norge Selektiv R	DNB Norge Selektiv E	No	
DNB SMB A	DNB SMB A	Yes	
DNB SMB N	DNB SMB A	No	
DNB SMB R	DNB SMB A	No	
FIRST Aksjer Norge	FIRST Aksjer Norge	Yes	
FIRST Aksjer Norge Kl.I	FIRST Aksjer Norge	No	
FIRST Aksjer Norge Kl.III	FIRST Aksjer Norge	No	
FIRST Generator A	FIRST Generator S	No	
FIRST Generator S	FIRST Generator S	Yes	
Globus Norge	Globus Norge	Yes	
Globus Norge II	Globus Norge	No	
Handelsbanken Norge	Handelsbanken Norge	Yes	
Handelsbanken Norge A10	Handelsbanken Norge	No	
Handelsbanken Norge Index	Handelsbanken Norge Index	Yes	
Handelsbanken Norge Index A9	Handelsbanken Norge Index	No	
KLP AksjeNorge Indeks	KLP AksjeNorge Indeks	Yes	
KLP AksjeNorge Indeks II	KLP AksjeNorge Indeks	No	
Landkreditt Utbytte A	Landkreditt Utbytte A	Yes	
Landkreditt Utbytte I	Landkreditt Utbytte A	No	
Nordea Kapital	Nordea Kapital	Yes	
Nordea Kapital II	Nordea Kapital	No	
Nordea Kapital III	Nordea Kapital	No	
Nordea SMB	Nordea SMB	Yes	
Nordea SMB II	Nordea SMB	No	
ODIN Norge A	ODIN Norge C	No	
ODIN Norge B	ODIN Norge C	No	
ODIN Norge C	ODIN Norge C	Yes	
ODIN Norge D	ODIN Norge C	No	
ODIN Norge II	ODIN Norge C	No	Not a share class by definition, but .9999 correlated returns, same fund manager, and almost identical information about the funds.
Pareto Aksje Norge A	Pareto Aksje Norge I	No	
Pareto Aksje Norge B	Pareto Aksje Norge I	No	
Pareto Aksje Norge C	Pareto Aksje Norge I	No	
Pareto Aksje Norge D	Pareto Aksje Norge I	No	
Pareto Aksje Norge I	Pareto Aksje Norge I	Yes	
Pareto Investment Fund A	Pareto Investment Fund A	Yes	
Pareto Investment Fund B	Pareto Investment Fund A	No	
Pareto Investment Fund C	Pareto Investment Fund A	No	
SEB Norway Focus	SEB Norway Focus	Yes	
SEB Norway Focus Fund HNWC NOK	SEB Norway Focus	No	
SEB Norway Focus Fund IC NOK	SEB Norway Focus	No	
SR-Bank Norge A	SR-Bank Norge A	Yes	
SR-Bank Norge B	SR-Bank Norge A	No	
SR-Bank Norge C	SR-Bank Norge A	No	
SR-Bank Norge D	SR-Bank Norge A	No	
Storebrand Norge	Storebrand Norge	Yes	
Storebrand Norge A	Storebrand Norge	No	
Storebrand Norge B	Storebrand Norge	No	
Storebrand Norge H	Storebrand Norge	No	Not a share class by definition, but .9911 correlated returns, same fund manager, and almost identical information about the funds.
Storebrand Norge I	Storebrand Norge	No	Not a share class by definition, but .9853 correlated returns, same fund manager, and almost identical information about the funds.
Storebrand Norge Institusjon	Storebrand Norge	No	Not a share class by definition, but .9900 correlated returns, same fund manager, and almost identical information about the funds.
Storebrand Verdi A	Storebrand Verdi A	Yes	
Storebrand Verdi N	Storebrand Verdi A	No	

Appendix III

Descriptive statistics for the factor returns for various (sub) periods. The table shows statistics for the factors. Numbers are reported per month in percentage (e.g. 0.0100 means 0.01 %). Data from Bernt Ødegaard, except RMW and CMA which is constructed with Kenneth French data (see Appendix C).

	Panel A: Sample 1981 - 2019					
	Mean	**Min**	**Max**	**Std**	**Skew**	**Kurt**
Rm	0.9460	-29.1327	19.7164	5.9764	-1.1001	3.8022
Rf	0.3175	0.0520	2.0740	0.2514	1.6202	4.2415
Rm-Rf	0.6285	-30.3957	18.6104	6.0129	-1.1517	3.9088
SMB	0.6512	-17.0784	22.2175	3.8238	0.0673	3.2068
HML	-0.0479	-16.6487	18.4563	4.3990	-0.3132	1.2162
PR1YR	0.9797	-16.7805	15.4272	4.5188	-0.3478	1.6044
RMW	0.4162	-4.7199	4.7749	1.5129	-0.2878	0.4741
CMA	0.2367	-5.2569	6.9349	1.6341	0.4593	1.9197
	Panel B: Sample 1981 - 1990					
	Mean	**Min**	**Max**	**Std**	**Skew**	**Kurt**
Rm	1.8224	-29.1327	19.7164	7.4065	-0.7485	2.1814
Rf	1.0724	0.8680	1.3580	0.1043	0.2819	-0.4176
Rm-Rf	0.7500	-30.3957	18.6104	7.4131	-0.7559	2.2563
SMB	0.9502	-12.6644	22.2175	4.7859	0.9453	2.6148
HML	1.2048	-12.0661	18.4563	4.4907	0.2773	1.4890
PR1YR	1.6862	-16.4162	14.5809	4.8254	-0.5913	1.5046
RMW	0.0432	-0.9606	0.7486	0.5775	-0.5125	-1.0370
CMA	0.0831	-1.5960	1.4925	1.1457	-0.1375	-1.4718
	Panel C: Sample 1991 - 2005					
	Mean	**Min**	**Max**	**Std**	**Skew**	**Kurt**
Rm	1.0474	-25.4165	14.2375	6.4847	-0.7551	1.3415
Rf	0.4553	0.1580	2.0740	0.2146	1.6262	9.9771
Rm-Rf	0.5921	-26.1135	13.8025	6.5359	-0.7678	1.3627
SMB	1.0995	-17.0784	22.1400	3.9738	-0.1526	5.0259
HML	0.0777	-16.6487	14.6609	5.4466	-0.5483	0.7173
PR1YR	0.5689	-16.7805	15.4272	5.2743	-0.1538	0.6303
RMW	0.3801	-4.7199	4.7749	1.6416	-0.2221	0.5875
CMA	0.4053	-5.2569	6.9349	2.0986	0.3082	0.7437
	Panel D: Sample 2006 - 2019					
Rm	0.8264	-27.1659	16.5207	5.4872	-1.5614	6.9328
Rf	0.1785	0.0520	0.6430	0.1211	1.6069	2.0631
Rm-Rf	0.6479	-27.8089	16.3497	5.5193	-1.6215	7.1757
SMB	0.3151	-11.0296	12.8176	3.6169	0.1021	1.3776
HML	-0.2049	-7.7997	9.1011	3.4391	0.2011	-0.3204
PR1YR	1.2350	-16.0945	12.0515	3.8438	-0.4675	2.6781
RMW	0.4438	-4.5808	3.2025	1.4165	-0.3401	0.1474
CMA	0.1172	-3.3478	3.8451	1.1852	0.1965	0.3357

Appendix IV

Correlation matrices for relevant (sub) periods. Data from Bernt Ødegaard, except RMW and CMA which is constructed with Kenneth French data (see Appendix C). RMW and CMA factors are not available prior to 1991.

Panel A: Sample 1981 - 2019

	EW Active	TNAVW Active	EW Index	Rm-Rf	SMB	HML	PR1YR	RMW	CMA
EW Active	1.0000								
TNAVW Active									
EW Index									
Rm-Rf	0.9460			1.0000					
SMB	-0.3147			-0.4307	1.0000				
HML	0.0210			0.0404	-0.1252	1.0000			
PR1YR	-0.1436			-0.1417	0.1204	-0.0326	1.0000		

Panel B: Sample 1981 - 1990

	EW Active	TNAVW Active	EW Index	Rm-Rf	SMB	HML	PR1YR	RMW	CMA
EW Active	1.0000								
TNAVW Active									
EW Index									
Rm-Rf	0.8860			1.0000					
SMB	-0.2579			-0.3814	1.0000				
HML	0.1524			0.1366	-0.1121	1.0000			
PR1YR	0.0959			0.0838	0.1245	0.1871	1.0000		

Panel C: Sample 1991 - 2005

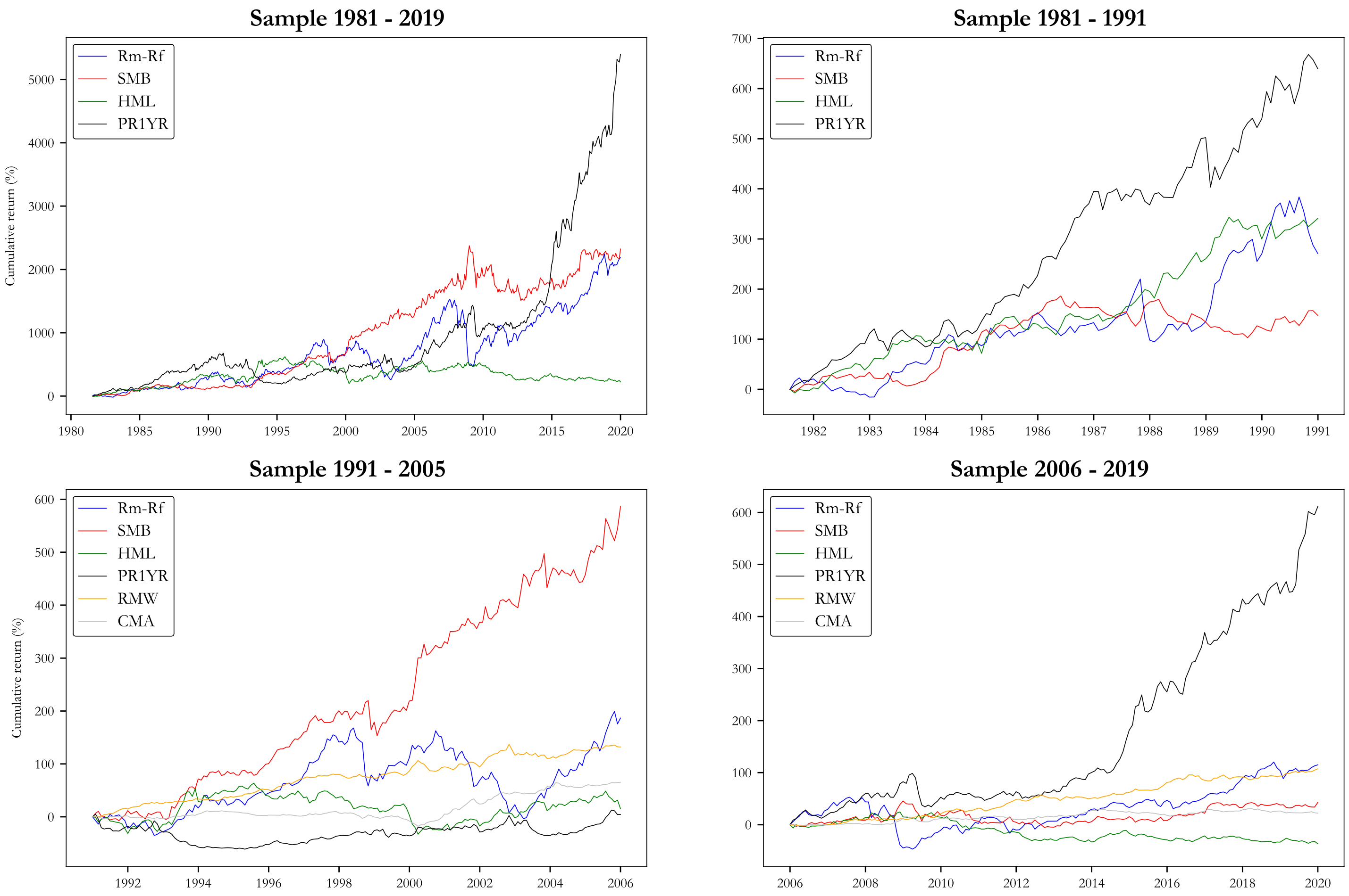
	EW Active	TNAVW Active	EW Index	Rm-Rf	SMB	HML	PR1YR	RMW	CMA
EW Active	1.0000								
TNAVW Active	0.9874	1.0000							
EW Index	0.9521	0.9575	1.0000						
Rm-Rf	0.9671	0.9767	0.9755	1.0000					
SMB	-0.1737	-0.2239	-0.2916	-0.3191	1.0000				
HML	0.0317	0.0915	0.0872	0.0848	-0.2371	1.0000			
PR1YR	-0.2367	-0.2634	-0.2456	-0.2551	0.1409	-0.1609	1.0000		
RMW	-0.0215	-0.0246	-0.0311	-0.0041	0.0212	0.0695	0.2156	1.0000	
CMA	-0.0244	0.0010	0.0030	0.0033	-0.0719	0.0489	-0.0398	-0.1875	1.0000

Panel D: Sample 2006 - 2019

	EW Active	TNAVW Active	EW Index	Rm-Rf	SMB	HML	PR1YR	RMW	CMA
EW Active	1.0000								
TNAVW Active	0.9979	1.0000							
EW Index	0.9746	0.9779	1.0000						
Rm-Rf	0.9844	0.9847	0.9853	1.0000					
SMB	-0.5947	-0.6034	-0.6787	-0.6521	1.0000				
HML	-0.1551	-0.1555	-0.1515	-0.1750	0.0465	1.0000			
PR1YR	-0.2442	-0.2454	-0.2084	-0.2284	0.1097	-0.0316	1.0000		
RMW	-0.0119	-0.0130	0.0059	-0.0159	-0.0181	-0.1327	0.1088	1.0000	
CMA	-0.0339	-0.0354	-0.0226	-0.0166	0.1149	-0.1376	0.1111	-0.3797	1.0000

Appendix V

Plots of the cumulative returns for the factors. Data from Bernt Ødegaard, except RMW and CMA which is constructed with Kenneth French data (see Appendix C). RMW and CMA factors are not available prior to 1991.



Appendix VI

A detailed study of active fund manager skills. The following six tables report the same models, estimates, and summary statistics as Table 5.1 for the three relevant (sub) periods and the two types of weighting. Each table is filtered on investor size.

Panel A: Sample 1981 - 2019 (EW)

	Investor size	AR	α	β_{MKT}	β_{SMB}	β_{HML}	β_{PR1YR}	β_{RMW}	β_{CMA}	adj R^2	N
CAPM	Small	0.1651	0.1528	0.8816						0.79	24 (5 452)
		(0.12)	(0.00)	(0.00)							
			(0.44)	(0.01)							
	Medium	0.1755	0.1195	0.9108						0.82	43 (9 676)
		(0.09)	(0.00)	(0.00)							
			(0.44)	(0.02)							
	Large	0.1733	0.1052	0.9242						0.84	51 (11 167)
		(0.09)	(0.00)	(0.00)							
			(0.45)	(0.03)							
	All	0.0882	0.0289	0.9400						0.85	78 (13 836)
		(0.33)	(0.34)	(0.00)							
			(0.71)	(0.05)							
Fama-French 3f	Small	-0.0068	0.0078	0.9419	0.1884	-0.0064				0.81	24 (5 452)
		(0.95)	(0.87)	(0.06)	(0.00)	(0.80)					
			(0.93)	(0.33)	(0.00)	(0.89)					
	Medium	0.0173	-0.0116	0.9651	0.1708	-0.0052				0.84	43 (9 676)
		(0.86)	(0.72)	(0.06)	(0.00)	(0.77)					
			(0.97)	(0.38)	(0.00)	(0.85)					
	Large	0.0217	-0.0126	0.9756	0.1597	0.0006				0.85	51 (11 167)
		(0.82)	(0.66)	(0.17)	(0.00)	(0.97)					
			(0.99)	(0.42)	(0.00)	(0.96)					
	All	-0.0846	-0.1230	0.9818	0.1717	-0.0173				0.87	78 (13 836)
		(0.31)	(0.00)	(0.16)	(0.00)	(0.19)					
			(0.67)	(0.45)	(0.00)	(0.81)					
Carhart	Small	0.0028	0.0261	0.9404	0.1868	-0.0066	-0.0110			0.81	24 (5 452)
		(0.98)	(0.60)	(0.05)	(0.00)	(0.79)	(0.39)				
			(0.92)	(0.36)	(0.00)	(0.89)	(0.87)				
	Medium	0.0262	0.0101	0.9625	0.1685	-0.0061	-0.0121			0.84	43 (9 676)
		(0.79)	(0.77)	(0.04)	(0.00)	(0.72)	(0.26)				
			(0.96)	(0.40)	(0.00)	(0.84)	(0.94)				
	Large	0.0318	0.0104	0.9732	0.1569	-0.0001	-0.0116			0.85	51 (11 167)
		(0.74)	(0.72)	(0.12)	(0.00)	(1.00)	(0.23)				
			(0.99)	(0.44)	(0.00)	(0.95)	(0.96)				
	All	-0.0692	-0.1027	0.9787	0.1695	-0.0188	-0.0148			0.87	78 (13 836)
		(0.41)	(0.00)	(0.09)	(0.00)	(0.14)	(0.10)				
			(0.70)	(0.46)	(0.00)	(0.79)	(0.85)				
Fama-French 5f	Small	0.0718	0.0524	0.9519	0.1917	-0.0099		-0.0526	-0.1271	0.82	24 (5 132)
		(0.31)	(0.26)	(0.11)	(0.00)	(0.69)		(0.05)	(0.01)		
			(0.80)	(0.42)	(0.00)	(0.87)		(0.55)	(0.38)		
	Medium	0.0853	0.0235	0.9729	0.1723	-0.0142		-0.0729	-0.1396	0.85	43 (9 343)
		(0.20)	(0.50)	(0.13)	(0.00)	(0.38)		(0.01)	(0.00)		
			(0.85)	(0.45)	(0.00)	(0.78)		(0.54)	(0.57)		
	Large	0.0858	0.0101	0.9833	0.1615	-0.0060		-0.0534	-0.1126	0.86	51 (10 834)
		(0.18)	(0.74)	(0.34)	(0.00)	(0.67)		(0.01)	(0.00)		
			(0.86)	(0.48)	(0.00)	(0.91)		(0.63)	(0.66)		
	All	0.0108	-0.0976	0.9873	0.1773	-0.0207		-0.0624	-0.0605	0.88	78 (13 347)
		(0.85)	(0.00)	(0.32)	(0.00)	(0.08)		(0.00)	(0.04)		
			(0.79)	(0.50)	(0.00)	(0.80)		(0.60)	(0.79)		

Appendix VI (Continued, part 2 of 6)

Panel B: Sample 1981 - 2019 (TNAV-W)											
	Investor size	**AR**	**α**	**β_{MKT}**	**β_{SMB}**	**β_{HML}**	**β_{PR1YR}**	**β_{RMW}**	**β_{CMA}**	**adj R^2**	**N**
CAPM	Small	-0.0530	0.0972	0.9055						0.88	24 (4 499)
		(0.55)	(0.00)	(0.00)							
			(0.49)	(0.00)							
	Medium	-0.0375	0.1001	0.9136						0.88	43 (7 669)
		(0.68)	(0.00)	(0.00)							
			(0.44)	(0.00)							
	Large	-0.0277	0.0850	0.9259						0.90	50 (8 871)
		(0.75)	(0.00)	(0.00)							
			(0.41)	(0.00)							
	All		0.0850	0.9259						0.90	51 (9 001)
			(0.00)	(0.00)							
			(0.41)	(0.00)							
Fama-French 3f	Small	-0.1716	-0.0310	0.9574	0.1565	0.0116				0.89	24 (4 499)
		(0.05)	(0.23)	(0.02)	(0.00)	(0.31)					
			(0.73)	(0.14)	(0.00)	(0.64)					
	Medium	-0.1681	-0.0358	0.9703	0.1680	0.0225				0.89	43 (7 669)
		(0.05)	(0.08)	(0.01)	(0.00)	(0.02)					
			(0.78)	(0.29)	(0.00)	(0.52)					
	Large	-0.1477	-0.0215	0.9730	0.1347	0.0261				0.91	50 (8 871)
		(0.08)	(0.27)	(0.01)	(0.00)	(0.00)					
			(0.95)	(0.24)	(0.00)	(0.48)					
	All		-0.0215	0.9729	0.1348	0.0261				0.91	51 (9 001)
			(0.27)	(0.01)	(0.00)	(0.00)					
			(0.95)	(0.24)	(0.00)	(0.48)					
Carhart	Small	-0.1566	-0.0318	0.9566	0.1578	0.0117	-0.0065			0.89	24 (4 499)
		(0.07)	(0.19)	(0.02)	(0.00)	(0.31)	(0.61)				
			(0.75)	(0.12)	(0.00)	(0.65)	(0.62)				
	Medium	-0.1542	-0.0345	0.9697	0.1689	0.0224	-0.0055			0.89	43 (7 669)
		(0.08)	(0.09)	(0.00)	(0.00)	(0.02)	(0.50)				
			(0.79)	(0.27)	(0.00)	(0.53)	(0.77)				
	Large	-0.1312	-0.0079	0.9699	0.1330	0.0250	-0.0088			0.91	50 (8 871)
		(0.12)	(0.63)	(0.00)	(0.00)	(0.00)	(0.25)				
			(1.00)	(0.21)	(0.00)	(0.49)	(0.84)				
	All		-0.0080	0.9699	0.1330	0.0250	-0.0088			0.91	51 (9 001)
			(0.62)	(0.00)	(0.00)	(0.00)	(0.25)				
			(1.00)	(0.21)	(0.00)	(0.49)	(0.84)				
Fama-French 5f	Small	-0.0074	0.0118	0.9619	0.1530	0.0071		-0.0506	-0.0551	0.91	24 (4 333)
		(0.90)	(0.62)	(0.03)	(0.00)	(0.51)		(0.04)	(0.02)		
			(0.94)	(0.14)	(0.00)	(0.68)		(0.32)	(0.62)		
	Medium	-0.0177	-0.0153	0.9744	0.1665	0.0180		-0.0313	-0.0409	0.90	43 (7 503)
		(0.78)	(0.50)	(0.02)	(0.00)	(0.04)		(0.08)	(0.08)		
			(0.96)	(0.31)	(0.00)	(0.57)		(0.60)	(0.78)		
	Large	0.0022	-0.0150	0.9768	0.1355	0.0221		-0.0151	-0.0427	0.91	50 (8 705)
		(0.97)	(0.48)	(0.02)	(0.00)	(0.00)		(0.25)	(0.05)		
			(0.96)	(0.26)	(0.00)	(0.50)		(0.81)	(0.84)		
	All		-0.0150	0.9768	0.1355	0.0221		-0.0152	-0.0427	0.91	51 (8 835)
			(0.47)	(0.02)	(0.00)	(0.00)		(0.25)	(0.05)		
			(0.96)	(0.26)	(0.00)	(0.50)		(0.80)	(0.84)		

Appendix VI (Continued, part 3 of 6)

Panel C: Sample 1991 - 2005 (EW)											
	Investor size	**AR**	**α**	**β_{MKT}**	**β_{SMB}**	**β_{HML}**	**β_{PR1YR}**	**β_{RMW}**	**β_{CMA}**	**adj R^2**	**N**
CAPM	Small	0.3061	0.2955	0.9719						0.85	17 (2 002)
		(0.03)	(0.00)	(0.25)							
			(0.30)	(0.38)							
	Medium	0.3457	0.2687	0.9732						0.86	32 (3 706)
		(0.01)	(0.00)	(0.05)							
			(0.34)	(0.36)							
	Large	0.3375	0.2501	0.9714						0.87	36 (4 145)
		(0.01)	(0.00)	(0.01)							
			(0.35)	(0.29)							
	All	0.2442	0.1099	0.9765						0.88	63 (6 363)
		(0.04)	(0.01)	(0.02)							
			(0.64)	(0.31)							
Fama-French 3f	Small	0.0321	0.0344	1.0056	0.2426	-0.0840				0.89	17 (2 002)
		(0.77)	(0.63)	(0.81)	(0.00)	(0.04)					
			(0.99)	(0.91)	(0.01)	(0.27)					
	Medium	0.0927	0.0424	0.9988	0.2078	-0.0794				0.89	32 (3 706)
		(0.38)	(0.35)	(0.93)	(0.00)	(0.00)					
			(0.95)	(0.75)	(0.01)	(0.26)					
	Large	0.0934	0.0428	0.9950	0.1920	-0.0711				0.90	36 (4 145)
		(0.37)	(0.34)	(0.66)	(0.00)	(0.00)					
			(0.95)	(0.63)	(0.01)	(0.29)					
	All	-0.0054	-0.1023	0.9975	0.1911	-0.0645				0.91	63 (6 363)
		(0.95)	(0.02)	(0.80)	(0.00)	(0.00)					
			(0.68)	(0.62)	(0.01)	(0.39)					
Carhart	Small	0.0206	0.0059	1.0074	0.2378	-0.0802	0.0380			0.89	17 (2 002)
		(0.85)	(0.93)	(0.76)	(0.00)	(0.05)	(0.10)				
			(0.97)	(0.87)	(0.01)	(0.28)	(0.73)				
	Medium	0.0804	0.0200	1.0017	0.2042	-0.0776	0.0277			0.90	32 (3 706)
		(0.45)	(0.65)	(0.90)	(0.00)	(0.00)	(0.06)				
			(0.99)	(0.77)	(0.01)	(0.27)	(0.71)				
	Large	0.0822	0.0233	0.9995	0.1897	-0.0716	0.0233			0.91	36 (4 145)
		(0.43)	(0.58)	(0.96)	(0.00)	(0.00)	(0.04)				
			(1.00)	(0.69)	(0.01)	(0.29)	(0.70)				
	All	-0.0149	-0.1102	0.9984	0.1893	-0.0662	0.0065			0.91	63 (6 363)
		(0.87)	(0.01)	(0.86)	(0.00)	(0.00)	(0.52)				
			(0.67)	(0.65)	(0.01)	(0.38)	(0.93)				
Fama-French 5f	Small	0.0751	0.0472	1.0034	0.2450	-0.0846		-0.0182	-0.0059	0.89	17 (2 002)
		(0.49)	(0.54)	(0.88)	(0.00)	(0.03)		(0.80)	(0.88)		
			(0.92)	(0.90)	(0.00)	(0.28)		(0.51)	(0.93)		
	Medium	0.1078	0.0396	0.9964	0.2139	-0.0778		-0.0429	0.0252	0.90	32 (3 706)
		(0.30)	(0.43)	(0.78)	(0.00)	(0.00)		(0.34)	(0.38)		
			(0.98)	(0.71)	(0.01)	(0.29)		(0.50)	(0.69)		
	Large	0.1068	0.0342	0.9941	0.1987	-0.0701		-0.0458	0.0277	0.91	36 (4 145)
		(0.30)	(0.46)	(0.57)	(0.00)	(0.00)		(0.23)	(0.29)		
			(0.99)	(0.59)	(0.01)	(0.32)		(0.52)	(0.63)		
	All	0.0066	-0.0955	0.9954	0.1991	-0.0638		-0.0571	0.0318	0.91	63 (6 363)
		(0.94)	(0.02)	(0.64)	(0.00)	(0.00)		(0.04)	(0.21)		
			(0.68)	(0.58)	(0.01)	(0.42)		(0.53)	(0.76)		

Appendix VI (Continued, part 4 of 6)

Panel D: Sample 1991 - 2005 (TNAV-W)											
	Investor size	**AR**	**α**	**β_{MKT}**	**β_{SMB}**	**β_{HML}**	**β_{PR1YR}**	**β_{RMW}**	**β_{CMA}**	**adj R^2**	**N**
CAPM	Small	0.1133	0.0719	0.9337						0.94	16 (1 214)
		(0.24)	(0.05)	(0.00)							
			(0.64)	(0.01)							
	Medium	0.1780	0.1557	0.9490						0.91	29 (1 887)
		(0.08)	(0.00)	(0.00)							
			(0.45)	(0.03)							
	Large	0.1796	0.1495	0.9469						0.92	33 (2 146)
		(0.07)	(0.00)	(0.00)							
			(0.49)	(0.03)							
	All	0.1795	0.1493	0.9470						0.92	34 (2 183)
		(0.07)	(0.00)	(0.00)							
			(0.49)	(0.03)							
Fama-French 3f	Small	-0.0587	-0.1156	0.9586	0.1421	0.0062				0.95	16 (1 214)
		(0.50)	(0.01)	(0.01)	(0.00)	(0.66)					
			(0.31)	(0.05)	(0.00)	(0.55)					
	Medium	-0.0165	-0.0684	0.9818	0.1762	0.0196				0.93	29 (1 887)
		(0.86)	(0.06)	(0.16)	(0.00)	(0.19)					
			(0.50)	(0.22)	(0.00)	(0.51)					
	Large	-0.0122	-0.0717	0.9784	0.1727	0.0161				0.93	33 (2 146)
		(0.89)	(0.04)	(0.06)	(0.00)	(0.22)					
			(0.46)	(0.21)	(0.00)	(0.61)					
	All	-0.0123	-0.0719	0.9785	0.1727	0.0159				0.93	34 (2 183)
		(0.89)	(0.04)	(0.05)	(0.00)	(0.21)					
			(0.46)	(0.21)	(0.00)	(0.61)					
Carhart	Small	-0.0518	-0.1055	0.9510	0.1415	0.0028	-0.0299			0.95	16 (1 214)
		(0.54)	(0.01)	(0.01)	(0.00)	(0.84)	(0.06)				
			(0.39)	(0.02)	(0.00)	(0.65)	(0.11)				
	Medium	-0.0204	-0.0710	0.9776	0.1734	0.0184	-0.0082			0.93	29 (1 887)
		(0.82)	(0.04)	(0.10)	(0.00)	(0.22)	(0.48)				
			(0.54)	(0.13)	(0.00)	(0.58)	(0.37)				
	Large	-0.0177	-0.0784	0.9745	0.1693	0.0159	-0.0023			0.93	33 (2 146)
		(0.84)	(0.01)	(0.03)	(0.00)	(0.23)	(0.85)				
			(0.49)	(0.13)	(0.00)	(0.66)	(0.44)				
	All	-0.0178	-0.0786	0.9745	0.1693	0.0158	-0.0024			0.93	34 (2 183)
		(0.84)	(0.01)	(0.03)	(0.00)	(0.22)	(0.85)				
			(0.49)	(0.13)	(0.00)	(0.66)	(0.44)				
Fama-French 5f	Small	0.0157	-0.0616	0.9556	0.1469	0.0102		-0.1363	0.0170	0.95	16 (1 214)
		(0.86)	(0.18)	(0.00)	(0.00)	(0.37)		(0.01)	(0.64)		
			(0.63)	(0.04)	(0.00)	(0.47)		(0.04)	(0.93)		
	Medium	0.0252	-0.0622	0.9789	0.1795	0.0210		-0.0460	0.0641	0.93	29 (1 887)
		(0.78)	(0.09)	(0.08)	(0.00)	(0.10)		(0.27)	(0.04)		
			(0.67)	(0.18)	(0.00)	(0.46)		(0.21)	(0.80)		
	Large	0.0278	-0.0672	0.9763	0.1754	0.0164		-0.0376	0.0566	0.93	33 (2 146)
		(0.75)	(0.06)	(0.03)	(0.00)	(0.15)		(0.31)	(0.04)		
			(0.61)	(0.18)	(0.00)	(0.57)		(0.23)	(0.82)		
	All	0.0277	-0.0675	0.9763	0.1754	0.0163		-0.0375	0.0570	0.93	34 (2 183)
		(0.75)	(0.05)	(0.02)	(0.00)	(0.14)		(0.30)	(0.04)		
			(0.61)	(0.18)	(0.00)	(0.57)		(0.23)	(0.82)		

Appendix VI (Continued, part 5 of 6)

Panel E: Sample 2006 - 2019 (EW)											
	Investor size	**AR**	**α**	**β_{MKT}**	**β_{SMB}**	**β_{HML}**	**β_{PR1YR}**	**β_{RMW}**	**β_{CMA}**	**adj R^2**	**N**
CAPM	Small	0.0892	0.0991	0.8681						0.82	24 (3 105)
		(0.29)	(0.06)	(0.00)							
			(0.59)	(0.01)							
	Medium	0.0776	0.0619	0.8932						0.85	43 (5 606)
		(0.31)	(0.09)	(0.00)							
			(0.64)	(0.01)							
	Large	0.0802	0.0592	0.9083						0.86	51 (6 658)
		(0.27)	(0.06)	(0.00)							
			(0.61)	(0.01)							
	All	0.0743	0.0540	0.9077						0.86	53 (6 892)
		(0.31)	(0.08)	(0.00)							
			(0.63)	(0.01)							
Fama-French 3f	Small	0.0140	0.0277	0.9310	0.1397	0.0444				0.83	24 (3 105)
		(0.86)	(0.61)	(0.02)	(0.00)	(0.01)					
			(0.88)	(0.24)	(0.04)	(0.38)					
	Medium	0.0121	-0.0017	0.9521	0.1293	0.0429				0.86	43 (5 606)
		(0.86)	(0.96)	(0.01)	(0.00)	(0.00)					
			(0.91)	(0.29)	(0.05)	(0.45)					
	Large	0.0187	0.0011	0.9631	0.1215	0.0440				0.87	51 (6 658)
		(0.78)	(0.97)	(0.04)	(0.00)	(0.00)					
			(0.86)	(0.35)	(0.06)	(0.40)					
	All	0.0124	-0.0046	0.9643	0.1255	0.0442				0.87	53 (6 892)
		(0.85)	(0.88)	(0.04)	(0.00)	(0.00)					
			(0.89)	(0.37)	(0.06)	(0.39)					
Carhart	Small	0.0440	0.0633	0.9262	0.1368	0.0420	-0.0249			0.83	24 (3 105)
		(0.57)	(0.21)	(0.01)	(0.00)	(0.01)	(0.14)				
			(0.89)	(0.25)	(0.04)	(0.38)	(0.85)				
	Medium	0.0399	0.0365	0.9465	0.1262	0.0402	-0.0259			0.86	43 (5 606)
		(0.57)	(0.30)	(0.00)	(0.00)	(0.00)	(0.03)				
			(0.88)	(0.29)	(0.05)	(0.45)	(0.83)				
	Large	0.0482	0.0394	0.9579	0.1179	0.0417	-0.0242			0.87	51 (6 658)
		(0.47)	(0.20)	(0.02)	(0.00)	(0.00)	(0.03)				
			(0.80)	(0.35)	(0.07)	(0.39)	(0.82)				
	All	0.0423	0.0342	0.9590	0.1222	0.0417	-0.0266			0.87	53 (6 892)
		(0.52)	(0.25)	(0.02)	(0.00)	(0.00)	(0.02)				
			(0.82)	(0.37)	(0.06)	(0.39)	(0.80)				
Fama-French 5f	Small	0.0228	0.0382	0.9375	0.1496	0.0368		-0.0131	-0.1589	0.83	24 (3 105)
		(0.76)	(0.41)	(0.03)	(0.00)	(0.01)		(0.64)	(0.00)		
			(0.87)	(0.29)	(0.03)	(0.43)		(0.90)	(0.47)		
	Medium	0.0270	0.0154	0.9584	0.1356	0.0295		-0.0428	-0.1890	0.86	43 (5 606)
		(0.69)	(0.66)	(0.02)	(0.00)	(0.00)		(0.10)	(0.00)		
			(0.87)	(0.34)	(0.04)	(0.53)		(0.99)	(0.44)		
	Large	0.0291	0.0085	0.9697	0.1276	0.0335		-0.0272	-0.1586	0.87	51 (6 658)
		(0.66)	(0.78)	(0.09)	(0.00)	(0.00)		(0.16)	(0.00)		
			(0.84)	(0.41)	(0.05)	(0.46)		(0.95)	(0.51)		
	All	0.0229	0.0043	0.9707	0.1315	0.0338		-0.0291	-0.1546	0.87	53 (6 892)
		(0.73)	(0.89)	(0.09)	(0.00)	(0.00)		(0.13)	(0.00)		
			(0.87)	(0.43)	(0.05)	(0.45)		(0.95)	(0.54)		

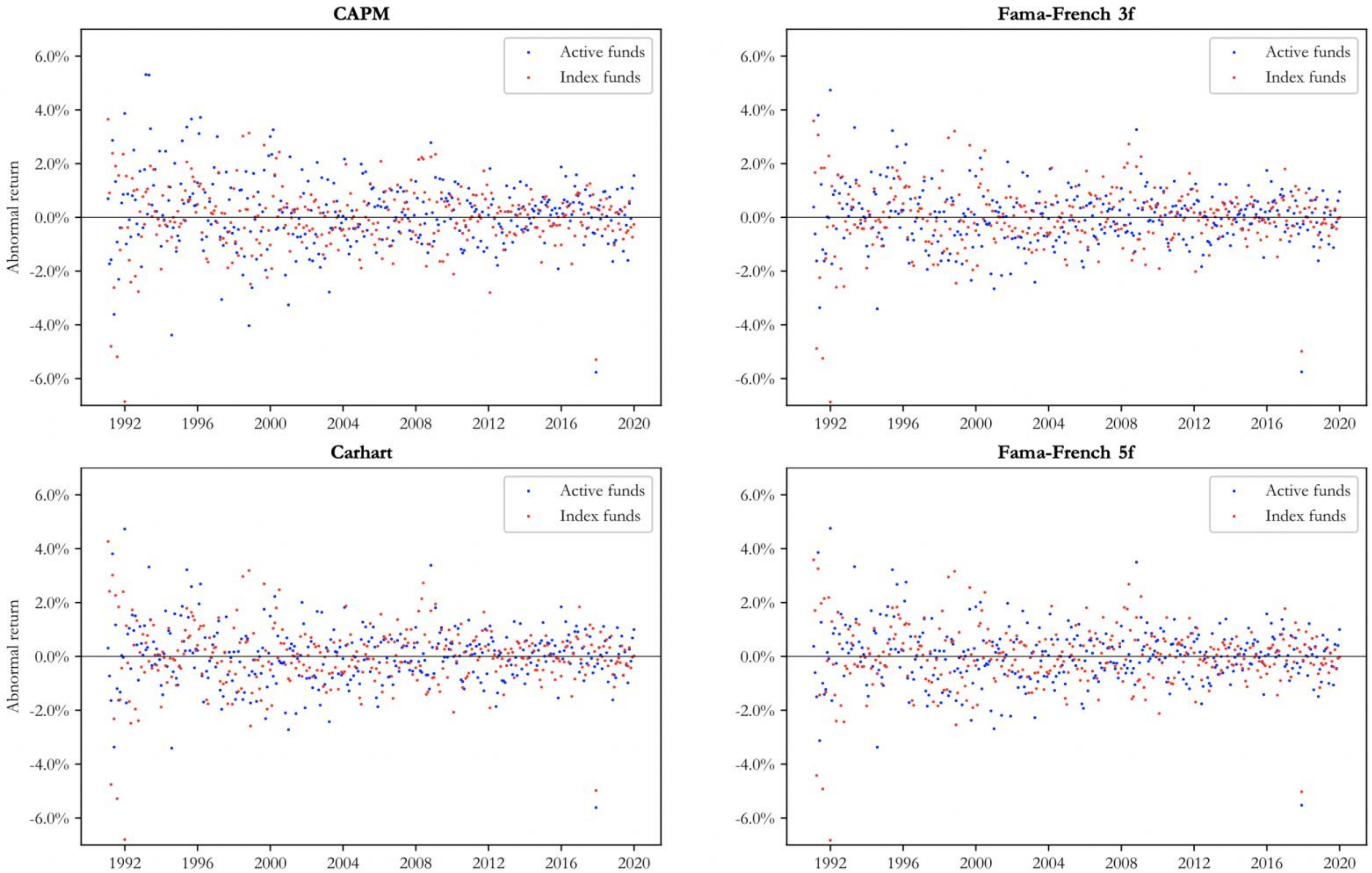
Appendix VI (Continued, part 6 of 6)

Panel F: Sample 2006 - 2019 (TNAV-W)

	Investor size	**AR**	**α**	β_{MKT}	β_{SMB}	β_{HML}	β_{PR1YR}	β_{RMW}	β_{CMA}	**adj R_2**	**N**
CAPM	Small	0.0150	0.0752	0.8852						0.89	24 (3 100)
		(0.85)	(0.02)	(0.00)							
			(0.58)	(0.00)							
	Medium	-0.0146	0.0410	0.8857						0.89	43 (5 597)
		(0.86)	(0.12)	(0.00)							
			(0.69)	(0.00)							
	Large	0.0032	0.0503	0.9086						0.90	50 (6 540)
		(0.96)	(0.03)	(0.00)							
			(0.53)	(0.00)							
	All	0.0031	0.0503	0.9085						0.90	51 (6 633)
		(0.97)	(0.02)	(0.00)							
			(0.53)	(0.00)							
Fama-French 3f	Small	-0.0528	0.0062	0.9450	0.1323	0.0382				0.90	24 (3 100)
		(0.46)	(0.85)	(0.01)	(0.00)	(0.01)					
			(0.88)	(0.16)	(0.03)	(0.32)					
	Medium	-0.0858	-0.0317	0.9491	0.1409	0.0393				0.90	43 (5 597)
		(0.25)	(0.26)	(0.00)	(0.00)	(0.00)					
			(0.95)	(0.21)	(0.02)	(0.34)					
	Large	-0.0434	-0.0011	0.9544	0.1003	0.0405				0.91	50 (6 540)
		(0.51)	(0.97)	(0.00)	(0.00)	(0.00)					
			(0.77)	(0.18)	(0.08)	(0.30)					
	All	-0.0435	-0.0011	0.9544	0.1003	0.0405				0.91	51 (6 633)
		(0.51)	(0.97)	(0.00)	(0.00)	(0.00)					
			(0.77)	(0.18)	(0.08)	(0.30)					
Carhart	Small	-0.0252	0.0284	0.9418	0.1307	0.0367	-0.0150			0.90	24 (3 100)
		(0.73)	(0.31)	(0.01)	(0.00)	(0.01)	(0.38)				
			(0.84)	(0.14)	(0.04)	(0.31)	(0.91)				
	Medium	-0.0484	0.0030	0.9440	0.1387	0.0369	-0.0252			0.90	43 (5 597)
		(0.52)	(0.90)	(0.00)	(0.00)	(0.00)	(0.02)				
			(0.91)	(0.19)	(0.02)	(0.34)	(0.60)				
	Large	-0.0073	0.0399	0.9477	0.0957	0.0376	-0.0238			0.91	50 (6 540)
		(0.91)	(0.05)	(0.00)	(0.00)	(0.00)	(0.01)				
			(0.62)	(0.14)	(0.10)	(0.30)	(0.73)				
	All	-0.0075	0.0398	0.9476	0.0958	0.0376	-0.0238			0.91	51 (6 633)
		(0.91)	(0.04)	(0.00)	(0.00)	(0.00)	(0.01)				
			(0.63)	(0.14)	(0.10)	(0.30)	(0.73)				
Fama-French 5f	Small	-0.0431	0.0186	0.9473	0.1394	0.0319		-0.0037	-0.1363	0.90	24 (3 100)
		(0.54)	(0.51)	(0.01)	(0.00)	(0.01)		(0.89)	(0.00)		
			(0.89)	(0.18)	(0.03)	(0.35)		(0.86)	(0.48)		
	Medium	-0.0613	-0.0076	0.9518	0.1473	0.0301		-0.0344	-0.1535	0.90	43 (5 597)
		(0.40)	(0.77)	(0.00)	(0.00)	(0.00)		(0.07)	(0.00)		
			(0.96)	(0.25)	(0.01)	(0.41)		(0.92)	(0.42)		
	Large	-0.0363	0.0028	0.9578	0.1058	0.0346		-0.0068	-0.1119	0.91	50 (6 540)
		(0.58)	(0.91)	(0.00)	(0.00)	(0.00)		(0.65)	(0.00)		
			(0.78)	(0.20)	(0.08)	(0.32)		(0.86)	(0.59)		
	All	-0.0363	0.0028	0.9578	0.1058	0.0346		-0.0069	-0.1120	0.91	51 (6 633)
		(0.58)	(0.91)	(0.00)	(0.00)	(0.00)		(0.64)	(0.00)		
			(0.78)	(0.20)	(0.08)	(0.32)		(0.86)	(0.59)		

Appendix VII

Plots comparing the active and index fund abnormal return performance. The y-axis measures the net abnormal returns of EW portfolios of active and index funds. All funds are included (i.e. no filters on minimum investment). Abnormal return is the sum of the alpha coefficient and the monthly residuals on a fund-level. The sample period is 1991 to 2019.



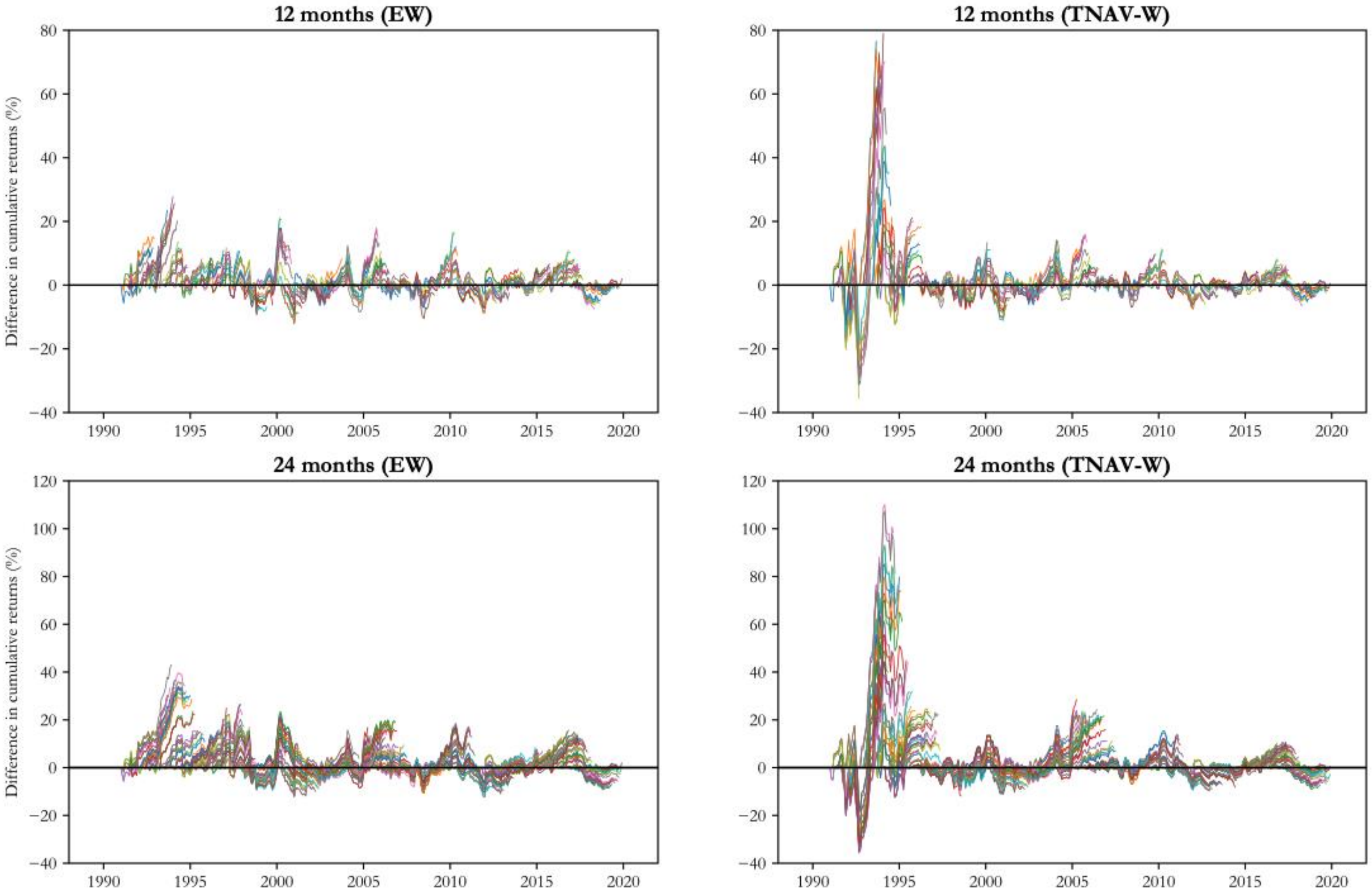
Appendix VIII

Stochastic dominance tests for the alphas and t(α). The table shows the results of first-order (FSD) and second-order (SSD) stochastic dominance tests on the alphas and t(α) for the two subperiods. These tests are the same as those used by Crane & Crotty (2018). The tables also include the number of observations (i.e. one per fund).

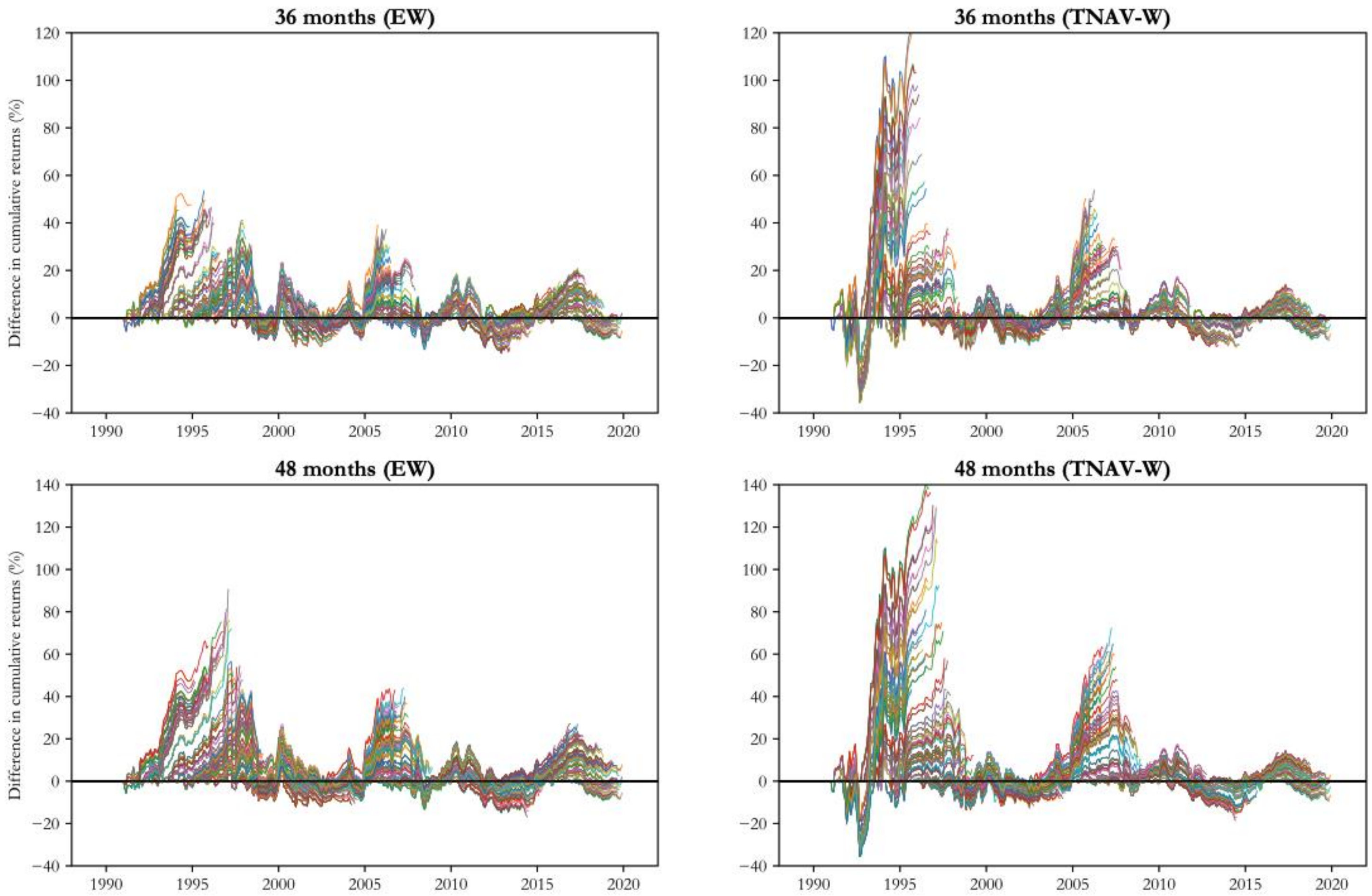
Panel A: Alphas for 1991 - 2005						
	FSD		**SSD**		**Observations**	
Model	**Index FSD Active**	**Active FSD Index**	**Index SSD Active**	**Active SSD Index**	**Index**	**Active**
CAPM	0.1600	0.9840	0.2700	1.0000	7	63
Fama-French 3f	1.0000	0.8780	1.0000	0.9880	7	63
Carhart	1.0000	0.9040	1.0000	0.9680	7	63
Fama-French 5f	1.0000	0.2700	1.0000	0.8240	7	63
Panel B: Alphas for 2006 - 2019						
	FSD		**SSD**		**Observations**	
Model	**Index FSD Active**	**Active FSD Index**	**Index SSD Active**	**Active SSD Index**	**Index**	**Active**
CAPM	0.0060	0.9980	0.8380	1.0000	11	53
Fama-French 3f	0.9820	0.7800	1.0000	0.9940	11	53
Carhart	0.3660	0.2500	1.0000	0.9600	11	53
Fama-French 5f	0.1000	0.8840	1.0000	0.9980	11	53
Panel C: t(α) for 1991 - 2005						
	FSD		**SSD**		**Observations**	
Model	**Index FSD Active**	**Active FSD Index**	**Index SSD Active**	**Active SSD Index**	**Index**	**Active**
CAPM	0.1720	1.0000	0.6620	1.0000	7	63
Fama-French 3f	1.0000	0.9340	1.0000	1.0000	7	63
Carhart	1.0000	0.9480	1.0000	1.0000	7	63
Fama-French 5f	1.0000	0.6200	1.0000	1.0000	7	63
Panel D: t(α) for 2006 - 2019						
	FSD		**SSD**		**Observations**	
Model	**Index FSD Active**	**Active FSD Index**	**Index SSD Active**	**Active SSD Index**	**Index**	**Active**
CAPM	0.0360	1.0000	0.1580	1.0000	11	53
Fama-French 3f	1.0000	0.8600	1.0000	1.0000	11	53
Carhart	0.9060	0.9980	1.0000	1.0000	11	53
Fama-French 5f	0.5300	1.0000	0.8400	1.0000	11	53

Appendix IX

Plots of Simulation A for various holding periods. The charts show the pairwise difference between the cumulative return from holding active funds versus index funds. The left chart shows the equally weighted (EW) portfolios, while the right chart weighs the funds according to their monthly TNAV. All cumulative returns are plotted, i.e. one line for each month as the starting month for the holding period. The sample period is 1991 to 2019. The charts are carefully explained in Appendix F.

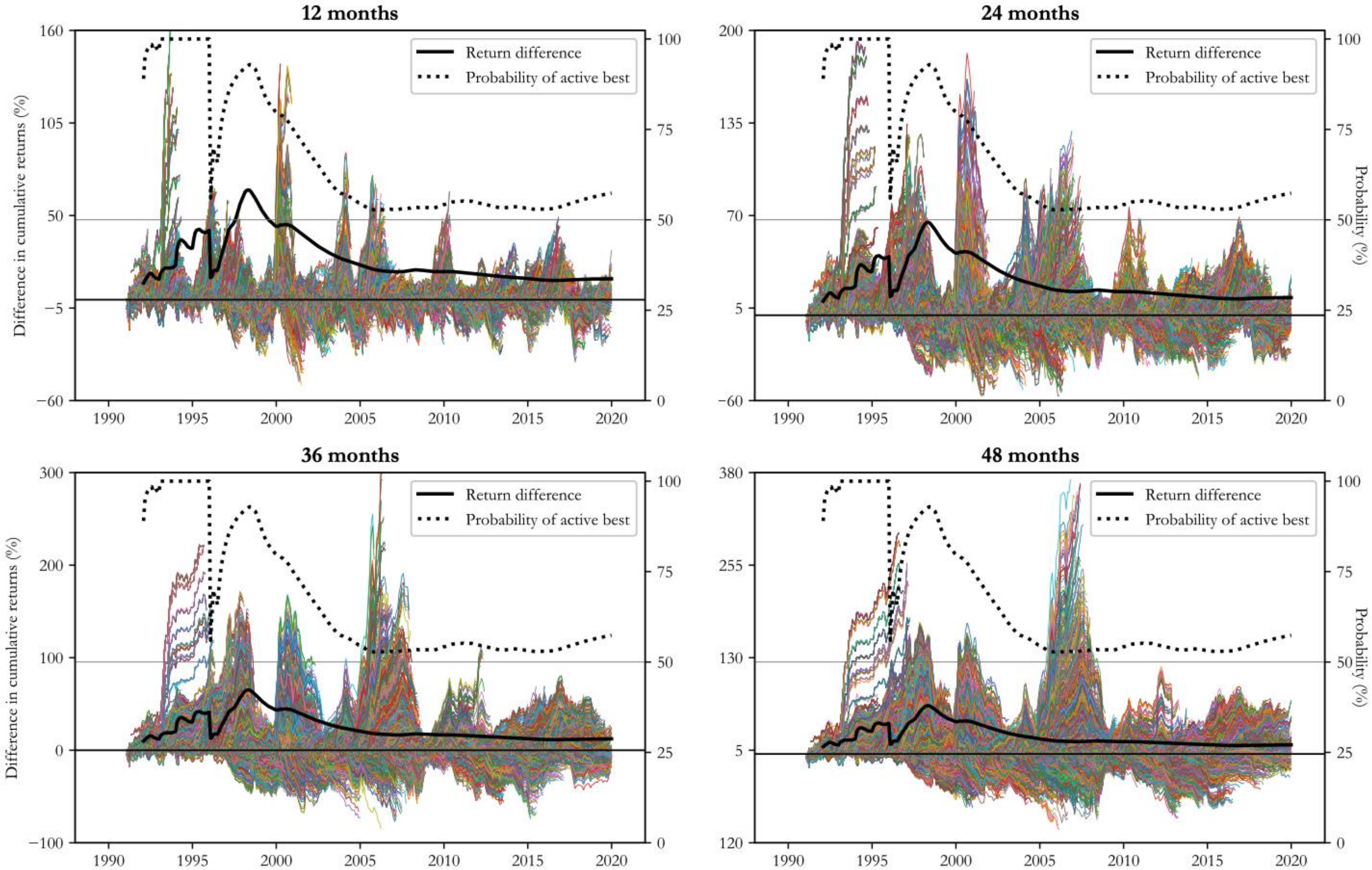


Appendix IX (Continued, part 2 of 2)



Appendix X

Plots of Simulation B for various holding periods. The charts show the pairwise difference between the cumulative return from holding active funds versus index funds for the simulated portfolios in Simulation B. All cumulative returns are plotted, i.e. one line for each month as a starting month for the holding period. The return difference (on the left-hand y-axis) and the probability of active outperforming index (on the right-hand y-axis) are plotted for the last month of the holding period, with a corresponding horizontal line for 0% cumulative return difference and 50% probability. The sample period is 1991 to 2019. The charts are carefully explained in Appendix F.



Appendix: Supplementary material

Appendix A

As discussed in the motivation, and as mentioned in other parts of the thesis, a series of reports and recommendations have been published in the media discussing whether investors should prefer active investment or the benchmark portfolio. A selection of the reports and conclusions are illustrated in Table A.1.

Most of the statements above are presented as hard evidence, but without reports documenting the methodology (e.g. sample period, included funds, benchmark portfolio, and the number of returns). As discussed in previous sections (e.g. Section 4.4.1 on Survivorship bias and Appendix D), the results are sensitive to the underlying assumptions. The lack of transparency, therefore, undermines the legitimacy of the statements. However, by directly contacting responsible parties for a selection of the reports above, we have gained essential insight into their methodology. The following section describes their methodology and illustrates their weaknesses.

Alfred Berg report - Published by Finansavisen April 24th, 2017 with the title "Norske aktive fond slår markedet"

Alfred Berg (AB) downloaded monthly returns on Norwegian Equity Mutual Funds between 1991 to 2016 from Oslo Børs Information (OBI). Next, they excluded all funds that had less than five years of returns and above 5 000 NOK minimum investment. The responsible fund manager could not confirm the stated minimum investment limit with complete accuracy but said that the intention was to exclude all mutual funds targeted at institutional investors, according to a phone call with him on April 7th, 2020. As a measure of performance, AB used the funds excess return over the OSEFX benchmark, treating active and index funds as one group, reporting the fraction of funds that outperformed during a rolling 5-year window. Mathematically, the 5-year (60 months) rolling average excess return ($\overline{ER}_{5y,i,T}$) for fund i at time T was computed as

$$\overline{ER}_{5y,i,T} = \frac{1}{60}\sum_{t=T-60}^{T} r_{i,t} - r_{OSEFX,t}$$

where r denotes the net return. Then, the fraction of funds that outperformed (FOP_T) its benchmark in time T was computed as

$$FOP_T = \frac{1}{N_T}\sum_{i=1}^{N_T} \mathbb{I}\left(\overline{ER}_{5y,i,T} > 0\right)$$

where N_T is the number of funds used to compute $\overline{ER}_{5y,i,T}$ and $\mathbb{I}$ denotes the indicator function.

The graph published in Finansavisen illustrates the percentage of funds that outperformed its benchmark over a rolling 5-year period between 1996 to 2016, as it can only be computed after the first 5 years of the sample (Henriksen, 2017). AB found that 56% of the mutual funds beat its benchmark in the sample period. Also, they find that the performance varies substantially over time, e.g. between 2009 to 2014 the fraction fell from 90% to below 30%. The article does not explicitly analyze whether active or index funds have been a better choice for investors, but it indirectly states that active funds perform better by saying that the fraction would increase if index funds were removed from the sample.

As AB chose to exclude funds with less than five years of return history, the result of the analysis is likely biased. Our data will not have this bias since we include funds that have been alive for less than five years (our simulation studies will have a similar bias for longer holding periods, but we analyze shorter holding periods and also single-month returns). Another weakness of their analysis stems from comparing the performance of funds to the OSEFX (which is the funds stated benchmark for many of the funds in their sample). While the fund managers strive to outperform its stated benchmark, a fund's holdings can deviate substantially from the benchmark index. If it does, the risk exposure will also deviate substantially, so measuring the performance relative to the stated benchmark is an imprecise way of adjusting for the risk exposure. We argue that it is better to compare the net returns of active funds and index funds directly and exclude the benchmark intermediary, as this more appropriately represents the net returns attainable by investors. The approach used by AB is similar to studying the abnormal returns in the CAPM benchmark model. Benchmark models have several weaknesses that are emphasized in Appendix D.

Table A.1. Selection of news articles in media between 2016 and 2020. The table reports the key essence from a selection of articles published by the media. The table emphasizes that it is difficult for the average investor to make sense of what fund type the investor is better off investing in.

Originator	Publisher	Title of article	Date published	Recommends active funds	Essence
Morningstar (MS)	Morningstar (MS)	Aktiv eller passiv i det norske markedet?	03.06.2016	No	By examining value-weighted domestic index funds and domestic active funds in Norway from 2000 to 2015, MS found that the passive alternative yield 7.7% annual returns to investors annually, versus 7.5% from active management.
Norwegian Fund and Asset Management Association & Morningstar	DN	Norske sparepenger renner inn i indeksfond	29.01.2017	Inconclusive	The asset manager from KLP claims that the majority of assets should be allocated to index funds. Conflicting this view, the asset manager from Arctic Fund Management claims the opposite. Regardless, the article emphasizes that assets are steadily moving towards index.
DNB	DNB	Fondsbingo. Fondskundene går i flokk - til bingolokalet	01.02.2017	Yes, although the wording of the article emphasizes the volatility of active funds.	By examining data from 2000-2016, DN finds that 28 out of 44 active funds have beat their benchmark. At the same time, the article emphasizes the volatility of active returns, and urge the investor to buy index funds, as these are more likely to yield returns close to the benchmark.
Alfred Berg	Finansavisen	Norske aktive fond slår markedet	24.04.2017	Yes	Active funds beat the benchmark by 56% from 1991 to 2006, on average, for holding periods longer than 5 years.

Table A.1. (Continued, part 2 of 4)

The Norwegian Consumer Council	E24	Forbrukerrådet har studert 20 år med data: - Dyre aksjefond ikke bedre enn billige	15.02.2018	Yes	Active funds beat their benchmark index with 0.86% per annum, on average, from 1998 to 2017.
Stavanger Asset Management and Noon Invest	Norsk Familieøkonomi	Derfor bør du velge indeksfond	19.11.2018	No	While an actively managed fund can cost up to 2.5 percent in fees, an index fund can cost as little as 0.00 to 0.30 percent. For savers who will leave their money untouched for more than five years, it may be worthwhile to choose a reasonable saving form - such as an index fund.
The Norwegian Consumer Council	E24	Forbrukerrådets aksjefond-gjennomgang: Mener flaks avgjør prestasjonen	14.12.2018	Yes	The article is a sequel of a rather comprehensive study from NCC. Norwegian mutual fund managers have generated risk-adjusted performance from 1998 to 2017, based on the CAPM. Although, the article that the funds' performance is not persistent.
Aftenposten	Aftenposten	Nesten tre av fire aksjefond påførte kundene ekstra tap i fjor	29.01.2019	Inconclusive. Although, the wording in the article emphasize the inferior performance of active funds in the short term	The number of times the return of active mutual funds were lower than their stated benchmark index: - 2018: 32 out of 45 - Last 3 years: 31 out of 45 - Last 5 years: 24 out of 42 - Last 10 years: 16 out of 38 - Last 15 years: 14 out of 35 5 out of 45 with a greater return than the benchmark in 2018 also had a greater return in 2017.

Table A.1. (Continued, part 3 of 4)

Finansavisen	Finansavisen	Indeksfond skviser ut aktive aksjefond	11.03.2019	Inconclusive	Money flow in favor of index funds in Norway. Fund managers from First Fondene claim that index funds simplify making profits from stocks that are not included in the index.
KLP Asset Management	Finansavisen	Stadig flere velger indeksfond	19.12.2019	No	The asset manager from KLP argues that "Norwegians (still) have too much of the savings in active funds. It's unwise. There is a high chance that you will pick a manager who fails to deliver excess returns. Index funds should be the cornerstone of your fund savings". It should be noted that KLP is one of the largest index fund providers in Norway.
DNB Asset Management	Finansavisen	Dette DNB-fondet skal gi deg bedre nattesøvn	11.03.2020	Yes. Particularly, "low volatility" funds.	The asset manager from DNB argues that index funds are "costly saving with high volatility", suggesting that investors should prefer active "low volatility" funds. While "low volatility" funds are not attainable for domestic funds in Norway, the article still contributes to perplexing the view of the equity fund investor.
Finansco	Finansavisen	Formuesforvalteren Finansco anbefaler kjøp av aktive, norske aksjefond	18.03.2020	Yes	The asset manager from Finansco states that "In general, we support index funds, but just for Norwegian equities, history shows that actively managed equity funds pay more to the investor than index funds over time". Further, he argues that active managers, particularly in bear markets, will outperform their passive counterparts.

Table A.1. (Continued, part 4 of 4)

Fund manager DNB Emerging Markets	DNB News	Hvor lenge må du spare i fond for å være sikker på gevinst?	24.04.2020	Yes	The probability of loss disappears over time. - 9.5 % probability after 5 years - 0.3% probability after 7 years

Aftenposten – Published by Aftenposten January 29th, 2019 with the title "Nesten tre av fire aksjefond påførte kundene ekstra tap i fjor"

Aftenposten extracted return data from Morningstar on the 10 biggest mutual fund suppliers in VFF and include 45 active mutual funds from 2003 to 2018. The funds represent 40.5% of the assets under management in Norway in 2018. As a measure of performance, Aftenposten took the annual return of active mutual funds and subtracted the annual return from the funds stated benchmark index. For instance, if the benchmark is up 10% in a given year, and the mutual fund is up 8%, the performance of the mutual fund will be -2%. Morningstar picks an "appropriate" benchmark index if the funds do not explicitly state one themselves. Aftenposten examined the performance for all funds in 2018 and for the last 3, 5, 10, and 15-year periods. The article reports the ratio of active funds that beat the benchmark for each holding period. For the mentioned holding periods, the share of funds that underperformed its benchmark was 32/45 (71%), 31/45 (69%), 24/42 (57%), 16/38 (42%), and 14/35 (40%), respectively.

Since the analysis represent less than half of the mutual funds available to investors (40.5%) on the Oslo Stock Exchange, the findings cannot be generalized to the Norwegian mutual fund industry. If, for instance, excluded funds earn inferior (superior) returns, their results will be positively (negatively) biased in favor of active management. Our data will not have this bias since we use data from both Morningstar and Oslo Børs Information, which represent 99.83% of all mutual funds that have been listed on the OSE from 1981 to 2019. For instance, while Aftenposten includes 45 active mutual funds in 2018, our sample contains 53 distinct active mutual funds and 90 fund classes. Also, similar to the critique of Alfred Berg in the above, we argue that comparing active returns to stated benchmarks does not answer whether passive or active management favor investors.

Norwegian Consumer Council (NCC) – Published by E24 February 15th 2018 with the title "Forbrukerrådet har studert 20 år med data: - Dyre aksjefond ikke bedre enn billige"

The NCC downloaded annual returns from Morningstar on Norwegian active mutual funds between 1998 to 2017 which included 47 funds and 733 observations in total. NCC calculated fund performance as the excess return over the fund's stated benchmark in percent. If the fund has not stated a benchmark index, NCC employs the benchmark that most funds use. They explain the procedure on page 10 and 11 of the report as

$$ER_{i,t} = \ln\left(\frac{1 + r_{i,t}}{1 + b_{i,t}}\right)$$

where $ER_{i,t}$ is fund i's excess return in year t, $r_{i,t}$ is fund i's performance in year t and $b_{i,t}$ is the stated (or handpicked) benchmark return of fund i in year t. The average performance of a fund for the entire sample period is then

$$\overline{ER}_i = \frac{1}{N}\sum_{t=1}^{N} ln\left(\frac{1 + r_{i,t}}{1 + b_{i,t}}\right)$$

NCC removed all funds with a minimum investment above 500 000 NOK from their sample. If a fund has several share classes, only the oldest class is used in the analysis. For fund-in-funds, they use the fund with the lowest minimum investment. Finally, only funds with 1 year of complete return history (i.e. the fund has been alive for the full year) are included. Overall, NCC found that active funds beat their benchmark by 0.86% per annum, on average.

Although the report of the NCC intends to "provide consumers with an unbiased and fact-based decision support when buying mutual funds", we identify several weaknesses in their methodology. Firstly, similar to the studies above, their sample does not include all mutual funds that have been available to Norwegian investors. While NCC includes 47 funds between 1998 to 2017, our sample includes 83 funds in the same period. Secondly, their study uses the funds' oldest share class. As highlighted in Section 4.2, different share classes need not generate the same return. For instance, Alfred Berg Aktiv I and II have an average monthly absolute difference of 1.09 percent in net returns per month, which is not a trivial difference for the mutual fund investor. Consequently, only including the oldest share class will positively (negatively) bias active management if the oldest share classes on average yield superior (inferior) returns. Our sample includes 145 share classes of active funds from 1998 to 2017 (using NAV weighted returns for funds with several classes), which is roughly three times the number of funds included by NCC. Thirdly, their study uses annual, as opposed to monthly, fund returns. Each fund will, therefore, have a maximum of 20 fund returns from 1998 to 2017. The returns in other months need not represent the return at the end of the year, which carry an uncertainty in the performance estimate, in particular in terms of the volatility. Thus, we argue that is more appropriate to use monthly returns. Notably, by assuming that all 47 funds are alive throughout the period, this would increase the number of observations by twelve-fold. Lastly, by the same token as the studies above, we argue that comparing active returns to stated benchmarks does not answer whether passive or active management favor investors.

Appendix B

Most of the previous research on mutual fund performance compare active funds to benchmark indices, not to actual index funds. In this appendix, we elaborate on the history and composition of such benchmark models.

The Capital Asset Pricing Model has been an important part of finance since its introduction in the 1960s (see e.g. Sharpe (1964), Lintner (1965) and Mossin (1966)). Later, Jensen (1968) proposed an extension to CAPM – a 1-factor model intended to estimate the abnormal performance of a fund. The alpha describes mutual fund return in excess of what is expected by the CAPM.

$$r_{i,t} - r_t^f = \alpha_i + \beta_{i,MKT} MKT_t + \epsilon_{i,t}$$

where $r_{i,t} - r_t^f$ is the excess return of fund i over the risk-free rate (r_t^f) at time t, α_i is the abnormal return (commonly referred to as Jensen's alpha), $\beta_{i,MKT}$ is the market's systematic risk factor, $MKT_t = r_t^m - r_t^f$ is the excess market return over the risk-free rate, $\epsilon_{i,t}$ is the idiosyncratic risk which can be diversified. A fund's performance is measured by alpha.

The 1-factor model has many problems explaining the cross-section of expected stock returns (e.g., the size and book-to-market equity anomalies, and, worst of all, the weak relation between average returns and β for stocks). Multifactor models seem to better explain the returns (Fama, 1991).

Fama and French (1993) proposed that three stock-market factors are common in stock-returns; (1) an overall market factor (MKT), (2) a factor related to firm size (SMB; the return of a portfolio long small-cap stocks and short large-cap stocks), and (3) a factor related book-to-market equity (HML; the return of a portfolio long high book-to-market stocks and short low book-to-market stocks), commonly referred to as the Fama and French 3-factor model

$$r_{i,t} - r_t^f = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \epsilon_{i,t}$$

Carhart (1997), building on the work of Fama and French, introduced an additional factor; one-year momentum

$$r_{i,t} - r_t^f = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,UMD} UMD_t + \epsilon_{i,t}$$

The one-year momentum factor (UMD; the return of a portfolio long past winners and short past losers) is motivated by the rationale that if some managers have the skill to beat their benchmark, we should expect persistence in their performance. Although most studies conclude that only a minority of fund managers have skills, they do find support for persistence in the performance of those fund managers (see e.g. Cremers & Petajisto (2009), Kosowski, Timmermann, Wermers & White (2006), Carhart (1997), and Daniel, Grinblatt, Titman & Wermers (1997)).

The appropriate benchmark model is a matter of extensive debate in the mutual fund literature (Crane & Crotty, 2018). Cremers, Petajisto, and Zitzewitz (2012) compared the standard Fama-French and Carhart models and found that both produce economically and statistically significant nonzero alphas for passive benchmark indices. Based on this finding, the authors proposed small methodological changes to the Fama-French factors, as well as another alternative model, to improve performance evaluation of actively managed portfolios. Fama and French (2015) later proposed a revised version of their model – now a 5-factor model including profitability (RMW; the return spread of the most profitable firms minus the least profitable) and investment factors (CMA; the return spread of firms that invest conservatively minus aggressively)

$$r_{i,t} - r_t^f = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,RMW} RMW_t + \beta_{i,CMA} CMA_t + \epsilon_{i,t}$$

Furthermore, Cederburg et al. (2018) found that the conventional factor-model regressions may lead to contaminating the evidence of manager skills, as they fail to account for changes in the portfolios' style exposures. They introduced a new method for evaluating conditional portfolio performance that may be used to assess managerial skill in security selection, factor timing, and volatility timing.

Appendix C

In this appendix, we describe our procedure for constructing the RMW and CMA factors used in the Fama-French five-factor model.

Bernt Arne Ødegaard publishes Norwegian factors for the CAPM, the Fama-French three-factor, and the Carhart four-factor models. He does not publish returns for the CMA and RMW factors in the Fama-French five-factor due to the lack of available accounting data for Norwegian firms before 2012/2013 (changes in accounting standards lead to sufficient data to compute them after 2012/2013). To our knowledge, no other party publishes Norwegian CMA and RMW factors.

To construct the CMA and RMW factors, we started by comparing the European and developing markets factor returns provided by Kenneth French (2020) with the factor data provided by Ødegaard. The European and developed market factors gave similar results, so we chose the European factors for our analysis. The Kenneth French factors are available from 1991.

Next, to exclude the part of CMA and RMW explained by the three Ødegaard factors, we regressed the European CMA and RMW factors (one at a time) on the Norwegian MKT, HML and UMD factors for with the 1991 to 2019 data. Formally

$$CMA_t = \hat{\alpha} + \hat{\beta}_{MKT} MKT_t + \hat{\beta}_{SMB} SMB_t + \hat{\beta}_{HML} HML_t + \hat{\epsilon}_t$$

for the CMA factor, and

$$RMW_t = \hat{\alpha} + \hat{\beta}_{MKT} MKT_t + \hat{\beta}_{SMB} SMB_t + \hat{\beta}_{HML} HML_t + \hat{\epsilon}_t$$

for the RMW factor.

We use the regression output not explained by MKT, HML, and UMD as our CMA and RMW factors (i.e., alpha plus the residuals).

Appendix D

In this appendix, we study the sensitivity of the chosen market benchmark on our analyses. In particular, we focus on Tables 5.2 and 5.3, where we draw inferences from benchmark models including index funds.

As discussed in Section 4.1.2, we use a combination of OBX and OSEFX as the market return in our benchmark models. The index funds are not directly tracking these two benchmarks, which may introduce a bias to our results. Table D.1 reports the benchmark index for the index funds that are still alive in our sample. The figure shows that 8 funds are tracking OSEBX, 2 are tracking OBX and 2 are tracking OSEFX. In terms of the fraction of returns in our full sample, ~50%, ~30%, and ~5% are for funds tracking OSEBX, OBX, and OSEFX among the index funds (and fund classes) that are alive today, respectively. It is not apparent which market index that is most appropriate for index funds in our benchmark models. For that reason, we proceed by analyzing the following benchmark market indices:

- **Rm:** Our market index. A combination of OBX and OSEFX as described in Section 4.1.2.
- **EW and VW**: The data is published by Bernt A. Ødegaard. The returns of the two indices are constructed from most stocks at the OSE. The least liquid and smallest stocks are filtered out. EW is equally weighted. VW is value-weighted.
- **Allshare**: The data is published by Bernt A. Ødegaard. It is constructed using the total index (TOTX) provided by OSE between 1993 and 1999 and the all-share index (OSEAX) from 1999 to 2019 (2020b, p. 28). The OSEAX consists of all shares listed on the OSE.
- **OBX**: The index is published by Bernt A. Ødegaard. The Total Return Index (OBX) consists of the 25 most traded stocks on the OSE (2020a).
- **OSEBX**: The data is published by the OSE (2020a). The OSE Benchmark index (OSEBX) compromises the most traded shares listed on the OSE. To construct the monthly returns from daily prices, we use the closing price for the last trading day of each month. The returns are computed as $return_t = \ln{(price_t / price_{t-1})}$. Note that the OSEBX return series is for 1996 to 2019.

Table D.3 reports the index fund performance for the benchmark we include in our main analysis and the alternative market benchmarks. In the CAPM model, the alpha return changes from negative to significantly positive when we replace our market index with either (purely) OBX or OSEBX and use the Fama-MacBeth p-values. The OSEBX benchmark also suggests a significant positive alpha using the Cuthbertson-Nitzsche p-values. The estimates for OBX and OSEBX differ substantially in magnitude which may be caused by the lack of data for OSEBX between 1991 and 1996. A similar story is apparent for the abnormal returns; the result shifts from negative to (significantly) positive when considering the OBX or OSEBX instead of our chosen market return.

As discussed above, most index funds track the OSEBX. The simple CAPM model suggests that index funds significantly outperform the OSEBX. In terms of abnormal returns, the model suggests an outperformance of 0.17% per month, corresponding to a whopping 2.06% annual outperformance over the sample period. When we control for additional factors in the Fama-French and Carhart models, the finding seems to be robust. The index funds in our sample have low exposures toward the additional factors in these models, which explains why the additional factors do not alter the results.

These results are somewhat difficult to explain, in particular for the OSEBX, but supports one of the main critiques of benchmark models. The results are sensitive to the choice of the market index. Moreover, as the theory states that the true market portfolio is unknown, we cannot state with certainty that we have chosen the correct market index in our main study.

Next, we turn our attention to the comparison of active and index alpha performance reported in Table 5.3. Table D.2 reports the same sensitivity analysis with respect to the choice of the market index as in D.3, but focuses on the comparison of abnormal performance between active and index funds. The alternative benchmark factors do also suggest that there are no significant differences in the abnormal returns generated by active and index funds.

Again, these results suggest that the benchmark models are highly sensitive to the choice of the market index. However, even though using OSEBX as the market return affects the results, the overall result is still in line with what we find using our chosen market index. No models report a significant difference between active and index funds.

Table D.1. Overview of index fund benchmarks. The figure includes the benchmark index for the index funds that are alive as of December 2019. * indicates that we could not retrieve the benchmark index.

Fund name	ISIN	Number of returns	Benchmark index
Alfred Berg Indeks Classic	NO0010700891	70	OSEBX
Alfred Berg Indeks I	NO0010242233	182	OSEBX
DNB Norge Indeks A	NO0010582976	112	OSEBX
Handelsbanken Norge Index	SE0011309525	16	OSEBX
Handelsbanken Norge Index A9	SE0011309533	16	OSEBX
KLP AksjeNorge Indeks	NO0010285042	171	OSEBX
KLP AksjeNorge Indeks II	NO0010455694	135	OSEBX
Nordea Norw Eq Mark Fund	NO0010325855	171	NA*
Nordnet Superfondet Norge	SE0005993110	66	NA*
PLUSS Index (Fondsforvaltning)	NO0010606098	290	OBX
Storebrand Indeks - Norge	NO0010704265	69	OSEBX
XACT OBX	SE0009723026	176	OBX
DIX Norway Restr NOK	DK0060955425	20	OSEFX
DIX Norway Restr NOK W	DK0060608461	56	OSEFX

Table D.2. Comparing active and index fund performance for alternative market benchmarks (1991 to 2019). The figure reports numbers in the same format as Table 5.3. The table uses EW for active funds. In Table 5.3, Rm is used as the market benchmark. * indicates that the sample period includes months where we do not have return data for OSEBX.

Sample 1991 - 2019					
	Market return	AR			
		Active	Index	Highest	p value
CAPM	Rm	0.1591	-0.0179	Active	(0.05)
	EW	-0.6236	-0.6845	Active	(0.48)
	VW	-0.7587	-0.9342	Active	(0.05)
	Allshare	0.0273	-0.1511	Active	(0.04)
	OBX	0.2153	0.0276	Active	(0.03)
	OSEBX	0.2586	0.1740	Active	(0.33)
Fama French	Rm	-0.0045	-0.0046	Active	(1.00)
	EW	-0.3063	-0.2380	Index	(0.37)
	VW	-0.9027	-0.8652	Index	(0.63)
	Allshare	-0.0660	-0.0675	Active	(0.98)
	OBX	0.0312	0.0074	Active	(0.76)
	OSEBX	0.1296	0.2028	Index	(0.32)
Carhart	Rm	-0.0013	-0.0073	Active	(0.94)
	EW	-0.3199	-0.2601	Index	(0.43)
	VW	-0.8356	-0.8200	Index	(0.84)
	Allshare	-0.0245	-0.0521	Active	(0.72)
	OBX	0.0560	0.0152	Active	(0.60)
	OSEBX	0.1572	0.2094	Index	(0.47)

Table D.3. Index fund performance for alternative market benchmarks. The figure reports numbers in the same format as Table 5.2. In Table 5.2, Rm is used as the market benchmark. * indicates that the sample period includes months where we do not have return data for OSEBX.

					Sample: 1991 - 2019					
	Market return	**AR**	**α**	**β_{MKT}**	**β_{SMB}**	**β_{HML}**	**β_{PR1YR}**	**R_2**	**adj R_2**	**N**
CAPM	Rm	-0.0179	-0.0226	0.9591				0.9483	0.9478	2 268
		(0.78)	(0.37)	(0.00)						
			(0.80)	(0.06)						
	EW	-0.6845	-0.6392	1.0832				0.7453	0.7427	2 268
		(0.00)	(0.00)	(0.03)						
			(0.02)	(0.19)						
	VW	-0.9342	-0.8898	1.0508				0.9444	0.9438	2 268
		(0.00)	(0.00)	(0.00)						
			(0.00)	(0.09)						
	Allshare	-0.1511	-0.0833	1.0165				0.9528	0.9523	2 268
		(0.02)	(0.08)	(0.24)						
			(0.51)	(0.21)						
	OBX	0.0276	0.0287	0.9730				0.9741	0.9739	2 268
		(0.55)	(0.04)	(0.02)						
			(0.67)	(0.05)						
	OSEBX	0.1740	0.1518	0.9749				0.9772	0.9769	2 071*
		(0.00)	(0.00)	(0.01)						
			(0.03)	(0.31)						
Fama French	Rm	-0.0046	-0.0076	0.9565	-0.0166	0.0457		0.9529	0.9514	2 268
		(0.94)	(0.84)	(0.00)	(0.32)	(0.00)				
			(1.00)	(0.06)	(0.56)	(0.12)				
	EW	-0.2380	-0.2382	1.0301	-0.5296	-0.0314		0.8809	0.8773	2 268
		(0.02)	(0.00)	(0.09)	(0.00)	(0.02)				
			(0.18)	(0.56)	(0.00)	(0.48)				
	VW	-0.8652	-0.8316	1.0349	-0.0554	0.0055		0.9486	0.9470	2 268
		(0.00)	(0.00)	(0.01)	(0.00)	(0.71)				
			(0.00)	(0.29)	(0.07)	(0.75)				
	Allshare	-0.0675	-0.0096	0.9882	-0.0905	-0.0310		0.9585	0.9572	2 268
		(0.28)	(0.84)	(0.38)	(0.00)	(0.01)				
			(0.82)	(0.53)	(0.00)	(0.14)				
	OBX	0.0074	0.0102	0.9759	0.0151	-0.0111		0.9761	0.9754	2 268
		(0.87)	(0.60)	(0.02)	(0.19)	(0.24)				
			(0.81)	(0.14)	(0.66)	(0.66)				
	OSEBX	0.2028	0.1739	0.9645	-0.0294	0.0035		0.9784	0.9776	2 071*
		(0.00)	(0.00)	(0.00)	(0.01)	(0.60)				
			(0.02)	(0.16)	(0.25)	(0.70)				
Carhart	Rm	-0.0073	-0.0049	0.9550	-0.0203	0.0466	0.0075	0.9536	0.9516	2 268
		(0.91)	(0.88)	(0.00)	(0.24)	(0.00)	(0.40)			
			(0.99)	(0.06)	(0.50)	(0.11)	(0.74)			
	EW	-0.2601	-0.2660	1.0329	-0.5294	-0.0286	0.0167	0.8826	0.8777	2 268
		(0.01)	(0.00)	(0.04)	(0.00)	(0.02)	(0.26)			
			(0.16)	(0.54)	(0.00)	(0.52)	(0.68)			
	VW	-0.8200	-0.7764	1.0269	-0.0521	0.0017	-0.0502	0.9513	0.9493	2 268
		(0.00)	(0.00)	(0.02)	(0.00)	(0.91)	(0.00)			
			(0.00)	(0.40)	(0.07)	(0.69)	(0.13)			
	Allshare	-0.0521	-0.0076	0.9892	-0.0839	-0.0315	-0.0150	0.9603	0.9586	2 268
		(0.40)	(0.86)	(0.33)	(0.00)	(0.01)	(0.28)			
			(0.89)	(0.59)	(0.00)	(0.14)	(0.65)			
	OBX	0.0152	0.0136	0.9759	0.0158	-0.0106	0.0001	0.9767	0.9756	2 268
		(0.74)	(0.49)	(0.01)	(0.17)	(0.26)	(0.99)			
			(0.83)	(0.17)	(0.66)	(0.67)	(0.95)			
	OSEBX	0.2094	0.1816	0.9634	-0.0300	0.0031	-0.0046	0.9786	0.9775	2 071*
		(0.00)	(0.00)	(0.00)	(0.01)	(0.63)	(0.32)			
			(0.02)	(0.16)	(0.23)	(0.74)	(0.63)			

Appendix E

The methods presented in Section 3.0 study single-month (net or alpha) returns. To incorporate the fact that most mutual investors hold mutual funds over multiple months, we include a simulation study of the cumulative returns in our analysis, explained in this appendix. More specifically, we use historical simulations based on the returns generated by the funds in our sample (i.e. no "artificial" simulations). The simulations are performed for holding periods of 12, 24, 36, 48, and 60 months using a type of the Monte Carlo simulation method.

In the first simulation, hereafter referred to as "Simulation A", we simulate the cumulative returns of an EW and TNAV-W portfolio to compare the performance of active and index funds. For instance, over a holding period of H months, the cumulative return starting in period t can be denoted as

$$CR_{H,t} = \left(\prod_{h=t}^{t+H}(1+\bar{r}_t)\right) - 1$$

where CR is the cumulative return, $\prod$ is the product operator, and $\bar{r}_t$ is the average return in period t (for either an EW or TNAV-W portfolio of active funds or an EW portfolio of index funds as discussed below).

We simulate the cumulative returns separately for active and index funds for all possible starting months, t, in the sample and store the returns in vectors

$$CR_{A,H} = \begin{bmatrix} CR_{A,H,1} \\ CR_{A,H,2} \\ \vdots \\ CR_{A,H,T-H} \end{bmatrix} \quad and \quad CR_{I,H} = \begin{bmatrix} CR_{I,H,1} \\ CR_{I,H,2} \\ \vdots \\ CR_{I,H,T-H} \end{bmatrix}$$

where A and I represent active and index funds, respectively, and T – H represents the last month t where we may compute a cumulative return for a holding period of H months given the T months in our sample.

Next, we elementwise compare the vectors to report summary statistics. Let

$$DCR = CR_{A,H} - CR_{I,H} = \begin{bmatrix} CR_{A,H,1} \\ CR_{A,H,2} \\ \vdots \\ CR_{A,H,T-H} \end{bmatrix} - \begin{bmatrix} CR_{I,H,1} \\ CR_{I,H,2} \\ \vdots \\ CR_{I,H,T-H} \end{bmatrix} = \begin{bmatrix} CR_{A,H,1} - CR_{I,H,1} \\ CR_{A,H,2} - CR_{I,H,2} \\ \vdots \\ CR_{A,H,T-H} - CR_{I,H,T-H} \end{bmatrix}$$

where DCR is short for the difference in cumulative returns. DCR is positive when the active fund portfolio has a higher cumulative return than the index fund portfolio, zero when the returns are equal, and negative when index funds have a higher cumulative return. Each element can be denoted by letting t represent the starting month (i.e. DCR_t), for instance $DCR_1 = CR_{A,H,1} - CR_{I,H,1}$. We may now compute the summary statistics as follows

$$P(Active > Index) = \frac{1}{T-H}\sum_{t=1}^{T-H} \mathbb{I}(DCR_t > 0)$$

where $P(Active > Index)$ is the approximated probability of a higher cumulative return associated with investing in active versus index funds and $\mathbb{I}$ is the indicator operation. Similarly, we compute the mean and standard deviation of the difference between the cumulative returns as

$$\overline{DCR} = \frac{1}{T-H}\sum_{t=1}^{T-H} DCR_t \quad and \quad \sigma(DCR) = \sqrt{\frac{\sum_{t=1}^{T-H}(DCR_t - \overline{DCR})^2}{T-H-1}}$$

We report the measures per year, also for holding periods succeeding one year, computed using the formulas for geometric mean and geometric standard deviation.

In addition to these summary statistics, we compute and report the average difference between the intra-holding period Sharpe ratios of the active and index fund portfolios

$$\overline{Sharpe}_{X,H} = \frac{1}{T-H}\sum_{t=1}^{T-H} Sharpe_{X,H,t}$$

where X denotes either the active or index fund portfolio, H denotes the holding period, t denotes the starting month, and

$$Sharpe_{X,H,t} = \frac{\bar{r}_{X,H,t} - \bar{r}_{f,H,t}}{\sigma(\bar{r}_{X,H,t})}$$

where $\bar{r}_{X,H,t}$ is the geometric average return of the portfolio of X over a H months holding period starting in month t, r_f is the risk-free rate, and σ is the geometric standard deviation. As an example, consider the difference in Sharpe ratios for a 12 month holding period ($H = 12$) over a 3-year period ($T = 36$). For the first starting month ($t = 1$), we compute the Sharpe ratio for active funds

$$Sharpe_{active,12,1} = \frac{\bar{r}_{active,12,1} - \bar{r}_{f,12,1}}{\sigma(\bar{r}_{active,12,1})}$$

which we repeat 24 times ($T - H = 36 - 12 = 24$), once for each starting month. Next, we compute the average as

$$\overline{Sharpe}_{active,12} = \frac{1}{24}\sum_{t=1}^{24} Sharpe_{active,12,t}$$

We repeat the process for index funds and report $\overline{Sharpe}_{active,12} - \overline{Sharpe}_{index,12}$.

In the second simulation, hereafter referred to as "Simulation B", we use a similar procedure, but now drawing a random active and a random index fund at each starting time t and comparing them pairwise. The procedure is repeated 100 times for each starting month. If a randomly picked fund is closed during the holding period, we assume that the investor reinvests a new, randomly drawn fund. Simulation A and B differ in that A study EW or TNAV-W portfolios of active versus index funds, while simulation B more precisely replicate the returns an investor earns by holding one fund throughout the holding period. In other words, simulation A study average cumulative returns while simulation B captures the wide variety of cumulative returns that investors have experienced over our sample period. We compute the same summary statistics as in Simulation A (the exact same formulas), but now with an additional dimension of 100 simulations per starting month.

In Simulation B, we also include a test of whether the intra-holding period net return difference of the randomly paired active and index funds are significantly different. For the same reason as in our benchmark-adjusted return testing, we use a two-sample paired t-test. We compute one test statistic for each iteration of the simulation (so, $100 \cdot (\mathrm{T} - \mathrm{H})$ test statistics for each holding period) and check whether either active or index funds have a significantly higher return (an upper-tailed test). That is, we compare each of the $100 \cdot (\mathrm{T} - \mathrm{H})$ test statistics with the critical value of the t distribution with $H - 1$ degrees of freedom and report the fraction of rejections at a 5% significance level. For the upper-tailed test, we reject the null if the test statistic is greater than the critical value. By chance, the fraction of rejections at a 5% level is expected to be around 5%. If the observed fraction of rejections deviates much from 5%, it is evidence in favor of either active or index funds (we test both ways).

Appendix F

In this appendix, we explain Figure 5.4 and 5.5 in detail. Figures explaining these charts follow on the next two pages.

The first chart plots the cumulative returns difference for an EW and TNAV-W portfolio of active funds minus an EW portfolio of index funds. The first step is to compute the weighted average of the net returns of active funds and index funds throughout the holding period, starting with the first month in the sample. Next, the cumulative return for active funds and index funds are computed separately for the given holding period. Then, the difference between the two cumulative returns is computed and plotted. Finally, the procedure is repeated for all starting months in the simulation. Figure F.1 illustrates the process step-by-step.

The second chart plots the cumulative return difference for a randomly drawn pair of an active fund and an index fund simulated 100 times per starting month, as well as the estimated probability of active funds having a higher cumulative return than index funds and the average difference in cumulative returns. As Figure F.1 shows, the first step is to add the 100 simulated return differences to the plot. Then, the lines for the probability and average difference are added, plotted on the last month of the holding period. Finally, the procedure is repeated for all starting months in the simulation.

Figure F.1. Explanation of the Simulation A plot (i.e. Figure 5.4). Step 0 is not included in Figure 5.4 but shows the two cumulative returns (lines) which the difference of is plotted, exemplified by the first starting month in the simulation. In other words, Step 1 plots the cumulative return of active funds (the green line in Step 0) minus the cumulative return of index funds (the red line in Step 0).

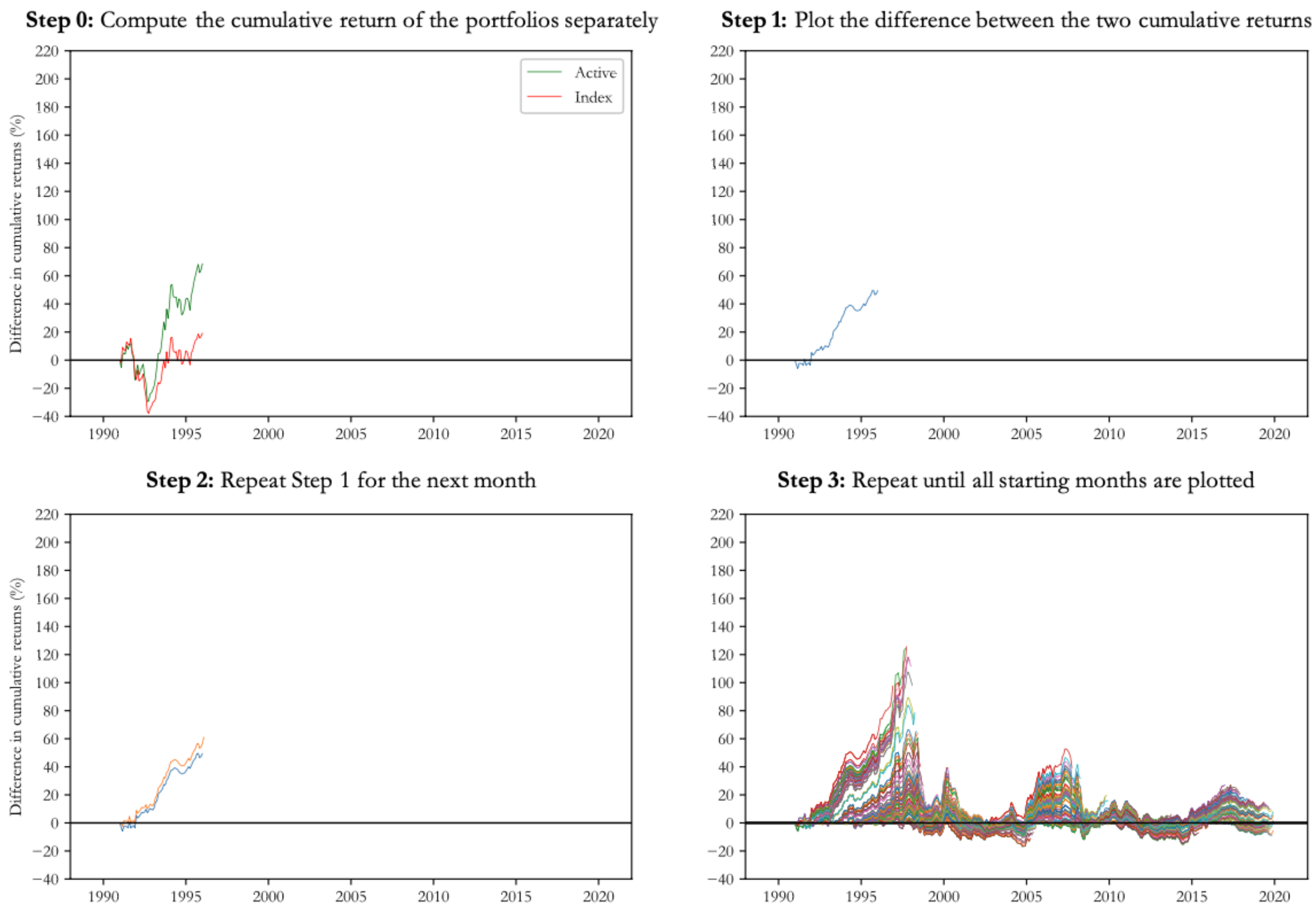
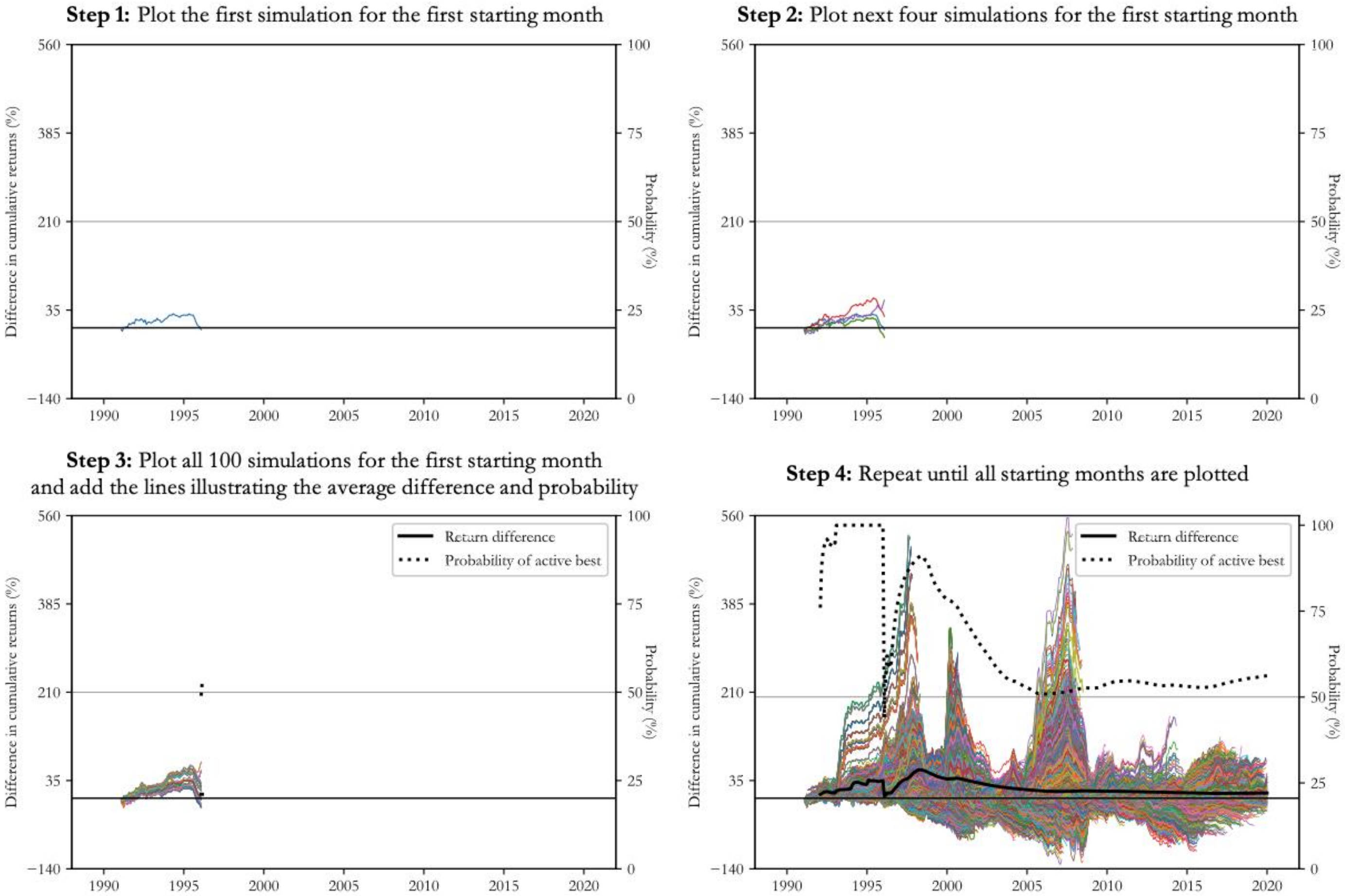


Figure F.2. Explanation of the Simulation B plot (i.e. Figure 5.5). The simulation randomly pairs an active fund and an index fund. To compute the difference between the cumulative returns, the same procedure as in Figure F.1 is conducted (but not shown in this figure). Step 1 plots the result of the first simulation of the first month and Step 2 the first five simulations. For illustrational purposes, these two plots have 15x thicker lines than the lines in Figure 5.5. Step 3 plots all 100 simulations for the first month with the two thick lines showing the probability of a higher return from the active fund and the average difference between the 100 pairwise cumulative returns. Step 4 extends step 3 to all starting months.



Appendix G

In our stochastic dominance analysis, we find that active funds first-order stochastically dominates active funds at a 5% level when considering net returns. This finding is somewhat surprising to us as we initially expected to find evidence suggesting that investors should prefer index funds over active funds. For a while, we were skeptical of the results and looked for ways to evaluate the robustness of our implementation of the test. In this appendix, we consider one way we sanity checked our results using the implementation of Whang (2019).

Whang (2019, p. 218-241) published MATLAB codes for Barret and Donald's (2003) test, both the multiplier method and the recentered bootstrap approach for simulating the p-values. We replicated our analysis using Whang's implementation. Table G.1 reports the results where we have grouped the p-values for the tests to make it visually easy to compare. The p-values are somewhat different in magnitude for some of the tests which may not be too surprising as the p-values have to be simulated. Most importantly, this robustness analysis shows that the Whang implementation rejects (and does not reject) the same null hypotheses as our implementation.

Table G.1. Summary of the robustness of our stochastic dominance implementation. The table reports p-values for three implementations of the Barret and Donald (2003) stochastic dominance test. The first is our implementation of the multiplier method in Python. The two others are Whang's (2003) implementation of the multiplier and recentered bootstrap methods in MATLAB. The p values are simulated 500 times for each test. The table includes the same samples and investor size groups as in Table 5.4.

Investor size	Our result	Multiplier method	Recentered bootstrap	Our result	Multiplier method	Recentered bootstrap
Panel A: FSD for sample 1991 - 2005						
	Index FSD Active			**Active FSD Index**		
Small	0.0140	0.0160	0.0020	0.4860	0.9420	0.4600
Medium	0.0140	0.0160	0.0180	0.5480	0.9660	0.4960
Large	0.0140	0.0300	0.0080	0.5820	0.9760	0.5420
All	0.0500	0.1180	0.0360	0.2920	0.6660	0.2700
Panel B: SSD for sample 1991 - 2005						
	Index SSD Active			**Active SSD Index**		
Small	0.1340	0.9980	0.1040	0.4020	1.0000	0.4320
Medium	0.0900	0.9420	0.0480	0.5180	1.0000	0.5260
Large	0.0560	0.9720	0.0600	0.5400	1.0000	0.5460
All	0.3220	1.0000	0.3580	0.2620	1.0000	0.2760
Panel C: FSD for sample 2006 - 2019						
	Index FSD Active			**Active FSD Index**		
Small	0.1040	0.2120	0.1020	0.5440	0.9720	0.5300
Medium	0.0900	0.3520	0.1360	0.5460	0.9800	0.5180
Large	0.0420	0.2060	0.0780	0.4740	0.9540	0.4960
All	0.0580	0.1680	0.0900	0.4840	0.9400	0.4900
Panel D: SSD for sample 2006 - 2019						
	Index SSD Active			**Active SSD Index**		
Small	0.5760	1.0000	0.5720	0.4180	1.0000	0.4640
Medium	0.5600	1.0000	0.5900	0.3920	1.0000	0.3900
Large	0.6040	1.0000	0.6320	0.3620	1.0000	0.3500
All	0.6300	1.0000	0.6280	0.3400	1.0000	0.3460

Appendix H

The mutual fund costs investors are typically charged with in Norway can be split into subscription fee ("tegningsgebyr"), redemption fee ("innløsningsgebyr"), and management fee ("løpende kostnader"). A minority of the funds do also charge a result-based fee ("resultatavhengig forvaltningshonorarer"). Our return data is computed from the funds TNAV, accounting for the management and result based fees (see definition of 'Total Return' in Morningstar Direct). The returns do not account for subscription and redemption fees. In this appendix, we study the effect of introducing subscription and redemption fees in our simulation analysis.

Data on mutual fund costs and fees in Norway is not easily available. The Finance Portal (2020) provides what we have identified as the best available data to students. In Table H.1, the subscription and redemption fees for the 62 funds available in the Finance Portal is reported. The average subscription fees are 0.31% and 0.09% for active and index funds, respectively, while for redemptions costs the numbers are 0.13% and 0.09%. The similarity in the average subscription costs is noteworthy. The data is not available as a time-series, only as a snapshot, potentially biasing the results of the analysis we present in this appendix. The analysis should still provide insights into the effect of today's cost level moving forward.

In Table H.2, we report the effect of introducing subscription and redemption fees on Simulation B. Different from the results reported in Figure A8, we deduct the subscription fee at the start of the period and the redemption fee at the end of the period. More precisely, as we draw pairwise active and index funds, we use the funds' actual costs and fees when available. We do not have data on the fees for all the funds. For those we do not have data on, we use the average of the fees for active and index funds, respectively. The results show that the difference in fees between active and index funds do affect the probability of active being best, but, for most of the groups over the two samples, the probability stays on the same side of the 50%-mark that suggests either fund type has a higher probability of giving the highest return. The main deviation stems from the 'All' group which had a probability of 51.33% for a holding period of 1 year between 2006 and 2019 before introducing subscription and redemption fees, and 49.90% after. Overall, these results suggest that the subscription and redemption fees investors face today are of a magnitude that affects the choice between active and index funds, in particular for holding periods up to 2 years, but that the expected excess cumulative return from choosing active funds remains at 0.34 to 1.24% per year in the most recent subsample, depending on the length of the holding period.

Table H.1. Summary of the subscription and redemption costs. The table reports data downloaded from the Finance Portal (2020) on June 8th, 2020. Index funds are presented in the top rows, while the rest of the data is for active funds.

Fund name	Fund type	Subscription fee	Redemption fee
Alfred Berg Indeks Classic	Index	0.0 %	0.0 %
DNB Norge Indeks A	Index	0.0 %	0.0 %
Handelsbanken Norge Index	Index	0.0 %	0.0 %
KLP AksjeNorge Indeks II	Index	0.0 %	0.0 %
Nordnet Superfondet Norge	Index	0.0 %	0.0 %
PLUSS Index (Fondsforvaltning)	Index	0.5 %	0.5 %
Storebrand Indeks - Norge	Index	0.2 %	0.2 %
XACT OBX	Index	0.0 %	0.0 %
Alfred Berg Aktiv	Active	0.0 %	0.0 %
Alfred Berg Gambak	Active	0.0 %	0.0 %
Alfred Berg Humanfond	Active	0.0 %	0.0 %
Alfred Berg Norge Classic	Active	0.0 %	0.0 %
Arctic Norwegian Equities Class A	Active	0.0 %	0.0 %
Arctic Norwegian Value Creation A NOK	Active	0.0 %	0.0 %
Arctic Norwegian Value Creation B NOK	Active	0.0 %	0.0 %
Arctic Norwegian Value Creation C NOK	Active	0.0 %	0.0 %
C WorldWide Norge	Active	3.0 %	1.0 %
Danske Invest Norge I	Active	2.0 %	0.3 %
Danske Invest Norge II	Active	1.5 %	0.3 %
Danske Invest Norge Vekst	Active	2.0 %	0.3 %
Delphi Norge	Active	0.0 %	0.0 %
DNB Norge A	Active	0.0 %	0.0 %
DNB Norge N	Active	0.0 %	0.0 %
DNB Norge R	Active	0.0 %	0.0 %
DNB Norge Selektiv A	Active	0.0 %	0.0 %
DNB Norge Selektiv N	Active	0.0 %	0.0 %
DNB Norge Selektiv R	Active	0.0 %	0.0 %
DNB SMB A	Active	0.0 %	0.0 %
DNB SMB N	Active	0.0 %	0.0 %
DNB SMB R	Active	0.0 %	0.0 %
Eika Norge	Active	2.0 %	0.5 %
FIRST Generator A	Active	0.2 %	0.2 %
FIRST Generator S	Active	0.2 %	0.2 %
FIRST Norge Fokus	Active	0.2 %	0.2 %
Fondsfinans Norge	Active	0.0 %	0.0 %
Fondsfinans Utbytte	Active	0.0 %	0.0 %
FORTE Norge	Active	0.0 %	0.0 %
FORTE Trønder	Active	0.0 %	0.0 %
Handelsbanken Norge	Active	0.0 %	0.0 %
Holberg Norge A	Active	0.0 %	0.0 %
KLP AksjeNorge	Active	0.0 %	0.0 %
Landkreditt Utbytte A	Active	0.5 %	0.2 %
Nordea Avkastning	Active	0.0 %	0.0 %
Nordea Kapital	Active	0.1 %	0.1 %
Nordea Norge Pluss	Active	0.1 %	0.1 %
Nordea Norge Verdi	Active	0.0 %	0.0 %
ODIN Norge B	Active	0.3 %	0.3 %
ODIN Norge C	Active	0.3 %	0.3 %
ODIN Norge D	Active	0.3 %	0.3 %
Pareto Aksje Norge A	Active	1.0 %	0.5 %
Pareto Aksje Norge B	Active	1.0 %	0.5 %
Pareto Investment Fund A	Active	1.0 %	0.5 %
PLUSS Aksje (Fondsforvaltning)	Active	0.5 %	0.5 %
PLUSS Markedsverdi (Fondsforvaltning)	Active	0.5 %	0.5 %
Sbanken Framgang Sammen	Active	0.0 %	0.0 %
SR-Bank Norge A	Active	0.0 %	0.0 %
SR-Bank Norge B	Active	0.0 %	0.0 %
Storebrand Norge	Active	0.0 %	0.0 %
Storebrand Norge Fossilfri	Active	0.2 %	0.2 %
Storebrand Vekst	Active	0.0 %	0.0 %
Storebrand Verdi A	Active	0.0 %	0.0 %
Storebrand Verdi N	Active	0.2 %	0.2 %

Table H.2. Summary of Simulation B with subscription and redemption costs. The table reports the same numbers as in Table 5.6. For the probability, we report both with and without the costs. The Sharpe ratio and t-tests are not affected by subscription and redemption fees as they are based on the intra-holding period net returns (not the holding period cumulative return) which is why they are not reported in this figure.

Panel A: Sample 1991 - 2005

Holding period (months)	Investor size	Probability of active > index		Difference in cumulative returns inc. fees		
		Original	Inc. Fees	Mean	Median	Std
12	Small	64.21 %	64.95 %	6.57 %	4.00 %	15.97 %
	Medium	66.51 %	65.11 %	7.47 %	3.70 %	19.63 %
	Large	67.26 %	66.10 %	7.03 %	3.78 %	17.90 %
	All	63.24 %	62.29 %	5.32 %	2.85 %	16.49 %
24	Small	64.01 %	64.46 %	5.76 %	3.49 %	16.42 %
	Medium	66.60 %	65.80 %	6.14 %	3.76 %	17.60 %
	Large	67.26 %	66.65 %	6.25 %	4.01 %	17.48 %
	All	63.24 %	62.62 %	4.33 %	2.80 %	16.01 %
36	Small	60.69 %	60.89 %	5.23 %	3.08 %	18.04 %
	Medium	65.15 %	64.56 %	5.90 %	3.66 %	19.48 %
	Large	64.37 %	63.84 %	5.65 %	3.41 %	19.16 %
	All	58.88 %	58.49 %	3.80 %	2.04 %	17.73 %
48	Small	60.63 %	60.92 %	5.30 %	2.10 %	19.96 %
	Medium	64.71 %	64.07 %	5.64 %	2.62 %	20.99 %
	Large	64.79 %	64.25 %	5.65 %	2.56 %	21.40 %
	All	57.27 %	56.78 %	3.52 %	1.25 %	19.58 %
60	Small	58.98 %	59.00 %	5.70 %	1.71 %	27.60 %
	Medium	64.02 %	63.10 %	6.22 %	2.06 %	29.95 %
	Large	62.77 %	61.99 %	5.98 %	1.94 %	29.27 %
	All	53.28 %	52.64 %	3.44 %	0.43 %	25.93 %

Panel B: Sample 2006 - 2019

Holding period (months)	Investor size	Probability of active > index		Difference in cumulative returns inc. fees		
		Original	Inc. Fees	Mean	Median	Std
12	Small	51.44 %	50.47 %	0.86 %	0.06 %	9.58 %
	Medium	51.25 %	49.27 %	0.34 %	-0.14 %	8.94 %
	Large	52.61 %	50.52 %	0.50 %	0.08 %	8.55 %
	All	51.33 %	49.90 %	0.40 %	-0.02 %	8.76 %
24	Small	49.46 %	49.29 %	0.66 %	-0.11 %	9.82 %
	Medium	50.88 %	48.68 %	0.46 %	-0.17 %	9.25 %
	Large	52.72 %	50.94 %	0.66 %	0.11 %	9.02 %
	All	52.53 %	51.03 %	0.58 %	0.10 %	8.81 %
36	Small	50.70 %	51.77 %	0.91 %	0.18 %	10.88 %
	Medium	54.42 %	53.32 %	0.74 %	0.37 %	10.02 %
	Large	56.89 %	57.15 %	1.05 %	0.68 %	9.73 %
	All	56.88 %	55.24 %	0.89 %	0.53 %	9.45 %
48	Small	54.55 %	54.20 %	0.99 %	0.40 %	12.09 %
	Medium	57.81 %	57.04 %	1.02 %	0.65 %	10.84 %
	Large	60.54 %	59.68 %	1.24 %	0.91 %	10.48 %
	All	59.18 %	58.38 %	1.12 %	0.78 %	10.39 %
60	Small	53.24 %	54.00 %	0.92 %	0.39 %	13.18 %
	Medium	57.71 %	56.81 %	0.89 %	0.61 %	12.06 %
	Large	62.20 %	61.07 %	1.14 %	0.89 %	11.38 %
	All	60.23 %	59.97 %	1.13 %	0.80 %	11.17 %